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Analytical methods to predict the surf-riding threshold and the wave-blocking threshold

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Abstract

For the safe design and operation of high-speed craft it is important to predict their behaviour in waves. There still exists a concern, however, in the framework of the International Maritime Organization (IMO) with regards to the stability criteria. In particular, for high-speed craft, the higher limit of operational speed resulting in wave blocking as well as the lower limit known as the surf-riding threshold are important features. Therefore, by applying the polynomial approximation to wave induced surge force including the nonlinear surge equation, an analytical formula in order to predict the wave blocking and surf-riding thresholds is proposed. Comparative results of the surf-riding threshold and wave blocking threshold utilizing the proposed formula and the numerical bifurcation analysis indicate fairly good agreement. In addition, previously proposed analytical formulae are inclusively examined. It is concluded that the analytical formulae based on a continuous piecewise linear approximation and Melnikov's method agrees well with the wave blocking threshold and the surf-riding threshold obtained by the numerical bifurcation analysis and the free-running model experiment. As a result, it is considered that these two calculation methods could be recommended for the early design stage tool for avoiding broaching and bow-diving.

1. Introduction

When a vessel runs in following and/or stern quartering seas, it is in danger of broaching [1), 2)] or bow-diving [3)], which generally precede a capsizing event. Broaching is a phenomenon in which a vessel cannot maintain the desired course despite engaging maximum steering.. Although several experimental [4)], numerical and analytical studies [5)] have shown that this phenomenon occurs even at low Froude numbers, that is in usual less than 0.3, this kind of broaching is avoidable by utilizing optimal rudder control [6)]. In general, broaching in the high-Froude region is considered to be more dangerous. Since one of the prerequisites of broaching is the phenomenon of surf-riding. The estimation of the surf-riding threshold is important in order to assess a vessel's safety in following and/or strern quartering seas. However, at higher Froude numbers, the surf-riding phenomenon disappears and, in contrast, a ship overtakes the wave. The transition region at which this occurs is called 'the wave blocking threshold'. The transition is considered to be the threshold of the bow diving, i.e. the upper limit of its occurrence. Therefore this threshold could provide meaningful parameters when running at higher Froude numbers. Bow diving occurs when a vessel's bow is continually immersed into an oncoming wave crest owing to the vessel's forward speed and amplitude of the wave relative to the bow. Once these phenomena, broaching and bow diving occur, passengers could be injured at the best, or at the worst, the ship could capsize from the resulting yaw & extreme induced roll motion. With regard to broaching, it is one of the three major capsizing scenarios incorporated in the new generation intact stability criteria which shall be added to the 2008 Intact Stability Code (IS code) by the International Maritime Organization (IMO). [7)].

The second generation intact stability criteria to be established at IMO consist of: vulnerability criteria and performance-based criteria. If a ship fails to pass the vulnerability criteria, its safety is to be assessed against the performance-based criteria, utilizing numerical simulation or its equivalent. Thus, it is important that any vulnerability criterion is easy to apply, ensures a conservative safety level, and is not based upon empirical data. A vulnerability criterion for surf-riding in regular following seas can be used in place of that for capsizing due to broaching because, as noted, surf-riding is the prerequisite to these phenomena and travelling in following seas is the most susceptible heading for surf-riding. In order to estimate the surf-riding threshold or wave blocking threshold in regular following seas, an analytical solution is obviously most suitable because it retains a theoretical background.

In this paper, the authors develop a generalized formula for predicting the surf-riding and wave blocking threshold by making use of Melnikov's method. From previous research [8], the authors propose an analytical formulae for estimating the wave blocking threshold based upon the continuous piecewise linear (CPL) approximation. Finally new formulae for predicting the surf-riding and wave blocking threshold based upon 3rd order polynomial approximation for the wave-induced surge force are proposed. Predictions using this approach are validated using results from free running model experiments.

2. Reducing the nonlinear surge equation

2.1 Basic autonomous surge equation

The co-ordinate system used in the formulae presented in this paper is illustrated in fig.1. An inertia co-ordinate system $o-\xi\zeta$ with the origin at a wave trough has the ξ axis pointing toward the wave direction. The ship fixed co-ordinate system, $G-xz$ with the origin at the centre of gravity of the ship has the x - axis pointing toward the bow from the stern and z - axis downward. Here the ship longitudinal velocity u is defined as $u=\dot{\xi}_G+c$, where c indicates the wave celerity. Initially the generalized form of the approximate polynomial nonlinear surge equation is to be calculated. This approach for the analysis has been used previously by the authors [8], and is thus briefly summarized. The equation representing nonlinear surge motion in this paper is described as follows:

$$(m+m_x)\ddot{\xi}_G+[R(u)-T(u;n)]-X_w=0 \tag{2.1}$$

In this equation, a dot denotes differentiation with respect to time t. Where R: the ship resistance in calm water, T: the propeller thrust, m: the ship mass, m_x: the added mass in the x direction, u: the instantaneous ship velocity in the x direction , n: the propeller rate. In this equation higher order terms such as thrust variation due to wave particle velocity are ignored. Assuming that the hull form is almost longitudinally symmetric, the Froude-Krylov force is represented as a first order approximation as follows:

$$X_w \approx f\sin(k\xi_G) \tag{2.2}$$

where wave number k is defined as $2\pi/\lambda$, and λ is the wavelength. Here the phase of the sinusoidal function representing X_w, is ignored (See Appendix). The resistance curve $R(u)$ and the thrust coefficient curve $K_T(J)$ can be approximated by n-th polynomial:

$$R(u) \approx \sum_{i=0}^{n} r_i u^i = r_0 + r_1 u + r_2 u^2 + \cdots \tag{2.3}$$

$$K_T(J) \approx \sum_{i=0}^{n} \kappa_i J^i = \kappa_0 + \kappa_1 J + \kappa_2 J^2 + \cdots \tag{2.4}$$

where each r_i and κ_i is chosen on the basis of a polynomial fit of the resistance curve and the thrust coefficient, obtained from the tank tests or the numerical calculation. Note that $J = u(1-w_p)/nD$. Then $T(u,n)$ becomes:

$$T_e(u;n) = (1-t_p)\rho n^2 D^4 K_T(J) = \sum_{i=0}^{n} \frac{(1-t_p)(1-w_p)^i \rho\kappa_i u^i}{n^{i-2} D^{i-4}} \tag{2.5}$$

Where t_p and w_p are the thrust deduction and wake fraction, respectively. These values are customarily taken at their still-water value. Here D and ρ are the propeller diameter and water density, respectively. Substituting these equations into Equation (2.1) yields;

$$(m+m_x)\ddot{\xi}_G + \sum_{i=1}^{n}\sum_{j=1}^{i} c_i \binom{i}{j} \dot{\xi}_G^i c_w^{i-j} + f \sin k\xi_G = T_e(c_w;n) - R(c_w) \tag{2.6}$$

where c_i denotes;

$$c_i \equiv -\frac{(1-t_p)(1-w_p)^i \rho\kappa_i}{n^{i-2} D^{i-4}} + r_i \tag{2.7}$$

Here c_w is wave celerity. This equation represents the approximate generalised expression of surge.

3. Prediction method of the surf-riding and wave blocking threshold

3.1 Brief review on the existing work

Application of Melnikov's method to Equation (2.6) has been conducted, and generalized results obtained by Maki et al. [8] can be shown as:

$$\sqrt{\pi}\,\frac{T_e(c_w;n) - R(c_w)}{f} = \sum_{i=1}^{n}\sum_{j=1}^{i} C_{ij} (-2)^j \,\Gamma\!\left(\frac{j+1}{2}\right) \Big/ \Gamma\!\left(\frac{j+2}{2}\right) \quad \text{,where } C_{ij} \equiv \frac{c_i}{fk^j}\binom{i}{j}\frac{(fk)^{j/2}}{(m+m_x)^{j/2}} c_w^{i-j} \tag{3.1}$$

Here Γ represents the Gamma function. Substitute $n=3$, and put $\kappa_3 = 0$ and taking account of $I_1 = 4$, $I_2 = \pi$ and $I_3 = 8/3$, following condition had been obtained;

$$\frac{T_e(c_w;n)-R(c_w)}{f}=-\frac{4\left(c_1+2c_2c_w+3c_3c_w^2\right)}{\pi\sqrt{fk(m+m_x)}}+\frac{2(c_2+3c_3c_w)}{k(m+m_x)}-\frac{32c_3\sqrt{f}}{3\pi[k(m+m_x)]^{3/2}} \quad (3.2).$$

Furthermore, Maki et al. [8)] obtained the formula predicting the surf-riding threshold by utilizing CPL approximation. This result is briefly summarized below. Using $n=3$ and $\kappa_3=0$ in Equation (2.6), this is reduced to:

$$(m+m_x)\ddot{\xi}_G+A_1(c_w\,;n)\dot{\xi}_G+A_2(c_w)\dot{\xi}_G^2+A_3\dot{\xi}_G^3+f\sin(k\xi_G)=T_e(c_w\,;\;n)-R(c_w) \quad (3.3)$$

This equation is completely identical to that obtained by Spyrou [9]. In Equation (3.3)**,** let the some terms be defined by $A_1(c_w\,;n),\,A_2(c_w),\,A_3,T(c_w\,;n),\,R(c_w)$ are as follows:

$$\begin{cases}A_1(c_w\,;n)=3r_3c_w^2+2(r_2-\tau_2)c_w+r_1-\tau_1n\\ A_2(c_w)=3r_3c_w+(r_2-\tau_2)\\ A_3=r_3\\ T_e(c_w\,;\;n)\equiv\tau_2c_w^2+\tau_1nc_w+\tau_0n^2\\ R(c_w)\equiv r_3c_w^3+r_2c_w^2+r_1c_w\end{cases} \quad (3.4)$$

Taking the quadratic regression of damping terms in Equation (3.3), Spyrou obtained the analytical formula to estimate the surf-riding threshold:

$$f=[R(c_w)-T(c_w\,;n)]\sqrt{\frac{k^2(m+m_x)^2}{4\gamma^2}+1} \quad (3.5),$$

[I suggest: *T(c_w;n),* for consistency, rather than *T(c_w,n)*]

where:

$$\gamma(n)=-\left[A_1(c_w\,;n)\sum_{i=1}^{I}\dot{\xi}_G^3+A_2(c_w)\sum_{i=1}^{I}\dot{\xi}_G^4+A_3\sum_{i=1}^{I}\dot{\xi}_G^5\right]/\sum_{i=1}^{I}\dot{\xi}_G^4 \quad (3.6)$$

On the other hand, Maki et al. [8)] approximated this damping term by the linear regression:

$$\beta(n)=\left[A_1(c_w\,;n)\int_I\dot{\xi}_G^2d\dot{\xi}_G+A_2(c_w)\int_I\dot{\xi}_G^3d\dot{\xi}_G+A_3\int_I\dot{\xi}_G^4d\dot{\xi}_G\right]/\int_I\dot{\xi}_G^2d\dot{\xi}_G \quad (3.7).$$

The above expression, however, leads to the unsolvable form, i.e. a nonlinear pendulum equation:

$$(m+m_x)\ddot{\xi}_G+\beta(n)\dot{\xi}_G+f\sin(k\xi_G)=T(c_w\,;n)-R(c_w) \quad (3.8)$$

To overcome this difficulty, the sinusoidal term is approximated using the CPL function, then the following formula for estimating the surf-riding threshold can be obtained.

$$-\alpha=2e^{\lambda_R\tau}[c_R\cos\lambda_I\tau-c_I\sin\lambda_I\tau] \quad (3.9).$$

Here $c_R=\mathrm{Re}[c_3],c_I=\mathrm{Im}[c_3],\lambda_R=\mathrm{Re}[\lambda_3]$ and $\lambda_I=\mathrm{Im}[\lambda_3]$ where:

$$\tau = \frac{1}{\lambda_R} \ln \left\| \frac{\alpha \sqrt{\Lambda_1^{\,2} + \Lambda_2^{\,2}}}{2\lambda_I \left(c_R^{\,2} + c_I^{\,2} \right)} \right\| \tag{3.10}$$

$$\Lambda_1 \equiv c_R \lambda_I + c_I \lambda_R + c_I \lambda_2 ,\ \Lambda_2 \equiv c_R \lambda_R - c_I \lambda_I + c_R \lambda_2 \tag{3.11}$$

$$\alpha \equiv \frac{1}{4} \lambda + \frac{\lambda \alpha_3}{4 \alpha_2} \tag{3.12}$$

$$\lambda_{1,2} \equiv \frac{-\alpha_1 \pm \sqrt{\alpha_1^{\,2} + 16\alpha_2 / \lambda}}{2},\ \lambda_{3,4} \equiv \frac{-\alpha_1 \pm \sqrt{\alpha_1^{\,2} - 16\alpha_2 / \lambda}}{2} \tag{3.13}$$

$$c_{3,4} = \pm \left[-\frac{1}{4} \lambda + \frac{\lambda \alpha_3}{4 \alpha_2} \right] \cdot \frac{(\lambda_1 + \lambda_{4,3})}{(\lambda_3 - \lambda_4)} \tag{3.14}$$

3.2 The wave blocking threshold

The method to predict the surf-riding threshold is almost identical to that of the wave blocking threshold. However, the difference being that the trajectory on an upper phase plane is employed in its formulation whereas for the surf-riding case it is on the lower plane. The detailed explanation of the methodology is omitted, however, the final results are illustrated below.

The generalized condition of the wave blocking threshold obtained by applying Melnikov's method is represented as:

$$\frac{T_e(c_w;n) - R(c_w)}{f} = \sum_{i=1}^{n} \sum_{j=1}^{i} \frac{C_{ij} 2^j}{\sqrt{\pi}} \Gamma\left(\frac{j+1}{2} \right) / \Gamma\left(\frac{j+2}{2} \right) \tag{3.15}$$

Assuming $n = 3$ and $\kappa_3 = 0$, then we can obtain:

$$\frac{T_e(c_w;n) - R(c_w)}{f} = \frac{4\left(c_1 + 2c_2 c_w + 3c_3 c_w^2\right)}{\pi \sqrt{f k (m + m_x)}} + \frac{2(c_2 + 3c_3 c_w)}{k(m + m_x)} + \frac{32 c_3 \sqrt{f}}{3\pi \left[k(m + m_x) \right]^{3/2}} \tag{3.16}$$

The following bifurcation condition is obtained by applying CPL approximation method:

$$-\alpha = 2e^{\lambda_R \tau} \left[c_R \cos \lambda_I \tau - c_I \sin \lambda_I \tau \right] \tag{3.17}$$

where:

$$\tau = \frac{1}{\lambda_R} \ln \left\| \frac{\alpha \sqrt{\Lambda_1^{\,2} + \Lambda_2^{\,2}}}{2\lambda_I \left(c_R^{\,2} + c_I^{\,2} \right)} \right\| \tag{3.18}$$

$$\Lambda_1 \equiv c_R \lambda_I + c_I \lambda_R + c_I \lambda_2 ,\ \Lambda_2 \equiv c_R \lambda_R - c_I \lambda_I + c_R \lambda_2 \tag{3.19}$$

$$\alpha \equiv -\frac{1}{4} \lambda + \frac{\lambda \alpha_3}{4 \alpha_2} \tag{3.20}$$

$$\lambda_{1,2} \equiv \frac{-\alpha_1 \pm \sqrt{\alpha_1^2 + 16\alpha_2 / \lambda}}{2}, \; \lambda_{3,4} \equiv \frac{-\alpha_1 \pm \sqrt{\alpha_1^2 - 16\alpha_2 / \lambda}}{2} \tag{3.21}$$

$$c_{3,4} = \pm\left[\frac{1}{4}\lambda + \frac{\lambda\alpha_3}{4\alpha_2}\right] \cdot \frac{(\lambda_1 + \lambda_{4,3})}{(\lambda_3 - \lambda_4)} \tag{3.22}$$

with the following defined as: $c_R = \mathrm{Re}[c_3], c_I = \mathrm{Im}[c_3], \lambda_R = \mathrm{Re}[\lambda_3]$ and $\lambda_I = \mathrm{Im}[\lambda_3]$. Both conditions, (3.15) and (3.17), are numerically solved using Newton's method.

3.3 Reduction of New Analytical Formula estimating the wave blocking and the surf-riding threshold

Although the formulae shown in the previous sections are obtained by using piecewise linear approximation for the sinusoidal term in equation (3.3), here we try to apply polynomial approximation. In equation (3.3), the non-dimensionalization is as follows:

$$\begin{cases} y = k\xi_G \\ \tau = \sqrt{\dfrac{fk}{m+m_x}}t \end{cases} \tag{3.23}$$

which yields:

$$\frac{d^2y}{d\tau^2} + \tilde{\beta}\frac{dy}{d\tau} + \sin y = \frac{r}{q} \tag{3.24}$$

where each coefficient is defined as:

$$\begin{cases} \tilde{\beta} = \dfrac{\beta(n)}{\sqrt{fk(m+m_x)}} \\ q = \dfrac{fk}{m+m_x} \\ r = \dfrac{[T_e(c_w\,;n) - R(c_w)]k}{m+m_x} \end{cases} \tag{3.25}$$

Here the sinusoidal function is approximated by the polynomial of a third order as follows:

$$\sin y \approx -\mu y(y-y_1)(y+y_1) \tag{3.26}$$

Finally equation (3.24) can be transformed as follows:

$$\ddot{y} + \tilde{\beta}\dot{y} - \mu y(y-y_1)(y+y_1) = \frac{r}{q} \tag{3.27}$$

It is worth noting that the periodicity of wave induced force does not disappear using this approximation. However, with careful scrutiny it can be seen that the approximation for one wave is sufficient since the heteroclinic orbit joining two saddles can be considered within one wave. Assuming $a_1 < a_2 < a_3$, an analytical factorization, such as Cardano's technique, yields:

$$\mu y(y-y_1)(y+y_1)+\frac{r}{q}=\mu(y-a_1)(y-a_2)(y-a_3) \quad (3.28).$$

Then equation (3.27) becomes:

$$\ddot{y}+\tilde{\beta}\dot{y}-\mu(y-a_1)(y-a_2)(y-a_3)=0 \quad (3.29).$$

Putting:

$$x=\frac{y-a_1}{a_3-a_1} \quad (3.30),$$

The following equation is obtained:

$$\ddot{x}+\tilde{\beta}\dot{x}+\tilde{\mu}x(1-x)(x-\tilde{a})=0 \quad (3.31),$$

where:

$$\begin{cases}\tilde{a}=\dfrac{a_2-a_1}{a_3-a_1}\\ \tilde{\mu}=\mu(a_3-a_1)^2\end{cases} \quad (3.32).$$

Here note $0<\tilde{a}<1$. As pointed out by Maki et al. [10], the state equation (3.29) is identical in form with the FHN (FitzHugh-Nagumo) equation except for some of the coefficients. Now we take the following ansatz[10]:

$$\dot{x}=\tilde{c}x(1-x) \quad (3.33).$$

Here $\ddot{x}$ can be calculated as:

$$\ddot{x}=\tilde{c}^2(1-x)(1-2x) \quad (3.34),$$

so that substituting equation (3.34) into equation (3.31) yields:

$$x(\tilde{\mu}-2\tilde{c}^2)+(\tilde{c}^2+\tilde{\beta}\tilde{c}-\tilde{\mu}\tilde{a})=0 \quad (3.35),$$

If above equation is satisfied for $x^{\forall}\in(0,1)$, following conditions:

$$\begin{cases}\tilde{\mu}-2\tilde{c}^2=0\\ \tilde{c}^2+\tilde{\beta}\tilde{c}-\tilde{\mu}\tilde{a}=0\end{cases} \quad (3.36),$$

are required to be satisfied. Eliminating $\tilde{c}$ from these equations yields:

$$\frac{\tilde{\mu}}{2}\pm\tilde{\beta}\sqrt{\frac{\tilde{\mu}}{2}}-\tilde{\mu}\tilde{a}=0 \quad (3.37).$$

This equation represents the condition of the surf-riding or the wave blocking threshold that is to be satisfied. In this equation, the upper sign corresponds to the wave blocking threshold while the lower sign corresponds to the surf-riding threshold. Now solving equation (3.37) with the iterative method, the time domain trajectory can be obtained as:

$$q^0(t) \equiv x = 1/\left[1+\exp\left(-\tilde{c}t+\tilde{d}\right)\right] \qquad (3.38),$$

where $\tilde{c} > 0$ and $\tilde{d} \in (-\infty, \infty)$ is an arbitrary constant determined by the initial condition. Equation (3.38) is alternatively represented as:

$$q^0(t) = \frac{1}{2}\left(1+\tanh\frac{\tilde{c}t-\tilde{d}}{2}\right) \qquad (3.39).$$

4. Verification of Several Analytical Formulae against Numerical Bifurcation Analysis

In order to verify the proposed formulae, comparative calculation between the formulae and numerical bifurcation analysis was carried out for the ONR tumblehome vessel [11]. The principal characteristics and the body plan of this vessel are shown in Table 1 and Figure 2, respectively. Fig.3 shows the initial wave blocking threshold using the piecewise linear approximation method, i.e. equation (3.17), and is compared with that obtained from the numerical bifurcation by using the CPL approximated wave-induced surge force. In this figure, the abscissa is the wavelength to ship length ratio, while the ordinate indicates the nominal Froude number, defined as the ship velocity in calm water with the same propeller revolutions. The numerical bifurcation analysis is based on Maki et al. [12]. Since there is no discernible difference between the two, it can be concluded that the proposed formula is consistent with the numerical bifurcation analysis for predicting the wave blocking threshold. Following the above, it is necessary to validate the results of the polynomial approximation method. The appropriate CPL curves are determined in order to keep the zero crossing points the same as those of the original function between $[-3\pi/2, 3\pi/2]$. As a result, the sinusoidal function can be represented as follows:

$$\sin y \approx -\frac{8}{3\pi^3} y(y-\pi)(y+\pi) \qquad (3.40).$$

The approximation result is shown in fig.4. Figure 5 and Figure 6 show the surf-riding threshold and the wave blocking threshold that are obtained by solving (3.17), respectively. The numerically obtained thresholds for the approximated polynomial surge equation are also plotted. In Figure 5 the abscissa is the wavelength to ship length ratio, while in fig.6 the abscissa is the wave-steepness. Since there is no discernible difference between the two for both thresholds, it can be concluded that the proposed formula (3.17) is consistent with the numerical bifurcation analysis for predicting the surf-riding threshold.

5. Validation of Several Analytical Formulae against Free-running Model Experiment

In order to validate experimentally the proposed formula, predictions computed by all the formulae were compared with results obtained from a free-running model experiment carried out in the seakeeping and manoeuvring basin of NRIFE (National Research Institution of Fishing Engineering) with a scale model of the ONR tumblehome vessel. In the experiment, the autopilot course χ_C was set to -5 degrees from the wave direction, because it is shown that the effect of a small deviation in the course on the surf-riding threshold is negligibly small [12]. Additionally a course of 0 degrees could cause a collision with the tank wall at the beginning of the model run. The definitions of the heading angle, χ , and the auto pilot course χ_C, are given in figure 7. Initially the model drifted near the wave maker and then the propellers and the autopilot controls were activated. The propeller revolutions were set in an attempt to control the specified nominal Froude number during the model runs and using a proportional autopilot with the rudder gain of 1.0.

Figure 8 shows the time series as an example of oscillatory motions for the upper Froude region, where the model overtakes the wave. In contrast, figure 9 indicates the time series as an example of surf-riding. It shows that the ship is captured on a wave and forced to run at the wave celerity. These two examples indicate that the wave blocking threshold may exist between a Froude number of 0.4 and 0.45. However the thresholds, particularly the wave blocking threshold, are affected by the initial conditions [13]. To exclude the dependence upon the initial conditions, model runs with various initial conditions are recommended as a task for future.

Figures 10-13 show a comparison of the predicted surf-riding threshold and wave blocking threshold, using the proposed three formulae. These being: the formula based on Melnikov's method; the formula based on the piecewise linear approximation (CPL); the formula based on the polynomial approximation; and the numerical bifurcation analysis [12], with the experimental results. In Figures 10 and 12 the abscissa is the wavelength to ship length ratio, while the abscissa is the wave-steepness in Figures 11 and 13. All the predicted thresholds agree well with the results from the experiments, where the threshold is between the runs showing oscillatory motion, and those showing surf-riding (solid circles and hollow squares in Figures 10-13 respectively). In comparison to the results obtained from the numerical bifurcation analyses, the CPL method constantly provides an overestimation (resp. underestimating) results in the Froude number for surf-riding (resp. wave blocking) threshold. This is because the CPL approximation underestimates the wave induced surge force [8]. Further it has been previously shown [8] that the slight underestimate from experimental results for the surf-riding threshold, in the Froude number for the predicted threshold, could be caused by the diffraction effect in the wave-induced surge force. On the other hand, the results obtained by the method based on the polynomial approximation qualitatively predict the tendency of the thresholds, but quantitative agreement with the results from the numerical bifurcation analysis seems to be inadequate. However, for this calculated condition, this method yields results on the conservative side of safety. Thus, this conservative tendency is considered to be preferable from a practical point of view for operational guidance in following seas. Further, as it is observed in the case of the prediction of the surf-riding threshold [8], the threshold predicted by the formula based on the Melnikov's method shows fairly good agreement with that obtained by the numerical bifurcation analysis and the experiments for the wave blocking case. Summarizing the above results, it is concluded that the formulae based on the Melnikov's method and CPL approximation method provide sufficiently accurate predictions and these two calculation methods could be used for the vulnerability criteria included in new generation intact ship stability code (IS code).

6. Conclusions

The main conclusions from this work are summarized as follows:

1. By applying two of the analytical approaches, i.e. Melnikov's method and the CPL approximation method, the analytical predictions for estimating the wave blocking threshold have been obtained.
2. Using an approximation of the wave-induced force by a polynomial function, the analytical formulae to

estimate the surf-riding and wave blocking threshold have been demonstrated.

3. All the proposed formulae have been numerically and experimentally validated. Further, as a result of the comparison with the free running model experiments it is concluded that the formulae based on the Melnikov's method and the CPL approximation method both provide sufficiently accurate results.

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Appendix. Proof of the reason why the phase of the sinusoidal function is ignored in the surge equation

The reason why surf-riding threshold is independent of the phase of surge force is explained here. Let the simplified surge equation;

$$(m+m_x)\ddot{\xi}_G + \beta(n)\dot{\xi}_G + f\sin k(\xi_G - \xi_P) = T(c_w,n) - R(c_w) \qquad (9),$$

where ξ_P represents the phase of wave induced surge force. When

$$\xi'_G = \xi_G - \xi_P \qquad (10),$$

the following can be obtained:;

$$(m+m_x)\ddot{\xi}'_G + \beta(n)\dot{\xi}'_G + f\sin k\xi'_G = T(c_w;n) - R(c_w) \qquad (11).$$

Comparison of (9) and (11) demonstrates that the phase ξ_P does not affect the surf-riding threshold.

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Tables

Table 1 Principal particulars of the ONR tumblehome vessel.

Items	Values
Length	154.0 m
Breadth	18.8 m
Draught	14.5 m
Block Coefficient	0.535

Figures legends

Figure.1 Co-ordinate systems

Figure.2 Body plan of the ONR tumblehome vessel.

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Figure 6 Comparison of the surf-riding threshold predicted using the Numerical Solution, and that predicted using the Analytical Solution with the polynomial approximation, for $\lambda / L = 1.0$.

Figure 7 Definition of the heading angle and the autopilot course with respect to the wave direction.

Figure 8 Time series of the oscillatory motion with $Fn = 0.45$ and $\chi_C = -5.0(\text{deg.})$ for $\lambda / L = 0.8$, $H / \lambda = 0.05$.

Figure 9 Time series of surf-riding with $Fn = 0.4$ and $\chi_C = -5.0(\text{deg.})$ for $\lambda / L = 0.8$, $H / \lambda = 0.05$.

Figure 10 Comparison of the predicted surf-riding threshold for the three methods with the experimental results as functions of wavelength to ship length ratio for a wave-steepness of 0.05.

Figure 11 Comparison of the surf-riding threshold between experimental results and several methods as functions of wave-steepnessfor a wavelength to ship length ratio of 0.8

Figure 12 Comparison of the wave blocking threshold for the three methods as functions of wavelength and ship length ratio with the experimental results for a wave-steepness of 0.05.

Figure 13 Comparison of the wave blocking threshold between experimental results and several methods as functions of wave-steepness, for a wavelength to ship length ratio of 0.8.

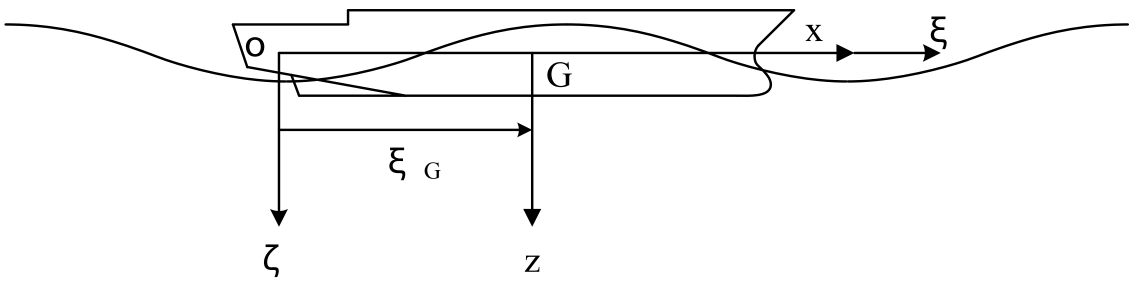


Fig.1 Co-ordinate systems.

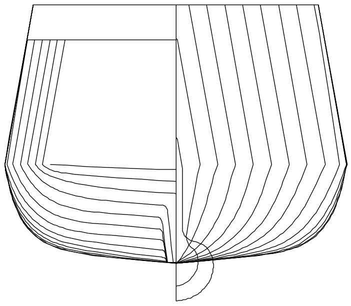


Fig.2 Body plan of the ONR tumblehome vessel.

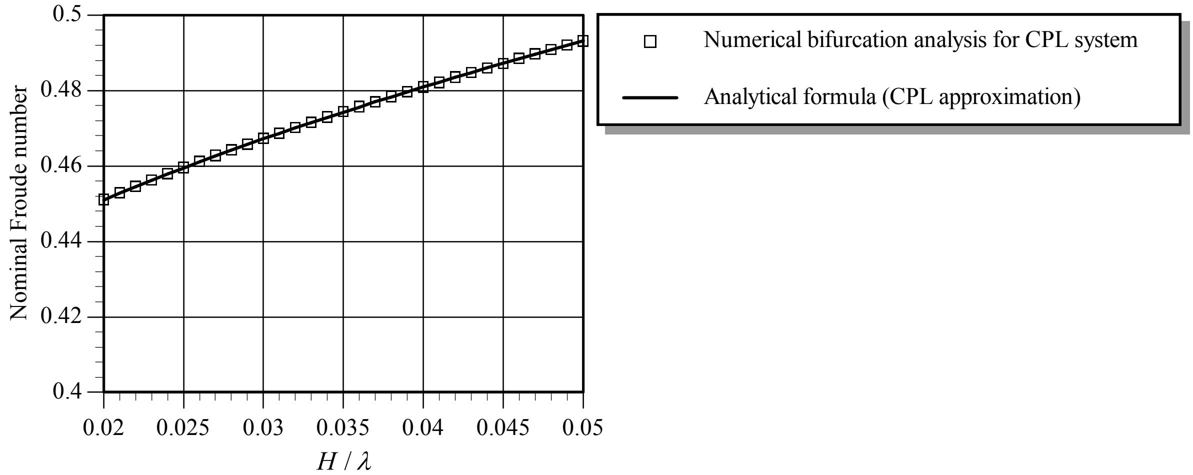


Figure 3 Comparison of the wave blocking threshold predicted using the Numerical Solution, and that predicted using the Analytical Solution with Piecewise Linear approximation, for $\lambda / L = 1.0$.

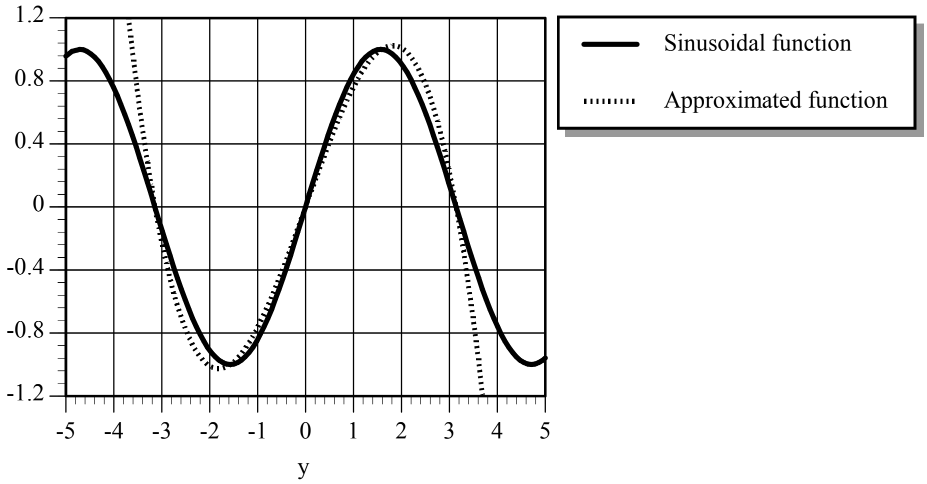


Figure 4   Linear, quadratic and cubic approximation of the sinusoidal function.

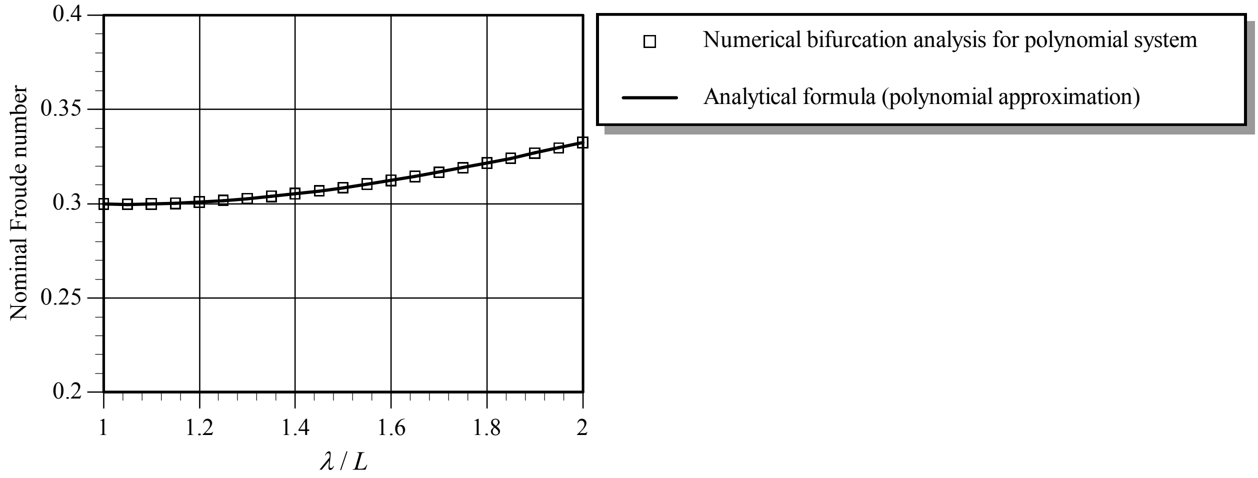


Figure 5 Comparison of the surf-riding threshold predicted using the Numerical Solution, and that predicted using the Analytical Solution with the polynomial approximation, for $H/\lambda = 0.05$.

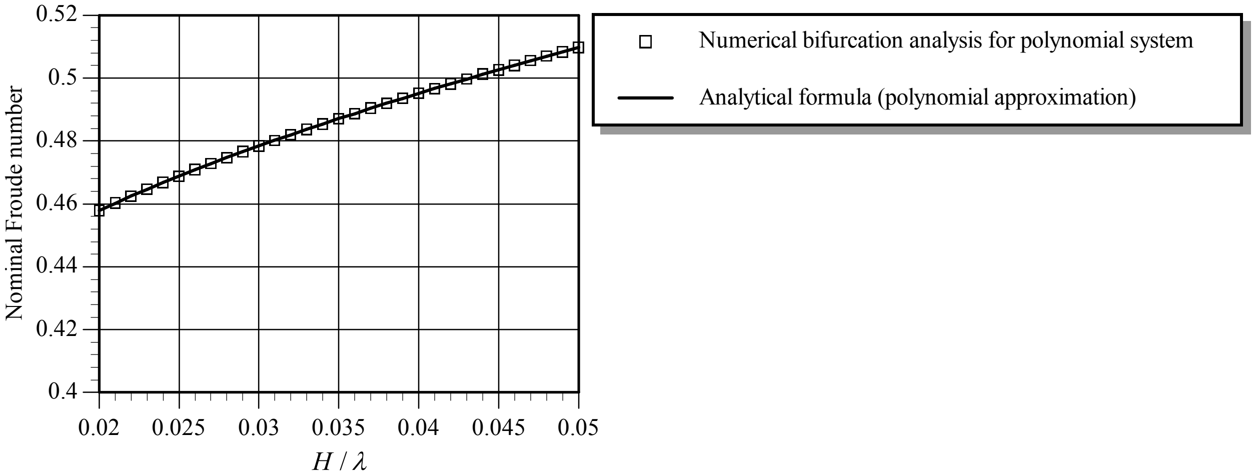
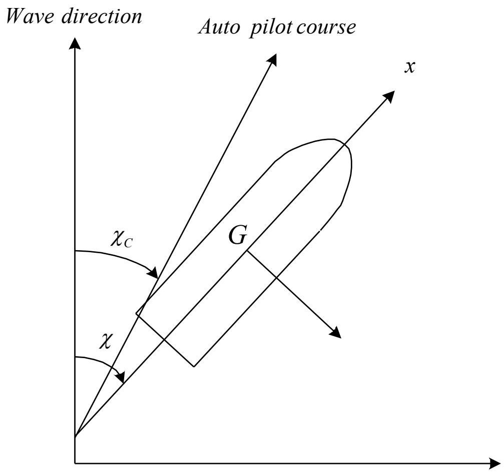


Figure 6 Comparison of the surf-riding threshold predicted using the Numerical Solution, and that predicted using the Analytical Solution with the polynomial approximation, for $\lambda / L = 1.0$.



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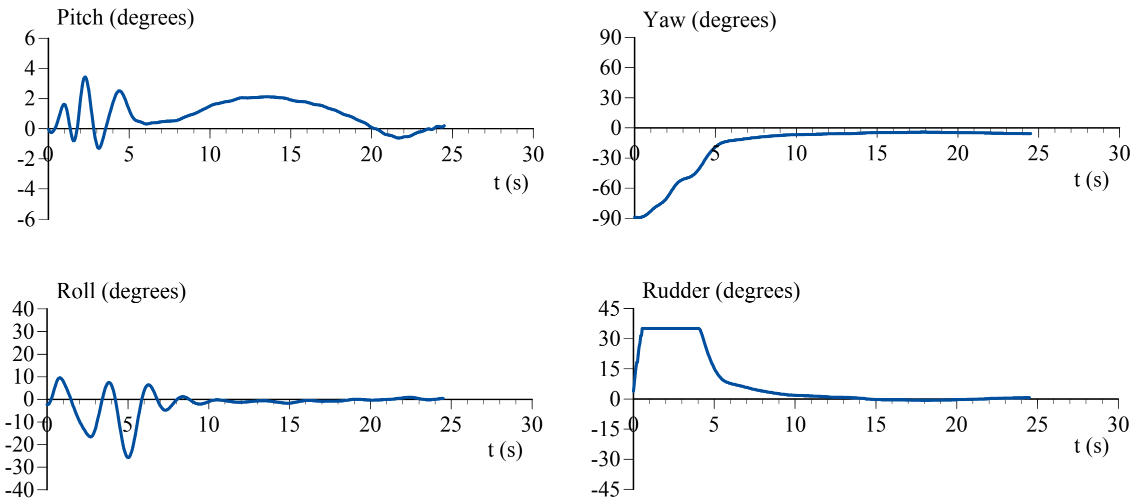

Figure 8 Time series of the oscillatory motion with $Fn = 0.45$ and $\chi_C = -5.0(\deg.)$ for $\lambda / L = 0.8$, $H / \lambda = 0.05$.

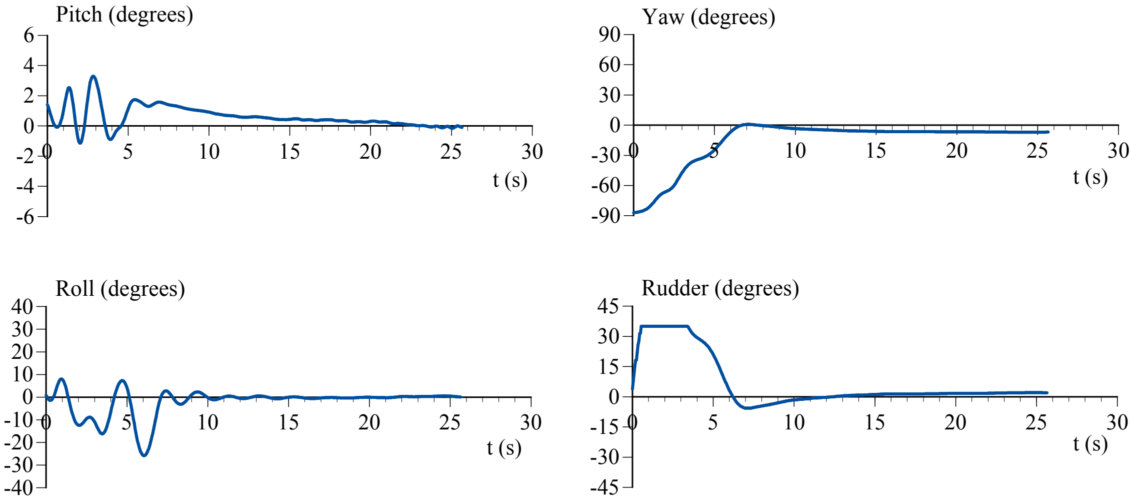

Figure 9 Time series of surf-riding with $Fn = 0.4$ and $\chi_C = -5.0$(deg.) for $\lambda / L = 0.8$, $H / \lambda = 0.05$.

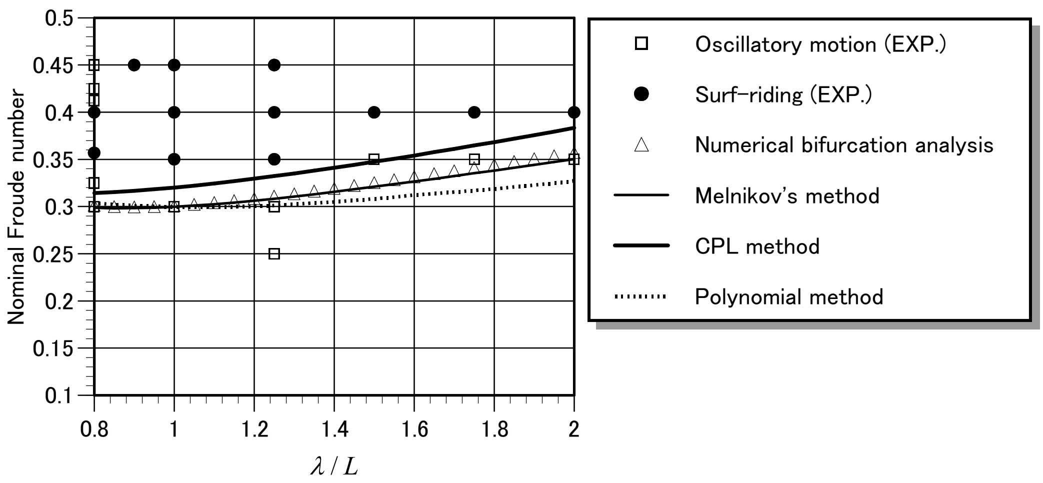

Figure 10　Comparison of the predicted surf-riding threshold for the three methods with the experimental results as functions of wavelength to ship length ratio for a wave-steepness of 0.05.

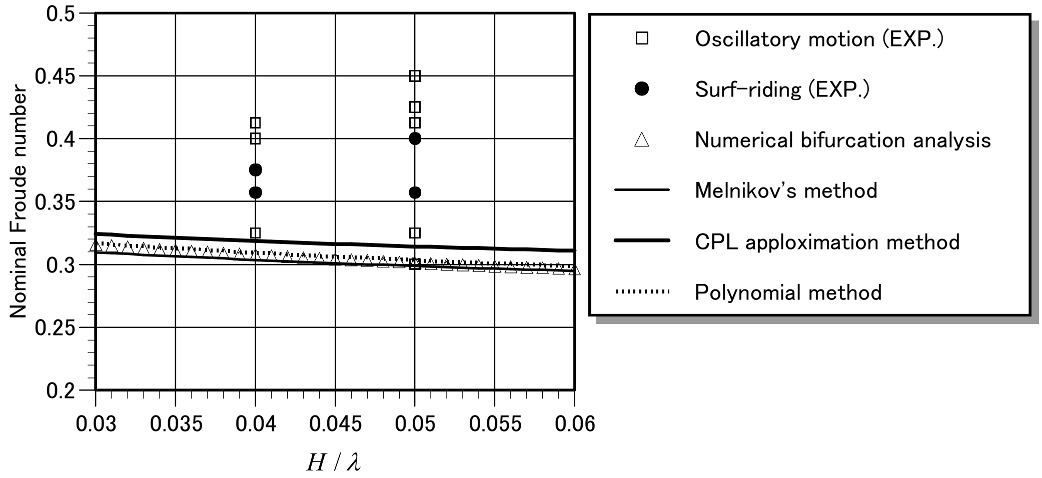

Figure 11 Comparison of the surf-riding threshold between experimental results and several methods as functions of wave-steepnessfor a wavelength to ship length ratio of 0.8.

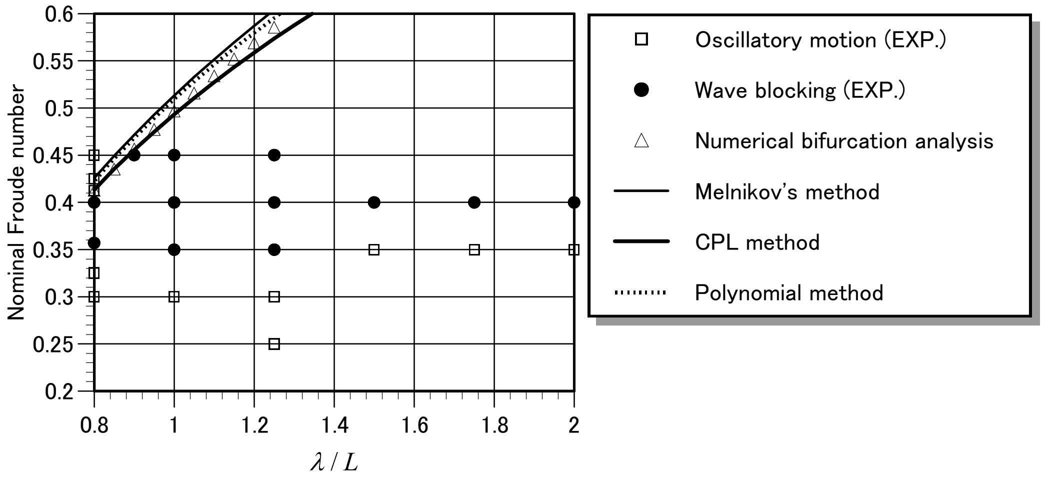

Figure 12 Comparison of the wave blocking threshold for the three methods as functions of wavelength and ship length ratio with the experimental results for a wave-steepness of 0.05.

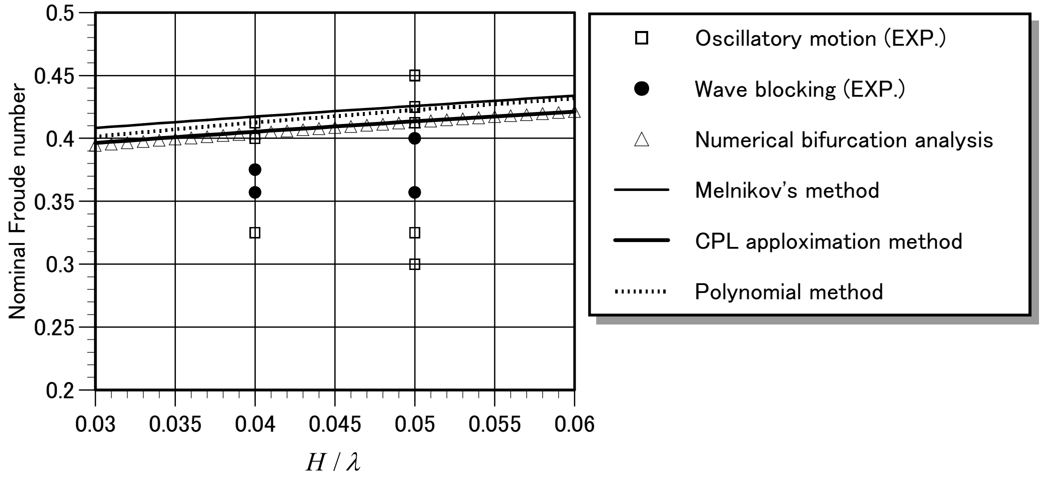

Figure 13 Comparison of the wave blocking threshold between experimental results and several methods as functions of wave-steepness, for a wavelength to ship length ratio of 0.8.