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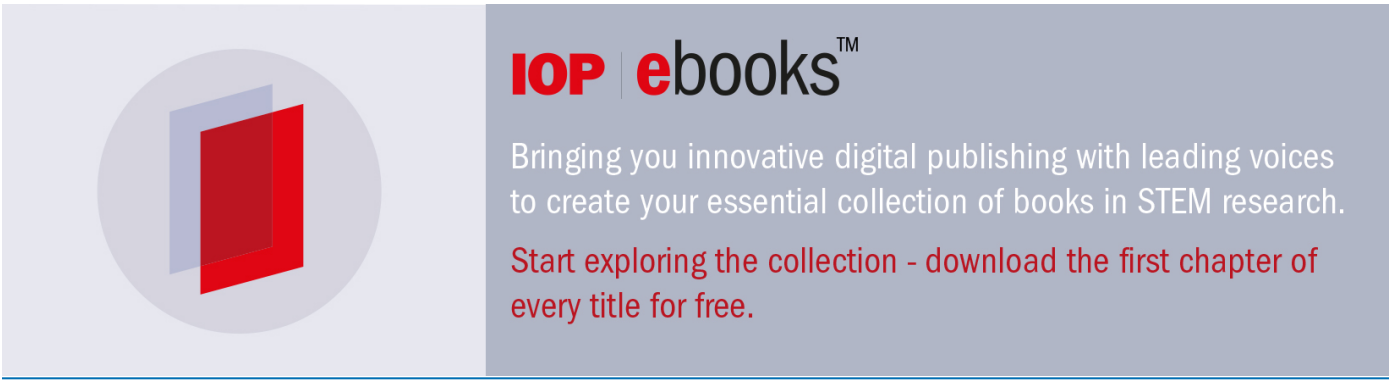

Eulerian methods for modelling gas-particle flows with low inertia particles

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Abstract. The comparison of two Eulerian methods for simulating low inertia particle flows in a moving gas is performed for the case of hyperbolic flow. The traditional Lagrangian approach is used as a reference. It is shown that for low inertia particles, results obtained by the two-fluid Eulerian approach and the Lagrangian approach are in a good agreement. For the Eulerian equilibrium approach a reasonable agreement with the Lagrangian approach is achieved only for very small particle response time values. The discrepancy increases significantly with the growth of the particle response time.

1. Introduction

Mathematical modelling of inertial particles transport in a moving gas is an important problem of fluid mechanics. Such models have applications in mechanical and chemical engineering [2]. One of the basic methods of modelling inertial particle flows is the Lagrangian approach, in the framework of which ordinary differential equations of particle motion are solved in the given fluid flow field [1]. A disadvantage of this approach is that the calculation of particle number density is numerically expensive. Knowledge of the number density of inertial particles in a moving gas is needed in many applications to account for the heat-mass transfer of a dispersed phase with a gas phase. Eulerian approaches are more convenient for coupling solutions for different phases. In this approaches the number density is inferred from the solution of transport equations for the carrier and dispersed phases on the same computational grid. For inertial particles their motion is under influence of the drag force. In the traditional Eulerian approach, that is known as the two-fluid approach [1, 4], a dispersed phase is represented as a continuum fluid with averaged characteristics of dispersed phase (number density, velocity) and the drag force is accounted using source terms in momentum conservation equations. Another way to model gas-particle flow with inertial particles is known as the equilibrium Eulerian approach. In the framework of this approach, the dispersed phase velocity is expressed through the velocity of the carrier phase [3, 5] under the assumption on the small particle response time. The comparison of two mentioned methods is the aim of this work. The results of calculating the number density of low inertia particles using two mentioned methods are compared. The traditional Lagrangian approach is used as a reference.

2. Mathematical models

The Lagrangian approach is based on solving motion equations for individual particles

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}, \quad \frac{d\mathbf{v}}{dt} = \frac{\mathbf{u} - \mathbf{v}}{\tau},$$

where $\mathbf{x}$ is the position, $\mathbf{v}$ is the velocity of particle, $\mathbf{u}$ is the velocity of gas, and τ is the particle response time. The Stokes drag law for particles is assumed. To calculate the particle number density on an Eulerian grid, the box counting is used. In this method, a lot of particles are tracked in a computational domain, and number density in a cell is calculated as normalized number of particles in that cell.

Eulerian methods for modelling particle flows are based on solving the continuity equation for disperse phase which is presented in the absence of sources and sinks of particles as

$$\frac{\partial n}{\partial t} + \nabla \cdot (\mathbf{v}n) = 0, \tag{1}$$

where $n(t, \mathbf{x})$ is the number density of particles, $\mathbf{v}(t, \mathbf{x})$ is the particle velocity field. For both considered Eulerian approaches it is assumed that all particles have the same velocity in a point of space (monokinetic assumption). The difference between the approaches is in the treatment of $\mathbf{v}(t, \mathbf{x})$.

2.1. Eulerian equilibrium approach.

For low inertia particles with a response time $\tau << 1$, the velocity field of the disperse phase can be expressed through the carrier phase velocity using an expansion over τ [5]

$$\mathbf{v} = \mathbf{u} - \tau\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) + O(\tau^2). \tag{2}$$

The advantage of this method is that it allows us to calculate the number density of particles without additional equations for disperse phase momentum.

2.2. Eulerian two-fluid approach.

The particle velocity field $\mathbf{v}(t, \mathbf{x})$ can be found by solving equations for disperse phase momentum

$$\frac{\partial (\mathbf{v}n)}{\partial t} + \nabla \cdot (\mathbf{v}\mathbf{v}n) = \frac{\mathbf{u} - \mathbf{v}}{\tau} n. \tag{3}$$

In contrast with the equilibrium method, this approach requires us to solve one additional equation for each of the space dimensions. This method can be used for particles with higher inertia. The volume fraction of dispersed phase is assumed to be sufficiently low so that the influence of the dispersed phase on the gas phase can be ignored. In this case transport equations for particles can be solved in the given gas flow field.

Since both Eulerian methods are based on the monokinetic assumption, they should not be used for flows with particles trajectory crossings, that are characterized by multiple particle velocities at one point. Such flows appear, for example, when particle inertia is sufficiently high and carrier phase flow changes direction rapidly. We will consider only low particle response time cases for which the monokinetic assumption can be justified.

3. Results and discussion

A finite volume method was used to solve equations (1) and (3). The first order explicit scheme in time and second-order scheme in space with a minmod slope limiter were used. For the two-fluid approach, an operator splitting technique with separate treating advective flux and the drag force term was used to stabilize the solution [6].

The described methods were tested using the case of hyperbolic fluid flow as a simplest model of colliding jets. The velocity field of this flow is defined as $\mathbf{u} = (u_x, u_y) = (x, -y)$. The number density of particles in hyperbolic flow was calculated using Lagrangian box counting and both Eulerian methods in the square region $[-1; 0] \times [-1; 0]$. For Eulerian methods the uniform grid of 512×512 finite volumes was used. Particles were initially placed on the bottom boundary region $[-0.1; -0.01]$ with unit number

density, zero x-velocity and unit y-velocity. On the right and top boundaries the number density and velocity was set to zero. On the left boundary zero flux conditions was used.
Contours of the particle number density calculated for $\tau = 0.1$ are shown in Fig. 1. The y-component of gas velocity decreases from 1 to 0 in vertical direction and the x-component of gas velocity increases in absolute value from 0 to 1 in horizontal direction. The particles move from bottom boundary with velocity $\mathbf{v} = (0, 1)$ towards the line $y = 0$, while slowing down in vertical direction and accelerating in horizontal direction under influence of the drag force. This leads to the compression of particle cloud on y-direction and its expansion in x-direction. As a result of the compression, the number density of particles increases at the front of the cloud. It can be seen in Fig. 1, that all three methods produce qualitatively similar results. Contours obtained using Lagrangian approach and two-fluid approach are in a good agreement. The main difference is that boundaries are blurred for the Eulerian approach as a result of numerical diffusion that is specific for solution of a pure advection equation. The two-fluid approach underestimates the highest value of number density, and the equilibrium method overestimates it in the front of particles flow.

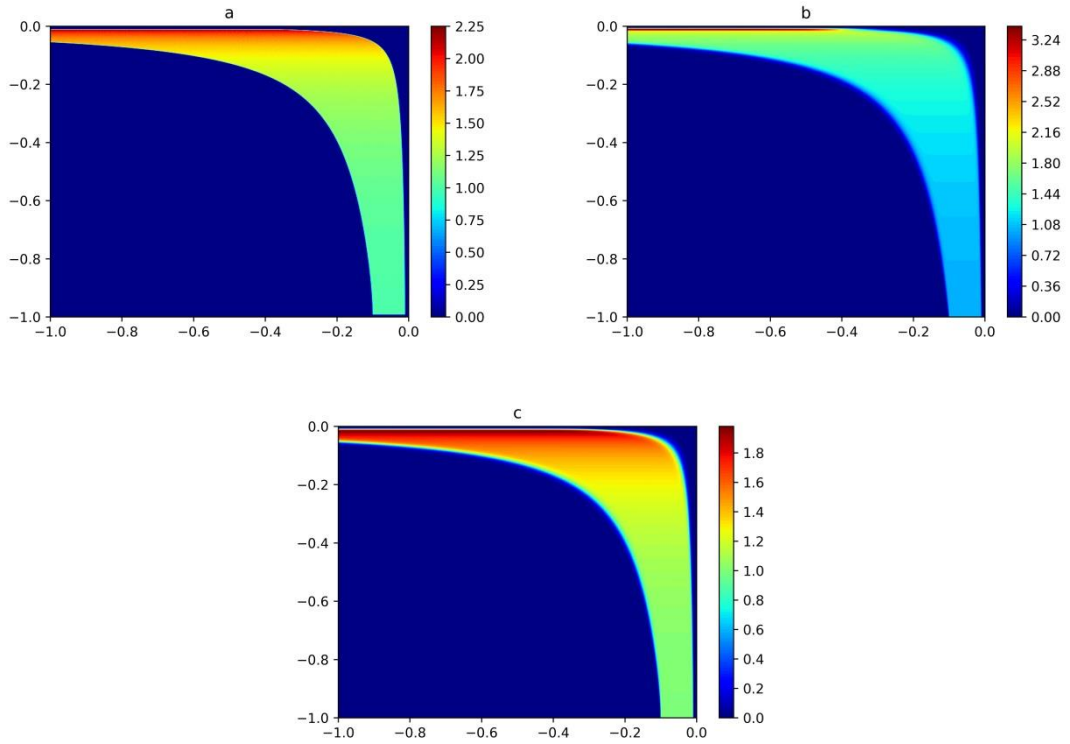

Figure 1. Contours of particle number density obtained by Lagrangian approach (a), Eulerian equilibrium approach (b), Eulerian two-fluid approach (c) for particles with $\tau = 0.1$.

The number density profiles along line $x = –0.1$ obtained using different methods for three values of τ (0.05, 0.1 and 0.2) are shown in Fig. 2. For all three values of τ, results obtained using Lagrangian box counting and the Eulerian two-fluid method are in a good agreement everywhere, except for the front of the cloud, where two-fluid method underestimates highest value of the number density and the sharpness of its decline at the front of the cloud. The Eulerian equilibrium method gives similar results for particles with the lowest response time ($\tau = 0.05$). In this case, the relative difference with the Lagrangian approach is about 2%. For particles with higher inertia (higher response time), the method leads to overestimated values of number density, compared to the other two methods. The relative error for $\tau = 0.1$ is 5%, for $\tau = 0.2$ it is 15%. Note that for the Lagrangian approach small oscillations

of solution are observed. This is inferred from calculation of number density on Eulerian grid. For higher values of τ oscillations are less visible.

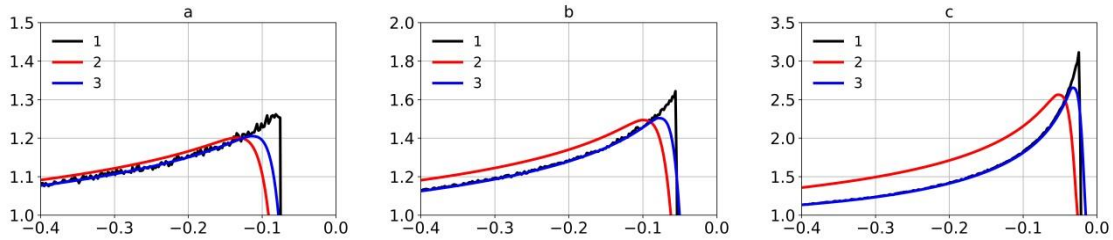

Figure 2. Particle number density along line $x = -0.1$ for different $\tau = 0.05$ (a), 0.1 (b), 0.2 (c) obtained using Lagrangian approach (1), Eulerian equilibrium approach (2), Eulerian two-fluid approach (3).

Contours of the absolute difference between number densities calculated by Lagrangian and the Eulerian methods are shown in Fig. 3. Results obtained from the Lagrangian approach are used as a reference solution. It can be seen, that the regions of highest errors are on boundaries and in the front of particle cloud. For the equilibrium method the error is more noticeable, which is associated with a vertical bias of particle cloud. This bias can be seen from Fig. 2. For low inertia particle flows, the Eulerian two-fluid method gives better prediction of the number density, compared to the equilibrium method.

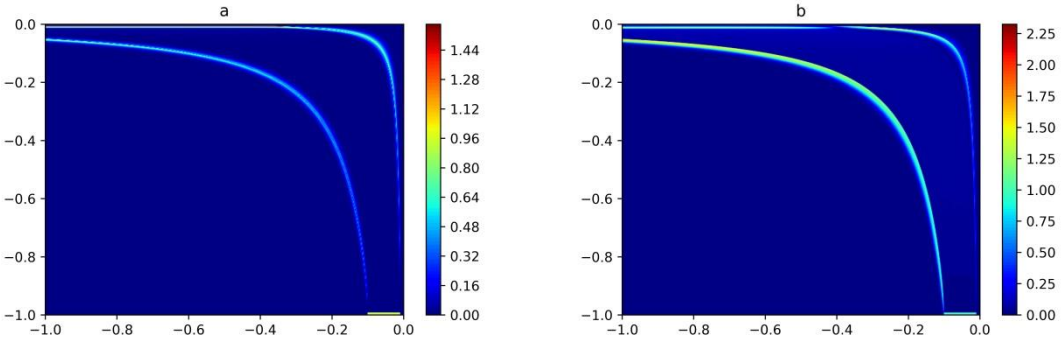

Figure 3. Contours of absolute error in particle number density calculated using Eulerian two-fluid approach (a) and Eulerian equilibrium approach (b).

4. Conclusions

The comparison of two Eulerian approaches for modeling gas-particle flow with low inertia particles is performed. The Lagrangian box counting method is used as a reference. It was shown, that in the case of low inertia particles the Eulerian two-fluid approach is in better agreement with the Lagrangian approach. A reasonable agreement for the equilibrium method is achieved only for very small particles response time ($\tau = 0.05$). For this method, the error in the calculation of the number density increases with growth of particles response time.

Acknowledgments

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