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MORREY SPACES ARE CLOSELY EMBEDDED BETWEEN VANISHING STUMMEL SPACES

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Abstract. We prove a new property of Morrey function spaces by showing that the generalized local Morrey spaces are embedded between weighted Lebesgue spaces with weights differing only by a logarithmic factor. This leads to the statement that the generalized global Morrey spaces are embedded between two generalized Stummel classes whose characteristics similarly differ by a logarithmic factor. We give examples proving that these embeddings are strict. For the generalized Stummel spaces we also give an equivalent norm.

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