IOP Publishing J. Phys. Commun. 3 (2019) 065007 https://doi.org/10.1088/2399-6528/ab291b

Journal of Physics Communications



OPEN ACCESS

RECEIVED
10 May 2019

REVISED
17 May 2019

ACCEPTED FOR PUBLICATION
12 June 2019

PUBLISHED
24 June 2019



PAPER

# Modelling a laser plasma accelerator driven free electron laser

B M Alotaibi[1,3], Sh M Khalil[1,2], B W J McNeil[3,4] and Piotr Traczykowski[3,4]

[1] Physics Department, Faculty of Science, Princess Nourah Bint Abdulrahman University, Riyadh, Kingdom of Saudi Arabia
[2] Plasma Physics & Nuclear Fusion Department, NRC, Atomic Energy Authority, Cairo, Egypt
[3] Department of Physics, University of Strathclyde, Glasgow, G4 0NG United Kingdom
[4] Cockcroft Institute, Warrington, WA4 4AD, United Kingdom

**E-mail:** bmalotaibi@pnu.edu.sa

**Keywords:** free-electron lasers, laser-plasma accelerator, puffin code, undulator

## Abstract

Free-electron lasers (FEL) are the brightest, coherent sources of short wavelength radiation from the VUV into the x-ray. There is much research interest in reducing the cost and the size of FELs by utilising new accelerator techniques. Laser-plasma accelerator (LPA) are a promising accelerator for next generation compact FEL light sources with many potential advantages due to the high acceleration gradient and large peak currents they offer. The electron beams of a LPA typically have a smaller transverse emittance, a large energy spread and tend to be of shorter duration and higher current than conventional Radio Frequency (RF) accelerators. In this paper, a FEL driven by an electron beam from a typical LPA was simulated using the 3D FEL simulation code Puffin. It is shown that lowering the homogenous electron beam energy spread increases the radiation energy output in a short undulator and , as become less than the FEL, or Pierce parameter ($\rho$), then the peak radiation energy increases and the saturation length reduces significantly as expected.

## Introduction

The FEL can create tunable, high-power sources from the hard x-ray to far infrared (FIR) with high brightness, coherent radiation, as has been demonstrated at many facilities worldwide [1]. Figure 1 shows a typical high gain FEL configuration with a highly relativistic electron beam propagating through an undulator or (wiggler) characterised by the dimensionless undulator parameter, $a_u = \frac{eB_u\lambda_u}{2\pi mc}$, where, $B_u$ is the RMS undulator magnetic field strength, $\lambda_u$ is the undulator period, $e$ and $m$ are the electron charge and rest-mass respectively. The resonant FEL wavelength is then given by $\lambda_r = \frac{\lambda_u}{2\gamma_0^2}(1 + a_u^2)$, where $\gamma_0$ is the mean Lorentz factor.

A fundamental scaling parameter for a FEL that determines the strength of FEL interaction in the 1D is the dimensionless Pierce parameter(also called the FEL parameter), defined as [2]:

$$\rho = \left[\frac{a_u^2[JJ]^2}{16}\frac{k_p^2}{k_u^2}\right]^{1/3} = \left[\frac{1}{8}\frac{I_{peak}}{I_A}\frac{a_u^2[JJ]^2}{\gamma_0^3\sigma_x^2k_u^2}\right]^{1/3} \quad (1)$$

where a planer undulator trajectory is assumed [3] with a Bessel function factor $[JJ] = [J_0(\xi) - J_1(\xi)]$ where $\xi = a_u^2/2(1 + a_u^2)$. The gain length of the interaction is $l_g = \lambda_u/4\pi\rho$. $k_p = \sqrt{2I_{peak}/(\gamma_0^3 I_A\sigma_x^2)}$ is the longitudinal plasma oscillation, $k_u$ is the undulator wavenumber, $I_A = ec/r_e \approx 17\ kA$ is the Alfven current , $r_e \approx 2.8 \times 10^{-15}m$ is the classical electron radius, $I_{peak}$ is the peak current , and $\sigma_x = \sigma_y$ is the rms transverse radius of the electron beam for a circular cross-sectional beam[2, 3] .

The normalised beam emittance $\varepsilon_n$ introduces an effective energy spread [4] in the resonant energies of the electron beam [2, 3] which may be written as:

$$\sigma_\epsilon = \frac{\epsilon_n a_u^2 k_u^2 \beta}{4\gamma_r(1 + a_u^2)} \quad (2)$$

where $\beta$ is the betatron function and $\gamma_r$ is the resonant energy for the radiation field. This may be combined with the homogenous electron beam energy spread $\sigma_\gamma$, to give a total effective energy spread of :

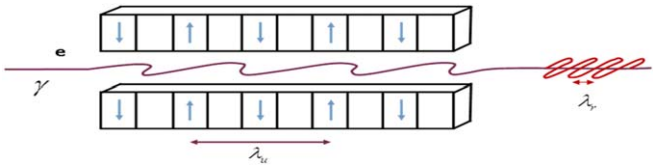


**Figure 1.** Schematic of an FEL amplifier with the electron passing through a planer undulator and emitting resonant undulator radiation.

**Table 1.** Output parameters from LPA.

| Parameters | Value parameters |
|---|---|
| Normalised emittance ($\varepsilon_n$) | 0.2 mm mrad |
| Normalised beam energy ($\gamma$) | 600 MeV |
| Peak current ($I_{peak}$) | 9.6 kA |
| Bunch charge current (Q) | 40 pC |
| RMS energy spread ($\sigma_\gamma$) | (0.0, 0.25, 0.5, 0.75, 1.0)% |

$$\sigma_{eff} = \sqrt{\sigma_\epsilon^2 + \sigma_\gamma^2}. \tag{3}$$

Some conditions are required for high gain FEL operation [1, 3, 5]:

The homogeneous energy spread must be less than the FEL parameter $\sigma_\gamma \leqslant \rho$.

The conflicting requirements of transverse matched beam radius and radiation diffraction require that the normalised beam emittance $\epsilon_n \leqslant \lambda_r \gamma / 4\pi$.

To balance the $a_u$-dependent Pierce parameter $\rho$ and the resulting gain length $l_g$, while minimizing $\lambda_u$ and obtaining a practical undulator gap $g$, undulator parameters $a_u = 2.3$ and $(\lambda_u = 15mm)$were used [6]. An undulator length of 2 m (determined from the estimated saturation length and $\lambda_u$) was chosen to allow for a compact set-up and to avoid refocusing optics between undulator modules enhancing FEL performance with a planar undulator design [6].

The following simulations were carried out using the code Puffin [7, 8] which is a non-averaged FEL simulation code that may be used to simulate FEL parameters typical of those generated by LPA output . In contrast in [6] they used Genesis code. GENESIS and PUFFIN are considered as high gain FEL simulation codes, including clear differences between them.

In spite of the agreement between GENESIS and PUFFIN was in general excellent, it is shown that [9] for a relatively small energy modulations, the GENESIS is a sufficient tool for modeling both HGHG and EEHG beams. However in more extreme settings, such as those with very large energy modulations, the assumptions of GENESIS could cause inaccurate results [9].

## Simulation results

A study was first carried out for a range of parameters using the Ming Xie formalism of [10, 11]. These analytical calculations of FEL performance, which does not require any significant computation, estimates important parameters, such as the gain length, while taking into account multiple electron beam and 3D effects, such as radiation diffraction of importance for short wavelength operation. The estimates so obtained are a quick and useful method for optimising FEL output and other parameters such as the gain length.

The FEL simulation code Puffin [7, 8] was also used in a steady-state mode, which has periodic boundary conditions applied over one wavelength of the radiation field/electron beam [12]. Full 3D-Puffin simulations were used to model a LPA driven FEL which assumed Gaussian distributions for the electron pulse duration and other electron parameters. An electron bunch with LPA–like parameters as given in table 1 was used for different values of uncorrelated energy spreads 0.0%–1.0% [6].

The- Ming Xie formalism was used to determine the effect of many beam parameters in table 1, such as electron beam emittance and energy spread on the undulator length to achieve high power saturated radiation output. Figure 2 shows contour plots of (**a**) Pierce parameter $\rho$ (**b**) RMS transverse sizes of the electron beam $\sigma_{x,y}$

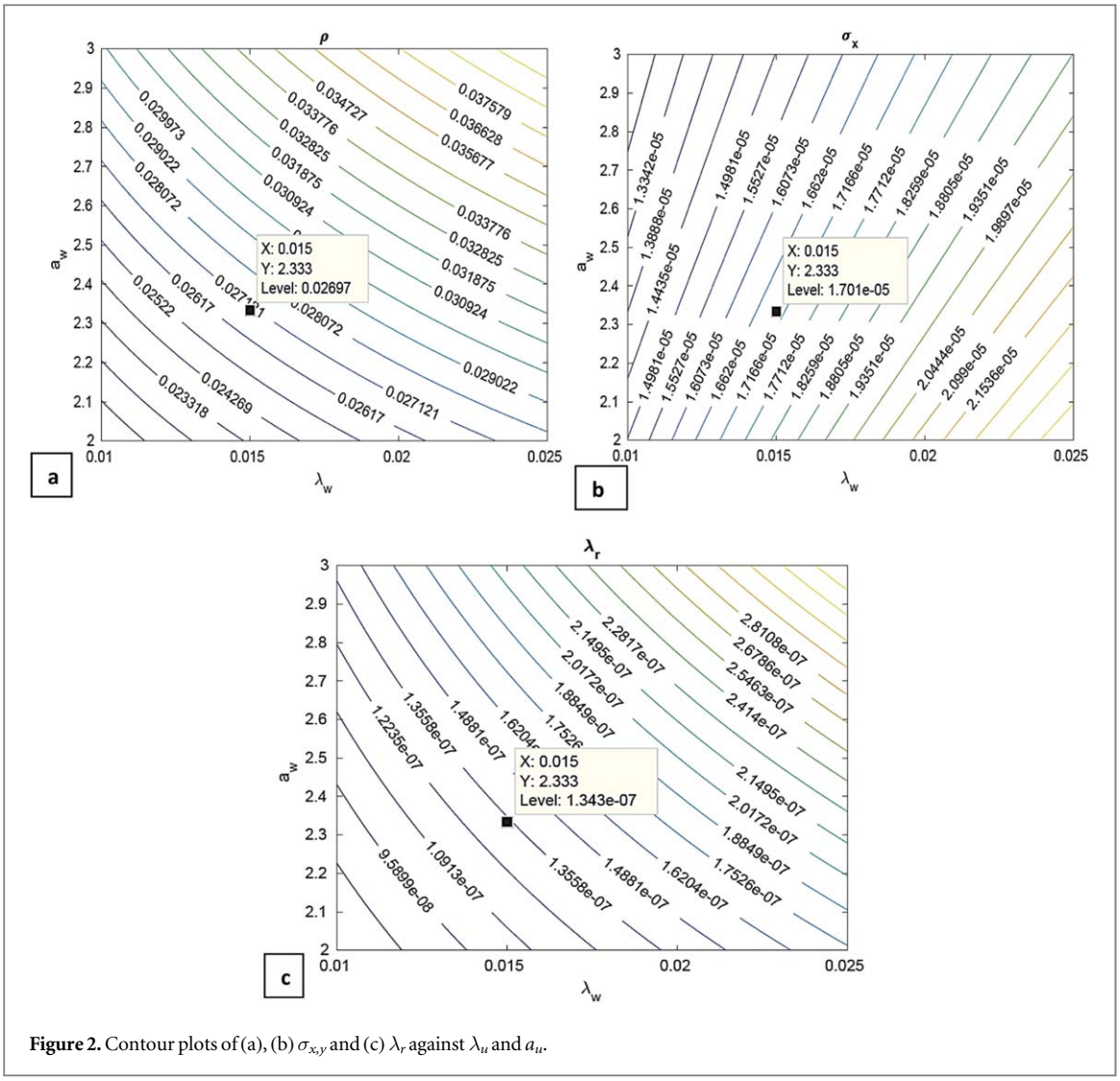


**Figure 2.** Contour plots of (a), (b) $\sigma_{x,y}$ and (c) $\lambda_r$ against $\lambda_u$ and $a_u$.

**Table 2.** Output parameters from Ming Xie formalism.

| Parameters | Value parameters |
|---|---|
| $\rho$ | 0.0269 |
| $L_{sat}$ | 0.83 m |
| $\lambda_r$ | $1.34 \times 10^{-7}$ m = 134 nm |
| $\sigma_x$, $\sigma_y$ | $1.7 \times 10^{-5}$ m |

and (**c**) resonant wavelength against $\lambda_u$ and $a_u$ using the parameters of table 1. This resulted in the parameters of table 2 being chosen.

These parameters were then used in the Puffin simulation code in steady-state mode.

$L_{sat}$ is the saturation length, which can be approximated by $L_{sat} \approx 20L_g$ [13]. Figure 3 shows a comparison between the radiation energy of different values of energy spreads $\sigma_\gamma$ in steady state mode .

## 3D laser-wakefield accelerator (LWA) driven FEL

After running Puffin in periodic (steady state) mode as above, it is seen that beam energy spreads of $\sigma_\gamma = (0.25\%–0.5\%)$ give a reasonable gain. A full Gaussian current electron pulse beam for these values of energy spreads are now simulated. A beam of peak current $I_{peak} = 9.6$ kA , charge of $Q = 40$ $pC$, and unchirped energy of 600 MeV was used [6]. A planar undulator period of $\lambda_u = 15$ mm with an undulator pole gap of

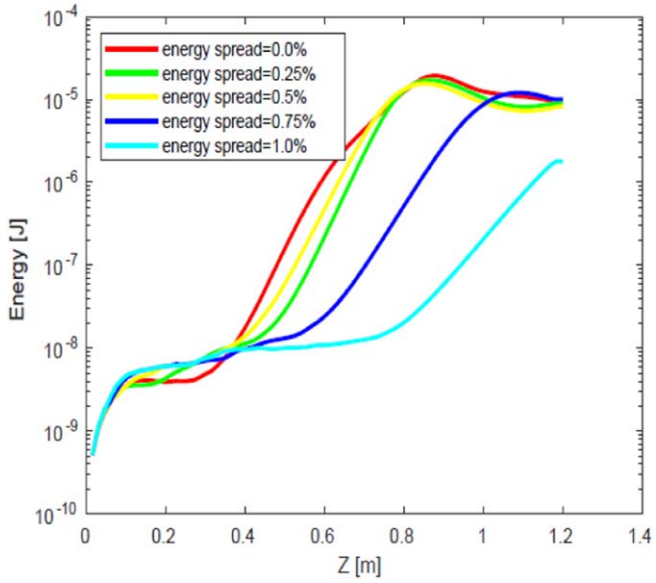


**Figure 3.** The radiation energy as a function of distance z through the undulator for different values of energy spread.

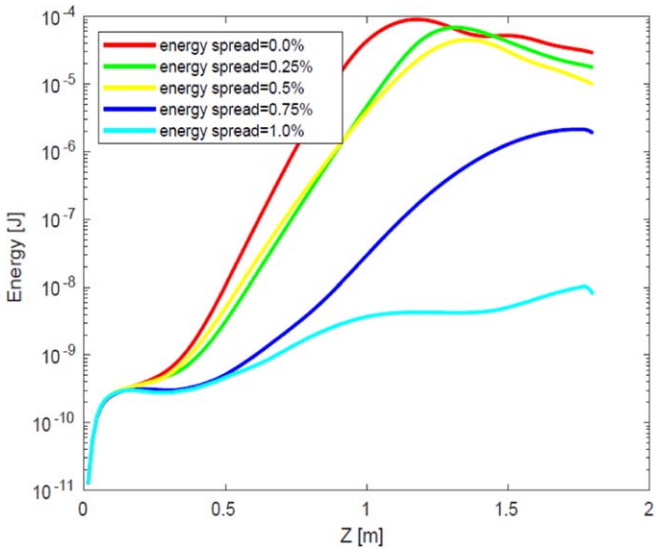


**Figure 4.** 3D Puffin simulation showing radiation energy as a function of distance z through the undulator in Gaussian distribution.

$g = 2.5$ mm was chosen to give an undulator parameter of $a_u = 2.3$ [6]. These beam and undulator parameters give an FEL parameter of $\rho = 0.0269$ as above with a resonant wavelength $\lambda_r = 134\ nm$.

The electron bunch length is calculated as $\sigma_e = \frac{cQ}{\sqrt{2\pi}\, I_{peak}} = 0.5\mu m$ which is similar to that of one cooperation length $l_c = \frac{\lambda_r}{4\pi\rho} = 0.4\ \mu m$ [7], so that the radiation output will then be in the weak superradiant mode of operation to give a single, short radiation pulse output [14]. Figure 4 shows the total radiated energy in the FEL for the Gaussian electron bunch. Figure 5 shows that if we have a sufficiently low value of energy spread then a relatively high peak energy can be achieved in a short undulator. Note that as the energy spread approaches the criterion $\sigma_\gamma \geqslant \rho$, then the peak energy reduces and the saturation length increases significantly as expected.

Figure 6 shows the intensity spectrum ($|\tilde{A}|^2$) plotted as function of scaled frequency $\omega/\omega_r$ for two values of energy spread=0.25% and 0.5% , for z=1.05 m through the undulator . It is seen that the maximum gain is at $\omega/\omega r \approx 0.99$ for 0.25% and $\omega/\omega_r \approx 0.97$ for 0.5%. This is in broad agreement with small shift from resonance ($\omega/\omega_r = 1$) for the peak that occurs due to the effect of the emittance ($\epsilon_n = \beta\gamma\varepsilon_{rms}$) [13], where $\varepsilon_{rms} = \frac{\lambda_r}{4\pi}$ is the geomertric rms emittance [4].

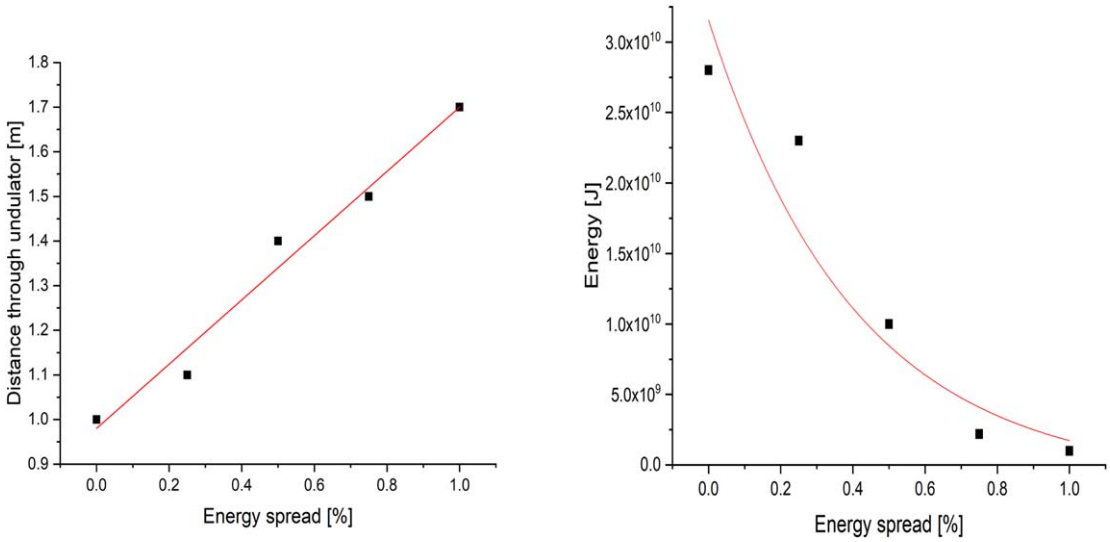


**Figure 5.** Energy [J] as a function of energy spread and undulator distance as a function of energy spread through the undulator in Gaussian distribution.

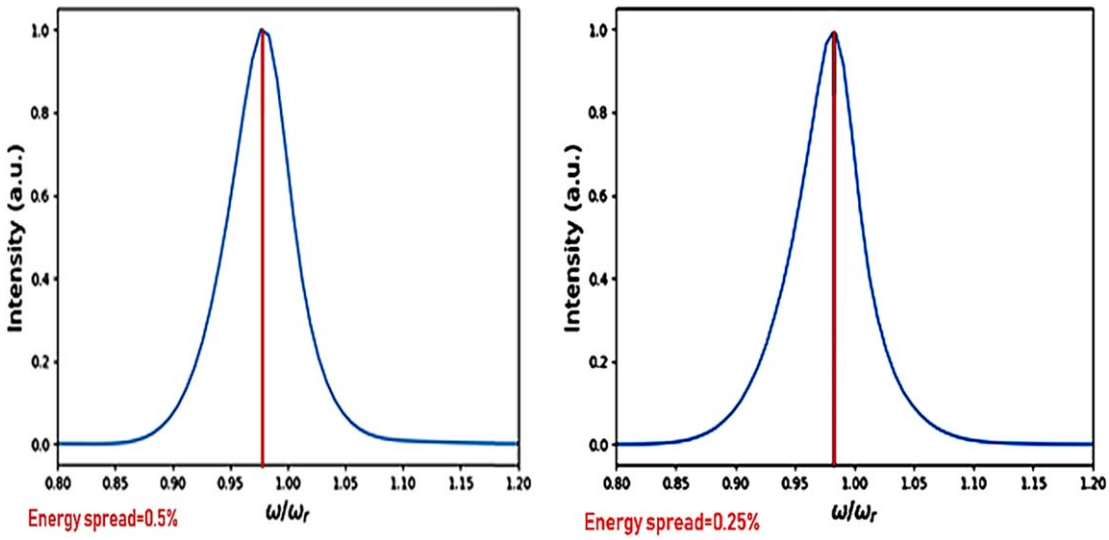


**Figure 6.** The intensity spectrum ($|\tilde{A}|^2$) is plotted as function of scaled frequency $f = \omega/\omega_r$ for two values of energy spread $\sigma_\gamma = 0.25\%$ and 0.5%.

Figure 7 shows the temporal power of the pulse as a function of $(ct - z)$ at saturation (z=1.05 m) for energy spread $\sigma_\gamma =$ (0.25% and 0.5%), clearly showing the reduced power when $\sigma_\gamma \geqslant \rho$.

Future work , will look to start-to-end FELs simulations using LPA in the range of soft x-ray (XFEL) between 1–10 nm [15, 16]. This will requrie high quality beam with high peak current, high energy and small transverse emittance within a relatively short undulator. This can be challenging in propagating, conditioning and matching the electron beams into the undulator due to , relatively large energy spreads and divergence of the electrons at the LPA exit [17].

## Conclusions

The work on plasma accelerators is not only limited to optimisation of the qualities of electron beams; FEL performance benefits from beams with a high current, small transverse emittance and small energy spread. The undulator design is based on recent achievements in the development of undulators to optimise the FEL performance.

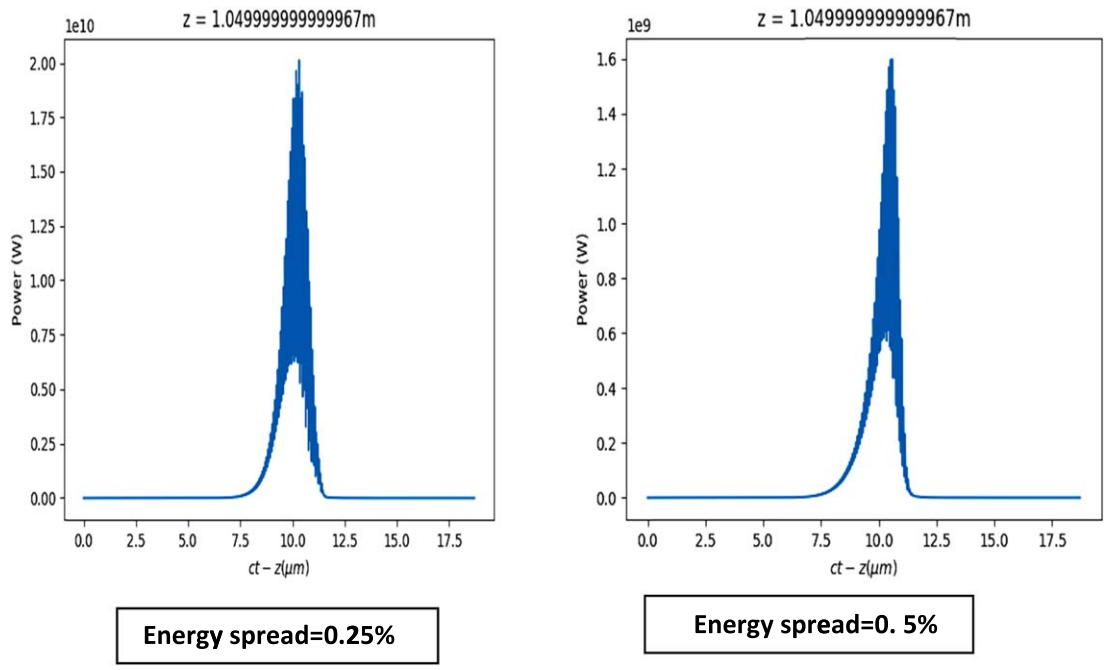


**Figure 7.** The temporal power of the pulse as a function of the scaled time coordinate ($ct-z$) at saturation (z = 1.05 m) for energy spread $\sigma_\gamma = 0.25\%$ and 0.5%.

The undulator length was limited however, saturation can be achieved in a shorter distance as seen from figure 4 with peak saturated output at 1–1.2 m. This may be great interest when considering modelling of a compact LPA Driven FEL.

To reach short radiation wavelength requires (i) shorter undulator period $\lambda_u$, or (ii) large e-beam energy $\gamma$. Radiation wavelength tuning is achieved by varying beam energy and/or undulator period, with the undulator parameter $a_u$.

In case of steady-state simulations (figure 4) we obtained the reasonable gain for Gaussian distributions as clear from (figure 5). However, in case of steady-state a higher order of energy gain is obtained compared to the Gaussian pulse mode where for the typical LWA parameters used here, a relatively short pulse was used of the order of the cooperation length. This generates a single superradiant pulse output [18] with a lower peak power output than in the steady state. This short, single pulse output is of interest in its own right where such output is often sought by FEL users.

For lower values of energy spread a higher peak energy is obtained at shorter distance along the undulator as expected .

The best results of were obtained when the energy spread $\sigma_\gamma$ was below 0.5% and it is clear that the energy spread has large impact on the energy gain and output in the FEL and must be minimised to $\sigma_\gamma < \rho$ in order to achieve acceptable FEL output efficiency.

## ORCID iDs

B M Alotaibi https://orcid.org/0000-0002-9499-2711

## References

[1] McNeil B W J and Thompson N R 2010 Review article: x-ray free-electron lasers *Nat. Photonics* **4** 814
[2] Bonifacio R, Pellegrini C and Narducci L 1984 Collective instabilities and high-gain regime in a free electron laser *Optics Commun.* **50** 373
[3] Huang Z and Kim K-J 2007 Review of x-ray free electron laser theory *Physical Review Special Topics, AB* **10** 034801
[4] Bonifacio R, Souza L D S and McNeil B W J 1992 Emittance limitations in the free electron laser *Optics Comm.* **93** 179
[5] Henderson J R, Campbell L T and McNeil B W J 2015 Free electron laser using 'beam by design *New J. Phys.* **17** 083017
[6] Maier A R, Meseck A, Reiche S, Schroeder C B, Seggebrock T and Gruner F 2012 Demonstration scheme for a laser-plasma-driven free-electron laser *Physical Review*, X **2** 031019
[7] Campbell L T and McNeil B W J 2012 Puffin: a three dimensional, unaveraged free electron laser simulation code *Phys. Plasmas* 19093119

[8] Campbell L T, Smith J D A, Traczykowski P and McNeil B W J 2018 Updated description of the FEL simulation code puffin *Proc. of IPAC2018, Vancouver, BC, Canada, 02 Photon Sources and Electron Accelerators, A06 Free Electron Lasers, THPMK112* http://ipac2018.vrws.de/papers/thpmk112.pdf
[9] Brian *et al* 2017 Comparing FEL codes for advanced configuration *38th Int. Free Electron Laser Conf., FEL (Santa Fe, NM, USA)* http://accelconf.web.cern.ch/AccelConf/fel2017/papers/mop016.pdf
[10] Xie M 2000 Exact and variational solutions of 3D eigenmodes in high gain FELs *Nuclear Instruments and Methods in Physics Research, section* A **445** 59
[11] Xie M 1995 Design optimization for an x-ray free electron laser driven by SLAC linac *Proc. of the 1995 Particle Accelerator Conf. (JACoW, Geneva)* https://accelconf.web.cern.ch/accelconf/p95/ARTICLES/TPG/TPG10.PDF
[12] Dornmair I *et al* 2016 Towards plasma-driven free electron laser *NIC Symposium* **48** 401
[13] Marco Venturini, Basics on FEL Physics; Undulators; High-Level Machine-Design Parameters, Course Materials, Lecture Mo2 - Rutgers University - New Jersey, 21-06-2015 http://uspas.fnal.gov/materials/15Rutgers/Lecture_Mo2.pdf
[14] Campbell L T and McNeil B W J 2012 A simple model for the generation of ultra-short radiation pulses *FEL2012: Proc. of the 34th Int. Free-Electron Laser Conf.* paper THPD41 26–31 *(Nara, Japan)* https://accelconf.web.cern.ch/accelconf/FEL2012/papers/thpd41.pdf
[15] Campbell L T and Maier A R 2017 Velocity dispersion of correlated energy spread electron beams in the free electron laser *New J. Phys.* **19** 033037 http://iopscience.iop.org/article/10.1088/1367-2630/aa6205
[16] Pellegrini C 2012 The history of x-ray free-electron lasers *Eur. Phys. J.* H **37** 659
[17] Khojoyan M, Briquez F, Labat M, Oulergue A L, Marcouille O, Marteau F, Sharma G and Couprie M E 2016 Transport studided of LPA electron beam towards the FEL amplification at COXINEL *Nuclear Instruments and Methods in Physics Research section* A **829** 260
[18] Bonifacio R, McNeil B W J and Pierini P 1989 Superradiance in the high-gain free electron laser *Phys. Rev.* A **40** 4467