UNIVERSIDADE da MADEIRA

University of Madeira
Faculty of Exact Sciences and Engineering
Curricular Unit: Hydraulics
Course: Bachelor/1st Cycle of Bologna - Civil Engineering

HYDRAULICS

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Technical Specifications

Title	Hydraulics: Practice
Authors	Sérgio António Neves Lousada Rafael Freitas Camacho Adhonys Alexander Rincon Rodrigues
Editors	University of Madeira
Edition	1st
Edition Years	2018
Volume	I
ISBN	978-989-8805-49-2
Support	Eletronic
Format	PDF

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List of symbols and abbreviations

Symbols

δ_{ij}	Symbol of Kronecker
∇	Gradient of deformation
a	Acceleration; Dimension; index
A	Area
b	Dimension; Index
c	Dimension; Index; Velocity of elastic waves propagation
C	Coefficient; Costs
C’	Factor of contraction
C_1	Initial costs
C_2	Annual energy costs
C_c	Contraction coefficient
C_d	Discharge or download coefficient
C_p	Specific heat
C_v	Specific heat; Coefficient of velocity
d	Dimension; Index
D	Diameter
E	Energy
Eu	Number of Euler
F	Force
f	Function
g	Gravitational acceleration
H	Charge; Energy
h	Height; Elevation; level
i	Inclination; Index; Slope
I	Moment of inertia
J	Unitary charge losses
k	Coefficient
K	Coefficient of rugosity
Kn	Number of Kundsen
L	Support; Length
l	Dimension; Index
M	Number of Mach; Total quantity of movement or total impulsion
n	Coefficient; Index; Rotation velocity
N	Number of rotations
p	Pressure
P	Perimeter; Position; Power; Pressure
Q	Flow rate; Matrix of transformation
q	Instantaneous Flow rate
r	Radius
R	Hydraulic radius or radius
S	Cross Section Area; Capacity of aspiration; Plain surface

s	Length of a Line
St	Number of Strouhal
T	Tensor of tensions; Temperature; Torque
t	Time
U	Average velocity or speed
v	Velocity
V	Velocity; Volume
We	Number of Weber
x	Coordinate
y	Height; Coordinate
Z	Topographic height
Δ	Variation
ΔH	Charge losses
Φ	Function
Ψ	Function
rot	Rotational
α	Angle; Coefficient
α'	Coefficient of Boussinesq
γ	Angle; Volumetric weight
δ	Coefficient of superficial tension
ε	Coefficient; Grade of reaction of turbines; Parameter of rugosity of Nikuradse
η	Coefficient; efficiency/yield; Specific rotation
θ	Angle
λ	Coefficient; Function; Charge loss;
μ	Coefficient of dynamic viscosity or absolute viscosity
ρ	Volumetric mass or density
σ	Superficial Tension
τ	Tension
υ	Coefficient of kinematic or relative viscosity
χ	Wet or slinked perimeter
ω	Angular velocity; Vorticity
ϵ	Tensor rate of deformation

ABBREVIATIONS

Approx..	Approximately
Cf.	According
CG	Gravity center
CGS	System Centimetre–Gram–Second
Cst.	Constant
Eq.	Equation
NA	Level of Water
NPSH	Net Positive Suction Head
RAM	*Região Autónoma da Madeira*
S.L. or F.S.	Free surface
SI	International system
T.Q.M.	Theorem of the amount of movement
Theo.	Theorem
US	United States
USBR	United States Bureau of Reclamation
VRP	Pressure reduction valve
WES	Waterways Experiment Station

EXERCISES

CHAPTER 1 – Introduction

1.1. The relative specific weight of a substance is 0.8. How much will be its specific weight?

1.2. Find the air mass and weight contained in a room $3.0\,m$ high and an area of $4.0\,m \times 5.0\,m$. What would be the mass and weight of an equal volume of water? ($g = 9.8\,m/s^2$)

1.3. In a pipe flows hydrogen ($k = 1.4; R = 4122\,m^2/s^2K$), at a section (1), $p_1 = 3 \times 10^5\,N/m^2\,(abs)$ and $T_1 = 30\,℃$. The temperature is constant throughout the pipe. What is the gas specific mass at a section (2), where $p_2 = 1.5 \times 10^5\,N/m^2\,(abs)$?

1.4. The kinematic viscosity of an oil is $0.028\,m^2/s$ and its relative specific weight is 0.85. Determine the dynamic viscosity in the unit systems MK_pS, CGS and SI. ($g = 10\,m/s^2$)

1.5. A square plate $1.0\,m$ long and weighing $20\,N$ slides in an inclined plane at an angle of $30°$ over a layer of oil. The plate velocity is $2\,m/s$, constant. What is the oil dynamic viscosity, if the thickness of the oil layer is $2\,mm$?

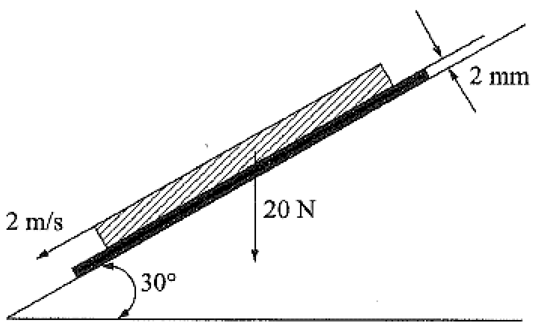

Figure 1 – Sliding Plate (Brunetti, 2008).

1.6. Two different parallel plates are separated by $2\,mm$. The upper plate moves at $4\,m/s$, while the lower one is fixed. If the space between those two plates is filled with oil ($\nu = 0.1\,St;\ \rho = 830\,kg/m^3$), what would be the shear or tangential tension acting on the oil?

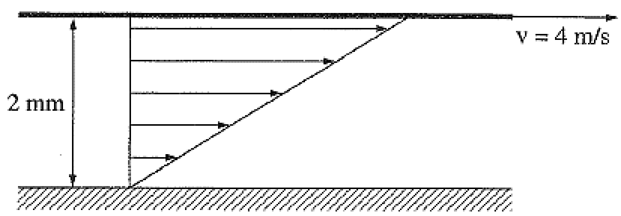

Figure 2 – Oil between two parallel plates (Brunetti, 2008).

CHAPTER 2 – Fundamental Equation of Fluids Movement

2.1. A gas flows in permanent regime in a section of the pipe in the figure. At section (1), $A_1 = 20\ cm^2$, $\rho_1 = 4\ kg/m^3$ and $v_1 = 30\ m/s$. At section (2), $A_2 = 10\ cm^2$ and $\rho_2 = 12\ kg/m^3$. What is the velocity at section (2)?

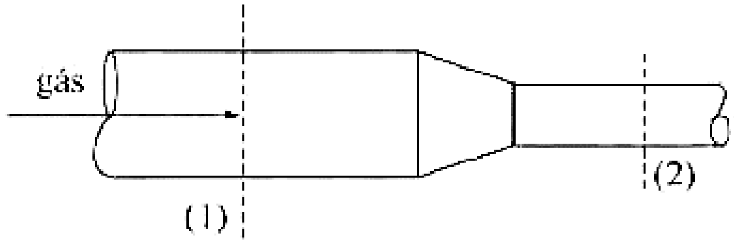

Figure 3 – Convergent pipe with a flowing gas (Brunetti, 2008).

2.2. The Venturi is a convergent/divergent pipe, as shown in the figure. Determine the velocity at the constricted section (or choke) of $5\ cm^2$, if the section at the entrance has $20\ cm^2$ and velocity of $2\ m/s$. (Incompressible Fluid).

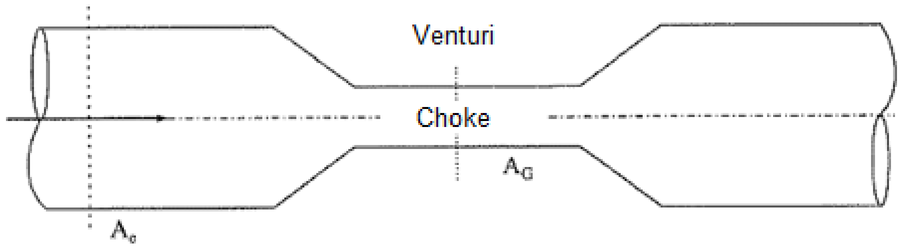

Figure 4 – Convergent/divergent pipe (Brunetti, 2008).

2.3. The air flows in a convergent pipe. The area at the major section of the pipe is $20\ cm^2$ and the minor section is $10\ cm^2$. The specific mass of the air in section (1) is $1.2\ kg/m^3$, while in section (2) is $0.90\ kg/m^3$. Being the velocity at section (1) $10\ m/s$, determine the flow rate in terms of volume, mass and weight, as well as the average velocity in section (2). ($g = 10\ m/s^2$)

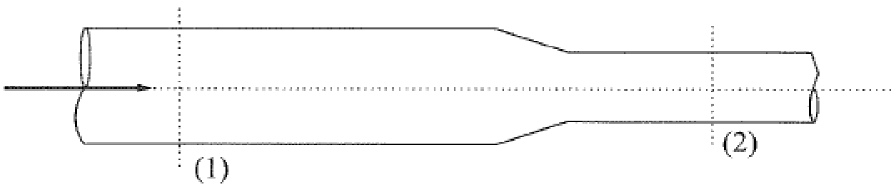

Figure 5 – Convergent pipe with air (Brunetti, 2008).

2.4. A pipe admits water ($\rho = 1000\ kg/m^3$) to a reservoir with a flow rate of $20\ L/s$. In the same reservoir is introduced oil ($\rho = 800\ kg/m^3$) by another pipe with a flow rate of $10\ L/s$. The homogeneous mixture is discharged through a pipe which section has $30\ cm^2$. Determine the specific mass of the mixture in the discharge pipe and its velocity.

2.5. At a plane Oxy, the velocity field is given by $v_x = 2xt$ and $v_y = y^2t$. Determine the acceleration at the origin and at point $P = (1,2)$ at instant $t = 5\ s$ (meassurement in cm).

2.6. A viscous and incompressible fluid flows between two vertical flat plates as shown in the figure. Assume a laminar, permanent and uniform flow.
a) Determine an expression for the gradient of pressures in the direction of flow by using Navier-Stokes's equations. Express dp/dy as a function of flow rate per length unit (q);
b) What would be the flow rate if $dp/dy = 0$.

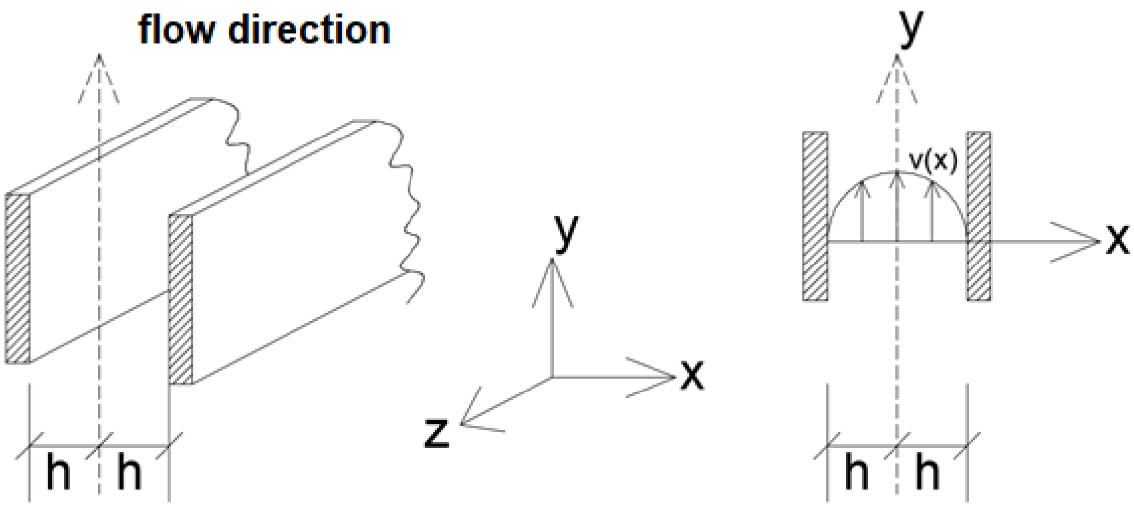

Figure 6 – Fluid flowing between two plates (source: Author).

CHAPTER 3 – Hydrostatics

3.1. The figure shows a scheme of a hydraulic press. Those two plugs have areas of $A_1 = 10\ cm^2$ and $A_2 = 100\ cm^2$. If a $200\ N$ force is applied at plug (1), what would be the force transmitted to plug (2)?

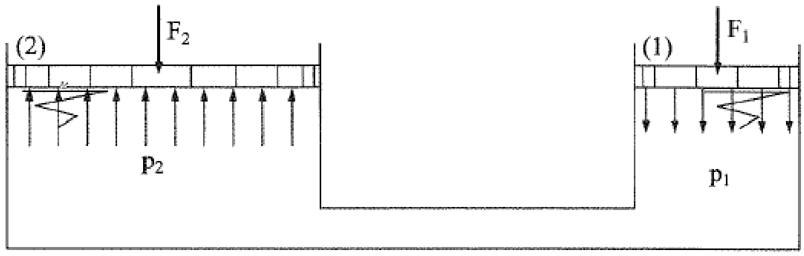

Figure 7 – Hydraulic Press (Brunetti, 2008).

3.2. Determine, based on a pressure value of $340\ mmHg$, the pressure values in kgf/cm^2 and psi at an effective or manometric scale, and in Pa and atm in an absolute scale. ($p_{atm} = 101.2\ kPa$)

3.3. A storage tank $12.0\ m$ deep is full of water. The top of it is open to the atmosphere. What is the absolute pressure at the bottom of it? What is the manometric pressure? ($g = 9.8\ m/s^2$)

3.4. Given the figure:
a) What is the reading on the manometer?
b) What is the force that act at the reservoir top?

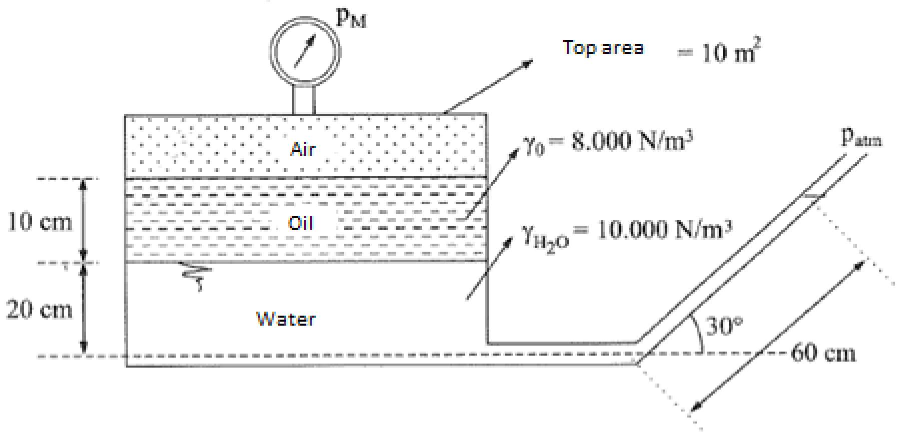

Figure 8 – Manometer (Brunetti, 2008).

3.5. A rectangular reservoir as shown in the figure is $4.5\ m$ long, $1.2\ m$ wide and $1.5\ m$ high. It contains $0.6\ m$ of water and $0.6\ m$ of oil. Calculate the forces caused by the liquids in the lateral walls and bottom. Given $\gamma_1 = 8500\ N/m^3$; $\gamma_2 = 10000\ N/m^3$.

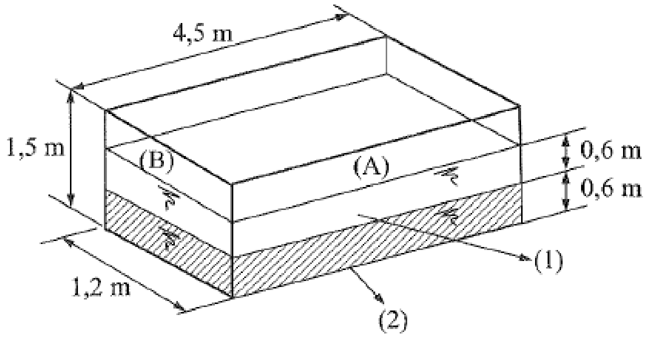

Figure 9 – Rectangular reservoir (Brunetti, 2008).

3.6. A cast iron cylinder with $30\ cm$ in diameter and $30\ cm$ high, is immersed in sea water ($\gamma = 10300\ N/m^3$). What is the impulsion caused by water in the cylinder? What is the impulsion, if the cylinder was made of wood ($\gamma = 7500\ N/m^3$)? In that case, what will be the immerse height of the cylinder?

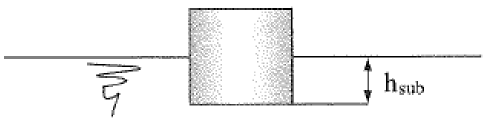

Figure 10 – Cylinder partially immersed (Brunetti, 2008).

CHAPTER 4 – Theorem of Bernoulli and applications

4.1. A Pitot tube is attached to a ship moving at $45\ km/h$. What would be the water height (h) in the vertical pipe? ($g = 10\ m/s^2$)

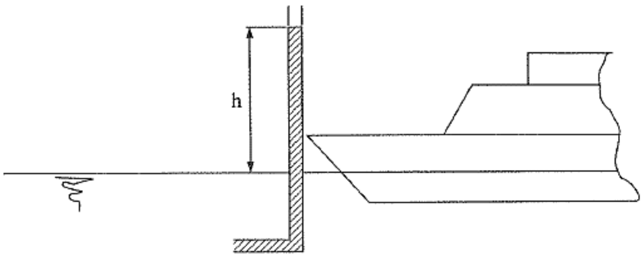

Figure 11 – Pitot tube attached to a ship (Brunetti, 2008).

4.2. Given the device shown in figure, calculate the water flow rate in the pipe. Given: $\gamma_{H_20} = 10^4\ N/m^3$; $\gamma_m = 6 \times 10^4\ N/m^3$; $p_2 = 20\ kPa$; $A = 10^{-2}\ m^2$; $g = 10\ m/s^2$; ignore losses and consider a uniform diagram of velocities.

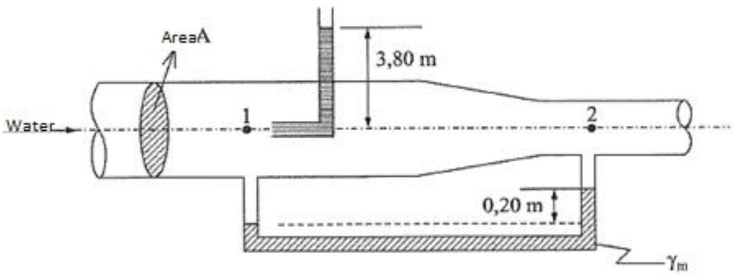

Figure 12 – Pipe with a Pitot tube (Brunetti, 2008).

4.3. In the pipe shown in the figure, the fluid is considered ideal. Given: $H_1 = 16\ m$; $p_1 = 52\ kPa$; $\gamma = 10^4\ N/m^3$; $D_1 = D_3 = 10\ cm$. Determine: ($g = 10\ m/s^2$)

a) The flow rate in terms of weight;

b) The height h_1 in the manometer;

c) The diameter of section (2).

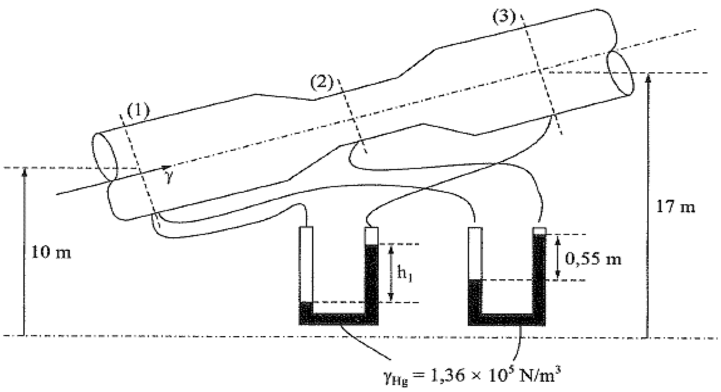

Figure 13 – Venturi Pipe (Brunetti, 2008).

4.4. A pump is installed at a PVC pipe that connects two reservoirs with free surfaces at elevations of $95\,m$ and $120\,m$, as shown in the figure. The pump works daily for $10\,hours$, at 85% efficiency (pump-motor), elevating a total of $720\,m^3$. At such conditions determine:

a) The pump's total height;
b) The daily energy consumption;
c) The water pressure, upstream of curve D.

Note: Consider reservoirs of great dimensions, $\gamma_{H_20} = 9800\,N/m^3$, $g = 9.8\,m/s^2$ and ignore localized charge losses.

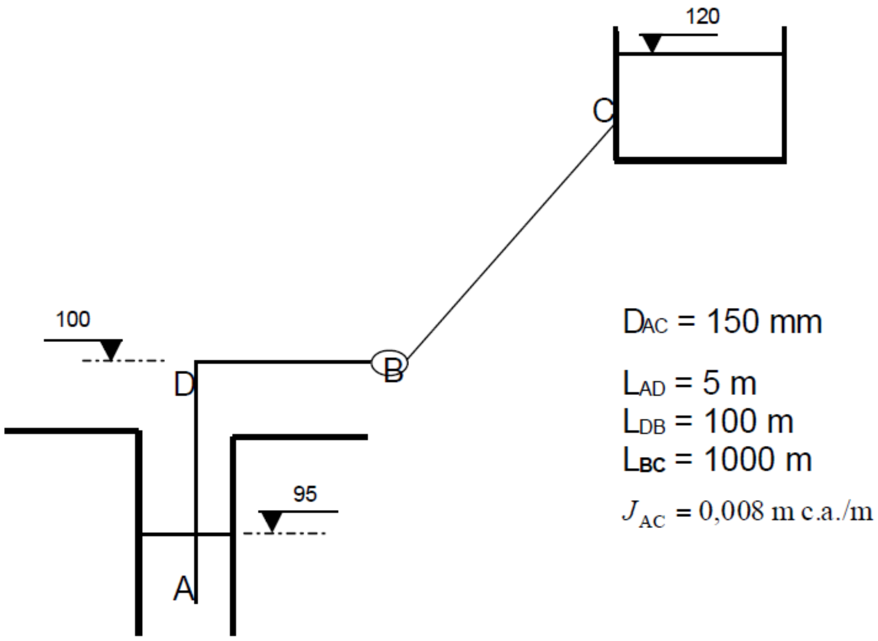

Figure 14 – Hydraulic circuit (Vasconcelos, 2005).

CHAPTER 5 – Theorem of amount of movement and applications

5.1. A jet diverter moves at $9\ m/s$. A nozzle of $5\ cm$ in diameter ejects oil at $15\ m/s$. Such jet acts in the diverter as shown the figure. The output angle is $60°$ and the oil specific weight is $8000\ N/m^3$. Calculate the force of the jet against the diverter. ($g = 10\ m/s^2$)

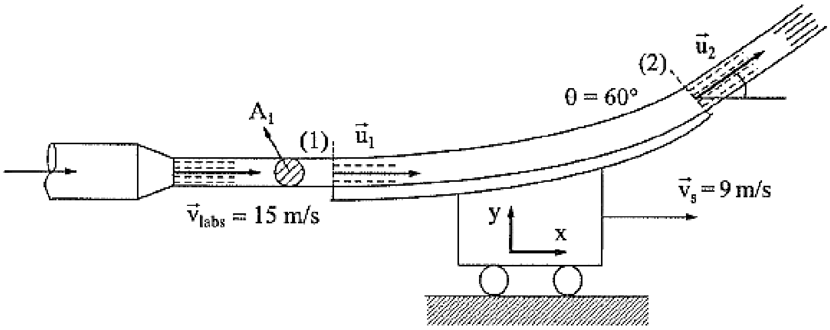

Figure 15 – Jet diverter (Brunetti, 2008).

5.2. Calculate the horizontal force applied at the support of the nozzle in the figure. Knowing that the water acts in the plate, flat and vertical, and it gets distributes in all directions equally, calculate the necessary force to keep the plate in its position (rest). Given: $p_1 = 150\ kPa$; $v_1 = 5\ m/s$; $D_1 = 10\ cm$; $D_2 = 5\ cm$; $\rho = 1000\ kg/m^3$.

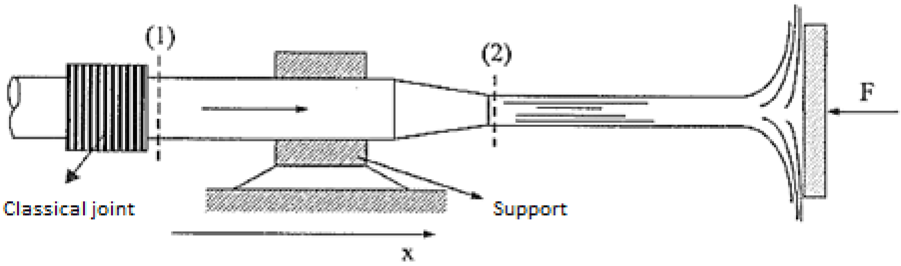

Figure 16 – Nozzle, jet and plate (Brunetti, 2008).

5.3. The water that leaves a reservoir of greats dimensions flows through a pipe of $15\ cm$ of diameter and acts in a fixed reflector blade, it is deviated $90°$, as shows the figure. Knowing that the developed horizontal push in the blade is $1000\ N$, determinate the power of the turbine. Given: $\rho = 1000\ kg/m^3$; the charge losses of turbine are ignored; $\eta_T = 70\%$.

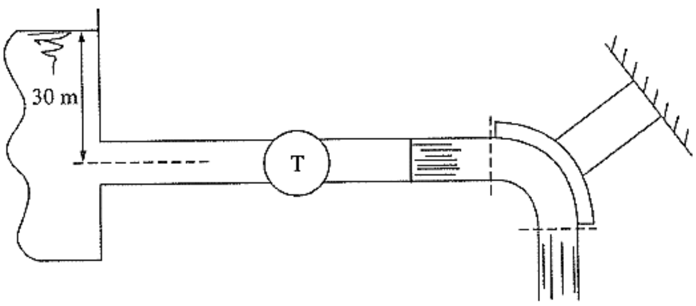

Figure 17 – Jet and fixed deflected blade (Brunetti, 2008).

5.4. Determine the power transmitted by a water jet in a Pelton type action turbine. Also determine the power transmission efficiency.

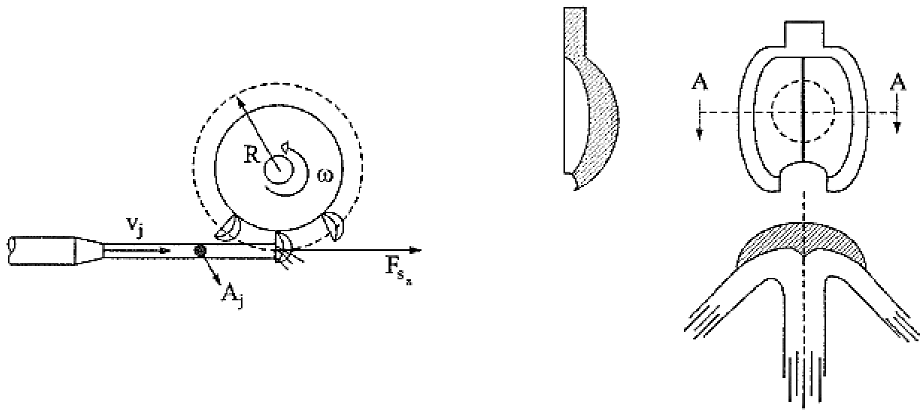

Figure 18 – Pelton turbine (Brunetti, 2008).

5.5. The turbine shown in the figure "extracts" a power of $2.9\ kW$ from flowing water. Ignoring losses at the choke, calculate the applied forces by water at the choke and turbine, respectively. Given: $\rho_{H_2O} = 1000\ kg/m^3$; $g = 10\ m/s^2$.

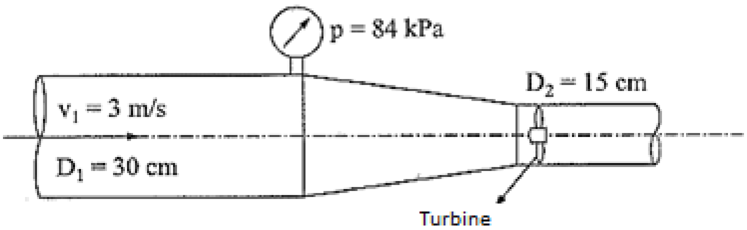

Figure 19 – Turbine installed in a choke (Brunetti, 2008).

5.6. The ship shown in the figure has a propulsion system that consist in a pump that suck waters at bow and ejects it at stern. All pipes have $5\ cm$ in diameter, and an output flow rate of $50\ L/s$. Calculate the propulsion force at departure, which means that at that moment the ship is at rest. Admit that the pressure at inputs and outputs is practically atmospheric ($\rho = 1000\ kg/m^3$).

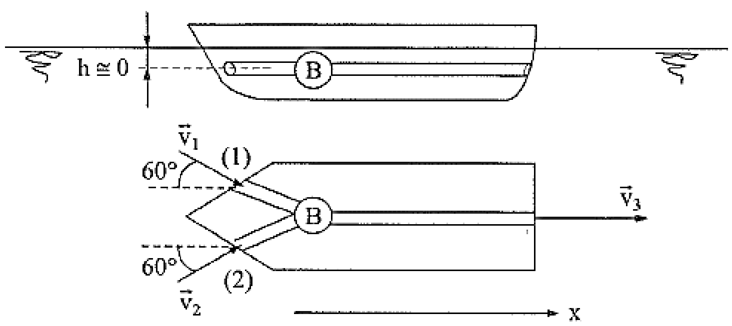

Figure 20 – Ship (Brunetti, 2008).

CHAPTER 6 – Flows through orifices and dischargers or spillways

6.1. Being $C_v = 0.9$ and $C_c = 0.6$, determine pressure p_1, knowing that the fluid is water and rises $3\ m$ in the Pitot tube. Determine the flow rate, knowing that the orifice area is $50\ cm^2$.

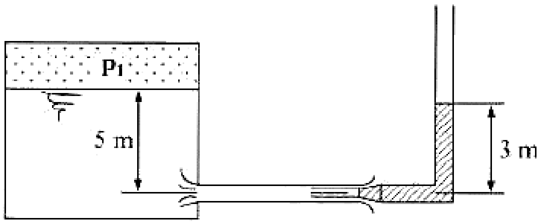

Figure 21 – Reservoir with orifice and Pitot tube (Brunetti, 2008).

6.2. The upper reservoir discharges water through an orifice with $C_d = 0.6$, to a reservoir that discharges through another orifice. The system is in equilibrium, so the water level does not change at any reservoir. What is the discharge coefficient in the second orifice? Given: $orifice\ (1)\ diameter$: $9\ cm$; $orifice\ (2)\ diameter$: $10\ cm$.

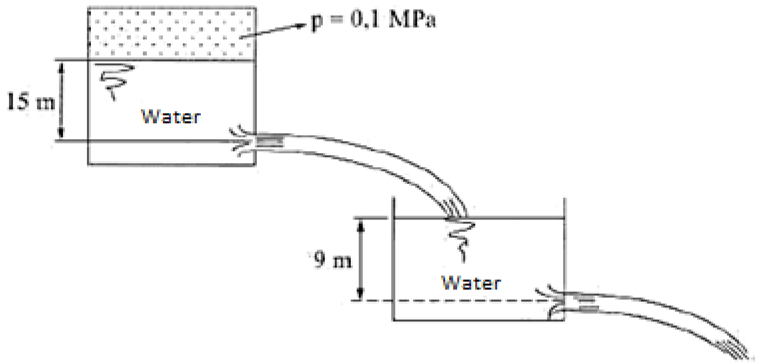

Figure 22 – Reservoir system (Brunetti, 2008).

6.3. At the device shown in the figure, calculate the water flow rate in the pipe. Given: $\gamma_{H_2O} = 10^4\ N/m^3$; $\gamma_m = 6 \times 10^4\ N/m^3$; $p_2 = 20\ kPa$; $A = 10^{-2}\ m^2$; $g = 10\ m/s^2$. Ignore losses and admit a uniform diagram of velocities at the section.

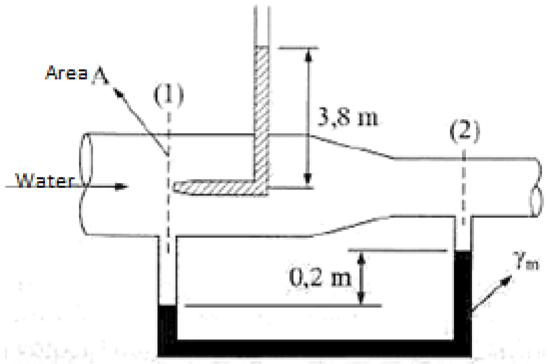

Figure 23 – Device (Brunetti, 2008).

6.4. At the bottom of the lower reservoir shown in the figure, which is initially empty, has a wooden cube with a $1\ m$ edge. From the upper reservoir flows water through a sharp edge orifice with a contraction coefficient of $C_c = 0.6$. Determine the value of the orifice velocity coefficient, in order to make the cube float in $20\ s$. Given: $\gamma_{wood} = 8000\ N/m^3$; $A_{orifice} = 0.1\ m^2$; $g = 10\ m/s^2$.

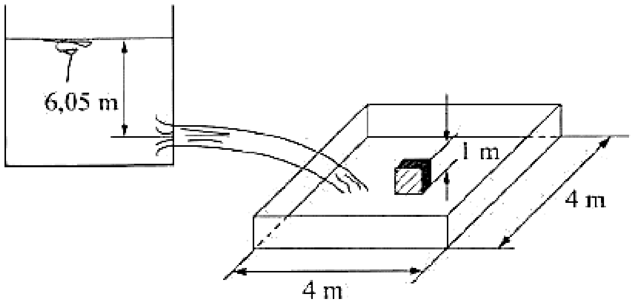

Figure 24 – System of reservoirs and a wooden cube (Brunetti, 2008).

6.5. A thin edge orifice with $7.5\ cm$ in diameter discharges at a flow rate of $28\ L/s$. A point in the jet trajectory is measured, obtaining $x = 4.7\ m$ for $y = 1.2\ m$. Determine the orifice velocity, contraction and flow rate coefficients.

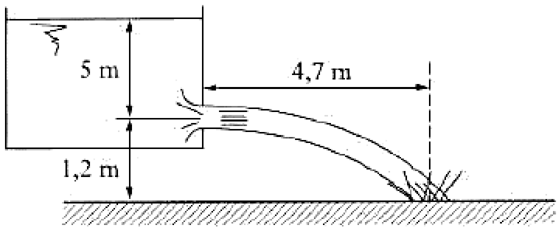

Figure 25 – Reservoir with thin edge orifice (Brunetti, 2008).

6.6. After operating for $5\ min$, the lower reservoir initially empty is full, then, the gate turns around A axis, due to the moment of $6 \times 10^4\ N.m$ produced by water. Determine the output orifice discharge coefficient of the upper reservoir. $Orifice\ area = 0.01\ m^2$.

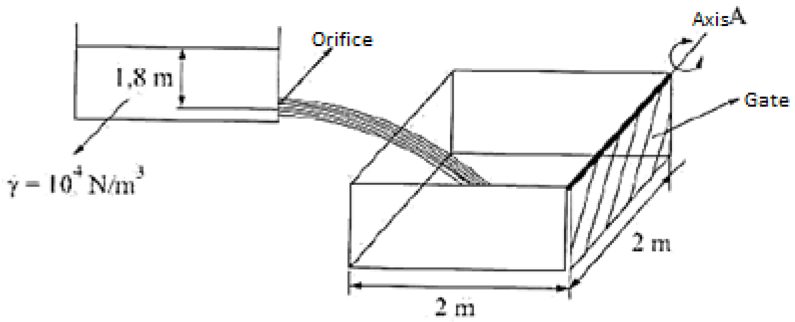

Figure 26 – Reservoirs system with gate (Brunetti, 2008).

CHAPTER 7 – Dimensional Analysis

7.1. Write the dimensional equation of kinematic viscosity in FLT (mass, length, and time) system of units.

7.2. The flow rate Q of an ideal liquid that flows to the atmosphere through a thin edge orifice located at a lateral wall of a reservoir, is function of the orifice diameter D, the fluid specific mass ρ and the pressure difference between surface and the center of the orifice. Determine the flow rate expression.

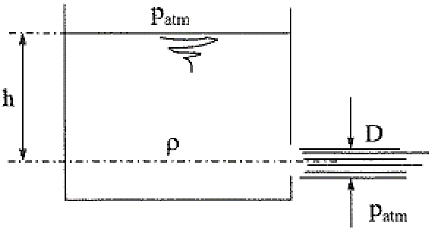

Figure 27 – Ideal liquid that flows to atmosphere (Brunetti, 2008).

7.3. The velocity v with which the fluid goes through the triangular spillway shown in the figure, is function of the gravitational acceleration g and the height h of the free surface of the liquid in relation to the triangle vertex. Determine the flow rate expression.

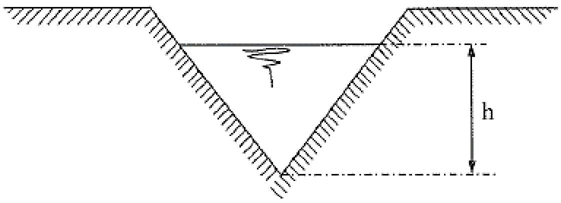

Figure 28 – Triangular spillway (Brunetti, 2008).

CHAPTER 8 – Similarity

8.1. It is necessary a $15\,N$ force to tow a plate $1.5\,m$ long and $15\,cm$ wide, totally immersed in water, at $6\,m/s$. What dimensions a similar plate must have, towed in the air at $30\,m/s$, to verify complete similarity? At those conditions, what force it is necessary to maintain the plate in movement? Given: $\rho_{H_2O} = 1000\,kg/m^3$; $\nu_{H_2O} = 10^{-6}\,m^2/s$; $\rho_{ar} = 1.2\,kg/m^3$; $\nu_{ar} = 10^{-5}\,m^2/s$. Representative function of the phenomenon: $f(F, v, L, \rho, \mu) = 0$.

8.2. Testing a model in a test tank, it was verified that the magnitudes involved in the phenomenon are: v, g, L, ν. The prototype will operate in water at $20℃$, of kinematic viscosity $\nu = 10^{-6}\,m^2/s$. It is known that the geometrical similarity scale is: $K_L = L_m/L_p = 1/2$. Choose, between the given fluids, the one to obtain complete similarity:

Table 1 – Fluids (adapted from Brunetti, 2008).

Fluid	$\nu = (m^2/s)$
Water at $20℃$	10^{-6}
Water at $50℃$	7×10^{-7}
Water at $90℃$	3.54×10^{-7}
Mercury	1.25×10^{-7}
Gasoline	5.12×10^{-7}
Kerosene	3.1×10^{-6}

8.3. The figure shows a sketch of a centrifugal pump in a section view. At a centrifugal pump, the manometric charge increases when making difficult the pass of the fluid, which means, flow rate. It also means that the same pump in different hydraulic installations can supply different flow rates and manometric charges, depending in the difficulty created to the flow. The figure shows the characteristic curve $H_B = f(Q)$ of a centrifugal pump, with a rotor $15\,cm$ in diameter and $3500\,rpm$. Remember, that the dimensionless characteristics parameters of a pump are $\phi = Q/nD^3$ and $\Psi = gH_B/n^2D^2$, because of the negligible effect of viscosity, it is not necessary to assume or include Re. Determine:

a) A universal curve for all similar pumps;

b) The characteristic curve $H_B = f(Q)$ of a similar pump, but with two times the diameter and half the rotation.

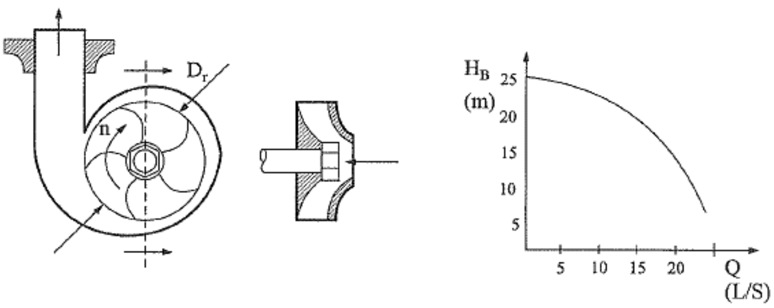

Figure 29 – Centrifugal Pump (section) and characteristic curve (Brunetti, 2008).

CHAPTER 9 – Flows under pressure

9.1. In the installation shown in the figure, the objective is to know the difference of heights Δh between those water reservoirs. Given: $power\ supplied\ to\ the\ fluid\ N = 0.75\ kW$; $diameter\ D = 3\ cm$; $Q = 3\ L/s$; $L_{1,2} = 2\ m$; $L_{3,6} = 10\ m$; $k_{s_1} = 1$; $k_{s_4} = k_{s_5} = 1.2$; $k_{s_6} = 1.6$; $\nu = 10^{-6}\ m^2/s$; $f = 0.02$; $\gamma = 10^4\ N/m^3$. Also determine the pipe rugosity and height h_0, in order to have a null effective pressure at the pump.

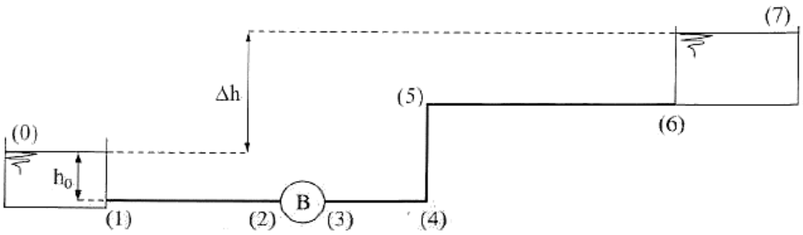

Figure 30 – Installation of two reservoirs and a pump (Brunetti, 2008).

9.2. In the figure, $H_1 = 56\ m$, $H_4 = 38\ m$ and the equivalent lengths of the singularities are $L_{eq_2} = 18\ m$ and $L_{eq_3} = 2\ m$. Determine:

a) The coefficient of distributed charge losses f;

b) The length of the installation between (1) and (4);

c) Singular charge losses due to valve (3).

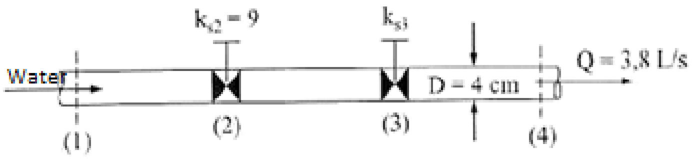

Figure 31 – Pipe with singularities (Brunetti, 2008).

9.3. At the installation are given: $reservoirs\ of\ great\ dimensions$; $f = 0.01$; $k_{s_2} = 2$; $\gamma = 10^4\ N$; $g = 10\ m/s^2$. Determine:

a) Flow rate in terms of volume;

b) Charge losses in the installation;

c) The value of x;

d) Replacing the elbow (2) by a turbine an keeping the other conditions, determine the power, knowing that $\eta_T = 90\%$.

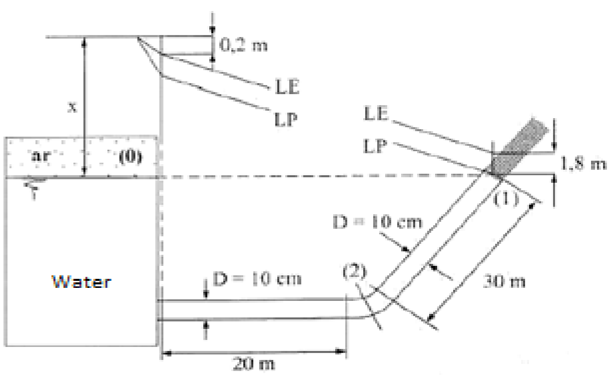

Figure 32 – Installation with reservoirs of great dimensions (Brunetti, 2008).

CHAPTER 10 – Permanent flows in pipes, conditioned by hydraulic machines

10.1. At the installation shown in the figure, the machine M_2 supply the fluid with an energy per unit of weight of $30\ m$ and the total system charge loss is $15\ m$. Determine:
a) Power of the machine M_1, being $\eta_{m1} = 0.8$;
b) Pressure at section (2) in mca;
c) Charge losses between $(2) - (5)$.
Given: $Q = 20\ L/s$; $\gamma = 10^4\ N/m^3$; $g = 10\ m/s^2$; $A = 10\ cm^2$ (pipes' section area).

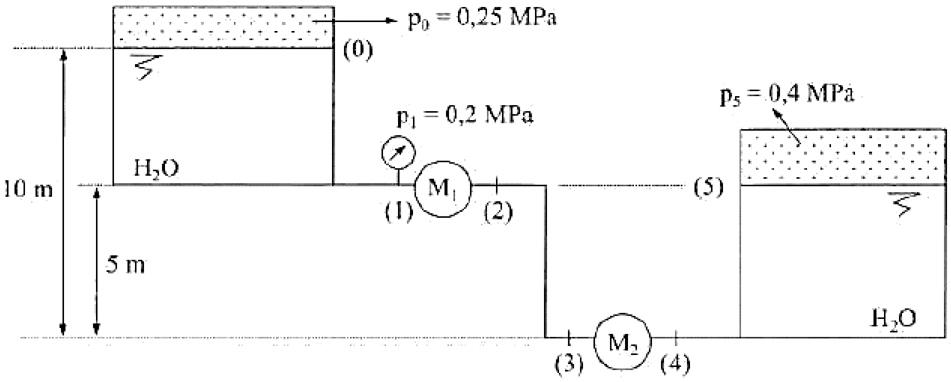

Figure 33 – Installation conditioned by 2 hydraulic machines (Brunetti, 2008).

10.2. In the installation shown in the figure, the machine flow rate is $16\ L/s$ and $H_{p1,2} = H_{p3,4} = 1\ m$. The manometer at section (2) indicates $200\ kPa$ and at section (3) indicates $400\ kPa$. Determine:
a) The flows direction;
b) The charge losses between $(2) - (3)$;
c) The type of machine and necessary power to exchange with the fluid in kW;
d) The air pressure in section (4) in MPa.

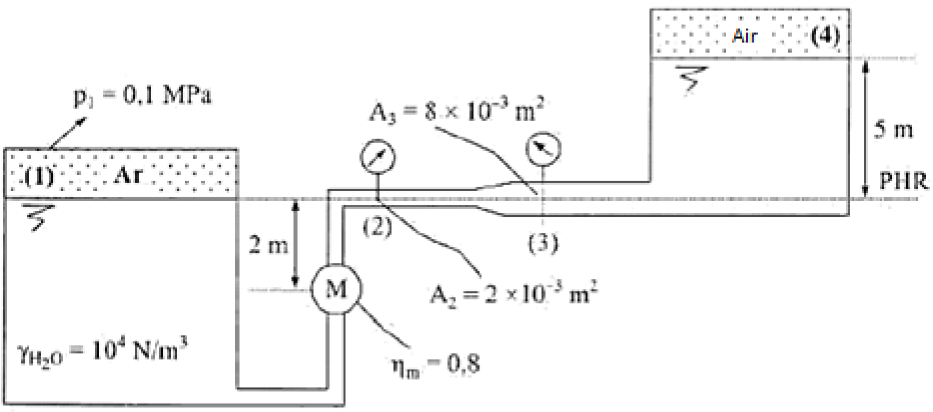

Figure 34 – Installation conditioned by a hydraulic machine (Brunetti, 2008).

10.3. In the installation shown in the figure, all pipes have same diameter ($D = 138\ mm$); The register at section (1) is set to a have half of the flow rate at section (2). For such condition, the pump manometric height is $8\ m$ and the charge losses are: $H_{p0,e} = \frac{1}{3}(v_e^2/2g)$; $H_{ps,1} = 5(v_1^2/2g)$; $H_{ps,2} = 1.5(v_2^2/2g)$. Ignoring the charge losses at the T (pump output), determine the power of the pump with an efficiency of 48%. ($\gamma_{H_2O} = 10^4\ N/m^3$; $g = 10\ m/s^2$).

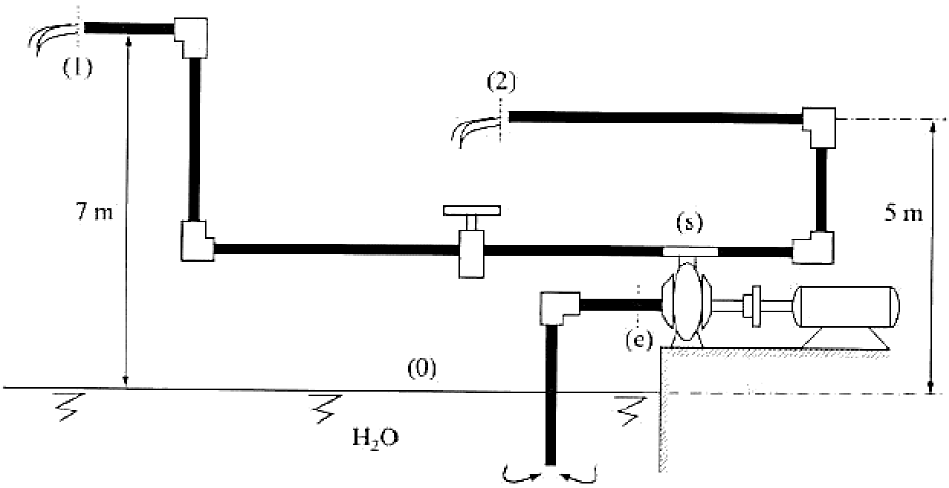

Figure 35 – Installation conditioned by a pump (Brunetti, 2008).

CHAPTER 11 – Flows with Free Surface

11.1. Consider a cross-section represented in the figure, where rugosity in the lower bottom is given by $K_S = 80\ m^{1/3}/s$ and at the higher bottom (flood) is $K_S = 40\ m^{1/3}/s$. The bottom slope is 0.2%.

a) Determine the channel water deepness (h), when present a flow rate of $25\ m^3/s$ considering uniform regime.

b) Determine the flow rate, considering uniform regime, at a situation where water rises $2.00\ m$ above the upper bottom (flood).

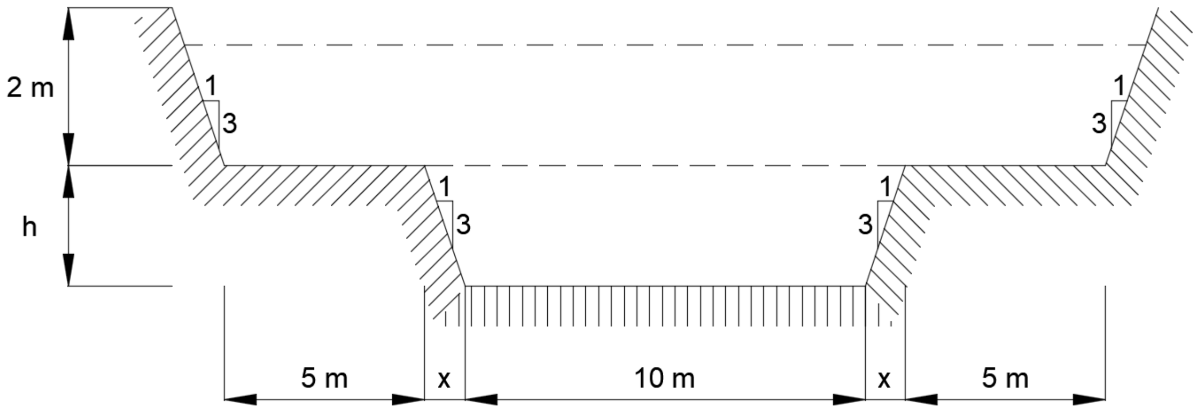

Figure 36 – Channel cross-section (source: Author).

11.2. Consider a trapezoidal channel with the following characteristics, where flows water at $15\ m^3/s$ in uniform regime: $base\ length = 2.00\ m$; $lateral\ walls\ slope = 1/1$; $walls'\ rugosity\ (concrete), C_B = 0.16$; $bottom\ slope = 3\ m/km$. Determine:

a) Critical height.

b) Normal height and the respective wet perimeter.

11.3. A $4.0\ m$ long normal threshold discharger or spillway, discharges a flow rate of $20\ m^3/s$. The hydraulic rebound that forms in the channel, downstream of the spillway, has an upstream height equal to $1.2\ m$. Determine:

a) Height at downstream of the rebound;

b) Energy losses at rebound;

c) Velocities at upstream and downstream.

SOLUTIONS

CHAPTER 1 – Introduction

1.1. Answer:

$$\gamma_r = \frac{\gamma}{\gamma_{H_2O}} \rightarrow \gamma = \gamma_r \cdot \gamma_{H_2O} = 0.8 \times 1000 = 800\ kgf/m^3 \cong 8000\ N/m^3$$

1.2. Answer:

$$The\ volume\ of\ the\ room\ is\ V = 3.0 \times 4.0 \times 5.0 = 60\ m^3$$

$$Air\ mass\ (m_{air}): m_{air} = \rho_{air} V = 1.2 \times 60 = 72\ kg$$

$$The\ air\ weight: P_{air} = m_{air} g = 72 \times 9.8 = 700\ N$$

$$The\ mass\ of\ a\ same\ volume\ in\ water: m_{water} = \rho_{water} V = 1000 \times 60 = 6.0 \times 10^4\ kg$$

$$The\ weight\ is: P_{water} = m_{water} g = (6.0 \times 10^4) \times 9.8 = 5.9 \times 10^5 N = 66\ ton$$

A room full of air weights the same as an adult of average size! The water is almost 1000 times denser than air; its mass and weight are major by that same multiplier. The weight of a room filled with water would make the floor of a normal house collapse.

1.3. Answer:

$$\frac{p_1}{\rho_1} = RT_1 \rightarrow \rho_1 = \frac{p_1}{RT_1}; T_1 = 30 + 273 = 303\ K$$

$$Then: \rho_1 = \frac{3 \times 10^5}{4122 \times 303} = 0.24\ kg/m^3$$

$$As: T_1 = T_2 \rightarrow \frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}\ ou\ \rho_2 = \rho_1 \frac{p_2}{p_1}$$

$$Therefore: \rho_2 = 0.24 \times \frac{1.5 \times 10^5}{3 \times 10^5} = 0.12\ kg/m^3$$

1.4. Answer:

$$\begin{cases} \mu = 2.38\ kgf \cdot s/m^2\ (MK_pS) \\ \mu = 233\ dina \cdot s/cm^2\ (CGS) \\ \mu = 23.3\ N \cdot s/m^2\ (SI) \end{cases}$$

1.5. Answer:

From theory:

$$\mu = \tau \frac{d\vec{r}}{d\vec{v}}$$

As $\vec{v} = constant \rightarrow \vec{a} = 0$, then $\sum \vec{F} = 0$, thus:

$$\tau = weight\ in\ the\ direction\ of\ the\ movement\ at\ the\ given\ velocity = G \times \tan\alpha$$
$$= 20 \times \tan 30°$$

Adopting a linear diagram of velocities:

$$\begin{cases} d\vec{r} = thickness\ of\ the\ oil\ layers = \varepsilon \\ d\vec{v} = velocity\ of\ the\ plate = v_0 \end{cases}$$

Then:

$$\mu = \tau \frac{\varepsilon}{v_0} = (20 \times \tan 30°) \cdot \frac{0.002}{2} \cong 10^{-2}\ N \cdot s/m^2$$

1.6. Answer:

$$\mu = \tau \frac{d\vec{r}}{d\vec{v}} \rightarrow \tau = \mu \frac{d\vec{v}}{d\vec{r}}$$

The dynamical viscosity (μ) can be calculated from kinematic viscosity (ν) attending to the ratio:

$$\nu = \frac{\mu}{\rho}$$

Then, in international unit system:

$$\mu = \nu\rho = 0.00001 \times 830 = 0.0083\ N \cdot s/m^2$$

Adopting a linear diagram of velocities:

$$\begin{cases} d\vec{r} = distance\ between\ plates = \varepsilon \\ d\vec{v} = velocity\ of\ the\ upper\ plate = v_0 \end{cases}$$

Then:

$$\tau = \mu \frac{v_0}{\varepsilon} = 0.0083 \cdot \frac{4}{0.002} = 16.6\ N/m^2$$

CHAPTER 2 – Fundamental Equation of Fluids Movement

2.1. Answer:

As:

$$Q_{m1} = Q_{m2}$$

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \rightarrow v_2 = v_1 \frac{\rho_1}{\rho_2} \frac{A_1}{A_2}$$

Therefore:

$$v_2 = 30 \times \frac{4}{12} \times \frac{20}{10} = 20\ m/s$$

2.2. Answer:

By the equation of continuity:

$$v_e A_e = v_G A_G$$

$$v_G = v_e \frac{A_e}{A_G} = 2 \cdot \frac{20}{5} = 8\ m/s$$

2.3. Answer:

As:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \rightarrow v_2 = v_1 \frac{\rho_1}{\rho_2} \frac{A_1}{A_2}$$

Therefore:

$$v_2 = 10 \times \frac{1.2}{0.9} \times \frac{20}{10} \approx 26.7\ m/s$$

The flow rates in terms of volume are given by:

$$\begin{cases} Q_1 = v_1 A_1 = 10 \times 0.002 = 0.02\ m^3/s \\ Q_2 = v_2 A_2 = 26.7 \times 0.001 = 0.0267\ m^3/s \end{cases}$$

The flow rates in terms of mass are given by:

$$\begin{cases} Q_{m1} = \rho_1 v_1 A_1 = \rho_1 Q_1 = 1.2 \times 0.02 = 0.024\ kg/s \\ Q_{m2} = \rho_2 v_2 A_2 = \rho_2 Q_2 = 0.9 \times 0.0267 \approx 0.024\ kg/s \end{cases}$$

$Q_{m1} = Q_{m2} = Q_m$, which means, the air mass is constant along the pipe.

The flow rate in terms of weight is given by:

$$Q_{G1} = Q_{G2} = Q_m \times g = 0.024 \times 10 = 0.24\ N/s$$

2.4. Answer:

The specific mass of the mixture is given by:

$$\rho_3 = \frac{Q_{m3}}{Q_3} = \frac{(Q_{m1} + Q_{m2})}{Q_3}$$

The flow rate in terms of mass are given by:

$$\begin{cases} Q_{m1} = \rho_1 Q_1 = 1000 \times 0.02 = 20\ kg/s \\ Q_{m2} = \rho_2 Q_2 = 800 \times 0.01 = 8\ kg/s \end{cases}$$

Then:

$$\rho_3 = \frac{20 + 8}{0.03} \approx 933\ kg/m^3$$

The velocity in (3) is given by:

$$v_3 = \frac{Q_3}{A_3} = \frac{0.03}{0.003} = 10\ m/s$$

2.5. Answer:

The movement is varied, because v_x and v_y are functions of time.

$$\begin{cases} a_x = \dfrac{\partial v_x}{\partial t} + v_x \dfrac{\partial v_x}{\partial x} + v_y \dfrac{\partial v_x}{\partial y} + v_z \dfrac{\partial v_x}{\partial z} \\ a_y = \dfrac{\partial v_y}{\partial t} + v_x \dfrac{\partial v_y}{\partial x} + v_y \dfrac{\partial v_y}{\partial y} + v_z \dfrac{\partial v_y}{\partial z} \end{cases}$$

$$\begin{cases} a_x = 2x + 2xt(2t) + y^2 t(0) = 2x + 4xt^2 \\ a_y = y^2 + 2xt(0) + y^2 t(2yt) = y^2 + 2y^3 t^2 \end{cases}$$

At instant $t = 5\ s$,

$$\begin{cases} a_x = 2x + 4x(25) = 102x \\ a_y = y^2 + 2y^3(25) = y^2 + 50y^3 \end{cases}$$

At point $P = (1,2)$,

$$\begin{cases} a_x = 102 \times 1 = 102 \\ a_y = (2)^2 + 50(2)^3 = 4 + 400 = 404 \end{cases}$$

Then:

$$\vec{a}_{(P,t)} = 102\vec{e}_x + 404\vec{e}_y$$

$$|\vec{a}| = \sqrt{(102)^2 + (404)^2} = 416\ cm/s^2$$

2.6. Answer:

In this problem exist only a component y of velocity, and it is in function only of x for a flow fully developed. Then:

$$\begin{cases} u = w = 0 \\ v = v(x) \end{cases}$$

In order to have a flow must exist a gradient of pressure in the direction of y, which means, $\partial p/\partial y \neq 0$.

Then, the equations of Navier-Stokes become:

$$In\ x:\quad 0 = \rho g_x - \frac{\partial p}{\partial x} \rightarrow \frac{\partial p}{\partial x} = 0 \quad as\ \ g_x = 0$$
$$In\ z:\quad 0 = \rho g_z - \frac{\partial p}{\partial z} \rightarrow \frac{\partial p}{\partial z} = 0 \quad as\ g_z = 0$$
$$In\ y:\quad 0 = \rho g_y - \frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} \qquad (1)$$

a) Replacing $g_y = -g$ in (1), and being $v = v(y)$:

$$(2): \frac{d^2 v}{dx^2} = \frac{\gamma}{\mu} + \frac{1}{\mu}\frac{dp}{dy}$$

Integrating a first time:

$$(3): \frac{dv}{dx} = \frac{\gamma}{\mu}x + \frac{1}{\mu}\frac{dp}{dy}x + C_1$$

Knowing that $\tau = 0$ in $x = 0$ and as $\tau = \mu\frac{dv}{dy}$, then $C_1 = 0$.

Integrating a second time:

$$(4): v = \frac{\gamma}{2\mu}x^2 + \frac{1}{2\mu}\frac{dp}{dy}x^2 + C_2$$

Being $v = 0$ in $x = h$, it is obtained:

$$0 = \frac{\gamma}{2\mu}h^2 + \frac{1}{2\mu}\frac{dp}{dy}h^2 + C_2, where:$$

$$C_2 = -\frac{h^2}{2\mu}\left[\frac{dp}{dy} + \gamma\right]$$

Replacing C_2 in (4), then:

$$\boxed{v = \frac{1}{2\mu}\left[\frac{dp}{dy} + \gamma\right](x^2 - h^2)}$$

The flow rate per unit of length is:

$$q = \int_{-h}^{+h} v\,dx = \frac{1}{2\mu}\left[\frac{dp}{dy} + \gamma\right]\left\{\int_{-h}^{+h} x^2\,dx - \int_{-h}^{+h} h^2\,dx\right\}\ ou$$

$$q = \frac{1}{2\mu}\left[\frac{dp}{dy} + \gamma\right]\left\{\frac{2}{3}h^3 - 2h^3\right\}, where:$$

$$\boxed{q = -\frac{2h^3}{3\mu}\left[\frac{dp}{dy} + \gamma\right]}$$

Obtaining:

$$\frac{dp}{dy} = -\frac{3\mu q}{2h^3} - \gamma\ or:$$

$$\therefore \boxed{\frac{dp}{dy} = -\left[\gamma + \frac{3\mu q}{2h^3}\right]}$$

b) For $dp/dy = 0$, it is obtained a flow caused only by gravity and given by:

$$\therefore \boxed{q = -\frac{2\gamma h^3}{3\mu}}$$

CHAPTER 3 – Hydrostatics

3.1. Answer:

The pressure transmitted by the plug (1) will be $p_1 = F_1/A_1$.

But based in the Pascal law, that pressure will be transmitted integrally to plug (2), therefore $p_2 = p_1$. Then:

$$p_2A_2 = p_1A_1 = F_2$$

As:

$$p_1 = \frac{200}{10} = 20\ N/cm^2$$

Then:

$$F_2 = 20 \times 100 = 2000\ N$$

Notice that is possible not only to transmit a force but also to increase it. In practice, this principle is used in hydraulic presses, control devices, brakes, etc.

3.2. Answer:

$$\begin{array}{lll} 760\ mmHg & \_\_\_ & 1.033\ kgf/cm^2 \\ 340\ mmHg & \_\_\_ & x\ kgf/cm^2 \end{array}$$

$$x = \frac{340 \times 1.033}{760} = 0.461\ kgf/cm^2$$

$$\begin{array}{lll} 760\ mmHg & \_\_\_ & 14.7\ psi \\ 340\ mmHg & \_\_\_ & y\ psi \end{array}$$

$$y = \frac{340 \times 14.7}{760} = 6.6\ psi$$

To determinate the absolute pressure, it is enough to remember that: $p_{abs} = p_{ef} + p_{atm}$.

$$\begin{array}{lll} 760\ mmHg & \_\_\_ & 101230\ Pa \\ 340\ mmHg & \_\_\_ & z\ Pa \end{array}$$

$$z = \frac{340 \times 101230}{760} = 45287\ Pa = 45.3\ kPa$$

Then, $p_{abs} = 45.3 + 101.2 = 146.5\ kPa\ (abs)$.

$760\ mmHg$ ____ $1\ atm$
$340\ mmHg$ ____ $u\ atm$

$$u = \frac{340 \times 1}{760} = 0.447\ atm$$

Then, $p_{abs} = 0.447 + 1 = 1.447\ atm\ (abs)$.

3.3. Answer:

$$The\ absolut\ pressure\ is: p = p_0 + \rho g h = (1.01 \times 10^5) + 1000 \times 9.8 \times 12.0 = 2.19 \times 10^5\ Pa = 2.16\ atm$$

$$The\ manometric\ pressure\ is: p - p_0 = (2.19 - 1.01) \times 10^5\ Pa = 1.18 \times 10^5\ Pa = 1.16\ atm$$

When a tank has a manometer, it is normally calibrated to measure manometric pressure and not absolute pressure. The pressure variation in the atmosphere within a few meters high can be ignored.

3.4. Answer:

a) Determination of p_M

Using the manometric equation, having in mind that γ of gases is small, so, it can be ignored the effect of the column of air when compared with other effects, and by working with an effective scale $p_{atm} = 0$, then:

$$p_M + \gamma_0 h_0 + \gamma_{H_2O} h_{H_2O} - \gamma_{H_2O} L \sin 30° = 0$$

$L \sin 30°$ is the difference of the column of water in the right section, because of the theorem of Stevin, which says that the pressure is independent from distances and only depends on the heights or elevations.

Then:

$$p_M = \gamma_{H_2O}\left(L \sin 30° - h_{H_2O}\right) - \gamma_0 h_0$$

$$p_M = 10000(0.6 \times 0.5 - 0.2) - 8000 \times 0.1$$

$$p_M = 200\ N/m^2$$

b) By the definition of pressure

$$F_{top} = p_M A = 200 \times 10 = 2000\ N$$

3.5. Answer:

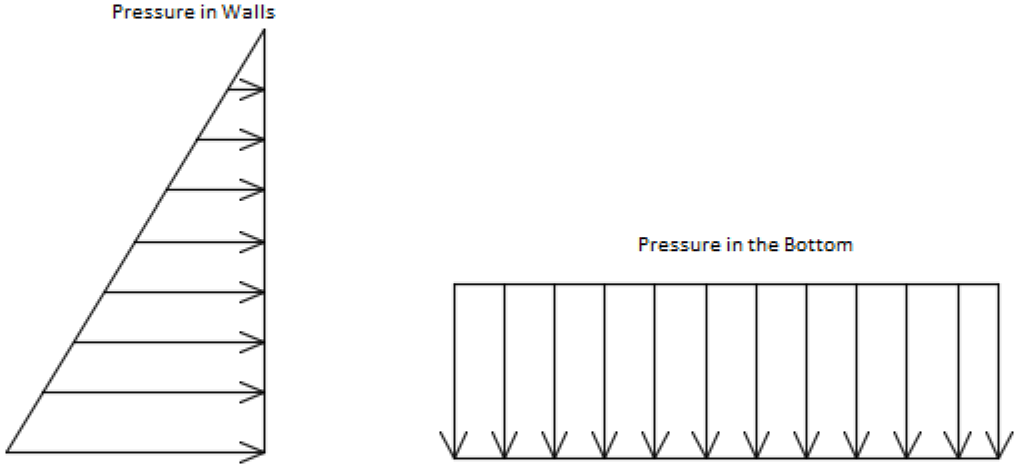

Figure 37 – Distribution of pressures (origin: Author).

$$F_A = \frac{(8500 \times 0.6) \times 0.6}{2} \times 4.5 + \frac{[(8500 \times 0.6 + 10000 \times 0.6) + (8500 \times 0.6)] \times 0.6}{2} \times 4.5$$

$$F_A = 6885 + 21870 = 28755\ N$$

$$F_B = \frac{(8500 \times 0.6) \times 0.6}{2} \times 1.2 + \frac{[(8500 \times 0.6 + 10000 \times 0.6) + (8500 \times 0.6)] \times 0.6}{2} \times 1.2$$

$$F_B = 1836 + 5832 = 7668\ N$$

$$F_{bottom} = (8500 \times 0.6 \times 1.2 \times 4.5) + (10000 \times 0.6 \times 1.2 \times 4.5)$$

$$F_{bottom} = 27540 + 32400 = 59940\ N$$

3.6. Answer:

For a cylinder totally immerse:

$$I = G = \gamma_{H2O} V_{total} = 10300 \times (\pi \times 0.15^2 \times 0.3) \approx 218\ N$$

$$I = G = \gamma_{wood} V_{total} = 7500 \times (\pi \times 0.15^2 \times 0.3) \approx 159\ N$$

Submerged height? – Considering forces in equilibrium and only the immersed section of the cylinder, then:

$$I = G \rightarrow \gamma_{H2O} V_{immr} = \gamma_{wood} V_{total} \rightarrow$$

$$\rightarrow V_{immr} = \frac{\gamma_{wood}}{\gamma_{water}} V_{total} \rightarrow \pi \times r^2 \times h_{immr} = \frac{\gamma_{wood}}{\gamma_{water}} \times \pi \times r^2 \times h_{total} \rightarrow$$

$$\rightarrow h_{immr} = \frac{\gamma_{wood}}{\gamma_{water}} \times \frac{\pi \times r^2 \times h_{total}}{\pi \times r^2} \rightarrow h_{immr} = \frac{\gamma_{wood}}{\gamma_{water}} h_{total} \rightarrow$$

$$\rightarrow h_{immr} = \frac{7500}{10300} \times 0.3 \approx 0.218\, m$$

CHAPTER 4 – Theorem of Bernoulli and applications

4.1. Answer:

$$h=\frac{\Delta p}{\rho g}=\frac{\frac{1}{2}\rho v_1^2}{\rho g}=\frac{v_1^2}{2g}\rightarrow h=\frac{\left(\frac{45\times 10^3}{60\times 60}\right)^2}{2\times 10}=7.8125\cong 7.8\ m$$

4.2. Answer:

Adopting the index (0) for the input of Pitot tube:

$$H_1=H_0\rightarrow z_1+\frac{p_1}{\gamma_{H_2O}}+\frac{v_1^2}{2g}=z_0+\frac{p_0}{\gamma_{H_2O}}+\frac{v_0^2}{2g}$$

As $z_1=z_0$, then:

$$\frac{p_1}{\gamma_{H_2O}}+\frac{v_1^2}{2g}=\frac{p_0}{\gamma_{H_2O}}+\frac{v_0^2}{2g}$$

At the input of the Pitot tube the velocity (v_0) is null and the piezometric height (p_0/γ_{H_2O}) is $3.8\ m$, then:

$$\frac{p_1}{\gamma_{H_2O}}+\frac{v_1^2}{2g}=3.8+0\rightarrow\frac{v_1^2}{2g}=3.8-\frac{p_1}{\gamma_{H_2O}}\rightarrow v_1^2=\left[\left(3.8-\frac{p_1}{\gamma_{H_2O}}\right)\times 2g\right]\rightarrow$$

$$\rightarrow v_1=\sqrt{\left[\left(3.8-\frac{p_1}{10000}\right)\times 20\right]}$$

Attending to the difference of the manometer $p_1>p_2$. By the manometric equation:

$$p_1+h\times\gamma_{H_2O}-h\times\gamma_m=p_2\rightarrow p_1=p_2+h\times\gamma_m-h\times\gamma_{H_2O}\rightarrow$$

$$\rightarrow p_1=p_2+h\times\left(\gamma_m-\gamma_{H_2O}\right)=20000+0.2\times(60000-10000)=30000\ N/m^2$$

Then:

$$v_1=\sqrt{\left[\left(3.8-\frac{30000}{10000}\right)\times 20\right]}=4\ m/s$$

Therefore:

$$Q=v\cdot A=4\times 10^{-2}\ m^3/s=40\ L/s$$

4.3. Answer:

a)

$$H_1 = H_3 \rightarrow z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_3 + \frac{p_3}{\gamma} + \frac{v_3^2}{2g}$$

As $v_1 = v_3$, then:

$$z_1 + \frac{p_1}{\gamma} = z_3 + \frac{p_3}{\gamma} \rightarrow 10 + \frac{52000}{10000} = 17 + \frac{p_3}{\gamma} \rightarrow \frac{p_3}{\gamma} = 10 + \frac{52000}{10000} - 17 = -1.8\ m$$

$$H_1 = H_3 = 16\ m = z_3 + \frac{p_3}{\gamma} + \frac{v_3^2}{2g} \rightarrow 16 = 17 - 1.8 + \frac{v_3^2}{2 \times 10} \rightarrow v_3 = 4\ m/s$$

$$Q_G = \gamma \cdot v \cdot A = 10000 \times 4 \times \left(\pi \frac{0.1^2}{4}\right) \cong 314\ N/s$$

b)

By the manometric equation:

$$p_1 + h_1 \times \gamma - h_1 \times \gamma_{Hg} - (z_3 - z_1) \times \gamma = p_3 \rightarrow$$

$$\rightarrow 52000 + h_1 \times 10000 - h_1 \times 136000 - (17 - 10) \times 10000 = -1.8 \times 10000 \rightarrow$$

$$\rightarrow h_1 \times 126000 = 52000 - 70000 + 18000 \rightarrow h_1 \times 126000 = 0 \rightarrow h_1 = 0\ m$$

c)

Starting from :

$$H_1 = H_2 = 16\ m = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

By the manometric equation, determine p_2/γ:

$$p_1 + h \times \gamma - h \times \gamma_{Hg} - (z_2 - z_1) \times \gamma = p_2 \rightarrow$$

$$\rightarrow 52000 + 0.55 \times 10000 - 0.55 \times 136000 - (z_2 - 10) \times 10000 = p_2 \rightarrow$$

$$\rightarrow p_2 = 52000 + 5500 - 74800 - 10000 z_2 + 100000 = 82700 - 10000 z_2 \rightarrow$$

$$\rightarrow \frac{p_2}{\gamma} = 8.27 - z_2$$

By Bernoulli's Theo.:

$$H_1 = H_2 = 16\, m = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} \rightarrow 16 = z_2 + 8.27 - z_2 + \frac{v_2^2}{2 \times 10} \rightarrow$$

$$\rightarrow v_2^2 = (16 - 8.27) \times 20 \rightarrow v_2 = \sqrt{154.6}\, m/s$$

Notice that $Q_1 = Q_2$, then:

$$4 \times \left(\pi \frac{0.1^2}{4}\right) = \sqrt{154.6} \times \left(\pi \frac{D_2^2}{4}\right) \rightarrow D_2 = \sqrt{\frac{4 \times 0.1^2}{\sqrt{154.6}}} \cong 0.057\, m = 5.7\, cm$$

4.4. Answer:

a)

The pump total height of elevation is the gains of charge at the pump necessary to transport the pretended flow rate to the pretended charge. The unitary charge loss is constant because along the pipe, the diameter, type of pipe and flow rate are also constant.

Applying the Theorem of Bernoulli between two reservoirs, it is obtained:

$$H_{Rm} - H_{Rj} = J_{AB}L_{AB} - H_t + J_{BC}L_{AC}, with\, J_{AB} = J_{BC}$$

$$H_t = H_{Rj} - H_{Rm} + J_{AB}L_{AB} + J_{BC}L_{AC}$$

$$H_t = 120 - 95 + 0.008 \times 105 + 0.008 \times 1000 = 33.84\, m.c.a.$$

The pump total height of elevation is $33.84\, m.c.a.$.

b)

The daily energy consumption in function of the power of the motor:

$$P_m = \frac{\gamma Q H_t}{\eta_B \eta_m}$$

The flow rate defined by the elevated daily volume and the interval of elevation:

$$Q = \frac{V}{t} = \frac{720}{10 \times 60 \times 60} = 0.02\, m^3/s$$

Then:

$$P_m = \frac{9800 \times 0.02 \times 33.84}{0.85} \cong 7803\ N \cdot \frac{m}{s} = 7803\ W = 7.803\ kW$$

The daily energy consumption is:

$$E = P_m t = 7.803 \times 10 = 78.03\ kWh$$

c)

The determination of the charge at upstream section of curve D and the average velocity at that section allows, by the definition of charge, to determine pressure at the section axis. The topographical elevation is given in the problem.

The application of the theorem of Bernoulli between the upstream reservoir and the section at upstream of curve D:

$$H_{Rm} - H_D = J_{AD} L_{AD} \rightarrow H_D = H_{Rm} - J_{AD} L_{AD} = 95 - 0.008 \times 5 = 94.96\ m.c.a.$$

The average velocity in the pipe is:

$$U = \frac{Q}{A} = \frac{0.02}{\frac{\pi \times 0.15^2}{4}} \cong 1.13\ m/s$$

The coefficient of pressure distribution is equal to $\beta = 1$ for straight and parallels current lines. The coefficient of Coriolis at a turbulence regime has a value of $\alpha = 1.15$.

The application of the concept of charge allows to obtain:

$$H_D = Z_D + \beta \frac{p_D}{\gamma} + \alpha \frac{U^2}{2g} \rightarrow 94.96 = 100 + 1 \frac{p_D}{\gamma} + 1.15 \frac{1.13^2}{2 \times 9.8} \rightarrow$$

$$\rightarrow \frac{p_D}{\gamma} = 94.96 - 100 - 0.08 = -5.12\ m.c.a \rightarrow p_D = -5.12 \times 9800 = -50176\ N/m^2$$

The pressure at upstream section of the curve is negative and equal to $50176\ N/m^2$.

CHAPTER 5 – Theorem of amount of movement and applications

5.1. Answer:

Having :

$$\begin{cases} F_{s_x} = \rho Q_{ap}(u_1 - u_2 \cos\theta) \\ F_{s_y} = \rho Q_{ap}(0 - u_2 \sin\theta_2) \end{cases}$$

Supposing that in modulus or magnitudes $u_1 = u_2 = u$, it is obtained:

$$\begin{cases} F_{s_x} = \rho A_j u(u - u\cos 60°) = \rho A_j u^2(1 - \cos 60°) \\ F_{s_y} = \rho A_j u(0 - u\sin 60°) = -\rho A_j u^2 \sin 60° \end{cases}$$

$$u = v_j - v_s = 15 - 9 = 6\ m/s$$

$$A_j = \frac{\pi D_j^2}{4} = \frac{\pi \times 5^2}{4} \times 10^{-4} \approx 1.96 \times 10^{-3}\ m^2$$

$$\rho = \frac{\gamma}{g} = \frac{8000}{10} = 800\ kg/m^3$$

Then:

$$\begin{cases} F_{s_x} = 800 \times 1.96 \times 10^{-3} \times 6^2(1 - 0.5) \approx 28.2\ N \\ F_{s_y} = -800 \times 1.96 \times 10^{-3} \times 6^2 \times 0.866 \approx -49\ N \end{cases}$$

Therefore:

$$F_s = \sqrt{28.2^2 + (-49)^2} \cong 56.5\ N$$

5.2. Answer:

$$\vec{F_s} = -[p_1 A_1 \vec{n}_1 + p_2 A_2 \vec{n}_2 + Q_m(\vec{v}_2 - \vec{v}_1)]$$

$$F_{s_{x,support}} = -[p_1 A_1(-1) + Q_m(v_2 - v_1)] \rightarrow F_{s_{x,support}} = p_1 A_1 + Q_m(v_1 - v_2)$$

With:

$$v_2 = v_1 \left(\frac{D_1}{D_2}\right)^2 = 5 \times \left(\frac{10}{5}\right)^2 = 20\ m/s$$

$$Q = v_1 \frac{\pi D_1^2}{4} = 5 \times \frac{\pi \times 0.1^2}{4} \approx 0.0393\ m^3/s \rightarrow Q_m = \rho Q = 1000 \times 0.0393 = 39.3\ kg/s$$

Then:

$$F_{s_{x,support}} = 150 \times 10^3 \times \frac{\pi \times 0.1^2}{4} + 39.3 \times (5 - 20) \approx 1178 - 589 = 589\ N$$

The necessary force to maintain the plate in rest:

$$|F| = \left|F_{s_{x,plate}}\right| = \rho v_2^2 A_2 = 1000 \times 20^2 \times \frac{\pi \times 0.05^2}{4} \cong 785\ N$$

5.3. Answer:

$$z_1 - H_T = \frac{v_2^2}{2g} \rightarrow H_T = z_1 - \frac{v_2^2}{2g}$$

$$F_{s_x} = \rho v_2^2 A_2 = \rho v_2^2 \frac{\pi D^2}{4} \rightarrow v_2 = \sqrt{\frac{4F_{s_x}}{\rho \pi D^2}} = \sqrt{\frac{4 \times 1000}{1000 \times \pi \times 0.15^2}} \approx 7.52\ m/s$$

$$H_T = 30 - \frac{7.5^2}{20} \approx 27.2\ m$$

$$Q = 7.52 \times \frac{\pi \times 0.15^2}{4} \approx 0.133\ m^3/s$$

$$N_T = \gamma Q H_T \eta_T = (10^4 \times 0.133 \times 27.2 \times 0.7) \times 10^{-3} \cong 25.3\ kW$$

5.4. Answer:

Notice the section AA corresponding to a jet diverter with an exit angle of θ. Considering only half of the blade the solution is:

$$\frac{\vec{F}_s}{2} = \frac{Q_{m_{ap}}}{2}(\vec{u}_1 - \vec{u}_2) \vee \vec{F}_s = Q_{m_{ap}}(\vec{u}_1 - \vec{u}_2)$$

Projecting in the direction of x:

$$F_{s_x} = Q_{m_{ap}}(u_1 - u_2 \cos\theta)$$

Suppose $u_1 = u_2 = u = v_j - v_s$, where $v_s = \omega R$, it is obtained:

$$F_{s_x} = Q_{m_{ap}}(v_j - v_s)(1 - \cos\theta)$$

In case of a turbine, it has a great number of blades and a relatively high angular velocity ω . This makes it possible to admit that at any moment there is a blade in the section shown in figure. Because of that, it is possible to admit that the jet flow rate is used in the transmission of power, in a form which it is possible to replace the apparent flow rate by the real flow rate.

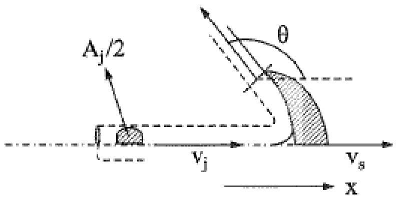

Figure 38 – Blade of a turbine (Brunetti, 2008).

Then, the equation of the turbine is:

$$F_{s_x} = Q_m(v_j - v_s)(1 - \cos\theta) \vee F_{s_x} = \rho A_j v_j (v_j - v_s)(1 - \cos\theta)$$

The power is given by $N = F_{s_x} v_s$, then:

$$N = \rho A_j v_j (v_j - v_s)(1 - \cos\theta) v_s$$

The efficiency of the power transmission from the jet to the turbine is obtained by comparing the power of the turbine with the power of the jet, that is given by:

$$N_j = \frac{\rho A_j v_j^3}{2}$$

Then, the efficiency is:

$$\eta = \frac{N}{N_j} = \frac{\rho A_j v_j (v_j - v_s)(1 - \cos\theta) v_s}{\frac{\rho A_j v_j^3}{2}} \vee \eta = \frac{2(v_j - v_s)(1 - \cos\theta) v_s}{v_j^2}$$

The maximum efficiency in function of the velocity v_s can be obtained by deriving η in relation of v_s and equating to 0:

$$\frac{d\eta}{dv_s} = \frac{2(1 - \cos\theta)}{v_j^2}\left(v_j - 2v_s\right) = 0 \vee v_s = \frac{v_j}{2}$$

Replacing the result in the expression of efficiency:

$$\eta_{max} = \frac{2\left(v_j - \frac{v_j}{2}\right)(1 - \cos\theta)\frac{v_j}{2}}{v_j^2} = \frac{1 - \cos\theta}{2}$$

It is observed that the ideal exit angle θ would be $180°$, but it is not possible because the jet would return over itself, acting in the next blade. In practice, the adopted angle θ is lower than $180°$.

5.5. Answer:

Reduction, $F_{s_{x,r}}$:

Determination of v_2 and p_2 based in the Theo. of Bernoulli, necessary to the resolution:

$$H_1 = H_2 \rightarrow \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = \frac{v_2^2}{2g} + \frac{p_2}{\gamma}$$

$$v_1 \frac{\pi D_1^2}{4} = v_2 \frac{\pi D_2^2}{4} \rightarrow v_2 = v_1 \left(\frac{D_1}{D_2}\right)^2 = v_1 \left(\frac{30}{15}\right)^2 = 4v_1 \rightarrow v_2 = 4 \times 3 = 12\ m/s$$

$$p_2 = p_1 + \gamma \frac{v_1^2 - v_2^2}{2g} = p_1 + \frac{\rho}{2}(v_1^2 - v_2^2) \rightarrow p_2 = 84000 + \frac{1000}{2}(3^2 - 12^2) = 16500\ Pa$$

$$\vec{F_s} = -[p_1 A_1 \vec{n}_1 + p_2 A_2 \vec{n}_2 + Q_m(\vec{v}_2 - \vec{v}_1)]$$

$$F_{s_{x,r}} = -[p_1 A_1(-1) + p_2 A_2(+1) + Q_m(v_2 - v_1)] = p_1 A_1 - p_2 A_2 + Q_m(v_1 - v_2)$$

Where:

$$Q_m = \rho v_1 \frac{\pi D_1^2}{4} = 1000 \times 3 \times \frac{\pi \times 0.3^2}{4} \approx 212\ kg/s$$

Then:

$$F_{s_{x,r}} = 84000 \times \frac{\pi \times 0.3^2}{4} - 16500 \times \frac{\pi \times 0.15^2}{4} + 212 \times (3 - 12) \cong 3738\ N$$

Turbine, $F_{s_{x,T}}$:

Determination of p_3 based in the Theo. of Bernoulli, necessary to the resolution:

$$H_2 - H_T = H_3 \rightarrow \frac{p_2}{\gamma} - H_T = \frac{p_3}{\gamma} \rightarrow p_3 = p_2 - \gamma H_T$$

Where:

$$N = \gamma Q H_T \rightarrow H_T = \frac{N}{\gamma Q}$$

$$Q = \frac{Q_m}{\rho} = \frac{212}{1000} = 0.212\ m^3/s$$

$$\gamma = \rho g = 1000 \times 10 = 10000\ N/m^3$$

$$H_T = \frac{2.9 \times 10^3}{10000 \times 0.212} \approx 1.37\, m$$

Resulting in:

$$p_3 = 16500 - 10000 \times 1.37 = 2800\, Pa$$

$F_{s_{x,T}}$, became:

$$\vec{F_s} = -[p_2 A_2 \vec{n}_2 + p_3 A_3 \vec{n}_3 + Q_m(\vec{v}_3 - \vec{v}_2)]$$

$$F_{s_{x,T}} = -[p_2 A_2(-1) + p_3 A_3(+1) + 0] = p_2 A_2 - p_3 A_3$$

As $A_2 = A_3$:

$$F_{s_{x,T}} = (p_2 - p_3)A = (16500 - 2800) \times \frac{\pi \times 0.15^2}{4} \cong 242\, N$$

5.6. Answer:

As there are two entrances and one exit, it must be applied the equation:

$$\vec{F_s} = -\sum p_i A_i \vec{n}_i + \sum_e Q_m \vec{v} - \sum_s Q_m \vec{v}$$

By the hypothesis referred to pressures, then:

$$\vec{F_s} = Q_{m_1}\vec{v}_1 + Q_{m_2}\vec{v}_2 - Q_{m_3}\vec{v}_3$$

Projecting in terms of x:

$$F_{s_x} = Q_{m_1} v_1 \cos 60° + Q_{m_2} v_2 \cos 60° - Q_{m_3} v_3$$

By the symmetry of the system:

$$Q_{m_1} = Q_{m_2} \wedge v_1 = v_2$$

Then:

$$F_{s_x} = 2Q_{m_1} v_1 \cos 60° - Q_{m_3} v_3$$

But, by the equation of Continuity:

$$Q_{m_1} = \frac{Q_{m_3}}{2} \wedge v_1 = \frac{v_3}{2}$$

Then:

$$F_{s_x} = 2\frac{Q_{m_3}}{2}\frac{v_3}{2}\cos 60° - Q_{m_3}v_3$$

Therefore, as $\cos 60° = 1/2$:

$$F_{s_x} = -\frac{3}{4}Q_{m_3}v_3$$

But:

$$Q_{m_3} = \rho Q_3 = 1000 \times 50 \times 10^{-3} = 50\ kg/s$$

And:

$$v_3 = \frac{4Q_3}{\pi D_3^2} = \frac{4 \times 50 \times 10^{-3}}{\pi \times 0.05^2} \approx 25.46\ m/s$$

Then:

$$F_{s_x} = -\frac{3}{4}50 \times 25.46 = -954.75\ N$$

The negative sign indicates that the propulsion force has an opposite direction of the adopted x axis.

CHAPTER 6 – Flows through orifices and dischargers or spillways

6.1. Answer:

$$\frac{p_1}{\gamma}+z_1=\frac{v_t^2}{2g}$$

$$\frac{v_r^2}{2g}=\frac{p}{\gamma}\rightarrow v_r=\sqrt{2\times 10\times 3}\approx 7.75\,m/s$$

$$v_t=\frac{v_r}{C_v}=\frac{7.75}{0.9}\approx 8.61\,m/s$$

$$\frac{p_1}{\gamma}=\frac{8.61^2}{2\times 10}-5\approx -1.29\,m\rightarrow p_1=-1.29\times 10^4=-12.9\,kPa$$

$$Q=C_vC_cv_tA_o=0.9\times 0.6\times 8.61\times 50\times 10^{-4}\cong 0.0232\,m^3/s=23.2\,L/s$$

6.2. Answer:

Upper Reservoir:

$$H_0=H_1$$

$$\frac{p_0}{\gamma}+z_0=\frac{v_{t_1}^2}{2g}$$

Then:

$$v_{t_1}=\sqrt{2g\left(\frac{p_0}{\gamma}+z_0\right)}=\sqrt{20\left(\frac{0.1\times 10^6}{10^4}+15\right)}\approx 22.36\,m/s$$

$$Q_{t_1}=v_{t_1}\frac{\pi D_{o_1}^2}{4}=22.36\frac{\pi\times 0.09^2}{4}\approx 0.1422\,m^3/s$$

$$Q_{r_1}=C_{d_1}Q_{t_1}=0.6\times 0.1422\approx 0.0853\,m^3/s=Q_{r_2}$$

$$Q_{r_2}=C_{d_2}v_{t_2}\frac{\pi D_{o_2}^2}{4}\rightarrow C_{d_2}=\frac{4Q_{r_2}}{v_{t_2}\pi D_{o_2}^2}\wedge v_{t_2}=\sqrt{2gz_2}=\sqrt{20\times 9}\approx 13.42\,m/s$$

$$C_{d_2}=\frac{4\times 0.0853}{13.42\times\pi\times 0.1^2}\cong 0.81$$

6.3. Answer:

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}=\frac{p_0}{\gamma}=3.8\,m$$

$$p_1+\gamma\times 0.2-\gamma_m\times 0.2=p_2$$

$$p_1=20000+0.2\times(6\times 10^4-10^4)=30000\,Pa$$

$$v_1=\sqrt{2g\left(\frac{p_0}{\gamma}-\frac{p_1}{\gamma}\right)}=\sqrt{20\times(3.8-3)}=4\,m/s$$

$$Q=v_1A=4\times 10^{-2}\,m^3/s=40\,L/s$$

6.4. Answer:

$$E=G$$

$$\gamma_{H_2O}V_{immr}=\gamma_{wood}V_{cube}$$

$$\gamma_{H_2O}A_{base}h_{immr}=\gamma_{wood}A_{base}h\rightarrow h_{immr}=h\frac{\gamma_{wood}}{\gamma_{H_2O}}=1\times\frac{8000}{10000}=0.8\,m$$

$$V=4\times 4\times 0.8=12.8\,m^3$$

$$Q=\frac{v}{t}=\frac{12.8}{20}=0.64\,m^3/s$$

$$Q=C_dA_ov_t\wedge v_t=\sqrt{2gh}=\sqrt{20\times 6.05}=11\,m/s$$

$$C_d=\frac{Q}{A_ov_t}=\frac{0.64}{0.1\times 11}\approx 0.582$$

$$C_v=\frac{C_d}{C_c}=\frac{0.582}{0.6}\cong 0.97$$

6.5. Answer:

$$v_r=x\sqrt{\frac{g}{2y}}=4.7\times\sqrt{\frac{10}{2\times 1.2}}\approx 9.6\,m/s$$

$$v_t=\sqrt{2gh}=\sqrt{20\times 5}=10\,m/s$$

$$C_v = \frac{v_r}{v_t} = \frac{9.6}{10} = 0.96$$

$$Q_t = v_t A_o = 10 \times \frac{\pi \times 0.075^2}{4} \approx 0.0442\, m^3/s$$

$$C_d = \frac{Q_r}{Q_t} = \frac{28 \times 10^{-3}}{0.0442} \cong 0.634$$

$$C_c = \frac{C_d}{C_v} = \frac{0.634}{0.96} \cong 0.66$$

6.6. Answer:

$$M = \bar{p}A \times \frac{2}{3}h = \gamma \frac{h}{2} hb \times \frac{2}{3}h = \frac{\gamma b h^3}{2} \rightarrow h = \sqrt[3]{\frac{3M}{\gamma b}} = \sqrt[3]{\frac{3 \times 6 \times 10^4}{10^4 \times 2}} \approx 2.1\, m$$

$$V = 2 \times 2 \times 2.1 = 8.4\, m^3 \rightarrow Q = \frac{V}{t} = \frac{8.4}{5 \times 60} = 0.028\, m^3/s$$

$$v_t = \sqrt{2gh} = \sqrt{20 \times 1.8} = 6\, m/s \rightarrow Q_t = v_t A_o = 6 \times 0.01 = 0.06\, m^3/s$$

$$C_d = \frac{Q}{Q_t} = \frac{0.028}{0.06} \cong 0.467$$

CHAPTER 7 – Dimensional Analysis

7.1. Answer:

The kinematic viscosity is given by:

$$\nu = \frac{\mu}{\rho}$$

By definition:

$$\rho = \frac{m}{V}$$

In FLT base, the mass is a derivate magnitude and it must be related to fundamental magnitudes. The equation that allows such ratio is the law of Newton:

$$F = ma \vee m = \frac{F}{a}$$

A force is a fundamental magnitude, then:

$$[F] = F$$

From kinematics, it is known that acceleration is the ratio between the distance and the time squared. Then:

$$[a] = \frac{L}{T^2} = LT^{-2}$$

By Geometry, it is known that the volume is the cubic distance or length:

$$[V] = L^3$$

Then:

$$[\rho] = \frac{F}{LT^{-2}L^3} = \frac{F}{L^4T^{-2}} \rightarrow [\rho] = FL^{-4}T^2$$

The dynamic viscosity μ can be obtained with:

$$\tau = \mu\frac{dv}{dy} \vee \mu = \frac{\tau}{\frac{dv}{dy}}$$

But $\tau = F_t/A$ and therefore:

$[\tau] = F/L^2 \vee [\tau] = FL^{-2}$

The gradient of velocity is:

$$\left[\frac{dv}{dy}\right] = \frac{LT^{-1}}{L} = T^{-1}$$

Therefore:

$$[\mu] = \frac{FL^{-2}}{T^{-1}} \vee [\mu] = FL^{-2}T$$

Thus:

$$[\nu] = \frac{FL^{-2}T}{FL^{-4}T^2} = F^0L^2T^{-1}$$

7.2. Answer:

$Q = f(D, \rho, p)$

$f(Q, D, \rho, p) = 0 \rightarrow f(\pi) = 0$

As exists only one dimensionless parameter, it is constant.

$$\left.\begin{aligned} &[Q] = L^3T^{-1} \\ &[D] = L \\ &[\rho] = FL^{-4}T^2 \\ &[p] = FL^{-2} \end{aligned}\right\} \begin{aligned} m = n - r = 4 - 3 = 1 \\ Base: \rho, p, D \end{aligned}$$

$$\pi = \rho^{\alpha_1} p^{\alpha_2} D^{\alpha_3} Q = (FL^{-4}T^2)^{\alpha_1} (FL^{-2})^{\alpha_2} L^{\alpha_3} L^3 T^{-1}$$

$$\pi = F^{\alpha_1+\alpha_2} L^{-4\alpha_1-2\alpha_2+\alpha_3+3} T^{2\alpha_1-1}$$

$$\left.\begin{aligned} &\alpha_1 + \alpha_2 = 0 \\ &-4\alpha_1 - 2\alpha_2 + \alpha_3 + 3 = 0 \\ &2\alpha_1 - 1 = 0 \end{aligned}\right\} \begin{aligned} &\alpha_2 = -1/2 \\ &\alpha_3 = -2 \\ &\alpha_1 = 1/2 \end{aligned}$$

$$\pi = \rho^{\frac{1}{2}} p^{-\frac{1}{2}} D^{-2} Q = \frac{Q\rho^{\frac{1}{2}}}{D^2 p^{\frac{1}{2}}}$$

$$Q = CD^2 \sqrt{\frac{p}{\rho}}$$

7.3. Answer:

$f(v, g, h) = 0$

$\pi = g^{\alpha_1} h^{\alpha_2} v \rightarrow \pi = L^{\alpha_1} T^{-2\alpha_1} L^{\alpha_2} L T^{-1} \rightarrow \pi = L^{\alpha_1+\alpha_2+1} T^{-2\alpha_1-1}$

$$\left.\begin{array}{l}\alpha_1 + \alpha_2 + 1 = 0 \\ -2\alpha_1 - 1 = 0\end{array}\right\} \begin{array}{l}\alpha_2 = -1/2 \\ \alpha_1 = -1/2\end{array}$$

$\pi = g^{-\frac{1}{2}} h^{-\frac{1}{2}} v \rightarrow v = \pi\sqrt{gh}$

$$Q = vA \rightarrow A = \frac{bh}{2} \rightarrow \tan\frac{\alpha}{2} = \frac{b}{2h} \rightarrow b = 2h\tan\frac{\alpha}{2} \rightarrow A = \frac{2h\tan\left(\frac{\alpha}{2}\right) \times h}{2} = h^2\tan\left(\frac{\alpha}{2}\right)$$

$$Q = \pi\sqrt{gh} \times h^2 \tan\left(\frac{\alpha}{2}\right) = C g^{\frac{1}{2}} h^{\frac{5}{2}}$$

CHAPTER 8 – Similarity

8.1. Answer:

$$Base: \rho, v, L$$

$$\pi_1 = Eu = \frac{F}{\rho v^2 L^2} \rightarrow k_F = k_\rho k_v^2 k_L^2$$

$$\pi_2 = Re = \frac{\rho v L}{\mu} = \frac{vL}{\nu} \rightarrow k_\nu = k_v k_L$$

$$k_\rho = \frac{1000}{1.2}; k_\nu = \frac{10^{-6}}{10^{-5}} = 0.1; k_v = \frac{6}{30} = 0.2; k_L = \frac{k_\nu}{k_v} = \frac{0.1}{0.2} = 0.5 = \frac{L_m}{L_p} \rightarrow L_p = 2L_m$$

$$Plate\ with \begin{cases} l = 1.5 \times 2 = 3\ m \\ b = 15 \times 2 = 30\ cm \end{cases}$$

$$k_F = \frac{1000}{1.2} \times 0.2^2 \times 0.5^2 \approx 8.33 = \frac{F_m}{F_p} \rightarrow F_p = \frac{F_m}{8.33} = \frac{15}{8.33} \cong 1.8\ N$$

8.2. Answer:

$$v, g, L, \nu \rightarrow Base: v, L$$

$$Dimensionless\ parameters \begin{cases} Fr = \dfrac{v^2}{Lg} \rightarrow k_v^2 = k_L k_g \\ Re = \dfrac{vL}{\nu} \rightarrow k_\nu = k_v k_L \end{cases}$$

$$k_v = \sqrt{k_L k_g} = \sqrt{\frac{1}{2} \times 1} \approx 0.707$$

$$k_\nu = 0.707 \times \frac{1}{2} \approx 0.353 \rightarrow \frac{\nu_m}{\nu_p} = 0.353 \rightarrow \nu_m = 3.53 \times 10^{-7}\ (water\ at\ 90℃)$$

8.3. Answer:

$$\begin{cases} \phi = \dfrac{Q}{nD^3} = \dfrac{Q}{\dfrac{3500}{60} \times 0.15^3} \approx 5.08Q \\ \Psi = \dfrac{gH_B}{n^2 D^2} = \dfrac{9.8H_B}{\left(\dfrac{3500}{60}\right)^2 \times 0.15^2} \approx 0.128H_B \end{cases}$$

For a prototype:

$$n_p = \frac{n_m}{2} = \frac{3500}{2} = 1750\,rpm \wedge D_p = 2D_m = 2 \times 0.15 = 0.3\,m$$

$$\begin{cases} Q_p = \phi n_p D_p^3 = \phi \times \dfrac{1750}{60} \times 0.3^3 = 0.7875\phi \\ H_{B_p} = \Psi \dfrac{n_p^2 D_p^2}{g} = \Psi \times \dfrac{\left(\dfrac{1750}{60}\right)^2 \times 0.3^2}{9.8} = 7.8125\Psi \end{cases}$$

With those expressions it is possible to build the following table and the pump's curves.

Table 2 – Characteristic, universal and similarity pump's curves (source: Author).

$Q\ (m^3/s)$	0	$5\text{x}10^{-3}$	$10\text{x}10^{-3}$	$15\text{x}10^{-3}$	$20\text{x}10^{-3}$
$H_B\ (m)$	25	24	23	20	14
$\phi\ (-)$	0	0.0254	0.0508	0.0762	0.1016
$\Psi\ (-)$	3.2	3.07	2.94	2.56	1.79
$Q_p\ (m^3/s)$	0	$20\text{x}10^{-3}$	$40\text{x}10^{-3}$	$60\text{x}10^{-3}$	$80\text{x}10^{-3}$
$H_{B_p}\ (m)$	25	24	23	20	14

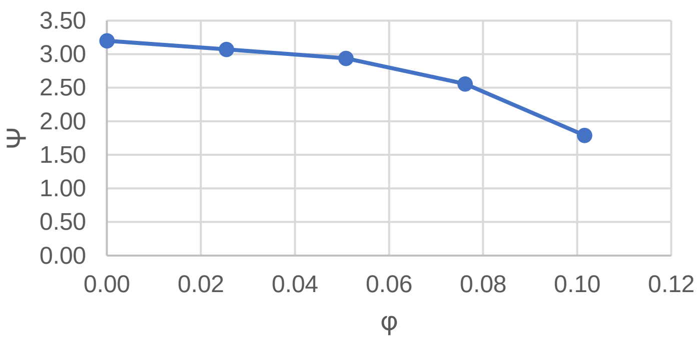

Figure 39 – Answer to a) (source: Author).

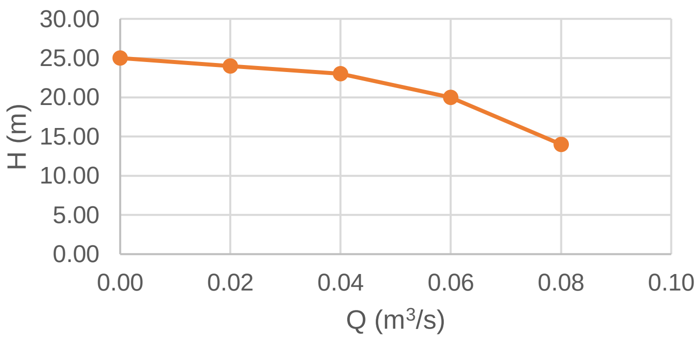

Figure 40 – Answer to b) (source: Author).

CHAPTER 9 – Flows under pressure

9.1. Answer:

$\Delta \boldsymbol{h} = ?$

$$H_0 + H_B = H_7 + H_{p_{0,7}}$$

$$z_1 - z_0 = \Delta h = H_B - H_{p_{0,7}}$$

$$N = \gamma Q H_B \rightarrow H_B = \frac{N}{\gamma Q} = \frac{0.75 \times 10^3}{10^4 \times 3 \times 10^{-3}} = 25\ m$$

$$H_{p_{0,7}} = \left(f\frac{L}{D_H} + \sum k_s\right)\frac{v^2}{2g}$$

$$v = \frac{4Q}{\pi D^2} = \frac{4 \times 3 \times 10^{-3}}{\pi \times 0.03^2} \approx 4.24\ m/s$$

$$H_{p_{0,7}} = \left(0.02 \times \frac{12}{0.03} + 5\right)\frac{4.24^2}{20} \approx 11.7\ m \rightarrow \Delta h = 25 - 11.7 = 13.3\ m$$

$\boldsymbol{k} = ?$

$$\begin{cases} Re = \dfrac{vD}{\nu} = \dfrac{4.24 \times 0.03}{10^{-6}} \approx 1.27 \times 10^5 \\ f = 0.02 \end{cases} \rightarrow Moody - Rouse: \frac{D_H}{k} = 2000 \rightarrow k = \frac{D_H}{2000} =$$

$$= \frac{0.03}{2000} = 1.5 \times 10^{-5}\ m$$

$\boldsymbol{h_0} = ?$

$$H_0 = H_2 + H_{p_{0,2}} \rightarrow z_0 = h_0 = \frac{v^2}{2g} + f\frac{L_{1,2}}{D_H}\frac{v^2}{2g} + k_{s_1}\frac{v^2}{2g} \rightarrow h_0 = \left(1 + f\frac{L_{1,2}}{D_H} + k_{s_1}\right)\frac{v^2}{2g} =$$

$$= \left(1 + 0.02 \times \frac{2}{0.03} + 1\right) \times \frac{4.24^2}{20} \cong 3\ m$$

9.2. Answer:

a)

$$f\frac{L_{eq_2}}{D}\frac{v^2}{2g} = k_{s_2}\frac{v^2}{2g} \rightarrow f = \frac{k_{s_2}D}{L_{eq_2}} = \frac{9 \times 0.04}{18} = 0.02$$

b)

$$H_{p_{1,4}} = f\frac{L_{tot}}{D}\frac{v^2}{2g} \rightarrow L_{tot} = \frac{2gDH_{p_{1,4}}}{fv^2}$$

$$H_{p_{1,4}} = H_1 - H_4 = 56 - 38 = 18\ m$$

$$v = \frac{4Q}{\pi D^2} = \frac{4 \times 3.8 \times 10^{-3}}{\pi \times 0.04^2} \approx 3\ m/s$$

$$L_{tot} = \frac{20 \times 0.04 \times 18}{0.02 \times 3^2} = 80\ m$$

$$L_{1,4} = L_{tot} - L_{eq} - L_{eq_3} = 80 - 18 - 2 = 60\ m$$

c)

$$h_{s_3} = f\frac{L_{eq_3}}{D}\frac{v^2}{2g} = 0.02 \times \frac{2}{0.04} \times \frac{3^2}{20} = 0.45\ m$$

9.3. Answer:

a)

$$\frac{v^2}{2g} = 1.8\ m \rightarrow v = \sqrt{20 \times 1.8} = 6\ m/s$$

$$Q = v\frac{\pi D^2}{4} = 6 \times \frac{\pi \times 0.1^2}{4} \approx 0.0471\ m^3/s = 47.1\ L/s$$

b)

$$H_{p_{0,1}} = h_{s_1} + h_{s_2} + h_f$$

$$h_{s_1} = 0.2\ m \rightarrow of\ the\ line\ of\ energy$$

$$h_{s_2} = k_{s_2}\frac{v^2}{2g} = 2 \times 1.8 = 3.6\ m$$

$$h_f = f\frac{L}{D}\frac{v^2}{2g} = 0.01 \times \frac{50}{0.1} \times 1.8 = 9\ m$$

$$H_{p_{0,1}} = 0.2 + 3.6 + 9 = 12.8\ m$$

c)

$$\frac{p_0}{\gamma} = \frac{v_1^2}{2g} + H_{p_{0,1}}$$

$$x = \frac{p_0}{\gamma} = 1.8 + 12.8 = 14.6\ m$$

d)

$$\frac{p_0}{\gamma} - H_T = \frac{v_1^2}{2g} + H_{p_{0,1}} - h_{s_2}$$

$$H_T = \frac{p_0}{\gamma} - \frac{v_1^2}{2g} - H_{p_{0,1}} + h_{s_2} = 14.6 - 1.8 - 12.8 + 3.6 = 3.6\ m$$

$$N_T = \gamma Q H_T \eta_T = 10^4 \times 0.0471 \times 3.6 \times 0.9 \times \frac{1}{1000} \cong 1.5\ kW$$

CHAPTER 10 – Permanent flow in pipes, conditioned by hydraulic machines

10.1. Answer:

a)

$$H_0 = \frac{v_0^2}{2g} + \frac{p_0}{\gamma} + z_0 = 0 + \frac{0.25 \times 10^6}{10^4} + 10 = 35\ m$$

$$v_1 = \frac{Q}{A_1} = \frac{20 \times 10^{-3}}{10 \times 10^{-4}} = 20\ m/s$$

$$H_1 = \frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{20^2}{20} + \frac{0.2 \times 10^6}{10^4} + 5 = 45\ m$$

$$H_1 > H_0 \rightarrow Flow\ from\ (5)\ to\ (0)$$

$$H_5 + H_{M_2} + H_{M_1} = H_0 + H_{p_{5,0}} \rightarrow \frac{p_5}{\gamma} + z_5 + H_{M_2} + H_{M_1} = \frac{p_0}{\gamma} + z_0 + H_{p_{5,0}} \rightarrow$$

$$\rightarrow H_{M_1} = \frac{0.25 \times 10^6}{10^4} + 10 + 15 - \frac{0.4 \times 10^6}{10^4} - 5 - 30 = -25\ m$$

$$H_T = 25\ m$$

$$N_T = \gamma Q H_T \eta_T = 10^4 \times 20 \times 10^{-3} \times 25 \times 0.8 \times \frac{1}{1000} = 4\ kW$$

b)

$$\frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + H_{M_1} = \frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1$$

$$\frac{p_2}{\gamma} = \frac{p_1}{\gamma} - H_{M_1} \rightarrow \frac{p_2}{\gamma} = \frac{0.2 \times 10^6}{10^4} - (-25) = 45\ mca$$

c)

$$H_5 + H_{M_2} = H_2 + H_{p_{5,2}}$$

$$H_{p_{5,2}} = \frac{p_5}{\gamma} + H_{M_2} - \frac{v_2^2}{2g} - \frac{p_2}{\gamma} = \frac{0.4 \times 10^6}{10^4} + 30 - \frac{20^2}{20} - 45 = 5\ m$$

10.2. Answer:

a)

$$v_2 = \frac{16 \times 10^{-3}}{2 \times 10^{-3}} = 8\ m/s \wedge v_3 = \frac{16 \times 10^{-3}}{8 \times 10^{-3}} = 2\ m/s$$

$$H_2 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} = \frac{8^2}{20} + \frac{200 \times 10^3}{10^4} = 23.2\ m$$

$$H_3 = \frac{v_3^2}{2g} + \frac{p_3}{\gamma} = \frac{2^2}{20} + \frac{400 \times 10^3}{10^4} = 40.2\ m$$

$$H_3 > H_2 \rightarrow orientation\ from\ (4)\ to\ (1)$$

b)

$$H_{p_{3,2}} = H_3 - H_2 = 40.2 - 23.2 = 17\ m$$

c)

$$H_2 + H_M = H_1 + H_{p_{2,1}}$$

$$H_M = \frac{p_1}{\gamma} - H_2 + H_{p_{2,1}} = \frac{0.1 \times 10^6}{10^4} - 23.2 + 1 = -12.2\ m\ (turbine)$$

$$N_T = \gamma Q H_T = (10^4 \times 16 \times 10^{-3} \times 12.2) \times 10^{-3} \cong 1.95\ kW$$

d)

$$\frac{p_4}{\gamma} + z_4 = H_3 + H_{p_{4,3}}$$

$$p_4 = \gamma\left(H_3 + H_{p_{4,3}} - z_4\right) = 10^4(40.2 + 1 - 5) \times 10^{-6} = 0.362\ MPa$$

10.3. Answer:

$$\gamma Q_0 H_0 + \gamma Q_0 H_B = \gamma Q_1 H_1 + \gamma Q_2 H_2 + \gamma Q_0 H_{p_{0,e}} + \gamma Q_1 H_{p_{s,1}} + \gamma Q_2 H_{p_{s,2}}$$

$$\begin{cases} Q_2 = 2Q_1 \\ Q_0 = Q_1 + Q_2 \end{cases} \rightarrow Q_0 = 3Q_1$$

$$\gamma 3Q_1 H_0 + \gamma 3Q_1 H_B = \gamma Q_1 H_1 + \gamma 2Q_1 H_2 + \gamma 3Q_1 H_{p_{0,e}} + \gamma Q_1 H_{p_{s,1}} + \gamma 2Q_1 H_{p_{s,2}}$$

$$3H_0 + 3H_B = H_1 + 2H_2 + 3H_{p_{0,e}} + H_{p_{s,1}} + 2H_{p_{s,2}}$$

$$\begin{cases} H_0 = 0 \\ H_B = 8 \\ H_1 = 7 + \dfrac{v_1^2}{2g} \\ H_2 = 5 + \dfrac{v_2^2}{2g} \\ H_{p_{0,e}} = \dfrac{1}{3}\dfrac{v_e^2}{2g} \\ H_{p_{s,1}} = 5\dfrac{v_1^2}{2g} \\ H_{p_{s,2}} = 1.5\dfrac{v_2^2}{2g} \end{cases} \rightarrow 3 \times 8 = 7 + \frac{v_1^2}{2g} + 10 + 2\frac{v_2^2}{2g} + \frac{v_e^2}{2g} + 5\frac{v_1^2}{2g} + 3\frac{v_2^2}{2g}$$

$$7 = 6\frac{v_1^2}{2g} + 5\frac{v_2^2}{2g} + \frac{v_e^2}{2g}$$

$$\begin{cases} v_e = 3v_1 \\ v_2 = 2v_1 \end{cases} \rightarrow 140 = 6v_1^2 + 20v_1^2 + 9v_1^2 \rightarrow 35v_1^2 = 140 \rightarrow v_1 = 2\ m/s \rightarrow v_e = 6\ m/s$$

$$Q_e = v_e\frac{\pi D_e^2}{4} = 6 \times \frac{\pi \times 0.138^2}{4} \approx 0.0897\ m^3/s$$

$$N_B = \frac{\gamma Q_e H_B}{\eta_B} = \frac{10^4 \times 0.0897 \times 8}{0.48} \times \frac{1}{1000} \cong 15\ kW$$

CHAPTER 11 – Flows with Free Surface

11.1. Answer:

a)

$$Q = KSR^{\frac{2}{3}}i^{\frac{1}{2}} \rightarrow 25 = 80 \times \left(\frac{10+10+2x}{2}\right) \times h \times \left[\frac{\left(\frac{10+10+2x}{2}\right) \times h}{10+2\sqrt{x^2+h^2}}\right]^{\frac{2}{3}} \times \left(\frac{0.2}{100}\right)^{\frac{1}{2}}$$

$$\frac{x}{h} = \frac{1}{3} \rightarrow x = \frac{h}{3}$$

$$80 \times \left(10+\frac{h}{3}\right) \times h \times \left[\frac{\left(10+\frac{h}{3}\right) \times h}{10+2\sqrt{\frac{10}{9}h^2}}\right]^{\frac{2}{3}} \times (0.002)^{\frac{1}{2}} - 25 = 0 \rightarrow h \cong 0.837\ m$$

b)

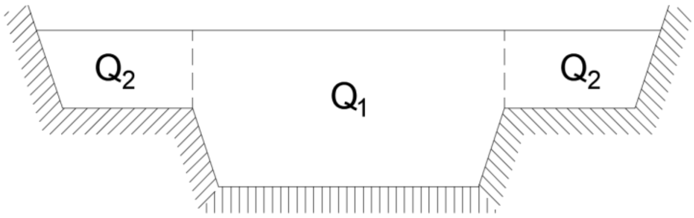

Figure 41 – Splitting of the channel (source: Author).

$$Q = Q_1 + 2Q_2$$

$$Q_1 = 80 \times 0.002^{\frac{1}{2}} \times \left[\left(\left(10+\frac{0.837}{3}\right) \times 0.837\right) + \left(\left(10+2\times\frac{0.837}{3}\right) \times 2\right)\right] \times \left\{\frac{\left[\left(\left(10+\frac{0.837}{3}\right) \times 0.837\right) + \left(\left(10+2\times\frac{0.837}{3}\right) \times 2\right)\right]}{\left[10+2\sqrt{\frac{10}{9}0.837^2}\right]}\right\}^{\frac{2}{3}} \approx 197\ m^3/s$$

$$Q_2 = 40 \times 0.002^{\frac{1}{2}} \times \left[\frac{\left(5+\frac{2}{3}\right)+5}{2} \times 2\right] \times \left\{\frac{\frac{\left(5+\frac{2}{3}\right)+5}{2} \times 2}{5+\sqrt{\left(\frac{2}{3}\right)^2+2^2}}\right\}^{2/3} \approx 25\ m^3/s$$

$$Q = 197 + 2 \times 25 = 247\ m^3/s$$

11.2. Answer:

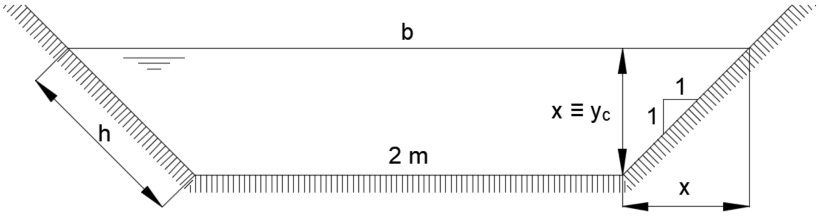

Figure 42 – Support sketch (source: Author).

a)

$$\frac{Q}{\sqrt{g}} = S\sqrt{y_c} \wedge y_c = \frac{S}{b}$$

$$\frac{S}{b} = \frac{\left(\frac{b+2}{2} \times x\right)}{b}, with\ b = 2 + 2x \rightarrow \frac{S}{b} = \frac{\left(\frac{2+2x+2}{2} \times x\right)}{2+2x} = \frac{2x+x^2}{2+2x}$$

Then:

$$\frac{15}{\sqrt{10}} = 2x + x^2 \times \sqrt{\frac{2x+x^2}{2+2x}} \rightarrow 2x + x^2 \times \sqrt{\frac{2x+x^2}{2+2x}} - \frac{15}{\sqrt{10}} = 0 \rightarrow x = y_c \cong 1.40\ m$$

b)

$$U = C \times \sqrt{R \times i} \rightarrow Q = S \times C \times \sqrt{R \times i}$$

$$R = \frac{S}{\chi} = \frac{2x+x^2}{2+2h} \wedge C = \frac{87\sqrt{R}}{C_B + \sqrt{R}} = \frac{87\sqrt{\frac{2x+x^2}{2+2h}}}{0.16 + \sqrt{\frac{2x+x^2}{2+2h}}}$$

$$h^2 = x^2 + x^2 \rightarrow h = \sqrt{2x^2} = x\sqrt{2}$$

Then:

$$(2x+x^2) \times \left(\frac{87\sqrt{\frac{2x+x^2}{2+2x\sqrt{2}}}}{0.16 + \sqrt{\frac{2x+x^2}{2+2x\sqrt{2}}}}\right) \times \sqrt{\frac{2x+x^2}{2+2x\sqrt{2}} \times 0.003} - 15 = 0 \rightarrow x = y_n \cong 1.30\ m$$

$$\chi = 2 + 2x\sqrt{2} = 2 + 2 \times 1.30 \times \sqrt{2} \cong 5.68\ m$$

11.3. Answer:

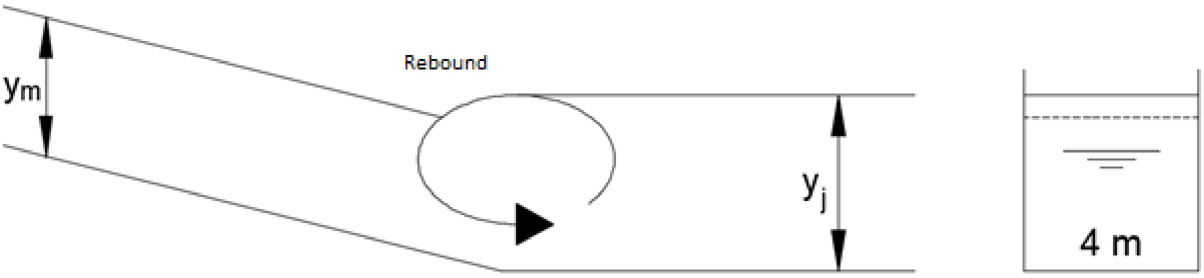

Figure 43 – Sketch of a spillway and a receptor channel (source: Author).

a)

$$\frac{y_m}{y_j} = \frac{1}{2}\left(\sqrt{1+8Fr}-1\right) \wedge Fr = \frac{Q^2 b}{gS^3} = \frac{20^2 \times 4}{9.8 \times \left(4 \times y_j\right)^3}$$

Then:

$$\frac{1.2}{y_j} = \frac{1}{2}\left(\sqrt{1+8\frac{20^2 \times 4}{9.8 \times \left(4 \times y_j\right)^3}}-1\right) \rightarrow \frac{1}{2}\left(\sqrt{1+8\frac{20^2 \times 4}{9.8 \times \left(4 \times y_j\right)^3}}-1\right)y_j - 1.2 = 0 \rightarrow$$

$$\rightarrow y_j \cong 1.547\ m$$

b)

$$\Delta E = E_m - E_j$$

$$E_m = y_m + \frac{Q^2}{2gS_m^2} = 1.2 + \frac{20^2}{2 \times 9.8 \times (1.2 \times 4)^2} \approx 2.086$$

$$E_j = y_j + \frac{Q^2}{2gS_j^2} = 1.547 + \frac{20^2}{2 \times 9.8 \times (1.547 \times 4)^2} \approx 2.08$$

Then:

$$\Delta E = 2.086 - 2.08 = 0.006\ m$$

c)

$$\begin{cases} U_\mathrm{m} = \dfrac{\mathrm{Q}}{S_\mathrm{m}} = \dfrac{20}{4 \times 1.2} \cong 4.17\ m/s \\ U_\mathrm{j} = \dfrac{\mathrm{Q}}{S_\mathrm{j}} = \dfrac{20}{4 \times 1.547} \cong 3.23\ m/s \end{cases}$$

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