Hypercyclic subspaces in Fréchet spaces

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Dedicated to the memory of Professor Miguel de Guzmán, who died on April 2004

Abstract

In this note, we show that every infinite-dimensional separable Fréchet space admitting a continuous norm supports an operator for which there is an infinite-dimensional closed subspace consisting, except for zero, of hypercyclic vectors. The family of such operators is even dense in the space of bounded operators when endowed with the strong operator topology. This completes earlier work of several authors.

1 Introduction and notation

Throughout this paper, the following standard notation will be used: $\mathbb{N}$ is the set of positive integers, $\mathbb{R}$ is the real line, and $\mathbb{C}$ is the complex plane. The symbols (m_k), (n_k) will stand for strictly increasing sequences in $\mathbb{N}$. If X, Y are (Hausdorff) topological vector spaces (TVSs) over the same field $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, then $L(X, Y)$ will denote the space of continuous linear mappings from X into Y, while $L(X)$ is the class of operators on X, that is, $L(X) = L(X, X)$. The strong operator topology (SOT) in $L(X)$ is the one where the convergence is defined as pointwise convergence at every $x \in X$.

A sequence $(T_n) \subset L(X, Y)$ is said to be *universal* or *hypercyclic* provided there exists some vector $x_0 \in X$ –called hypercyclic for the sequence (T_n)– such that its

orbit $\{T_n x_0 : n \in \mathbb{N}\}$ under (T_n) is dense in Y. It is obvious that if there is a hypercyclic sequence $(T_n) \subset L(X, Y)$ then Y must be separable. In the (perhaps most interesting) case $X = Y$, an operator $T \in L(X)$ is called *hypercyclic* if the sequence (T^n) of its iterates –that is, $T^1 = T$, $T^2 = T \circ T$ and so on– is hypercyclic. See the excellent surveys [21] and [22] for concepts and results concerning universality.

In 1969 Rolewicz [33] proved that no finite-dimensional TVS X supports a hypercyclic operator, and showed that any multiple λB ($|\lambda| > 1$) of the backward shift $B : (\alpha_0, \alpha_1, \alpha_2, \ldots) \mapsto (\alpha_1, \alpha_2, \alpha_3, \ldots)$ on the sequence spaces l_p ($1 \leq p < \infty$) or c_0 possesses a dense orbit, so exhibiting the first example of a hypercyclic operator on a Banach space. He asked in [33] whether any infinite-dimensional separable *Banach space* supports a hypercyclic operator. During the late nineties, this question was solved in the affirmative, independently, by Ansari [1] and the author [3] (the proof of Ansari is also applicable on a wide class of TVSs). Finally, in 1998 Bonet and Peris [13] gave the same affirmative answer for *Fréchet spaces*. We recall that an F-space is a complete and metrizable TVS, while a Fréchet space is a locally convex F-space. The proofs of [1], [3] and [13] are all based on a result of Salas [34, Theorem 3.3], who proved that when $(w_n)_{n \geq 0}$ is any bounded sequence of positive scalars then the operator $I + B_\omega$ –where I denotes the identity operator and B_ω is the weighted backward shift generated by the weights w_n– is hypercyclic on l_p ($1 \leq p < \infty$) or c_0. We point out that there exist separable complete locally convex spaces that do not support any hypercyclic operators, see [13, Proposition 6].

A natural problem that arises in this context is that of the *size* and the *structure* of the set $HC(T)$ of hypercyclic vectors for a hypercyclic operator T on a TVS X. First, $HC(T)$ is dense because T has dense range and $T^n x \in HC(T)$ for all $x \in HC(T)$ and all $n \in \mathbb{N}$. Second, if X is Baire, metrizable and separable then $HC(T)$ is even residual, that is, its complement in X is of first category (more generally, if X is Baire, Y is metrizable separable and $(T_n) \subset L(X, Y)$ is a hypercyclic sequence, then its set $HC((T_n))$ of hypercyclic vectors is residual in X). It is evident that $HC(T)$ is not a linear space, but recently Wengenroth [36] has shown the existence of a *dense linear manifold* $M \subset X$ such that $M \setminus \{0\} \subset HC(T)$, so completing earlier work by Herrero [24], Bourdon [14] and Bès [8], see also [4].

In a complementary line, the study of the existence of *hypercyclic subspaces* (the terminology is due to Chan [16]), that is, of infinite-dimensional closed subspaces $Z \subset X$ such that $Z \setminus \{0\} \subset HC(T)$ (or $\subset HC((T_n))$ if (T_n) is a hypercyclic sequence) was initiated by Montes and the author [6] in 1995. They proved that if $G \subset \mathbb{C}$ is a

domain that is not isomorphic to $\mathbb{C} \setminus \{0\}$ then there is a hypercyclic subspace for the sequence (C_{φ_n}) of composition operators on $H(G)$ (= the Fréchet space of holomorphic functions on G, endowed with the compact-open topology) generated by a sequence (φ_n) of automorphisms of G acting properly discontinuously on G. This means that, given a compact set $K \subset G$, there is $n \in \mathbb{N}$ with $\varphi_n(K) \cap K = \emptyset$. In particular, this applies on sequences of euclidean translations in $\mathbb{C}$ or of noneuclidean translations in the open unit disk $\mathbb{D}$. Later, Montes, León and González provided a series of papers ([29], [26], [27], [19], see also [31, Section 2]) where some conditions for an operator on a Banach space X to have a hypercyclic subspace are established. Among other results, they proved that if T satisfies the Hypercyclicity Criterion (see Definition 2.1) for some $(n_k) \subset N$ and there exists an infinite-dimensional closed subspace $X_0 \subset X$ such that $T^{n_k} x \to 0$ $(k \to \infty)$ for all $x \in X_0$, then T has a hypercyclic subspace. In [29, Theorem 3.4] it is shown that not every hypercyclic operator on Hilbert space possesses a hypercyclic subspace. Nevertheless, *every separable infinite-dimensional Banach space supports an operator that admits a hypercyclic subspace*, see [26, Corollary 2.2].

Let us conclude the history of these recent results. Chan [15] gave an alternative simpler proof of the mentioned result of Montes and co-workers in the Hilbert space context. Later, Chan and Taylor [17] and, independently, Montes and Romero [30] extended the arguments of Chan to the setting of Banach spaces. In 2002, Chan [16] showed that hypercyclic operators on Hilbert space are SOT-dense in the space of operators. Even more, he was able to prove that the family of operators on a (separable, infinite-dimensional) Hilbert space X *supporting a hypercyclic subspace* is still SOT-dense in $L(X)$. Finally, Bès and Chan [9, Theorem 2] have recently noticed that the hypercyclic operators on any separable, infinite-dimensional) *Fréchet* space are SOT-dense.

In view of the aforementioned statements, the following problem arises naturally: *Given an infinite-dimensional separable Fréchet space X, is the family of operators admitting a hypercyclic subspace SOT-dense in $L(X)$?* The aim of this short note is to furnish a rather general answer by showing (see Section 2) that the denseness of such family is guaranteed whenever X admits a continuous norm. The space $H(G)$ is a typical example of a non-normable space admitting a continuous norm.

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2 Supporting operators with hypercyclic subspaces

Before giving the main result of the present paper (Theorem 2.5), we recall a notion which has turned out to be extremely useful in applications, see [18] and [21].

Definition 2.1. Assume that X and Y are separable F-spaces. Assume that $(T_n) \subset L(X, Y)$, that $T \in L(X)$ and that $(n_k) \subset \mathbb{N}$. We say that (T_n) satisfies the *Hypercyclicity Criterion* (HC) (or the Universality Criterion) with respect to (n_k) whenever there exist dense subsets $D_1 \subset X$, $D_2 \subset Y$ such that $T_{n_k} x \to 0$ $(k \to \infty)$ for all $x \in D_1$, and for every $y \in D_2$ there is a sequence $(v_k) \subset X$ such that $v_k \to 0$ and $T_{n_k} v_k \to y$ $(k \to \infty)$. The sequence (T_n) satisfies the HC provided that the last property holds for some sequence (n_k). Accordingly, T is said to satisfy the HC (with respect to (n_k)) if the sequence (T^n) of its iterates satisfies the HC (with respect to (n_k), respectively).

If (T_n) satisfies the HC then it is hypercyclic, see [21]. The reader is referred to [7], [11], [5] and [2] for equivalent properties to the HC.

We will make use of four auxiliary results. The first one is due to Grivaux [20, Theorem 1.3 and Remark 1.4]. It is in fact a modification of [34, Theorem 3.3] and reads as follows.

Lemma 2.1. *Let X be one of the sequence spaces l_p $(1 \le p < \infty)$ or c_0, and $(w_n)_{n \ge 0}$ be any bounded sequence of positive scalars. Consider the operator S defined on X by*

$$S(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \ldots) = (w_0 \alpha_1, w_1 \alpha_3, w_2 \alpha_5, \ldots).$$

Then the operator $I + S$ is hypercyclic on X. In fact, it satisfies the HC.

The second auxiliary statement is a very useful one due to Martínez-Giménez and Peris [28, Lemma 2.1]. It is a stronger version of the so-called Hypercyclic Comparison Principle due to J.H. Shapiro [35, p. 111].

Lemma 2.2. *Assume that X, Y are F-spaces, that $T \in L(X)$, $\widetilde{T} \in L(Y)$, $Q \in L(Y, X)$, that Q has dense range and that $Q\widetilde{T} = TQ$. If $\widetilde{T}$ satisfies the HC then T also does.*

The third auxiliary statement is a modified version of Theorem 3.5 of the very recent paper [12] by Bonet, Martínez-Giménez and Peris. In our lemma, condition

(iii) below has replaced the more restrictive one "$\lim_{k\to\infty} T_{m_k} x = 0$ for each $x \in X_0$" given in [12, Theorem 3.5].

Lemma 2.3. *Assume that $(m_k) \subset \mathbb{N}$, that X and Y are Fréchet spaces and that $(S_n) \subset L(X, Y)$. Suppose that the following properties are satisfied:*

(i) *The space X admits a norm which is continuous.*

(ii) *The sequence (S_n) satisfies the HC with respect to (m_k).*

(iii) *There exists a closed infinite-dimensional subspace $X_0 \subset X$ such that, for every $x \in X_0$, the sequence $(S_{m_k} x)$ converges in Y.*

Then (S_{m_k}) (hence (S_n)) admits a hypercyclic subspace.

Proof. Exactly as in [12, proof of Theorem 3.5], conditions (i) and (ii) allow us to assert the existence of an operator K on X such that $\{S_{m_k} K x : k \in \mathbb{N}\}$ is dense in Y for all $x \in X \setminus \{0\}$ and, in addition, the operator $I + K$ has closed range and finite-dimensional kernel. Therefore $X_1 := (I + K)(X_0)$ is a closed infinite-dimensional subspace of X. If $x_1 \in X_1 \setminus \{0\}$, then there exists $x_0 \in X_0 \setminus \{0\}$ such that $x_1 = x_0 + K x_0$. Let $u(x_0) := \lim_{k\to\infty} S_{m_k} x_0 \in Y$. Fix $y \in Y$ and let (n_k) be a subsequence of (m_k) satisfying $S_{n_k} K x_0 \to y - u(x_0)$ as $k \to \infty$. Hence, by linearity,

$$S_{n_k} x_1 = S_{n_k} x_0 + S_{n_k} K x_0 \to u(x_0) + (y - u(x_0)) = y \quad (k \to \infty).$$

In other words, the sequence $\{S_{m_k} x_1 : k \in \mathbb{N}\}$ is dense in Y. Consequently, X_1 is a hypercyclic subspace for (S_{m_k}). $\qquad\square$

The fourth and last auxiliary result is a nice one due to Hadwin, Nordgren, Radjavi and Rosenthal ([23], see also [10, Theorem 1 and the note following it]), and it was cleverly used by Bès and Chan in [10, Corollary 6] to give a new proof of [9, Theorem 2] together with some new results.

Lemma 2.4. *Let X be an infinite-dimensional locally convex space, and let T in $L(X)$ be fixed. The following statements are equivalent:*

(i) *The set of conjugates $\{STS^{-1} : S \text{ invertible}\}$ of T is SOT-dense in $L(X)$.*

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(ii) *For all $n \in \mathbb{N}$, there exist $h_1, \ldots, h_n$ in X so that the set $\{h_1, \ldots, h_n, Th_1, \ldots, Th_n\}$ is linearly independent.*

If X is a TVS then the linear span of a subset A of X is denoted by $\mathrm{span}(A)$. If A is an absolutely convex subset of X and p_A is the Minkowski functional of A, X_A will stand for $\mathrm{span}(A)/\mathrm{ker}p_A$ endowed with the norm induced by p_A. The topological dual of X will be denoted by X'. We are now ready to establish our assertion, which concludes the paper.

Theorem 2.5. *Suppose that X is a separable infinite-dimensional Fréchet space admitting a continuous norm. Then X supports an operator which possesses a hypercyclic subspace. Even more, the family of such operators is SOT-dense in $L(X)$.*

Proof. The second assertion of the theorem is obtained from the first one together with Lemma 2.4: Just take into account that the family under consideration is invariant under conjugation, and that every hypercyclic operator has at least one dense (so linearly independent, because X is infinite-dimensional) orbit $\{x_0, Tx_0, T^2x_0, \ldots\}$; then Lemma 2.4 applies if we choose $h_j := T^{2j-2}x_0$ $(j = 1, \ldots, n; \ n \in \mathbb{N})$.

Hence it is enough to exhibit an operator $T \in L(X)$ possessing a hypercyclic subspace. Our approach takes advantage of the clever method given by Bonet and Peris in [13, Lemmas 2–3].

It is known that the countable product of lines $\omega := \mathbb{K}^{\mathbb{N}}$, endowed with the product topology, is a Fréchet space but it does not admit a continuous norm, so $X \neq \omega$. Then by [13, Lemma 2 and proof of Theorem 1] there are sequences $(x_n)_{n\geq 0} \subset X$ and $(f_n)_{n\geq 0} \subset X'$ satisfying the following conditions:

(a) (x_n) converges to 0 in X, and the closed absolutely convex hull B of (x_n) satisfies that X_B is a Banach space which is dense in X.

(b) (f_n) is X-equicontinuous in X'.

(c) $f_m(x_n) = 0$ if $m \neq n$ and $\{f_n(x_n) : n \geq 0\} \subset (0, 1)$.

Consider the operator T on X defined as $T := I + S$, where

$$Sx := \sum_{n=0}^{\infty} \frac{f_{2n+1}(x)}{2^n} x_n \quad (x \in X).$$

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Exactly as in [13, proof of Lemma 3], we obtain that

$$B = \left\{ \sum_{n=0}^{\infty} \alpha_n x_n : \sum_{n=0}^{\infty} |\alpha_n| \leq 1 \right\} \tag{1}$$

and that the mapping

$$Q : \alpha = (\alpha_n)_{n \geq 0} \in l_1 \mapsto \sum_{n=0}^{\infty} \alpha_n x_n \in X_B$$

is linear, continuous and surjective. By Lemma 2.1, the operator

$$\widetilde{T} : (\alpha_n)_{n \geq 0} \in l_1 \mapsto \left(\alpha_0 + f_0(x_0)\alpha_1, \alpha_1 + \frac{f_1(x_1)}{2}\alpha_3, \alpha_2 + \frac{f_2(x_2)}{2^2}\alpha_5, \ldots \right) \in l_1$$

satisfies the HC, since the weights $w_n := \frac{f_n(x_n)}{2^n}$ are positive and form a bounded sequence.

Let us show that T also satisfies the HC. If we consider Q as a mapping $Q : l_1 \to X$ then Q is also continuous. Indeed, such mapping is the composition of $Q : l_1 \to X_B$ with the canonical inclusion $X_B \to X$. This inclusion is (linear and) continuous, because if $\{u_k := \sum_{n=0}^{\infty} \alpha_{nk} x_n\}_{k \in \mathbb{N}} \subset X_B$ is a sequence tending to zero in the topology of X_B then it also tends to zero in the topology of E. Let us explain why this is so. By (1) we have that $\lambda B = \{\sum_{n=0}^{\infty} \alpha_n x_n : \sum_{n=0}^{\infty} |\alpha_n| \leq \lambda\}$ for all $\lambda > 0$. Due to (c), the series expansion $x = \sum_{n=0}^{\infty} \alpha_n x_n$ of each $x \in X_B$ in terms of some sequence $(\alpha_n) \subset \mathbb{K}$ is unique. So if $\lambda > 0$ and $x \in \lambda B$ we get $\sum_{n=0}^{\infty} |\alpha_n| \leq \lambda$. Therefore $\sum_{n=0}^{\infty} |\alpha_n| \leq p_B(x)$. But $\lim_{k \to \infty} p_B(u_k) = 0$, whence $\lim_{k \to \infty} \sum_{n=0}^{\infty} |\alpha_{nk}| = 0$. On the other hand, X is locally convex, so in order to show that (u_k) tends to cero in X it is enough to prove that $u_k \to 0$ in the Mackey topology of X, that is, that $\lim_{k \to \infty} \sup_{\varphi \in A} |\varphi(u_k)| = 0$ for every equicontinuous subset $A \subset X'$ (see for instance [25, Proposition 7, pages 200–201]). Since (x_n) is bounded (because $x_n \to 0$) in X, it holds that if A is equicontinuous then there is a constant $K \in (0, +\infty)$ such that $\sup_{\varphi \in A, n \geq 0} |\varphi(x_n)| \leq K$. Thus,

$$\sup_{\varphi \in A} |\varphi(u_k)| = \sup_{\varphi \in A} \left| \varphi \left(\sum_{n=0}^{\infty} \alpha_{nk} x_n \right) \right|$$

$$\leq \sum_{n=0}^{\infty} |\alpha_{nk}| \sup_{\substack{\varphi \in A \\ n \geq 0}} |\varphi(x_n)| \leq K \sum_{n=0}^{\infty} |\alpha_{nk}| \to 0 \quad (k \to \infty),$$

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from which one derives that the left hand side tends to zero as $k \to \infty$, as desired. Consequently, $Q : l_1 \to X$ is linear and continuous. In addition, it has dense range (due to $Q(l_1) = X_B$ and (a)) and $TQ = Q\widetilde{T}$ on l_1. Recall that $T \in L(X)$, $\widetilde{T} \in L(l_1)$ and that $\widetilde{T}$ satisfies the HC. This and Lemma 2.2 (with $Y := l_1$) drive us to assert that T satisfies the HC.

On the other hand, the restriction of the operator $\widetilde{T}$ to the subspace $Z := \{\alpha \in l_1 : \alpha_{2n+1} = 0$ for all $n \geq 0\}$ is the identity operator. Consider the linear manifold $Z_0 := Q(Z) \subset X_B \subset X$. Since $TQ = Q\widetilde{T}$ on l_1, the fact that the restriction of $\widetilde{T}$ to Z is the identity yields that T is the identity on Z_0, i.e. $Tx = x$ for all $x \in Z_0$. By continuity, we get $Tx = x$ for all $x \in X_0$, where

$$X_0 := \mathrm{closure}_X(Z_0). \tag{2}$$

From (c) one easily derives that the sequence (x_n) is linearly independent. Therefore $\mathrm{span}(\{x_{2n} : n \geq 0\})$ is infinite-dimensional, hence by (2) and by the inclusions $\mathrm{span}(\{x_{2n} : n \geq 0\}) \subset Z_0 \subset X_0$ we obtain that X_0 is a closed infinite-dimensional subspace of X.

Finally, if $x \in X_0$ then $Tx = x$, so $S_n x = x$ for all $n \in \mathbb{N}$ and all $x \in X_0$, where we have set $S_n := T^n$ ($n \in \mathbb{N}$). Hence, trivially, $\lim_{n\to\infty} S_n x$ exists ($= x$) for every $x \in X_0$. Therefore $(S_{m_k} x)$ converges in X for each $x \in X_0$, where $(m_k) \subset \mathbb{N}$ is a sequence with respect to which T –or, that is the same, the sequence (S_n)– satisfies the HC. An application of Lemma 2.3 with $Y := X$ concludes the proof. $\qquad \square$

References

[1] S.I. ANSARI, Existence of hypercyclic operators on topological vectors spaces, *J. Funct. Anal.* **148** (1997), 384–390.

[2] T. BERMÚDEZ, A. BONILLA and A. PERIS, On hypercyclicity and supercyclicity criteria *Bull. Austral. Math. Soc.* **70** (2004), 45–54.

[3] L. BERNAL-GONZÁLEZ, On hypercyclic operators on Banach spaces, *Proc. Amer. Math. Soc.* **127** (1999), 1003–1010.

[4] L. BERNAL-GONZÁLEZ, Universal images of universal elements, *Studia Math.* **138** (2000), 241–250.

[5] L. BERNAL-GONZÁLEZ and K.-G. GROSSE-ERDMANN, The Hypercyclicity Criterion for sequences of operators, *Studia Math.* **157** (2003), 17–32.

[6] L. BERNAL-GONZÁLEZ and A. MONTES-RODRÍGUEZ, Non-finite dimensional closed of universal functions for composition operators, *J. Approx. Theory* **82** (1995), 375–391.

[7] J. BÈS, *Three problems on hypercyclic operators*, Thesis, Kent State University, 1998.

[8] J. BÈS, Invariant manifolds of hypercyclic vectors for the real scalar case, *Proc. Amer. Math. Soc.* **127** (1999), 1801–1804.

[9] J.P. BÈS and K. CHAN, Denseness of hypercyclic operators on a Fréchet space, *Houston J. Math.* **29** (2003), 195–206.

[10] J.P. BÈS and K. CHAN, Approximation by chaotic operators and by conjugate classes, *J. Math. Anal. Appl.* **284** (2003), 206–212.

[11] J.P. BÈS and A. PERIS, Hereditarily hypercyclic operators, *J. Funct. Anal.* **167** (1999), 94–113.

[12] J. BONET, F. MARTÍNEZ-GIMÉNEZ and A. PERIS, Universal and chaotic multipliers on spaces of operators, *J. Math. Anal. Appl.* **297** (2004), 599–611.

[13] J. BONET and A. PERIS, Hypercyclic operators on non-normable Fréchet spaces, *J. Funct. Anal.* **159** (1998), 587-596.

[14] P.S. BOURDON, Invariant manifolds of hypercyclic operators, *Proc. Amer. Math. Soc.* **118** (1993), 845–847.

[15] K.C. CHAN, Hypercyclicity of the operator algebra for a separable Hilbert space *J. Operator Theory* **42** (1999), 231–244.

[16] K.C. CHAN, The density of hypercyclic operators on a Hilbert space, *J. Operator Theory* **47** (2002), 131–143.

[17] K.C. CHAN and R.D. TAYLOR, Hypercyclic subspaces of a Banach space, *Integr. Equ. Oper. Theory* **41** (2001), 381–388.

[18] G. GODEFROY and J. H. SHAPIRO, Operators with dense, invariant, cyclic vector manifolds, *J. Funct. Anal.* **98** (1991), 229–269.

[19] M. GONZÁLEZ, F. LEÓN-SAAVEDRA and A. MONTES-RODRÍGUEZ, Semi-Fredholm Theory: Hypercyclic and Supercyclic subspaces, *Proc. London Math. Soc.* **81** (2000), 169–189.

[20] S. GRIVAUX, Hypercyclic operators with an infinite dimensional closed subspace of periodic points, *Rev. Mat. Complutense* **16** (2003), 383–390.

[21] K.G. GROSSE-ERDMANN, Universal families and hypercyclic operators, *Bull. Amer. Math. Soc.* (N.S.) **36** (1999), 345–381.

[22] K.G. GROSSE-ERDMANN, Recent developments in hypercyclicity, *Rev. R. Acad. Cien. Serie A. Mat.* **97** (2003), 273–286.

[23] D.W. HADWIN, E.A. NORDGREN, H. RADJAVI and P. ROSENTHAL, Most similarity orbits are strongly dense, *Proc. Amer. Math. Soc.* **76** (1979), 250–252.

[24] D.A. HERRERO, Limits of hypercyclic and supercyclic operators, *J. Funct. Anal.* **99** (1991), 179–190.

[25] J. HORVÁTH, *Topological vector spaces and distributions*, Vol. 1, Addison-Wesley, Reading, 1966.

[26] F. LEÓN-SAAVEDRA and A. MONTES-RODRÍGUEZ, Linear structure of hypercyclic vectors, *J. Funct. Anal.* **148** (1997), 524–545.

[27] F. LEÓN-SAAVEDRA and A. MONTES-RODRÍGUEZ, Spectral theory and hypercyclic subspaces, *Trans. Amer. Math. Soc.* **353** (2000), 247–267.

[28] F. MARTÍNEZ-GIMÉNEZ and A. PERIS, Chaos for backward shift operators, *Int. J. Bifurcation and Chaos* **12** (2002), 1703–1715.

[29] A. MONTES-RODRÍGUEZ, Banach spaces of hypercyclic vectors, *Michigan Math. J.* **43** (1996), 419–436.

[30] A. MONTES-RODRÍGUEZ and M.C. ROMERO-MORENO, Supercyclicity in the operator algebra, *Studia Math.* **150** (2002), 201–213.

[31] A. MONTES-RODRÍGUEZ and H.N. SALAS, Supercyclic subspaces, *Bull. London Math. Soc.* **35** (2003), 721–737.

[32] H. PETERSSON, Hypercyclic subspaces for Fréchet space operators, *preprint* 2004.

[33] S. ROLEWICZ, On orbits of elements, *Studia Math.* **32** (1969), 17–22.

[34] H. SALAS, Hypercyclic weighted shifts, *Trans. Amer. Math. Soc.* **347** (1995), 993–1004.

[35] J.H. SHAPIRO, *Composition operators and classical function theory*, Springer-Verlag, New York, 1993.

[36] J. WENGENROTH, Hypercyclic operators on non-locally convex spaces, *Proc. Amer. Math. Soc.* **131** (2003), 1759–1761.

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