## Engineering Applications of Computational Fluid Mechanics

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# Investigation of submerged structures' flexibility on sloshing frequency using a boundary element method and finite element analysis 

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#### Abstract

In this study, the boundary element method-finite element method (BEM-FEM) model is employed to investigate the sloshing and flexibility terms of elastic submerged structures on the behavior of a coupled domain. The methods are finite element and boundary elements which are utilized for structural dynamic and sloshing modeling, respectively. The applied models are used to assess dynamic parameters of a fluid-structure system. Based on the proposed model, a code is developed which can be applied to an arbitrary two- and three- dimensional tank with an arbitrarily shaped elastic submerged structure. Results are validated based on the existing methods represented in the literature and it is concluded that the absolute relative deviation is lower than $2 \%$. Finally, the interactive influences of submerged components which are more meaningful are investigated.


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## KEYWORDS

fluid-structure interaction; boundary element method (BEM); reduced order modeling; submerged elastic structure; finite element method (FEM)

## 1. Introduction

Free board motion of liquid formed bilateral sloshing in a container. There are several challenging engineering problems which are related to sloshing behavior in liquid reservoir tanks, pools, or wings. The source of disturbance in the container of liquid is actually hydrodynamic pressure and its distribution on the wall of tankers. It is possible for the free board of a liquid to move in various ways including planar, rotary, uniplanar, and chaotic with its corresponding natural frequencies. The natural frequencies are dependent on several parameters such as gravity, geometry of the tanker, and axial acceleration.

Several studies have concentrated on the dynamic behavior of sloshing in liquid tanks and the related dynamic behavior of the fluid and the other subjects, like the management of fluid in micro and zero gravity conditions and controlling or optimizing sloshing elevation (Abramson \& Silverman, 1966; Ibrahim, 2005). In limber structures, the flexibility terms of structures are influential on sloshing. For instance, Tan, Xiong, Xing, and Toyoda (2006) conducted a study on the characteristics of a tank which was filled partially with liquid by utilizing a substructure approach in proceedings of hydro elasticity and investigated the coupled sensitivity.

In another study, Gavrilyuk, Hermann, Lukovsky, Solodun, and Timokha (2008) applied a model on a conical tank to evaluate fluid-structure interaction and the effects of sloshing. Kim, Koh, \& Kwahk (1996) utilized the Rayleigh-Ritz approach and represented an analytical solution for the rectangular reservoir. In their research, just a pair of walls was considered as flexible. In another study (Bauer \& Eidel, 2004), research was carried out on a two-dimensional rectangular container. The container was partially filled and had a plate which was elastic. They concluded that liquid frequency increases with the length of the covering beam.

A conventional technique for decreasing the disturbance effects of sloshing is an additional sub-structure adoption, which is known as a baffle or submerged structure within the tank. It can sharply affect the dynamic response of the systems in the fluid-structure domain. There is little research focused on sloshing frequencies in a tank filled with liquid which contains single or multiple submerged structures or baffles. Noorian, FirouzAbadi, and Haddadpour (2012) studied the sloshing and structural vibration interaction behavior in baffled tanks. Their study represents a model for coupled sloshing and structural problems. Armenio and La Rocca (1996)

[^0]obtained results numerically and experimentally for a rectangular tank which had a vertical bottom-mounted internal baffle. Gavrilyuk, Lukovsky, Trotsenko, and Timokha (2006), based on the analytical method, surveyed the sloshing behavior in a vertical cylindrical tank with a thin rigid-ring horizontal baffle. Isaacson and Premasiri (2001) applied an energy method and predicted hydrodynamic damping which was induced by vertical and horizontal baffles and oscillating horizontally. Evans (1987) evaluated the influences of a thin vertical baffle from either above or below on the rectangular tank of water using Eigen-function expansion approaches. Choun and Yun (1996) and Choun and Yun (1999) performed a study of the linearized behavior of free surfaces in a tank, which was rectangular, with rigid submerged ones. Watson and Evans (1991) exploit Eigen-function and utilized the Galerkin expansion method to evaluate the dynamic response of sloshing in a rectangular tank with submerged structures. Their problem was considered as rigid walls with a submerged structure that was centrally installed on the floor. Mitra and Sinhamahapatra (2007) through finite-element model which derived base on the pressure formulation, assessed the influences of a bottom-mounted unit with rectangular geometry on the sloshing dynamics in a rigid container. The influence of having one end fixed of a submerged structure with a cylindrical shape within fluid domain was investigated by Askari \& Daneshmand (2010). In their study, structure and fluid modeled as a shell element and potential theory, respectively. Various effects of other parameters on the dynamic characteristics of the system were evaluated and analyzed; in addition, a reliable and fast solution was represented in the study. Obtained results indicated that under a typical earthquake, the sloshing amplitude and hydrodynamic pressure had a reducing trend. Esmailzadeh, Lakis, Thomas, \& Marcouiller, (2008) represented a model which was three-dimensional to model structures which were curved and contained and/or submerged in fluid. Coupling of a cylinder-shaped container, which was partially fluid-filled, with an internal body, is calculated by Askari, Daneshmand, and Amabili (2011). In this study, cylindrical body affection, which was considered rigid, on bulging frequencies and its corresponding modes were evaluated. Hasheminejad and Aghabeigi (2009) adopt the linear potential theory to study the free lateral sloshing inside an elliptical vessel which was half-full horizontal.

Several numerical methods are utilized in order to investigate sloshing dynamics. Rebouillat \& Liksonov (2010) reviewed numerical methods in the sloshing problem and estimated the amplitude of the sloshing wave, pressure exerted on the walls, frequency, and the influence of sloshing on the container stability. In
addition, liquid sloshing in a baffled and un-baffled container which was partially filled with liquid was evaluated in their study. Biswal and Bhattacharyya (2010) and Biswal, Bhattacharyya, and Sinha $(2004,2006)$ applied the finite element method (FEM) in order to assess the effect of some type of baffles on the dynamic responses of structures which were coupled with liquid sloshing in tanks with a cylindrical shape.

In this paper, the use of the boundary element method (BEM) as an efficient tool for the indication of the sloshing effect on engineering problems is completely revealed. Using this approach, boundaries instead of a whole domain are discretized so the computational time and cost are reduced significantly in comparison with other conventional methods like finite element. Nakayama and Washizu (1981) utilized BEM for analyzing a two-dimensional sloshing problem. Moreover, Hwang, Kim, Seol, Lee, and Chon (1992) represented a solution for the dynamic acting of sloshing in a spherical tank which was partially filled with liquid. The study was conducted for the conditions in which excitation was induced by a panel approach on the basis of source distribution along the tank boundaries. Using the boundary element method, a three-dimensional rigid baffled tank was investigated by Firouz-Abadi, Haddadpour, Noorian, and Ghasemi (2008). Christensen and Brunty (1997) also adopted this method to analyze the hydrodynamics of slosh. Dutta and Laha (2000) employed a low-order BEM method to characterize liquid sloshing elevations in three-dimensional tanks. In addition, using the BEM technique with liquid viscosity suppositions the sloshing disturbances in a two-dimensional moving container modeled by Zang, Xue, and Kurita (2000). Donescu \& Virgin (2001) illustrated the application of an implicit BEM for nonlinear free surfaces with floating bodies. The impact of some baffle configuration on linearized free surface dynamics by the variational BEM approach was evaluated by Gedikli and Ergüven (1999). Internal structure influences on liquid two-dimensional tanks using multiple domain BEM technique was studied by Zhu and Saito (2000). Chen, Hwang, and Ko (2007) performed some investigation on the liquid tanks, with cylindrical and cubic shapes, under harmonic and seismic excitation using BEM.

In order to decrease computational cost and time, it is useful to apply the natural frequency and modes of a dynamic system. Utilizing this approach, called reduced order modeling (ROM), in various problems showed its ability in analyzing fluid dynamics. Several researchers have employed this approach in order to analyze linear and nonlinear sloshing beside the flexibility considerations of tank compartments (Faltinsen \& Timokha, 2002; Faltinsen, Rognebakke, Lukovsky, \& Timokha, 2000;

Firouz-Abadi, Haddadpour, \& Ghasemi, 2009; Ohayon, 2004). In recent years computational fluid dynamic (CFD) can be considered like the previously mentioned method to model fluid (Ramezanizadeh, Alhuyi Nazari, Ahmadi, \& Chau, 2019). Volume of Fluid (VOF) multiphase modeling along with CFD simulation are also another tools for evaluation of liquid sloshing in reservoir tanks (Sanapala, Velusamy, \& Patnaik, 2016; Singal, Bajaj, Awalgaonkar, \& Tibdewal, 2014).

In the current paper, the investigation of the role of elastic submerged structures on sloshing behavior especially in elastic three-dimensional forms is presented. A reduced order BEM-FEM model is applied (Noorian et al., 2012). Using the BEM, the equation of the free surface is presented. Moreover, the finite-element method is employed to govern structural equations. Since the represented method is a synthesis of BEM and FEM, it has the advantages of both approaches. By using BEM, computational time and cost reduces; in addition, the robustness of finite-element method is another advantage of this approach. This method utilizes the modal analysis to derive reduced equations of the coupled systems. Moreover, the represented model is compared with the literature and is applied to investigate the coupling effects between the flexibility of submerged structures and free surface oscillation in three dimensional forms.

## 2. Governing equations

Because of the complexity and large order of coupled equations for structure-liquid systems, the reduced order approach is utilized. In the following section, the equations are derived for both domains. Afterwards, by using the BEM-FEM method, a solution is represented.

### 2.1. Flow field governing equation

By assuming inviscid, incompressible and irrotational flow, the internal flow field can be derived from the Laplace equation as (Ibrahim, 2005):

$$
\begin{equation*}
\nabla^{2} \phi=0 \tag{1}
\end{equation*}
$$

Boundary conditions on the interface between liquid and structure and on the free surface which must be considered for the solution of Equation (1) is represented in next section.

### 2.2. Boundary conditions

Here, the Bernoulli equation is considered for modeling free surface boundary conditions as (Ibrahim, 2005):

$$
\begin{equation*}
\mathrm{g} \delta+\frac{\partial \phi}{\partial \mathbf{n}}=0 \tag{2}
\end{equation*}
$$

In the above equation, $g$ and $\delta$ represent gravity acceleration and free surface elevation, respectively. By considering small motion and excitation amplitude, boundary condition is linearized for the free surface of the tank and represented as:

$$
\begin{equation*}
\left.\frac{\partial \phi}{\partial \mathbf{n}}\right|_{\text {free surface }}=\dot{\delta}_{\text {free surface }} \tag{3}
\end{equation*}
$$

In the above equation $\frac{\partial}{\partial \mathbf{n}}$ is the outward normal derivative operator. By applying the Bernoulli equation for the free surface, Equation (2) can be rewritten as:

$$
\begin{equation*}
\left.\frac{\partial \phi}{\partial \mathbf{n}}\right|_{\text {free surface }}=-\frac{1}{\mathrm{~g}} \ddot{\phi} \tag{4}
\end{equation*}
$$

In addition, by assuming the impermeability condition of the components and the tank wall, the boundary condition for the structure-liquid interface is obtained as:

$$
\begin{equation*}
\left.\frac{\partial \phi}{\partial \mathbf{n}}\right|_{\text {wall }}=\dot{\delta}_{\text {structure }} \tag{5}
\end{equation*}
$$

In the above equation, $\delta_{\text {structure }}$ shows the structural displacement of the wall of the tank and the submerged component.

### 2.3. Developing a boundary element model

Based on reference (Ibrahim, 2005), the solution of the Laplace equation using a unit source which generates a velocity potential field specifically for a two-dimensional region can be expressed as:

$$
\begin{equation*}
\stackrel{*}{\phi}=\ln (1 / r) / 2 \pi \tag{6}
\end{equation*}
$$

where $r$ refers to the source point distances. Using Green's second relationship, the potential field formed the equation as follows:

$$
\begin{equation*}
C_{i} \phi_{i}=\int(\stackrel{*}{\phi} q-\stackrel{*}{q} \phi) d s \tag{7}
\end{equation*}
$$

In which $q=\frac{\partial \phi}{\partial \mathbf{n}}, \stackrel{*}{q}=\frac{\partial \stackrel{*}{\phi}}{\partial \mathbf{n}}, C_{i}$ is a coefficient that is equal to 0.5 for a flat boundary and 1 for internal domain of the flow. Discretizing the boundary of the field by the isoperimetric elements, the matrix form of Equation (7), can be represented as:

$$
\begin{equation*}
\mathbf{A} \Phi-\mathbf{B Q}=\mathbf{0} \tag{8}
\end{equation*}
$$

In this equation, $\mathbf{A}$ and $\mathbf{B}$ refer to the influence matrices while $\boldsymbol{\Phi}$ and $\mathbf{Q}$ containing the vectors form the potential and its derivative value at all nodes in the boundary of the domain, respectively.

Decomposing the matrices into two blocks results in a block related to the free surface and a block which is
related to the walls and submerged components. Using the mentioned consideration, Equation (8) can be rewritten as:

$$
\left[\begin{array}{ll}
\mathbf{A}_{11} & \mathbf{A}_{12}  \tag{9}\\
\mathbf{A}_{21} & \mathbf{A}_{22}
\end{array}\right]\left[\begin{array}{l}
\boldsymbol{\Phi}_{\mathrm{fs}} \\
\boldsymbol{\Phi}_{\mathrm{w}}
\end{array}\right]=\left[\begin{array}{ll}
\mathbf{B}_{11} & \mathbf{B}_{12} \\
\mathbf{B}_{21} & \mathbf{B}_{22}
\end{array}\right]\left[\begin{array}{l}
\mathbf{Q}_{\mathrm{fs}} \\
\mathbf{Q}_{\mathrm{w}}
\end{array}\right]
$$

### 2.4. Fluid structure interaction based on reduced order modeling

By using natural mode shapes $\overline{\mathbf{W}}_{\mathrm{i}}$, the displacement fields of the structures are expanded as:

$$
\begin{equation*}
\delta=\sum \overline{\mathbf{W}}_{\mathrm{i}} \eta_{\mathrm{i}} \tag{10}
\end{equation*}
$$

Substituting Equation (10) into Equation (5), Equation (9) can be written as:

$$
\begin{align*}
& \mathbf{A}_{11} \boldsymbol{\Phi}_{\mathrm{fs}}+\mathbf{A}_{12} \boldsymbol{\Phi}_{\mathrm{w}}=\mathbf{B}_{11} \mathbf{Q}_{\mathrm{fs}}-\mathbf{B}_{12} \sum \overline{\mathbf{W}}_{\mathrm{i}} \dot{\eta}_{\mathrm{i}}  \tag{11}\\
& \mathbf{A}_{21} \boldsymbol{\Phi}_{\mathrm{fs}}+\mathbf{A}_{22} \boldsymbol{\Phi}_{\mathrm{w}}=\mathbf{B}_{21} \mathbf{Q}_{\mathrm{fs}}-\mathbf{B}_{22} \sum \overline{\mathbf{W}}_{\mathrm{i}} \dot{\eta}_{\mathrm{i}} \tag{12}
\end{align*}
$$

Thus $\boldsymbol{\Phi}_{\mathrm{w}}$ equals:

$$
\begin{equation*}
\boldsymbol{\Phi}_{\mathrm{w}}=\mathbf{A}_{22}^{-\mathbf{1}}\left(-\mathbf{A}_{21} \boldsymbol{\Phi}_{\mathrm{fs}}+\mathbf{B}_{21} \mathbf{Q}_{\mathrm{fs}}-\mathbf{B}_{22} \sum \overline{\mathbf{W}}_{\mathrm{i}} \dot{\boldsymbol{\eta}}_{\mathrm{i}}\right) \tag{13}
\end{equation*}
$$

Substituting Equation (12) into Equation (13) results in:

$$
\begin{equation*}
\mathbf{Q}_{\mathrm{fs}}=\mathbf{D} \boldsymbol{\Phi}_{\mathrm{fs}}+\mathbf{E} \sum \overline{\mathbf{W}}_{\mathrm{i}} \dot{\eta}_{\mathrm{i}} \tag{14}
\end{equation*}
$$

where $\mathbf{D}$ and $\mathbf{E}$ are introduced as:

$$
\begin{align*}
& \mathbf{D}=\left(\mathbf{B}_{11}-\mathbf{A}_{12} \mathbf{A}_{22}^{-1} \mathbf{B}_{21}\right)^{-1}\left(\mathbf{A}_{11}-\mathbf{A}_{12} \mathbf{A}_{22}^{-1} \mathbf{A}_{21}\right)  \tag{15}\\
& \mathbf{E}=\left(\mathbf{B}_{11}-\mathbf{A}_{12} \mathbf{A}_{22}^{-1} \mathbf{B}_{21}\right)^{-1}\left(\mathbf{B}_{12}-\mathbf{A}_{12} \mathbf{A}_{22}^{-1} \mathbf{B}_{22}\right) \tag{16}
\end{align*}
$$

Hence, the free surface boundary equation is extracted as:

$$
\begin{equation*}
\ddot{\boldsymbol{\Phi}}_{\mathrm{fs}}+\mathrm{gD} \boldsymbol{\Phi}_{\mathrm{fs}}+\mathrm{gE} \sum \overline{\mathbf{W}}_{\mathrm{i}} \dot{\eta}_{\mathrm{i}}=0 \tag{17}
\end{equation*}
$$

Solution to Equation (17) is considered as $\boldsymbol{\Phi}=\overline{\boldsymbol{\Phi}} e^{i \omega t}$. Using the properties of right or left eigenvectors and also expanding $\boldsymbol{\Phi}_{\mathrm{fs}}$ into the summation form $\boldsymbol{\Phi}_{\mathrm{fs}}=\sum \overline{\phi_{\mathrm{i}}} \boldsymbol{\xi}_{\mathrm{i}}$ Equation (17) turns into:

$$
\begin{equation*}
\overline{\boldsymbol{\Phi}} \ddot{\xi}+\mathrm{g} \mathbf{D} \overline{\boldsymbol{\Phi}} \xi+\mathrm{gE} \overline{\mathbf{W}} \dot{\eta}=\mathbf{0} \tag{18}
\end{equation*}
$$

Pre-multiplying both sides of Equation (20) by the left eigenvectors yields the following equation:

$$
\begin{equation*}
\mathbf{S} \ddot{\boldsymbol{\xi}}_{\mathrm{i}}+\mathrm{g} \boldsymbol{\Xi} \xi_{\mathrm{i}}+\mathrm{g} \Gamma \sum \overline{\mathbf{W}}_{i} \dot{\boldsymbol{\eta}}_{i}=0 \tag{19}
\end{equation*}
$$

where: $S=\overline{\boldsymbol{\Psi} \boldsymbol{\Phi}},(\overline{\boldsymbol{\Psi}}$ and $\overline{\boldsymbol{\Phi}}$ are right and left eigenvectors), $\Xi=\overline{\mathbf{\Psi}} \mathbf{D} \overline{\boldsymbol{\Phi}}, \Gamma=\overline{\mathbf{\Psi}} \mathbf{E}$. By modeling the elastic tank and submerged components base on finite-element, and
ignoring the structural damping effect, the structural dynamics model can be written as:

$$
\begin{equation*}
\mathbf{M}_{s} \ddot{\mathbf{W}}_{\mathbf{s}}+\mathbf{K}_{s} \mathbf{W}_{\mathbf{s}}=\mathbf{F} \tag{20}
\end{equation*}
$$

In the above equation $\mathbf{F}$ is the external load vector. In addition, $\mathbf{M}_{s}$ and $\mathbf{K}_{s}$ refer to the structural mass and stiffness matrices, respectively. Also $\mathbf{W}_{\mathbf{s}}$ is a nodal structural vector. By applying the model represented in Equation (20), Equation (21) is obtained as:

$$
\begin{equation*}
\ddot{\eta}+\omega^{2} \eta=\bar{F} \tag{21}
\end{equation*}
$$

where $\omega$ shows the natural frequencies of the structure; $\bar{F}$ can be rewritten as $\int_{0}^{l} \overline{\mathbf{W}}$.Pds ( $\mathbf{P}$ is the dynamic pressure of fluid due to the sloshing stemming from the unsteady Bernoulli equation and equals $-\rho_{f} \dot{\boldsymbol{\Phi}}_{\mathrm{w}}$, where $\rho_{f}$ is the density of the fluid. Therefore, $\overline{\mathbf{F}}$ can be presented in matrix form as:

$$
\begin{equation*}
\overline{\mathbf{F}}=\rho_{f} \overline{\mathbf{H}} \dot{\boldsymbol{\Phi}}_{\mathrm{w}} \tag{22}
\end{equation*}
$$

where $\overline{\mathbf{H}}$ is produced by evaluating $\mathbf{P}$ by linear elements produced with the modal matrix, and $\dot{\boldsymbol{\Phi}}_{\mathrm{w}}$ is obtained from the combination of Equation (12) and Equation (13) as follows:

$$
\begin{align*}
\dot{\Phi}_{\mathrm{w}}= & \mathbf{A}_{22}{ }^{-1}\left(\left(\mathbf{B}_{21} \mathbf{D}-\mathbf{A}_{21}\right) \dot{\Phi}_{\mathrm{fs}}\right. \\
& \left.+\left(\mathbf{B}_{21} \mathbf{E}-\mathbf{B}_{22}\right) \sum \overline{\mathbf{W}}_{\mathbf{i}}(\mathbf{x}) \ddot{\eta}_{\mathbf{i}}(\mathbf{t})\right) \tag{23}
\end{align*}
$$

Hence, the composition of the reduced orders governing equations for both fluid and structure fields in the matrix form gives:

$$
\left[\begin{array}{cc}
\mathbf{S} & \mathbf{0}  \tag{24}\\
\mathbf{0} & \mathbf{G}_{\mathbf{s}}
\end{array}\right]\left[\begin{array}{l}
\ddot{\boldsymbol{\xi}} \\
\ddot{\eta}
\end{array}\right]+\left[\begin{array}{cc}
\mathbf{0} & \mathrm{g} \cdot \Gamma \\
\mathbf{C}_{\mathbf{f}} & \mathbf{0}
\end{array}\right]\left[\begin{array}{l}
\dot{\boldsymbol{\xi}} \\
\dot{\eta}
\end{array}\right]+\left[\begin{array}{cc}
\mathrm{g} \cdot \Xi & \mathbf{0} \\
\mathbf{0} & \Omega
\end{array}\right]\left[\begin{array}{l}
\boldsymbol{\xi} \\
\eta
\end{array}\right]=0
$$

where

$$
\begin{aligned}
\mathbf{G}_{s} & =\left(\overline{\mathbf{H}} \mathbf{A}_{22}^{-1}\left(\mathbf{B}_{21} \mathbf{E}-\mathbf{B}_{22}\right) \overline{\mathbf{W}}+\mathbf{I}\right), \mathbf{C}_{\mathbf{f}} \\
& =\overline{\mathbf{H}} \mathbf{A}_{22}{ }^{-1}\left(\mathbf{B}_{21} \mathbf{D}-\mathbf{A}_{21}\right) \Omega=\omega^{2}
\end{aligned}
$$

## 3. Numerical results

In addition to experimental and analytical approaches, numerical methods are widely used to solve engineering problems (Alizadeh et al., 2018; Ghalandari, Mirzadeh Koohshahi, Mohamadian, Shamshirband, \& Chau, 2019; Ramezanizadeh et al., 2019). In this section, some test cases which are presented in (Mitra \& Sinhamahapatra, 2007) are compared with the present model, ANSYS,


Figure 1. Problem configuration and nomenclature.
and existing models in the literature beside the study of two- and three-dimensional frequency comparisons. Afterwards, the influence of the elasticity of a rectangular submerged structure on the sloshing frequencies of a liquid-filled rigid container is investigated. Furthermore, a new test case involving an LNG tank with a flexible bottom-mounted rectangular submerged structure is presented.

### 3.1. Verification example

The first test case, as shown in Mitra and Sinhamahapatra (2007) is a rigid rectangular tank with a rigid rectangular block mounted at the bottom (Figure 1). Computations are carried out with a 15 m width block located at the center of the tank floor of a 30 m width with 13 m liquid depth for several block heights (h). The comparison between ANSYS, the evaluated boundary element model, and results presented in Mitra and Sinhamahapa$\operatorname{tra}$ (2007) in different $\mathrm{h} / \mathrm{d}$ ratios for rigid tank is listed in Table 1. Results indicate appropriate agreement between the present model, ANSYS, and the literature for all the test cases Figure 2.

Also, comparison of two- and three-dimensional sloshing frequencies shows good agreement between ANSYS FEM results and frequencies which are estimated from the presented model in rigid structures (Table 2, Figure 3).

Table 1. Comparison between sloshing frequencies of the present model, ANSYS, and ref. Mitra and Sinhamahapatra (2007).

|  |  |  |  | Mitra and Sin- <br> hamahapatra |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| UINT(RAD/Sec) | $\mathrm{h} / \mathrm{d}$ | Present | Yun (1996) <br> (2007) | ANSYS |  |
|  | 0.2 |  |  |  |  |
| MODE1 |  | 0.9127 | 0.897 | 0.898 | 0.9094 |
| MODE2 |  | 1.428 | 1.415 | 1.415 | 1.4612 |
| MODE3 |  | 1.7647 | 1.744 | 1.746 | 1.7644 |
|  | 0.4 |  |  |  | 0.842 |
| MODE1 |  | 0.8396 | 0.849 | 1.406 | 1.4306 |
| MODE2 |  | 1.3964 | 1.404 | 1.74 | 1.7588 |
| MODE3 |  | 1.7564 | 1.742 |  |  |
|  | 0.6 |  |  | 0.731 | 0.731 |
| MODE1 |  | 0.7251 | 1.3109 | 1.343 | 1.347 |
| MODE2 |  | 1.7215 | 1.733 | 1.734 | 1.3437 |
| MODE3 | 0.8 |  |  |  | 1.73 |
|  |  | 0.5333 | 0.55 | 0.552 | 0.5383 |
| MODE1 |  | 1.0691 | 1.19 | 1.192 | 1.1044 |
| MODE2 | 1.5522 | 1.73 | 1.732 | 1.5809 |  |
| MODE3 |  |  |  |  |  |

Table 2. Two-dimensional (2D) and three-dimenional (3D) frequencies of FEM and BEM ( $\mathrm{rad} / \mathrm{sec}$ ).

| ANSYS |  | BEM |  | Frequencies |
| :--- | :--- | :--- | :--- | :--- |
| 3D | 2D | 3D | 2D |  |
| 3.7655 | 3.6985 | 3.6575 | 3.6519 | Semi-circular section |



Figure 3. Three-dimensional mesh of container.

### 3.2. Example 2

As shown in Figure 4, a second test case is the same tank as the previous one, but with a submerged elastic rectangular structure and $\mathrm{h} / \mathrm{d}$ ratio 0.8 . The three dimensional elastic submerged structure is mounted in the middle of the tank. The elasticity of the submerged structure is


Figure 2. Three-dimensional lowest first three modes of sloshing.


Figure 4. Problem configuration and dimensions.

Table 3. Natural frequencies of Structural model (rad/s).

| 1 | 1.8358 |
| :--- | ---: |
| 2 | 4.9269 |
| 3 | 11.8579 |

modeled with the solid elements. The mode shapes of structural vibration are extracted by utilizing solid 187 in ANSYS software; natural frequencies of the structure are listed in Table 3. An efficient ROM which is created by third first one of sloshing and structural mode shape has been employed to evaluate the FSI natural frequencies. The dimensions and material properties of the tank and submerged elastic are presented below:

$$
\begin{aligned}
E & =200 \mathrm{GPa}, v=0.3, t=5 \mathrm{~cm} \\
\rho_{s} & =7800 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{f}=1000 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

where $E$ is the modulus of elasticity; $v$ is the Poisson ratio; $t$ is the thickness of the submerged structure and $\rho_{s}$ and $\rho_{f}$ are the structure and fluid densities, respectively.

The flexibility parameter of the submerged structure is dimensionless and can be defined as:

$$
\begin{equation*}
f=\frac{\rho_{f}\left(1-v^{2}\right) W^{2}}{E \omega_{0}^{2}}\left(\frac{W}{t}\right)^{3} \tag{25}
\end{equation*}
$$

In Equation (25), $W$ and $\omega_{0}$ are width and the first sloshing frequency of the submerged structure, respectively. The variation of sloshing and structural vibration frequencies for various flexibility parameters of the submerged structure is shown in Figure 6. Based on the obtained results, the sloshing frequencies are sensitive to the flexibility parameter. For low flexibility parameters, frequencies of sloshing and structure are fully decoupled from each other. However, in the higher values of flexibility parameters, slosh and structural dynamics get coupled. This coupling is influenced by both structural and sloshing frequencies, meaning that if the two system frequencies are close, the coupling occurs earlier.

Frequencies are presented in the non-dimension form. $\omega_{c}$, and $\sigma$ refer to the imaginary and real part of coupled system frequencies, respectively Figures 4-6.

In Figure 3, from 1e3 to 1 e 6 flexibility interval, instability can be seen. This type of dynamic instability may only happen in high flexibility parameters and result in positive damping values in the coupled system eigenvalues (Figure 7(a) and 7(b)) where $\sigma$ is the real part of eigenvalue system. The first three sloshing frequencies in rigid and flexible structures at different heights of the submerged structure is shown in Figure 8. In Figure 8 n is number of mode.

As shown in Figure 8 and Figure 9 the structural effects on the coupled domain in high value flexibility are more tangible.


Mode 3


Figure 5. The first third mode of the submerged structure.


Figure 6. Variation of the lowest sloshing and structural dynamic frequencies for a tank with $\mathrm{h} / \mathrm{d}=0.8 \mathrm{vs}$. flexibility parameter.

### 3.3. Example 3

In the third case, a LNG carrier with a submerged elastic bottom-mounted structure is considered. The specifications of the investigated tank are as below:

$$
\begin{aligned}
L & =13.8 m, h=2.55 m, W=11 m \\
W_{w a} & =W_{w b}=1.4 m, h_{w a}=h_{w b}=1.4 m \\
W_{u b} & =W_{u a}=0
\end{aligned}
$$



Figure 8. Variation of sloshing frequencies versus rigid and flexible submerged structures.

$$
\begin{aligned}
E & =200 \mathrm{GPa}, v=0.3, t=2 \mathrm{~cm}, \\
\rho_{s} & =7800 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{f}=1000 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

The influence of the flexibility parameter for $\mathrm{h} / \mathrm{d}=0.66$ on the FSI model is shown in Figure 11. Based on obtained results, for high values of flexibility parameters, the coupling effects appeared. In addition, sloshing and structural dynamics are completely coupled and the system dynamics should be expressed using coupled mode shapes (Figures 10-11).


Figure 7. (a) Zoomed region in imaginary domain; (b) Zoomed region in real domain.


Figure 9. Width effects on coupled frequencies.


Figure 10. LNG configuration and nomenclature.


Figure 11. Variation of the lowest frequencies for a tank with $h / d=0.66$ vs. flexibility parameter.

## 4. Conclusion

In this study, a general BEM-FEM model, along with an efficient reduced order method for the investigation of sloshing in tanks with flexible submerged structures, is utilized. The proposed model was compared with ANSYS and the literature, which indicates its good agreement with previous studies. The presented model was also applied to a 3D system in such a way that the results also show good agreement to ANSYS. Also, the influence of the flexibility effect of the submerged structure on the frequencies of system and damping was investigated. In addition, frequency-flexibility curves were represented in order to get better insight into the conditions during which the coupling of sloshing and structural dynamics happens. Based on obtained results, utilizing flexible submerged structures with various thicknesses, height, and width can lead to sharp changes in the dynamics of the coupled system. So, because of the different natures of failure, future study could be focused on the nonlinearity effect of submerged structures and its effect on coupled systems behavior with BEM-FEM models.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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