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# Optimal Auditing under Learning and Sympathy: A Game Theory Approach

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**Abstract** In this paper, a game-theoretic framework is proposed to address the problem of fraud deterrence and detection taking into consideration fundamental principles and threats that may affect the performance of an audit process. Based on evidence from social psychology and on the outcomes of statistical sampling theory on audit quality, a mathematical model based on game theory is proposed providing reasonable assurance that the financial statements as a whole are free from material misstatements. It is proven that the auditing/fraud detection game between two new engagement parties has a unique mixed strategy equilibrium, between an experienced auditor and a client has a unique pure strategy equilibrium, whereas in the long-run the game converges to a pure strategy equilibrium that is governed by sympathy. Furthermore, a closed form solution to the optimal auditor's replacement problem is extracted. The validity of the proposed scheme is tested on empirical data and modeling results comply with the International Federation Accountants Code of Ethics that requires the key audit partner to be rotated after a predefined period.

**Keywords** Auditing · Decision analysis · Finance · Behavioural OR · Game theory · Quality control

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## 1 Introduction

The main objective of an audit of financial statements is to enable the auditor to express an opinion about whether the financial statements are prepared, in all material respects, in accordance with an applicable financial reporting framework (ISA 200 2009). To achieve this, audits should be planned and performed with an attitude of professional scepticism recognizing that circumstances may exist causing financial statements to be materially misstated ISA 200 (2009). Professional scepticism is a concept of crucial importance in audit practice and an intrinsic part of the audit process (Anugerah et al., 2011). However, a fundamental prerequisite for its success is the professional account to be compliant with the principles of *integrity*, *objectivity*, *professional competence and due care*, *confidentiality*, and *professional behaviour* (IESBA 2011).

Unfortunately, professional scepticism can be affected by several factors which is turn might degrade the performance of the auditing process. Typical examples include the cases where an auditor does not appropriately evaluate the results of the audit process leading to failed reporting, or due to self-interest it's judgement has been affected. Advocacy and familiarity threats where an auditor can either promote a client's opinion or due to a close relationship can act with sympathy may also affect audit quality. These threats can be further exaggerated under long-term interactions between auditors and small clients (Li 2010).

To eliminate these threats or mitigate them below a specific threshold, several solutions have been proposed in the literature of business economics. However, the majority of the existing approaches have studied the impact of these threats on the audit quality segmentally[1]. For example, the impact of ethics on audit quality has been studied by Shaub and Lawrence (1996) where it is shown that ethics has a positive relationship to auditors' professional skepticism. For an excellent review on the subject the reader is referred to (Nelson 2009). Other studies, use logistic regression models to test the relation between audit firm tenure and audit quality (Carcello and Nagy 2004; Ghosh and Moon 2005; Fargher et al., 2008; Jenkins and Velury 2008; Jackson et al., 2008; Stanley and DeZoort 1996; Corbella et al., 2015; Garcia-Blandon and Ma Argiles 2015) and it is argued that both the knowledge to identify and the ability to develop processes to address audit issues rely on professional maturity (McCoy et al., 2011). Another research area tries to develop mechanisms that are mainly focused on evidence gathering for detecting fraud using statistical sampling theory (Matsamura and Tucker 1992; Bolton 2002). To the authors' opinion, an efficient decision making framework that could assist the auditors to identify, evaluate, and mitigate fraudulent financial reporting should jointly address all the threats mentioned above. In addition to this, it should be able to determine the optimal audit plan and provide reasonable

[1] A review on the factors affecting the quality of the audit process is provided by Sutton (1993)

assurance that the financial statements taken as a whole are free from material misstatements.

One of the first research efforts on the subject has been recently analyzed by Anugerah et al., (2011), where the method of statistical hypothesis testing is used to evaluate the relation between ethics, expertise, and experience in the auditors' professional scepticism. However, a critical issue that is not analyzed is how the interactions between auditors and clients affect the behavior of the auditee firm. Motivated by the pioneer paper of Fellingham and Newmam (1985) researchers started to analyze the interactions between auditors and clients in such a way that the auditor's strategy has an impact on the behavior of the auditee. Specifically, in this scheme, the client has the option to choose between high and low effort to eliminate misstatements in financial reports, while the auditor may select between high and low audit effort to detect misstatements. Inspired by this approach significant research has been carried out on the fraud detection problem employing game theory (Cook et al., 1997; Coates et al., 2002; Patterson and Noel 2003; Eleftheriou 2013; Fischbacher and Stefani 2007; Laitinen and Laitinen 2015; Carpenter 2007; Bowlin et al., 2009; Wilks and Zimbelman 2004; Anastasopoulos and Asteriou 2013; Anastasopoulos et al., 2013).

Unlike previous approaches, in the current paper a holistic framework is proposed to address the problem of fraud deterrence and detection taking into consideration fundamental principles and threats that may affect the performance of an audit process. Based on evidence from social psychology principles that are used to model the impact of auditor's tenure on the auditee firm, as well as on the outcomes of statistical sampling theory on audit quality, a mathematical model based on game theory is proposed to combat the limitations of the existing approaches. The proposed game-theoretic scheme considers two players, the auditor and the client. The client has the choice either to commit fraud or not while the auditor has the choice to adopt an audit plan that is based either on a basic set of audit procedures or on an extended set of risk assessment procedures. Factors such as familiarity and experience that severely affect the outcome of the audit process are analytically modeled based on evidence from social psychology.

The present study extends the literature in several ways. First of all, a unified decision theory model is developed to jointly confront the basic threats that are involved in an auditing process and provide reasonable assurance for that the financial statements are free from material misstatement. This is achieved by analyzing the dynamics of auditing strategies over time under varying payoffs using game theory. Second, the possible equilibrium points are investigated. It is proven that the auditing/fraud detection game between:

a) two new engagement parties; has unique mixed strategy equilibrium,
b) an experienced auditor and a client; has unique pure strategy equilibrium that is predominately affected by the auditor increased abilities to detect frauds

c) two old engagement parties; the audit process switches to new pure strategy equilibrium that is governed by sympathy.

Third, it provides a closed form solution for the optimal auditor's period replacement. The results obtained comply with the International Federation of Accountants (IFAC) Code of Ethics for Professional Accountants that requires the key audit partner to be rotated after a predefined period.

The rest of the paper is organized as follows. In Section 2, the impact of auditor's tenure on the client specific knowledge acquisition and intimacy are modeled based on social psychology principles. A generalized version of the audit/fraud detection game using classic game theory is presented in Section 3, whereas a set of risk assessment procedures together with the experimental validation of the proposed scheme is provided in Section 4. Finally, Section 5 concludes the paper.

## 2 Related Work

During the last years there has been an increasing demand for improving audit quality and questions were raised about how this objective can be achieved. Proponents of mandatory auditor rotation argue that audit quality is diminished with long auditor tenure. For example, Vanstraelen (2000) discovered a negative relationship between auditor tenure and the probability of qualified opinions. For this reason, considering the possible negative repercussions of long term services engagements on auditor independence, regulators have correlated the audit quality with auditor tenure (Azizkhani et al 2007). Typical examples include the standards and guidelines of the IFAC and the Institute of Chartered Accountants in Australia that require the rotation of the key audit partner after a predefined period, normally no more than seven years and five years, respectively (IESBA 2011).

On the other hand, the opponents of mandatory rotation theory state that there are no convincing empirical evidence from the auditing research literature that justifies these actions. They also argue that audit quality is improved over time, since for longer audit engagements, the auditor acquires an in-depth understanding on activities and relevant internal control systems of the audited entity (Azizkhani et al 2007). For example, Carcello and Nagy (2004) found empirical evidence that fraudulent financial reporting occurs in the early years of an auditor-client relationship quite often. The same conclusions are drawn by Geiger and Raghunandan (2002), who identified that reporting failures occur more often in the first five years of an auditor-client relationship. Similarly, the AICPA documents allegations of audit failures to be increased almost by factor three for first- or second-year audits. Focusing on the user perspective, Ghosh and Moon (2005) found evidence that investors and rating agencies rely on audited financial reports more strongly as auditor tenure increases. Moreover, Myers et al., (2003) found extreme accounting choices to be constrained more strongly under a high auditor's tenure. Finally, Mansi et al., (2004) reported that costs of debt decrease with auditor tenure.

However, to the authors' opinion this is partial justification for the view that auditing tenure and rotation theories are not conflicting, but rather complementary ways of understanding, planning and performing an audit of financial statements. In the following sections the impact of auditor's tenure on the client specific knowledge acquisition and intimacy are modeled based on social psychology principles. Then, based on game theory a unified framework is proposed that identifies the optimal auditing strategies to obtain reasonable assurance about whether the financial statements are free from material misstatement.

## 3 Modeling Learning and Sympathy

### 3.1 Auditors Skills Acquisition Curves

Intrinsic to the concept of skill acquisition is the notion that performance quality improves over time, that is, people are becoming better as they persist in the activity under examination (Green 2003). This observation is closely related to the arguments espoused by the opponents of mandatory rotation theory who believe that audit quality improves over time. The fact that skills develop over time raises an obvious question about the course of skill acquisition, especially in auditing. To this end, the accurate modeling of the course of that improvement is a key issue to be addressed.

Despite the vast body of literature in the area of knowledge acquisition, especially in cognitive and motor skill development (Lane 1987; Mazur and Hastie 1978; Newell and Rosenbloom 1981; Nanda and Adler 1977), the relevant research in auditing is limited (Earley 2001; Dillard and Roberts 2010). In one such study, Beck and Wu (2006) modeled the auditors learning process through a recursive relation of the form: $n_{t+1} = n_t + \kappa$. where $n_t$ denotes auditors knowledge about the clients business model at time period $t$. This formula states that, given $n_t$, the auditors knowledge about the evolution of the clients business model will be increased in the period $t + 1$ by $\kappa$ leading to improved audit quality. An important aspect is that this continual improvement in audit quality is characterized by a "diminishing returns" effect in which during the initial practice trials large gains are observed, whereas after a successful number of practice trials the size of the increment in performance quality is reduced (Green 2003). The large body of research on acquisition of cognitive and motor skills has centered on two distinct families of curves, the *exponential* and *power* functions, both of which are consistent with the general pattern of decelerating performance increments mentioned above.

In the present study, knowledge acquisition in auditing is related to the detection probability of a fraud and follows the general exponential function describing a learning curve like the dashed line of Fig. 1. A key assumption is that, the probability of fraud detection converges to 1 as the number of auditing periods tends to infinity. This is aligned with the studies of Beck and

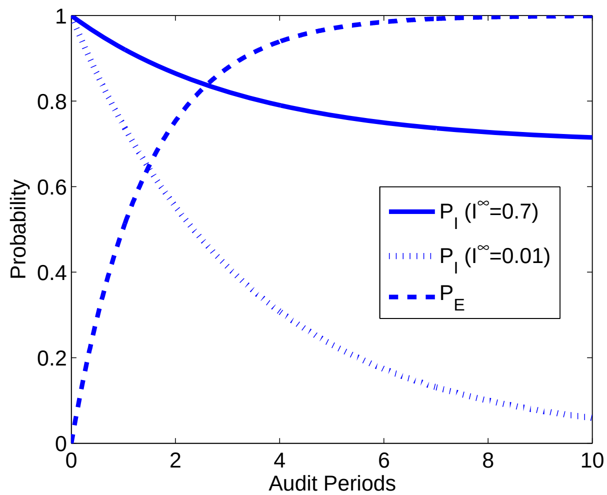



Fig. 1: Auditor's skill acquisition and sympathy curves representing performance improvements and closeness, respectively, over the course of practice

Wu (2006) where it is shown that under specific assumptions[2] the auditor in the long run will completely learn about the clients status. To mathematically model this process, the following equation is considered:

$$p_E(t) = 1 - E_0 e^{-\alpha_L t} \tag{1}$$

where $p_E$ is a measure of performance quality i.e. detection probability of a fraudulent reporting by an experienced auditor, $E_0$ represents performance level on the first practice trial, for example, the probability of failed audit after the engagement of a new audit, $e$ is the base of the natural logarithm (or approximately 2.718), $\alpha_L$ reflects the auditor's learning ability in the new environment, and $t$ is the number of auditing periods.

### 3.2 Modeling Sympathy

According to biobehavioral psychologists, closeness gives people a greater amount of confidence that they know their partner's behavior which, in turn, leads to a bias toward believing their partner's behavior is truthful. This leads to a lower rate of deception detection accuracy (Levine and McCornack 1992; McCornack and Parks 1986; Stiff et al., 1992). In terms of auditing, this observation may lead to a deterioration of audit quality over time since the auditors are gradually becoming less independent from the client firm. This is verified

[2] It is assumed that the clients earnings distributions over time are identically and independently distributed

by Bazerman et al., (1997) where, for example, it is stated that "*under current institutional arrangements, it is psychologically impossible for auditors to maintain their objectivity*". This closer relationship with the managers potentially increases the risk of collusion and may prevent the auditors from making objective decisions. This situation is referred to as *Non-Independence (NI)* and may lead to biased auditor's judgments in favor of their own and their client's interest (Bazerman et al., 1997).

These effects can be mathematically modeled by appropriately modifying the exponential law of closeness suggested by Marshall (1992). Specifically, the probability an auditor to report a detected fraud is related to the probability of being *Independent (I)*, $p_I$, described through following equation:

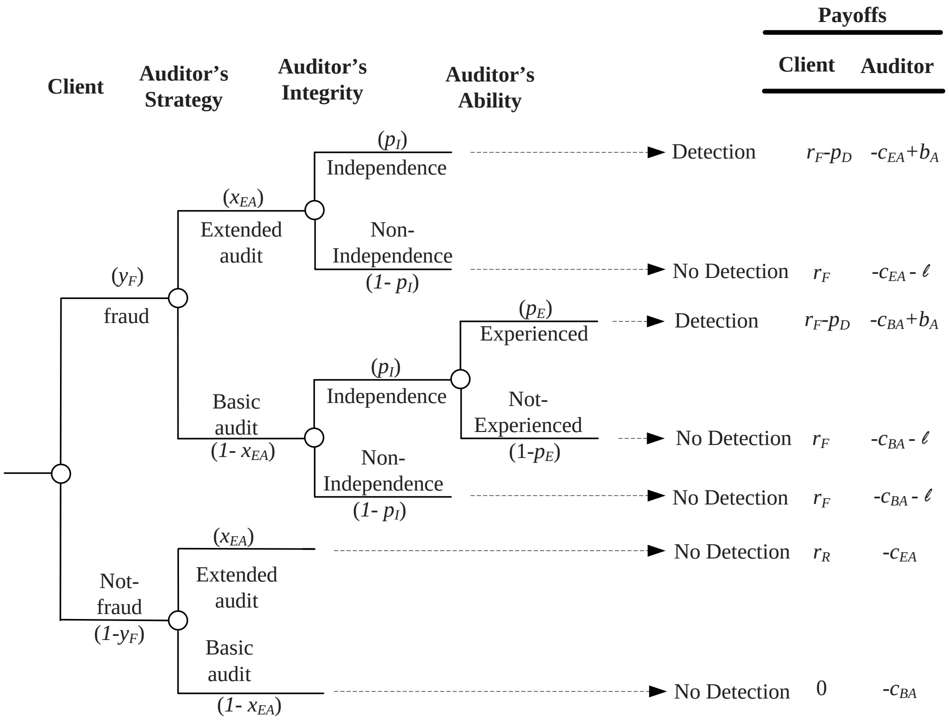
$$p_I(t) = I^{\infty} + (1 - I^{\infty})\, e^{-\alpha_I t} \tag{2}$$

In (2), $\alpha_I$ is a parameter describing the auditor's personal characteristics, $t$ is the social distance between the engagement parts that is related to the number of auditing periods and $I^{\infty}$ is the probability an auditor being independent after a large number of auditing periods. This probability depends on several factors including the opportunities available to the auditor to generate future revenues through the prospect of retaining the client, mental and body similarity, attractiveness etc. The exact evaluation of this parameter is out of the scope of the present work, however, for a detailed discussion on the subject, along with evidence from laboratory tests the reader is referred to (Sally 2000).

## 4 Problem Formulation

### 4.1 Modeling Payoffs Functions

In this Section, the auditing/fraud detection problem is modeled using non-cooperative game theory. This game belongs to the general class of coordination games in which all players involved can be benefited provided that they are acting in a mutual consistent manner (Russel 1998). In the present study, two types of players are considered that is, the auditor and the client. Throughout the auditing process, the auditor needs to obtain reasonable assurance that the financial statements as a whole are free from material misstatements (ISA 240 2009). Misstatement in the financial statements can be caused either inadvertently or intentionally (fraudulent reporting). Although standard audit procedures can be effectively used to identify unintentionally caused material misstatements, in some cases they can be proved inefficient for the detection of misstatements caused by fraud. A possible solution to this problem is the auditor to apply an extended set of risk assessment procedures. These procedures can reduce the risks of misstatements resulting from fraudulent financial reporting to an acceptable low level (ISA 315 2009). However, their main disadvantage is that they are more costly since they require additional auditing effort.




$p_D$ : Client pays an additional punitive penalty
$r_F$ : Value of the stolen assets
$r_R$ : Increase of clients' reputation
$c_{EA}$ : Cost of Extended Audit
$c_{BA}$ : Cost of Basic Audit
$\ell$ : The auditors suffer a loss if an intentional error due to fraud goes undetected
$b_A$ : Bonus for detecting a fraud
$x_{EA}$ : Probability of Extended Audit
$y_F$ : Probability of Fraudulent reporting
$p_I$ : Probability of Independence

Fig. 2: The Two-Player Audit/Fraud Detection Game in extensive-form

Extending this rationale, the auditor can either adopt a Basic Audit ($BA$) or an Extended Audit ($EA$) plan following the guidelines of ISA 315 (2009). The auditor's pure-strategy set can be defined as $S_A = \{BA, EA\}$. On the other hand, the client can either select to commit fraud ($F$) or not ($NF$). In this case, the client's pure-strategy set is $S_C = \{NF, F\}$. At this point it should be noted that that material misstatements due to inadvertent error can be detected by both auditing procedures. However, intentional errors (frauds) can be detected either by $EA$ or by an experienced auditor who has adopted $BA$. Specifically, if the auditor has remained for several auditing periods to the same auditee firm then, the learning process of the auditor may lead to an increasing ability to detect material misstatements in the financial statements of the auditee. Therefore, it is very likely that frauds will be detected by an experienced auditor, even if $BA$ has been adopted. However, an increase in tenure may result in increasing empathy between the engagement parties (Fairchild 2007). As already mentioned, closeness gives people a greater amount of confidence that they know their partner's behavior which, in turn, leads to a bias toward believing their partner's behavior is truthful. In terms of auditing, this observation may lead to a deterioration of audit quality over time since the auditors are gradually becoming less independent from the audited entity. Therefore, even if the financial statements of an auditee are materially misstated, a sympathetic yet experienced auditor will be incapable of interpreting this information as negative.

A graphical representation of the audit/fraud detection game between the two parties is provided in Fig. 2, where two basic scenarios are observed:

a) If the client does not commit a fraud, the payoff for the auditor is $-c_{EA}$ ($-c_{BA}$) and for the client is $r_R$ (0) in case the auditor selects $EA$ ($BA$). It is clear that $-c_{BA} > -c_{EA}$ since the cost of $EA$, namely $c_{EA}$, is higher that the cost $c_{BA}$ of $BA$ due to additional auditing efforts. On the contrary, an extended audit that will not reveal misstatements will improve client's reputation. Therefore, the client will receive a bonus reward, $r_R > 0$.
b) If the client commits a fraud then, this incident may be detected either by $EA$ or by an experienced auditor who has adopted $BA$. The auditor will then receive a bonus $b_A$ due to its successful efforts to unveil frauds. Therefore, in the former case, the payoff for the auditor and the client is $-c_{EA} + b_A$ and $r_F - p_D$, while in the latter $-c_{BA} + b_A$ and $r_F - p_D$, respectively. It is clear that, $-c_{EA} + b_A < -c_{BA} + b_A$ since $EA$ is more costly compared to $BA$ due to additional auditing efforts. Furthermore, in both cases, a penalty $r_F - p_D$ ($p_D > r_F$) will be imposed to the client.

Unfortunately, there are some cases in which frauds are not detected. If the auditor does not have adequate experience to detect frauds, then due to the failure of the auditing procedures to unveil frauds, the auditor will pay $\ell$ ($\ell < 0$), as penalty, while the client will gain $r_F$, where $r_F$ is the value of the stolen assets. Similarly, if the auditor is not independent, frauds will not be detected regardless of the auditing strategies employed. The client will again receive a payoff $r_F$ while the auditor will pay as penalty $-c_{EA} + \ell$ or $-c_{EA} + \ell$ in case of $EA$ or $BA$, respectively. The payoff matrices $\mathbf{A}$, $\mathbf{B}$ for the auditor and the client, respectively, are defined as follows:

$$\mathbf{A} = \begin{pmatrix} a_{BA}^{NF} & a_{BA}^{F} \\ a_{EA}^{NF} & a_{EA}^{F} \end{pmatrix} \tag{3a}$$

$$\mathbf{B} = \begin{pmatrix} b_{BA}^{NF} & b_{BA}^{F} \\ b_{EA}^{NF} & b_{EA}^{F} \end{pmatrix} \tag{3b}$$

where $a_h^k$ denotes the payoff to the auditor and $b_h^k$ the payoff to the client when the auditor uses pure strategy $h \in S_A$ and the client pure strategy $k \in S_C$. Based on Fig. 2, these elements can be written in the following form:

$$a_{BA}^F = p_I\left[p_E\left(-c_{BA}+b_A\right)+\left(1-p_E\right)\left(-c_{BA}-\ell\right)\right]-\left(c_{BA}+\ell\right)\left(1-p_I\right) \quad (4a)$$
$$a_{EA}^F = p_I(-c_{EA}+b_A)+\left(1-p_I\right)\left(-c_{EA}-\ell\right) \quad (4b)$$
$$b_{BA}^F = p_I\left[p_E\left(r_F-p_D\right)+\left(1-p_E\right)\left(r_F\right)\right]+\left(r_F\right)\left(1-p_I\right) \quad (4c)$$
$$b_{EA}^F = p_I(r_F-p_D)+\left(1-p_I\right)\left(r_F\right) \quad (4d)$$

Substituting (1)-(2) into ($4a$)-($4d$) yields

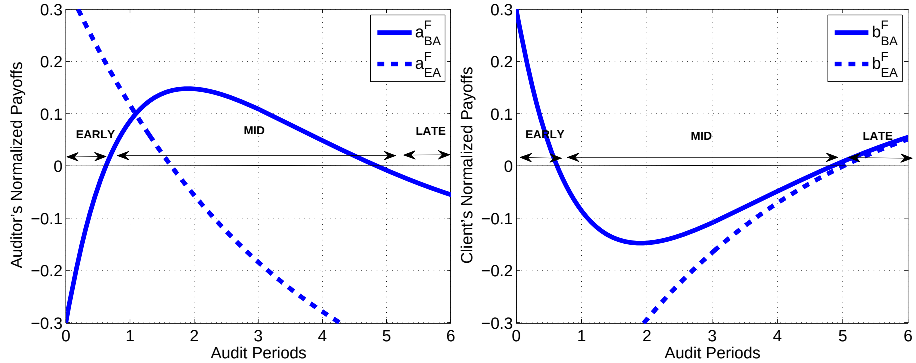
$$a_{BA}^F(t) = -(c_{BA}+\ell)+(b_A+\ell)p_I(t)p_E(t) \quad (5a)$$
$$a_{EA}^F(t) = -(c_{EA}+\ell)+(b_A+\ell)p_I(t) \quad (5b)$$
$$b_{BA}^F(t) = r_F - p_D p_I(t)p_E(t) \quad (5c)$$
$$b_{EA}^F(t) = r_F - p_D p_I(t) \quad (5d)$$

From ($5a$)-($5d$) it is observed that the payoffs for the auditor and the client depend on $p_I(t)$ and $p_E(t)$, which, in turn, are functions of $t$. A graphical representation of these equations, assuming that $p_D$ is much higher than $r_F$, is provided in Fig. 3. In the lower part of Fig. 3, the payoffs for the engagement parts are depicted for the ideal scenario where the probability an auditor being independent after a large number of auditing periods is high. On the contrary, in the upper part of Fig. 3, the same parameters are examined based on the rational assumption that the probability of an auditor being independent in the long run is low.

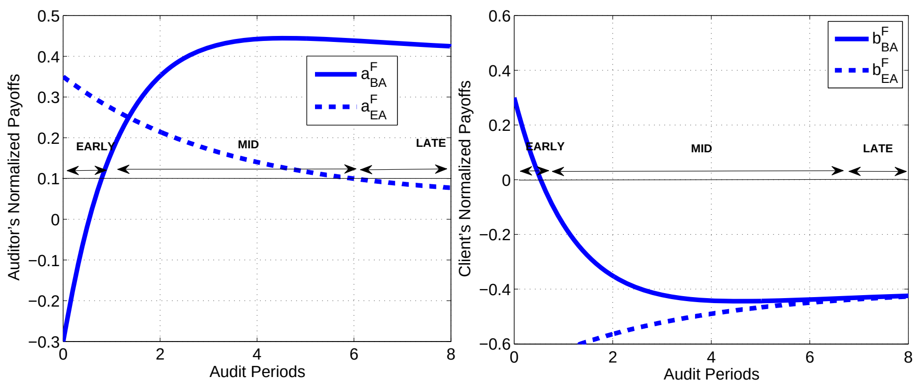
Depending on the time instant where the audit/fraud detection problem is analyzed (Early, Mid or Late phase) the following observations are drawn regarding the evolution of the payoffs for the auditor and the client over time:
1) *Early Phase*: During the early phase of the audit process ($t \to 0$) the payoffs for the auditor and the client, respectively, are given by

$$\lim_{t\to 0} a_{BA}^F(t) = -(c_{BA}+\ell)+(b_A+\ell)\lim_{t\to 0} p_I(t)p_E(t) = -c_{BA}-\ell < 0 \quad (6a)$$
$$\lim_{t\to 0} a_{EA}^F(t) = -(c_{EA}+\ell)+(b_A+\ell)\lim_{t\to 0} p_I(t) = -c_{EA}+b_A > 0 \quad (6b)$$
$$\lim_{t\to 0} b_{BA}^F(t) = r_F - p_D\lim_{t\to 0} p_I(t)p_E(t) = r_F > 0 \quad (6c)$$
$$\lim_{t\to 0} b_{EA}^F(t) = r_F - p_D\lim_{t\to 0} p_I(t) = r_F - p_D < 0 \quad (6d)$$

Given that, the psychological incentives for both parties are negligible, if the client selects to commit a fraud, the payoff for the auditor in case $EA$ is adopted is positive since the audit process will turn out to be successful. On the contrary, the payoff for the auditor in case $BA$ is adopted is negative since frauds will not be detected.
2) *Mid Phase*: During this phase of the audit process, the auditor has gained




(a) Probability of an auditor being independent after a large number of audit periods is low ($I^\infty \rightarrow 0$)




(b) Probability of an auditor being independent after a large number of audit periods is high ($I^\infty \rightarrow 1$)

Fig. 3: Evolution of payoffs over time

significant experience on the auditee firm. Therefore, the probability for detecting a fraud using BA is high ($a^F_{BA}(t) > 0$). This observation is illustrated in Fig. 3a where it is seen that $a^F_{BA}(t)$ is a concave function having it's maximum at $t_0$, whose value is determined from the solution of $\left.\frac{\partial a^F_{BA}(t)}{\partial t}\right|_{t=t_0} = 0$. Similarly, $b^F_{BA}(t)$ takes negative values due to the high probability of a BA to unveil frauds. Again, using the second derivative test, it can be easily shown that $b^F_{BA}(t)$ is a convex function having it's minimum at $\left.\frac{\partial b^F_{BA}(t)}{\partial t}\right|_{t=t_0} = 0$.
3) *Late Phase*: After a large number of auditing periods ($t \rightarrow +\infty$) the payoffs

for the auditor and the client will converge to the following values

$$\lim_{t\to+\infty} a^F_{BA}(t) = -(c_{BA}+\ell) + (b_A+\ell)I^\infty \tag{7a}$$

$$\lim_{t\to+\infty} a^F_{EA}(t) = -(c_{EA}+\ell) + (b_A+\ell)I^\infty \tag{7b}$$

$$\lim_{t\to+\infty} b^F_{BA}(t) = r_F - p_D I^\infty \tag{7c}$$

$$\lim_{t\to+\infty} b^F_{EA}(t) = r_F - p_D I^\infty \tag{7d}$$

Based on $I^\infty$, two major cases are examined:

a) For high values of $I^\infty$, both $b^F_{EA}(t)$ and $b^F_{BA}(t)$ are negative because fraudulent reporting will be always detected by an experienced and independent auditor.
b) For low values of $I^\infty$, both $b^F_{EA}(t)$ and $b^F_{BA}(t)$ will be positive. This is justified by the fact that due to biased auditor's judgments, deliberate misreporting will be not detected. As discussed in Bazerman et al., (1997), this is the most common case since auditors may find psychologically impossible to remain impartial and objective.

### 4.2 Problem Solution

Based on the analysis presented in the previous section, it is deduced that the audit process should be planed for entire horizon taking into account the following propositions:

**Proposition 1** *The auditing/fraud detection game between two new engagement parties has a single mixed strategy equilibrium.*

*Proof* When a new audit engagement is accepted, the impact of auditor's tenure on independence is negligible. Therefore, based on $(6b)$-$(6d)$ it may be easily observed that the best response of the auditor to the strategies $NF$ and $F$ is $BA$ and $EA$, respectively, while the best response of the client to the strategies $BA$ and $EA$ is $F$ and $NF$, respectively. Therefore, a pair of strategies in which each strategy is the best response compared to the other one in that pair does not exist (Dutta 1999).

Even though the game has no pure strategy equilibria, it has a single mixed strategy equilibrium. A mixed strategy for player $i,\ i = A,\ C$ is a probability distribution over his set $S_i$ of pure strategies (Fudenberg and Levin 1998). Since the set $S_i,\ i = A,\ C$ is finite, any mixed strategy $\mathbf{x}$ and $\mathbf{y}$ for the auditor and the client, respectively, may be represented as vector with their $h^{th}$ coordinate, $x_h, y_h$ denoting the probability assigned by the auditor and the client to pure strategy $h \in S_A$ and $h \in S_C$, respectively. It should be also noted that $x_h,\ y_h \geq 0$ and $\sum_{h\in S_A} x_h = 1$, $\sum_{h\in S_C} y_h = 1$. The expected payoffs of the two players are denoted by $u_A(\mathbf{x},\mathbf{y}) = \mathbf{x}\cdot\mathbf{A}\mathbf{y}$, and $u_C(\mathbf{x},\mathbf{y}) = \mathbf{y}\cdot\mathbf{B}^T\mathbf{x}$

and are given by:

$$u_A(\mathbf{x},\mathbf{y}) = x_{BA}\left[a^F_{BA}y_F + a^{NF}_{BA}y_{NF}\right] + x_{EA}\left[a^F_{EA}y_F + a^{NF}_{EA}y_{NF}\right] \quad (8a)$$
$$u_C(\mathbf{x},\mathbf{y}) = y_F\left[b^F_{BA}x_{BA} + b^F_{EA}x_{EA}\right] + y_{NF}\left[b^{NF}_{BA}x_{BA} + b^{NF}_{EA}x_{EA}\right] \quad (8b)$$

The auditor and the client choose $x_{BA}$ and $y_F$ that maximize $(8a)$ and $(8b)$, respectively. Their first order conditions are:

$$\frac{\partial u_A}{\partial x_{BA}} = a^F_{BA}y_F + a^{NF}_{BA}(1-y_F) - a^F_{EA}y_F - a^{NF}_{EA}(1-y_F) = 0 \quad (9a)$$
$$\frac{\partial u_C}{\partial y_F} = b^F_{BA}x_{BA} + b^F_{EA}(1-x_{BA}) - b^{NF}_{BA}x_{BA} - b^{NF}_{EA}(1-x_{BA}) = 0 \quad (9b)$$

Solving the set of equations $(9a)$-$(9b)$ for $y_i$ and $x_i$, respectively, the following mixed strategy equilibrium is derived:

$$(x^*_{BA}, x^*_{EA}) = \left(\frac{\delta}{\gamma+\delta}, \frac{\gamma}{\gamma+\delta}\right) \quad (10a)$$
$$(y^*_F, y^*_{NF}) = \left(\frac{\alpha}{\alpha+\beta}, \frac{\beta}{\beta+\alpha}\right) \quad (10b)$$

where

$$\alpha = a^{NF}_{BA} - a^{NF}_{EA} = -c_{BA} - (-c_{EA}) = c_{EA} - c_{BA} > 0 \quad (11a)$$
$$\beta = a^F_{EA} - a^F_{BA} = \ell + b_A - (c_{EA} - c_{BA}) > 0 \quad (11b)$$
$$\gamma = b^{NF}_{BA} - b^F_{BA} = -r_F < 0 \quad (11c)$$
$$\delta = b^F_{EA} - b^{NF}_{EA} = r_F - p_D - r_R < 0 \quad (11d)$$

From this solution the following conclusions are drawn

a) It is verified that the game has no pure strategy equilibria since the probabilities for the auditor and the client to select $BA$ or $EA$ and $NF$ and $F$, respectively, always take values in the range $(0,1)$.
b) The probability for the auditor to select $BA$ increases with $\delta$ and decreases with $\gamma$. Recall that $\delta$ is a decreasing function of the penalty that is imposed to the client when a fraudulent report is detected. $\delta$ also depends on the reputation that a company earns in case that at the end of an extended audit frauds are not detected by the auditor.
c) The probability for the client to select not to fraud increases with $\beta$, where $\beta$ depends on the bonus $b_A$ an auditor earns when a fraudulent report is detected.

**Proposition 2** *During the mid phase, the auditing/fraud detection game between an experienced auditor and the client has a single pure strategy equilibrium that is* $(x^*_{BA}, y^*_{NF}) = (1,1)$. *This equilibrium is asymptotically stable in* $x_{BA}, y_{NF} \in \Upsilon \backslash \{x^*_{BA}, y^*_{NF}\}$ *with* $\Upsilon$ *being a neighborhood region around* $x_{BA} = x^*_{BA}, y_{NF} = y^*_{NF}$.

*Proof* After few auditing rounds, the auditor has gained significant experience on the auditee firm. Therefore, the probability for detecting a fraud using either BA or EA is high. However, since $EA$ is more costly compared to $BA$ due to additional auditing efforts, the auditor will adopt $BA$. Algebraically, this event is expressed through $a_{BA}^{F} > a_{EA}^{F}$ and it is graphically illustrated in the left part of Figs. 3a and 3b. Thus, the best response of the auditor to the strategies $NF$ and $F$ is $BA$, respectively. On the other hand, the best response of the client to the strategies $EA$ and $BA$ is $NF$ since he knows *a priori* that if he commits a fraud then this action will be detected.

To proof the second claim, the evolutionary dynamics of the auditing/fraud detection game will be analyzed around the equilibrium $(BA, NF)$. The strategies for both players evolve over time according to the replicator dynamics system (Anastasopoulos and Asteriou 2013):

$$\dot{x}_{BA} = \left[\sum_{k\in\{NF,F\}} a_{BA}^{k} y_k - u_A(\mathbf{x},\mathbf{y})\right] x_{BA} \tag{12$a$}$$

$$\dot{y}_{NF} = \left[\sum_{h\in\{BA,EA\}} b_{h}^{NF} x_h - u_C(\mathbf{x},\mathbf{y})\right] y_{NF} \tag{12$b$}$$

Equations (12$a$)-(12$b$) state that strategies offering higher payoff than average grow whereas strategies with lower payoff shrink. After some algebraic manipulations, (12$a$)-(12$b$) take the following form:

$$\dot{x}_{BA} = x_{BA} x_{EA} \mathcal{F}(t, y_{NF}) \tag{13$a$}$$

$$\dot{y}_{NF} = y_{NF} y_F \mathcal{G}(t, x_{BA}) \tag{13$b$}$$

where $\mathcal{F}(t, y_{NF}) = [-\beta(t) + (\alpha(t) + \beta(t))\, y_{NF}]$ and $\mathcal{G}(t, x_{BA}) = [-\delta(t) + (\gamma(t) + \delta(t))\, x_{BA}]$ with $\alpha(t), \beta(t), \gamma(t), \delta(t)$ defined in the left part of (11$a$)-(11$d$):

$$\alpha(t) = c_{EA} - c_{BA} \tag{14$a$}$$

$$\beta(t) = -(c_{EA} - c_{BA}) + (b_a + \ell)p_I(t)(1 - p_E(t)) \tag{14$b$}$$

$$\gamma(t) = -r_F + p_D p_I(t) p_E(t) \tag{14$c$}$$

$$\delta(t) = r_F - r_R - p_D p_I(t) \tag{14$d$}$$

Then, the stability of the system (13$a$)-(13$b$) around the equilibrium point will be examined. However, due to its non-linear time-varying nature techniques based on *Lyapunovs direct* or *indirect method* are not sufficient for inferring stability properties of the audit/fraud detection game. An alternative approach is to infer asymptotic stability by examining the higher order derivatives of Lyapunov functions using the time varying version of the following theorem Butz (1999):

**Theorem 1** *Consider the nonlinear system $\dot{x} = f(t,x)$ and an m-vector function $\mathcal{V}(t,x)$ of the following form:*

$$\mathcal{V}(t,x) = [\mathcal{V}_1(t,x), \mathcal{V}_2(t,x), \ldots, \mathcal{V}_m(t,x)]^T \tag{15}$$

*whose time derivative $\dot{\mathcal{V}}(t,x)$ along the solutions of $\dot{x} = f(t,x)$ satisfies the following differential inequality*

$$\begin{bmatrix} \dot{\mathcal{V}}_1(t,x) \\ \dot{\mathcal{V}}_2(t,x) \\ \vdots \\ \dot{\mathcal{V}}_m(t,x) \end{bmatrix} \leq \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & -a_{m-1} \end{bmatrix} \begin{bmatrix} \mathcal{V}_1(t,x) \\ \mathcal{V}_2(t,x) \\ \vdots \\ \mathcal{V}_m(t,x) \end{bmatrix} \tag{16}$$

*with $a_0 > 0$ and $a_j \geq 0, j = 1,2,...,m-1$. In addition, assume $\dot{\mathcal{V}}(t,x^*) = 0$ and all the roots of the following characteristic equation are negative real numbers:*

$$s^m + a_{m-1}s^{m-1} + \cdots + a_1 s + a_0 = 0. \tag{17}$$

*If $\mathcal{V}_1(t,x)$ is positive definite function satisfying the following conditions:*

$$\mathcal{V}_1(t,x^*) = 0, \mathcal{V}_1(t,x) > 0, \forall x \in \Upsilon \backslash \{x^*\}, \tag{18}$$

*then the equilibrium point $x^*$ is asymptotically stable.*

A Lypapunov candidate function of the system $(13a)$-$(13b)$ satisfying the above properties is given by:

$$\begin{aligned} \mathcal{V}_1(t,x_{BA},y_{NF}) = \quad & \delta_1 \left( x_{BA} - x^*_{BA} - x^*_{BA} \ln\left(\frac{x_{BA}}{x^*_{BA}}\right)\right) \\ & + \delta_2 \left( y_{NF} - y^*_{NF} - y^*_{NF} \ln\left(\frac{y_{NF}}{y^*_{NF}}\right)\right) \end{aligned} \tag{19}$$

with $\delta_1 > 0$, $\delta_2 > 0$. The time derivative of $\mathcal{V}_1$ can be written in the following form:

$$\begin{aligned} \dot{\mathcal{V}}_1(t,x_{BA},y_{NF}) &= \delta_1 \left( \dot{x_{BA}} - x^*_{BA}\frac{\dot{x_{BA}}}{x_{BA}}\right) + \delta_2 \left( \dot{y_{NF}} - y^*_{NF}\frac{\dot{y_NF}}{y_{NF}}\right) \\ &= \delta_1 \left(x_{BA} - x^*_{BA}\right)\frac{\dot{x_{BA}}}{x_{BA}} + \delta_2 \left(y_{NF} - y^*_{NF}\right)\frac{\dot{y_{NF}}}{y_{NF}} \end{aligned} \tag{20}$$

Substituting $(13a)$-$(13b)$ into (21) and setting $x^*_{BA} = 1, y^*_{NF} = 1$ yields:

$$\dot{\mathcal{V}}_1(t,x_{BA},y_{NF}) = \delta_1 \left(x_{BA} - 1\right) x_{EA}\mathcal{F}(t,y_{NF}) + \delta_2 \left(y_{NF} - 1\right) y_F \mathcal{G}(t,x_{BA}) \tag{21}$$

or

$$\begin{aligned} \dot{\mathcal{V}}_1(t,x_{BA},y_{NF}) &= -\delta_1 \left(1 - x_{BA}\right)^2 \mathcal{F}(t,y_{NF}) - \delta_2 \left(1 - y_{NF}\right)^2 \mathcal{G}(t,x_{BA}) \\ &= \mathcal{V}_2(t,x_{BA},y_{NF}) \end{aligned} \tag{22}$$

In the next step, the first derivative of $\mathcal{V}_2(t, x_{BA}, y_{NF})$ will be evaluated. After straightforward algebra yields:

$$\begin{aligned}\dot{\mathcal{V}}_2(t, x_{BA}, y_{NF}) \leq &- \delta_1 (1 - x_{BA})^2 \left[-2\mathcal{F}^2(t, y_{NF}) + \dot{\mathcal{F}}(t, y_{NF})\right] \\ &- \delta_2 (1 - y_{NF})^2 \left[-2\mathcal{G}^2(t, x_{BA}) + \dot{\mathcal{G}}(t, x_{BA})\right]\end{aligned} \tag{23}$$

Using (22) and (23), the following linear combination is evaluated:

$$\begin{aligned}&\dot{\mathcal{V}}_2 + a_1\dot{\mathcal{V}}_1 + a_0\mathcal{V}_1 \leq \\ &\quad - \delta_1 (1 - x_{BA})^2 \left[-2\mathcal{F}^2(t, y_{NF}) + \dot{\mathcal{F}}(t, y_{NF}) + a_1\mathcal{F}(t, y_{NF}) - a_0\right] \\ &\quad - \delta_2 (1 - y_{NF})^2 \left[-2\mathcal{G}^2(t, x_{BA}) + \dot{\mathcal{G}}(t, x_{BA}) + a_1\mathcal{G}(t, x_{BA}) - a_0\right]\end{aligned} \tag{24}$$

Theorem 1 holds when the right part of (24) equals to zero:

$$-2\mathcal{F}^2(t, y_{NF}) + \dot{\mathcal{F}}(t, y_{NF}) + a_1\mathcal{F}(t, y_{NF}) - a_0 = 0 \tag{25$a$}$$

$$-2\mathcal{G}^2(t, x_{BA}) + \dot{\mathcal{G}}(t, x_{BA}) + a_1\mathcal{G}(t, x_{BA}) - a_0 = 0 \tag{25$b$}$$

However, given that $x_{BA}, y_{NF} \in \Upsilon$, throughout the mid auditing phase, $\mathcal{F}, \mathcal{G}$ can be successfully approximated by the following equations:

$$\mathcal{F}(t, y_{NF}) = \alpha - \epsilon(b_A + \ell)p_I(t)(1 - y_{NF}) \approx \alpha \tag{26$a$}$$

$$\begin{aligned}\mathcal{G}(t, x_{BA}) &= -r_F + r_R(1 - x_{BA}) + p_D p_I(t)(1 - \epsilon x_{BA}) \\ &\approx -r_F + p_D p_I(t)\end{aligned} \tag{26$b$}$$

Substituting (26$b$)-(26$c$) into (25$b$)-(25$b$) and assuming $\mathcal{F} \neq \mathcal{G}$ yields

$$a_0 = -2\alpha^2 + +a_1\alpha = \alpha(a_1 - 2\alpha) \tag{27$a$}$$

$$a_1 = -\frac{\dot{\mathcal{F}} - \dot{\mathcal{G}}}{\mathcal{F} - \mathcal{G}} + 2(\mathcal{F} + \mathcal{G}) \tag{27$b$}$$

(23) and (24) can be now written as a system of differential inequalities as follows:

$$\begin{bmatrix}\dot{\mathcal{V}}_1 \\ \dot{\mathcal{V}}_2\end{bmatrix} \leq \begin{bmatrix}0 & 1 \\ -a_0 & -a_1\end{bmatrix}\begin{bmatrix}V_1 \\ V_2\end{bmatrix} \tag{28}$$

Note that $[\mathcal{V}_1(t, x^*_{BA}, t^*_{NF}), \mathcal{V}_2(t, x^*_{BA}, y^*_{NF})] = [0, 0]$ whereas $\mathcal{V}_1$ is local positive definite function. The conditions of Theorem 1 are satisfied when the roots of the characteristic equation $s^2 + a_1 s + a_0 = 0$ are both negative real numbers. This holds when the following conditions are satisfied:

$$a_0 = -2\alpha^2 + a_1\alpha = \alpha(a_1 - 2\alpha) > 0 \tag{29$a$}$$

$$a_1 = -\frac{\dot{\mathcal{F}} - \dot{\mathcal{G}}}{\mathcal{F} - \mathcal{G}} + 2(\mathcal{F} + \mathcal{G}) > 0 \tag{29$b$}$$

$$\Delta = a_1^2 - 4a_0 = a_1^2 - 4a_1\alpha + 8\alpha^2 > 0 \tag{29$c$}$$

Solving the above inequalities yields

$$a_1 > 2\alpha \tag{30}$$

Thus, (30) is a sufficient condition for local asymptotical stability of the equilibrium state $(x^*_{BA} = 1, y^*_{NF} = 1)$.

**Proposition 3** *In the long run, the auditing/fraud detection game between a less independent auditor and the client has a single pure strategy equilibrium that is* $(x^*_{BA}, y^*_{NF}) = (1, 0)$. *This equilibrium is asymptotically stable in* $x_{BA}, y_{NF} \in \Upsilon\backslash\{x^*_{BA}, y^*_{NF}\}$ *with* $\Upsilon$ *being a neighborhood region around* $x_{BA} = x^*_{BA}, y_{NF} = y^*_{NF}$.

*Proof* After long term presence of the auditor in the auditee firm $(t \to \infty)$ it is observed that the best response of a less independent auditor $(I^\infty < (r_F - r_R)/p_D)$ to the strategies $NF$ and $F$ is $BA$, respectively, while the best response of the client to the strategies $BA$ and $EA$ is $F$. Therefore, in long term the game converges to $(BA, F)$. The same conclusion can be reached using the mixed strategy equilibrium method. Specifically, both the auditor and the client select their strategies in order to maximize (8a) and (8b), respectively. However, after straightforward algebra yields:

$$\frac{\partial u_A}{\partial x_{BA}} = -c_{BA} + c_{EA} > 0 \tag{31$a$}$$

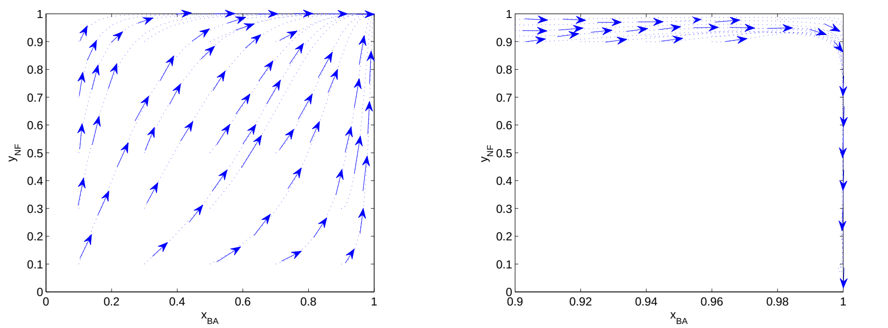
$$\frac{\partial u_C}{\partial y_F} = r_F - p_D I^\infty - r_R x_{EA} \geq r_F - r_R - p_D I^\infty > 0 \tag{31$b$}$$

From $(31a)$-$(31b)$ it is readily obtained that $u_A$ and $u_C$ are a strictly increasing functions of $x_{BA}$ and $y_F$, respectively. Given that, $0 \leq x_{BA}, y_F \leq 1$, their maximum value is reached at $(x_{BA}, y_F) = (1, 1)$. Therefore, the equilibrium point for auditing/fraud detection game between old engagement parties is $(x^*_{BA}, x^*_{EA}) = (1, 0)$ and $(y^*_{NF}, y^*_F) = (0, 1)$. To prove the stability of this solution equilibrium point a similar approach to Proposition 2 is adopted. The Lyapunov function for the system formed by $(12a)$-$(12b)$ around $(1, 0)$ is given by:

$$\mathcal{V}(t, x_{BA}, y_{NF} = \delta_1 \left( x_{BA} - x^*_{BA} - x^*_{BA} \ln \left( \frac{x_{BA}}{x^*_{BA}} \right) \right) + \frac{1}{2} \delta_2 \left( y_{NF} - y^*_{NF} \right)^2 \tag{32}$$

It is clear that $\mathcal{V}(t, x^*_{BA}, y^*_{NF}) = 0$ whereas $\mathcal{V}(t, x_{BA}, y_{NF}) > 0$ for all $(x_{BA}, y_{NF}) \in \Upsilon$ with $(x_{BA}, y_{NF}) \neq (x^*_{BA}, y^*_{NF})$. It remains to be seen that $\dot{\mathcal{V}}(t, x_{BA}, y_{NF}) < 0$ for all $(x_{BA}, y_{NF}) \in \Upsilon$ with $(x_{BA}, y_{NF}) \neq (x^*_{BA}, y^*_{NF})$. After some algebraic manipulations it can be easily shown that the time derivative of (32) is negative.

**Proposition 4** *In the long run, the auditing/fraud detection game between an independent auditor and the client has a single pure strategy equilibrium that is* $(x^*_{BA}, y^*_{NF}) = (1, 1)$.




(a) After few auditing periods the game converges to the stable solution $x_{BA}, y_{NF} = (1, 1)$.

(b) Mid phase local stability of the strategy set $x^1_{BA}, y_{NF} = (1, 1)$, Late phase the client switches from NF to F

Fig. 4: Phase diagram of the fraud/audit detection game between the client and a) an independent auditor and b)a less independent auditor

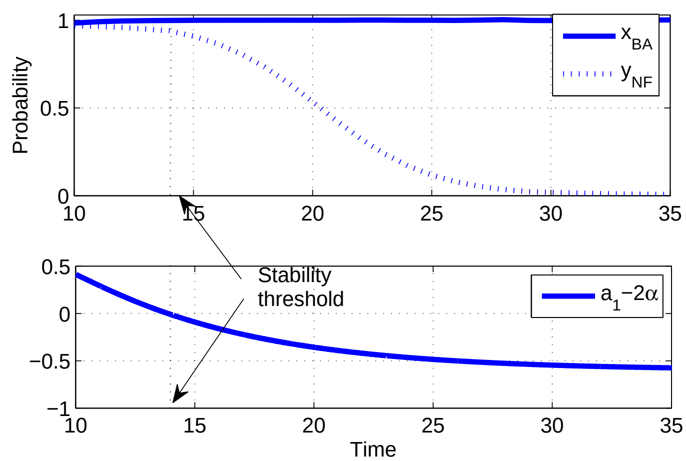



Fig. 5: a) Evolution of players' strategies over time, b) Stability condition over time: Above 13.85, the stability condition (30) is not satisfied and the client switches from NF to F

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*Proof* The proof is similar to the proof of Proposition 3.

The phase diagram of the audit/fraud detection game for the case of an independent and a less independent auditor is presented in Fig. 4a and 4b, respectively. For the case where the auditor is not affected by sympathy, the system in the long run will converge to the Nash Equilibrium point that attracts solutions from all initial conditions. However, in case of a less independent auditor, throughout the mid phase the system will be attracted by the $x_{BA}, y_{NF} = (1, 1)$ whereas after a specific time threshold will converge to $x_{BA}, y_{NF} = (1, 0)$. This is explained by the fact that beyond this time threshold the condition (30) is violated and the equilibrium point $x_{BA}, y_{NF} = (1, 1)$ becomes unstable (see Fig. 5).

## 5 Numerical Results and Discussions

### 5.1 Performance metric

At this point is should be mentioned that an important metric to be analyzed is the probability of failed audit, $P_{FA}$. An audit fails, if the client commits a fraud and, due to self-serving bias, this incident is not appropriately reported Bazerman et al., (1997). The following cases might occur:

- **Case 1** $(F; BA; Non-Experienced)$: the auditor selects $BA$ and his experience is not adequate to detect frauds. The probability this event to take place is given by the following equation:

$$P_1 = y_F x_{BA} \left(1 - p_E\right) \tag{33}$$

- **Case 2** $(F; BA; Experienced; Non-Independence)$ or $(F; EA; Non-Independence)$: the auditor no matter what audit methodology will select, frauds will be not reported since his opinion has been affected by his long period presence (non-independence) in the auditee firm. This case appears with probability given by:

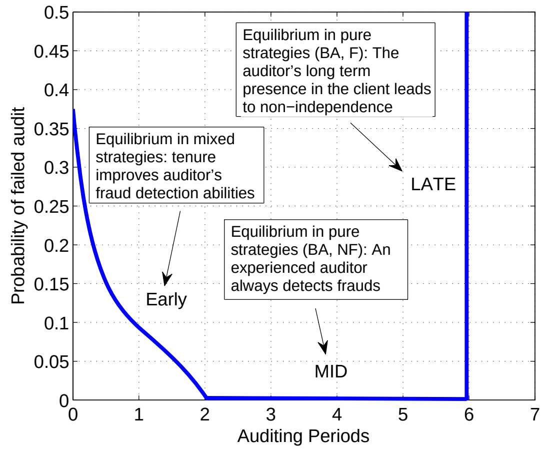
$$P_2 = y_F x_{BA} p_E \left(1 - p_I\right) + y_F x_{EA} p_E \left(1 - p_I\right) \tag{34}$$

Based on (33), (34) the probability for an audit to fail is described through the following equation:

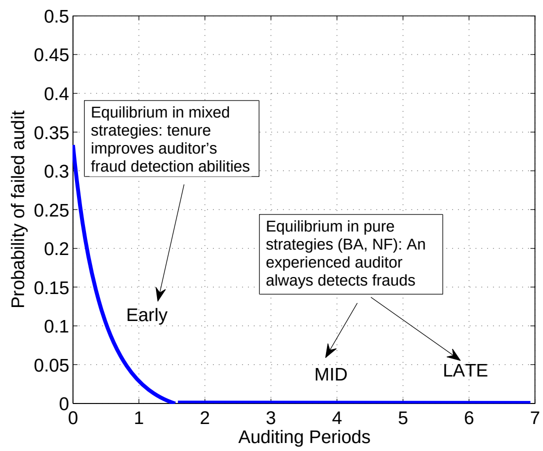
$$P_{FA} = y_F \cdot \left[x_{EA} \cdot \left(1 - p_I\right) + x_{BA} \left(1 - p_E + p_E \left(1 - p_I\right)\right)\right] \tag{35}$$

### 5.2 Results

So far, the fraud detection problem taking into account the impact of auditor's tenure on the quality of audit has been analyzed using game theory. In this section, a numerical example is provided to illustrate the theoretical results. Specifically, the evolution of the probability of failed audit over time is depicted in Figure 6. In Figure 6a the case of a non independent auditor is examined $(I^{\infty} \to 0)$. It is observed that in the first two auditing periods the probability of failed audit is reduced due to the increase of the auditor's ability to detect frauds. In the subsequent auditing period the auditee firm knows that even if the financial statements are materially misstated as a result of fraud, then, this




(a) $I^{\infty} \to 0$




(b) $I^{\infty} \to 1$

Fig. 6: Evolution of the probability of failed audit over time

action will be detected by the auditor. Therefore, the financial statements are prepared, in all material respects, in accordance with an applicable financial reporting framework. Unfortunately, after a few auditing periods, sympathy may affect auditor's professional independence and objectivity leading to an increased probability of failed audit. On the other hand, if the ideal case of an independent auditor is considered, then the learning effect has a favorable impact on audit quality leading in the long run to zero probability of failed audit (see Figure 6b).

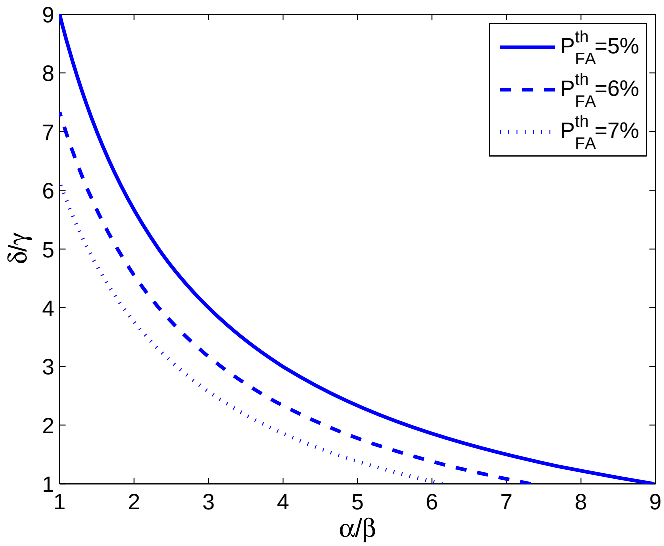



Fig. 7: Payoff values for $\alpha/\beta$ and $\delta/\gamma$ for different values of $P_{FA}^{th}$

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In Figure 6 it is also observed that the probability of failed audit is relatively high (about 40% during the early stage of the audit process). However, in practice, this probability should be kept below a specific threshold, $P_{FA}^{th}$, that usually takes values in the range $5-7\%$. Thus, the payoffs of both players should be selected in a way that satisfies the following inequality expressing the quality of audit:

$$P_{FA}^{*} = y_F \cdot [x_{EA} \cdot (1-p_I) + x_{BA}\,(1-p_E+p_E\,(1-p_I))] \leq P_{FA}^{th} \tag{36}$$

During the engagement of a new audit, the impact of sympathy on the audit quality is negligible ($P_I \to 1$). Therefore, (36) may be successfully approximated by the following equation

$$P_{FA}^{*} = y_F \cdot x_{BA}\,(1-p_E) = \frac{E_0 e^{-\alpha_L t}}{(\beta/\alpha+1)(\gamma/\delta+1)} \leq P_{FA}^{th} \tag{37}$$

From (37) it is deduced that the equilibrium probability of failed audit is an increasing function of $\alpha$, $\delta$, a decreasing function of $\beta$, $\gamma$ and also decreases with time. The contour plot of parameters $\alpha/\beta$ and $\delta/\gamma$ for different values of $P_{FA}^{th}$ is depicted in Figure 7. It is observed that a stricter threshold for the probability of failed audit may be achieved by increasing the reward gained by the auditor when a fraudulent report is detected or the bonus of the client when the financial reports are free of material misstatements. Moreover, it is clear that, given $P_{FA}^{th}$, a trade-off exists between $\alpha/\beta$ and $\delta/\gamma$. It is also observed from Figure 7 that the probabilities for the client to select $NF$ and for the

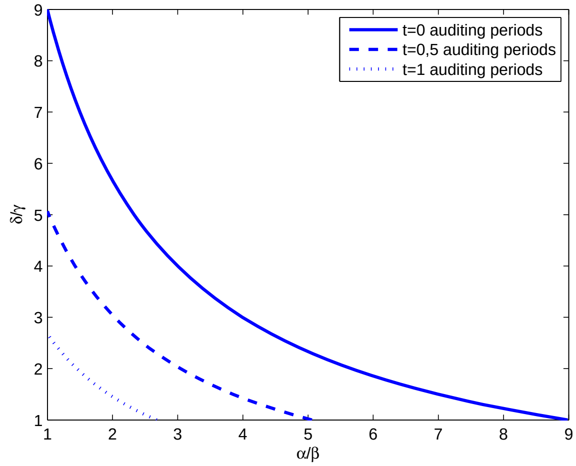


Fig. 8: Payoff values for $\alpha/\beta$ and $\delta/\gamma$ for different auditing periods ($P_{FA}^{th} = 5\%$)

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auditor $EA$ increase with $\alpha/\beta$ and $\delta/\gamma$, respectively. Therefore, it is deduced that the more determined the auditor is to reveal fraud, the less probable is for the client to commit a fraud. Finally, as deduced from Figure 8 with the increase of the auditor's tenure on the auditee firm, the same threshold for the probability of failed audit may be achieved by reducing the reward gained by the auditor when a fraudulent report is detected or the bonus of the client when the financial reports are free of material misstatements.

So far, the risk assessment policy has been limited to audit procedures between two new engagement parts where the impact of sympathy is negligible. Unfortunately, after long term audit engagements the quality of audit may be significantly mitigated since the auditor's actions are governed by sympathy. In this case, the probability of failed audit is an increasing function of Independence. To avoid this undesirable effect, replacement of the auditor is required.

The optimal rotation time or different values of the auditor's personal characteristics is depicted in Figure 9. It is observed that the less independent the auditor is the sooner should be replaced i.e. for large values of $\alpha_I$ the auditor, despite how experienced he is, should be replaced, at most, after three auditing periods. On the contrary, an independent auditor with increased knowledge acquisition skills should be replaced after a high number of auditing periods (above 8).

Finally, in order to support the predictions of the proposed theoretical model, the empirical data used in Fairchild et al., (2009) have been adopted. Fairchild et al., (2009), identified instances of fraudulent financial reporting by examining the gross number of qualified reports included in the FAME

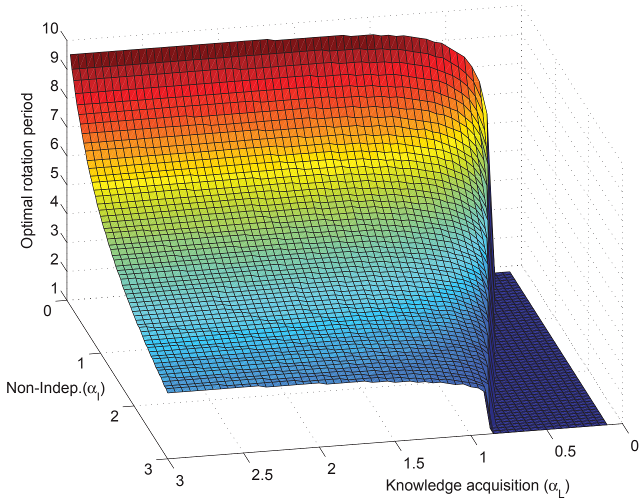


Fig. 9: Optimal rotation time for the auditor for different values of the auditor's personal characteristics ($P_{FA} = 10\%, I^{\infty} \to 0$)

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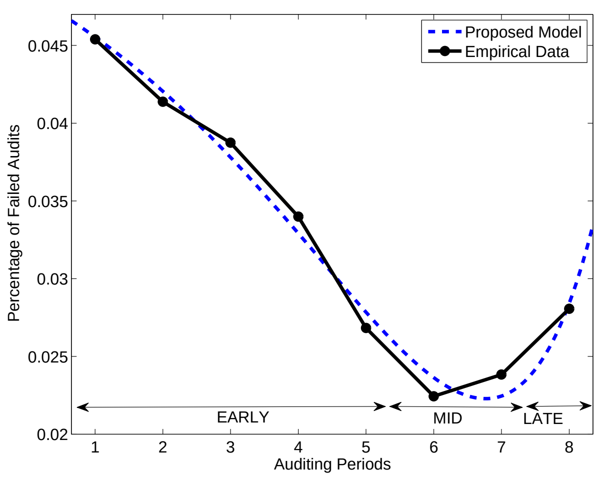


Fig. 10: Experimental validation of the proposed scheme

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database for the time period 1995-2005. Their analysis has been carried out taking into account the following conditions:

i. A company in order to be included in the sample must have a qualified report in the final period of filing and 8-10 accounting years must be also present.
ii. A company must have the same auditor for at least a 9 year period and at least one Qualified report in that period.

From the overall sample, the authors in Fairchild et al., (2009) identified 684 companies satisfying the above criteria with mean tenure 5.62 years. In order to integrate these empirical findings in the proposed scheme, it has been assumed that the audit process concerning the companies included in FAME database has been performed for an audit risk nominal level equal to 5%. Based on this assumption, $P_{FA}$ can be empirically estimated through the fraction of the number of non-qualified reports over the total number companies, multiplied by the audit risk level. The evolution of the probability of failed audit over time, both for theoretical model and the empirical data is depicted in Figure 10. It is observed that the curve describing the theoretical model is very close to the empirical data with coefficient of determination, $R^2 = 0.971$. The theoretical model has been evaluated for $I^{\infty} = 0.141$, $\alpha_I = 0.054$ and $\alpha_L = 0.285$, where $I^{\infty}$, $\alpha_I$ and $\alpha_L$ have been estimated using the well known least squares fitting technique. As expected, during the early phase of the audit process, $P_{FA}$ is reduced over time due to the client-specific learning effect. However, after few auditing rounds, physiological effects lead to an increase of the probability of failed audit.

## 6 Concluding Remarks

The paper studied the problem of fraud deterrence and detection in financial reporting taking into consideration all the fundamental principles and threats that may affect the performance of an audit process. In contrast to the existing schemes that segmentally studied the threats that could lead to a failed audit, in the current approach, a novel scheme has been proposed based on social psychology principles. The problem has been formulated and studied using game theory. A unified model has been developed to jointly confront the basic threats that are involved in an auditing process and provide reasonable assurance for that the financial statements are free from material misstatement. It has been proved that the auditing/fraud detection game between two new engagement parties has unique mixed strategy equilibrium, between an experienced auditor and a client has unique pure strategy equilibrium, whereas between two old engagement parties the audit process switches to new pure strategy equilibrium that is governed by sympathy. Finally, a closed form solution for the optimal auditor's period replacement has been extracted. The validity of the proposed scheme was examined using empirical data. The results obtained comply with the IFAC Code of Ethics for Professional Accountants that requires the key audit partner to be rotated after a predefined period.

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