This model of visual enumeration is based on three assumptions:

1. Percepts of the numerosity of integer numbers of tokens do not necessarily correspond to integer values, and vary stochastically about a mean value from trial to trial.
2. Whenever more than one process is available to inform numerosity responses, observers will rely on the process that produces the larger percept.
3. Any single enumeration process can be described by a function relating actual and perceived numerosity whose first derivative is a completely monotonic function: $(-1)^n f^{(n)}(x) \geq 0$.

The model consists of three parts.

1. A pair of equations that explain the relationship between a latent continuous function relating actual and perceived numerosity, and overt integer responses. Accuracy for enumerating x visual tokens is

$$f(x;\mu,\sigma^2) = \begin{cases} \mathbf{\Phi}\left(\frac{(x+1/2)-\mu}{\sigma}\right) - \mathbf{\Phi}\left(\frac{(x-1/2)-\mu}{\sigma}\right) & \text{if } x > 1 \\ \mathbf{\Phi}\left(\frac{(x+1/2)-\mu}{\sigma}\right) - \mathbf{\Phi}\left(\frac{0-\mu}{\sigma}\right) & \text{if } x = 1 \end{cases}$$

where $\mathbf{\Phi}$ is the cumulative distribution function for the normal distribution $\mathcal{N}(\mu,\sigma^2)$. The parameters can be defined as functions of x, e.g. $\mu = kx^\lambda$ and $\sigma^2 = ax$, that define the latent function.

Given equal numbers of trials, the probability of a response equal to any element of the domain of numerosities presented, $X = \{1,2,3,\dots,n\}$, is

$$f(x;\mu,\sigma^2) = \sum_{i \in X} \frac{\mathbf{\Phi}\left(\frac{(x+1/2)-\mu_i}{\sigma_i}\right) - \mathbf{\Phi}\left(\frac{\text{lower}-\mu_i}{\sigma_i}\right)}{n} \qquad \text{lower} = \begin{cases} x - 1/2 & \text{if } x > 1 \\ 0 & \text{if } x = 1 \end{cases}$$

where e.g. $\mu_i = ki^\lambda$ and $\sigma_i^2 = ai$.

2. Specification of techniques for estimating actual-perceived numerosity functions from enumeration experiment data using cumulative link models (CLMs).

Following Christensen & Brockhoff (2013, Journal de la Société Française de Statistique), a CLM is a linear model for an ordinal response variable Y_i that can fall in J categories, for n stimulus conditions. It models the transformation of cumulative probabilities γ_{ij} via a link function

$$P(Y_i \leq j) = \gamma_{ij} = \mathbf{\Phi}\left(\frac{\theta_j - \mathbf{x}_i^\top \boldsymbol{\beta}}{\exp(\mathbf{z}_i^\top \boldsymbol{\zeta})}\right) \qquad i = 1,\dots,n \qquad j = 1,\dots,J$$

where θ_j are ordered thresholds

$$-\infty \equiv \theta_0 \leq \theta_1 \leq \cdots \leq \theta_{J-1} \leq \theta_J \equiv \infty$$

For each stimulus condition beta and zeta weights can be estimated to model stimulus effects on the mean response and its variability. When the stimulus conditions are numerosities in an enumeration experiment, the beta weights represent a latent variable corresponding to perceived numerosity.

3. A statistical analysis criterion for identifying domains of numerosities that correspond to distinct psychological processes, as opposed to a single process. For a given domain of numerosities, this is a test for violations of complete monotonicity of the derivative of the function described by the CLM beta weights.