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Proposed structure for large quantum interference effects

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In this letter we propose and analyze a new semiconductor structure that can be fabricated by present day technology and can lead to large quantum interference effects with potential device applications.

Oscillations in the conductance as a function of magnetic field due to quantum interference (Aharonov–Bohm effect) have recently been observed both in metallic rings[1] and in semiconductor microstructures.[2] The magnitude of the oscillations is less than 1%, typically much less. In this letter we propose a new semiconductor structure [Fig. 1(a)] that should exhibit large (up to 100% in principle) oscillations in the conductance in a magnetic field or in an electric field (electrostatic Aharonov–Bohm effect). This structure can be fabricated by etching and regrowth[3] using present day semiconductor technology and has potential applications as a quantum interference transistor requiring extremely low gate voltages (~ 1 mV or less). The current through the device is determined by the quantum interference between two parallel quantum wells and can be controlled with a third terminal by changing the relative phase between the two paths. This is very different from other quantum devices like the resonant tunneling diode[4] which involves electron transport perpendicular to the quantum wells. The quantum effects in such structures arise from the interference between the multiple reflections from various interfaces.

The basic idea is as follows. The structure consists of two parallel GaAs quantum wells separated by an AlGaAs barrier. The barrier is thinner at the two ends. The parameters are chosen such that there is considerable tunneling between the wells at the ends but hardly any in the central region. The large tunneling at the ends makes the symmetric state $|S\rangle$ lower in energy than the antisymmetric state $|A\rangle$ as shown in Fig. 1(b) and most of the electrons occupy this state if the carrier density is such that the Fermi level is located as shown in Fig. 1(b). The wave function $\psi(\mathbf{r}, t)$ for electrons at the ends can then be written as

$$\psi(\mathbf{r}, t) = |S\rangle \exp i[k_x x + k_y y - Et/\hbar]. \tag{1a}$$

In the central region there is little tunneling so that the two states are degenerate. Here we can write the wave function $\psi(\mathbf{r}, t)$ as a linear combination of the wave functions $|1\rangle$ and $|2\rangle$ in the two (isolated) channels.

$$\psi(\mathbf{r}, t) = [C_1(x)|1\rangle + C_2(x)|2\rangle]\exp i[k_y y - Et/\hbar]. \tag{1b}$$

Since the electron is in a symmetric state at the left end, it will transmit nearly symmetrically into the channels so that at $x = 0$,

$$C_1(0) \simeq C_2(0). \tag{2}$$

Let k_1 and k_2 be the wave vectors in the x direction along channels 1 and 2 respectively:

$$C_1(L) = C_1(0)e^{ik_1L}, \tag{3a}$$

$$C_2(L) = C_2(0)e^{ik_2L}. \tag{3b}$$

At the right end ($x = L$) the symmetric component propagates out freely while the antisymmetric component is completely reflected. Thus the transmission coefficient $|T|^2$ for electrons from the left end to the right end is given by

$$|T|^2 = \left|\frac{C_1(L) + C_2(L)}{C_1(0) + C_2(0)}\right|^2. \tag{4}$$

Using Eqs. (2) and (3), we get from Eq. (4)

$$|T|^2 = \cos^2(k_1 - k_2)L/2. \tag{5}$$

We are assuming that the length L is shorter than a mean free path so that both elastic and inelastic scattering can be neglected. With a mobility of 10^5 cm^2/V s and a velocity of 10^7 cm/s the mean free path is $\sim 0.4\ \mu$m.

The conductance G of the structure for small voltages is proportional to the transmission coefficient $|T|^2$.[4]

$$G = e^2 W \int \frac{dk_x dk_y}{2\pi^2} \frac{\hbar k_x}{m^*} |T|^2 \left[-\frac{\partial f}{\partial E}\right], \tag{6}$$

where m^* is the effective mass and W is the width of the structure in the y direction. The transmission coefficient $|T|^2$ and hence the conductance G can be controlled if we can control $(k_1 - k_2)$. The principle described above is very similar to optical interference experiments in which a polarized beam is split in two parts, given different phase shifts and then analyzed. The functions of the two end regions of the structure shown in Fig. 1 are analogous to those of the "polarizer" and the "analyzer," respectively.

If the two channels are identical, then $k_1 = k_2$ and the transmission coefficient is equal to 1. If we apply a magnetic field B_y so that there is a vector potential $\mathbf{A} = (B_y z, 0, 0)$, then we can show that

$$k_1 - k_2 = eB_y d/\hbar, \tag{7a}$$

where

$$d = \langle 1|z|1\rangle - \langle 2|z|2\rangle. \tag{7b}$$

The wave functions in the two channels are also skewed somewhat by the magnetic field which can be neglected to first order. From Eqs. (5) and (7) we get

$$|T|^2 = \cos^2(e\Phi/2\hbar), \tag{8a}$$

where

$$\Phi = B_y dL. \tag{8b}$$

A similar effect is obtained if the average potential in channel 1 is different from that in channel 2 by an amount ΔV; this is accomplished with a transverse electric field $\mathscr{E}_z$. We then have

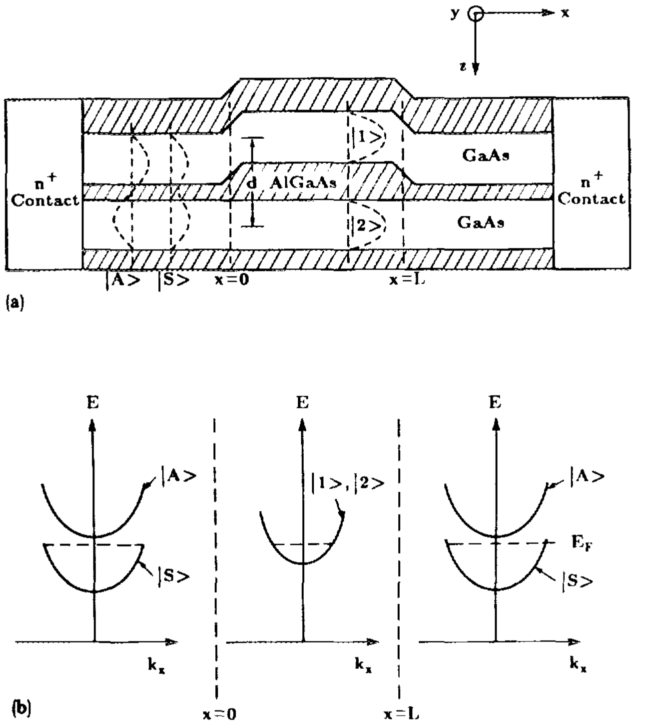

FIG. 1. Proposed structure. (a) Configuration: The current flow is in the x-y plane along the two GaAs quantum wells. $|S\rangle$ and $|A\rangle$ represent the wave functions in the z direction for the two lowest transverse modes in the end regions. In the central regions the eigenmodes $|1\rangle$ and $|2\rangle$ are isolated and degenerate. (b) E vs k_x at different points for the different transverse eigenmodes.

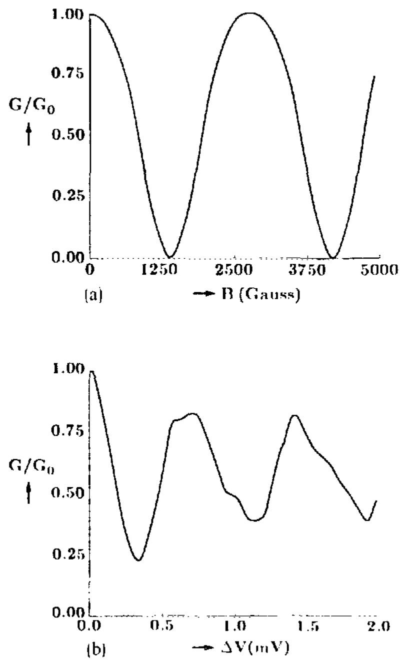

FIG. 2. Normalized conductance of the structure in Fig. 1 ($L = 5000$ Å) as a function of (a) magnetic field, B_y (assuming $d = 300$ Å) and (b) potential difference ΔV between the channels (assuming $v_f = 10^7$ cm/s).

$$e\Delta V = (\hbar^2/2m^*)(k_1^2 - k_2^2). \tag{9}$$

Assuming that $k_1 \simeq k_2 = k$, we get from Eq. (9)

$$k_1 - k_2 = e\Delta V/\hbar v, \tag{10a}$$

where

$$v = \hbar k/m^*. \tag{10b}$$

From Eqs. (5) and (10a)

$$|T|^2 = \cos^2(e\Delta V\tau_t/2\hbar), \tag{11}$$

where $\tau_t = L/v$ is the transit time of electrons through the channels. It is evident from Eqs. (8a) and (11) that the transmission coefficient of electrons can be modulated using either a magnetic field B_y or an electric field $\mathscr{E}_z$. An important point of difference between the two cases is that with a magnetic field, T is the same for all the electrons while with an electric field T depends on k_x through the transit time τ_t. Consequently the overall conductance modulation, which is determined by an average over all the electrons as indicated in Eq. (6), is smaller for the electrostatic Aharonov–Bohm effect compared to the magnetic one (Fig. 2).

The simple description presented above is qualitatively correct but it is complicated somewhat by multiple reflections. These can be accounted for by using scatter matrices; a similar approach has been used in the past to analyze one-dimensional metal rings.[5–7]

$$\begin{Bmatrix} C_1^-(0) \\ C_1^+(L) \\ C_2^-(0) \\ C_2^+(L) \end{Bmatrix} = \begin{bmatrix} 0 & \exp(ik_1^-L) & 0 & 0 \\ \exp(ik_1L) & 0 & 0 & 0 \\ 0 & 0 & 0 & \exp(ik_2^-L) \\ 0 & 0 & \exp(ik_2L) & 0 \end{bmatrix} \begin{Bmatrix} C_1^+(0) \\ C_1^-(L) \\ C_2^+(0) \\ C_2^-(L) \end{Bmatrix}. \tag{12}$$

Here the superscripts + and − are used to denote the amplitudes of wave functions propagating in the positive and negative x directions, respectively. For an electric field $\mathscr{E}_z$,

$$k_1 = k_1^- = k + (e\Delta V/2\hbar v), \tag{13a}$$

$$k_2 = k_2^- = k - (e\Delta V/2\hbar v), \tag{13b}$$

where $\Delta V = \mathscr{E}_z d$. For a magnetic field B_y,

$$k_1 = k_2^- = k + (eB_yd/2\hbar), \tag{14a}$$

$$k_2 = k_1^- = k - (eB_yd/2\hbar). \tag{14b}$$

The amplitudes C_S and C_A of the symmetric and antisymmetric components are defined by

$$C_S = (1/\sqrt{2})(C_1 + C_2), \tag{15a}$$

$$C_A = (1/\sqrt{2})(C_1 - C_2). \tag{15b}$$

The antisymmetric component is completely trapped between $x = 0$ and $x = L$ and cannot propagate in the two end regions with enhanced tunneling. Assuming that the reflection coefficient for the antisymmetric state is -1 at either end we get from Eq. (12)

$$\begin{Bmatrix} C_S^-(0) \\ C_S^+(L) \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} C_S^+(0) \\ C_S^-(L) \end{Bmatrix}, \tag{16}$$

where

$$a = e^{ikL} \cos(k_1 - k_2)L/2, \tag{17a}$$

$$b = e^{ikL} \sin(k_1 - k_2)L/2, \tag{17b}$$

$$b' = e^{ikL} \sin(k_1^- - k_2^-)L/2, \tag{17c}$$

$$S_{11} = S_{22} = -bb'/(1 - a^2), \tag{18a}$$

$$S_{21} = S_{12} = a[1 + b^2/(1 - a^2)]. \tag{18b}$$

If we assume that the symmetric component flows freely to and from the end regions without any reflection, then the transmission coefficient T is given by

$$T = S_{12}.$$

Figures 2(a) and 2(b) show the conductance G calculated as a function of the magnetic field B_y and the voltage difference ΔV respectively. In these calculations we assumed $L = 5000$ Å and $d = 300$ Å. The temperature is assumed low enough that $(-\partial f/\partial E)$ in Eq. (6) can be replaced by $\partial(E - E_f)$; the Fermi velocity v_f was assumed to be 10^7 cm/s. As discussed earlier, the conductance G goes to zero for certain values of B_y but is not zero for any value of ΔV because the phase shift is not uniform for all electrons. It is interesting to note that the conductance can be decreased by 75% with a voltage of only ~ 1 mV. This effect has potential applications as a quantum interference transistor requiring extremely low gate voltages. In practice, the symmetric component will not flow freely to and from the ends but will suffer some reflections. This can be taken into account using the scatter matrix in Eq. (16). We find that the plots in Fig. 2 are distorted significantly when the reflections are large and the results depend on the phase of the reflection coefficient.

To summarize, in this letter we have proposed and analyzed a new semiconductor structure that can be fabricated by present day technology and should exhibit large quantum interference effects with potential device applications.

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