2018

"On the Dynamics of a Three Stage Single Acting Reciprocating Compressor"

Sinan "Pişirici"
Dalgakıran Makina San. ve Tic. A.Ş., Istanbul, Turkey, sinan.pisirici@dalgakiran.com

Eren "Çakır"
Dalgakıran Makina San. ve Tic. A.Ş., Istanbul, Turkey, eren.cakir@dalgakiran.com

Haluk "Erol"
Istanbul Technical University, Mechanical Engineering Faculty, Gümüşsuyu, 34439, Istanbul, Turkey., erolha@itu.edu.tr

On the Dynamics of a Three Stage Single Acting Reciprocating Compressor

Sinan Pişirici[1]*, Eren Çakır[1], Haluk EROL[2]

[1]Dalgakıran Compressor, Research & Development,
Istanbul, Turkey
sinan.pisirici@dalgakiran.com, eren.cakir@dalgakiran.com

[2]Istanbul Technical University, Mechanical Engineering Faculty,
Istanbul, Turkey
erolha@itu.edu.tr

* Corresponding Author

ABSTRACT

The growing demand in the market for highly efficient, more reliable and less expensive compressors have activated the manufacturers to develop state-of-the-art analytical tools to predict, evaluate and optimize the existing as well as creating the new designs. Development of a mathematical tool for predicting the dynamic behavior saves considerable time and reduces the unnecessary number of prototypes as well as the development costs. In the present study, a non-linear dynamic model of a three stage single acting reciprocating compressor is developed to predict the entire motion of the compressor. The analytical model prepared for simulation of the motion is initially tested in a limited experiments for validation and an experimental setup is prepared. Pressure transducers are carefully mounted to record the pressure variation within the cylinder head. Using this data the steady state PV-diagram is constructed and the pressure variation within the cylinder head is obtained as function of the crank angle. Further modifications are being made in the setup to identify the angular position of the crank shaft and to record the angular speed of the crank shaft as a function of time. With these modifications the analytical model results are compared with the experimental findings and good agreement was observed. Then, a number of parametric studies are performed and the results are presented in the study.

1. INTRODUCTION

Reducing dynamic loads and speed fluctuations in piston compressors will also reduce the vibration levels of the compressor. Reducing the vibration levels reduces the possible damage caused by vibration and affects the compressor life (Hanlon, 2001). In addition, lowering the vibration levels means lowering the noise level of the machine.

Many studies on single-cylinder compressors offer a useful approach to solving this problem. However, in order to reveal the dynamic loads and angular velocity fluctuations of three-stage w-shape reciprocating compressors, a study involving the equation of motion is required. Therefore, a mathematical model was created by performing a parametric study for a w-shape three stage piston compressor.

In order to obtain the mathematical model, the necessary equations for the w-type three-stage compressor were obtained by referring to the studies made for the single-cylinder compressors in the literature (Shigley, J.E. *et al.*, 1981). The displacement, velocity, and acceleration equations for each piston are obtained depending on the crankshaft angle. In addition, cylinder internal pressure variations in the stages are modeled depending on the piston positions. Using the obtained cylinder internal pressures, the mathematical model of the compressor's torque requirement is obtained depending on the crankshaft angle. However, in order to solve the equation of motion, a mathematical model of the torque variance depending on the angular velocity of the electric motor that drives the compressor has been obtained. The mathematical model of the equation of motion is obtained through these studies. The generated model is solved with MATLAB software and parametrically presented. Thanks to this model,

machine torque diagrams and P-V diagrams of stages are obtained, the equation of motion is solved, and the angular speed change is obtained. At this point, the obtained data reveals the dynamic structure of w-type three stage reciprocating compressor to a great extent.
In the model verification tests, cylinder internal pressures, angular velocity and torque values were measured simultaneously depending on the crank angle, and the model was verified.
In the future, this study aims to shed light on the design of more efficient, quieter and more reliable reciprocating compressor.

2. MATHEMATICAL MODEL

In order to demonstrate the kinematics of the three-stage W type reciprocating compressor, the position, velocity and acceleration equations of each piston must be obtained.

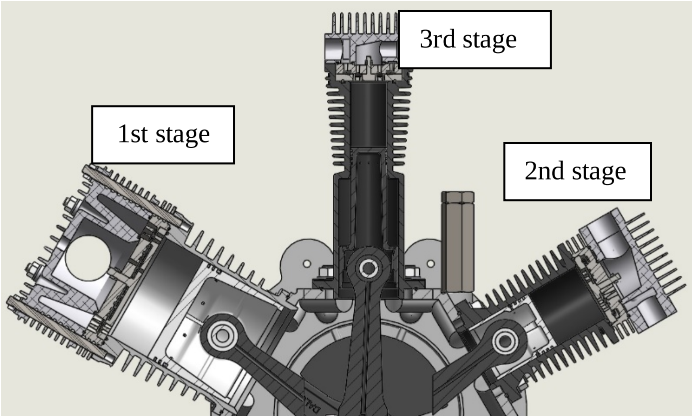

Figure 1: Three stage single acting reciprocating compressor

It is assumed that the angle of the crankshaft is 0 ° when the third piston is in the top dead center before starting to obtain the equations. Machine rotation direction is clockwise. The rotation angle of the first stage piston is greater than the crankshaft angle by the angle between each cylinder. In the same way, the rotation angle of the second stage piston is smaller than the crankshaft angle by the angle between the cylinders. In this way, the positions of the stage pistons can be written as follows where φ the angle between the cylinders is.

$$x_p = \left[r + \frac{r^2}{4l}\right] - \left[r * cos(q+\varphi) + \frac{r}{4l} * cos2(q+\varphi)\right] \tag{1}$$

In order to examine the dynamic behavior of the machine, the equation of motion of the machine needs to be obtained and solved. This will determine the characteristics that affect the machine's performance, such as changes in cylinder internal pressures, moment curves, angular velocity.
The total moment of inertia of the system, J_T, is equal to the reduced moment of inertia of the stages plus the moment of inertia of the electric motor, flywheel and crankshaft, expressed as:

$$J_T = J_1 + J_2 + J_3 + J_m + J_v + J_k \tag{2}$$

The reduced moment of inertia of the stages is expressed as follows:

$$J_{1,2,3} = (m_{bp} + m_p) * K_p{}^2 \tag{3}$$

Each K_p is the ratio of the each piston speed to the crankshaft angular velocity.

$$K_p = \frac{\dot{x}_p}{\dot{q}} \tag{4}$$

The motor torque equation based on the angular velocity is obtained by the curve fitting method on the motor torque curve.

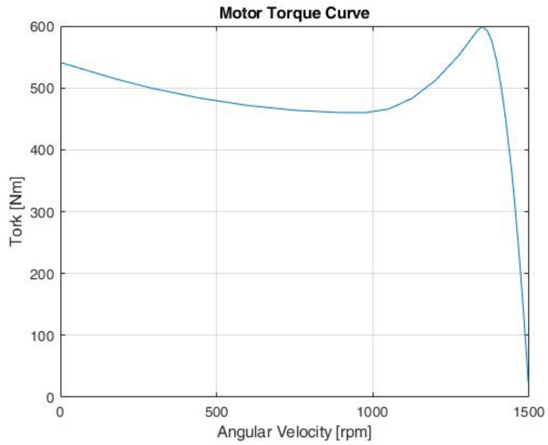

Figure 2: Motor torque curve by curve fitting method

The internal pressure model of the stages can be obtained by writing the 4 sections of the P-V diagram as a piecewise function.

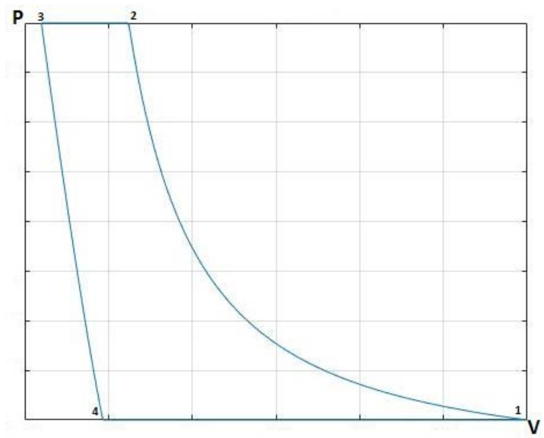

Figure 3: Basic P-V Diagram

The cylinder internal pressure of each stage is modeled parametrically with the functions to be written for each section in the diagram (Boles, M.A., *et. al.*, 2006).

$$P_{1-2} = P_s * \left(\frac{V_1}{(2 * r - x_p) * A + V_3}\right)^n \tag{5}$$

$$P_{2-3} = P_d \tag{6}$$

$$P_{3-4} = P_d * \left(\frac{V_3}{(2 * r - x_p) * A + V_3}\right)^n \tag{7}$$

$$P_{4-1} = P_s \tag{8}$$

The obtained equations are expressed as a single function depending on the crankshaft angle as follows.

$$P_c = \begin{cases} P_{3-4}, & (q+\varphi)_{mod2\pi} < \pi; \quad P_{3-4} > P_i \\ P_{4-1}, & (q+\varphi)_{mod2\pi} < \pi; \quad P_{3-4} \leq P_i \\ P_{1-2}, & \pi < (q+\varphi)_{mod2\pi} < 2\pi; \quad P_{1-2} < P_o \\ P_{2-3}, & \pi < (q+\varphi)_{mod2\pi} < 2\pi; \quad P_{1-2} \geq P_o \end{cases} \tag{9}$$

Thanks to the cylinder internal pressures obtained, compressor torque can be modeled.

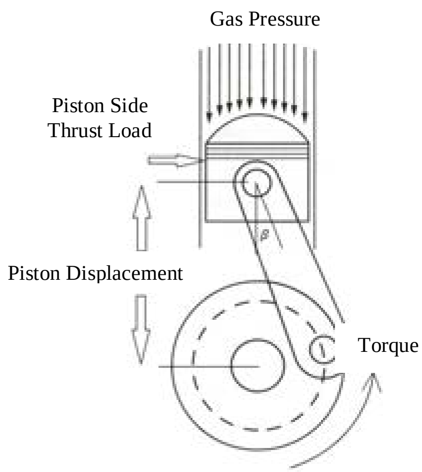

Figure 4: Piston-crank mechanism

Depending on the crankshaft angle, the angle of the connecting rod to each piston axis is written as follows.

$$\beta = \arcsin\left(\frac{r}{l} * \sin(q + \varphi)\right) \tag{10}$$

The moments at which each stage acts on the crankshaft can also be modeled as follows.

$$\tau_k = P_c * \mathrm{A} * tan\beta * (x_p + l - r) \tag{11}$$

Compressor torque is obtained by adding up each stage moment.

$$\tau_k = \tau_{k1} + \tau_{k2} + \tau_{k3} \tag{12}$$

The mathematical model of all expressions in the equation of motion is obtained parametrically. By solving the obtained differential equation, the angular velocity change of the machine, the torque curve, the P-V diagrams of the stages are obtained.
The general expression of the equation of motion is;

$$J_T(q) * \ddot{q} + \frac{1}{2} * \frac{dJ_T}{dq}(q) * \dot{q}^2 = \tau_m(\dot{q}) - \tau_k(q) \tag{13}$$

3. EXPERIMENTAL SETUP

Up to this point, the dynamic mathematical model of the three-stage reciprocating compressor has been obtained and solved. The P-V diagrams of the stages, the torque curve of the machine and the angular velocity graphs are obtained. An experimental setup was established to verify the obtained data with the tests.
In order to obtain the P-V diagrams of the stages, the cylinder internal pressures were measured from the dead volume area of the valves (Bhakta A. *Et Al.*, 2012). The preferred pressure transmitter for the experiment must have high sensitivity at high temperatures, since the air outlet temperature at the area to be measured is 180 °C.
Since the pressure are measured from the dead volume in the valve discharge area, there is a serious space problem for the pressure transmitter to be installed.

As a result of the investigations, it was decided to use Keller PAA-M5 HB type pressure transmitters which fulfill all these conditions. Transmitters with measurement options of 10bar for 1st stage, 30bar for 2nd stage and 50bar for 3rd stage are used.

Table 1: Technical specifications of pressure transmitters

Operating temperature range	-40°C - 200°C
Signal measurement range	0-50kHz
Accuracy	±0,1 %FS
Measuring pressure	10-30-50bar

According to the valve designs in the stages, the places where the transmitters are to be installed are determined.

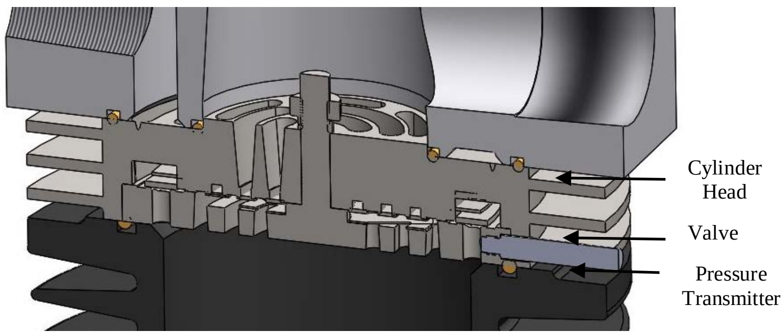

Figure 5: Pressure transmitter assembly

In order to measure the machine moment and angular velocity, a Burst 8661 model torque meter with encoder is used.

Table 2: Technical specifications of torque meter

Measurement Range	0 - 1000Nm
Accuracy	±0,05 %FS
Signal measurement range	0-25kHz
Angle measurement range	0,088°

The torque meter is mounted between the bare pump of the compressor and the motor via two flexible couplings.

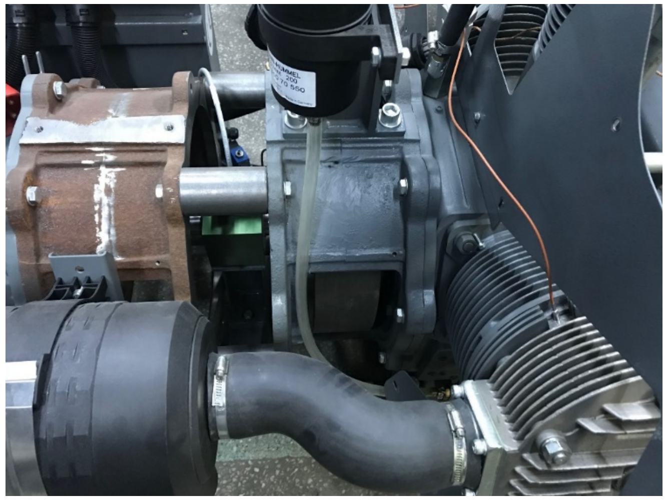

Figure 6: Torque meter assembly

Once the pressure transmitters and the torquemeter have been installed, the test setup is ready to take measurements. Pressure, torque, and angular velocity measurements were performed at a sampling frequency of 20kHz with a Dewesoft DEWE 43 data acquisition device.

4. MODEL VALIDATION

The input parameters including the physical properties of the Dalgakıran Compressor W166 W-type three-stage reciprocating compressor are shown below.

Table 3: Compressor physical parameters

First Stage Cylinder Diameter	160
Second Stage Cylinder Diameter	85
Third Stage Cylinder Diameter	52
Suction Capacity [l/min]	2767
Working Pressure [bar]	40
Shaft Power [kW]	27.4
Stroke [mm]	94
Angle Between Cylinders [deg]	60
Moment of inertia of crankshaft [kg*m^2]	0.095
Moment of inertia of motor [kg*m^2]	0.3034
Moment of inertia of flywheel [kg*m^2]	0.6

Stage pressures obtained by tests on reciprocating compressor with these physical properties are compared with model outputs.

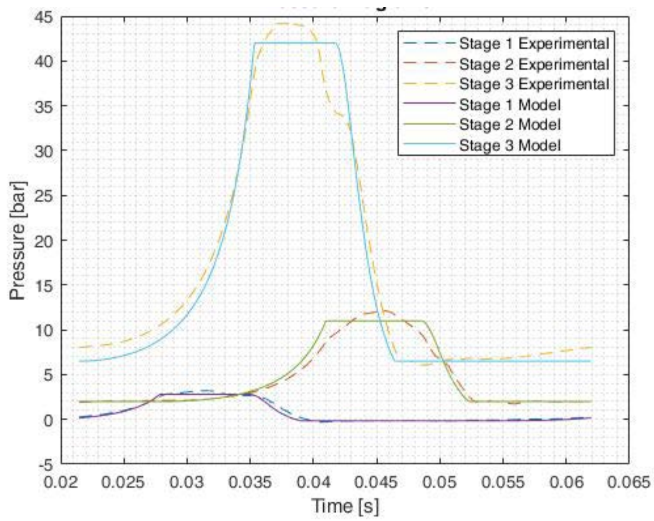

Figure 7: Pressure diagrams of the stages

The test results of the P-V diagrams are consistent with the model outputs. The main reason for overshoots in the pressure diagram is that the valve spring, adhesion and friction forces are not included in the model. Since the small difference in the P-V diagram has little effect on the outputs, it is not necessary to incorporate these forces into the model.

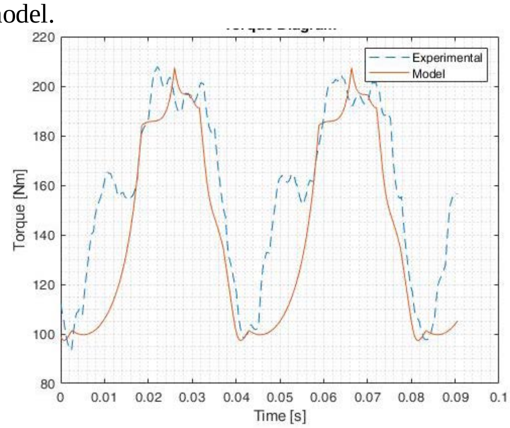

Figure 8: Torque Diagrams

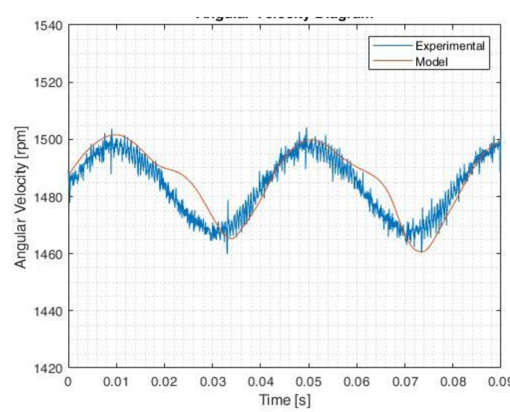

Figure 9: Angular Velocity Diagram

The torque curve and the angular velocity curve obtained from the model and experiment are similar. The difference between the two curves was observed because the friction moments were added to the model constant.

5. PARAMETRIC STUDIES

As a result of verifying the mathematical model with P-V diagrams, torque and angular velocity graphs, optimization of the dynamic loads of the reciprocating compressor has been studied. Depending on the counterweight values, the dynamic loads are plotted.

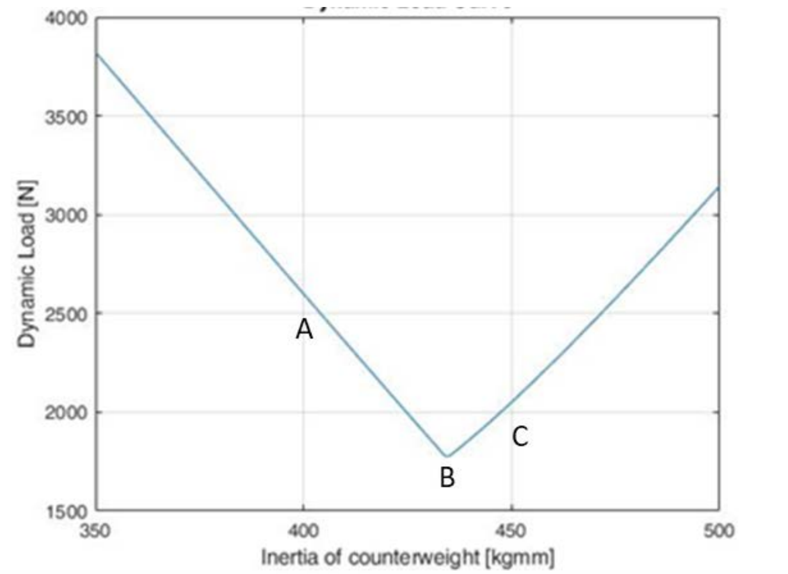

Figure 10: Dynamic load curve

The dynamic loads obtained for counterweight values corresponding to points A, B, C are:

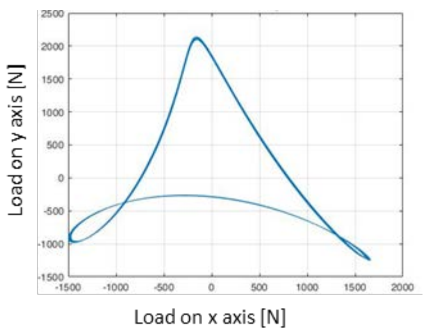

Figure 11: Counterweight of point B

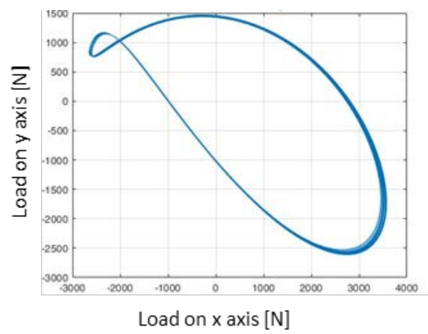

Figure 12: Counterweight of point A

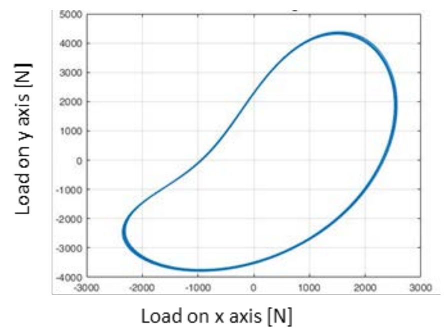

Figure 13: Counterweight of point C

The effect of changing the dynamic loads on the piston compressor is also measured by vibration tests. The results obtained in the vibration tests were similar to those in the counterweight load result diagram.

Table 4: Vibration test results

Measurement Point	Counter Weight	x [mm/s]	y [mm/s]	z [mm/s]
Crank Case	Point. A	13,6	14,2	8,3
Cylinder Head		18,4	19,2	9,6
Crank Case	Point. B	6,2	6,6	4,1
Cylinder Head		7,4	7,7	5,3
Crank Case	Point. C	11,5	12,5	7,9
Cylinder Head		15,3	18,6	8,9

6. CONCLUSION

The obtained theoretical and experimental results were examined and the model improvement studies were made and verified. In this way, a highly accurate numerical model can be used for all piston compressors to be designed from now on.
Dynamic loads, P-V diagrams of stages, angular velocity change and moment curves are modeled for a three-stage piston compressor. By modeling and calculating dynamic loads, optimum crankshaft and counterweight design can be made. At this point, improvement in machine vibration performance is observed.
Through the obtained numerical model, designs can be realized without experimental studies. This will provide both economic and temporal benefits. All these studies constitute an important milestone for reciprocating compressors to be designed in the future.

NOMENCLATURE

J_1	Reduced moment of inertia of 1st stage
J_2	Reduced moment of inertia of 2nd stage
J_3	Reduced moment of inertia of 3rd stage
J_k	Moment of inertia of crankshaft
J_m	Moment of inertia of motor
J_v	Moment of inertia of flywheel
l	Length of connecting rod
m_p	Total weight of piston, ring and pin
m_k	Weight of crankshaft with counter weight
m_b	Weight of connecting rod
r	Radius of crank
x_p	Piston displacement
q	Angle of crank
φ	Angle between cylinder
β	Angle of connecting rod

REFERENCES

Bhakta A. Et Al., (2012). “A Valve Design Methodology for Improved Reciprocating Compressor Performance” International Compressor Engineering Conference, 1252: 1-8.

Boles, M.A., Çengel Y.A., (2006). Thermodynamics and Engineering Approach, Fifth Edition, Mc Graw-Hill Book Co., New York.

Elhaj, M., Gu F., (2008). “Numerical simulation and experimental study of a two-stage reciprocating compressor for condition monitoring” Mechanical Systems and Signal Processing, 22: 374-389.

Hanlon, P.C., (2001). Compressor Handbook, Mc Graw-Hill Book Co., New York.

Lee, F.Y., Huang Y.C., (2012). “Dynamic Balancing Modal Analysis and Vibration Suppressing Design for Reciprocating Compressor Crankshaft” Applied Mechanics and Materials, 157: 996-999.

Lin, D.Y., Huang Y.C., (2014). “Dynamic Motion and Modal Analysis Simulation fort he Balance Mass of the Crankshaft of a Two-cylinder Reciprocating Air Compressor” Applied Mechanics and Materials, 511: 696-699.

Olgun, M., (2010). Tek Silindirli Bir Dizel Motorun Atalet Kuvvetlerinin Analizi ve Dengeleme Hesaplamaları, Yüksek Lisans Tezi, İTÜ Fen Bilimleri Enstitüsü, İstanbul

Shigley, J.E., Vicker, J.J., (1981). Theory of Machines and Mechanisms, International Edition, Mc Graw-Hill Book Co., Singapore.