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Effects of Gas Compressibility and Piston Secondary Motion on Leakage in the Piston-Cylinder Clearance of Reciprocating Compressors

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ABSTRACT

Leakage can significantly affect the performance of low-capacity reciprocating compressors, reducing the mass flow rate and increasing energy consumption. In reciprocating compressors, leakage takes place mainly in the piston-cylinder clearance and is brought about by the piston motion and pressure difference between the compression chamber and the shell internal environment. This paper reports a numerical analysis of leakage in the piston-cylinder clearance of a low-capacity reciprocating compressors based on the Reynolds equation for compressible fluid flow. A simulation model is developed and applied throughout the compression cycle to assess the effect of the clearance geometry, piston velocity and piston secondary motion on the leakage and compressor performance. A simplified version of the model considering the piston concentric in the cylinder is also adopted to assess the effect of the piston secondary motion on leakage. The results show that the compressibility effects are significant and have to be considered in the analysis and that the piston secondary motion can increase gas leakage by 90%.

1. INTRODUCTION

Gas leakage is a potential source of inefficiency of small reciprocating compressors used in household refrigeration systems. It can lower the mass flow rate provided by the compressor, reducing its volumetric efficiency, and increase energy consumption, decreasing its isentropic efficiency. Gas leakage takes place in the incomplete sealing of valves and in the piston-cylinder clearance. According to Silva and Deschamps (2015), a gap of 1µm in valves of small reciprocating compressors can reduce the volumetric and isentropic efficiencies by 2.7% and 4.4%, respectively. Leakage in the piston-cylinder clearance are expected to be even more critical and must be carefully taken into account in the design of high-efficiency compressors. Leakage in the piston-cylinder clearance is due to the pressure difference between the compression chamber and the internal environment of the compressor shell, and also affected by the piston motion. In most reciprocating compressors, the cylinder-piston clearance is filled with a mixture of refrigerant gas and lubricating oil. Hence, leakage is more detrimental to oil-free compressors because, in addition to its lubricating function, the oil also acts as a sealing element for the piston-cylinder clearance.

There are several studies related to leakage in clearances. Zuk and Smith (1969) presented an analytical model to assess leakage in small gaps between parallel and small tilt angle plates by applying the compressible flow formulation of Reynolds equation with static boundaries. Ferreira and Lilie (1984) proposed an analytical model to estimate leakage in the piston-cylinder clearance, considering the piston motion and incompressible fluid flow formulation, but disregarding inertial forces. In fact, Yuan et. al (1992) simulated the flow in small clearances with different flow formulations and found that inertial effects are negligible for clearances smaller than 6 μ m.

Although the piston axial motion is the only required motion for the compression process, radial forces arising from the drive mechanism give rise to radial piston motion, also known as piston secondary motion. The calculation of such motion is important for reliability issues as well as for prediction of gas leakage. Prata *et al.* (2000) developed a numerical model to calculate the piston secondary motion considering only oil in the piston-cylinder clearance. Their analysis adopted equations for solving the piston and connecting rod dynamics, and the two-dimensional Reynolds equation for incompressible fluid flow in order to calculate the pressure field in the clearance. The authors investigated parameters such as the wrist-pin position, clearance and oil viscosity on the piston trajectory, oil leakage and power consumption. Later, Grando *et al.* (2006) extended the analysis by adopting a two-phase formulation so as to consider the presence of oil and refrigerant in the clearance, calculating the mixture properties via the void fraction parameter. The authors estimated gas leakage based on the amount of refrigerant dissolved in the leaking oil.

Rigola *et al.* (2009) numerically predicted the piston secondary motion, by solving the piston and connecting rod dynamics and adopting the two-dimensional Reynolds equation for incompressible fluid flow. However, differently from the model developed by Prata *et al.* (2000), the authors assumed the piston-cylinder clearance to be completely filled with refrigerant fluid. A comparative analysis of gas leakage was carried out for the refrigerants R-134a, R-600a and R-744. More recently, Lohn and Pereira (2014) developed a three-dimensional simulation model for predictions of gas leakage in the piston-cylinder clearance, which were compared with the results of two analytical models: (i) Zuk and Smith (1969) and (ii) Ferreira and Lilie (1984). Lohn and Pereira (2014) obtained results with compressible and incompressible fluid flow formulations and found that the former is more suitable for the analysis. They also showed that the piston radial misalignment significantly affects the leakage.

The studies available in the literature on the effects of piston secondary motion are mainly focused on the fluid film lubricating in the piston-cylinder clearance, and few of them predict gas leakage either by using incompressible fluid formulation (Rigola *et al.*, 2009) or with a costly multiphase flow model (Grando *et al.*, 2006). The present paper presents a numerical analysis of gas leakage in the piston-cylinder clearance based on the two-dimensional Reynolds equation, taking into account compressibility effects and piston secondary motion. The piston secondary motion was solved using the same approach described in Prata *et al.* (2000), by assuming the clearance filled with lubricating oil. However, gas leakage rate for each position of the piston throughout the compression cycle is predicted considering the most critical scenario of clearance filled with gas. This approach requires smaller computational cost than models assuming multiphase flow, allowing the model to be coupled with the simulation model of the compression cycle. The analysis considers different compressor rotational speeds and clearance sizes, allowing estimates of the leakage impact on the compressor performance.

2. NUMERICAL MODEL AND SOLUTION PROCEDURE

Figure 1 shows the flow geometry and solution domain considered in the study. The region between the piston and the cylinder of length L and clearance h represents the solution domain. The pressure in the compression chamber is higher than the pressure in the compressor internal environment for most of the compression cycle duration. This pressure difference and the piston motion bring about the flow in the piston-cylinder clearance. When the piston moves towards the cylinder head, gas is dragged into the compression chamber and vice-versa when the piston moves in the other direction.

The following hypotheses were assumed for the flow in the clearance: i) inertial effects are negligible; ii) the pressure does not vary in the r direction; iii) the piston-cylinder clearance h is much smaller than the piston and cylinder diameters, so the annular gap can be treated as a flat channel; iv) the gas behaves as an ideal gas; v) the gas dynamic viscosity does not vary with z (although it varies from one crank angle to the next); vi) isothermal flow and viii) subsonic flow condition.



Figure 1: Flow geometry and solution domain.

With the hypotheses i)-viii), and by replacing the density by pressure via the isothermal flow condition of an ideal gas (i.e., $p/\rho = \text{constant}$), the governing equation for the compressible flow in the clearance can be modelled by the following simplified form of the Reynolds equation:

$$\frac{1}{R^2} \frac{\partial}{\partial \theta} \left(ph^3 \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(ph^3 \frac{\partial p}{\partial z} \right) = -6\mu V_{\rm p} \frac{\partial (ph)}{\partial z} \tag{1}$$

with p and μ representing the static pressure and the fluid dynamic viscosity, respectively, V_p the piston velocity and R the cylinder radius. Further details about the derivation of the Reynolds equation can be found in Hamrock *et al.* (2004).

Fluid refrigerants do not behave as an ideal gas in typical operating conditions found in household refrigeration. However, we adopted this hypothesis for simplicity since the main objective of our study is to analyze the effect of gas compressibility and piston secondary motion on leakage.

In order to determine the clearance field

$$h(\theta, z) = c - [e_{t} - z/L(e_{t} - e_{b})] \cos \theta$$
(2)

the diametric clearance c and the piston top and bottom eccentricity e_t and e_b (figure 2) must be known. This is achieved with the model proposed by Prata *et al.* (2000), which considers that the clearance is filled entirely by lubricating oil. Such calculations are performed beforehand, determining the piston trajectory, that is, the eccentricities e_t and e_b , as a function of the crank angle. Then, this information is used as an input parameter of the leakage simulation model.

The problem can be significantly simplified if we assume that piston and cylinder are concentric, thus h is constant, and the pressure depends only on the z coordinate. In this case, the governing equation is:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(p\frac{\mathrm{d}p}{\mathrm{d}x}\right) = \frac{6\mu V_{\mathrm{p}}}{h^2}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{3}$$



Figure 2: Piston eccentric position.

The differential equations (1) and (3) were numerically solved with the finite volume method (FVM) by dividing the solution domain into equally spaced control volumes to form the computational mesh. The differential equations are discretized in each control volume, resulting in a system of algebraic equations.

We estimated the pressures at the boundaries of the volumes via the arithmetic mean of the pressures at the neighboring points. The pressure field is calculated by solving the system of algebraic equations with the solver provided by the Eigen library (Guennebaud and Jacob, 2004). Since equations (1) and (3) are non-linear, an initial pressure field is required to determine the coefficients of the algebraic equations. Then, an iterative solution procedure is employed to update the coefficients until reaching the convergence criterion.

Once the pressure field is known, the leakage mass flow rate \dot{m}_1 can be obtained at any section of the domain parallel to the $r\theta$ plane by using:

$$\dot{m}_{\rm l} = \int_0^{2\pi} \left(-\frac{\rho h^3}{12\mu} \frac{\partial p}{\partial z} + \frac{\rho V_{\rm p} h}{2} \right) R \mathrm{d}\theta \tag{4}$$

with ρ being the gas density. The first and second terms inside the parentheses of Equation (4) are the contributions to the leakage promoted by the pressure difference and piston motion, respectively. We evaluate equation (4) numerically, estimating the pressure derivative through a second-order approximation.

Two boundary conditions for pressure are needed in the numerical model. One of them is the pressure in the compressor internal environment, which is constant throughout the compression cycle. The second is the pressure in the compression chamber (p_{cc}), which is evaluated at each crank angle (wt) with a simulation model described in Link and Deschamps (2011). This simulation model also evaluates the piston velocity, dynamic viscosity and density used in equation (4), throughout the compression cycle. In turn, the model for the flow in the piston-cylinder clearance returns the leakage required to evaluate the compression cycle. Figure 3 illustrates the coupling between these two models.



Figure 3: Schematic of the solution procedure.

3. RESULTS AND DISCUSSION

3.1 Predictions with the 1D model

The results presented hereafter were obtained for a low-capacity reciprocating compressor with displacement of 3 cm³, ratio L/R = 1.72, operating with R-600a and evaporating temperature of -23.3°C and condensing temperature of 54.4°C (LBP condition). The effects of compressibility and compressor speed can be investigated by applying the 1D model that assume the piston concentric within the cylinder. For such model the compression cycle was solved for small increments of crank angle ($\Delta wt = 0.001^\circ$). A total of 250 control volumes were employed to discretize the solution domain, so as to guarantee that the difference between predictions and estimates via the Richardson extrapolation was less than 0.01%. The model adopted to simulate the compression cycle model did not solve valve dynamics and heat transfer at the compression chamber walls.

The results of the 1D model were initially validated through comparisons with predictions of a 3D model developed with the CFD code Ansys FLUENT v.14, considering a pressure difference $\Delta p = 7$ bar, diametric clearance $c = 8 \mu m$ and different instantaneous piston velocities. Positive velocities represent that the piston is moving towards the bottom dead center, and the opposite for negative values. As can be seen from Figure 4, the results for leakage mass flow rate (\dot{m}_1) of both models are in good agreement, with differences within 1%.

The compressor rotational speed (*n*) or drive frequency determines the piston velocity, which is also responsible for the flow in the piston-cylinder clearance. In order to investigate the effect of the piston velocity on the leakage throughout the compression cycle, the 1D model was applied with different values of rotational speed. Figure 5 shows the results of these simulations for $c = 5 \ \mu m$ and LBP condition. The 'no velocity' curve represents the results of equation (4) with $V_p = 0 \ m/s$. The vertical axis represents the relative leakage mass flow rate (\dot{m}_1^*), defined as the leakage mass flow divided by the mass flow rate an ideal compressor without leakage could supply. Positive values of \dot{m}_1^* represent gas leaking from the compression chamber and vice-versa. The results show that the piston velocity acts to reduce the leakage rate in the compression and discharge processes (crank angles between 0° and 180°) and the opposite happening in the expansion and suction processes (crank angles between 0° and 180°). In fact, the piston moves towards the top dead center in the compression process and therefore dragging gas into the compression chamber through viscous friction, which reduces \dot{m}_1^* . The opposite effect occurs when the piston moves towards the bottom dead center. Figure 5 also shows that the most significant effect of the piston motion occurs at the crank angle around 135°, when it reaches its maximum velocity. The total leakage throughout the cycle is reduced by roughly 5% when n = 5400 rpm.

The effect of gas compressibility on leakage was investigated by comparing the results of the simulation models for compressible and incompressible fluid flow formulations. The calculations considered the LBP condition and different rotational speeds. Figure 6 shows results of leakage mass flow rate for two diametric clearances (c = 5 and 13 µm). It is clear that predictions with the incompressible fluid formulation overestimates leakage. For instance, considering n = 1800 rpm, the leakage predicted with the incompressible fluid formulation is approximately twice as big as the value predicted by the compressible fluid model.



Figure 4: Comparison between results of the 1D and 3D models.



Figure 5: Relative instantaneous gas leakage rate for different rotational speed; $c = 5 \mu m$, LBP condition: (a) n = 1800 rpm; (b) n = 5400 rpm.

The volumetric efficiency and isentropic efficiency are commonly used to assess compressor performance. The volumetric efficiency is defined as the ratio between the actual mass flow rate, \dot{m} , and the ideal mass flow rate, \dot{m}_{th} , as indicated in equation (5). The ideal mass flow rate would result if there was no reduction of mass flow rate caused by different aspects, such as leakage, cylinder clearance volume, suction gas superheating and flow viscous friction.

$$\eta_{\rm v} = \frac{\dot{m}}{\dot{m}_{\rm th}} \tag{5}$$

The isentropic efficiency is defined as the ratio between the compression power required by an isentropic process and the actual indicated power, i.e.:

$$\eta_{\rm s} = \frac{\dot{W}_{\rm th}}{\dot{W}} \tag{6}$$

Figure 7 shows the results of efficiency reduction due to leakage for different rotational speed and clearances, considering the LBP condition. The efficiency reduction is defined herein as the absolute difference between the compressor efficiencies with and without gas leakage (subscript 'nl'), as indicated in equation (7). As can be seen in Figure 7, the effect of leakage on the reduction of both efficiencies is more significant for smaller drive speed and larger clearances. In the worst case, the efficiency reduction can reach more than 8% for the volumetric efficiency and more than 22% for the isentropic efficiency.

$$\Delta \eta_{v} = |\eta_{v} - \eta_{v,nl}|, \quad \Delta \eta_{s} = |\eta_{s} - \eta_{s,nl}| \tag{7}$$



Figure 6: Predictions of leakage obtained with compressible and incompressible fluid flow formulations for different rotational speeds: (a) $c = 5 \mu m$; (b) $c = 13 \mu m$.



Figure 7: Reduction of volumetric efficiency (a) and isentropic efficiency (b) due to leakage for different rotational speeds and clearances.

3.2 Predictions with the 2D model

In order to assess the influence of piston secondary motion on leakage for the geometry illustrated in figure 8, we first determined the piston trajectory with a model described by Prata et al. (2000). The piston trajectory was calculated for different piston wrist-pin positions (z_p) and different clearances (c). The results are shown in figure 9, where $\varepsilon = e/c$ is the dimensionless eccentricity.

Once the piston trajectory is known, leakage of gas is predicted throughout the compression cycle using the 2D leak model. A 30x30 mesh was adopted after analysis of truncation error. Figure 10 shows the percentage difference between results of leakage predicted with the 2D and 1D models. It is worth noting that leakage is greatly intensified by the piston secondary motion. This effect becomes more important as the eccentricities increase, that is, for larger clearances and when the wrist-pin is located below the piston center. Considering $c = 13 \,\mu\text{m}$ or $z_p/L = 0.75$, leakage due to the piston secondary motion is approximately 90% greater than leakage for the concentric piston.



$z_{\rm CM}/L$	0.442
$z_{\rm P}/L$	0.56
piston mass	19.7 g
connecting rod mass	24 g
$C_{\rm CM}/C_{\rm rod}$	0.7
piston moment of inertia	1460 g mm ²
connecting rod moment of inertia	9678 g mm ²
п	1800 rpm

where:

CM: center of mass

 $z_{\rm CM}$: position of piston center of mass

C_{rod}: connecting rod length

 $C_{\rm CM}$: position of connecting rod center of mass

Figure 8: Geometry used in the piston secondary motion analysis.



Figure 9: Piston dimensionless eccentricity (filled marker: top, empty marker: bottom); n = 1800 rpm, LPB condition: (a) different z_p/L ratios and $c = 9 \ \mu$ m; (b) different clearances and $z_p/L = 0.56$.

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Figure 10: Percentage variation of leakage due to piston secondary motion (n = 1800 rpm, LPB condition): (a) different pin positions and $c = 9 \mu m$; (b) different clearances and $z_p/L = 0.56$.

6. CONCLUSIONS

A simulation model based on the compressible fluid flow formulation of the Reynolds equation was developed to predict gas leakage in the piston-cylinder clearance. The results of the model were validated through comparisons with predictions obtained with a three-dimensional model prepared with a CFD commercial code. A simulation model for the compression cycle was then coupled to the leakage model, allowing the analysis of leakage in the piston-cylinder clearance throughout the compression cycle. The effects of compressor speed, gas compressibility and piston secondary motion on gas leakage were investigated. We found that the piston velocity plays a minor role on leakage, with a small reduction of the overall leakage in the cycle even when the piston velocity is increased. The present study also showed that predictions with incompressible fluid formulation strongly overestimates leakage. Concerning the compressor volumetric and isentropic efficiencies, leakage was found to be more detrimental at small rotational speeds and large clearances. For the compressor adopted in the analysis, reductions of up to 8% and 22% were predicted for the volumetric and isentropic efficiencies, respectively. It was shown that leakage is significantly increased by the piston secondary motion and, as the piston eccentricity is increased, it can be approximately 90% greater than the leakage that would occur if the piston motion was concentric in the cylinder.

NOMENCLATURE

С	Nominal diametric clearance	[µm]	n	Rotational speed	[rpm]
С _{СМ}	Connecting rod center of mass	[mm]	p	Gas static pressure	[Pa]
C_{rod}	Connecting rod length	[mm]	p_{cc}	Compression chamber pressure	[bar]
e_b	Piston top eccentricity	[µm]	R	Piston radius	[mm]
e_t	Piston bottom eccentricity	[µm]	V_p	Piston velocity	[m/s]
h	Radial clearance	[µm]	Ŵ	Indicated power	[W]
L	Piston length	[mm]	\dot{W}_{th}	Isentropic compression power	[W]
ṁ	Compressor mass flow rate	[kg/s]	wt	Crank angle	[°]
\dot{m}_l	Leakage mass flow rate	[kg/s]	Z _{CM}	Position of piston center of mass	[mm]
\dot{m}_l^*	Relative leakage mass flow rate	[kg/s]	Z_P	Compression chamber pressure	[mm]
\dot{m}_{th}	Ideal mass flow rate	[kg/s]			

Greek symbols

ε_b	Dimensionless top eccentricity	[-]	η_s	Isentropic efficiency	[-]
ε_t	Dimensionless bottom eccentricity	[-]	μ	Gas dynamic viscosity	[Pa.s]
η_v	Volumetric efficiency	[-]	ρ	Gas density	$[kg/m^3]$

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