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Xiaodong Hou

*School of Electrical and Computer Engineering, Purdue University; Center for High Performance Buildings, Ray W. Herrick Laboratories, hou39@purdue.edu*

Yingying Xiao

*School of Electrical and Computer Engineering, Purdue University; Center for High Performance Buildings, Ray W. Herrick Laboratories, xiao106@purdue.edu*

Jaewan Joe

*Lyles School of Civil Engineering, Purdue University; Center for High Performance Buildings, Ray W. Herrick Laboratories, jjoe@purdue.edu*

Jie Cai

*Aerospace and Mechanical Engineering, The University of Oklahoma, jcai@ou.edu*

Panagiota Karava

*Lyles School of Civil Engineering, Purdue University; Center for High Performance Buildings, Ray W. Herrick Laboratories, pkarava@purdue.edu*

*See next page for additional authors*

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**Authors**

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# An Agent-based Control Implementation for the Optimal Coordination of Multiple Rooftop Units

Xiaodong HOU<sup>1\*</sup>, Yingying XIAO<sup>1</sup>, Jaewan JOE<sup>2</sup>, Jie CAI<sup>4</sup>, Panagiota KARAVA<sup>2</sup>, Jianghai HU<sup>1</sup>, and James E. BRAUN<sup>3</sup>

<sup>1</sup> School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN, USA  
{hou39, xiao106, jianghai}@purdue.edu

<sup>2</sup> Lyles School of Civil Engineering, Purdue University, West Lafayette, IN, USA  
{jjoe, pkarava}@purdue.edu

<sup>3</sup> School of Mechanical Engineering, Purdue University, West Lafayette, IN, USA  
{cai40, jbraun}@purdue.edu

<sup>4</sup> Aerospace and Mechanical Engineering, The University of Oklahoma, Norman, OK, USA  
jcai@ou.edu

\* Corresponding Author

## ABSTRACT

Model predictive control (MPC) has been a well-studied advanced supervisory control approach for optimizing the operations of building heating, ventilation and air-conditioning (HVAC) systems, with the objectives of reducing energy consumption and delivering better comfort. However, centralized MPC designs are often 1) not scalable to the increasing sizes of the building systems, 2) not adaptive to subsystem addition/attrition, i.e., ‘Plug-and-Play’ implementation. Agent-based approaches, such as distributed model predictive control (DMPC), are attractive alternatives. In this paper, taking a multiple rooftop units (RTU) coordination problem as case study, we experimentally investigate the energy saving potential by implementing an agent-based DMPC strategy to coordinate the operations of multiple “virtual” RTUs with diverse unit efficiencies (COP) in an open space with multiple thermal zones. The operations of three RTUs are emulated by two groups of variable air volume (VAV) diffusers and a separate VAV box, that can be individually controlled to provide continuously changing sensible cooling rates into respective zones. Three laptop computers are dispatched into the three thermal zones as local agents. A server computer connected to both the Building Automation System (BAS) and the outside internet is responsible for predicting various exogenous inputs and exchanging information with the local agents. Experimental results show that the proposed agent-based DMPC design and implementation is able to achieve over 20% cost savings, in terms of electricity consumption charge with Time-of-Use pricing schedules, while at the same time maintaining local occupancy comfort.

## 1. INTRODUCTION

Office and commercial buildings often consist of multiple thermal zones, with either coupled or decoupled dynamics. Many of these small to medium sized buildings are conditioned by multiple packaged units, such as RTUs. However, traditional control strategies are neither capable of recognizing imbalanced loads across zones or efficiency differences between units, nor able to provide flexibility of customized conditioning based on occupancy preferences.

Many different advanced control or optimization methods have been utilized for building control and energy management problems. Among them, the model predictive control (MPC) approach (Ma et al., 2012) (Oldewurtel et al., 2012) has become increasingly studied due to its ability of incorporating weather and disturbance information into the optimization of the operation of HVAC systems. In addition, distributed model predictive control (DMPC) (Ma, Anderson, & Borrelli, 2011) (Hou, Xiao, Cai, Hu, & Braun, 2017) or agent-based control approaches (Cai, Kim, Jaramillo, Braun, & Hu, 2016) are more effective in dealing with buildings with many thermal zones compared to centralized MPC, due to the improved scalability.

This study takes the Purdue Living Lab 3 (LL3) as a testbed, which is representative of buildings with large open space and multiple thermal zones, and investigates the cost savings potential (in terms of electricity bill reduction) by coordination between different HVAC equipment and utilizing building thermal mass for load shifting. In particular, we experimentally study a virtual optimal RTU coordination problem. The RTU coordination problem has received a lot of

attention recently (Kim, Braun, Cai, & Fugate, 2015) (Putta, Kim, Cai, Hu, & Braun, 2015). To be discussed in detail later, “virtual” refers to the fact that there is no real RTUs in the testbed, instead, multiple groups of VAV diffusers and VAV box are utilized to emulate the functionality of RTUs in an open plan building: providing continuously adjustable cooling/heating to different zones with certain capacities and efficiencies. The coordinations between different RTUs and load shifting are achieved by implementing an agent-based DMPC algorithm.

In order to implement the proposed agent-based optimal coordination algorithm, the first step is to obtain a control-oriented multi-zone model that is able to capture the thermal behavior of LL3 and accommodate an efficient controller design. We propose a new resistance-capacitance (RC) network multi-zone model that is suitable for not only LL3, but also other large open spaces with multiple air nodes and asymmetric airflow exchange rates.

This study is a continuation of the work in (Hou, Xiao, Cai, Hu, & Braun, 2016), where the building model was not data-driven, and only simulation results were presented. The rest of the paper is organized as follows. In Section 2, we introduce the case study testbed, the building thermal model, and the virtual RTU model. Section 3 outlines the formulation of the agent-based distributed model predictive control approach using a parallel distributed optimization algorithm. Experiment setup and implementation results of proposed algorithm are presented in Section 4 in comparison with a baseline strategy. Finally, some concluding remarks are given in Section 5.

## 2. CASE STUDY AND MODEL DESCRIPTION

### 2.1 Case Study Description

LL3 (Figure 1 left) is a large open-space student office, located on the top floor of the Center for High Performance Buildings at West Lafayette, IN, USA. The room was originally conditioned by a centralized air handling unit through three VAV boxes with terminal units of one large rectangular diffuser and eight standard square ceiling diffusers.

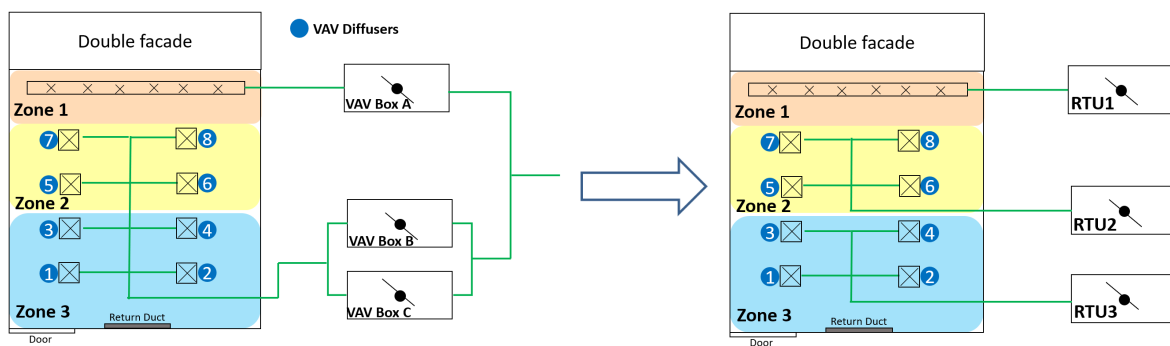


Figure 1: LL3 Layout

The typical static ceiling or wall diffuser setup does not offer the flexibility of personalized local control and comfort delivery. In the case of LL3, there is a significant load imbalance in summer as the south-facing double facade accumulates much more solar radiation than the part of the room near the door. With this in mind, we retrofitted the HVAC system by replacing the eight traditional ceiling diffusers with eight VAV diffusers, which can be individually controlled to allow continuous and localized comfort control through the Building Automation System (BAS). The eight VAV diffusers are grouped into two clusters, serving Zone 2 (diffusers 1, 2, 3, 4) and Zone 3 (diffusers 5, 6, 7, 8), respectively; the zone served by the VAV box A through traditional rectangular diffuser is denoted as Zone 1.

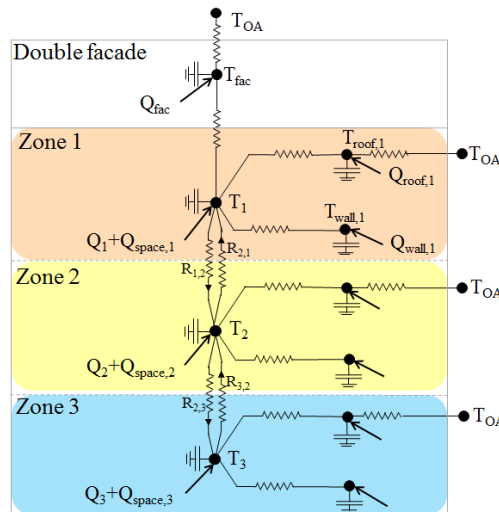
Two RTD sensors are installed into each zone, one in the east column and the other in the west column. The average of two readings are treated as the sensing temperature for the corresponding zone. Local zone level PID control strategies are implemented in the BAS to adjust the openings of the two groups of VAV diffusers as well as the VAV Box A such that each zone sensing temperature is maintained at its setpoint. We can think of the installed thermal sensors and the PID logic behind as three virtual thermostats that are controlling the operations of the VAV diffusers and box.

By adjusting the openings of the VAV diffusers and VAV box, we are essentially emulating three RTUs cycling on and off, such that different amounts of cooling/heating are provided to individual zones. Therefore, the retrofitted LL3 setup

can be used as a virtual testbed for the coordination problem of multiple RTUs (Figure 1 right) with various efficiencies and capacities as long as different virtual RTU models are provided. This open-space multi-zone configuration is commonly encountered in many small to medium-sized commercial buildings such as open plan restaurants, banks, stores, etc.

## 2.2 Building Model

In LL3, the double facade is separated from the three zones by glass windows while there are dynamic couplings between adjacent zones. Another feature of the room is that the return air duct is located on the back wall (bottom in Figure 2) in Zone 3 near the door and thus there is significant air flow from Zone 1 to 3 but the airflow in the opposite direction is relatively small. Therefore, the coupling resistances connecting two adjacent zones have different values in different directions, as indicated by the two directional resistors in Figure 2 ( $R_{i,i+1} \neq R_{i+1,i}$ ).



**Figure 2: Model Structure of LL3**

A continuous time state-space representation of the above model is formulated in (1),

$$\begin{aligned} \dot{x} &= Ax + Bu + Fw, \\ y &= Cx, \end{aligned} \quad (1)$$

where the state variable  $x \in \mathbb{R}^{10}$  consists of 10 temperatures, one for the double facade air and three for each zone ( $T_i, T_{\text{roof},i}, T_{\text{wall},i}, i = 1, 2, 3$ ); the output  $y \in \mathbb{R}^4$  represents the three measurable zone temperatures ( $T_i, i = 1, 2, 3$ ) and double facade temperature; the control input  $u \in \mathbb{R}^3$  is a vector of the controllable sensible cooling/heating rates ( $Q_i, i = 1, 2, 3$ ) into three zones provided by RTUs while the disturbance  $w \in \mathbb{R}^{12}$  consists of all uncontrollable disturbances, namely, ambient temperature, solar radiation, internal heat gains from lighting, plug load and occupants;  $A, B, F, C$  are matrices with proper dimensions. For model training and validation, all disturbances come from sensor measurements except the heat gain of occupants, which is estimated based on the occupancy schedule of the room.

Since Zone 2 is affected by Zone 1 through the resistance  $R_{1,2}$  and by Zone 3 through  $R_{3,2}$ , we can estimate the RC values of Zone 2 separately by treating the measured adjacent zones' temperatures,  $T_1$  and  $T_3$ , as boundary conditions. The same is also true for Zone 1 and Zone 3. Therefore, the RC values for the three zones can be estimated individually by treating adjacent zones' measured temperatures as boundary conditions. The identification results of the three zones are then integrated together to form the overall system model in (1).

Here we use Zone 2 as an example to demonstrate the identification of individual zones, of which the dynamics are represented by

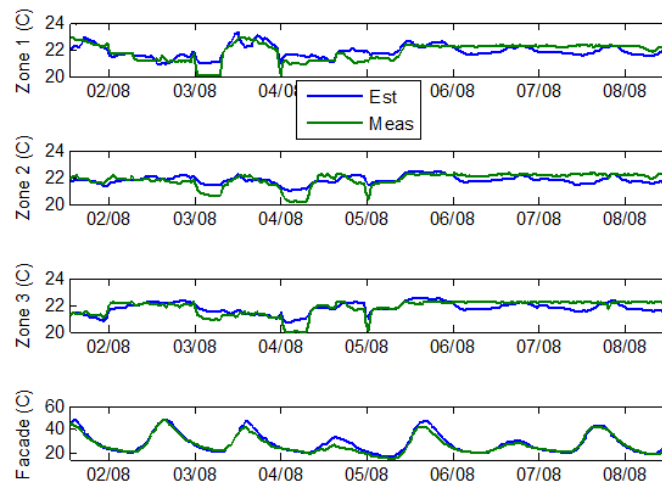
$$\begin{aligned} \dot{x}_2 &= A_2x_2 + B_2u_2 + F_2w_2, \\ y_2 &= C_2x_2, \end{aligned} \quad (2)$$

where  $A_2, B_2, F_2, C_2$  are constructed from the corresponding entries in  $A, B, F$  and  $C$ , respectively. For  $w_2$ , besides the afore-mentioned uncontrollable disturbances in  $w$ , it also includes the air temperatures  $T_1$  and  $T_3$ . Correspondingly,  $F_2$  depends on the portion of  $F$  relevant to Zone 2 and the coupling resistances  $R_{1,2}$  and  $R_{3,2}$ . By discretization with a sampling time of 0.5 hour and concatenating all vectors over time  $N$ , we have

$$\begin{aligned} X_2 &= \Omega_2 x_2(0) + \Phi_2 U_2 + \Psi_2 W_2, \\ Y_2 &= \Gamma_2 X_2, \end{aligned}$$

where  $X_2 = (x_2(1), \dots, x_2(N))$ ,  $U_2 = (u_2(1), \dots, u_2(N))$ ,  $W_2 = (w_2(1), \dots, w_2(N))$  and  $Y_2 = (y_2(1), \dots, y_2(N))$  is the estimated Zone 2 temperature, which should be close to its real measurement  $\tilde{Y}_2$ , while  $\Omega_2, \Phi_2, \Psi_2, \Gamma_2$  are constant matrices built up from  $A_2, B_2, F_2, C_2$ . Then the objective is to minimize  $\|Y_2 - \tilde{Y}_2\|_2^2$  by optimizing the values of internal resistances/capacitances of Zone 2 as well as  $R_{1,2}$  and  $R_{3,2}$ . This problem is solved with the function `lsqnonlin` in MATLAB, with initial guesses of parameters derived from building construction parameters.

The data collected from July 17, 2017 to August 8, 2017 is used for warm-up (8 days), model training (7 days) and validation (7 days). The validation result is shown in Fig. 3 and the root mean square deviation(RMSD) for each zone temperature is  $0.46^\circ\text{C}$ ,  $0.41^\circ\text{C}$ ,  $0.39^\circ\text{C}$  and  $2.12^\circ\text{C}$  respectively.



**Figure 3: Model Validation (Blue Lines: Estimated Temperatures, Green Lines: Measured Temperatures)**

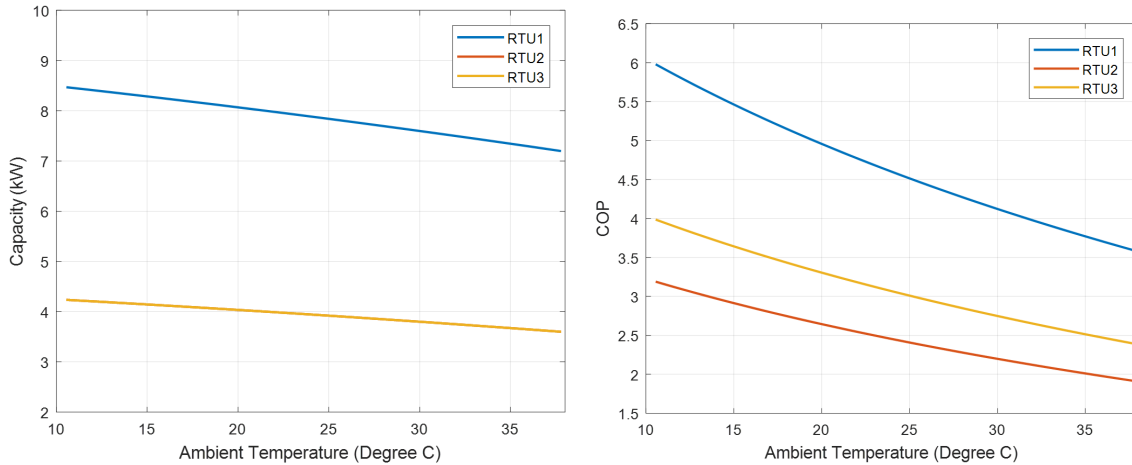
### 2.3 Virtual RTU Model

It has been discussed that the two groups of VAV diffusers in Zone 2 & 3, and the VAV Box A for Zone 1, can be thought as three independent RTUs controlled by three virtual thermostats. We call the RTU in Zone  $i$  as RTU  $i$ . In addition, we assume RTU 1 is the largest and most efficient unit, while RTU 2 and 3 are relatively smaller in size, and less efficient. The plot of total sensible capacity (latent capacity or dehumidification is not considered in this study) and  $COP$  of all three RTUs under different  $T_{amb}$  is given in Figure 4 to demonstrate their efficiency and capacity differences.

The virtual RTU models are correlation-based (Cai, 2015) and developed from manufacturer catalogue data with capacities scaled accordingly. The model has the following structure

$$[Power, PLR] = RTU(Q_{sen}, T_{amb}, T_{wb}, T_{db}), \quad (3)$$

where  $Q_{sen}$  is the sensible cooling rate provided by RTU into the zone during the current sampling time;  $T_{amb}$  is the ambient temperature;  $T_{wb}/T_{db}$  are the nominal wet/dry bulb temperatures entering the RTU evaporator coils and are assumed to be constants  $15.3/28.9^\circ\text{C}$ ;  $Power$  is the power consumption and  $PLR$  is the part-load ratio based on the total sensible capacity. The units are more efficient (higher  $COP$ ) in general when operating under lower ambient temperatures. At part load conditions, RTUs cycle on and off to match  $Q_{sen}$ . Degradation curves are used to capture the efficiency decrease caused by cycling. For fixed  $T_{amb}, T_{wb}$  and  $T_{db}$ ,  $Power$  and  $PLR$  are convex functions in  $Q_{sen}$ . Since  $T_{wb}/T_{db}$  are assumed to be constants, they will be dropped from the model for notational simplicity.



**Figure 4: RTU Characteristics** Left: *Capacity vs.  $T_{amb}$*  (RTU 2 & 3 have the save curve) Right: *COP vs.  $T_{amb}$*

### 3. AGENT-BASED OPTIMAL RTU COORDINATION

Traditionally, multiple RTUs in a large open space are controlled by multiple thermostats with fixed setpoint schedules that are pre-configured by users, without considering: 1) efficiency differences in RTUs; 2) load variations across different zones; 3) thermal interactions between adjacent zones. However, the efficiency differences in units and thermal interactions between zones give us opportunities for electricity bill reduction through coordination if certain flexibility is allowed in terms of setpoints and occupancy comfort.

If we assume that occupancy comfort is satisfied as long as the sensing temperature is within certain upper and lower bounds, then instead of fixed setpoint schedules, the setpoint for each thermostat can be dynamically adjusted over time, to achieve the coordination between RTUs' operations. Then the problem becomes how to optimally decide the setpoint temperature trajectories for each thermostat and zone. Notice that this is a supervisory level optimal control problem as it assumes that at the zone level, individual setpoint can be tracked by operating each local RTU via PID control or other strategies. As discussed in (Hou et al., 2016), this problem can be formulated and solved as an agent-based DMPC problem.

#### 3.1 Objective Function

The objective function for the MPC problem is the total energy bill corresponding to the RTUs' operations in certain prediction horizon  $N$ ,

$$\sum_{k=1}^N \left( P_e(k) \cdot \sum_{i=1}^3 RTU_{i,power}(Q_i(k), T_{amb}(k)) \right), \quad (4)$$

where  $P_e(k)$  is the Time-of-Use (TOU) electricity price,  $Q_i(k)$  is the sensible cooling rate provided by RTU  $i$  into zone  $i$ , and  $RTU_{i,power}(Q_i(k), T_{amb}(k))$  denotes the current power consumption rate of RTU  $i$ . From this objective function, we notice that another cost savings potential comes from shifting the loads to a period with lower electricity price or higher RTU efficiency.

#### 3.2 Thermal Dynamics Constraint

The discrete time state space model of LL3's dynamics is obtained by discretization of (1) with a  $0.5h$  sampling time. We can further concatenate the dynamics during an  $N$  step prediction horizon as

$$X = \Omega x(0) + \Phi Q + \Psi W, \quad Y = \Gamma X \quad (5)$$

where  $X = (x(1), \dots, x(N))$ ,  $Q = (Q(0), \dots, Q(N-1))$ ,  $Q(k) = (Q_1(k), Q_2(k), Q_3(k))$ ,  $Y = (y(1), \dots, y(N))$  and  $W = (w(0), \dots, w(N-1))$ .  $\Omega$ ,  $\Phi$ ,  $\Psi$  and  $\Gamma$  are constant matrices of proper dimensions constructed from  $A, B, C, F$ .

### 3.3 Optimization Formulation

The objective function (4) can be written as  $f(Q) = \sum_{i=1}^3 f_i(Q_i)$  where  $f_i(Q_i) = \sum_{k=1}^N (P_e(k) \cdot RTU_{i,power}(Q_i(k), T_{amb}(k)))$ , and  $Q_i = (Q_i(0), \dots, Q_i(N-1))$  because  $P_e(k)$  and  $T_{amb}(k)$  are predictable from utility service and weather forecast, respectively. Then, the centralized optimization problem can be represented as

$$\underset{Q, X}{\text{minimize}} \quad \sum_{i=1}^3 f_i(Q_i) \quad (6)$$

$$\text{subject to} \quad X = \Omega x(0) + \Phi Q + \Psi W, \quad Y = \Gamma X \quad (7)$$

$$Y_{i,min} \leq Y_i \leq Y_{i,max},$$

$$RTU_{i,PLR}(Q_i(k), T_{amb}(k)) \leq 1, \quad k = 0, \dots, N-1,$$

where  $Y_i = (y_i(1), \dots, y_i(N))$ , and  $RTU_{i,PLR}(Q_i(k), T_{amb}(k))$  denotes the current *PLR* of RTU *i*.  $[Y_{i,min}, Y_{i,max}]$  is the local comfort interval for zone *i*'s temperature. Because  $X$  and  $Y$  are affine transformations of  $Q$ , we can eliminate them from the decision variables and obtain an equivalent formulation

$$\underset{Q_i}{\text{minimize}} \quad \sum_{i=1}^3 f_i(Q_i) \quad (8)$$

$$\text{subject to} \quad \sum_{i=1}^3 D_i Q_i \leq d, \quad (9)$$

$$RTU_{i,PLR}(Q_i(k), T_{amb}(k)) \leq 1, \quad k = 0, \dots, N-1, \quad (10)$$

where  $D_i$  and  $d$  are proper matrices and vector, respectively. Constraint (10) is convex because  $RTU_{i,PLR}(Q_i, T_{amb})$  is a convex function with respect to  $Q_i$ , which can be equivalently written as  $Q_i \in \mathcal{Q}_i$ . Each  $f_i$  is a convex function in  $Q_i$  since  $RTU_{i,power}(Q_i(k), T_{amb}(k))$  is convex in  $Q_i(k)$ . Notice that constraint (9) couples local variables  $Q_i$  together thus the above problem is not readily separable. We call (9) the coupling constraint and (10) local constraints.

### 3.4 Agent-based Distributed Optimization Solution

Dual decomposition (Farokhi, Shames, & Johansson, 2014) and Gauss-Seidel Alternating Directions Method of Multipliers (Boyd, Parikh, Chu, Peleato, & Eckstein, 2011) are two existing distributed optimization methods designed for problems in the form (8). However, they have certain disadvantages as discussed in (Hou et al., 2017). In this study, we employ the so called Proximal Jacobian ADMM (parallel ADMM) method. First we introduce slack variables  $Q_0$  to convert the coupling inequality constraint in (9) to an equality constraint

$$\underset{Q_i}{\text{minimize}} \quad \sum_{i=0}^3 f_i(Q_i) \quad (11)$$

$$\text{subject to} \quad \sum_{i=0}^3 D_i Q_i = d, \quad Q_i \in \mathcal{Q}_i,$$

where  $f_0(Q_0) = \text{const.}$ ,  $D_0$  is the identity matrix, and  $Q_0$  is the positive orthant. The agent-based distributed optimization algorithm for solving (11) is summarized in Algorithm 1.

In Algorithm 1,  $\mathcal{L}_i(Q_i, \lambda) = f_i(Q_i) + \lambda^\top D_i Q_i$  is the part of the Lagrangian function of problem (11) that is dependent on  $Q_i$ ;  $\varphi(Q) = \frac{\rho}{2} \|\sum_{i=0}^3 D_i Q_i - d\|^2$ . In addition,  $i-$ (resp.  $+$ ) denotes indices smaller (resp. larger) than  $i$ . In Algorithm 1, each agent first updates their local decision variables  $Q_i$  in parallel according to (12), then a central coordinator collects the updated  $Q_i$  from all of the agents and uses them to update the dual variable  $\lambda$  according to (13). Parameters  $\alpha_i$  and  $\rho$  need to satisfy certain conditions to ensure convergence to some optimal solution of (11). We refer the interested readers to (Hou et al., 2017) for more details. Once Algorithm 1 is terminated, we obtain the optimal cooling rates trajectories  $Q_i^*$  for all the RTUs. Then, the optimal setpoint trajectories for the thermostats can be recovered via the identified building model

$$X^* = \Omega x(0) + \Phi Q^* + \Psi W, \quad Y^* = \Gamma X^*.$$



**Algorithm 1** Agent-based Distributed Algorithm1: Initialize  $(Q^0, \lambda^0)$ , set  $v = 0$ ;2: **repeat**3:   Update  $Q_i$  (in parallel) according to

$$Q_i^{v+1} = \operatorname{argmin}_{Q_i \in \mathcal{Q}_i} (\mathcal{L}_i(Q_i, \lambda^v) + \frac{\alpha_i}{2} \|Q_i - Q_i^v\|^2 + \varphi(Q_{i-}^v, Q_i, Q_{i+}^v)); \quad (12)$$

4:   Update  $\lambda$  according to

$$\lambda^{v+1} = \lambda^v + \rho \left( \sum_{i=0}^3 D_i Q_i^{v+1} - d \right). \quad (13)$$

5:    $v \leftarrow v + 1$ ;6: **until** some stopping criterion is satisfied.**4. EXPERIMENT IMPLEMENTATION****4.1 Experiment Setup**

The proposed agent-based DMPC algorithm for the optimal coordination of multiple virtual RTUs is implemented in LL3. Prediction horizon is set to be 12 hours. It is assumed that the occupied hours are 10am-10pm. Different zone temperature comfort intervals are set for the three zones (summarized in Table 1). Specifically, since there are no permanent occupants in Zone 1, it has a higher upperbound during the occupied hours. However, Zone 2 & 3 are more heavily occupied, hence tighter comfort intervals need to be maintained during the occupied hours. During unoccupied hours, night setback is implemented. The TOU price is assumed to have a peak value of \$0.16/kWh from 12pm to 6pm, and an off-peak value of \$0.067/kWh during other times. All local optimization problems are solved in MATLAB with CVX (Grant & Boyd, 2014). During the DMPC implementation, unmeasured state variables need to be estimated. Since the only coupling between adjacent zones are through their zone temperatures, a decentralized Kalman Filter was developed so that each zone can estimate local unmeasured states using a standard Kalman Filter by treating adjacent zone temperatures as exogenous inputs.

**Table 1**

	<b>Occupied Hours (10am-10pm)</b>	<b>Unoccupied Hours (10pm-10am)</b>
<b>Zone 1</b>	[21°C, 24.5°C]	[20°C, 27°C]
<b>Zone 2</b>	[21°C, 23°C]	
<b>Zone 3</b>	[21°C, 23°C]	

To demonstrate the cost savings potential of the proposed DMPC method, we compare its performance against a baseline strategy, for which the thermostats' setpoint schedules are set to be the upper-bounds of their corresponding comfort intervals. If a zone temperature is lower than its current setpoint, no cooling will be given and zone temperature is allowed to float freely inside the comfort interval. The baseline controllers were simulated using the history disturbance information from the same days of the DMPC experiment.

**4.2 Prediction of Exogenous Inputs**

The DMPC implementation requires the prediction of exogenous inputs (ambient temperature, solar radiation, occupants, lighting and plug load). The ambient temperature prediction is obtained from National Oceanic and Atmospheric Administration (NOAA) website at the Purdue University Airport station (1.5 miles away from testbed, LL3). Solar radiation (GHI & GSI) is inferred from sky cover percentage prediction (also obtained from NOAA) using the model from (Zhang, Huang, & Lang, 2002). Occupancy, lighting and plug load are predicted by averaging the corresponding history data for the same period in the previous three weeks. The comparison between predicted ambient temperature, solar radiation, and plug loads with their respective measurements from September 21, 2018 to September 27, 2018 are given in Figure 5, which shows that our predictions are reasonably accurate.

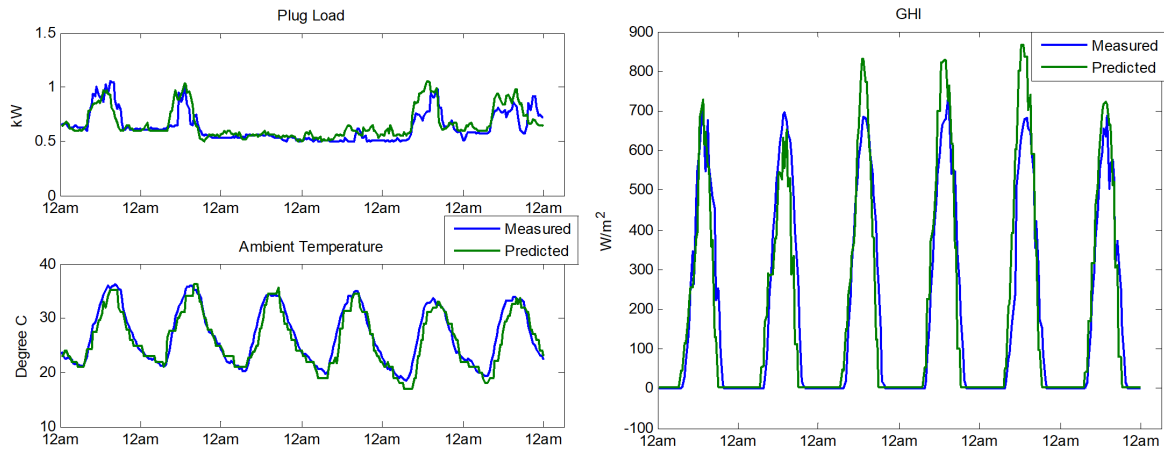


Figure 5: Exogenous Inputs Prediction

### 4.3 Communication Prototype and Information flow

The agent-based distributed optimization algorithm in Algorithm 1 requires three agents, representing each of the three zones, and a central coordinator. Three laptop computers are dispatched to the three zones, acting as the agents and are responsible for updating local decision variables as well as exchanging information with the central coordinator. The central coordinator is a server computer that has Internet connection for weather forecasting and also communicates with the BAS by sending command signals (setpoints for all three zones in our case) and collecting sensor data.

The MATLAB TCP/IP object and protocol was utilized for the communication between agents and coordinator, since the optimizations at local agents are run in MATLAB with CVX. At the beginning of each iteration, the coordinator sends to the agents: the updated Lagrange multiplier  $\lambda^v$  from the previous iteration, the TOU electricity price  $P_e(k)$  for the current prediction horizon and the predicted trajectories of other disturbances (solar, occupants, lighting, plug load, ambient temperature); and agents send their updated optimal local decision variables  $Q_i^v$  back to the coordinator. After convergence, the coordinator will recover the optimal setpoints for all three zones during the next sampling step and send them to the BAS. One limitation of the MATLAB TCP/IP object is that it does not allow broadcasting, which means that the coordinator can only exchange information with one agent at each time. More advanced communication protocols that allow broadcasting from the coordinator to the agents will take better advantage of the parallel updating structure. This will be investigated in future studies.

### 4.4 Experiment Results

After a 3 day warm-up period, the experiment and simulation were tested for 7 days in parallel (from October 4, 2017 to October 11, 2017). The zone temperature trajectories as well as the optimal sensible cooling trajectories are given in Figure 6 (baseline simulation) and Figure 7 (optimal coordination with agent-based DMPC). In both figures, the gray dashed line denotes the same zone temperature comfort lowerbound for all three zones ( $20^\circ\text{C}/21^\circ\text{C}$  during unoccupied/occupied hours); and the blue (red) dashed line denotes the zone temperature comfort upperbound for Zone 1 (2 & 3) during occupied hours.

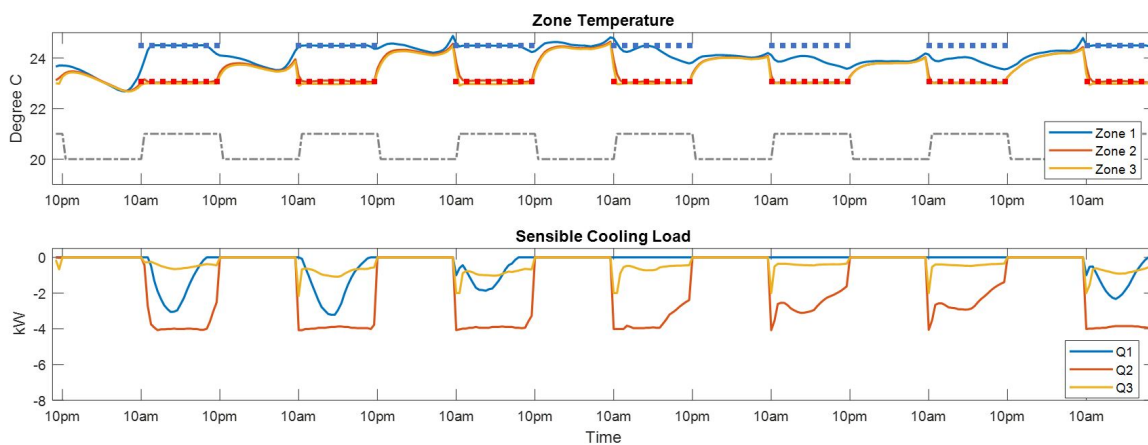
From the baseline simulation results, we can observe that during unoccupied hours, when setpoint setbacks are implemented, no cooling is provided by any of the three RTUs and the zone temperatures are allowed to free-float. During occupied hours, all three zone temperatures are well maintained at their respective comfort upperbounds ( $24.5^\circ\text{C}$  for Zone 1 and  $23^\circ$  for Zone 2 & 3). Between Zone 2 & 3, even though they have the same setpoint schedule, RTU 2 is doing more work because of the directional airflow from south to north. The load for RTU 1 is smaller than that of RTU 2 because Zone 1 has higher setpoints.

If we look at the experiment results with the proposed agent-based DMPC algorithm, two different types of coordinations can be observed.

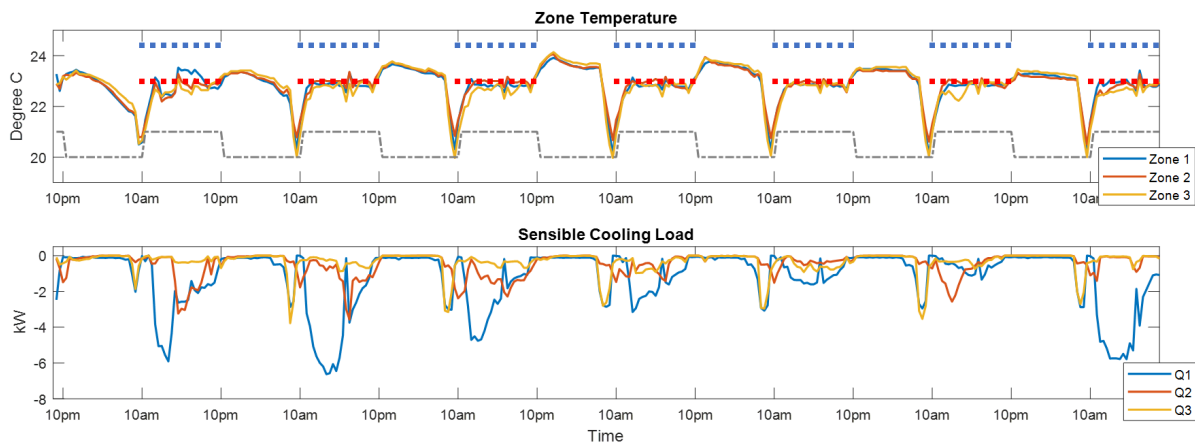
1. **“Coordination in Space”**: During occupied hours, while the setpoint temperatures for Zone 2 & 3 are still their

comfort upperbound, the setpoint temperature for Zone 1 is set to 23°C, which is significantly lower than its comfort upperbound, 24.5°C. The reason behind this is two fold: a) by lowering the setpoint for Zone 1, the most efficient unit RTU 1 is more heavily utilized, which helps reduce the load for the less efficient units, RTU 2 & 3; b) the directional airflow from south to north (Zone 1 → Zone 2 → Zone 3) is being taken advantage of by providing more cooling directly into Zone 1. From the sensible cooling load profile we can verify that RTU 1 is “helping out” RTU 2 & 3 by providing more cooling than what it would if only selfishly using the highest setpoint temperature.

2. **“Coordination in Time”**: Similar to the baseline case, zone temperatures are free-floating during most of the unoccupied hours and no coolings are provided from RTUs. However, every morning from around 6am to 10am, there are significant pre-coolings provided by all three RTUs as the zone temperature setpoints decrease to their lowerbound. The reason is that the RTUs are trying to cool down the building thermal mass when the electricity price is low (before 10am), and units are more efficient due to lower ambient temperatures during early mornings. By doing this, extra cooling energies are stored in the building thermal mass and will be released into the zones during occupied hours so that the peak loads for all three RTUs are reduced.



**Figure 6: Baseline Simulation**



**Figure 7: Optimal Coordination with Agent-based DMPC Experiment**

The electricity bill for the baseline case and the optimal coordination with agent-based DMPC algorithm is summarized in Table 2. Through coordination between different zones and RTUs and shifting loads to early mornings when electricity price and ambient temperature are low, 28% of the total electricity bill is reduced from the baseline.

Table 2

	Baseline	Optimal Coordination with Agent-based DMPC
<b>Total Sensible Cooling (kWh)</b>	330.88	375.69
<b>Electricity Bill (\$)</b>	25.75	18.46 (↓28.31%)

## 5. CONCLUSIONS

This study experimentally investigates the electricity bill reduction potential of coordinating between multiple virtual RTUs in a large open space with multiple thermal zones. An agent-based DMPC algorithm is implemented in the testbed to achieve coordinations in both space and time. Compared to the baseline strategy with which thermostats selfishly choose setpoints as their comfort upperbounds, 28% of the total electricity bill is saved with the proposed method. Although the coordination problem of 3 zones/RTUs could also be solved with a centralized MPC approach, the proposed method is more suitable for extension to larger number of zones or building clusters, as it provides much better scalability. One future direction is to add demand charges into the cost function and investigate how thermostats and RTUs respond differently.

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