# Purdue University Purdue e-Pubs

International High Performance Buildings Conference

School of Mechanical Engineering

July 2018

# Model Predictive Control Of Inverter Air Conditioners Responding to Real-Time Electricity Prices In Smart Grids

Maomao Hu *The Hong Kong Polytechnic University, Hong Kong S.A.R. (China),* chace.hu@connect.polyu.hk

Fu Xiao The Hong Kong Polytechnic University, Hong Kong S.A.R. (China), linda.xiao@polyu.edu.hk

Follow this and additional works at: https://docs.lib.purdue.edu/ihpbc

Hu, Maomao and Xiao, Fu, "Model Predictive Control Of Inverter Air Conditioners Responding to Real-Time Electricity Prices In Smart Grids" (2018). *International High Performance Buildings Conference*. Paper 303. https://docs.lib.purdue.edu/ihpbc/303

Complete proceedings may be acquired in print and on CD-ROM directly from the Ray W. Herrick Laboratories at https://engineering.purdue.edu/ Herrick/Events/orderlit.html

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.

# Model Predictive Control of Inverter Air Conditioners Responding to Real-Time Electricity Prices in Smart Grids

### Maomao HU<sup>1</sup>, Fu Xiao<sup>1, 2</sup>\*

<sup>1</sup>Department of Building Services Engineering, The Hong Kong Polytechnic University Kowloon, Hong Kong

<sup>2</sup>Research Institute for Sustainable Urban Development, The Hong Kong Polytechnic University, Kowloon, Hong Kong

chace.hu@connect.polyu.hk, linda.xiao@polyu.edu.hk

\* Corresponding Author

### ABSTRACT

The rapid development of smart grids has imposed a new requirement on residential appliances, i.e. being demand response-enabled (DR-enabled). One of the key features of DR-enabled appliances is to automatically respond to realtime electricity pricing (RTP) so as to effectively shift peak power demands from high-RTP to low-RTP periods and reduce electricity costs. Advanced DR control methods are essential to develop DR-enabled appliances. Residential air conditioners (ACs) are the major contributors to home electricity bills and electrical grids. Inverter AC becomes popular in today's homes due to the higher energy efficiency at part-load conditions. In this paper, we aim to apply the model predictive control (MPC) method to inverter AC to make it RTP-responsive. The MPC method can simultaneously consider multiple influential variables including weather condition, occupancy and RTP to achieve the optimization of energy consumption or electricity cost. Considering the computational efficiency for real-time online control, a simple-structured grey-box room thermal model is developed. We also develop a steady-state physical model of inverter AC and generate its performance maps for online applications. A TRNSYS-MATLAB co-simulation testbed is developed to test the performances of the MPC controller. Test results show that compared with PID control, the MPC-based DR controller helps to improve the thermal comfort at the beginning of occupancy, reduce peak power demands and total electricity costs.

# **1. INTRODUCTION**

Supply and demand imbalance is a critical issue faced by current electrical grids. Buildings are responsible for around 40% of the total energy consumptions worldwide, and consume over 70% of the total electrical energy in the USA (Somasundaram et al., 2014) and over 90% of the total electricity in Hong Kong (Electrical and Mechanical Services Department, 2017). As the major end-users of electricity, buildings have great responsibilities and potential to provide power reductions during on-peak hours. Since heating, ventilation and air conditioning (HVAC) systems account for a large portion of the total building electricity use, their power consumption has a direct impact on power grids. Unlike the commercial buildings, due to the lack of advanced DR control methods, residential appliances are still facing challenges to make automatic DR during peak demand hours, particularly residential ACs. DR control of ACs is normally formulated as an optimal trade-off between electricity cost and thermal comfort. Conventional control methods such as the on-off control, PID control and fuzzy logic control (FLC) for building energy systems are incompetent to fulfill the DR control task involving multiple influential factors to be considered (Henze, Felsmann, & Knabe, 2004; Ma, Kelman, Daly, & Borrelli, 2012). An advanced optimal control method, therefore, is needed to fulfill the automatic DR control of residential ACs which involves multiple influential variables.

Optimal control, also known as supervisory control, has been used in the control of HVAC systems, which aims at minimizing the energy consumption or operating cost while satisfying thermal comfort when subject to the changing indoor and outdoor conditions as well as the characteristics of building and HVAC systems (Wang & Ma, 2008). Optimal/supervisory control has also been used for DR control of HVAC systems. One of the major features of optimal

DR control is that the dynamic electricity pricing, as an additional exogenous variable, is considered in the optimization process. Chen et al. (Chen, Wu, & Fu, 2012) used stochastic optimization and robust optimization approaches to optimize the operation scheduling of six typical residential appliances based on real-time electricity prices. The objective was to minimize the whole-day electricity payment without largely sacrificing thermal comfort. A mixed-integer linear programming problem was formulated and solved by Hubert et al. (Hubert & Grijalva, 2012) to minimize electricity costs. Their work showed that advanced scheduling controllers implemented in HEMS were valuable to fully achieve the DR benefits. Thomas et al. (Thomas et al., 2012) developed an intelligent AC controller which can provide the optimal comfort and cost trade-offs for the residents by scheduling the AC on/off status.

However, three major research gaps exist in the previous studies. First, the residential ACs were all single-speed ACs with the on-off control method. The on-off control has a big disadvantage of undesired current peaks during state transitions (Aswani, Master, Taneja, Culler, & Tomlin, 2012). The single-speed ACs are also gradually replaced by the inverter ACs which have gained an increasing market share in recent years due to its improved efficiency at partload conditions. Second, two types of dynamic retail electricity prices are widely used by most utilities in the United States, i.e. Day Ahead Price (DAP) and Real-Time Price (RTP). DAP is usually calculated at hourly intervals and announced to the end-consumers one day ahead. RTP is determined every 5 minutes based on the current electricity supply and demand of grid nodes. Previous studies of DR from residential appliances mainly targeted at DAP. There is a lack of real-time optimal DR control methods responding to RTP for residential appliances. Third, the optimal DR control method in the literature is local-controller-based supervisory control in which the command signals for actuators such as rotational speed of motor and opening degree of valve are controlled by classical local controllers, e.g. on-off controller and PID controller. The high-level controller, i.e. supervisory controller, is used to output the optimal set-point for the low-level local controllers. In order to respond to RTP at the interval of 5 minutes, it is impractical to adjust the temperature set-point of local controller every 5 minutes. Thus, we attempt to directly regulate the operating frequencies of the compressor in inverter AC in this study to minimize the energy use and operating cost with the assistance of optimization techniques.

MPC is a flexible and intuitive approach for constrained control which was initially applied in the process industries such as petroleum refineries and chemical plants in the late seventies and has been successfully developed in many other fields over the last decade (Camacho & Alba, 2013; Morari & Lee, 1999; Qin & Badgwell, 2003). In recent years, MPC techniques have also been used in the built environment. Ma et al. (Ma, Borrelli, et al., 2012; Ma, Matuško, & Borrelli, 2015) used both deterministic and stochastic MPC approaches to control large building cooling system consisting of chilled water plans, cooling towers and thermal storage tanks. The experiment results show the MPC controller can effectively improve the efficiency of central plants and reduce the energy costs. Oldewurtel et al. (Oldewurtel et al., 2012) used stochastic MPC technique to control building climate and investigated its energy saving potential for various building types using large-scale simulations. Feng et al. (Feng, Chuang, Borrelli, & Bauman, 2015) applied MPC to radiant slab systems with evaporative cooling sources. In contrast to rule-based methods, the MPC-based method reduced the energy consumption of cooling tower by 55% and energy consumption of pumps by 25%. To conclude, existing studies prove that MPC is more energy-efficient and cost-efficient than the conventional control methods in terms of building climate control. In this paper, we aim at using the MPC method to directly control the operating frequency of inverter AC responding to RTP from electric utilities or third-party load aggregators. Test results show that the MPC controller is able to improve thermal comfort at the beginning of occupancy, shift peak power demands and reduce electricity costs.

# 2. OFFLINE SYSTEM MODEL DEVELOPMENT

Before being used for online predictive control, control-oriented room thermal model and AC model are required to be developed and identified. The models should be readily used for the purpose of control while capturing the essential dynamics of the target system which requires insights into both the HVAC engineering and the control engineering.

#### 2.1 Room thermal model

2.1.1 Model development: The building models used in popular building simulation tools such as TRNSYS, EnergyPlus and eQUEST are not suitable for the development of model-based control methods since the complex building models are usually not computationally efficient. In our previous study (Hu & Xiao, 2017; Hu, Xiao, & Wang, 2017), we have developed, identified and validated a grey-box RC room thermal model as illustrated in Figure 1. The RC model is a grey-box room thermal model which can be trained using the collected data from smart in-home

sensors. With the simple structure, the RC model can be easily implemented in the model-based controller to predict the room thermal dynamics. The energy balance for the external/internal walls, indoor air and internal thermal mass are given by Equations (1) - (4).



Figure 1: Schematic of the grey-box RC room thermal model.

$$C_w \frac{dT_{w,ext}}{dt} = \frac{T_o - T_{w,ext}}{R_{w,o}} + \frac{T_{w,int} - T_{w,ext}}{R_w} + f_{solar,w} A_w I_{solar}$$
(1)

$$C_{w}\frac{dT_{w,int}}{dt} = \frac{T_{w,ext} - T_{w,int}}{R_{w}} + \frac{T_{in} - T_{w,int}}{R_{w,in}}$$
(2)

$$C_{in}\frac{dT_{in}}{dt} = \frac{T_m - T_{in}}{R_{in,m}} + \frac{T_{w,int} - T_{in}}{R_{w,in}} + \frac{T_o - T_{in}}{R_{win}} + f_{inter,in}Q_{inter} + Q_{HVAC}$$
(3)

$$C_m \frac{dT_m}{dt} = \frac{T_{in} - T_m}{R_{in,m}} + f_{solar,m} A_{win} I_{solar} + f_{inter,m} Q_{inter}$$
(4)

where *R* and *C* represent the overall heat resistance and capacitance; *T* denotes temperature; subscripts *in*, *o*, *w*, *int*, *ext*, *win* and *m* indicate indoor air, outdoor air, exterior wall, internal wall surface, external wall surface, window and internal mass, respectively;  $Q_{inter}$  denotes the internal heat gain;  $I_{solar}$  denotes global solar radiation; *A* denotes geometric area; *f* denotes the conversion coefficients for the heat gains, which are also identified together with *R* and *C*. The detailed model identification methods can be found in our previous work (Hu et al., 2017).

2.1.2 Model linearization and discretization: The room thermal dynamics as described by the differential equations (1) - (4) is a MIMO system. State space model is commonly used when a MIMO system is considered since it can explicitly express the relationships between the outputs and inputs. Also, it can result in convex optimization problems which in general can be well solved by state-of-the-art optimization software. In order to make the model more realistic, white Gaussian noise is added to the system process. Equations (1) - (4) then can be equally converted into a continuous-time state-space model with unknown stochastic noise, as shown in Equation (5).

$$dT = (AT + Bu + Ed)dt + dw(t)$$
(5)

Where the system state  $T = \begin{bmatrix} T_{w,ext} & T_{w,int} & T_{in} & T_m \end{bmatrix}^T$ ; the input vector  $u = Q_{HVAC}$ ; the disturbance vector  $d = \begin{bmatrix} T_o & I_{solar} & Q_{inter} \end{bmatrix}^T$ ; A, B and E are the system, input and disturbance matrices, respectively; w(t) is a Wiener process, which is a stochastic process with independent normal distributed increments.

In real-life applications, the stochastic continuous-time state-space model needs to be discretized, and the stochastic discrete-time state-space model can be given by Equations (6) - (7).

$$T_{k+1} = A_d T_k + B_d u_k + E_d d_k + w_k (6)$$

$$y_k = C_d T_k + v_k \tag{7}$$

where  $A_d$ ,  $B_d$  and  $E_d$  are the corresponding matrices of the discrete-time state-space model which depend on the sampling time; The observed output vector  $y_k = T_{in}$ ; The random variables  $w_k$  and  $v_k$  represent the process and measurement noise, respectively, which are assumed to be independent, white and with normal distribution probabilities, i.e.  $w_k \sim N(0, Q)$  and  $v_k \sim N(0, R)$ ; Q and R are covariance matrices of process noise and measurement noise, respectively. Equations (6) – (7) are embedded in the MPC controller to predict the system evolutions.

2.1.3 Model identification: A residential building is built in TRNSYS to test the RC room thermal model in this study. The residential room ( $L \times W \times H$ : 4.8m×3.6m×3m) is located in Hong Kong and has one south-facing exterior wall

 $(3.6\text{m}\times3\text{m})$  and one east-facing exterior wall  $(4.8\text{m}\times3\text{m})$ . Both exterior walls have single glazed windows and the window-wall-ratio of each wall is 0.2. The overall heat transfer coefficients of the exterior wall and the single glazed window are 2.57 and 5.69 W/(m<sup>2</sup>·K), respectively. Instead of using the default TMY weather data, real historical weather data at the interval of 1 minute from Hong Kong observatory are used for simulation tests. Four weeks (1 June to 29 June 2015) of indoor air temperature data at 1-minute intervals generated from TRNSYS are used to identify the room thermal model. The identification results are:  $C_w = 8,381,606$  J/K,  $C_{in} = 871,887$  J/K,  $C_m = 13,904,351$  J/K,  $R_{win} = 0.0051$  K/W,  $R_w = 0.0060$  K/W,  $R_{w,o} = 0.0010$  K/W,  $R_{w,in} = 0.0041$  K/W,  $R_{in,m} = 0.0023$  K/W,  $f_{solar,w} = 0.4834$ ,  $f_{solar,m} = 0.0012$ ,  $f_{inter,in} = 3.0367$ ,  $f_{inter,m} = 0.9976$ . Figure 2 shows the weather disturbances and the indoor air temperature data predicted by TRNSYS ( $T_{in,TRN}$ ) and RC model ( $T_{in,RC}$ ), respectively. Prediction results show that the RC room thermal model is able to predict the indoor air temperature in a relatively high degree of accuracy.



Figure 2: Weather disturbances and indoor air temperature predicted by TRNSYS and RC model, respectively.

With the identified values of RC and the sampling time of 5 minutes in our case study, the matrices in the discrete-time state-space model, i.e. Equations (6) - (7), can be determined as follows:

$$A_{d} = 10^{-2} \begin{pmatrix} 95.809 & 0.5757 & 0.0023 & 0.0001 \\ 0.5757 & 98.573 & 0.7543 & 0.0600 \\ 0.0223 & 7.2508 & 73.899 & 13.012 \\ 0.0001 & 0.0362 & 0.8159 & 99.119 \end{pmatrix}_{4\times4}; \quad E_{d} = 10^{-4} \begin{pmatrix} 361.21 & 3.4146 & 0.0001 \\ 3.7494 & 0.0101 & 0.0415 \\ 581.53 & 0.0004 & 9.0293 \\ 2.8933 & 0.0013 & 0.2591 \end{pmatrix}_{4\times3};$$

# 2.2 Inverter AC model

2.2.1 Steady-state physical model of inverter AC: AC models can be generally classified into transient models and steady-state models. The refrigerant dynamics in residential ACs are much quicker than the thermal dynamics of the room. The refrigerant re-distributions in different AC components normally accomplish in a short period, around 100 seconds (He, Liu, & Asada, 1997; Rasmussen, 2005). Therefore, a steady-state AC model is developed to predict the coupled dynamic behaviors of the room and the AC under time-varying internal and external conditions. The steady-state performances of inverter ACs in terms of cooling capacity (Q), coefficient of performance (COP) and power consumption (P) can be determined from compressor frequencies ( $N_{comp}$ ) as well as indoor and outdoor air temperature ( $T_{out}$  and  $T_{in}$ ). Since the AC manufacturers seldom provide enough performance data in the full range of operation conditions, the physical modeling method is developed to predict the performance data of typical ductless split inverter ACs. For simplification in control applications, only the four major components, i.e. condenser, evaporator, variable speed compressor, and electronic expansion valve, are modeled in our study. Other minor components such as accumulator, refrigerant pipeline, sub-cooler and receiver are not considered here. Details of the component-wise modeling of inverter AC can be found in our previous paper (Hu & Xiao, 2018).

2.2.2 Identification and validation of inverter AC model: There have been a number of experimental studies of steadystate performances of ACs. Gayeski (Gayeski, 2010) carried out quite elaborate experiments for a Mitsubishi splittype inverter air conditioner with a rated cooling capacity of 2.5kW. It is one of the most popular types of residential inverter ACs and the main specifications can be found in our previous paper (Hu & Xiao, 2018). The experimental results in (Gayeski, 2010), however, did not contain the complete performance data. The same split-type inverter air conditioner was chosen for performance identifications under a wide range of typical operating conditions using physical modeling techniques. To validate the model, 40 sets of experimental data by Gayeski (Gayeski, 2010) are compared with the modeled data under the same operating conditions. Among the compared data, the ranges of indoor air temperature, outdoor air temperature and compressor speed are  $23 - 38^{\circ}$ C,  $24 - 36^{\circ}$ C, and 19-95 Hz, respectively. Figure 3 shows the comparisons between the modeled data and the experimental data. The deviations are mainly in the range of ±15%. The mean absolute percentage errors (MAPEs) between the predicted and tested cooling capacity and COP are 5.65% and 11.94%, respectively. Figure 4 shows the performances of the inverter AC at the compressor speeds of 30Hz, 60Hz and 90Hz predicted by the model.



Figure 3: Comparisons between the modeled data and experimental data.



Figure 4: Performances of the inverter AC at the compressor speeds of 30Hz, 60Hz and 90Hz, respectively.

# **3. ONLINE MODEL PREDICTIVE CONTROL**

# 3.1 Preparation/prediction of exogenous input variables

3.1.1 Weather conditions and occupancy: Weather conditions such as outdoor air temperature and solar radiation have effects on the room thermal dynamics and AC performances, which may come from local observatory or from prediction models. The integrated building system model in MPC makes predictions of system dynamics using the weather forecast. Occupancy prediction plays significant role in the model-based building climate control. The occupancy pattern determines the internal heat gain and the control range of indoor air temperature over the prediction horizon. In our case studies, the weather forecast is acquired from the local observatory and the occupancy is prespecified for simplicity purpose.

3.1.2 RTP and dynamic power thresholds using RTP: Inverter AC is able to run at a wide range of frequencies, e.g. 20-100 Hz, which may lead to the large variation of the power consumptions. To reduce the peak power demands in electrical grids, RTP is normally increased by grid operators to indicate DR program participants to reduce the power consumptions during peak demand or other critical events. To effectively respond to the increased RTP, we can set a

lower operating power threshold for electric appliances to provide DR resources. For this reason, a "Price-to-PowerThreshold" model for inverter ACs is formulated in this study. Cumulative probability distribution of RTP, obtained by analyzing the historical RTP database, can help us to develop the model. The cumulative probability of RTP refers to the probability when the RTP is less than or equal to a specified value of RTP. RTP at 5-minute interval is chosen in this study for the online MPC of inverter ACs. The historical nodal real-time electricity prices of the Electric Reliability Council of Texas (ERCOT) in June 2016 are used for the MPC analysis (Electricity Reliability Council Of Texas (ERCOT)). Figure 5 shows the probability and cumulative probability distributions of RTPs in the ERCOT retail market in June 2016, respectively. The marked point (24.97, 70%) means the probability when the RTP is less or equal to 24.97 cents/kWh is 70%, i.e.  $RTP_{cp=0.7} = 24.97$ .



Figure 5: Probability and cumulative probability distributions of RTPs in the ERCOT retail market in June 2016.

Using cumulative probability distribution of RTP, we propose a strategy to determine dynamic power thresholds for inverter ACs, which can be mathematically described by Equation (8).

$$P_{thsh,i} = \begin{cases} 100\% \cdot P_{max} & \text{if } RTP_i \leq RTP_{cp=0.7} \\ 85\% \cdot P_{max} & \text{if } RTP_{cp=0.7} < RTP_i \leq RTP_{cp=0.85} \\ 70\% \cdot P_{max} & \text{if } RTP_{cp=0.85} < RTP_i \leq RTP_{cp=0.95} \\ 55\% \cdot P_{max} & \text{if } RTP_i \geq RTP_{cp=0.95} \end{cases}$$
(8)

where  $P_{max}$  refers to the maximum operating power of an inverter AC;  $P_{thsh,i}$  denotes the dynamic power threshold of AC at the time step *i*;  $RTP_{cp=0.7}$ ,  $RTP_{cp=0.85}$  and  $RTP_{cp=0.95}$  are the real-time electricity prices when the cumulative probabilities (*cp*) are 0.7, 0.85 and 0.95, respectively. The dynamic power threshold is used as a time-varying constraint in the MPC controller to limit the maximum operating power of AC during DR events.

#### 3.2 Optimization problem formulation and solving

The objective of the MPC-based DR controller is to minimize the electricity cost for operating the inverter AC while keeping the indoor air temperature in the pre-specified ranges. The optimization problem formulation can be given by Equations (9) - (14). Prediction interval and prediction horizon play significant roles in the performance of MPC controllers. In order to protect the compressor of inverter AC, the compressor is allowed to change the rotational speed every 5 minutes. The prediction interval of the MPC controller in our study is therefore set as 5 minutes. The prediction horizon is set as 3 hours to be able to make full preparations for the upcoming disturbances.

$$\min_{P_1, P_2, \dots, P_{N-1}} \sum_{k=1}^{N-1} P_k \Delta t \cdot RTP_k + \rho_e e_k \tag{9}$$

subject to

$$y_k = C_d T_k + v_k \tag{11}$$

(10)

 $T_{k+1} = A_d \cdot T_k + B_d \cdot COP_k \cdot P_k + E_d \cdot d_k + w_k$ 

$$y_{lb,k} - e_k \le y_k \le y_{ub,k} + e_k \tag{12}$$

$$e_k \ge 0 \tag{13}$$

$$P_k = 0 \text{ or } P_{min} \le P_k \le P_{thsh,k} \tag{14}$$

where N is the prediction horizon;  $\Delta t$  is the prediction interval;  $P_k$  and  $COP_k$  are the AC power consumption and COP at the time step k, respectively;  $T_k$ ,  $y_k$ , and  $d_k$  are the state vector, output vector, disturbance vector in the state-space system model, respectively;  $w_k$  and  $v_k$  are the process and measurement noise, respectively, which are assumed to be independent, white and with normal distribution probabilities.  $A_d$ ,  $B_d$ ,  $C_d$ ,  $D_d$  are the state-space matrices of the discrete-time state-space model. The objective function, Equation (9), aims at minimizing the electricity cost over the prediction horizon. As long as the constraints are not violated, the optimal power consumption remains at 0 kW, i.e. the AC is turned off. Equations (10) - (11) depict the system dynamics in a state-space form. The input variable, cooling capacity ( $Q_{HVAC,k}$ ), is represented by the product of  $COP_k$  and power consumption ( $P_k$ ). Equation (12) is a dynamic soft constraint which keeps the indoor air temperature in the dynamic ranges. During unoccupied hours, the AC is supposed to be turned off and the optimized power consumption remains at 0 kW. To achieve that goal, the temperature range is normally set wide, e.g.  $20 - 30^{\circ}$ C, making the indoor air temperature difficult to exceed the constraint. When the room is occupied, the indoor air temperature should remain between the upper bound and the lower bound to ensure the occupant comfort. However, sometimes the temperature constraint may not be satisfied in practice even if the AC runs at the full cooling capacity. This situation may lead to the failure of the optimization solving. To address this issue, we can relax the MPC problem by introducing a slack variable  $e_k$ . The hard constraint is then converted as a soft constraint which has a similar function as the dead-band plays in the on-off control. To ensure most of the time the preset temperature demand is met, i.e.  $e_k = 0$ , an especially large penalty  $\rho_e$  is normally imposed on slack variable  $e_k$  in the objective function. Equation (14) is a time-varying power constraint to limit the maximum power consumption of AC using the RTP from electric utilities. For an inverter AC, it has the minimum operating frequency, e.g. 20 Hz, and the corresponding minimum power consumption P<sub>min</sub>. Therefore, the power consumption should range between  $P_{min}$  and  $P_{thsh,k}$  or equal zero, i.e. turned off.

#### 3.3 Control signal implementation and state estimation using Kalman filter

Actuator in inverter AC is the variable-speed compressor which takes the frequency as the control signal. The optimization result, power signal, should be converted into frequency signal before the real implementation. The performance maps of inverter ACs can be inversely used to determine the corresponding frequency. The essence of the model predictive control is a feedback control, which is accomplished by iterative optimizations over moving prediction horizons. The optimization result at each time step is a trajectory of command signals. Only the first signal is implemented and the rest signals are disposed. At next time step, the optimization is repeated starting from the new measured/estimated current state. In our case, the system state includes four variables, i.e.  $T = [T_{w,ext} \quad T_{w,int} \quad T_{in} \quad T_m]^T$ . However, only the indoor air temperature  $T_{in}$  is measured. To address this issue, Kalman filter is commonly used to estimate the immeasurable variables and filter the noise from the measurements (Bishop & Welch, 2001; Simon, 2006). The Kalman filter is an algorithm that provides an efficient computational solution to estimate the state of a process in a way that minimizes the mean squared error. The solution is recursive in that each updated estimate of the state is computed from the previous estimate and the new measured data.

#### 4. TEST RESULTS AND DISCUSSIONS

#### 4.1 A TRNSYS-MATLAB co-simulation testbed and test conditions

Building energy simulators such as EnergyPlus and TRNSYS are commonly used to dynamically simulate building thermal performance and energy performance of its HVAC system. However, the embedded controller models are programmed based on conventional control methods, e.g. on-off and PID controllers. In this study we connect TRNSYS to MATLAB in which advanced predictive control mechanism can be readily developed. Both the frequency-based power consumption model of inverter AC and its frequency-based controller model (e.g. PID and MPC) are developed in MATLAB. The output of the controller is the operating frequency of the compressor motor. With the operating frequency, the inverter AC determines corresponding cooling capacity and delivers it to the building component in TRNSYS. In this study, TRNSYS 18 (32-bit) is used to connect to MATLAB 2014a (32-bit) via the inherent component (Type 155) in TRNSYS under Windows 10 64-bit operating system.

Weather forecast from local observatory, occupancy profiles and RTPs from electric utilities are used as the exogenous inputs of MPC controller. Exogenous variables on five typical summer days are shown in Figure 6, which are used to test the performances of MPC controller in this study. It can been seen that RTP at 5-min interval from ERCOT retail market varies during the whole day and normally reaches the peak around 16:00, which indicates the occurrence of DR events. A fluctuant occupancy pattern is strategically predefined to test the controllability of MPC under time-

changing circumstance of occupancy. The room is assumed to be occupied during 12:00 - 18:00 and 20:00 - 08:00. The simulation starts from the 8:00 in the morning.



Figure 6: Weather conditions and RTP from ERCOT on five typical summer days.

#### 4.2 Basic case using conventional PID control

To demonstrate the advantages of MPC controller for inverter ACs, the system performance is first tested using conventional PID control method. A PID controller is chosen here to control the inverter AC to maintain the indoor temperature at the set-point. The indoor air temperature set-point is set as  $24 \,^{\circ}$ C when the occupants are awake (12:00 – 18:00 and 20:00 – 24:00), and  $26 \,^{\circ}$ C when the occupants are asleep (24:00 - 08:00). In the PID case, RTPs are only used for electricity charges and not involved in the control method. The simulation time step is set as 1 minute. As shown in Figure 7, the PID controller can fulfill the control objective most of the time. However, some issues can be found as follows. In terms of thermal comfort, when the room changes from unoccupied to occupied, the inverter AC takes a period of time to remove the accumulated heat gains, which may cause the thermal discomfort of the occupants. Besides, the thermal comfort cannot be satisfied even if the AC runs under its largest cooling capacity. As shown around 16:00 on Day 3 and Day 4, the indoor air temperatures have large deviations from the set-point, although the AC operates under the maximum power consumption. This is because of the inadequacy of AC cooling capacity under bad outdoor weather conditions. In terms of power consumption and cost, the compressor of inverter AC may still run at high frequencies when RTP is much higher than the normal price during DR events. This may lead to peak power consumptions in the electrical grid when a large population of residential ACs are considered. Due to high electricity prices during peak demand periods, it will also cause the accompanying cost ineffectiveness.

#### 4.3 MPC-based DR Controller responding to RTP

The MPC controller can fully address the thermal comfort issue at the beginning of occupancy by automatically and optimally precooling the room in advance. As can be seen from Figure 7, on Day 1 the MPC controller makes a prediction around 10:00 and realizes that the indoor air temperature will exceed the upper bound 24°C after the residents come back from work or school if no action is taken. Hence, the inverter AC automatically precooled the room in advance to improve the thermal comfort at the beginning of occupation. The start-up time and duration of precooling depend on the future weather conditions and thermal evolution of the room. It can be seen that in the PID case the temperature deviations during 12:00 - 14:00 from the set-point on Day 3 and Day 4 are larger than the temperature deviation on Day 1 due to the worse weather conditions. Therefore, it takes a longer time period to precool the room on Day 3 and Day 4 for MPC controller. We can also find that the temperature in MPC case fluctuates around the upper temperature bound. This is because the prediction cannot be absolutely accurate due to the unknown noise in the real case, which may lead to the deviations of MPC outputs. However, most of temperature deviations are in the acceptable range of  $\pm 0.5^{\circ}$ C.

The MPC-based DR controller is also price-responsive and helps to reduce peak power demands during high-RTP hours and reduce total electricity costs. Since the maximum operating power around 16:00 is limited due to the high RTP, the MPC-based DR controller makes the AC to precool the room for a longer time period to a lower temperature point. In this way, even when the AC cooling capacities around 16:00 are limited, the thermal comfort can still be satisfied due to the previously stored cooling. Compared to the PID case, the average power consumptions during 15:00 to 17:00 are reduced by 14.46% (Day 1), 33.15% (Day 2), 36.83% (Day 3), 40.53% (Day 4) and 25.66% (Day 5) respectively, in the MPC-based DR control case. Since the MPC-based DR controller helps to efficiently shift the

power demands from high-RTP periods to low-RTP periods, the all-day electricity costs are reduced by 5.74% (Day 1), 15.03% (Day 2), 17.67% (Day 3), 24.27 (Day 4) and 5.29% (Day 5), respectively. It is worth mentioning that the potential of electricity cost reduction is influenced by the weather disturbances and the shape of RTP profile. For example, compared with Day 3 and Day 4, the weather on Day 1 is not that hot and the RTP shape is flatter, so the MPC-based DR controller provides less DR potential.



Figure 7: System performances (i.e. indoor air temperature, power consumption, and electricity costs) under the PID control and MPC-based DR control.

# **5. CONCLUSIONS**

The development of electrical grids imposes a new function on residential AC, i.e. being DR-enabled. The essence of being DR-enabled is able to respond to dynamic RTP from electrical utilities or third-party load aggregators. To achieve that goal, we aim to use MPC approach to directly control the operating frequency of inverter AC while considering all the influential variables including weather condition, occupancy and RTP. In order to improve the online computational efficiency, a simple-structured RC room thermal model is developed, linearized, discretized and identified. Besides, we develop the steady-state physical model of inverter AC and generate its performance maps for fast online applications. A conventional MPC controller without DR function and MPC-based DR controller are designed and tested on a TRNSYS-MATLAB co-simulation testbed. Test results show that compared with PID control, the MPC-based DR controller for inverter AC is able to improve the thermal comfort at the beginning of occupancy, reduce the peak power demands and reduce total electricity costs.

## REFERENCES

- Aswani, A., Master, N., Taneja, J., Culler, D., & Tomlin, C. (2012). Reducing transient and steady state electricity consumption in HVAC using learning-based model-predictive control. *Proceedings of the IEEE*, 100(1), 240-253.
- Bishop, G., & Welch, G. (2001). An introduction to the Kalman filter. *Proc of SIGGRAPH, Course*, 8(27599-23175), 41.

Camacho, E. F., & Alba, C. B. (2013). Model predictive control: Springer Science & Business Media.

- Chen, Z., Wu, L., & Fu, Y. (2012). Real-time price-based demand response management for residential appliances via stochastic optimization and robust optimization. *Smart Grid, IEEE Transactions on*, 3(4), 1822-1831.
- Electrical and Mechanical Services Department. (2017). *Hong Kong Energy End-use Data 2017*. Retrieved from Hong Kong:
- Electricity Reliability Council Of Texas (ERCOT). (2017). Historical RTM Load Zone and Hub Prices. Electric Reliability
   Council of Texas.
   Retrieved from from http://mis.ercot.com/misapp/GetReports.do?reportTypeId=13061&reportTitle=Historical%20RTM%20Loa d%20Zone%20and%20Hub%20Prices&showHTMLView=&mimicKey
- Feng, J. D., Chuang, F., Borrelli, F., & Bauman, F. (2015). Model predictive control of radiant slab systems with evaporative cooling sources. *Energy and Buildings*, 87, 199-210.
- Gayeski, N. T. (2010). Predictive pre-cooling control for low lift radiant cooling using building thermal mass. Massachusetts Institute of Technology,
- He, X.-D., Liu, S., & Asada, H. H. (1997). Modeling of Vapor Compression Cycles for Multivariable Feedback Control of HVAC Systems. *Journal of Dynamic Systems, Measurement, and Control, 119*(2), 183-191. doi:10.1115/1.2801231
- Henze, G. P., Felsmann, C., & Knabe, G. (2004). Evaluation of optimal control for active and passive building thermal storage. *International Journal of Thermal Sciences*, 43(2), 173-183. doi:10.1016/j.ijthermalsci.2003.06.001
- Hu, M., & Xiao, F. (2017). Investigation of the Demand Response Potentials of Residential Air Conditioners Using Grey-box Room Thermal Model. *Energy Procedia*, 105, 2759-2765. doi:10.1016/j.egypro.2017.03.594
- Hu, M., & Xiao, F. (2018). Price-responsive model-based optimal demand response control of inverter air conditioners using genetic algorithm. *Applied Energy*, 219, 151-164. doi:https://doi.org/10.1016/j.apenergy.2018.03.036
- Hu, M., Xiao, F., & Wang, L. (2017). Investigation of demand response potentials of residential air conditioners in smart grids using grey-box room thermal model. *Applied Energy*, 207(Supplement C), 324-335. doi:<u>https://doi.org/10.1016/j.apenergy.2017.05.099</u>
- Hubert, T., & Grijalva, S. (2012). Modeling for residential electricity optimization in dynamic pricing environments. *Smart Grid, IEEE Transactions on, 3*(4), 2224-2231.
- Ma, Y., Borrelli, F., Hencey, B., Coffey, B., Bengea, S., & Haves, P. (2012). Model predictive control for the operation of building cooling systems. *Control Systems Technology, IEEE Transactions on, 20*(3), 796-803.
- Ma, Y., Kelman, A., Daly, A., & Borrelli, F. (2012). Predictive control for energy efficient buildings with thermal storage. *IEEE Control System Magazine*, 32(1), 44-64.
- Ma, Y., Matuško, J., & Borrelli, F. (2015). Stochastic model predictive control for building HVAC systems: Complexity and conservatism. *IEEE Transactions on Control Systems Technology*, 23(1), 101-116.
- Morari, M., & Lee, J. H. (1999). Model predictive control: past, present and future. *Computers & Chemical Engineering*, 23(4), 667-682.
- Oldewurtel, F., Parisio, A., Jones, C. N., Gyalistras, D., Gwerder, M., Stauch, V., . . . Morari, M. (2012). Use of model predictive control and weather forecasts for energy efficient building climate control. *Energy and Buildings*, 45, 15-27. doi:10.1016/j.enbuild.2011.09.022
- Qin, S. J., & Badgwell, T. A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7), 733-764. doi:10.1016/s0967-0661(02)00186-7
- Rasmussen, B. P. (2005). Dynamic modeling and advanced control of air conditioning and refrigeration systems. (PhD), UIUC,
- Simon, D. (2006). Optimal state estimation: Kalman, H infinity, and nonlinear approaches: John Wiley & Sons.
- Somasundaram, S., Pratt, R., Akyol, B., Fernandez, N., Foster, N., Katipamula, S., . . . Taylor, Z. (2014). Reference guide for a transaction-based building controls framework. *Pacific Northwest National Laboratory*.
- Thomas, A. G., Jahangiri, P., Wu, D., Cai, C., Zhao, H., Aliprantis, D. C., & Tesfatsion, L. (2012). Intelligent residential air-conditioning system with smart-grid functionality. *Smart Grid, IEEE Transactions on, 3*(4), 2240-2251.
- Wang, S., & Ma, Z. (2008). Supervisory and Optimal Control of Building HVAC Systems: A Review. HVAC&R Research, 14(1), 3-32. doi:10.1080/10789669.2008.10390991

## ACKNOWLEDGEMENT

The research work presented in this paper is financially supported by a research grant (G-YBTB) in the Hong Kong Polytechnic University (PolyU) and the Strategic Focus Area (SFA) Scheme (1-BBW7) of Research Institute for Sustainable Urban Development (RISUD) in PolyU. The support is gratefully acknowledged.