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TR-EE 91-20
May 1991

April 15th 1991

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Abstract

In this paper we address the problem of controlling multiple robots manipulating a rigid object cooperatively when the robots and load parameters are uncertain. We propose a controller that takes into account the dynamics of both the load and the manipulators. The linearity of the dynamics of the robots and the load, with respect to the unknown parameters, is exploited during the derivation of the parameter adaptation scheme. In order to design a control and update laws that do not require the measurements of the of the joint accelerations or the load acceleration, the dynamics of both the robots and the load are filtered through a stable first order filter. Then two prediction error vectors are defined as the difference between the measured filtered dynamics and the predicted filtered dynamics of both the robots and the load. The least-squares estimation method is used to estimate the parameters of the multi-robot system from the prediction errors. We then develop a controller that is based on the cancellation of the nonlinearities. The proposed controller guarantees global asymptotic tracking of the robot and load trajectories and also guarantees the asymptotic tracking of the internal forces trajectories.

1. Introduction

Recently considerable amount of research has focused on the problem of cooperative control and coordination of multiple robots. Interest in multi-robot systems has arisen because several tasks require the use of two or more robots. Examples of such tasks include the joining and securing of large pipes for the construction of space structures, picking up and carrying heavy loads, and grasping odd shaped loads. Cooperative robots may be used in hazardous or unsafe environments such as in space, in deep waters and in radioactive environments. By using more than one robot the manipulation capability and the workspace of the system are further increased. However the multi-robot systems are more difficult to control than single robots. Additional problems arise as the parameters of the robots and the manipulated load may not be known exactly.

It is important to review some major developments in the single robot adaptive control literature because of their relevance to the multiple robots case.

Ortega and Spong [10] presented a summary of several recent papers on adaptive control for rigid robots. Craig et. al. [4] proposed an adaptive control law which is basically a modification of the a computed torque control scheme. Slotine and Li [14] exploited the structural properties of the rigid manipulator dynamics to design an adaptive controller capable of general trajectory tracking control. They tested their design on a two degree of freedom semi-direct drive robot. They experimentally showed that the parameters of the robot which were assumed to be unknown initially can be estimated within a very short time. They also showed that their controller has the same level of robustness to unmodeled dynamics as the PD controlled system. Better tracking performance than the PD or the computed torque schemes were also obtained.

Bayard and Wen [2] introduced a class of asymptotically stable adaptive control laws for robotic manipulators. In their analysis they used a parameterization of the dynamics of the robot which is based on physical quantities. They also used the property that the dynamics are linear with respect to the unknown parameters of the manipulator. They proposed an energy like Lyapunov function that retains the nonlinear character and structure of the dynamics. The authors proved that their approach leads to asymptotically stable adaptive systems. Their approach does not require the convergence of the parameter estimates, invertibility of the mass matrix estimate or the measurement of joint accelerations.

Seraji [13] proposed a decentralized adaptive control algorithm for robot manipulators. His controller is based on independent joint control. A PID controller and position velocity acceleration feedforward controller both with adjustable gains are used to control each joint independently. Sadegh and Horowitz [12] presented a control scheme that uses the desired trajectory outputs to update the parameters and the controller. They also address the robustness properties of the proposed scheme. Middleton and Goodwin [8] examined the use of predictive adaptive control laws to a rigid link manipulator. They used a computed torque controller in conjunction with linear estimation techniques to prove the global convergence of the adaptive system.

There were two important developments for the adaptive control of single robots which are relevant to our work on the control of multiple robots. The first development was the use of linearity of the uncertain parameters of the robot with respect to the dynamics of the robot. The second development is the use of the predictive adaptive control to design control laws for rigid robots. We will use these two developments to design an adaptive controller for the multi-robot system.

Few control schemes for cooperative multiple robots manipulating a common load have been proposed. Tarn et al. [16] presented a control method that is based on the linearization of the robot models with nonlinear feedback and nonlinear transformation; they then use the theory of linear optimal control to design a robust controller. Yun et al. [17] used exact linearization and output decoupling to design a controller for the two robot manipulators. Zheng and Luh [20] considered the dual arms system as a closed dynamic chain and then used master/slave method to design the control laws. Yoshikawa and Zheng [19] proposed a cooperative dynamic hybrid control method; this method takes into consideration both the manipulator dynamics and the object dynamics. Guo and Ahmad [1] looked at the control problems when the robots have compliant joints.

Hsu et al [6] developed a control algorithm for the coordinated manipulation of multifingered robot hand. Their control guarantees the convergence of the load motion and internal forces to the desired values respectively.

Walker et al. [18] developed an algorithm for the control of two robots handling a load of unknown mass. Their controller achieves global convergence of the load motion and *the internal forces* to their respective desired trajectories. Their controller basically amounts to an open loop controller for the internal forces and a closed loop controller for the positions. The attractive feature of the proposed control law is that the computational algorithm is the same for the two manipulators.

Hu and Goldenberg [7] addressed the problem of multiple robots manipulating a load in contact with the environment. First they decomposed the multi-robot system into three substems, one for the position error another one for the contact forces and a third one for the internal forces. Then they used these subsystems errors and Popov's hyperstability theory to derive the update laws to estimate the unknown parameters. The proposed control law guarantees the global asymptotic convergence to the desired position, internal forces and contact forces trajectories.

Carignan used reduced order models for the tracking control of two manipulator arms handling a load. An adaptive controller was designed to track a desired trajectory using a reduced model of the system instead of the full order model of the system. Carignan's results were very encouraging, however a lot of work remains to be done on the use of reduced order models.

Zribi and Ahmad [21] and [22] proposed an adaptive controller for the multi-robot system manipulating a rigid object cooperatively. Their controller takes into account the dynamics of the manipulators and the load and does not require feedback of joint acceleration or the inversion of the inertia or the Jacobian matrices. They also considered the effects of bounded disturbances on the system. A control law which guarantees the convergence of the tracking error to a bounded set is also given.

In this paper we address the problem of controlling multiple robots handling an object cooperatively. In section 2 we develop the dynamic models of the manipulators and the load. In section 3 we exploit the linearity of the dynamics with respect to the unknown parameters to derive the parameters update laws. The dynamics of both the manipulators and the load are filtered through a first order stable filter so that the control and update laws do not require the measurements of the accelerations. We define two prediction error vectors for the load and the manipulators. These prediction errors are the difference between the measured filtered dynamics and the predicted filtered dynamics. In section 4 we use these prediction errors with a Least-squares estimation method to estimate the parameters of the multi-robot system. We then develop a controller which guarantees the asymptotic convergence of the object motion and the internal forces exerted by the manipulators on the load to their respective desired trajectories.

2. Cooperative Multi-Robot System Model

2.1 Dynamics Model

The general dynamic model for k cooperative multi-robot system has been investigated thoroughly in the literature, and is also described in the below for completeness. We first start by stating few assumptions that will be used in the subsequent derivation.

Assumptions:

(1) The manipulators are rigidly grasping the load.

(2) The number of joints of the ith robot, n_i, is equal to six and all the manipulators are non redundant.

(3) All the manipulators can independently move along each axis and therefore all the robots are nonsingular along the desired trajectories.

The dynamic equation of the ith manipulator in cooperative manipulation is given as:

$$H_i(q_i)\ddot{q}_i + N_{1i}(q_i,\dot{q}_i) = \tau_i - J_i(q_i)^T F_i \qquad i=1, ..., k \tag{1}$$

where, $q_i(t) \in R^{n_i}$ is the vector of joint displacements, and n_i is the number of joints of the ith robot. The inertia matrix of the ith robot is $H_i(q_i) \in R^{n_i \times n_i}$, this is a positive definite and symmetric matrix. The vector of centrifugal, Coriolis and gravity forces is $N_{1i}(q_i,\dot{q}_i) \in R^{n_i}$; the manipulator Jacobian is $J_i(q_i) \in R^{n_i \times n_i}$. The control input torque for the ith robot is $\tau_i \in R^{n_i}$. The forces/moments applied by the ith manipulator on the object at the point of contact is $F_i \in R^6$ and it can be written in terms of the contact forces $f_i \in R^3$ and contact moments $\eta_i \in R^3$, such that $F_i = \begin{bmatrix} f_i^T & \eta_i^T \end{bmatrix}^T$ for $i = 1,...,k$.

Now we will group the dynamics of the k robots to get,

$$H(q)\ddot{q} + N_1(q,\dot{q}) = \tau - J^T(q)F \tag{2}$$

Let $n = \sum_{i=1}^{i=k} n_i = 6k$ where n_i is the number of joints of the ith robot. Then it should be noted that $H \in R^{n \times n}$, $N_1 \in R^n$, $\tau \in R^n$, $J \in R^{n \times n}$, $F \in R^n$, $q \in R^n$. Here $H(q)$ is a diagonal matrix whose diagonal elements are $H_i(q_i)$, (i.e $H = diag(H_i)$), also $J(q)$ is a diagonal matrix whose diagonal elements are $J_i(q_i)$, (i.e $J = diag(J_i)$). Further,

$$N_1(q,\dot{q}) = \begin{bmatrix} N_1(q_1,\dot{q}_1) \\ . \\ . \\ . \\ N_k(q_k,\dot{q}_k) \end{bmatrix}, \quad q = \begin{bmatrix} q_1 \\ . \\ . \\ . \\ q_k \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ . \\ . \\ . \\ \tau_k \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} F_1 \\ . \\ . \\ . \\ F_k \end{bmatrix} \tag{3}$$

The equations of motion of the object (load) are obtained from the Newton-Euler mechanics (equations balancing forces and moments).

$$M\ddot{x} + Mg = \sum_{i=1}^{i=k} f_i \tag{4}$$

$$I\dot{\omega} + \omega \times (I\omega) = \sum_{i=1}^{i=k} (\eta_i + r_i \times f_i) \tag{5}$$

Where the position of the center of mass of the object expressed in world coordinate frame is $x(t) \in R^3$. The rotational velocity of the center of mass of the object in world coordinate frame is $\omega(t) \in R^3$, and the gravitational forces acting on the object is $g(x) \in R^3$. The mass matrix $M \in R^{3\times3}$ is a diagonal matrix whose diagonal elements are the mass of the load; $I \in R^{3\times3}$ is the inertia matrix of the load. The position of the end effector of the ith manipulator with respect to the object center of mass, expressed in the world coordinate frame, is $r_i(t) \in R^3$.

The motion of the load expressed by equations (4) and (5) can be re-written as:

$$S\ddot{z} + N_2(z,\dot{z}) = GF \tag{6}$$

Where $G \in R^{6\times n}$ is the grasp matrix, and it is defined as

$$G = \begin{bmatrix} T_1 & T_2 & \cdots & T_k \end{bmatrix} \tag{7}$$

The matrix $T_i \in R^{6\times 6}$ is obtained from equations (4) and (5) and it is given as

$$T_i = \begin{bmatrix} I_{3\times3} & 0 \\ \Omega_i(r_i) & I_{3\times3} \end{bmatrix} \quad \text{and} \quad \Omega_i(r_i) = \begin{bmatrix} 0 & -r_{iz} & r_{iy} \\ r_{iz} & 0 & -r_{ix} \\ -r_{iy} & r_{ix} & 0 \end{bmatrix}$$

where $I_{3\times3}$ is the 3 by 3 identity matrix, and $r_i = [\, r_{ix}, r_{iy}, r_{iz}]^T \in R^3$ represents the vector from the center of mass of the object to the contact point between the object and the ith manipulator.

Also the matrices $S \in R^{6\times6}$ and $N_2 \in R^{6\times6}$ are defined such that,

$$S = \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \quad \text{and} \quad N_2 = \begin{bmatrix} Mg \\ \omega\times(I\omega) \end{bmatrix} \tag{8}$$

The angular velocity of the center of mass of the object expressed in world coordinate frame is $v(t) \in R^3$. The position and orientation vector of the center of mass of the object expressed in world coordinate frame is $z(t) \in R^6$ and the velocity vector is $\dot{z}(t) = \begin{bmatrix} v^T(t) & \omega^T(t) \end{bmatrix}^T \in R^6$. Notice that $\dot{z}(t) = \begin{bmatrix} \dot{x}^T(t) & \omega^T(t) \end{bmatrix}^T$.

2.2 Kinematic Model

Occasionally, we might be interested in controlling the manipulators in some predefined Cartesian task space such that:

$$z_i(t) = K_i(q_i(t)) \qquad i = 1,\ldots,k \tag{9}$$

where $K_i(.) : R^{n_i} \rightarrow R^6$ is the transformation from the joint angle space of $q_i(t)$ to the task space containing $z_i(t)$, and $z_i(t) \in R^6$ is the position and orientation of the point of contact of the ith manipulator, with the load, expressed in the world coordinate frame. Notice that the angular velocity of the point of contact of the ith manipulator, with the load, expressed in world coordinate frame is $v_i(t) \in R^3$, the rotational velocity of the point of contact of the ith manipulator, with the load, expressed in world coordinate frame is $\omega_i(t) \in R^3$, $\dot{z}_i(t) = \begin{bmatrix} v_i^T(t) & \omega_i^T(t) \end{bmatrix}^T \in R^6$.

If we differentiate equation (9) with respect to time, and if we define $J_i(q_i)$ to be the differential map of the $q_i(t)$ space to $z_i(t)$ space (i.e. $J_i = \frac{\partial K_i}{\partial q_i}$), then we can write

$$\dot{z}_i(t) = J_i(q_i(t))\dot{q}_i(t) \qquad i = 1,\ldots,k \tag{10}$$

If these equations are stacked into a single vector by forming the J_i into a block diagonal matrix, and concatenating the $\dot{q}_i$'s into one vector $\dot{q}$, we get

$$v_c = J\dot{q} \tag{11}$$

where $v_c = \left[\dot{z}_1^T \ \dot{z}_2^T \ \ldots\dot{z}_k^T\right]^T$ is the vector velocity of the contact points, and $J = \text{diag}(J_i)$.
If we write F_o as the total force acting on the center of mass of the object, then F_o correspond to the left hand side of equation (6), hence equation (6) can be written as,

$$F_o = GF \tag{12}$$

Now from the duality between the forces and the velocities [11], we can write

$$G^T\dot{z} = v_c \tag{13}$$

where $\dot{z}$ is the velocity of the center of mass of the object.
Thus for the k robots system, we combine equations (11) and (13) to get

$$G^T\dot{z} = J\dot{q} \tag{14}$$

where G is the grasp matrix for the multi-robot system, and J is the Jacobian of the system. Now if we differentiate the above equation, we get

$$\ddot{q} = J^{-1}G^T\ddot{z} - J^{-1}\dot{J}\dot{q} \tag{15}$$

2.3 Definition of Position and Internal Forces Errors

We require the load to follow a predefined twice differentiable trajectory $z_d(t) \in C^2$ hence the trajectory tracking error of the load, $e(t) \in R^6$, is:

$$e(t) = z(t) - z_d(t) \tag{16}$$

The end effector force of the ith manipulator, F_i, can be decomposed into two forces, the motion force and the internal force. The internal force does not cause any motion of the load. However we must control this force in order to prevent excessive compressive or expansive forces being applied to the load.

Let F_I represent the internal grasping forces and let $F_{I,d}(t)$ represent the desired internal grasping forces. Thus the internal grasping forces error is,

$$e_f = F_I - F_{I,d} \tag{17}$$

Notice that *the internal forces and the internal forces errors* do not contribute to the motion of the load.

3. Properties and Preliminary Definitions of the Multi-robot System

3.1 Properties

Few important properties of the inertia matrix, the centrifugal/Coriolis matrix and the gravity vector which will be used in the developments are now given.

Property 1 : Linearity in the robots parameters:

The first property which will be used in the subsequent development is the linearity of H(q) and $N_1(q)$ with respect to the manipulators dynamic parameters P_1.

$$H(q)\ddot{q} + N_1(q,\dot{q}) = Y_1(q,\dot{q},\ddot{q})P_1 \tag{18}$$

where $P_1 \in R^{r_1}$ is a vector of r_1 robot parameters which are constants for the given manipulators. These parameters will be estimated by the proposed adaptive scheme. The regressor matrix $Y_1(q,\dot{q},\ddot{q}) \in R^{n \times r_1}$ represents the structure of the robots dynamics, hence its elements are combinations of the elements of the inertia matrix and the centrifugal, Coriolis and the gravity vector.

Property 2 : Linearity in the load parameters:

Another property deals with the linearity in the load parameters,

$$S\ddot{z} + N_2(z,\dot{z}) = Y_2(z,\dot{z},\ddot{z})P_2 \tag{19}$$

where $P_2 \in R^{r_2}$ is a vector of r_2 load parameters which are constants for a given load. These parameters will be estimated by the proposed adaptive scheme. The regressor matrix $Y_2(z,\dot{z},\ddot{z}) \in R^{6 \times r_2}$ represents the structure of the load dynamics.

3.2 Preliminaries and Definition of Variables

Let $\hat{P}_1$ be the vector of estimates of the parameters of the robots, then the error vector in the estimates of the robots parameters is $\tilde{P}_1 = \hat{P}_1 - P_1$. Similarly, we can write the parameter estimation error vector for the load as $\tilde{P}_2 = \hat{P}_2 - P_2$. Notice that we can write

$$\hat{H}(q)\ddot{q} + \hat{N}_1(q,\dot{q}) = Y_1(q,\dot{q},\ddot{q})\hat{P}_1 \tag{20}$$

Where $\hat{H}$ is the estimate of the inertia matrix H(q) and $\hat{N}_1$ is the estimate of the Coriolis, centrifugal and the gravity vector.

Also notice that $\tilde{H}(q)\ddot{q} + \tilde{N}_1(q,\dot{q})\dot{q} = Y_1(q,\dot{q},\ddot{q})\tilde{P}_1$, where $\tilde{H}$ is the error in the inertia matrix, and $\tilde{N}_1$ is the error in the Coriolis, centrifugal and gravity vector.

Similarly we can write

$$\hat{S}\ddot{z} + \hat{N}_2 = Y_2\hat{P}_2 \tag{21}$$

where $\hat{S}$ is the estimate of S and $\hat{N}_2$ is the estimate of N_2.

We want to design control and parameter update laws that does not require the measurements of the acceleration vectors $\ddot{q}$ and $\ddot{z}$, thus we will filter the dynamics of both the multi-robot system and the dynamics of the load. To achieve this we multiply both sides of the equation we want to filter by $\lambda_f/(D + \lambda_f)$, where λ_f is a known positive constant and D is the Laplace operator [15].

Equation (2) and (18) can be combined so that we can write

$$Y_1P_1 = \tau - J^TF \tag{22}$$

Now we filter equation (22), to get

$$Y_{1f}P_1 = \tau_f - (J^TF)_f \tag{23}$$

where,

$$Y_{1f} \triangleq (\frac{\lambda_f}{D+\lambda_f})Y_1 \tag{24}$$

$$\tau_f \triangleq (\frac{\lambda_f}{D+\lambda_f})\tau \tag{25}$$

$$(J^TF)_f \triangleq (\frac{\lambda_f}{D+\lambda_f})(J^TF) \tag{26}$$

The prediction error for the manipulators parameters will be defined as,

$$e_{p_1} \triangleq \tau_f - (J^TF)_f - Y_{1f}\hat{P}_1 = -Y_{1f}\tilde{P}_1 \tag{27}$$

Similarly filtering equation (19) we get

$$Y_{2f}P_2 = (GF)_f \tag{28}$$

where Y_{2f} is defined as

$$Y_{2f} \triangleq (\frac{\lambda_f}{D+\lambda_f})Y_2 \tag{29}$$

and

$$(GF)_f \triangleq (\frac{\lambda_f}{D+\lambda_f})(GF) \tag{30}$$

The prediction error for the load parameters will be defined as,

$$e_{p_2} \triangleq (GF)_f - Y_{2f}\hat{P}_2 = -Y_{2f}\tilde{P}_2 \tag{31}$$

Finally we will state two lemmas which will be used in the derivation of the control law.

Lemma 1:
If V_1 is a matrix and V_2 is a vector, then we have

$$\frac{D+\lambda_f}{\lambda_f}(V_1V_2) = \frac{1}{\lambda_f}\dot{V}_1V_2 + V_1\frac{D+\lambda_f}{\lambda_f}(V_2)$$

the proof for lemma 1 is obvious and follows directly from the definition.

Lemma 2:
Let

$$e = H(s)r$$

where H(s) is an n×m strictly proper, exponentially stable transfer function. Then $r \in L_2$ implies that $e \in L_2 \cap L_\infty$, $\dot{e} \in L_2$, e is continuous, and $e \to 0$ as $t \to \infty$. If, in addition, $r \to 0$ as $t \to \infty$, then $\dot{e} \to 0$.

Lemma 2 is taken from Desoer and Vidyasagar [5], see also Ortega and Spong [10].

4. Parameter Estimation and Control law Design

Theorem 1: Parameter Estimation

The below update laws guarantee that $\hat{P}_1$, $\hat{P}_2$ are bounded, and that $e_{p_1}, e_{p_2} \in L_2$ regardless of the control law.

$$\dot{\hat{P}}_1(t) = \Gamma_1(t) Y_{1f}^T e_{p_1} \tag{32}$$

$$\dot{\Gamma}_1(t) = -\Gamma_1(t) Y_{1f}^T Y_{1f} \Gamma_1(t) \tag{33}$$

and the load parameters are updated as,

$$\dot{\hat{P}}_2(t) = \Gamma_2(t) Y_{2f}^T e_{p_2} \tag{34}$$

$$\dot{\Gamma}_2(t) = -\Gamma_2(t) Y_{2f}^T Y_{2f} \Gamma_2(t) \tag{35}$$

Where the time varying gains $\Gamma_1 \in R^{r_1 \times r_1}$, $\Gamma_2 \in R^{r_2 \times r_2}$ and $\Gamma_1(t_0)$ and $\Gamma_2(t_0)$ are symmetric positive definite matrices.

Proof:

Consider the following Lyapunov function candidate:

$$V(t) = \tilde{P}_1^T \Gamma_1(t)^{-1} \tilde{P}_1 + \tilde{P}_2^T \Gamma_2(t)^{-1} \tilde{P}_2 \tag{36}$$

If we differentiate V with respect to time, we get

$$\dot{V}(t) = 2\tilde{P}_1^T \Gamma_1(t)^{-1} \dot{\hat{P}}_1 + \tilde{P}_1^T \dot{\Gamma}_1(t)^{-1} \tilde{P}_1 + 2\tilde{P}_2^T \Gamma_2(t)^{-1} \dot{\hat{P}}_2 + \tilde{P}_2^T \dot{\Gamma}_2(t)^{-1} \tilde{P}_2 \tag{37}$$

and noting that $\frac{d}{dt}(\Gamma_i^{-1}\Gamma_i) = 0$ for (i=1,2), we get

$$\dot{\Gamma}_1^{-1} = -\Gamma_1^{-1}\dot{\Gamma}_1\Gamma_1^{-1} = \Gamma_1^{-1}(\Gamma_1 Y_{1f}^T Y_{1f} \Gamma_1)\Gamma_1^{-1} = Y_{1f}^T Y_{1f} \tag{38}$$

$$\dot{\Gamma}_2^{-1} = -\Gamma_2^{-1}\dot{\Gamma}_2\Gamma_2^{-1} = \Gamma_2^{-1}(\Gamma_2 Y_{2f}^T Y_{2f} \Gamma_2)\Gamma_2^{-1} = Y_{2f}^T Y_{2f} \tag{39}$$

Thus,

$$\begin{aligned} \dot{V}(t) &= 2\tilde{P}_1^T \Gamma_1^{-1}\Gamma_1 Y_{1f}^T e_{p_1} + \tilde{P}_1^T Y_{1f}^T Y_{1f} \tilde{P}_1 + 2\tilde{P}_2^T \Gamma_2^{-1}\Gamma_2 Y_{2f}^T e_{p_2} + \tilde{P}_2^T Y_{2f}^T Y_{2f} \tilde{P}_2 \\ &= -2e_{p_1}^T e_{p_1} + e_{p_1}^T e_{p_1} - 2e_{p_2}^T e_{p_2} + e_{p_2}^T e_{p_2} \end{aligned} \tag{40}$$

Hence we have

$$\dot{V}(t) = -e_{p_1}^T e_{p_1} - e_{p_2}^T e_{p_2} = -\tilde{P}_1^T Y_{1f}^T Y_{1f} \tilde{P}_1 - \tilde{P}_2^T Y_{2f}^T Y_{2f} \tilde{P}_2 \tag{41}$$

Therefore $\dot{V}(t) \le 0$ and the estimator is always stable. Thus $\hat{P}_1$, $\hat{P}_2$ are bounded, and e_{p_1} , $e_{p_2} \in L_2$.

This concludes the proof of theorem 1.

In the following we will assume that the parameters $\hat{P}_1$ and $\hat{P}_2$ are in some convex region C_τ such that $\hat{H}$ and $\hat{S}$ are positive definite matrices. (i.e. $\hat{P}_1$ and $\hat{P}_2 \in C_\tau$ / $\hat{H}$ and $\hat{S} > 0$).

Theorem 2: Control Law

The adaptive control law given below (equations 42-47) used in conjunction with the update laws given in Theorem 1 guarantees the global convergence of the cooperative multi-robot system tracking errors (i.e. $e(t) \to 0$ and $e_f(t) \to 0$ as $t \to \infty$).

$$\tau = \tau_1 + \tau_2 + \tau_3 + \tau_4 \tag{42}$$

where τ_1, τ_2, τ_3 and τ_4 are defined as

$$\tau_1 = J^T(F_{I,d} - K_I \int e_f dt) \tag{43}$$

$$\tau_2 = -\hat{H}_T(-\ddot{z}_d + K_v\dot{e} + K_p e) + (-\hat{H}J^{-1}\dot{J}\dot{q} + \hat{N}_1 + J^T G^+ \hat{N}_2) \tag{44}$$

$$\tau_3 = -\frac{1}{\lambda_f} J^T \dot{J}^{-T} e_{p_1} - \frac{1}{\lambda_f} J^T G^+ \hat{A} \dot{\hat{A}}^{-1} (GJ^{-T} e_{p_1} + e_{p_2}) \tag{45}$$

$$\tau_4 = \frac{1}{\lambda_f} J^T G^+ Y_{2f} \dot{\hat{P}}_2 + \frac{1}{\lambda_f} Y_{1f} \dot{\hat{P}}_1 \tag{45a}$$

$$\hat{H}_T \triangleq \hat{H}J^{-1}G^T + J^T G^+ \hat{S} \tag{46}$$

$$\hat{A} \triangleq GJ^{-T}\hat{H}_T = GJ^{-T}\hat{H}J^{-1}G^T + \hat{S} \tag{47}$$

and the adaptation laws are given by equations (32), (33), (34) and (35).

The matrices K_v, K_p, K_I are defined to be constant symmetric positive definite matrices; these matrices are design parameters.

Proof:

Applying the inverse filter to equation (31), we get,

$$\frac{D + \lambda_f}{\lambda_f}(e_{p_2}) = GF - Y_2\hat{P}_2 - \frac{1}{\lambda_f} Y_{2f}\dot{\hat{P}}_2 \tag{48}$$

From equation (48) we can solve for F, to get

$$F = G^+[\frac{D + \lambda_f}{\lambda_f}(e_{p_2}) + Y_2\hat{P}_2 + \frac{1}{\lambda_f} Y_{2f}\dot{\hat{P}}_2] + F_I \tag{49}$$

where F_I denotes the internal grasping forces, and $G^+ = G^T(GG^T)^{-1}$.

Also applying the inverse filter to equation (27), we get,

$$\frac{D+\lambda_f}{\lambda_f}(e_{p_1}) = \tau - J^T F - Y_1\hat{P}_1 - \frac{1}{\lambda_f}Y_{1f}\dot{\hat{P}}_1 \tag{50}$$

Now combining equations (49) and (50), we get

$$\begin{aligned}\frac{D+\lambda_f}{\lambda_f}(e_{p_1}) + J^T G^+ \frac{D+\lambda_f}{\lambda_f}(e_{p_2}) &= \tau - Y_1\hat{P}_1 - \frac{1}{\lambda_f}Y_{1f}\dot{\hat{P}}_1 - J^T F_I \\ &\quad - J^T G^+[Y_2\hat{P}_2 + \frac{1}{\lambda_f}Y_{2f}\dot{\hat{P}}_2]\end{aligned} \tag{51}$$

Replacing τ, τ_1 and τ_4 by their values from equations (42), (43) and (45a), we get

$$\begin{aligned}\frac{D+\lambda_f}{\lambda_f}(e_{p_1}) + J^T G^+ \frac{D+\lambda_f}{\lambda_f}(e_{p_2}) &= \tau_1 + \tau_2 + \tau_3 - Y_1\hat{P}_1 - J^T G^+ Y_2\hat{P}_2 - J^T F_I \\ &= (\tau_2 - Y_1\hat{P}_1 - J^T G^+ Y_2\hat{P}_2) \\ &\quad + \tau_3 - J^T(e_f + K_I\textstyle\int e_f dt)\end{aligned} \tag{52}$$

Using (20), (21) and (15), we get

$$\begin{aligned}-Y_1\hat{P}_1 - J^T G^+ Y_2\hat{P}_2 &= -\hat{H}\ddot{q} - \hat{N}_1 - J^T G^+(\hat{S}\ddot{z} + \hat{N}_2) \\ &= -\hat{H}(J^{-1}G^T\ddot{z} - J^{-1}\dot{J}\dot{q}) - \hat{N}_1 - J^T G^+(\hat{S}\ddot{z} + \hat{N}_2) \\ &= -(\hat{H}J^{-1}G^T + J^T G^+\hat{S})\ddot{z} - (-\hat{H}J^{-1}\dot{J}\dot{q} + \hat{N}_1 + J^T G^+\hat{N}_2) \\ &= -\hat{H}_T\ddot{z} - (-\hat{H}J^{-1}\dot{J}\dot{q} + \hat{N}_1 + J^T G^+\hat{N}_2)\end{aligned} \tag{53}$$

If we combine (53) and (44), we get

$$\tau_2 - Y_1\hat{P}_1 - J^T G^+ Y_2\hat{P}_2 = -\hat{H}_T(\ddot{e} + K_v\dot{e} + K_p e) \tag{54}$$

Now combining equations (52) and (54), we have,

$$\begin{aligned}\frac{D+\lambda_f}{\lambda_f}(e_{p_1}) + J^T G^+ \frac{D+\lambda_f}{\lambda_f}(e_{p_2}) &= -\hat{H}_T(\ddot{e} + K_v\dot{e} + K_p e) \\ &\quad + \tau_3 - J^T(e_f + K_I\textstyle\int e_f dt)\end{aligned} \tag{55}$$

Multiplying equation (55) by GJ^{-T}, we get

$$\begin{aligned}GJ^{-T}\frac{D+\lambda_f}{\lambda_f}(e_{p_1}) + \frac{D+\lambda_f}{\lambda_f}(e_{p_2}) &= -\hat{A}(\ddot{e} + K_v\dot{e} + K_p e) \\ &\quad + GJ^{-T}\tau_3 - G(e_f + K_I\textstyle\int e_f dt)\end{aligned} \tag{56}$$

Now recall the fact that the internal grasping forces lie in the null space of G, thus we can conclude that

$$G(e_f + K_I \int e_f dt) = 0 \tag{57}$$

Now substituting τ_3 by its value from equation (45), equation (56) becomes:

$$GJ^{-T}\frac{D+\lambda_f}{\lambda_f}(e_{p_1}) + \frac{D+\lambda_f}{\lambda_f}(e_{p_2}) = -\hat{A}(\ddot{e} + K_v\dot{e} + K_p e) - \frac{1}{\lambda_f}G\dot{J}^{-T}e_{p_1}$$
$$-\frac{1}{\lambda_f}\hat{A}\dot{\hat{A}}^{-1}(GJ^{-T}e_{p_1} + e_{p_2}) \tag{58}$$

Equation (58) becomes:

$$\frac{D+\lambda_f}{\lambda_f}(GJ^{-T}e_{p_1} + e_{p_2}) = -\hat{A}(\ddot{e} + K_v\dot{e} + K_p e)$$
$$-\frac{1}{\lambda_f}\hat{A}\dot{\hat{A}}^{-1}(GJ^{-T}e_{p_1} + e_{p_2}) \tag{60}$$

Now multiply both sides of equation (60) by $\hat{A}^{-1}$, and use the identity given by Lemma 1, we get,

$$\frac{D+\lambda_f}{\lambda_f}[\hat{A}^{-1}(GJ^{-T}e_{p_1} + e_{p_2})] + (\ddot{e} + K_v\dot{e} + K_p e) = 0 \tag{61}$$

If we let

$$u \triangleq -\frac{1}{\lambda_f}\hat{A}^{-1}(GJ^{-T}e_{p_1} + e_{p_2}) \tag{62}$$

Then equation (61) becomes

$$\ddot{e} + K_v\dot{e} + K_p e = (D + \lambda_f)u \tag{63}$$

Also it should be noted that $u \in L_2$ because of the following:
(1) The exact value of A, $(A = GJ^{-T}HJ^{-1}G^T + S)$, is a positive definite matrix because both H and S are positive definite matrices. We assumed that we are working in convex region of $\hat{P}_1$ and $\hat{P}_2$ such that $\hat{H}$ and $\hat{S}$ are positive definite, which implies that $\hat{A}$ is positive definite and thus $\hat{A}^{-1}$ is bounded.
(2) GJ^{-T} which is a function of the kinematic parameters of the manipulators (which are assumed to be known) is bounded.
(3) In theorem 1 we proved that $e_{p_1} \in L_2$ and $e_{p_2} \in L_2$.
Hence using Lemma 2 and the facts that $u \in L_2$ and $\hat{A}^{-1}$ is bounded, we can conclude that $e \to 0$ as $t \to \infty$.
In the following we will prove that $u \to 0$ as $t \to \infty$ and therefore $\dot{e} \to 0$ as $t \to \infty$. Middleton and Goodwin [8], proved that the regressor matrix Y_f and $\dot{Y}_f$ are bounded for rigid link manipulator systems. Therefore Y_{1f} and Y_{2f} will also satisfy the same property because they are also represented by same rigid mechanical dynamics. Hence using the same arguments as Middleton and Goodwin (we omit the proofs here, see page 13-14 of [8]), it can be shown that Y_{1f}, $\dot{Y}_{1f}$, Y_{2f} and $\dot{Y}_{2f}$ are bounded. Thus e_{p_1} is bounded because $e_{p_1} = -Y_{1f}\tilde{P}_1$. Now $\dot{\hat{P}}_1 = \Gamma_1 Y_{1f}^T e_{p_1}$ is bounded because Γ_1, Y_{1f} and e_{p_1} are bounded. Also, $\dot{e}_{p_1} = -\dot{Y}_{1f}\tilde{P}_1 - Y_{1f}\dot{\hat{P}}_1$ is bounded. Therefore we

can conclude e_{p_1} is uniformly continuous. Since e_{p_1} is uniformly continuous and $e_{p_1} \in L_2$, we have $e_{p_1} \to 0$ as $t \to \infty$.

Similarly e_{p_2} is bounded because $e_{p_2} = -Y_{2f}\tilde{P}_2$. Now $\dot{\hat{P}}_2 = \Gamma_2 Y_{2f}^T e_{p_2}$ is bounded because Γ_2, Y_{2f} and e_{p_2} are bounded. Also, $\dot{e}_{p_2} = -\dot{Y}_{2f}\tilde{P}_2 - Y_{2f}\dot{\hat{P}}_2$ is bounded. Therefore we can conclude that e_{p_2} is uniformly continuous. Since e_{p_2} is uniformly continuous and $e_{p_2} \in L_2$, we have $e_{p_2} \to 0$ as $t \to \infty$. Hence $u = -\frac{1}{\lambda_f}\hat{A}^{-1}(GJ^{-T}e_{p_1} + e_{p_2})$ tends to zero as t tends to infinity. Finally we can conclude from Lemma 2 that $\dot{e} \to 0$ as $t \to \infty$.

Thus we can conclude that $\hat{P}_1$, $\hat{P}_2$, $\dot{\hat{P}}_1$, $\dot{\hat{P}}_2$ are bounded and also e_{p_1}, e_{p_2}, e, $\dot{e}$ tend to zero as t tends to infinity.

To prove that e_f tends to zero, we should note that equation (55) can be written as:

$$\frac{D+\lambda_f}{\lambda_f}(e_{p_1}) + J^T G^+ \frac{D+\lambda_f}{\lambda_f}(e_{p_2}) = -\hat{H}_T(\ddot{e} + K_v\dot{e} + K_p e) - \frac{1}{\lambda_f}J^T\dot{J}^{-T}e_{p_1}$$
$$-\frac{1}{\lambda_f}J^T G^+ \hat{A}\dot{\hat{A}}^{-1}(GJ^{-T}e_{p_1} + e_{p_2}) - J^T(e_f + K_I\int e_f dt) \quad (65)$$

Now because e_{p_1}, e_{p_2}, e and $\dot{e}$ tend to zero as t tends to infinity we have

$$J^T(e_f + K_I\int e_f dt) \to 0 \quad (66)$$

However J^T is not singular, thus we have

$$e_f + K_I\int e_f dt \to 0 \quad (67)$$

Finally because K_I is a positive definite matrix, we have $e_f \to 0$ as $t \to \infty$.

This concludes the proof of theorem 2.

5. Conclusion

In this paper, we proposed an adaptive control scheme for the multi-robot system during cooperative motion. The proposed controller takes into account the dynamics of the object and the dynamics of the manipulators. The linearity of the dynamics of both the robots and the load with respect to the parameters, were exploited during the derivation of the controller. The dynamics of both the robots and the load were filtered so that the control law does not require the measurements of the joint accelerations and the load acceleration. The proposed controller guarantees convergence of the actual position to the desired one and the convergence of the internal grasping forces to the desired internal grasping forces. Least-squares estimation was used to estimate the parameters. Future work on using different methods for parameter estimation, and robustness of the least-squares estimator is underway.

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