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ADAPTIVE CONTROL OF FLEXIBLE JOINT ROBOTS DERIVED FROM ARM ENERGY CONSIDERATIONS

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1. Abstract

Almost all industrial robots exhibit joint flexibility due to mechanical compliance of their gear boxes. In this paper we outline a design of an adaptive controller for flexible joint robots based on the arms energy.

The desired actuator trajectory in a flexible joint robot is dependent not only on the desired kinematic trajectory of the link but also on the link dynamics. Unfortunately, link dynamic parameters are unknown in most cases, as a result the desired actuator trajectory is also unknown. To overcome this difficulty, a number of control schemes have suggested the use of acceleration and link jerk feedback. In this paper we describe a control scheme which does not use link jerk or acceleration. The control law we derive is based on the energy of the arm deviating from the desired trajectory and it has two stages with two corresponding adaptation laws. The first stage drives the actuator and the joints to a desired manifold, the second controller then seeks to drive the joints to their desired trajectory. On application of our first controller there is an apparent structural reduction of the order of the system. This apparent reduction in the structure is exploited by our second stage controller. Our control scheme does not require link acceleration or jerk measurements, and the numerical differentiation of the velocity signal, or the inversion of the inertial matrices are also unnecessary. Simulations are presented to verify the validity of the control scheme. The superiority of the

proposed scheme over existing rigid robot adaptive schemes is also illustrated through simulation.

2. Introduction

Many of today's rigid robots are driven by actuators with high gear ratios, the load due to the arm at the actuator is reduced by a factor of n_g, where, $n_g > 1$, is the gear ratio. In fact, inertia of the arm experienced by the actuator is reduced by $(1/n_g^2)$, and as the actuator acceleration is n_g times the joint acceleration, the overall load is reduced by $(1/n_g)$. Thus the load experienced by robots with high gear ratios are dominated by actuator dynamics, link dynamics are secondary. Recent trend is towards high-technology direct-drive robots. Here, the actuators are directly connected links and the lack of high gear ratios and increasing demand for high-speed operation, requires the control system to compensate for the dominant nonlinear link dynamics. Robots which move fast (apparently with reasonable manufacturing cycle times) and or carry large loads have additional problems. It is experimentally found that most gearing systems are compliant, as a result, actuators are connected to the robot links through effectively flexible shafts. The presence of high gear ratios reduces the effective load experienced at the motors, and the absence of gearing adds to the complexity of the control problem. Experimental evidence indicates that joint flexibility should be accounted for in both modeling and control of manipulators (Ahmad 1988) (Widmann et al 1987) (Ghorbel et al 1989) The presence of joint flexibility in the direct-drive high-speed actuators can be modeled by a "linear" torsional spring. This flexibility may be attractive in practical applications especially when the robot must make contact with an unknown surface.

Numerous techniques to control Flexible Joint Robots have been suggested [Widmann et al 1987, De-Luca 1988, Fu et al 1989, Khorasani 1989, Spong et al 1987, Ghorbel et al 1989). One approach is based on the idea of feedback linearization, which requires the measurement of joint acceleration and jerk to be used in the feedback loop (De-Luca 1988), (Spong et al 1987). Another method is based on the concept of reduced order system and requires the restriction of the system to a suitable integral manifold in the state space (Khorasani 1989).

We propose a controller which drives the FJR to track a desired trajectory in two stages. The first stage drives the actuator to the desired actuator trajectory, while the second stage drives the arm to the its desired trajectory. Similar to the work on rigidly jointed robots (Slotine et al 1987, 1988), (Koditschek 1987), our controller design starts by selecting a Lyapunov function which is similar to the energy of the FJR. Our control scheme does not require link jerk, or acceleration feedback or the inversion of the inertia matrix, in addition parameter adaptation is easily accommodated.

At this time, the only adaptive control scheme for flexible joint robots that we are aware of that uses position and velocity feedback is the one derived from singular perturbation arguments by Ghorbel, Spong and Hung (Ghorbel 1989). Inorder to derive an adaptive scheme from a singular perturbation argument several assumptions are necessary, these include sufficient joint stiffness and that it is possible to ignore the higher order terms in the singular perturbation expansion. Assumptions such as these are not necessary in our derivations.

An important problem in adaptive control is that of parameter convergence, providing a sufficiently rich tracking signal has sometimes been assumed to be adequate conditions for parameter convergence. However tracking a persistently exciting trajectory does not mean that all of the unknown parameters of a certain manipulator can be estimated. In general, the maximum number of parameters that may be estimated depends on the trajectory used for estimation and on the kinematic structure of the manipulator (Khosla 1989). These unknown parameters could be categorized as uniquely identifiable, identifiable in linear combinations only, or unidentifiable. Typically, only those dynamic parameters that affect the force/torque equations of at least one joint can be identified.

3. Manipulator Models

Experimental investigations of industrial robots with harmonic drive transmission and other forms of gearing indicate that joint flexibility contributes significantly to the overall dynamics of the system (Ahmad 1988), (Spong 1987). The dynamic equations of the flexible joint robots are given as :

$$\tau = D_m \ddot{q}_m + B_m \dot{q}_m + K_s(q_m - q) \quad (1)$$

$$0 = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + K_s(q - q_m) \tag{2}$$

where, an n-link manipulator becomes a 2n-degrees of freedom system:

D_m : Diagonal motor inertia matrix $\in \mathbb{R}^{n\times n}$
B_m : Diagonal motor damping matrix $\in \mathbb{R}^{n\times n}$
K_s : Diagonal drive shaft stiffness matrix $\in \mathbb{R}^{n\times n}$
q_m : Vector of sensed motor angles $\in \mathbb{R}^{n\times 1}$
$D(q)$: Link inertia matrix $\in \mathbb{R}^{n\times n}$
$C(q,\dot{q})$: Centrifugal and Coriolis terms matrix $\in \mathbb{R}^{n\times n}$
$g(q)$: Gravitational vector term $\in \mathbb{R}^{n\times 1}$
q : Vector of link joint angles $\in \mathbb{R}^{n\times 1}$

Matrices D_m, B_m, K_s, are positive definite matrices. Further, $D(q)$ is symmetric, positive definite and both $D(q)$ and $D^{-1}(q)$ are both bounded as function of q (Spong 1987), (Ghorbel et al 1989). When K_s tends toward infinity, the robot is considered to have rigid joints (i.e. $q=q_m$). The dynamic equations which represent the rigidly jointed robot, with the same inertial and coriolis matrices as the FJR defined above, are:

$$\tau = [D(q)+D_m]\ddot{q} + [C(q,\dot{q})+B_m]\dot{q} + g(q) \tag{3}$$

Some properties of the rigid model concerning the inertia matrix, Coriolis and centrifugal force matrix were discussed by Koditschek. Those properties remain valid for the flexible model (Ghorbel et al 1989). The first most important property shows that $D(q)$ and $C(q,\dot{q})$ are not independent, but the matrix $(\dot{D}-2C)$ is skew symmetric, this can be easily derived from the Lagrangian formulation of the manipulator dynamics (see Appendix A). The second property confirms that the individual terms of the right hand side of equation (2), excluding the $K_s(q-q_m)$ term, could be represented by a linear relationship between a suitably selected set of unknown manipulator and load parameters (Slotine et al 1987), (Ghorbel et al 1989), (Spong 1987), in other words equation (2) could be rewritten as:

$$0 = Y(\ddot{q},\dot{q},q)\,P + K_s\,(q - q_m) \tag{4}$$

where $Y(\ddot{q},\dot{q},q) \in \mathbb{R}^{n\times r}$, is called the regressor matrix of known functions, and $P \in \mathbb{R}^{r\times 1}$ is a vector of unknown parameters.

4. Trajectory Model

Let $q_d(t) \in C^4$ denote a desired link trajectory in which case $q_d(t), \dot{q}_d(t), \ddot{q}_d(t), \dddot{q}(t)$ are all bounded and continuously differentiable. The set of desired motor trajectory can be derived using equation (4). The diagonal stiffness matrix, $K_s \in \mathbb{R}^{n \times n}$, can be written as $K_s = \mathrm{Diag}\,[\, k_{s1}, k_{s2}, \ldots, k_{sn} \,]$, where $k_{si} > 0$, for $i = 1,2,\ldots,n$, represents the spring constant of the i^{th} drive shaft. Since all of these constants are positive and K_s is a diagonal matrix, as a result matrix K_s is invertible and positive definite.

We assume the link parameters and the load handled by the end effector are time invariant, i.e.

$$P = \text{Constant vector, thus, } \dot{P} = \ddot{P} = 0 \tag{5}$$

The above assumption is valid in a large class of applications. The desired motor trajectory may now be computed as follows:

$$q_{md}(t) = K_s^{-1} Y(\ddot{q}_d, \dot{q}_d, q_d)P + q_d(t) \tag{6}$$

$$\dot{q}_{md}\,(t) = K_s^{-1} \dot{Y}(\ddot{q}_d, \dot{q}_d, q_d)P + \dot{q}_d(t) \tag{7}$$

$$\ddot{q}_{md}\,(t) = K_s^{-1} \ddot{Y}(\ddot{q}_d, \dot{q}_d, q_d)P + \ddot{q}_d(t) \tag{8}$$

The subscript "d" is used to denote the desired trajectory.

Notice that the desired motor trajectory $q_{md}(t)$, $q_{md}(t)$ and $\ddot{q}_{md}(t)$ are dependent on the desired link trajectory $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ and also on the unknown parameters P and the link dynamic structure represented by Y_d, $\dot{Y}_d$ and $\ddot{Y}_d$. This makes it difficult to design a control law which utilizes the desired motor position and velocity.

Using equations (6), (7) and (8), removing subscripts d, and using equation (1) and (2), we can rewrite equation (1) in-link coordinates q as:

$$\tau = D_m K_S^{-1} D(q) q^{(4)} + N(q, q^{(1)}, q^{(2)}, q^{(3)}) \tag{8a}$$

$$= Y^*(q, q^{(1)}, q^{(2)}, q^{(3)}, q^{(4)}) P^* \tag{8b}$$

where, N (.,.,.,.) is a nonlinear function $\in \mathbb{R}^n$, $q^{(i)}$ is the i^{th} time derivative of q. From the structure of equation (8a) we can see that the FJR can be stabilized by feeding back a nonlinear function of the link position, velocity, acceleration and jerk. Notice that the fourth order dynamics in the link coordinates can also be written in the regressor matrix form in terms of some suitably selected vector of unknown parameters P^*.

5. Selection of an Energy based Lyapunov function

If E is the total energy of the robot links and actuators, i.e. E is the sum of the kinetic and potential energies of the actuator and linkages:

$$E=\tfrac{1}{2}\dot{q}_m^t D_m \dot{q}_m+\tfrac{1}{2}\dot{q}^t D\dot{q}+\tfrac{1}{2}(q-q_m)^t K_s(q-q_m)+\Phi(q) \tag{9}$$

where, $\Phi(q)$ is the gravitational potential energy of the linkage. Then the power input to the FJR is through the actuator and is given as:

$$\frac{dE}{dt}=(\tau_m-B_m\dot{q}_m)^t\dot{q}_m \tag{10}$$

Notice that when $\Phi(q) = 0$, $E(\dot{q}_m,\dot{q},q_m,q)$ becomes a quadratic in q, $\dot{q}$, q_m, and $\dot{q}_m$. Notice also, if we set $\tau_m = B_m\dot{q}_m-\Omega^t\dot{q}_m$, then

$$\frac{dE}{dt}=-\dot{q}_m^t\Omega\dot{q}_m \leq 0 \tag{11}$$

where, $q_m \in \mathbb{R}^n$, and $\Omega \in \mathbb{R}^{n\times n} > 0$, such that $x^t\Omega x = 0$, if and only if $x = 0$.

We can conclude that, with an appropriate rate feedback, we may track a static joint trajectory. This exposition shows why most FJR with appropriate damping will track a static rate trajectory, i.e. $\lim_{t\rightarrow\infty}(\dot{q}_m - \dot{q}_{md})\rightarrow 0$. This exposition indicates to us that if we select a Lyapunov function similar to E given in (9), we may stabilize the FJR along a nonstatic link trajectory by suitable position and velocity feedback.

Excluding the potential energy of the FJR, the energy of the robot arm along a prespecified trajectory is :

$$E(t)=\tfrac{1}{2}\dot{q}_d^t D\dot{q}_d+\tfrac{1}{2}(q_d-q_{md})^t K_s(q_d-q_{md})+\tfrac{1}{2}\dot{q}_{md}^t D_m\dot{q}_{md} \quad . \tag{12}$$

Likewise, the energy in the system which causes the FJR to deviate from the desired trajectory is given as :

$$V(t)=\tfrac{1}{2}\dot{e}^t D\dot{e}+\tfrac{1}{2}(e-e_m)^t K_s(e-e_m)+\tfrac{1}{2}\dot{e}_m^t D_m\dot{e}_m \tag{13}$$

where, we define the error terms as: $e = (q_d - q)$ and $e_m = (q_{md} - q_m)$. Throughout the trajectory it is desired to have $\frac{dV}{dt} \leq 0$, furthermore $\dot{V}(t)$ and $V(t)$ should be dependent on e and e_m as well as $\dot{e}_m$ and $\dot{e}$. We can make

$V(t)$ dependent on $\dot{e}$, e, $\dot{e}_m$ and e_m by selecting:

$$V(t)=\tfrac{1}{2}\dot{e}_m^t D_m \dot{e}_m+\tfrac{1}{2}\dot{e}^t D\dot{e}+\tfrac{1}{2}(e-e_m)^t K_s(e-e_m)$$

$$+\tfrac{1}{2}e^t K_p e+\tfrac{1}{2}e_m^t K_{pm} e_m \tag{14}$$

where, $K_p \in \mathbb{R}^{n\times n}$, $K_{pm} \in \mathbb{R}^{n\times n}$ are some positive gain matrices. The derivation of τ_m to make $\dot{V}(t) < 0$ and proportional to the variables e, e_m, $\dot{e}$ and $\dot{e}_m$ will be addressed in the next section.

6. Control and Adaptation Law Design

As the dynamic parameters of the arm are unknown, we can define the parameter error vector as $e_p = \tilde{P} - P$, where $\tilde{P}$ is the estimated parameter vector. Notice $\dot{e}_p = \dot{\tilde{P}}$, as $\dot{P} = 0$. Based on the estimated value of the parameter vector $\tilde{P}$, we obtain an estimate of the desired motor position as $\tilde{q}_{md}$ using equation (6). Similarly, we can compute the estimated motor velocity and acceleration. We can define the following motor error as $\tilde{e}_m=(\tilde{q}_{md}-q_m)$. Similar terms for $\dot{\tilde{e}}_m$ and $\ddot{\tilde{e}}_m$ can be defined. Based on the above Lyapunov function (14), we can find the energy of the trajectory deviating from the desired trajectory as :

$$V(t)=\tfrac{1}{2}\dot{\tilde{e}}_m^t D_m \dot{\tilde{e}}_m+\tfrac{1}{2}\dot{e}^t D\dot{e}+\tfrac{1}{2}(e-\tilde{e}_m)^t K_s(e-\tilde{e}_m)$$

$$+\tfrac{1}{2}e^t K_p e+\tfrac{1}{2}\tilde{e}_m^t K_{pm} \tilde{e}_m+\tfrac{1}{2}e_p^t M e_p \tag{15}$$

The last term in (15) is added to account for parameter adaptation, where $K_p, K_{pm} \in \mathbb{R}^{n\times n}$, and $M \in \mathbb{R}^{r\times r}$ are some positive gain matrices.
For convenience let us define:

$$D(q)\ddot{q}_d+C(q,\dot{q})\dot{q}_d+g(q)=\Psi(\ddot{q}_d,\dot{q}_d,\dot{q},q)P \tag{16}$$

where, $\Psi \in \mathbb{R}^{n\times r}$, and

$$Y_d = Y(\ddot{q}_d,\dot{q}_d,q_d) \tag{17}$$

where, $Y_d \in \mathbb{R}^{n\times r}$, and let Λ be some positive diagonal matrix $\in \mathbb{R}^{n\times n}$, then we let

$$\Gamma(\ddot{q}_d,\dot{q}_d,\dot{q},q)P=D(q)[\ddot{q}_d+\Lambda\dot{e}]+C(q,\dot{q})[\dot{q}_d+\Lambda e]+g(q) \tag{18}$$

where, $\Gamma \in \mathbb{R}^{n\times r}$. Furthermore let us define the following variables:

$s \in \mathbb{R}^n$, $s=(\dot{e}+\Lambda e)$. Let us also define a region where $\ddot{\tilde{e}}_{mi} = \ddot{\tilde{q}}_{mdi} - \ddot{q}_{mi}$ as:

$$\mu_{min}(i) \leq \ddot{\tilde{e}}_{mi} \leq \mu_{max}(i) \text{ for } i=1,2,...,n \tag{19}$$

where, $\mu_{min}(i)$, and $\mu_{max}(i)$ are real scalars. Let us also set vector $\lambda = (\lambda_1,...\lambda_n)^t \in \mathbb{R}^n$ be defined such that:

$$\lambda_i = \tfrac{1}{2} D_{mi}\{Sgn(s_i)[\mu_{min}(i) - \mu_{max}(i)] + \mu_{min}(i) + \mu_{max}(i)\} \tag{20}$$

for i=1,2,...,n .

$$\text{where, } Sgn(s_i) = \begin{cases} +1 & \text{if } s_i > 0 \\ -1 & \text{if } s_i < 0 \\ 0 & \text{if } s_i = 0 \end{cases} \tag{20a}$$

Theorem 1:

The system given by the dynamical model (1) and (2), subjected to the following two stage control and adaptation laws, achieves desired trajectory tracking.

$$\tau = D_m \ddot{\tilde{q}}_{md} + B_m \dot{q}_m + K_s(\tilde{q}_{md} - q_d) + K_{pm}\tilde{e}_m + K_{dm}\dot{\tilde{e}}_m$$

$$+ \frac{\dot{\tilde{e}}_m}{\|\dot{\tilde{e}}_m\|^2}[\dot{e}^t((\Psi - Y_d)\tilde{P} + K_p e) + \dot{e}^t K_d \dot{e} + e^t K_p e + \tilde{e}_m^t K_{pm} \tilde{e}_m] \tag{21}$$

if $\|\dot{\tilde{e}}_m\|^2 > \epsilon > 0$, for a scalar ϵ, and

$$\text{otherwise, } \tau = \Gamma(\ddot{q}_d, \dot{q}_d, \dot{q}, q)\tilde{P} + D_m \ddot{\tilde{q}}_{md} + B_m \dot{q}_m + K_d s - \lambda \tag{22}$$

where, K_{dm}, $K_d \in \mathbb{R}^{n \times n}$ are some positive gain matrices. Corresponding to the two stage control laws we have the following adaptation laws.

$$\text{if } \|\dot{\tilde{e}}_m\|^2 > \epsilon, \quad \dot{\tilde{P}}(t) = M^{-1}\Psi^t(\ddot{q}_d, \dot{q}_d, \dot{q}, q)\, \dot{e} \tag{23}$$

and otherwise,

$$\dot{\tilde{P}}(t) = M^{-1}\Gamma^t(\ddot{q}_d, \dot{q}_d, \dot{q}, q)[\dot{e} + \Lambda e] \tag{24}$$

Proof of Theorem 1:

Differentiation of the candidate Lyapunov function V(t) in equation (15) yields the following:

$$\dot{V}(t)=\dot{\tilde{e}}_m^t[D_m\ddot{\tilde{e}}_m+K_s(\tilde{e}_m-e)+K_{pm}\tilde{e}_m]$$

$$+\dot{e}^t[D\ddot{e}+\tfrac{1}{2}\dot{D}\dot{e}+K_pe+K_s(e-\tilde{e}_m)]+e_p^tM\dot{e}_p-\dot{e}^t(\tfrac{1}{2}\dot{D}-C)\dot{e} \qquad (25)$$

In order to simplify equation (25), we have subtracted the term $\dot{e}^t(\frac{1}{2}\dot{D}-C)\dot{e} = 0$, see (Appendix A). Simplifying equation (25) and substituting the dynamic equation of the FJR given by (1) and (2), we have

$$\dot{V}(t)=\dot{\tilde{e}}_m^t\{D_m\ddot{\tilde{q}}_{md}+K_s(\tilde{q}_{md}-q_d)+K_{pm}\tilde{e}_m-[D_m\ddot{q}_m+K_s(q_m-q)]\}$$

$$+\dot{e}^t\{D\ddot{q}_d+C\dot{q}_d+K_s(q_d-\tilde{q}_{md})-[D\ddot{q}+C\dot{q}+K_s(q-q_m)+g(q)]$$

$$+K_pe+g(q)\}+e_p^tM\dot{e}_p$$

$$=\dot{\tilde{e}}_m^t[D_m\ddot{\tilde{q}}_{md}+K_s(\tilde{q}_{md}-q_d)+K_{pm}\tilde{e}_m+B_m\dot{q}_m-\tau]$$

$$+\dot{e}^t[D\ddot{q}_d+C\dot{q}_d+K_s(q_d-\tilde{q}_{md})+K_pe+g(q)]+e_p^tM\dot{e}_p \qquad (26)$$

Let us now assume that $\|\dot{\tilde{e}}_m\|^2 > \epsilon>0$, where ϵ is a suitably small number determined to guarantee the numerical stability of the simulation. Then by substituting the controller (21) into (26), using the definition of Ψ given by equation (16), and by using the fact that $K_s(q_d-\tilde{q}_{md})=-Y_d\tilde{P}$ derived from (6), we get:

$$\dot{V}(t)=\dot{\tilde{e}}_m^t\{-K_{dm}\dot{\tilde{e}}_m-\frac{\dot{\tilde{e}}_m}{\|\dot{\tilde{e}}_m\|^2}[\dot{e}^t((\Psi-Y_d)\tilde{P}+K_pe)+\dot{e}^tK_d\dot{e}+e^tK_pe+\tilde{e}_m^tK_{pm}\tilde{e}_m]\}$$

$$+\dot{e}^t[\Psi P-Y_d\tilde{P}+K_pe]+e_p^tM\dot{e}_p$$

$$=-\dot{\tilde{e}}_m^tK_{dm}\dot{\tilde{e}}_m-\dot{e}^tK_d\dot{e}-e^tK_pe-\tilde{e}_m^tK_{pm}\tilde{e}_m-\dot{e}^t\Psi e_p+e_p^tM\dot{e}_p$$

$$=-\dot{\tilde{e}}_m^tK_{dm}\dot{\tilde{e}}_m-\dot{e}^tK_d\dot{e}-e^tK_pe-\tilde{e}_m^tK_{pm}\tilde{e}_m+e_p^t[M\dot{e}_p-\Psi^t\dot{e}] \qquad (27)$$

Since, $\dot{e}_p=\dot{\tilde{P}}-\dot{P}$, and as, $\dot{P}=0$ (robot arm parameters are time invariant), we can substitute the adaptation law (23) into (27) and the final expression for the derivative of the Lyapunov function is given as:

$$\dot{V}(t) = -\dot{\tilde{e}}_m^t K_{dm} \dot{\tilde{e}}_m - \dot{e}^t K_d \dot{e} - e^t K_p e - \tilde{e}_m^t K_{pm} \tilde{e}_m \leq 0 \tag{28}$$

Which guarantees the convergence of $\dot{\tilde{e}}_m$, $\dot{e}$, $\tilde{e}_m$, and e as time goes to infinity.

Let us now consider the case when $\|\dot{\tilde{e}}_m\|^2 \leq \epsilon$. We cannot use τ defined in equation (21) as $\|\dot{\tilde{e}}_m\|^2 \longrightarrow 0$, then $\tau \longrightarrow$ large value. At this stage the motor is tracking the estimated actuator trajectory in velocity, but a steady state error may exist between the actual and desired motor position. Therefore we should use the second stage control given by (22).

Notice now as $||\dot{\tilde{e}}_m||^2 \longrightarrow 0$, **a structural reduction in the system is** apparent as the "Lyapunov" function V(t) in (15) resembles that of a rigid robot, as the first term is zero. We will exploit this property in the second stage control. The dynamic equations (1) and (2) can be combined by simple addition to obtain a single system equation:

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + D_m \ddot{q}_m + B_m \dot{q}_m \tag{29}$$

Let us define a surface, $s=(\dot{e}+\Lambda e) \in \mathbb{R}^n$, along which we desire the link trajectory to track. Let us consider the Lyapunov function candidate W(t):

$$W(t)=½(\dot{e}+\Lambda e)^t D(\dot{e}+\Lambda e) + ½e_p^t M e_p \tag{30}$$

Differentiating W(t) with respect to time, substituting the defined value of Γ given by (18), and using the dynamic equation (29) leads us to:

$$\dot{W}(t)=s^t[D(\ddot{e}+\Lambda\dot{e})+½\dot{D}(\dot{e}+\Lambda e)] + e_p^t M \dot{e}_p$$

$$=s^t[D\ddot{q}_d + D\Lambda\dot{e} + C(q,\dot{q})(\dot{q}_d+\Lambda e) - (D\ddot{q}+C(q,\dot{q})\dot{q})] + e_p^t M \dot{e}_p$$

$$=s^t[D(\ddot{q}_d+\Lambda\dot{e}) + C(q,\dot{q})(\dot{q}_d+\Lambda e) + g(q)$$

$$+D_m\ddot{q}_m + B_m\dot{q}_m - \tau] + e_p^t M \dot{e}_p$$

$$=s^t[\Gamma P + D_m \ddot{\tilde{q}}_{md} - D_m \ddot{\tilde{e}}_m + B_m \dot{q}_m - \tau] + e_p^t M \dot{e}_p \tag{31}$$

Substituting for τ from equation (22) into (31) yields,

$$\dot{W}(t)=s^t[-\Gamma e_p - K_d s - D_m \ddot{\tilde{e}}_m + \lambda] + e_p^t M \dot{e}_p$$

$$=-s^t\Gamma \dot{e}_p - s^t K_d s - s^t[D_m \ddot{\tilde{e}}_m - \lambda] + e_p^t M \dot{e}_p$$

$$= -s^t K_d s - s^t[D_m \ddot{\tilde{e}}_m - \lambda] + e_p^t[M\dot{e}_p - \Gamma^t s] \tag{32}$$

Notice that, $\dot{e}_p(t) = \dot{\tilde{P}}(t)$, since $\dot{P}=0$, now let us substitute $\dot{\tilde{P}}(t)$ given by equation (24) into (32), it yields:

$$\dot{W}(t) = -s^t K_d s - s^t[D_m \ddot{\tilde{e}}_m - \lambda] + e_p^t[MM^{-1}\Gamma^t s - \Gamma^t s]$$

$$= -s^t K_d s - \sum_{i=1}^{n} s_i (D_{mi} \ddot{\tilde{e}}_{mi} - \lambda_i) < 0 \tag{33}$$

A substitution for the values of λ_i's from equation (20) guarantees that $\dot{W}(t)$ is upper bounded by zero and decreases for any nonzero ($s = \dot{e} + \Lambda e$), s converges to zero with time going to infinity for positive gain matrices Λ, and K_d. Consequently, this implies that both $\dot{e}(t)$ and $e(t)$ decreases as time goes to infinity.

7. Simulation Results For A Two-Link Planar FJR.

We now describe the computer simulation for a two-link planar manipulator with revolute joints (see Figure 1). The linkage are composed of two identically uniform beams which are infinitely rigid, with actuators mounted at the joints (see Figure 2). We assume that the load carried by the end-effector is a part of the second link. From equations (1) and (2), the dynamic equations of the two link manipulator are given as :

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix} d_{m1} & 0 \\ 0 & d_{m2}\end{bmatrix}\begin{bmatrix}\ddot{q}_{m1}\\ \ddot{q}_{m2}\end{bmatrix} + \begin{bmatrix} b_{m1} & 0 \\ 0 & b_{m2}\end{bmatrix}\begin{bmatrix}\dot{q}_{m1}\\ \dot{q}_{m2}\end{bmatrix} + \begin{bmatrix} k_{s1} & 0 \\ 0 & k_{s2}\end{bmatrix}\begin{bmatrix} q_{m1}-q_1\\ q_{m2}-q_2\end{bmatrix} \tag{34}$$

and,

$$\begin{bmatrix}0\\ 0\end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + \begin{bmatrix} g_1(q)\\ g_2(q)\end{bmatrix} + \begin{bmatrix} k_{s1} & 0 \\ 0 & k_{s2}\end{bmatrix}\begin{bmatrix} q_1-q_{m1}\\ q_2-q_{m2}\end{bmatrix} \tag{35}$$

where the coefficients can be derived from the Lagragian formulation (similar to that in Paul's book 1986). Notice that $g \in \mathbb{R}^1$ is the gravitational acceleration and it is assumed to be 9.81 ms^{-2}.

$d_{11}=I_1+I_2+(m_1+4m_2)l_1^2+m_2l_2^2+4m_2l_1l_2\cos(q_2)$

$d_{12}=d_{21}=I_2+m_2l_2^2+2m_2l_1l_2\cos(q_2)$

$d_{22}=I_2+m_2l_2^2$

$c_{11}=-4m_2l_1l_2\dot{q}_2\sin(q_2)$

$c_{12}=-2m_2l_1l_2\dot{q}_2\sin(q_2)$

$c_{21}=2m_2l_1l_2\dot{q}_1\sin(q_2)$

$c_{22}=0$, and $I_j=.33m_jl_j^2+.01m_jl_j^2$ for $j=1,2$

$g_1(q)=g[(m_1+2m_2)l_1\cos(q_1)+m_2l_2\cos(q_1+q_2)]$

$g_2(q)=gm_2l_2\cos(q_1+q_2)$

For notational convenience let us define

$$S_1=\sin q_1 \quad C_1=\cos q_1 \quad C_{12}=\cos(q_1+q_2)$$

$$S_2=\sin q_2 \quad C_2=\cos q_2 \quad S_{12}=\sin(q_1+q_2)$$

We can rewrite the manipulator dynamics in the regressor form with the unknown parameters appearing linearly as :

$$\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}\ddot{q}_1 & (\ddot{q}_1+\ddot{q}_2) & (\alpha C_2-\beta S_2) & gC_1 & gC_{12}\\ 0 & (\ddot{q}_1+\ddot{q}_2) & (\ddot{q}_1C_2+\dot{q}_1^2S_2) & 0 & gC_{12}\end{bmatrix}\begin{bmatrix}p_1\\p_2\\p_3\\p_4\\p_5\end{bmatrix}$$

$$+\begin{bmatrix}k_{s1} & 0\\ 0 & k_{s2}\end{bmatrix}\begin{bmatrix}q_1-q_{m1}\\ q_2-q_{m2}\end{bmatrix} \tag{36}$$

$$\text{where, } \alpha=2\ddot{q}_1+\ddot{q}_2 \text{ and } \beta=2\dot{q}_1\dot{q}_2+\dot{q}_2^2 \tag{37}$$

Furthermore, the unknown parameter vector P is given as:

$$P=\begin{bmatrix}p_1\\p_2\\p_3\\p_4\\p_5\end{bmatrix}=\begin{bmatrix}I_1+m_1l_1^2+4m_2l_1^2\\ I_2+m_2l_2^2\\ 2m_2l_1l_2\\ l_1(m_1+2m_2)\\ m_2l_2\end{bmatrix} \tag{38}$$

Therefore the vector functions of unknown parameters $P \in \mathbb{R}^5$ and the regressor matrix $Y(\ddot{q},\dot{q},q) \in \mathbb{R}^{2\times5}$ are well defined. After choosing the desired links trajectory, we use equations (6), (7) and (8) to derive the desired motor trajectory.

The control law given in equation (21) and (22) and the adaptation laws given (23) and (24) were used with the following definitions of Ψ and Γ :

$$\Psi=\begin{bmatrix} \ddot{q}_{1d} & (\ddot{q}_{1d}+\ddot{q}_{2d}) & (\alpha_1 C_2-\beta_1 S_2) & gC_1 & gC_{12} \\ 0 & (\ddot{q}_{1d}+\ddot{q}_{2d}) & (\ddot{q}_{1d} C_2+\dot{q}_{1d}\dot{q}_1 S_2) & 0 & gC_{12} \end{bmatrix} \tag{39}$$

$$\text{where, } \alpha_1 = 2\ddot{q}_{1d} + \ddot{q}_{2d} \text{ and } \beta_1 = 2\dot{q}_{1d}\dot{q}_2 + \dot{q}_{2d}\dot{q}_2 \tag{40}$$

and

$$\Gamma=\begin{bmatrix} (\ddot{q}_{1d}+a) & \gamma_1 & (\alpha_2 C_2-\beta_2 S_2) & gC_1 & gC_{12} \\ 0 & \gamma_2 & (\ddot{q}_{1d}+a)C_2+(\dot{q}_{1d}+c)\dot{q}_1 S_2 & 0 & gC_{12} \end{bmatrix} \tag{41}$$

$$\text{where, } \alpha_2 = 2(\ddot{q}_{1d}+a)+\ddot{q}_{2d}+b \text{ , } \beta_2 = 2(\dot{q}_{1d}+c)\dot{q}_2 + (\dot{q}_{2d}+d)\dot{q}_2$$

$$\text{and } \gamma_1=(\ddot{q}_{1d}+\ddot{q}_{2d}+a+b) \text{ , } \gamma_2=(\ddot{q}_{1d}+\ddot{q}_{2d}+a+b) \tag{42}$$

assuming a, b, c, and d are derived from :

$$\Lambda\dot{e}=\begin{bmatrix} a \\ b \end{bmatrix} \text{ and } \Lambda e=\begin{bmatrix} c \\ d \end{bmatrix} \tag{43}$$

We selected a sampling period of 10×10^{-3} seconds corresponding to a servo rate of 100 Hz. We selected, $\epsilon = 1$, and second controller was activated when, $\|\dot{e}_m\|^2 < \epsilon$. The value of ϵ is quite large and is selected to ensure numerical stability. The following bounds were used in the definition of λ_i $-2\leq \|\ddot{e}_{mi}\|\leq 2$, for $i = 1,2$. Table S-1 shows the numerical values of the parameter vector P, and the known motors parameters.

parameter	value
$K_{s1}=k_{s2}$	50 N m rad^{-1}
$D_{m1}=d_{m2}$	.05 kg m^2
$D_{m1}=b_{m2}$	.05 N m sec rad^{-1}
P_1	1.66
P_2	0.42
P_3	0.63
P_4	3.75
P_5	1.25
$l_1=l_2$	.25 m
$m_1=m_2$	5 kg

Table S-1 : Actual parameters values

Three different cases were simulated to show the improvement obtained over current adaptive control schemes for robotic manipulators. The need for adaptive control is also illustrated through simulations.

As seen from table (S-1), the robot considered here has extremely flexible joints. A load of 5 kg, when the arm is fully extended and parallel to the horizontal plane, results in the inner joint q_1 to deflect by 1 rad or 57.3°. Current industrial robots have joint stiffness in excess of several hundred Nm rad^{-1}. Notice also, this manipulator is not light and each link has a weight of 5 kg.

In the below simulations, we assume the manipulator is initially at rest with $q_1 = -90°$, and $q_2 = 0°$. The desired trajectory is given by:

$$q_{1d}(t) = [-\frac{\pi}{2} + 0.3\sin(\pi t)]\text{rad}. \tag{44}$$

$$q_{2d}(t) = [-0.3 + 0.3\cos(\pi t)]\text{rad}. \tag{45}$$

Case#1:

In order to show that current rigid robot adaptive schemes are ineffective when applied to FJR. In this case, we applied the elegant adaptive control schemes suggested by (Slotine et al 1987) to the FJR described in table (S-1). As this controller was derived on the assumption that the joints are rigid, equation (3) was used for the rigid robot model and the rigid robot control law was:

$$\tau=\Gamma(\ddot{q}_d,\dot{q}_d,\dot{q},q)\tilde{P}+D_m(\ddot{q}_d+\Lambda e)+B_m\dot{q}+K_d(\dot{e}+\Lambda e) \quad (46)$$

and the adaptation law was:

$$\dot{\tilde{P}}(t)=M^{-1}\Gamma^t(\ddot{q}_d,\dot{q}_d,\dot{q},q)[\dot{e}+\Lambda e] \quad (47)$$

The controller gains were found to be

$$K_p=K_{pm}=\begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix} \quad K_d=K_{dm}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad (48)$$

$$\Lambda=\begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix} \quad M=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad (49)$$

The response of the manipulator to Slotine and Li's adaptive control law is shown in Figures (3a, 3b, and 3c). Figure 3a shows the link angle responses, the motor responses are shown in Figure 3b, and parameter $\tilde{P}_1$ is shown in Figure 3c. Notice all the parameters behave similarly to $\tilde{P}_1$, shown in Figure 3c. From Figures 3a and 3b, it can be seen that unacceptable link and motor responses are obtained before the system goes unstable. Figure 3c shows that the parameters vary wildly before diverging.

We expect that all other rigid robot adaptive control schemes would also produce unstable responses when applied to control FJR's with such low joint stiffness. These simulations indicate clearly the need to develop new adaptive control schemes for the FJR. Note that the rigid control law (46) gives acceptable responses for very large joint stiffness.

Case#2:

In order to show the need for adaptation and the effectiveness of the derived control scheme, we applied the control scheme given by equations (21) and (22). We assumed the parameter vector $\tilde{P}=[2,1,1,3.5,1]$, The parameters are different from their actual values given in table S-1, the response of the FJR to the control scheme without the adaptation is given in Figures (4a, and 4b). We can see the tracking errors of the links in Figure 4a, and the tracking errors of the motors in Figure 4b. Notice that the scheme given in equations (21) and (22) is more effective in tracking the FJR trajectory than Slotine and Li's scheme, which gave unstable responses. The controller gains were found to be

$$K_p = K_{pm} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \quad K_d = K_{dm} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \tag{50}$$

$$\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad M = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \tag{51}$$

We can see a significant steady state errors develop, clearly this is undesirable in many applications. In order to compensate for the steady state tracking error, it is desirable to employ an adaptive control scheme. Notice, even if $\tilde{P}$ was determined such that, $\tilde{P}-P=0$, the need for adaptation is not eliminated as the robot may pick up unknown loads and therefore alter the P vector. This would once again result in steady state tracking error.

Case#3:

In order to show the effectiveness of the results derived in this paper given by equations (21) through (24), we applied our control scheme to the FJR. The response is given in Figures (5a, 5b, and 5c). Figure 5a shows the responses for the links, while Figure 5b shows the responses of the motors, and Figure 5c shows the estimates of the parameters. We can see that the motor and the link tracking errors go to zero. The parameters also do not diverge, although they do not converge to their exact actual values, they oscillate about their true values. The controller gain matrices given in equations (50) and (51) were used for this case. Clearly, the response of the manipulator to the adaptive FJR scheme described in this paper is significantly better than applying rigid robot adaptive schemes as seen in case #1. Notice also the adaptive scheme has superior performance over the non-adaptive control law simulated in case #2, which developed significant steady state errors.

8. Conclusion

In this paper we have presented an adaptive control scheme for the FJR without employing linearization techniques such as (Fu et al 1989). Acceleration and jerk measurements, as well as inertia matrix inversion were not needed. Adaptive controller for the FJR was derived using Lyapunov's second method. From the simulation results, it is clear that the improvement in the tracking and parameter estimation is significant over rigid robot adaptive schemes, and therefore it is necessary to account for joint flexibility effects when deriving control schemes for industrial robots with such compliance.

We required a rich reference signal and good initial guesses of the parameters. It is obvious that some correction scheme could be added to the derived adaptation law to improve the robustness of our controller in the presence of bounded disturbances or unmodeled dynamics (Ioannou 1986). Experimental work will also be necessary to verify the practicality of our scheme. It is important to point out that most industrial robots use feedback sensors mounted on the actuator and in order to compensate for joint flexibility additional sensors must be mounted to measure the joint angles and velocities.

9. Acknowledgement

We would like to thank Professor Martin Corless for initial expository discussions on the area of flexibly jointed systems and Lyapunov stability theory.

10. Appendix A : To show $[\dot{D} - 2C]$ is Skew Symmetric.

Here we will show that $D(q)$ and $C(q,\dot{q})$ are not independent, but the matrix $(\dot{D}-2C)$ is skew symmetric (Slotine et al 1988), (Ghorbel et al 1989), (Spong 1987). This can be easily derived from the Lagrangian formulation of the manipulator dynamics. In order for a square matrix W to be skew-symmetric, we need $W^t = -W$. From equation (2), we can represent the $(kj)^{th}$ element of $C(q,\dot{q})$ by

$$c_{kj} = \sum_{i=1}^{n} [\frac{\partial d_{kj}}{\partial q_i} - ½\frac{\partial d_{ij}}{\partial q_k}]\, \dot{q}_i \tag{A1}$$

where, d_{kj} is the $(kj)^{th}$ element of the inertia matrix $D(q)$ Now, by interchanging the (i,j) indices and using the symmetry property of D (q), we note:

$$\sum_{i,j}^{n} \frac{\partial d_{kj}}{\partial q_i} = ½\sum_{i,j}^{n} [\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}] \tag{A2}$$

Therefore, we can substitute (A2) into (A1), and:

$$c_{kj} = \sum_{i=1}^{n} ½[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}]\ \dot{q}_i \tag{A3}$$

Let, $W(q,\dot{q})=[\dot{D}(q)-2C(q,\dot{q})]$, then the $(jk)^{th}$ element of W is:

$$w_{kj} = \dot{d}_{kj} - 2c_{kj}$$

$$= \sum_{i=1}^{n} [\frac{\partial d_{kj}}{\partial q_i} - (\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k})]\ \dot{q}_i$$

$$= \sum_{i=1}^{n} [\frac{\partial d_{ij}}{\partial q_k} - \frac{\partial d_{ki}}{\partial q_j}]\ \dot{q}_i \tag{A4}$$

Since D(q) is symmetric, it is clear from (A4) that, $w_{kj} = -w_{jk}$. Therefore, $W(q,\dot{q})$ is skew symmetric, furthermore the diagonal entries of W are zero as:

$$w_{jj} = \sum_{i=1}^{n} [\frac{\partial d_{ij}}{\partial q_j} - \frac{\partial d_{ji}}{\partial q_j}]\ \dot{q}_i = 0 \tag{A5}$$

Again by the symmetry property of D(q), (A5) is straight forward.

Now we can conclude that $W(q,\dot{q})=[\dot{D}(q)-2C(q,\dot{q})]$ is skew symmetric with zero diagonal entries, which yields

$$\dot{q}^t\ [\dot{D}(q) - 2C(q,\dot{q})]\ \dot{q} = 0 \tag{A6}$$

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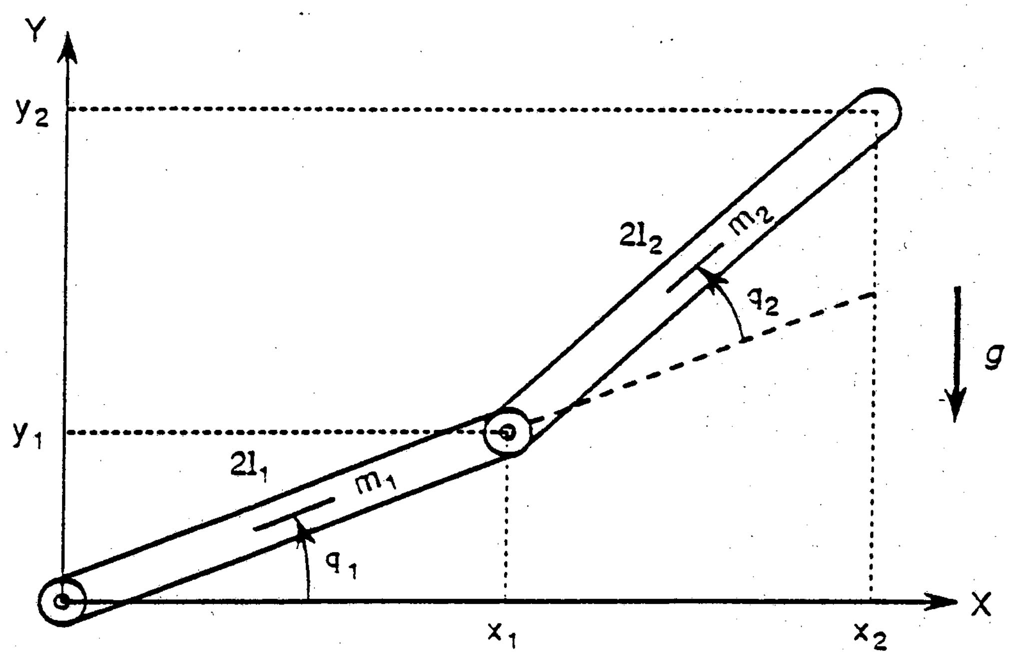


Figure 1. The Two Link Planar Manipulator.

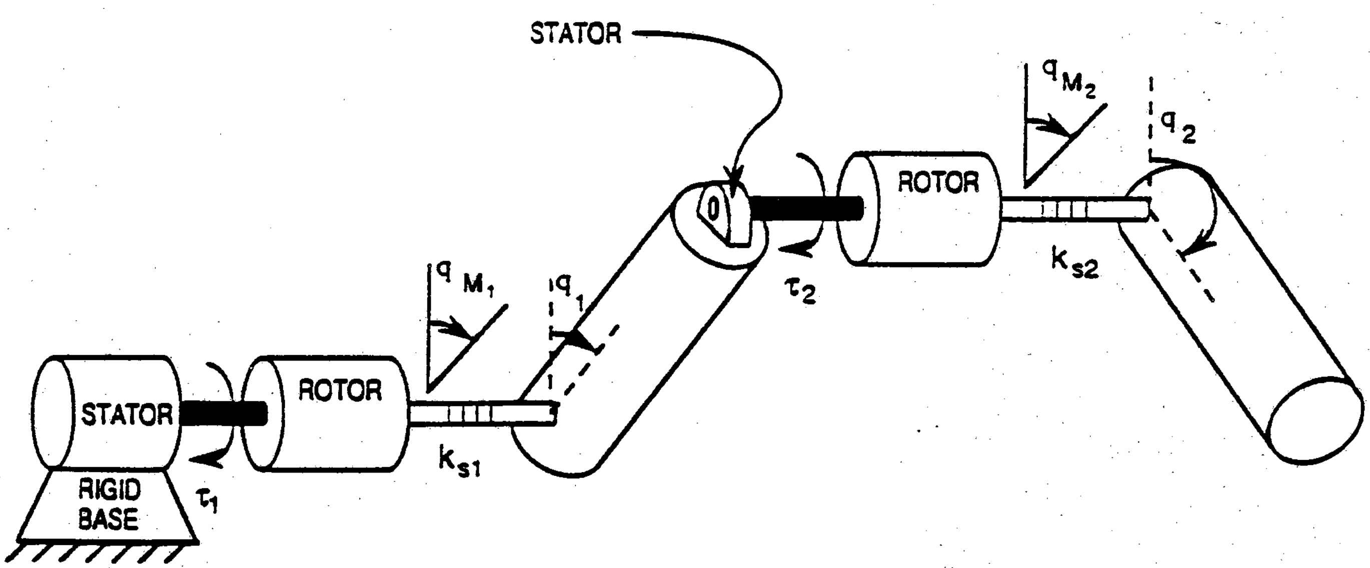


Figure 2. Conceptual Diagram of Two Link Compliant Manipulator.

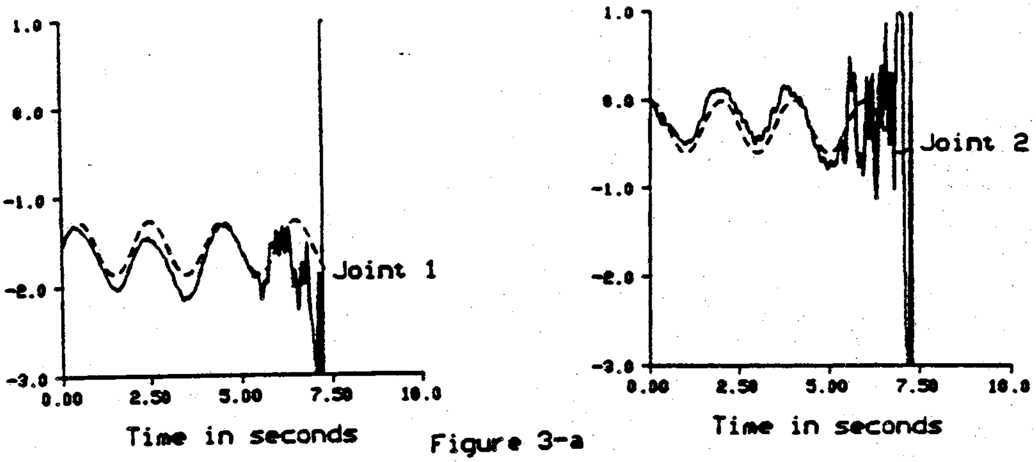


Figure 3-a

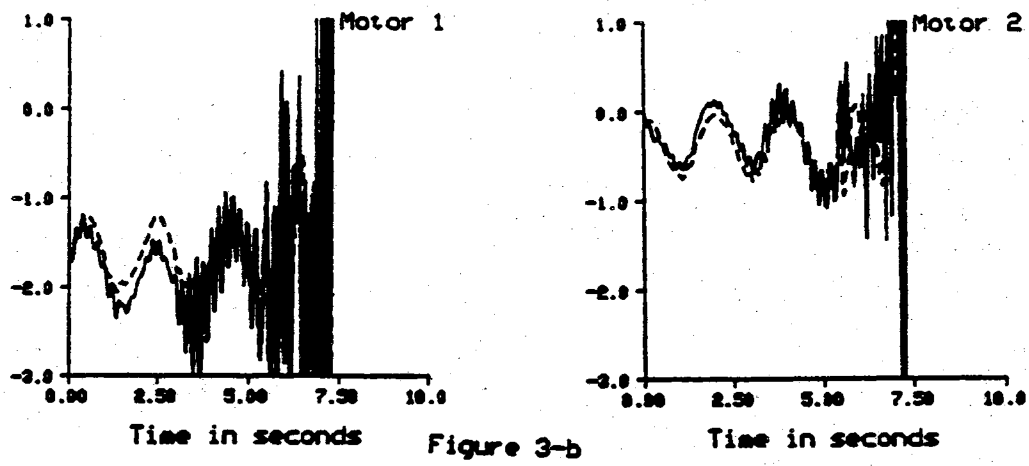


Figure 3-b

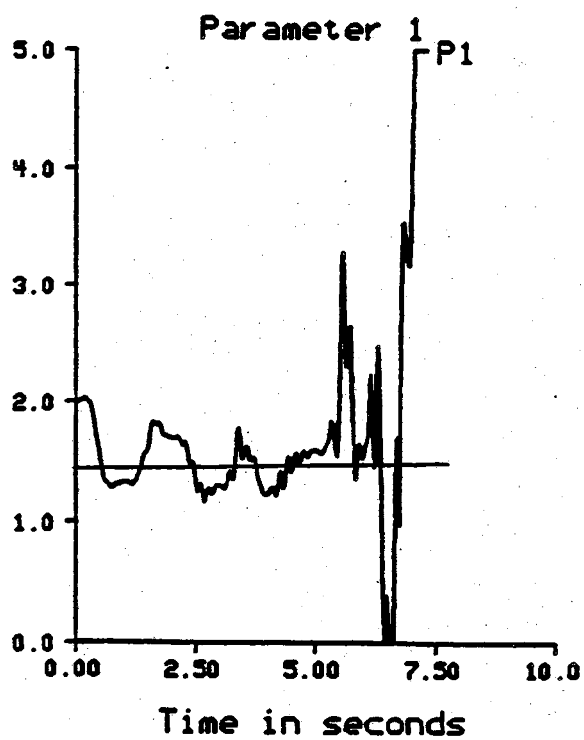
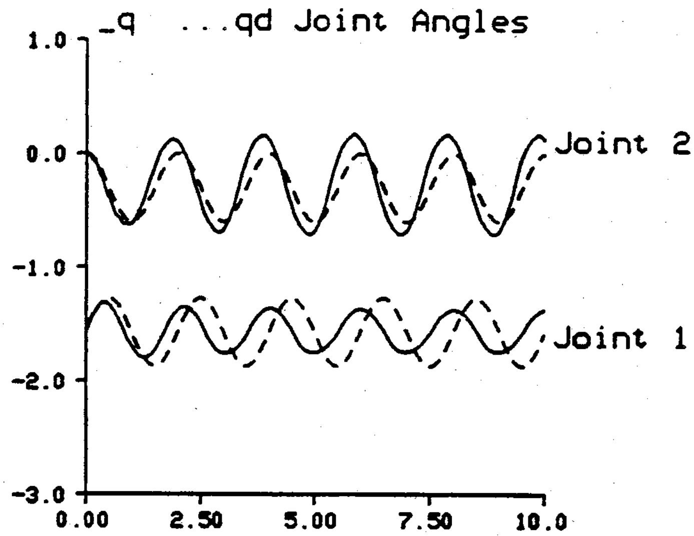
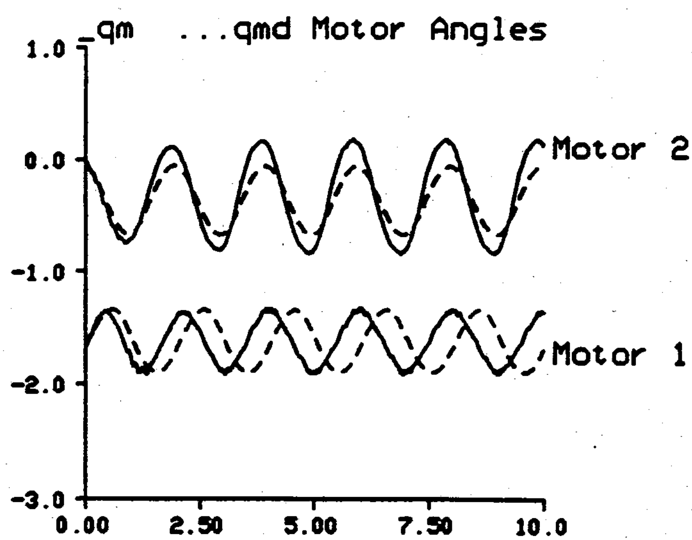


Figure 3-c

Case #1: Current rigid robot adaptive scheme appplied to FJR.



Time in seconds
Figure 4-a



Time in seconds
Figure 4-b

Case #2: Derived Control scheme without adaptation

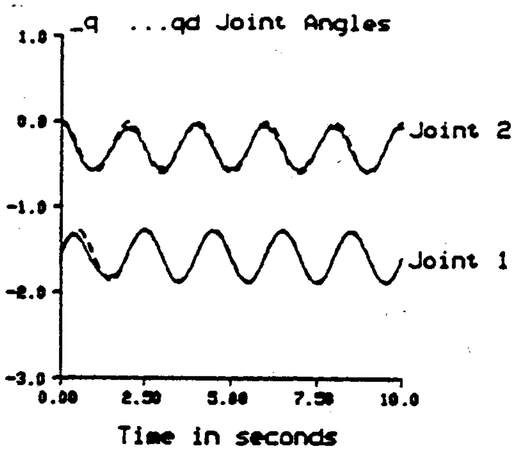


Figure 5-a

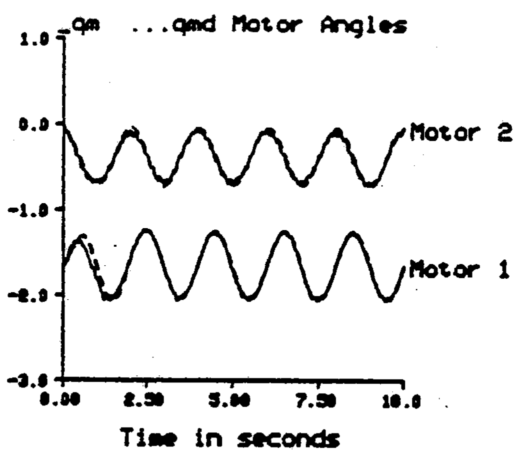


Figure 5-b

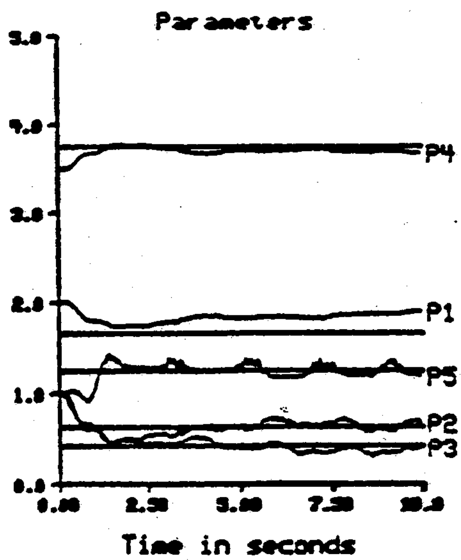


Figure 5-c

Case #3: Derived Adaptive Control scheme