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FINAL REPORT VOL. I
PROJECT 4335
TASK 40621

PURDUE UNIVERSITY
SCHOOL OF ELECTRICAL ENGINEERING

Information Efficiency of Binary Communication Systems

J. C. Hancock, Principal Investigator
E. M. Sheppard

May, 1962
Lafayette, Indiana

COMMUNICATIONS LABORATORY
AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE

FINAL REPORT

VOLUME I

CONTRACT AF 33(616) - 8283

Project 4335

Task 40621

PRF 2906

INFORMATION EFFICIENCY OF BINARY COMMUNICATION SYSTEMS

for

U. S. AIR FORCE

WRIGHT AIR DEVELOPMENT DIVISION

WRIGHT-PATTERSON AIR FORCE BASE

DAYTON, OHIO

by

J. C. Hancock, Principal Investigator

E. M. Sheppard

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Lafayette, Indiana

May, 1962

It has long been recognized that widebanding techniques are particularly suited for the estimation of channel parameters when multipath presents a problem. Volume III considers the use of widebanding techniques for this purpose and considers the optimum open loop system to compensate for multipath effects. The results of this study are related to previous studies in this area.

The principal investigator wishes to acknowledge the many fruitful discussions with the project monitor, Mr. R. B. Russell, as well as his associates. Many of the ideas conveyed herein, particularly in Volume III, are outgrowths of these discussions.

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ABSTRACT

Most of the work which has been done with binary communication systems up until now has assumed operation in a symmetric mode. This work is concerned with the problem of evaluating various combinations of modulation and detection in both symmetric and non-symmetric modes of operation.

The most frequently used criterion for describing performance in a binary system is total probability of error. A discussion of this and other criteria such as realizable rate and minimum energy per bit factors is given. A new criterion called information efficiency is defined which is based on realizable information rate on a per symbol basis. The primary advantage of this criterion is that it gives a truer indication of performance than probability of error in the case of unsymmetric operation.

Several types of conventional binary systems are analyzed and compared under the conditions that additive gaussian white noise is the only perturbing influence. Systems considered include amplitude shift keying or a carrier on-off type of modulation with linear envelope detection and with synchronous detection, phase shift keying of a phase reversal type of modulation with both synchronous and phase comparison detection schemes. Performance curves showing information efficiency and probability of error as functions of signal-to-noise ratio are given.

A similar type of analysis is given for a group of matched filter systems which includes both coherent and non-coherent matched filter detection of amplitude and frequency shift keyed signals in the face of

gaussian white noise and the coherent matched filter detection of phase shift keyed signals. Also included are some results concerning the use of differentially coherent detection of phase shift keyed signals.

The response of various systems to variations in decision thresholds is examined and it is shown that phase shift keyed systems are superior in this respect.

The optimum detection of amplitude shift keyed signals requires a variable threshold level for different conditions at the detector input. The case of fixed threshold systems is examined and it is shown that a fixed threshold limits the maximum attainable performance of the system and that there is a distinct trade-off between this maximum possible performance at high signal-to-noise ratios and good performance (i.e., near optimum) at low signal-to-noise ratios.

The problem of Rayleigh fading is discussed and indications of fading on the performance of the various systems is given.

Finally, all of the systems discussed are compared on the same basis by using a time bandwidth product which allows the signal-to-noise ratios on which the conventional system analysis is based to be converted to an energy per symbol to noise spectral density ratio, which is the basis for matched filter analysis.

LIST OF IMPORTANT SYMBOLS

LIST OF IMPORTANT SYMBOLS (continued)

Chapter I

Introductory Remarks

1.1 Introduction

The purpose of this chapter is to outline the problems considered and the results obtained in the following seven chapters, and to relate them to previous work in the same area.

1.2 Efficiency in a Communication System

In order to examine the relative merits of different communication systems, it becomes necessary to form some basis for comparison. The criterion by which a system is judged will depend on the purpose and manner in which the system is operated. For example, if one wishes to compare an analog system using amplitude modulation with one using frequency modulation for the transmission of speech, the natural criterion to use is signal-to-noise ratio since this quantity may be related to a human being's ability to correctly detect what is being transmitted. With the advent of modern communication theory as postulated by Shannon[1] and others, the analog communication system is giving way to the more efficient digital or pulse code modulation techniques. In such systems, analog information is sampled and quantized, and the transformed information is transmitted in digital form. In the work which follows, all information will be reduced to binary form before transmission and later decoded at the receiver. Thus it is of concern to describe the performance of the binary link in this process.

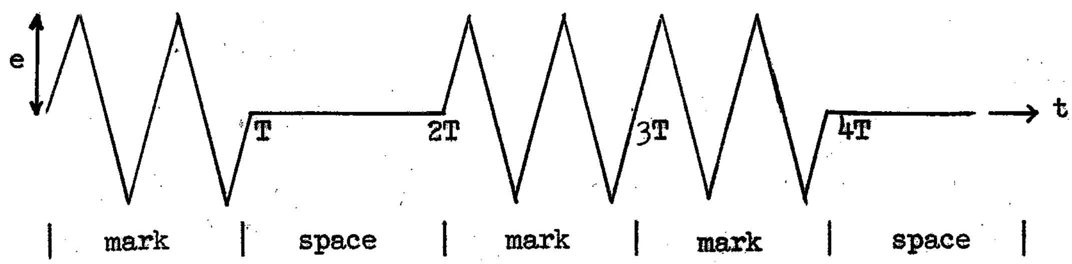
Development of a criterion for such a link is the primary object of Chapter II. Several well known criteria are examined for merit and a new one called information efficiency is presented. Information efficiency is a quantity which is related to realizable rate on a per symbol basis and is a measure of how efficiently each transmitted symbol is being used. Information efficiency is shown to be a truer criterion of goodness than probability of error in the case of a non-symmetric system, and is the primary basis on which system performance is judged in later chapters.

1.3 The Types of Modulation Considered

In this section the three types of modulated signal which are considered are defined.

I Amplitude Shift Keying (ASK)

In this type of modulation, a carrier signal is used in an off-on manner. Thus if a mark is to be transmitted, an rf pulse having a baud length of T seconds will be transmitted, and if a space is to be sent, then no signal will be sent for T seconds (see Fig. 1.1). For a signal of this type, the average signal power is one-half the power when



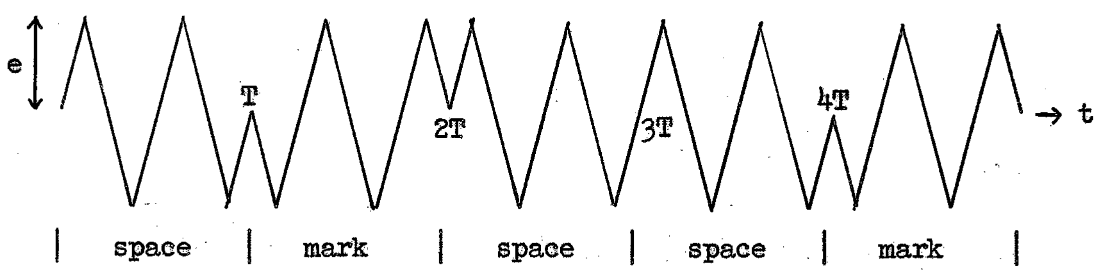
A TYPICAL ASK SIGNAL

Figure 1.1

a mark is transmitted and equal to $e^2/4$ (note: this is based on the assumption that a mark and a space are equally probable). The normalized correlation coefficient between a mark and a space (ρ_{10}) is zero, and the average energy per baud is $E = \frac{e^2T}{4}$.

II Phase Shift Keying (PSK)

For this type of signal, the information content of the transmitted waveform lies in the phase. Thus a chain of rf pulses (of baud length T) are transmitted, and each pulse has a phase of either 0^o or 180^o. If the phase is 0^o, a space has been transmitted and if it is 180^o, then a mark was sent (see Fig. 1.2). The average signal power is $e^2/2$, and is independent of the probability of transmitting either a



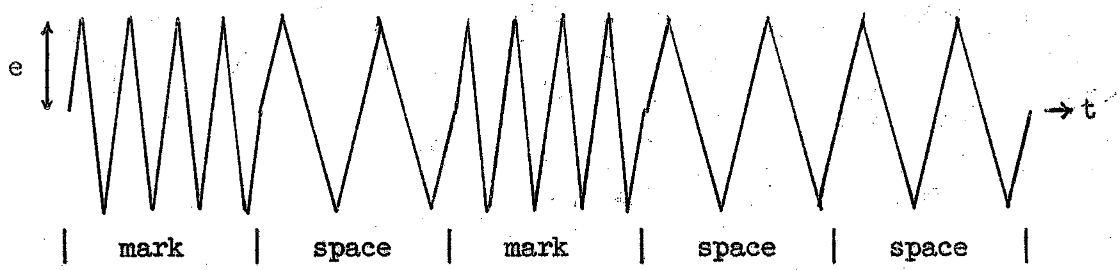
A TYPICAL PSK SIGNAL

Figure 1.2

mark or a space. The average energy per baud is $E = \frac{e^2T}{2}$ and $\rho_{10} = -1$. Note that this signal is equivalent to an amplitude modulated signal with a suppressed carrier.

III Frequency Shift Keying (FSK)

In this case, rf pulses of differing frequencies are transmitted to represent a mark and a space (see Fig. 1.3). For such a waveform the



A TYPICAL FSK SIGNAL

Figure 1.3

average signal power is $e^2/2$ and the average energy per baud is $\frac{e^2T}{2}$ (both of these quantities are independent of the probabilities of transmitting a mark or a space). The normalized cross correlation coefficient between a mark and a space is a function of the separation between the two frequencies used for a mark and a space. Since the frequency separations used in practical FSK systems are large, it is reasonable to assume that $\rho_{10} = 0$.[2] This assumption is used in the work which follows.

1.4 The Performance of Binary Symmetric Systems in the Face of Gaussian White Noise

In Chapters III and IV several types of binary communication systems are analyzed. The systems considered in Chapter III are of the more conventional type where the analysis is based on receiver input signal-to-noise ratios. Included are an ASK system with linear envelope detection, an ASK system using synchronous detection, a PSK system employing synchronous detection, and a PSK system using phase comparison detection. The analysis of these systems for probability of error is based heavily on the work of Rice[3] which describes the statistical nature of a sine wave plus gaussian noise. Both of the PSK systems discussed have been analyzed for probability of error by Cahn.[4,5] The results of Chapter III carry the analysis of these systems on to the concept of information efficiency.

Chapter IV treats six matched filter systems. These systems have been analyzed and compared on a probability of error basis by several people, one of the earliest being Reiger[6] in 1953. Chapter IV carries the analysis of these systems one step further, that is, the results are presented in terms of information efficiency. The analysis used is also the basis for further work in Chapters V, VI and VII where various systems are considered in various non-symmetric modes of operation. The systems discussed in Chapter IV are: ASK systems using matched filters with both coherent and non-coherent detection, a PSK system with coherent matched filter detection (this is the optimum binary system for a system perturbed by gaussian noise), a differential phase coherent system, and FSK systems with both coherent and non-coherent matched filter detection.

1.5 Threshold Sensitivity in a Binary System

The results of Chapter V are new, and describe the effects of improper threshold settings in the decision process of various binary systems. Although it has been well established that the optimum mode of operation for a binary system is a symmetric one, the probability of building a system and actually operating it in a symmetric fashion is very small. Thus, due to practical considerations, all systems will actually operate in a non-symmetric manner.

The analysis of Chapter V examines the effects of this dissymmetry. Threshold sensitivity curves showing the degradation of performance due to the use of improper threshold levels are shown, and comparisons are made by means of a threshold sensitivity factor.

1.6 Fixed Threshold Systems and Fading

For many of the systems considered a proper threshold level is not a function of signal strength, however, in all of the ASK systems considered this is not the case. Chapter VI deals with this class of systems and their performance in the case of varying signal strength and fixed threshold. The results show a significant trade-off between maximum attainable performance and the quality of low-level performance.

The situation described above is apt to arise due to the presence of fading in the channel. A simple model for Rayleigh fading is assumed and the performance of various systems in the presence of fading is indicated.

1.7 Comparison of Systems and Conclusions

Chapter VII gives a comparison of all of the systems discussed. Although the various matched filter systems have been compared before on a probability of error basis, the results of Chapter VII bring together the conventional systems of Chapter III with the matched filter systems of Chapter IV. This is done by converting the signal-to-noise ratios of Chapter III to energy-to-noise ratios as used in Chapter IV.

This is done by means of a time-bandwidth product as described in Chapter II. The comparisons are based on information efficiency.

In addition, Chapter VII gives comparisons of fading performance of symmetric systems in the presence of fading.

Finally, Chapter VIII gives some conclusions regarding the work of Chapters I through VII and suggestions for the continuation of this research.

Chapter II

Binary Communication Links

2.1 Introduction

In this chapter, a communication system is defined in terms of a binary channel. While the channel is the heart of the system, it does not include the coding and decoding processes necessary to convert input information to a binary form at the transmitter and reconvert the binary information to the desired form at the receiver output. There is a discussion of several criteria of goodness for binary channels including probability of error, information efficiency, rate and Sanders'[7] β factors or the minimum energy per bit criterion. A discussion of time-bandwidth product and its function in comparing matched filter systems with more conventional types is given.

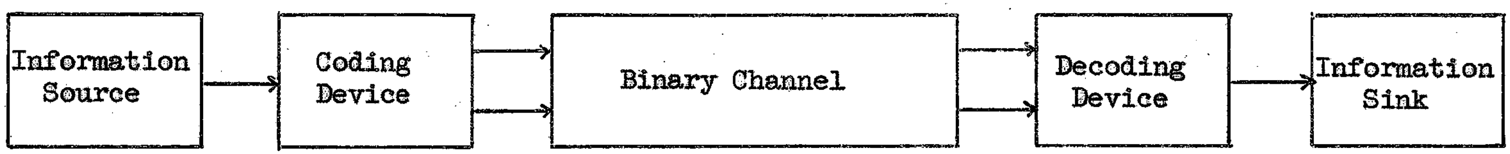
2.2 The General Binary Channel

In the analysis which follows, the term "channel" will refer to an entire binary communication link less the input coding and output decoding devices (see Fig. 2.1). The binary channel may be subdivided into a modulator and transmitter, a transmission path wherein gaussian white noise is added to the transmitted signal and fading takes place due to multipath conditions, a receiver and demodulator, and a decision device (see Fig. 2.2).

The binary channel described above may be characterized mathematically by a flow diagram of the type shown in Fig. 2.3, where

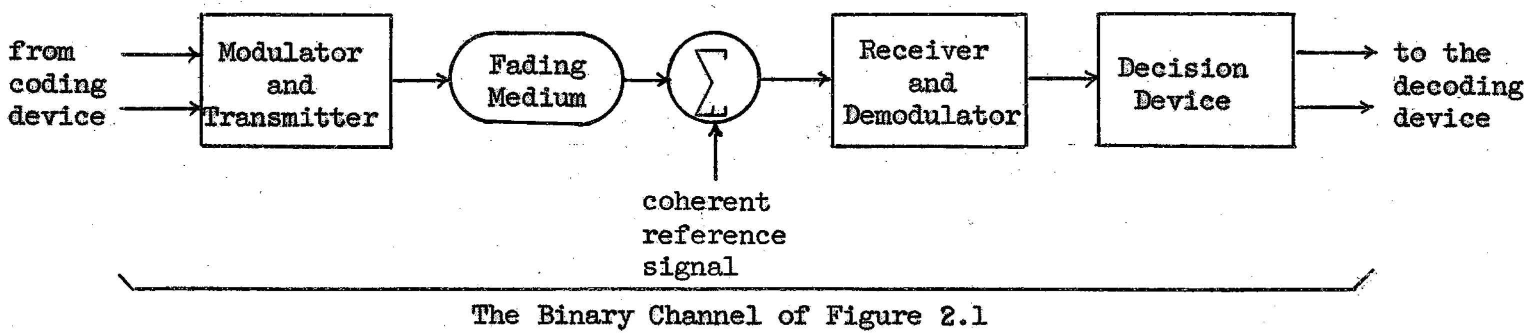
$P(0_t)$ = the probability of a space being sent

$P(1_t)$ = the probability of a mark being sent



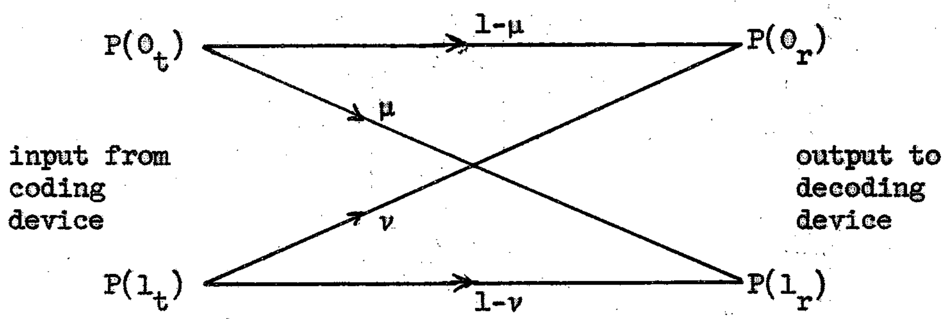
A BINARY COMMUNICATION SYSTEM

Figure 2.1



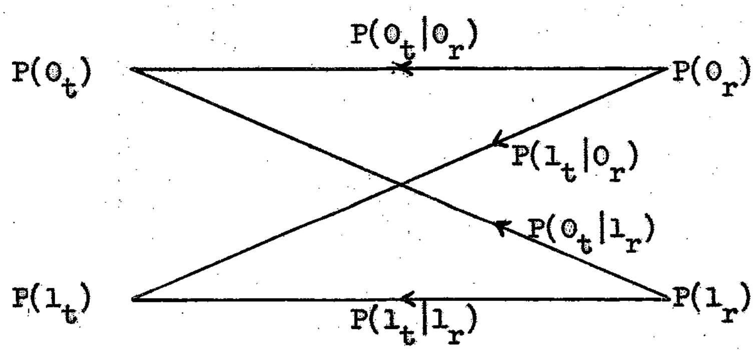
A BINARY CHANNEL

Figure 2.2



FLOW DIAGRAM FOR A
BINARY CHANNEL

Figure 2.3



REVERSE FLOW DIAGRAM FOR A
BINARY CHANNEL

Figure 2.4

$P(0_r)$ = the probability of a space being received

$P(1_r)$ = the probability of a mark being received

$\mu = P(1_r|0_t)$ = the probability of a transmitted space being received as a mark

$\nu = P(0_r|1_t)$ = the probability of a transmitted mark being received as a space.

The channel may also be characterized by the reverse flow diagram shown in Fig. 2.4, where the transitional probabilities are,

$$P(0_t|0_r) = \frac{1}{1 + \frac{\nu}{1-\mu}\frac{P(1_t)}{P(0_t)}} \tag{2-1}$$

$$P(0_t|1_r) = \frac{1}{1 + \frac{1-\nu}{\mu}\frac{P(1_t)}{P(0_t)}} \tag{2-2}$$

$$P(1_t|0_r) = \frac{1}{1 + \frac{1-\mu}{\nu}\frac{P(0_t)}{P(1_t)}} \tag{2-3}$$

$$P(1_t|1_r) = \frac{1}{1 + \frac{\mu}{1-\nu}\frac{P(0_t)}{P(1_t)}} \tag{2-4}$$

Thus a binary communication system may be thought of as a binary channel whose characteristics (i.e., the transitional probabilities) are determined by the choice of modulation and detection employed and by the perturbing influences which are present. Note that the channel, as defined above, does not include any error detecting and/or correcting coding or decoding processes.

2.3 Criteria for the Comparison of Binary Systems

In order to compare various binary systems, it is first necessary to determine a criterion of goodness upon which to base the comparison. There are several possible choices and no one of them is ideal for all purposes. What follows is a discussion of four criteria for binary channels, giving both the advantages and disadvantages of each criterion, and their relation to each other.

I Probability of Error

Probability of error is the simplest and most frequently used criterion to describe a binary channel. The total probability of error is

$$P_e = P(0_t)P(1_r|0_t) + P(1_t)P(0_r|1_t). \tag{2-5}$$

A symmetric system is defined as one in which the transitional probabilities of error are equal. Since

$$P(0_t) + P(1_t) = 1, \tag{2-6}$$

it follows that

$$P_e = P(0_r|1_t) = P(1_r|0_t). \tag{2-7}$$

Although P_e represents how often a mistake may be expected on the average, it does not give an indication of the actual information rate which can be realized in terms of error-free information transferred from transmitter input to receiver output.[1] Such an indication may be obtained by computing the information loss in the channel or the equivocation and subtracting it from the input information rate as discussed below. Neither is P_e a direct measure of how efficient a system is in terms of information transferred compared to that possible with an ideal system (i.e., one in which no errors occur), since information rates for both cases must first be calculated using the transitional probabilities of error.

One the other hand, the transitional probabilities of error must be calculated regardless of whether P_e or information rate is desired. P_e is the most easily calculated of all the criteria being considered and this simplicity is a very desirable characteristic in itself.

II Information Efficiency

While P_e specifies the average error rate for a system, it would be desirable to specify system performance in terms of realizable rate. Since information transfer is the fundamental purpose of a communication system, the rate at which this transfer takes place is the truest criterion of the system's effectiveness. Information efficiency is based on such a quantity.

The rate (on a per symbol basis) at which information can be transferred by a digital communication system is given by[1]

$$\text{Rate/symbol} = H(x) - H(x|y), \tag{2-8}$$

where $H(x)$ is the entropy of, or uncertainty associated with, the source feeding the channel and $H(x|y)$ is the equivocation or the loss of information due to using a channel where errors occur.

Information efficiency (η) is defined as

$$\eta = \frac{H(x) - H(x|y)}{H(x)} \times 100. \tag{2-9}$$

η is a normalized rate on a per symbol basis. It gives the percentage of source information (per symbol) which is correctly transferred by a digital communication link, and thus gives an idea of how efficiently each transmitted symbol is being used.

For the binary channel described in Section 2.2,

$$H(x) = -\left[P(0_t)\log_2 P(0_t) + P(1_t)\log_2 P(1_t)\right] \tag{2-10}$$

and

$$\begin{aligned} H(x|y) = P(0_t)&\left[(1-\mu)\log_2\left(1 + \frac{\nu}{1-\mu}\frac{P(1_t)}{P(0_t)}\right)\right. \\ &\left.+ \mu\log_2\left(1 + \frac{1-\nu}{\mu}\frac{P(1_t)}{P(0_t)}\right)\right] \\ &+ P(1_t)\left[(1-\nu)\log_2\left(1 + \frac{\mu}{1-\nu}\frac{P(0_t)}{P(1_t)}\right)\right. \\ &\left.+ \nu\log_2\left(1 + \frac{1-\mu}{\nu}\frac{P(0_t)}{P(1_t)}\right)\right]. \end{aligned} \tag{2-11}$$

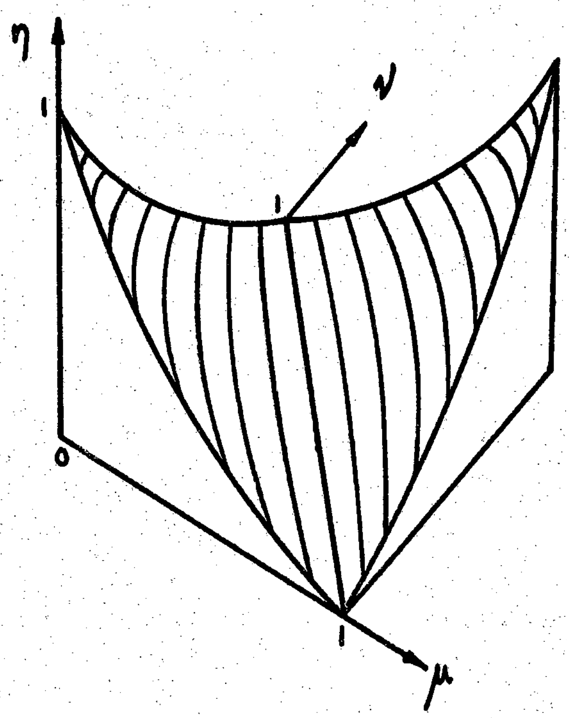
As may easily be seen, the expression for information efficiency in the general case is rather unwieldy to handle. If the simplifying assumption that $P(0_t) = P(1_t) = 1/2$ is made, then $H(x)$ and $H(x|y)$ reduce to

$$H(x) = 1 \tag{2-12}$$

and

$$\begin{aligned} H(x|y) = 1/2&\left[(1-\mu)\log_2\left(1 + \frac{\nu}{1-\mu}\right)\right. \\ &+ \mu\log_2\left(1 + \frac{1-\nu}{\mu}\right) \\ &+ (1-\nu)\log_2\left(1 + \frac{\mu}{1-\nu}\right) \\ &\left.+ \nu\log_2\left(1 + \frac{1-\mu}{\nu}\right)\right]. \end{aligned} \tag{2-13}$$

A model of information efficiency in such a channel is shown in Fig. 2.5.



INFORMATION EFFICIENCY IN A BINARY CHANNEL

FIGURE 2.5

For the remainder of the work which follows, this assumption of symmetry at the input will be used.

Still further simplification results if the channel is constrained to be symmetric. Under these circumstances,

$$H(x) = 1, \tag{2-14}$$

and

$$H(x|y) = -\left[P_e \log_2 P_e + (1-P_e) \log_2 (1-P_e)\right]. \tag{2-15}$$

Thus in the symmetric case, the information efficiency of a system is related to P_e in a straightforward manner.

There are two principal advantages in using information efficiency as a criterion of performance for binary channels. The first has already been stated and is, that system effectiveness is measured in terms of information rate instead of error rate.

The second advantage becomes evident when a system is operating in a non-symmetric mode. Two systems can operate with two different sets of transitional probabilities of error such that P_e is the same in both cases, but the information efficiencies for the two cases may be quite different. For example, suppose that $P(1_r|0_t) = 0.05$, $P(0_r|1_t) = 0.35$ and $P(0_t) = 0.50$. $P_e = 0.20$ and $\eta = 32.384$. Another system operating in the symmetric mode with $P_e = P(0_r|1_t) = P(1_r|0_t) = 0.20$ would have $\eta = 27.807$. Therefore, although P_e would indicate that the systems are equivalent, there is more than a 16 per cent difference in their capabilities in terms of the maximum rate that can be realized with each system. Since rate is the ultimate goal of a communication system, η is a superior criterion of performance for the non-symmetric channel.

A significant result here is that if P_e is held constant in a binary channel, the maximum information efficiency, and hence the maximum rate, is realized at the point of greatest dissymmetry. This may be easily seen in Fig. 2.5. If P_e is constrained to be a constant, then all possible operating points for the channel lie on a line perpendicular to the line $\mu = \nu$. From the concave shape of the surface, it can be seen that the points of maximum information efficiency fall where $\mu = 0$ or $\nu = 0$.

For the above reasons, it is felt that in general, η is a better index of performance than P_e. The concept of information efficiency may be applied to all digital systems and is not restricted to the binary case. Since it is easy to relate P_e and η for the symmetric case, the system performance curves of Chapters III, IV and VII show both η and P_e for the equivalent symmetrical system.

It should be noted here that in order to realize the rates discussed above, it would be necessary to employ an optimum coding scheme.[8] Since the problem of optimum codes has not in general been solved, the analysis which follows will not include coding and decoding operations (see Fig. 2.1).

III Rate

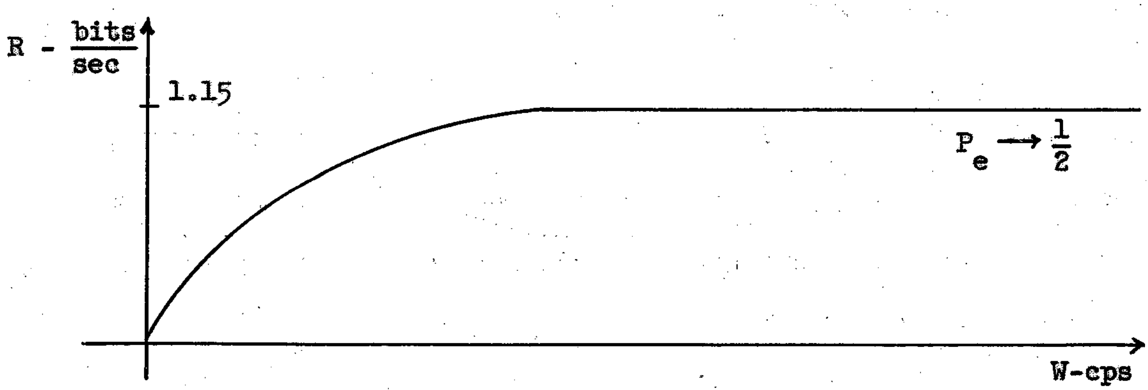
While information efficiency is actually a normalized rate, it is rate on a per symbol rather than a time basis. To obtain the information rate (R) on a time basis, the symbol rate m must be introduced where m is the number of symbols transmitted per second. Thus

$$R = m \frac{\eta}{100} \text{ bits/second} \qquad (2\text{-}16)$$

(note: in all of the work which follows, it will be assumed that information rates will be measured in bits).

Although it would be quite desirable to use information rate (bits/sec) in comparing various systems, there are several drawbacks. In the first place symbol rate m is directly proportional to bandwidth and as will be shown later in this chapter, bandwidth comparisons are often difficult to make.

Another factor is that maximum rates do not necessarily coincide with minimum probability of error. In fact, if rate in bits/sec is plotted as a function of bandwidth W for an ASK system employing synchronous detection, the curve shown in Fig. 2.6 is obtained. Maximum rate occurs as W approaches infinity where the probability of error is in the neighborhood of 0.5.



RATE AS A FUNCTION OF BANDWIDTH FOR AN ASK SYSTEM

Figure 2.6

Another consideration here is that of intersymbol influence and multipath. Both of these factors place upper limits on system symbol rates m.[9,10]

Thus there are several practical reasons for not using information rate (bits/sec) for a criterion even though it is the principal objective sought in a communication system.

IV β Factors

The last criterion to be examined is the so-called β factor as defined by Sanders.[7] The purpose of a β factor is to give the required energy per bit that a given system requires for a given noise level and is a function of η. Thus

$$\beta = \frac{E/N_o}{R} = \frac{E/N_o}{m\eta} \times 100. \tag{2-17}$$

It should be noted that since β is formulated on the basis of rate, the same restrictions apply to its use as noted above in the discussion of rate.

A modified β factor (β') may be defined on a per symbol basis and thereby remove some of the difficulties encountered above.

$$\beta' = \frac{E/N_o}{\eta} \times 100 \tag{2-18}$$

Note that in order to determine values of β' for some systems it is necessary to define a TW product for the system; this is discussed below.

2.4 TW Product

Basically there are two types of binary channels which will be considered in the analysis which follows. The first category comprises the more conventional type of systems, such as carrier on-off systems which employ synchronous detection or envelope detection and phase reversal systems with coherent detection, where the analysis is based on the signal-to-noise ratio at the receiver input. The second type of system analyzed is that which employs matched filters or correlation techniques in the receiver. In these systems, performance is dependent on the input ratio of energy per baud to the noise spectral density. In order to draw valid comparisons between these two types of systems, it is first necessary to be able to convert signal-to-noise ratios into energy-to-noise ratios and vice versa.

The signal-to-noise ratio for a receiver input signal perturbed by additive gaussian white noise is

$$\frac{S}{N} = \frac{S}{2N_o W} \tag{2-19}$$

where

S = the average signal power

N_o = the noise spectral density defined on a double sided basis

W = the bandwidth of the input signal in cps.

If E is the energy contained in each baud and T is the duration of each baud, then

$$\frac{S}{N} = \frac{E/T}{2N_o W} = \frac{E}{N_o} \frac{1}{2TW} . \tag{2-20}$$

In order to compare systems it is necessary to define a TW product for each type of modulation. It should be noted that for any waveform of finite duration, the bandwidth is infinite in an exact sense. It therefore becomes necessary to define an arbitrary bandwidth in order to get a finite TW product. TW products for ASK and PSK systems are derived in Appendix I. A TW product for FSK systems is not defined due to the difficulty of assigning bandwidth to an FSK signal.

Chapter III

The Performance of Conventional Systems in the Symmetric Mode

3.1 Introduction

In this chapter an analysis is made of the more conventional types of binary systems when perturbed by gaussian white noise and operating in the symmetric mode. Included are: ASK systems employing envelope detection, ASK systems using synchronous detection, PSK systems with coherent detection and PSK systems using phase comparison detection. FSK systems employing frequency discriminators are discussed but not analyzed for reasons given later. All of these systems are analyzed on a signal-to-noise ratio basis and the results include plots of η and P_e as functions of signal-to-noise ratio.

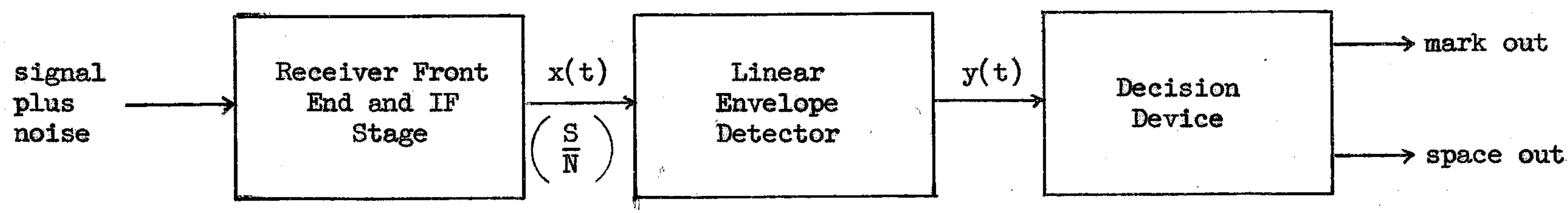
3.2 ASK Systems

Analyzed here are two systems employing amplitude shift keying or a carrier on-off type of modulation as described in Chapter I.

I Linear Envelope Detection with Threshold Decision

This is the simplest to instrumentate of all the various binary systems considered. This system consists of a receiver front end and IF stage which feeds into a linear envelope detector. The output of the detector is applied to a decision device which samples the output and renders a decision on the basis of the voltage at the time of sampling. A block diagram of this system is shown in Fig. 3.1.

In analyzing this system and the others which follow, only the detection and decision processes will be considered. Although the



AN ASK RECEIVER USING ENVELOPE DETECTION

Figure 3.1

receiver will necessarily have a front end and IF stage which are noisy and therefore degrade the system's performance, this same situation will occur in all of the systems analyzed and therefore is of no consequence in comparing systems.

If the input to the linear envelope detector, $x(t)$, is composed of an ASK signal as described in Chapter I plus gaussian white noise, then the output of the detector, $y(t)$, may be described by one of the probability density functions[3] given below,

$$p(y|0) = \frac{y}{2N_oW} \epsilon^{-\frac{y^2}{4N_oW}}, \qquad y \geq 0 \tag{3-1}$$

$$p(y|0) = 0, \qquad y < 0$$

and

$$p(y|1) = \frac{y}{2N_oW} \epsilon^{-\left[\frac{y^2}{4N_oW} + \frac{S}{N_oW}\right]} I_o\left(\frac{y\sqrt{S}}{N_oW}\right), \quad y \geq 0 \tag{3-2}$$

$$p(y|1) = 0, \qquad y < 0.$$

Where,

S = The average signal power at the detector input. (Note that since it has been assumed that a mark and a space are equally probable, S is equal to half of the signal power at the detector input when a mark is being transmitted.)

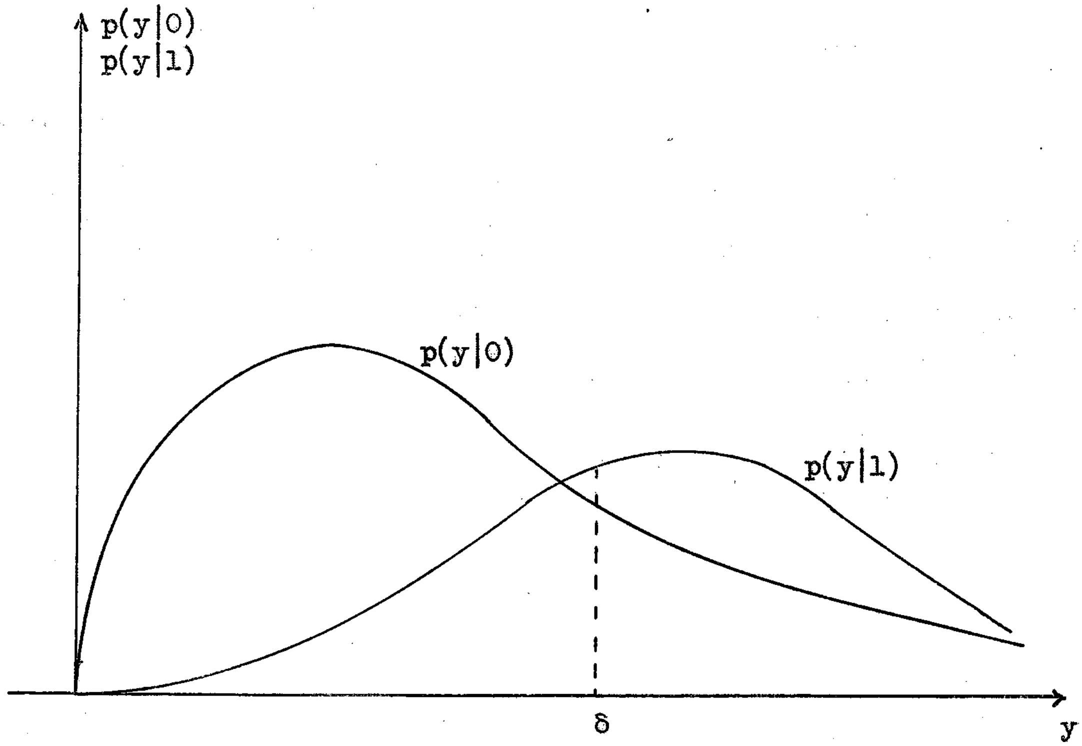
N_o = The spectral density of the gaussian white noise at the detector input defined on a double sided basis.

W = the bandwidth of the signal at the detector input.

The input signal-to-noise ratio is

$$\left(\frac{S}{N}\right) = \frac{S}{2N_oW} \tag{3-3}$$

A sketch of these density functions is shown in Fig. 3.2, $p(y|0)$ is the Rayleigh density function and $p(y|1)$ is a modified Rayleigh density function.



OUTPUT DENSITY FUNCTIONS FOR ENVELOPE DETECTOR

Figure 3.2

The purpose of the decision device shown in Fig. 3.1 is to announce whether a mark or a space has been received on the basis of a present voltage level $y = \delta$ (see Fig. 3.2). If the decision level (δ) is known, the transitional probabilities of error may be computed and are

$$P(0_r|1_t) = \int_0^{\delta} p(y|1)dy \tag{3-4}$$

$$P(1_r|0_t) = \int_{\delta}^{\infty} p(y|0)dy . \tag{3-5}$$

For the system to operate in the symmetric mode, the transitional probabilities of error must be equal and,

$$P_e = \int_0^{\delta} p(y|1)dy = \int_{\delta}^{\infty} p(y|0)dy. \qquad (3\text{-}6)$$

This equation may be reduced by a substitution of variables to

$$P_e = \int_0^{\lambda} \epsilon^{-\left[x + \frac{S}{N_oW}\right]} I_o\left[\sqrt{\frac{4xS}{N_oW}}\right] dx = \epsilon^{-\lambda}, \qquad (3\text{-}7)$$

where

$$\lambda = \delta^2/4\, N_oW. \qquad (3\text{-}7a)$$

To compute P_e, the integral equation (3-7) must first be solved for λ, given the input signal-to-noise ratio, and then P_e computed.

It is significant to note that in order to determine P_e for the symmetric system, it is only necessary to know the signal-to-noise ratio (see Eq. 3-7). If, however, the actual decision level (δ) for symmetric operation at a given signal-to-noise level is desired, it is necessary to know the actual noise power as well as the signal-to-noise ratio (see Eq. 3-7a). That is λ is a function of signal-to-noise ratio only whereas δ is a function of both λ and the noise power ($2N_oW$).

The above results illustrate two important characteristics common to all ASK systems. The first is that the optimum decision level is dependent on the signal-to-noise ratio and need not always be the same as will be the case in some systems which are discussed later. The second is that not only the signal-to-noise ratio but the actual values of the signal and noise powers must be known to determine this optimum level.

The integral equation 3-7 can not be evaluated in closed form. It may, however, be converted to a double summation as shown below (note: it was assumed here that $N_oW = 1$),

$$\epsilon^S = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{\lambda^{(m+n)} S^m}{m!(m+n)!} \tag{3-8}$$

and

$$P_e = \epsilon^{-\lambda}. \tag{3-9}$$

A second approach is to solve the integral equation 3-7 using numerical techniques and a digital computer*. The later solution was used here and P_e and η were computed as functions of input signal-to-noise ratio. These results are shown in Fig. 3.3. Discussion of results will be held to a minimum in this chapter since all the results are compared and commented on in Chapter VII.

II Synchronous Detection with Threshold Decision

In this case, the output of the IF stage and a coherent reference signal are applied to a multiplier as shown in Fig. 3.4. The product is then put through a low-pass filter and processed by the decision device.

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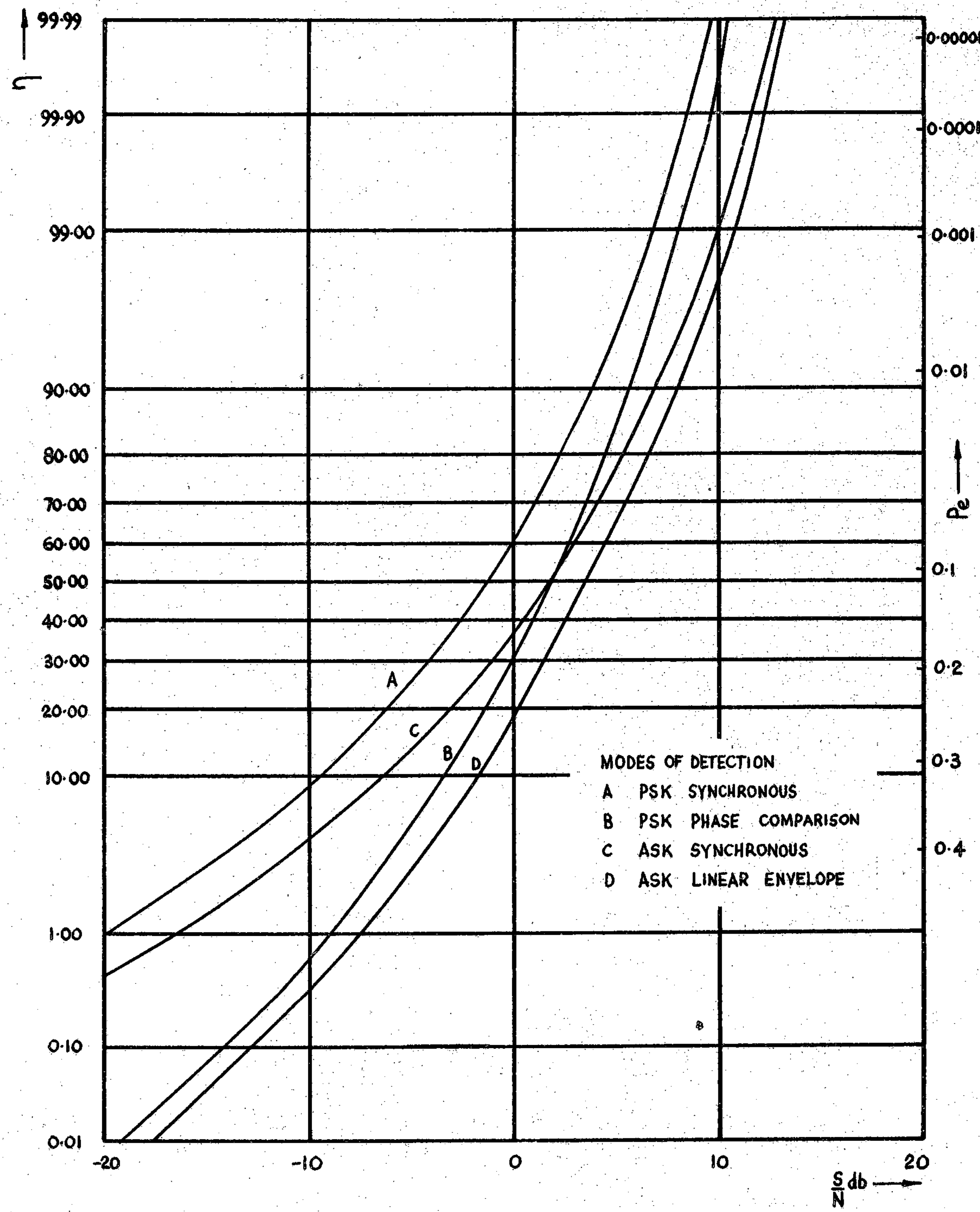
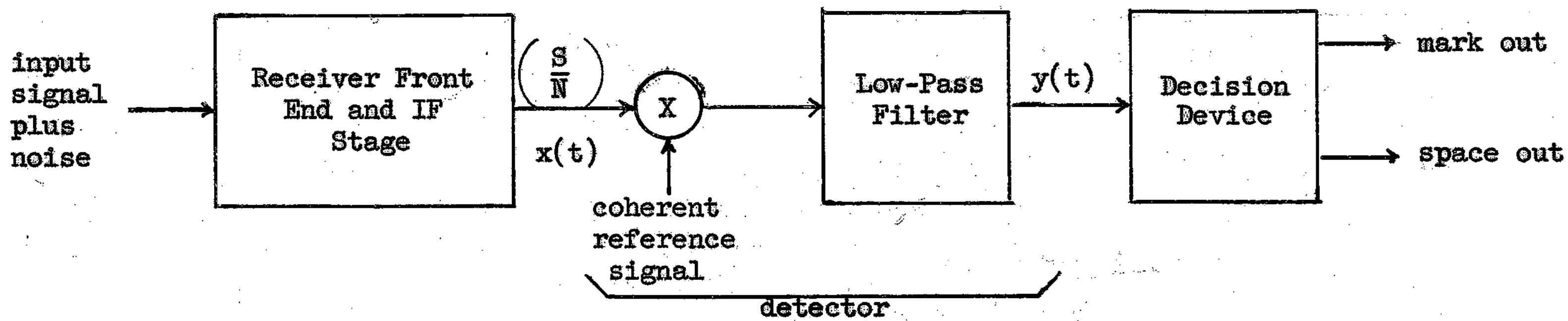
*It should be noted here that most of the computations in this and following chapters have been carried out using a digital computer. An approximation for the error function was obtained from Hastings[11] and the integrations were performed by means of Simpson's rule. The zeroth order Bessel function with imaginary argument was approximated by a truncated series for arguments less than 10 and by the following approximation for arguments greater than 10,

$$I_o(x) = \frac{\epsilon^x}{2\pi x}\left(1 + \frac{1}{8x} + \frac{9}{128x^2}\right).$$



INFORMATION EFFICIENCY OF CONVENTIONAL BINARY SYSTEMS

FIGURE 3.3



AN ASK RECEIVER USING SYNCHRONOUS DETECTION

Figure 3.4

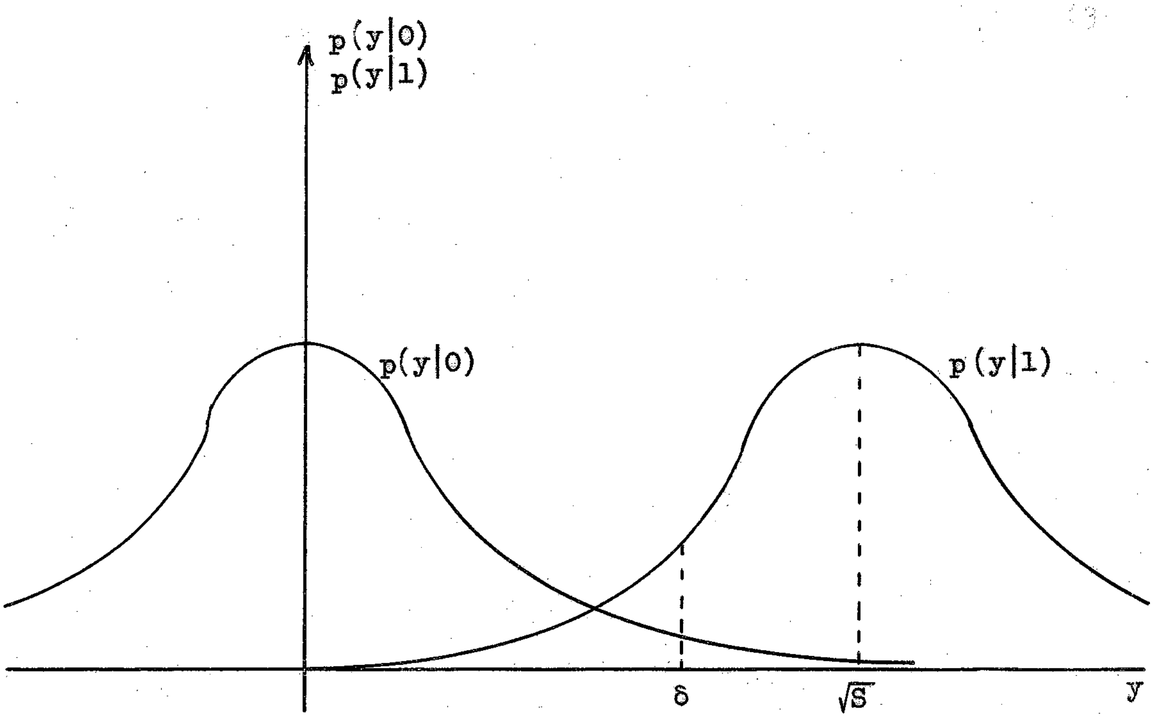
If the amplitude of the input signal is e when a mark is transmitted, then the input to the low-pass filter looks like $e \cos^2 \omega_c t$ which has a d-c component of $e/2 = \sqrt{S}$.

The output of the filter is gaussian and may be described by one of the following density functions,

$$p(y|1) = \frac{1}{\sqrt{\pi N_o W}} \epsilon^{\frac{-(y-\sqrt{S})^2}{N_o W}} \qquad (3\text{-}10)$$

$$p(y|0) = \frac{1}{\sqrt{\pi N_o W}} \epsilon^{-\frac{y^2}{N_o W}}, \qquad (3\text{-}11)$$

depending on whether a mark or a space has been transmitted. A sketch of these density functions is shown in Fig. 3.5. As in the case of the



OUTPUT DENSITY FUNCTIONS FOR A SYNCHRONOUS DETECTOR

Figure 3.5

linear envelope detector, the decision is based on a preset voltage level (δ). The transitional probabilities of error are,

$$P(0_r|1_t) = \int_{-\infty}^{\delta} p(y|1)dy \tag{3-12}$$

$$P(1|0) = \int_{\delta}^{\infty} p(y|0)dy. \tag{3-13}$$

For operation in the symmetric mode, $\delta = \sqrt{S}/2$ and P_e reduces to

$$P_e = 1/2 \left[1 - \text{erf}\left(\sqrt{\frac{S}{4N_oW}}\right)\right]. \tag{3-14}$$

This system displays the two characteristics of ASK systems previously mentioned. Although P_e is dependent on signal-to-noise ratio for optimum (i.e., symmetric) operation, the actual signal power must be known in order to set the proper decision level.

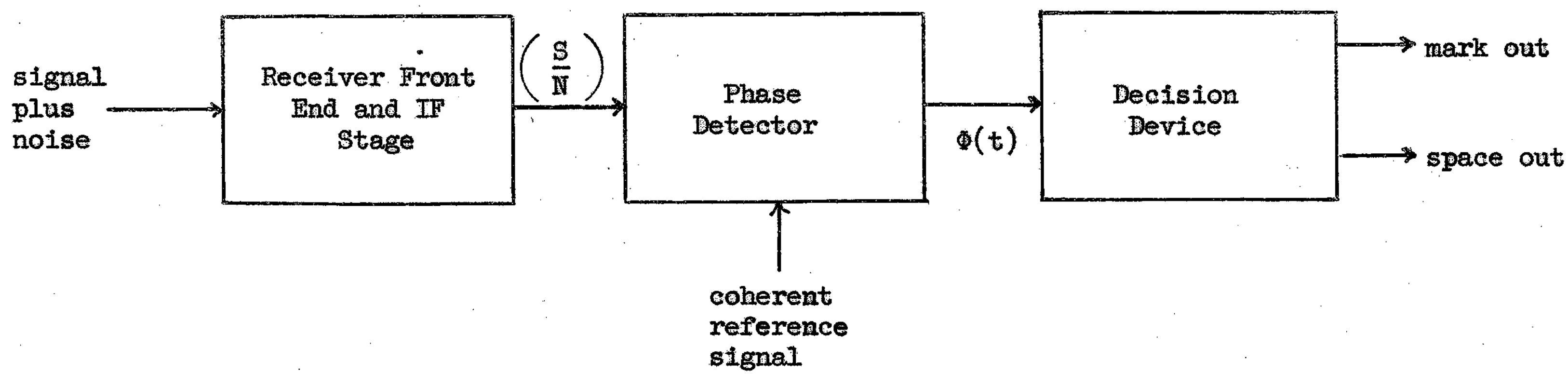
P_e and η as functions of signal-to-noise ratio for this system are shown in Fig. 3.3. It can be seen that synchronous detection is always better than envelope detection, but the difference becomes negligible as signal-to-noise ratio increases.

3.3 PSK Systems

In this section, two systems employing PSK or phase reversal modulation as described in Chapter I are analyzed.

I Synchronous Detection and Threshold Decision

In this system the phase of the incoming signal is compared with that of a coherent reference signal by means of a phase detector. A voltage which is proportional to the phase difference between the two signals is then fed to a decision device (see Fig. 3.6).



A PSK RECEIVER USING SYNCHRONOUS DETECTION

Figure 3.6

If the input to the phase detector is a PSK signal, as described in Chapter I, plus gaussian white noise, then the phase at the output of the detector (Φ) may be described by integrating the joint probability density function for envelope and phase of a sine wave plus gaussian noise to obtain the following probability density function,

$$p(\Phi|0) = \frac{1}{2\pi}\,\epsilon^{-\frac{S}{2N_oW}}\left[1 + \sqrt{\frac{2\pi S}{N_oW}}\cos\Phi\,\epsilon^{\frac{S}{2N_oW}\cos^2\Phi}\,\Phi\left(\sqrt{\frac{S}{N_oW}}\cos\Phi\right)\right] \tag{3-15}$$

$$p(\Phi|1) = \frac{1}{2\pi}\,\epsilon^{-\frac{S}{2N_oW}}\left[1 - \sqrt{\frac{2\pi S}{N_oW}}\cos\Phi\,\epsilon^{\frac{S}{2N_oW}\cos^2\Phi}\,\Phi\left(\sqrt{\frac{S}{N_oW}}\cos\Phi\right)\right] \tag{3-16}$$

where

$$\Phi(x) = \frac{1}{2\pi}\int_{-\infty}^{x}\epsilon^{-u^2/2}\,du, \tag{3-17}$$

and

$$\frac{S}{2N_oW} = \text{the signal-to-noise ratio at the detector output.}$$

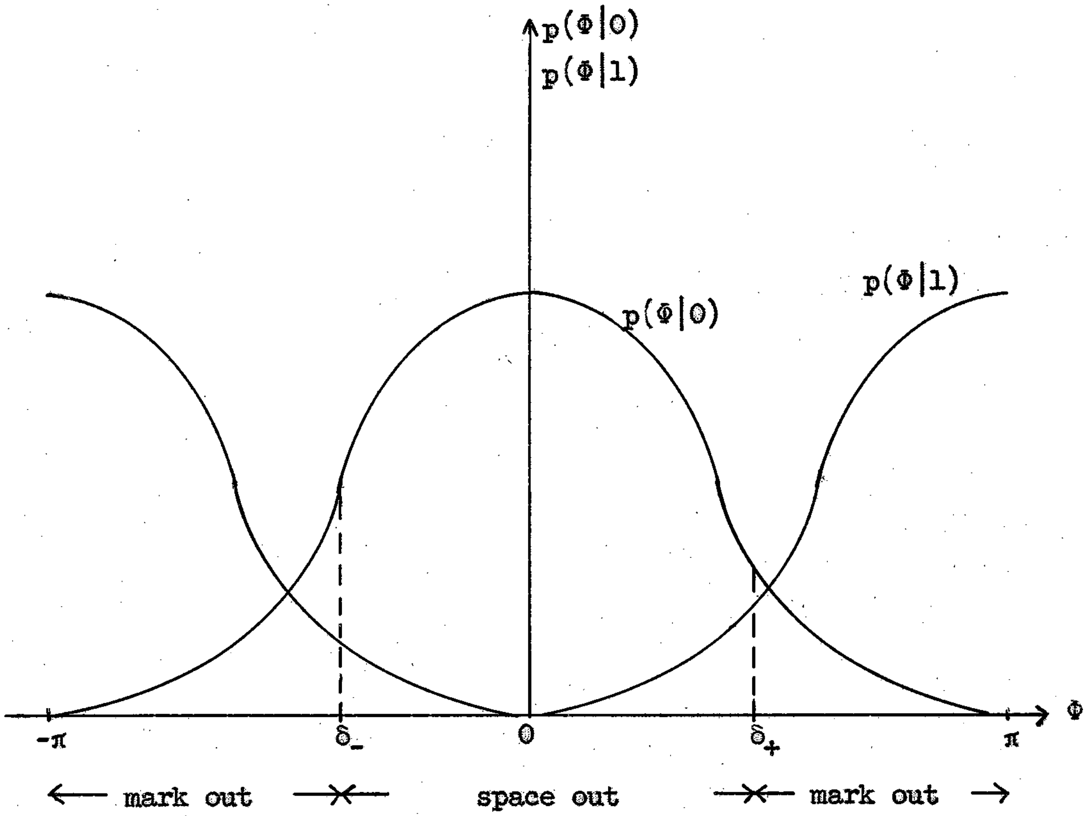
A sketch of these density functions is shown in Fig. 3.7.

The decision in this case is based on whether Φ lies in the space out or mark out regions which are determined by the values of δ_+ and δ_- (see Fig. 3.7).

The transitional probabilities of error are

$$P(0_r|1_t) = \int_{\delta_-}^{\delta_+} p(\Phi|1)\,d\Phi \tag{3-18}$$

$$P(1_r|0_t) = \int_{-\pi}^{\delta_-} p(\Phi|0)\,d\Phi + \int_{\delta_+}^{\pi} p(\Phi|0)\,d\Phi. \tag{3-19}$$



OUTPUT DENSITY FUNCTIONS FOR PSK SYNCHRONOUS DETECTION

Figure 3.7

For operation in the symmetric mode, $\delta_- = -\pi/2$ and $\delta_+ = \pi/2$. This is a result of the fact that

$$p(\Phi|1) = p(\Phi+\pi|0). \tag{3-20}$$

The density function shown in Fig. 3.7 will change with signal-to-noise ratio but Eq. 3-20 will always hold and therefore the optimum values for δ_+ and δ_- will not vary with signal strength, noise power or signal-to-noise ratio. This is in sharp contrast to the case in an ASK system. The relation shown in Eq. 3-20 also means that one need only know one density function in order to determine system performance (see Eq. 3-21 below).

The probability of error in the symmetric case then reduces to

$$P_e = \int_{-\pi/2}^{\pi/2} p(\Phi|1)d\Phi \tag{3-21}$$

$$P_e = 1 - \int_{-\pi/2}^{\pi/2} p(\Phi|0)d\Phi , \tag{3-22}$$

which reduces to

$$P_e = 1/2 \left[1 - \mathrm{erf} \left(\sqrt{\frac{S}{2N_oW}} \right) \right] . \tag{3-23}$$

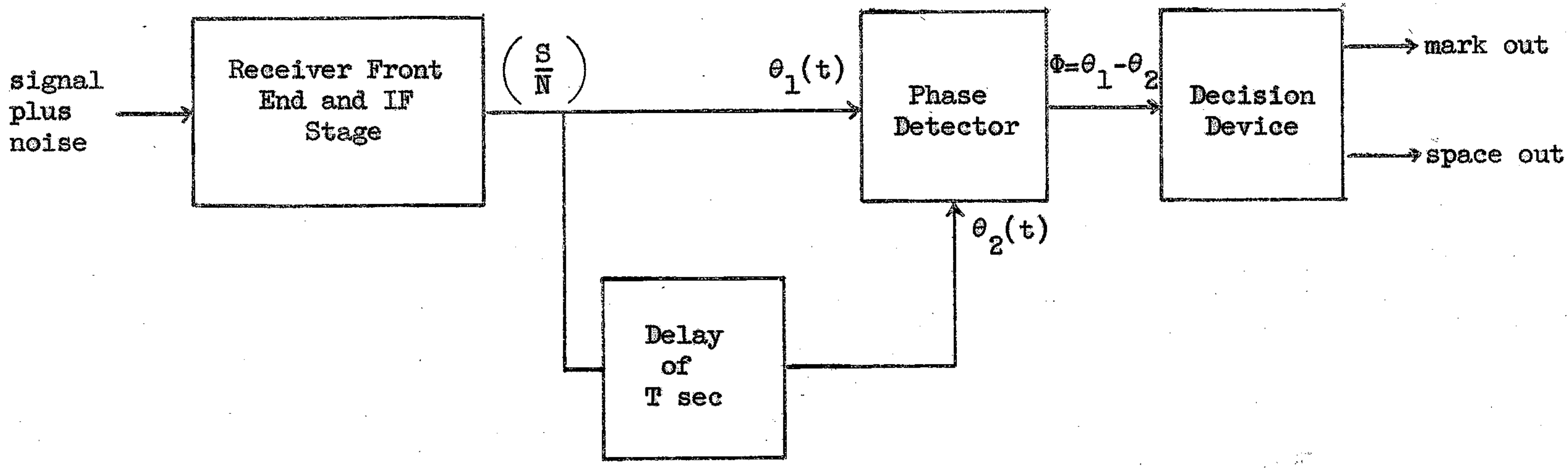
P_e and η as functions of signal-to-noise ratio for this system are shown in Fig. 3.3. It can be seen from the curves of Fig. 3.3 that this system is 3db better than an ASK system employing synchronous detection. This is true for all signal-to-noise ratios and may be verified by comparing Eq. 3-14 and Eq. 3-23.

II Phase Comparison and Threshold Decision

In this system, the output of the IF stage is split and one part applied to a delay line having a delay equal to one baud length. The other part of the IF output (θ_1) is applied to a phase detector which uses the output of the delay line (θ_2) as a reference (see Fig. 3.8). The output of the phase detector is processed by the decision device which determines whether or not a phase reversal has taken place.

If the input to the system is the same as for the case just treated above, then the probability density function for Φ may be obtained by convolving $p(\Phi|0)$ (Eq. 3-15) with itself.

$$p_d(\Phi) = \int_{-\pi}^{\pi} p(\Phi|0)p(\Phi+\Psi|0)d\Psi .$$



A PSK RECEIVER EMPLOYING PHASE COMPARISON DETECTION

Figure 3.8

A closed form solution for the above is not available; however, C. R. Cahn[5] has numerically evaluated the density function of Φ and its related distribution function for signal-to-noise ratios of from -10db to +19 db.

The analysis here is similar to that used in the previous case and only one density function is needed. The decision levels for the system to be symmetric are $\pm\, \pi/2$, and the probability of error is,

$$P_e = 1 - \int_{-\pi/2}^{\pi/2} p_d(\Phi)d\Phi.$$

Cahn[4] has worked out this case and P_e reduces to

$$P_e = 1/2\, \epsilon^{-\frac{S}{2N_oW}}.$$

As in the case of coherent detection, the optimum decision levels are independent of signal power and signal-to-noise ratio.

P_e and η as functions of signal-to-noise ratio for this system are shown in Fig. 3.3. The performance of this system is very close to that of a PSK system with synchronous detection for high signal-to-noise ratios; however, at signal-to-noise ratios below 2db, the performance of this system falls below that of an ASK synchronous detection scheme. For very small signal-to-noise ratios, it is similar to envelope detection in performance. This system has the substantial advantage of not requiring a reference signal at the receiver.

3.4 FSK Systems

At the present time there is no analysis for a frequency discriminator which is suitable for a wide range of input signal-to-noise ratios. There is an analysis for the high signal-to-noise ratio case,[12] but this is rather restrictive. The principal problem lies in specifying probability density functions at the output of a non-linear device with a gaussian input. For this reason, the performance of systems employing frequency discriminators is not included here. This comment also applies to the detection of PSK signals by related methods.

Chapter IV

The Performance of Matched Filter Systems in the Symmetric Mode

4.1 Introduction

In this chapter an analysis is made of various binary systems employing matched filters[12] (or the equivalent cross-correlation techniques) in their detection processes, and operating in the face of gaussian white noise. The analysis is restricted at present to operation in the symmetric mode. The systems discussed are as follows: ASK systems with matched filter coherent and non-coherent detection, PSK systems with matched filter coherent detection, a differentially coherent detection scheme, and FSK systems with matched filter coherent and non-coherent detection. Because of the nature of a matched filter system, the analysis which follows is done on an input energy per baud to noise spectral density ratio basis rather than the signal-to-noise ratio used in Chapter III. In Chapter VII a comparison between all systems will be made by using a TW product to convert signal-to-noise ratio to energy-to-noise ratio as discussed in section 2.4. Results include plots of η and P_e as functions of E/N_o and plots of β' as a function of P_e.

4.2 ASK Systems

In this section two ASK systems using matched filters in their detection processes are analyzed. The primary difference between the two is in how the output of the matched filter is treated. In the coherent case, the output is sampled at precisely the correct time (i.e., the time when the output signal is a maximum). In the non-coherent case the optimum received results in using a linear envelope detector before sampling takes place.[13] The ASK modulation is as described in Chapter I.

I Coherent Detection

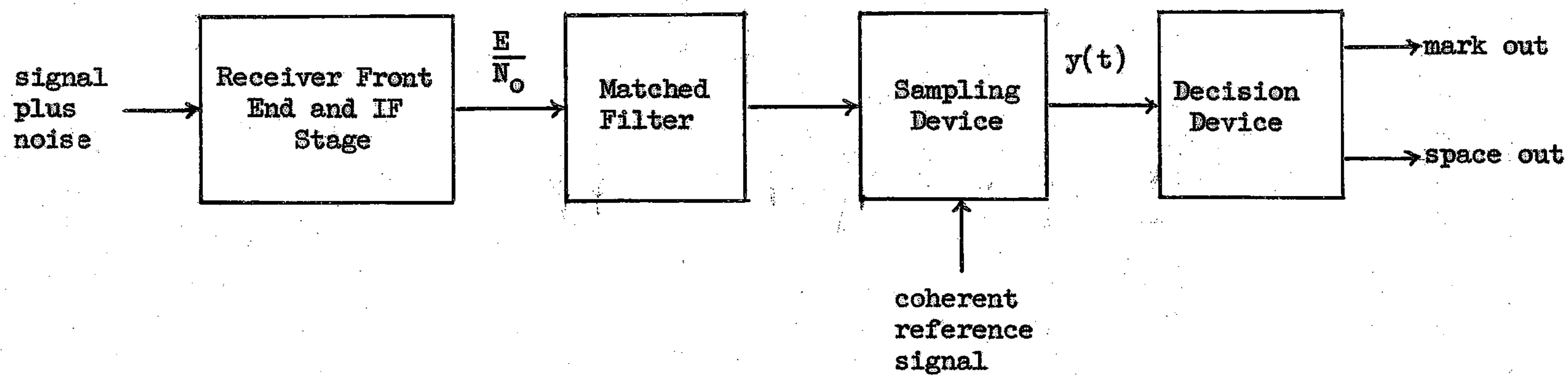
In this system, the receiver front end and IF stage feeds into a matched filter as shown in Fig. 4.1. The output of the matched filter is then sampled at a time when the signal-to-noise ratio should be maximum and a decision rendered on the basis of $y_s(t)$.

The signal-to-noise ratio at the sampler output is $\frac{E'}{N_o}$ if a mark has been transmitted,[12] where E' is the energy per baud when a mark is sent and is twice the average energy per baud (E) since the probability of transmitting a mark is the same as for a space. If a mark is sent in the absence of any noise, then the output of the sampler is 2E. The variance at the filter output is N_oE regardless of whether a mark or a space was transmitted. Since the filter is linear, the density functions describing $y_s(t)$ are gaussian. They are

$$p(y|0) = \frac{1}{\sqrt{2\pi N_o E}} \epsilon^{-\frac{y^2}{2N_o E}} \quad (4\text{-}1)$$

$$p(y|1) = \frac{1}{\sqrt{2\pi N_o E}} \epsilon^{-\frac{(y-2E)^2}{2N_o E}} \quad (4\text{-}2)$$

A sketch of these density functions is shown in Fig. 4.2. The decision device renders its decision on the basis of a preset voltage level δ. The transitional probabilities of error are

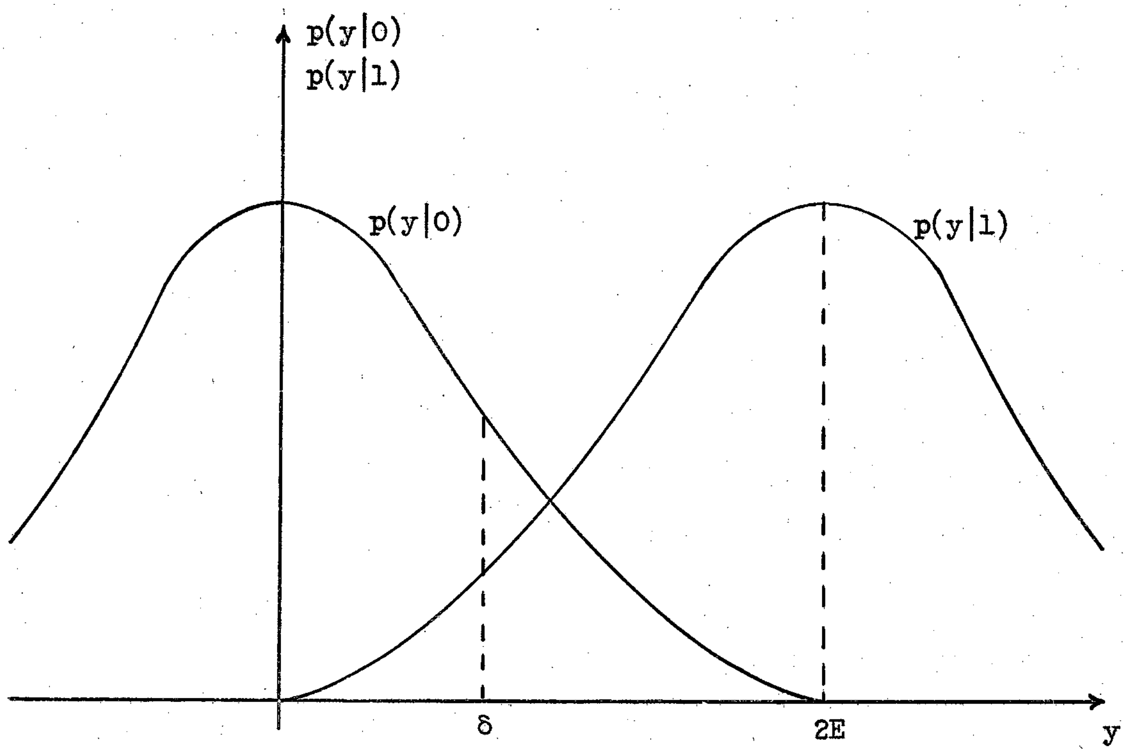


A COHERENT MATCHED FILTER RECEIVER
FOR ASK OR PSK SIGNALS

Figure 4.1

$$P(0_r|1_t) = \int_{-\infty}^{\delta} p(y|1)dy \tag{4-3}$$

$$P(1_r|0_t) = \int_{\delta}^{\infty} p(y|0)dy. \tag{4-4}$$



OUTPUT DENSITY FUNCTIONS FOR AN ASK
COHERENT MATCHED FILTER SYSTEM

Figure 4.2

For optimum operation the system will be constrained to be symmetrical and $\delta = E$. This follows from the fact that

$$p(y|1) = p(y-2E|0). \tag{4-5}$$

Thus, P_e reduces to

$$P_e = 1/2\left[1 - \text{erf}\left(\sqrt{\frac{E}{4N_o}}\right)\right]. \tag{4-6}$$

It should be noted that the two characteristic traits of an ASK system, discussed in Chapter III, are again present. That is, calculating the optimum performance of the system depends only on E/N_o, but in order to set a proper threshold level, the signal strength must be known.

Curves of η and P_e are shown for this system in Fig. 4.3 and a plot of β' versus P_e is given in Fig. 4.4. A full discussion of the results obtained in this chapter will be given in Chapter VII.

It should be noted here that for a coherent matched filter system operating in the symmetric mode, the probability of error is[13]

$$P_e = 1/2 \left[1 - \text{erf} \left(\sqrt{\frac{E}{4N_o} (1-\rho_{12})} \right) \right] \tag{4-7}$$

where

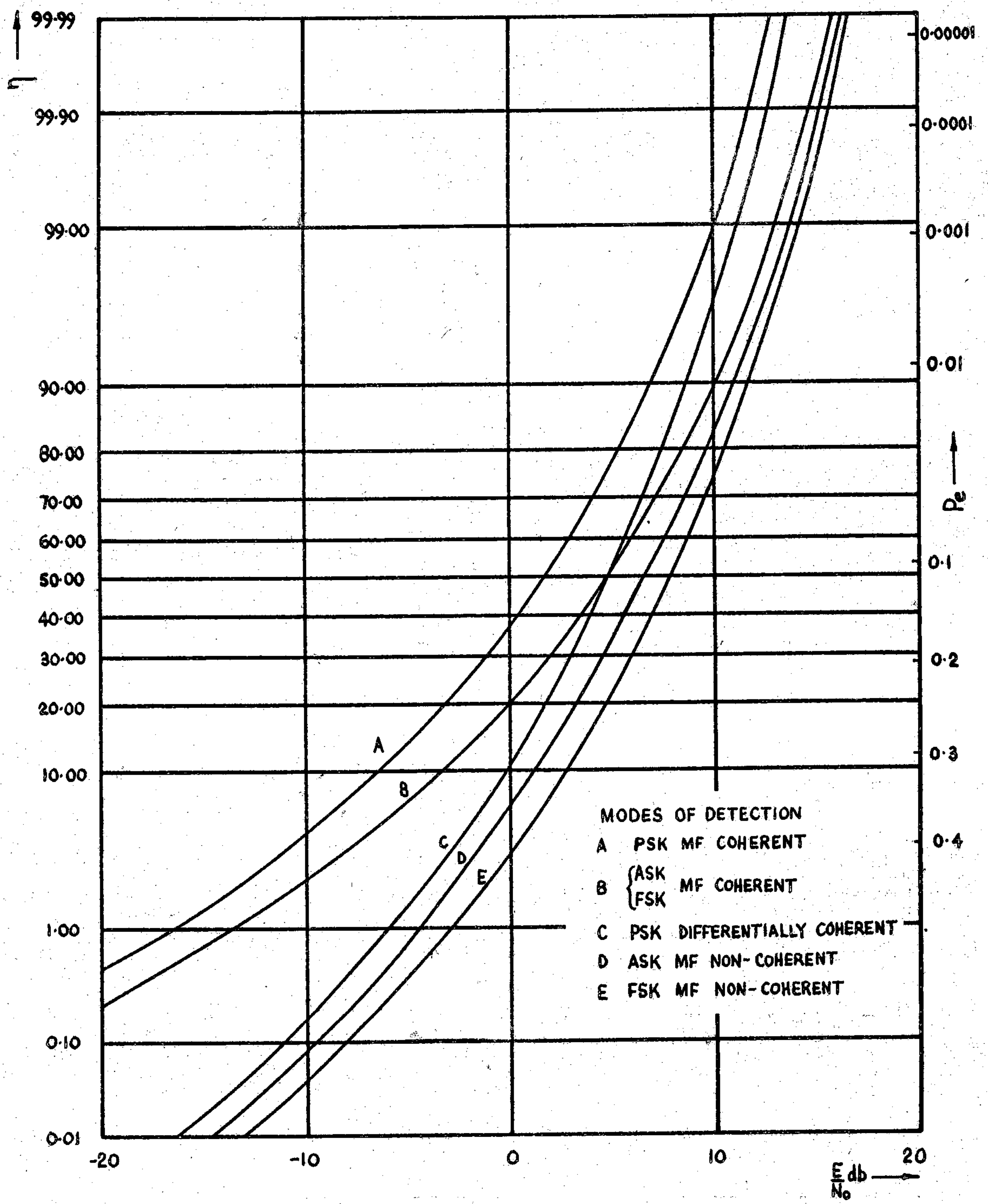
ρ_{12} = the normalized correlation coefficient between the transmitted signals used for a mark and a space.

This general result may be applied here where $\rho_{12} = 0$, to get Eq. 4-6, as well as in two of the cases which follow. However, since the results obtained here will be used later in the analysis of these same systems operating in a non-symmetric mode, a specific derivation will be given for each of these systems as well.

II Non-Coherent Detection

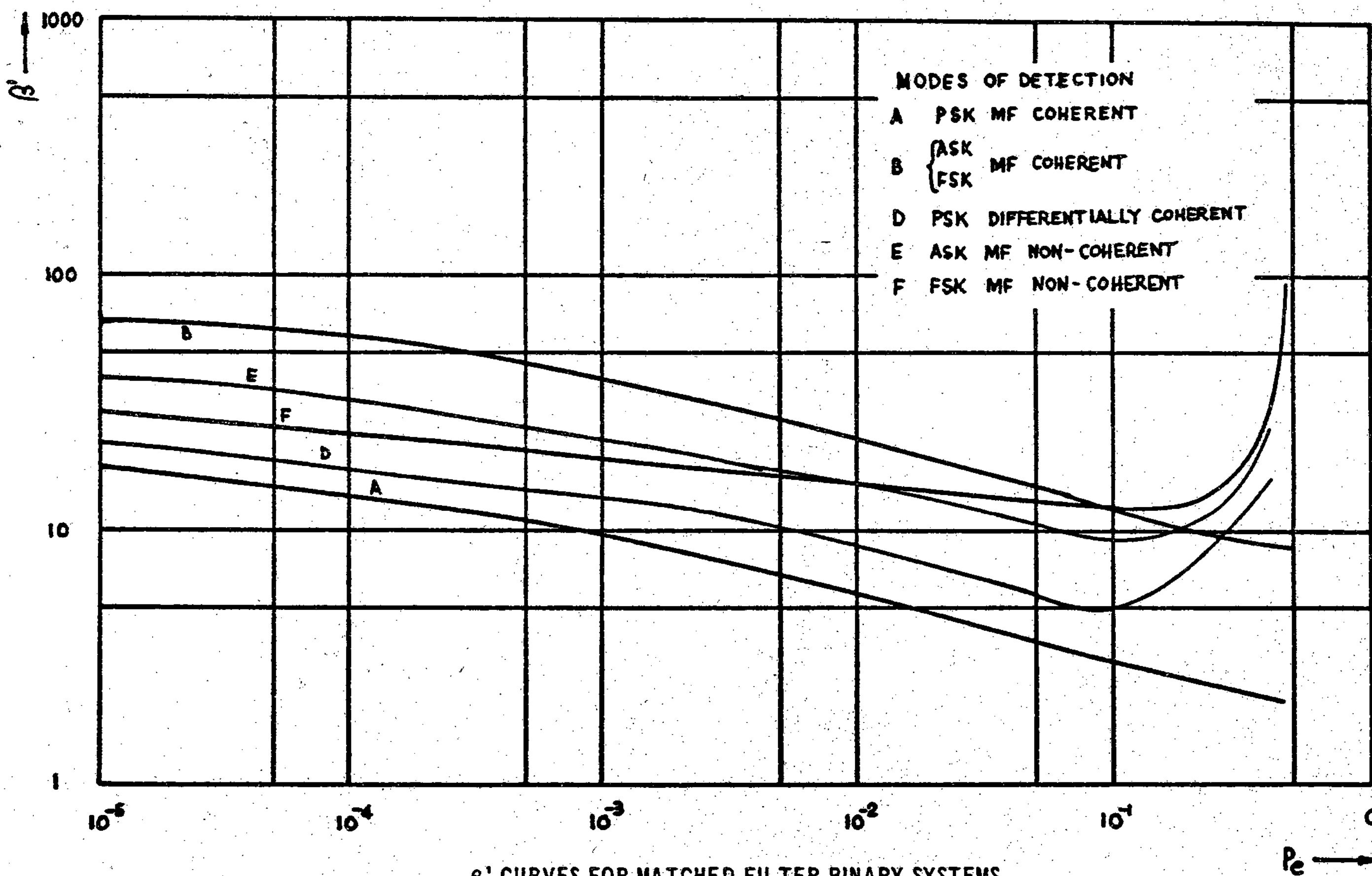
For this system, sufficient phase information is not available and the optimum system results in a matched filter followed by a linear envelope detector[2] (see Fig. 4.5).

The signal-to-noise ratio at the matched filter output is again $\frac{2E}{N_o}$. The output of the linear envelope detector may then be described



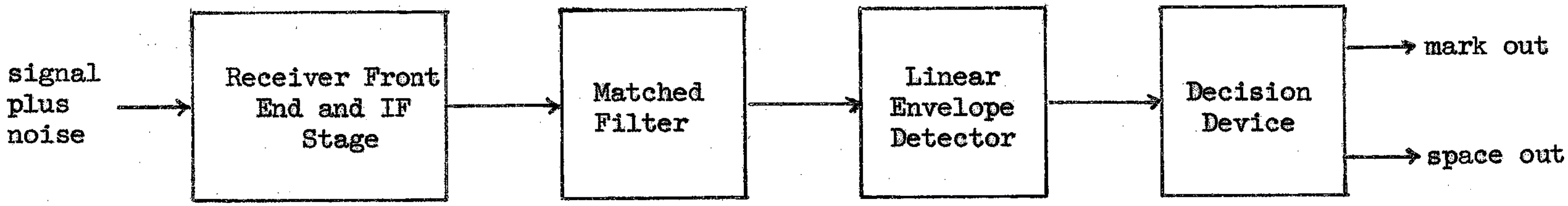
INFORMATION EFFICIENCY OF MATCHED FILTER BINARY SYSTEMS

FIGURE 4.3



β' CURVES FOR MATCHED FILTER BINARY SYSTEMS

FIGURE 4.4



A NON-COHERENT RECEIVER FOR AN ASK SYSTEM

Figure 4.5

by the two density functions given below,

$$p(y|0) = \frac{y}{2N_oE}\,\epsilon^{-\frac{y^2}{4N_oE}}, \qquad y \geq 0 \qquad (4\text{-}8)$$

$$p(y|0) = 0, \qquad y < 0$$

$$p(y|1) = \frac{y}{2N_oE}\,\epsilon^{-\left(\frac{y^2}{4N_oE} + \frac{E}{N_o}\right)} I_o\left(\frac{y}{N_o}\right), \qquad y \geq 0 \qquad (4\text{-}9)$$

$$p(y|1) = 0, \qquad y < 0.$$

These are similar to those obtained in Chapter II (see Fig. 3.2). If the decision device operates on the basis of a threshold (δ), then the conditional probabilities of error are,

$$P(0_r|1_t) = \int_0^\delta p(y|1)dy \qquad (4\text{-}10)$$

$$P(1_r|0_t) = \int_\delta^\infty p(y|0)dy. \qquad (4\text{-}11)$$

For operation in the symmetric mode,

$$\int_0^\delta \frac{y}{2N_oE}\,\epsilon^{-\left[\frac{y^2+4E^2}{4N_oE}\right]} I_o\left(\frac{y}{N_o}\right)dy = \int_\delta^\infty \frac{y}{2N_oE}\,\epsilon^{-\frac{y^2}{4N_oE}}\,dy. \qquad (4\text{-}12)$$

By substituting variables and integrating the right half of the above equation,

$$P_e = \int_0^\lambda \epsilon^{-\left[x + \frac{E}{N_o}\right]} I_o\left(2\sqrt{\frac{E}{N_o}\,x}\right)dx = \epsilon^{-\lambda}, \qquad (4\text{-}13)$$

where

$$\lambda = \frac{\delta^2}{4N_o E} . \qquad (4\text{-}13a)$$

As in the case of a linear envelope detector alone, P_e must be determined by first solving Eq. 4-13 for λ and then computing P_e. As in all ASK systems it is necessary to know both the signal and the noise as well as their ratio in order to operate the receiver in an optimum fashion. The performance of this system is shown in Fig. 4.3 and 4.4 where η and P_e versus E/N_o and β' versus P_e are given. From Fig. 4.3 it can be seen that the performance of this system is very close to that of the coherent system above for high values of E/N_o but falls off rapidly as E/N_o decreases.

4.3 PSK Systems

Here, PSK coherent matched filter detection is analyzed. The PSK waveform is as described in Chapter I and the normalized correlation between a mark and a space is -1. This is of special interest since this represents the best possible binary system with respect to probability of error.[13] The non-coherent detection of PSK signals is not analyzed since the envelopes of the mark and space waveforms at the output of the matched filter are indistinguishable from one another and hence this system will not work. This fact also follows from the fact that a linear envelope detector destroys the phase of the signal wherein its information content lies. Results for a differentially coherent detection scheme are also given.

I Coherent Detection

The block diagram of the coherent PSK receiver is identical to that of the coherent ASK receiver shown in Fig. 4.1. In this case the energy per baud is the same regardless of whether a mark or a space has been sent and is equal to the average energy per baud. The signal-to-noise ratio at the filter output is $^{E}/N_o$ and the output is gaussian since the filter is linear. The output may be described by the two following density functions,

$$p(y|0) = \frac{1}{\sqrt{2\cdot 2\pi N_o E}}\ \epsilon^{-\frac{(y+E)^2}{2N_o E\cdot 2}} \qquad (4\text{-}14)$$

$$p(y|1) = \frac{1}{\sqrt{2\, 2\pi N_o E}}\ \epsilon^{-\frac{(y-E)^2}{2N_o E\cdot 2}}\ . \qquad (4\text{-}15)$$

A sketch of these two density functions is shown in Fig. 4.6. The transitional probabilities of error are,

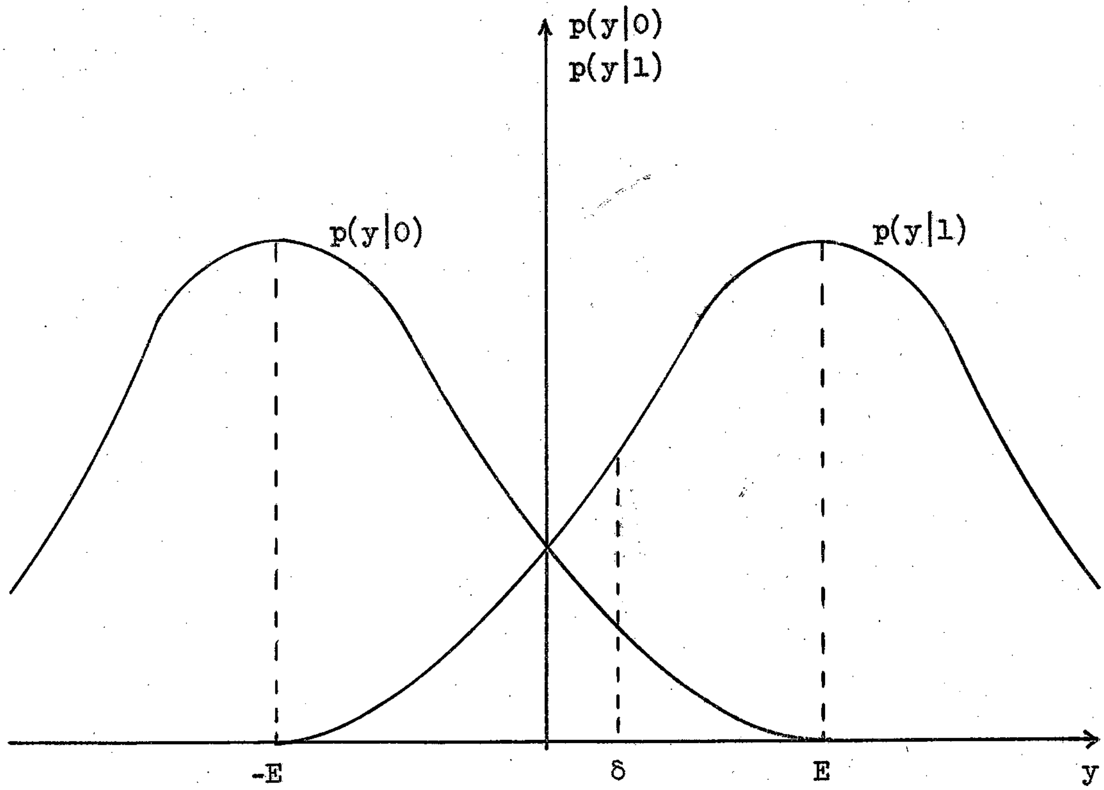
$$P(0_r|1_t) = \int_{-\infty}^{\delta} p(y|1)dy \qquad (4\text{-}16)$$

$$P(1_r|0_t) = \int_{\delta}^{\infty} p(y|0)dy\ . \qquad (4\text{-}17)$$

By reasons of symmetry, the optimum value of δ is zero and P_e reduces to

$$P_e = 1/2\left[1 - \text{erf}\left(\sqrt{\frac{E}{2N_o}}\right)\right]. \qquad (4\text{-}18)$$

Note that this result may also be obtained from Eq. 4-7 when $\rho_{12} = -1$ as is the case here.



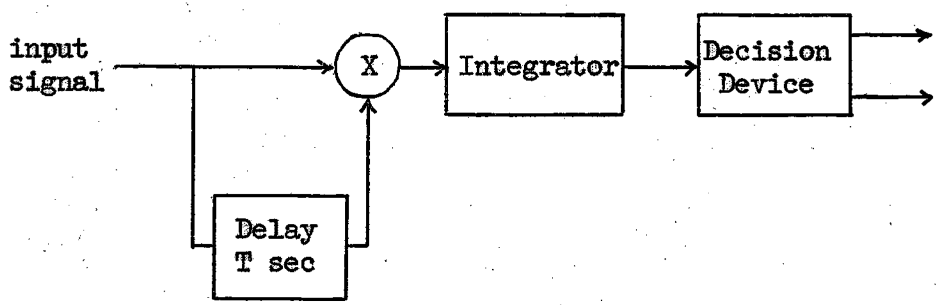
OUTPUT DENSITY FUNCTIONS FOR A PSK
MATCHED FILTER SYSTEM

Figure 4.6

Again, as was the case for conventional PSK systems, the optimum decision level is not a function of signal or noise strength. The performance curves for this system are shown in Fig. 4.3 and 4.4. The performance of this system is better than that of any other. Comparison of Eq. 4-18 and 4-6 show that this system is always 3db better than an ASK system using a coherent matched filter.

II Differentially Coherent Detection

In this system, the reference signal at the receiver is provided by the previous baud by means of a delay line. The setup is similar to that used for phase comparison detection (see Fig. 3.8) except that the two signals are multiplied and integrated as in a correlation receiver (see Fig. 4.7). The probability of error for this system in a symmetric



PSK DIFFERENTIALLY COHERENT DETECTION

Figure 4.7

mode has been derived by Lawton[14] and is

$$P_e = 1/2\ e^{-E/N_o} . \tag{4-19}$$

This type of operation has been equivalently realized in the kineplex system[15] and as may be seen from Fig. 4.3 the operation of this system approaches that of the ideal system for high values of E/N_o.

4.4 FSK Systems

In this section two FSK matched filter systems will be analyzed. They are the coherent case where the proper sampling times are known and the non-coherent case where envelope detection is used. In both cases, the FSK waveform is as described in Chapter I.

I Coherent Detection

Since there are two distinct waveforms which are not correlated (i.e., $\rho_{12} = 0$) it is necessary to use a bank of two matched filters in parallel (see Fig. 4.8). The filter outputs are sampled at times of maximum expected signal-to-noise ratio at their outputs. The sampled outputs are then subtracted and fed to a decision device. The decision is rendered on the basis of a preset threshold δ.

The signal-to-noise ratio at each matched filter output (after proper sampling) is either E/N_o or 0 depending whether a mark or a space was transmitted. Since the filters are linear and the input is gaussian, the output may be described by the density functions below,

$$p(y_1|1) = \frac{1}{\sqrt{2\pi N_o E}}\,\epsilon^{-\frac{(y_1-E)^2}{2N_oE}} \tag{4-20}$$

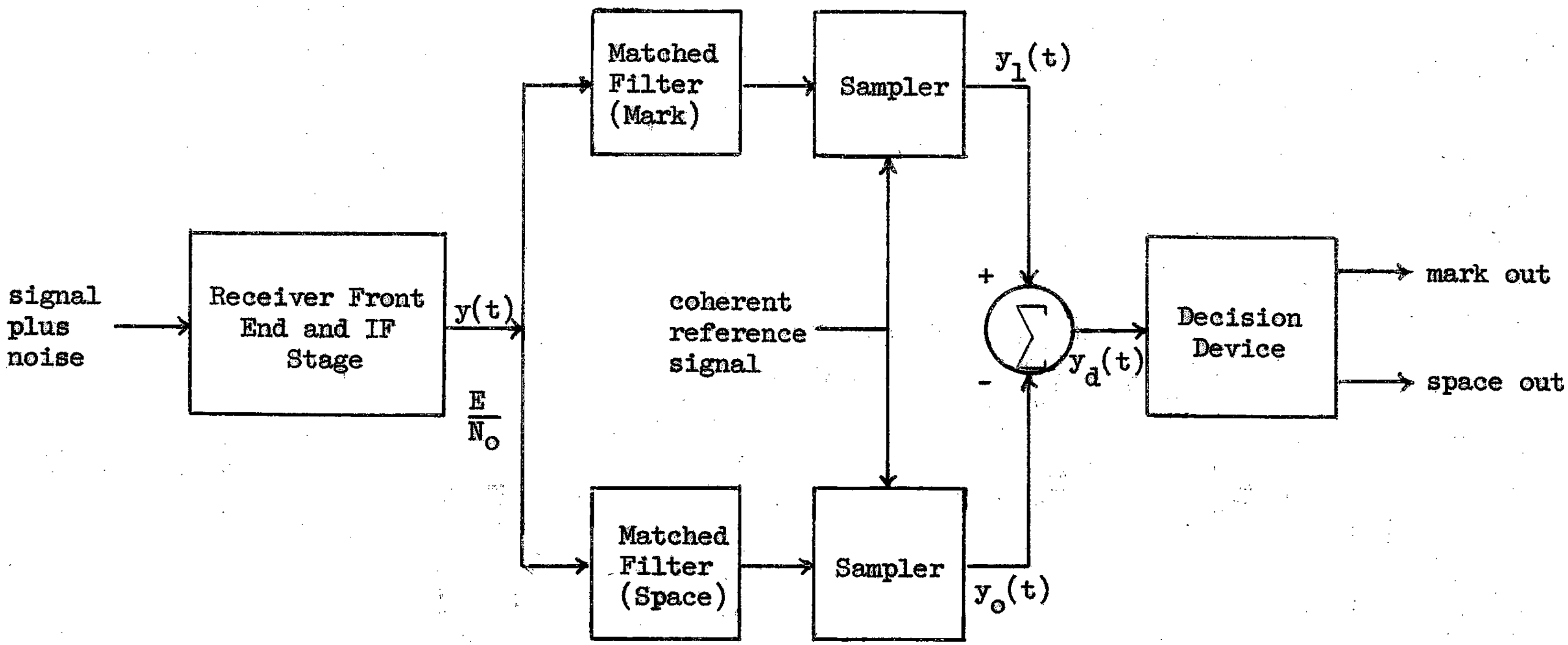
$$p(y_1|0) = \frac{1}{\sqrt{2\pi N_o E}}\,\epsilon^{-\frac{y_1^2}{2N_oE}} \qquad (4\text{-}21)$$

$$p(y_o|1) = \frac{1}{\sqrt{2\pi N_o E}}\,\epsilon^{-\frac{y_o^2}{2N_oE}} \qquad (4\text{-}21)$$

$$p(y_o|0) = \frac{1}{\sqrt{2\pi N_o E}}\,\epsilon^{-\frac{(y_o-E)^2}{2N_oE}}. \tag{4-23}$$

Since y_1 and y_o are gaussian and uncorrelated (see Appendix II) they are independent and therefore $y_1 - y_o = y_d$ is a gaussian random variable. The density functions for y_d are,

$$p(y_d|0) = \frac{1}{\sqrt{4\pi N_o E}}\,\epsilon^{-\frac{(y_d+E)^2}{4N_oE}} \tag{4-24}$$



A COHERENT MATCHED FILTER RECEIVER
FOR FSK SIGNALS

Figure 4.8

$$p(y_d|1) = \frac{1}{\sqrt{4\pi N_o E}} \, \epsilon^{-\frac{(y_d - E)^2}{4N_o E}} \, . \qquad (4\text{-}25)$$

These closely resemble in shape the output density functions for the coherent PSK receiver shown in Fig. 4.6.

The transitional probabilities of error are,

$$P(0_r|1_t) = \int_{-\infty}^{\delta} p(y_d|1) dy_d \qquad (4\text{-}26)$$

$$P(1_r|0_t) = \int_{\delta}^{\infty} p(y_d|0) dy_d \, . \qquad (4\text{-}27)$$

The optimum decision level is 0 since the two density functions are symmetric about the origin. For this case P_e reduces to

$$P_e = 1/2 \left[1 - \text{erf} \left(\sqrt{\frac{E}{4N_o}} \right) \right] . \qquad (4\text{-}28)$$

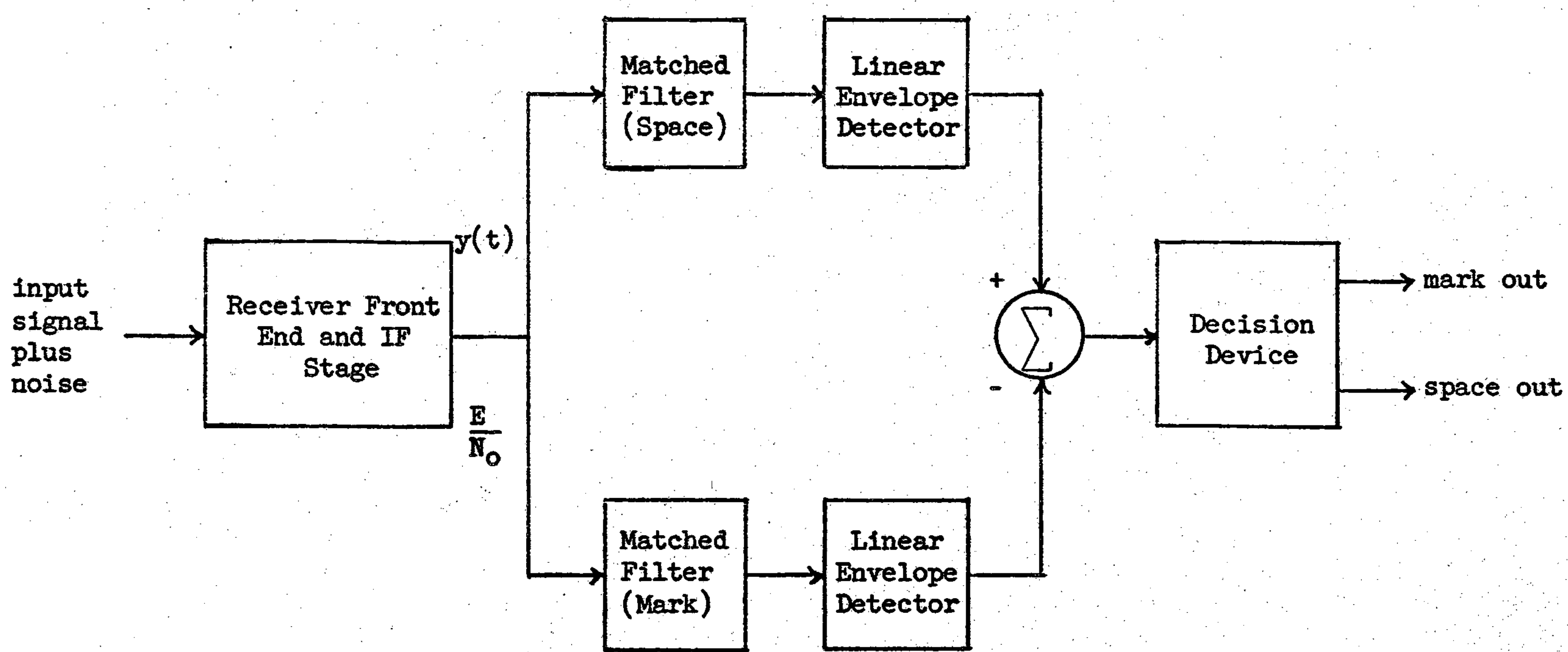
Note that again this same result follows from Eq. 4-7.

Performance curves for this system are shown in Fig. 4.3 and 4.4. Note that the results here are identical to those for an ASK system using coherent matched filter detection ($\rho_{10} = 0$).

II Non-Coherent Detection

In this case, the samplers at the matched filter outputs are replaced by linear envelope detectors (see Fig. 4.9). The detector outputs may be described by the following density functions.[3]

$$p(y_1|1) = \frac{y_1}{N_o E} \, \epsilon^{-\frac{y_1^2 + E^2}{2N_o E}} \, I_o \left[\frac{y_1}{N_o} \right] \qquad (4\text{-}29)$$



A NON-COHERENT MATCHED FILTER RECEIVER
FOR FSK SIGNALS

Figure 4.9

$$p(y_1|0) = \frac{y_1}{N_oE} \epsilon^{-\frac{y_1^2}{2N_oE}} \tag{4-30}$$

$$p(y_o|1) = \frac{y_o}{N_oE} \epsilon^{-\frac{y_o^2}{2N_oE}} \tag{4-31}$$

$$p(y_o|0) = \frac{y_o}{N_oE} \epsilon^{-\frac{y_o^2+E^2}{2N_oE}} I_o\left[\frac{y_o}{N_o}\right] . \tag{4-32}$$

Since y_1 and y_o are independent, their joint density functions are

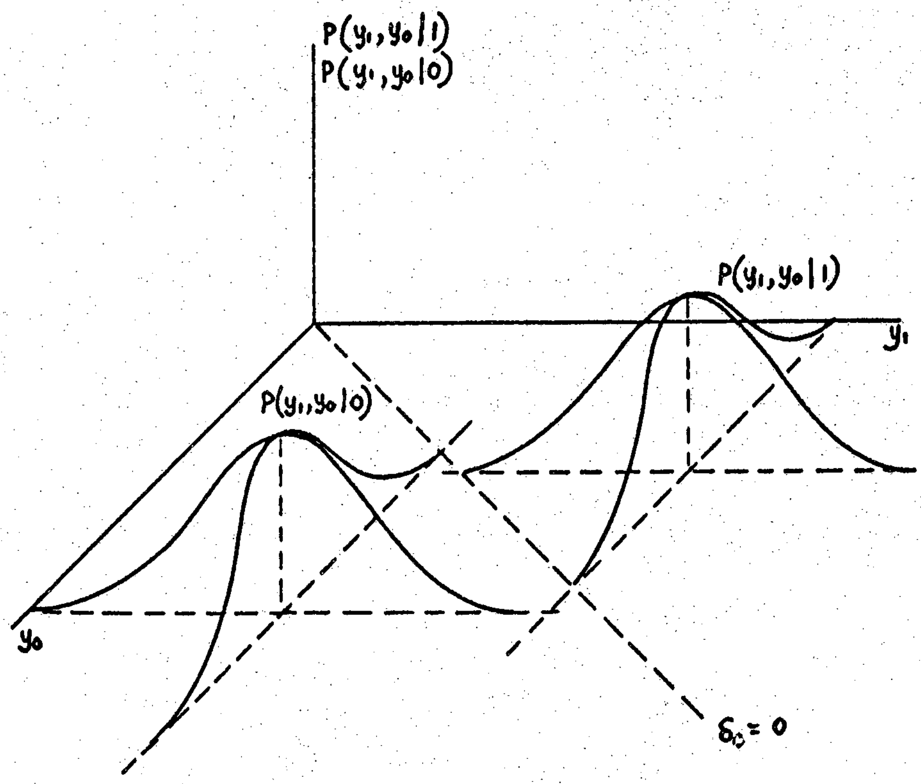
$$p(y_1,y_o|1) = \frac{y_oy_1}{N_o^2E^2} \epsilon^{-\frac{y_o^2+y_1^2+E^2}{2N_oE}} I_o\left[\frac{y_1}{N_o}\right] \tag{4-33}$$

$$p(y_1,y_o|0) = \frac{y_oy_1}{N_o^2E^2} \epsilon^{-\left(\frac{y_o^2+y_1^2+E^2}{2N_oE}\right)} I_o\left[\frac{y_o}{N_o}\right] . \tag{4-34}$$

A sketch of these is shown in Fig. 4.10. The transitional probabilities of error are

$$P(1_r|0_t) = \iint_{\text{mark}} p(y_1,y_o|0)dy_1dy_o \tag{4-35}$$

$$P(0_r|1_t) = \iint_{\text{space}} p(y_o,y_1|1)dy_ody_1 . \tag{4-36}$$



OUTPUT DENSITY FUNCTIONS FOR NON-COHERENT FSK DETECTION

FIGURE 4.10

If $\delta = 0$, then

$$P_e = \int_0^\infty \int_{y_1}^\infty \frac{y_1 y_o}{N_o^2 E^2} \epsilon^{-\frac{y_o^2 + y_1^2 + E^2}{2N_o E}} I_o\left[\frac{y_1}{N_o}\right] dy_o dy_1 \tag{4-37}$$

$$= \int_0^\infty \frac{y_1}{N_o E} \epsilon^{-\frac{2y_1^2 + E^2}{2N_o E}} I_o\left[\frac{y_1}{N_o}\right] dy_1 \tag{4-38}$$

which reduces to[6],

$$P_e = 1/2\ \epsilon^{-\frac{E}{4N_o}} . \tag{4-39}$$

Performance curves for this system are shown in Fig. 4.3 and 4.4. The performance of this system falls 3db below the differentially coherent PSK system (see Eq. 4-19 and 4-39). At high E/N_o, this system approaches the coherent ASK and FSK matched filter systems in performance.

Chapter V

The Effects of Improper Threshold Settings

5.1 Introduction

In this chapter, the systems analyzed in Chapter III and IV are examined for sensitivity to threshold variations. These systems operate in an optimum manner only when a proper decision level or threshold is used. What follows is an analysis of what happens when non-optimum thresholds occur. The sensitivity to threshold is shown by means of plots of η/η_{opt} as a function of δ. A threshold sensitivity factor α is defined and the different systems are compared.

5.2 A Threshold Sensitivity Factor

In order to show the effect of improper threshold settings on a binary communication system, the information efficiency for the system with various decision levels is compared with the information efficiency when the optimum threshold is used. The results are plots of η/η_{opt} versus δ as shown in Fig. 5.1 through 5.8. From these plots an idea of a given system's sensitivity to threshold variation may be obtained. However, it would be desirable to compare the sensitivities of various systems. The range of δ is not the same for all systems, and even the units of δ may differ (e.g., volts for an ASK system with synchronous detection and radians for a PSK system with synchronous detection). Therefore, different sensitivity curves can not be simply superimposed on one another. One solution is to define a sensitivity factor which describes the sharpness, or lack thereof, of the peaks on the η/η_{opt}

curves in a manner similar to the way Q describes the sensitivity of an RLC circuit.

A factor which does this is α, which is defined as follows,

$$\alpha = \frac{\sigma}{\delta_s} \qquad (5\text{-}1)$$

where

σ = distance between the points on the threshold sensitivity curve where $\eta/\eta_{opt} = 0.95$.

δ_s = the separation between detector outputs (or conversely decision device inputs) for a mark and space if no noise is present.

Thus α is like 1/Q for an RLC circuit in that the broader the peak, the higher α will be and the less sensitive the system is to threshold variation.

In order to make comparisons meaningful, identical input conditions are assumed for each class of system. Although there are many possible choices, the assumptions made for conventional systems analyzed on a signal-to-noise ratio are that the input noise power is two watts on a one ohm basis (i.e., $N_oW = 1$)* and that the input energy-to-noise ratio is 10db. These assumptions will be used in the remainder of this chapter.

---

*Although these assumptions are somewhat large for practical systems, the relative merits of each system should not change with the use of smaller values for N_o and N_oW.

5.3 ASK Systems

This category consists of the four ASK systems analyzed in sections 3.2 and 4.2. They are analyzed with regard to threshold sensitivity as follows.

I Linear Envelope Detection

For this system, the transitional probabilities of error as given in Chapter III and modified by the assumptions stated in section 5.2 are,

$$\mu = P(1_r|0_t) = \int_{\delta} \frac{y}{2} \epsilon^{-\left(\frac{y^2}{4}\right)} dy \tag{5-2}$$

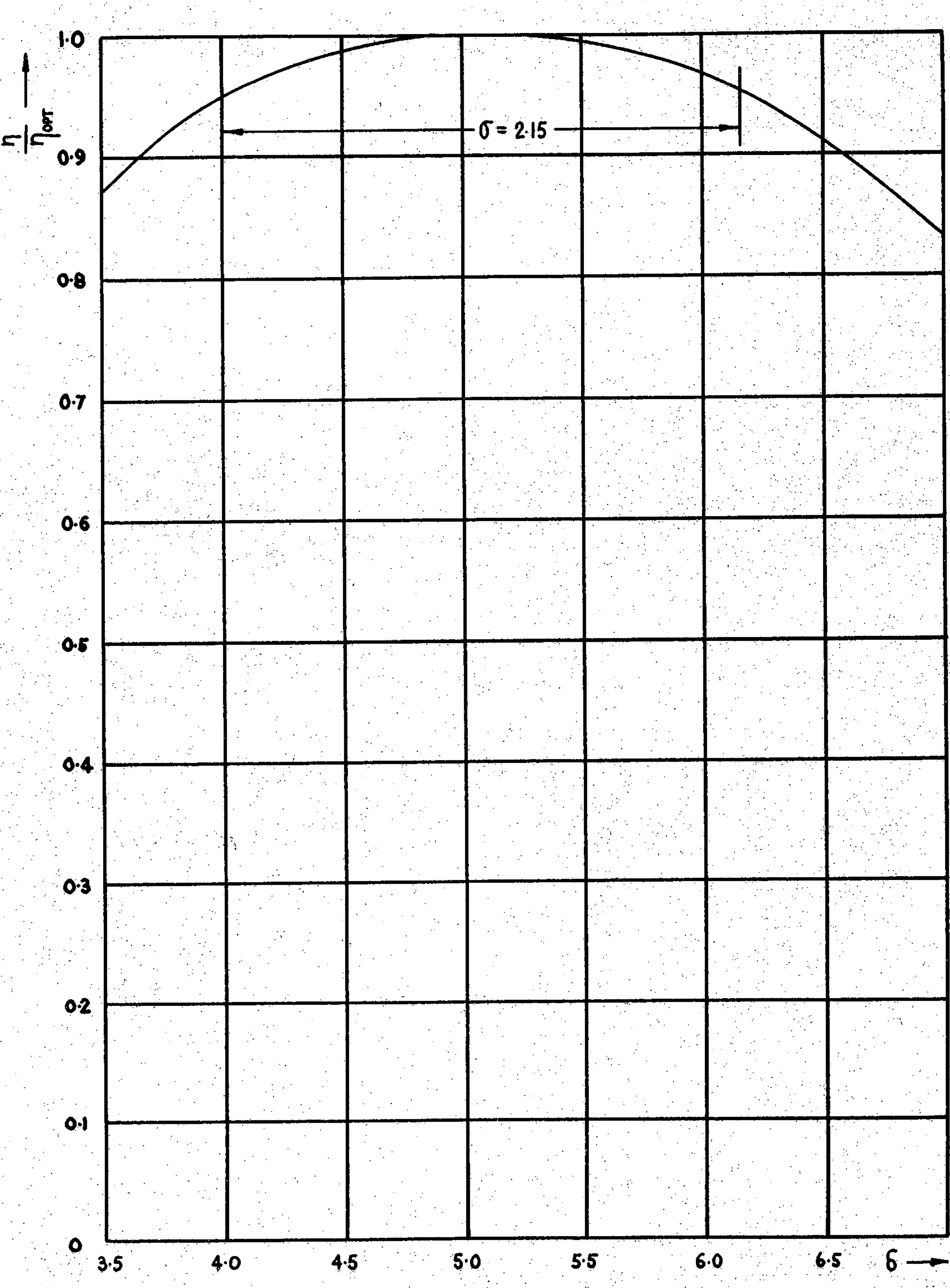
$$\nu = p(0_r|1_t) = \int_0^{\delta} \frac{y}{2} \epsilon^{-\left(\frac{y^2}{4} + 20\right)} I_o\left(y\sqrt{20}\right) dy\ . \tag{5-3}$$

The right part of the first equation can be integrated and μ reduces to

$$\mu = \epsilon^{-\frac{\delta^2}{4}}\ . \tag{5-4}$$

The optimum value of δ occurs when the transitional probabilities of error are equal and is 5.01. If δ is varied from 3.5 to 7.0, the threshold sensitivity curve shown in Fig. 5.1 is obtained.

If a space is transmitted and no noise is present, the detector output will be 0. If a mark is transmitted under the same circumstances, the detector output will be 8.87 (note: for an ASK system S is the average power at the detector input and one-half of the input power when a mark is transmitted). Therefore $\delta_s = 8.87$, and from Fig. 5.1, $\sigma = 2.15$ so that $\alpha = 0.243$ for this system at the specified operating point.



THRESHOLD SENSITIVITY FOR AN ASK SYSTEM WITH ENVELOPE DETECTION

FIGURE 5.1

Another method of evaluating threshold sensitivity of a system which should be mentioned is the derivative of η with respect to δ. For the general binary channel,

$$\frac{d\eta}{d\delta} = -50 \left(\frac{d\mu}{d\delta}\right) \log_2\left(1 + \frac{1-\mu-\nu}{\mu(1+\mu+\nu)}\right) + \left(\frac{d\nu}{d\delta}\right)\log_2\left(1 + \frac{1-\mu-\nu}{\nu(1+\mu-\nu)}\right). \tag{5-5}$$

For an ASK system with envelope detection as discussed above, this becomes

$$\frac{d\eta}{d\delta} = -25\delta\epsilon^{-\frac{\delta^2}{4}}\left[\log_2\left(\frac{1+\Theta}{1+\frac{1}{\Phi}}\right) + \epsilon^{-20} I_o\left(\delta\sqrt{20}\right)\log_2\left(\frac{1+\frac{1}{\Theta}}{1+\Phi}\right)\right] \tag{5-6}$$

where

$$\Theta = \frac{\int_0^\delta \frac{y}{2}\,\epsilon^{-\left(\frac{y^2}{4}+20\right)} I_o\left(y\sqrt{20}\right) dy}{1-\epsilon^{-\frac{\delta^2}{4}}} \tag{5-7}$$

$$\Phi = \frac{\epsilon^{-\frac{\delta^2}{4}}}{1-\int_0^\delta \frac{y}{2}\,\epsilon^{-\left(\frac{y^2}{4}+20\right)} I_o\left(y\sqrt{20}\right) dy}\,. \tag{5-8}$$

As may easily be seen, the above result is rather unwieldy, and since the integrations must be done numerically in either case, it is just as simple to calculate a threshold sensitivity curve and evaluate α which gives a better picture of the situation regarding threshold sensitivity.

II Synchronous Detection

In this case the transitional probabilities of error under the conditions imposed in section 5.2 are from Chapter III,

$$\mu = \frac{1}{\sqrt{\pi}} \int_{\delta}^{\infty} \epsilon^{-y^2} dy \tag{5-9}$$

$$\nu = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\delta} \epsilon^{-(y-\sqrt{20})^2} dy. \tag{5-10}$$

These may also be given in terms of the error function and are

$$\mu = 1/2\,[1-\mathrm{erf}(\delta)], \qquad 0 \leq \delta < \infty \tag{5-11}$$

$$\mu = 1/2\,[1+\mathrm{erf}(-\delta)], \qquad -\infty < \delta < 0 \tag{5-12}$$

$$\nu = 1/2\,[1-\mathrm{erf}(\sqrt{20}-\delta)], \qquad -\infty < \delta \leq \sqrt{20} \tag{5-13}$$

$$\nu = 1/2\,[1+\mathrm{erf}(\delta-\sqrt{20})], \qquad \sqrt{20} < \delta < \infty . \tag{5-14}$$

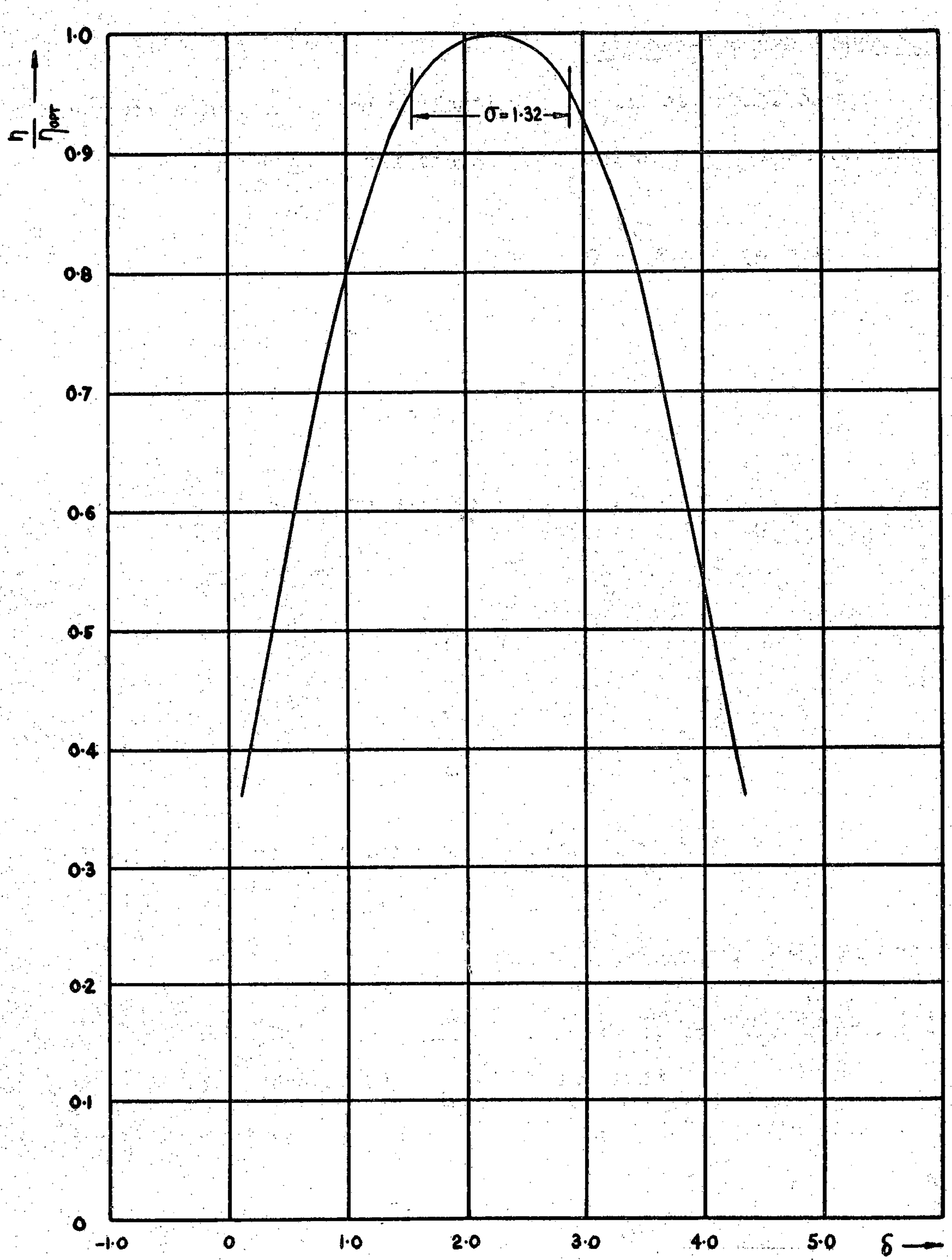
The threshold sensitivity curve for this case is shown in Fig. 5.2. The optimum value of δ is 5, $\sigma = 1.32$, $\delta_s = 20$ and $\alpha = 0.295$ for the specified operating point.

III Matched Filter Coherent Detection

Here the analysis resembles that for synchronous detection. The transitional probabilities of error at the assumed operating point are from section 4.2,

$$\mu = \frac{1}{\sqrt{\pi 40}} \int_{\delta}^{\infty} \epsilon^{-\frac{y^2}{40}} dy \tag{5-15}$$

$$\nu = \frac{1}{\sqrt{\pi 40}} \int_{-\infty}^{\delta} \epsilon^{-\frac{(y-20)^2}{40}} dy \tag{5-16}$$



THRESHOLD SENSITIVITY FOR AN ASK SYSTEM WITH SYNCHRONOUS DETECTION

FIGURE 5.2

which reduce to

$$\mu = 1/2\ [1-\mathrm{erf}(\delta/\sqrt{40})], \qquad 0 \le \delta < \infty \tag{5-17}$$

$$\mu = 1/2\ [1+\mathrm{erf}(-\delta/\sqrt{40})], \qquad -\infty < \delta < 0 \tag{5-18}$$

$$\nu = 1/2\ [1-\mathrm{erf}(\sqrt{10}-\delta/\sqrt{40})], \ -\infty < \delta \le 20 \tag{5-19}$$

$$\nu = 1/2\ [1+\mathrm{erf}(\delta/\sqrt{40} - \sqrt{10})], \ 20 < \delta < \infty . \tag{5-20}$$

The threshold sensitivity curve for this system is shown in Fig. 5.3. The optimum value of δ is 10, $\sigma = 4.6$, $\delta_s = 20$ and $\alpha = 0.230$.

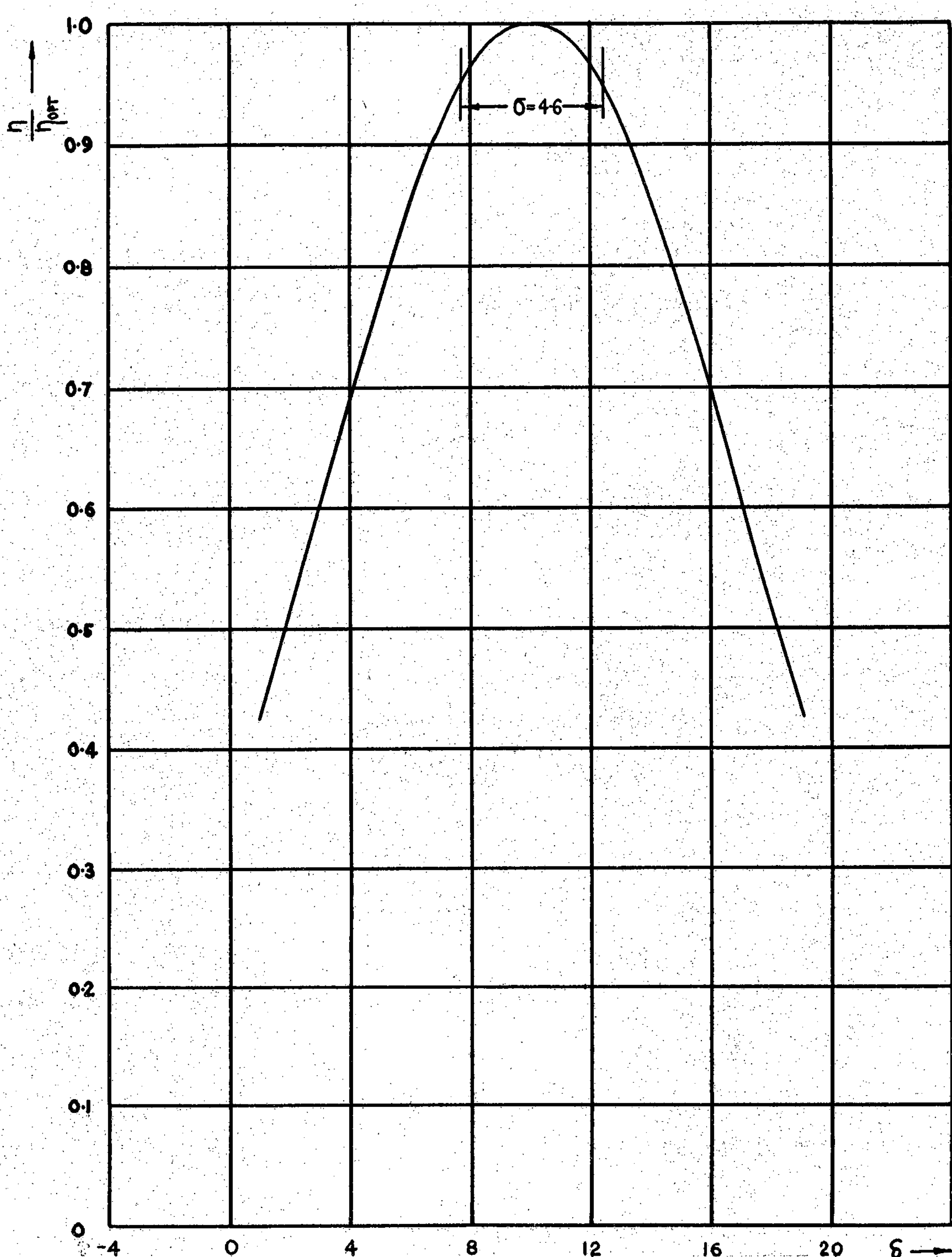
IV Matched Filter Non-Coherent Detection

The analysis in this case is very similar to that of the straight envelope detector above. The transitional probabilities of error for such a system at the operating point specified in section 5.2 are (see Chapter IV),

$$\mu = \int_{\delta}^{\infty} \frac{y}{20}\, \epsilon^{-\left(\frac{y^2}{40}\right)} dy = \epsilon^{-\frac{\delta^2}{40}} \tag{5-21}$$

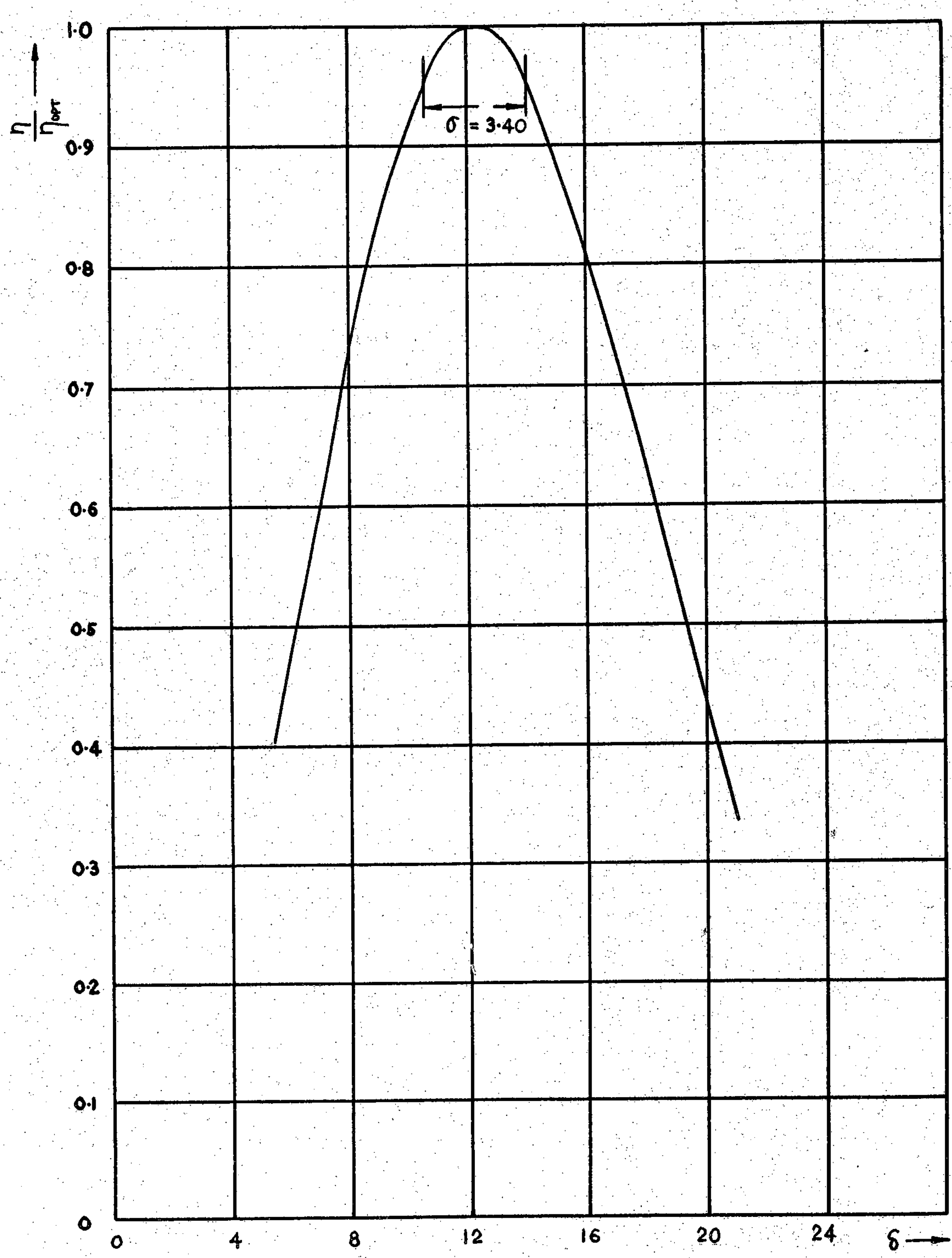
$$\nu = \int_{0}^{\delta} \frac{y}{20}\, \epsilon^{-\left(\frac{y^2}{40} + 10\right)} I_o(y)dy . \tag{5-22}$$

The threshold sensitivity curve for this case is shown in Fig. 5.4. The optimum value of δ is 12.03, $\sigma = 3.4$, $\delta_s = 20$ and $\alpha = 0.170$.



THRESHOLD SENSITIVITY FOR AN ASK SYSTEM WITH COHERENT MATCHED FILTER DETECTION

FIGURE 5.3



THRESHOLD SENSITIVITY FOR AN ASK SYSTEM WITH NON-COHERENT MATCHED FILTER DETECTION

FIGURE 5.4

5.4 PSK Systems

Considered here are the three PSK systems discussed in Chapters III and IV.

I Synchronous Detection

The transitional probabilities of error for this case are given by Eq. 3-17 and 3-18, and are specified in terms of $p(\Phi|1)$ and $p(\Phi|0)$. Since $p(\Phi|0) = p(\Phi+\pi|1)$, the transitional probabilities can both be specified in terms of $p(\Phi|0)$ as follows below.

$$\mu = P(1_r|0_t) = 1 - P(0_r|0_t) \tag{5-23}$$

$$= 1 - \int_{\delta_-}^{\delta_+} p(\Phi|0)d\Phi \tag{5-24}$$

and

$$\nu = P(0|1) = \int_{\delta_-}^{\delta_+} p(\Phi|1)d\Phi \tag{5-25}$$

by letting $\Phi = \pi - \Theta$

$$\nu = -\int_{\pi-\delta_-}^{\pi-\delta_+} p(\pi-\Theta|1)d\Theta \tag{5-26}$$

but

$$p(\pi-\Theta|1) = p(\pi+\Theta|1) = p(\Theta|0) \tag{5-27}$$

since the density function is an even function of Θ, and ν reduces to

$$\nu = \int_{\pi-\delta_+}^{\pi-\delta_-} p(\Theta|0)d\Theta \; . \tag{5-28}$$

If the assumption that $\delta_+ = -\delta_- = \delta$ is made, then

$$\mu = 1 - 2\int_0^{\delta} p(\Phi|0)d\Phi \tag{5-29}$$

$$\nu = 2\int_{\nu-\delta}^{\pi} p(\Phi|0)d\Phi \ . \tag{5-30}$$

Using the above assumption and those of section 5.2, the transitional probabilities of error become,

$$\mu = 1 - \frac{\delta\epsilon^{-10}}{\pi} + \sqrt{\frac{40}{\pi}}\ \epsilon^{-10}\int_0^{\delta} \cos\Phi\ \epsilon^{10\cos^2\Phi}\ \Phi\,(\sqrt{20}\cos\Phi)d\Phi \tag{5-31}$$

$$\nu = \frac{\delta\epsilon^{-10}}{\pi} + \sqrt{\frac{40}{\pi}}\ \epsilon^{-10}\int_{\pi-\delta}^{\pi} \cos\Phi\ \epsilon^{10\cos^2\Phi}\ \Phi\,(\sqrt{30}\cos\Phi)d\Phi \ . \tag{5-32}$$

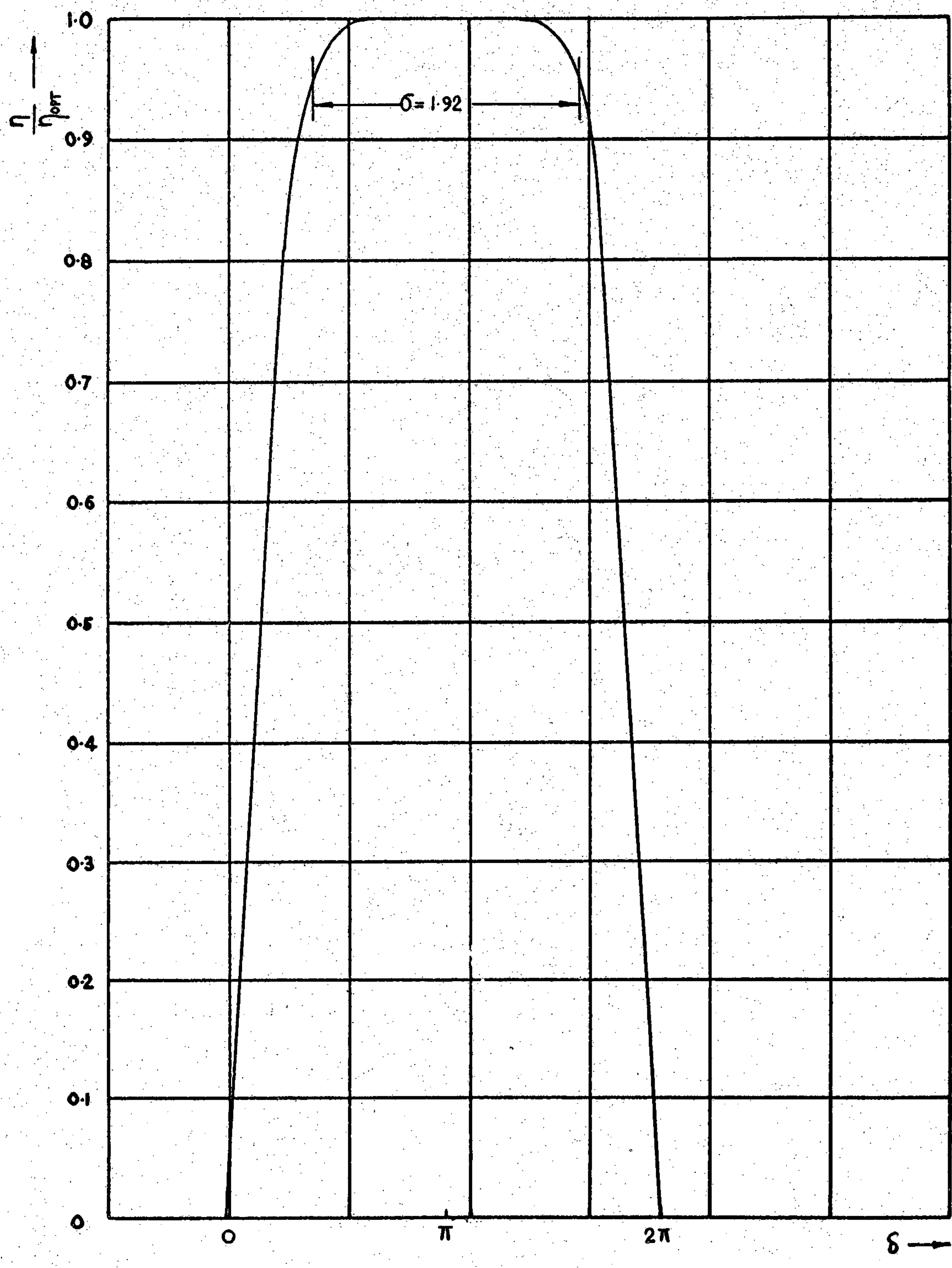
A threshold sensitivity curve for this case is shown in Fig. 5.5, $\delta_{opt} = \pi/2$, $\sigma = 1.92$, $\delta_s = \pi$ and $\alpha = 0.612$.

<u>II Phase Comparison Detection</u>

The situation with a phase comparison detector is similar to the synchronous case discussed above. Accordingly the transitional probabilities of error are

$$\mu = 1 - 2\int_0^{\delta} p_d(\Phi)d\Phi \tag{5-33}$$

$$\nu = 2\int_{\pi-\delta}^{\pi} p_d(\Phi)d\Phi \ . \tag{5-34}$$



THRESHOLD SENSITIVITY FOR A PSK SYSTEM WITH SYNCHRONOUS DETECTION

FIGURE 5.5

As above it is assumed that $\delta_+ = -\delta_- = \delta$. The density function $p_d(\Phi)$ must be obtained by convolving $p(\Phi|0)$ with itself, which has been done by Cahn.[5] Using Cahn's results and numerically integrating, the threshold sensitivity curve shown in Fig. 5.6 is obtained. The optimum threshold occurs when $\delta = \pi/2$, $\sigma = 1.48$, $\delta_s = \pi$, and $\alpha = 9.472$.

III Matched Filter Coherent Detection

This case is similar to the ASK systems using synchronous detection or coherent matched filter techniques. The transitional probabilities of error at the specified operating point are from Chapter IV,

$$\mu = \frac{1}{\sqrt{20\pi}} \int_{\delta}^{\infty} \epsilon^{-\frac{(y+10)^2}{20}} dy \tag{5-35}$$

$$\nu = \frac{1}{\sqrt{20\pi}} \int_{-\infty}^{\delta} \epsilon^{-\frac{(y-10)^2}{20}} dy \tag{5-36}$$

which reduce to

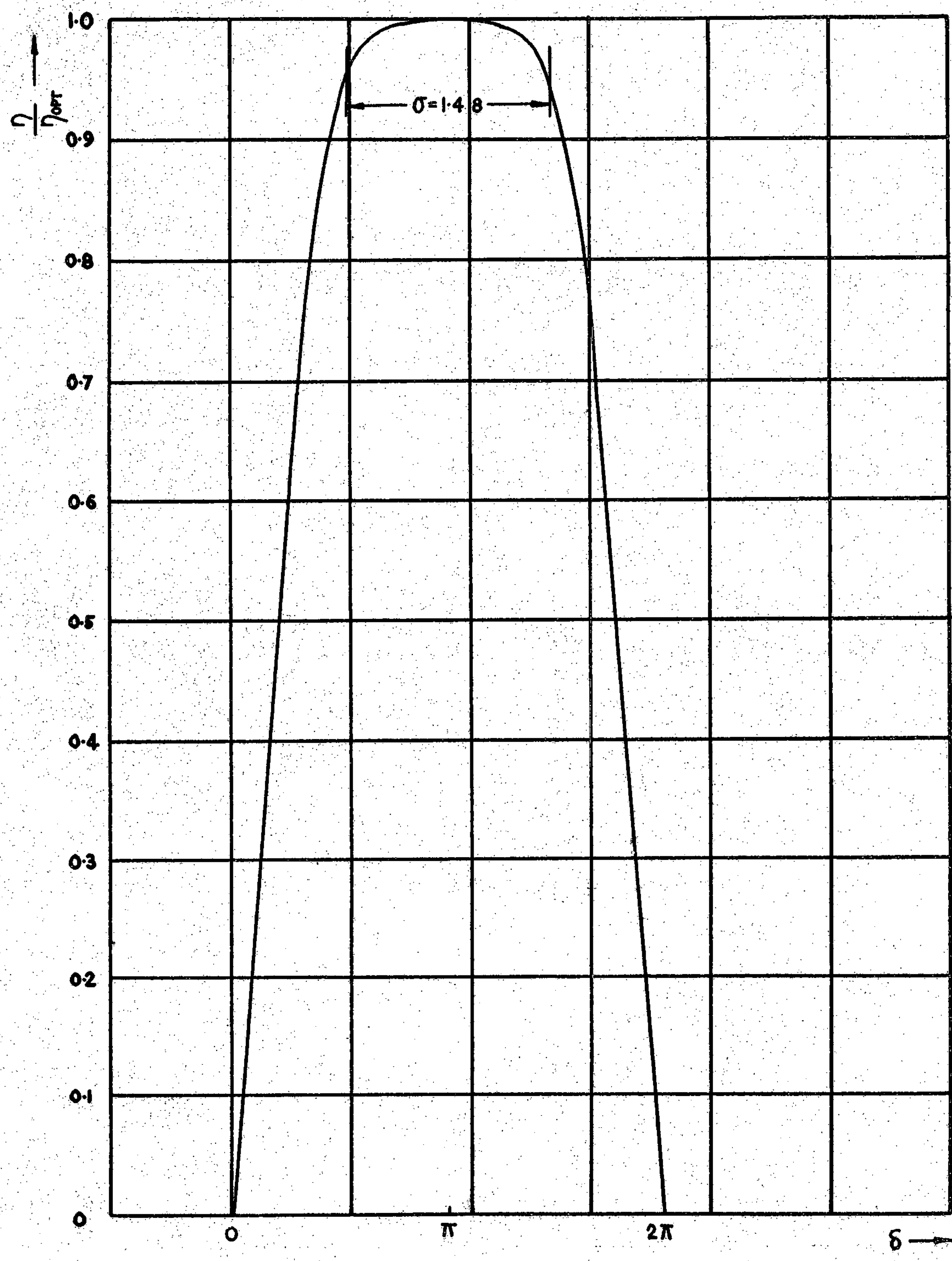
$$\mu = 1/2\ [1-\mathrm{erf}(\delta/\sqrt{20} + \sqrt{5})], \qquad 10 \leq \delta < \infty \tag{5-37}$$

$$\mu = 1/2\ [1+\mathrm{erf}(\sqrt{5} - \delta/\sqrt{20})], \qquad \infty < \delta < 10 \tag{5-38}$$

$$\nu = 1/2\ [1-\mathrm{erf}(\sqrt{5} - \delta/\sqrt{20})], \qquad 10 \leq \delta < \infty \tag{5-39}$$

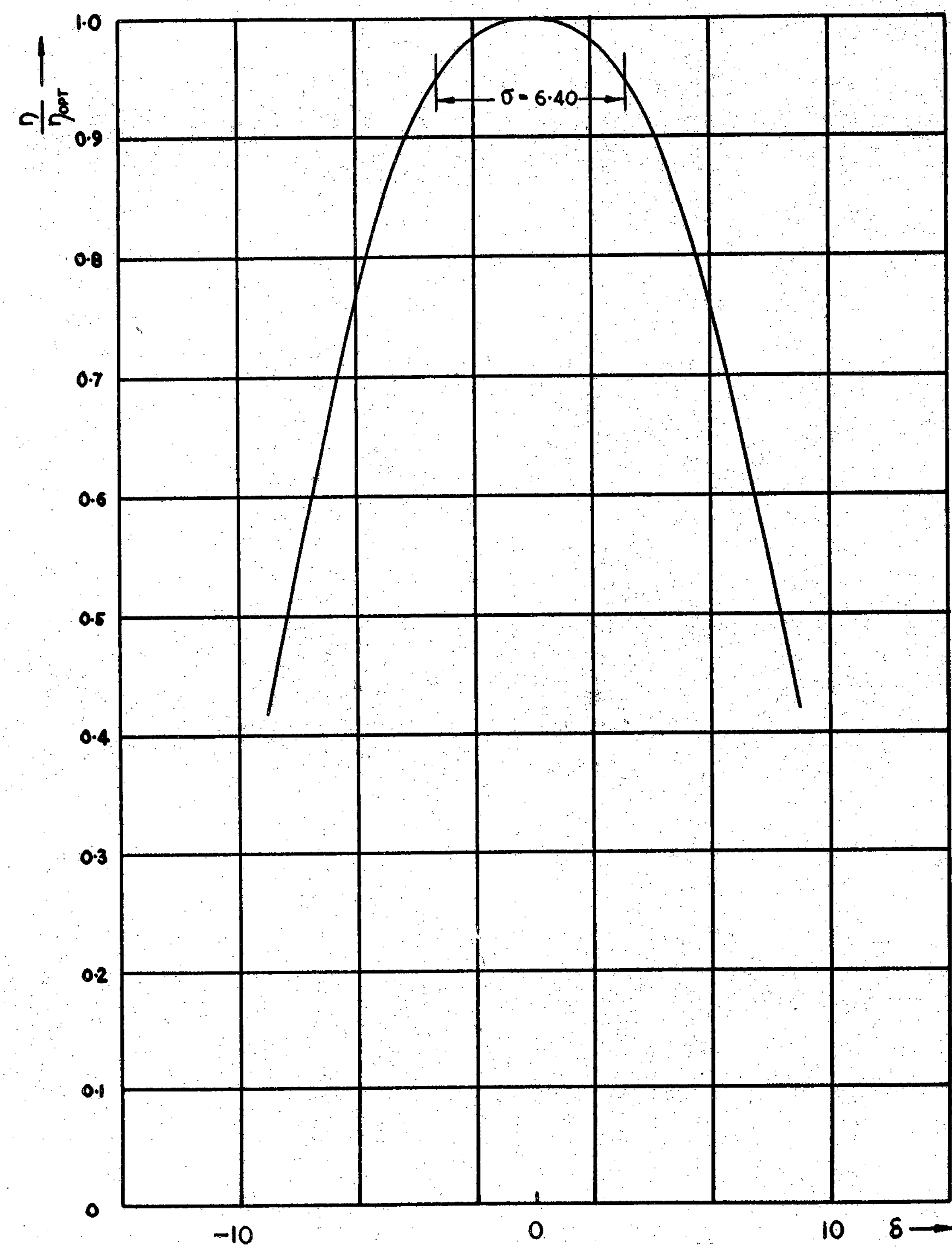
$$\nu = 1/2\ [1+\mathrm{erf}(\delta/\sqrt{20} + \sqrt{5})], \qquad \infty < \delta < 10. \tag{5-40}$$

The threshold sensitivity curve is plotted for δ varying from -10 to +10 in Fig. 5.7. The optimum threshold is zero, $\sigma = 6.4$, $\delta_s = 20$, and $\alpha = 0.320$.



THRESHOLD SENSITIVITY FOR A PSK SYSTEM WITH PHASE COMPARISON DETECTION

FIGURE 5.6



THRESHOLD SENSITIVITY FOR A PSK SYSTEM WITH COHERENT MATCHED FILTER DETECTION

FIGURE 5.7

5.5 FSK Systems

Analyzed here are the two matched filter systems discussed in section 4.3.

I MF Coherent Detection

Here again the detector outputs are gaussian, and from Eq. 4-26 and 4-27,

$$\mu = \frac{1}{\sqrt{40\pi}} \int_{\delta}^{\infty} \epsilon^{-\frac{(y_d+10)^2}{40}} dy_d \tag{5-41}$$

$$\nu = \frac{1}{\sqrt{40\pi}} \int_{-\infty}^{\delta} \epsilon^{-\frac{(y_d-10)^2}{40}} dy_d \tag{5-42}$$

for the specified operating point. These reduce to,

$$\mu = 1/2\ [1\text{-erf}(^{\delta}/2\sqrt{10} + 2.5)], \qquad 10 \leq \delta < \infty \tag{5-43}$$

$$\mu = 1/2\ [1\text{+erf}(2.5\text{-}^{\delta}/2\sqrt{10})], \qquad \infty < \delta < 10 \tag{5-44}$$

$$\nu = 1/2\ [1\text{-erf}(^{\delta}/2\sqrt{10} - 2.5)], \qquad 10 \leq \delta < \infty \tag{5-45}$$

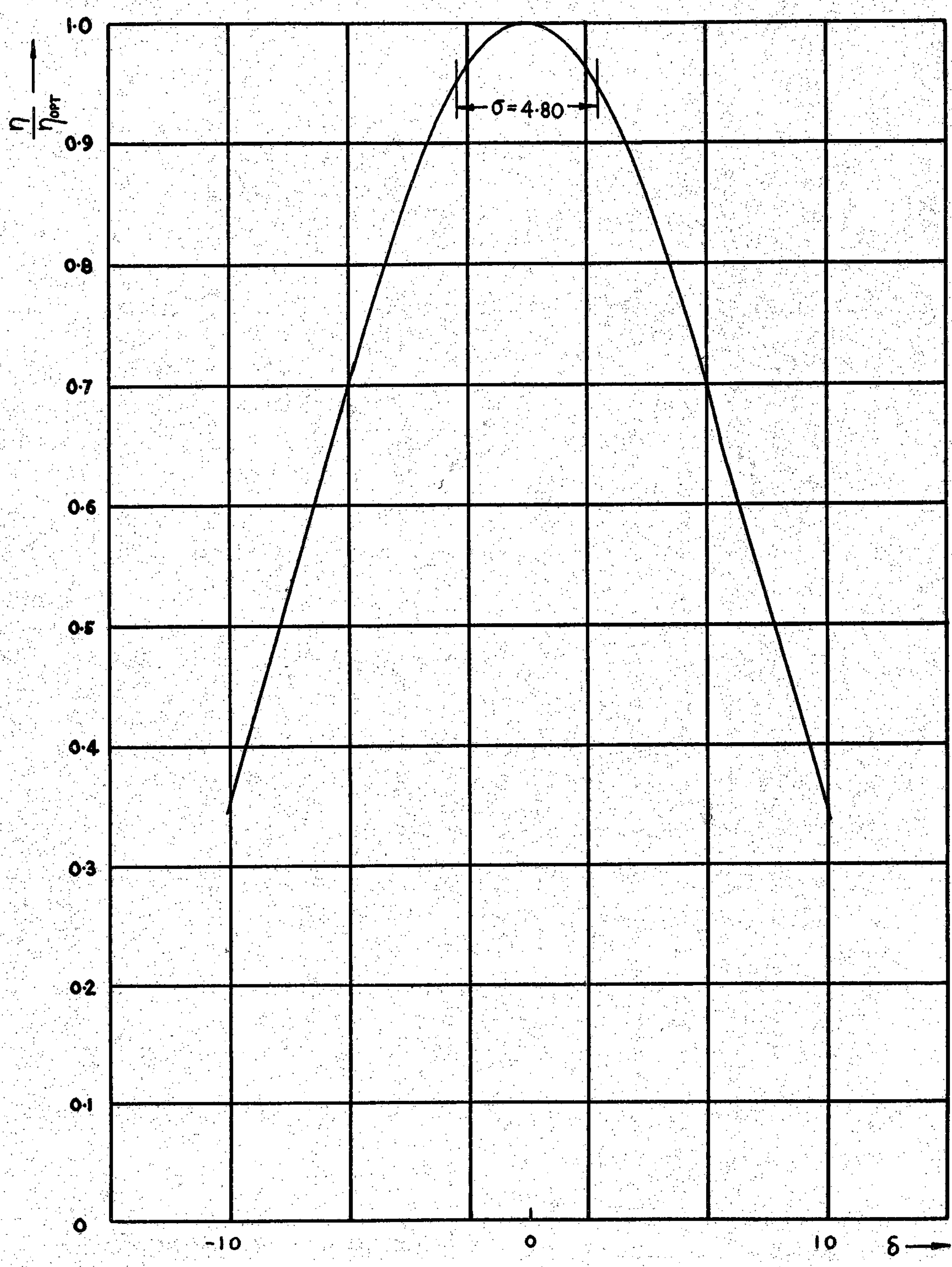
$$\nu = 1/2\ [1\text{+erf}(2.5\text{-}^{\delta}/2\sqrt{10})], \qquad \infty < \delta < 10\ . \tag{5-46}$$

A threshold sensitivity curve is shown in Fig. 5.8. $\delta_{opt} = 0$, $\sigma = 4.8$, $\delta_s = 20$, and $\alpha = 0.240$.

II MF Non-Coherent Detection

In this case, the transitional probabilities of error from Chapter IV and letting $E = 10$ and $N_o = 1$ are

$$\mu = \iint_{mark} \frac{y_o y_1}{100}\, \epsilon^{-\frac{(y_o{}^2+y_1{}^2+100)}{20}} I_o(y_1) dy_o dy_1 \tag{5-47}$$

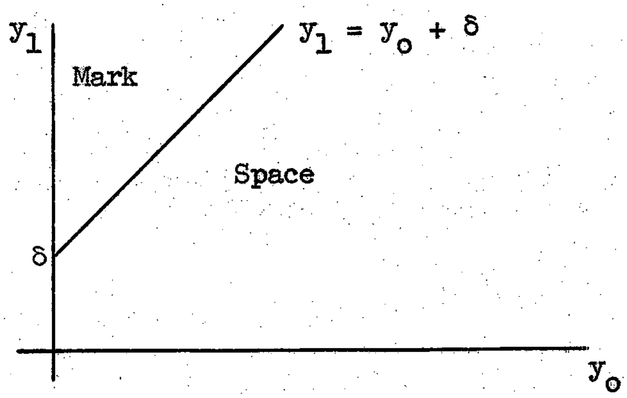


THRESHOLD SENSITIVITY FOR AN FSK SYSTEM WITH COHERENT MATCHED FILTER DETECTION

FIGURE 5.8

$$\nu = \iint\limits_{\text{space}} \frac{y_o y_1}{100} \epsilon^{-\frac{(y_o^2+y_1^2+100)}{20}} I_o(y_1) dy_o dy_1 \; . \qquad (5\text{-}48)$$

In order to relate the above equations to a decision level or threshold, the mark and space regions must be defined, which is done in Fig. 5.9.



MARK AND SPACE REGIONS FOR AN FSK SYSTEM

Figure 5.9

Unfortunately, the integration of Eq. 5-47 and 5-48 does not reduce to single integrals in a simple fashion except for the symmetric case where $\delta = 0$. Thus in order to compute a threshold sensitivity curve, it would be necessary to compute μ and ν numerically, which has not been done in this case.

5.6 A Comparison of Systems

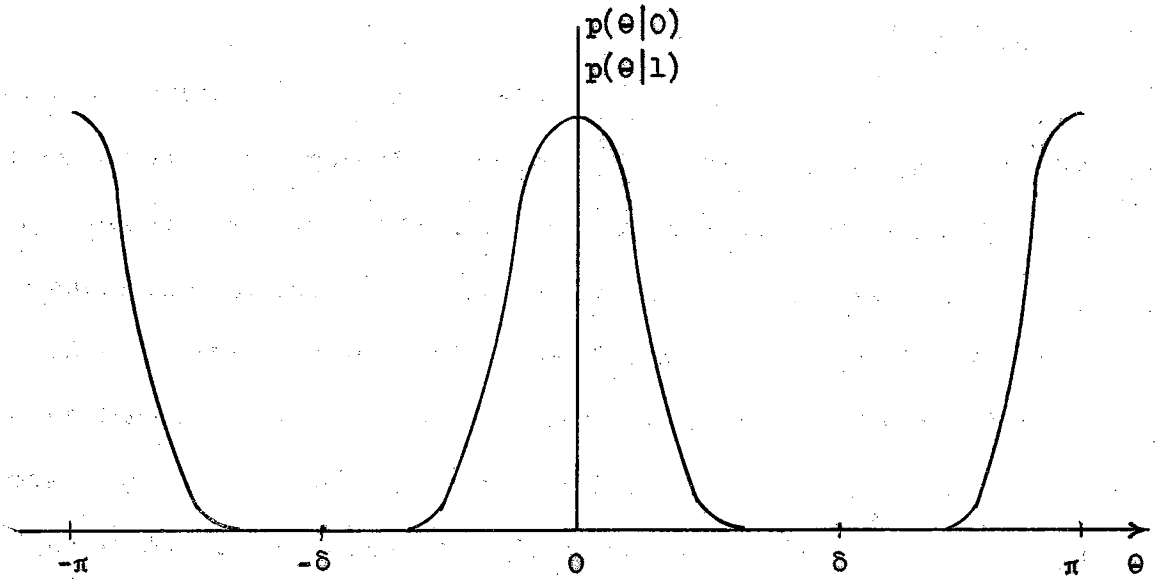
Table 5.1 shows δ_s, σ and α for all of the conventional systems analyzed above. From this table it can be seen that a PSK system employing synchronous detection has by far the highest value of α among all systems considered. From Fig. 3.3 it can be seen that for a signal-to-noise ratio of 10db, the information efficiency for this system

System	δ_s	σ	α
ASK-Linear Envelope Detection	8.87	2.15	0.242
ASK-Synchronous Detection	4.47	1.32	0.295
PSK-Synchronous Detection	3.14	1.92	0.612
PSK-Phase Comparison Detection	3.14	1.48	0.472

A COMPARISON OF THRESHOLD SENSITIVITY
IN CONVENTIONAL SYSTEMS

Table 5.1

approaches 100 per cent. The probability density functions which describe the detector output are a function of signal-to-noise ratio and for S/N = 10db most of the area under them is concentrated about 0 and $\pm\pi$ (see Fig. 5.10).



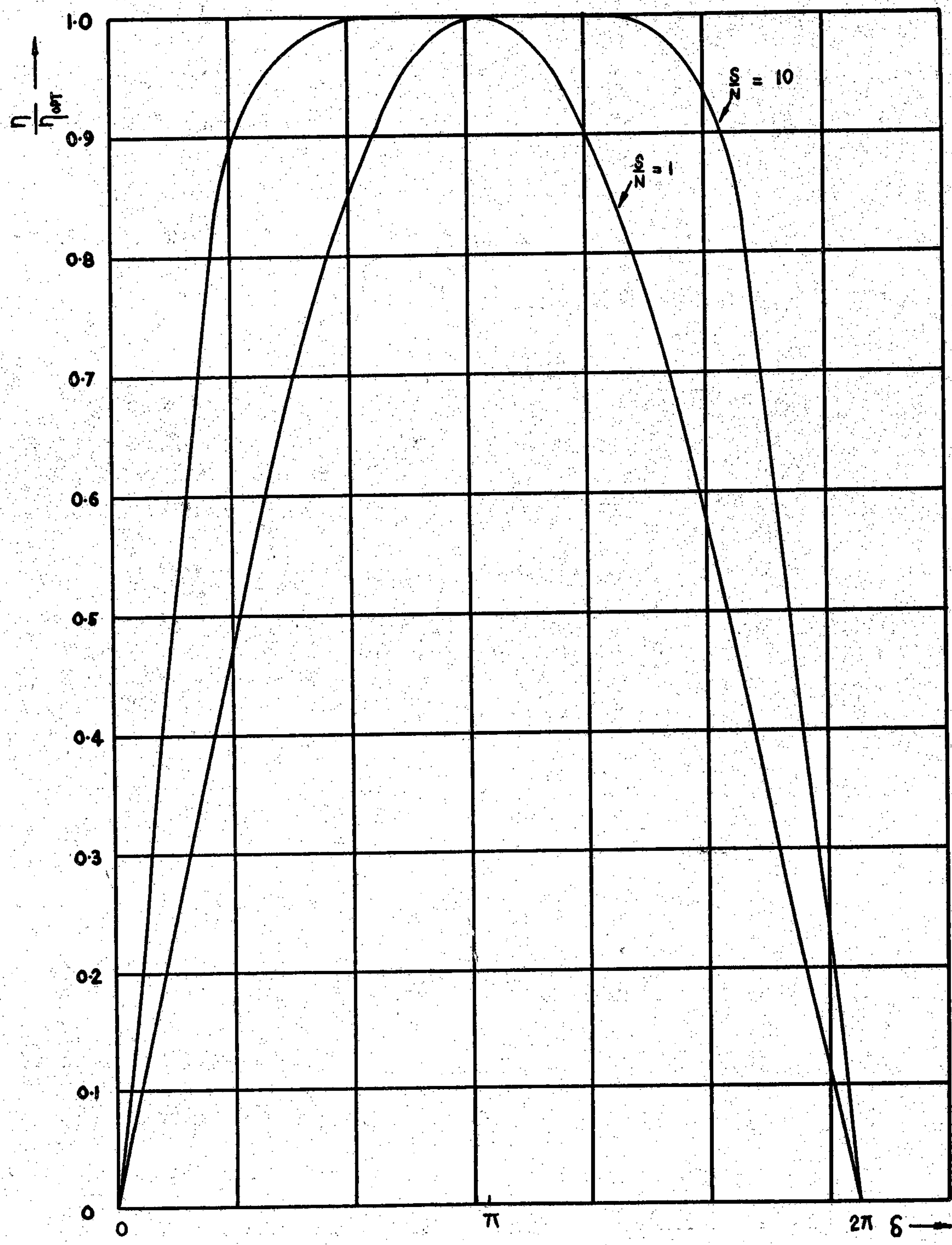
OUTPUT DENSITY FUNCTIONS FOR A PSK SYSTEM

Figure 5.10

Thus rather substantial variations in δ can be tolerated without appreciably affecting the values of μ and ν, and hence η. If a lower signal-to-noise ratio is chosen, the density functions are more like those indicated in Fig. 3.7, and variations in δ are more influential on the values of μ, ν and η. This is shown in Fig. 5.11, where threshold sensitivity curves for a PSK synchronous detection system are shown for signal-to-noise ratios of 0db and 10db. The second curve is the same curve as Fig. 5.5. From this figure it can be seen that the threshold sensitivity factor for such a system is enhanced by operating it at a higher signal-to-noise ratio (e.g., $\alpha = 0.612$ at S/N = 10db versus $\alpha = 0.207$ at S/N = 0db). Note that in this system, $\delta_s = \pi$ regardless of S/N, and therefore α changes only with σ. This is not the case with most other systems where δ is a function of signal strength as discussed below.

The system having the next best threshold sensitivity is a PSK system using phase comparison detection. Although the performance of this system is not quite as good as that of the synchronous detection scheme discussed above, it does have the rather substantial advantage of not requiring a coherent reference signal at the receiver. As in the case above, $\delta_s = \pi$, and is independent of the input signal and noise conditions. Thus for the same reasons stated above, this system's performance in the face of threshold variations will be better as the input signal-to-noise ratio is increased.

The next best system is ASK-synchronous detection which has an α of about one-half that of the PSK-synchronous detection scheme discussed above. It is interesting to compare the threshold sensitivity



THRESHOLD SENSITIVITY AS A FUNCTION OF S/N FOR A PSK SYSTEM WITH SYNCHRONOUS DETECTION

FIGURE 5.11

of this system with different input conditions. Fig. 5.12 shows threshold sensitivity curves for this system with an input noise power of 2 watts in both cases and input signal powers of 2 and 20 watts (on a 1 ohm basis) respectively. In this case, δ_s changes with input signal power and α decreases as the input signal-to-noise ratio increases, and the system becomes more sensitive to threshold variations.

The most sensitive, and hence the least desirable, system (in terms of threshold variations) is an ASK-linear envelope detection scheme.

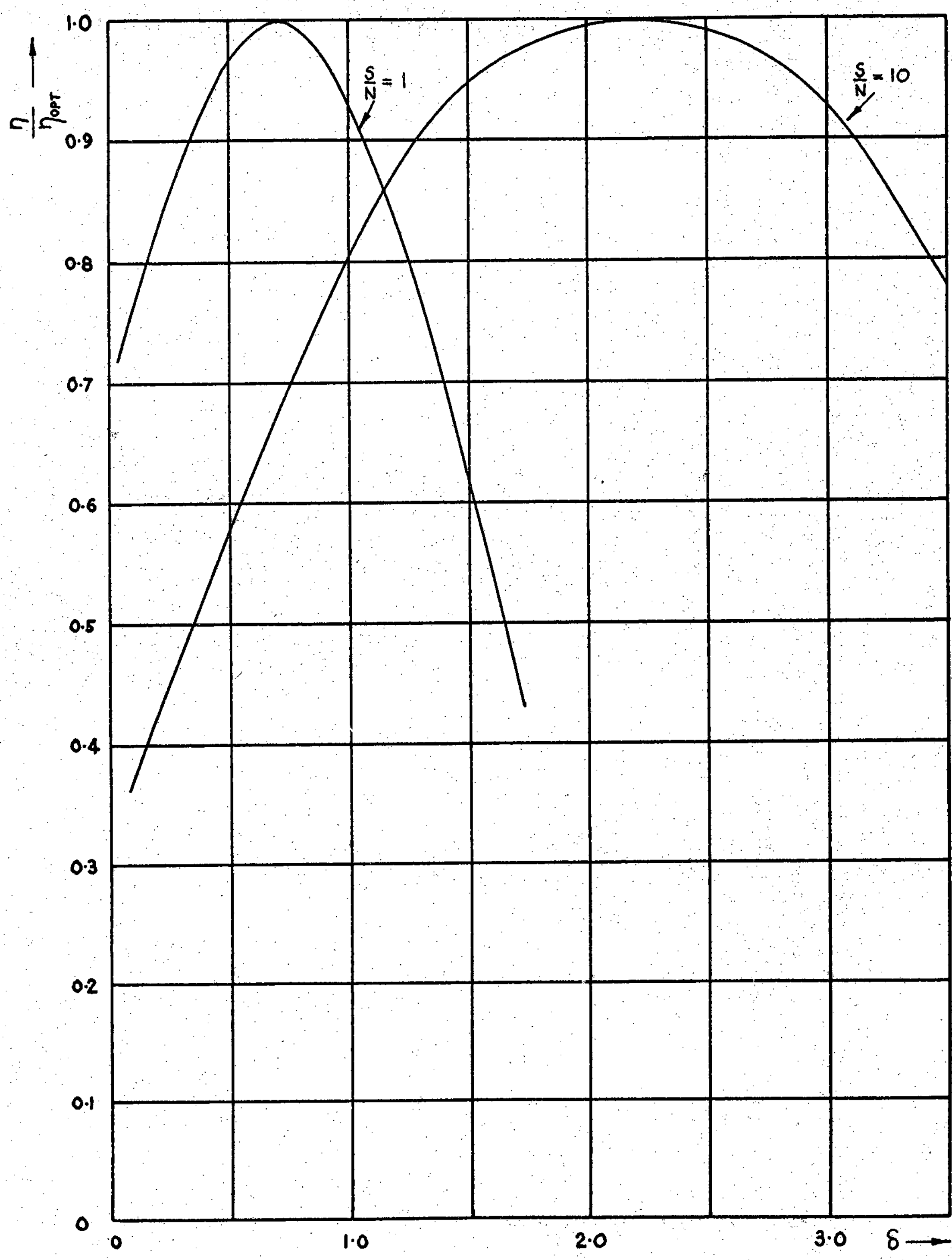
The matched filter system's results are shown in Table 5.2. The

System	δ_s	σ	α
ASK-MF-Coherent Detection	20.00	4.60	0.230
ASK-MF-Non-Coherent Detection	20.00	3.40	0.170
PSK-MF-Coherent Detection	20.00	6.40	0.320
FSK-MF-Coherent Detection	20.00	4.80	0.240

A COMPARISON OF THRESHOLD SENSITIVITY
IN MATCHED FILTER SYSTEMS

Table 5.2

best system in terms of threshold sensitivity is the PSK-MF coherent detection scheme. As in the case of ASK synchronous detection discussed above, δ_s is a function of the input conditions. This will be true, in fact, for all of the matched filter systems considered. The ASK-MF coherent detection scheme and the FSK-MF coherent detection scheme are almost identical in performance, and again the poorest scheme involves linear envelope detection (i.e., ASK-Non-Coherent Detection).



THRESHOLD SENSITIVITY AS A FUNCTION OF S/N FOR AN ASK SYSTEM WITH SYNCHRONOUS DETECTION

FIGURE 5.12

Chapter VI

The Effects of Signal Power Variation and its Relation to Fading

6.1 Introduction

In this chapter, the effects of varying signal strength while maintaining a fixed threshold level are examined. This is especially interesting in the case of ASK systems, since optimum thresholds are a function of signal strength. The results of this analysis and those of Chapters III and IV are then applied to the case of Raleigh Fading. Fading performance is indicated by curves showing the minimum information efficiency which may be expected for a specified percentage of the operating time. The sensitivity of a given system to fading is also examined and a fading performance factor is discussed.

6.2 The ASK System with a Fixed Threshold

Since the decision threshold in an ASK system is a function of both input signal and noise power, a detector with a fixed threshold will be optimum at only one point, while all of the PSK and FSK systems that have been analyzed have thresholds which are independent of input signal and noise conditions. For this reason it is interesting to examine the class of ASK systems with fixed thresholds. What follows is an analysis of sub-optimum operation of the four ASK detectors that were discussed in Chapters III and IV. In order to reduce the number of variables, it has been assumed that $N_oW = 1$ and hence, $S/N = S/2$ for the conventional systems and that $N_o = 1$ for the matched filter systems. Although these values are not typical of those encountered in physical systems, the relative merits of the various systems will remain the same for other system parameters.

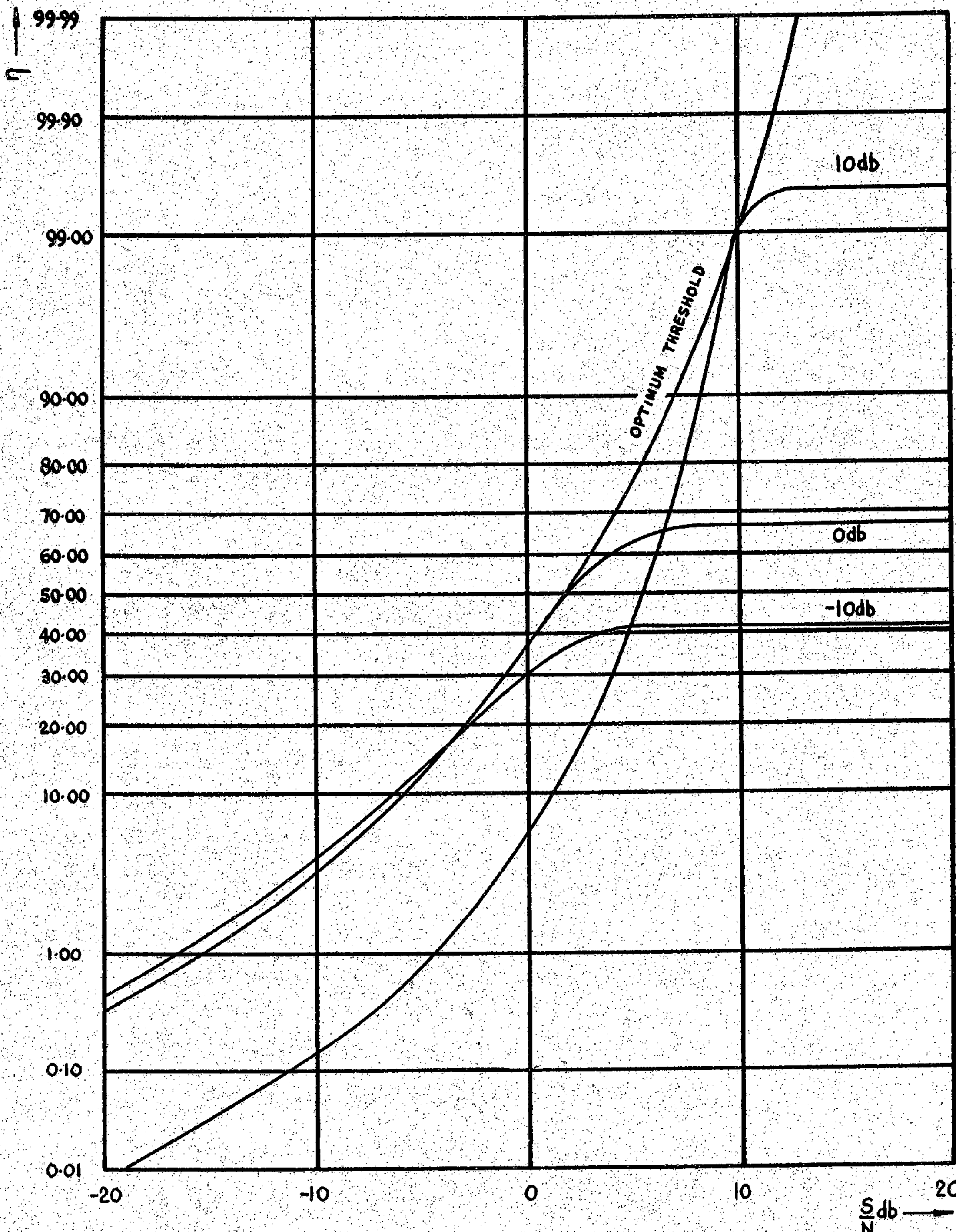
I Synchronous Detection

In the case where synchronous detection is used, the transitional probabilities of error (if $N_o W = 1$) are from Chapter III,

$$\mu = \frac{1}{\sqrt{\pi}} \int_{\delta}^{\infty} \epsilon^{-y^2} dy \tag{6-1}$$

$$\nu = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\delta} \epsilon^{-[y-\sqrt{2(S/N)}]^2} dy . \tag{6-2}$$

In order for the system to operate in an optimum manner, the decision level must be set at $\delta = \sqrt{S}/2$. If the input signal-to-noise ratio is 0db, then $\delta_{opt} = 0.707$. If the input signal-to-noise ratio is now varied from -20db to +20db with the threshold fixed, then the information efficiency varies as is shown in Fig. 6.1. It can be seen that the maximum information efficiency which may be obtained under these circumstances is about 68 per cent. The reason for this is that once the value of δ and $N_o W$ are fixed, the probability of a transmitted space being received as a mark becomes a constant regardless of the input signal-to-noise ratio. From Fig. 2.5 it can be seen that if μ is held constant, the maximum obtainable information efficiency occurs when $\nu = 0$ or 1 and may be considerably less than 100 per cent. This is in distinct contrast to the behavior of the symmetric systems of Chapters III and IV where there is no limit (under 100 per cent) to the attainable information efficiency, given a sufficiently high signal-to-noise ratio.



INFORMATION EFFICIENCY OF A FIXED THRESHOLD ASK SYSTEM WITH SYNCHRONOUS DETECTION

FIGURE 6.1

This same system has been analyzed with the threshold set for optimum operation at -10db and +10db and the results are shown in Fig. 6.1. From this figure it can be seen that for a fixed threshold, the higher the signal-to-noise ratio for which δ yields optimum operation, the higher the information efficiency, which can be obtained from the system, will become. At the same time, the performance below optimum signal-to-noise ratio becomes poorer. For example, if the threshold is set for optimum operation at S/N = 10db (i.e., δ = 5), then an information efficiency of about 99.5 per cent can be obtained for signal-to-noise ratios greater than or equal to +12db, but the performance of the fixed threshold system is about 12db below that of an optimum system at η = 1 per cent. If on the other hand the system is set for optimum operation at S/N = 0db, then an information efficiency of only about 68 per cent can be obtained, but the fixed threshold system performance is only about 1db below that of an optimum system at η = 1 per cent. If δ is optimum for S/N = -10db, the maximum information efficiency only approaches 42 per cent but the system performance is virtually the same as that of the optimum system below -10db.

From the above it is clear that if an ASK synchronous detector is to be constrained to fixed threshold operation, choosing the most suitable value for δ involves a distinct trade-off between maximum attainable information efficiency at high signal-to-noise ratios and non-optimum performance at lower signal-to-noise ratios. That is, if a fixed threshold detector is to be used over a wide range of signal strengths, one must choose between good performance at high signal-to-noise ratios with its correspondingly poor performance at low signal levels, and the opposite case where low level performance is nearly optimum but high level performance is severely limited.

II Linear Envelope Detection

In this case, the transitional probabilities of error are from Chapter III,

$$\mu = \epsilon^{-\delta^2/4} \tag{6-3}$$

$$\nu = \int_0^{\delta} \frac{y}{2} \epsilon^{-\left(\frac{y^2}{4} + S\right)} I_o(y\sqrt{S})dy \,. \tag{6-4}$$

For optimum operation of this system at -10db, 0db, and +10db, the values of δ are 1.21, 1.44, and 2.68. For each of these values of δ, η as a function of signal-to-noise is plotted in Fig. 6.2. The general pattern of operation is the same as for the synchronous detector and the same comments as above apply.

III Matched Filter Coherent Detection

From Chapter IV the transitional probabilities of error for this system are,*

$$\mu = \frac{1}{\sqrt{2\pi}} \int_{\delta}^{\infty} \epsilon^{-y^2/2} \, dy \tag{6-5}$$

$$\nu = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\delta} \epsilon^{-\frac{(y-2E)^2}{2}} dy \,. \tag{6-6}$$

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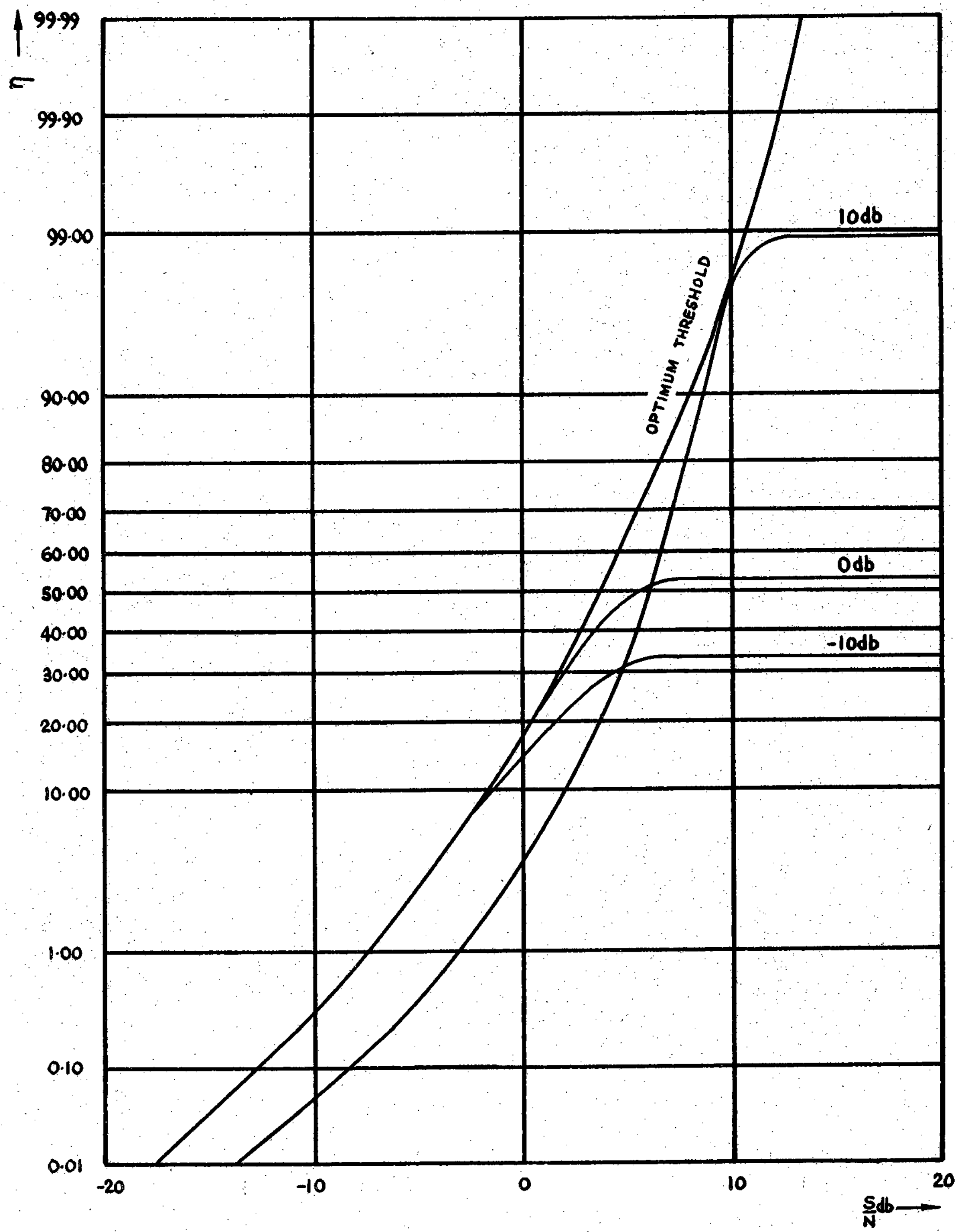
*Note: in this case and the one which follows it, it is assumed that the filters are not truly matched in the sense of both a time structure and amplitude. Here a time structure match only is assumed.



INFORMATION EFFICIENCY OF A FIXED THRESHOLD ASK SYSTEM WITH LINEAR ENVELOPE DETECTION

FIGURE 6.2

The optimum values of δ for E/N_o = -10db, 0db, and +10db are 0.05, 0.5, and 5 respectively. η as a function of E/N_o is plotted for each of these values in Fig. 6.3. Again the general pattern of operation is similar to the first case above.

IV Matched Filter Non-Coherent Detection

In this case μ and ν are from Chapter IV,

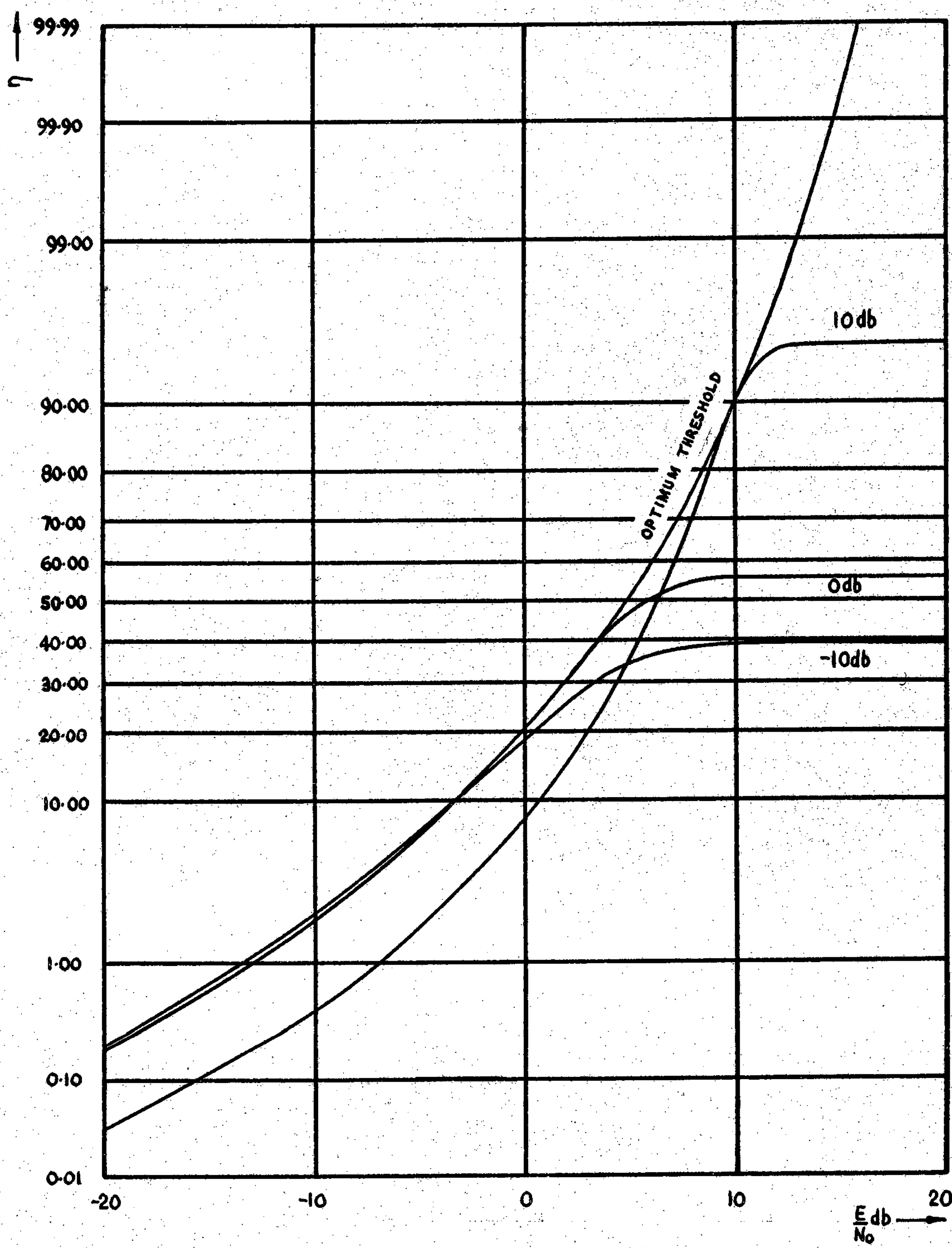
$$\mu = \epsilon^{-\delta^2/2} \tag{6-7}$$

$$\nu = \int_0^\delta y\,\epsilon^{-\left(\frac{y^2}{2} + E\right)} I(y\sqrt{2E})dy\ . \tag{6-8}$$

The values for δ_{opt} at E/N_o = -10db, 0db, and +10db are 1.75, 2.32, and 5.00. The performance curves for these values are shown in Fig. 6.4. The comments of case I above again apply.

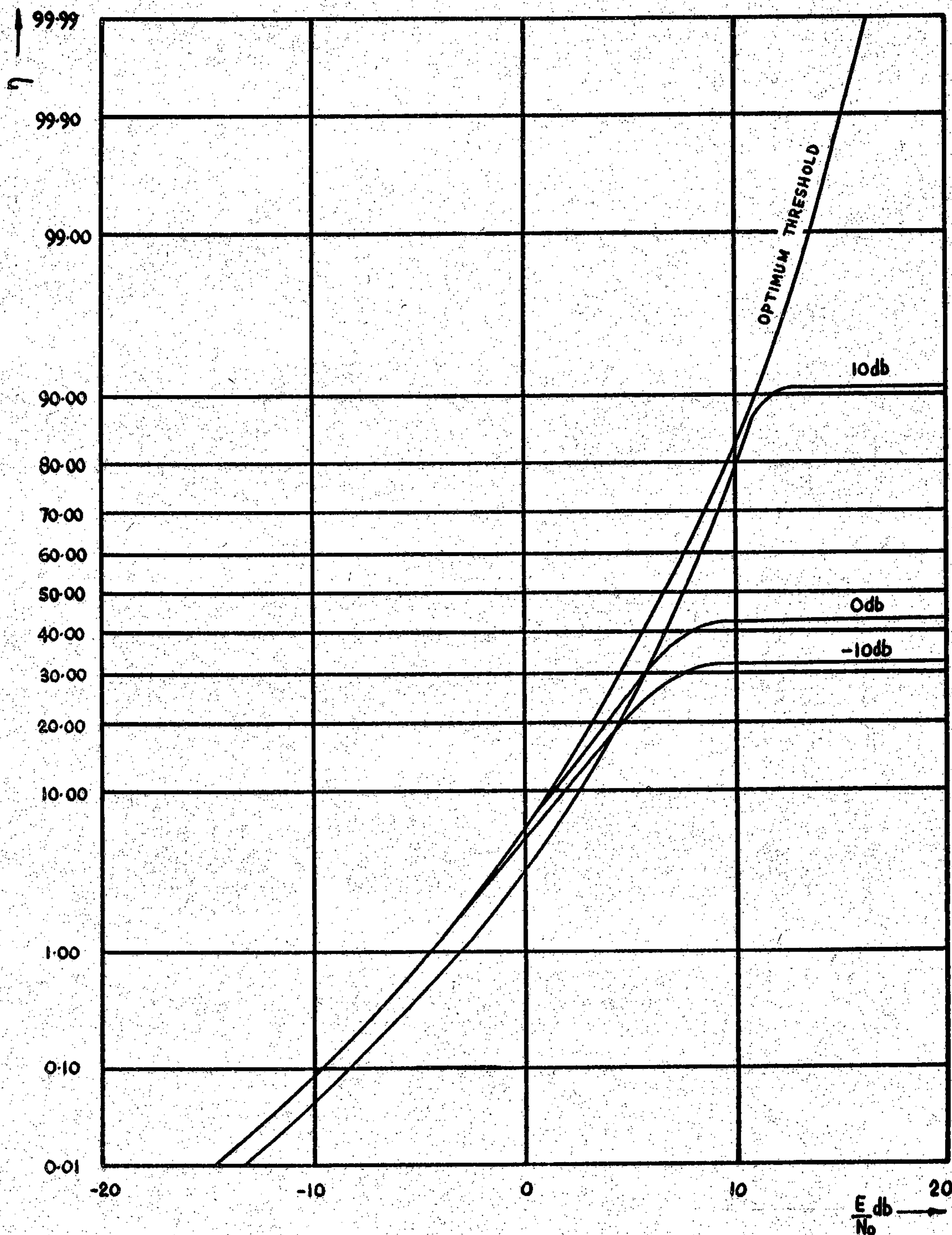
6.3 Fading

In the previous section, several ASK systems with fixed threshold and variable input signal strengths are analyzed. The question which naturally follows is why examine such a case in the first place. The primary reason is the problem of fading. Due to the effects of multipath, the signal strength at the receiver will be a random variable which essentially fades with time. A great deal of work has been done in this area[2, 13] and it is not intended here to analyze the causes of fading. Instead, a relatively simple fading model will be assumed and performance of the various systems, which have already been considered, will be examined in the light of this fading model.



INFORMATION EFFICIENCY OF A FIXED THRESHOLD ASK SYSTEM WITH COHERENT MATCHED FILTER DETECTION

FIGURE 6.3



INFORMATION EFFICIENCY OF A FIXED THRESHOLD ASK SYSTEM WITH NON-COHERENT MATCHED FILTER DETECTION

FIGURE 6.4

The fading model which will be used here is that receiver input signal strength is a random variable, characterized by a Rayleigh probability density function.[2]

The approach to fading which is used below consists of plotting the minimum information efficiency which can be expected from a system where fading is present versus the percentage of the time for which this minimum value holds, given the average signal strength. Normalized plots of η_{min}/η are also computed and show a system's sensitivity to fading.

If the average signal power at the receiver input is S, then $x = \sqrt{S}$ is the signal strength which is Rayleigh distributed and,

$$p(x) = \frac{x}{\sigma^2} \epsilon^{-\frac{x^2}{2\sigma^2}} \quad , \qquad x \geq 0 \tag{6-9}$$

$$p(x) = 0 \quad , \qquad x < 0$$

where

σ^2 = variance of x.

Transforming Eq. 6-9 by letting $S = x^2$,

$$p(S) = \frac{1}{\bar{S}} \epsilon^{-S/\bar{S}} \quad , \qquad S \geq 0 \tag{6-10}$$

$$p(S) = 0 \quad , \qquad S < 0$$

where

$\bar{S}$ = the average signal power over a long period of time.*

*Note that S is average signal power but may be computed by simply averaging the signal power when a mark is transmitted with the signal power when a space is transmitted. $\bar{S}$ is the average of S where variations in S occur due to the presence of fading in the channel, and is an average taken over a period of time greatly exceeding two baud lengths.

Thus if signal strength is considered to be Rayleigh distributed, then the average signal power is exponentially distributed as shown by Eq. 6-10.

The percentage of the time (X) when S exceeds a given value $\mathcal{S}$ is given by

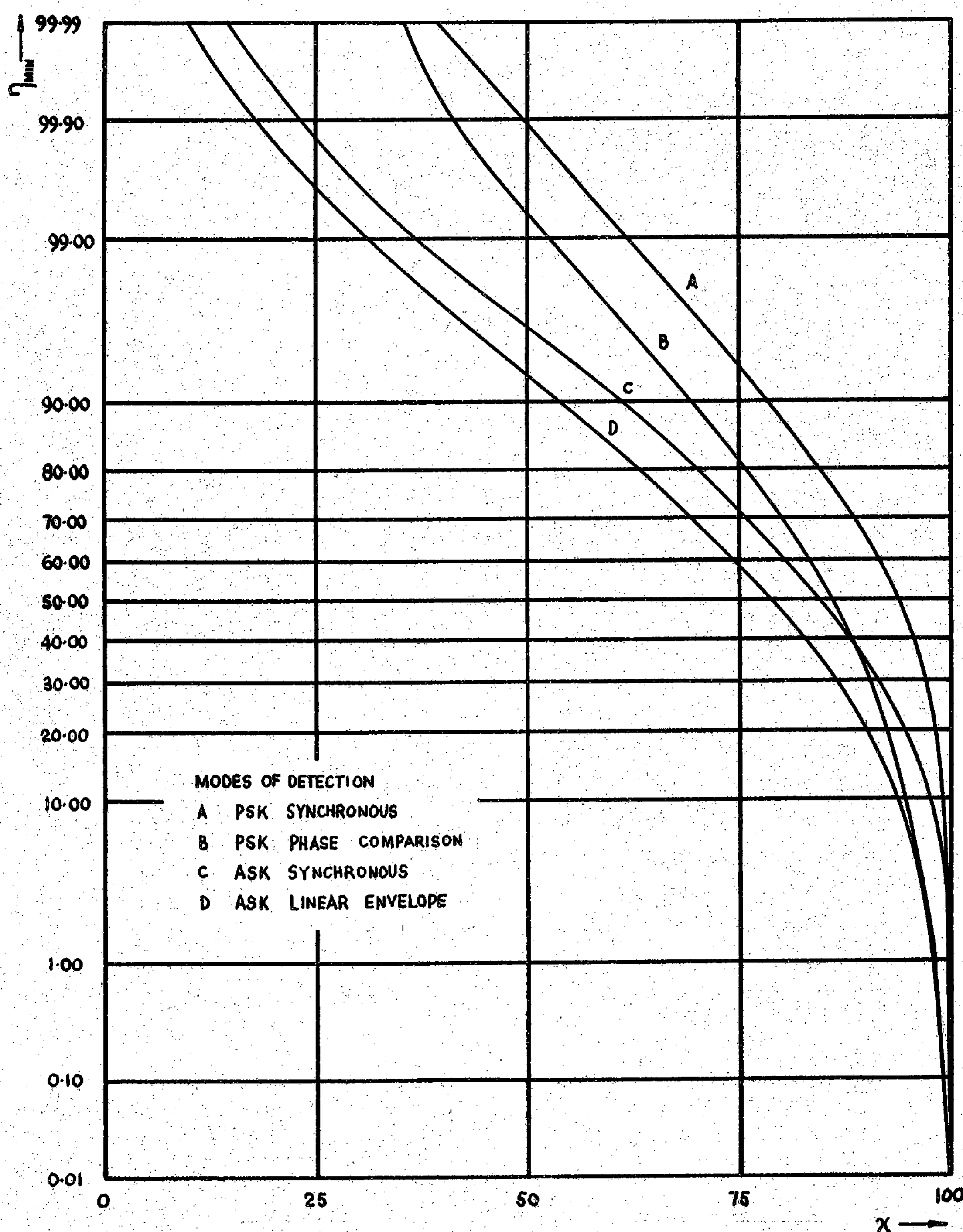
$$X = \int_{\mathcal{S}}^{\infty} p(S)dS \tag{6-11}$$

which reduces to

$$X = \epsilon^{-\mathcal{S}/\overline{S}} . \tag{6-12}$$

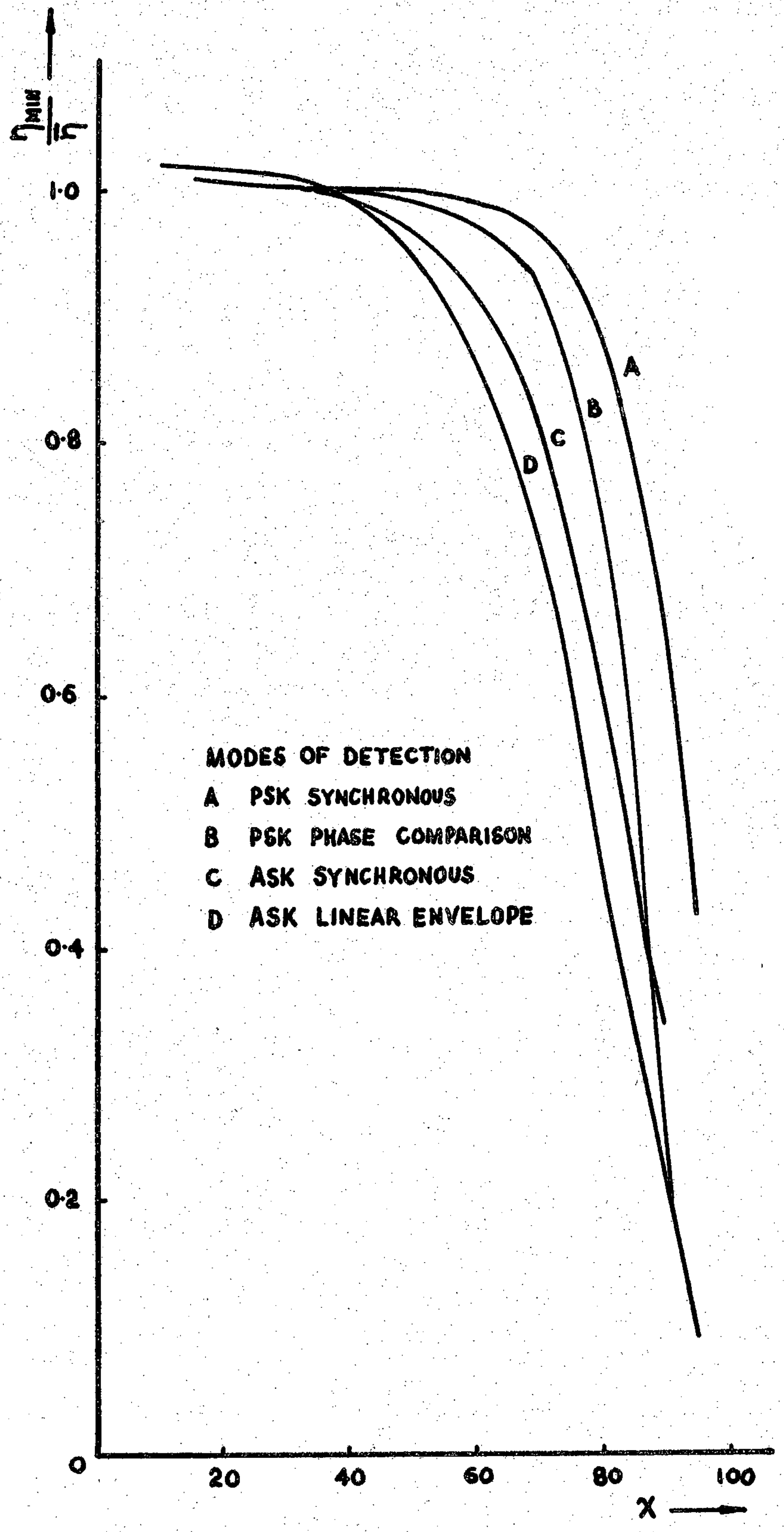
Thus for a given value of $\mathcal{S}$ and $\overline{S}$, a minimum value of information efficiency (η_{min}) can be determined since in all of the cases which have been discussed η is a monotonically increasing function of signal-to-noise ratio. Therefore plots of η_{min} as a function of X can be made. In order to reduce the number of variables present, and make comparisons on a common basis, it has been assumed that $N_oW = 1$ and that $\overline{S}/2 = +10$db. Fig. 6.5 shows plots of η_{min} as a function of X for the systems discussed in Chapter III. Fig. 6.6 shows $\eta_{min}/\overline{\eta}$ as a function of X for these same systems. Fig. 6.7 and 6.8 show fading performance curves for a fixed threshold ASK system with synchronous detection, and linear envelope detection respectively.

In the case of matched filter systems, signal strength is proportional to $\sqrt{E}$ and a similar analysis can be carried out. It is assumed here that $N_o = 1$ and that $\overline{E} = +10$db. Fig. 6.9 and 6.10 show fading performance curves and fading sensitivity curves for the systems of



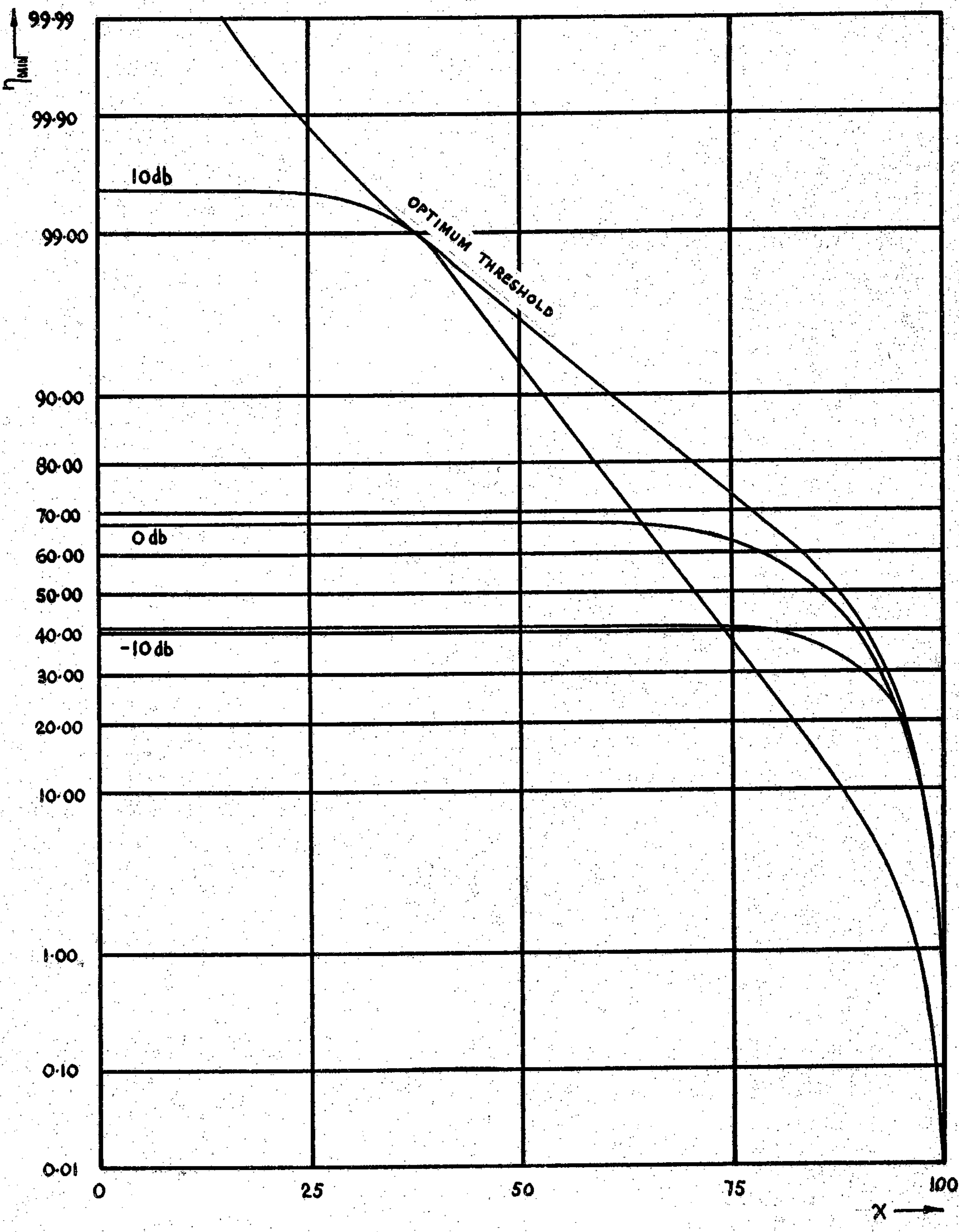
FADING PERFORMANCE CURVES FOR CONVENTIONAL BINARY SYSTEMS

FIGURE 6.5



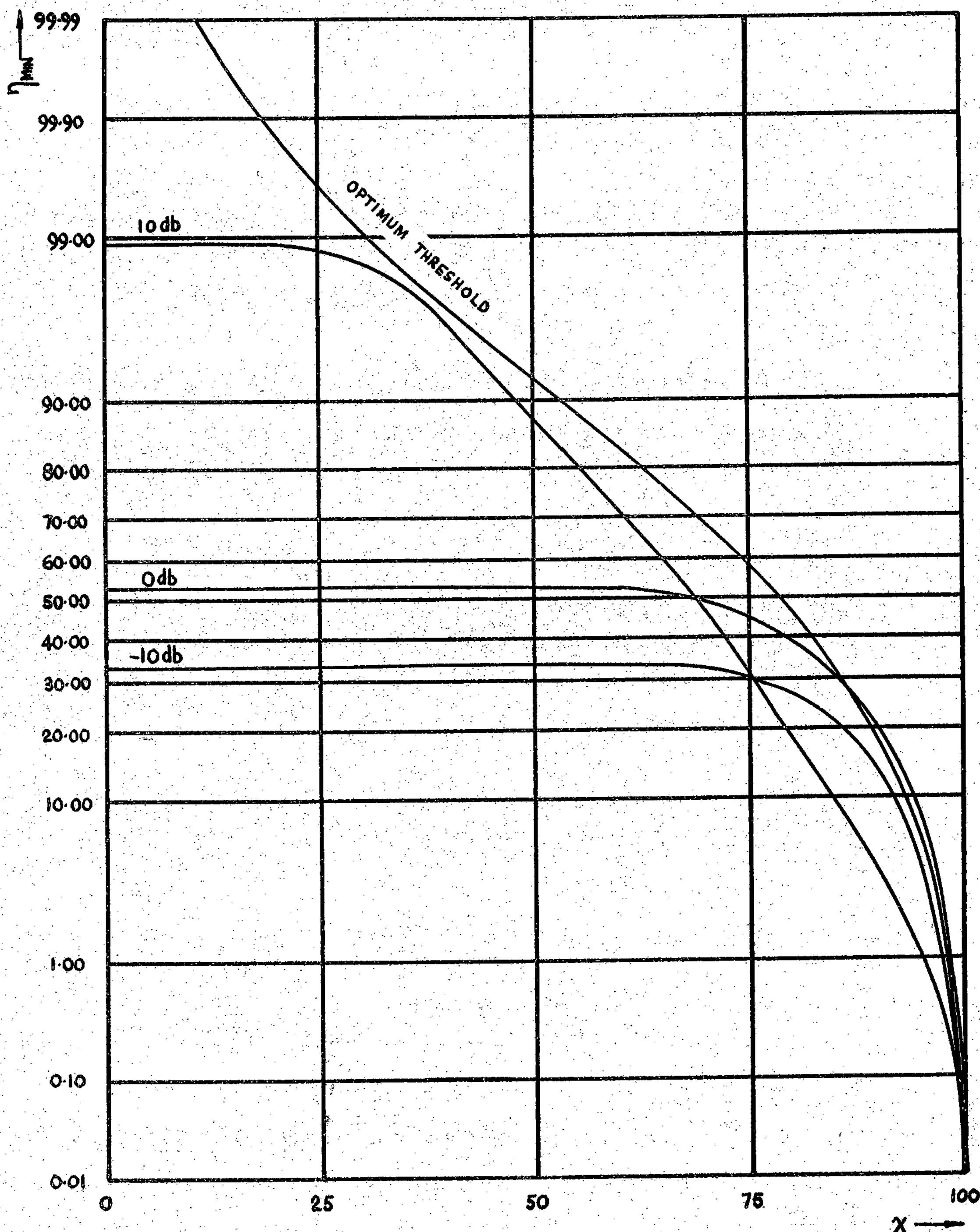
FADING SENSITIVITY CURVES FOR CONVENTIONAL BINARY SYSTEMS

FIGURE 6.6



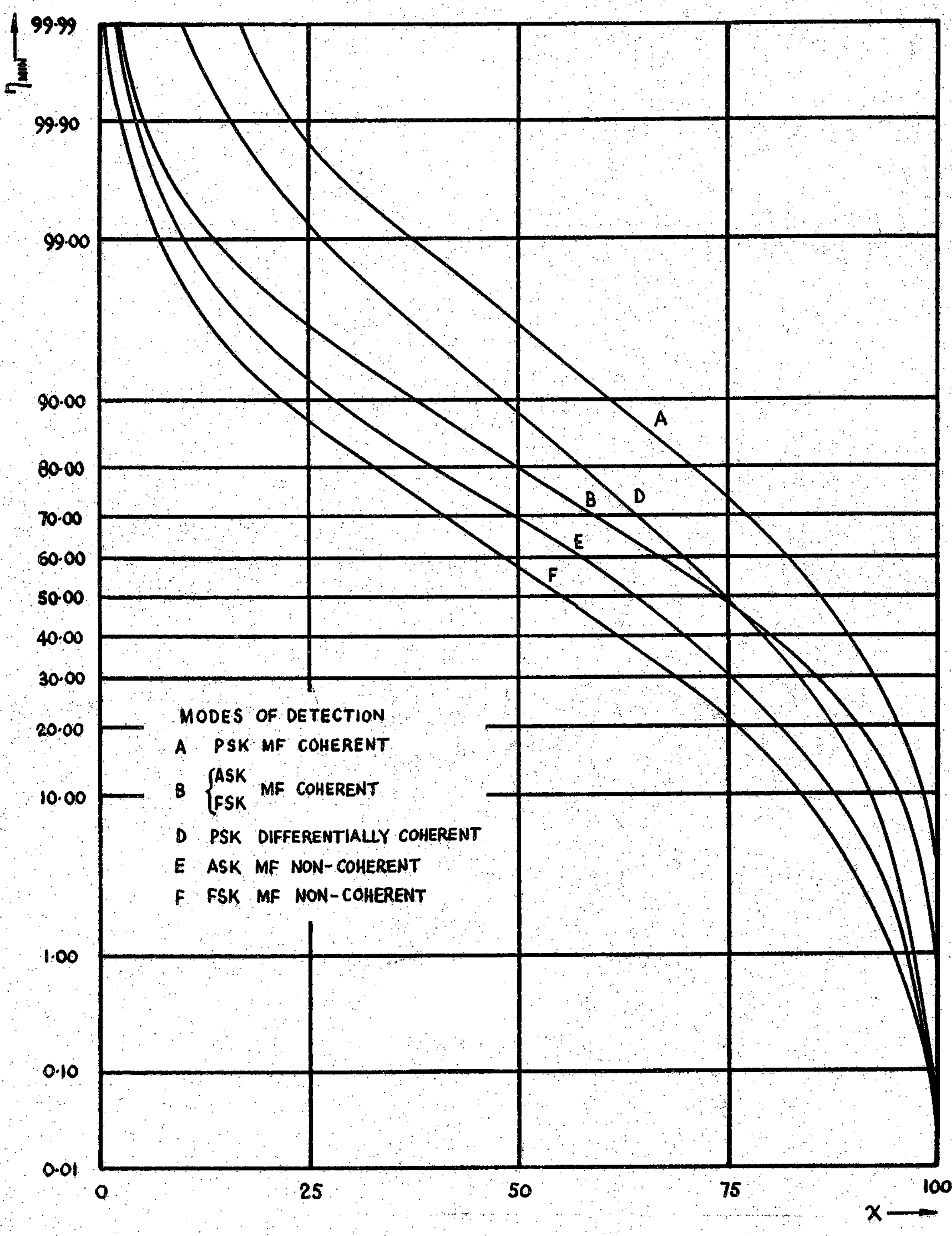
FADING PERFORMANCE CURVES FOR A FIXED THRESHOLD ASK SYSTEM WITH SYNCHRONOUS DETECTION

FIGURE 6.7



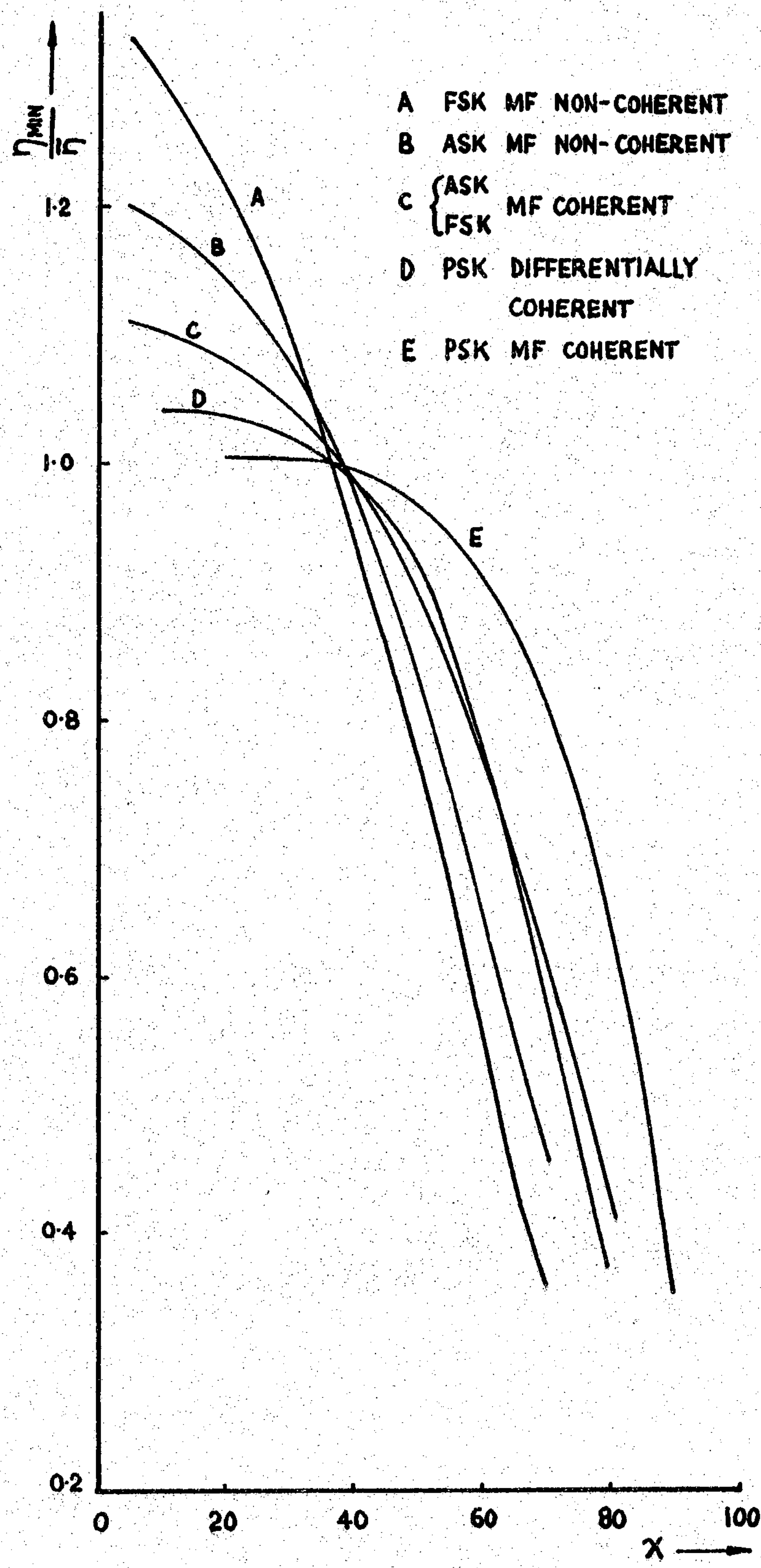
FADING PERFORMANCE CURVES FOR A FIXED THRESHOLD ASK SYSTEM WITH LINEAR ENVELOPE DETECTION

FIGURE 6.8



FADING PERFORMANCE CURVES FOR MATCHED FILTER BINARY SYSTEMS

FIGURE 6.9



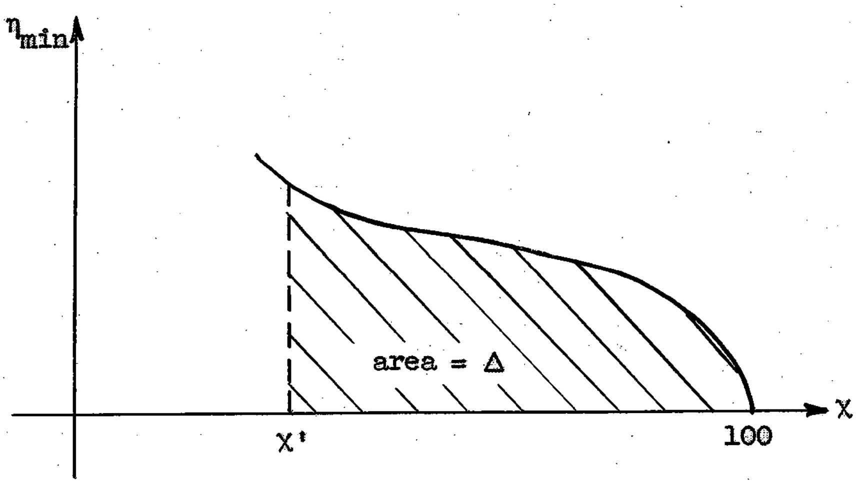
FADING SENSITIVITY CURVES FOR MATCHED FILTER BINARY SYSTEMS

FIGURE 6.10

Chapter IV. Fig. 6.11 and 6.12 show fading performance curves for the two matched filter fixed threshold systems discussed in section 6.2

6.4 A Fading Performance Factor

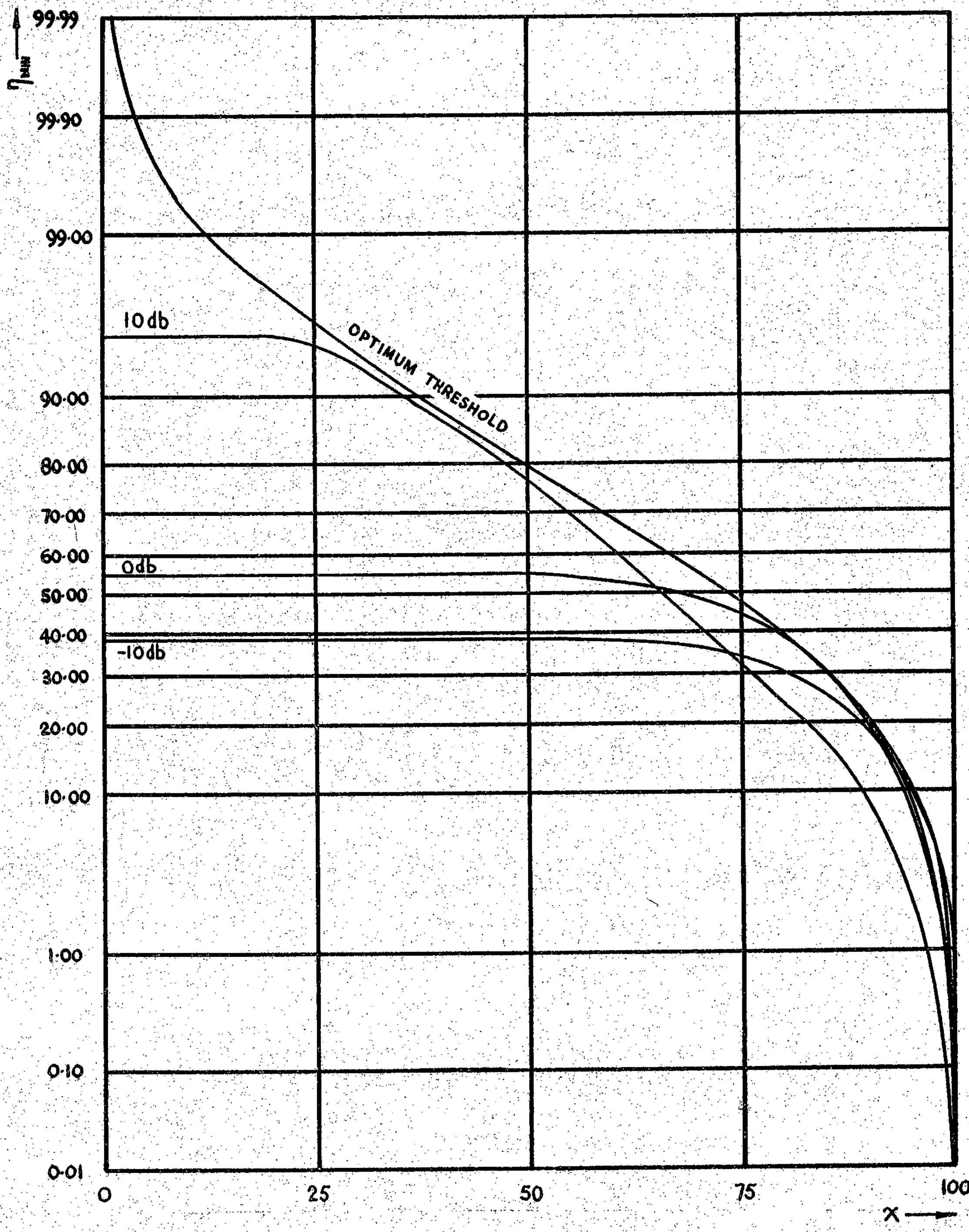
In comparing various systems with respect to their performance in the face of gaussian white noise and Rayleigh fading, the situation sometimes is made more difficult by two of the curves crossing. For example, in Fig. 6.7, fading performance curves for a fixed threshold ASK system employing synchronous detection are shown. The curves for a 10db threshold and a 0db threshold cross and the question arises as to which system is better and by how much. One method of answering this question is by computing a fading performance factor (Δ), where Δ is the area under a fading performance curve from a suitable value of $X = X'$ up to $X = 100$ (see Fig. 6.13).



A FADING PERFORMANCE FACTOR

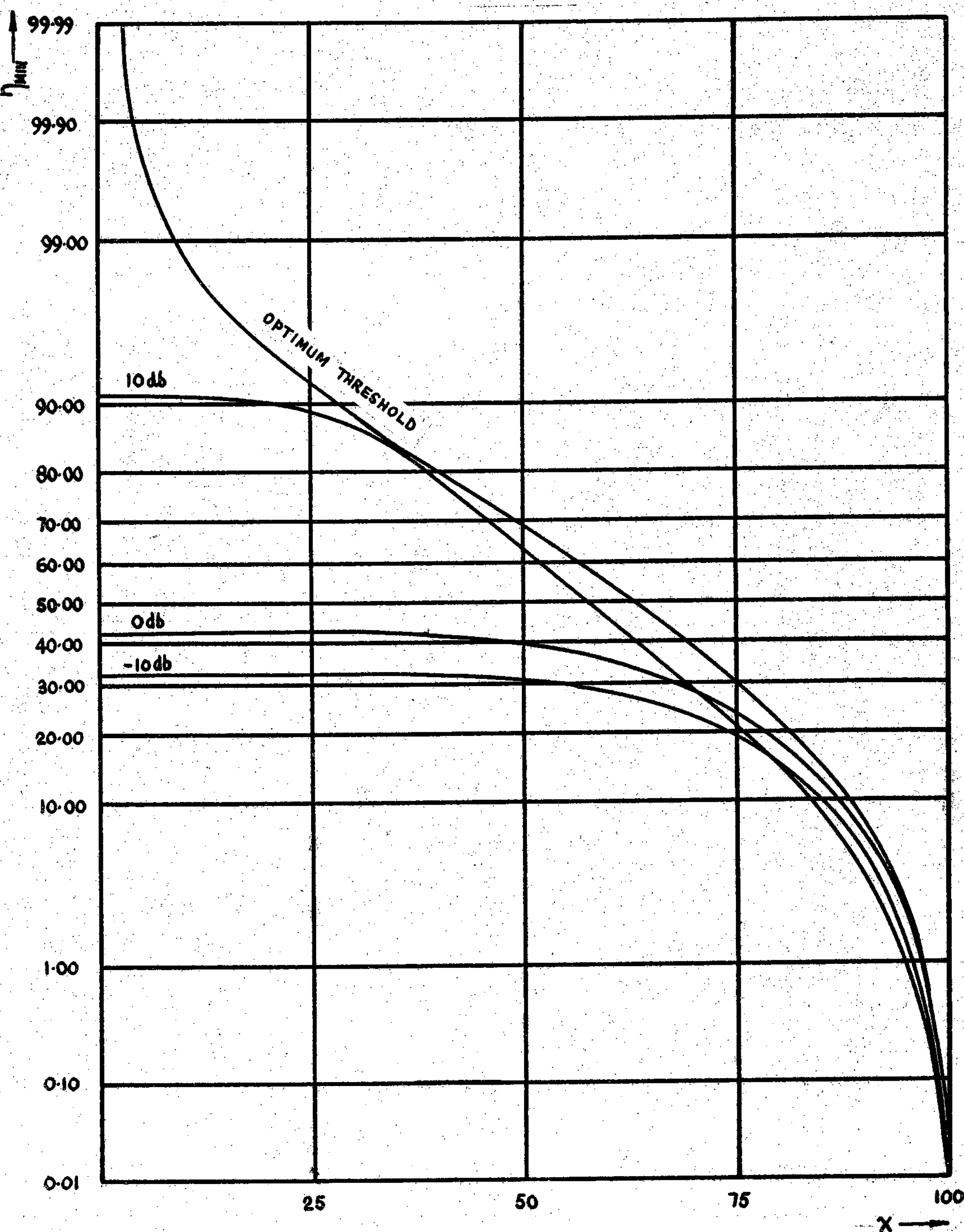
Figure 6.13

The reason for selecting X' other than zero is that in many cases, η_{min} goes to ∞ as X goes to zero and the integration becomes difficult.



FADING PERFORMANCE CURVES FOR A FIXED THRESHOLD ASK SYSTEM WITH COHERENT MATCHED FILTER DETECTION

FIGURE 6.11



FADING PERFORMANCE CURVES FOR A FIXED THRESHOLD ASK SYSTEM WITH NON-COHERENT MATCHED FILTER DETECTION

FIGURE 6.12

For the example above, $X' = 15$ is a suitable value and $\Delta = 38.7$ for the ideal system, $\Delta = 33.9$ for an optimum threshold at 10db, $\Delta = 30.1$ for an optimum threshold at 0db, and $\Delta = 25.3$ for an optimum threshold at -10db. Thus a figure of merit may be placed on various systems which is useful in comparing various systems. It should be noted that Δ will be used for comparative purposes only and the actual magnitude is of no consequence. In the next chapter, all of the systems analyzed above are put on a common basis (i.e., E/N_o) and fading performance curves and factors given.

Chapter VII

A Comparison of Binary Systems

7.1 Introduction

In this chapter, the results of Chapters III, IV, V and VI are correlated and the various binary systems which have been discussed are compared with each other. The performance curves for the conventional systems of Chapter III are converted to an energy-to-noise ratio basis by means of a TW product, and compared with the matched filter systems of Chapter IV. Finally, a comparison is made of all systems in the presence of fading. This is done first with optimum systems and then the class of fixed threshold ASK systems is compared with other PSK and FSK systems.

7.2 A Comparison of Symmetric Systems Perturbed by Gaussian White Noise

In this section, all of the systems analyzed in Chapters III and IV will be compared. In order to make such a comparison it is necessary to convert the signal-to-noise ratios of Chapter II to equivalent energy-to-noise ratios or vice versa. This can be done by using a TW product as discussed in section 2.4. The relation is given by Eq. 2-20,

$$\frac{S}{N} = \frac{E}{N_o} \frac{1}{2TW} . \qquad (2\text{-}20)$$

Since no TW product has been determined for an FSK system, a conversion of $\frac{S}{N}$ to $\frac{E}{N_o}$ has been made. The TW product for an ASK and a PSK system as derived in Appendix I are both the same and equal to two. Therefore for either a PSK or an ASK system,

$$\frac{E}{N_o} = 4\,\frac{S}{N}\,. \tag{7-1}$$

Using this result the total probability of error for the various systems considered becomes:

ASK-Linear Envelope Detector

$$P_e = \int_0^{\lambda} \epsilon^{-\left[x + \frac{E}{2N_o}\right]} I_o\left(\sqrt{\frac{2E}{N_o}\,x}\right) dx = \epsilon^{-\lambda} \tag{7-2}$$

ASK-Synchronous Detection

$$P_e = 1/2\left[1\text{-erf}\left(\sqrt{\frac{E}{8N_o}}\right)\right] \tag{7-3}$$

PSK-Synchronous Detection

$$P_e = 1/2\left[1\text{-erf}\left(\sqrt{\frac{E}{4N_o}}\right)\right] \tag{7-4}$$

PSK-Phase Comparison Detection

$$P_e = 1/2\ \epsilon^{-\frac{E}{4N_o}} \tag{7-5}$$

ASK-MF-Coherent Detection

$$P_e = 1/2\left[1\text{-erf}\left(\sqrt{\frac{E}{4N_o}}\right)\right] \tag{7-6}$$

ASK-MF-Non-Coherent Detection

$$P_e = \int_0^{\lambda} \epsilon^{-\left[x + \frac{E}{N_o}\right]} I_o\left(\sqrt{\frac{4E}{N_o}\,x}\right) dx = \epsilon^{-\lambda} \tag{7-7}$$

PSK-MF-Coherent Detection

$$P_e = 1/2 \left[1\text{-erf}\left(\sqrt{\frac{E}{2N_o}}\right)\right] \qquad (7\text{-}8)$$

PSK-Differentially Coherent Detection

$$P_e = 1/2\ \epsilon^{-\frac{E}{2N_o}} \qquad (7\text{-}9)$$

PSK-MF-Coherent Detection

$$P_e = 1/2 \left[1\text{-erf}\left(\sqrt{\frac{E}{4N_o}}\right)\right] \qquad (7\text{-}10)$$

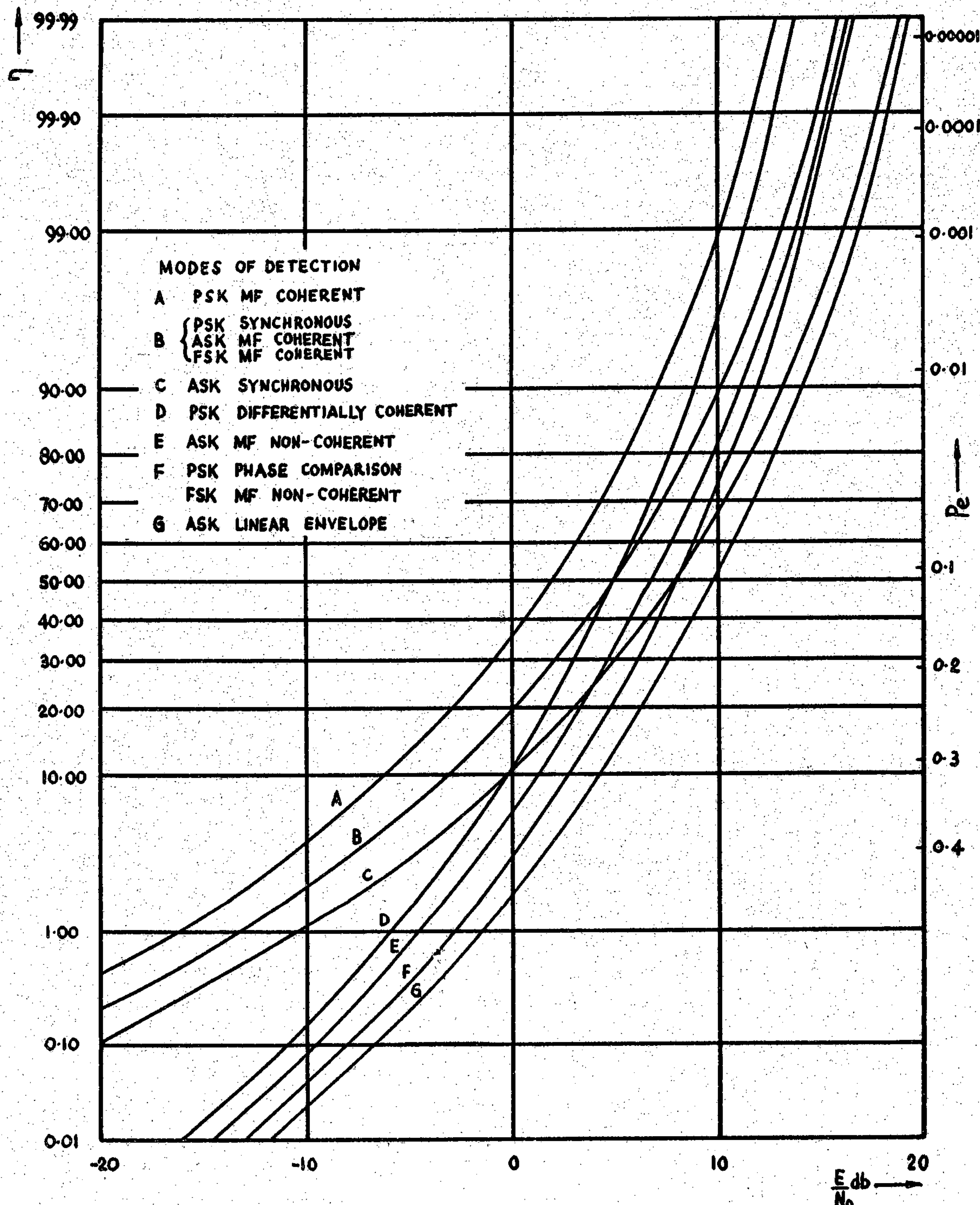
FKS-MF-Non-Coherent Detection

$$P_e = 1/2\ \epsilon^{-\frac{E}{4N_o}} \qquad (7\text{-}11)$$

A plot of η versus E/N_o for all of these systems is shown in Fig. 7.1.

There are several points which should be made here. The first is that several of the systems considered are equivalent to each other. Thus a PSK system with synchronous detection is equivalent to an ASK matched filter system with coherent detection and an FSK matched filter system using coherent detection. The second group of equivalent systems consists of an FSK matched filter system with non-coherent detection and a PSK system using phase comparison detection.

Another fact which should be noted is that regardless of the energy-to-noise ratio there are fixed differences between some of the systems or groups of systems. For example, a PSK matched filter system with coherent detection (the ideal binary system) is always 3db better than the next group of systems which includes PSK synchronous detection



INFORMATION EFFICIENCY OF BINARY SYSTEMS

FIGURE 7.1

and matched filter coherent detection of both an ASK and FSK signal. An ASK system with synchronous detection is another 3db worse, or 6db below the performance of an ideal system. A PSK differentially coherent detector is always 3db better than the PSK phase comparison, FSK non-coherent matched filter group. There is also a 3db difference between an ASK system with a matched filter and linear envelope detector and one having just a linear envelope detector alone.

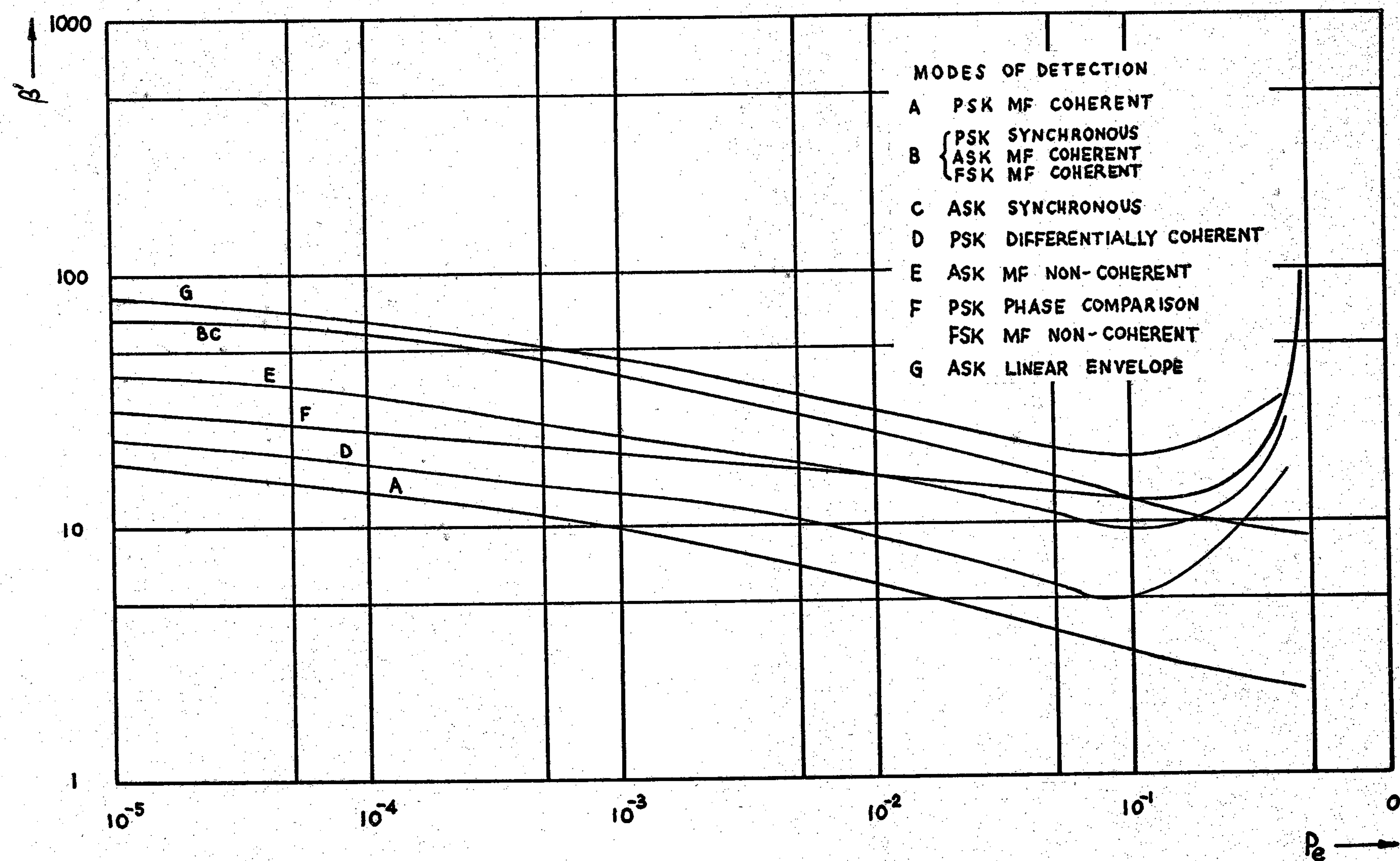
Other items of interest include the convergence of various systems and/or groups of systems at high energy-to-noise ratios. Thus the PSK differentially coherent detection system approaches the ideal system at high values of E/N_0. The PSK synchronous detection, ASK matched filter coherent detection, and the FSK matched filter coherent detection group, the ASK matched filter system with non-coherent detection, and the PSK phase comparison group all have converging performance curves at high energy-to-noise ratios. In a similar manner, the performance curves for ASK systems employing synchronous detection and linear envelope detection converge at high energy-to-noise ratios. Thus what is left at high energy-to-noise ratios are three groups of systems led by the ideal system and followed by a second group 3db down from the ideal and a third group 6db below the ideal system.

It is interesting to note that the ideal system requires a coherent reference signal at the receiver whereas the differentially coherent PSK system does not. Thus for high values of E/N_0 the differentially coherent system appears to offer a substantial advantage over other systems. Of the systems in the second group, phase comparison detection is quite attractive for the same reason. However, at low values of E/N_o all of the non-coherent systems fare considerably worse than even the poorest of the coherent systems.

Fig. 7.2 shows the β' curves for the systems discussed above. The β' curve for an ASK system employing synchronous detection is almost exactly the same as that of the ASK-MF-Coherent Detection group and only one curve is given. As above, the PSK-MF system has the lowest β' and is therefore the best system. Note that in general, β' decreases as P_e increases up to a P_e of 0.1 to 0.4. Thus it would appear that the best performance occurs at a P_e in this range; however, as in the case of rate versus bandwidth (Fig. 2.6) the increased performance requires sophisticated coding techniques.

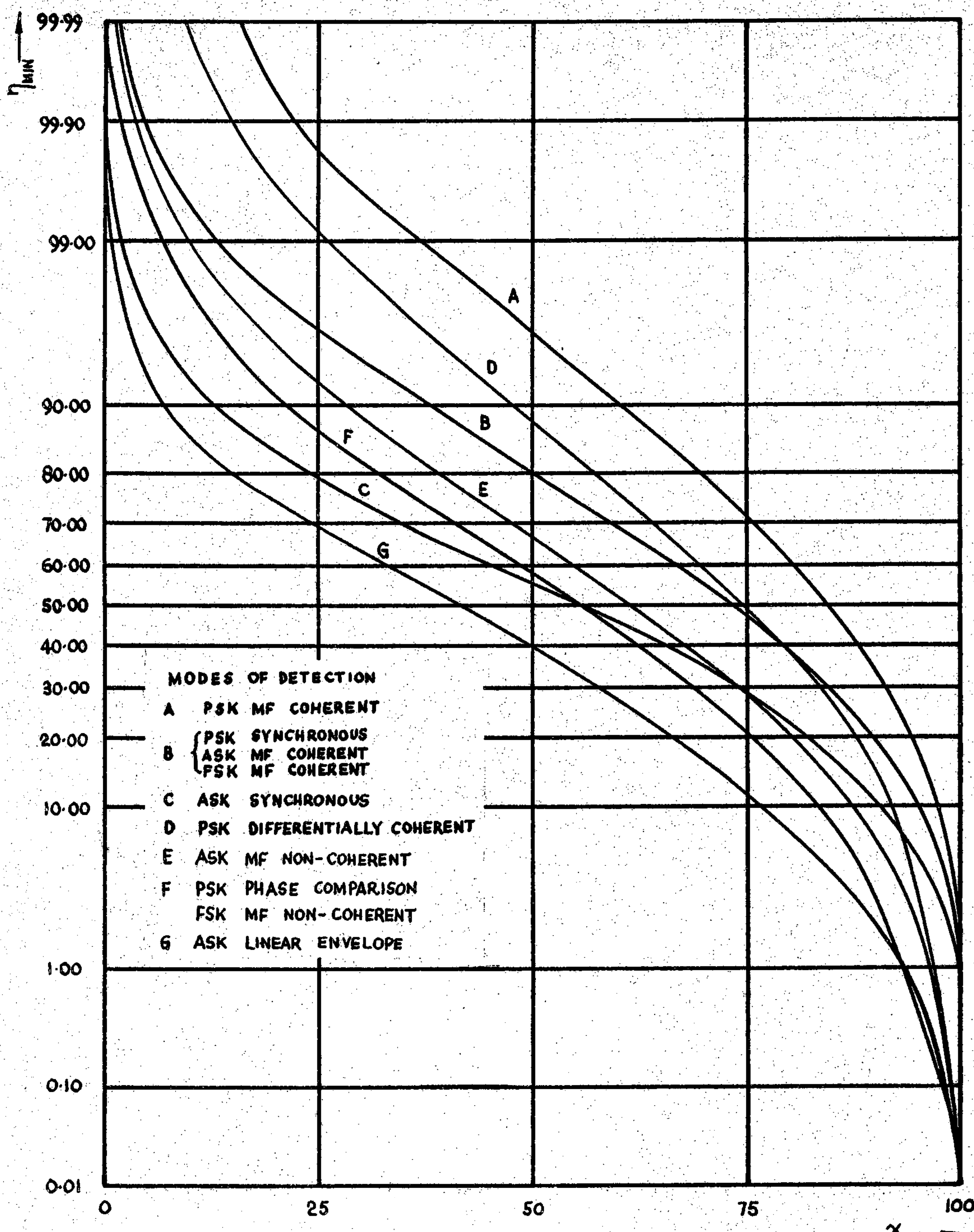
7.3 A Comparison of Symmetric Systems in the Presence of Fading

In order to compare the fading performance of the symmetric systems of Chapters III and IV, a set of curves giving η_{min} as a function of X has been drawn (see Fig. 7.3) from the common base (E/N_o) efficiency curves shown in Fig. 7.1 (note: $\bar{\eta}$ = 10db). Table 7.1 shows the value of Δ for all of the systems shown on the basis of $X' = 20$. With a Δ of 35.1, the PSK-MF system represents the best performance for the fading model which has been chosen. The next best system is the PSK-Differentially Coherent system. The next best systems are the group which includes ASK and FSK coherent matched filter detection and PSK synchronous detection. Following this group is ASK non-coherent matched filter detection and then ASK synchronous detection. The PSK phase comparison, FSK non-coherent matched filter group has a somewhat lower Δ and finally, ASK envelope detection represents the worst case. As before, the differentially coherent system appears quite attractive.



β' CURVES FOR BINARY SYSTEMS

FIGURE 7.2



FADING PERFORMANCE CURVES FOR BINARY SYSTEMS IN A SYMMETRIC MODE

FIGURE 7.3

System	Δ
ASK-Linear Envelope Detection	21.0
ASK-Synchronous Detection	24.8
PSK-Phase Comparison Detection	24.0
PSK-MF-Non-Coherent Detection	24.0
ASK-MF-Non-Coherent Detection	26.3
ASK-MF-Coherent Detection	29.5
FSK-MF-Coherent Detection	29.5
PSK-Synchronous Detection	29.5
PSK-MF-Coherent Detection	35.1
PSK-Differentially Coherent Detection	30.6

A COMPARISON OF SYSTEMS
IN THE PRESENCE OF FADING

Table 7.1

Chapter VIII

Conclusions and Suggestions for Further Research

8.1 Introduction

The purpose of this chapter is to present some conclusions regarding the results which have been obtained and to suggest some lines for the continuation of research.

8.2 Conclusions

In examining the results which have been obtained, it appears that the concept of information efficiency has proved to be a satisfactory one. Aside from having the advantages mentioned in Chapter II, it is convenientsince the equivalent P_e for a symmetric system is easily plotted on the same graph as η.

It is felt that the results of Chapter V are of special importance since they describe the performance of sub-optimum systems. This category includes any physical realization of the systems which have been discussed since the probability of realizing a system with exactly the proper threshold is essentially zero.

The results of analyzing an ASK fixed threshold class of systems are important in that they clearly show that difficulty involved with using such a system. From the curves shown it is clear that if a high level of performance is desired for high signal strengths then one must be prepared to sacrifice low signal performance.

In regard to the results concerning fading performance, it should be noted that the character of the fading model is of a fairly simple

simple nature. A more sophisticated model might include a random phase aspect as well as Rayleigh amplitude variation.

The comparisons of conventional systems with matched filter systems by the use of a TW product appears to be quite satisfactory. By using this procedure, it is possible to evaluate all systems on a common basis.

Of all the systems examined, PSK with differentially coherent detection appears to be one of the most attractive. For reasonably high signal-to-noise ratios, this system approaches the performance of the ideal matched filter system while at the same time having the rather substantial advantage of requiring no coherent reference signal at the receiver. A realization of this scheme may be found in the Kineplex system.[15]

8.3 Suggestions for Further Research

There are several directions in which the study of binary systems could continue. The first and one of the most important is in the area of analyzing systems in the presence of fading. Turin[2] has performed such an analysis using a fading model which includes a random phase as well as a Rayleigh amplitude variation. He has analyzed an FSK system under these conditions and it would be of interest to extend this to other systems.

Another area for study would be the effect of using redundant codes on overall performance. In the analysis which has been done here, the use of such codes has not been considered. Still further sophistication would be obtained if the sampling and quantization processes as well as the recovery processes were included.

Finally, it would be of interest to investigate the effects of perturbing various systems with colored noise. This problem was investigated to some extent and the conclusions are that in the case of conventional systems, it is the variance of the noise and not its color which is of importance. However, in the case of a matched filter system, the ideal receiver requires the use of a prewhitener and a study of the effects of using such a device on overall performance should be of interest.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Shannon, C. E.: "A Mathematical Theory of Communication," Bell System Technical Journal, vol. 27, pp. 379-423, 623-656; July-October, 1948.

2. Turin, G. L.: "Error Probabilities for Binary Symmetric Ideal Reception through Nonselective Slow Fading and Noise," Proceedings of the IRE, vol. 46, pp. 1603-1619; September, 1958.

3. Rice, S. O.: "Mathematical Analysis of Random Noise," Bell System Technical Journal, vols. 23 and 24, pp. 282-332, 46-156.

4. Cahn, C. R.: "Performance of Digital Phase Modulated Communication Systems," IRE Transactions on Communication Systems, vol. CS-7, pp. 3-6; May, 1959.

5. Cahn, C. R.: "Performance of Digital Phase Modulated Communication Systems," Ramo-Woolridge Corporation, Los Angeles, California, Technical Report No. M110-9U5; April, 1959.

6. Reiger, S.: "Error Probabilities of Binary Data Transmission Systems in the Presence of Random Noise," Convention Record of the IRE, part 8, pp. 72-79; 1953.

7. Sanders, R. W.: "Communication Efficiency Comparison of Several Communication Systems," Proceedings of the IRE, vol. 48, pp. 575-588; April, 1960.

8. Hancock, J. C. and Holsinger, J. L.: "Some Useful Coding Techniques for Binary Communication Systems," Purdue University, School of Electrical Engineering, TR-EE62-1; January, 1962.

9. Hancock, J. C. and Aein, J.: "Correlation Receivers Minimizing Intersymbol Interference," Purdue University, School of Electrical Engineering, TR-EE62-7; April, 1962.

10. Hancock, J. C. and Lindsey, W. C.: "A Wideband Adaptive Communications System," Purdue University, School of Electrical Engineering, TR-EE62-6; May, 1962.

11. Hastings, C.: Approximations for Digital Computers, Princeton University Press, 1957.

12. Hancock, J. C.: An Introduction to the Principles of Communication Theory, McGraw-Hill, 1961.

13. Baghdady, E. J.: Lectures on Communication System Theory, McGraw-Hill, 1961.

14. Lawton, J. G.: "Theoretical Error Rates of 'Differentially Coherent' Binary and 'Kineplex' Data Transmission Systems," Proceedings of the IRE, vol. 47, pp. 333-334; February, 1959.

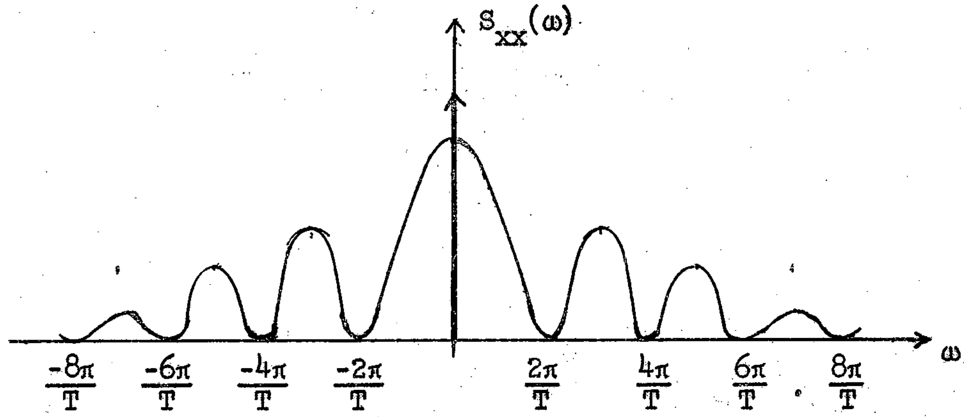
15. Dolez, M. L., Heald, E. T. and Martin, D. L.: "Binary Data Transmission Techniques for Linear Systems," Proceedings of the IRE, vol. 45, pp. 656-661; May, 1957.

APPENDICES

APPENDIX I

TW Products for ASK and PSK Signals

The ASK signal described in Chapter I may be characterized by a carrier of frequency ω_c which is amplitude modulated by a random coin flip wave which takes on values of 0 or 1 with equal probability. The spectral density for such a wave is of the form $\left(\frac{\sin x}{x}\right)^2$ (see Fig. I.1).

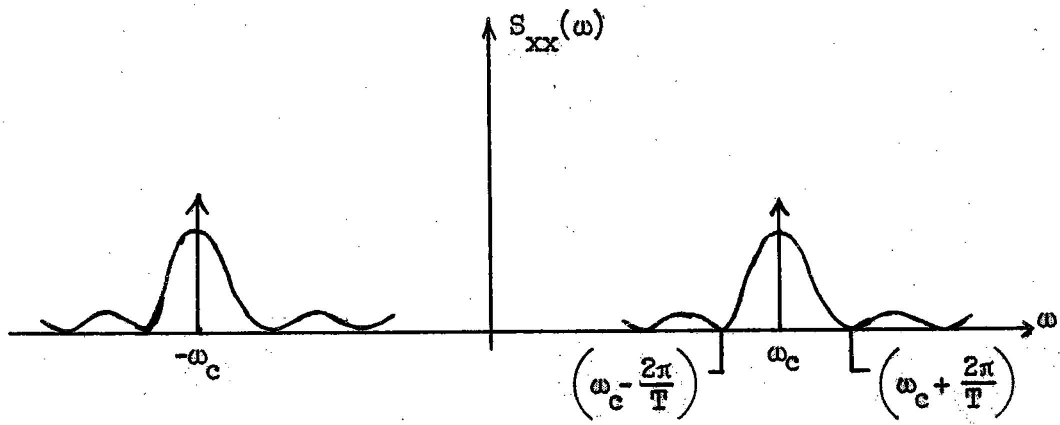


SPECTRAL DENSITY OF A RANDOM COIN FLIP WAVE

Figure I.1

The modulation process translates this spectrum about $\pm\ \omega_c$ (see Fig. I.2). Now if the bandwidth of the ASK signal is defined as the distance between zeros and centered about ω_c, then TW = 2.

The TW product for a PSK signal as described in Chapter I is also equal to 2. This follows as a result of the fact that the PSK signal is also a suppressed carrier ASK signal and has essentially the same spectrum as discussed above (only the carrier component is missing).



SPECTRAL DENSITY OF AN ASK SIGNAL

Figure I.2

APPENDIX II

On the Statistical Independence of Sample Points

The outputs of the two matched filters which are part of the FSK detector shown in Fig. 4.8 may be described by

$$y_0 = \int_0^T y(t)\ S_o(t)dt \tag{II-1}$$

$$y_1 = \int_0^T y(t)\ S_1(t)dt \tag{II-2}$$

where $S_1(t)$ and $S_0(t)$ are the mark and space waveforms unperturbed by noise. Both y_0 and y_1 will be gaussian (see Eq. 4-20--23) and hence it is only necessary to show them to be uncorrelated in order to establish independence. This may be done as follows.

$$E[y_0 y_1] = E\left[\iint y(t)S_1(t)y(\tau)S_0(\tau)dtd\tau\right] \tag{II-3}$$

Assume

$$y(t) = S_1(t) + n(t) \tag{II-4}$$

where n(t) is gaussian white noise.

$$E[y_0 y_1] = E\left[\iint\Big(S_1(t)+n(t)\Big)\Big(S_1(\tau)+n(\tau)\Big)\ S_1(t)S_0(\tau)dtd\tau\right] \tag{II-5}$$

$$= E\left[\iint\Big(S_1^2(t)S_1(\tau)S_0(\tau)+n(t)S_1(\tau)S_1(t)S_0(\tau) +n(\tau)S_1^2(t)S_0(\tau)+n(t)n(\tau)S_1(t)S_0(\tau)\Big)\ dtd\tau\right] \tag{II-6}$$

Now ρ_{1o} has been assumed to be zero and therefore the first term in Eq. II-6 drops out. Reversing the order of the expectation and integration, Eq. II-6 becomes

$$E(y_0 y_1) = \iint \Big[E[n(t)S_1(\tau)S_1(t)S_0(\tau)] + E[n(\tau)S_1^2(t)S_0(\tau)] + E[n(t)n(\tau)S_1(t)S_0(\tau)] \Big] dt d\tau. \quad \text{(II-7)}$$

If the noise is assumed to have a mean value of zero, then the first and second terms of the integrand in Eq. II-7 drop out since S_0 and S_1 are constants in terms of taking the expectation. The last remaining term reduces to

$$E(y_0 y_1) = \iint N_o \delta(t-\tau) S_1(t) S_0(\tau) dt d\tau \quad \text{(II-8)}$$

$$= N_o \int S_1(t) S_0(t) dt. \quad \text{(II-9)}$$

But since $\rho_{10} = 0$, $E(y_0 y_1) = 0$. Thus the conditions of establishing that the variables are uncorrelated and gaussian have been met and they are independent.

ERRATA

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Page		
9	"coherent reference signal" should read "gaussian white noise"	
60	$\int_{\delta}^{\infty} \frac{y}{2} \epsilon^{-\left(\frac{y^2}{4}\right)} dy$	(5-2)
70,72,79	abcissa of Fig. 5.5, 5.6, 5.11 should read "$0, \pi/2, \pi$" instead of "$0, \pi, 2\pi$"	
92	$X = \epsilon^{-\delta/\bar{S}}$	(6-12)
112 line 7	"convenientsince" should read "convenient since"	
112 line 15	"that difficulty" should read "the difficulty"	