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A Comparison of Two Instructional Sequences in an Intelligent Tutoring Program on Multiplicative Concepts and Problem Solving of Students with Mathematics Difficulties

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A COMPARISON OF TWO INSTRUCTIONAL SEQUENCES IN AN INTELLIGENT TUTORING PROGRAM ON MULTIPLICATIVE CONCEPTS AND PROBLEM SOLVING OF STUDENTS WITH MATHEMATICS DIFFICULTIES

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To my husband and parents

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TABLE OF CONTENTS

Page

Page

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

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One of the crucial goals of the National Councils of Teachers Mathematics standards (2000) was to have *all* students, including students with mathematics difficulties (MD), to succeed in establishing a higher-order thinking in mathematic. However, there has been a lack of research on developing differentiated mathematics instructions necessary for students with MD to learn about multiplication concept. This study examines the differential effects of two instructional sequences taught in an intelligent tutor system that is designed to nurture students' multiplicative concept to enhance their problem solving skills. A total of 18 third (n = 13) and fourth (n = 5) grade students with MD were assigned to one of the two treatment conditions (modules A-**B**-C-D-E and A-C-**B**-D-E) by switching the order of mixed unit coordination (MUC) task. A repeated measure ANOVA design involved achievement measures. The results indicate that both instructional sequences improved, from pretest to posttest, their performance on the MR criterion test and COMPS near-transfer test. However, the alternative instructional sequence was more efficient for students with MD to establish multiplicative concepts as they took less number of sessions to achieve the same level of

word problem solving skills. The results of this study suggests that students with MD can be expected to establish conceptual understanding of multiplication and show better performance on their multiplicative word problem solving when instructional framework is tailored towards their needs.

CHAPTER 1. INTRODUCTION

1.1 Background

Mathematical proficiency has been considered by U.S. educational policies as one of the vital components essential for individual fulfillment, active citizenship, and career-readiness in today's highly technical society (National Education Goals Panel, 1999). Increasing evidence indicates that advanced mathematical thinking skills are necessary in today's competitive workforce. Various state and federal educational policy makers encouraged a reform in the national educational framework and promoted systemic changes to ensure equitable educational opportunities and high levels of educational achievement for all students. For instance, the Science, Technology, Engineering and Mathematics (STEM) education initiative was established to focus on improving students' mathematical thinking as well as to apply it to other related academic areas (e.g., science and engineering) to expand the pipeline of students set to enter college and the workforce. Thus, mathematics is a practical subject essential in life and needs to be learned properly.

The demand for advanced mathematical thinking, such as problem solving and reasoning skills, has also led to a renewed interest in reforming the current mathematics curriculum. This is especially important in the early school years for all students, including those struggling in mathematics, so that they can be successful both in academic and occupational opportunities in the future (Maccini & Ruhl, 2000; Mercer

Jordan, & Miller, 1994; Xin, 2008). One of the key shifts in mathematics reform called for the need to enhance students' understanding of mathematical concepts, and to improve their ability to associate between mathematical ideas and models to solve mathematics word problems (Common Core State Standards Initiative [CCSSI], 2012; National Council of Teachers of Mathematics [NCTM], 2000; National Research Council [NRC], 2001). The synergy between a solid conceptual understanding and procedural fluency skill may lead these students to be one step closer to solving word problems with various contexts and larger numbers. The initial step of acquiring advanced mathematical thinking requires the development of multiplicative reasoning by constructing the concepts of multiplication (Confrey, 1994; Harel & Sowder, 2005). Thus, mathematics reform calls for the need to seek adaptive pedagogies that could potentially facilitate *all* students, including those with mathematical difficulties, in developing conceptual knowledge of multiplication.

1.2 The Current Problem

The concept of multiplication is a foundational skill which children must learn in early elementary school in order to proceed to advanced mathematical thinking. According to the National Council of Teachers of Mathematics [NCTM], students develop the concepts of multiplication and division as early as third grade (2000). By fourth or fifth grade, most students are expected to have established the conceptual understanding of multiplication, have mastered multiplicative reasoning, and to be able to solve various types of multiplication word problems (Xin, Tzur, & Si, 2008). However, many U.S. elementary students encounter difficulties in developing mathematics problem-solving and reasoning skills (National Assessment Educational Progress

[NAEP], 2011). Previous international comparison studies over the past decades have revealed lower levels of mathematics problem-solving performance of U.S. students when compared to those from other countries (National Research Council [NRC], 2001). According to the most recent international comparison assessments of student mathematics achievement (The Trends in International Studies Mathematics and Science Study [TIMSS], 2011), which examined more than 12,000 fourth grade students from 57 countries, American fourth grade students' average mathematics proficiency gradually increased since 1995, and is now in the top fifteen education systems in mathematics (Mullis, Martin, Foy, & Arora, 2012). However, only 13% of the students were able to solve higher-level mathematical tasks (e.g., word problems and reasoning), which required them to solve various complex situations involving whole numbers and to explain their reasoning based on their mathematics concepts. While advanced mathematical thinking is difficult for normal-achieving students, approximately 6-7% of students who demonstrate low performance in mathematics will experience even more hardship in achieving higher mathematical thinking standards (No Child Left Behind [NCLB], 2002).

Students with mathematical difficulties (MD) manifest poorer performance in transitioning to advanced mathematical thinking, leading them to remain considerably lower in mathematics abilities than their normal-achieving peers (Bottge, Rueda, Laroque, Serlin, & Kwon, 2007; Gagnon & Maccini, 2001; Woodward & Brown, 2006). One of the major reasons is that most students with MD experience difficulty transitioning from the concept of addition to the concept of multiplication (Harel & Confrey, 1994). The act of multiplying the numbers is more than performing a repeated addition (Steffe, 1994).

Students are reconceptualizing the numbers and the notion of units, which requires significant cognitive processing. Students with MD may require specialized instruction to undergo such complex information processing to construct multiplication concepts (Swanson, 1993).

Helping students with MD to establish multiplication concepts has received critical attention from researchers and educators. However, little is known about how children with MD develop multiplicative concept reasoning. Given that many American students struggle with mathematics problem solving and reasoning, several questions are posed about how low-achieving students construct multiplicative reasoning and how the existing differentiated instruction would nurture their multiplicative reasoning. In particular, what are some indicators that signify students with good multiplicative reasoning skills? What are some difficulties that students with MD undergo during the conceptual leap between addition/subtraction to multiplication/division but that normal-achieving students do not experience? And what are the current best practices of problem solving with a focus on promoting mathematics conceptual understanding for students with MD?

1.2.1 Development of Multiplicative Concept

Multiplicative concept has been a critical constructive resource for children to establish multiplication. Previous multiplicative concept studies have focused on students' counting schemes in identifying students' progress in constructing the concept of multiplication (Steffe, 1988; Steffe & Cobb, 1988). A multiplication counting scheme indicates students' counting acts while solving multiplication word problems. According to the constructivist point of view, normal-achieving children undergo numerous adjustments to re-establish their counting acts through their personal experience of counting (Steffe & Cobb, 1994). They gradually refine the notion of units from singletons (ones) to ultimately constructing composite units (CU): “2x3 is two units of three” (Steffe, 1992). A CU is a set of an equal quantity of singletons (Clark and Kamii, 1996; Steffe, 1988; Steffe & Cobb, 1988). Students in this stage can (a) mentally understand that three ones are taken as one three (Steffe, 1992), (2) coordinate the units between the CU and ones while solving word problems, and (3) represent the operation in an abstract manner (i.e., math sentence) in order to enhance their multiplicative reasoning skills.

1.2.2 Existing Interventions

Interventions used in the field of special education have mostly emphasized improving students’ problem-solving accuracy in an explicit manner. A number of interventions have applied cognitive perspective to teach word problem solving to children with MD (Montague, 1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986). During cognitive/metacognitive instruction, children follow the guidance of four to seven sequential steps when solving problems. Previous studies have comparable critical solving steps: (1) read the problem, (2) think about the solution process, (3) write the math sentences, (4) solve the problem by computing, and (5) recheck the solution process and the final answer. Other interventions have involved a series of representations (concrete-semi-concrete-abstract) to assist students with MD to depict problem situations from the concrete to the abstract level (Miller & Mercer, 1993).

Other interventions have been designed to approach teaching word problems through teaching or broadening problem schema, which has been shown to be effective for students with MD (Jitendra & Hoff, 1996; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Marshall, Barthuli, Brewer, & Rose, 1989). The schema-based instruction (SBI) engages students in conducting semantic analyses of word problems to identify the common problem schemata for solution planning. During SBI, students map the problem features into the corresponding schema diagrams and determine the operation to use by applying a set of solution rules.

Xin and colleagues (Xin, 2012, Xin, 2008; Xin, Wiles, & Lin, 2008; Xin & Zhang, 2009) further developed the Conceptual Model-based Problem Solving (COMPS) approach. The COMPS approach engages students in representing word problems in mathematical model equations on the basis of an analysis of underlying problem structure. With the COMPS approach, mathematical model equations directly *drive* the solution plan including the selection of operations. The COMPS emphasizes the connection between mathematical ideas with the purpose of facilitating students' generalized problem-solving skills.

More recently, the Computer-Assisted Instruction (CAI) has been used as a tool to effectively convey problem-solving instruction on the basis of cognitive learning theory. The strategies used in CAI programs typically involved schema-based (Chang, Sung, & Lin, 2006; Leh & Jitendra, 2012), cognitive/metacognitive strategies (Seo & Bryant, 2012; Shiah, Mastropieri, Scruggs, & Fulk, 1994-1995), and COMPS (Xin, 2012) approach. However, there is a lack of CAI programs that address a broader set of mathematics problem solving. Thus, there is still room for the development of educational programs for higher mathematical thinking (e.g., concept-based approach and mathematical reasoning) that is aligned with the reform agenda.

Overall, the existing problem-solving interventions in the field of special education are characterized as an explicit instruction primarily emphasizing performance outcome (Woodward & Brown, 2006). The recent development of the problem-solving interventions focuses on helping students enhance problem-solving performance through symbolic modeling and cognitive factors, which are often taught through explicit strategy instruction. Given that the recent reform promotes constructivist learning paradigm (Woodward, 2004), special educators need to reexamine the current practices used to teach problem-solving skills and explore effective ways to incorporate the constructivist view of learning to ensure that students with MD are taught using an instruction method consistent with the mathematics reform agenda (Woodward & Montague, 2002).

1.3 The Alternatives

Constructivist instruction is student-focused self-exploration, where mathematics learning revolves around students constructing their own representation of reality (Piaget, 1973). Unlike direct instruction with teacher prompts, students in a constructivist-learning environment are encouraged to develop their mathematics conceptual understanding based on their own experience. Due to the nature of constructivist instruction, teachers need to provide a minimum of prompts, and instead, to challenge students to think of how they solved the problem (Reid, 2002; Rosales, Vicente, Chamoso, Munez, & Orrantia, 2012). One of the crucial difficulties facing mathematics education for students with MD is how to help them enhance higher order thinking in mathematics (Baxter, Woodward, & Olson, 2001). Findings from qualitative methods by

Baxter and colleagues (2001) reported that most struggling students have lack of chance to engage in class mathematics discussions as normal-achieving students dominate the conversation. Struggling students further struggle during the current reform-based instruction, as it demands higher cognitive load while learning about complex concepts and reasoning skills. Thus, further instructional interventions tailored towards these students' needs should be developed (Baxter et al., 2001; Baxter, Woodward, Voorhies, & Wong, 2002). Despite the demand, there is still lack of instructional approach educators could apply to help students with MD enhance their mathematics reasoning skills.

1.3.1 Please Go and Bring Me (PGBM)-COMPS Tutor Program

As the outcome of a collaborative work that integrates research-based practices from mathematics education and special education, a PGBM-COMPS intelligent tutor (Xin, Tzur, & Si, 2008) was developed to nurture MR skills in an explicit manner for students with MD. On one hand, the constructivist learning environment provided students a chance to explore multiplicative concept. On the other hand, COMPS approach (Xin et al., 2013) explicitly models word problem underlying structures (*story-grammar*). The PGBM-COMPS tutor was made of two parts: (1) the "Please Go and Bring Me..." (PGBM) turn-taking games that help students build multiplicative concept and (2) the COMPS (Xin, 2012) part, which engages students to represent mathematics word problems using the mathematical models. Various mathematics activities within the PGBM turn-taking games provide students with the chance to learn *multiplicative double counting* (mDC)—a fundamental idea for a multiplicative concept. During PGBM, students are encouraged to make the distinctions between singletons and CUs as they go through the conceptual understanding at both the concrete and symbolic levels (Tzur et al., 2012). The sequence of mathematical tasks is guided by Steffe's (1992) developmental stages of the multiplication concept underlying the hypothetical learning trajectory (HLT). COMPS fosters understanding of the problem structure and representation in mathematical models (Xin, 2012). The intelligent tutor has five modules: A, B, C, D, & E. Module A deals with mDC; Module B deals *with mixed unit coordination* (MUC) tasks. Module C deals with *quotitive division* (QD) tasks; and Module D with *partitive division* (PD) tasks. Finally, module E deals with *multiplicative comparison* (MC) tasks.

Xin and colleagues have conducted a series of studies (e.g., Ma et al., 2014; Park et al., 2013; Xin, Hord, et al., 2012; Xin et al., 2013), which field-tested the impact of the PGBM-COMPS programs on enhancing the performance of students with MD. Findings from these studies showed that the PGBM-COMPS tutor program elicited positive outcomes on enhancing overall multiplicative problem-solving performance of students with MD. In particular, a group comparison study (Xin et al., 2013) indicated that the students who received the PGBM-COMPS instructional approach showed a significantly greater improvement rate on the word problem-solving performance than those who received traditional instruction from their teachers. Although the students with MD seemed to benefit from the PGBM-COMPS approach on their overall multiplication/division word problem performance, the field notes and the process data (e.g., Xin, Hord, et al., 2012) showed that students with MD struggled on solving problems in Module B. In particular, Module B involves MUC problem. The findings from Xin, Hord, et al. (2012) were supported by another study from the same research group project by Park et al. (2013). These results may suggest that mastering mDC tasks (e.g., students count 2, 4, 6, 8 when solving for the total number of cubes of 2 towers of 4 cubes in each) are insufficient for students with MD to perform MUC tasks. The issue has grown in importance in light of the possible differences in the conceptual learning processes of multiplication between students with and without MD.

According to Steffe's teaching experiment (1992), normal-achieving students were able to apply previous knowledge of CU and coordination between the two units (i.e., ones and CUs) through multiplicative scheme tasks to solve MUC. Although the divisional scheme was not yet established, normal-achieving children acted by iterating-up the CUs (e.g., double counting) to divide ones equally to construct CUs. That is, children counted a collection of units of 3 for example, 3, 6, 9, 12, 15, and found out that they could make 5 towers of 3 cubes each. The act of reversing the iterating-units forward or backward to solve unfamiliar tasks is "a precursor to partitioning a totality as required for division" (Tzur et al., 2012, p. 160). Because of the nature of their strategic knowledge deficiency, students with MD struggle to further expand their knowledge of unit coordination to solve for divisional scheme tasks, which is a crucial ability to reconstruct their learned knowledge and apply it while solving novel tasks (Steffe & Cobb, 1994).

Unlike other multiplicative schemes, MUC scheme (e.g., Tom has 6 towers with 3 cubes in each. Sam gave Tom 15 more cubes. How many towers of 3 would Tom have in all?; hereafter 6T3 +15) entails two-step problems where students need to use both divisional (i.e., 15 cubes= 5T3) and unit coordination (i.e., 6T3 + 5T3) schemes to solve for the solution. Thus, it requires children to hold on to the answer from the first step and apply it to the second step to derive the final solution. The above process is especially difficult for students with MD as they struggle with working memory deficits (Geary et al., 2007; Siegel & Ryan, 1989; Swanson, 1993). In the discussion, Xin, Hord, et al. (2012) suggested that appropriate scaffolding would be needed for students with MD on MUC tasks in order for them to organize and retain the information. Thus, the need for an adaptive sequence of multiplication schemes is apparent as students with MD undergo difficulties with the existing scheme, possibly due to their mathematics cognition deficits as well as their lack of ability to adapt their existing strategies to solve novel tasks.

1.4 Present Study

This study will compare the existing module sequence in the current PGBM-COMPS tutor (A-**B**-C-D-E) with an alternative instructional sequence of A-C-**B**-D-E. An analysis of variance (ANOVA, 2 groups x 4 times) with repeated measures of time (pretest, posttest, and one follow-up test) will be used to compare which sequence of mathematical tasks is more effective for students with MD in enhancing multiplicative concepts and problem solving. This study will contribute to the literature by (1) examining the effectiveness of the PGBM-COMPS tutor program in improving the multiplicative reasoning and problem solving accuracy of students with MD, and (2) indicating the differentiated mathematics task sequence for students with MD.

1.4.1 Research Questions

In order to meet the above goals, the following questions will be addressed in this study:

1. What are the differential effects of the two instructional sequences (i.e., A-**B**-C-D-E vs. A-C-**B**-D-E) on the mathematics performances students with MD measured by the percentage of correct responses on a criterion test designed to evaluate students' multiplicative reasoning and problem solving (MR-test)?
2. Are students in both groups able to maintain the newly-acquired problem-solving skills after the completion of the PGBM-COMPS tutor program measured by the percentage of correct responses on a criterion test (MR-test)?
3. Are students able to transfer the constructed multiplicative concept to novel word problems, measured by a comprehensive multiplicative word problem solving test (COMPS test, Xin, Wiles, & Lin, 2008), that entail similar structure but are different from problems used during the intervention?
4. To what extent does the word problem-solving performance of students with MD differ with that of the normative reference (NR) group prior to and following the intervention on the MR criterion test?
5. To what degree are the instructions in the PGBM-COMPS tutor program perceived as useful, effective support for multiplicative word problem solving by the students and their classroom teachers?

Null hypotheses. From the above four questions, the following null hypothesis and alternative hypothesis were developed:

1. (a) There are no statistically significant differences between the two groups (A-**B**-C-D-E vs. A-C-**B**-D-E) following the intervention based on the percentage of correct response for the MR criterion test. (b) There is no statistically significant effect on time (i.e., pretest, posttest, maintenance, and follow-up tests) for both groups based on the percentage of correct responses for the MR criterion test. (c) There is no statistically significant interaction between group and time.
2. (a) There are no statistically significant differences between the two groups (A-**B**-C-D-E or A-C-**B**-D-E) following the intervention based on the percentage of correct response for the MR criterion test. (b) There is no statistically significant effect on time (i.e., pretest, posttest, maintenance, and follow-up tests) for both groups based on the percentage of correct responses for the MR criterion test. (c) There is no statistically significant interaction between group and time.
3. (a) There are no statistically significant differences between the two groups (A-**B**-C-D-E or A-C-**B**-D-E) following the intervention based on the percentage of correct response for the COMPS test. (b) There is no statistically significant effect on time (i.e., pretest, posttest, maintenance, and follow-up tests) for both groups based on the percentage of correct responses for the COMPS test. (c) There is no statistically significant interaction between group and time.

4. There is no statistically significant effect on time (i.e., pretest & posttest) for both groups based on the percentage of correct responses for the MR criterion test.

Alternative hypotheses.

1. (a) There are statistically significant differences between the two groups (modules A-**B**-C-D-E vs. A-C-**B**-D-E) following the intervention based on the percentage of correct response for the MR criterion test. The hypothesis is as follows: the mean posttest for the A-C-**B**-D-E group will be greater than the mean posttest score for the A-**B**-C-D-E group. (b) There is a statistically significant effect on time (i.e., pretest, posttest, maintenance, and follow-up tests) for both groups based on the percentage of correct responses for the MR criterion test. The hypothesis is as follows: the mean score of the posttest will be greater than the mean score of the pretest. (c) There is a statistically significant interaction between group and time on the percentage of correct responses for the MR criterion test. The hypothesis is as follows: the increase in the mean score from pretest to posttest, maintenance test, and to follow-up test for the A-C-**B**-D-E group will be similar to the increase in the mean score for the A-**B**-C-D-E group.
2. There is statistically significant effect on time (i.e., posttest, maintenance test, and follow-up tests) for both groups based on the percentage of correct responses for the MR criterion test. The hypothesis is as follows: there is no significant change of performance from posttest to maintenance test to follow-up test for both groups.
3. (a) There is statistically significant differences between the two groups (A-**B**-C-D-E or A-C-**B**-D-E) following the intervention based on the percentage of correct response for the COMPS test. The hypothesis is as follows: the mean posttest for the

A-C-**B**-D-E group will be greater than the mean posttest score for the A-**B**-C-D-E group. (b) There is statistically significant effect on time (i.e., pretest, posttest, maintenance, and follow-up tests) for both groups based on the percentage of correct responses for the COMPS test. The hypothesis is as follows: the mean score of the posttest will be greater than the mean score of the pretest. (c) There is statistically significant interaction between group and time. The hypothesis is as follows: the increase in the mean score from pretest to posttest, maintenance test, and to follow-up test for the A-C-**B**-D-E group will be similar to the increase in the mean score for the A-**B**-C-D-E group.

4. There is a statistically significant effect on time (i.e., pretest, posttest) for both groups based on the percentage of correct responses for the MR criterion test. The hypothesis is as follows: the mean score of the posttest will be greater than the mean score of the pretest of the MR criterion test.

CHAPTER 2. LITERATURE REVIEW

2.1 Students with Mathematics Difficulties

Under IDEA, students with specific learning disabilities (SLD) are defined as "a disorder in one or more of the basic psychological processes involved in understanding or using languages spoken, or written, which may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations" (U.S. Office of Education, 1977, p. 65083). One of the major diagnostic criterions for students with SLD is the discrepancy between a child's IQ and achievement. In particular, a child who has normal IQ level, based on the child's age, but fails to achieve at that level (Raymond, 2000, Vaughn & Fuchs, 2003). One of the areas of SLD is mathematics disability. According to the Diagnostic and Statistical Manual of Mental Disorders (DSM-IV-TR, 2000), students are diagnosed as mathematics disability when they are "substantially below that expected given the person's chronological age, measured intelligence, and age-appropriate education" (p. 53) measured by standardized tests (Zentall, 2014).

An estimated 5-8% of school-age children are at-risk of mathematics disabilities (Geary, Hoard, Byrd-Craven, & DeSoto, 2004). These students are often referred as students with mathematics difficulties (MD). Many researchers have strived towards developing ways to best define students with (MD) (e.g., Fuchs, Fuchs, & Vaughn, 2008; Geary et al., 2007; Mazzocco & Myers, 2003; Murphy, Mazzocco, Hanich, & Early, 2007). With a divergence of terms used to describe MD (Hannell 2005), there is still no one agreeable operating definition of MD among the researchers. Diagnostic criteria for mathematics learning difficulties employed in the United States often include a student's performance in class (Geary, 2007; Jordan & Hanich, 2000) as well as a student's standardized achievement scores in mathematics (Geary, 1990). A common approach to measure and define students with MD is through the use of cut off scores (Geary et al., 2007). Some researchers have used a strict criterion cut off score, ranging between the 15^{th} to 35^{th} percentile (Fuchs, et al., 2008; Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009), whereas other researchers have attempted to further differentiate between mathematics difficulty and disabilities by having a range of scores from lenient to restrictive. For instance, Geary and colleagues (2007) suggested that researchers use $< 30^{th}$ percentile for lenient and restrictive ($< 5^{th}$ or 10^{th} percentile). As the purpose of this study was to examine the intervention developed particularly for students with MD, I purposefully use the term mathematics difficulty and have chosen the lenient 35^{th} percentile as the cut-off score for this study (Jordan, Hanich, & Kaplan, 2003).

More recently, the identification process of learning disability has been transitioning away from the traditional IQ-achievement discrepancy model and moving towards the "Response to Intervention" (RtI) model. RtI constitutes a multi-tiered approach to identifying children with learning disabilities who did not respond to evidence-based interventions (Fuchs & Fuchs, 2006; Hughes & Dexter, 2011; Lembke, Hampton, & Byers, 2012). The RtI model identifies potential at-risk students through early screening before the onset of extensive academic deficits.

During RTI, students who are identified as at-risk based on the universal screening measure go through three tiers of instruction. As they move on to further tier level, students are provided with more intensive progress monitoring and intervention. During Tier 1 instruction, the teacher monitors students' progress weekly to bi-weekly (Fuchs & Fuchs, 2006). If they do not show sufficient response, students move on to Tier 2 instruction where small group instruction and more progress monitoring by using curriculum-based measurement (CBM) are provided to them. Students who do not respond to Tier 2 move on to Tier 3 level. Tier 3 instruction provides students with more individualized instructions with special education services (Hughes & Dexter, 2011). According to Fuchs and colleagues (2007), those who are unresponsive towards Tier 2 are likely to be identified as having a learning disability.

Fuchs and Deshler (2007) suggested that students should be screened as early as kindergarten or first grade. Based on the results of students' screening and progress monitoring, appropriate evidence-based interventions should be used to target the particular weak areas in mathematics. Hughes and Dexter (2011) illustrate a positive impact on academics, including mathematics. Although more recent attention has been focused on the area of mathematics, there is still a lack of literature on the use of RTI on this subject (Hughes & Dexter, 2011; Lembke et al., 2012). Furthermore, Hughes and Dexter (2011) emphasized a lack of research dealing with the evidence-based interventions for higher level thinking skills that could potentially be applied within the response to intervention (RTI) systems of service delivery.

2.1.1 Characteristics of Students with MD

Students with mathematics difficulties (MD) share similar characteristics to students with mathematics learning disabilities (MLD). For every 2-3 years of academic learning, students with MLD often improve by 1 year's worth of learning (Zentall, 2007). Difficulty with higher-order thinking skills is considered as one of the crucial deficits of MLD. Students with MLD undergo various academic problems, including conceptual understanding (Montague, 1997) and problem-solving skills. Conceptual knowledge is established when children link together bits of discrete information, obtained through personal observation and experience outside the formal learning environment (Goldman & Hasselbring, 1997). Children's conceptual knowledge expands as they make links among information. However, students with MLD as well as MD struggle with conceptual knowledge because they are not successful in creating these links.

Numerosity skills. According to Butterworth, Varma, and Laurillard (2011), one of the core deficits students with MD face is the concept of numerosity (sense of numbers). Numerosity skills are often developed naturally as early as 3 months old (Piazza et al., 2010). In particular, students with MD struggle to understand the meaning of the numerical quantities (Butterworth et al., 2011), which leads them to struggle to understand the meaning of math facts (Butterworth et al., 2011). Students with MD have poor understanding of the concept of number sets and poor ability to manipulate sets to solve problems (Geary et al., 2007). For instance, when solving for 18 + 4, students with MD are less likely to decompose the 4 into two sets of 2 to find the total (Koontz & Berch, 1996).

Cognitive skills. Mathematics requires various cognitive skills, including working memory (Butterworth et al., 2011; Swanson & Lee, 2001). Working memory is defined as "the ability to hold a mental representation of information in mind while simultaneously engaging in other mental processes" (Geary et al., 2007). According to Baddeley and Hitch's model, working memory consists of central executive, phonological processing, and visuo-spatial information (Baddeley, 1986, 1996). While the role of central executive processing is to arrange decision-making and flow of information, phonological processing and visual-spatial information store domain-specific information (Meyer et al., 2010).

Students with MD are well known for their deficiency in working memory span tasks compared to normal students (Swanson, 1993; Swanson & Lee, 2001; Zentall 2014). Poor working memory may compromise their problem-solving skills (Passolunghi and Pazzaglia, 2004). One of the reasons is due to their low phonological processing skills (Swanson, 1993). As such, students with MD struggle to store the verbal information in mind while simultaneously performing in other mental processes, such as decoding the text and identifying the solution strategy (Swanson & Lee, 2001).

2.2 Students' Development of Multiplicative Concepts

Multiplication word problem solving can be permanently enhanced as students build their multiplication concepts. Establishing multiplication concepts occurs when a child makes a conceptual leap from addition to multiplication. This leap is made when students reconceptualize numbers and units through exploration to ultimately internalize numbers in an abstract manner (Steffe, 1994). Lamon (1996) indicated that more sophisticated understanding of unit structure is one of the crucial mechanisms by which multiplicative reasoning develops. Previous literature on how normal achieving students develop multiplicative concepts, studies mainly the changes of the conception of the unit as students' counting schemes are examined (Clark & Kamii, 1996; Steffe, 1988; Steffe & Cobb, 1988). As defined in the literature, a scheme is "a psychological construct for inferring into the mental realms of thinking and learning" (Tzur et al., 2012, p. 156). A multiplication counting scheme indicates students' counting acts while solving multiplication word problems. According to the constructivist point of view, students undergo numerous adjustments to re-establish their counting acts through their personal experience of counting (Steffe & Cobb, 1994). Children who established multiplication concepts can coordinate between ones and a composite unit (CU).

Overall, establishing CU involves two major counting schemes: unit coordination and unit segmenting schemes (Steffe, 1992). Steffe and Cobb's study (1988) indicated that young children obtain the ability to coordinate with the two units (i.e., ones & CU) as they internalize the numbers by undergoing major developmental number sequences. Students continually refine the notion of units from singletons: "1, 2... 3, 4... 5, 6" to iterable units: "2, 4, 6" (one iterated two times leads to one two, which can also be broken down to two ones) to forming composite units (CU): "2x3 is two units of three" (Steffe, 1992). Students in this stage can mentally understand that three ones are taken as one three (Steffe, 1992). As children reach a level where they are able to construct CU in their minds, building the concept of CU through unit coordination, normal-achieving students begin constructing the scheme of segmenting the units through decomposing CU to ones. According to Lamon (1996), "growth in sophistication of the unitizing process, signified by the use of more composite units or larger units, should be reflected in students' partitioning process" (p. 172). Steffe (1992) conducted teaching experiments exploring the developmental changes of unit segmentation scheme with 8-year-old normal achieving children as they underwent numerous adjustments of the number sequences. Thus, those who have established multiplication concepts have enough understanding of the relation between the two quantities to be able to flexibly interchange between the acts of unitizing and decomposing based on the CU (Steffe & Cobb, 1988). However, no empirical studies have examined the development of multiplicative scheme of students with MD.

2.3 Existing Interventions in Special Education

While the field of general education has been focusing on how children develop concepts, the field of special education is more intrigued by developing intervention that would enhance their problem-solving skill. Mathematical word problem-solving instructions for students with MD focus on explicit strategy training to enhance students' acquisition, maintenance, and generalization of the learned skills (Hord & Xin, 2013). The following section discusses the different word problem-solving strategies.

2.3.1 Interventions of Multiplicative Word Problem Solving

Schema-based instruction (SBI). The problem-solving instruction that addressed SBI as a strategy emphasized the identification of crucial elements (e.g., key words) that relate to the problem schema (Marshall, 1990; Riley et al., 1983) and demonstrated a positive effect on students' word problem solving performance with a wide range of age groups from elementary (Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Jitendra & Hoff, 1996) to college level (Zawaiza & Gerber, 1993). A schematic approach focuses on semantic analysis of word problems and identification of shared problem schemata. In this dissertation study, problem type will be defined as a set of the problems incorporating the same schema. The schematic approach allowed students to identify the correct problem type based on their interpretation of the situation and key words in the word problem (problem situation) and then choose the correct representational diagram and the operation sign to solve the problem (Xin & Jitendra, 1999). The number of problem types can be categorized by available operation signs for students. For instance, some schema studies categorize addition, subtraction, multiplication, and division word problems into five different problem types: change, group, compare, restate, and vary (Marshall et al., 1989), while other studies categorize addition and subtraction word problems into three different types: change, compare, and parts and total (Jitendra & Hoff, 1996).

According to Jitendra and Hoff (1996), the SBI strategy improved students' one-step addition and subtraction word problem performances. In particular, Jitendra and colleagues (1998) investigated the differential effects of two problem-solving instructional approaches, SBI and general strategy instruction, on solving one-step addition and subtraction problems with 34 elementary students who were at-risk or had mild disabilities. The students in the SBI group were encouraged to complete the following training phases: (a) identify the problem type based on the story situations, (b) map the important elements problem onto the schema diagram, (c) identify the key word to determine the appropriate operation depending on the problem type, and (d) solve the problem. Results indicated that students receiving SBI significantly outperformed the general instruction group on an immediate posttest, a delayed posttest, and a generalization test.

In related work conducted with low-performing elementary students, Fuchs and colleagues (2004) additionally found that merging SBI strategy and guided schema-based sorting practice (e.g., what kind of problem is this? Is this a transfer problem? What kind of transfer problem is it?) showed more promising results than that of an SBI-only strategy for students with and without disabilities.

Conceptual model-based problem solving approach (COMPS). Building on existing research, Xin and colleagues (Xin, 2012, Xin, Wiles, & Lin, 2008, Xin et al., 2011) recently developed the COMP approach that focuses on understanding and representing word problems in mathematical model equations. This approach prevents children from relying on rules to determine the choice of operation for solution. The COMPS approach helps students to apply their conceptual understanding of multiplication, for instance, by representing the problems in a COMPS diagram, and developing a solution plan driven by model equations.

Preliminary results (e.g., Xin, Wiles, & Lin, 2008) showed that the COMPS approach has facilitated students' mathematics problem solving performance on researcher-developed criterion tests as well as the problem-solving subtest of the norm-referenced standardized tests, such as KeyMath Revised Normative Update (KMR- NU; Connolly, 1998) and Stanford Achievement Test-10th edition (SAT-10; Pearson Inc., 2004). According to Xin and Zhang (2009), the COMPS approach also showed positive effects on facilitating students' complex real-life mathematics word problem solving skills that involve rounding up or down a solution involving a decimal, irrelevant problems, pictograph problems, and multi-step problems.

In particular, Xin et al. (2011) examined the differential effects of two problem-solving instructional approaches, COMPS and general heuristic instructional approach (GHI), on enhancing the multiplication reasoning and problem-solving skills of 29 third and fourth grade students with learning problems in mathematics problem-solving skills. The students in the COMPS condition were asked to solve multiplication word problems (i.e., EG and MC problem type) using the conceptual model diagram and four-step problem-solving cognitive strategy called DOTS (Xin, Wiles, & Lin, 2008). Using the DOTS checklist, students (a) detect the problem type (i.e., Equal Group or Multiplicative Comparison problems), (b) organize the information using the conceptual model, (c) transform the diagram to a math equation, and (d) solve for the unknown quantity and check work. The students in the GHI condition were asked to solve multiplication problems using a five-step problem-solving checklist, "SOLVE (Search-Organize- Look- Visualize-Evaluate)" (p. 836). Using the SOLVE checklist, students were asked to (a) search the question, (b) organize the information by highlighting the key words, (c) look for a strategy and identify which operation to use, (d) visualize the problem situation, and (e) evaluate the answer. Results showed that students receiving COMPS instruction significantly outperformed those receiving GHI instruction on the researcher-developed criterion test as well as the problem-solving subtest of a norm-referenced standardized test.

Cognitive /Metacognitive instruction. During the past decade or so, many interventions have applied the cognitive perspective to teach word problem solving of children with MD using direct modeling (Woodward, 2004). One of the interventions used in the field of special education is cognitive/metacognitive instruction where students learn several cognitive steps to understand problem-solving procedures. The cognitive steps usually involve dividing the word problem-solving process into four to eight sequenced steps, with cues presented depending on the content. All of these studies entailed comparable critical solving steps: (a) read the problem, (b) think about the solution process, (c) write the math sentences, (d) solve the problem by computing, and (e) recheck the solution process and final answer. The metacognitive strategy was used to support children's self-regulation based on the given cognitive strategy.

A number of studies investigated the effectiveness of the cognitive/metacognitive strategies on the mathematical word problem-solving performance of students with LD (Case, Harris, and Graham, 1992; Montague; 1992; Montague et al., 1993; Montague & Bos, 1986). For instance, Case and colleagues (1992) conducted a preliminary study that investigated the effectiveness of the cognitive and meta-cognitive instruction for enhancing the addition and subtraction word problem-solving skills to four elementary students with LD. The children were explicitly taught the five-step strategy (Read the problem aloud, Look for important words and circle them, Draw pictures to illustrate the situation, Write down the math sentence, Write down the answer) accompanying it with self-regulation strategy (i.e., self-assessment, self-recording, self-instruction). Initially, the teacher explicitly modeled the procedure with a think-aloud strategy. As the instruction further progresses, students had more responsibility in applying the strategy to obtain 100% accuracy on the targeted word problems. Overall, the results indicated that all students improved their performance on addition and subtraction word problems during the intervention phase. Montague (1992) and Montague and colleagues (1993) further conducted research on the effects of integrating cognitive and meta-cognitive strategies with explicit instruction on the word problem solving performance of students with LD. Results showed substantial improvement not only on posttest, but also on generalization and maintenance tests.

Concrete-semi-concrete-abstract (CSA) sequence. Throughout the years, many prominent math educators (e.g., Bruner, 1996; Piaget & Inhelder, 1958) believed that math is learned by going through a sequence of "enactive, iconic, and symbolic stages" (as cited from Miller & Mercer, 1993, p. 89). This is particularly crucial for students with MD as many of them fail to construct abstract concept as they are required to "understand theoretical properties and think beyond what a person can touch or see" (Witzel, Mercer, & Miller, 2003, p. 121). CSA sequence instruction, emphasizing the use of manipulative and pictorial representations, has been shown to be effective in developing students with MD to build mathematical conceptual knowledge (Allsopp, 1999; Butler et al., 2003; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Miller & Mercer, 1993; Underhill, Uprichard, & Heddens, 1980). Using those representations, the CSA instructional approach allows students to illustrate their mathematical concepts at one of the three cognitive operational levels (concrete, semi-concrete, and abstract). The goal of the CSA approach is to have children further build on their current cognitive level of mathematical conceptual understanding and advance in levels until they are able to demonstrate the concept and solve the problems in an abstract manner. During the concrete level, students were asked to use concrete manipulative devices (e.g. Unifix cubes) to solve for the problems. According to Miller and Hudson (2007), the instruction progresses to the semi-concrete level when students have reached a pre-specified mastery level (e.g., 80% accuracy on independent practice tasks) by using the concrete objects. During the semi-concrete level, students were asked to use any sorts of iconic imagery (e.g. drawing pictures or tallies) to illustrate their understanding. The instruction progresses to the abstract level when students have reached a pre-specified mastery level (e.g., 80% accuracy on independent practice tasks) by using the pictures. During the abstract level, students were asked to use mathematic symbols (e.g. operations, numbers) to illustrate their understanding.

Computer-assisted instruction (CAI). Computer-assisted instruction (CAI) is an alternative tool that could deliver instructional content tailored towards the content areas and the needs of students (Larkin & Chabay, 1992). Researchers in the field of education have been incorporating mathematics instruction into CAI programs for students with learning disabilities (Babbitt & Miller, 1996). While previous CAI programs were developed to enhance students' computational skills (Babbitt & Miller, 1996) by providing a drill-and-practice environment (Okolo, 1992), recent CAI programs have been developed to improve students' word problem solving skills. According to the findings from research synthesis by Jitendra and Xin (1997), CAI for problem solving skills provided similar positive outcomes as the instructions delivered by the teacher (i.e., not significantly different). Furthermore, findings from meta–analytic reviews by Seo and Bryant (2009) reported that the current CAI studies in computation and word problem skills for students with LD demonstrated small but positive effect sizes with non-significant math gains.

CAI programs for mathematics word problem solving have the capability of providing ongoing prompts and feedback based on students' performances (Kappa, 2001). Furthermore, they bring forth more personalized instruction by recording a student's learning by indicating error rate, error patterns, time, and progress monitoring (Chang, Sung, & Lee, 2006), leading towards its use as a self-study tool (Abidin & Hartley, 1998). The chance to learn based on students' pace would allow them to have control over their own learning, which would further increase their motivation (Chen & Liu, 2007).

Academic improvements with technology rely primarily on the effectiveness of the instructional approaches used during tutor-student interaction (Clark, 1983). Studies show that word problem CAI programs developed for struggling students incorporated evidence-based practices such as schema-based (Chang, Sung, & Lin, 2006; Leh & Jitendra, 2012), cognitive/metacognitive strategies (Seo & Bryant, 2012; Shiah, Mastropieri, Scruggs, & Fulk, 1994-1995), and COMPS (Xin, 2012). However, there were fewer CAI programs focusing on higher mathematical thinking (e.g., concept-based approach and mathematical reasoning).

Thus, there is a lack of research on exploring possible reform-based interventions developed for students with MD despite the crucial need for them to catch up with the recent reform agenda. Thus, further research is needed to find ways to successfully engage students with MD in a constructivist-learning setting, as a means to gain experience with "the process of gathering, discovering, and creating knowledge in the course of some activity" (Romberg, 1992, p.61).

2.4 Collaborative Work between General Mathematics and Special Education

Much of the research has focused on how normal-achieving children develop multiplicative concepts. More recently, the multiplicative concept studies have expanded due to increasing interest in exploring differentiated instruction for students with MD in order to nurture multiplicative concepts (Ma et al., 2014; Park et al., 2013; Xin, 2012; Xin et al., 2013).

As an outcome of a collaborative work that integrates best practices from mathematics education and special education, Xin, Tzur, and Si (2008) developed an intelligent tutor (PGBM-COMPS) that nurtures multiplicative reasoning to students with LD or those at-risk of LD, particularly in mathematics problem solving. The PGBM-COMPS tutor program entails both a constructivist view of learning and conceptual model-based problem-solving approach to establish fundamental multiplicative reasoning concepts in an explicit manner. As part of the PGBM-COMPS program, the "Please Go and Bring Me…" (PGBM) turn-taking game nurtures children's creation and differentiation between the unit of ones and the CU (Tzur, Xin, Si, Woodward, & Jin, 2009). During the PGBM component, the program asks students to manipulate the Unifix cubes to form same-sized towers while solving various PGBM activities, which are aligned with the six multiplicative schemes. The Conceptual Model-based Problem Solving (COMPS, Xin, 2012) allows students to generalize their understanding of the two units to a more abstract mathematical model. During COMPS, students translate the mathematical relation in a word problem to a conceptual model equation to solve various real-life Equal Group (EG) and Multiplicative Comparison (MC) word problems.

2.4.1 The Teaching Experiment

Elaborating on the hypothetical learning trajectory of multiplication applied on Steffe's constructivist teaching experiments (e.g., Steffe, 1992) and based on a teaching experiment from the Nurturing Multiplicative Reasoning in Students with Learning Difficulties (NMRSD) project, Tzur et al. (2012) developed a developmental framework that separates multiplicative development into six stages and providing sample tasks that are linked to each scheme.

The **first scheme** focuses on a child constructing *Multiplicative Double Counting* (mDC, Woodward, et al., 2009). The child's goal is to identify the total of ones through simultaneous counting of CUs and ones that comprise each CU. For example, students may say: 2, 4, 6, 8 to solve two times four. Students may use their hands to double count by using one hand to keep track of the ones and the other hand to keep track of the CU. According to Kouba (1989), double counting may require more abstract processing skills, as students need to keep track of two counting sequences.

The **second scheme** involves *Same Unit Coordination* (SUC), where students operate on CUs. This scheme typically asks students to find the sums of or differences between the two sets of CUs. For instance, it may ask: "Sam brought $7T_3$ and then Tom brought $4T_3$; How many towers do they have in all?" or "Sam brought $7T_3$; Tom brought a few more; Together, Sam and Tom have $11T_3$; How many towers did Tom bring?" (Tzur et el., 2012). Under this scheme, the child needs to pay attention to the CU and figure out the sum or difference.

The **third scheme** involves *Unit Differentiation and Selection* (UDS, McClintock, Tzur, Xin, & Si, 2011). This scheme encourages children to indicate the similarities and differences between two sets of CUs, and to find the differences in ones. For instance, "Sam has $7T_3$ and Tom has $4T_3$; How are our collections similar? Different? How many more cubes do you have?" (Tzur et el., 2012). There are two possible ways children could solve UDS. They can find the difference in 1s by either operating both sets on 1s and then obtaining the difference (e.g., (7x3) – (4x3)) or operating both sets on CU to find the difference of CU, which they would then multiply by the unit rate (e.g., 3 (7-4)) to operate on 1s. Unit rate represents the number of singletons in each CU (Xin, 2012). According to Tzur et al. (2012), the UDS scheme is the onset of distinguishing between CU and 1s.

The **fourth scheme** involves *Mixed-Unit Coordination* (MUC, Tzur et al., 2009). During MUC, children are coordinating and segmenting the units (Steffe, 1992). For instance, the scheme may ask: "Sam has $7T_3$; Tom gave Sam 18 *more cubes*; How many towers of 3 would Sam have in all?" (Tzur et al., 2012). MUC encourages children to segment the 1s into CU based on a given unit rate (18 cubes= $6T_3$) and add the total number of towers of both collections. According to Tzur and colleagues (2012), MUC provides a basis for partitioning a totality, as required for division.

The **fifth scheme** involves *Quotitive Division* (QD). QD encourages children to iterate the CUs or segment the 1s into equal-sized groups. For instance, it may ask: "Jessica has 36 cubes. She wants to make towers of 4 cubes in each. How many towers will she make?" QD considers division as an inverse operation to multiplication (Tzur, et al., 2012).

The final, **sixth scheme** is *Partitive Division* (PD). PD involves fair sharing where students are asked to equally distribute 1s to the given number of CUs for the solution.

For instance, PD may ask: "You want to put 36 cubes in 9 equal towers. How many cubes will you have in each tower?" (Tzur et al., 2012). Children may initially solve PD by distributing all given 1s to each CU one by one and gradually recognize that each round of distribution of 1s would make a CU. Once they have established this concept, children can solve PD tasks by using double counting to find the unit rate (# of items in each CU) without performing the distribution.

The above sequence of multiplicative tasks was arranged as such for a few reasons. First, Tzur et al. (2012) proposed that the above sequence of multiplicative tasks unique to students with MD in constructing the multiplicative schemes to establish the concept of multiplication. Second, the task in each proceeding scheme could challenge students to establish more complex multiplicative schemes by having them solve novel multiplicative situations using their existing multiplicative schemes (assimilation) and modifying the existing multiplicative scheme (accommodation; Steffe & Cobb, 1988). According to Piaget, assimilation and accommodation are general processes that children go through to establish a more challenging multiplicative scheme (i.e., adaptation; Vergnaud, 1994). In the context of multiplication, adaptation is defined as a child's ability to flexibly modify the current multiplicative scheme to solve for a novel task (Steffe & Cobb, 1994). As students establish multiplicative schemes by undergoing numerous adaptation process, their number sequence ability advance (Tzur et al., 2012). That is, various multiplicative tasks strengthens their understanding of CU as well as their ability to coordinate both CU and ones to solve multiplication word problems.

However, not all students undergo the same learning trajectory to establish multiplicative concepts. Simon and Tzur (2004) noted that the trajectory of learning mathematics would constantly need to be crafted based on students' existing knowledge in mathematics and cognitive abilities. Thus, the effect of the above sequence of multiplicative schemes, based on the teaching experiment involving a few students with LD needs more empirical research to support this hypothesis. That is, it is still unclear how the six multiplicative schemes tasks would be best sequenced to benefit students with MD.

2.4.2 Findings from Preliminary Studies of the PGBM-COMPS Tutor Program

Xin and colleagues (Ma et al., 2014; Park et al., 2013; Xin, Hord, et al., 2012, Xin et al, 2013) conducted several studies exploring the impact of the PGBM-COMPS intelligent tutor on enhancing the problem-solving ability of students with MD. The outcomes of preliminary studies that investigated the PGBM-COMPS program using single-subject design (Ma et al., 2014; Park et al., 2013; Xin, Hord, et al., 2012) indicated that there seems to be a functional relationship between the intervention of this tutor and students' performance on both a researcher-developed multiplicative reasoning (MR) criterion test and a comprehensive multiplicative word problem-solving test (Xin, Tzur, & Si, 2008).

In addition to single subject design studies, Xin and colleagues (2013) conducted a randomized group comparative study which investigated the differential effects of two problem-solving instructional approaches, PGBM-COMPS and traditional teacher-delivered instruction (TDI), on enhancing multiplication reasoning and problem-solving skills with 17 third and fourth grade students with learning difficulties in mathematics. The students in the PGBM-COMPS group were asked to work on the five modules of the

PGBM-COMPS intelligent tutor system. During the tutoring program, students were encouraged to manipulate the cubes and towers as they went through each multiplicative scheme tasks. They were also asked to complete the following sequential steps during the COMPS component: (a) detect the problem type (i.e., EG or MC), (b) organize the information using the conceptual model, (c) transform the diagram to a math equation, and (d) solve for the unknown quantity and check work (Xin, Wiles, & Lin, 2008). The students in the TDI group worked on the same word problem tasks as the PGBM-COMPS group taught by the two third or fourth grade classroom teachers. These teachers used typical word problem-solving strategies used in the classroom such as using (a) repeated addition or subtraction to solve multiplication or division problems, (b) guess and check or key word strategies to decide which operation to use for the answer, and/or (c) multiplication or division directly with no further explanation. Results indicated that students in the PGBM-COMPS group had a significantly higher improvement rate on their multiplicative problem-solving performance than the students in the TDI group. More importantly, only the students in the PGBM-COMPS group significantly improved their performance, from pre- to posttest, on a far transfer norm-referenced assessment (Xin et al, 2013).

2.4.3 Struggling Students' Response to the Six Multiplicative Schemes

Although the above preliminary studies show that overall, students' performance was enhanced after the intervention with the PGBM-COMPS tutoring system, more in-depth analyses of the process data from the single-subject studies clarify how students with MD progress across each of the modules of the PGBM-COMPS tutoring system. For instance, Xin, Hord, et al. (2012) indicated that students with LD and those at-risk for LD showed variations in performance across six multiplicative schemes. In this study, in order to monitor students' learning progress in multiplicative schemes during the intervention phase (working with the tutor program), students were asked to complete two probes after they completed each of the five modules in the PGBM-COMPS program. Each of the two equivalent probes consisted of six items that were similar to the tasks presented in each of the modules. Results suggested that all three participants did not perform as well on MUC problems (Module B) as on the other module problems. In particular, none of the participants obtained above 20% correct in the MUC probes. Xin, Hord, et al. (2012) noted that the knowledge obtained through the schemes prior to the MUC (i.e., mDC, SUC, and UDS) may be insufficient for low-performing students to make the conceptual leap to MUC.

2.4.4 Mixed Unit Coordination (MUC)

According to research with normal-achieving students (Steffe, 1992), the *Mixed Unit Coordination* (MUC, Tzur et al., 2009) scheme is introduced prior to engaging in divisional schemes (i.e., QD & PD), as it serves as a foundation of division that encourages students to decompose the total number of 1s (Tzur et al., 2012) by modifying the iterative scheme (i.e., double counting). The MUC task introduces children to the initial steps of division by requiring them to adapt the pre-existing iterative scheme knowledge that they obtained from the previous multiplicative schemes (i.e., mDC, SUC, & UDS). Specifically, the MUC task encourages children to flexibly modify an iterative scheme to solve for a divisional scheme (i.e., unit-segmentation), which they have not yet established. The MUC task triggers such adaptation by providing situations where children need to perform both unit-coordination and unit-segmentation and invert the two counting acts for the solution (Steffe, 1992). Thus, the MUC scheme enables the transition from a multiplicative to a divisional scheme.

During the MUC task, children need to identify which unit to operate on (ones or CU) and coordinate with the two units (i.e., CU & ones), and then segment the ones into CUs. There are two ways children could approach the MUC task by reversing the unit segmenting and unit-coordinating schemes for the solution. First, if the question asks: "Sam has $7T_3$. Tom gave Sam 18 *more cubes*. How many towers of 3 would Sam have in all?", students could operate with units of ones by converting the towers of the first collection into cubes to find the total number of cubes (i.e., 7T3= 21 cubes; 21 cubes+18 cubes= 39 cubes) and segment the ones into CU according to the given unit rate (i.e., segment 29 cubes into towers of three; 7T3). The second approach is to operate with units of CU by segmenting the cubes of the second collection to identify the number of units of threes they can make out of 18 cubes (i.e., 6 towers of three) and then find the total number of CUs (i.e., 13 towers of three). Both approaches require children to determine which unit to operate on (e.g. ones or CU) and coordinate the two units to solve for the solution as well as segment the ones into CUs.

Steffe (1992) conducted a study on 8-year-old normal-achieving children's development of the multiplicative scheme and described the use of this counting scheme for solving the MUC task. He found that high achievers, who were not yet exposed to the divisional scheme, frequently used repeated addition to represent the ones in order to continue to form a CU identical to that of the previous set. Students obtained the answer by adding the number of CUs on both sets. Using the above MUC task, for example, a student may first recognize that the first set is already organized into units of three and continue segmenting the ones to counting-by-threes (e.g., 3, 6, 9, 12, 15, 18; 6 towers). That is, the child modified the iterative scheme and used it to find out how many iterations of CU he would need to construct to get known number of ones. The act of iterating-up-to unit is one of the indicators that students have constructed "flexibility of iterative scheme" (Steffe and Cobb, 1994, p.54). This ability allows children to flexibly adapt the iterative scheme to iterate-up the units when attempting to solve novel tasks within a variety of settings (Steffe & Cobb, 1994).

Although the current placement of the MUC task may facilitate some children's to establishment of MUC scheme through flexibly modifying an iterative scheme to solve for a divisional scheme, the current placement of the MUC tasks has been shown to be difficult for students with learning disabilities (LD) and those with MD to adapt their pre-existing knowledge of the iterative scheme to novel situations by themselves without receiving direct guidance from the teacher (Ma et al., 2014; Park et al., 2013; Xin, Hord, et al., 2012). That is, students with MD experience difficulty assimilating and accommodating mDC, SUC, and UDS schemes to solve for the MUC task. Weakness in the child's executive functioning skills might have contributed to his or her low performance on adaptation skills (ability to apply acquired knowledge to novel tasks), one of the crucial characteristics of students with MD. Particularly, students with MD lack ability to review long-term memory and match previously learned information to the newly encountered task (Bottge, 2001).

Xin, Hord, et al. (2012) also stated that the MUC scheme may be difficult for students with MD as it involves a two-step problem (e.g., I have 8 towers of 9 cubes in each and 45 more single cubes. If I put the 45 single cubes in towers of 9 cubes each, how many towers of 9 will I end up with?), where they need to operate multiplicatively through unit segmentation (e.g., 45÷9 = 5; $5T_9$) and addition (e.g., $8T_9 + 5T_9$). According to many researchers (e.g., Bottge, 2001; Swanson & Lee, 2001), students with MD struggle to solve for two-step word problems due to their cognitive disadvantages. Because of their disabilities, children with MD often have restricted amount of processing capacity available to solve for complex mathematics word problems. Solving MUC tasks exceeds their available processing capacity, as they require identifying the two units (ones and composite unit) and coordinating these two units multiple times for the solution.

Due to inadequate cognitive processing capacity, children with MD also have a considerable difficulty performing working memory (Siegel & Ryan, 1989; Swanson, 1993), leading them to struggle with successfully grasping the result from the first step and then applying it to the second step for the final solution, which is a necessary skill needed to solve for the MUC task.

Based on the above limitations, children with MD might benefit from direct guidance on divisional scheme (QD) prior to solving the MUC task. Establishing the division concept prior to MUC task could facilitate their understanding of dividing ones (cubes) into CU (equal-sized towers) to find the number of equal-sized towers using a collection of cubes. Furthermore, the development of divisional scheme could provide more exposure to new ways to operate and coordinate the two units. This experience may help reduce the heavy cognitive load required to successfully solve MUC task. Thus, further research is needed to explore differentiated instructions that would allow students with MD to construct fundamental multiplication concepts “by linking new learning to previously acquired concepts” (Montague, 1997, p.164).

CHAPTER 3. METHODOLOGY

This study was conducted within the larger context of the National Science Foundation-funded project, *Nurturing Multiplicative Reasoning in Students with Learning Disabilities/Difficulties* project[1] (NMRSD; Xin, Tzur, & Si, 2008), in which the PGBM-COMPS intelligent tutor program has been developed.

3.1 Research Design

This study applied a pretest–posttest, comparison group design with to compare the differential effects of the two instructional sequences (i.e., A-**B**-C-D-E and A-C-**B**-D-E) on solving multiplication and division word problems by third and fourth grade elementary students with mathematics difficulties (MD). To calculate the needed sample size for this study, a power analysis conducted with an alpha level of .05 and an effect size of 1.25 per existing research (e.g., Xin et al., 2011). This power analysis indicated that each group should consist with a minimum of nine participants in order to obtain a power of .87 for 2 x 4 repeated measure analyses of variance (ANOVA, Friendly, 2000). Thus, nine participants in each group should provide sufficient statistical power to indicate any difference in word problem performance between the two groups.

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3.2 Participants and Settings

Institutional Review Board (IRB) approval was secured prior to the recruitment of participants. Participants were third and fourth grade students with Mathematics Difficulties (MD) from one elementary school in the Midwestern United States. The school was selected based on the following criteria: (a) a school with a minimum of 10 students for both 3rd and 4th grade to have a sufficient number of students with MD in each grade level, (b) a school that provided an after school program, and (c) a school with a minimum of eight computers with Internet access. All of the participants received the assigned interventions in an after-school program (November 2014-March 2015). Table 1 indicates demographic information of the participants' gender, grade, age, ethnicity, classification, IQ scores, and standardized achievement scores in math and reading. According to the state academic standards (2014), students in these two grade levels are expected to solve real-life multiplication and division word problems involving equal groups by forming the equation with a symbol for the unknown number (3.AT.2 & 4.AT.3). Criteria for selecting the participants were as follows: (1) recommended by the school teachers as those who struggled in mathematics problem solving, (2) performance on the researcher-developed MR criterion test below 60% correct, and (3) performance on the problem-solving subtest of the Stanford Achievement Test-10th edition (SAT-10, Pearson Inc., 2004) below 35 percentile. The current convention in the field of special education suggests that students scoring below 35th percentile be considered as at-risk in mathematics word problem solving (Jordan, Hanich, & Kaplan, 2003). Thus, students who tested below the 35th percentile on the SAT word problem-solving subtest were considered having MD in mathematics problem solving.

Table 1. *Demographics Table*

Variable	Experimental Group A-C-**B**-D-E	Comparison Group A-**B**-C-D-E	Total
Gender			
Male	4	5	9
Female	5	4	9
Grade			
3rd	7	6	13
4th	2	3	5
Mean	3.2	3.3	3.3
Age in months Mean (SD)	106.7 (8.2)	111 (8.4)	
Ethnicity			
Caucasian	1	2	3
African American	1	1	2
Hispanic	6	6	12
Multiracial	1	0	1
Classification[a]			
LD	0	0	0
LI	1	2	3
OHI	0	2	2
NL	8	5	13

	A-**C**-B-D-E Group		A-**B**-C-D-E Group		Total	
IQ[b]	Mean (SD)	*n*	Mean (SD)	*n*	Mean (SD)	*n*
Verbal	86 (9)	7	82 (10.2)	8	84 (9.6)	15
Performance	87 (9.1)	7	82 (18.4)	8	85 (13.4)	15
Full Scale	86 (4.5)	7	81 (13.1)	8	83 (10.6)	15
Achievement[c] (Percentage)						
Math	34 (11.5)	9	36 (10.7)	9	35 (11.2)	18
Reading	30 (9.1)	8	29 (13.6)	9	29 (11.3)	17

Note. [a] LD= Learning Disability; LI: Language Impairment; OHI: Other Health Impairment; NL: Not Labeled. [b] IQ scores were obtained from Otis Lennon Standardized Assessment Test (OLSAT) School Ability Index (SAI) scores. [c]Achievement scores for both math and reading were obtained from the Acuity Test.

Although both grade levels expect students to solve multiplication/division problems, the fourth grade students will have had more exposure to learning multiplication/division than the third grade students by the time the experiment is in progress. To increase the accuracy in terms of identifying students with learning difficulties in mathematics problem solving, this study used grade-appropriate SAT scores as a primary measure to identify students with learning difficulties. That is, the third grade students were not penalized for having lower skill set compared to the fourth grade students. Previous longitudinal research in the field of special education indicated that children who were identified with MD in first grade continuously show deficits in academic achievement and cognitive characteristics through fourth grade (Vukovic & Siegel, 2010).

A total of 18 elementary students with MD meeting the above criteria were recruited in this study. This study used a stratified random-sampling procedure based on students' pretest scores (e.g., SAT-10 and MR criterion test) to randomly assign 18 participants into two comparison conditions, with 9 students in each condition: (a) the instructional sequence that follows modules A-**B**-C-D-E (comparison group), and (b) the instructional sequence that follows modules A-C-**B**-D-E (experimental group). In addition, 13 third grade and 11 fourth grade normal-achieving students were recruited in the same school where the students with MD were also recruited for the study to be served as the normative reference (NR) group, who completed MR pretest and posttest. They were recruited by referral from classroom teachers in each grade level and scored within the mid range (between the 40^{th} and 60^{th} percentile) on their mathematics achievement tests. The NR group served as a benchmark for the experimental group to compare the progress rate. Thus, the NR group did not receive any intervention in this study.

All of the assessment and intervention sessions with the intelligent tutor were conducted in the participating school's library during the after-school program. The library was equipped with desktop computers on top of the table, round tables in the middle of the room, chairs, and unifix cubes. Each desktop had Internet access with a Windows operating system that had access to the PGBM-COMPS tutor program. All of the desktops were equipped with a mouse and a headset. A vacant classroom, near the computer lab, was used for testing purposes.

3.3 Measurement

The dependent variables used in this study consisted of measures of students' multiplicative word problem solving performance, students' attitude toward mathematics and their own mathematics achievement, and students' satisfaction with the appointed instructional sequence in the PGBM-COMPS tutor program.

3.3.1 Multiplicative Reasoning (MR) Criterion Test

As part of the NMRSD project (Xin, Tzur, & Si, 2008), a ten-item multiplicative reasoning (MR) criterion test (Purdue Research Foundation, 2011) was developed to assess students' multiplication concepts and problem solving acquisition, maintenance, and follow-up. The MR criterion test was developed based on the literature from both mathematics and special education, as well as input from both mathematics education researchers and educators. According to Xin et al. (2013), the test-retest reliability of the MR criterion test was .89. The MR test consisted of various multiplicative problems that were designed to assess the concepts of multiplicative double counting (mDC), same unit coordination (SUC), unit differentiation and selection (UDS), mixed unit coordination

(MUC), quotitive division (QD), and partitive division (PD). Please refer to Table 3 for sample problem of all types. The problem context varied ranging from towers and cubes to real-life contexts (e.g., money) using two to three-digit numbers. In particular, the MR criterion test included two mDC problems, one SUC problem, two MUC problems, three QD problems, and two PD problems. Problems in MR test can be found in Appendix A. The participants will be required to provide numerical solution and written responses to justify their answers.

3.3.2 Stanford Achievement Test (SAT-10)

The *mathematical problem solving* subtest of the *Stanford Achievement Test-10th Edition* (SAT-10, Harcourt, 2004) was used as a participant selection criterion as well as a far-transfer measure. The SAT-10 is a standardized achievement test that is norm-referenced and criterion-referenced, which has undergone extensive reliability and validity. While the SAT-10 has two alternate forms, Form A was given to all of the participants throughout the study. The *problem-solving* subtest of the SAT-10 aligns with the National Assessment of Educational Progress (NAEP), and measures mathematics concepts as well as processes in accordance with the National Council of Teachers of Mathematics *Principle and Standards for School Mathematics* (PSSM, Pearson Inc., 2004). The mathematics concepts assessed include number sense and operation, relationship and algebra, and measurement. The mathematics processes skills assessed computation and representation, estimation, and reasoning and problem solving, which are necessary to solve problems at the grade level (Pearson Inc., 2004). The third grade level *problem-solving* subtest consisted of 46 items, and the fourth grade level included 48 items. This far-transfer test was administered before and after the tutor instruction to assess the students' ability to solve problem solving to more challenging word problems with larger numbers and varied situations. The internal consistency of the problem solving subtests for Grades 3 and 4 were 0.91 and 0.90, respectively. The alternate-form reliability of the problem solving subtests for these grades ranged from 0.74 (Grade 4) to 0.85 (Grade 3).

3.3.3 Comprehensive Multiplicative Word Problem Solving (COMPS)

The COMPS test developed by Xin, Wiles, and Lin (2008) was used as a near-transfer measure to assess students' ability to solve multiplicative word problem with various contexts (see Appendix B). The COMPS test includes six Equal Groups (EG) and six Multiplicative Comparison (MC) problem types. The sample problems can be found in Table 5. As shown in Table 5, items in the COMPS test involve a range of real-life contexts and one- to three-digit numbers. The COMPS test included four alternative forms. Four alternative forms of COMPS tests (Xin, Wiles, & Lin, 2008) were used for pretest, immediate posttest, and two more follow- up tests. According to Xin, Si, et al. (2012), the alternate form reliability of this test was .84. Internal consistency of this test (Form A) was .86 (Xin, Si, et al., 2012).

3.3.4 Students' Attitude and the Survey Questionnaire

Test of mathematical abilities-2nd edition (TOMA-2). *An Attitude toward Math (AT)* subtest of the *Test of Mathematics Abilities,* 2nd Edition (TOMA-2, Brown, Cronin, & McEntire, 1994) was used to evaluate any possible changes in students' attitude about mathematics before and after the PGBM-COMPS tutor program. This subtest consists of 15 items asking the students about their perception of mathematics instructions and their own achievement. Students would respond to each of the items using a four-point Likert scale ranging from Strongly Disagree to Strongly Agree (Yes, definitely; Closer to Yes; Closer to No; No, definitely), with 4 indicating Strongly Agree and 1 indicating Strongly Disagree. For each item, students marked one of the four choices. The purpose of administering TOMA-2 is to identify the participants' attitude towards general mathematics and their achievement in mathematics.

Satisfaction questionnaire. A five-item questionnaire was developed to assess the social validity of the PGBM-COMPS intelligent tutoring program (see Appendix C). The satisfaction questionnaire was given to the participants and to the school teacher, who served as the session supervisor, following the intervention. The questionnaire enabled the researcher to solicit opinions from both the participants and the school teachers who was be involved in this study about the tutor program. The items asked about the usefulness of the tutor program (e.g., Do you think the program helped you do better on multiplication word problems?) as well as their acceptability (e.g., would you use this program in the future? Would you recommend this to your colleagues as well as other students with mathematics difficulties in your school?). The items were developed using the framework by Xin (2003). The statements in the questionnaire were developed based on Johnson and Christensen's (2011) principles of questionnaire construction. Both the participants and the school teachers answered each question using a four-point Likert scale ranging from Strongly Disagree (1) to Strongly Agree (4) was used. Follow up interviews were conducted after the completion of the questionnaires to further investigate their responses.

3.4 Procedures

Table 2 presents a summary of overall procedures that will be organized for this study.

Table 2. *Overall Procedural Checklist*

	Pretest	**Intervention**	**Posttest**	**Maintenance Test**	**Follow-up Test**
Comparison Group (A-**B**-C-D-E)	MR/COMPS/ SAT	PGBM-COMPS*	MR/COMPS/SAT	MR/COMPS	MR/ COMPS
Experimental Group (A-C-**B**-D-E)	MR/COMPS/ SAT	PGBM-COMPS*	MR/COMPS/SAT	MR/COMPS	MR/ COMPS
NR* Group	MR	-	MR	-	-

Note. NR= Normative Reference; PGBM-COMP= Please Go and Bring Me- Conceptual model-based Understanding Intelligent Tutoring System; SAT= Stanford Achievement Test Problem solving subtest (SAT-10).

Both comparison and experimental groups completed the following: (1) TOMA-2, criterion test (MR), near-transfer (COMPS) test, and far-transfer (SAT-10) test respectively prior to working on the PGBM-COMPS tutor program, (2) the PGBM-COMP tutor program with respective module sequences, and (3) the posttest (MR), maintenance (MR), near-transfer (COMPS), and far-transfer (SAT-10) tests following the PGBM-COMPS tutor program. Maintenance test was given one to two weeks following the termination of the tutor program, and the follow-up test was conducted three to five weeks following the PGBM-COMPS tutor. In addition to the tests listed in the table, TOMA-2 was given to both groups before and after working on the tutor program. The satisfaction questionnaire was given to both groups upon completion of the intervention program. The NR group completed the MR pretest and posttest during the same time as the other two groups.

3.4.1 Testing Procedures

The testing in this study was conducted in an adequate workspace so that the physical environment was conducive to the participants' concentration (e.g., free from noise and any interruptions). During testing sessions, the seating was arranged so that the students were seated far away from each other to prevent any temptation to observe each other's test materials. During the paper testing administration, participants were asked to read the problems carefully and to show their work. Plenty space was given on test sheet for students to show their problem solving process. Scratch papers, pencils were given with the test sheet. They were reminded to try their best while solving the problems. The examiner assisted in reading the problems if a student requests it. However, no prompts were given regarding their solution process and/or solution at any time. Students were provided with sufficient time to complete the tests. A calculator was allowed throughout the experiment to accommodate their arithmetic calculations.

The far-transfer testing procedure followed as directed in the SAT-10 manual (Harcourt Assessment Inc., 2004). Apart from the standard procedures used for all the rest of the tests (e.g., read the problems carefully and mark your answer), the administrator went through a sample problem with the students to show how to mark the answer on the answer sheet. Along with the materials used during the other tests, students were given a paper ruler with inch and centimeter markings. The testing procedure of the TOMA-2 *An Attitude Toward Math (AT)* subtest was followed as directed in the TOMA-2 manual (Brown et al., 1994). One of the major roles of the examiner was to go over the two sample items together and make sure that the student will be able to mark an "x" that is the closest to the way he/she felt about each statement. Lastly, the exit questionnaire survey was answered immediately after the posttest. The procedure of this questionnaire survey was identical to that of the TOMA-2 *An Attitude Toward Math (AT)* subtest.

3.4.2 Scoring Procedures

As for the MR criterion test, each correctly solved problem was awarded one point. When a problem involved a set of sub-questions, the points were evenly distributed to each of the sub-questions. For example, if there were two questions within a problem, each sub-question was worth 0.5 point. As for the reasoning "why" supplemental question on the MR criterion test, each student response was scored according to the student's response in an original handwritten sentence, a mathematics operation sentence, and/or correct answer. When a student solved the second problem shown in Appendix A, "Do you think you will say the number 84 if you continue counting seven cubes in the towers?", and the student answers "yes" for the main problem and put "7x12=84," or "84" a full point was given. If a student wrote "I do not know" or other incorrect reasoning, no credits were given. If a student wrote "I just added" or gives another ambiguous reason, the research assistant further asked the student to give more information right after completing the assessment to clarify their answers.

As for the COMPS near-transfer test, each correctly solved problem was awarded one point. When a student provided correct mathematics operation sentence with an incorrect answer, a full point was given. The main focus of this assessment was to measure students' ability to form correct mathematics equation using the correct operation rather than their computational skills. The SAT scoring was consistent with scoring procedure indicated in the SAT-10 manual (Harcourt Assessment Inc., 2004). The TOMA-2 scoring was consistent with the scoring procedure indicated in the TOMA-2 manual (Brown et. al., 1994).

3.4.3 Instructional Procedures

The intervention was conducted over a period of 16 weeks during the afterschool (2:45pm - 3:45pm). There were four sessions per week (one session per day, Monday to Thursday), each session lasting approximately 60 minutes. Participants worked with the PGBM-COMPS tutor program individually using the desktops in the school's participating lab. These desktops had Internet access and Microsoft Windows operating systems. Prior to the study, a research assistant majoring in computer science visited the school to ensure that all the desktops in the lab are able to access to the PGBM-COMPS tutor program.

Five research assistants (four in special education, one in educational psychology) and one undergraduate assistant (majoring in mathematics education) served as the session supervisors. In addition, one research assistant in computer science was responsible for technical issues (e.g., computer malfunction and/or error). One school teacher also served as the session supervisor in charge of gathering the students for the after school program and monitoring students during each session. All session supervisors went through a one-hour training session on assessment administration and PGBM-COMPS instructional strategy. Each session supervisor (a) monitored two participants during the intervention, (b) wrote the program log for progress monitoring purposes, (c) detected program malfunctions and redirect the students back to the program, and (d) wrote field notes whenever additional human assistance occurred during the session. During the first four intervention sessions, the session supervisor introduced the turn-taking game PGBM (Tzur et al., 2012) to the participants using physical cubes prior to using the PGBM-COMPS tutor program. The purpose of playing the PGBM game with concrete manipulatives was to help students get familiar with the game before they use the computer program where virtual manipulative was used. To play this game, each session supervisor worked with five participants as a group. Thus, the game was played between the session supervisor and the participants in a ratio of one-to-five, respectively. The session supervisor placed a box of Unifix cubes in another part of the classroom. During the PGBM game, the session supervisor's role was the sender and the participants' role was to be the bringer. The sender asked the bringer to build equal-sized towers (e.g., "Could you bring two towers of 3 cubes"; 2T3). After all of the towers are made, the sender asked a group of participants the following questions: (1) How many towers did you bring? (2) How many cubes are in each tower? (3) How many cubes are there in all?, and (4) How did you figure it out?

Each participant logged into his or her account and began working on the program based on the assigned instructional sequences. All of the participants were engaged in the intervention four times a week, with each session lasting approximately 20-25 minutes. During the one-hour afterschool program, the two groups each took turns working on computer for a half an hour time slot. The order for each group to work on the computer will be counterbalanced. That is, one day, the comparison group worked on computer first (while the other group engage in homework) and vice versa.

3.4.4 PGBM-COMPS tutor program

Following the tutor session with concrete manipulatives, both groups engaged with the PGBM-COMPS intelligent tutor program. The tutor program is a game-based instructional program developed for third and fourth grade students with learning disabilities and is designed to enhance their mathematical concepts and problem-solving skills through a hybrid instructional approach (constructivist and explicit instructional approaches). Incorporating research-based instructional practices, PGBM-COMPS immerse students in an interactive hands-on experience of using virtual manipulatives (cubes) to promote multiplicative conceptual understanding. The tutor program also applies animations to visually illustrate the problem situation as well as a mathematical model to explicitly represent the relationship among the numbers. Depending on the nature of the task, students input their answers in various ways including making towers with cubes and inputting the correct answer and/or the label. Various types of feedback, including corrective feedback, are incorporated into the PGBM-COMPS tutor program to allow students to understand the correctness of their performance, while hints provide students further prompts when they fail to come up with the solution on their first trial. All of the directions and feedback are delivered both in written text and by voice.

Both groups worked on the PGBM-COMPS program during the intervention phase. The difference between the two groups was the sequence of the modules in the tutor program. Specifically, the comparison group went through the A-**B**-C-D-E instructional sequence whereas the experimental group went through the program using the A-C-**B**-D-E. The PGBM-COMPS tutor program consists of five modules (A, B, C, D, E). Figure 1 presents the five modules in the program (adapted from NMRSD project

Concept Map, Xin, Tzur, and Si, 2008). As shown in Figure 1, the PGBM and COMPS components go hand in hand.

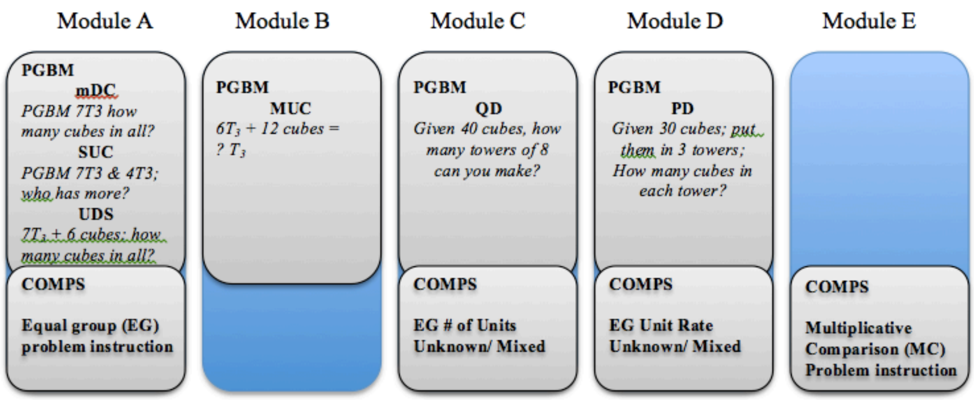

Figure 1. Five Modules in the PGBM-COMPS Tutor Program (© NMRSD Project; Xin, Tzur, & Si, 2008)

Note. 7T3= 7 towers of 3 cubes in each; this concept map is adapted from the NMRSD project and it is copyrighted by the NMRSD project. All rights reserved. No part of this concept map may be used for any purposes without prior permission from the Project Director (yxin@purdue.edu).

PGBM component. The PGBM component focused on developing multiplication concepts by going through the six multiplicative schemes (i.e., mDC, SUC, UDS, MUC, QD, & PD). Table 3 presents sample tasks in each module applied in the NMRSD-COMPS tutor program.

Table 3. *Sample Tasks in the NMRSD-COMPS Tutor Program (adapted from the PGBM-COMPS Tutor Program*; *Xin, Tzur, & Si, 2008)*

Module	Multiplicative Scheme/ Problem Type	Sample Task
A	mDC	Pretend I asked you to bring 4 towers. Each tower has 8 cubes in it. How many cubes would you bring in all?
	SUC	You have 11 towers. Each tower has 6 cubes. I have some more. Together we have 14 towers. How many towers do I have?
	UDS	John has collection of 10 towers with 8 cubes in each. Sarah has collection of 7 towers with 8 cubes in each. Who has more cubes? How many more cubes does John have than Sarah?
B	MUC	Tom has a collection of 4 towers of 6 cubes in each. If Tom brought 18 more cubes, how many towers of 6 would Tom have?
C	QD	I have 40 cubes. I want to divide into towers of 10 cubes each and bring back the pile. How many towers will I bring back to the pile?
D	PD	I want to make 6 towers with 12 cubes. How many cubes will there be in each tower?

Note. mDC= Multiplicative double counting; SUC= Same Unit Coordination; MUC= Mixed Unit Coordination; QD= Quotitive Division; PD= Partitive Division

Module A provides tasks designed to give them the skills of *multiplicative double counting* (mDC). When working with mDC (e.g., PGBM 7T4; How many cubes in all?), students identify the two units (e.g., # of towers, and total # of cubes) and count two number sequences. Using the knowledge of double counting, students would go through *same unit coordination* (SUC) and *unit differentiation and selection* (UDS) schemes. While solving SUC (e.g., 6T3 + 3T3; how many towers in all?) encourages students to operate with CU [towers], UDS (e.g., 8T3 + 6 cubes; How many cubes in all?) encourages them to operate with ones [cubes]. Module B includes *multiplicative mixed unit coordination* (MUC) tasks (e.g., 8T7 + 14 cubes = ? T7). Students need to differentiate the CU and ones they are operating on while comparing the two collections of towers and cubes. Students first choose the correct unit for the solution, whether it is the number of cubes [the ones] or the number of towers [the CU], and perform coordinating and segmenting the operating units. It is important to note that MUC is the only multiplicative scheme, which requires multiple steps of problem solving. Module C contains *quotitive division* (QD) tasks where students divide the cubes into equal-sized groups to figure out the number of groups of cubes. Module D provides *partitive division* (PD) tasks where students solve the problems through equally distributing the ones to the given number of CU for the solution.

Overall, there are four different types of tasks for each of the schemes (i.e., mDC, SUC, UDS, MUC, QD, PD) in the PGBM component. Thus, each scheme is divided into four blocks in the PGBM-COMPS tutor program. The tasks in block1 encourage students to manipulate with virtual representations of concrete cubes and towers to solve for the tasks. As students undergo each block, the tasks become progressively more challenging and abstract. The tasks in block 2 involve towers that the software brings and covers, and then it presents four questions for students to solve the task. When students do not have visible towers to operate on when answering the four questions, they would imagine the towers (or mentally re-present them in their mind’s eye). Eventually, a third type of task is presented, in which the student is asked to pretend—imagine in the abstract—that there are some number of towers composed of an equal number of cubes (e.g., 7 towers with 4 cubes in each). During block 3, students were asked to solve the task with no cubes. Lastly, the tasks in block 4 include novel contexts other than cubes and towers

Promotion criteria for PGBM. In general, the tutor program promoted students based on their previous performance. In particular, the tutor applied a criterion of “three consecutive correct answers” to promote students from one scheme to the next one (e.g., mDC → SUC). In order to understand which instructional sequence is more effective, two additional criteria were used in this study. First, if students struggle to solve three consecutive problems, they would be moved to the next block within the module. Second, students who exceed the maximum number of sessions they are allowed to work on would be moved to the next block. The number of sessions students were able to work on each of the modules were as follows: module A (24 sessions); module B (4 sessions); module C (4 sessions); module D (4 sessions), and module E (4 sessions). These numbers of sessions were determined based on the previous field-testing experience. It is important to note that module A required more sessions to complete as it consisted of many tasks, including PGBM activity, three scheme tasks (i.e., mDC, SUC, UDS), and COMPS component.

While the PGBM-COMPS tutor promoted students from one scheme to the next one based on the criterion stated above, the promotion also occurred within a scheme using different number layers and cognitive level of operation when solving for problems/tasks. For each scheme, promotion proceeded based on the four number layers (e.g., within mDC, from Layer 1 to Layer 2 to Layer 3 to Layer 4). Table 4 provides the numbers used for each number layer. The layer began with numbers that are easy for a child to conceptualize the operation (coordination) on the two units (see Table 4). That is, the numbers in Layer 1 did not necessarily require students to think mathematically in order to operate with CUs. This is due to the fact that they can mentally recall these numbers without a challenge. As students work towards higher layers, they were asked to work on tasks with larger numbers.

Table 4. *Number for Number Layer (adapted from NMRSD Project, Xin, Tzur, & Si, 2008)*

Layer	Sample Task
Layer 1	whole number: 2, 5, or 10
Layer 2	Whole number of 3 and 4
Layer 3	whole number of 6, 7, 8, and 9
Layer 4	all numbers in the second decade (11, 12, 13, 14, 15, 16, 17, 18, 19, 20)

Another promotion criterion within the scheme was the cognitive level of operation. As stated above regarding the block structure of PGBM component, the tasks become progressively more challenging and abstract. However, if students experience difficulty solving any task, the tutor will indicate that the cognitive level of operation represented in the current task is beyond the students' ability to reason. To ensure appropriate learning for all leaners, the same task the student was struggling with will be represented in a lower cognitive level of operation. For example, if students struggle to solve mDC task in an abstract manner, they will then be given a virtual representation of the physical cubes and towers to solve the problems. Eventually, students will need to solve the task at the abstract level of operation for each of the six schemes.

COMPS component. At the end of Module A, Module C and Module D, the COMPS component challenges the students to represent real-world equal group (EG) problems in COMPS diagram equations (e.g., UR x # of Units = Product, Xin, 2012) and then to solve the problem using the diagram equation. Ultimately, this hybrid approach between the two components would help students to connect the fundamental multiplicative concepts the students learned from the PGBM part of the program and the mathematical models they would use to solve real-world problems with large numbers.

Then, Module E focused on solving Multiplicative Comparison (MC) multiplication word problems. The EG problem type dealt with a number of equal units, whereas the MC problem type compared the two given numbers where one quantity is a multiple/part of the other quantity (Xin, 2012). Table 5 shows samples of the EG and MC problems. As shown in Table 5, both EG and MC problems had three variations based on the placement of the unknown number.

Table 5. *EG and MC Multiplicative Word Problems* (*adapted from Xin, 2012)*

Problem Type	Sample Problems
Equal Groups (EG)	
Unit Rate (UR) Unknown	It costs a total of $400 to buy 50 math books. If each book costs the exact same price, how much does each math cost?
Number of Units unknown	There are 72 marbles. If the Susan wants to put 9 marbles in each bag, how many bags does she need?
Product unknown	Bob's grandmother cooked 4 batches of cookies. Each batch has 13 cookies in it. How many cookies did she bake?
Multiplicative Compare (MC)	
Referent Unit unknown	Bob finished 182 math problems for homework. Bob finished 13 times as many problems as John. How many problems did John finish?
Multiplier Unknown	A farmer named Bob has 238 cows on his dairy farm. Another farmer named John has only 17 cows on his farm. The number of cows Bob has is how many times the number of cows John has?
Product Unknown	Tom baked 7 muffins. John backed 6 times as many muffins as Tom. How many muffins did John bake?

The COMPS instruction (Xin, 2012) was carried out by four different phases: (1) introducing the concept of equal groups (i.e., same number of items in each group), (2) representing the EG/MC problem situation in the conceptual model equation, (3) developing the solution plan that is driven by the conceptual model representation, and (4) practice solving EG/MC problems in various contexts. Students went through these four phases for EG problems (Module A, C, and D) and then for MC problems (Module E).

Introduction to the concept of equal groups (EG). During this phase of instruction, the tutor provided a short presentation depicting the concept of equal groups, a crucial component in multiplicative reasoning (Xin, Si, et al., 2012) through grouping the items equally (see Figure 2). Both equal group examples and non-equal group examples were introduced to the students during this phase. When a non-equal group example is presented, students were asked to correct it to make it into an equal group problem. Students were asked to solve mDC problem using cubes and towers similar to the PGBM component. The tutor program later engaged students to solve mDC problem and emphasize that the towers made are equal-sized towers, which would represent equal groups.

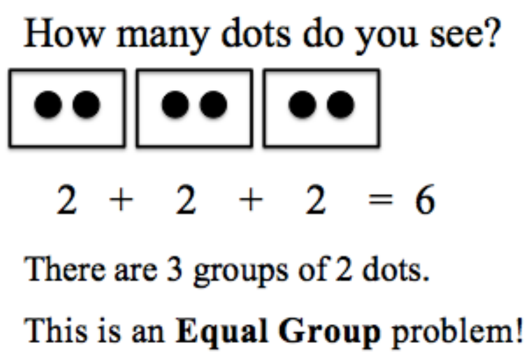

Figure 2. The Equal Group Concept (adapted from Xin, Si, et al., 2012)

Representation of the EG problem situation. During the second phase of instruction, students were asked to identify the three elements (i.e., *unit rate, # of units, and product*) in the given EG problems to complete the EG conceptual model equation. The first three tasks were story problems where all of the numbers for the three elements will be known. This was to encourage students to focus on mapping a complete representation so that they could have sufficient time to comprehend mathematical relation among the three elements. The concrete modeling from the computer simulation allowed students to establish the connection between the visual representations of a concrete object to the abstract level by understanding "(1) unit rate as the number of items in each group, (2) the # of units as the number of groups, and (3) the product as the total number of items in all groups" (Xin, Si, et al., 2012, p.77; see Figure 3).

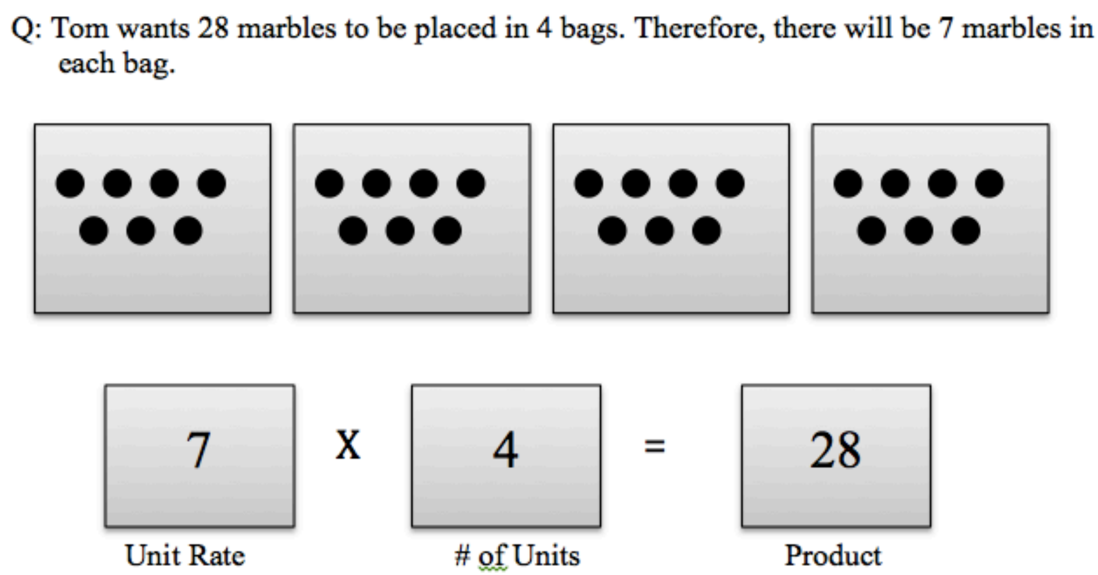

Figure 3. Equal Group Problem Representation (adapted from Xin, 2012)

Developing the solution plan using the conceptual model representation. After students had sufficient experience filling in the numbers in the conceptual model, the tutor challenged students to solve the problems by finding the unknown number. Students were asked to use the letter "a" to represent the unknown number. During this phase, students were encouraged to transition from the conceptual model to a mathematics equation. Students then learned procedural steps to solve for unknown numbers. If the product was unknown, students would multiply the two factors as indicated in the model equation. If one of the two factors was unknown, students divided the product by the known factor (Xin, Si, et al., 2012). A calculator was provided on the screen for students to use when finding for the unknown number. During this phase, students solved various real-life contexts with larger numbers.

After students completed the three stages of EG problem solving (i.e., introduction of EG, representation, representation using COMPS model), they went through the three stages of MC problem solving (e.g., The height of the doghouse is 4 feet. The house is 3 times as tall as the doghouse. The height of the house is 12 feet.). As stated earlier, MC problem solving involved a comparison sentence that contains “one quantity as a multiple or a part of the quantity” (Xin, 2012, p. 14). Using the same conceptual model from the EG problem, students were asked to figure out the meaning behind the MC problem situation. The labels used in the MC problem model were the following: “(referent) unit”, “multiplier (i.e., multiple or part),” and “product” (Xin, 2012, p. 123). In the above MC problem, for instance, the height of the house is compared to the height of the doghouse. Thus, the doghouse is the referent unit (see Figure 4).

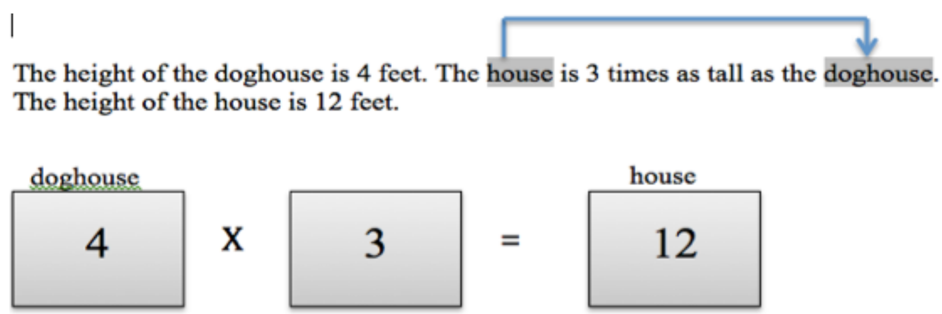

Figure 4. MC Problem Representation (adapted from Xin, 2012)

Practice solving EG/MC problems. Next, the students practiced solving EG and MC problems. During this phase, students were introduced to a four-step DOTS (Detect-

Organize-Transform-Solve) checklist (Xin, 2012, p. 107) to support their problem solving process (see Figure 5). In step 1, students were to "detect the problem type" (i.e., EG, MC, or addition/subtraction). Step 2 asked students to "organize the information using the conceptual model". By filling in the diagram, students were encouraged to figure out the meaning and the underlying structure within the context of the MC problem situation. Step 3 encouraged students to use the completed conceptual model in a mathematics equation. This diagram functioned as intermediate scaffolding, which ultimately allowed students to form a math sentence when solving the problems. Step 4 asked students to "solve for the unknown quantity" and to "check their answer" by using the onscreen calculator.

DOTS Checklist
Detect the problem type
Organize the information using the conceptual model diagram
Transform the diagram into a meaningful math equation
Solve for the unknown quantity in the equation and check your answer

Figure 5. DOTS Problem Solving Checklist (adapted from Xin, 2012)

Promotion criteria for COMPS. Similar to the PGBM component, the tutor applied criterion of "three consecutive correct answers" to promote students from one block to the next one (e.g., COMPS A block 1 →COMPS A block 2). Specifically, students were promoted to the next block when they attain a score of 100% correct on mapping and solution.

A-C-B-D-E Instructional sequence condition. The participants worked on the same tutor program as those with the A-**B-**C-D sequential condition, except that the MUC task was introduced after module C. Each participant will work with the computer tutor one-on-one. Similar to the A-**B-**C-D-E group, the same research assistants monitored each session.

3.5 Fidelity of Implementation

Several steps were taken to ensure the procedures were implemented as expected. Prior to the study, the five session supervisors met with the project coordinator or the author and memorized the script for assisting students when the PGBM-COMPS tutor program malfunctioned or when students ask questions regarding math problems. The session supervisors also role-played with each other to practice applying the script in various situations. During the intervention, session supervisors observed their assigned students to ensure that students went through all of the mathematical contents in the PGBM-COMPS program in assigned sequence (i.e., the comparison group went through modules A-**B**-C-D-E sequence and the experimental group went through modules A-C-**B**-D-E). In addition to observation, session supervisors also used students' progress data, reported from the database for each student's account, to ensure that the participants followed the intervention as planned. Prior to every school visit, each session supervisor visited the project's progress report website to conduct a daily-based progress and record the date of which the child completed the module on his/her school visit log. The information facilitated session supervisors to reconfirm their observation and helped keep track of each participant's pace while working on the program.

3.6 Inter-rater Reliability

The researcher scored all of the tests using the answer key. Unaware of the purpose of the study, a research assistant re-scored 30% of each test given to the participants in this study. Inter-rater reliability was computed by dividing the number of agreements by the total number of agreements and disagreements and multiplying by 100%. Inter-rater ability was 97%.

CHAPTER 4. RESULTS

4.1 Pretreatment Group Equivalency

A one-way analysis of variance (ANOVA) for standard instructional sequence (A-**B**-C-D-E) and experimental instructional sequence (A-C-**B**-D-E) on students' MR criterion and COMPS pretest performance was performed to examine pretreatment group equivalency. The comparison group (A-**B**-C-D-E) solved an average of 1.44 problems (SD = 1.69) during the pre-intervention on MR criterion test while the experimental group (A-C-**B**-D-E) solved an average of 1.11 problems (SD = 1.34) correctly. Results indicated no significant difference between the two groups on the MR pretest ($F(1, 16) = .22$, $p = .65$). As for the COMPS pretest performance; the comparison group obtained an average of 1.33 problems correct (SD = 2.40) while the experimental group obtained 1.22 problem correct (SD = 1.64). Similar to the MR criterion test, there was no statistically significant difference between the two groups on the COMPS pretest ($F(1, 16) = .01$, $p = .91$). To further evaluate pretreatment group equivalency, a simple statistical analysis was conducted by comparing the average percentile ranks of both groups' pretreatment Standardized Achievement Test (SAT) word problem solving subtest. While students in the comparison group achieved a mean of the 16th percentile rank, those in the experimental group achieved a mean of the 16th percentile rank, assuring pretreatment equivalency between the two groups' word problem solving performance.

4.2 Acquisition and Maintenance of the PGBM-COMPS Tutor Program

A 2 (Comparison and Experimental groups) x 4 (Time: pretest, posttest, maintenance test, follow-up test) repeated measures of ANOVA was conducted to compare the effects of two instructional sequences in the PGBM-COMPS tutor program (i.e., module A-**B**-C-D-E vs. A-C-**B**-D-E) on students' multiplicative word problem solving performance. All of the participants in this study completed all of the tests. Descriptive statistics for the two groups at four times can be found in Table 6.

Table 6. *Students' Performance on MR Criterion Test*

	Comparison Group (A-B-C-D-E)			**Experimental Group (A-C-B-D-E)**			
	M	*n*	SD	M	*n*	SD	*d*
Pretest	1.44	9	1.69	1.11	9	1.34	- 0.22
Posttest	8.67	9	1.30	9.22	9	.94	+ 0.48
Maintenance	8.67	9	.87	8.83	9	.35	+ 0.24
Follow-Up	8.72	9	.97	9.28	9	.57	+ 0.70

Note. d = Cohen's d value (experimental group mean minus the comparison group divided by the pooled standard deviation); a positive *d* indicates a favorable effect for the experimental group and a negative *d* indicates a favorable effect for the comparison group.

Results revealed that the main effect of time in MR criterion test performance was (F(3, 14) = 213.75, p = .00, partial η^2 = .979), which indicated that both groups showed significant improvement (positive) across four times. In addition, the main effect for group showed no statistically significant difference in MR criterion test performance (F(1, 16) = .39, p = .54, partial η^2 = .024). This result indicates that, overall, both groups did not significantly differ in their MR criterion test performance across four times. Furthermore, there was no statistically significant interaction between time and group (F(3, 14) = .66, p = .59, partial η^2 = .124). That is, the two instructional sequences did not have differential effects on students' word problem solving performance across four times. In other words, the two groups improved their performance at a similar rate across four times. Figure 6 depicts comparable performance between the comparison and experimental group.

As shown in Figure 6, both groups showed similar performance during the pretest. Following the PGBM-COMPS tutor program in the assigned instructional sequence, participants in both groups substantially improved their performance (M = 8.67, SD = 1.30 for the comparison group; M = 9.22, SD = .94 for the experimental group). Although both groups showed similar rates of group mean increase from pretest to posttest on the MR criterion test, the experimental group showed a relatively higher mean increase than the comparison group. While the comparison group increased to an average of 72.3% correct, the experimental group increased to an average of 81.1% correct. Furthermore, positive effect sizes for MR post (d = .48), maintenance (d = .24), and follow-up tests (d = .70) indicate an overall small to medium differential effects between two groups during posttreatment assessment, favoring the experimental group.

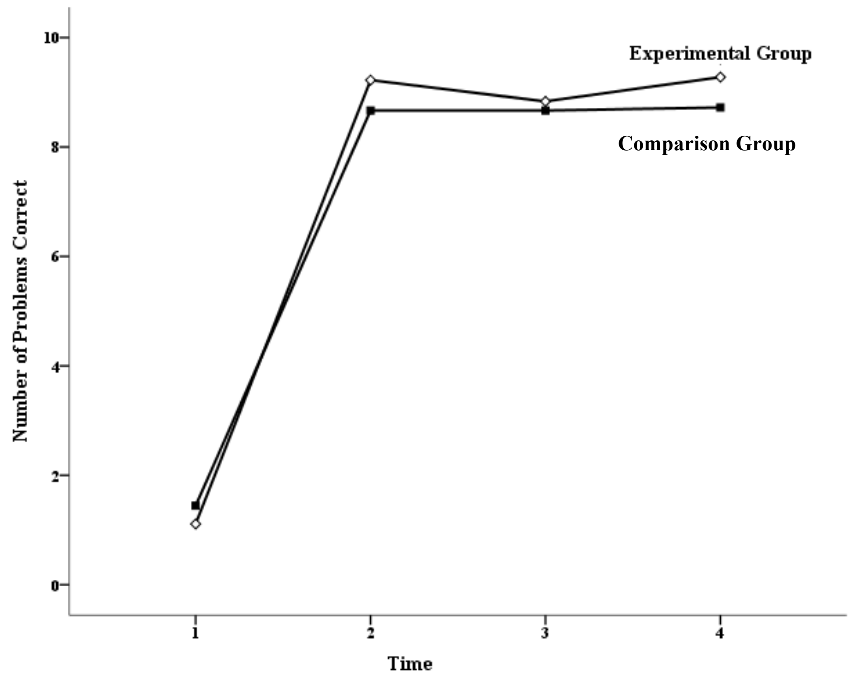

Figure 6. Two Groups' Performance on the MR Criterion Test

To further investigate the significance level differences between each time on both groups' word problem solving performance, a post hoc analysis with a Bonferroni adjustment was conducted. According to Perneger (1998), this statistical adjustment is used when multiple dependent or independent statistical analyses are being conducted simultaneously on each data set. Because four MR criterion tests were compared simultaneously in this study, a Bonferroni adjustment adjusts the p level by dividing the standard p value (.05) by four to have $p < 0.0125$ as the minimum alpha level. Thus, $p < 0.05$ indicated in the post hoc test results correspond to $p < 0.0125$. The result revealed that students' word problem solving performance improved significantly from pretest to posttest (Mean difference = -7.67, 95% CI [-8.89, -6.44], $p = .00$), but not from posttest to maintenance test (Mean difference = .19, 95% CI [-.51, .90], $p = 1.00$) and maintenance test to follow-up test (Mean difference = - .25, 95% CI [- .70, .20], $p = .69$)

for both groups. These results indicate that both groups improved their problem-solving performance following either the A-**B**-C-D-E sequence of instruction or the A-C-**B**-D-E sequence of instruction, and they maintained their improved posttest performance during maintenance and follow-up phases.

4.3 Near-transfer Effects of Word Problem Solving Performance

A 2 (Comparison and Experimental groups) x 4 (Time: pretest, posttest, maintenance test, follow-up test) repeated measures ANOVA were conducted to investigate the effects of two instructional sequences in PGBM-COMPS tutor programs (i.e., module A-**B**-C-D-E vs. A-C-**B**-D-E) on students' near-transfer word problem solving performance using the COMPS test. Descriptive statistics for students by group across four times can be found in Table 7.

Table 7. *Students' Performance on the COMPS Test*

	Comparison Group (A-B-C-D-E)			**Experimental Group (A-C-B-D-E)**			
	M	*n*	SD	M	*n*	SD	*d*
Pretest	1.33	9	2.40	1.22	9	1.64	- 0.05
Posttest	10.78	9	.83	10.78	9	.97	0
Maintenance	10.67	9	.87	11.22	9	.67	+ 0.71
Follow-Up	11.11	9	.78	11.00	9	.71	- 0.15

Note. d = Cohen's d value (experimental group mean minus the comparison group divided by the pooled standard deviation); a positive *d* indicates a favorable effect for the experimental group and a negative *d* indicates a favorable effect for the comparison group.

While the main effect of time showed a statistically significant difference on the COMPS test performance ($F(3, 14) = 177.07$, $p = .00$, partial $\eta^2 = .974$), the groups showed no statistically significant difference on COMPS ($F(1, 16) = .07$, $p = .80$, partial $\eta^2 = .004$). That is, both groups' performance on the near-transfer word problem solving measure significantly improved across four times. Results also showed no statistically significant interaction between time and group ($F(3, 14) = .54$, $p = .66$, partial $\eta^2 = .104$). That is, the two instructional sequences did not have differential effects on students' progress in solving near-transfer problems across four times. Figure 7 illustrates the two groups' performance across four times.

As shown in Figure 7, both groups showed similar performance during the pretest (M = 1.33, SD = 2.40 for the comparison group; M = 1.22, SD = 1.64 for the experimental group. See Table 7). Although the average number of correct responses during the pretest for the comparison group was slightly higher than the average for the experimental group during the pretest, the difference between the two mean scores was not statistically significant. After completing the PGBM-COMPS tutor program in the assigned instructional sequence, participants in both groups showed significant improvement on their near-transfer word problem solving performance (M = 10.78, SD = .83 for the comparison group; M = 10.78, SD = .97 for the experimental group). Both groups also showed a similar mean increase from pretest to posttest on the COMPS test. While the percentage of mean increase for the comparison group was 78.8%, the percentage of mean increase for the experimental group was 79.7%. Although both groups showed similar rates of group mean increase from pretreatment to posttreatment on the COMPS test, positive effect sizes for COMPS post ($d = 0$) and maintenance ($d = .71$) and a negative effect size for follow-up test ($d = - .15$) indicated an overall differential effects between two groups. While there were negligible to small differential effects during the posttest and follow-up test, the effect size for the maintenance test indicated a stronger differential effect between both groups, favoring the experimental group. Overall, both groups maintained their improved posttest performance during the maintenance and follow-up tests.

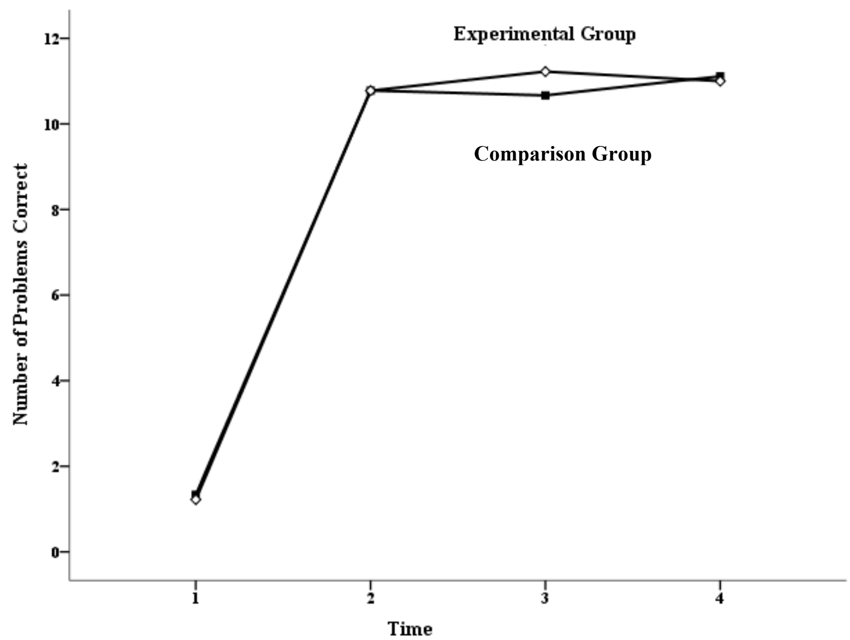

Figure 7. Two Groups' Performance on Near-transfer Problems

To further examine the significance level differences between time on both groups' word problem solving performance, a post hoc analysis with a Bonferroni adjustment was conducted. The results revealed that students' near-transfer performance in both groups significantly improved from pretest to posttest (Mean difference = -9.50, 95% CI [-11.22, -7.78], p = .00), but not from posttest to maintenance test (Mean difference = - .17, 95% CI [-1.16, .82], p = 1.00) and maintenance test to follow-up test (Mean difference = - .11, 95% CI [- .90, .68], p = 1.00). These results indicate that both groups improved their problem-solving performance on the near transfer test following either the A-**B**-C-D-E sequence of instruction or the A-C-**B**-D-E sequence of instruction.

In addition, both group students' improved posttest performance was stable during maintenance and follow-up tests.

4.4 Normative Reference Comparison

To compare the multiplicative word problem solving of the students in the experimental group to that of the normative reference group (NR) before and after the intervention, a 2 (Experimental Group and NR) x 2 (Time: pretest and posttest) repeated ANOVA was conducted. The NR group consisted of 13 third grade students. As 77.8% of students in the experimental group were in the third grade (Mean age: 9.3), we used a total of 13 third grade students (Mean age: 9) from an average-performing class (all the students in that class) as the NR group (note: in the participating school, students were grouped into high-performing class, average performing class, low performing class, as well as special education class based on achievement level). Thus, nine students in the experimental group and 13 third grade students in the NR group were involved in this analysis. The analysis regarding the 4th grade normative comparison group will be discussed later in the discussion section. Table 8 gives descriptive statistics of the two groups' multiplicative word problem solving performance across two times.

Table 8. *Two Groups' MR Criterion Word Problem Solving Performance Across Time*

	Experimental group (A-C-B-D-E)			**3rd grade NR Group**			**4th grade NR Group**		
	M	*n*	SD	M	*n*	SD	M	*n*	SD
Pretest	1.11	9	1.34	2.42	13	1.57	5.36	11	1.67
Posttest	9.22	9	.94	4.81	13	1.68	6.23	11	1.98

Results indicate a statistically significant main effect of time ($F(1, 20) = 109.54$, $p = .00$, partial $\eta^2 = .846$) and of group ($F(1, 20) = 16.43$, $p = .00$, partial $\eta^2 = .451$). More importantly, the results showed a statistically significant interaction between time and group ($F(1, 20) = 32.61$, $p = .00$, partial $\eta^2 = .620$), indicating differential effects of instruction on students' performance across two times. Results from Paired-Sample T Test indicate a statistically significant performance change from pretest to posttest for the experimental group ($t(8) = -12.9$, $p = .00$) and for the 3rd grade NR group ($t(12) = -3.36$, $p = .01$). Figure 8 illustrates differences in the performance between the three groups across two times.

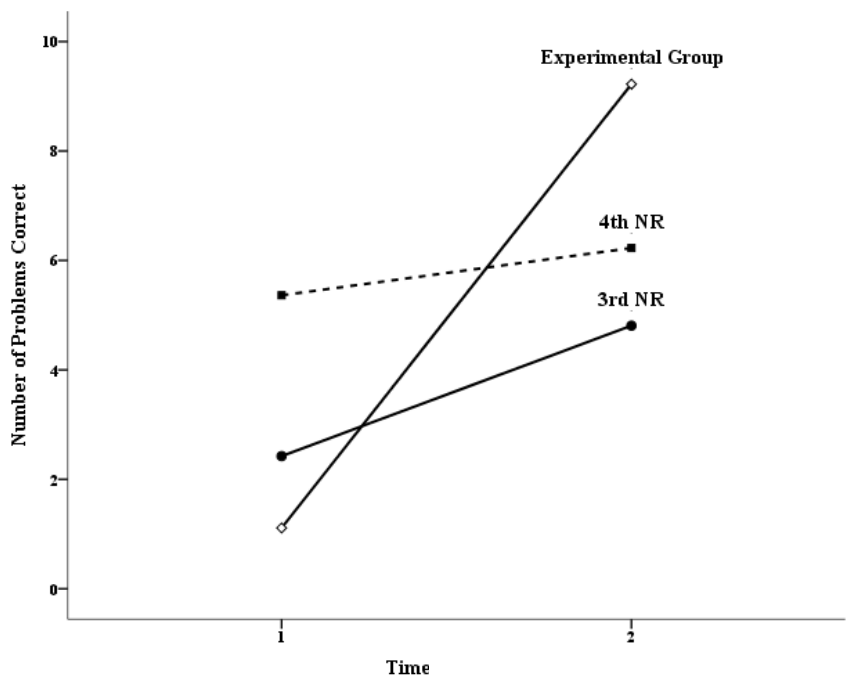

Figure 8. Three Groups' Performance on MR Criterion Test

As shown in Figure 8, the third-grade NR group had a slightly higher group mean than the experimental group on the MR criterion test (M = 1.11, SD = 1.34 for the experimental group; M = 2.42, SD = 1.57 for the 3rd NR group), although the difference between the two groups was not statistically significant. However, the experimental group had a higher mean increase (81.1%) than the 3rd grade NR group (23.9%) from pretest to posttest.

4.5 Efficiency of Two Instructional Sequences

To compare the efficiency of the two instructional sequences in learning the multiplicative word problem solving performance, an independent-samples t-test was conducted to compare the number of the intervention sessions it took for students in the two groups to complete the PGBM-COMPS program. It is important to note that the tutor program allowed participants to progress at their own pace. Thus, the number of sessions (each session is fixed in duration of 25 to 30 minutes) of the intervention phase was served as an indicator of student efficiency in learning multiplicative word problem solving. The results show a statistical significant difference in the number of sessions it took the two groups to finish the tutor program ($t(16) = 2.19$, $p = .04$). The descriptive statistics also indicates that the experimental group (M = 26.11, SD = 5.33) took fewer sessions than those in the comparison group (M = 31.56, SD = 5.25). That is, students who went through the PGBM-COMPS tutor program using the alternative instructional sequence completed the tutor program five days less on average than those in the comparison group.

4.6 Perceptions of Mathematics

In regards to students' perception in mathematics prior to the PGBM-COMPS program, Table 9 summarizes students' pre-TOMA likert-scale survey responses (note: the percentages shown to interpret the pre and post TOMA results combined strongly agree and agree responses together and disagree and strongly disagree together). Approximately 27.8% students suggested that it was not fun to work math problems in the Likert-scale survey questions from TOMA with the rest disagreeing (72.2%). As for the students' perceptions of their ability to solve math problems, slightly more than half of the students (55.6%) reported that they were not better at math than their peers, while 44.4% reported they were better at math compared to their peers. The majority of the students (72.2%) indicated mathematics was interesting and exciting with five students disagreeing (27.8%). Furthermore, 10 out of 18 students (55.6%) believed math tests were usually easy for them and the rest of the students (44.4%) reported math tests were difficult for them.

Half of the students (50%) reported liking to talk or read about problems in math books, and the other half of the students disagreed. In a follow-up interview, students who did not like to talk or read about the problems in math books believed math problems need to be solved quietly by themselves. These students also thought that talking was unnecessary when solving mathematics problems because finding out the correct answer and writing it in the book is the most important thing to do. Lastly, 27.8% of the students reported that they use math a lot outside of school with the majority of students disagreeing (72.2%).

Table 9. *Pre-TOMA Survey Responses*

	Strongly Agree	Agree	Disagree	Strongly Disagree
It's fun to work math problems.	10 55.6%	3 16.7%	0	5 27.8%
I'm better at math than most of my friends.	4 22.2%	4 22.2%	5 27.8%	5 27.8%
Math is interesting and exciting.	11 61.1%	2 11.1%	2 11.1%	3 16.7%
Math tests are usually easy for me.	4 22.2%	6 33.3%	4 22.2%	4 22.2%
When we use math books, I like to talk or read about the problems we do.	7 38.9%	2 11.1%	2 11.1%	7 38.9%
I use math a lot outside of school.	5 27.8%	0	2 11.1%	11 61.1%

Note. N = 18

Post-TOMA survey. Following the PGBM-COMPS program, all students believed that solving mathematics problems was fun (see Table 10 for a summary of students' post-TOMA survey responses). More than half of the students (55.6%) indicated that they were better at math than most of their friends with 44.4% of students disagreeing. The majority of students (94.4%) responded that math was interesting and exciting and that math tests were usually easy for them (83.3%). As for students' perception of the usefulness of mathematics following the intervention, the majority of students (77.8%) reported that they like to talk or read about the mathematics problems when using math books. In addition, more than two thirds of the students (83.3%) suggested that they use math a lot outside of school.

Table 10. *Post-TOMA Survey Responses*

	Strongly Agree	Agree	Disagree	Strongly Disagree
It's fun to work math problems.	11 61.1%	7 38.9%	0	0
I'm better at math than most of my friends.	7 38.9%	3 16.7%	7 38.9%	1 5.6%
Math is interesting and exciting.	13 72.2%	4 22.2%	1 5.6%	0
Math tests are usually easy for me.	5 27.8%	10 55.6%	1 5.6%	2 11.1%
When we use math books, I like to talk or read about the problems we do.	11 61.1%	3 16.7%	3 16.7%	1 5.6%
I use math a lot outside of school.	8 44.4%	7 38.9%	2 11.1%	1 5.6%

Note. N = 18

Perceptions of the PGBM-COMPS program. After completing the PGBM-COMPS program, students completed an exit survey (see Appendix C). Overall, all of the students in the comparison and experimental group enjoyed working with the PGBM-COMPS program (see Table 11). Furthermore, all of the students in both groups believed that the tutor program helped them to understand and solve multiplication/division problems. For instance, one student stated during the follow-up interview, "Well at school I'm getting better at math."

In regards to the perceptions of the two instructions embedded in the PGBM-COMPS program, approximately 88.9% of students in each group believed that the *Please Go and Bring Me* (PGBM) game was helpful when solving multiplication/division problems with 11.1% of students in each group disagreeing. Thus, there was no difference on their perceptions of the PGBM game between the two groups. As for the *COMPS* instruction, all of the students in both groups believed that the EG diagram was helpful when solving multiplication/division problems. According to the follow-up interview, the majority of students believed the EG diagram to be particularly helpful when solving problems with larger numbers. One student said, " It tells you to solve difficult questions with big numbers, but we got to solve them… so that is like learning." In fact, most of the students believed that their understanding of unit rate and the EG diagram were mostly used in their math class. For example, one student stated during the follow-up interview, "[I used] like unit rate… on the computer…what you have been showing us…. unit rate and stuff in class."

Slightly more than half of the students in each group (55.6% for the comparison group; 66.7% for the experimental group) reported that they were using what they learned in the PGBM-COMPS program in their classroom all the time or often, and 33.3% of the students in each group replied sometimes. In the follow-up interview, students described how they applied what they learned during math class: "Well in my math class, they told us [to solve problems] with the one with dividing, and I already know what division means because the program helped me." There were some things [problems] about the towers. Sometimes I don't really get that question, [but] I already know because you guys [the program] already helped me." Yet, one student (11.1%) in the comparison group reported that he never used what they learned through the tutor program.

Table 11. *Exit Questionnaire Responses*

	Very much		Sometimes		Not really		Not at all	
	Comp-arison group	Experimental group	Comp-arison group	Experimental group	Comp-arison group	Experimental group	Comp-arison group	Experimental group
How much did you enjoy the computer program?	4 44.4%	8 88.9%	5 55.6%	1 11.1%	0	0	0	0
How helpful was the computer program in understanding and solving multiplication/division problems?	6 66.7%	6 66.7%	3 33.3%	3 33.3%	0	0	0	0
How helpful was the *Please Go and Bring Me* game in the program when solving multiplication/division problems?	3 33.3%	4 44.4%	5 55.6%	4 44.4%	1 11.1%	1 11.1%	0	0
How helpful was the *EG* diagram in the program when solving multiplication/division problems?	5 55.6%	8 88.9%	4 44.4%	1 11.1%	0	0	0	0
	All the time		**Often**		**Sometimes**		**Never**	
	Comp-arison group	Experimental group	Comp-arison group	Experimental group	Comp-arison group	Experimental group	Comp-arison group	Experimental group
Are you using what you have learned in the program in your classroom?	2 22.2%	2 22.2%	3 33.3%	4 44.4%	3 33.3%	3 33.3%	1 11.1%	0
Will you recommend the program to your friends?	3 33.3%	2 22.2%	3 33.3%	3 33.3%	3 33.3%	4 44.4%	0	0

He explained that his math teacher rarely asked him to use the EG diagram while solving multiplication and division word problems. Furthermore, he stated that his math teacher rarely asked him to explain why multiplication/division needed to be used. Lastly, all of the students reported that they want to recommend the PGBM-COMPS program to their peers.

Teachers' perceptions. Following the data collection, four teachers were interviewed about their perceptions of the PGBM-COMPS program using the same exit survey that was given to the students. All of the teachers believed that the students either enjoyed the tutor program very much or sometimes. In addition, three out of four teachers believed that the computer program was either very helpful or sometimes helpful to students in understanding and solving multiplication/division problems. As for the two instructional strategies in the tutor program, three out of four teachers believed that the *Please Go and Bring Me* (PGBM) game was helpful. One teacher reported that the PGBM game was not really helpful. According to the follow-up interview, she felt some students took a longer time to understand the purpose of building equal towers with the given cubes. She strongly believed that more explicit instruction is needed for these students to understand the purpose of the PGBM game activity. In regards to the COMPS instruction, three out of four teachers reported that the EG diagram was helpful to students when solving multiplication/division problems. These teachers explicitly stated that they have seen their students frequently use the diagram during their math class. One teacher who disagreed was skeptical about the EG diagram because the ordering of the factor in the diagram differed from the diagram taught in their mathematics textbook. While the EG diagram asked students to identify the Unit Rate (number of items in each group), the number of units (number of groups), and the product (total number of items) respectively, the diagram they teach during mathematics class asked students to identify the number of units first and then the unit rate. Thus, the teacher believed that this inconsistency might confuse the students. Lastly, all of the teachers believed that the students used what they have learned in the computer program in their classrooms.

CHAPTER 5. DISCUSSION

The present study was designed to compare the differential effects of the two instructional sequences (i.e., A-**B**-C-D-E and A-C-**B**-D-E) for teaching multiplication/division word problem solving skills to students with mathematics difficulties (MD). In a school district with an afterschool math program in place for third and fourth grade students, the study was conducted as an attempt to seek a better instructional sequence that would meet the needs of students with MD while solving multiplicative word problems. Overall, the results indicate that the alternative instructional sequence (A-C-**B**-D-E) not only leads to positive achievement of mathematics problem solving outcomes but also increases students' learning efficiencies and their ability to solve for complex multiplication/division word problems.

5.1 Effects of the tutor program on MR Criterion Test

Results of the MR criterion test indicate that there were no differential effects of the two instructional sequences (i.e., A-**B**-C-D-E and A-C-**B**-D-E) on students' multiplication and division word problem solving performance. Furthermore, both groups' performances were maintained at a 2- and 3- week follow up. Both groups showed positive improvements on the MR criterion test items at a similar rate after the intervention. Both groups' maintenance and follow-up test performance on the MR criterion test indicate that, regardless of the instructional task sequence, the PGBM-

COMPS tutor program facilitated students' maintaining of their performance for a lengthy period of time. Overall, these results are in line with those of previous studies on the effectiveness of the PGBM-COMPS tutor program on multiplicative word problem solving by third and fourth grade elementary students with MD (Ma et al., 2014; Park et al., 2013; Xin, 2012; Xin et al., 2013).

5.1.1 Potential Differential Acquisition Effect

The results of this study show that students who went through the alternative instructional sequence obtained higher mean increases from pretest to posttest than those who went through the standard instructional sequence. The effect size (Cohen's d = .48), calculated on the basis of both groups' pre-posttest gain scores, indicates a moderate differential effect. A possible explanation for the positive effect size is that the students in the experimental group, who followed the alternative instructional sequence, were more likely to solve the *Mixed Unit Coordination* (MUC) tasks. While all of the students in the experimental group (100%) solved the MUC tasks on the MR criterion posttest, fewer students in the comparison group (66.7%) solved them. Similarly, those who went through the alternative instructional sequence obtained higher mean increases during maintenance and follow-up phases than those who went through the standard instructional sequence. While a small differential effect was shown (d = .24) between the two groups on the maintenance, a larger differential effect was shown (d = .70) between the two groups on the follow-up phase. It may be that the alternative sequence benefitted students with MD from better sustaining their learned problem-solving skills.

During the study, an opportunity arose to interview a student in each group while solving MUC tasks during Module B. Though deduction through such an interview cannot be made, sharing its outcome of the difference in the MUC solving approaches between the two groups may be noteworthy.

Annie, a female, Caucasian student with a mathematics difficulty placed in the experimental group (A-C-**B**-D-E), illustrated the most advanced problem-solving strategy during her first trial in solving the MUC tasks. Following her learning of quotitive division problem solving during Module C, Annie's approach in solving the MUC tasks was typical of the other students in the experimental group. Her MUC solving approach during the interview was as follows:

[Q]: Tom has a collection of 4 towers with 6 cubes in each. Tom brings 30 more cubes. Tom wants to put the 30 cubes into towers with 6 cubes in each, and put them under the red cover. How many towers of 6 cubes each will Tom have altogether?

R: Okay so how did you.. So you have 30 cubes over here, and you have to make towers of six. You did that by…?
A: I did that by… um dividing 30 divided by six.
R: And then you got…
A: And then I got Five.
R: Okay. So what's the final answer for this question? So there's five towers of six cubes. And so how many total towers of six cubes do we have?
A: Nine.
R: Nine. Okay. How did you get nine?
A: Because five plus four equals nine.
R: Okay.

Like other students in this group, she learned the quotitive division (QD) scheme prior to solving the mixed unit coordination (MUC) tasks. Annie first approached the MUC tasks by dividing the number of given cubes in the second set of a collection to form equal-sized towers. She then added the existing towers from the first collection to find the total number of towers. As Annie assimilated and accommodated her prior knowledge of the divisional scheme, she successfully solved the MUC scheme tasks. It is important to note that Annie's solving approach showed her ability of assimilation and accommodation in solving the novel problems.

Tom, a male, Caucasian student with a mathematics difficulty placed in traditional instructional sequence (A-**B**-C-D-E), performed a more primitive approach commonly used by students in the comparison group when solving MUC tasks. Like many other students in this group, he solved MUC tasks based on the multiplicative double counting (mDC) scheme. During module B, Tom notably used repeated addition to solve MUC tasks. His solving approach during the interview was as follows:

[Q]: Tom has a collection of 4 towers with 6 cubes in each. Tom brings 30 more cubes. Tom wants to put the 30 cubes into towers with 6 cubes in each, and put them under the red cover. How many towers of 6 cubes each will Tom have altogether?

T: Twenty-one [cubes].
R: Twenty-one cubes. So you have six [cubes] here and six [cubes] there. Oh, I think you mistakenly drew this. You only have five (cubes) over here.
T: Twenty-four.
R: Twenty-four? Okay. So you had twenty-four cubes right so far? Why don't you keep going?
T: I don't know.
R: Okay. I just want to let you know that we have thirty cubes, and you made towers of six. Do you think we can make another tower of six? You used twenty-four [cubes], and we have thirty. So do you think we can make another tower of six?
T: 1, 2, 3,4, 5, 6.
R: Then how many total cubes do we have over here?
T: Thirty.
R: Very good. So how many towers of six did you make?
T: Five.
R: Very good. Are we done?
T: No.
R: Good. How many total towers do you have?
T: Four.

Unlike Annie, Tom used double counting to find the number of equal towers, counting by six. During this process, he kept tracking the remaining number of cubes to find out when to stop double counting. As shown in the above interview, Tom successfully made four towers of six cubes in each using 24 cubes. However, he was unaware of the remaining six cubes and that another tower of six cubes could be made until the research assistant prompted him by asking whether he could made another equal-sized tower. According to Tom's interview above, he felt unsure about when to stop adding six cubes, and said that it was difficult keeping track of the remaining cubes. In addition, Tom struggled to solve the second step of the MUC task, which was to add towers. Although Tom figured the number of towers during the first step of the problem, he struggled to proceed to the final step, which is finding the total number of the towers. Tom needed further prompting that would break down the problem solving processes.

While Tom, in the comparison group, applied the double counting scheme, the only scheme they learned prior to MUC tasks, Annie, in the experimental group, applied the divisional scheme when trying to partition the single cubes into equal-sized towers. Although both approaches lead to the solution, experimental group students' knowledge of the quotitive divisional (QD) scheme could have contributed to their better performance in solving the MUC tasks in the MR criterion test. According to Fischbein and colleagues (1985), the double counting method is considered a primitive approach to multiplication. There were some limitations on the use of repeated addition to solve the first step of the MUC tasks, which is a QD problem. According to Hitch and McAuley (1991), it is common for students with MD to make errors during counting procedures mainly due to their limited working memory, leading them unable to monitor their progress during the calculation procedures. Furthermore, those who used repeated addition to solve MUC tasks was more likely to forget to proceed to the second step to solve for the final answer, which was to add the existing towers of the first collection.

These results are consistent with those of Xin and colleagues (2013) who stated that students struggle to solve MUC tasks may be due to their lack of ability to divide the given number of cubes to form equal-sized towers (Xin et al., 2013). Thus, students with MD struggle to manipulate their mDC, SUC, and UDS schemes to solve for the MUC tasks. Clearly, double counting (mDC) and the ability to operate with CU (SUC) and ones (UDS) skills were insufficient for the students to solve for the MUC tasks. Students with MD are well known to have problems with the acquisition and generalization due to their disadvantages in memory and cognitive processing (Kroesbergen & Van Luit, 2003). Without introducing the concept of division explicitly, students with MD experience difficulty identifying the units and flexibly interchanging the two units to solve for novel tasks.

The MUC tasks were more approachable for students with MD when they went through the quotitive division (QD) scheme tasks. Introducing the concept of partition prior to the MUC tasks may facilitate the process of accommodation, leading towards a better adaptation in the MUC tasks for students with MD. According to Steffe & Cobb (1994), students who use division operation are able to mentally partition a collection of given items into equal groups without the counting acts. Students at this stage have the ability to explicitly reverse the unit coordination by partitioning the total number of cubes by unit rate (UR). By understanding the QD scheme, students required less cognitive load to mentally partition the cubes into equal-sized towers as they explicitly learned to divide the product by the number of cubes in each tower. The QD scheme also positively affected students' ability to flexibly coordinate with the two units (i.e., unit rate and # of units), which led them to arrive at the final solution to find the total number of equal-sized towers. The result of this study suggests that explicit learning of the divisional scheme, prior to solving MUC tasks, could better accommodate students with MD to adapt to the new scheme (i.e., MUC).

5.1.2 Differential Efficiency Effect

Results of the efficiency comparison of the two instructional sequences further support the use of the alternative instructional sequence over the standard instructional sequence for students with MD. Given the fixed duration of each session, the experimental group completed the program approximately five days earlier than the comparison group. The alternative sequence, as opposed to the standard sequence, enabled the students to make the same improvements in a relatively shorter period of time. A possible explanation for this might be that students in the experimental group needed a shorter period of time to solve the MUC tasks with the use of the divisional scheme. Furthermore, students' ability to mentally operate division prevented them from making errors during the counting activity, which led to consistent accuracy in solving MUC tasks and earlier promotion to the next module.

Furthermore, students' early exposure to the QD scheme may have had a positive effect on their ability to adapt to other novel schemes introduced after MUC tasks (i.e., PD and MC). By establishing QD scheme, students in the experimental group had a stronger ability to identify the two units (ones and composite unit) and coordinate these two units, which are crucial skills to transition from multiplicative scheme to divisional scheme.

5.1.3 Effects on Closing the Achievement Gap

Results of the normative comparison with the MR criterion test are encouraging. A large mean difference (4.41) during posttest between the experimental group and the third grade normative reference (NR) group, favoring the experimental group, provides further insights into the effectiveness of the alternative sequence in the PGBM-COMPS tutor program. Although both groups showed improvement between pretest and posttest, the experimental group outperformed third grade normal-achieving students on the MR criterion test. It is important to note that instructional grouping was in effect in the elementary school when this study was conducted. That is, students were grouped into different ability classes on the basis of their academic performance. The normal-achieving students in this study were from a class where students are between the 40^{th} and 60^{th} percentiles on their mathematics achievement.

These results indicate that the PGBM-COMPS tutor program helped close the performance gap between the participating students with MD and their normal-achieving peers on multiplication/division word problem-solving skills. An interesting issue to note is that the third grade normal-achieving students' performance on the MR criterion pretest was relatively low (24.2% correct). Although they performed better than students with MD, the difference in the pretest mean between the two groups' MR performance was small (see Figure 8). One of the possible reasons for their unexpected low performance on MR criterion tests during the pretest might be that the third grade normal-achieving students were not yet exposed to various multiplicative schemes while solving multiplication word problems. According to the teacher who taught this level of mathematics class, the students were beginning to practice solving one-digit multiplication basic facts and word problems when this study was conducted. Due to limited resources, the study did not further examine the strategies the teacher used to implement the curriculum to teach students in the mathematics class. According to the mathematics textbook used by third grade students in the elementary school, the intended computational curriculum involved in understanding the concept of multiplication using equal-groups, area array, and equal jumps on the number line models. The curriculum also involved establishing fluency of multiplication and division facts using whole numbers from zero to ten (enVisionMath Common Core, 2012). As for the problem solving skills, the third grade students were expected to solve real-world problem solving using whole numbers up to 100. The strategies promoted in the textbook include illustrations, metacognitive strategy, and/or mathematics equation in the context of equal groups, arrays, and measurement (enVisionMath Common Core, 2012). While the COMPS model (Xin, 2012) led students to solve various word problems by using a mathematical model, which required students' conceptual understanding of equal groups and the relationship among the three numbers, the textbook used the context of equal groups to visually represent the problem situation and encouraged them to multiply the two numbers.

The experimental group also outperformed fourth grade normal-achieving students. This result is somewhat surprising, as a majority of the experimental group were third grade students, who had less exposure to solving multiplication/division word problems than the fourth grade students. In addition to the intended curriculum on multiplication they learned during their previous year, the fourth grade students were expected to multiply a whole numbers up to four digit numbers by two-digit numbers by using property of operation and place value strategies. As for the problem-solving skills, the fourth grade students were expected to solve real-life multiplication and division word problems using bar diagrams, metacognitive strategy, and the relationship between addition and multiplication or between subtraction and division (enVisionMath Common Core, 2012). The fourth grade students also work on representing and solving multiplicative comparison word problems using the same strategies as listed above (enVisionMath Common Core, 2012).

A possible explanation for their low performance on the MR criterion test might be that the third and fourth grade normal-achieving students were not exposed to all of the multiplication/division word problem types covered in the MR criterion test. While their textbooks covered problem types involving mDC, QD, PD, and MC schemes, they did not cover problem types involving SUC, UDS, and MUC schemes. Thus, a few word problems presented in the MR criterion test might have been unfamiliar to the normal-achieving students.

Granted that both groups eventually solved the quotitive division problems, and that the only difference was the sequence of when it was learned, students from both groups positively performed at a similar rate. However, the duration needed for students from the experimental group to complete the PGBM-COMPS intelligent tutor program was shorter than the comparison group. These findings suggest that while both instructional sequences bring positive outcomes to students with MD, the alternative instructional sequence seems to be a more efficient approach for students with MD to establish multiplicative concepts as they took less number of sessions to achieve the same level of word problem solving skills. In particular, the alternative sequence prepared the students with MD to solve challenging word problems (i.e., two-step problems), including MUC tasks, by explicitly teaching quotitive divisional scheme (QD) prior to solving MUC tasks. This finding supports the previous research on the positive effects of explicit and strategic instruction (Montague, 1992; Montague et al., 1993).

5.2 Effects of the tutor program on COMPS Test

The study also used a repeated measure ANOVA across time to investigate the differences between the two multiplicative instructional task sequences (i.e., A-**B**-C-D-E and A-C-**B**-D-E) based on students' near-transfer word problem solving performance, measured by a comprehensive near-transfer multiplicative word problem solving test (i.e., COMPS test). The results analyses indicate that there were no significant differences between the two instructional groups on their near-transfer performance. That is, both groups showed an increase in their near-transfer performance across four times at a similar rate. Overall, the negligible to small effect sizes on the post-treatment further indicate no significant difference between the two groups. Further, the results were maintained at a two- and three- week follow up. These results are in accord with recent studies examining the effects of the PGBM-COMPS tutor program on the improvement of students' near-transfer performance (Park et al., 2013).

Interestingly, the effect size on the maintenance phase, favoring the experimental group, was higher than those for the post-test and the follow-up phases (see Table 7). However, a pairwise comparison analysis confirmed that there was no significant difference between the two groups during the maintenance phase. Thus, it is difficult to claim that the experimental group maintained significantly better on the COMPS test than the comparison group. The above results indicate that the PGBM-COMPS tutor program, regardless of the instructional sequences, facilitated the students in transferring the learned multiplicative schemes to solve for novel word problems that entail similar problem structure but different story contexts.

There are two likely reasons for the no significant differences between two groups' near-transfer performance. First, it may be that both groups completed the COMPS part of the instruction in the PGBM-COMPS tutor program. Although the order of which they learned quotitive division (QD) was different, both groups went through all of the multiplicative schemes (i.e., QD and PD) that brought direct effects to solving the items on the COMPS test. Secondly, the COMPS test consisted of one-step multiplication and division word problems similar to the ones commonly shown in their mathematics textbook. The COMPS test did not included *Mixed Unit Coordination* (MUC) problems. Thus, potential difference in understanding in MUC between the two groups may not have critically affected their near-transfer performance.

5.3 Effects of the TOMA Test

Overall, results on the Attitude towards Math subtest in the Test of Mathematical Abilities (TOMA) and follow-up interviews provide support for the benefits of their attitudes about mathematics for students with mathematics disabilities (MD) after completing the PGBM-COMPS tutor program. There seems to be a positive change in students' perceptions of mathematics before and after the intervention phase. That is, the PGBM-COMPS tutor program positively affected students' attitude toward mathematics.

Pre-survey results revealed how most struggling students perceive mathematics in general. Many students with MD are prominent for their lack of self-esteem in solving mathematics word problems. Many participants believed that mathematics was fun and an interesting subject, but they also believed that they were not better at mathematics compared to their peers. In addition, they had a lack of understanding of how mathematical knowledge could be applied in everyday activity. The pre-survey also reflected the need for more opportunities for students with MD to engage in reform-based mathematics curriculum. For instance, many students in this study did not consider discussing their solving process as part of mathematics learning activities. This is not surprising as the use of explicit and strategic instructions with students with MD has been well documented in the field of special education (Montague, 1992; Montague et al., 1993). Students with MD are continuously accustomed to the traditional notion of solving word problems, where finding the correct numerical answer is the ultimate goal. Because of their lack of experience in process-focused mathematics instruction, it is challenging for students with MD to engage in mathematics discussions where they reason. Baxter, Woodward, and Olsen (2001) reported that low-achieving students contribute less during mathematics discussions, as normal-achieving students generally dominate the conversation.

Overall, the post-survey results indicate that the PGBM-COMPS tutor program has a positive effect on students' perception towards mathematics problem solving. The students seemed to have enhanced motivation, as more students believed that math was interesting (from 72.2% to 94.4%) and fun (from 72.2% to 100%). Following the intervention session, many students seemed to have gained a substantial amount of confidence in problem solving following the intervention sessions. More students (from 44.4% to 55.6%) believed that they were better at mathematics than most of their friends compared to their pretest TOMA response. Furthermore more students (from 55.6% to

83.3%) reported that mathematics was easy when compared to their pretest TOMA response. Their responses on the post TOMA survey also suggest that more students (from 50% to 77.8%) were transitioning their understanding of the act of problem solving from a conventional notion to a reform-based notion of problem solving. That is, many students were starting to believe that solving word problems requires not only finding the answer but also explaining how they solved a problem. Lastly, the post-survey results indicate that the PGBM-COMPS tutor program has a positive effect on students' ability to connect mathematics with their daily lives.

As for students' perception of the PGBM-COMPS tutor program, a majority of students seemed to have enjoyed working with the tutor program. Overall, both groups believed the PGBM-COMPS tutor program to be helpful in understanding and solving multiplication/division problems. Many students also believed that the tutor program prepared them to be successful in their mathematics classes. However, according to the observation during the intervention session, there was a slight difference between the two groups in their perception of the PGBM-COMPS tutor program when they were working on module B, which consisted of the MUC tasks. Many students in the comparison group expressed their frustration in solving the MUC tasks, as they were unable to proceed to the next problem fast enough. Many students in the comparison group were spending a significant amount of time on each MUC task as they struggled to find the solution. Thus, it seems possible that students with MD experience more limitations and frustrations on the MUC tasks when going through the PGBM-COMPS tutor program with the instructional sequence of A-**B**-C-D-E.

5.4 Limitation and Future Research

In our attempts to compare the differences in multiplicative word problem solving performance for the two groups, this study had several limitations. First, the findings are derived from a modest sample size that was needed to perform statistical analysis. Thus, a caution is due here, as the findings might not be extrapolated to all students with mathematics difficulties (MD). Second, this study is limited given that the students' progress on word problem-solving performance was determined on the sole basis of their accuracy of responses. The MUC scheme establishment may have affected aspects of mathematics knowledge other than their response accuracy.

For example, the study did not conduct a further analysis on students' progress on the conceptual development. As many researchers (e.g., Steffe, 1994; Vergnaud, 1988) have indicated, establishing multiplicative schemes takes a lengthy period of time. Steffe (1994) noted that, "any knowledge that involves carrying out actions or operations cannot be instilled ready-made into students or children but must, quite literally, be actively built up by them" (p. 4). The intervention phase in this study provided students only 11 weeks to learn six types of multiplicative schemes and was restricted by the time to complete the PGBM-COMPS tutor program. A longitudinal study should be undertaken to investigate how students with MD progress in their conceptual development as they go through the PGBM-COMPS tutor program. Moreover, further research is needed to investigate whether any differences exist between the two instructional sequences on students' maintenance of the multiplicative scheme knowledge over time. The above results could help further clarify whether there are any differential effects between the two instructional sequences on their establishment of the multiplicative schemes.

Thus, greater efforts are needed to research in greater depth the instructional support as well as accommodations needed for students with MD to enhance their higher-order thinking skills.

In addition, it is important to bear in mind the possible threat to external validity. In this study, the PGBM-COMPS tutor program delivered the instruction, and the researchers mainly assisted at times of technical difficulties and during the assessment phase. Although the classroom teachers engaged in this study, they directly worked with the researchers to notify them when students came across technical difficulties. Thus, it is difficult to claim that the results of this study could be generalized when teachers apply the tutor program in their mathematics classroom without direct assistance.

In order for educators to benefit from the use of the PGBM-COMPS tutor program to improve students' word problem solving skills, they need to understand overall structure and the functionality of the tutor program as well as their role in monitoring students' interaction with the intelligent tutor program. Therefore, further efforts need to be made to implement professional development for the teachers, which would help them to understand the theory and application of the PGBM-COMPS instructional program in order to have a better knowledge of implementing this tutoring program in their classroom settings. There have been an increasing number of CAI programs developed for students with MD to enhance their mathematics problem solving skills. However, there is a lack of computer-assisted tutoring program that incorporate the constructivist learning/instructional pedagogy; as such it is critical to focus on the professional development component in order to better prepare the synergistic role of the teachers in using such intelligent tutor program in their classroom settings. In addition to the knowledge of the tutor program teachers' attitudes towards computer-assisted instruction (CAI) should also be of special interest. A number of researchers have reported how teachers' perspectives about the use of CAI have a significant role on students' success in learning from CAI (Dorman, 1998).

Lastly, while conducting the study, students were gradually less motivated to work on the program by the time when they were particularly working on module C (i.e., QD). Many of the students were frequently asking how much they had completed and when they would be done with the tutor program. One of the primary reasons for their lack of motivation may be due to the lack of a progress-monitoring feature in the current version of the PGBM-COMPS program. That is, students were unable to see their overall progress while working on the tutor program In fact, to address this drawback, the researchers in this study begin each session by telling the students where they were currently working and how many more blocks were left for them to complete the tutor program by using the main menu screen. The above activities enabled students to understand how much they had accomplished and how much further progress needed to be made to finish the tutor program. The research assistants also applied a progress chart for each student to visually keep track of their progress using smiley stickers. When they completed each block, students received three smiley stickers to put on top of their name. Additional stickers were given to the students when they showed good behavior while working on the tutor program. For future enhancement of the PGBM-COMPS tutor program, more effort is needed in developing such features towards motivating students.

CHAPTER 6. CONCLUSION

The present study was designed to explore differentiated multiplicative instructional sequence that aims to nurture multiplicative concept to enhance the word problem solving performance of students with mathematics difficulties (MD). In particular, this study compared the differential effects of two instructional sequences taught in the PGBM-COMPS intelligent tutor system on the performance of students with MD. The present study makes several noteworthy contributions to the reform-based mathematics instructions in the field of special education. The first finding interests the learning of multiplicative word problem solving skills through a combination of constructivist and the Conceptual Model-based Problem Solving instructional approaches (COMPS, Xin, 2012) to teach students with MD the multiplicative concept in an explicit manner. With the right amount of knowledge of the PGBM-COMPS program, the results of this study suggest that teachers could use it as an intervention in their classrooms to help students with MD to enhance their multiplicative concepts as well as their overall performance on various multiplicative word problems.

The second finding stresses the importance of the appropriate instructional sequence of multiplicative scheme tasks to accommodate students with MD. Although both instructional sequences (modules A-**B**-C-D-E and A-C-**B**-D-E) had positive effects on the students' multiplicative word problem solving skills, further comparisons between the two groups' performance on the MUC tasks and their efficiency in learning multiplication word problem solving suggest that the alternative instructional sequence better accommodates students with MD who are working through challenging multiplicative schemes, leading them to become better mathematics problem solvers. In particular, the alternative sequence of the multiplicative schemes enabled students with MD to solve MUC tasks after they solved the quotitive divisional (QD) scheme tasks. The students' knowledge of the QD scheme (prior to the MUC tasks) accommodated their disadvantages in cognitive processing and working memory during the process of solving the MUC scheme tasks by explicitly learning the divisional scheme.

Therefore, by sequencing the multiplicative scheme tasks to accommodate students with MD, conceptual understanding of multiplication could be taught by using the constructivist approach alongside the explicit teaching of conceptual model-based problem-solving (COMPS) strategies. Consequently, students with MD could have more opportunities to establish higher order, complex mathematics thinking skills with the right amount of scaffolding.

The third finding suggests that CAI programs could improve the mathematics word problem solving skills for students with MD. Many teachers struggle in a mathematical reasoning interaction with students with LD, as teachers have long been recommended to use the systematic model of teaching (i.e., explicit and concrete based strategies) when teaching mathematics to students with LD (Woodward, 2004). It is still uncertain how mathematics in special education should adopt the constructivist approach, as the explicit instruction, an approach different from the constructivist approach, has been used to teach students who are struggling in mathematics (Baker, Gersten, & Lee, 2002). The adoption of the PGBM-COMPS program seems to overcome obstacles to delivering a reform-based instruction tailored towards the needs for students' learning. In addition, the CAI enabled academically diverse students to progress without compromising other students' abilities to reach their highest level of mathematics performance. While integrating CAI into students' learning has many benefits, the study suggests that the instructional features of the computer program play a crucial role in the students' learning experience (Seo & Bryant, 2009). As discussed earlier, no progress-monitoring feature was embedded into the current version of the PGBM-COMPS program so students could not keep track of their progress. Thus, with the appropriate instructions and features, students with MD appear to profit from learning multiplicative word problem solving skills with CAI.

Finally, this study suggests that students with MD benefit from the PGBM-COMPS tutor program by improving their perception of mathematics in general. Both instructional sequences of multiplicative scheme tasks taught with the tutor program seem to generate positive attitudes towards mathematics among students with MD. It appears that the PGBM game activity and Conceptual Model-based Problem Solving instructional approach (COMPS; Xin, 2012) play a critical role in students' confidence level in solving multiplicative word problems.

In conclusion, the discovery-based mathematics teaching in itself is inadequate to promote mathematics learning of students with MD (Woodward, 2004). Overall, this study strengthens the idea that the differentiated instruction tailored towards the students' needs may be effective in improving their multiplicative word problem solving skills.

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APPENDICES

Appendix A Multiplicative Reasoning (MR) Criterion Test

1. Grandma baked 27 cookies. She has 3 grandchildren: Manuel, Erika, and Anna. She gave all cookies to the children, and each grandchild received the same number of cookies. How many cookies did each grandchild get?

2. There are 28 students in Ms. Franklin's class. During reading, she puts all students in groups of 4. She asked a student (Steve): "How many groups will I make?" Steve said: "32. Because 28+4 is 32." Do you think that Steve is correct? Why?

3. A clown at the circus sells balloons in bunches. To make each bunch, he tied 5 balloons together. He made 13 bunches. How many balloons does he have in all 13 bunches?

4. Tonya bought some new shirts. Each new shirt has 6 buttons. There are 42 buttons in all. How many new shirts did Tonya buy?

5. Rachael has built 13 towers with 2 cubes in each. Mary has built 7 towers with 4 cubes in each. Who has more towers, Rachael or Mary? How many more towers does she have?

6. Pretend that you have made many towers, each made of 7 cubes. How many cubes are in every tower? How many cubes are in the first 4 towers? So we can count those by seven, "7, 14, 21, 28 …" Do you think you will say the number 84 if you continue counting cubes in the towers? Why?

7. Maria made birthday bags. She wants each bag to have 6 candies. After making 3 **bags**, she still had 12 candies left. How many **bags** will she have altogether after putting these 12 candies in bags?

8. Tom's father bought 6 pizzas. Each pizza had 4 slices. Tom's mother bought a few more pizzas. Then, there were 9 pizzas. How many more **slices** did Toms' mother bring?

9. Ali wants to buy a T-shirt that costs $6.50. He has a jar full of quarters. How many quarters will he need to buy the T-shirt?

One (1) dollar = 100 cents
One (1) quarter = 25 cents
There are 4 quarters in a dollar

10. After an art class, there were 78 crayons out on the tables. There are 6 boxes for the crayons. Ms. Brown puts the same number of crayons in each box. How many crayons would she put in each box?

(Purdue Research Foundation, 2011)

Appendix B 4th Grade COMPS Test Form A

1. It costs a total of $384 to buy 24 pizzas. How much does each pizza cost?

2. Francis received a total of $116 for his birthday. He wants to buy some schoolbooks. Each schoolbook costs $29. How many schoolbooks can he buy?

3. It takes 15 stamps to mail one package. How many stamps would you need to send 22 packages?

4. Mrs. Bond ordered 165 sheets of crepe paper to be shared equally among 5 art classes. How many sheets will each class get?

5. Pretend you are going to bake some chocolate chip cookies. Each cookie needs 26 chocolate chips. How many chocolate chips will you need if you want to bake 15 cookies?

6. There are 572 people in a school. There are 22 people in each classroom. How many classrooms are there in the school?

7. Brendan has been on the basketball team for 18 days. His friend Kali has been on the basketball team 27 times as long as Brendan. How long has Kali been on the basketball team?

8. Elliot has 187 pennies in a jar. Elliot has 11 times as many pennies as his sister Sandra. How many pennies does Sandra have?

9. Peter has 360 points. Caley has 20 points. Peter has how many times as many points as Caley?

10. Larry has 12 baseball cards. His friend Angel has 16 times as many baseball cards as Larry. How many baseball cards does Angel have?

11. Roger has answered 78 of his homework questions. If Roger has answered 3 times as many homework questions as his classmate Julian, then how many homework questions has Julian answered?

12. A farmer named Bob has 196 cows on his dairy farm. Another farmer named John has only 28 cows on his farm. The number of cows Bob has is how many times the number of cows John has?

(Xin, Wiles, & Lin, 2008)

Appendix C Exit Questionnaire

Please circle one statement that gives you the best picture of your experience of the computer tutor program.

A. How much did you enjoy the computer program?

1. enjoyed it very much	2. sometimes enjoyed it, sometimes not	3. did not enjoy it	4. did not enjoy it at all

B. How helpful was the computer program in understanding and solving multiplication/division problems?

1. very helpful	2. sometimes helpful	3. not really helpful	4. not helpful at all

C. How helpful was the *Please go and Bring Me* game in the computer program when solving multiplication/division problems?

1. very helpful	2. sometimes helpful	3. not really helpful	4. not helpful at all

D. How helpful was the *EG* diagram in the computer program when solving multiplication/division problems?

1. very helpful	2. sometimes helpful	3. not really helpful	4. not helpful at all

E. Are you using what you have learned in the computer program in your classroom?

1. never	2. sometimes	3. often	4. all the time

F. Will you recommend the computer program to your friends?

1. never	2. sometimes	3. often	4. all the time

(adapted from Xin, 2003)

VITA

VITA

Educational Background

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Professional Experience

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Managed database and analyzed the data to nurture multiplicative reasoning of students with learning disabilities/difficulties (NMRSD, Project Director/Supervisor: Dr. Yan Ping Xin)

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Implemented screen designs for computer-assisted instruction program and carried out the interventions at two elementary schools for the project on Nurturing Multiplicative Reasoning of Students with learning Disabilities/difficulties (NMRSD, Project Director/Supervisor: Dr. Yan Ping Xin)

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URT (Undergraduate Research Training)

December 2010

College of Education Ambassador Award

Grants

Purdue University West Lafayette, IN

August 2013 - August 2014 PRF Grant through Purdue University

Fall 2011-2012 Dean’s Graduate Student Travel Support

Publications

<u>Juried</u>

Park, J. Y, Xin, Y. P., Liu, J, Hord, C., Tzur, R., Si, L., Cordova, M., & Ruan, L. Y. (2013). Exploring the effects of intelligent tutoring system on multiplicative reasoning and problem solving of Students with Learning Disabilities. Paper accepted to present at the 2013 American Educational Research Association (AERA) Annual Meeting, San Francisco, CA.

Xin, Y. P., Si, L., Hord, C., Zhang, D., Cetintas, S, & **Park. J. Y.** (2012). The effects of computer-assisted instruction in teaching conceptual model-based problem solving. *Learning Disabilities: A Multidisciplinary Journal, 18*(2), 71-85.

Xin, Y. P., Si, L., Hord, C., Zhang, D., Cetinas, S., & **Park, J. Y.** (2012). Conceptual model-based problem solving that facilitates algebra readiness: An exploratory study with computer-assisted instruction. *Learning Disabilities: A Multidisciplinary Journal*, *18*(2). 71-85.

Xin, Y. P., Zhang, D., **Park, J. Y**., Tom, K., Whipple, A., & Si, L. (2011). A comparison of two mathematics problem-solving strategies: Facilitate algebra-readiness. *The Journal of Educational Research*, *104,* 1-15.

Cetintas, S., Si, L., Xin, Y. P., Zhang, D., & **Park, J. Y.** (2011). Automatic detection of irrelevant sentences in mathematical word problems. Educational Data Mining Journal.

Cetintas, S, Si, L., Xin, Y. P., Zhang, D., **Park, J. Y.**, & Tzur, R. (2010). "A Joint Probabilistic Classification Model of Relevant and Irrelevant Sentences in Mathematical Word Problems**".** *Journal of Educational Data Mining, 2*(1), 83-101.

Cetintas, S., Si, Luo., Xin, Y. P., Zhang, D., & **Park, J. Y.** (2009). Automatic text classification of mathematical word problems. In Proceedings of the 22nd International FLAIRS Conference (pp. 27-32). AAAI Press.

Work Submitted or In Progress

Park, J. Y. & Kastberg, S. (in progress). The approaches general and special education teachers make while engaging students with learning difficulties on proportional reasoning.

Park, J. Y. & Xin, Y. P. (in progress). Exploring the effects of an intelligent tutoring system on multiplicative reasoning and problem-solving of students with learning disabilities.

Park, J. Y. & Yakubova, G. (submitted). Teaching money skills for independent purchasing to students with developmental disabilities. *Teaching Exceptional Children*.

National Conferences & Presentations

Park, J. Y., Xin, Y. P., Tzur, R., Si, L., & Hord, C. (2014, July). A Comparison of instructional sequence in intelligent tutor-assisted math problem-solving intervention program. Poster Presentation at The Psychology of Mathematics Education (PME-38), Vancouver, Canada.

Ma, X., Xin, Y. P., Tzur, R., Si, L., Yang, X., **Park J. Y.**, Liu, J., & Ding, R. (2014, July). The effect of an intelligent tutor on math problem-solving of students with learning disabilities. Paper presented at the 2014 The Psychology of Mathematics Education (PME-38), Vancouver, Canada.

Park, J. Y., Xin, Y. P., Liu, J, Hord, C., Tzur, R., Si, L., Cordova, M., & Ruan, L. Y. (May, 2013). Exploring the effects on intelligent tutoring system on multiplicative reasoning and problem-solving of students with learning disabilities. Paper presented at the 2013 American Educational Research Association (AERA) Annual Meeting, San Francisco, CA.

Xin, Y. P., Hord, C., Liu, J., **Park, J. Y.**, Tzur, R., Si, L., Cordova, M., & Ruan, L. Y. (May, 2013). A comparison of teacher-delivered instruction and an intelligent tutor-assisted math problem-solving intervention program. Paper accepted at the 2013 American Educational Research Association (AERA) Annual Meeting, San Francisco, CA.

Xin, Y. P., **Park, J. Y.**, Hord, C., Liu, J., Bugdayci, A., Tzur, R., & Si, L. (April, 2012). Nurture multiplicative reasoning and problem solving through concept construction: The effect of an intelligent tutor. CEC Annual Convention and Expo, Denver, CO.

Xin, Y. P., Hord, C., **Park, J. Y.**, Liu, J., Tzur, R., Bugdayci, A., & Si, L. (April, 2012). Make explicit the reasoning behind math problem solving: Explore the effect of an intelligent tutor. AERA Annual Meeting, Vancouver, British Columbia, Canada.

Local Presentations

Park, J. Y. (September 2012). *Characteristics of Math Learning Disabilities and Intervention*. Invited presentation to EDCI 584 (Teaching Mathematics to Diverse Learners) lecture. Purdue University.

TEACHING

College Level

January 2013- May 2013 **Purdue University**

Instructor, Assessment Techniques

Planned and taught EDPS 361, Use Assessment Techniques in Special Education for undergraduate teacher education students. Responsibilities included lecturing, grading, and instructing students on conducting related assessments to students.

August 2012- December 2012 **Purdue University**

Instructor, Mild Disabilities

Planned and taught EDPS 270, Characteristics of Individuals with Mild Disabilities, for undergraduate teacher education students. Responsibilities included lecturing, grading, and instructing students on technology integration and related topics.

August 2011- May 2012 **Purdue University**

Graduate Teaching Assistant, Block II

Taught one of the recitation classes and worked as a lecture assistant for EDPS 235 & 265, Learning and Motivation & The Inclusive Classroom, with the university faculty members. Responsibilities included planning and leading discussions relevant to the topics from the lecture.

K-12 Teaching

Lafayette, Indiana

January Spring 2010 – To Present **Glenn Acre Elementary School**

PGBM-NMRSD After-school Program Instructor

Taught and assisted elementary students with learning disabilities and those at-risk enhance their multiplicative reasoning skills while using the developed computer program.

January 2008 - May 30, 2008 **Edgelea & Miami Elementary School**

COMPS Afterschool Program Assistant

Taught elementary students with learning difficulties math problem solving using Conceptual-model based solving.

West Lafayette, Indiana

2011-2012 **Beacon Academy**

Student's Performance evaluator

Evaluated student's math, reading comprehensions, and written language using norm-referenced and criterion-reference assessments.

2010 **Klondike Elementary School**

Student Teacher

Taught half-time kindergarten class for a total of forty students. Responsibilities included teaching lessons, planning lessons, and prepare teaching materials.

SERVICE/ENGAGEMENT/LEARDERSHIP

Memberships

American Educational Research Association

Council for Exceptional Children
Divisions of Learning Disabilities, Research, and Technology and Media

Psychology of Mathematics Education (PME)