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"Smart" Foams for Active Absorption of Sound (written version)

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"SMART" FOAMS FOR ACTIVE ABSORPTION OF SOUND

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ABSTRACT

A porous layer can absorb a significant amount of acoustic energy only if its thickness is comparable to the wavelength of the incident sound. Thus, a porous layer inevitably becomes a less effective sound absorber as the frequency is decreased. In this paper, it will be shown through theoretical calculations that the low frequency performance of a finite-depth layer of elastic porous material may be enhanced by applying an appropriate force to the solid phase at the front surface of the layer. In particular, it will be shown that at any angle of incidence the solid phase may be forced so as to create a perfect impedance match with an incident plane wave, thus causing the sound to be completely absorbed. Note that the success of the approach suggested here requires a significant degree of coupling between the motion of the solid and fluid phases of the porous material. Thus, it may be expected that partially reticulated, polyurethane foams will be susceptible to this approach, owing to the degree of viscous and inertial coupling between their fluid and solid phases.

INTRODUCTION

High performance, compact, and lightweight noise control treatments are needed for weight-critical applications such as aircraft fuselage linings. In this article, the feasibility of developing a "smart" foam which combines the inherent high frequency noise control capability of an acoustic foam treatment with the low frequency noise control capability of an active noise control treatment is investigated.

The smart foam configuration investigated here comprises a thin layer of partially reticulated (i.e., high specific flow resistance) acoustic foam that is rigidly constrained at its back surface and whose solid phase is driven by a force applied at the layer's front surface. Other forcing configurations are possible (e.g., application of a force at the back surface of the foam layer), but the analysis of alternative configurations is very similar to that presented here.

In this paper, the force, displacement, and power requirements for a smart foam (based on the physical properties of a typical polyurethane foam) are presented. Those results will be used as a starting point for subsequent investigations to be focussed on determining the most effective method of foam actuation, the control-system requirements, and the performance limitations of smart foams. In the first section of the present paper, a theory governing wave propagation in elastic porous materials is described briefly. Next, the boundary conditions applicable to an actuated foam are described. Based on those boundary conditions the forces required to achieve perfect absorption can be calculated, and typical results are shown here. Note that an early form of this paper was presented elsewhere [1].

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WAVE PROPAGATION IN ELASTIC POROUS MATERIALS

In this section, a theory governing sound propagation in partially reticulated foam is presented.

The study of sound propagation in porous materials has a long history, perhaps beginning with a theory presented by Rayleigh in 1896 [2]. Rayleigh's model consisted of a rigid solid with identical pores (either stacked cylinders or parallel slits) oriented perpendicular to the material's surface. The Rayleigh theory does not account for motion of the solid phase or tortuosity of the pores (i.e., deviation of the pores from straight and constant diameter), but it does allow for viscosity and heat transfer energy losses in the pores.

In 1949, Zwikker and Kosten [3] introduced a theory that allowed for motion of the solid phase of the foam. The Zwikker and Kosten theory is more advanced than the Rayleigh theory in that it allows solid phase motion and includes both viscous and momentum coupling (due to pore tortuosity) between the solid and fluid phases. The theory accounts for simultaneous viscous and thermal losses in the fluid phase. The Zwikker and Kosten theory predicts the existence of two wave types that propagate in both the air and in the solid phase of the porous material.

In 1956, Biot [4,5] published a theory which includes the features of the Zwikker and Kosten theory but which also accounts for the shear stiffness of the solid phase. The Biot theory thus predicts the existence of three bulk wave types: two dilatational waves like those of the Zwikker and Kosten theory and one shear wave type. In its original form the Biot theory accounts for viscous losses, but it does not consider thermal losses in the fluid phase.

Biot's primary interest was geophysics (perhaps explaining the omission of thermal losses in the fluid phase) while Zwikker and Kosten's was noise control: two different nomenclatures were the result. The Biot nomenclature is used in this article, as it is widely used in recent literature, e.g. [6]. Reconciliation of the two nomenclatures leads to some overlap of variable names in this article. For example, ε is a strain while ε' is the tortuosity; σ_x, σ_y, and σ_z are stress tensors while σ is the flow resistivity; and ω is frequency while ω' is a rotational strain.

The challenge in applying any of the above theories to the study of partially reticulated foams is to define appropriate functions for the various coupling coefficients, elastic moduli, and densities. In this regard, the references cited above are incomplete. Many researchers have made significant contributions in the application of the general theory of sound propagation in porous elastic materials to partially reticulated foam. A listing of these contributions is not presented here owing to space limitations but may be found in several sources including: Bolton[7], Bolton and Green [8], Green [9], Shiau [10] and Bolton and Shiau [11].

FOAM THEORY

Biot [4, equation 6.7] gives the dynamic equations for a fluid-saturated, isotropic, porous solid with viscous dissipation as:

$$N\nabla^2\mathbf{u} + \nabla[(A+N)e + Q\varepsilon] = \frac{\partial^2}{\partial t^2}(\rho_{11}\mathbf{u} + \rho_{12}\mathbf{U}) + b\frac{\partial}{\partial t}(\mathbf{u} - \mathbf{U}), \tag{1}$$

$$\nabla[Qe + R\varepsilon] = \frac{\partial^2}{\partial t^2}(\rho_{12}\mathbf{u} + \rho_{22}\mathbf{U}) - b\frac{\partial}{\partial t}(\mathbf{u} - \mathbf{U}), \tag{2}$$

where $\mathbf{u}$ is the displacement vector of the solid phase, $\mathbf{U}$ is the displacement vector of the fluid phase, e is the volumetric strain of the solid phase, ε is the volumetric strain of the fluid phase, t is time, $N = E_1 / 2(1 + \nu)$ is the shear modulus, $E_1 = E_m(1 + j\eta)$, ν is the Poisson's ratio, E_m

is Young's modulus for the solid phase, $j = (-1)^{1/2}$, η is the *in vacuo* loss factor used to account for hysteretic damping of the solid phase, $A = \nu E_1 / (1 + \nu)(1 - 2\nu)$ is a Lame' constant, $Q = (1 - h)E_2$ is a coefficient relating fluid and solid phase strain, $E_2 = \rho c^2 / [\, 1+2(\gamma-1)\mathrm{T}((N_{pr})^{1/2}\lambda_c(-j)^{1/2}) / ((N_{pr})^{1/2}\lambda_c(-j)^{1/2}\,)]$ is the fluid's complex bulk modulus of elasticity (made complex to account for heat transfer to the pore walls), ρ is the fluid (air) density, c is the fluid's speed of sound, γ is the fluid's ratio of specific heats, N_{pr} is the fluid's Prandlt number, $\lambda_c = 8\omega\rho\varepsilon' / h\sigma$, ω is the angular frequency, ε' is the tortuosity, σ is the flow resistivity, $\mathrm{T}(x) = \mathrm{J}_1(x) / \mathrm{J}_0(x)$, J_0 and J_1 are the Bessel functions of the first kind of zero and first order, respectively, h is the foam porosity, $R = hE_2$, $\rho_{11} = \rho_1 + \rho_a$, $\rho_{22} = \rho_2 + \rho_a$, $\rho_{12} = -\rho_a$, ρ_1 is the bulk density of the solid phase, $\rho_c = \rho / [\, 1 - 2\mathrm{T}(\lambda_c(-j)^{1/2}\,) / (\lambda_c(-j)^{1/2})\,]$, $\rho_2 = h\rho$, $\rho_a = \rho_2(\varepsilon'\mathrm{Re}\{\rho_c/\rho\} - 1)$, and $b = -\omega\rho\varepsilon' h\mathrm{Im}\{\rho_c/\rho\}$. In this article, bold type is used for vectors, and no special notation is used for complex variables as most of the variables are complex. Notation of the form u_x is used to denote the component of **u** in the x-direction. Further explanation of the above material parameters may be found in [7-11].

By applying the divergence operator to equations (1) and (2), and by using the relationships $\nabla\bullet\mathbf{u} = e$ and $\nabla\bullet\mathbf{U} = \varepsilon$, it is possible to obtain [4, equation 7.1]:

$$\nabla^2(Pe + Q\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{11}e + \rho_{12}\varepsilon) + b\frac{\partial}{\partial t}(e - \varepsilon), \tag{3}$$

$$\nabla^2(Qe + R\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}e + \rho_{22}\varepsilon) - b\frac{\partial}{\partial t}(e - \varepsilon), \tag{4}$$

where $P = A + 2N$. Then by applying the curl operator to equations (1) and (2) and using the relationships $\nabla\times\mathbf{u} = \omega'$ and $\nabla\times\mathbf{U} = \Omega$ one obtains [4, equation 7.2]:

$$N\nabla^2\omega' = \frac{\partial^2}{\partial t^2}(\rho_{11}\omega' + \rho_{12}\Omega) + b\frac{\partial}{\partial t}(\omega' - \Omega), \tag{5}$$

$$\mathbf{0} = \frac{\partial^2}{\partial t^2}(\rho_{12}\omega' + \rho_{22}\Omega) - b\frac{\partial}{\partial t}(\omega' - \Omega), \tag{6}$$

where ω' and Ω are the rotational strain vectors for the solid and fluid phases, respectively.

Combination of equations (3) and (4) yields (under the assumption of $e^{j\omega t}$ time dependence):

$$\nabla^4 e + A_1\nabla^2 e + A_2 e = 0, \tag{7}$$

$$\nabla^4 \varepsilon + A_1\nabla^2 \varepsilon + A_2 \varepsilon = 0, \tag{8}$$

where

$$A_1 = \frac{-R(-\rho_{11}\omega^2 + jb\omega) + 2Q(-\rho_{12}\omega^2 - jb\omega)\ P(-\rho_{22}\omega^2 + jb\omega)}{RP - Q^2}, \tag{9}$$

$$A_2 = \frac{-(-\rho_{12}\omega^2 - jb\omega)^2 + (-\rho_{11}\omega^2 + jb\omega)(-\rho_{22}\omega^2 + jb\omega)}{RP - Q^2}. \tag{10}$$

The above differential equations allow two propagating plane wave solutions of the form $e^{\pm jk_{1,2}x}$, where the two wavenumbers are:

$$k_{1,2} = \sqrt{\frac{A_1 \pm \sqrt{A_1^2 - 4A_2}}{2}}. \tag{11}$$

Thus, two types of dilatational bulk waves may propagate in an elastic porous material. Biot refers to the faster and slower of the two wave types as dilatational waves of the first and second kind, respectively. Bolton and Green [8] call the faster and slower wave types the frame-borne and airborne waves, respectively. The names frame-borne and airborne refer to the fact that for foams the behavior of the two wave types is determined primarily by the physical properties of solid and fluid phases, respectively. The names frame-borne and airborne are, however, somewhat misleading as both types of dilatational waves travel in both the solid and fluid phases of the foam as shown by the differential equations (7) and (8).

Return now to the shear (rotational) strain relationships. By combining equations (5) and (6) one can obtain (again assuming a time dependence of $e^{j\omega t}$):

$$N\nabla^2\omega' = \left[(-\rho_{11}\omega^2 + jb\omega) - \frac{(-\rho_{12}\omega^2 - jb\omega)^2}{(-\rho_{22}\omega^2 + jb\omega)}\right]\omega', \tag{12}$$

$$N\nabla^2\Omega = \left[(-\rho_{11}\omega^2 + jb\omega) - \frac{(-\rho_{12}\omega^2 - jb\omega)^2}{(-\rho_{22}\omega^2 + jb\omega)}\right]\Omega. \tag{13}$$

Equations (12) and (13) allow a propagating plane wave solution of the form $e^{\pm jk_t x}$, where

$$k_t = \sqrt{N^{-1}\left[(-\rho_{11}\omega^2 + jb\omega) - \frac{(-\rho_{12}\omega^2 - jb\omega)^2}{(-\rho_{22}\omega^2 + jb\omega)}\right]}. \tag{14}$$

Thus, only one type of bulk shear (rotational) wave is possible. By combining equations (5) and (6) it can be shown that at a single frequency, ω, the rotational strain of the solid and fluid phases are directly proportional:

$$\Omega = \omega'\left[\frac{\omega^2\rho_{12} + j\omega b}{-\omega^2\rho_{22} + j\omega b}\right]. \tag{15}$$

BOUNDARY CONDITIONS

Consider the response of an infinite slab of foam to an oblique incidence, harmonic, plane wave as show in Figure 1. The velocity potential of the incident plane wave is:

$$\Phi_i = e^{j(\omega t - k_x x - k_y y)}, \tag{16}$$

where $k_x = k\sin\theta$, $k_y = k\cos\theta$, $k = \omega/c$ is the wavenumber in air, and θ is the angle of incidence. The assumption of unit amplitude for the velocity potential is arbitrary and presents no problem as long as the behavior of sound propagation in the air and foam can be assumed to be linear. The linearity assumption is implicit in the development of the foam theory above.

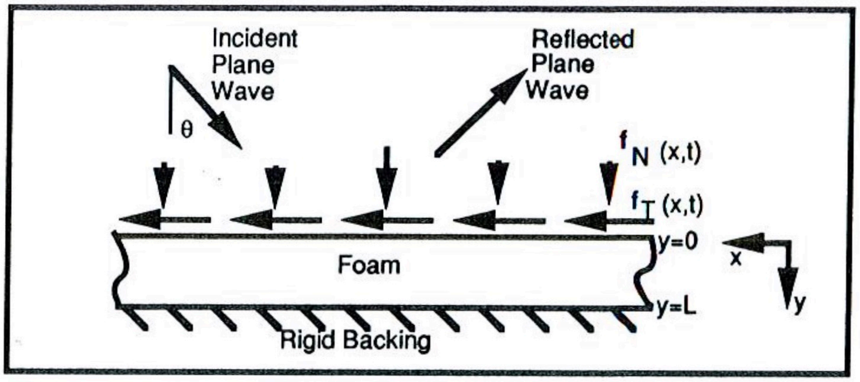

Figure 1 - Forced Foam Configuration.

In accordance with Snell's law, all of the wave components in the porous material must have the same trace wavenumber, k_x. Under these conditions and under the assumption of plane strain (a result of the foam's assumed infinite extent and orientation to the incident plane wave), the dilatational strain in the solid phase can be assumed to have the form:

$$e = e^{j(\omega t - k_x x)}(C_1 e^{-jk_{1y}y} + C_2 e^{jk_{1y}y} + C_3 e^{-jk_{2y}y} + C_4 e^{jk_{2y}y}) \tag{17}$$

where C_1, C_2, C_3, and C_4 are arbitrary constants whose values depend on the boundary conditions; $k_{1y} = (k_1^2 - k_x^2)^{1/2}$; and $k_{2y} = (k_2^2 - k_x^2)^{1/2}$. Motion of the fluid phase is coupled to that of the solid phase per equations (3) and (4), and it can be shown that:

$$\varepsilon = e^{j(\omega t - k_x x)}(b_1 C_1 e^{-jk_{1y}y} + b_1 C_2 e^{jk_{1y}y} + b_2 C_3 e^{-jk_{2y}y} + b_2 C_4 e^{jk_{2y}y}), \tag{18}$$

where

$$b_{1,2} = \frac{-[R(-\omega^2\rho_{11} + j\omega b) - Q(-\omega^2\rho_{12} - j\omega b)] - (RP - Q^2)k_{1,2}^2}{R(-\omega^2\rho_{12} - j\omega b) - Q(-\omega^2\rho_{22} + j\omega b)}. \tag{19}$$

The rotational (shear) strain in the solid phase is:

$$\omega' = 0\mathbf{i} + 0\mathbf{j} + e^{j(\omega t - k_x x)}(C_5 e^{-jk_{ty}y} + C_6 e^{jk_{ty}y})\mathbf{k}, \tag{20}$$

where **i**, **j**, and **k** are the unit vectors in the x-, y- and z-directions, respectively, C_5 and C_6 are arbitrary constants whose values depend on the boundary conditions, and $k_{ty} = (k_t^2 - k_x^2)^{1/2}$. The rotational strain of the fluid phase is proportional to that of the solid phase per equation (15), so that

$$\Omega = 0\mathbf{i} + 0\mathbf{j} + g e^{j(\omega t - k_x x)}(C_5 e^{-jk_{ty}y} + C_6 e^{jk_{ty}y})\mathbf{k}, \tag{21}$$

where

$$g = \frac{\omega^2\rho_{12} + j\omega b}{-\omega^2\rho_{22} + j\omega b}. \tag{22}$$

Thus, the response of the foam is known if the six constants C_1 through C_6 can be determined by boundary conditions.

The boundary conditions are most conveniently formulated in terms of velocities and stresses while equations (17) through (22) are expressed in terms of strain. The required stress-strain relationships for the foam are given by Biot [4, equation 2.12]:

$$\sigma_x = 2Ne_x + Ae + Q\varepsilon, \tag{23}$$

$$\sigma_y = 2Ne_y + Ae + Q\varepsilon, \tag{24}$$

$$\sigma_z = 2Ne_z + Ae + Q\varepsilon, \tag{25}$$

$$\tau_{xy} = N(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}), \tag{26}$$

$$\tau_{xz} = N(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}), \tag{27}$$

$$\tau_{yz} = N(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}), \tag{28}$$

$$s = Qe + R\varepsilon, \tag{29}$$

$$\text{with } e = e_x + e_y + e_z = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}, \tag{30}$$

where the directions of the tensors are shown in Figure 2. For this problem, $\tau_{xz} = \tau_{yz} = e_y = e_z = \varepsilon_y = \varepsilon_z = 0$. Strain and displacements in the foam are related by $\nabla \cdot \mathbf{u} = e$, $\nabla \cdot \mathbf{U} = \varepsilon$, $\nabla \times \mathbf{u} = \omega'$, and $\nabla \times \mathbf{U} = \Omega$. Single frequency displacement and velocity are related by $\mathbf{v} = j\omega\mathbf{u}$ and $\mathbf{V} = j\omega\mathbf{U}$.

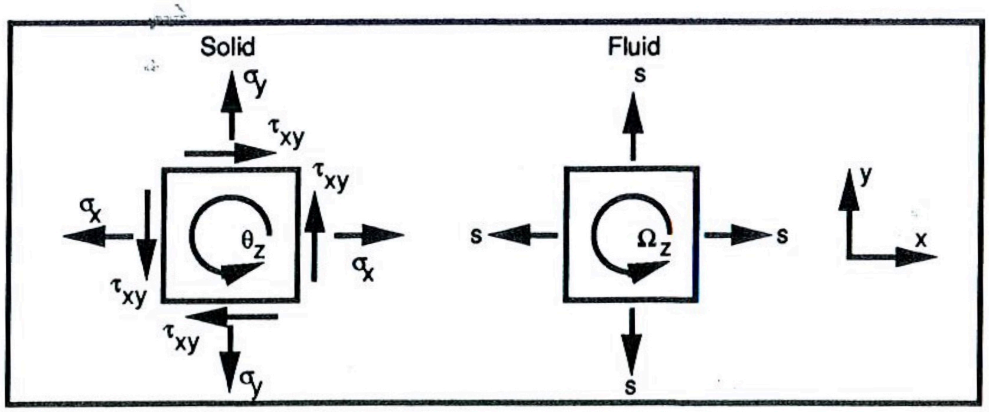

Figure 2 - Assumed Tensor Directions.

In the air outside the foam, the velocity, $\mathbf{v}_a$, is given by the velocity potential definition:

$$\mathbf{v}_a = \nabla\Phi_i, \tag{31}$$

and the pressure and velocity are related by the linearized momentum equation:

$$\rho\frac{\partial \mathbf{v}_a}{\partial t} = -\nabla p_a, \tag{32}$$

where ρ is the air density, and p_a is the air pressure.

By using the relationships given above, the boundary conditions (in terms of stresses and displacements) can be used to find the unknown coefficients C_1 through C_6.

The configuration to be analyzed consists of a rigidly backed foam slab with a force per unit area of the form

$$\mathbf{f}(x,t) = F_T e^{j(\omega t - k_x x)}\mathbf{i} + F_N e^{j(\omega t - k_x x)}\mathbf{j} + 0\mathbf{k} = f_T\mathbf{i} + f_N\mathbf{j} \tag{33}$$

applied to the face of the foam as shown in Figure 1. Here F_T and F_N represent tangential and normal forces applied to the solid phase of the foam at the surface of the foam. Note that it has been assumed that the force is applied at the same frequency and trace wavenumber as the incident plane wave. It can be shown that these requirements must be satisfied if there is to be no reflected wave.

At the rigid boundary, $y = L$, three boundary conditions must be satisfied. The displacements of the solid phase normal and tangential to the backing must be zero, and the normal displacement of the fluid phase must be zero:

$$(i)\ \ u_x = 0, \tag{34}$$

$$(ii)\ \ u_y = 0, \tag{35}$$

$$(iii)\ \ U_x = 0. \tag{36}$$

At the $y = 0$ boundary, the normal and tangential forces on a very thin layer (assumed massless) must balance, and the volume velocities outside the foam and at the surface of the foam must balance [8]. Therefore at $y = 0$

$$(iv)\ \ -hp_a = s\ , \tag{37}$$

$$(v)\ \ -(1-h)p_a - (1-h)f_N = \sigma_y\ , \tag{38}$$

$$(vi)\ \ v_{ay} = j\omega(1-h)u_y + j\omega h U_y \tag{39}$$

$$(vii)\ \ \tau_{xy} = -(1-h)f_T\ , \tag{40}$$

where p_a, v_{ay}, and Φ_i are known from equations (16), (31), and (32).

There are seven boundary conditions to be satisfied and eight unknowns (C_1 through C_6, F_N, and F_T); therefore, no unique solution exists. Note that perfect absorption of the incident sound field is enforced simply by requiring that the total field in the exterior region be equal to the incident field alone: i.e., equation (16). If it assumed that $F_T = 0$ (no tangential control force), there are seven equations and seven unknown, and a unique solution exists. Similarly, if $F_N = 0$, a unique solution exists. The physical interpretation of this system of linear equations is that it should be possible to control the foam with a normal force alone, a tangential force alone, or an infinite number of combinations of normal and tangential forces.

RESULTS

In this section, the theory developed above is used to calculate the required force, displacement, and power to cause perfect absorption of the incident sound field for a specific smart foam configuration. The foam is a 25 mm thick slab which is assumed to be infinite in lateral extent. The foam is forced by either a normal or tangential force applied to the open front face. The back face of the foam is attached to a rigid backing. The incident sound is a harmonic plane wave traveling at an angle θ from the surface normal, and it is assumed that the smart foam is functioning so that no reflected wave is produced. Unless stated otherwise, the foam is assumed to have a porosity, h, of 0.9, a bulk density of the solid phase, ρ_1, of 30 kg/m^3, an *in vacuo* bulk Young's modulus, E_m, of 8x10^5 N/m^2, an *in vacuo* loss factor, η, of 0.265, a bulk Poisson's ration, ν, of 0.4, a flow resistivity, σ, of 25x10^3 mks rayls/m, and a structure factor, ε', of 7.8. Those values are assumed to be representative of a typical partially reticulated polyurethane foam [9]. The fluid is air at standard temperature and pressure.

Figure 3 shows the absorption coefficient (ratio of absorbed to incident acoustic intensity) for the foam without the application of the control force. The theory for the unforced foam calculation is not presented here but may be found in [7].

Control force, displacement, and power requirements for the smart foam are plotted in Figures 4 through 6. The power per unit area is calculated as $0.5(1 - h)\mathrm{Re}\{F_N(jwu_y)^*\}$ for normal force application and $0.5(1 - h)\mathrm{Re}\{F_t(j\omega u_x)^*\}$ for tangential force application. At low frequencies, the force, displacement, and power requirements are high; therefore, the useable frequency range of a smart foam is displacement-limited at low frequency, but probably no more so that for a conventional loudspeaker active control system of equal area. As the frequency approaches that of the first peak in the absorption coefficient curve, the power and force requirements drop significantly. Control at higher frequencies is not an issue as the smart foam relies on the inherent high frequency passive absorption the foam.

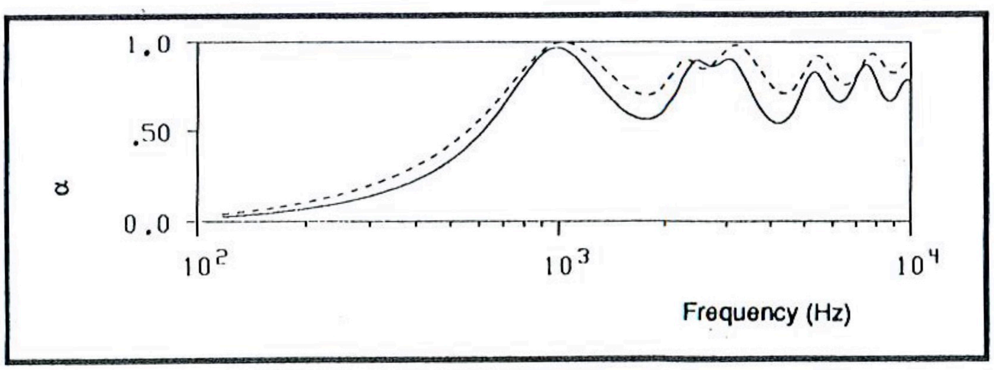

Figure 3 - Absorption Coefficents for the Foam without a Control Force. Normal Incidence ____. $\theta = 45$ degrees _ _ _.

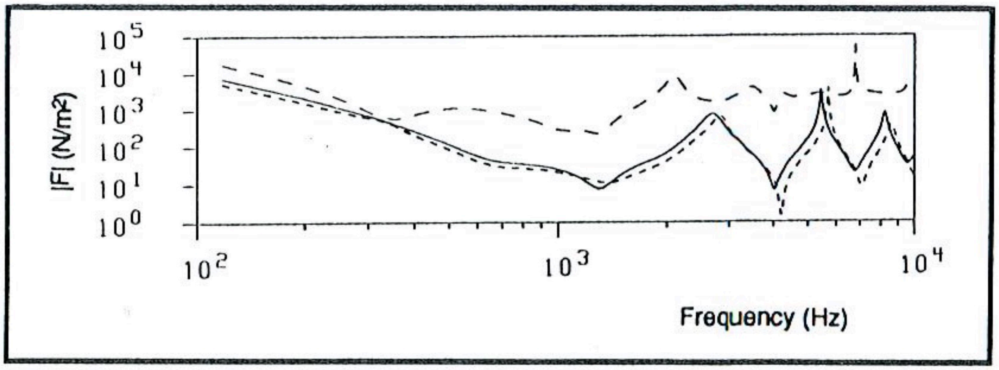

Figure 4 - The Required Control Force Amplitude. Normal Incidence ____. Normally Applied Force, θ = 45 degrees _ _ _. Tangentially Applied Force, θ=45 degrees __ __.

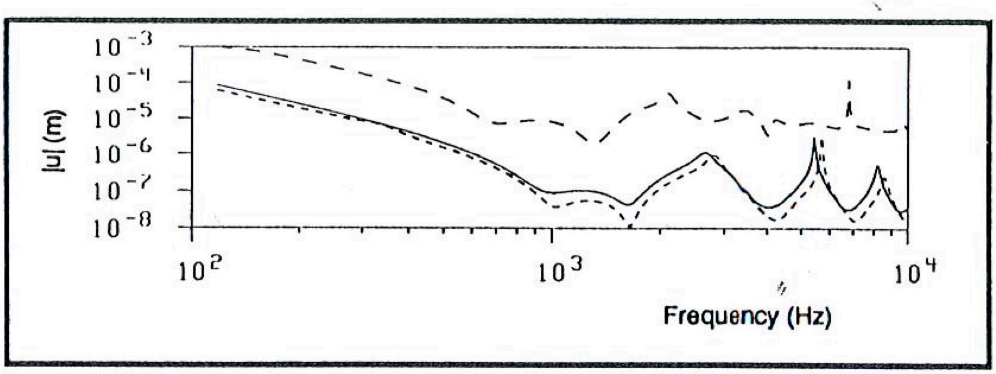

Figure 5 - The Required Control Displacement Amplitude. Normal Incidence ____. Normally Applied Force, θ = 45 degrees _ _ _. Tangentially Applied Force, θ=45 degrees __ __.

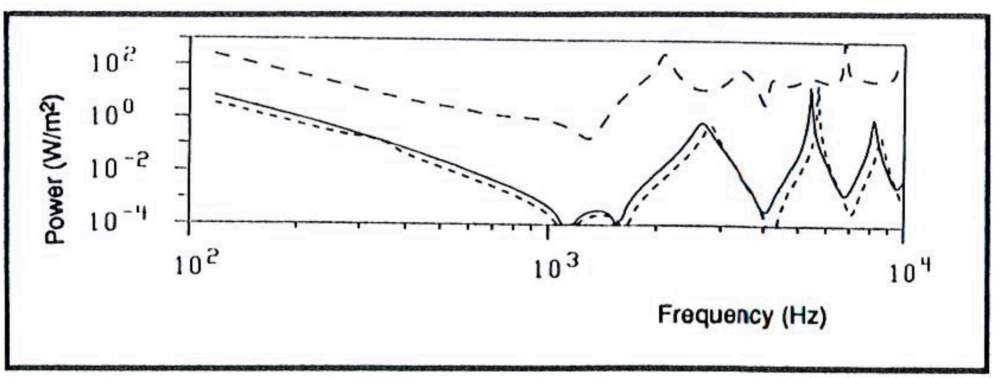

Figure 6 - The Required Control Power. Normal Incidence ____. Normally Applied Force, θ = 45 degrees _ _ _. Tangentially Applied Force, θ=45 degrees __ __.

Also plotted in Figures 4 through 6 are results for sound incident at 45° with a control force applied either normally or tangentially to the surface. The results are similar both quantitatively and qualitatively to those obtained for normal incidence sound, though, as a rule, the tangentially applied control force requires greater force to be applied through a greater distance.

CONCLUSIONS

A theory has been presented which allows the required force, displacement, and control power to be calculated for smart foams, and the results are presented for a specific smart foam configuration.

It was shown that the useable frequency range of a smart foam is displacement-limited at low frequency, but probably no more so than for a conventional loudspeaker active control system of identical area. Also, relatively stiff, high flow resistivity, partially reticulated foams typical of those used for passive sound absorption provide a reasonable compromise between low power consumption at low frequency and good passive sound absorption at higher frequency. Normal and tangential control force application are shown to be equivalent in effect.

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