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Optimal Design of Multi-Layer Microperforated Sound Absorbers

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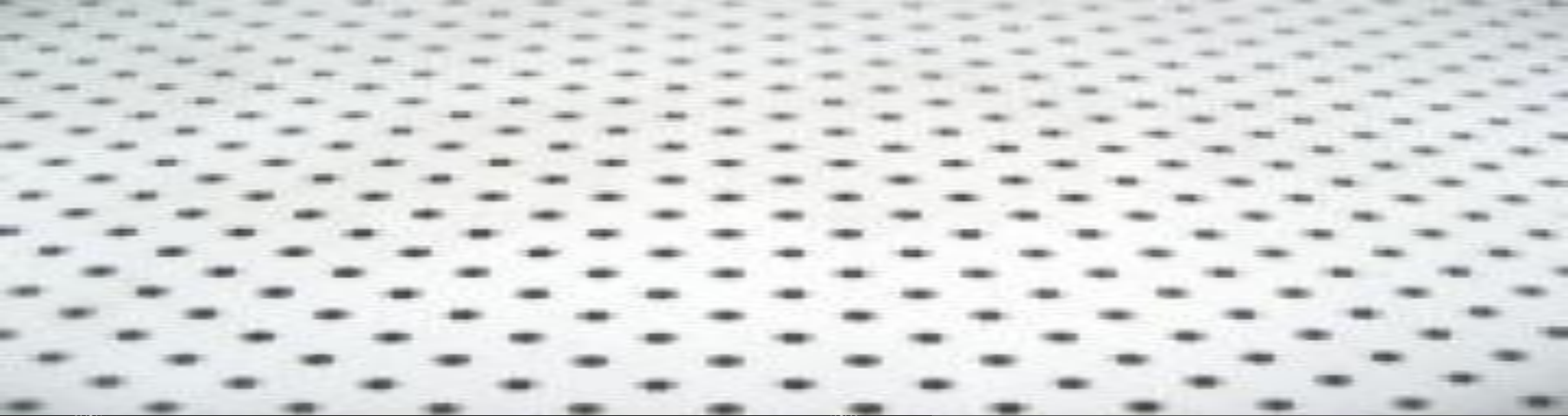
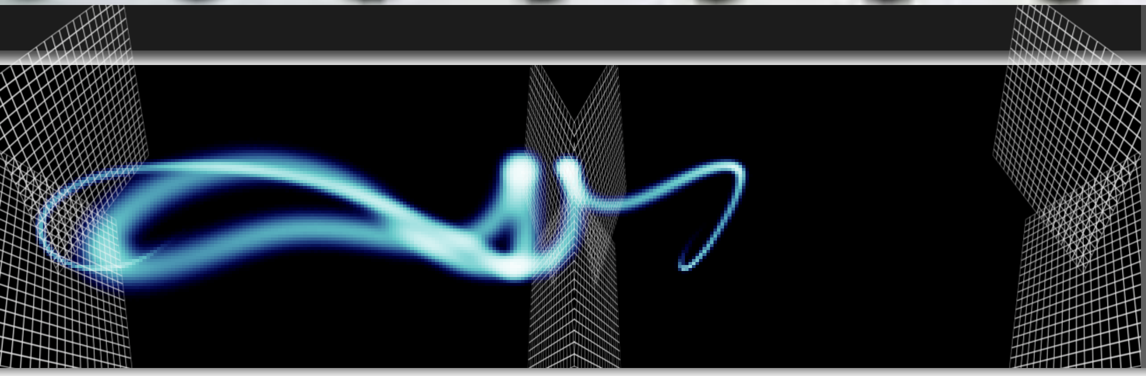
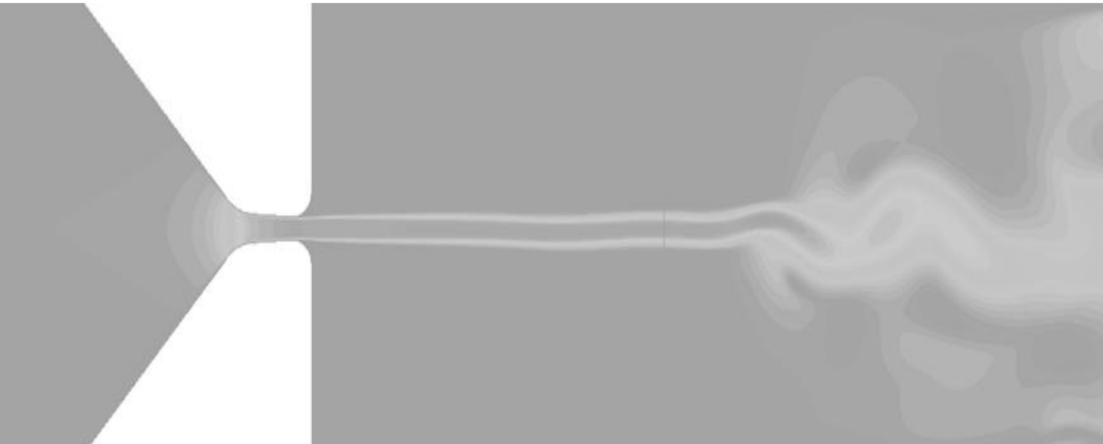




Optimal design of multi-layer microperforated sound absorbers



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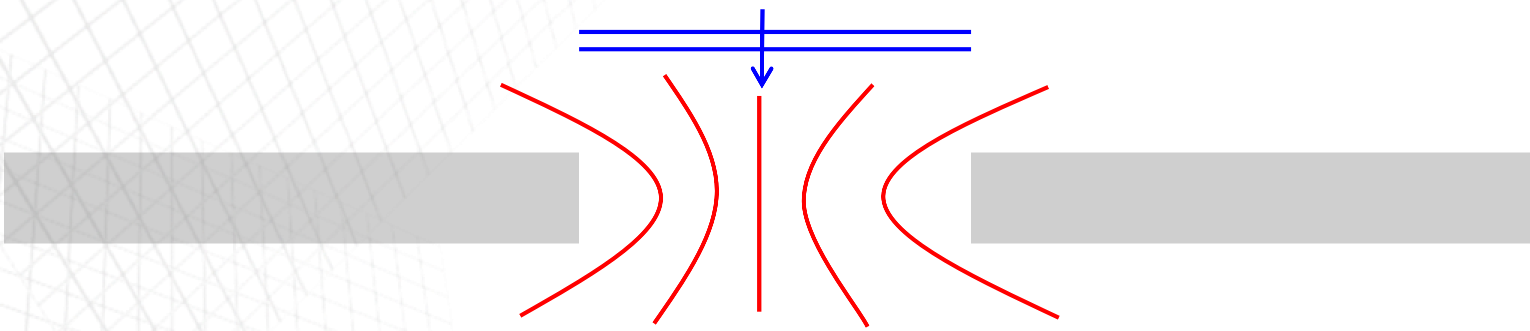
- **Microperforated material**

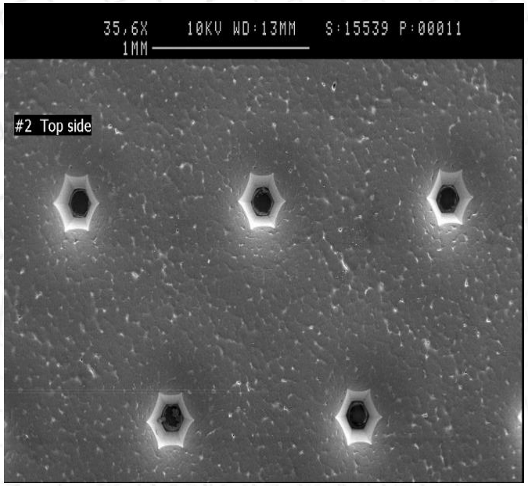
  Thin film with 100 microns scale holes

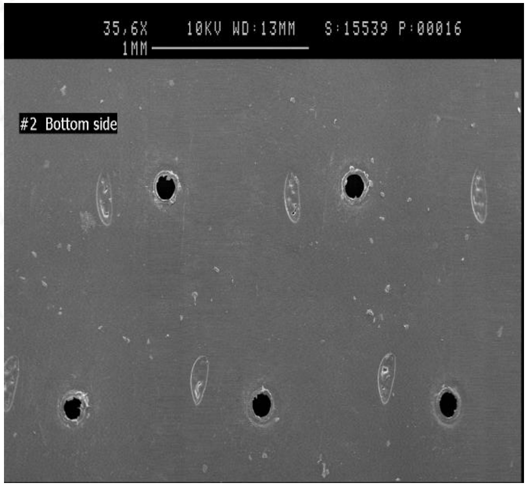
  Clean, light → one alternative to fibrous sound absorbing materials

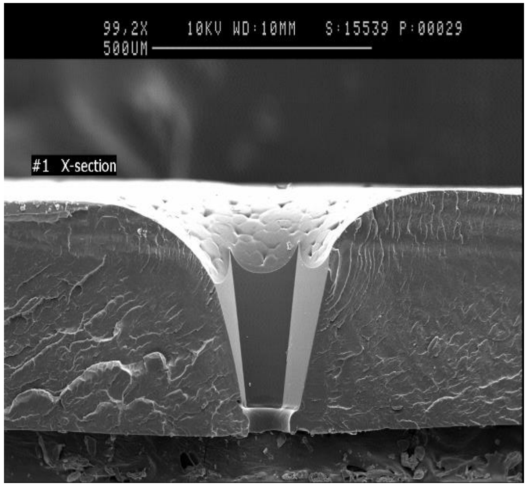


- **Viscous Dissipation**
  - By shearing within hole
  - Within shearing fluid exterior to the hole

Objective







- Effect of hole angle of tapered MPP
- Effect of film motion (finite mass/unit area)
- Multilayer panel to be effective over a broad range

Guo model[1)] (cylindrical hole)

Transfer Impedance:

$$Z_{Guo} = \frac{j\rho\omega L}{\sigma}\left[1 - \frac{2}{k\sqrt{-j}}\frac{J_1(k\sqrt{-j})}{J_0(k\sqrt{-j})}\right]^{-1} + \frac{\alpha 2 R_s}{\sigma} + \frac{j\delta\rho\omega}{\sigma}$$

Error Correction Factor for Resistance

Error Correction Factor for Reactance

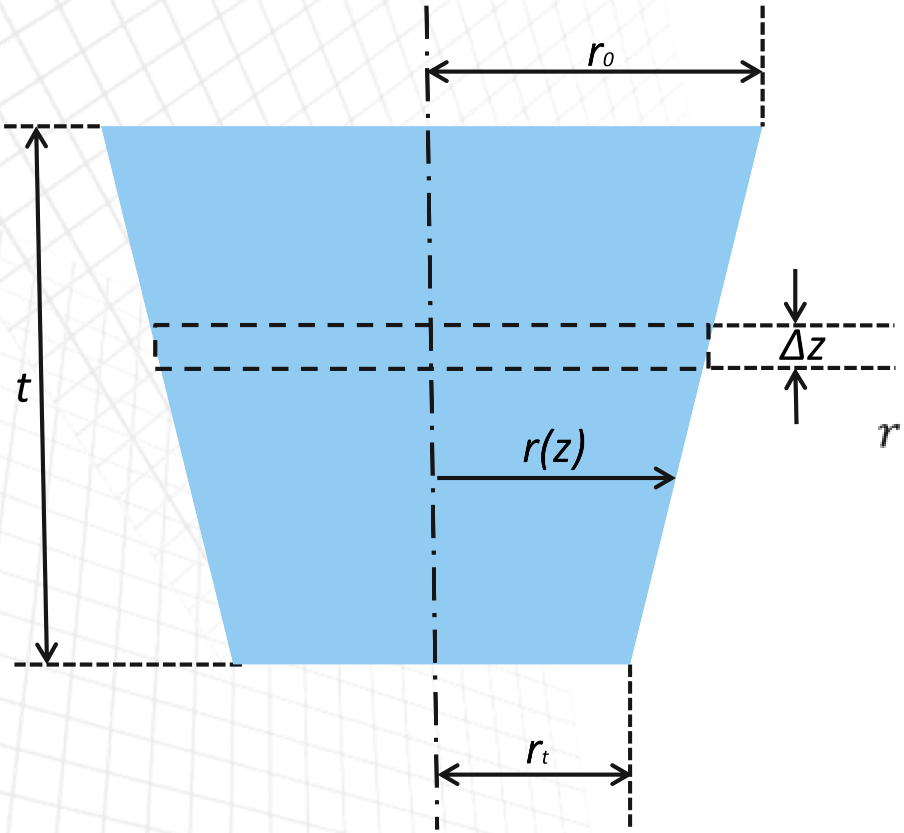
$$k = d\sqrt{\frac{\omega\rho_0}{4\eta}}$$

$$R_s = \frac{\sqrt{2\omega\rho_0\eta}}{2}$$

$\alpha = 2$ when smooth end

$\alpha = 4$ when sharp end

Randeberg's method[2)] (tapered hole)

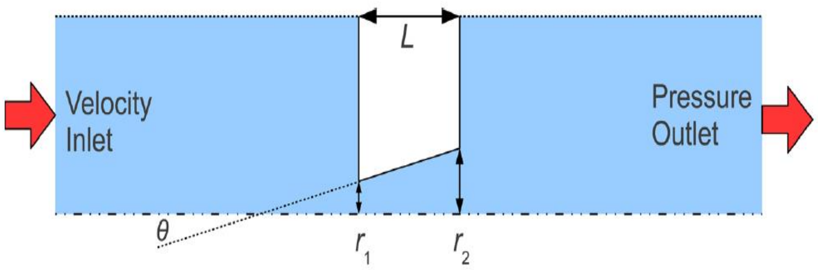


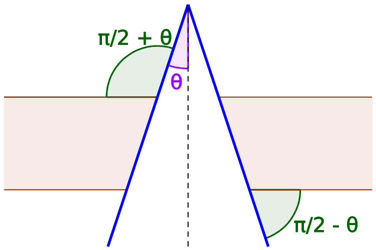
Based on Guo's model, Randeberg used integration method. (used *a* = *4 for sharp edge)*

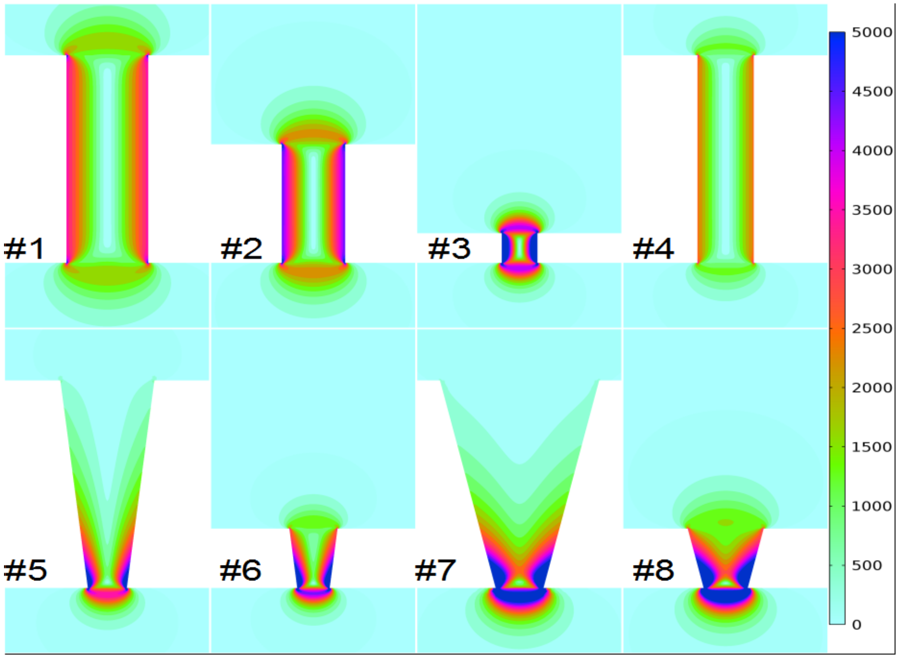
$$r = Re\left\{\sum_{n=1}^{N} \frac{j\omega\Delta z}{\sigma_n c}\left[1 - \frac{2}{k_n\sqrt{-j}}\frac{J_1(k_n\sqrt{-j})}{J_0(k_n\sqrt{-j})}\right]^{-1}\right\} + \frac{\alpha R_s}{\sigma_1 \rho c} + \frac{\alpha R_s}{\sigma_N \rho c}$$

$$k_n = r_n\sqrt{\omega\rho/\eta}$$

Herdtle's method[3)]



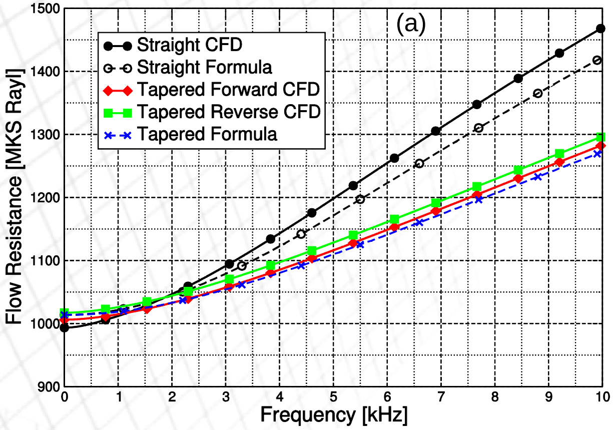


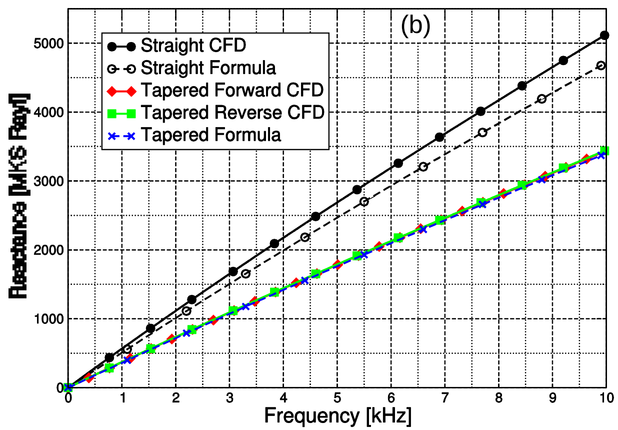


Based on result of incompressible CFD calculations with various shapes of tapered hole, equation for transfer impedance with error correction factor was formulated.

Modeling Approach

Herdtle's method[3)]





$$Z_{taper} = j\rho\omega\int_0^L \frac{1}{\sigma_x}\left[1-\frac{2}{k_x\sqrt{-j}}\frac{J_1(k_x\sqrt{-j})}{J_0(k_x\sqrt{-j})}\right]^{-1} dx$$

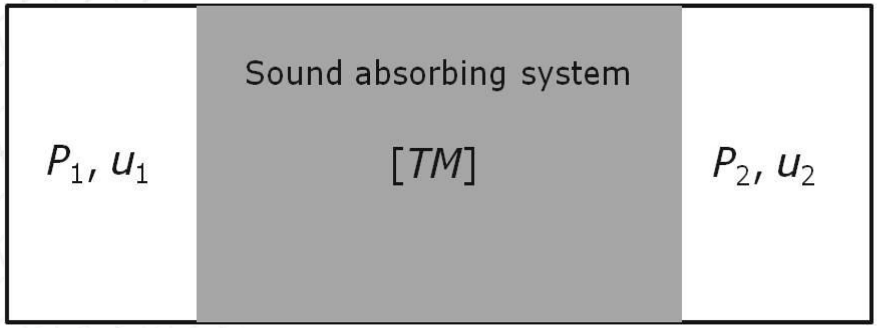
$$r_x = r_1 + (r_2 - r_1)x / L$$

$$k_x = r_x\sqrt{\omega\rho / \eta}$$

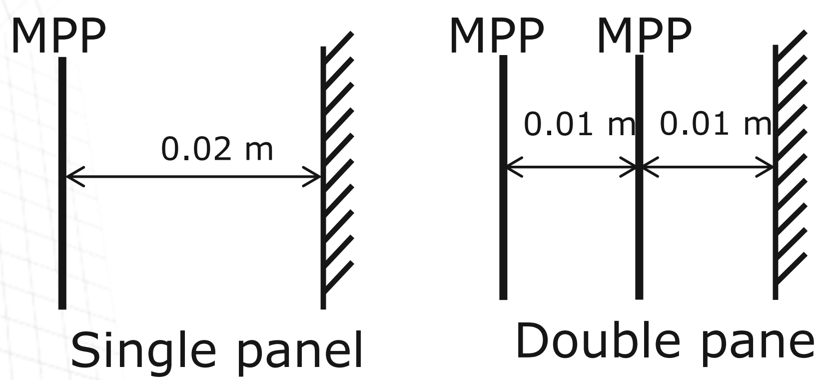
$$\sigma_x = \pi r_x^{\,2} / A$$

$$Z_* = \frac{\frac{L}{3}(r_1^2 + r_1 r_2 + r_2^2) + \beta(\frac{\pi - 2\theta}{\pi} r_1^3 + \frac{\pi + 2\theta}{\pi} r_2^3)}{\frac{L}{3}(r_1^2 + r_1 r_2 + r_2^2)} Z_{taper}$$

Transfer Matrix Method



$$\begin{bmatrix} P_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} TM_{11} & TM_{12} \\ TM_{21} & TM_{22} \end{bmatrix}_{total} \begin{bmatrix} P_2 \\ u_2 \end{bmatrix} = [\mathrm{TM}]_1 [\mathrm{TM}]_2 [\mathrm{TM}]_3 \cdots [\mathrm{TM}]_n \begin{bmatrix} P_2 \\ u_2 \end{bmatrix}$$

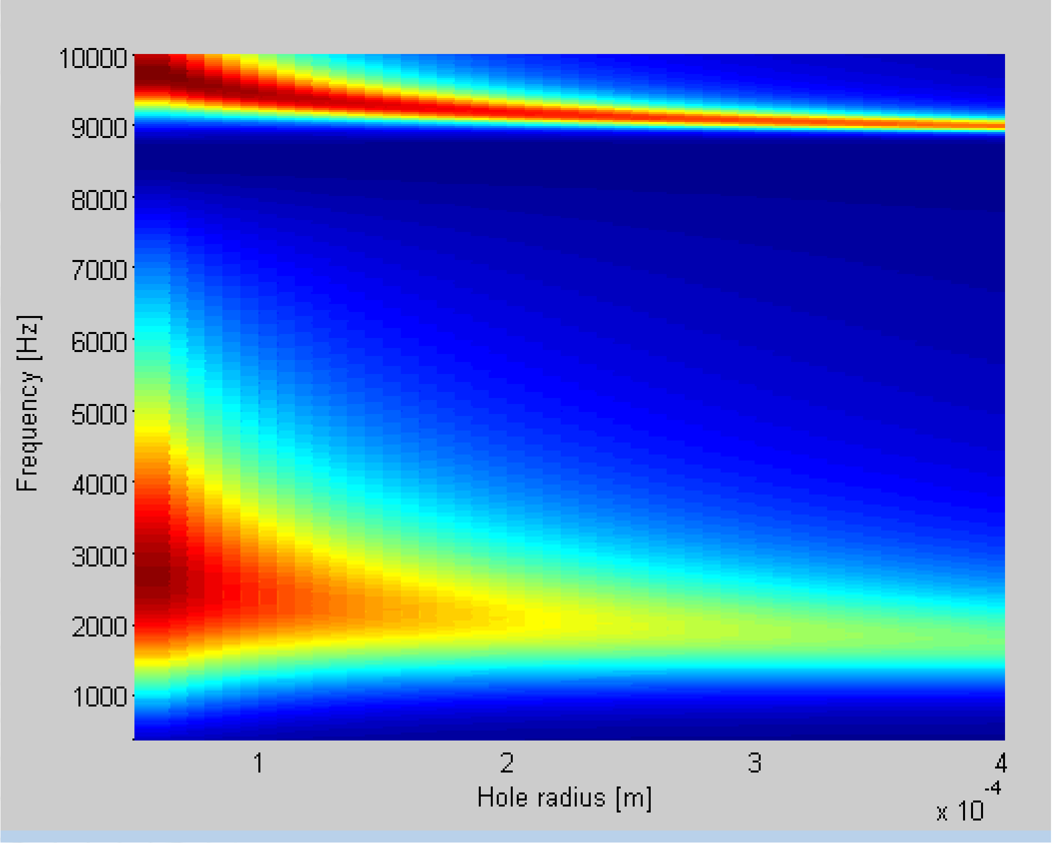


$$TM_{mpp} = \begin{bmatrix} 1 & Z_{mpp} \\ 0 & 1 \end{bmatrix} \quad TM_{air} = \begin{bmatrix} \cos kD & j\rho c \sin kD \\ \frac{j}{\rho c} \sin kD & \cos kD \end{bmatrix}$$

Transfer Matrix Method

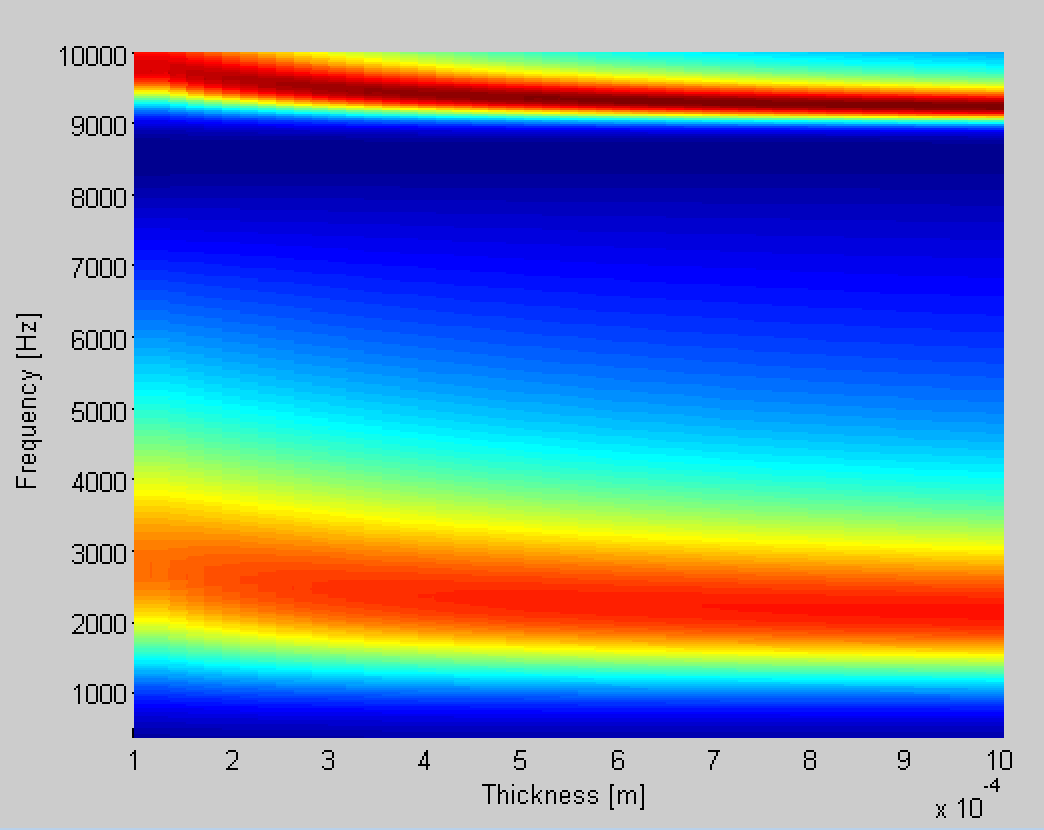
- Transfer Matrix of single and double panel
  - Single panel $TM_{single} = TM_{mpp} TM_{air}$
  - Double Panel $TM_{double} = TM_{mpp} TM_{air} TM_{mpp} TM_{air}$
- If the end of the system is hard surface
  - Impedance $Z = \dfrac{TM_{11}}{TM_{21}}$
  - Reflection coefficient $R = \dfrac{Z - \rho c}{Z + \rho c}$
  - Absorption Coefficient $\alpha = 1 - |R|^2$

- **Absorption Coefficient (0.02 m)**
  - **t=0.0004 m, σ = 0.02, θ = 15°, m = 1 kg/m²**



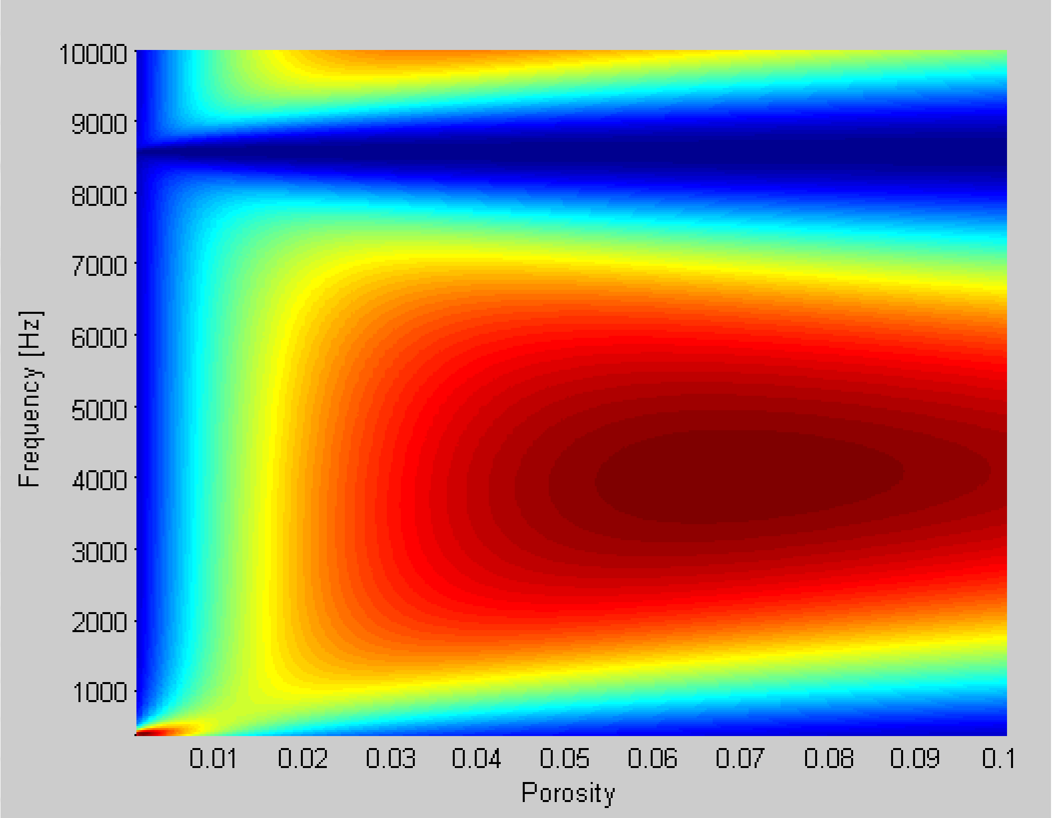
Effect of thickness

- **Absorption Coefficient (0.02 m)**
  - **r=0.0001 m, σ = 0.02, θ = 15°, m = 1 kg/m²**

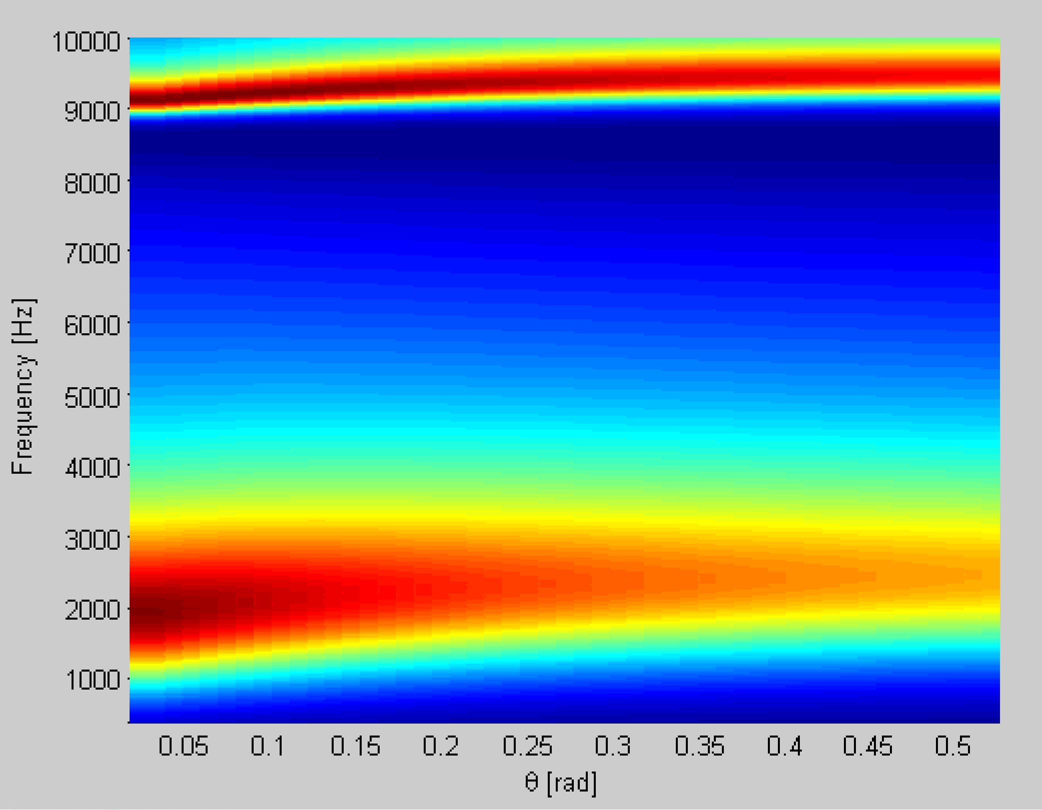


Effect of porosity

- Absorption Coefficient (0.02 m)
  - r = 0.0001, m t=0.0004 m, θ = 15°, m = 1 kg/m²

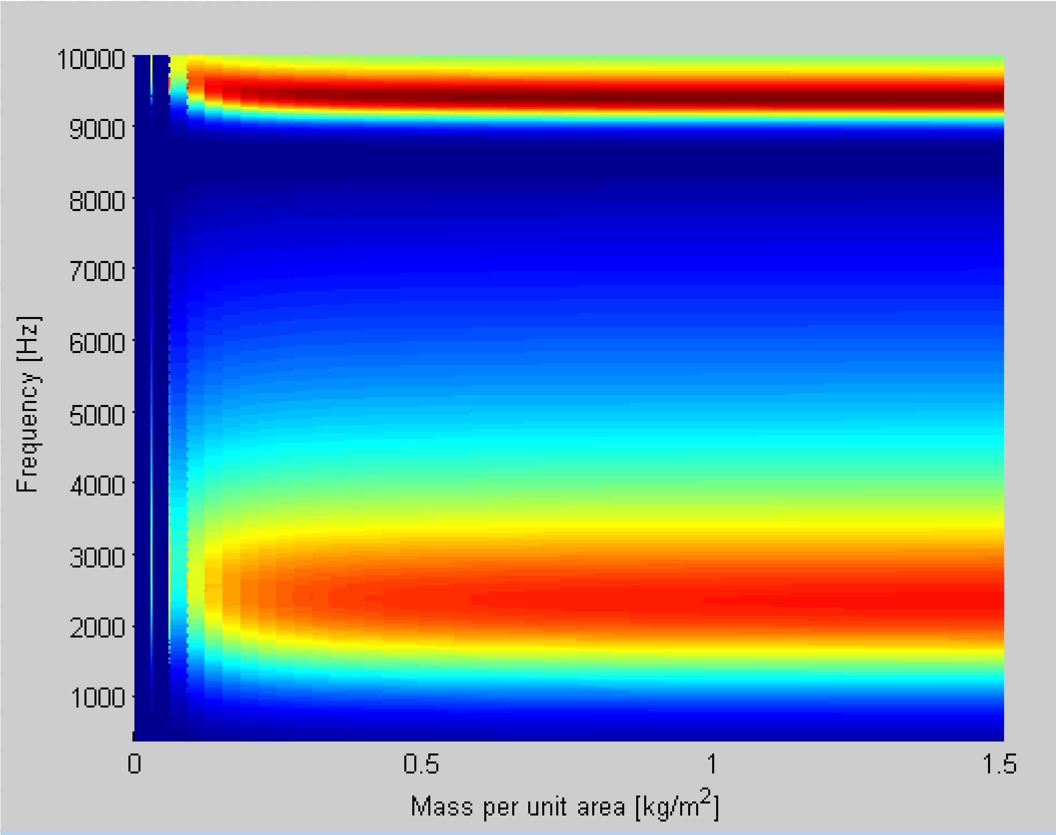


- **Absorption Coefficient (0.02 m)**
  - **r = 0.0001 m, t=0.0004 m, σ = 0.02, m = 1 kg/m²**

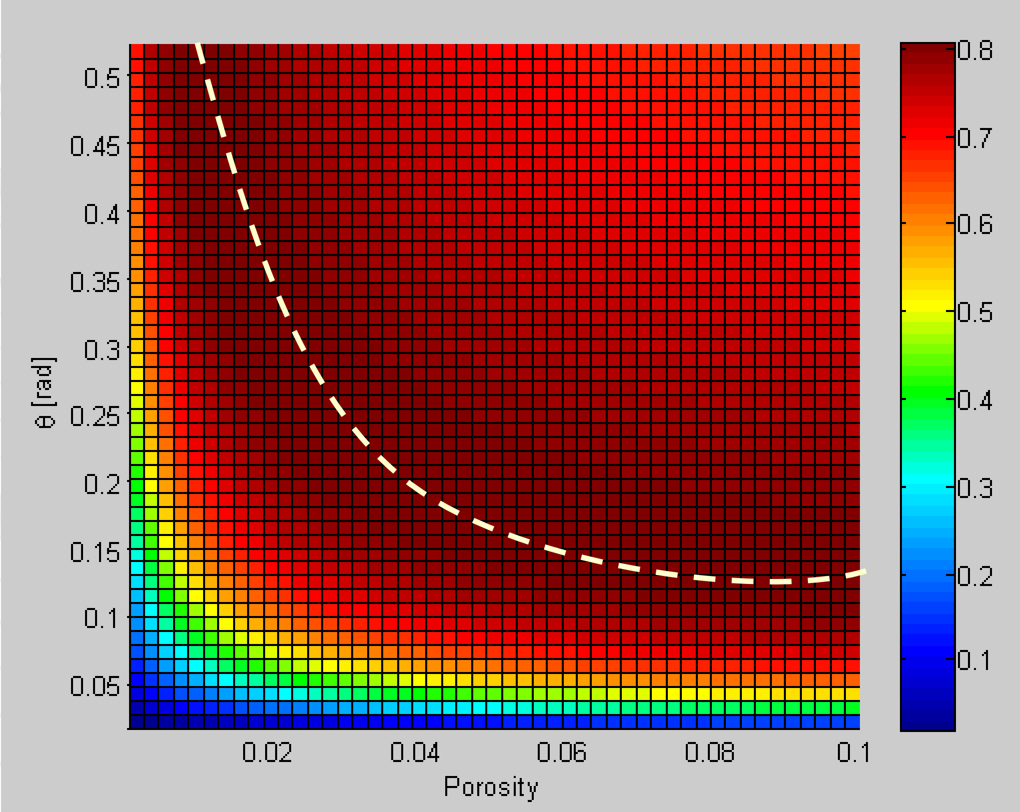


Effect of mass

- **Absorption Coefficient (0.02 m)**
  - **r = 0.0001 m, t=0.0004 m, σ = 0.02, θ = 15°**

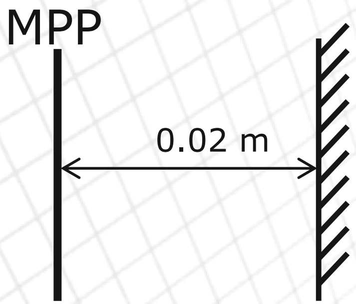


- **Summation of Absorption Coefficient within 400 Hz to 10000 Hz range**
  - **r = 0.0001 m, t=0.0004 m, m = 1 kg/m²**

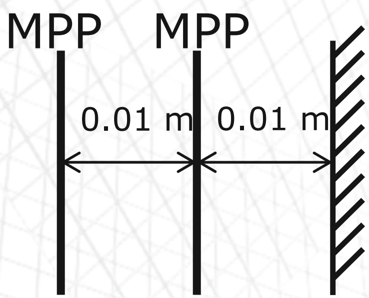


- Maximum value line (white)
- Equivalent MPP can be made with smaller porosity but by higher perforation angle.

Optimization



Single panel



Double panel

- Assumption
  - Considered only normal incidence case
  - For flexible case, panel can move only back and forth
  - MPP are exactly same for double panel
  - Air space between MPP and hard backing of single panel is 0.02 m and air space between two layers and between the second layer and hard backing of double panel is 0.01 m.
- Constraints
  - Diameter of small hole is 0.1 – 0.4 mm
  - Thickness of panel is 0.1 – 1 mm
  - Porosity is 0.001 - 0.1
  - Angle is 1° - 30 °
  - Mass is 0.5 to 1.5 kg/m^2

- Levenberg Algorithm was used.
- Optimized in 400 to 10000 Hz
- $\sum(1-\alpha)^2$ was used for error function.
- $Z_p = \frac{Z_{mpp} + j\omega m}{j\omega m Z_{mpp}}$ was used for transfer impedance of flexible case.

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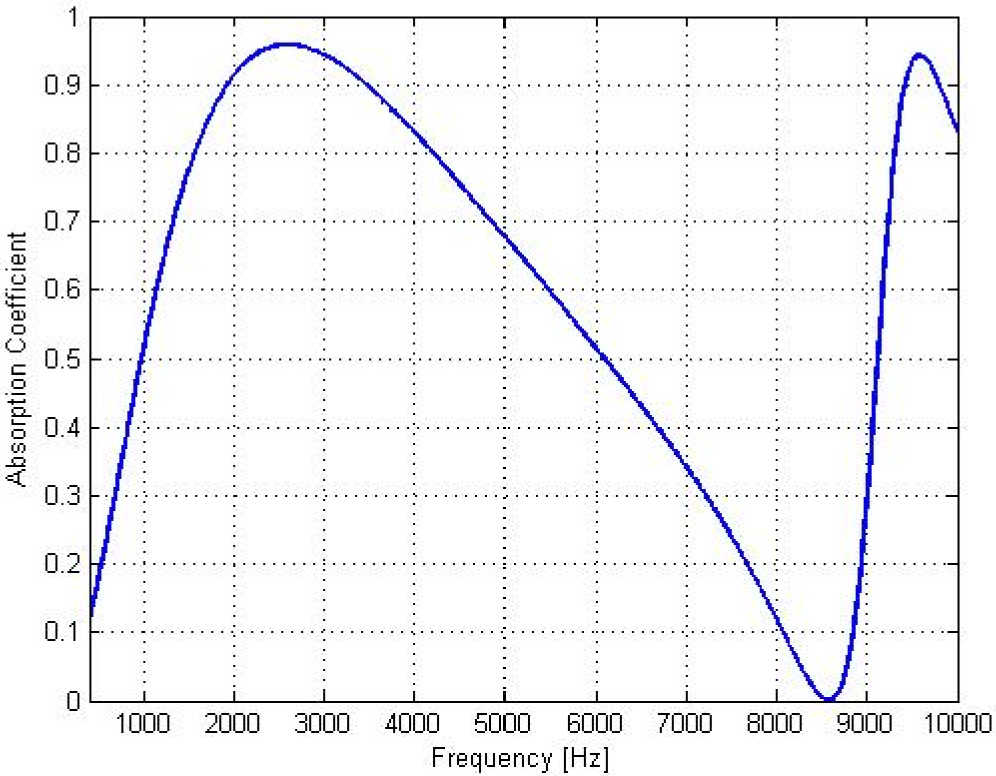
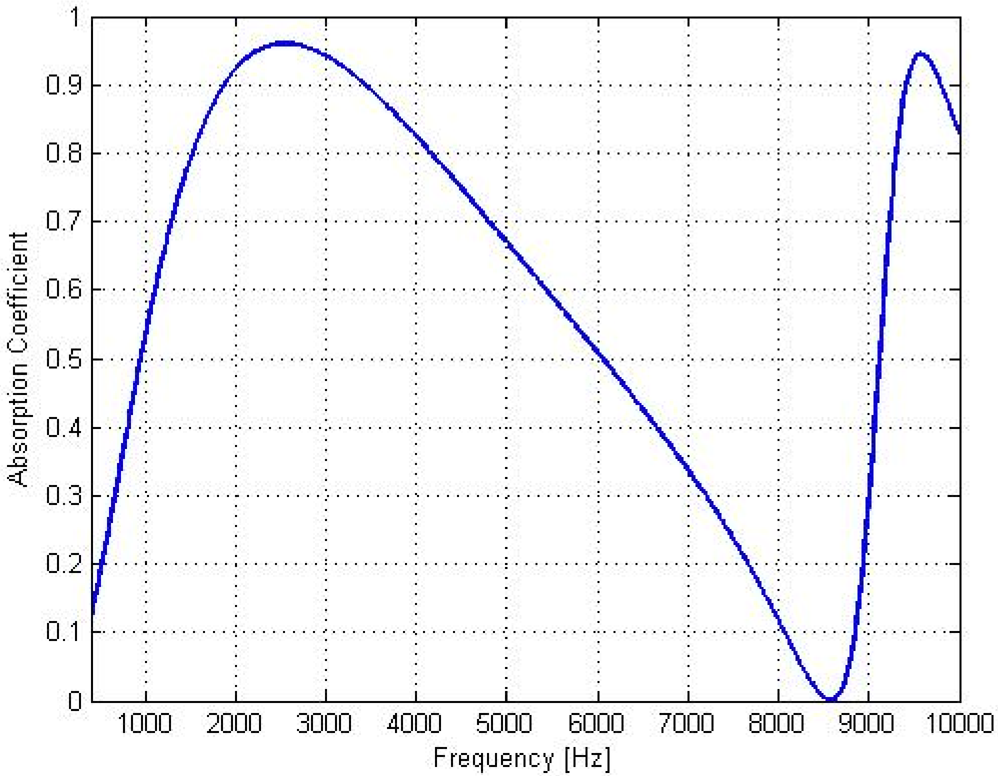
❖ Single rigid case (D = 0.02 m)

r [m]	t [m]	σ	θ [rad]
0.00005	0.0001	0.01397	0.01745



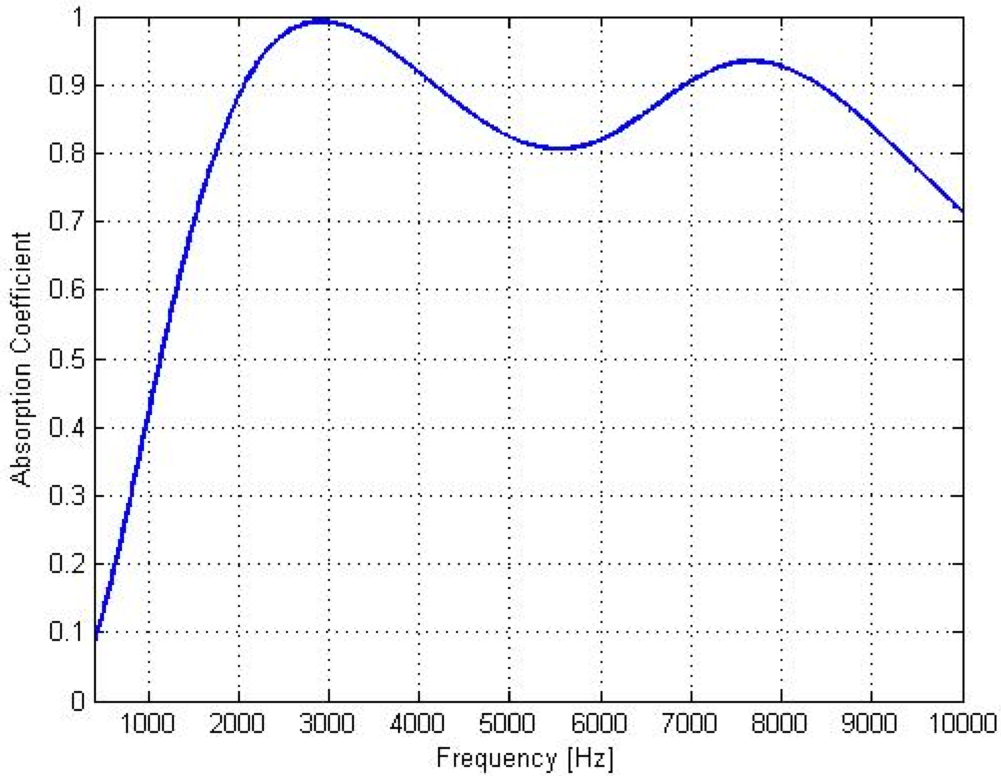
❖ Single flex case (D = 0.02 m)

r [m]	t [m]	σ	θ [rad]	m [kg/m²]
0.00005	0.0001	0.01376	0.01745	1.5



- Double rigid case (D = 0.01 m, l = 0.01 m)

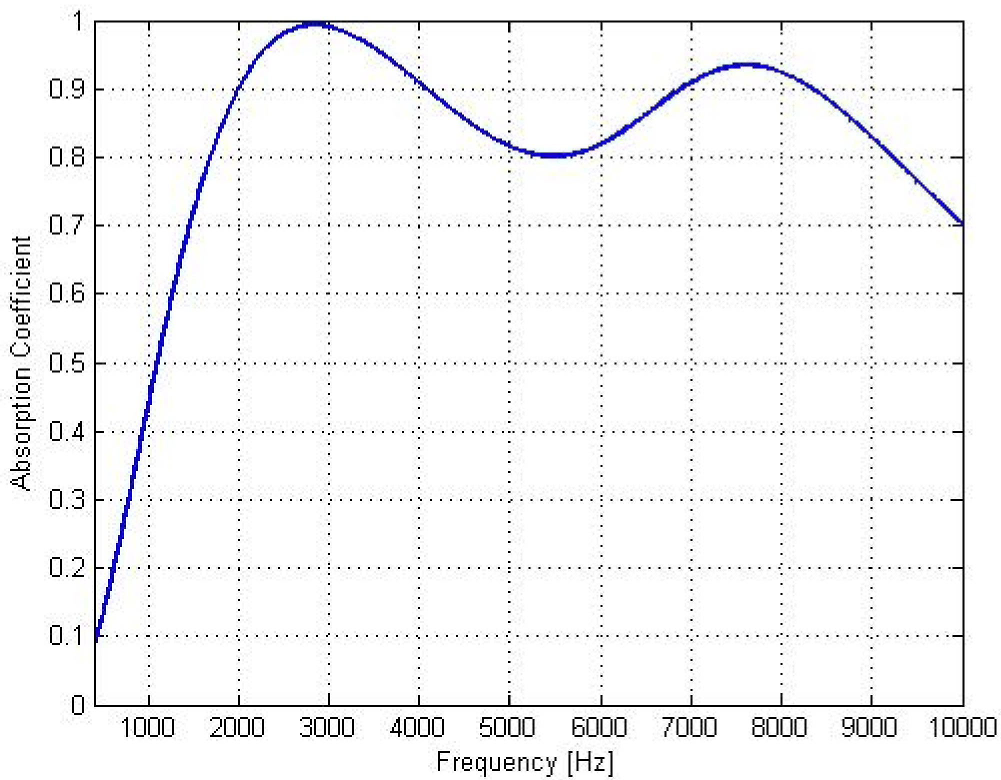
r [m]	t [m]	σ	θ [rad]
0.00005	0.0001	0.02512	0.01745



Optimization

- **Double flex case (D = 0.01 m, l = 0.01 m)**

r [m]	t [m]	σ	θ [rad]	m [kg/m²]
0.00005	0.0001	0.02497	0.01745	0.5



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Conclusion

❖ **The performance of Microperforated panel with tapered hole is determined by the radius of the hole, film thickness, porosity, angle, and mass per unit area.**

❖ **Angle of hole and porosity can be traded off, and larger angle yields the same performance with fewer holes.**

❖ **Optimization result shows that proper combination can give good performance, but over a broad range with single panel.**

❖ **Optimization result of double panel shows much better performance than single panel.**

Reference

1. Y. Guo, S. Allam, and M. Abom, “Micro-perforated Plates for Vehicle Applications,” Proc. INTER-NOISE 2008, Shanghai, China, 2008.

2. R. T. Randeberg, “Perforated panel absorbers with viscous energy dissipation enhanced by orifice design,” Ph.D. thesis, Norwegian University of Science and Technology, 2000.

3. Thomas Herdtle, J. Stuart Bolton and Nicholas N. Kim, Jonathan H Alexander, Ronald W. Gerdes, “Transfer impedance of microperforated materials with tapered holes,” J Acoust. Sco Am. 134(6), Pt. 2, December 2013