The role of symmetry in computational models of 3D vision

A consensus has grown up among experts in Psychophysics during the last hundred years that the human being's percepts are inferences, which are based on a minimum, or simplicity principle that is applied to the currently-available sensory data. When the perceptual inference is expressed within a deterministic framework of the regularization methods, the percept corresponds to the minimum of a cost function that combines the *a priori* simplicity constraints with the sensory data. Alternatively, one can use a probabilistic framework in which the perceptual inference is based on a Bayesian posterior. The Bayesian formulation is essentially equivalent to the regularization methods and the choice between probabilistic and deterministic models is simply a matter of the modeler's preference. One of the main advantages of Bayesian inference, when compared to regularization methods, is that it naturally allows for the updating of the priors. Note well, however, there is little, possibly even no, empirical evidence suggesting that priors are ever updated in 3D human vision.

Perceptual inferences play the critical role in establishing *veridical* perceptual representations of the 3D environment, where by "veridical" we simply mean that the percept agrees with what is "out there." These veridical representations cannot be achieved without making use of symmetries, much like those known in Physics, where they are essential for characterizing our physical world and for deriving the Conservation Laws (Noether, 1918). But, unlike in Physics, the important role that symmetry plays in Psychophysics has only been demonstrated, and its important role has only been explained, within the last ten years.

Symmetry is one of the most fundamental concepts in both Physics and Mathematics. "A thing is symmetrical if there is something we can do to it so that after we have done it, it looks the same as it did before" (Feynman et al., 1963). A human face is mirror-symmetrical because the face *in* a mirror looks the same as the face *facing* the mirror. Alternatively, the symmetry of objects can be defined by redundancy: one half of a symmetrical object is identical, or similar, to the other half. Symmetry in Mathematics is defined by an equivalence relation within a group of transformations (Rosen, 2008). A set of transformations is a *group* if it satisfies four axioms: closure, identity, inverse and associativity. When the equivalence is defined by symmetries of objects, scenes or events, the symmetry becomes a source of redundancy, a property that is absolutely essential in 3D vision.

Without symmetry, 3D vision as we know it, would be impossible. Specifically, the general case of an optical projection from a 3D scene to a 2D retinal image is many-to-one, and, as such, it violates all group axioms. Without groups, there are no invariants in the 2D image: all permanent geometrical and physical properties of 3D objects, scenes and events are destroyed in the projection to the 2D image. If you think that you have found *invariants* in a 2D image, it means that your 3D stimuli are characterized by symmetries. It turns out that the redundancy represented by the symmetries of objects, scenes and events fundamentally changes the problem of 3D inferences in vision. Specifically, the mapping from a 3D world characterized by symmetries to a 2D image becomes one-to-one and it forms a group that is characterized by invariants. Mirror-symmetry is one of the most important types of symmetries in 3D vision because all animals are mirror-symmetrical. Other types of symmetries, namely rotational and translational, are used to describe plants and parts of objects. But there is even more. Rigid motion and binocular vision are also examples of 3D symmetry, but note that they are rarely discussed as such.

The *mathematical formalism* described here that starts with symmetries in the physical world and leads to veridical mental representations, consisting of invariant characteristics, such as the 3D shapes of objects, as well as their positions and movements, corresponds to the *mathematical formalism* used by physicists where Conservation Laws, which are invariant characteristics of physical events, are derived from the symmetries of the Laws of Nature. In both Perception and Physics, symmetries and conservations are linked by a least-action, or minimum, principle that is a form of a *constrained optimization.* In this new approach, perceptual veridicality has a special status, namely, it is a natural consequence of how the visual system actually works. So, in this approach, perceptual veridicality is justified on theoretical grounds. The empirical status of veridicality has always been obvious because successful observers *must* have accurate information about their environment to plan and implement effective behaviors. Failures of veridicality only happen when the stimulus lacks the necessary symmetries. Unfortunately, this is all too common in experiments done in laboratories. It almost never happens elsewhere.