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Technical Note

A simple mathematical model for the nonlinear compliance of arteries

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Sometimes mathematical and computational models of the cardiovascular system must take into account for the nonlinear compliance of blood vessels, especially arteries, the pressure within which can vary greatly. This short technical note presents the derivation of a simple formula for arterial compliance based on the assumption that arterial walls act as classical Fungian biomaterials, described on page 223 of Fung's original book (Y. C. Fung, Biomechanics: mechanical properties of living tissues, Springer-Verlag, New York, 1981).

For a cylindrical blood vessel the pressure domain analog to the stress-strain relationship of a classical Fungian biomaterial is

$$P = P_{ref}\left(e^{b(V-V_{ref})/V_{ref}}\right).$$

Here P is the instantaneous transmural distending pressure for the vessel. Atmospheric pressure is zero. The reference pressure, P_{ref}, refers to the pressure at which a fixed experimental value for dynamic compliance of the artery has been determined, such as normal arterial diastolic pressure. The reference pressure P_{ref} is analogous to the reference tension T* in Fung's original model. V is the instantaneous volume of the vessel, V_{ref} is the reference volume, b is a dimensionless constant, and e is the base of the natural logarithms.

To get nonlinear dynamic compliance at an arbitrary positive pressure, P , we can recast the above equation as

$$\ln\left(\frac{P}{P_{ref}}\right) = \frac{b\left(V - V_{ref}\right)}{V_{ref}}\text{ , or}$$

$$V = V_{ref} + \frac{V_{ref}}{b}\cdot\ln\left(\frac{P}{P_{ref}}\right).$$

The dynamic compliance at pressure, P , is

$$\frac{dV}{dP} = \frac{V_{ref}}{b} \cdot \frac{1}{\frac{P}{P_{ref}}} \cdot \frac{1}{P_{ref}} = \frac{V_{ref}}{b} \cdot \frac{1}{P}.$$

When $P = P_{ref}$ we have $\frac{dV}{dP} = \frac{V_{ref}}{b} \cdot \frac{1}{P_{ref}} \equiv C_{ref}$,

and $\frac{V_{ref}}{b} = C_{ref} P_{ref}$.

So

$$\frac{dV}{dP} = C_{ref} \cdot \frac{P_{ref}}{P}$$

as a function of pressure only. Thus we can convert constant compliances, C_{ref}, into nonlinear, pressure dependent ones, using

$$\frac{dV}{dP} = C_{ref} \cdot \frac{P_{ref}}{P}.$$