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Unbalanced Analysis of Variance Comparing Standard and Proposed Approximation Techniques for Estimating the Variance Components

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UNBALANCED ANALYSIS OF VARIANCE COMPARING STANDARD AND PROPOSED APPROXIMATION TECHNIQUES FOR ESTIMATING THE VARIANCE COMPONENTS

by

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James Philip Pugsley

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ABSTRACT

Unbalanced Analysis of Variance Comparing Standard and Proposed Approximation Techniques for Estimating the Variance Components

by

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This paper considers the estimation of the components of variation for a two-factor unbalanced nested design and compares standard techniques with proposed approximation procedures. Current procedures are complicated and assume the unbalanced sample size to be fixed. This paper tests some simpler techniques, assuming sample sizes are random variables. Monte Carlo techniques were used to generate data for testing of these new procedures.

(58 pages)

CHAPTER I

DESCRIPTION OF PROBLEM

Many practical research designs involve the collection of data in which the factors of interest are "nested." A nested design is a concept in which each factor appears in one and only one factor. This is similar to a tree, in that the leaves are nested to branches, and these branches are nested to the tree trunk. For another example, think of a batching process where raw materials are purchased in large, fixed lot sizes. Batches are produced from smaller quantities of this incoming lot and then the individual product is packaged from these batches. This concept is illustrated in Figure 1.

One characteristic of nested designs is that they often involve unequal sample sizes. In industrial situations, this is usually a consequence of the fact that individual products can be defective. If the events "being defective" are independent, they are said to have a binomial distribution, sometimes referred to as a Bernoulli process.

The current methods of analysis for unbalanced nested designs are complicated and computationally difficult.

The current analysis procedures are based on the assumption that the unbalanced sample sizes are fixed, i.e., if the experiment is rerun the same sample sizes will occur. The proposed method relaxes the fixed sample size assumption and assumes that they are random variables.

This thesis investigates the properties of this new method of analysis and compares them with standard methods when sample sizes are random variables.

A two factor unbalanced nested design will be examined utilizing Monte Carlo simulation techniques to generate the data. The generated data will be used to assess new simplified procedures.

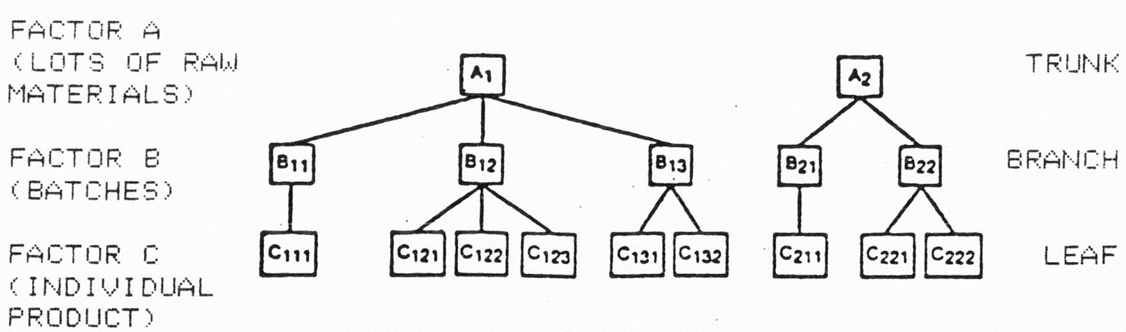

Figure 1. Unbalanced Nested Design

CHAPTER II

REVIEW OF THE LITERATURE

Many current activities regarding the computation of variance components from unbalanced designs are summarized by Harville (1976), although his emphasis in this paper are on maximum likelihood estimation approaches. Harville (1967) presents work for an unbalanced one-way design assuming the sample sizes are random variables from a poisson distribution. Harville (1968) expands his prior work to a two-way completely random classification, this again assumed the sample sizes to be random variables from a poisson distribution. The estimates of the components of variation were still complex creating a desire for a more straight forward approach.

Many other papers have been written discussing computation of the component of variations, such as, Olsen, Seely, and Birkes (1976), Harville (1974), Rao (1972), Searle (1971), Thompson (1969), Harville (1969), Klotz, Milton, and Zacks (1969), Russell and Bradley (1958), and Henderson (1953) to name just a few. With exception of the early work of Harville (1967, 1968) none of the above addressed the concept of sample size being a random variable.

The decision was made to use Monte Carlo techniques and develop a much simplier approach, while at the same time relaxing the fixed sample size assumption. The poisson distribution used by Harville (1967, 1968) seems restrictive in applications while assuming sample sizes were random variables form Bernoulli distribution appear to have more general applications.

CHAPTER III

OUTLINE OF PROJECT

The current analysis procedures for analysis of an unbalanced nested design are complicated. The assumption of a fixed sample size does not seem reasonable and should be relaxed. The relaxation of the fixed sample size assumption can also simplify the difficult analysis techniques.

A Monte Carlo study was designed based on the sample size being a Bernoulli (Binomial) random variable. The Bernoulli distribution fits many practical applications. This distribution allows for inclusion or deletion of an observation based on a known probability. It is reasonable to assume that a known proportion of the observations or cells in a nested design will be missing, given a specified target or intended sample size. This allows the sample size to be considered a random variable. The example shown in Figure 1 could be one of building rocket motors. It is known and predictable that a certain proportion of the batches of propellant will be lost. It is also known and predictable that a given proportion of the individual product, the test samples, will be missing.

The above scenario is common and happens frequently in applied work. Models of this type conform to a Bernoulli distribution. Various probabilities for missing cells and missing observations were examined. The expected mean squares (E(MS)) are listed in Table 1, under the assumption that the sample sizes are fixed.

Table 1. Unbalanced Nested Design E(MS)

SOURCE	D.F.	SS	MS	E(MS)
A	a-1	SS(A)	MS(A)	$\sigma^2_e + k_2\sigma^2_{B/A} + k_3\sigma^2_A$
B/A	$\sum_i (b_i - 1)$	SS(B/A)	MS(B/A)	$\sigma^2_e + k_1\sigma^2_{B/A}$
C/B/A	$\sum_i \sum_j (c_{ij} - 1)$	SS(C/B/A)	MS(C/B/A)	σ^2_e

The components of variation are usually computed utilizing the mean squares (MS) in conjunction with the E(MS) where the various estimates are:

$$\hat{\sigma}^2_e = MS(C/B/A) \qquad \text{eq. 1}$$

$$\hat{\sigma}^2_{B/A} = [MS(B/A) - \hat{\sigma}^2_e]/k_1 \qquad \text{eq. 2}$$

$$\hat{\sigma}^2_A = [MS(A) - (\hat{\sigma}^2_e + k_2 \hat{\sigma}^2_{B/A})]/k_3 \qquad \text{eq. 3}$$

The k_i's are defined for the standard method in Table 2, p. 10.

A new method is proposed to estimate the k_i's. An examination of various methods of computing the sums of squares are explored. There are four ways to compute the overall mean, two ways to compute the cell means and one way to compute the sample averages. The stability of these was unknown so all the estimates were tried. These estimates were only examined with the new method of computing the k_i's.

The unbalanced nested design model to be assessed is a two factor model:

$$X_{ijk} = \delta_{ij} \eta_{ijk} (\mu + A_i + B_{ij} + C_{ijk})$$

where

$i = 1,2,...,a$

$j = 1,2,...,b_i$

$k = 1,2,...,c_{ij}$

and

$$\delta_{ij} = \begin{cases} 1 \text{ if } B_{ij} \text{ is present} \\ 0 \text{ if it is missing} \end{cases}$$

$$\eta_{ijk} = \begin{cases} 1 \text{ if } C_{ijk} \text{ is present} \\ 0 \text{ if it is missing} \end{cases}$$

so that

$$p(\delta_{ij} = 0) = \pi_b$$

$$p(\eta_{ijk} = 0) = \pi_c$$

The π_b indicates that a branch is missing from the tree or an entire batch is missing while π_c designates that a leaf is missing from the branch or a test sample is missing from a batch.

Given that

$$A_i \sim NID(0, \sigma^2_A)$$

$$B_{ij} \sim NID(0, \sigma^2_{B/A})$$

$$C_{ijk} \sim NID(0, \sigma^2_e)$$

δ_{ij} from factor B follows a Bernoulli random distribution with the probability of π_b being equal to zero and η_{ijk} follows a Bernoulli random distribution with π_c being equal to zero. The individual observation follow a random normal distribution with mean zero and variance σ_i^2. The Analysis of Variance (ANOVA) Table (Table 1) is provided to give a basis for further discussion.

Expected values were used to obtain a new method of estimating the k_i's (see Appendix A for derivation). It was noted that the following relationships existed:

$$E(c_{ij}) = (1 - \pi_c)c$$

$$E(b_i) = (1 - \pi_b)b$$

$$E(c_{i.}) = (1 - \pi_b)(1 - \pi_c)bc$$

$$E(c_{..}) = (1 - \pi_b)(1 - \pi_c)abc$$

$$E(b_.) = (1 - \pi_b)ab$$

so that

$$E[MS(A)] = \sigma_e^2 + (1 - \pi_c)c\,\sigma_{B/A}^2 + (1 - \pi_b)(1 - \pi_c)bc\,\sigma_A^2$$

$$E[MS(B/A)] = \sigma_e^2 + (1 - \pi_c)c\,\sigma_{B/A}^2$$

$$E[MS(C/B/A)] = \sigma_e^2$$

and

$$c_{..}/b_{.} \simeq (1 - \pi_c)c$$

$$c_{..}/a = (1 - \pi_b)(1 - \pi_c)bc$$

Using the above results now gives rational for a new method of estimating the k_i's.

Besides the standard methods of computation for the k_i factor (Snedecor and Cochran 1976; Fryer 1966) a new method is being proposed. The two methods are presented in Table 2.

Table 2. Definition of k_i

STANDARD METHOD	NEW METHOD
$k_1 = \sum_i \sum_j c_{ij}^2 (1/c_{ij} - 1/c_{i.})/(b_{.} - a)$	$k_1 = c_{..}/b_{.}$
$k_2 = \sum_i \sum_j c_{ij}^2 (1/c_{i.} - 1/c_{..})/(a - 1)$	$k_2 = c_{..}/b_{.}$
$k_3 = \sum_i c_{i.}^2 (1/c_{i.} - 1/c_{..})/(a - 1)$	$k_3 = c_{..}/a$

The new method of estimating the k_i's is much simplier and it will be shown to provide estimates that are at least as good as those utilizing the computationally difficult standard techniques. Notice that the new method estimating $k_1 = k_2$ is simply the average number of samples per non empty cell and that k_3 is the average number of non empty cells times the average number of samples per non empty cells. This provides a very simple method of estimating the coefficients necessary to compute the variance components.

In addition to the comparison between methods of computing k_i a comparison was made of various simplifying methods of computing the sums of squares. These include all possible ways to estimate sample means, cell means and overall means. The computations of these sums of squares are presented in Table 3.

All combinations of l and m were examined for the sums of squares but only the new method of estimating the k_i's were considered in the estimation of the variance components. It was not known which would be the best. The mathematical derivations are not sufficiently tractable so all possible combinations were tried. All are relatively simple and had the potential of being better estimates.

Table 3. Simplified Sums of Squares

SOURCE OF VARIATION	SUMS OF SQUARES
A	$SS(A) = c_{..}/a \sum_{i} (\bar{X}^{(l)}_{i..} - \bar{X}^{(m)}_{...})^2$
B/A	$SS(B/A) = c_{..}/a \sum_{i} \sum_{j} (\bar{X}_{ij.} - \bar{X}^{(l)}_{i..})^2$
C/B/A	$SS(C/B/A) = (c_{..} - b_{.})/b_{.i} \sum_{i} \sum_{j} s^2_{ij}$

where

$l = 1,2$

$m = 1,2,3,4$

and

$$\bar{X}_{ij.} = \sum_{k} X_{ijk}/c_{ij}$$

$$\bar{X}^{(1)}_{i..} = \sum_{j} \sum_{k} X_{ijk} / \sum_{j} c_{ij}$$

$$\bar{X}^{(2)}_{i..} = \sum_{j} (\sum_{k} X_{ijk}/c_{ij})/b_{i} = \sum_{j} \bar{X}_{ij.}/b_{i}$$

$$\bar{X}^{(1)}_{...} = \sum_{i} \sum_{j} \sum_{k} X_{ijk} / \sum_{i} \sum_{j} c_{ij}$$

$$\bar{X}^{(2)}_{...} = \sum_i \left(\sum_j \sum_k X_{ijk} \Big/ \sum_j c_{ij}\right)/a = \sum_i \bar{X}_{i..}/a$$

$$\bar{X}^{(3)}_{...} = \sum_i \sum_j \left(\sum_k X_{ijk}/c_{ij}\right) \Big/ \sum_i b_i = \sum_i \sum_j \bar{X}_{ij.}/b_{.}$$

$$\bar{X}^{(4)}_{...} = \sum_i \left(\sum_j \left(\sum_k X_{ijk}/c_{ij}\right)/b_i\right)/a = \sum_i \left(\sum_j \bar{X}_{ij.}/b_i\right)/a$$

Notice from Table 3 there are four possible ways to compute the overall mean, two ways to compute the cell means and one way to compute the sample means.

The estimates of the variance components were truncated, that is if $\sigma^2_i < 0$ then $\sigma^2_i = 0$.

The values for the Bernoulli random variables were:

$$\pi_c = 0.0,\ 0.2,\ 0.4$$

$$\pi_b = 0.0,\ 0.1,\ 0.2$$

The assumption used here was that a sample should have a larger possibility of being missing than that of an entire cell.

We will be interested in certain functions of the variances. These are:

$$\phi_1 = \frac{\sigma_e^2 + c\sigma_{B/A}^2}{\sigma_e^2}$$

and

$$\phi_2 = \frac{\sigma_e^2 + c\sigma_{B/A}^2 + bc\sigma_A^2}{\sigma_e^2 + c\sigma_{B/A}^2}$$

Now, let

$$\theta_i = \frac{1}{\phi_i}$$

The values of θ_i are the power of the test parameter, $\pi(\theta_i)$, as defined by Graybill (1976), i.e.

$$\pi(\theta) = \int_{\theta F_{\alpha, n_i n_j}}^{\infty} F(\omega : n_i, n_j)\, d\omega \quad \text{for } 0 < \omega < 1$$

Thus, these estimates are invariant for constant values of the ϕ_i. The values used in this study for ϕ_1 and ϕ_2 were:

$\phi_1 = 1, 4, 16$

and $\phi_2 = 1, 4, 16$

This now provides a more general examination of the problem and allows the study of ratios. The values considered for the study were based on $\sigma_e^2 = 1$ (see page 6). The various values for the other variance components are computed and given in Table 4 where:

a = 5 = target sample size for factor A

b = 4 = target sample size for factor B

c = 3 = target sample size for factor C

This converts to a 3^4 factorial set of simulation runs with 100 replications at each observation. The comparison criteria will be the mean square errors (MSE) as defined in equation 4.

$$MSE = \frac{1}{I}\sum_{i=1}^{I} (\hat{\sigma}^2_{ij} - \sigma^2_j)^2 \qquad \text{eq. 4}$$

The target values are for I = 100. This measures the estimated variance component against its theoretical value. The MSE comparisons are given in Table 5 for σ_e^2, Table 6 for $\sigma^2_{B/A}$ and Table 7 for σ^2_A.

Table 4. Input Components of Variation

ϕ_1	ϕ_2	σ^2_e	$\sigma^2_{B/A}$	σ^2_A
1	1	1	0	0.00
1	4	1	0	0.25
1	16	1	0	1.25
4	1	1	1	0
4	4	1	1	1
4	16	1	1	5
16	1	1	5	0
16	4	1	5	4
16	16	1	5	20

The definition of terms in Tables 5, 6 and 7 are as follows: S_i indicates the standard method of computing the sums of squares and the standard method of computing the k_i's, the SN_i indicates the standard method of computing the sums of squares and the new method of computing the k_i's, and $N_{i,j}$ indicates the various simplifying methods of computing the sums of squares and the new method of computing the k_i's. The methods of computing the simplifying methods are given in Table 3.

Table 5. MSE Comparisons for the

Estimates of σ_e^2

π_b	π_c	ϕ_1	ϕ_2	$\sigma^2_{B/A}$	σ^2_A	S_1	$N_{1,1}$
0	0	1	1	0	0	0.0450	0.0450
0	0	1	4	0	0.25	0.0560	0.0560
0	0	1	16	0	1.25	0.0508	0.0508
0	0	4	1	1	0	0.0597	0.0597
0	0	4	4	1	1.00	0.0621	0.0621
0	0	4	16	1	5.00	0.0512	0.0512
0	0	16	1	5	0	0.0618	0.0618
0	0	16	4	5	4.00	0.0423	0.0423
0	0	16	16	5	20.00	0.0541	0.0541
0	0.2	1	1	0	0	0.0837	0.0954
0	0.2	1	4	0	0.25	0.0662	0.0755
0	0.2	1	16	0	1.25	0.0798	0.0884
0	0.2	4	1	1	0	0.0643	0.0735
0	0.2	4	4	1	1.00	0.0833	0.0940
0	0.2	4	16	1	5.00	0.0639	0.0754
0	0.2	16	1	5	0	0.0760	0.0692
0	0.2	16	4	5	4.00	0.0626	0.0724
0	0.2	16	16	5	20.00	0.0787	0.0825
0	0.4	1	1	0	0	0.1524	0.1631
0	0.4	1	4	0	0.25	0.1179	0.1902
0	0.4	1	16	0	1.25	0.0950	0.1743
0	0.4	4	1	1	0	0.1618	0.2098
0	0.4	4	4	1	1.00	0.1373	0.1631
0	0.4	4	16	1	5.00	0.1383	0.1572
0	0.4	16	1	5	0	0.1449	0.1755
0	0.4	16	4	5	4.00	0.1502	0.1577
0	0.4	16	16	5	20.00	0.1476	0.2058
0.1	0	1	1	0	0	0.0746	0.0746
0.1	0	1	4	0	0.25	0.0549	0.0549
0.1	0	1	16	0	1.25	0.0631	0.0631
0.1	0	4	1	1	0	0.0567	0.0567
0.1	0	4	4	1	1.00	0.0607	0.0607
0.1	0	4	16	1	5.00	0.0531	0.0531
0.1	0	16	1	5	0	0.0598	0.0598
0.1	0	16	4	5	4.00	0.0589	0.0589
0.1	0	16	16	5	20.00	0.0571	0.0571
0.1	0.2	1	1	0	0	0.1041	0.1136
0.1	0.2	1	4	0	0.25	0.0735	0.0773
0.1	0.2	1	16	0	1.25	0.0718	0.0834
0.1	0.2	4	1	1	0	0.0920	0.0906
0.1	0.2	4	4	1	1.00	0.1092	0.1121
0.1	0.2	4	16	1	5.00	0.0701	0.0778
0.1	0.2	16	1	5	0	0.1203	0.1164
0.1	0.2	16	4	5	4.00	0.0719	0.0825
0.1	0.2	16	16	5	20.00	0.0621	0.0773
0.1	0.4	1	1	0	0	0.1614	0.1972
0.1	0.4	1	4	0	0.25	0.1410	0.1771
0.1	0.4	1	16	0	1.25	0.1607	0.1873
0.1	0.4	4	1	1	0	0.1049	0.1905
0.1	0.4	4	4	1	1.00	0.0953	0.1781
0.1	0.4	4	16	1	5.00	0.1125	0.1729
0.1	0.4	16	1	5	0	0.1208	0.1450
0.1	0.4	16	4	5	4.00	0.1741	0.1521
0.1	0.4	16	16	5	20.00	0.1516	0.1707

Table 5. (Continued) MSE Comparisons for the

Estimates of σ^2_e

π_b	π_c	ϕ_1	ϕ_2	$\sigma^2_{B/A}$	σ^2_A	S_1	$N_{1,1}$
0.2	0	1	1	0	0	0.0717	0.0717
0.2	0	1	4	0	0.25	0.0602	0.0602
0.2	0	1	16	0	1.25	0.0718	0.0718
0.2	0	4	1	1	0	0.0652	0.0652
0.2	0	4	4	1	1.00	0.0529	0.0529
0.2	0	4	16	1	5.00	0.0589	0.0589
0.2	0	16	1	5	0	0.0751	0.0751
0.2	0	16	4	5	4.00	0.0690	0.0690
0.2	0	16	16	5	20.00	0.0774	0.0774
0.2	0.2	1	1	0	0	0.1021	0.1145
0.2	0.2	1	4	0	0.25	0.0899	0.1139
0.2	0.2	1	16	0	1.25	0.0686	0.1058
0.2	0.2	4	1	1	0	0.1058	0.1277
0.2	0.2	4	4	1	1.00	0.0593	0.0784
0.2	0.2	4	16	1	5.00	0.0825	0.0981
0.2	0.2	16	1	5	0	0.0999	0.0919
0.2	0.2	16	4	5	4.00	0.0705	0.0805
0.2	0.2	16	16	5	20.00	0.0612	0.0774
0.2	0.4	1	1	0	0	0.1614	0.1882
0.2	0.4	1	4	0	0.25	0.1877	0.1618
0.2	0.4	1	16	0	1.25	0.1489	0.1936
0.2	0.4	4	1	1	0	0.1435	0.1978
0.2	0.4	4	4	1	1.00	0.1643	0.1869
0.2	0.4	4	16	1	5.00	0.1606	0.1877
0.2	0.4	16	1	5	0	0.1714	0.1973
0.2	0.4	16	4	5	4.00	0.1871	0.1676
0.2	0.4	16	16	5	20.00	0.1604	0.1544

Table 6. MSE Comparisons for the

Estimates of $\sigma^2_{B/A}$

π_b	π_c	ϕ_1	ϕ_2	$\sigma^2_{B/A}$	σ^2_A	S_2	SN_2	$N_{2,1}$	$N_{2,2}$
0	0	1	1	0	0	0.0118	0.0118	0.0118	0.0118
0	0	1	4	0	0.25	0.0128	0.0128	0.0128	0.0128
0	0	1	16	0	1.25	0.0109	0.0109	0.0109	0.0109
0	0	4	1	1	0	0.3049	0.3049	0.3049	0.3049
0	0	4	4	1	1.00	0.2232	0.2232	0.2232	0.2232
0	0	4	16	1	5.00	0.2812	0.2812	0.2812	0.2812
0	0	16	1	5	0	3.5170	3.5170	3.5170	3.5170
0	0	16	4	5	4.00	4.4060	4.4060	4.4060	4.4060
0	0	16	16	5	20.00	3.9510	3.9510	3.9510	3.9510
0	0.2	1	1	0	0	0.0181	0.0173	0.0541	0.0476
0	0.2	1	4	0	0.25	0.0135	0.0130	0.0451	0.0389
0	0.2	1	16	0	1.25	0.0179	0.0172	0.0428	0.0352
0	0.2	4	1	1	0	0.2321	0.2223	0.2636	0.2432
0	0.2	4	4	1	1.00	0.3706	0.3479	0.4498	0.4040
0	0.2	4	16	1	5.00	0.2970	0.2847	0.3288	0.3117
0	0.2	16	1	5	0	5.2140	5.0440	5.3170	5.1000
0	0.2	16	4	5	4.00	4.5240	4.3790	5.2520	4.9720
0	0.2	16	16	5	20.00	5.0840	4.9720	4.8020	4.7200
0	0.4	1	1	0	0	0.0360	0.0332	0.1555	0.1358
0	0.4	1	4	0	0.25	0.0393	0.0356	0.1906	0.1639
0	0.4	1	16	0	1.25	0.0376	0.0338	0.1568	0.1356
0	0.4	4	1	1	0	0.3762	0.3499	0.6548	0.5537
0	0.4	4	4	1	1.00	0.4454	0.4122	0.8052	0.6868
0	0.4	4	16	1	5.00	0.4736	0.4367	0.5972	0.5094
0	0.4	16	1	5	0	6.7400	6.0330	6.5260	5.7360
0	0.4	16	4	5	4.00	4.5280	4.2050	5.0380	4.4990
0	0.4	16	16	5	20.00	3.9630	3.6790	4.7900	4.2160
0.1	0	1	1	0	0	0.0523	0.0523	0.0523	0.0523
0.1	0	1	4	0	0.25	0.0109	0.0109	0.0109	0.0109
0.1	0	1	16	0	1.25	0.0108	0.0108	0.0108	0.0108
0.1	0	4	1	1	0	0.2903	0.2903	0.2903	0.2903
0.1	0	4	4	1	1.00	0.2304	0.2304	0.2304	0.2304
0.1	0	4	16	1	5.00	0.2330	0.2330	0.2330	0.2330
0.1	0	16	1	5	0	3.9330	3.9330	3.9330	3.9330
0.1	0	16	4	5	4.00	4.0490	4.0490	4.0490	4.0490
0.1	0	16	16	5	20.00	4.0460	4.0460	4.0460	4.0460
0.1	0.2	1	1	0	0	0.0859	0.0834	0.1148	0.1037
0.1	0.2	1	4	0	0.25	0.0197	0.0185	0.0561	0.0471
0.1	0.2	1	16	0	1.25	0.0270	0.0256	0.0756	0.0636
0.1	0.2	4	1	1	0	0.3152	0.2989	0.3979	0.3542
0.1	0.2	4	4	1	1.00	0.2917	0.2785	0.3003	0.2812
0.1	0.2	4	16	1	5.00	0.3702	0.3559	0.3614	0.3399
0.1	0.2	16	1	5	0	5.3600	5.1550	5.0440	4.7230
0.1	0.2	16	4	5	4.00	6.4100	6.1680	6.1630	5.7190
0.1	0.2	16	16	5	20.00	4.1500	3.8490	4.5660	4.2160
0.1	0.4	1	1	0	0	0.1131	0.1044	0.2646	0.2297
0.1	0.4	1	4	0	0.25	0.0503	0.0461	0.1642	0.1432
0.1	0.4	1	16	0	1.25	0.0520	0.0469	0.1890	0.1656
0.1	0.4	4	1	1	0	0.4345	0.4115	0.5699	0.4962
0.1	0.4	4	4	1	1.00	0.4520	0.4195	0.8331	0.7123
0.1	0.4	4	16	1	5.00	0.4544	0.4136	0.5922	0.4818
0.1	0.4	16	1	5	0	6.1580	5.7060	6.8010	6.1750
0.1	0.4	16	4	5	4.00	4.7640	4.6090	5.3160	4.8050
0.1	0.4	16	16	5	20.00	4.8140	4.5510	4.5800	3.8560

Table 6. (Continued) MSE Comparisons for the Estimates of $\sigma^2_{B/A}$

π_b	π_c	ϕ_1	ϕ_2	$\sigma^2_{B/A}$	σ^2_A	S_2	SN_2	$N_{2,1}$	$N_{2,2}$
0.2	0	1	1	0	0	0.0591	0.0591	0.0591	0.0591
0.2	0	1	4	0	0.25	0.0210	0.0210	0.0210	0.0210
0.2	0	1	16	0	1.25	0.0129	0.0129	0.0129	0.0129
0.2	0	4	1	1	0	0.2424	0.2424	0.2424	0.2424
0.2	0	4	4	1	1.00	0.2912	0.2912	0.2912	0.2912
0.2	0	4	16	1	5.00	0.4196	0.4196	0.4196	0.4196
0.2	0	16	1	5	0	3.8580	3.8580	3.8580	3.8580
0.2	0	16	4	5	4.00	5.9020	5.9020	5.9020	5.9020
0.2	0	16	16	5	20.00	5.3510	5.3510	5.3510	5.3510
0.2	0.2	1	1	0	0	0.0757	0.0709	0.1282	0.1147
0.2	0.2	1	4	0	0.25	0.0232	0.0217	0.0594	0.0521
0.2	0.2	1	16	0	1.25	0.0230	0.0218	0.0566	0.0473
0.2	0.2	4	1	1	0	0.5064	0.4785	0.5909	0.5355
0.2	0.2	4	4	1	1.00	0.3300	0.3158	0.3975	0.3625
0.2	0.2	4	16	1	5.00	0.3917	0.3697	0.5390	0.4762
0.2	0.2	16	1	5	0	6.5680	6.2500	6.4380	6.1900
0.2	0.2	16	4	5	4.00	4.9220	4.7160	6.3360	5.6670
0.2	0.2	16	16	5	20.00	5.9920	5.7000	5.8150	5.4160
0.2	0.4	1	1	0	0	0.1411	0.1305	0.3123	0.2638
0.2	0.4	1	4	0	0.25	0.0485	0.0443	0.1795	0.1501
0.2	0.4	1	16	0	1.25	0.0471	0.0425	0.1771	0.1478
0.2	0.4	4	1	1	0	0.5603	0.5096	0.8100	0.6724
0.2	0.4	4	4	1	1.00	0.4885	0.4389	0.6652	0.5630
0.2	0.4	4	16	1	5.00	0.5969	0.5463	0.7920	0.6570
0.2	0.4	16	1	5	0	7.9560	7.1960	8.0540	6.9580
0.2	0.4	16	4	5	4.00	9.7130	8.3670	10.3900	8.9500
0.2	0.4	16	16	5	20.00	5.7400	5.2420	7.9020	6.5840

Table 7. MSE Comparisons for the

Estimates of σ^2_A

π_b	π_c	ϕ_1	ϕ_2	$\sigma^2_{B/A}$	σ^2_A	S_3	SN_3	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$	$N_{3,4}$	$N_{3,5}$	$N_{3,6}$	$N_{3,7}$	$N_{3,8}$
0	0	1	1	0	0	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015
0	0	1	4	0	0.25	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386
0	0	1	16	0	1.25	1.0160	1.0160	1.0160	1.0160	1.0160	1.0160	1.0160	1.0160	1.0160	1.0160
0	0	4	1	1	0	0.0553	0.0553	0.0553	0.0553	0.0553	0.0553	0.0553	0.0553	0.0553	0.0553
0	0	4	4	1	1.00	0.4976	0.4976	0.4976	0.4976	0.4976	0.4976	0.4976	0.4976	0.4976	0.4976
0		4	16	1	5.00	12.7300	12.7300	12.7300	12.7300	12.7300	12.7300	12.7300	12.7300	12.7300	12.7300
0	0	16	1	5	0	0.8309	0.8309	0.8309	0.8309	0.8309	0.8309	0.8309	0.8309	0.8309	0.8309
0	0	16	4	5	4.00	12.4700	12.4700	12.4700	12.4700	12.4700	12.4700	12.4700	12.4700	12.4700	12.4700
0	0	16	16	5	20.00	228.7000	228.7000	228.7000	228.7000	228.7000	228.7000	228.7000	228.7000	228.7000	228.7000
0	0.2	1	1	0	0	0.0027	0.0027	0.0026	0.0026	0.0027	0.0027	0.0030	0.0030	0.0029	0.0029
0	0.2	1	4	0	0.25	0.0481	0.0477	0.0473	0.0469	0.0471	0.0470	0.0491	0.0486	0.0485	0.0485
0	0.2	1	16	0	1.25	0.9894	0.9814	0.9427	0.9388	0.9397	0.9394	0.9552	0.9525	0.9542	0.9538
0	0.2	4	1	1	0	0.0652	0.0695	0.0647	0.0640	0.0662	0.0659	0.0570	0.0564	0.0554	0.0552
0	0.2	4	4	1	1.00	0.8470	0.8420	0.8975	0.8861	0.8849	0.8843	0.8875	0.8753	0.8775	0.8774
0	0.2	4	16	1	5.00	10.8000	10.7200	10.8100	10.6700	10.6700	10.6600	10.6700	10.5200	10.5400	10.5300
0	0.2	16	1	5	0	1.0160	1.0890	1.1020	1.0880	1.0960	1.0950	0.9730	0.9616	0.9599	0.9562
0	0.2	16	4	5	4.00	13.2700	13.2500	13.3400	13.2300	13.3000	13.2600	12.6500	12.5500	12.5700	12.5300
0	0.2	16	16	5	20.00	170.5000	169.0000	163.7000	163.0000	162.8000	162.9000	165.7000	165.2000	165.3000	165.3000
0	0.4	1	1	0	0	0.0070	0.0071	0.0900	0.0850	0.0090	0.0090	0.0147	0.0134	0.0130	0.0127
0	0.4	1	4	0	0.25	0.0705	0.0700	0.0724	0.0710	0.0711	0.0710	0.0726	0.0716	0.0721	0.0717
0	0.4	1	16	0	1.25	0.9936	0.9695	0.9788	0.9609	0.9737	0.9607	0.9889	0.9760	0.9845	0.9750
0	0.4	4	1	1	0	0.1101	0.1162	0.1065	0.1041	0.1098	0.1081	0.1183	0.1184	0.1151	0.1144
0	0.4	4	4	1	1.00	0.7003	0.6924	0.6920	0.6829	0.6897	0.6837	0.6751	0.6664	0.6659	0.6624
0	0.4	4	16	1	5.00	25.4900	24.8600	23.9000	23.2300	23.3200	23.2500	22.3400	21.6700	21.7700	21.6400
0	0.4	16	1	5	0	1.1170	1.1990	1.1060	1.0570	1.1110	1.0880	1.0310	1.0100	0.9860	0.9771
0	0.4	16	4	5	4.00	15.7000	15.5200	17.2300	16.5400	16.7200	16.5600	16.8600	15.9300	16.2200	15.9200
0	0.4	16	16	5	20.00	230.2000	224.9000	269.1000	259.0000	260.7000	258.9000	261.4000	251.8000	254.0000	251.9000

Table 7. (Continued) MSE Comparisons for the

Estimates of σ^2_A

π_b	π_c	ϕ_1	ϕ_2	$\sigma^2_{B/A}$	σ^2_A	S_3	SN_3	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$	$N_{3,4}$	$N_{3,5}$	$N_{3,6}$	$N_{3,7}$	$N_{3,8}$
0.1	0	1	1	0	0	2.2890	2.2890	2.2900	2.2900	2.2900	2.2900	2.2900	2.2900	2.2900	2.2900
0.1	0	1	4	0	0.25	0.0675	0.0672	0.0673	0.0671	0.0673	0.0671	0.0673	0.0671	0.0673	0.0671
0.1	0	1	16	0	1.25	1.0230	1.0120	0.9811	0.9712	0.9811	0.9712	0.9811	0.9712	0.9811	0.9712
0.1	0	4	1	1	0	0.0327	0.0323	0.0376	0.0364	0.0376	0.0364	0.0376	0.0364	0.0376	0.0364
0.1	0	4	4	1	1.00	0.7375	0.7287	0.7123	0.7005	0.7123	0.7005	0.7123	0.7005	0.7123	0.7005
0.1	0	4	16	1	5.00	15.0200	14.8700	14.1000	14.0000	14.1000	14.0000	14.1000	14.0000	14.1000	14.0000
0.1	0	16	1	5	0	1.6420	1.6150	1.7010	1.6520	1.7010	1.6520	1.7010	1.6520	1.7010	1.6520
0.1	0	16	4	5	4.00	18.1700	17.8500	20.2400	19.7000	20.2400	19.7000	20.2400	19.7000	20.2400	19.7000
0.1	0	16	16	5	20.00	300.1000	297.2000	288.6000	283.8000	288.6000	283.8000	288.6000	283.8000	288.6000	283.8000
0.1	0.2	1	1	0	0	1.7070	1.6920	1.6390	1.6320	1.6310	1.6310	1.6600	1.6550	1.6550	1.6560
0.1	0.2	1	4	0	0.25	0.0583	0.0574	0.0641	0.0621	0.0633	0.0620	0.0710	0.0684	0.0703	0.0684
0.1	0.2	1	16	0	1.25	0.9559	0.9372	1.0620	1.0280	1.0630	1.0300	1.1390	1.0930	1.1290	1.0910
0.1	0.2	4	1	1	0	0.1212	0.1255	0.1201	0.1167	0.1207	0.1177	0.1123	0.1096	0.1106	0.1082
0.1	0.2	4	4	1	1.00	0.7116	0.6949	0.7161	0.7035	0.7129	0.7052	0.7048	0.6913	0.7012	0.6899
0.1	0.2	4	16	1	5.00	14.6100	14.1900	17.1200	16.6300	17.1200	16.6300	17.9600	17.4600	17.9700	17.4600
0.1	0.2	16	1	5	0	0.9522	1.0140	1.0970	1.0510	1.0900	1.0680	1.0250	0.9619	0.9719	0.9447
0.1	0.2	16	4	5	4.00	21.7800	21.6800	21.0400	20.6900	20.8300	20.7400	22.3900	21.7200	21.8300	21.6500
0.1	0.2	16	16	5	20.00	341.8000	334.9000	324.8000	317.6000	321.9000	317.5000	321.1000	312.6000	318.0000	312.7000
0.1	0.4	1	1	0	0	2.3060	2.2540	2.7120	2.6040	2.6270	2.6040	2.6410	2.5410	2.5660	2.5410
0.1	0.4	1	4	0	0.25	0.0863	0.0834	0.0914	0.0867	0.0887	0.0868	0.0900	0.0848	0.0867	0.0845
0.1	0.4	1	16	0	1.25	0.7327	0.7172	0.7946	0.7672	0.7775	0.7679	0.7868	0.7589	0.7724	0.7573
0.1	0.4	4	1	1	0	0.0851	0.0880	0.1159	0.1037	0.1123	0.1057	0.1260	0.1129	0.1171	0.1101
0.1	0.4	4	4	1	1.00	1.4680	1.4550	1.4380	1.3670	1.4250	1.3760	1.4850	1.4020	1.4530	1.3940
0.1	0.4	4	16	1	5.00	16.1700	15.7000	15.7000	15.2600	15.4400	15.3200	16.0900	15.5000	15.6200	15.4400
0.1	0.4	16	1	5	0	1.2770	1.3890	1.6610	1.4520	1.6910	1.5010	1.4090	1.2930	1.3780	1.2510
0.1	0.4	16	4	5	4.00	18.0100	17.1500	22.9500	20.5700	23.3800	20.5700	25.2900	22.9400	25.7300	22.9400
0.1	0.4	16	16	5	20.00	183.5000	175.2000	192.4000	180.6000	187.8000	180.6000	186.2000	174.2000	181.7000	174.2000

Table 7. (Continued) MSE Comparisons for the Estimates of σ^2_A

π_b	π_c	ϕ_1	ϕ_2	$\sigma^2_{B/A}$	σ^2_A	S_3	SN_3	$N_{3,1}$	$N_{3,2}$	$N_{3,3}$	$N_{3,4}$	$N_{3,5}$	$N_{3,6}$	$N_{3,7}$	$N_{3,8}$
0.2	0	1	1	0	0	3.0030	2.9740	2.8950	2.8460	2.8950	2.8460	2.8950	2.8460	2.8950	2.8460
0.2	0	1	4	0	0.25	0.0754	0.0742	0.0753	0.0735	0.0753	0.0735	0.0753	0.0735	0.0753	0.0735
0.2	0	1	16	0	1.25	0.7137	0.6989	0.7532	0.7366	0.7532	0.7366	0.7532	0.7366	0.7532	0.7566
0.2	0	4	1	1	0	0.1266	0.1209	0.1932	0.1760	0.1932	0.1760	0.1932	0.1760	0.1932	0.1760
0.2	0	4	4	1	1.00	0.8796	0.8589	0.9189	0.9038	0.9189	0.9038	0.9189	0.9038	0.9189	0.9038
0.2	0	4	16	1	5.00	16.3000	15.8100	19.5200	18.2900	19.5200	18.2900	19.5200	18.2900	19.5200	18.2900
0.2	0	16	1	5	0	1.1810	1.1530	1.5800	1.4910	1.5800	1.4910	1.5800	1.4910	1.5800	1.4910
0.2	0	16	4	5	4.00	11.4100	11.0500	11.2800	10.8000	11.2800	10.8000	11.2800	10.8000	11.2800	10.8000
0.2	0	16	16	5	20.00	294.9000	286.3000	306.3000	296.0000	306.3000	296.0000	306.3000	296.0000	306.3000	296.0000
0.2	0.2	1	1	0	0	3.4280	3.3580	3.2600	3.1870	3.2320	3.1860	3.2240	3.1380	3.1930	3.1380
0.2	0.2	1	4	0	0.25	0.1045	0.1009	0.1119	0.1035	0.1097	0.1039	0.1170	0.1081	0.1136	0.1075
0.2	0.2	1	16	0	1.25	0.6998	0.6816	0.7593	0.7317	0.7487	0.7302	0.8003	0.7708	0.7913	0.7721
0.2	0.2	4	1	1	0	0.0779	0.0754	0.1230	0.1047	0.1219	0.1062	0.1282	0.1094	0.1239	0.1078
0.2	0.2	4	4	1	1.00	0.7441	0.7167	1.2010	1.0680	1.2070	1.0690	1.1760	1.0580	1.1870	1.0570
0.2	0.2	4	16	1	5.00	18.1200	17.2800	16.9600	16.1500	16.6100	16.1700	16.3000	15.5700	15.9700	15.5600
0.2	0.2	16	1	5	0	1.2680	1.3230	1.5020	1.3760	1.6340	1.4110	1.6160	1.5260	1.6600	1.4900
0.2	0.2	16	4	5	4.00	16.9400	16.5200	18.9700	18.0000	18.9400	18.1600	19.8400	18.8600	19.5300	18.7400
0.2	0.2	16	16	5	20.00	163.8000	151.2900	248.3000	236.9000	244.4000	237.3000	247.2000	233.5000	241.4000	233.1000
0.2	0.4	1	1	0	0	1.8410	1.7570	1.9360	1.8160	1.8890	1.8160	1.8770	1.7530	1.8290	1.7530
0.2	0.4	1	4	0	0.25	0.1069	0.1004	0.1347	0.1232	0.1315	0.1239	0.1554	0.1422	0.1500	0.1414
0.2	0.4	1	16	0	1.25	1.9380	1.8030	1.5950	1.5310	1.5780	1.5350	1.5460	1.4800	1.5290	1.4750
0.2	0.4	4	1	1	0	0.2086	0.2819	0.4281	0.3424	0.4031	0.3473	0.4710	0.3848	0.4330	0.3793
0.2	0.4	4	4	1	1.00	0.8765	0.8605	1.1450	1.0500	1.1050	1.0540	1.1420	1.0270	1.1010	1.0220
0.2	0.4	4	16	1	5.00	13.0400	12.2200	13.6800	12.4300	12.9200	12.4100	13.5800	12.2600	12.8200	12.2700
0.2	0.4	16	1	5	0	2.1510	2.2580	3.8300	3.1790	3.6600	3.2360	3.4130	2.8130	3.1110	2.7660
0.2	0.4	16	4	5	4.00	14.8100	14.1400	18.2000	16.9000	17.2800	17.0500	20.5700	18.8000	19.3100	18.6700
0.2	0.4	16	16	5	20.00	207.9000	197.4000	196.3000	189.2000	192.5000	189.0000	201.0000	192.7000	197.4000	192.9000

CHAPTER IV

RESULTS

The analysis was broken into three parts, these were the various methods of estimating (1) σ_e^2, (2) $\sigma_{B/A}^2$ and (3) σ_A^2 and assessing which estimating methods provide the better estimates. The criteria to assess the estimates was based on the smallest mean square errors. The Wilcoxon signed rank test [Ostle,(1966)] was used in all cases to make the above assessment as to which procedures had the smallest MSE, with an $\alpha = 0.05$.

Two methods of estimating σ_e^2 were examined, the standard method of computing the sums of squares, S_1, and the simplifying method of computing the sums of squares, $N_{1,1}$. Table 3, page 12, defines the simplying method. The standard procedure had MSE's significantly smaller than the simplifying method.

Four methods of estimating the component of variation $\sigma_{B/A}^2$ were examined, these methods are given in Table 8.

The "l" methods of computing the simplified sums of squares are given in Table 3. The results of the test of hypothesis are given in Table 9, with the standard method of estimating the sums of squares paired with the new method of estimating the k_i's being superior to the other methods.

There were ten ways of estimating the component of variation σ_A^2, these methods are summarized in Table 8. The "l" and "m" methods of computing the simplified sums of squares are given in Table 3. An examination of Table 9 clearly shows that the standard method of estimating the sums of squares with the new method of estimating the k_i's is at least as good as any method tried and significantly better than the current standard procedure.

Table 8. Definition of Procedures

		SUMS OF SQUARES			k_i COEFFICIENTS	
			SIMPLIFYING			
COMPONENT	TERM	STANDARD	l =	m =	STANDARD	NEW METHOD
σ_e^2	S_1	X				
	$N_{1,1}$		- X	-		
$\sigma_{B/A}^2$	S_2	X			X	
	SN_2	X				X
	$N_{2,1}$		1	-		X
	$N_{2,1}$		2	-		X
σ_A^2	S_3	X			X	
	SN_3	X				X
	$N_{3,1}$		1	1		X
	$N_{3,2}$		1	2		X
	$N_{3,3}$		1	3		X
	$N_{3,4}$		1	4		X
	$N_{3,5}$		2	1		X
	$N_{3,6}$		2	2		X
	$N_{3,7}$		2	3		X
	$N_{3,8}$		2	4		X

Table 9. Results of Wilcoxon Signed Rank Test

HYPOTHESIS	RESULTS*
$S_1 = N_{1,1}$	$S_1 < N_{1,1}$
$S_2 = SN_2$	$SN_2 < S_2$
$SN_2 = N_{2,1}$	$SN_2 < N_{2,1}$
$SN_2 = N_{2,2}$	$SN_2 < N_{2,2}$
$S_3 = SN_3$	$SN_3 < S_3$
$SN_3 = N_{3,1}$	$SN_3 < N_{3,1}$
$SN_3 = N_{3,2}$	no difference
$SN_3 = N_{3,3}$	$SN_3 < N_{3,3}$
$SN_3 = N_{3,4}$	no difference
$SN_3 = N_{3,5}$	$SN_3 < N_{3,5}$
$SN_3 = N_{3,6}$	no difference
$SN_3 = N_{3,7}$	$SN_3 < N_{3,7}$
$SN_3 = N_{3,8}$	no difference

* At $\alpha = 0.05$

CHAPTER V

CONCLUSIONS

The standard method of computing σ_e^2 is significantly better than the simplified method. The standard method of computing the sums of squares with the new method of computing the k_i coefficients is superior to any other method for estimating $\sigma_{B/A}^2$. The standard method of computing the sums of squares paired with the new method of computing the k_i coefficients are at least as good as any simplifying methods of computing σ_A^2 and better than the current standard method. These lead to the overall conclusion that the standard method of computing the sums of squares with the new method of computing the k_i's should be used in computing the components of variation. This method is not only simplier to use but allows for the consideration of the sample size to be a random variable, as it should be considered.

It is interesting to note from Table 9 that a pattern emerges with respect to computing σ_A^2, that is the

$X^{(2)}_{...}$ and $X^{(4)}_{...}$ methods of computing the simplified sums of squares appear to give better estimates of the components of variation than $X^{(1)}_{...}$ and $X^{(3)}_{...}$. This is true regardless of the method of computing $X_{i..}$.

The conclusions reached here suggest that further analysis involving other designs should be examined to determine if similar estimates of the components of variation are universal. Another positive aspect of the new method of computing the k_i coefficients is the ability to compute exact F test, this needs to be examined.

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APPENDIXES

Appendix A.

Derivation of Expected Values

The expected values for the cell and observation sample sizes were computed. The following assumptions were made:

1. Sample sizes are independent binomial random variables
2. π_i = probability of missing cell or observation
3. $0 \leq b_i \leq b$
4. $0 \leq c_{ij} \leq c$

$$E[b_i] = \sum_{b_i=0}^{b} b_i \binom{b}{b_i} (1 - \pi_b)^{b_i} \pi_b^{(b-b_i)}$$

$$= \sum_{b_i=1}^{b} \frac{b!}{(b_i - 1)!(b - b_i)!} (1 - \pi_b)^{b_i} \pi_b^{(b-b_i)}$$

$$= b(1 - \pi_b) \sum_{b_i=1}^{b-1} \binom{b-1}{b_i-1} (1 - \pi_b)^{(b_i-1)} \pi_b^{(b-b_i)}$$

let $y = b_i - 1$ and $m = b - 1$

$$E[b_i] = b(1 - \pi_b) \sum_{y=0}^{m} \binom{y}{m} (1 - \pi_b)^{y} \pi_b^{m-y}$$

noting that

$$(c + d)^k = \sum_{i=0}^{k} \binom{k}{i} c^i d^{k-i}$$

so that

$$\sum_{y=0}^{m} \binom{m}{y} (1 - \pi_b)^y \pi_b^{m-y}$$

$$= [(1 - \pi_b) + \pi_b]^m = 1$$

and

$$E[b_i] = (1 - \pi_b)b$$

Similary

$$E[c_{ij}] = \sum_{c_{ij}=0}^{c} c_{ij} \binom{c}{c_{ij}} (1 - \pi_c)^{c_{ij}} \pi_c^{c-c_{ij}}$$

$$= (1 - \pi_c)c$$

$$E[b_.] = E[b_1 + b_2 + ... + b_a]$$

$$= [(1 - \pi_b)b + (1 - \pi_b)b + ... + (1 - \pi_b)b]$$

$$= (1 - \pi_b)ab$$

$$E[c_{i.}] = E[c_{i1} + c_{i2} + ... + c_{ib_i}]$$

$$= (1 - \pi_c)c + (1 - \pi_c)c + ... + (1 - \pi_c)c$$

$$= E[c_{ij}]E[b_i]$$

$$= (1 - \pi_b)(1 - \pi_c)bc$$

$$E[c_{..}] = E[c_{11} + \ldots + c_{1b_1} + c_{21} + \ldots + c_{2b_2} + \ldots + c_{a1} + \ldots + c_{ab_a}]$$

$$= E[c_{ij}]E[b_i]a$$

$$= (1 - \pi_b)(1 - \pi_c)abc$$

It should now be observed that the expected values for the k_i's for a balanced nested design are:

$$k_1 = k_2 = c$$

$$k_3 = bc$$

By substituting the expected values for b_i and c_{ij} in the unbalanced nested design for b and c in the balanced nested design the values of the expected mean squares listed on page 9 are obtained. The expected values for the new method of estimating the k_i coefficients listed on page 10 are now derived.

$$E\left[\frac{c_{..}}{a}\right] = \frac{1}{a} E[c_{..}]$$

$$= \frac{1}{a}(1 - \pi_b)(1 - \pi_c)abc$$

$$= (1 - \pi_b)(1 - \pi_c)bc$$

$$E\left[\frac{c_{..}}{b_{.}}\right] = E(c_{..})E\left[\frac{1}{b_{.}}\right]$$

$$= (1 - \pi_b)(1 - \pi_c)abc\left(E\left[\frac{1}{b_{.}}\right]\right)$$

The evaluation of $E\left[\frac{1}{b_{.}}\right]$ is as follows:

$$E\left[\frac{1}{b_{.}}\right] = \sum_{b_i=1}^{b} \frac{1}{b_1 + b_2 + \ldots + b_a} \binom{b}{b_i} (1 - \pi_b)^{b_i} \pi_b^{b-b_i}$$

$$= \frac{1}{a} \sum_{b_i=1}^{b} \frac{1}{b_i} \binom{b}{b_i} (1 - \pi_b)^{b_i} \pi_b^{b-b_i}$$

$$\simeq \frac{1}{a}\sum_{b_i=0}^{b} \frac{(b+1)b!(1-\pi_b)^{b_i+1}}{b(b_i+1)b_i!(b-b_i)!(1-\pi_b)}\pi_b^{b-b_i}$$

this provides a reasonable approximation for

$$0 \leq \pi_b \leq 0.25$$

and

$$b > 1$$

so that

$$E\left[\frac{1}{b_.}\right] = \left[\frac{1}{ab(1-\pi_b)}\sum_{b_i=0}^{b+1}\frac{(b+1)!}{(b_i+1)!(b-b_i)!}\right]$$

$$\left[(1-\pi_b)^{b_i+1}\pi_b^{b-b_i}\right]$$

$$= \frac{1}{ab(1-\pi_b)}\sum_{b_i=0}^{b+1}\binom{b+1}{b_i+1}(1-\pi_b)^{b_i+1}\pi_b^{b-b_i}$$

$$= \frac{1}{ab(1-\pi_b)}$$

this now gives

$$E\left[\frac{c_{..}}{b_.}\right] \simeq \frac{(1-\pi_b)(1-\pi_c)abc}{(1-\pi_b)ab}$$

$$= (1 - \pi_c)c$$

Appendix B.

Main Program

```
      DIMENSION X(9,9,9),SC(9,9),SSC(9,9),SB(9)
      DIMENSION NC(9,9),XK(2,3),NB(9),XMSA(9),XMSB(3)
      DIMENSION XMSC(2),XMEAN(24),SIGM(24),XMSE(24)
      DIMENSION XBARB(9),XBARC(9,9),XBDDD(4),B(9),SIGA(18)
      DIMENSION SIGB(4)
      DIMENSION SIGMA(9,9),PC1(3),PB1(3),VBB1(3),VAA1(9)
      DATA PC1/0.,.2,.4/
      DATA PB1/0.,.1,.2/
      DATA VBB1/0.,1.,5./
      DATA VAA1/0.,.25,1.25,0.,1.,5.,0.,4.,20./
      VCC=1.
      II=5
      JJ=4
      KK=3
      DATA ZM/Z80000000/
C
C  RANDUM SEED 0<IXX<100000 MUST BE ODD INTEGER
C
      IIX=48193
C
C  SETS NUMBER OF CASES TO BE RUN AT ONE TIME
C
      DO 2000 MN1=1,3
      DO 2000 MN2=1,3
      MN5=0
      DO 2000 MN3=1,3
      DO 2000 MN4=1,3
      MMM=0
      IXX=IIX
      MN5=MN5+1
      VBB=VBB1(MN3)
      PB=PB1(MN2)
      PC=PC1(MN1)
      VAA=VAA1(MN5)
C
C  NUMBER OF TIMES FOR EACH CASE SET AT 100
C
      DO 1000 IJKLMN=1,100
C
C  INITIALIZE ALL VARIABLES TO ZERO
C
      CALL INITAL(SC,SSC,SB,NC,NB,XK,XMSA,XMSB,XMSC,SIG,
```

```
     1SIGMA,XBARC,XBARB,B,XBDDD,II,JJ)
      SA=0.
      NA=0
      XBARA=0.
      VA=SQRT(VAA)
      VB=SQRT(VBB)
      VC=SQRT(VCC)
      DO 10 I=1,II
C
C  CALL RANDOM NUMBER GENERATOR FOR RANDOM
C  RECTANGULAR 0 TO 1
C
      CALL RANDU(IIX,IY,YFL)
C
C  CALL NCTRI TO COMPUTE NORMAL RANDOM NUMBER
C
      CALL NCTRI(YFL,X1,C,IER)
      DO 10 J=1,JJ
      CALL RANDU(IIX,IY,YFL)
      CALL NCTRI(YFL,X2,C,IER)
      DO 10 K=1,KK
      CALL RANDU(IIX,IY,YFL)
      CALL NCTRI(YFL,X3,C,IER)
      X(I,J,K)=X1*VA+X2*VB+X3*VC
   10 CONTINUE
      DO 50 I=1,II
      DO 50 J=1,JJ
C
C  DETERMINE IF CELL OR OBSERVATION IS TO BE INCLUDED BY
C  USING BERNOULLI PROBABILITIES
C
      CALL RANDU(IIX,IY,YFL)
      IF(YFL.LT.PB)GO TO 30
      DO 20 K=1,KK
      CALL RANDU(IIX,IY,YFL)
      IF(YFL.LT.PC)X(I,J,K)=ZM
   20 CONTINUE
      GO TO 50
   30 CONTINUE
   40 X(I,J,K)=ZM
   50 CONTINUE
      DO 80 I=1,II
      DO 70 J=1,JJ
      DO 60 K=1,KK
      IF(X(I,J,K).EQ.ZM)GO TO 60
C
C  COMPUTE CELL AND SAMPLE SIZES
C  COMPUTE SUMS AND SUMS OF SQUARES
C
      NC(I,J)=NC(I,J)+1
```

```
      SC(I,J)=SC(I,J)+X(I,J,K)
      SSC(I,J)=SSC(I,J)+X(I,J,K)*X(I,J,K)
   60 CONTINUE
      SB(I)=SB(I)+SC(I,J)
   70 CONTINUE
      SA=SA+SB(I)
   80 CONTINUE
C
C  COMPUTE DEGREES OF FREEDOM
C
      DFA=0.
      DFB=0.
      DFC=0.
      CALL DGFREE(NC,NB,NA,DFA,DFB,DFC,A,B,II,JJ)
C
C  COMPUTE THE K(I) COEFFICIENTS
C
      CALL COEFF(XK,DFA,DFB,DFC,NC,NB,NA,II,JJ)
      ST1=0.
      DO 110 I=1,II
      IF(NB(I).EQ.0)GO TO 110
C
C  COMPUTE MEANS SQUARES
C
      XMSA(1)=XMSA(1)+SB(I)*SB(I)/NB(I)
      DO 100 J=1,JJ
      IF(NB(I,J).EQ.0)GO TO 100
      XMSB(1)=XMSB(1)+SC(I,J)*SC(I,J)/NC(I,J)
      DO 90 K=1,KK
      IF(X(I,J,K).EQ.ZM)GO TO 90
      XMSC(1)=XMSC(1)+X(I,J,K)*X(I,J,K)
      ST1=ST1+X(I,J,K)
   90 CONTINUE
  100 CONTINUE
  110 CONTINUE
      XMSC(1)=(XMSC(1)-XMSB(1))/DFC
      XMSB(1)=(XMSB(1)-XMSA(1))/DFB
      IF(NA.EQ.0)GO TO 1000
      XMSA(1)=(XMSA(1)-ST1*ST1/NA)/DFA
      XBDDD(1)=SA/NA
      DO 200 I=1,II
      IF(NB(I).EQ.0)GO TO 200
      XBDDD(2)=XBDDD(2)+SB(I)/NB(I)
  200 CONTINUE
      XBDDD(2)=SBDDD(2)/A
      DO 215 I=1,II
      DO 210 J=1,JJ
      IF(NC(I,J).EQ.0)GO TO 210
      XBDDD(3)=XBDDD(3)+SC(I,J)/NC(I,J)
  210 CONTINUE
```

```
 215 CONTINUE
     XBDDD(3)=XBDDD(3)/(DFA+DFB+1.)
     DO 225 I=1,II
     IF(B(I).LE.0.)GO TO 225
     DO 220 J=1,JJ
     IF(NC(I,J).EQ.0)GO TO 220
     XBDDD(4)=XBDDD(4)+SC(I,J)/NC(I,J)/B(I)
 220 CONTINUE
 225 CONTINUE
     XBDDD(4)=XBDDD(4)/A
     MMM=MMM+1
     L1=1
     DO 235 LL=1,4
     L1=L1+1
     DO 230 I=1,II
     IF(NB(I).EQ.0)GO TO 230
     XMSA(L1)=XMSA(L1)+(SB(I)/NB(I)-XBDDD(LL))*(SB(I)
    1/NB(I)-XBDDD(LL)
 230 CONTINUE
 235 CONTINUE
     DO 255 LL=1,4
     L1=L1+1
     DO 250 I=1,II
     IF(B(I).LE.0.)GO TO 250
     SC1=0.
     DO 240 J=1,JJ
     IF(NC(I,J).EQ.0)GO TO 240
     SC1=SC1+(SC(I,J)/NC(I,J))/B(I)
 240 CONTINUE
     XMSA(L1)=XMSA(L1)+(SC1-XBDDD(LL))*(SC1-XBDDD(LL))
 250 CONTINUE
 255 CONTINUE
     DO 260 I=2,9
 260 XMSA(I)=XMSA(I)*(DFA+DFB+DFC+1.)/A/DFA
     DO 280 I=1,II
     IF(NB(I).EQ.0)GO TO 280
     DO 270 J=1,JJ
     IF(NC(I,J).EQ.0)GO TO 270
     XMSB(2)=XMSB(2)+(SC(I,J)/NC(I,J)-SB(I)/NB(I))
    1(SC(I,J)/NC(I,J)-SB(I)/NB(I))
 270 CONTINUE
 280 CONTINUE
     DO 310 I=1,II
     SC1=0.
     IF(B(I).LE.0.)GO TO 310
     DO 290 J=1,JJ
     IF(NC(I,J).EQ.0)GO TO 290
     SC1=SC1+(SC(I,J)/NC(I,J))/B(I)
 290 CONTINUE
     DO 300 J1=1,JJ
```

```
      IF(NC(I,J1).EQ.0)GO TO 300
      XBSB(3)=XMSB(3)+(SC1-SC(I,J1)/NC(I,J1))*
     1(SC1-SC(I,J1)/NC(I,J1))
  300 CONTINUE
  310 CONTINUE
      XMSB(2)=XMSB(2)*(DFA+DFB+DFC+1.)/(DFA+DFB+1.)/DFB
      XMSB(3)=XMSB(3)*(DFA+DFB+DFC+1.)/(DFA+DFB+1.)/DFB
      NN=0
      DO 330 I=1,II
      DO 330 J=1,JJ
      IF(NC(I,J).EQ.0)GO TO 330
      IF(NC(I,J).EQ.1)GO TO 320
      SIGMA(I,J)=(SSC(I,J)-SC(I,J)*SC(I,J)/NC(I,J))/
     1(NC(I,J)-1)
  320 NN=NN+1
      XMSC(2)=XMSC(2)+SIGMA(I,J)
  330 CONTINUE
      XMSC(2)=XMSC(2)/NN
      M=0
C
C  COMPUTE COMPONENTS OF VARIATION
C
      SIGB(1)=(XMSB(1)-XMSC(1))/XK(1,1)
      IF(SIGB(1).LT.0.)SIGB(1)=0.
      SIGB(2)=(XMSB(1)-XMSC(1))/XK(2,1)
      IF(SIGB(2).LT.0.)SIGB(2)=0.
      SIGB(3)=(XMSB(2)-XMSC(2))/XK(2,1)
      IF(SIGB(3).LT.0.)SIGB(3)=0.
      SIGB(4)=(XMSB(3)-XMSC(2))/XK(2,1)
      IF(SIGB(4).LT.0.)SIGB(4)=0.
      SIGA(1)=(XMSA(1)-XMSC(1)-XK(1,2)*SIGB(1))/XK(1,3)
      IF(SIGA(1).LT.0.)SIGA(1)=0.
      SIGA(2)=(XMSA(1)-XMSC(1)-XK(2,2)*SIGB(2))/XK(2,3)
      IF(SIGA(2).LT.0.)SIGA(2)=0.
      DO 340 J=2,9
      I=J+1
      SIGA(I)=(XMSA(J)-XMSC(2)-XK(2,2)*SIGB(3))/XK(2,3)
      IF(SIGA(I).LT.0.)SIGA(I)=0.
  340 CONTINUE
      DO 350 J=2,9
      I=I+1
      SIGA(I)=(XMSA(J)-XMSC(2)-XK(2,2)*SIGB(4))/XK(2,3)
      IF(SIGA(I).LT.0.)SIGA(I)=0.
  350 CONTINUE
      DO 360 I=1,18
C
C  COMPUTE AVERAGE COMPONENT OF VARIANCE
C  COMPUTE THE VARIANCE OF THE AVERAGE
C  COMPUTE THE MEAN SQUARE ERROR
C
```

```
      XMEAN(I)=XMEAN(I)+SIGA(I)
      SIGM(I)=SIGM(I)+SIGA(I)*SIGA(I)
      XMSE(I)=XMSE(I)+(SIGA(I)-VAA)*(SIGA(I)-VAA)
  360 CONTINUE
      J=0
      DO 370 I=19,22
      J=J+1
      XMEAN(I)=XMEAN(I)+SIGB(J)
      SIGM(I)=SIGM(I)+SIGB(J)*SIGB(J)
      XMSE(I)=XMSE(I)+(SIGB(J)-VBB)*(SIGB(J)-VBB)
  370 CONTINUE
      J=0
      DO 380 I=23,24
      J=J+1
      XMEAN(I)=XMEAN(I)+XMSC(J)
      SIGM(I)=SIGM(I)+XMSC(J)*XMSC(J)
      XMSE(I)=XMSE(I)+(XMSC(J)-VCC)*(XMSC(J)-VCC)
  380 CONTINUE
 1000 CONTINUE
      DO 390 I=1,24
      SIGM(I)=(SIGM(I)-XMEAN(I)*XMEAN(I)/MMM)/(MMM-1)
      XMEAN(I)=XMEAN(I)/MMM
      XMSE(I)=XMSE(I)/MMM
  390 CONTINUE
C
C  PROVIDE PRINT OUT OF INFORMATION
C
      WRITE(7,9000)
 9000 FORMAT('1',///,21X,'XBAR',16X,'COMPONENT',/,17X,
     1'CALCULATION',17X,'OF',31X,'MEAN SQUARE',/,5X,
     2'ANALYSIS',4X,'...',1X,'I..',1X,'IJ.',1X,
     3'COEFFICIENT,1X,'VARIATION',7X,'MEAN',6X,
     4'VARIANCE',6X,'ERROR',/)
      WRITE(7,9001)XMEAN(1),SIGM(1),XMSE(1)
 9001 FORMAT(5X,'STANDARD',16X,'STANDARD',8X,'A',6X,
     13F12.6)
      WRITE(7,9002)XMEAN(2),SIGM(2),XMSE(2)
 9002 FORMAT(5X,'STANDARD',16X,'APPROXIMATE',5X,'A',6X,
     13F12.6)
      L=2
      DO 410 I=1,2
      DO 410 J=1,2
      DO 410 K=1,4
      L=L+1
      WRITE(7,9003)K,J,I,XMEAN(L),SIGM(L),XMSE(L)
  410 CONTINUE
 9003 FORMAT(5X,'APPROXIMATE ',I2,I4,I4,2X,'APPROXIMATE',
     15X,'A',6X,3F12.6)
      WRITE(7,9014)
      WRITE(7,9004)XMEAN(19),SIGM(19),XMSE(19)
```

```
9004 FORMAT(5X,'STANDARD',16X,'STANDARD',8X,'B',6X,)
    13F12.6)
     WRITE(7,9010)XMEAN(20),SIGM(20),XMSE(20)
9010 FORMAT(5X,'STANDARD',16X,'APPROXIMATE',5X,'B',6X,
    13F12.6)
     J=20
     DO 420 I=1,2
     J=J+1
     WRITE(7,9011)I,XMEAN(J),SIGM(J),XMSE(J)
 420 CONTINUE
9011 FORMAT(5X,'APPROXIMATE  -   -',I4,2X,'APPROXIMATE',
    15X,'B',6X,3F12.6)
     WRITE(7,9014)
     WRITE(7,9012)XMEAN(23),SIGM(23),XMSE(23)
9012 FORMAT(5X,'STANDARD',32X,'C',6X,3F12.6)
     WRITE(7,9013)XMEAN(24),SIGM(24),XMSE(24)
9013 FORMAT(5X,'APPROXIMATE',29X,'C',6X,3F12.6)
     WRITE(7,9005)VAA,VBB,VCC
     WRITE(7,9009)PB,PC
     WRITE(7,9006)MMM
     WRITE(7,9008)IXX
     WRITE(7,9015)II,JJ,KK
9005 FORMAT(/,5X,'INPUT VARIANCE OF A = ',F10.5
    1       /,5X,'INPUT VARIANCE OF B = ',F10.5
    2       /,5X,'INPUT VARIANCE OF C = ',F10.5)
9006 FORMAT(//,5X,'NOTE:',I4,' REPLICATIONS WERE USED')
9007 FORMAT(/)
9008 FORMAT(//,5X,'NOTE: RANDOM NUMBER GENERATOR SEED =
    1',I8)
9009 FORMAT(//,5X,'PROBABILITY OF B MISSING = ',F10.5
    1        /,5X,'PROBABILITY OF C MISSING = ',F10.5)
9014 FORMAT(/)
9015 FORMAT(//,5X,'TARGET SAMPLE SIZES:  A = ',I2,3X,
    1'B= ',I2,3X,'C = ',I2)
2000 CONTINUE
     STOP
     END
```

Appendix C.

Subroutine RANDU

```
      SUBROUTINE RANDU(IX,IY,YFL)
C
C  COMPUTES RANDOM RECTANGULAR NUMBERS
C
      IY=IX*65539
      IF*IY)5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*.4656613E-9
      IX=XY
      RETURN
      END
```

Appendix D.

Subroutine NCTRI

```
      SUBROUTINE NCTRI(P,X,C,IE)
C
C  COMPUTES RANDOM NORMAL NUMBERS
C
      IE=0
      X=.99999E+74
      C=X
      IF(P)1,4,2
    1 IE=-1
      GO TO 12
    2 IF(P-1.0)7,5,1
    4 X=-.999999E+74
    5 C=0.0
      GO TO 12
    7 C=P
      IF(C-.5)9,9,8
    8 C=1.0-C
    9 T2=ALOG(1.0/(C*C))
      T=SQRT(T2)
      X=T-(2.515517+0.802853*T+.010328*T2)/(1.0+1.432788*T
     1+0.189269*T2+0.001308*T*T2)
      IF(P-0.5)10,10,11
   10 X=-X
   11 C=0.3989423*EXP(-X*X/2.0)
   12 RETURN
      END
```

Appendix E.

Subroutine DGFREE

```
      SUBROUTINE DGFREE(NC,NB,NA,DFA,DFB,DFC,A,B,II,JJ)
C
C  COMPUTES DEGREES OF FREEDOM
C
      DIMENSION NC(9,9),NB(9),B(9)
      DO 2 I=1,II
      DO 1 J=1,JJ
      NB(I)=NB(I)+NC(I,J)
      IF(NC(I,J).EQ.0)GO TO 1
      DFC=DFC+NC(I,J)-1
      DFB=DFB+1
      B(I)=B(I)+1.
    1 CONTINUE
      NA=NA+NB(I)
      IF(NB(I).EQ.0)GO TO 2
      DFA=DFA+1
    2 CONTINUE
      A=DFA
      BI=DFB
      CIJ=DFC+BI
      DFB=DFB-DFA
      DFA=DFA-1
      RETURN
      END
```

Appendix F.

Subroutine COEFF

```
      SUBROUTINE COEFF(XK,DFA,DFB,DFC,NC,NB,NA,II,JJ)
C
C  COMPUTES THE K(I) COEFFICIENTS
C
      DIMENSION XK(2,3),NC(9,9),NB(9)
      DO 2 I=1,II
      DO 1 J=1,JJ
      IF(NC(I,J).EQ.0.OR.NB(I).EQ.0.OR.NA.EQ.0)
     1GO TO 1
      XK(1,1)=XK(1,1)+NC(I,J)*NC(I,J)*(1./NC(I,J)-1./
     1NB(I))/DFB
      XK(1,2)=XK(1,2)+NC(I,J)*NC(I,J)*(1./NB(I)-1./NA)
     1/DFA
    1 CONTINUE
      IF(NB(I).EQ.0.OR.NA.EQ.0)GO TO 2
      XK(1,3)=XK(1,3)+NB(I)*NB(I)*(1./NB(I)-1./NA)/DFA
    2 CONTINUE
      DF=DFA+DFB+DFC+1
      XK(2,1)=DF/(DFA+DFB+1)
      XK(2,2)=XK(2,1)
      XK(2,3)=DF/(DFA+1)
      RETURN
      END
```

Appendix G.

Subroutine INITAL

```
      SUBROUTINE INITAL(SC,SSC,SB,NC,NB,XK,XMSA,XMSB,XMSC,
     1SIG,SIGMA,XBARC,XBARB,B,XBDDD,II,JJ)
C
C  INITIALIZES THE VARIABLES
C
      DIMENSION SC(9,9),SSC(9,9),SB(9),XBDDD(4)
      DIMENSION NC(9,9),NB(9),XK(2,3),XMSA(9),XMSB(3)
      DIMENSION SIG(4,4),SIGMA(9,9),XBARB(9),XBARC(9,9)
      DIMENSION XMSC(2),B(9)
      DO 2 I=1,II
      DO 1 J=1,JJ
      SC(I,J)=0.
      SSC(I,J)=0.
      SIGMA(I,J)=0.
      NC(I,J)=0
      XBARC(I,J)=0.
    1 CONTINUE
      B(I)=0.
      SB(I)=0.
      NB(I)=0
      XBARB(I)=0.
    2 CONTINUE
      DO 3 I=1,2
      DO 3 J=1,3
      XK(I,J)=0.
    3 CONTINUE
      DO 4 I=1,2
      XMSC(I)=0.
    4 CONTINUE
      DO 5 I=1,3
      DO 5 J=1,4
      SIG(I,J)=0.
    5 CONTINUE
      DO 6 I=1,3
    6 XMSB(I)=0.
      DO 7 I=1,9
    7 XMSA(I)=0.
      DO 8 I=1,4
    8 XBDDD(I)=0.
      RETURN
      END
```

VITA

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Master of Science

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