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LEARNING LOGIC: A MIXED METHODS STUDY TO EXAMINE THE EFFECTS OF CONTEXT ORDERING ON REASONING ABOUT CONDITIONALS

by

Christina W. Lommatsch

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Education
(Mathematics Education and Leadership)

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UTAH STATE UNIVERSITY
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2018

ABSTRACT

Learning Logic: A Mixed Methods Study to Examine the Effects of Context Ordering on Reasoning About Conditionals

by

Christina W. Lommatsch, Doctor of Philosophy

Utah State University, 2018

Major Professor: Patricia Moyer-Packenham, Ph.D.
Department: School of Teacher Education and Leadership

Logical statements are prevalent in mathematics, the sciences, law, and many areas of everyday life. The most common logical statements are conditionals, which have the form “If H…, then C…,” where “H” is a hypothesis (or condition) to be satisfied and “C” is a conclusion to follow. Reasoning about conditionals is a skill that is only superficially understood by most individuals and depends on four main conditional contexts (e.g., intuitive, abstract, symbolic, or counterintuitive). The purpose of this study was to test a theory about the effects of context ordering on reasoning about conditionals. To test the theory, the researcher developed, tested, and revised a virtual manipulative educational mathematics application, called the Learning Logic App.

This study employed a convergent parallel mixed methods design to answer an overarching research question and two sub-questions. The overarching research question was “How does the order of teaching four conditional contexts influence reasoning about conditionals?” The two subquestions examined this influence on reasoning in terms of performance and perceptions. This study involved two phases. During Phase I, 10 participants interacted with the Learning Logic App in a clinical setting. The researcher used information gathered in Phase I to revise the Learning Logic App for Phase II. During Phase II, 154 participants interacted with the Learning Logic App in a randomly assigned context ordering in an online setting. In both phases, the researcher collected quantitative and qualitative data. After independent analyses, the researcher made meta-inferences from the two data strands. The results of this study suggest that context ordering does influence learners’ reasoning. The most beneficial context ordering for learners’ performance was symbolic-intuitive-abstract-counterintuitive. The most beneficial context ordering for learners’ perceptions was intuitive-abstract-counterintuitive-symbolic. Based on these results, the researcher proposed a new context ordering: symbolic-intuitive-abstract-counterintuitive-symbolic. This progression incorporates a catalyst at the beginning (symbolic context) which aids the learner in reassessing their prior knowledge. Then, the difficulty of the contexts progresses from easiest to hardest (intuitive-abstract-counterintuitive-symbolic). These findings are important because they provide an instructional sequence for teaching and learning to reason about conditionals that is beneficial to both learners’ performance and their perceptions.

(172 pages)

PUBLIC ABSTRACT

Learning Logic: A Mixed Methods Study to Examine the Effects of Context Ordering on Reasoning About Conditionals

Christina W. Lommatsch

Logical statements are prevalent in mathematics, the sciences, law, and many areas of everyday life. The most common logical statements are conditionals, which have the form "If H…, then C…," where "H" is a hypothesis (or condition) to be satisfied and "C" is a conclusion to follow. Reasoning about conditionals is a skill that is only superficially understood by most individuals and depends on four main conditional contexts (e.g., intuitive, abstract, symbolic, or counterintuitive). The purpose of this study was to test a theory about the effects of context ordering on reasoning about conditionals. To test the theory, the researcher developed, tested, and revised a virtual manipulative educational mathematics application, called the Learning Logic App.

This study employed a convergent parallel mixed methods design to answer an overarching research question and two subquestions. The overarching research question was "How does the order of teaching four conditional contexts influence reasoning about conditionals?" The two subquestions examined this influence on reasoning in terms of performance and perceptions. This study involved two phases. During Phase I, 10 participants interacted with the Learning Logic App in a clinical setting. The researcher used information gathered in Phase I to revise the Learning Logic App for Phase II. During Phase II, 154 participants interacted with the Learning Logic App in a randomly assigned context ordering in an online setting. In both phases, the researcher collected quantitative and qualitative data. After independent analyses, the researcher made meta-inferences from the two data strands. The results of this study suggest that context ordering does influence learners' reasoning. The most beneficial context ordering for learners' performance was symbolic-intuitive-abstract-counterintuitive. The most beneficial context ordering for learners' perceptions was intuitive-abstract-counterintuitive-symbolic. Based on these results, the researcher proposed a new context ordering: symbolic-intuitive-abstract-counterintuitive-symbolic. This progression incorporates a catalyst at the beginning (symbolic context) which aids the learner in reassessing their prior knowledge. Then, the difficulty of the contexts progresses from easiest to hardest (intuitive-abstract-counterintuitive-symbolic). These findings are important because they provide an instructional sequence for teaching and learning to reason about conditionals that is beneficial to both learners' performance and their perceptions.

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Christina Marie Watts Lommatsch

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CHAPTER I

INTRODUCTION

Logical inference abilities are essential to deep mathematical understanding (Harel & Sowder, 2007) and can be applied in almost every academic discipline: from theorems in chemistry, physics, and biology to programming in computer science to classifications in geography, philosophy, and religion to argumentation in law. Logical inference abilities have their foundation in reasoning about conditionals. Conditionals are statements of the form “if H, then C,” where H is the hypothesis (or condition) and C is the conclusion. The prevalence of conditionals in academics and in everyday life supports the need for citizens to understand the underlying logical structures found in conditionals.

Background and Problem Statement

Reasoning about conditionals can be a difficult task and becomes more demanding depending on the structures of the hypothesis and conclusion and/or the type of reasoning involved (Zandieh, Roh, & Knapp, 2014). Furthermore, reasoning about conditionals rarely develops spontaneously (G. J. Stylianides & Stylianides, 2009) and differs depending on the context of the conditional. Different contexts can include those that are intuitive, abstract, symbolic, or counterintuitive. These factors can lead to several types of fallacious reasoning, such as *child’s logic* (O’Brien, 1974) where an individual assumes the bidirectionality of a conditional. For example, the transformation of the intuitive context conditional “If it is a fish, then it swims” into two conditionals: “If it is a fish, then it swims” and “If it swims, then it is a fish,” would imply that everything that can swim is a fish. This illogical leap can be harmless and somewhat entertaining as in the example above, but becomes a much larger concern when it appears in an individual's interpretation of tax law or ballot measures on which she or he intends to vote. Thus, it is essential for individuals to be able to correctly reason about conditionals.

In addition to the benefits of understanding conditionals for their direct applications, research has shown that there are correlations of correct conditional reasoning and higher-level reasoning in other fields. For example, Kılıç and Sağlam (2014) demonstrated that individuals who could correctly reason about conditionals had higher abilities in comprehension of genetics concepts than their peers who were unable to reason correctly about conditionals. Kılıç and Sağlam further noted that individuals who were unable to correctly reason about conditionals had a much higher tendency to use rote memorization techniques rather than a conceptual understanding approach to learning the science material. Consequently, learning to reason about conditionals may be beneficial not only in direct applications, but also in positively influencing an individual's learning strategies in other areas. Research has also shown that reasoning about conditionals is highly context-dependent (e.g., Cosmides & Tooby, 1992; Thompson, 2000), that there appears to be a progression of difficulty from one context to another (e.g., Christoforides, Spanoudis, & Demetriou, 2016; Vamvakoussi, Van Dooren, & Verschaffel, 2013), and that learners do not naturally progress from reasoning correctly in one context to another (e.g., Artman, Cahan, & Avni-Babad, 2006; G. J. Stylianides & Stylianides, 2009). Additionally, this type of reasoning is foundational to (e.g., Herman, Loui, Kaczmarczyk, & Zilles, 2012) or at least correlated with (Kılıç & Sağlam, 2014) success in other subject areas. Therefore, the overall problem is to determine how individuals should be exposed to conditionals in different contexts and how learners can be encouraged to transition from reasoning in one type of context to another.

Significance of Study

There is a need for educational programs that teach individuals to reason about conditionals. This is not only important for improving an individual's reasoning about conditionals themselves, but also for improving the potential positive influence this reasoning may have on learning in other subject areas and on an individual's ability to be a productive citizen in a legalistic society. This study examined the effects of context ordering on reasoning about conditionals. This is important to the field because prior research has not examined effects of context ordering on reasoning about conditionals.

Research shows that learners acquire knowledge through the process of developmental progressions (Clements & Sarama, 2004). Developmental progressions are "levels of thinking; each more sophisticated than the last, which lead to achieving the mathematical goal. That is, the developmental progression describes a typical path children follow in developing understanding and skill about that mathematical topic" (Clements & Sarama, 2010, p. 2). This study is significant because progressions which are more beneficial for learning to reason about conditionals can provide guidance about the order in which instructors should present conditional contexts. The understanding of these progressions can improve efficiency of instructional time and can apply to other subject areas that build upon reasoning about conditionals as their foundation, such as higher order logic or computer programming. The findings from this study can inform educational programs whose reach may extend to improving an individual's reasoning in a multitude of other subject areas.

Research Questions

The purpose of this study was to test a theory about the effects of context ordering on reasoning about conditionals. To test the theory, the researcher developed, tested, and revised a virtual manipulative educational mathematics application, called the Learning Logic App. An overarching research question and two subresearch questions guided this study.

> *Overarching Research Question*: How does the order of teaching four conditional contexts influence reasoning about conditionals? Reasoning was measured by learners' performance on the Conditional Logic Assessment and the Learning Logic App and learners' perceptions gathered through interviews and surveys.
>
> *Research Question 1*: How does the order of teaching four conditional contexts influence learners' performance on the Conditional Logic Assessment and the Learning Logic App?
>
> *Research Question 2*: What are learners' perceptions about how the order of teaching four conditional contexts influenced their performance on reasoning about conditionals?

Summary of Research Design

The research design for this study was a convergent parallel mixed methods design (Creswell & Plano Clark, 2011) with two phases that included the collection of quantitative and qualitative data. Quantitative data included pre- and post-assessment scores on the Conditional Logic Assessment (CLA) and score logs from the Learning Logic App. The CLA is an assessment designed to examine a learner's reasoning about conditionals in multiple contexts. These data answered Research Question 1. Qualitative data included video recordings of the participants' interactions with the Learning Logic App and participants' responses during semi-structured interviews in Phase I and on a computer survey in Phase II. These data sources provided insights about how learners interacted with the Learning Logic App and about participants' perceptions of how the context orderings influenced their performance on reasoning about conditionals to answer Research Question 2. In Phase I, participants completed the Conditional Logic Assessment as a pre-assessment. Then, the participants interacted with the Learning Logic App for approximately 18 minutes. The participants returned to the Conditional Logic Assessment as a posttest. Finally, participants participated in a semi-structured interview with questions about the app to reveal their awareness of features and perceived affordance access. In Phase II, the second set of participants began by completing the Conditional Logic Assessment as a pre-assessment. They were then randomly assigned to one of 23 context orderings to interact with the Learning Logic App. Next, they completed the Conditional Logic Assessment as a post-assessment. Finally, the participants provided their responses on a computer survey about their perceptions of the context ordering and their awareness of features and perceived affordance access in the app. Creswell and Plano Clark (2011) state that the strength of the mixed methods research design is that both data types improve the larger design.

The quantitative data analysis included descriptive statistics, visual and graphical analysis, and a multiple regression. The qualitative data analysis included two rounds of coding for each qualitative data source, initially with open or processing coding (as appropriate for the data source) followed by axial coding. At the completion of these two independent analyses, the researcher merged and compared the results to answer the Overarching Research Question.

Assumptions and Scope of Study

Assumptions

The researcher followed the constructivist theory of learning, meaning that it was assumed that while learning, individuals constructed new knowledge based on their prior knowledge structures (Clements & Battista, 1990). Additionally, the researcher assumed that participants had at least a middle school level of science knowledge and literacy skills. The researcher assumed that the participants did their best to complete assessments and answered openly and honestly in interviews and on surveys. This was encouraged by including anonymity in the data collection process. Also, the researcher assumed that the participants did not experience an excessive amount of fatigue which would impact their performance on any part of the study. The assumptions for running a multiple linear regression were considered when conducting the quantitative analysis.

Delimitations

This study examined two logical structures modus ponens (direct reasoning) and modus tollens (contrapositive reasoning). Only participants who had not previously studied logic formally, such as in a university course, were included. Individuals with prior knowledge of logic may have had conceptions or misconceptions which could influence the way that they construct new knowledge about logic, and which may require a different theory of learning. In this study, participants interacted with the Learning Logic App for one 18-minute session. The researcher chose this time limit based on her experience with a pilot study where participants reported fatigue after completing the assessments and the app interactions for a longer period of time. The researcher also shortened the assessments to address pilot participants' feedback to reduce the fatigue issue. Several recent studies examining the use of games to teach logic have also used a short period of time (e.g., 10-20 minutes) for participants to interact with the logic games (Hicks & Milanese, 2015; Schäfer et al., 2013).

Limitations

The sample for this study was limited to a large university in the intermountain west. This population was from a demographic group that was primarily Caucasian. Additionally, there was a single researcher as the sole coder of the data due to limitations of resources and time. Therefore, the bias of the single researcher inevitably influenced the qualitative analysis.

Definitions of Terms

The following terms were defined for this study.

Conditionals are statements of the form "If H, then C," where H is the hypothesis (or condition) and C is the conclusion.

Reasoning is mathematical reasoning where (1) the process results in a factual statement, (2) the process is novel to the reasoner, and (3) is "founded on the intrinsic mathematical properties of the components in the task" (Bergqvist, 2007, p. 352).

A *logical structure* is an argument form with two premises: the first premise is the conditional itself “If H, then C,” the second premise is which part of the conditional is or is not satisfied.

Modus ponens is a logical structure where the conditional is assumed to be true and that the hypothesis is satisfied. This is associated with direct reasoning.

Modus tollens is a logical structure where the conditional is assumed to be true and the conclusion is falsified. Reasoning here uses the logically equivalent form of the conditional “If not C, then not H.” This is associated with contrapositive reasoning.

Conditional contexts are the settings in which conditionals are situated. Possible conditional contexts include intuitive, abstract, symbolic, and counterintuitive.

Intuitive context conditionals are defined as conditionals where a learner may have prior knowledge that would positively influence their ability to correctly reason about the conditional such as a well-known social rule (Cosmides & Tooby, 1992).

Abstract context conditionals are defined as conditionals where a learner has no prior knowledge that may influence their ability to correctly reason about the conditional (Wason, 1968).

Symbolic context conditionals are defined as conditionals where the learner must reason with mathematics expressions which have no mathematical meaning. Learners may apply fallacious mathematical meaning.

Counterintuitive context conditionals are defined as conditionals where the learner must reason about statements that are counter to their prior knowledge (De Neys & Franssens, 2009; De Neys & Van Gelder, 2009).

CHAPTER II

LITERATURE REVIEW

Learning about logic and its foundational component, reasoning about conditionals, has been empirically studied in various forms for over 50 years. This has been a topic of interest for many researchers with regard to the relationship it has with several theories of thinking and learning. For example, the mental model theories and dual-processing theories (Johnson-Laird, 1983, 2013) where dichotomous models allow for the attribution of true or false values follow strict grammatical and semantic rules, much like the strict rules defining truth in the different logical structures of conditional reasoning. Consequently, several researchers have studied the teaching of reasoning about conditionals with educational programs ranging from tedious rote memorization to the more successful educational games.

The first section of this chapter presents an overview of the research literature on conditionals. The second section of the chapter discusses the literature on educational app design. Finally, the third section presents a conceptual framework based on the research literature that integrates a constructivist lens.

Overview of Conditionals Research

This section of the literature review on conditionals is presented in two parts. The first part is an historical overview of research related to reasoning about conditionals. The second part addresses the research on teaching reasoning about conditionals.

Historical Overview of Reasoning about Conditionals

Historically, the research on reasoning about conditionals has fallen into four main contexts, categorized as abstract, intuitive, symbolic, and counterintuitive. Recent research has focused on comparing individual's reasoning in each of these four contexts and how learners transition among these contexts. There are few studies which have studied only one of the four contexts. These studies are discussed in detail in the first four sections followed by a section that discusses all studies that have compared reasoning in two or more contexts. Then the research about transitioning to reasoning in different contexts is discussed.

Abstract context. The most well-known study related to reasoning about conditionals is the Wason Selection Task, proposed and administered by P. C. Wason in 1966. The Selection Task asks subjects to consider four envelopes, laid flat, that have a letter on one side and a number on the other. The subjects were asked to "select just the envelopes definitely needed to be turned over to find out whether they violate the rule" from the four envelopes labeled "D," "C," "5," and "4" (Wason, 1968). The rule, that is the conditional, proposed is "If an envelope has a D on one side, then it has a 5 on the other side" (Wason, 1968). The Task involves direct reasoning, modus ponens, and contrapositive reasoning, modus tollens. Wason found that less than 10% of the undergraduate participants correctly chose both D (the correct solution for direct reasoning) and 4 (the correct solution for contrapositive reasoning). Nearly all participants correctly selected D, however, it was the second card, 4, which would validate the contrapositive of the statement, that most participants failed to select. The

Selection Task requires participants to reason about two different logical structures, direct reasoning and contrapositive reasoning, with each of these placed in an abstract context. This means that a participant would have no prior knowledge which could influence their reasoning about the conditional.

Studying participants' responses to this abstract conditional clearly showed that participants had not naturally developed some aspects of reasoning about the conditionals. Prior to Wason's study, Piaget had proposed that the development of direct reasoning about conditionals in their abstract form only developed at the age of 11 years or older (Piaget, 1970), placing children who had attained this type of reasoning in the formal operational stage.

Heavy critique of both the Wason Selection Task and Piaget's stages has indicated that the tasks used to measure reasoning about conditionals by both Wason and Piaget failed to precisely measure the reasoning about conditionals as originally intended (Inglis & Simpson, 2004; Wagner-Egger, 2007). For example, Wagner-Egger found that when college students were presented with a question equivalent to the Wason Selection Task, that was situated in a different context (intuitive) where they could use prior knowledge to aid their contrapositive reasoning, participants performed significantly differently than they had on the Wason Selection Task. Thus, while the Wason Selection Task may measure a participant's ability to reason about a conditional in an abstract context, it fails to generalize to a participants' ability to reason about a conditional in other contexts, such as the intuitive context tested by Wagner-Egger. These critiques have led to further research into the intricacies involved in reasoning about conditionals in other contexts.

Intuitive context. Cosmides and Tooby (1992) changed the context of the abstract task posed by Wason to a more intuitive context. In their study, the Wason Selection Task conditional became “If a person is drinking beer, then he must be over 21 years old” with the options of checking a person who is “drinking beer, drinking coke, 25 years old, 16 years old” (Cosmides & Tooby, 1992, p. 182). The correct solution of checking the age of the person drinking beer and the 16-year-old’s drink is much more evident thanks to the social contracts inherent in the question. That is, a participant can use their prior knowledge of alcohol consumption laws and the associated social situations to aid their direct and contrapositive reasoning. Cosmides and Tooby concluded that questions involving these social contracts positively influence subjects’ ability to correctly reason about conditionals. Similarly, in Wolf and Shigaki's (1983) study of gifted children, direct logical reasoning abilities in intuitive conditionals improved markedly after the age of 9 or 10. This influence of intuitive contexts could be advantageous in teaching some topics where a teacher may use a class project to help build this intuitive background knowledge so that the students can more easily reason about statements related to the topic. However, the goal of skill transfers to contexts outside of the intuitive must also be carefully considered. If learners can only successfully reason about conditionals in familiar contexts they may be unable to further generalize the concepts. This limits learners to an empirical proof scheme as described by Harel and Sowder (2007) when faced with contexts other than those which are intuitive.

Symbolic context. Understanding of the underlying logical structures of conditionals appears to be a key aspect of transferring knowledge of reasoning about conditionals to contexts other than the intuitive ones studied by Cosmides and Tooby (1992) and Thompson (2000). A. J. Stylianides, Stylianides, and Philippou (2004) conducted a study comparing participants' responses to conditionals posed both in verbal (here, equivalent to the intuitive) and symbolic contexts. The participants were divided into two groups: a group of undergraduate mathematics majors and a group of undergraduate education majors. The mathematics majors had been explicitly trained to understand the underlying logic found in conditionals. The groups performed equally on the intuitive context conditionals, but the mathematics majors performed significantly better on the symbolic conditionals. Hence, a next step in transfer from intuitive and abstract contexts to more applied contexts may be a symbolic context in which the intuitive nature of the context is removed, but with symbolic expressions as the content.

Counterintuitive context. A further step in the application of this reasoning about conditionals is the application of the reasoning when an individual's prior knowledge conflicts with the necessary logical deduction. When this conflict exists, a learner must engage in what is known as "belief inhibition" to overcome the conflict and successfully follow the underlying logic to the conclusion of the conditional statement. Researchers De Neys and Franssens (2009) conducted experiments which indicated that participants' reasoning about conditionals was negatively impacted when cued beliefs conflicted with the associated logic. For example, the syllogism "All whales are mammals. All mammals have hair. Whales have hair" would generally cue an individual to envision a whale, which generally does not have any hair in their concept image (Tall

& Vinner, 1981). This is contrary to the associated logic within the syllogism which implies the final statement "Whales have hair" must be true given the first two conditionals and the fact.

De Neys and Van Gelder (2009) conducted a second study which addressed how belief inhibition affects an individual's reasoning about intuitive and counterintuitive conditionals across their lifetime. The study indicated that difficulties with belief inhibition decreased from the age of 12 years up through early adulthood but the difficulties quickly began to increase thereafter. This is consistent with the findings of Rafetseder, Schwitalla, and Perner (2013) who found that prior to the age of 12 years old, most children are unable to correctly reason about conditionals involving belief inhibition. Thus, belief inhibition appears to progress from difficult in early life, easier in early adulthood, to difficult again in late adulthood. It is possible that an individual in early adulthood who becomes aware of the cognitive changes they will be experiencing may be able to combat the increased difficulty by learning to reason about the underlying logic.

Comparison of reasoning in multiple contexts. Recent research has focused on how individuals reason when conditionals are presented in one context as compared to another. Several researchers have demonstrated that learners perform better when conditionals are presented in intuitive contexts rather than in abstract, symbolic, or counterintuitive contexts. Thompson (2000) examined participants' reasoning about conditionals in three separate contexts: abstract (represented by the Wason Selection Task), intuitive (represented by social contract questions like those found in Cosmides &

Tooby's, 1992, study), and factual (where statements were listed as facts, but no intuition was associated). Similar to Cosmides and Tooby's findings, participants most correctly answered the questions related to the intuitive context conditionals, but again generally failed to correctly respond to the contrapositive piece of the Wason Selection Task. This further supports a student's natural understanding of direct reasoning about conditionals when they are placed in intuitive contexts, but continued lack of understanding of reasoning in abstract contexts. Similarly, Case (2013) found that students who were familiar with (that is, had some sort of intuition about) particular theorems in calculus were better able to reason about the conditionals than when the conditionals were presented with meaningless abstract terms. Christoforides et al. (2016) compared intuitive, symbolic, and abstract contexts, and found that intuitive contexts made correct reasoning about the converse and inverse (generally difficult logical structures) easier for young children than when the contexts were abstract or symbolic.

Several researchers have also examined conditionals as they are posed in the counterintuitive context as compared with the intuitive context. Vamvakoussi et al. (2013) found that when presented with counterintuitive conditionals, participants performed worse than on those conditionals in the intuitive or symbolic contexts. Babai, Levyadun, Stavy, and Tirosh (2006) noted that high school students who were presented with counterintuitive questions had significantly longer response times than those who were presented with intuitive questions. In a similar study, De Neys and Franssens (2009) found that counterintuitive conditionals also influenced response times on questions unrelated to conditionals; with response times increasing if the participant was presented with a counterintuitive question prior to completing the unrelated lexical decision questions as compared to when they were presented with an intuitive context conditional prior to completing the unrelated lexical decision task.

These studies indicate that reasoning, which is sufficient in an intuitive or abstract context, may not be sufficient when a learner encounters conditionals which are presented in the symbolic or counterintuitive contexts. In addition, correct reasoning about more complex logical statements may be easier to achieve given an intuitive context. However, this reasoning may not transfer to the other contexts due to interference from an individual's prior knowledge about the content of the logical statement or the context in which it is conveyed.

Transitioning from context to context. Transitioning learners from successfully reasoning in one context to the next requires an examination of what kind of reasoning is occurring in each context and how that reasoning may differ from reasoning in another context. Empirical understanding, that is understanding based on experience and observation of an idea, is not uncommon to the average individual. There are almost no theorems in everyday life that can be proven definitively true due to the abundance of factors that must be considered in the applied world. This follows with the idea that "there is an exception to every rule." Indeed, that "rule" has its own exception in that the rules of logic and mathematics are not considered as rules unless they are true in absolutely every case. Consequently, many learners, when first approaching the study of mathematics or logic, apply the same reasoning of empirical justifications as sufficient to prove a theorem or idea as "mostly true." Unfortunately, for these learners, their empirical justifications of mathematics theories, which are often outside the intuitive context in which they can naturally reason, are often lacking in rigor. Researchers have found that when novices attempt to prove a statement in a symbolic context or when they examine proof attempts (sample work that attempts to prove a conjecture, but does not constitute a complete or rigorous proof) they are very likely to accept empirical justifications as rigorous methods of proof (Demiray & Bostan, 2015; İmamoğlu & Toğrol, 2015; Lachmy & Koichu, 2014; A. J. Stylianides, 2007b; G. J. Stylianides & Stylianides, 2009). Experts, on the other hand, expected rigorous deductive arguments for any proofs or conjectures made within their subject area, but were willing to accept empirical justifications when presented with proofs that were outside of their domain (Weber, 2008, 2013), reverting back to the more simplistic reasoning used in intuitive contexts. These studies illustrate the idea that learners will revert to using an intuitive or empirical justification in unfamiliar contexts.

Research also indicates that without the aid of an outside catalyst, learners will retain their empirical justifications as sufficient for a method of reasoning or proving in any context (Artman et al., 2006; G. J. Stylianides & Stylianides, 2009). G. J. Stylianides and Stylianides facilitated this transition from the less sophisticated empirical justification to more formal reasoning by presenting undergraduate mathematics education majors with a “monstrous counterexample.” The “monstrous counterexample” is the idea of a seemingly true mathematics formula or pattern whose counterexample is quite inconceivably out of the range of what an empirical justification would be able to prove or disprove and creates a sincere cognitive conflict which the learner must overcome. For example, the researchers gave students the conjecture: “The expression 1 $+1141n^2$ for n a natural number never equals a square number.” The students tested out a few cases using their method of empirical justification and concluded that the conjecture was indeed true. Then the researchers revealed that while the conjecture was true for the natural numbers 1 through 30,693,385,322,765,657,197,397,207, the next natural number yields a square number when inputted to the expression. This truly monstrous counterexample surprised the students and led them to conjecture that even their most intense forms of empirical justifications would not be feasibly sufficient and that they must begin looking for other more formal methods of reasoning. Leung and Lew (2013) conducted a similar study confirming that the examination of counterexamples improved reasoning about conditionals, but indicated that younger students required more guidance in correctly forming counterexamples due to their difficulties in reasoning about the contrapositive (modus tollens).

Some research studies have examined ways in which students can be trained in the rules pertaining to conditionals. A. J. Stylianides (2007a) examined Deborah Ball’s classroom of third-grade students and observed the large positive impact that the instructor’s facilitation of mathematical discourse had on the children’s ability to transition away from their natural empirical justifications. Leighton (2006) trained students in reasoning about conditionals in the symbolic context and noted that the success in training appeared to be domain-specific (that is, specific to the content of the questions). Hub (2017) conducted an intensive teaching experiment to guide students to reinvent their reasoning about intuitive mathematical conditionals and found the participants shifted to visual set-theoretic representations of conditional reasoning.

Several studies also reported on the barriers they encountered while attempting to train students in reasoning about conditionals and proofs. In particular, five studies noted that the students' opinion of the purpose of proof as compared to the amount evidence they personally needed to qualify something as "proved" correlated with the quality of their own proofs (Conner, 2007; Güler & Dikici, 2014; Övez & Özdemir, 2014; Perkowski, 2013; Steele & Rogers, 2012). This indicates that careful attention must be paid to providing learners with the reasoning behind more formal proving methods. This could be achieved through the presentation of tasks which trigger a true cognitive conflict as suggested by G. J. Stylianides and Stylianides (2009) above.

The use of empirical justifications for reasoning about conditionals forms the baseline of several researchers' proposed progressions of reasoning about conditionals. G. J. Stylianides and Stylianides (2009) proposed a progression from less to more sophisticated reasoning and included the requirement of some sort of catalyst so that students have sufficient "intellectual need" to transition from one level of sophistication to the next. Lee (2016) presented his progression of proof construction within the symbolic context as a continuum from irrelevant inferences to deductive proofs. In addition to suggesting that students must progress from informal to more formals types of reasoning, researchers also suggest that the reasoning must be able to be completed in a variety of contexts, not just those in which the learners have some familiarity or form of intuition (Babai et al., 2006).

Each of these proposed progressions support the idea that a novice learner will maintain empirical justifications regardless of context at the least sophisticated levels of a progression. To transition the learner from this type of reasoning, an outside catalyst of some form must be presented to move the learner to more formal reasoning. When considering these various progressions together, the research literature supports a larger progression where learners transition to more and more formal reasoning in each context and through each of the four contexts: intuitive, abstract, symbolic, and counterintuitive.

This research shows the importance of purposefully designing educational programs to teach reasoning about conditionals, with attention paid to the contexts in which conditionals are presented. Research is needed to determine the best progression of contexts that learners should be exposed to so that they can develop their reasoning about conditionals that aligns with what research shows about the development of this mathematical topic.

Teaching and Learning about Conditionals

Rote memorization of logical equivalencies has been the most common way in which reasoning about conditionals has been presented. In undergraduate level mathematics texts, students are often given a set of truth tables to be memorized and a set of exercises to practice with (e.g., Esty & Esty, 2008). While this method may work for those who are well-vested in learning logical equivalencies, it is not a preferred method for the students who are commonly heard saying “After this, I’ll never have to take another math class again.” Hence, several researchers have attempted to create innovative and accessible ways in which to present the material associated with conditionals, most which involve some form of game. This section provides a summary of those games reported in the research literature.

Games to teach logic. Researchers have designed games to teach propositional logic and conditional reasoning for many years. Many of these games involve a game board of some variety (Allen, 1963; Hicks & Milanese, 2015; Lane, 1983; McFeetors & Mason, 2009). These games are limited in that an expert must be present to verify the solutions in the initial learning phase. Learners are also limited by the physical size and space of the game materials (Lane, 1983). Apart from these limiting factors, these same researchers found that, if implemented in a rigorous manner, the games had benefits for both learning and motivation for the students (Allen, 1963).

To further the motivational aspects in both enjoyment and learning and to overcome the limitations of physical manipulatives, more recent developments have been made in creating and testing computer games to teach these logical reasoning skills. Most of these games focus primarily on the symbolic and abstract elements of conditionals and logic (Costabile, De Angeli, Roselli, Lanzilotti, & Plantamura, 2003; Eysink, Dijkstra, & Kuper, 2002; Pareto, 2014; Schäfer et al., 2013). For example, Lester (1975) used computer terminals to demonstrate logical structures to subjects. Subjects were provided with a list of rules that could be applied to a string of 1s and 0s to create new strings. The goal was to create a target string given the initial string displayed on the terminal. Lester then measured the subjects' abilities to apply these rules, involving direct reasoning about the conditionals, in the most efficient manner. The results showed that there was a significant difference in performance among the age groups, grades 1-12, with younger children performing the worst.

Kalish (2010) found that while younger children could memorize the associated patterns found in these board games and computer programs, they had difficulties appropriately applying the reasoning to other contexts. The purely abstract nature of the contexts presented in each of these instructional programs, such as blocks on a board and geometric shapes on a computer, have been shown to effectively train subjects within their respective environments, but have also shown to have poor transfer to applications in their real-world counterparts (Eysink et al., 2002; Lane, 1983). These real-world counterparts, in which reasoning about conditionals would be applied, such as in science, programming, language, and law, are not limited to the symbolic and abstract contexts and are rarely presented in the straightforward "if…, then…" form. Thus, the question of transfer of knowledge of reasoning about conditionals to contexts other than those posed within a game environment is a pressing one which must be addressed by any educational program.

These studies illustrate the role technology can, and should, play in teaching reasoning about conditionals. With the aid of scaffolded learning environments facilitated by technology, learners can transition from successfully reasoning only about conditionals in intuitive contexts to conditionals in abstract, symbolic, and counterintuitive contexts. Currently, there are few applications or programs that purport to teach reasoning about conditionals, and, to the researcher's knowledge, none that explicitly teach both modus ponens and modus tollens in contexts other than abstract or symbolic. Consequently, the app used in this study allowed the researcher to examine the effects of ordering of all four contexts (intuitive, abstract, symbolic, and counterintuitive), which was not possible with any other currently available educational program.

Educational App Design

Design of educational apps has been studied from a wide variety of perspectives and for nearly every school subject area from literacy and social studies to mathematics and science. This section of the chapter presents general design guidelines for apps and their theoretical underpinnings and more detailed design guidelines for the type of app, a virtual manipulative, used in this study.

General Design Guidelines for Educational Apps

General design guidelines for educational apps include applications of the constructivist learning theory and the availability of multiple dimensions of depth and breadth for accessibility and growth within the educational app environment. Each of these are discussed in detail below.

Constructivist theory of learning. The constructivist theory of learning, which asserts that knowledge is constructed based on prior knowledge and requires active engagement (Clements & Battista, 1990), has been embraced by many instructional technology designers. For example, Tam (2000) provided a set of suggestions outlining how a constructivist paradigm can better engage those involved in distance education due to the individualizable and efficient nature of current technology. However, Petraglia (1998) warns that epistemological assumptions of constructivism are often unintentionally abused in instructional technology design. For example, the core assumption that each individual’s prior knowledge will be unique is rarely factored into instructional designs. A proposed correction to this oversight is the use of theory to create a low enough floor (to borrow Burke and Kafai’s [2014] terminology) in the instructional design, so that it can be ensured that each individual’s prior knowledge base contains (at a minimum) the assumed prerequisite knowledge. Thus, an educational app employing a constructivist paradigm would repeatedly build upon a learner’s unique knowledge base to progress the learner towards the intended outcome.

This repeated building upon a learner’s knowledge base can be achieved through what Bruner (1983) describes as the combinatorial activity inherently involved in play. This combinatorial play can be facilitated by educational app designers by providing an unlimited number game objects which learners can manipulate to test out one mental model or another. Additionally, designers can allow for an open-ended number of tasks within the app environment allowing again for combinatorial play and small shifts in learning through productive struggle (Watts et al., 2016).

Dimensions of depth and breadth. Papert (1980), Resnick et al. (2009), and Burke and Kafai (2014) all discuss three principles for usability in programming languages and game design environments (such as Scratch, App Inventor, or Unity) that can be easily applied to game and app design. These principles are illustrated through a metaphor describing a house that has

> Low floors [so that the game] is intuitive enough to allow new users to acclimate to it gradually and with a degree of confidence…. High ceilings [so that the game] allows more experienced users to create constructs… that can grow increasingly complex and nuanced as [a player’s] proficiency increases…. Wide walls [so that the game]…allows its users to create a wide range of constructs. (Burke & Kafai, 2014, p. 697)

Educational app designers can include low floors in multiple ways, such as with interactive tutorials at the beginning of the app, with scaffolding that is slowly faded throughout the play experience (Belland, 2016), or with multiple entry points. High ceilings can be implemented through infinite and random generation or progressively more difficult levels, challenges, or ill-structured problems. Wide walls can be implemented by loosening the restrictions on what activities a player can complete during gameplay through mechanisms such as a "sandbox" level or area where players can define their own structures within the environment. An example of a sandbox level in a tangram game would be one where a player has access to an unlimited number of tangram pieces and allowed to produce any shape or figure that they choose.

Specific Design Guidelines for Virtual Manipulatives

Each of the general guidelines described above apply to any subcategory of educational apps. This section discusses guidelines that are specific to the design of virtual manipulatives. A virtual manipulative is "an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge" (Moyer-Packenham & Bolyard, 2016, p. 13). The researcher chose a virtual manipulative educational app for this study because of the ease with which it incorporates each of the general guidelines above as well as the potential for consideration of embodied cognition. Embodied cognition is the idea that "cognitive processes are deeply rooted in the body's interaction with the world" and that individuals "off-load cognitive work onto the environment" (Wilson, 2002, p. 625), such as in a game environment.

Specific design guidelines for the design of virtual manipulative educational apps include applications of affordance theory and potentially the use of developmental progressions. Each of these theory applications is addressed below.

Affordance theory. The roots of affordance theory began with a definition from the field of biology. In this context, Gibson (1979) defined affordances as what the environment "*offers* to the animal, what it *provides* or *furnishes*…. It implies the complementarity of the animal and the environment [emphasis in original]" (p. 127). This definition has been further refined to "cues of the potential uses of an artefact by an agent in a given environment" by Burlamaqui and Dong (2014). With this refinement, there is a natural application of affordance theory to the design of educational apps.

Some of first steps in applying affordance theory to the design of educational apps were taken by Moyer-Packenham and Westenskow (2013, 2016) who identified five categories of affordances in virtual manipulative mathematics apps that promoted student learning: motivation, simultaneous linking, efficient precision, focused constraint, and creative variation. Affordances in these categories (e.g., motivation) may mitigate effects of common mathematical challenges for learners, such as mathematics anxiety. The affordance category of focused constraint parallels the game design strategy of oversimplification (Squire, 2011), which allows learners to more critically analyze particular parts of game play. Consequently, research has shown that the purposeful inclusion of affordances from these categories is beneficial for the learner (Moyer-

Packenham & Westenskow, 2013, 2016). App designers must also be aware of the possibility that affordances, such as these, which are intended to be helpful may in implementation be helpful, be a hindrance, or not be attended to at all by the learner (Bullock, Moyer-Packenham, Shumway, MacDonald, & Watts, 2015; Moyer-Packenham et al., 2016).

Affordances are important in this study because an educational application that is used without guidance by a researcher or teacher must be able to independently communicate the purposes of the artefact to the learner. In Phase II of this study, participants engaged with the app online and were not guided by a researcher or teacher. Thus, the use of affordance theory in this study served to ensure that any possible helping or hindering aspects of the artefact could be considered in the interpretation of the results. This allowed the researcher to control for potentially confounding variables in measuring changes of reasoning introduced by the artefact, such as features which accidentally promote incorrect reasoning (e.g., Moyer-Packenham et al., 2016).

Developmental progressions. Developmental progressions have been used in a diverse set of subject areas such as mathematics (Clements & Sarama, 2004), atomic molecular theory (Smith, Wiser, Anderson, & Krajcik, 2006), modern genetics (Duncan, Rogat, & Yarden, 2009), and the nature of matter (Stevens, Delgado, & Krajcik, 2010). A developmental progression is described by Clements and Sarama (2010) as "levels of thinking; each more sophisticated than the last, which lead to achieving the mathematical goal. That is, the developmental progression describes a typical path children follow in developing understanding and skill about that mathematical topic" (p. 1). Clements and

Sarama (2007) further specify that developmental progressions are “most propitiously characterized within a specific domain or topic” (p. 464). And that domain-specific progressions are: “the objects and actions [the students] have developed in that domain [which] are the main determinant of the thinking within each progression, although hierarchic interactions occur at multiple levels within and between topics, as well as with general cognitive processes” (Clements & Sarama, 2007, p. 464).

These progressions aid in assessing small shifts in the learner’s level of thinking about a particular content domain on a continuum of understanding. Developmental progressions are important to this study because the researcher proposed that the four contexts (intuitive, abstract, symbolic, and counterintuitive) form a continuum of understanding of reasoning about conditionals. This developmental progression was inspected through the examination of the effects of ordering of the four contexts on individuals’ ability to reason about conditionals.

App Design Process

A final note must be made about the iterative process of the development of any app, educational or otherwise. Zimmerman (2003) describes the process of app development as a “design methodology based on a cyclic process of prototyping, testing, analyzing, and refining a work in progress” and describes his own development of three educational apps which each went through several cycles of design, testing, and refinement. These cycles allow educational app designers to identify potential bugs in the educational apps or unintended results, such as the unintentionally hindering affordances described by Bullock et al. (2015). Therefore, in this study, there were two cycles of app development. Phase I of the study involved a smaller participant group which aided in identifying bugs in the app and allowed for revision before the use of the app by the larger participant group in Phase II.

Conceptual Framework

This final section of the chapter presents a conceptual framework that provides a representation of some of the abstract ideas previously discussed and gives a theoretic structure of assumptions for this study. The framework features components for the constructivist theory of learning, a developmental progression, affordance theory, and iterative app design. Figure 1 depicts a graphic of the framework.

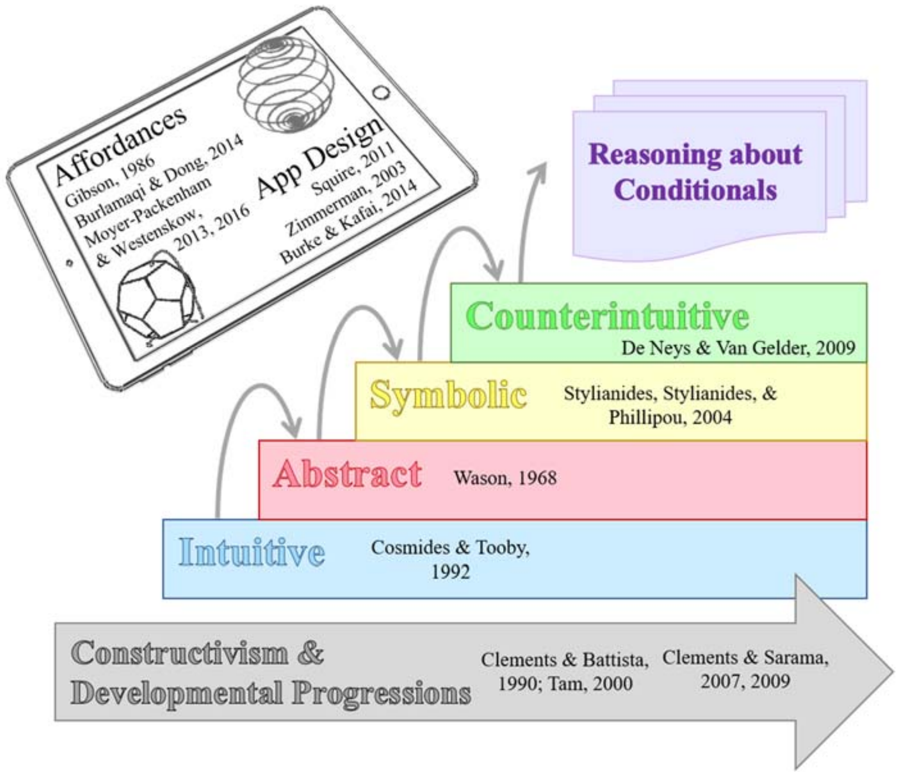


Figure 1. Conceptual framework.

This conceptual framework is depicted with an arrow, a staircase, and a touch-screen tablet. The arrow represents the theoretical foundations of this study with constructivism and developmental progressions as a foundation for the staircase. The staircase represents the proposed developmental progression based on the research on reasoning about conditionals and the progression of conditional contexts. The touch-screen tablet represents the research on educational app design and affordances.

Arrow

At the bottom of the conceptual framework is an arrow showing that constructivism and the developmental progressions are foundational to the development of the app and the progression of learning. Constructivism is important to this study because, when learning about conditionals, learners' prior knowledge may positively or negatively influence their reasoning, implying that the learners construct their knowledge based on prior structures. Developmental progressions are important to this study because research implies that there is a natural progression of difficulty when learning about conditionals. The app design in this study was based on a developmental progression to follow this natural progression of constructing knowledge based on prior knowledge structures, as is suggested by the theory of constructivism.

Staircase

The staircase progression depicts the conditional contexts to illustrate the building upon of previous knowledge inherent in the proposed developmental progression. The research literature on reasoning about conditionals implies that there is relationship among the four different contexts, with intuitive being the easiest and counterintuitive being the most difficult. Several researchers have examined these relationships (e.g., Christoforides et al., 2016; Vamvakoussi et al., 2013) but the research literature is missing the connection of all four contexts in a developmental progression. The author conjectured that progressing through the successively more difficult contexts (intuitive, abstract, symbolic, and counterintuitive) would have a positive influence on a learner's overall reasoning about conditionals. This idea stems from the notion that learner's prior knowledge influences their reasoning about conditionals. This provides a base for the theory about the effects of context ordering, because knowledge gained at one level in the developmental progression will influence the learner's construction of new knowledge at the next level in the developmental progression.

Touch-Screen Tablet

The touch-screen tablet includes elements for app design and affordances because the author employed this research to inform the design choices made in the app, such as the consideration of virtual manipulative affordance categories in designing the gameplay mechanisms. The literature on educational app design indicates that the design process should include iterative cycles of development, testing, and refining (Zimmerman, 2003). App design can also include simple game mechanics with "low floors" and "high ceilings" (Burke & Kafai, 2014; Squire, 2011). The intertwining spirals of the loxodrome within the touch-screen tablet represent these components, which show the cycles as well as the "low floors" and "high ceilings." The research on educational app design also includes aspects of affordance theory, which is particularly important for a stand-alone app, such as the app used in this study. The dodecahedron in the touch-screen tablet represents this component. For one learner, the polyhedron may afford something to stand on, for another it may be something to crawl through, and a third may not notice the polyhedron at all.

Conclusion

This chapter has presented a review of the literature on reasoning about conditionals and guidelines for educational app design as well as a conceptual framework. The research on conditional contexts shows that learners do not spontaneously learn to reason about conditionals, specifically when considering contexts other than the intuitive context or reasoning other than direct reasoning. The contexts in which conditionals are placed have a strong influence on the associated reasoning and must be carefully considered in the development of any study examining reasoning about conditionals. Currently, there is a lack of research examining a learner's progression from the intuitive contexts, as presented by Cosmides and Tooby (1992), to the farther reaching symbolic contexts and counterintuitive contexts, which prepare the learner for application of the knowledge. Therefore, this study examined the effects of context ordering on reasoning about conditionals. This research provides further insights into teaching reasoning about conditionals, whether in a direct instruction fashion or a less traditional technology-enabled game environment, to aid learners in understanding not only the direct applications of reasoning about conditionals, but also in applications to other subject areas and everyday life.

CHAPTER III

METHODS

The purpose of this study was to test a theory about the effects of context ordering (e.g., intuitive, abstract, symbolic, or counterintuitive) on reasoning about conditionals. To test this theory, the researcher developed, tested, and revised an educational mathematics app, the Learning Logic App, based on research from the literature on educational app design and virtual manipulatives. Based on the review of the literature, the researcher hypothesized that the ordering of the contexts in which the conditionals are presented would influence the effectiveness of the mathematics app for students' reasoning about conditionals.

Research Design

The design of this study was a convergent parallel mixed methods design (Creswell & Plano Clark, 2011) with two phases. A convergent parallel mixed methods design involves the concurrent collection of quantitative and qualitative data, separate data analyses, and the merging of the data sets for interpretation (Creswell & Plano Clark, 2011, p. 73). This design was chosen to provide a more complete understanding of participants' experiences with the different context orderings through the complementarity of the quantitative and qualitative data (Tashakkori & Teddlie, 2010). In Phase I of the study, participants interacted with the Learning Logic App with the proposed developmental progression ordering of contexts (intuitive, abstract, symbolic, and counterintuitive) and were interviewed about their experiences. In Phase II of the study, participants were randomly assigned to one of 23 context orderings to interact with the Learning Logic App and provided information about their app experiences in a survey.

The following research questions were the focus of this study.

> *Overarching Research Question*: How does the order of teaching four conditional contexts influence reasoning about conditionals? Reasoning was measured by performance on the Conditional Logic Assessment and the Learning Logic App and learners' perceptions gathered through interviews and surveys.
>
> *Research Question 1*: How does the order of teaching four conditional contexts influence learners' performance on the Conditional Logic Assessment and on the Learning Logic App?
>
> *Research Question 2*: What are learners' perceptions about how the order of four conditional contexts influenced their performance on reasoning about conditionals?

Participants

The participants in this study were adults (over the age of 18 years) enrolled in an undergraduate program at a large public university in the intermountain west. Two sets of participants were recruited: 10 students for Phase I and 157 students for Phase II. Prior to recruitment, the researcher obtained IRB approval (protocol #7860, see Appendix A).

Recruitment

Participants volunteered for the study through the university's SONA System, a system for recruiting undergraduate student participants. Undergraduate students are required to participate in research as part of their coursework requirements at the university. The SONA system is a web-based human subjects pool management software at the researcher's university where undergraduate students can choose from a list of research studies in which to participate. Students receive SONA points for completing the studies. SONA points are one option to gain class credit in a number of undergraduate psychology courses as an assignment at the researcher's university. For example, in a freshman level introductory psychology course, a course which has upwards of 1000 students enrolled in the Spring 2017 semester, students may either participate in research studies from the SONA system or complete reviews of research articles to gain credit towards a class assignment. It is important to note that students are not required to complete studies to successfully complete their course assignments. This study was placed in the SONA system with a value of 1.5 SONA points for Phase I and 1.0 SONA point for Phase II. Before beginning the study, all participants were screened for prior experience with formally studying logic, such as in a university course. If participants had formally studied logic, they were excluded from the study.

Study Phases

In Phase I of the study, ten participants were recruited and data saturation was reached, that is "sampling additional cases [did] not provide any new information" (Collins, 2003). These participants were video-recorded as they interacted with the Learning Logic App and were interviewed about their experiences after interacting with the app. Because Phase I did not address the random assignment to context orderings, a larger sample size was not needed.

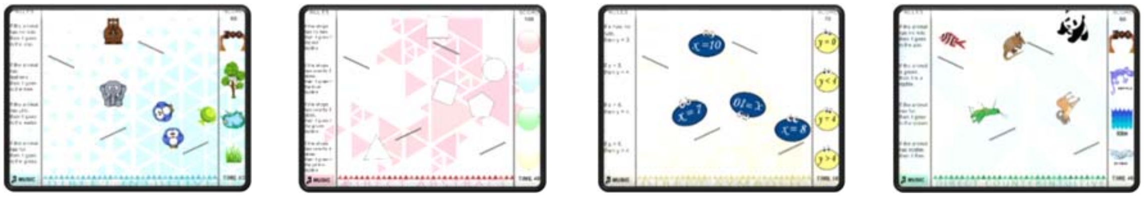
In Phase II of the study 157 participants were recruited. Once 150 students participated, the researcher ran a multiple regression analysis and determined no more participants needed to be recruited. These participants interacted with the Learning Logic

App and provided information about their experiences in an online survey after interacting with the app.

Learning Logic App

The treatment used in this study was an educational app, called the Learning Logic App. The Learning Logic App employs a simply designed sorting game to expose players to conditionals in the four contexts: intuitive, abstract, symbolic, and counterintuitive. The rules of the game require players to reason about conditionals in each of these contexts with two different logical structures: direct reasoning (modus ponens) and contrapositive reasoning (modus tollens). Figure 2 depicts each of the four contexts and two types of reasoning in the app. The Learning Logic App was designed

Direct Intuitive | Direct Abstract | Direct Symbolic | Direct counterintuitive



Contrapositive intuitive | Contrapositive abstract | Contrapositive symbolic | Contrapositive counterintuitive

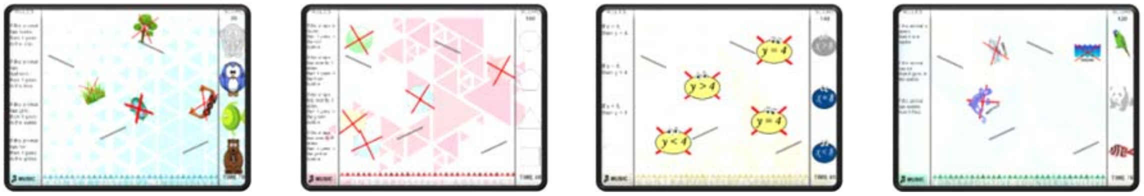


Figure 2. Screenshots of the four contexts and two reasoning types of the Learning Logic App.

based on the research about conditional reasoning and educational app design, such as incorporating “low floors” (Burke & Kafai, 2014) and simultaneous linking affordances (Moyer-Packenham & Westenskow, 2013, 2016). The following sections describe the research-based components of the app, the levels of the app, the gameplay, and the hardware and software specifications.

Research Informing Game Design

As the purpose of the Learning Logic App is as an educational app, the research chose nearly all aspects of the app based on previous research about game design and virtual manipulatives. In terms of the structure of the game, the app includes a sorting task, an interactive tutorial, and automatic advancement to the next level based on time. In terms of game features, the game incorporates an open-ended number of tasks, audio feedback, a timing feature, and motivational features. The simplicity of a sorting task was chosen for this app because of the high effect sizes associated with sorting tasks (Marzano, 2003) and to provide a “low floor” (Burke & Kafai, 2014), that is an easy entry point, for all players. Additionally, the app begins with an interactive tutorial, which introduces the player to the mechanics of gameplay, to mitigate some possible effects of high technological distance (Tucker, 2016) and further lowering the initial difficulty level. The increasing difficulty of the contexts and the logical structures creates a “high ceiling” (Burke & Kafai, 2014), that is the ability for complex tasks to be presented in this environment. “Wide walls” (Burke & Kafai, 2014) and the creative variation category of affordances (Moyer-Packenham & Westenskow, 2013, 2016) were not included in this design due to the directed nature of the progression in the app and the choice to use a focused constraint to draw learners' attention to the context ordering of the progression. The transition from one level to the next is dictated by a fixed interval of time (Belland, 2016), which allows players to interact with an open-ended number of character sorts in the time period (Watts et al., 2016), but ensures that they still move on to experience the other contexts and logical structures in higher levels.

App Levels

Each level of the app contains conditionals placed in one of four contexts: intuitive (I - blue), abstract (A - red), symbolic (S - yellow), and counterintuitive (C - green). Within each context, the app presents one of the two logical structures: direct reasoning (modus ponens) or contrapositive reasoning (modus tollens). Altogether, there are 8 levels including levels with direct reasoning first and contrapositive reasoning second in each context. Figure 3 shows all eight levels with the contexts and logical

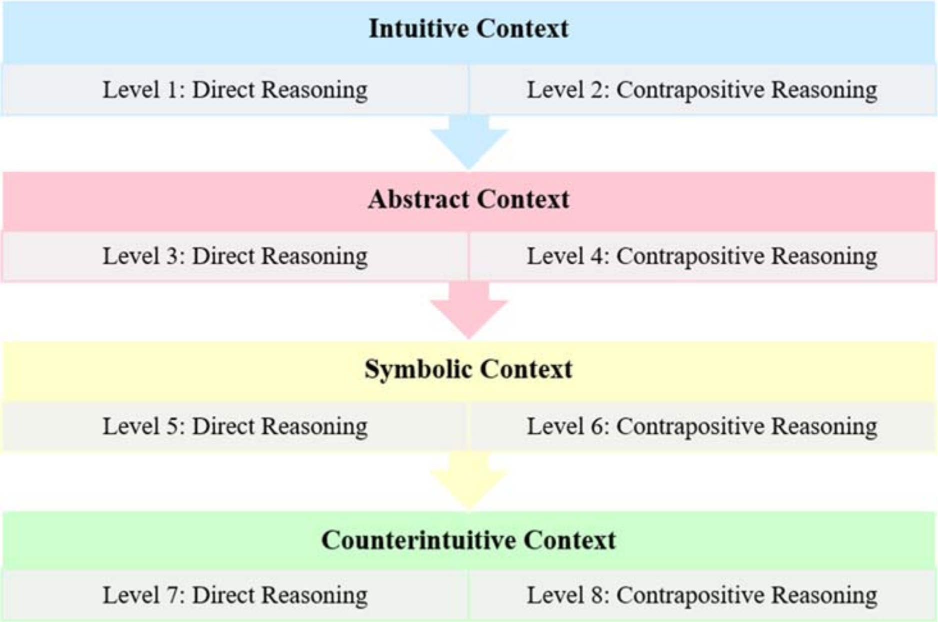


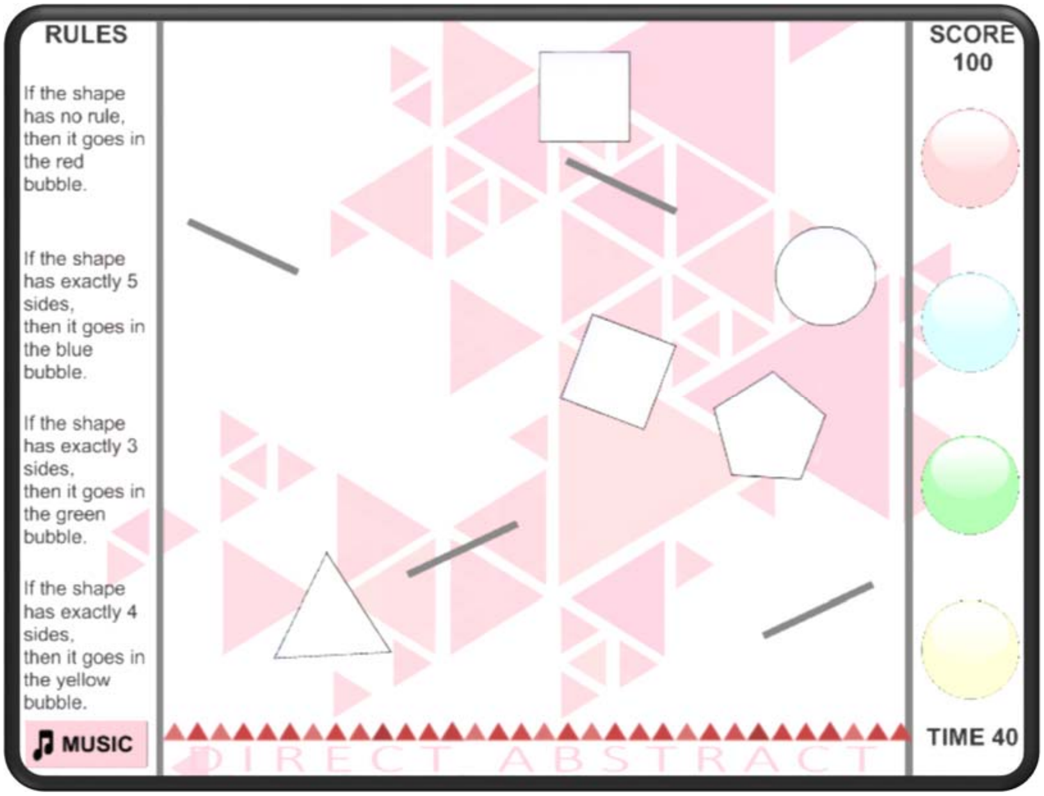
Figure 3. Depiction of all 8 levels of the Learning Logic App with the proposed context ordering of intuitive-abstract-symbolic-counterintuitive (IASC).

structures. The ordering of the contexts was as depicted for Phase I and was randomized for Phase II.

Gameplay

A player's interaction with the Learning Logic App begins with launching the app and pressing "Start" to commence the game. In Phase II, participants then entered their SONA ID numbers and their assigned context ordering. Players first experienced an interactive tutorial for direct reasoning, then began level 1. Players then continued to an interactive tutorial for contrapositive reasoning before beginning level 2. In total, there are 8 levels for players to interact with and players advanced from one level to the next based on a fixed time interval of 100 seconds. After completing the levels, the players saw their high scores for each level.

Within each level, there are characters, which are independent game objects in the app that player can manipulate. For example, in Figure 4, which depicts the abstract level, the characters are a square, a triangle, a pentagon, and a circle. Participants sort (by touching/clicking and dragging) the oncoming characters from the top of the screen to the appropriate zone on the right before they crash at the bottom of the screen and the player loses points. This sorting task is the primary game mechanic. A correct interaction with the app would be the sorting of a character into the correct zone as determined by the logical structure and the given conditionals for that level. An incorrect interaction would be the sorting of a character into an incorrect zone based on the conditionals and logical structure for that level. For example, in an abstract context level with direct reasoning, the characters are geometric shapes that can be sorted, such as in Figure 4. A conditional



If the shape has no rule, then it goes in the red bubble.

If the shape has exactly 5 sides, then it goes in the blue bubble.

If the shape has exactly 3 sides, then it goes in the green bubble.

If the shape has exactly 4 sides, then it goes in the yellow bubble.

Figure 4. Abstract level of the Learning Logic App.

in this level could be “If the shape has exactly 3 sides, then it goes in the green bubble.” A pentagon character would not be sorted into the green bubble because it does not have exactly three sides and the app would require the player to follow one of the other conditionals to complete a correct interaction. The researcher expected the player to be able to classify the shapes based on the number of sides and then sort the shape accordingly. However, no prior knowledge about geometric shapes should aid the player in correctly sorting a triangle into the green bubble. In this example, the players must reason directly, that is with modus ponens, which research indicates should be an easy task (Cosmides & Tooby, 1992; Wason, 1968). Later levels require more complex reasoning, that is contrapositive reasoning, about the conditionals.

As the game play advances, the app paces the incoming number of characters based on the player's score. This adaptive feature allows beginning players to spend more time thinking about each character interaction and for advanced players to be continually motivated by the gaming aspects once they have begun to master the concepts.

Hardware and Software Specifications

The researcher coded the Learning Logic App in the Unity game development platform in the programming language C#. It was deployed on a touch-screen tablet with an Android operating system (Nougat 7.0) for Phase I and online for Phase II, where participants used their own (nontouch-screen) devices. Participants accessed the app through a link in Qualtrics. The app included a script which automatically stored a score log file of each player's interactions with the app, including: the time the interaction occurred, the change in score, with details on what actions changed the score (e.g., which character and which zone caused the score change).

Procedures

The sections below describe the pilot study that occurred prior to this study and the two phases of this study. Figure 5 shows where this study was situated in the overall development of the Learning Logic App. The figure shows the first version of the app, version 0.0 was created during the pilot study. The figure also shows versions 1.0 and 2.0 of the app, used during Phase I and Phase II of this study, respectively. Version 3.0 will be developed in future work. The details of the pilot study and the two phases of this study are discussed below.

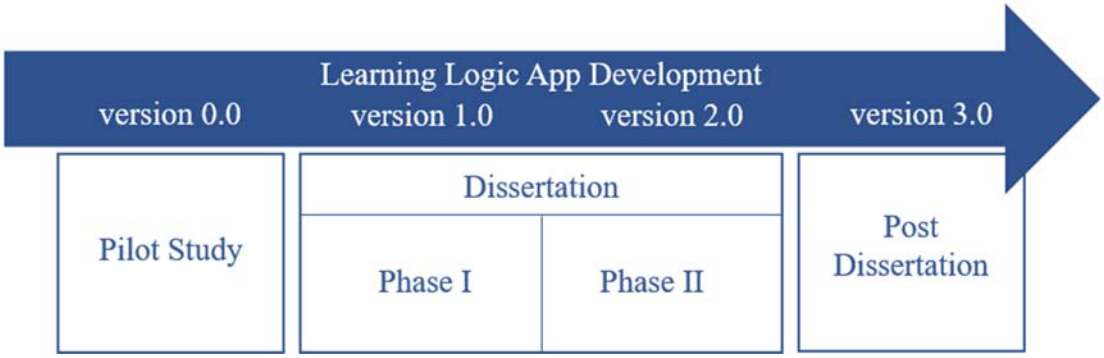


Figure 5. Illustration of the two phases in the design of this study as they are situated in the Learning Logic App development process.

Pilot Study

Prior to this study, the researcher conducted a pilot study (IRB # 7718) with 20 students enrolled in a graduate level course in the Instructional Technology and Learning Sciences Department. The purpose of this pilot study was to do initial development and testing of the app and to pilot the Conditional Logic Assessment and computer survey with a group of college student participants. All pilot study participants completed the pre- and post-assessments (i.e., the Conditional Logic Assessment), interacted with the Learning Logic App, and completed a computer survey on desktop or laptop computers.

During the pilot study, participants reported that the Conditional Logic Assessment (CLA) was interesting, but too long. The researcher modified CLA to address this concern for this study removing time-intensive questions involving listing combinatorial answers. Additionally, eight dichotomous questions were added to increase the reliability of the CLA. The computer survey questions elicited rich feedback from the participants about the Learning Logic App. The researcher also modified the computer survey to address pilot participant feedback.

Based on pilot participant feedback, the researcher modified the symbolic context level of the Learning Logic App to have a slower speed and a built-in feature which allows players to indicate that they have completed reading any text prior to the action of the game (re)commencing. Also, the adaptive element of changing speed, based on the players' interactions, ensured that the game mechanics did not inhibit reasoning. The results of the pilot study provided important information to inform revisions to the app and the design of this study.

Phase I

Phase I of the study answered Research Questions 1 and 2. During Phase I of the study, participants reviewed the study description in the SONA system. Participants who had not formally studied logic, such as in a university course, then scheduled a one-hour block of time to come to a clinical interview room on the university campus.

Upon arrival at the clinical interview room, participants were introduced to the study and completed the informed consent form in Qualtrics on the researcher's touch-screen tablet to maintain participant confidentiality. The researcher then turned on the wall-mounted camera to capture an over-the-shoulder view of the participants' interactions. Participants began the CLA as a pre-assessment, contained in the Qualtrics survey. After completing the CLA, participants put on a wearable GoPro camera using a chest harness, which captured the player's view of the app. The participants began playing the Learning Logic App on the touch-screen tablet. Participants in Phase I used version 1.0 of the Learning Logic App, which was a revised version of the app based on feedback from the pilot study. At the beginning of the Learning Logic App, participants reviewed the app’s instructions and interacted with the tutorials for 2-3 minutes. Participants then interacted with each of the 8 levels of the Learning Logic App for 100 seconds, for a total of 13 minutes of app play. The total app interaction time, including the tutorial and all 8 levels, was approximately 18 minutes. The score log for each participant was automatically stored by the app in a Google spreadsheet.

After completing the Learning Logic App, the participants returned to the CLA as a post-assessment, contained in the Qualtrics survey. After completing the post-assessment, the participant set aside the tablet and the researcher verbally interviewed the participant using a semistructured interview protocol (see Appendix C). Questions during this interview focused on the app’s game design, features, affordances, and technological distance.

The interview was recorded on both the GoPro camera and the wall-mounted camera. After the interview, the participant took off the GoPro camera and the researcher turned off the wall-mounted camera. The data collected during this phase included: pre- and post-assessment scores on the Conditional Logic Assessment, video recordings of the participants playing the Learning Logic App, score logs from the Learning Logic App, and participant responses to the semi-structured interview questions.

Phase II

Phase II of the study answered Research Questions 1 and 2 and the Overarching Research Question. In Phase II, participants completed the study entirely online. The participants reviewed the study description in the SONA system. Those participants who had not formally studied logic, such as in a university course, were immediately directed to follow a link to a Qualtrics survey to participate in the study. First, the participants reviewed the informed consent information and digitally signed the form. Next, the participants completed the CLA as a pre-assessment.

The participants in Phase II were randomly assigned by Qualtrics to one of 23 context orderings for their interaction with the Learning Logic App. This was a randomized trial (Gall, Gall, & Borg, 2011). The researcher chose this design because it provides "strong assurance that observed effects are caused by the experimental treatment and not by extraneous variables" (p. 380). Participants in Phase II used version 2.0 of the Learning Logic App, which was a revised version of the app based on feedback from the Phase I interviews. There are 24 possible permutations of the four contexts in the Learning Logic App, but 23 possible permutations were used. The Symbolic-Abstract-Counterintuitive-Intuitive (SACI) ordering was excluded as a condition because it follows the same context ordering as the CLA and may have caused a carryover effect (B. H. Cohen, 2013). This means that the participants' familiarity with the pre-assessment may have influenced their performance on the Learning Logic App and on the post-assessment.

As in Phase I, participants first completed the tutorial of the Learning Logic App for 2-3 minutes. Participants then interacted with each of the 8 levels for 100 seconds, for a total of 13 minutes of app interaction. The total app interaction time, including the tutorials and all 8 levels, was approximately 18 minutes. The participants then returned to the CLA as a post-assessment. Finally, the participants took the computer survey to provide feedback on the app's game design, features, affordances, and technological distance (see Appendix D). The score logs were automatically stored in a Google spreadsheet for each participant. The data collected during this phase included: pre- and post-assessment scores on the CLA, score logs, and the participants' responses on the computer surveys.

In Phase II, participants either used a desktop or laptop computer. Nearly all technical difficulties that participants reported were with using a touchpad to quickly select and maneuver the characters into the correct zone. Seven participants e-mailed the researcher during data collection with issues in completing the study due to technological difficulties or finding how to access the study. The researcher quickly resolved these issues (within 12 hours) and all participants were able to successfully interact with the app.

Data Sources

There were four data sources for this study: pre- and post-assessment scores on the Conditional Logic Assessment, score logs, video recordings (Phase I), and participant responses about the design, features, affordances, and technological distance in the app (Phase I interviews and Phase II computer surveys). Each of these data sources and the rationale for using them is described in detail below. Table 1 provides an overview of the research questions, data sources, and data analysis for Phase I and Phase II. In Phase I, the data sources included the quantitative pre- and post-assessment scores on the CLA and score logs as well as the qualitative video recordings and participant responses (semistructured interviews). In Phase II, the data sources included the quantitative pre-

Table 1

Overview of Research Questions, Data Sources, and Data Analysis Alignment

Research question	Data sources	Data analysis
Overarching Research Question: How does the order of teaching four conditional contexts influence reasoning about conditionals?	All sources listed below	Merging of quantitative and qualitative data analysis results to create meta-inferences
Research Question 1: How does the order of teaching four conditional contexts influence learners' performance on the Conditional Logic Assessment and the Learning Logic App?	Pre- and post-assessment scores on the Conditional Logic Assessment (CLA)	Descriptive Statistics (Phase I) Multiple regression analysis (Phase II)
	Score log	Visual and graphical analysis
Research Question 2: What are learners' perceptions about how the order of four conditional contexts influenced their performance on reasoning about conditionals?	Video Recordings (Phase I)	Process coding, axial coding
	Participant responses: Semi-structured interviews (Phase I)	Open coding, axial coding
	Computer survey (Phase II)	Open coding, axial coding

and post-assessment scores on the CLA and score logs as well as the qualitative participant responses (computer surveys).

Conditional Logic Assessment

The Conditional Logic Assessment (CLA) is an assessment designed by the researcher to assess an individual's reasoning about conditionals in different contexts and with different logical structures. The CLA is a composite of questions from the research literature on teaching and learning reasoning about conditionals. The CLA includes 32 questions broken into three sections. The researcher increased the number of questions from the pilot study to improve reliability of the CLA. The researcher drew each section of questions from different researchers' work in assessing reasoning about conditionals.

These three sections present conditionals in four different contexts: intuitive, abstract, symbolic, and counterintuitive. The CLA was used to measure change in reasoning about conditionals in each of the four contexts. The complete CLA is included in Appendix B.

Wason, Watts, and Esty section. The first section of the assessment includes questions from Wason (1968) and Watts and Esty (2013). Four questions are derived from the Wason Selection Task (Wason, 1968). In the original implementation of the Wason Selection Task, participants were presented with four envelopes laid flat with a letter on one side and a number on the other and a rule stating: “If a card has a vowel on one side, then it has an even number on the other side.” The participants were then asked which cards would they need to flip over to check that the rule has been followed. In the CLA, the questions were presented in a similar manner with one rule and each question posing a different scenario, just as each of the cards posed a different scenario. Instead of being asked if they need to flip the card over to check the rule, participants are asked if given the scenario, such as the card has an “A” on it, “what can be deduced?” Eight questions of the CLA are drawn from a study conducted by Watts and Esty with conditional statement questions in the abstract and symbolic contexts. In each of the abstract context questions, participants’ prior knowledge did not assist them in correctly answering the questions. However, in the symbolic context, participants’ prior knowledge may or may not interfere with their ability to correctly answer the questions. As with the Wason questions, each of the eight questions in these two contexts poses a rule and a scenario asking the participant “what can be deduced?” For example, “Suppose this statement is true: ‘If $k > 4$ then, $j > 12$’. If ‘$j = 6$’ is also true, what can be deduced?” The correct answer here is “k is less than or equal to 4.”

Cosmides and Tooby section. The second section of the assessment includes six questions from Cosmides and Tooby’s (1992) research on conditionals in intuitive contexts. In these questions, participants’ prior knowledge about the given scenario will most likely positively influence their ability to correctly answer these questions. For example, “A person is sitting at the bar drinking water. Do you need to check this person's age to see if they are following the law? Yes, check this person's age. OR No, do not check this person's age.” The correct solution here is “No, do not check this person’s age.”

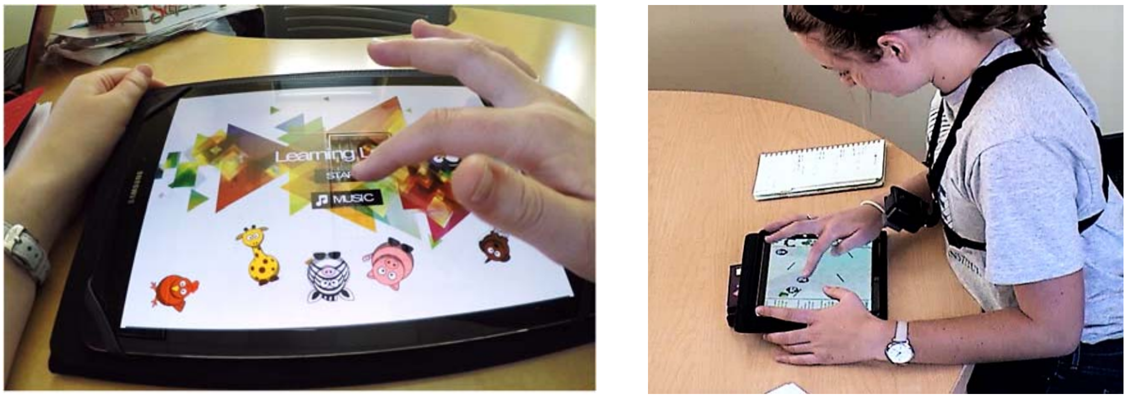
De Neys section. The third section of the assessment includes eight questions from the research of De Neys and Van Gelder (2009) and De Neys and Franssens (2009). These conditionals include both those that are in intuitive contexts as well as those that are in counterintuitive contexts. In the counterintuitive context, participants’ prior knowledge may inhibit their ability to correctly answer these questions. For example, “Suppose this statement is true: ‘All vehicles have wheels.’ If ‘A boat is a vehicle’ is also true, can the following statement be deduced? ‘A boat has wheels.’ Yes, the statement can be deduced. OR No, the statement cannot be deduced.” The correct answer here is “Yes, the statement can be deduced.”

Score Logs

A C# script in the Learning Logic App generated the score logs. The score logs record every change in a player’s score as well as the time the change occurred and the cause for the change in score, such as a correctly (or incorrectly) placed character or a character that crashed at the bottom of the screen because the player did not sort quickly enough to save it. The score logs served as a complementary data source to the participant responses. Data from the score logs allowed the researcher to collect information about how the game mechanics were influencing game play and complemented the video recordings of the game play. With these data, the researcher was able to gauge the appropriateness of the difficulty level and revised the game between Phase I and Phase II. Finally, these data allowed the researcher to determine if participants were actively playing with the Learning Logic App throughout the entire interaction period during Phase II, when all interactions occurred via computer.

Video Recordings

The researcher recorded video only in Phase I. There were two video recording perspectives captured in this study (see Figure 6). The player's perspective was recorded using a chest-mounted GoPro camera worn by the participant. An over-the-shoulder perspective was recorded using a wall-mounted camera. The researcher chose these two perspectives for several reasons. First, the two sources provided backups of the video data in case one camera should have failed or a video file should have become corrupted. Second, the two video sources provided multiple perspectives which provided both the "narrow shot" to display important details and the "wide shot" to capture all of the participant's activity (Roschelle, 2000, p. 717). These data complemented the score logs, so the researcher could further understand any anomalies in score as well as observe any phenomena not perceived by the participants about the app's influence on their reasoning about conditionals during Phase I.



Player's perspective

Over the shoulder perspective

Figure 6. Player and over-the-shoulder video recording perspectives.

Participant Responses

In both phases, the researcher collected participant responses concerning their perceptions of game design, features, affordances, and technological distance of the Learning Logic App. In Phase I, the researcher interviewed the participants using a semi-structured interview. In Phase II, a computer survey collected the responses with questions that duplicated those asked in the face-to-face interviews. Questions came from four categories: game design, features, affordances, and technological distance. Game design questions included questions about the player's overall impression of the Learning Logic App, suggestions for improvement, and player's enjoyment. Features questions included questions about awareness of features, helping or hindering features, and how features assist with the educational task. Affordances questions included questions about four of the five affordances identified by Moyer-Packenham and Westenskow (2013, 2016): motivation, simultaneous linking, efficient precision, and focused constraint. Technological distance questions included questions about potential difficulties with the gestures necessary to interact with the app (such as dragging and dropping).

Phase I semistructured interviews. In Phase I, the researcher conducted semi-structured interviews to collect data about the Learning Logic App version 1.0 and about participants' perceptions of their interactions. The researcher verbally interviewed the participants after they had completed the pre-assessment, played with the Learning Logic App, and completed the post-assessment. The semi-structured interviews took between 15 and 20 minutes. Throughout the interview, the researcher asked the participant for clarification on any responses, if necessary, to help ensure validity of the data. The complete semi-structured interview protocol is included in Appendix C. The responses from participants during these interviews informed the refinements to the Learning Logic App to create version 2.0.

Phase II computer surveys. In Phase II, computer surveys collected data about the Learning Logic App version 2.0. Participants completed the computer surveys after they had completed the pre-assessment, played the app, and the post-assessment. The computer surveys were in the Qualtrics form. The complete computer survey is included in Appendix D. Unlike the semi-structured interviews of Phase I, there was no ability to follow up or to ask further probing questions because participants completed the surveys anonymously. After collecting all responses, the researcher synthesized and summarized the responses from participants. The participants' responses about the app will inform refinements of the Learning Logic App to create version 3.0.

Data Analysis

The data analysis included independent quantitative and qualitative analyses and a comparison of the merged results for convergence or divergence. The researcher analyzed five data sources for this study: pre- and post-assessment scores on the CLA, score logs, video recordings (Phase I only), and participant responses about the design, features, affordances, and technological distance in the app. The sections below describe the analysis of each data source organized by phase.

Phase I

In Phase I, the data sources included the quantitative pre- and post-assessment scores on the CLA and score logs as well as the qualitative video recordings and participant responses (semi-structured interviews).

Conditional logic assessment. In Phase I, the quantitative analysis of the CLA pre- and post-assessment scores was an exploratory analysis. The researcher exported, organized, and cleaned, the data in a Microsoft Excel spreadsheet. The researcher computed descriptive statistics, such as measures of central tendency and indicators of dispersion, for the pre- and post-assessment scores, as well as the change scores, (post-assessment score subtracted from the pre-assessment score). The researcher did not compute inferential statistics due to the small sample size of this phase. This analysis produced quantitative descriptors of how participants' reasoning about conditionals in different contexts was changing after interacting with the Learning Logic App.

Participant responses. Participants responses were collected with semi-structured interviews in Phase I. The researcher transcribed the semi-structured interview responses and entered them into NVivo, a qualitative data analysis program. The transcripts were first open coded followed by a second round of axial coding. Open coding is the “interpretive process in which data is broken down analytically” and it “stimulates generative and comparative questions to guide the researcher upon return to the field” (Corbin & Strauss, 1990, p. 12). This allowed the researcher to return for a second round of axial coding which “extends the analytic work from initial coding” and “describes a category’s properties (i.e., characteristics or attributes) and dimensions (the location of a property along a continuum or range) and explores how the categories and subcategories related to each other” (Saldaña, 2015, p. 291). This analysis produced themes which the researcher used to determine what types of refinements to make to the Learning Logic App version 1.0 to improve it for the next phase of the study. These data also provided information about the influence, or lack thereof, of the proposed context ordering on participants’ reasoning about conditionals.

Video recordings. The video recordings of participants playing the Learning Logic App were also entered into NVivo. The video data were first process coded followed by a second round of axial coding. Process coding “uses gerunds (‘-ing’ words) exclusively to connote observable and conceptual action in the data” (Saldaña, 2015, p. 296). The second round of axial coding further described the relations of categories and the dimensions and properties of each category found in the process coding. This second source of data about the participants’ experience playing the Learning Logic App revealed needed refinements of which the participants were not aware, such as calibration of the collision between characters and zones. The researcher, who also coded the app, was able to recognize these difficulties and their cause through the video recordings. As with the semi-structured interview responses, this analysis produced themes which the researcher used to inform the refinements of version 1.0 of the Learning Logic App to create version 2.0.

Score logs. The researcher exported the score log data into a Microsoft Excel spreadsheet to be cleaned for analysis. In the spreadsheet, the columns denoted time and score. Each participant had two columns: one for the time of the interaction that caused the change in score and one for the score. The researcher plotted the data in line graphs for a visual analysis and compared with other participants. This enabled the researcher to examine the score log data in its entirety (Dickinson, 2010). The visual representations produced by this analysis also provided information on potential causes of technological distance that were unclear from the video recordings. The researcher used this information to determine the needed refinements for the Learning Logic App, such as one character causing a score change in two zones rather than one.

Phase II

In Phase II, the data sources included the quantitative pre- and post-assessment scores on the CLA and score logs as well as the qualitative participant responses (computer surveys).

Conditional logic assessment. In Phase II, the quantitative analysis of the CLA pre- and post-assessment scores involved a multiple regression. Before running the regression, the data were tested for linear relationships, multicollinearity, and outliers to ensure the assumptions of the regression were met. The independent variables were the pre-assessment score and the context ordering. The dependent variable was the post-assessment score. A multiple regression was chosen to allow for prediction of the post-assessment score (the dependent variable) by both the pre-assessment score and the context ordering (the two independent variables). The context ordering could have been treated as a discrete categorical variable or a continuous interval variable. For the context ordering to be considered as a continuous ordinal variable, each of the 24 permutations of the four contexts was measured using Spearman's rho, r_s, which is a correlation for ranked data (B. H. Cohen, 2013) against the proposed context ordering of Intuitive-Abstract-Symbolic-Counterintuitive (IASC). Using the Spearman's rho rank correlation, the similarity of context orderings is defined based on the pairwise distance of each context in an ordering as compared with the proposed IASC. For example, given the IASC context ordering and the reverse context ordering, CSAI, the pairwise distance for the first space is 3, the second space is 1, the third space is 1, and the fourth space is 3. The calculation for Spearman's rho then yields, $r_s = -1.0$. All 24 context orderings are grouped by their Spearman's rho in Table 2. The ordering Symbolic-Abstract-Counterintuitive-Intuitive (SACI) was excluded from the context orderings to prevent a carryover effect (B. H. Cohen, 2013) from the CLA.

To determine whether the context ordering should be treated as a categorical or ordinal variable, the change scores (post-assessment subtracted from pre-assessment) were plotted against the Spearman's rho value and visually analyzed to determine if the data appeared to be linear or discrete. Because the data appeared discrete, the context ordering was treated as a categorical variable.

Table 2

All 24 Context Orderings and Their Spearman's rho with the Proposed Context Ordering, Intuitive-Abstract-Symbolic-Counterintuitive

Spearman's r_s	Context orderings			
$r_s = 1.0$	IASC			
$r_s = 0.8$	AISC	IACS	ISAC	
$r_s = 0.6$	AICS			
$r_s = 0.4$	ASIC	ICAS	ISCA	SIAC
$r_s = 0.2$	ICSA	SAIC		
$r_s = 0.0$	ACIS	SICA		
$r_s = -0.2$	ASCI	CIAS		
$r_s = -0.4$	ACSI	CAIS	CISA	SACI[a]
$r_s = -0.6$	SCIA			
$r_s = -0.8$	CASI	CSIA	SCAI	
$r_s = -1.0$	CSAI			

[a] Context ordering, which was not part of random assignment because it mirrored the organization of the Conditional Logic Assessment.

After running the regression, the researcher tested the data for homoscedasticity and normality of residuals to meet the assumptions necessary for a multiple regression. The researcher performed these tests in R and the multiple regression in SPSS. The method of multiple regression was chosen because it allows for the prediction of the dependent variable based on two independent variables (J. Cohen, Cohen, West, & Aiken, 2003). This analysis produced an adjusted R^2, an *F*-ratio, *p* values, and correlation coefficients, which the researcher used to determine the fit of the model to the data and the influence of each independent variable on the dependent variable. These data provided information about the influence of context ordering on an individual's ability to reason about conditionals.

For this exploratory regression analysis, the researcher needed 150 participants to ensure that the numbers were sufficient for each context ordering. The researcher initially ran the regression after 154 participants had completed the study. Based on the results of the regression, the researcher concluded no more participants needed to be recruited.

Participant responses. In Phase II, the computer survey recorded responses in in Qualtrics and the researcher exported them into NVivo. Like the semi-structured interview transcripts, the computer surveys were first open coded (Corbin & Strauss, 1990) then axially coded (Saldaña, 2015). From the researcher's experience with the pilot study, she expected that most responses would be concise allowing the researcher to manage the large quantity of responses to each question. These data were supplementary to the quantitative data in Phase II and aided in triangulation and validation of the data (Nastasi, Hitchcock, & Brown, 2010). These responses also informed further refinements of the Learning Logic App which will be made in future work.

Score logs. The researcher exported, cleaned, and analyzed the score logs as in Phase I. In this phase, the score logs were a means of maintaining internal validity. Because the participants completed this phase of the study anonymously and entirely online, there was no opportunity for follow-up or clarifying questions. Therefore, anomalies, such as the participant who encountered technical difficulties and was unable to interact with levels 5-8, were recorded in the score logs and identifiable by the researcher. This mitigated the potential effects of active app engagement as a confounding variable.

Convergent Mixed Methods Analysis

After independently completing the two types of analyses, the researcher merged the results during interpretation to consider the Overarching Research Question of "How does the order of teaching four conditional contexts influence reasoning about conditionals?" as is implied by a convergent mixed methods design (Creswell & Plano Clark, 2011). These separate analyses provided "an understanding of the phenomenon under investigation" and were combined into "meta-inferences" (Teddlie & Tashakkori, 2009, p. 266). A meta-inference is described as "an overall conclusion, explanation, or understanding developed through an integration of the inferences obtained from the qualitative and quantitative strands of mixed methods study" (Tashakkori & Teddlie, 2010, p. 101). This aligns with Creswell and Plano Clark's (2011) recommendation for researchers to "analyze quantitatively and…qualitatively and then merge the two sets of results" (p. 71). The analysis of the quantitative data in this study provided inferences about the influence of context ordering on participants' performance in reasoning about conditionals. The analysis of the qualitative data in this study provided inferences about the influence of context ordering on participants' perceptions in reasoning about conditionals as well as what other factors, such as the design of the app, may have influenced this change. The meta-inferences from these two strands, that is, the merging of results, allowed the researcher to answer the Overarching Research Question.

CHAPTER IV

RESULTS

The purpose of this study was to test a theory about the effects of context ordering on reasoning about conditionals. The study included two phases where participants interacted with the Learning Logic App. The app teaches conditionals in four different contexts: intuitive, abstract, symbolic, and counterintuitive. Phase I involved 10 participants who interacted with version 1.0 of the Learning Logic App and provided feedback that allowed the researcher to develop version 2.0 of the app. Phase II involved 154 participants who interacted with version 2.0 of the Learning Logic App. Phase II participants were randomly assigned to one of 23 context orderings (i.e., orderings that the participants experienced the four contexts). Both Phase I and Phase II produced quantitative and qualitative data. In the sections below, Phase I results are presented first, followed by Phase II results. Finally, the mixed methods results are presented at the conclusion of the chapter.

Phase I Results

In Phase I, 10 participants came to a clinical interview room on campus to participate in the study. The participants completed pre- and post-assessments, interacted with the Learning Logic App v. 1.0, and responded to questions about their experience in a semi-structured interview. Phase I addressed Research Questions 1 and 2. Research Question 1 examined how the different context orderings influenced participants' performance on the Conditional Logic Assessment (CLA) and the Learning Logic App.

Research Question 2 addressed how participants perceived the influence of the context orderings on their learning about conditionals. All participants in this phase of the study experienced the app in the intuitive-abstract-symbolic-counterintuitive (IASC) context ordering.

Quantitative Results for Phase I

The quantitative results for Phase I include descriptive statistics from the CLA and a visual analysis of the score logs. These results provide information to answer Research Question 1 addressing how participants' interactions with the Learning Logic App in the IASC order influenced their performance.

Descriptive statistics from the CLA. The CLA contained questions from each of the four conditional contexts: intuitive, abstract, symbolic, and counterintuitive. The researcher computed descriptive statistics to examine participants' performance for each context of questions for both the pre- and post-assessments and for the change between pre- and post-assessments (see Table 3). The researcher did not compute inferential statistics due to the small sample size in this phase of the study.

As shown in Table 3, the participants attained the highest performance scores on the intuitive context questions in both the pre-assessment (85%) and the post-assessment (73%). This aligns with previous research on learning about conditionals (Cosmides & Tooby, 1992). However, this is the only context where the average change from the pre- to post-assessment was negative. In contrast to prior research (e.g., De Neys & Van Gelder, 2009), participants attained the second highest performance scores on the counterintuitive context questions, which was expected to be the most difficult context.

Table 3

Descriptive Statistics from the Conditional Logic Assessment for the Pre-Assessment, Post-Assessment, and Change Scores for Phase I

	Prescore		Postscore		Change Score	
Context	*M*	*SD*	*M*	*SD*	*M*	*SD*
Intuitive context	.85	.16	.73	.25	-.12	.24
Abstract context	.50	.10	.58	.12	.08	.16
Symbolic context	.35	.17	.55	.16	.20	.23
Counterintuitive context	.52	.30	.70	.24	.18	.24
All contexts	.60	.13	.66	.24	.06	.09

N = 10.

Participants made the greatest gains, on average, on the counterintuitive and the symbolic contexts (20% and 18%, respectively). The symbolic context was the most difficult for the participants (Pre: 35%, Post: 55%). Overall, all participants except one improved (*n* = 6) or maintained (*n* = 3) their score from the pre- to post-assessment.

The relatively high pre-assessment scores on the intuitive context questions left little room for improvement on the post-assessment. On average there was a decrease from pre- to post-assessment for this context. This may be explained as a regression to the mean or could possibly be due to assessment fatigue as the intuitive context questions were presented last in the CLA.

Visual analysis of the score logs. The score logs for the Learning Logic App provided documentation of how each player's score changed as they played the app. A C# script within the Learning Logic App automatically stored the score logs in a Google Sheet. The researcher plotted these data, with time along the horizontal axis and score along the vertical axis, to allow for a visual analysis (Dickinson, 2010) of each participant's interactions with the Learning Logic App.

At the beginning of every level, participants began with a score of 100. Each time the participant sorted a character, or let a character fall to the floor at the bottom of the screen, the score changed. After 100 seconds, the player transitioned to the next level. Figure 7 shows examples of four participants' interactions with the app through all eight levels. These four examples show the typical range of participant responses. All participants in Phase I interacted with the Learning Logic App in the IASC ordering (intuitive-abstract-symbolic-counterintuitive), meaning that participants first completed

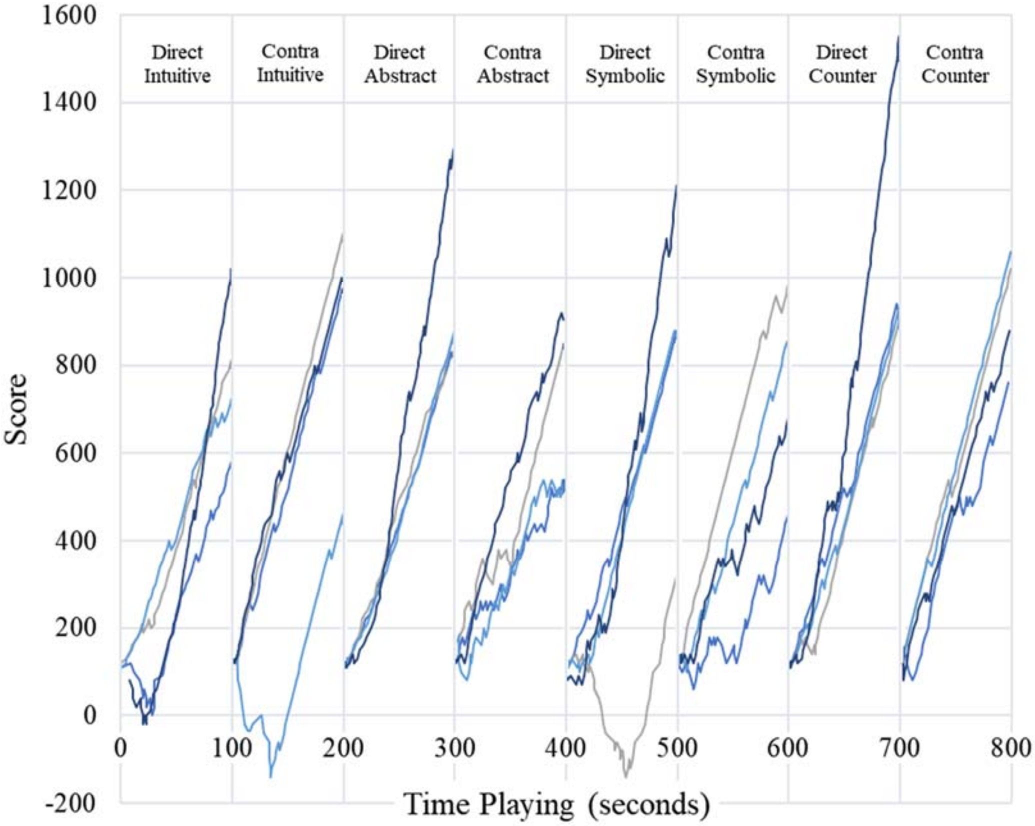


Figure 7. Representative score logs of four participants in Phase I. Each 100 second section represents a different level of the Learning Logic App.

the intuitive direct reasoning level and then the corresponding intuitive contrapositive reasoning level, with the same logical structure order for the other three contexts.

Overall, participants had increasing scores with a few mistakes at the beginning of each level and fewer mistakes as the level progressed. Participants generally performed better on the direct reasoning levels than the contrapositive reasoning levels.

The score logs for these participants indicated their understanding by the end of each level. This can be seen in the lines on the graph in Figure 7, which show increasing scores from the beginning to the end of each level. Of note in Figure 7 are the two outliers in the contrapositive intuitive level (100-200 seconds in light blue) and the direct symbolic level (400-500 seconds in grey) shown on the graph as negative scores. The shape of these graphs illustrates where a participant had initial difficulty understanding the reasoning structure or context within the level. The outlier in the contrapositive intuitive level resulted from the participant not initially understanding the contrapositive structure as exhibited by the participant’s comments while playing. The outlier in the symbolic direct level was due to the participant initially having difficulty understanding how the numbers (the characters) related to the equations within the rules, also exhibited through the participant’s comments while playing. In both cases, the score logs show that the participants understood the logical structure or context by the end of the level.

Qualitative Results for Phase I

In Phase I, two data sources provided qualitative results: the video recordings of participants’ interactions with the Learning Logic App and participants’ responses to the semi-structured interview questions. These data sources helped to answer Research

Question 2, which addressed the participants' perceptions about how the context order influenced their performance on reasoning about conditionals.

Video recordings. The process coding of the video recordings of the participants' interactions with the Learning Logic App revealed that all participants successfully interacted with all levels of the app. Every participant was "easily sorting" characters by the end of each level. Participants spent the most time understanding the symbolic levels as indicated by the process codes "sorting slower" and "taking a longer time to read rules." All process codes and their frequencies are included in Appendix E. Participants performed better and appeared more comfortable with the direct reasoning levels as compared with the contrapositive reasoning levels. Some participants had difficulties with the technical precision required to sort into the zones, coded as "difficulty sorting into intended zone." The researcher addressed the difficulty with technical precision in the revision of version 1.0 of the app.

In the contrapositive levels, the researcher observed that the first two participants implemented a strategy of alternating between two zones for sorting characters. This was possible and a good gaming strategy due to the fact that characters in the contrapositive levels can be sorted to multiple zones. However, this enabled the players to reduce the intended cognitive load for the logical structure. The researcher added a "grey-out" feature to the app for the remaining eight participants to maintain the cognitive load. The change to this feature is described in detail in the app revisions section.

Semi-structured interviews. Participants responded positively to the order of the contexts and the Learning Logic App. The semistructured interviews centered around the participants' perceptions about how their understanding of conditionals in different contexts changed after interacting with the Learning Logic App and their perceptions of the Learning Logic App itself.

Participants' perceptions of different contexts and ordering. All ten participants interacted with the Learning Logic App in the same order: intuitive-abstract-symbolic-counterintuitive (IASC). The researcher asked the participants their perceptions of how the context order influenced their understanding. Seven of the ten participants reported positive feelings towards the order of the contexts. Participants felt that the contexts began simply and became more complex. One participant stated:

> I feel like it was good to have the animals [intuitive context] in the first one, I thought that was really good because it is, you start to understand like what you're trying to make those connections. And then the shapes [abstract context], same thing. It was a little more difficult because you had to think about it a little harder. And then the numbers [symbolic context] was really hard. And then doing it totally opposite [contrapositive structure] was even a step higher. So I thought the order was really good.

While nearly all the participants indicated that they liked the order of the contexts, several felt that the most difficult levels were the two symbolic context levels (direct and contrapositive) rather than the counterintuitive context levels. When asked if she would change the order of any of the contexts, one participant said, "I think if they did switch, it would be the last two," meaning the symbolic and the counterintuitive contexts.

The order of the two logical structures for each context, direct and contrapositive in succession, appeared to aid participants in processing the reasoning for each context as a whole. The contrapositive levels were the most challenging for all participants. When asked her favorite part of the Learning Logic App, one participant replied, "Doing the opposite of the rules [contrapositive levels], because it took me a second. But once I got it, it was like really rewarding. You were like yeah!" Another participant noted: "Those contrapositives are hard! That was kind of fun actually. Oh I like those direct ones. I'm good at those, actually I enjoyed that a lot." None of the participants suggested reorganizing the app based on logical structure (i.e., completing all four contexts with the direct structure and then completing all four contexts with the contrapositive structure).

Two participants specifically mentioned that they felt the Learning Logic App positively influenced their answers on the post-assessment. One participant stated, "It made some of those questions a lot easier. It was weird, I don't know why. I was like, oh, this is weird. But I think I know it." Another participant said,

> The app did help me like, I dunno, like think through that reasoning I guess…. On the first pre-assessment whatever, I didn't put any 'nothings'. But then on the second one, I did put 'nothings.' So, I think that it totally changed my answers.

Overall, the participants perceived the context ordering IASC as beneficial to their understanding of conditionals in different contexts. This was exhibited through the codes of "helped me learn" and "encouraged thinking."

Participants' perceptions of the Learning Logic App. All participants reacted positively towards their interactions with and the design of the Learning Logic App. Eight of the ten participants rated their enjoyment of the app (on a scale of 1 to 10) with an 8 or higher. Two participants rated the app as a five. When asked how well the Learning Logic App kept their attention (on a scale of 1 to 10), all participants responded 9 or 10.

Participants found the sounds which indicated correct or incorrect sorting of the characters helpful. One participant said, “I did actually notice the sounds of like, oh you missed one, or oh, you got it, yay. They were good to identify and distinguish between.” These feedback sounds acted as a motivational affordance, encouraging this participant to stay engaged. The participants also reported that the background music was either fun or not even noticeable. There were no negative responses to the sounds within the app.

Participants suggested improvements to several aspects of the Learning Logic. These aspects included length of the levels, sensitivity of the sorting zones, and the theme of the app. The first participant played the app with 180 seconds per level. The researcher and the participant both noted that the 180 seconds per level did allow for her to gain understanding of each context and logical structure. However, the 180 seconds was so long that the participant had memorized where each character went and no longer actively engaging with (or was interested in) the logical structure. For the subsequent nine participants, the researcher reduced the length of the levels to 100 seconds per level. The remaining participants noted that the shortened length of the levels felt appropriate. One participant stated, “I think it was good. Good enough to, like, get you going, and so you could like figure out the pattern.” Another participant said: “Oh, it was perfect. It went by fast. It didn't feel like it was too long.”

All participants made at least one incorrect character sort due to the sensitivity of the zones and the technical precision required to be within the sensitivity tolerance. In other words, the participants moved the characters too close to the zone above or below their intended zone, which the app then registered as an incorrect interaction. One participant explained, “Sometimes you'd go to a category, and it would accidentally read it as a separate one, and you'd get it wrong. You just had to make sure you were getting, like, directly on that." The researcher adjusted this sensitivity in version 2.0 of the app.

Several of the participants noted the theme felt "kiddish," primarily due to the characters used in the app (e.g., cartoon animals and numbers with eyes). However, this did not appear to be a deterrent from the enjoyment of the app. In contrast, some participants appreciated the "cute animations." The researcher used the feedback provided by the participants to make specific refinements to the Learning Logic App to create version 2.0 for Phase II of the study.

Revisions to the Learning Logic App

The semi-structured interviews and the video recordings provided information for the refinement of the Learning Logic App version 1.0 to create version 2.0. The researcher made refinements to the user experience and the back-end development which changed some of the app's affordances. All app refinements are listed in Appendix F.

The refinements made to the user experience included: reduction of seconds per level, various aesthetic changes (e.g., including the level name at the bottom of the screen), and the addition of a grey-out feature. The researcher reduced the time from 180 seconds to 100 seconds per level, based on the participants' feedback about the length of the levels and from the researcher's observation that participants' strategy appeared to stabilize within the 100 seconds. The researcher also made small aesthetic changes, such as the adjustment of font size, and larger changes, such as the clarification of the contrapositive concept in the tutorial.

One important intended affordance of the app was the high cognitive demand of different levels. To preserve the intended difficulty of the contrapositive structure, the researcher added a grey-out feature to prevent players from using the same zone twice in a row. In the contrapositive structure, the “not” characters can be sorted into all but one zone (their corresponding “not” zone). Consequently, before the addition of the grey-out feature, players could reduce the difficulty of the contrapositive structure by alternating between two of the zones. The grey-out feature deactivated the most recently used zone so that a player had to choose one of the other possible zones. To indicate this visually, the zone image changed to greyscale to remind the player. This feature made it so the player could not memorize a single association for each character to replace the intended contrapositive reasoning, which requires the player to remember several associations for one character.

The refinements made to the backend development included: adjustment of zone sensitivity, re-organization of score log format, and adjustment to the adaptive speed. The adjustment of the zone sensitivity was made by reducing the size of the colliders for the characters and the zones, giving the player more room to maneuver characters into the intended zone. Reorganization of the score log data allowed for easier parsing of the data, which was a needed feature for the large number of participants that would be part of the study in Phase II. The researcher also adjusted the adaptive speed in consideration of the participants in Phase II who would be using laptop or desktop computers to participate in the study. Laptops and desktops make the dragging of characters slower than with the touchscreen tablet technology.

The refinements described above and in Appendix F comprise the changes made to the Learning Logic App to create version 2.0. These refinements improved understandability, reduced technological distance (Tucker, 2016), and reduced potential participant fatigue.

Phase II Results

In Phase II, students participated in the study completely online. A total of 157 participants completed the pre-assessment, played the Learning Logic App v. 2.0, completed the post-assessment, and filled out the computer survey. One participant attempted to sign up for the study but was not over the age of 18 years and could not participate. In this phase of the study, participants interacted with the Learning Logic App in a randomly assigned context ordering because the purpose of this phase was to examine the influence of context orderings on participants' conditional reasoning. Phase II addressed Research Question 1 about participant performance by using quantitative results, Research Question 2 about participants' perception using qualitative results, and the Overarching Research Question through the mixed methods results.

To accurately answer the research questions in this study, it was necessary for participants to complete the pre-assessment, post-assessment, and the computer survey and to fully interact with the Learning Logic App. Based on these criteria, the researcher excluded three participants from the data analysis for not interacting with the Learning Logic App for all the eight levels. The researcher made this decision by examining the Learning Logic App score logs and discovering levels where three participants did not attempt to sort any characters. Therefore, the data analyses included a total of 154 participants for this phase of the study.

Quantitative Results for Phase II

The quantitative results for this phase of the study include descriptive statistics of the CLA scores, a multiple regression of the CLA scores, and a visual analysis of the score logs. The quantitative results address Research Question 1 examining the influence of the randomly assigned context orderings on participants' performance on the CLA and the Learning Logic App.

Descriptive statistics from the CLA. Initially, we visually examined the data to get a preliminary sense of any trends. Figure 8 shows a summary dot plot of the average change from pre-assessment to post-assessment for each of the 23 context orderings. The horizontal axis of this graph demarks the 23 context orderings used in Phase II. The context orderings are arranged from lowest mean change to highest mean change from pre- to post-assessment. The error bars on the graph represent one standard deviation.

For all participants, there was an average change from pre- to post-assessment of 0.92 (indicated by the dashed line in Figure 8). Notably, only three context orderings had an average change greater than two (SIAC: *M* = 3.6, ASCI: *M* = 3.0, CIAS: *M* = 2.14). Four context orderings had a negative average change (see red squares in Figure 8; CSAI: *M* = -1.14, ICSA: *M* = -0.71, ACSI: *M* = -0.57, SICA: *M* = -0.33). When examining the error bars of the plot, only four context orderings had a positive change when considering one standard deviation less than the mean (see green circles in Figure 8). All other context orderings (indicated by blue triangles and red squares) drop below zero when considering one standard deviation below the mean.

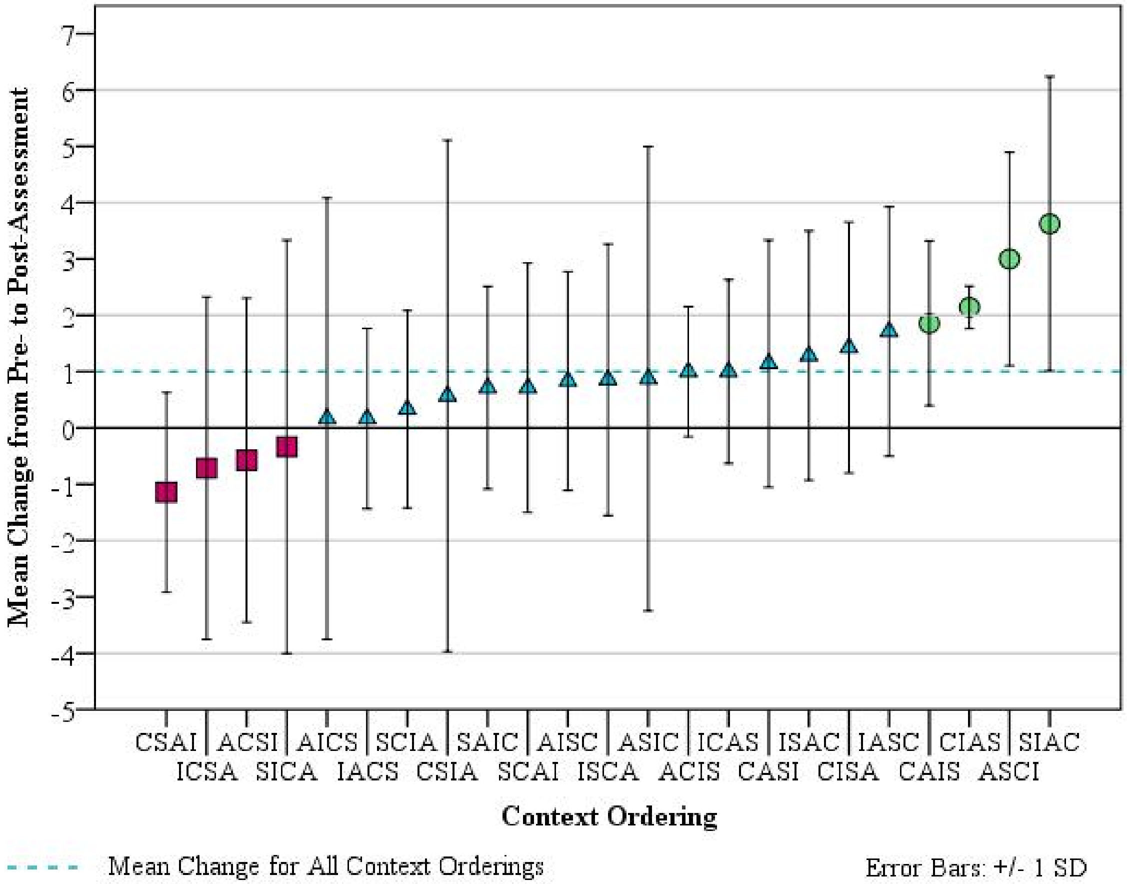


Figure 8. Summary dot plot of the change from pre- to post-assessment organized by mean change for all context orderings with one standard deviation error bars. The blue dotted line indicates the mean change for all context orderings. The red squares indicate context orderings which showed a decrease on average, the blue triangles indicate context orderings which showed a decrease on average, with one standard deviation dipping below zero; and the green circles indicate context orderings which showed an increase on average with one standard deviation still above 0.

The mean and standard deviation for the pre-assessment scores, post-assessment scores, and change between the pre-assessment and post-assessment for each context ordering are provided in Table 4. The contexts are ordered from the lowest to the highest average change. As can be seen in Table 4, the average pre-assessment score ranged between 17.17 and 22.86, the average post-assessment score ranged between 17.33 and 24.14, and the average change score ranged between -1.14 and 3.63.

Table 4

Descriptive Statistics from the Conditional Logic Assessment for the Pre-Assessment, Post-Assessment, and Change Scores for Phase II

	Prescore		Postscore		Change Score	
Context ordering	*M*	*SD*	*M*	*SD*	*M*	*SD*
CSAI	22.42	4.50	21.29	3.88	-1.14	1.64
ICSA	20.71	2.61	20.00	3.89	-0.71	2.81
ACSI	19.57	1.59	19.00	1.50	-0.57	1.09
SICA	20.67	1.89	20.33	2.42	-0.33	1.50
AICS	20.17	2.14	20.33	1.87	0.17	1.60
IACS	17.17	2.27	17.33	2.61	0.17	0.65
SCIA	19.50	1.20	19.83	1.38	0.33	0.72
CSIA	19.71	2.17	20.29	1.90	0.57	1.72
SAIC	20.14	2.05	20.86	1.82	0.71	0.68
SCAI	20.00	1.36	20.71	0.81	0.71	0.84
AISC	21.67	2.06	22.50	2.26	0.83	0.79
ISCA	18.14	1.93	19.00	1.60	0.86	0.91
ASIC	19.13	2.07	20.00	1.76	0.88	1.46
ACIS	16.86	2.24	17.86	2.04	1.00	0.44
ICAS	19.00	3.70	20.00	3.34	1.00	1.51
CASI	19.14	1.68	20.19	1.64	1.14	0.83
ISAC	22.86	2.04	24.14	2.13	1.29	0.84
CISA	17.86	2.56	19.29	2.27	1.43	0.84
IASC	17.43	1.69	19.14	1.28	1.71	0.84
CAIS	20.43	1.73	22.29	1.87	1.86	0.55
CIAS	17.57	1.13	19.71	1.21	2.14	0.14
ASCI	18.33	1.36	21.33	1.17	3.00	0.78
SIAC	19.38	1.92	23.00	1.49	3.63	0.93
All orderings	19.48	4.77	20.47	4.53	0.99	2.52

N = 154.

As proposed in Chapter III, the context ordering variable could be considered a continuous interval variable, if metricized with the Spearman's rho value, or a discrete categorical variable. To determine if the context ordering should be treated as an interval or a categorical variable, the researcher plotted the change scores against each context orderings' Spearman's rho and computed a Pearson's correlation coefficient. Figure 9 shows the plot of the change scores and the context orderings as measured by the Spearman's rho. No significant correlation existed between the change score and the spearman's rho value for the context orderings (*r* = 0.09, *N* = 154, *p* = 0.27). This means that it is very unlikely that there was a relationship between participants' change in performance and the Spearman's rho categorization of the context orderings. Thus, the researcher treated the context ordering variable as a discrete categorical variable for the multiple regression analysis.

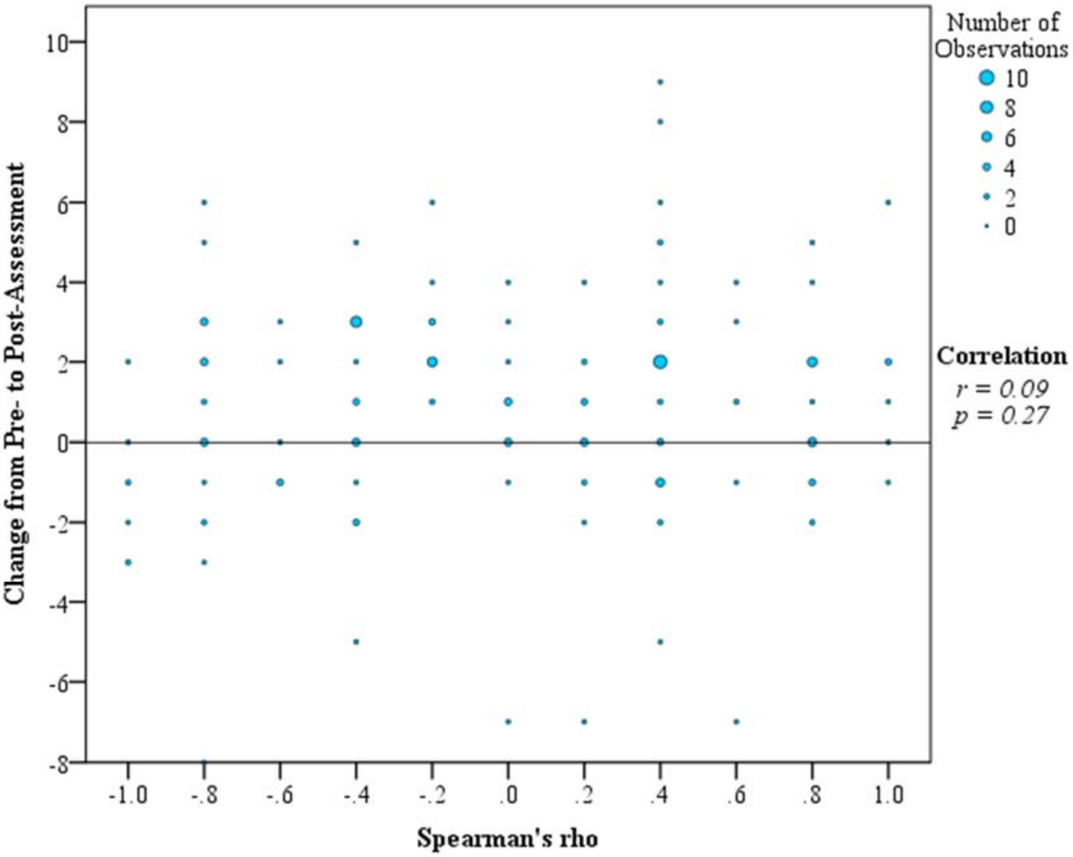


Figure 9. Scatter plot of the change scores (vertical axis) and the Spearman's rho for the context orderings.

Multiple regression of the CLA and context ordering. Multiple regression was used to model participants' CLA post-assessment scores based on their CLA pre-assessment scores and their randomly assigned context orderings (dummy-coded). In the dummy-coding of the context orderings, the CSAI ordering was designated as the reference category due to it being hypothesized as the least beneficial context ordering. This context ordering and pre-assessment score combine to account for 73.7% of the variance of the post-assessment score, $R^2_{\text{adj}} = .737$, $F(23, 130) = 19.68$, $p < .001$. To ensure that no severe violations to the multiple regression assumptions occurred, the researcher examined the residual diagnostic plots (see Appendix G). There was no evidence of violation of the assumptions of normality of variance, homogeneity of variance, or outliers.

Two context orderings and the pre-assessment score displayed significant effects on the post-assessment score (pre-assessment: $b = 0.81$, $\beta = 0.86$, $p < .001$; SIAC ordering: $b = 3.81$, $\beta = 0.19$, $p = .003$; ASCI ordering: $b = 2.99$, $\beta = 0.13$, $p = .029$). No other context orderings resulted in statistically significantly different effects from the reference context ordering of CSAI. Regression coefficients and p values for all ordering conditions from the model are provided in a table in Appendix H. This indicates that each additional question answered correctly on the pre-assessment (an increase of one unit in the pre-assessment score) is associated with an increase of 0.81 units on the post-assessment. Because prior knowledge should not have been hindered by the Learning Logic App interaction, the researcher anticipated this positive influence of the pre-assessment on the post-assessment.

Holding the pre-assessment scores constant, the SIAC ordering was associated with a 3.81-point increase in a participant's post-assessment score compared to the CSAI ordering. That is, it is expected that participants in the SIAC condition will correctly answer three to four more questions on the post-assessment than those participants in the CSAI ordering. In the ASCI ordering, the model predicts participants will correctly answer three more questions on the post-assessment (*b* = 2.99) than participants in the CSAI ordering. When examining both of these results, in terms of the standardized coefficients (SIAC: $\beta = 0.19$, ASCI: $\beta = 0.13$), the effects are considered medium and small, respectively (J. Cohen, 1988). It is important to note here that the number participants in each context ordering may have limited the power of the regression analysis.

The results of the multiple regression indicate that participants' prior knowledge (as measured by the pre-assessment) strongly influenced the post-assessment scores. In addition, the SIAC and ASCI orderings were the most positively influential on participants' post-assessment scores.

Visual analysis of the score logs. As in Phase I, the score logs recorded changes in each participant's score while playing all levels of the Learning Logic App. In Phase II, the level order of the Learning Logic App depended upon the participants' random assignment to one of 23 context orderings. Each score log plots the participants' score against time. Every level began with a score of 100 points and lasted for 100 seconds. Figure 10 presents four representative score logs which illustrate the four trends found across all the context orderings.

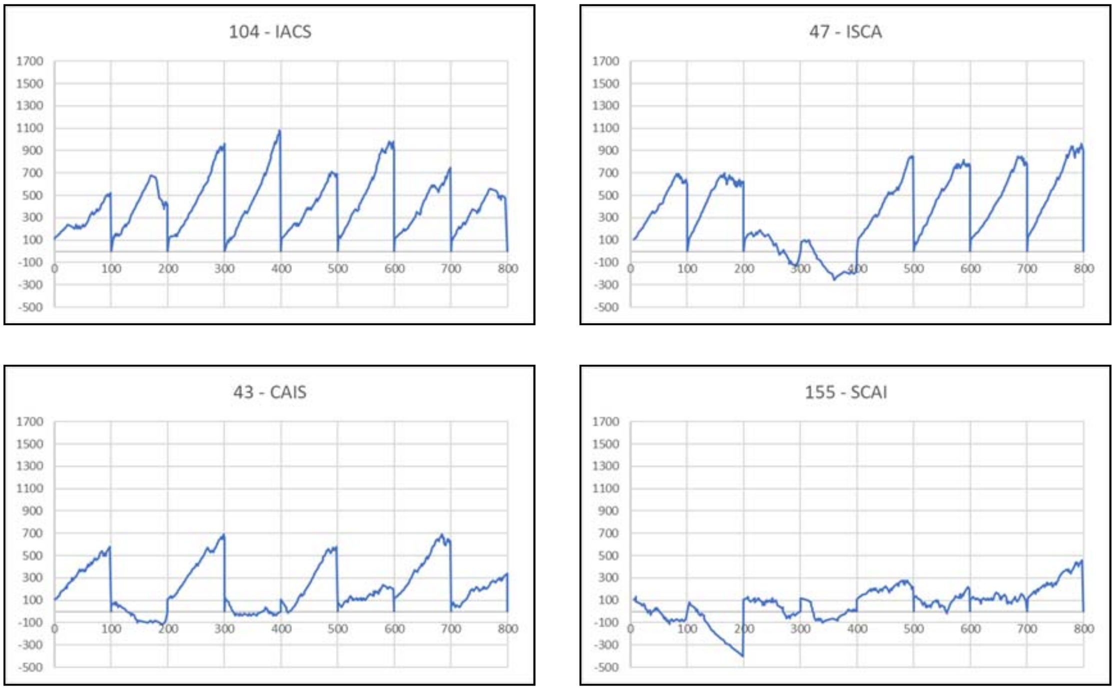


Figure 10. Representative score logs of four participants in Phase II. Each 100 second section represents a different level of the Learning Logic App. The context ordering is given at the top of each graph.

The top left score log of Figure 10 depicts a participant who experienced very little struggle while interacting with the app. A large portion of participants (39%) had similar experiences while interacting with the app. These participants exhibited their understanding by ending each level with high scores, regardless of context or task. The top right score log of Figure 10 depicts a participant who had considerable difficulty with the symbolic levels (levels 3 and 4) as compared with levels in the other contexts. This is the first time the participant would have encountered the most challenging symbolic context. This pattern appeared in 26% of participants. Interestingly, this pattern is unique to the symbolic context. The bottom left score log of Figure 10 depicts a participant who performed similarly across all contexts, but struggled with the contrapositive reasoning (levels 2, 4, 6, and 8). Other participants who exhibited similar patterns (12%) struggled with the contrapositive reasoning in one, two, or three of the levels while still performing well on the direct reasoning. Finally, the bottom right score log of Figure 10 depicts a participant who experienced a large amount of struggle throughout all eight levels with a little bit of improvement towards the final two levels (intuitive context). Few participants exhibited this level of struggle for all levels (7%). These participants struggled on all levels, regardless of context or task. The remaining 16% of participants exhibited patterns which included multiple trends described above (e.g., difficulties on the symbolic context and the contrapositive reasoning) or exhibited no discernable pattern at all.

When examining the score logs by the context ordering, the researcher did not find any themes for specific context orderings. This is likely because the score logs revealed areas where participants had noteworthy struggles, but they did not provide information about how the participants' understanding had changed after completing the entire sequence. Each context ordering had a mix of participants who struggled a lot, those who struggled a little, those who had difficulty with the symbolic context and/or contrapositive reasoning, and those who did not. In nearly all cases, participants were able to improve by the time they reached the eighth level.

Qualitative Results for Phase II

Phase II participants provided qualitative feedback through a computer survey which they completed after the post-assessment. The computer survey contained similar questions to those in the semi-structured interview from Phase I. The questions in the survey asked participants' perceptions of the different contexts and the order in which they experienced them as well as perceptions of the Learning Logic App. The results are discussed below in accordance with these two topics.

Perceptions of different context orderings and contexts. Participants provided their perceptions about four aspects of the different contexts and ordering on the computer survey: opinion of the order they experienced the contexts, a rating of how the order helped or hindered their learning, a ranking of the contexts from easiest to hardest, and how their experience with the app influenced their performance on the post-assessment. Trends within the responses are discussed separately below, first for the entire sample, and then for the separate context orderings.

Participants were asked their impressions of the order of the four contexts as they experienced them in the Learning Logic App. Approximately two-thirds of the participants felt that the order of the levels they experienced was good (50%) or okay (15%), while 13% of participants felt the order should be changed. Interestingly, the only context ordering where all participants thought the order was "good" and most felt the order was "progressive" was the IACS ordering (intuitive-abstract-counterintuitive-symbolic, this ordering was very similar to the proposed ordering IASC, $r_s = 0.8$). One participant in the IACS ordering said, "I thought they were presented in an order that was effective and that built upon each other." Another IACS participant said, "It got tougher and tougher each level, which was perfect." In contrast, most participants in the SCIA and the CSIA context orderings felt that the order of contexts "didn't flow as well as it could have" or that "they were in a weird level [order] that got a little easier every time." These were context orderings where the two most challenging contexts, symbolic and counterintuitive, came first and second (r_s = -0.6 and r_s = -0.8, respectively). A participant in the ASIC ordering (which is somewhat similar to the proposed ordering; r_s = 0.4) suggested the order be changed:

> I think intuitive would have been nice to have first because it's something, obviously, we should be familiar with while I have a hard time recalling rules for colors and shapes at such a high pace and it left me a little frustrated.

In addition to their written responses, participants rated how the context ordering impacted their learning (0 = very hindering, 5 = not helpful or hindering, 10 = very helpful). Across all context orderings, the participants rated the ordering of the levels as 6.19, which would be considered a little helpful to their learning on the given scale. Once these results were considered by the context orderings, the results were more distinctive. The IACS ordering participant group rated the ordering of the level the most helpful out of all the context orderings (7.67). This aligns with the IACS participants written responses about the order of the four contexts which indicated that participants felt the context ordering was good and progressive. The average ratings from the CAIS, AICS, CIAS, and ICSA participant groups (a mix of context orderings which ranged from r_s = -0.4 to r_s = 0.6) were not far behind (7.57, 7.4, 7.14, and 7, respectively). Participants in the ICAS, CSAI, and CASI context orderings (r_s = 0.4, r_s = -0.8, and r_s = -1.0, respectively) rated the level ordering as a 5 (neutral to their learning) on average. The only context ordering with an average rating indicating that the participants felt their learning was hindered by the ordering was CSIA (4.86).

Participants also ranked the difficulty of the contexts from easiest to most difficult (1 = easiest to 4 = hardest). Overall, participants consistently rated the symbolic context as the most difficult (average rating of 3.48) and the intuitive context as the easiest (average rating of 1.86). The participants rated the abstract and counterintuitive contexts nearly equally with the abstract context slightly more difficult (average rating of 2.42) followed by the counterintuitive context (average rating of 2.24). When examining these results by context ordering, participants in all context orderings but one (CIAS) indicated that the symbolic context was the most difficult. Participants in two thirds of the context orderings also indicated the intuitive context was easiest. There were mixed results as to the comparative difficulty of the abstract and counterintuitive contexts.

When asked if they felt the Learning Logic App influenced their responses on the post-assessment, 51% of participants responded “yes,” 32% responded “no,” and 13% said “it influenced some questions.” Participants in 14 of the 23 context orderings felt that the app influenced their responses on the post-assessment. In the ASCI ordering (r_s = -0.2), participants had the most positive responses, with all participants indicating that the app influenced their responses on the post-assessment. A participant in the IACS ordering (in which most participants said yes) responded, “Yes! I noticed I understood ALL the questions in full and changed a couple answers.” In contrast, participants in five of the context orderings (AISC, CIAS, ICAS, ICSA, and SAIC) had mixed responses about the influence of the app on their post-assessment, with responses such as “a little bit,” “maybe,” or “not really.” In the remaining three context orderings (CISA, CSIA, and ISAC), participants expressed that the app did not influence their post-assessment performance. One participant indicated that she “tried to implement it, but I think my answers were the same as before.”

Across these four aspects pertaining to participants' perceptions of the context orderings and contexts, there were three major themes. First, the participants positively perceived the IACS ordering. Second, the participants consistently perceived the symbolic context as the most challenging. Third, participants consistently perceived the CSIA ordering as hindering to their learning or not influencing their responses on the post-assessment. Interestingly, the IACS ordering is very similar to the proposed context ordering, IASC ($r_s = 0.8$). In contrast, the poorly perceived CSIA ordering is quite dissimilar ($r_s = -0.8$). Perceptions of the four context orderings with the highest average change.

The four context orderings with the highest average change from pre- to post-assessment are shown as green circles in Figure 8. Participants in these context orderings had a mix of perceptions. These four context orderings, in order of highest to lowest average, were SIAC, ASCI, CAIS, and CIAS. Participants who experienced the SIAC ordering had positive perceptions of the context ordering and the app's influence on their understanding. One SIAC participant said, "It made me think in a different way, it gave me a challenge." Another SIAC participant stated, "It challenges me to think about what the rule was in that first round [direct reasoning task], but then in the second round [contrapositive reasoning task] to recognize which rule it would follow without the x through it. I just thought it was a fun challenge." The SIAC ordering was the only context ordering of these four context orderings where the participants had consistent perceptions, and those perceptions were positive.

The participants who experienced the ASCI ordering had a range of perceptions with some participants expressing frustration to others who felt like they learned quite a bit. For example, one ASCI participant said, “I didn't like how it messed with logic” and “It was very frustrating.” In contrast, another ASCI participant said, “I was impressed with what I learned just in a short amount of time.” The perceptions of the other ASCI participants were similarly dispersed and did not reveal any consistent theme for the context ordering that was either positive or negative. The other two context orderings, CAIS and CIAS were not significantly different from the CSAI ordering, according to the multiple regression. Similar to the ASCI ordering, participants who experienced the CAIS ordering and the CIAS ordering had a wide range of perceptions of the context orderings and their influence on the participants’ understanding. For the participants who experienced the CAIS ordering, some participants felt the ordering was beneficial while others felt it was just confusing. For example, one CAIS participant said, “logic was very confusing,” while another CAIS participant said, “It was fun, and the pre and posttests helped me understand what I learned about logic.” Other participants in the CAIS ordering appeared to feel indifferent, expressing that “the activity was alright” and “the animation was a little boring.” Similar to the ASCI and CIAS orderings, the participants assigned to the CIAS context ordering had inconsistent perceptions across the group. Some CIAS participants had positive perceptions, but with qualifiers to their statements. For example, one participant stated, “I thought it was interesting, but I would not like to play it again” while another said, “It was good, but a little stressful.” Other participants had neutral to negative perceptions. One CIAS participant said he felt there was “no clear purpose” and that the “app was ok, not super effective.” Although these four context orderings produced the highest performance results, the SIAC ordering was the only context ordering which had a consistent theme among the participants' perceptions.

Perceptions of the Learning Logic App. The computer survey also asked questions about participants' perceptions of the Learning Logic App including: participants' overall opinions of the app, their enjoyment, feature awareness, use of strategies, and perception of the purpose of the app. The researcher examined each set of responses for all participants as well as by the context ordering. Learning Logic App perceptions which were influenced by the context ordering are presented first, followed by perceptions which were consistent across all context orderings.

Learning Logic perceptions influenced by context ordering. Participants' expressed positive overall impressions of the Learning Logic App. The most common theme among the responses related to the app encouraging participants' thinking or learning (21% of participants). One participant stated, "I was impressed with what I learned in just a short amount of time." Other common responses fit the themes of interesting (17%), good (16%), confusing (14%), fun (12%), technical issues (11%), and difficult (10%). Other responses from less than 10% of the participants included words and phrases such as challenging, stressful, long, boring, and too fast. After examining these results by the context orderings, only the IACS ordering participants had consistent positive responses. The IACS ordering participants made comments such as, "It made me think really hard but I thought it was enjoyable" and "I really liked it, and I felt I increased my knowledge about logic from it." In all other context orderings, participants had a mix of feelings ranging from those who had positive feelings to those who felt it was “highly stressful” and that “the game was very long and that it was very difficult.”

Across all context orderings, participants rated the app as 5.54 out of 10. When examining the app ratings as factored by the 23 context orderings, the highest average rating came from participants in the IACS ordering (8 out of 10), followed by participants in the SIAC ordering (6.7 out of 10). In the IACS ordering, one participant explained her reasoning for choosing the rating 10 out of 10 by saying, “I actually had fun and it made me feel as if I made connections in my brain stronger.” Similarly, a participant in the SIAC ordering (who rated the app 7 out of 10) stated, “It was a little bit stressful, and the equations were a bit hard to remember the rules of, but it was kind of fun and helped me understand what the pre-assessment wanted.” The lowest average rating came from participants in two different context orderings: ACIS and ACSI (4 out of 10 each). These are the only two context orderings where the participants first played the abstract levels and then the counterintuitive levels before the intuitive and symbolic levels. Several of these participants reported they felt the app was “repetitive” or “confusing.”

In addition to rating the app overall, participants rated their enjoyment of the app and how well the app kept their attention on a one to ten scale. For both questions, it was again participants in the IACS ordering who responded with the highest ratings (enjoyment = 8, attention = 8.17). Participants in the SIAC and SICA also highly rated enjoyment and attention respectively (SIAC: enjoyment = 7, SICA: attention = 8). When rating enjoyment, the ACIS and ACSI orderings received the lowest ratings (3.5, 3.6). For the attention rating, the CSAI and the AISC orderings were rated the lowest by the participants (5.17, 4.83). These results closely align with the participants’ overall ratings of the Learning Logic App.

Learning Logic perceptions consistent across context orderings. The participant responses which were consistent across all context orderings included favorite and least favorite parts of the app, awareness and opinion of app features, participant perceptions of the topic or idea of the app, and strategies used during gameplay. Participants were asked about their favorite and least favorite parts of the Learning Logic App. Overall, participants most enjoyed the character designs (18%), playing the game (17%), and the challenge provided in the content of the game (11%). One participant said, “I really liked the concept and difficulty of it.” The element of cognitive challenge was an important affordance built into the design of the game. Another participant stated he liked the “difficulty of putting aside known facts.” Less common responses included enjoyment of the “animals” levels (intuitive and counterintuitive), the different contexts, and the learning. Participants’ least favorite parts of the app included the increased speed of the characters appearing as participants’ scores increased (17%), the symbolic levels (15%), the technical precision required to play the app with a touchpad (15%), and one participant (> 1%) particularly disliked the armadillo character.

Participants were asked about their awareness of features in the app. Most participants were aware of the sounds (42%) and the music (26%), but had mixed opinions about these features. Regarding the sounds, a feature included to provide feedback, one participant said, “The sound effects that told me if I touched the spikes, and whether I got it right or wrong, were very helpful.” However, another participant said, “The doink noise [sound of impact with spikes at the bottom] was frustrating.”

Similarly, the participants had mixed responses to the music, a feature intended to afford motivation. One participant stated, “The music helped me focus because it distracted me from other background noises.” While another participant said, “The music also gave me anxiety.” Other features the participants mentioned include the helping feature of time pausing when new rules appeared and the hindering feature of new rules requiring a tap to un-pause time. No other features were mentioned by more than 5% of the participants.

The “grey-out” feature of the Learning Logic App provided an additional dimension of challenge to the contrapositive levels. This feature was intended to afford a focused constraint, so that participants focused on reasoning about the contrapositive structure rather than a gaming strategy. Nearly half (46%) of the participants felt the feature helped them in the game. Some participants felt this feature was helpful to developing their reasoning, while others felt it was helpful to the gaming strategy. For example, one participant commented on the feature’s influence on his reasoning saying: “It provided a greater challenge but it was fun to have to think through it.” Another participant spoke to her change in gaming strategy: “I loved greying out the animals! It became habit to go to the same two choices, but having one of them greyed out forced me to rapidly re-evaluate my options. That was very good.” One quarter of participants (25%) felt that the features provided a challenge. One participant remarked, “It added a little extra difficulty, but exercised the brain well as a result.”

When asked which idea or topic the Learning Logic App was designed to teach, over half of participants (61%) said “logic” or “deductive reasoning.” There were several participants (10%) that said the main idea of the app related to how prior knowledge influenced their reasoning saying things such as “overcoming typical thinking,” “looking past personal bias,” and “that common sense confuses us when we get new directions.” Participants also suggested “adapting” (8%), “sorting” (7%), and “quick thinking” (6%) as the idea or topic within the app. As a follow-up question, participants further elaborated on which aspects of the app helped them learn the idea or topic they described. The most common responses included the gaming medium (14%), the instructions (10%), and the contrapositive levels (6%). One participant eloquently stated, “*In totum*, the game provided sufficient education.”

To gain a deeper understanding of participants’ thought processes during gameplay, participants described the strategies they used while playing the Learning Logic App. Overall, there was an interesting balance between participants who used memorization (24%), created their own rules or associations (15%), or focused on gaming strategies (12%). One participant described a strategy of creating her own rules, “In my head I would start making my own rules. It stopped being because fish have scales and started being fish goes here.” In contrast another participant described a memorization strategy as, “Repeating the sequence of the things on the right [zones]… for the shape one [abstract context], I would repeat ‘5, 3, 4 sides’ and it made it easier to put the objects [characters] into their category.”

Overall, these results indicate that the participants experienced varying depths of knowledge levels while interacting with the Learning Logic App. Additionally, these responses imply that participants attended to and appreciated different features and characteristics of the app, but there was not a clear distinction in their feedback based on the context orderings the participants experienced.

Mixed Methods Results

This section discusses the meta-inferences derived from the quantitative and qualitative results to address the Overarching Research Question on how the order of teaching four conditional contexts influences reasoning about conditionals. These results merge the results on participants' performance and their perceptions. The section is organized by convergent results (i.e., results that appeared in both the quantitative and qualitative results) and divergent results (i.e., results that appeared in only the quantitative or the qualitative results). Figure 11 depicts the data sources used to determine the participants' performance and perceptions.

Convergent results. There are three meta-inferences derived from the quantitative and qualitative results. The first meta-inference relates to the positive impact of the SIAC ordering on learners' performance on and perceptions of reasoning about conditionals. Participants in the SIAC ordering had significant improvements on the CLA. Additionally, SIAC ordering participants highly rated the Learning Logic App overall and their enjoyment of the app. These strong indicators from both the quantitative

Performance results	*Perception results*
Pre- and post-assessment scores on the CLA	Computer survey questions about the context ordering and contexts
Score logs of the Learning Logic App	Computer survey questions about the Learning Logic App

Figure 11. Data sources contributing to the participants' performance and perception results.

and qualitative results seem to suggest a positive benefit of the SIAC ordering for teaching reasoning about conditionals.

The second meta-inference concerns the negative influence of the CSAI ordering on participants' performance and perceptions. Overall, CSAI participants performed the worst, with an average negative change from pre- to post-assessment. CSAI participants' perceptions of the Learning Logic App indicated that the ordering was the worst at engaging the participants as compared to all other context orderings. These parallel results point to a potential inadequacy in the CSAI ordering for teaching reasoning about conditionals.

The final meta-inference regards the challenges participants encountered with the symbolic context. The score logs of the Learning Logic App clearly show participants struggling with the symbolic context levels when they did not struggle with the levels for the other three contexts. Participants' awareness of their difficulties with the symbolic context were exhibited through their consistent ratings of the symbolic level as the most difficult. Additionally, participants' dislike of the symbolic context appeared as a theme in response to the questions about the "least favorite parts of the Learning Logic App."

Divergent result. There were two notable results that diverged in the quantitative and qualitative results. The first divergence reflects differing outcomes in the qualitative and quantitative results for the IACS ordering. For this context ordering, a disparity emerged between participants' strong praises for the IACS ordering and the mediocre to poor performance of the participants on the post-assessment. Participants in the IACS ordering had the most positive perceptions of their context ordering, the app overall, their enjoyment of the app, and how well the app kept their attention when compared with all other groups. However, when examining the average change in performance for IACS participants, the results indicated nearly a zero change. Thus, while participants liked the IACS ordering, it did not appear to benefit their reasoning about conditionals.

The second divergent result pertains to the influence of the ASCI ordering on the participants' performance and perceptions. While participants assigned to the ASCI ordering had significant improvements from pre- to post-assessment, the participants' perceptions were varied. Participants in this context ordering had the highest rating of the app influencing their performance on the pre-assessment. However, these same participants had a variety of responses (both positive and negative) on all other questions related to understanding, specifically the question pertaining to participants' perceptions of the ordering. Consequently, participants' performance was not aligned with their self-reported perceptions.

Conclusion

The meta-inferences derived from merging the quantitative and qualitative results suggest that the SIAC ordering may be the most beneficial context ordering for learners wanting to improve their reasoning about conditionals. In contrast, the results suggest that the CSAI ordering may be the least beneficial. Additionally, the qualitative results indicated that participants preferred the IACS ordering, but the quantitative results indicated that the IACS ordering provided very little performance benefits. Finally, the qualitative results for the ASCI ordering were not consistent enough to provide support for the associated quantitative results. The results also show that learners may need more assistance or practice with the symbolic context than the other three contexts when learning about conditionals. The results are especially important when considering the order in which the contexts should be taught, the amount of instructional time that should be dedicated to each context, and the potential transfer of conditional reasoning in all four contexts to other areas both within and outside of mathematics.

CHAPTER V

DISCUSSION

The purpose of this study was to test a theory about the effects of context ordering on reasoning about conditionals. To test this theory, the researcher developed, tested, and revised a virtual manipulative educational mathematics application, called the Learning Logic App, using four main contexts: intuitive, abstract, symbolic, and counterintuitive. Conditionals are increasingly prevalent in education thanks to the rise of computer programming in schools, STEM interdisciplinary initiatives, and the promotion understanding rather than memorization in mathematics classrooms. Additionally, everyday citizens encounter conditionals when voting on ballot measures, reading scientific news articles, and interpreting tax law. Reasoning about these conditionals, however, is a challenging task for nearly all individuals. This task is made more complex depending on the context (such as, a familiar or symbolic) of the conditional (Zandieh et al., 2014) and the underlying logical structures. Consequently, there is a need for effective strategies for teaching and learning of reasoning about conditionals with different contexts and logical structures.

The overarching research question guiding this study was: “How does the order of teaching four conditional contexts influence reasoning about conditionals?” Two subquestions examined the influence of context order on participants’ performance and perceptions. This discussion of the results has four sections. The first section discusses the four different contexts of the conditionals. The second section discusses the influence of context order on learners’ performance and learners’ perceptions. The third section discusses some recommendations for researchers and educators. The fourth and fifth sections discuss the limitations of the study and suggestions for future research.

Four Conditional Contexts

Each of the four contexts (intuitive, abstract, symbolic, and counterintuitive) in this study involved the same underlying logical structures. However, learners' performance and perceptions differed greatly across the four contexts. The results showed that the symbolic context was the most difficult for learners. Learners repeatedly struggled with the symbolic context in the pre-assessment and in the post-assessment. Additionally, the score logs of the Learning Logic App revealed a trend of struggle for many participants when they played the two symbolic context levels, regardless of if they encountered these levels at the beginning, middle, or end of the game. Through the interviews and computer surveys, participants also repeatedly expressed that they perceived the symbolic context as the most difficult of all four contexts.

The symbolic context's comparative difficulty with the intuitive and abstract contexts aligns with previous research studies comparing learners' performance in these contexts (Case, 2013; Christoforides et al., 2016; Stylianides et al., 2004; Thompson, 2000). However, based on the literature, the researcher did not expect the symbolic context to be more difficult for learners than the counterintuitive context. There are several possible explanations for this result.

First it is possible, as suggested by De Neys and Van Gelder's (2009) study of the effects of belief inhibition across individuals' lifespan, that the undergraduate participants in this study are at the stage of life when belief inhibition (the skill necessary for reasoning in the counterintuitive context) comes most naturally. This means that participants whose age differs significantly from the sample in this study may find the counterintuitive context more difficult than the symbolic due to their increased difficulties with belief inhibition.

Alternatively, it is possible that the participants had very influential prior notions about the symbols used in the app and in the assessments, which conflicted with how the symbols were used in the symbolic context questions of the Conditional Logic Assessment and the symbolic context levels of the Learning Logic App. This is similar to the results of Mutodi's (2016) study, which found that "learners stick to procedurally driven symbols at the expense of conceptual and contextual understanding" (p. xi). Consequently, participants would need to mentally maneuver through the intended intricacies of the symbolic context as well as employ their skills of belief inhibition (which are required by the counterintuitive context) within one context. If so, it would be expected that the symbolic context (which would contain aspects of both the symbolic and counterintuitive contexts) would be more difficult than the counterintuitive context alone.

Based on this result, the researcher recommends that more instructional time be allocated for the symbolic context as learners will most likely experience the most struggle with conditionals posed in the symbolic context. Without this extra instructional emphasis, these struggles may influence learners' ability to transfer their knowledge to symbol-laden statements of mathematics theorems, such as the Mean Value Theorem in Calculus or the Rational Zero Theorem in College Algebra.

Influence of Context Order on Learners' Performance and Perceptions

The main focus of this study was the influence of context orderings on learners' reasoning about conditionals. To examine this, participants were randomly assigned to a context ordering to determine its effects on their performance and perceptions. The sections below discuss the context orderings that were most influential for the participants, both positively and negatively.

Symbolic-Intuitive-Abstract-Counterintuitive Ordering

The results of the quantitative analysis indicated that the symbolic-intuitive-abstract-counterintuitive (SIAC) ordering positively influenced learners' reasoning about conditionals. Participants in the SIAC ordering had significant improvements from pre- to post-assessment. Additionally, participants in this context ordering highly rated their experience with the Learning Logic App, which indicated positive perceptions of the context ordering. These results mean that, for participants in this study, the SIAC ordering was most beneficial to the learners, when measured by learners' performance and perceptions. While this context ordering does not align with the researcher's proposed IASC ordering, it is similar to the IASC ordering with the most difficult context (symbolic) moved to the beginning (see Figure 12).

Because of the participants' difficulty with the symbolic context, it is very possible that the symbolic context acted as a catalyst for the learners prompting them to

Figure 12. Comparison of the hypothetical IASC ordering and the SIAC ordering.

reevaluate their conceptions of the conditionals (see Figure 12). The idea of a catalyst to prompt conceptual re-evaluation stems from G. J. Stylianides and Stylianides' (2009) study where researchers attempted to sway students from purely empirical justifications to deductive reasoning by using a catalyst, called a “monstrous counterexample.” Their monstrous counterexample was an example where an obvious pattern failed but only after so many iterations as to be outside of practical empirical evaluation (e.g., the pattern failed for an extremely large number but worked for all preceding smaller numbers). In the present study, when the participants interacted with the symbolic context levels first, their struggle with the symbolic level may have had a catalyzing effect encouraging them to reevaluate their underlying conceptions about how to reason about conditionals.

Following their interaction with the symbolic levels, the participants would be able to practice their newly formed conceptions of how to reason about conditionals in an order that progressed from easy to difficult (i.e., the intuitive, abstract, and counterintuitive levels). Consequently, the researcher recommends that, to effectively influence a learner's reasoning about conditionals, it may be beneficial to use some form of catalyst, such as the symbolic context used here, at the beginning of an instructional sequence of reasoning about conditionals to prompt the learner to re-analyze any prior (potentially flawed) conceptualizations on which they may be depending.

The results also point to the possibility that the counterintuitive context, or contexts not examined in this study, could be considered as an effective catalyst for learners. While there were no significant performance results for context orderings beginning with the counterintuitive context, future research could examine what contexts could act as the most effective catalyst for learners. Different learners may respond more readily to different catalysts.

Counterintuitive-Symbolic-Abstract-Intuitive Ordering

In contrast to the results for the SIAC ordering, the quantitative analysis indicated that the counterintuitive-symbolic-abstract-intuitive (CSAI) ordering negatively influenced learners' reasoning about conditionals. Participants in this context had an average decrease from the pre- to post-assessment. Participants in the CSAI ordering felt that they had a difficult time keeping their attention focused while playing the Learning Logic App. Additionally, several of these participants expressed annoyance or confusion after interacting with this context ordering. Based on the literature, the researcher had proposed that the context ordering of intuitive-abstract-symbolic-counterintuitive (IASC) would be the most beneficial to participants and similarly that the reverse, CSAI (see Figure 13), would be the least. Consequently, the researcher expected that this context ordering would be the least beneficial and possibly even hindering to learners' reasoning about conditionals.

The decrease from pre- to post-assessment for the CSAI participants could have been caused by their interaction with the Learning Logic App incorrectly influencing their reasoning about conditionals. Or possibly because the participants became frustrated or fatigued with the app and subsequently performed poorly on the post-assessment. It is also possible that beginning with the counterintuitive context, followed immediately by the most difficult symbolic context, did not allow the learners to apply their newly

Figure 13. Comparison of the hypothetical IASC ordering and the CSAI ordering.

disrupted reasoning pattern to either of the more challenging contexts (counterintuitive and symbolic). Or it is possible that the amount of confusion the participants experienced did not allow them to develop any new conceptualizations of reasoning about conditionals at all.

Because this context ordering is the exact reverse of the proposed progression of IASC, using the CSAI ordering conflicts with the underlying theories which prompted the proposal of the IASC ordering. As Clements and Sarama (2004) noted, progressions are designed with “levels of increasing sophistication, complexity, abstraction, power, and generality” (p. 83). The underlying theory of developmental progressions stems from the idea that “a critical mass of ideas from each level must be constructed before thinking…becomes ascendant in the child’s mental actions and behavior” (Sarama, Clements, Barrett, Van Dine, & McDonel, 2011, p. 668). Using the CSAI ordering for teaching reasoning about conditionals presents decreasing levels of sophistication, complexity, abstraction, power, and generality and consequently does not allow for the accumulation of ideas from the less complex levels. Thus, the researcher recommends that the CSAI ordering not be utilized in teaching reasoning about conditionals because it does not follow the developmental progression within this topic.

Intuitive-Abstract-Counterintuitive-Symbolic Ordering

In contrast to both the SIAC and CSAI ordering results, the performance and perceptions of participants in the intuitive-abstract-counterintuitive-symbolic (IACS) ordering were notably divergent. While the IACS participants reported the most enjoyment and attentiveness of all the context orderings and felt the ordering to be appropriate, these participants had almost no gains from pre- to post-assessment. When discussing the context ordering specifically, the participants described the context ordering as “progressive,” feeling that it went from easy to hard. These results mean that while participants in this context had very positive perceptions of the IACS ordering, their performance did not exhibit positive gains. Interestingly this ordering closely resembles the proposed IASC ordering but with the perceived most difficult context (symbolic) placed last (see Figure 14).

As noted in several previous research studies (e.g., Artman et al., 2006; O’Brien, 1974; G. J. Stylianides & Stylianides, 2009; Wagner-Egger, 2007), many learners have misconceptions when it comes to reasoning about conditionals. One possible explanation

Figure 14. The hypothetical IASC ordering and the IACS ordering.

for why the IACS ordering does not assist the learners in correcting these misconceptions could be that this progression (see Figure 14) does not provide enough cognitive disruption for the learner to correct the prior incorrect reasoning about conditionals. Consequently, learners may rely on prior conceptualizations with support from the easier contexts at the beginning of the ordering, allowing for success in the app and reinforcement of prior reasoning structures. Then when reaching the difficult contexts at the end of the ordering, participants may be prompted to alter their conceptions, but are provided no opportunity to practice and receive confirmation for these changes to their conceptions.

This result is similar to one found by Chen, Schneps, and Sonnert (2016), where researchers examined the order effects of presenting a simplified and then a realistic model of the solar system (or vice versa). The researchers found that the students exposed to the realistic model first and then the simplified model made gains from each model, but students exposed to the simplified model first and then the realistic model did not make gains from the second realistic model. Researchers posited that when introduced to the simplified model first, the simplified model "which requires less cognitive load, anchors students' understanding, and they appear reluctant to change their conceptualization when exposed to a model that requires a higher cognitive load" (Chen et al., 2016, p. 815). Thus, for the IACS ordering participants, the initial low cognitive load of the intuitive context may have re-anchored the participants understanding in their prior knowledge and may have made the participants reluctant to change their conceptualization with the later higher cognitive load of the counterintuitive and symbolic contexts. Consequently, participants experienced no substantial change in their reasoning about conditionals. Therefore, while this context ordering is progressively complex and difficult, it fails to provide sufficient influence on the learner's performance in reasoning about conditionals.

Recommendations for Researchers and Educators

This study provides important implications for researchers and educators. First, based on participants' performance gains with the SIAC ordering and participants' positive perceptions of the IACS ordering, the researcher recommends the consideration of a new context ordering for researching and teaching reasoning about conditionals, which includes participants' interactions with the symbolic context twice. This new context ordering is: symbolic-intuitive-abstract-counterintuitive-symbolic (SIACS, see Figure 15). This context ordering incorporates the catalytic effects of the symbolic context being presented first and the progressive difficulty and enjoyability of the IACS ordering being presented immediately thereafter. This newly proposed ordering capitalizes on the positive potential influences on both the learners' performance and perceptions of reasoning about conditionals. Second, for educators, the researcher recommends careful allocation of instructional emphasis with extra time reserved for students to interact with the symbolic context. The persistent difficulties participants in this study encountered with the symbolic context indicate that the equal emphasis given to all contexts in the design of the Learning Logic App either did not allow sufficient

Figure 15. Proposed symbolic-intuitive-abstract-counterintuitive-symbolic (SIACS) context ordering for teaching reasoning about conditionals.

time for the participants to reform their conceptualizations or did not effectively change their symbolic conceptualizations at all. Consequently, the researcher recommends that the app design be modified to include a feature or features, such as increased exposure time, that provide a focused constraint affordance regarding the reasoning involved within the symbolic context.

Limitations

As with all studies, there were limitations that affect the generalizability of these results. The four main limitations were the sample, the length of the intervention, the online administration of the second phase of the study, and the number of dummy variables in the multiple regression.

The sample was limited to undergraduates from a single university in the intermountain west. Because these students were drawn from typically first-or second-year courses, it is likely that the age distribution clustered around the early- to mid-20s age range. The students were in courses requiring them to participate in research studies for course credit. This study may have elicited different results if the participants had significantly differed in age or interest in learning about logic or playing an online app.

The amount of time participants were engaged in the intervention in this study was relatively short (18 minutes). Consequently, the researcher did not expect to find large gains in understanding in the participants' reasoning about conditionals. However, previous research indicates that it is possible to find small to medium changes in relatively short intervention times (10-20 minutes; Hicks & Milanese, 2015; Schäfer et al., 2013).

The online administration of Phase II of the study introduced possible outside distractions and timing factors as compared with the clinical interviews in Phase I. It is possible that participants in Phase II may have been distracted while completing the assessments or interacting with the Learning Logic App. Additionally, Phase II participants may have taken breaks between the main sections of the study, that is the pre-assessment, Learning Logic App, post-assessment, and computer survey. Consequently, there may have been a gap between a participants' completion of the intervention and completion of the post-assessment, where outside factors (e.g., memory loss or conceptual consolidation over time) could have influenced their post-assessment performance.

In the multiple regression, while the sample was sufficient for providing a broad view of the influence of the various context orderings, the sample is somewhat sparse for providing results about the magnitude of influence of specific context orderings. This is due to the number of dummy variables which leads to low power. It is possible that the multiple regression results appear by chance owing to the number of *t*-tests required. A Bonferroni adjustment for multiple comparisons would require a *p*-value to be less than .002 to be significant. While the SIAC ordering has a *p*-value slightly larger ($p = .003$), it should still be considered significant due to the overly severe corrections of Bonferroni adjustment with this number of variables (Perneger, 1998). These results provide guidance for future research where a subset of all possible context orderings can be more purposefully selected for comparison.

Suggestions for Future Research

The results of this study provide insight for future research on reasoning about conditionals. The newly recommended context ordering of symbolic-intuitive-abstract-counterintuitive-symbolic (SIACS) requires research to verify the benefits of this new ordering for learners. Future research could more closely examine the potentially catalyzing effects of the symbolic context. Due to the substantial difficulties participants had with the symbolic context, the researcher suggests an examination of participants' difficulties with reasoning in this context and the underlying conceptualizations. Because of the prevalence of symbolic representations in mathematics, computer science, engineering, and other STEM fields, this topic certainly merits further study. Future research could also examine other potential catalysts that could be used to efficiently disrupt dependence on prior incorrect conceptualizations.

In terms of the treatment used in this study, the Learning Logic App, the researcher recommends further study of the effects of the intervention with repeated exposure to the app as well as alignment with some form of instructional support.

Specifically, the researcher recommends a study design where the Learning Logic App is interacted with first followed by corresponding instruction as suggested by Denham (2017). This would then allow for a longitudinal analysis of the influences of the Learning Logic App and the context orderings. Finally, the researcher also suggests an examination of the potential for far transfer of the reasoning gained from the Learning Logic App. For example, this could include the evaluations of a learners' performance in reasoning about everyday appearances of conditionals in legal text or news articles or the academic appearances of conditionals in subjects outside of mathematics such as biology, physics, and computer programming.

Conclusion

This study showed how the order of four conditional contexts could influence learners' reasoning about conditionals. The mixed methods examination of learners' reasoning through the perspectives of performance and perceptions allowed the researcher to measure change in performance and to make inferences about the learners' shifts in conceptualization of reasoning about conditionals. The results showed that the (SIAC) ordering had the most positive influence on learners' reasoning about conditionals and the most positive perceptions. This finding, as related to the literature on conditionals, led to the suggestion of a new context ordering: symbolic-intuitive-abstract-counterintuitive-symbolic (SIACS). The researcher hypothesizes that this new ordering may be more beneficial to learners because of the catalytic effects of the difficult symbolic context presented at the beginning of ordering and revisited later at the end of the ordering.

These findings for improving individuals’ reasoning about conditionals using the Learning Logic App and the SIACS ordering advance the research literature in reasoning about conditionals in different contexts and in transitioning learners’ from correctly reasoning in one context to the next. These results can provide educators with more a more efficient instructional program to improve learners’ reasoning about conditionals and aid learners to become citizens who can interpret and capitalize on the conditionals they encounter in everyday life.

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APPENDICES

Appendix A

Institutional Review Board Letter of Approval

UtahState University
OFFICE OF RESEARCH AND GRADUATE STUDIES

Institutional Review Board

USU Assurance: FWA#00003308 Expedite #6 & #7

Letter of Approval

FROM: Melanie Domenech Rodriguez, IRB Chair

Nicole Vouvalis, IRB Administrator

To: Patricia Moyer-Packenham, Christina Lommatsch

Date: August 23, 2017

Protocol #:7860

Title: Learning Logic

Risk: Minimal risk

Your proposal has been reviewed by the Institutional Review Board and is approved under expedite procedure #6 & #7 (based on the Department of Health and Human Services (DHHS) regulations for the protection of human research subjects, 45 CFR Part 46, as amended to include provisions of the Federal Policy for the Protection of Human Subjects, November 9, 1998):

#6: Collection of data from voice, video, digital, or image recordings made for research purposes.
#7: Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

This approval applies only to the proposal currently on file for the period of one year. If your study extends beyond this approval period, you must contact this office to request an annual review of this research. Any change affecting human subjects must be approved by the Board prior to implementation. Injuries or any unanticipated problems involving risk to subjects or to others must be reported immediately to the Chair of the Institutional Review Board.

Prior to involving human subjects, properly executed informed consent must be obtained from each subject or from an authorized representative, and documentation of informed consent must be kept on file for at least three years after the project ends. Each subject must be furnished with a copy of the informed consent document for their personal records.

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Appendix B

Conditional Logic Assessment

Esty-Watts Questions

To "deduce" means to use logic properly to draw conclusions that follow from given statements. Sometimes the given statements are not enough to deduce anything, in which case give the answer "nothing". Here are two examples:

Example 1: John is taller than Mike. John is shorter than Bill. What can you deduce about Bill's height?

Answer: Bill is taller than Mike.

Example 2: John and Mike live on the same street. Mike likes rap music. What can you deduce about John's music preference?

Answer: nothing

Symbolic Context

1. Suppose this statement is true: "If k > 4 then, j > 12".

If "j = 13" is also true, what can be deduced?

2. Suppose this statement is true: "If k > 4 then, j > 12".

If "j = 6" is also true, what can be deduced?

3. Suppose this statement is true: "If k > 4 then, j > 12".

If "k = 8" is also true, what can be deduced?

4. Suppose this statement is true: "If k > 4 then, j > 12".

If "k = 2" is also true, what can be deduced?

Abstract Context

5. Suppose this statement is true: "If the rud is swity, then the rud is vergley".

If "The rud is not swity" is also true, what can be deduced?

6. Suppose this statement is true: "If the rud is swity, then the rud is vergley".

If "The rud is not vergley" is also true, what can be deduced?

7. Suppose this statement is true: "If the rud is swity, then the rud is vergley".

If "The rud is vergley" is also true, what can be deduced?

8. Suppose this statement is true: "If the rud is swity, then the rud is vergley".

If "The rud is swity" is also true, what can be deduced?

9. Suppose this statement is true: "If a card has a vowel on one side, then the card has an even number on the other side".

If "The card has a B on one side" is also true, what can be deduced?

10. Suppose this statement is true: "If a card has a vowel on one side, then the card has an even number on the other side".

If "The card has a E on one side" is also true, what can be deduced?

11. Suppose this statement is true: "If a card has a vowel on one side, then the card has an even number on the other side".

If "The card has a 4 on one side" is also true, what can be deduced?

12. Suppose this statement is true: "If a card has a vowel on one side, then the card has an even number on the other side".

If "The card has a 7 on one side" is also true, what can be deduced?

Cosmides & Tooby Questions

Intuitive Context

In this scenario, you are checking to see if people at a bar are following this law:

"If a person is drinking beer, then he/she must be 21 years of age or older."

13. A person is sitting at the bar drinking a beer. Do you need to check this person's age to see if they are following the law?

Yes, check this person's age. OR No, do not check this person's age.

14. A person is sitting at the bar drinking water. Do you need to check this person's age to see if they are following the law?

Yes, check this person's age. OR No, do not check this person's age.

15. A person is sitting at the bar who is 25 years old. Do you need to check this person's drink to see if they are following the law?

Yes, check this person's drink. OR No, do not check this person's drink.

16. A person is sitting at the bar who is 16 years old. Do you need to check this person's drink to see if they are following the law?

Yes, check this person's drink. OR No, do not check this person's drink.

De Neys Questions

Intuitive Context and Counterintuitive Context

17. Suppose this statement is true: “All mammals have hair”.

If the animal is a whale (a mammal), can the following statement be deduced?

“Whales have hair.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced.

18. Suppose this statement is true: “All land animals like water”.

If there is a cat, which does not like water, can the following statement be deduced?

“Cats are not land animals.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced.

19. Suppose this statement is true: “All birds have feathers”.

If the animal is an eagle (a bird), can the following statement be deduced?

“Eagles have feathers.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced.

20. Suppose this statement is true: “All dogs bark”.

If the animal is a coyote that barks, can the following statement be deduced?

“Coyotes are dogs.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced.

21. Suppose this statement is true: “All things that have a motor need oil.”

If "Bicycles need oil" is also true, can the following statement be deduced?

“Bicycles have a motor.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced.

22. Suppose this statement is true: “All flowers need water.”

If “Roses need water” is also true, can the following statement be deduced?

“Roses are flowers.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced.

23. Suppose this statement is true: “All vehicles have wheels.”

If “A boat is a vehicle” is also true, can the following statement be deduced?

“A boat has wheels.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced.

24. Suppose this statement is true: “All things made out of wood can be used as fuel.”

If “Gasoline is not made out of wood” is also true, can the following statement be deduced?

“Gasoline cannot be used as fuel.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced.

25. Suppose this statement is true: “All African countries are warm.”

If “Spain is warm” is also true, can the following statement be deduced?

“Spain is an African country.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced

26. Suppose this statement is true: “All foods that have sugar are sweet.”

If “Sour candies have sugar” is also true, can the following be deduced?

“Sour candies are sweet.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced

27. Suppose this statement is true: “All animals that growl are felines.”

If “Dogs growl” is also true, can the following statement be deduced?

“Dogs are not felines.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced

28. Suppose this statement is true: “All green living things are plants.”

If “A parrot is green” is also true, can the following statement be deduced?

“Parrots are plants.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced

29. Suppose this statement is true: “All bears with white hair are polar bears.”

If “The bear has yellow hair” is also true, can the following statement be deduced?

“The bear is not a polar bear.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced

30. Suppose this statement is true: “All shiny stones are diamonds.”

If “A ruby is shiny” is also true, can the following statement be deduced?

“A ruby is a diamond.”

Yes, the statement can be deduced. OR No, the statement cannot be deduced

In this scenario, you are an anthropologist studying the Kaluame, a warring people who live in small villages. A ruthless chieftain, named Big Kiku, offers starving newcomers outcast from other villages the following deal:

“If you get a tattoo on your face, then I’ll give you a cassava root.”

31. A newcomer has gotten a tattoo on their face. Would you need to see if this person

received a cassava root to see if Big Kiku held up his side of the deal?

Yes, check if the person received a cassava root. OR

No, do not check if the person received a cassava root.

32. A newcomer has not gotten a tattoo on their face. Would you need to see if this person received a cassava root to see if Big Kiku held up his side of the deal?

Yes, check if the person received a cassava root. OR

No, do not check if the person received a cassava root.

Appendix C

Semistructured Interview Protocol

Researcher: Thank you for completing the assessments and playing the Learning Logic App. The final portion of this study will be an interview about the Learning Logic App. [Researcher will have tablet with Learning Logic App available if participants would like to look at it for reference. GoPro and wall-mount cameras will continue recording.]

Game Design

1. What was your overall impression of the Learning Logic App? (Likert from 1 to 10)
    a. Why did you choose this rating?
    b. What would it take to get the rating to 10?
2. What was your favorite part of the Learning Logic App?
    a. Why?
3. What was your least favorite part of the Learning Logic App?
    a. Why?
4. What suggestions do you have for improving the Learning Logic App?
5. If your suggestions were implemented, would you recommend the Learning Logic App to a friend?
6. What did you think about the time length of each level (3 minutes)? (Likert, make shorter, just right, make it longer)
    a. How much time would you suggest to change it to?
7. What did you think about the order of the levels?
    a. Was the order of the levels helpful, neutral, or hindering to your learning? (Likert)

Features

1. Were there any features, such as sounds, animations, or other game mechanisms, that you noticed while interacting with the Learning Logic App?
2. Were any of these features helpful to you while you were interactions the Learning Logic App?
3. Did any of these features hinder you (that is, get in your way or cause confusion) while you were interacting with the Learning Logic App?
4. What idea or topic do you think the Learning Logic App was designed to teach?
5. Were there any features that helped you learn that idea or topic?

Affordances

1. Motivation:
    a. Did you find the Learning Logic App enjoyable? (Likert 1-10)
    b. Did the Learning Logic App keep your attention? (Likert 1-10)
2. Simultaneous Linking:
    a. Did you notice any part of the Learning Logic App where two objects were simultaneously linked? For example, where text and an image were linked together to indicate two representations of an idea.
3. Efficient Precision:
    a. What did you think of the fact that you had an open-ended number of sprites/characters you could interact with in each level?

4. Focused Constraint:
    a. What did you think of the Learning Logic App's built-in requirement to interact with each level for 3 minutes?

Technological Distance

1. Did you experience any technical difficulties while completing the Learning Logic App? (E.g., difficulty dragging the characters, text speed that is too fast to read, etc.).
    a. [Yes] Could you tell me more?

Researcher: Do you have any closing comments that you would like to share about the Learning Logic App? [after response]. Thank you for taking the time to participate. [Ensure SONA number is correctly recorded, turn off GoPro, show participant how to leave building].

Appendix D

Computer Survey

Game Design

1. What was your overall impression of the Learning Logic App?
    a. Open response
    b. (Likert from 1- dislike a great deal to 10 – like a great deal)
    c. Why did you choose this rating?
    d. What would it take to get the rating to 10?
2. What was your favorite part of the Learning Logic App?
    a. Open response
    b. Why?
3. What was your least favorite part of the Learning Logic App?
    a. Open response
    b. Why?
4. What suggestions do you have for improving the Learning Logic App?
    a. Open response
    b. If your suggestions were implemented, would you recommend the Learning Logic App to a friend?
5. What strategies did you use while playing the app?
6. What did you think about the time length of each level (100 seconds)? Would you change it to a different length?
7. What did you think about the order of the levels?
    a. Open response
    b. Was the order of the levels helpful, neutral, or hindering to your learning?

(Likert 1- very hindering to 10 – very helping)

8. Rank the difficulty of the levels with 1 being the easiest and 4 being the hardest.

Features

9. Were there any features, such as sounds, animations, or other game mechanisms, that you noticed while interacting with the Learning Logic App?
10. Did any features helpful to you while you were interacting with the Learning Logic App?
11. Did any features hinder you (that is, get in your way or cause confusion) while you were interacting with the Learning Logic App?
12. In the contrapositive levels, where the characters were "not", what did you think of the zones greying out after being used once?
13. What idea or topic do you think the Learning Logic App was designed to teach?
14. Were there any features that helped you learn that idea or topic?
15. Did the app influence how you completed the post-assessment?

Affordances

16. Did you find the Learning Logic App enjoyable? (Likert 1-10)
17. Did the Learning Logic App keep your attention? (Likert 1-10)

Technological Distance

16. Did you experience any technical difficulties while completing the Learning Logic App? (E.g., difficulty dragging the characters, text speed that is too fast to read, etc.).
    a. [Yes] Could you tell me more?

Appendix E

Process Codes and Frequencies from Phase I

Table E1

Process Codes and Frequencies from Phase I Video Analysis

Process codes	Number of participants coded (*N* = 10)
Easily sorting	10
Avoiding bumpers	2
Sorting slower (symbolic)	8
Taking a longer time to read the rules (symbolic)	8
Difficulty sorting into intended zone	5
Getting most right	4
Sorting to two zones (contrapositive)	2

Note. This table indicates that the participant was coded at least once for the process code. The participant may have repeated the action in multiple levels or just one.

Appendix F

Learning Logic App Refinements

User Experience

1. Change of level length from 180 seconds to 100 seconds
2. Addition of grey-out rule for contrapositive levels
3. Adaptive speed refinement for online experience due to more challenging mechanical aspects of clicking and dragging as compared with touch screen interactions
4. Fields to enter SONA number and assigned context order
5. Clarified tutorial to emphasize tap to continue and arrows to indicate dragging the penguin and not a box characters
6. Simplified level introductions with easier to read text
7. Added names of levels to the bottom of each level screen
8. Increased font size of rules in the left pane
9. Changed highlight of new rules to a more visually appealing yellow rather than hot pink
10. Reformatted end of game screen to display high scores beneath screen shots of the levels and to provide the end of game code for online players to continue the online survey
11. Added noise to indicate new rules spawning

Back End Development

1. Reformatted score logs to allow for easier data parsing
2. Adjustment of character and zone colliders for improved precision
3. Reduced speed of character spawns to match the mechanical difficulty of clicking and dragging on a computer rather than dragging and dropping on the tablet
4. Changed contrapositive rules to one large block rather than individual blocks

Appendix G

Residual Diagnostic Plots

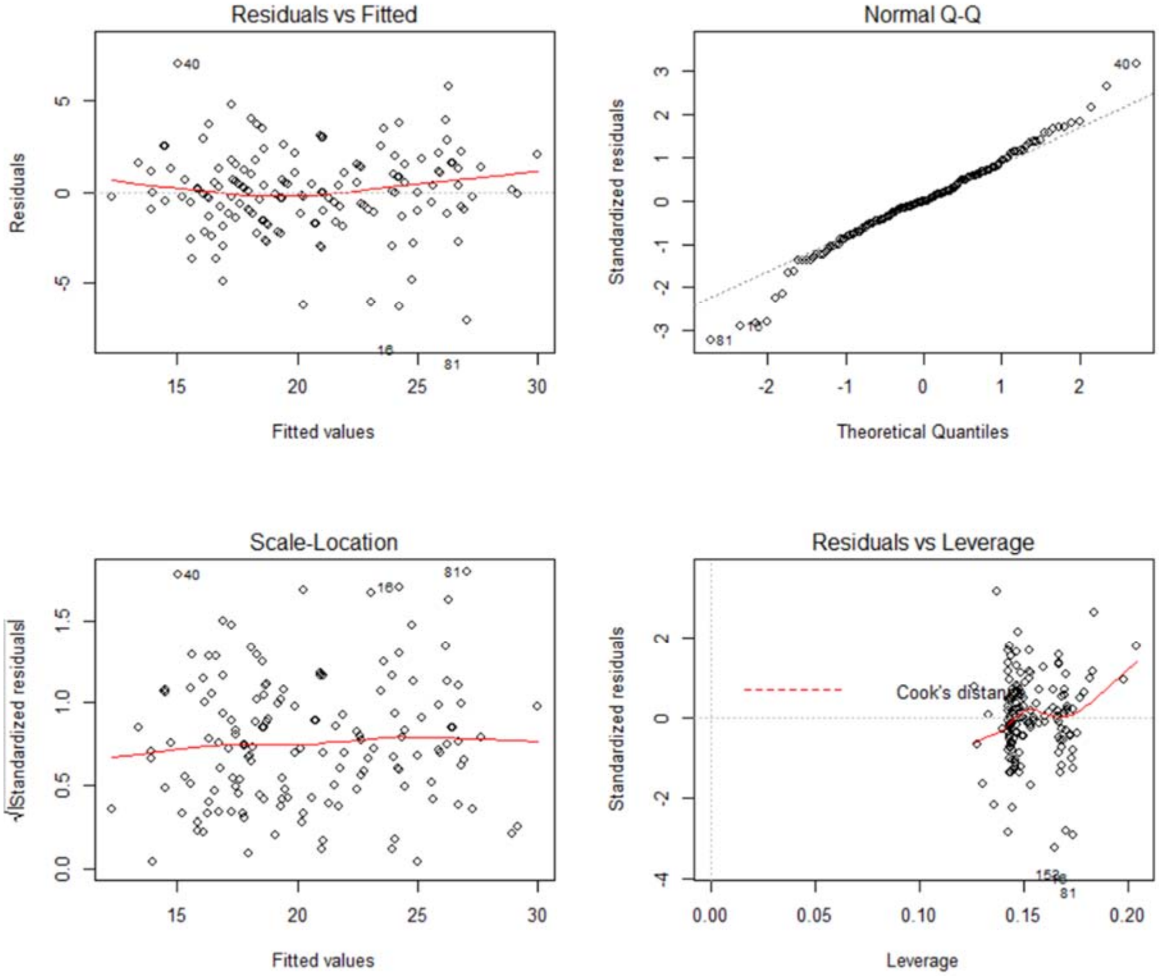

Figure G1. Residual diagnostic plots of the multiple regression model.

Appendix H

Multiple Linear Regression Model Coefficients

Table H1

Multiple Regression Coefficients and Statistics

Variable	Unstandardized coefficient, *b*	Standardized coefficient, β	*t*	Significance, *p*
Pre-Assessment	0.813	0.858	19.526	.001***
Context orderings				
SIAC	3.811	0.188	3.023	0.003**
ASCI	2.991	0.128	2.213	0.029*
CAIS	2.24	0.103	1.731	0.086
ISAC	2.124	0.098	1.646	0.102
CIAS	1.991	0.092	1.522	0.13
IASC	1.536	0.071	1.173	0.243
AISC	1.448	0.062	1.081	0.282
CISA	1.33	0.061	1.018	0.31
CASI	1.285	0.059	0.989	0.324
SAIC	1.044	0.048	0.806	0.422
SCAI	1.017	0.047	0.785	0.434
ASIC	1.014	0.05	0.804	0.423
ICAS	0.907	0.039	0.674	0.501
CSIA	0.821	0.038	0.633	0.528
ISCA	0.813	0.038	0.623	0.535
ICSA	0.761	0.033	0.567	0.572
ACIS	0.715	0.033	0.544	0.587
SCIA	0.543	0.023	0.403	0.687
AICS	0.501	0.021	0.373	0.71
SICA	0.094	0.004	0.07	0.944
IACS	-0.061	-0.003	-0.045	0.964
ACSI	-0.349	-0.016	-0.269	0.789

* $p < .05$.
** $p < .01$.
*** $p < .001$.

CURRICULUM VITAE

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EDUCATION

Ph.D. Expected 2018
Utah State University
Specialization: Curriculum and Instruction
Concentration: Mathematics Education and Leadership

M.S. May 2013
Montana State University
Master of Science in Mathematics

B.S. May 2011
Montana State University
Bachelor of Science in Mathematics

EMPLOYMENT HISTORY

Graduate Research Assistant (2014-present)
Utah State University, College of Education & Human Services, School of Teacher Education of Leadership, Logan, Utah
Research responsibilities include assisting professors on various research projects in mathematics education such as collecting data from study participants, coding video observations of participant actions, finding emergent themes and participating in writing group papers and presentations.

Graduate Teaching Assistant (2014-present)
Utah State University, College of Science, Department of Mathematics and Statistics and Department of Teacher Education and Leadership, Logan, Utah
Teaching assistant responsibilities include teaching Linear Algebra and Differential Equations, College Algebra, College Mathematics Preparation, and Beginning Algebra in the mathematics department. In the department of teacher education and leadership,

teaching responsibilities include teaching six online courses for the elementary mathematics endorsement.

Tutor.com Tutor (2010 – Present)
Assisting a diverse set of students through an online web portal in courses ranging from pre-algebra (4th grade) through introductory real analysis.

Tutor.com Mentor (2015)
As a mentor for Tutor.com, I advised a team of 20 mathematics and physics online tutors. My advisement included preparing new tutors for working in an online environment, reviewing tutoring sessions for correct mathematical content and pedagogy, time management tips, and encouraging experienced tutors in continued improvement.

Adjunct Instructor (2014)
ITT Technical Institute, Salt Lake City, Utah
Instructor of introductory course that included beginning algebra through a basic introduction to matrices. The courses were designed around five different career programs.

Graduate Teaching Assistant (2013-2014)
University of Kansas, College of Liberal Arts and Sciences, Department of Mathematics, Lawrence, Kansas
Teaching assistant responsibilities include teaching two recitation sections of engineering calculus and teaching a single lecture section of business calculus. In the engineering calculus course, I occasionally taught the 500-student lecture as a substitute lecturer. In the business calculus course, I designed my “Calculus in Your Career” project that successfully elevated students’ understanding of related rates and optimization.

Graduate Teaching Assistant (2011-2013)
Montana State University, College of Letters and Sciences, Department of Mathematical Sciences, Bozeman, Montana
Taught several college mathematics courses with 20-30 students per section: Precalculus, Engineering Calculus, and the Language of Math. While teaching I co-developed a “How to Succeed in Math Course” workshop

Math Learning Center Tutor (2009 – 2011)
Tutoring students in developmental mathematics through differential equations in a drop-in tutoring center.

TRiO tutor (2009)
Tutor students with learning disabilities in a one-on-one environment in both developmental level mathematics and higher-level mathematics.

AWARDS & PROFESSIONAL RECOGNITION

- *Fellowship*: Frederick Q. Lawson Fellowship ($10,000) Utah State University (2017-2018)
- Graduate Researcher of the Year, School of Teacher Education and Leadership, Utah State University (2017)
- Distinguished Service Award, Utah State University Student Association (2017, 2016)
- *Scholarship*: Graduate Research and Teaching Assistantship, Utah State University (2014-present)
- *Scholarship*: Madison and Lila Self Fellowship Finalist Scholarship, University of Kansas (2013-2014)
- Math Tutor of the Month for Tutor.com (June 2013)
- Outstanding Graduate Teaching Assistant, Montana State University (2013)
- NRM Journal Prize for Student Presentation, Natural Resource Modeling, World Conference on Natural Resource Modeling, Ottawa, Canada (2013)
- *Scholarship*: Graduate Teaching Assistantship, Montana State University (2011-2013)
- Outstanding Graduating Senior (2011)
- Member of Pi Mu Epsilon, Math Honor Society (2010-2011)
- Member of Mortar Board, Senior Honor Society (2010-2011)
- Montana University System Scholarship (2008-2011)
- *Scholarship*: National Merit Semi-Finalist Scholarship (2008)

RESEARCH

Research Interests

Technology and app development and implementation in mathematics education
Mathematics representations and manipulatives (physical, pictorial, symbolic, and virtual)

Research Projects

Affordances of Virtual Manipulatives (2016 – Present). Develop iPad-based interview protocols; conduct interviews with participants; collect, code, and analyze data; collaboratively write and present results. Utah State University (with PI Dr. Patricia Moyer-Packenham and the Virtual Manipulatives Research Group).
Learning Progression Based Games (2016 – Present). Cross-Institutional Collaboration with North Carolina State University creating an educational app called Codebreakers! to teach children simplifying mathematical numerical expressions. I am the primary point of contact at USU in the collaboration with doctoral students at North Carolina State University in the design and development of the app. Utah State University and North Carolina State University (with PIs Dr. Patricia Moyer-Packenham and Dr. Tiffany Barnes).

Captivated! Young Children's Learning Interactions with iPad Mathematics Apps (2014

– 2016). Analyze data from interviews of children’s interactions with iPad Mathematics Apps with respect to learning progressions and affordances. Outcomes of the project include 6 manuscripts published/under review. I led one of the papers and a book chapter on learning progressions and participated in one professional presentation. I will participate in at least two more professional presentations in Spring 2017. Utah State University (with PI Dr. Patricia Moyer-Packenham and the Virtual Manipulatives Research Group)

Undergraduate Math Students’ Proficiency with Multiplication Facts (2014 – 2015). Collect and analyze data on undergraduate students’ proficiency with multiplication facts. Outcomes of this project include one published manuscript and two professional presentations. Utah State University (with PI Dr. Cathy Callow-Heusser)

PUBLICATIONS

Journal Articles (Refereed)

Lommatsch, C. W. (2017). Calculus in your career: Putting the “relate” back in related rates. *Mathematics Teacher*, *111*(2), 112-118.

Tucker, S.I., Lommatsch, C. W., Moyer-Packenham, P.S., Anderson-Pence, K.L., & Symanzik, J. (2017). Kindergarten children’s interactions with touchscreen mathematics virtual manipulatives: An innovative mixed methods analysis. *International Journal of Research in Education and Science*, *3*(2), 646-665.

Bullock, E. P., Shumway, J. F., Watts, C. M., & Moyer-Packenham, P. S. (2017). Affordance Access Matters: Preschool Children’s Learning Progressions While Interacting with Touch-Screen Mathematics Apps. *Technology, Knowledge and Learning*, 1-27.

Watts, C. M., Moyer-Packenham, P. S., Tucker, S. I., Bullock, E. P., Shumway, J. F., Westenskow, A., Boyer-Thurgood, J., Anderson-Pence, K., Mahamane, S., & Jordan, K. (2016). An examination of children’s learning progression shifts while using touch screen virtual manipulative mathematics apps. *Computers in Human Behavior*, *64*, 814–828.

Moyer-Packenham, P. S., Bullock, E. K., Shumway, J. F., Tucker, S. I., Watts, C. M., Westenskow, A., Anderson-Pence, K. L., Maahs-Fladung, C., Boyer-Thurgood, J., Gulkilik, H., & Jordan, K. (2016). The role of affordances in children’s learning performance and efficiency when using virtual manipulative mathematics touch-screen apps. *Mathematics Education Research Journal*, *28*(1), 79–105.

Callow-Heusser, C. A., Bagley, J., & Watts, C. M. (2015). Why should students know basic math facts? Because multiplication fact skills predict grades in college math courses. *Utah Mathematics Teacher*, *8*, 56-60.

Watts, C. M., Cao, J., Panza, C., Dugaw, C., Colwell, M., & Burroughs, E. A. (2012). Modeling the effects of predator exclosures on a Western Snowy Plover population. *Natural Resource Modeling*, *25*(3), 529-547.

Book Chapters (Refereed)

Lommatsch, C. W., Tucker, S. I., Moyer-Packenham, P. S., Symanzik, J. (in press, 2017). Heatmap and Hierarchical Clustering Analysis to Highlight Changes in Young Children's Developmental Progressions Using Virtual Manipulative Mathematics Apps. In N. Calder, K. Larkin, & N. Sinclair (Eds.). *Using mobile technologies in the teaching and learning of mathematics. Mathematics Education in the Digital Era*. Springer.

Conference Proceedings (Refereed)

Bullock, E. P., Moyer-Packenham, P. S., Shumway, J. F., Watts, C., & MacDonald, B. (2015, March). Effective teaching with technology: Managing affordances in iPad apps to promote young children's mathematics learning. In D. Rutledge & D. Slykhuis (Eds.), *Proceedings of the Society for Information Technology and Teacher Education International Conference* (pp. 2357-2364), Las Vegas, Nevada.

Unpublished Manuscripts

Moyer-Packenham, P. S., Lommatsch, C. W., Litster, K., Ashby, J. Bullock, E., Roxburgh, A., Shumway, J., Speed, E., Covington, B., Hartmann, C., Clarke-Midura, J., Skaria, J., Westenskow, A., MacDonald, B., Symanzik, J., & Jordan, K. (under review, 2018). *The role of design features in the affordances of digital math games*.

Lommatsch, C. W., Esty, W. W. (in preparation, 2018). *Assessing conditional logic: Alternatives to Wason*. Unpublished manuscript.

Lommatsch, C. W. (in preparation, 2018). *Teaching and learning proof and logic: A review of the literature*. Unpublished manuscript.

UNIVERSITY TEACHING

Utah State University, Logan, Utah (2014-present)
College of Science, Department of Mathematics and Statistics

Courses Taught – Utah State University
Math 2250. Linear Algebra and Differential Equations (Spring 2017)
-- Linear systems, abstract vector spaces, matrices through eigenvalues and eigenvectors, solution of ode's, Laplace transforms, first order systems.

Math 1050. College Algebra (Spring 2016, Fall 2016)
-- Functions: graphs, transformations, combinations, and inverses. Polynomial, rational, exponential, logarithmic functions, and applications. Systems of equations and matrices. Partial fractions.

Math 0995. College Mathematics Preparation (Fall 2015)
-- Review of introductory algebra concepts. Topics include: manipulating and simplifying expressions; solving equations and inequalities; graphing equations and inequalities. Real world applications including linear, quadratic, polynomial, rational, exponential, and radical functions.

Math 0990. Beginning Algebra (Fall 2014, Spring 2015)
-- A first course in algebra. Real numbers; algebraic expressions; graphing and solving equations and inequalities; operations on polynomials; factoring polynomials; rational expressions and equations; and systems of equations.

College of Education and Human Services, School of Teacher Education and Leadership

Courses Taught – Utah State University

TEAL 6551. Assessment and Intervention (Fall 2017, online)
-- This course provides practicing teachers a deeper understanding of the various types of assessment and their appropriate use for guiding instruction, intervention and evaluation of student learning.

TEAL 6525. Data Analysis and Problem Solving (Fall 2017, online)
-- This course provides practicing teachers a deeper understanding of probability and data representation and analysis.

TEAL 6524. Geometry and Measurement (Fall 2017, online)
-- This course provides practicing teachers a deeper understanding of the geometry and measurement context that exists in the state core and instructional strategies to facilitate the instruction of this content.

TEAL 6523. Algebraic Reasoning (Spring 2018, online)
-- This course provides practicing teachers a deeper understanding of algebraic expressions, equations, functions, real numbers, and instructional strategies to facilitate the instruction of this content for elementary students.

TEAL 6522. Rational Numbers and Proportional Reasoning (Fall 2017, online)
--To provide practicing teachers a deeper understanding of rational numbers, operations with rational numbers, and proportionality, and instructional strategies to facilitate the instruction of this content for elementary students.

TEAL 6521. Numbers and Operations (Fall 2017, online)
--This course, for K-8 teachers, will cover the content of Number and Operations to develop comprehensive understanding of our number system and relate its structure to computation, arithmetic, algebra, and problem solving.

TEAL 6300. Online Elementary Math Teachers Academy (Fall 2017, online)
-- This is an exploration of current topics and methods in mathematics education. In the past, topics have included Common Core mathematics content, relevant mathematics in rural settings, and integration of mathematics and children's literature. Students choose three current topics per credit hour.

ITT Technical Institute, Salt Lake City, Utah (2014)

Courses Taught – ITT Technical Institute

Math 1210. Math 1 (Fall 2014)
-- This course focuses on fundamental mathematical concepts including quadratic, polynomial and radical equations, linear functions and their graphs, systems of linear equations, functions and their properties, and matrices. Activities include solving problems and using appropriate technological tools.

University of Kansas, Lawrence, Kansas (2013-2014)
College of Liberal Arts and Sciences, Department of Mathematics

Courses Taught – University of Kansas
Math 121. Calculus I. (Fall 2013)
-- Differentiation and integration of algebraic and trigonometric functions. Applications to physical sciences and engineering.

Math 115. Calculus I. (Spring 2014)
-- Elementary differential and integral calculus, with applications in management and the biological sciences.

Montana State University, Bozeman, Montana (2011-2013)
College of Letters and Sciences, Department of Mathematical Sciences

Courses Taught – Montana State University
M 171. Calculus I (Fall 2012)
-- Functions, elementary transcendental functions, limits and continuity, differentiation, applications of the derivative, curve sketching, and integration theory.

M 147. Language of Mathematics (Spring 2013)
-- Reading comprehension and writing skills in the language of mathematics; vocabulary, grammar, syntax and logic; emphasis on understanding, expressing, proving, and thinking mathematical thoughts.

M 151. Precalculus (Fall 2011, Spring 2012, Summer 2012)
-- Functions, graphs, and the use symbols for expressing mathematical thoughts. Polynomials, rational, exponential, logarithmic, and trigonometric functions.

M 149. Secrets of the Infinite (Fall 2009, Spring 2010, Fall 2010, Spring 2011)
-- Intriguing problems, puzzles, and paradoxes studied from an historical perspective. Hands-on thought experiments follow mathematical ideas as they evolved from ancient beginnings into their modern contexts. Topics vary by semester. (Assisted as an undergraduate fellow).

GRANT INVOLVEMENT

USU STARS! GEAR UP Grant (2015 – Present). Professional development work with mathematics teachers from across the state who are teaching in STARS! GEAR UP schools in professional development aimed to assist them in their course design, planning, and implementation. I also develop the Utah Council of Teachers of Mathematics Conference Presentations for the upper grade levels in the first STARS! GEAR UP cohort. In addition, I work with the GEAR UP App Camp for girls where I train high school girls in mentoring and programming and facilitate the App Camp for middle school girls where the high school girls serve as mentors. The girls are introduced to a variety of careers available to them in computer science. (PI Eric Packenham, Utah State University)

AspireIT App Camp (2016 – 2017). App Camp which provides mentoring and programming training for high school girls who then mentor middle school boys and girls in the App Camp. During App Camp, the students develop a variety of apps and are exposed to various careers in computer science. I contributed designing and implementing the curriculum, mentor leadership activities, mentor training, and camper activities as well conducting associated research activities. (PI Vicki Allen, Utah State University)

GRANTS FUNDED

- Travel Grant, School of Teacher Education and Leadership (TEAL) ($300). Utah State University (2018). Presentation at American Educational Research Association (AERA) Annual Meeting.
- Graduate Enhancement Award ($4,000). Utah State University (2017). Awardees show a history of using knowledge gained through educational opportunities to contribute to Utah State University campus, to local and national communities, and to their professional field to foster lasting change.
- Graduate Research and Collaborative Opportunities Grant (GRCO) ($1,000). Utah State University (2017). Awarded project was the development, testing, and implementation of an educational app, Learning Logic for the teaching of logical inference skills.
- Travel Grant, School of Teacher Education and Leadership (TEAL) ($300). Utah State University (2017). Presentation at the National Council of Teachers of Mathematics (NCTM) Annual Meeting.
- Travel Grant, School of Research and Graduate Studies ($300). Utah State University (2017). Presentation at the National Council of Teachers of Mathematics (NCTM) Annual Meeting.
- Travel Grant, School of Teacher Education and Leadership (TEAL) ($300). Utah State University (2017). Presentation at American Educational Research Association (AERA) Annual Meeting.
- Travel Grant, School of Research and Graduate Studies ($300). Presentation at American Educational Research Association (AERA) Annual Meeting. (2016). Utah State University.
- Travel Grant, School of Teacher Education and Leadership (TEAL) ($200). *Presentation at 26th International Conference of the Society for Information Technology and Teacher Education (SITE).* (2015). Utah State University.
- Travel Grant, School of Research and Graduate Studies ($200). *Presentation at 26th International Conference of the Society for Information Technology and Teacher Education (SITE).* (2015). Utah State University.
- Travel Grant, Natural Resource Modeling (NRM). *Presentation at World Conference*

on Natural Resource Modeling). (2011). Montana State University.
- Travel Grant, Undergraduate Scholars Program. *Presentation at Pikes Peak Region Undergraduate Mathematics Conference.* (2011). Montana State University.
- Research Grant, National Science Foundation (NSF). *National Science Foundation Research Experience for Undergraduates Grant, Award Number 0755582.* (2010). Humboldt State University.

PRESENTATIONS

International Presentations

Watts, C. M. (2011, June) *Modeling the effects of predator exclosures on a western snowy plover population*. Paper presentation and poster presentation, 2011 World Conference on Natural Resource Modeling, Ottawa, Ontario, Canada.

National Presentations

Watts, C. M. (2017, April). *How Many Elephants Fit on the Moon? Using Technology to Address Ill-Structured Problems.* Workshop Presentation, National Council of Teachers of Mathematics Annual Meeting, San Antonio, Texas.

Litster, K. & Watts, C. M. (2017, April). *Virtual Cookies: Free Virtual Resources to Increase Participation, Discussion, and Collaboration.* Workshop Presentation, National Council of Teachers of Mathematics Annual Meeting, San Antonio, Texas.

Moyer-Packenham, P. S., Bullock, E. P., Shumway, J. F., Tucker, S. I., Watts, C., Westenskow, A., Anderson-Pence, K. L., Boyer-Thurgood, J. (2017, April). *Affordances of Virtual Manipulative Math Apps: How They Help and Hinder Young Children's Learning.* Research Presentation, American Educational Research Association Annual Meeting, San Antonio, Texas.

Watts, C. M., Moyer-Packenham, P. S., Tucker, S. I., Bullock, E. P., Shumway, J. F., Westenskow, A., Boyer-Thurgood, J., Anderson-Pence, K., Mahamane, S., & Jordan, K. (2017, April). *Learning Progression Shifts: How Touch-Screen Virtual Manipulative Mathematics App Design Promotes Children's Productive Struggle*. Poster Presentation, American Educational Research Association Annual Meeting, San Antonio, Texas.

Moyer-Packenham, P. S., Shumway, J. F., Bullock, E. K., Anderson-Pence, K. L., Tucker, S. I., Westenskow, A., Boyer-Thurgood, J., Gulkilik, H., Watts, C. M., & Jordan, K. (2016, April). *Using virtual manipulatives on iPads to promote young children's mathematics learning*. Paper session, American Educational Research Association Annual Meeting, Washington, D.C.

Bullock, E. P., Moyer-Packenham, P. S., Shumway, J. F., Watts, C., & MacDonald, B. (2015, March). *Effective teaching with technology: Managing affordances in iPad apps to promote young children's mathematics learning*. Short paper presentation, Society for Information Technology and Teacher Education International Conference, Las Vegas, NV.

Watts, C. M. & Cao, J. (2011, January). *Modeling the effects of predator exclosures on a western snowy plover population*. Poster Presentation, Joint Mathematics Meeting 2011, New Orleans, LA.

State & Regional Presentations

Watts, C. M. (2016, November). *Technology for solving ill-structured problems*. IGNITE Presentation, Utah Council of Teachers of Mathematics Conference, Salt Lake City, UT.

Watts, C. M. (2016, November). *Model Engagement: Math Modeling in the Classroom with GEAR UP (9-12)*. Workshop Presentation, Utah Council of Teachers of Mathematics Conference, Salt Lake City, UT.

Litster, K. & Watts, C. M. (2016, March). Virtual cookies do not taste the same as physical ones. Poster presentation, Scholarship of Teaching and Engagement Conference, Orem, UT.

Callow-Heusser, C. & Watts, C. M. (2015, November). *Multiplication misconceptions: How multiplication fact knowledge predicts algebra grades in college developmental math courses*. Workshop presentation, Utah Council of Teachers of Mathematics Conference

Callow-Heusser, C. & Watts, C. M. (2015, November). *The effects of mindset on mathematical performance*. Workshop presentation, Utah Council of Teachers of Mathematics Conference, Lehi, UT.

Watts, C. M. (2014, November). *'Calculus in your career'… and in your classroom*. Workshop presentation, Utah Council of Teachers of Mathematics Conference, Layton, UT.

Watts, C. M. (2011, April). *Modeling the effects of predator exclosures on a western snowy plover population*. Poster presentation, Student Research Celebration, Bozeman, MT.

Watts, C. M. (2011, March). *Modeling the effects of predator exclosures on a western snowy plover population*. Poster presentation and paper presentation, Eighth Annual Pikes Peak Regional Undergraduate Mathematics Conference, Colorado Springs, CO.

Professional Presentations Pending

Lommatsch, C. W. (2018, April). *The Learning Logic App: Testing the effects of context ordering on reasoning about conditionals*. American Educational Research Association Annual Meeting, New York City, NY.

Lommatsch, C. W., Moyer-Packenham, P. S., & Litster, K. (2018, April). *Differences in children's affordance awareness and access between novice and experienced learners*. In symposium, Young children learning with mobile devices: Research on design and implementation. American Educational Research Association Annual Meeting, New York City, NY.

Moyer-Packenham, P. S., Lommatsch, C. W., Lister, K., Ashby, J., Bullock, E., Shumway, J., & MacDonald, B. (2018, April). *Affordances of digital games for mathematics learning in grades 3-6*. American Educational Research Association Annual Meeting, New York City, NY.

Bullock, E., Shumway, J. F., Lommatsch, C. W., Moyer-Packenham, P. S. (2018, April). *Preschool children's learning progressions while interacting with touch-screen mathematics apps and how affordance access matters*. American Educational Research Association Annual Meeting, New York City, NY.

Moyer-Packenham, P. S., Lommatsch, C. W., Litster, K., Ashby, M. J., & Roxburgh, A. (2018, March). *The Role of Design Features in the Affordances of Digital Math Games*. Research Presentation, Society for Information Technology and Teacher Education (SITE), Washington, D.C.

Moyer-Packenham, P. S., Litster, K., Lommatsch, C. W., Ashby, M. J., & Roxburgh, A. (2018, March). *Mediators of Learning in Game-Based Mathematics Apps.* Research Presentation, Society for Information Technology and Teacher Education (SITE), Washington, D.C.

LEADERSHIP ACTIVITIES

Director of Graduate Studies of the Utah State University Student Association (2016-2017)
As the Graduate Director, I conduct the graduate council meetings where graduate representatives from each college at Utah State University meet to discuss potential programs for and issues facing students in their college. Activities have included revision of mandatory teaching workshops, development of mental health awareness workshops, and design of social activities. I also oversee the application process and award of two university-wide graduate student awards.

Graduate Vice President of the College of Education and Human Services Student Council, Utah State University (2015-2017)
As the graduate vice president of the CEHS student council, I am the liaison between the graduate and undergraduate students within the college. I also work with the council in outreach and service activities designed to involve students and faculty in research and career development.

Member of the Graduate Student Council, Utah State University (2015-2017)
As a member of the Graduate Student Council for the university, I provide input for policy decisions affecting students university-wide and help coordinate events for outreach into the graduate student community.

President of the Association for Women in Mathematics Student Chapter, University of Kansas (2013- 2014)
As president of the AWM student chapter, I helped encourage women in sciences and mathematics through outreach programs to local young women interested in the sciences and support women currently in the field. I organized speakers, workshops, and outreach opportunities for members of our chapter.

Co-founder of the Montana State University Graduate Summit (2012-2013)
Proposed, planned, and conducted the first Graduate Summit at Montana State University. The symposium is designed to expose STEM graduate students to the career opportunities in academic, industrial, and alternative fields and the steps students should take both now and in the future to be successful. I aided in writing the funding proposal, selected the speakers and panel members, coordinated the vendors, managed the day of the event, and created a statistical analysis of the survey administered at the event.

Co-creator of the Undergraduate Math Success Workshop (2012-2013)

Organized and conducted the first “How to be Successful in a Math Course” workshop for undergraduates at Montana State University. This workshop helped undergraduates learn math specific study and time management skills, as well as exposed them to resources available not only at the university but also in online notes and videos.

Montana State University Flute Choir (2008-2013)
Participation in the MSU Flute Choir. Performing concerts at Montana State University and retirement homes.

Montana State University Mortar Board (2010-2011)
Secretary. Mortar Board is a national honor and service society. Main projects included Reading is Leading, renovating Danforth Park on MSU campus, and working with Habitat for Humanity.

Member of the Dean’s Student Council for College of Letters and Sciences (2010-2011)
A nominated position to discuss and promote the needs of the undergraduates within the College of Letters and Sciences.

Member of the Undergraduate Math Council (2010-2011)
A nominated position to discuss budgeting math department funds to support student resources in tutoring, research, and travel.

Montana State University Math Club (2008-2011)
Secretary. Founding member of a club designed to bring together the departments’ undergraduates, while also encouraging outside departments to join and appreciate mathematics.

SERVICE ACTIVITIES

Reviewer for the Journal for Research in Mathematics Education, (2017-present)
Review articles related to mathematics education research for publication.

Reviewer for the Educational Researcher, (2016-present)
Review articles related to educational research for publication.

Reviewer for the Mathematics Teacher, (2015-present)
Review articles and books related to the teaching of mathematics at the secondary and post-secondary levels for publication.

Book review for Mathematics Teacher, (2017)
Review of the book Significant Figures by Ian Stewart. Wrote a brief review and corrected mathematics typesetting errors.

Reviewer for the National Council of Teachers of Mathematics Annual Meeting, (2016) Review articles and submissions related to the teaching of mathematics at the secondary and post-secondary levels for paper and poster presentations.

Judge for USU Physics Day, MESA Prosthetic Arm Competition, (2015)
Judged high school students' performance in prosthetic arm dexterity competition. Students constructed their prosthetic arms and competed as a team.

Judge for Maria Montessori Academy Science and Engineering Fair, (2015)
Judged 6th, 7th, and 8th grade science fair projects to choose students who would continue to the regional competition. Students were evaluated on their adherence to the Scientific Method or the Engineering Process

PROFESSIONAL AFFILIATIONS

National Council for Teachers in Mathematics (NCTM) (2015-present)
American Educational Research Association (AERA) (2015-present)
Society for Information Technology and Teacher Education (SITE) (2015-present)