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Octonionic Maxwell Equations

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Abstract

An introduction to Quaternions and Octonions is given, and the Maxwell Equations are formulated in terms of each. The conventional, source-free relativistic theory of eight dimensional electromagnetism is introduced and examined. Similarly, the source-free Octonionic Maxwell Equations are developed, and it is shown that the seven dimensional electric and magnetic fields—pure Octonions—each admit plane wave solutions. An Octonionic Faraday tensor is constructed and compared with the conventional Faraday tensor, and it is shown that, in the source-free case, the conventional and Octonionic theories are equivalent.

1 An Introduction to Quaternions and Octonions

The general form of a complex number is $a + bi$, where a and b are real numbers, and $i = \sqrt{-1}$. Quaternions can be thought of as an extension to the complex numbers, and their general form is $a + bi + cj + dk$, where a , b , c , and d are real numbers, $i^2 = j^2 = k^2 = -1$, and $ijk = -1$. We note from this rule of multiplication that Quaternions are not, in general, commutative: $ijk = -1 = i^2$ implies that $jk = i$, $ijk = -1 = k^2$ implies that $ij = k$, which two conditions reveal that $jij = i$, or that, multiplying both sides by j , $-ij = ji$. Using these formulas, first discovered by William Rowan Hamilton in 1835 [1], one can construct a unique multiplication table for the Quaternions.

The general form of an Octonion is similar to that of a Quaternion, only with eight components instead of four. By convention, subscripts of the letter e are used to denote square roots of -1 , with e_0 used to denote the real number 1. Thus, the general form of an octonion is $a_0e_0 + a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4 + a_5e_5 + a_6e_6 + a_7e_7$, with each a_i a real number. Like Quaternions, Octonions are not commutative. In addition, Octonions are not associative [1], making them notoriously difficult to work with. Octonions are, however, alternative [2], which is considered a weak form of associativity: for Octonions X and Y , $X(XY) = (XX)Y$, and $(XY)Y = X(YY)$. The Real numbers, Complex numbers, Quaternions, and Octonions form the only normed division algebras [1]. Quaternions are an extension to the Complex numbers in the sense that the Quaternions can be constructed from the Complex numbers using the Cayley-Dickson construction [1]. It is in this sense that the Octonions are an extension to the Quaternions, and that the Complex numbers are

an extension to the Real numbers.

Where the rules of multiplication of Quaternions define a unique multiplication table for the Quaternions, there is no unique multiplication table for the Octonions. While there is a rigorous approach to explaining this result, we adopt a more intuitive explanation. We invoke the strong correlation between pure Quaternions and vectors in Cartesian coordinates, which is expounded upon in the next section: let i , j , and k define the three orthogonal directions in Cartesian space. We can begin by pointing in any direction and defining that to be the i direction. Now, we can point in any direction that is perpendicular to the i direction and define that direction to be the j direction. This leaves us with only one choice for the direction of k , since the k direction must be orthogonal to both i and j . Now, we repeat the experiment in seven dimensions. Having chosen our first two directions, which we now call e_1 and e_2 , we now find that we have more than one choice for the direction of e_3 , since there are more directions than one which are orthogonal to a two-dimensional plane in seven dimensional space. Notwithstanding the difficulty in truly comprehending seven spatial dimensions, we find that this non-uniqueness in the orthogonality of a line with a plane in seven dimensions is the starting point of the non-uniqueness in the multiplication table for the Octonions.

In this study, due in large part to the difficult nature of performing calculations with Octonions, MapleSoft™ has been used to make direct calculations: in particular, the “DifferentialGeometry” package was used¹. Relevant calculations made using this software have been included as an appendix to this report.

2 Differential Operators and the Maxwell Equations

We will now consider the multiplication of two Quaternions. For Quaternions of the form $a + bi + cj + dk$, the so-called “imaginary” part of a Quaternion is $bi + cj + dk$, while the “real” part is just a . Thus the multiplication of two Quaternions Q_1 and Q_2 is the sum of the real and imaginary parts of the product: $Q_1Q_2 = \text{Re}(Q_1Q_2) + \text{Im}(Q_1Q_2)$. A Quaternion whose real part is zero is called a “pure” Quaternion.

As it happens, there is a strong correlation between pure Quaternions and ordinary three-dimensional vectors in Cartesian coordinates, as was alluded to in the preceding section—correlated in the sense of the transformation $i \rightarrow \hat{x}$, $j \rightarrow \hat{y}$, and $k \rightarrow \hat{z}$. The imaginary part of the product of two pure Quaternions is correlated with the cross-product of two vectors in xyz space, and the real part of the product of two pure Quaternions is correlated with the Dot product. Just as the x , y , and z directions are orthogonal in the sense that the dot product of any two of them which are different is zero, i , j , and k are orthogonal in the sense that the real part of the product of any two of them which are different is also zero.

¹The “DifferentialGeometry” package is the product of the “Differential Geometry Software Project,” and is maintained at Utah State University by faculty members Dr. Anderson and Dr. Torre. It is available at <https://digitalcommons.usu.edu/dg/>

We now ask whether classical, source-free Electromagnetism can be formulated in terms of Quaternions instead of ordinary vector calculus. We define the differential operator to be the pure Quaternion

$$\nabla = \frac{\partial}{\partial x_1}i + \frac{\partial}{\partial x_2}j + \frac{\partial}{\partial x_3}k$$

which resembles the Cartesian differential operator. With this differential operator, and with the dot and cross product defined in terms of the Quaternion product, we define the curl of a pure Quaternion to be the cross product of the differential operator and the Quaternion; similarly, we define the divergence of a pure Quaternion to be the dot product of the differential operator and the Quaternion. We pose the source-free, Maxwell equations for Quaternions as

$$\nabla \cdot \mathbf{E} = 0 \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

$$\nabla \times \mathbf{B} = \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} \tag{4}$$

Where \mathbf{E} and \mathbf{B} are meant to represent the Electric and Magnetic fields, respectively. They are pure Quaternions with coordinate-dependant coefficients—“Quaternion fields,” meant to resemble Vector fields. As it turns out, Maxwell originally formulated his equations in terms of Quaternions, later reverting to Vectors as Vectors became the convention [3].

Now, we ask whether 7 + 1 dimensional Electromagnetism can be expressed in terms of Octonions. We say 7 + 1 instead of 8 to distinguish the single time coordinate (which corresponds to the e_0 direction) from the seven spatial coordinates. There is already a theory of 7 + 1 dimensional Electromagnetism, which is formulated in terms of the Faraday Tensor, which will be explained in a coming section. However, we propose a theory in terms of Octonions, whose source-free equations will have the form of equations 1-4, though with Octonionic fields in place of Quaternionic fields, and with

$$\nabla := \frac{\partial}{\partial x_1}e_1 + \frac{\partial}{\partial x_2}e_2 + \frac{\partial}{\partial x_3}e_3 + \frac{\partial}{\partial x_4}e_4 + \frac{\partial}{\partial x_5}e_5 + \frac{\partial}{\partial x_6}e_6 + \frac{\partial}{\partial x_7}e_7$$

3 The Wave Equation

With the source-free Maxwell equations given, it is useful to find solutions for the Electric and Magnetic Fields. We will show that both fields satisfy the wave equation. We begin by taking the Curl of both sides of equations 2 and 4:

$$\nabla \times \nabla \times \mathbf{E} = -\nabla \times \frac{\partial \mathbf{B}}{\partial t} \tag{5}$$

$$\nabla \times \nabla \times \mathbf{B} = \nabla \times \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} \tag{6}$$

Now, we consider some very useful facts about our differential operators. Even with Octonions, scalars factor out, the curl operator commutes with partial derivatives, and the Curl of the Curl is the Gradient of the Divergence less the Vector Laplacian:

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \quad (7)$$

$$\nabla \times \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} = \mu\epsilon \frac{\partial}{\partial t} (\nabla \times \mathbf{E}) \quad (8)$$

where the gradient is defined as in ordinary Cartesian coordinates—which it can be, seeing as the divergence of an Octonion or Octonionic field has no imaginary component. Therefore, equations 5 and 6 reduce to the following:

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) \quad (9)$$

$$\nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mu\epsilon \frac{\partial}{\partial t} (\nabla \times \mathbf{E}) \quad (10)$$

We can now substitute from equations 1-4 to obtain wave equations for both the Electric and Magnetic fields, which we emphasize is in terms of Octonions.

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (11)$$

$$\nabla^2 \mathbf{B} = \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} \quad (12)$$

4 Wave Equation Solution to Maxwell Solution

It can be shown that the following are solutions to equations 11 and 12²:

$$\mathbf{E} = \tilde{\mathbf{E}} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} \quad (13)$$

$$\mathbf{B} = \tilde{\mathbf{B}} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} \quad (14)$$

Now we have introduced the “wave vector” and “position vector” as \mathbf{k} and \mathbf{x} , respectively, as well as $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$. They are each pure octonions with arbitrary, constant coefficients. These solutions to their respective wave equations can also be constrained to become solutions to the Maxwell equations, which we show now. We begin by taking the divergence of equations 13 and 14, which actually turn out to be exactly what one would expect:

$$\nabla \cdot \mathbf{E} = i\mathbf{k} \cdot \mathbf{E} \quad (15)$$

$$\nabla \cdot \mathbf{B} = i\mathbf{k} \cdot \mathbf{B} \quad (16)$$

²It was later realized that there is some ambiguity in using i in these solutions, as it may correlate to one of the unit vectors in the Vector Space of Octonions: i.e. we may have $i = e_1$. In this case, it may be appropriate to use a solution of the form $\cos(\mathbf{k} \cdot \mathbf{x} - \omega t)$. However, the distinction is nearly inconsequential to what will become the proposed function relating the components of the Octonionic and conventional Faraday tensors, and so the complex exponential form will be used, seeing as it was used in the original calculations.

However, the divergence of both fields are zero by the Maxwell equations: hence, both $\mathbf{k} \cdot \mathbf{E}$ and $\mathbf{k} \cdot \mathbf{B}$ are zero, meaning that the wave vector \mathbf{k} must be orthogonal to both \mathbf{E} and \mathbf{B} . We can impose another crucial condition, and that is using equation 2. The following can be shown:

$$\nabla \times \mathbf{E} = i\mathbf{k} \times \mathbf{E} \quad (17)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -i\omega \mathbf{B} \quad (18)$$

Thus, by equation 2, we get the following:

$$\mathbf{B} = \frac{1}{\omega} \mathbf{k} \times \mathbf{E} \quad (19)$$

Thus, we can write the Magnetic field in terms of the Electric field, which we find of use later.

5 The Octonionic Faraday Tensor

We can now construct an electromagnetic tensor for the Cctonions ($\tilde{F}^{\mu\nu}$), and we will do so by borrowing the formulas from the conventional 3 + 1 dimensional theory:

$$\tilde{F}^{0i} = -\tilde{F}^{i0} = E^i \quad (20)$$

$$\tilde{F}^{ij} = \epsilon_{ijk} B^k \quad (21)$$

which we modify to the following:

$$\tilde{F}^{0i} = -\tilde{F}^{i0} = E^i \quad (22)$$

$$\tilde{F}^{ij} = -\text{Re}(\text{Im}(e_i e_j) B) \quad (23)$$

This gives us a tensor whose indices are shown below in the form of a matrix, in a basis of our choosing. The subscripts in this case indicate the component of the given quantity.

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 \\ -E_1 & 0 & B_3 & -B_2 & B_5 & -B_4 & -B_7 & B_6 \\ -E_2 & -B_3 & 0 & B_1 & B_6 & B_7 & -B_4 & -B_5 \\ -E_3 & B_2 & -B_1 & 0 & B_7 & -B_6 & B_5 & -B_4 \\ -E_4 & -B_5 & -B_6 & -B_7 & 0 & B_1 & B_2 & B_3 \\ -E_5 & B_4 & -B_7 & B_6 & -B_1 & 0 & -B_3 & B_2 \\ -E_6 & B_7 & B_4 & -B_5 & -B_2 & B_3 & 0 & -B_1 \\ -E_7 & -B_6 & B_5 & B_4 & -B_3 & -B_2 & B_1 & 0 \end{bmatrix} \quad (24)$$

Because we can write the components of the Magnetic field in terms of the Electric field, we can express $\tilde{F}^{\mu\nu}$ in terms of the components of \mathbf{k} and \mathbf{E} , which we suppress due to the longevity of the expression.

6 The Source-Free Conventional Maxwell Equations

We now turn our attention to the conventional approach to 7 + 1 dimensional electromagnetism, where there is no cross product. The Maxwell equations have only a tensorial representation, which was the primary motivation behind constructing an Octonionic tensor. In tensor notation, the conventional, source-free Maxwell equations are as follows:

$$F^{\mu\nu}{}_{;\nu} = 0 \quad (25)$$

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0 \quad (26)$$

The solution of equation 26 is the following:

$$F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu} \quad (27)$$

We call \mathbf{A} the vector potential, and now invoke equation 25 to constrain the nature of \mathbf{A} .

$$F^{\mu\nu}{}_{;\nu} = (\partial^\nu A^\mu - \partial^\mu A^\nu)_{;\nu} = \partial_\nu \partial^\nu A^\mu - \partial_\nu \partial^\mu A^\nu = \partial_\nu \partial^\nu A^\mu - \partial_\mu \partial^\nu A^\nu = 0 \quad (28)$$

However, in the Lorenz gauge, the second term in the last expression vanishes, and we are left with

$$\partial_\nu \partial^\nu A^\mu = 0 \quad (29)$$

Which is nothing more than the wave equation. We choose a solution of the form

$$A^\nu = \tilde{\mathbf{A}} e^{i(k \cdot x - \omega t)} \quad (30)$$

Here, $\tilde{\mathbf{A}}$ is a vector with arbitrary, constant coefficients. Now, the Faraday tensor can be written as follows, via equation 27, where n and m are indices ranging from 1 to 7:

$$F^{0,\nu} = -ik^\nu A^0 - i\omega A^\nu \quad (31)$$

$$F^{\mu,0} = ik^\mu A^0 + i\omega A^\mu \quad (32)$$

$$F^{n,m} = ik^m A^n - ik^n A^m \quad (33)$$

$$F^{\mu,\nu} = F^{0,\nu} + F^{\mu,0} + F^{n,m} \quad (34)$$

7 Comparison

Now, we consider whether the components of the Octonionic and conventional Faraday Tensors can be equated. Because both tensors have each entry multiplied by the same exponential expression, what remains to be seen is whether there is a solution for the coefficients $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{E}}$. After analyzing what becomes a rather large system of equations, we propose that the following formula is a solution:

$$E^\nu = -i\omega A^\nu - ik^\nu A^0 \quad (35)$$

This formula has an inverse, though as first sight it may appear as though it does not, due to the seven components on the left and eight on the right. However, recall that \mathbf{A} was chosen to be divergenceless: this implies that

$$i\omega A^0 = ik^n A^n$$

which means that there are only seven independent components of \mathbf{A} . Using the conventional relation between A^0 and the scalar potential V , we then generalize our formula in equation (35) as follows, in correlation with the 3 + 1 dimensional case:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V \tag{36}$$

8 Conclusion and Further Study

We conclude that source-free, 7 + 1 dimensional Electrodynamics can be formulated either conventionally or in terms of Octonions. Where there are interesting differences between the two theories, such as the existence of a Magnetic field, both theories make the same predictions about Electrodynamics in 7 + 1 dimensional space-time.

Now, there is a question as to whether the general Maxwell equations can be formulated in terms of Octonions. Since the source-free equations admit plane wave solutions, it is thought that the general equations may have solutions which are superpositions (possibly infinite) of the plane wave solutions, demonstrating that the general Maxwell equations can indeed be formulated in terms of Octonions. However, that question is left for another study.

References

- [1] Baez, John C. The Octonions. Bulletin of the American Mathematical Society, vol. 39, no. 02, 21 Dec. 2001, pp. 145206., doi:10.1090/s0273-0979-01-00934-x.
- [2] Schafer, Richard D. An Introduction to Nonassociative Algebras. Dover Publications, 2017, Project Gutenberg, <http://www.gutenberg.org/ebooks/25156>
- [3] Doran, Chris, and Anthony N. Lasenby. Geometric Algebra for Physicists. Cambridge Univ. Press, 2013.

9 Appendix

Below is the appendix, which includes all relevant calculations to this report. It has been organized in sections and contains minimal commentary to aid in the study thereof.

```

> with(LieAlgebras):
with(Tensor):
> L1:=AlgebraLibraryData("Octonions",O):
> Oct:=DGsetup(L1,['e0,e1,e2,e3,e4,e5,e6,e7'],[omega]);
Oct := Algebra: O (1)

O > Query(O,"Associative");
false (2)

O > Basis:=[e1,e2,e3,e4,e5,e6,e7];
Basis := [e1, e2, e3, e4, e5, e6, e7] (3)

O > extBasis:=[e0,e1,e2,e3,e4,e5,e6,e7];
extBasis := [e0, e1, e2, e3, e4, e5, e6, e7] (4)

O > MultiplicationTable();

```

O	e0	e1	e2	e3	e4	e5	e6	e7
e0	e0	e1	e2	e3	e4	e5	e6	e7
e1	e1	-e0	e3	-e2	e5	-e4	-e7	e6
e2	e2	-e3	-e0	e1	e6	e7	-e4	-e5
e3	e3	e2	-e1	-e0	e7	-e6	e5	-e4
e4	e4	-e5	-e6	-e7	-e0	e1	e2	e3
e5	e5	e4	-e7	e6	-e1	-e0	-e3	e2
e6	e6	e7	e4	-e5	-e2	e3	-e0	-e1
e7	e7	-e6	e5	e4	-e3	-e2	e1	-e0

(5)

Jacobi identity

In this section, we discover that the traditional Jacobi identity fails.

```

O > A:=DGzip([seq(a[i],i=1..7)],Basis,"plus");
B:=DGzip([seq(b[i],i=1..7)],Basis,"plus");
C:=DGzip([seq(c[i],i=1..7)],Basis,"plus");
A := a1 e1 + a2 e2 + a3 e3 + a4 e4 + a5 e5 + a6 e6 + a7 e7
B := b1 e1 + b2 e2 + b3 e3 + b4 e4 + b5 e5 + b6 e6 + b7 e7
C := c1 e1 + c2 e2 + c3 e3 + c4 e4 + c5 e5 + c6 e6 + c7 e7 (1.1)

```

```

O > evalDG(DGIm(A.DGIm(B.C))+DGIm(C.DGIm(A.B))+DGIm(B.DGIm(C.A)));
(3 a2 b4 c7 - 3 a2 b5 c6 + 3 a2 b6 c5 - 3 a2 b7 c4 - 3 a3 b4 c6 - 3 a3 b5 c7 + 3 a3 b6 c4 + 3 a3 b7 c5 - 3 a4 b2 c7
+ 3 a4 b3 c6 - 3 a4 b6 c3 + 3 a4 b7 c2 + 3 a5 b2 c6 + 3 a5 b3 c7 - 3 a5 b6 c2 - 3 a5 b7 c3 - 3 a6 b2 c5
- 3 a6 b3 c4 + 3 a6 b4 c3 + 3 a6 b5 c2 + 3 a7 b2 c4 - 3 a7 b3 c5 - 3 a7 b4 c2 + 3 a7 b5 c3) e1 - (3 a1 b4 c7
- 3 a1 b5 c6 + 3 a1 b6 c5 - 3 a1 b7 c4 - 3 a3 b4 c5 + 3 a3 b5 c4 + 3 a3 b6 c7 - 3 a3 b7 c6 - 3 a4 b1 c7
+ 3 a4 b3 c5 - 3 a4 b5 c3 + 3 a4 b7 c1 + 3 a5 b1 c6 - 3 a5 b3 c4 + 3 a5 b4 c3 - 3 a5 b6 c1 - 3 a6 b1 c5
- 3 a6 b3 c7 + 3 a6 b5 c1 + 3 a6 b7 c3 + 3 a7 b1 c4 + 3 a7 b3 c6 - 3 a7 b4 c1 - 3 a7 b6 c3) e2 + (3 a1 b4 c6
+ 3 a1 b5 c7 - 3 a1 b6 c4 - 3 a1 b7 c5 - 3 a2 b4 c5 + 3 a2 b5 c4 + 3 a2 b6 c7 - 3 a2 b7 c6 - 3 a4 b1 c6
+ 3 a4 b2 c5 - 3 a4 b5 c2 + 3 a4 b6 c1 - 3 a5 b1 c7 - 3 a5 b2 c4 + 3 a5 b4 c2 + 3 a5 b7 c1 + 3 a6 b1 c4
- 3 a6 b2 c7 - 3 a6 b4 c1 + 3 a6 b7 c2 + 3 a7 b1 c5 + 3 a7 b2 c6 - 3 a7 b5 c1 - 3 a7 b6 c2) e3 + (3 a1 b2 c7
- 3 a1 b3 c6 + 3 a1 b6 c3 - 3 a1 b7 c2 - 3 a2 b1 c7 + 3 a2 b3 c5 - 3 a2 b5 c3 + 3 a2 b7 c1 + 3 a3 b1 c6
- 3 a3 b2 c5 + 3 a3 b5 c2 - 3 a3 b6 c1 + 3 a5 b2 c3 - 3 a5 b3 c2 - 3 a5 b6 c7 + 3 a5 b7 c6 - 3 a6 b1 c3
+ 3 a6 b3 c1 + 3 a6 b5 c7 - 3 a6 b7 c5 + 3 a7 b1 c2 - 3 a7 b2 c1 - 3 a7 b5 c6 + 3 a7 b6 c5) e4 - (3 a1 b2 c6

```

(1.2)

$$\begin{aligned}
& + 3 a_1 b_3 c_7 - 3 a_1 b_6 c_2 - 3 a_1 b_7 c_3 - 3 a_2 b_1 c_6 + 3 a_2 b_3 c_4 - 3 a_2 b_4 c_3 + 3 a_2 b_6 c_1 - 3 a_3 b_1 c_7 \\
& - 3 a_3 b_2 c_4 + 3 a_3 b_4 c_2 + 3 a_3 b_7 c_1 + 3 a_4 b_2 c_3 - 3 a_4 b_3 c_2 - 3 a_4 b_6 c_7 + 3 a_4 b_7 c_6 + 3 a_6 b_1 c_2 \\
& - 3 a_6 b_2 c_1 + 3 a_6 b_4 c_7 - 3 a_6 b_7 c_4 + 3 a_7 b_1 c_3 - 3 a_7 b_3 c_1 - 3 a_7 b_4 c_6 + 3 a_7 b_6 c_4) e5 + (3 a_1 b_2 c_5 \\
& + 3 a_1 b_3 c_4 - 3 a_1 b_4 c_3 - 3 a_1 b_5 c_2 - 3 a_2 b_1 c_5 - 3 a_2 b_3 c_7 + 3 a_2 b_5 c_1 + 3 a_2 b_7 c_3 - 3 a_3 b_1 c_4 \\
& + 3 a_3 b_2 c_7 + 3 a_3 b_4 c_1 - 3 a_3 b_7 c_2 + 3 a_4 b_1 c_3 - 3 a_4 b_3 c_1 - 3 a_4 b_5 c_7 + 3 a_4 b_7 c_5 + 3 a_5 b_1 c_2 \\
& - 3 a_5 b_2 c_1 + 3 a_5 b_4 c_7 - 3 a_5 b_7 c_4 - 3 a_7 b_2 c_3 + 3 a_7 b_3 c_2 - 3 a_7 b_4 c_5 + 3 a_7 b_5 c_4) e6 - (3 a_1 b_2 c_4 \\
& - 3 a_1 b_3 c_5 - 3 a_1 b_4 c_2 + 3 a_1 b_5 c_3 - 3 a_2 b_1 c_4 - 3 a_2 b_3 c_6 + 3 a_2 b_4 c_1 + 3 a_2 b_6 c_3 + 3 a_3 b_1 c_5 \\
& + 3 a_3 b_2 c_6 - 3 a_3 b_5 c_1 - 3 a_3 b_6 c_2 + 3 a_4 b_1 c_2 - 3 a_4 b_2 c_1 - 3 a_4 b_5 c_6 + 3 a_4 b_6 c_5 - 3 a_5 b_1 c_3 \\
& + 3 a_5 b_3 c_1 + 3 a_5 b_4 c_6 - 3 a_5 b_6 c_4 - 3 a_6 b_2 c_3 + 3 a_6 b_3 c_2 - 3 a_6 b_4 c_5 + 3 a_6 b_5 c_4) e7
\end{aligned}$$

Orthogonality

Here, we define orthogonality.

$$\mathbf{O} > \text{evalDG}(\text{DGRe}(\mathbf{A} \cdot \text{DGIm}(\mathbf{A} \cdot \mathbf{B}))); \quad 0 \ e0 \quad (2.1)$$

$$\mathbf{O} > \text{evalDG}(\text{DGRe}(\text{DGIm}(\mathbf{A} \cdot \mathbf{B}) \cdot \mathbf{B})); \quad 0 \ e0 \quad (2.2)$$

Magnitude

Here, we verify different formulas for the magnitude.

This is $(\text{abs}(\mathbf{AxB}))^2$:

$$\mathbf{O} > \mathbf{AxB} := \text{DGIm}(\mathbf{A} \cdot \mathbf{B}); \quad (3.1)$$

$$\begin{aligned}
\mathbf{AxB} := & (a_2 b_3 - a_3 b_2 + a_4 b_5 - a_5 b_4 - a_6 b_7 + a_7 b_6) e1 - (a_1 b_3 - a_3 b_1 - a_4 b_6 - a_5 b_7 + a_6 b_4 + a_7 b_5) \\
&) e2 + (a_1 b_2 - a_2 b_1 + a_4 b_7 - a_5 b_6 + a_6 b_5 - a_7 b_4) e3 - (a_1 b_5 + a_2 b_6 + a_3 b_7 - a_5 b_1 - a_6 b_2 \\
& - a_7 b_3) e4 + (a_1 b_4 - a_2 b_7 + a_3 b_6 - a_4 b_1 - a_6 b_3 + a_7 b_2) e5 + (a_1 b_7 + a_2 b_4 - a_3 b_5 - a_4 b_2 \\
& + a_5 b_3 - a_7 b_1) e6 - (a_1 b_6 - a_2 b_5 - a_3 b_4 + a_4 b_3 + a_5 b_2 - a_6 b_1) e7
\end{aligned}$$

$$\mathbf{O} > \text{evalDG}(-\text{DGRe}(e1 \cdot \mathbf{AxB})); \quad (a_2 b_3 - a_3 b_2 + a_4 b_5 - a_5 b_4 - a_6 b_7 + a_7 b_6) e0 \quad (3.2)$$

$$\mathbf{O} > \text{comp} := \text{GetComponents}(\mathbf{AxB}, \text{Basis}); \quad (3.3)$$

$$\begin{aligned}
\text{comp} := & [a_2 b_3 - a_3 b_2 + a_4 b_5 - a_5 b_4 - a_6 b_7 + a_7 b_6, -a_1 b_3 + a_3 b_1 + a_4 b_6 + a_5 b_7 - a_6 b_4 - a_7 b_5, a_1 b_2 \\
& - a_2 b_1 + a_4 b_7 - a_5 b_6 + a_6 b_5 - a_7 b_4, -a_1 b_5 - a_2 b_6 - a_3 b_7 + a_5 b_1 + a_6 b_2 + a_7 b_3, a_1 b_4 - a_2 b_7 \\
& + a_3 b_6 - a_4 b_1 - a_6 b_3 + a_7 b_2, a_1 b_7 + a_2 b_4 - a_3 b_5 - a_4 b_2 + a_5 b_3 - a_7 b_1, -a_1 b_6 + a_2 b_5 + a_3 b_4 \\
& - a_4 b_3 - a_5 b_2 + a_6 b_1]
\end{aligned}$$

$$\mathbf{O} > \text{sqmagAxB} := \text{sum}((\text{comp}[i])^2, i=1..nops(\text{comp})); \quad (3.4)$$

$$\begin{aligned}
\text{sqmagAxB} := & (a_2 b_3 - a_3 b_2 + a_4 b_5 - a_5 b_4 - a_6 b_7 + a_7 b_6)^2 + (-a_1 b_3 + a_3 b_1 + a_4 b_6 + a_5 b_7 - a_6 b_4 \\
& - a_7 b_5)^2 + (a_1 b_2 - a_2 b_1 + a_4 b_7 - a_5 b_6 + a_6 b_5 - a_7 b_4)^2 + (-a_1 b_5 - a_2 b_6 - a_3 b_7 + a_5 b_1 \\
& + a_6 b_2 + a_7 b_3)^2 + (a_1 b_4 - a_2 b_7 + a_3 b_6 - a_4 b_1 - a_6 b_3 + a_7 b_2)^2 + (a_1 b_7 + a_2 b_4 - a_3 b_5 \\
& - a_4 b_2 + a_5 b_3 - a_7 b_1)^2 + (-a_1 b_6 + a_2 b_5 + a_3 b_4 - a_4 b_3 - a_5 b_2 + a_6 b_1)^2
\end{aligned}$$

The following is the proposed formula for the magnitude (squared):

$$\mathbf{O} > \text{AdotB} := \text{GetComponents}(\text{evalDG}(-\text{DGRe}(\mathbf{A} \cdot \mathbf{B})), [e0])[1]; \quad (3.5)$$

$$\text{AdotB} := a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 + a_5 b_5 + a_6 b_6 + a_7 b_7 \quad (3.5)$$

```
O > compA:=GetComponents(A,Basis);
sqmagA:=sum((compA[i])^2,i=1..nops(compA));
      compA := [a1, a2, a3, a4, a5, a6, a7]
      sqmagA := a1^2 + a2^2 + a3^2 + a4^2 + a5^2 + a6^2 + a7^2
```

$$(3.6)$$

```
O > compB:=GetComponents(B,Basis);
sqmagB:=sum((compB[i])^2,i=1..nops(compB));
      compB := [b1, b2, b3, b4, b5, b6, b7]
      sqmagB := b1^2 + b2^2 + b3^2 + b4^2 + b5^2 + b6^2 + b7^2
```

$$(3.7)$$

```
O > costhetasq:=((AdotB)^2)/(sqmagA * sqmagB);
sinthetasq:=1-costhetasq;
```

$$\text{costhetasq} := \frac{(a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 + a_5 b_5 + a_6 b_6 + a_7 b_7)^2}{(a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 + a_7^2) (b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2)}$$

$$\text{sinthetasq} := 1 - \frac{(a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 + a_5 b_5 + a_6 b_6 + a_7 b_7)^2}{(a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 + a_7^2) (b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2)}$$

$$(3.8)$$

```
O > magsq:=simplify((sqmagA) * (sqmagB) * sinthetasq);
magsq := (b2^2 + b3^2 + b4^2 + b5^2 + b6^2 + b7^2) a1^2 - 2 b1 (a2 b2 + a3 b3 + a4 b4 + a5 b5 + a6 b6 + a7 b7) a1 + (
      b1^2 + b3^2 + b4^2 + b5^2 + b6^2 + b7^2) a2^2 - 2 b2 (a3 b3 + a4 b4 + a5 b5 + a6 b6 + a7 b7) a2 + (b1^2 + b2^2 + b4^2
      + b5^2 + b6^2 + b7^2) a3^2 - 2 b3 (a4 b4 + a5 b5 + a6 b6 + a7 b7) a3 + (b1^2 + b2^2 + b3^2 + b5^2 + b6^2 + b7^2) a4^2
      - 2 b4 (a5 b5 + a6 b6 + a7 b7) a4 + (b1^2 + b2^2 + b3^2 + b4^2 + b6^2 + b7^2) a5^2 - 2 b5 (a6 b6 + a7 b7) a5 + (
      b1^2 + b2^2 + b3^2 + b4^2 + b5^2 + b7^2) a6^2 - 2 a6 a7 b6 b7 + a7^2 (b1^2 + b2^2 + b3^2 + b4^2 + b5^2 + b6^2)
```

$$(3.9)$$

```
O > simplify(magsq-sqmagAxB);
      0
```

$$(3.10)$$

Vector Procedures: curl, etc

This section defines procedures that were central to this project. One must execute this section before examining the "Maxwell's Equations" section.

```
O > Z:=DGzip([seq(z[i](x0,x1,x2,x3,x4,x5,x6,x7),i=0..7)],extBasis,"plus");
Z := z0(x0,x1,x2,x3,x4,x5,x6,x7) e0 + z1(x0,x1,x2,x3,x4,x5,x6,x7) e1 + z2(x0,x1,x2,x3,x4,x5,x6,
      x7) e2 + z3(x0,x1,x2,x3,x4,x5,x6,x7) e3 + z4(x0,x1,x2,x3,x4,x5,x6,x7) e4 + z5(x0,x1,x2,x3,x4,
      x5,x6,x7) e5 + z6(x0,x1,x2,x3,x4,x5,x6,x7) e6 + z7(x0,x1,x2,x3,x4,x5,x6,x7) e7
```

$$(4.1)$$

```
O > Curl:= proc(G)
      evalDG(add(evalDG(add(evalDG(diff(GetComponents(DGIm(G),Basis)[j],x||i)*
      DGIm(e||i.e||j)),i=1..7)),j=1..7));
      end proc;
```

$$(4.2)$$

```
Curl := proc(G)
      DifferentialGeometry:-evalDG( add( DifferentialGeometry:-evalDG( add( DifferentialGeometry:-
      evalDG( ( diff( DifferentialGeometry:-GetComponents( DifferentialGeometry:-DGIm(G), Basis) [j], x
      || i ) ) * DifferentialGeometry:-DGIm(e||i . e||j) ), i=1..7) ), j=1..7) )
```

```
end proc
O > Div:= proc(G)
      evalDG(-1*add(evalDG(add(evalDG(diff(GetComponents(DGIm(G),Basis)[j],
      x||i)*DGRe(e||i.e||j)),i=1..7)),j=1..7));
```

```

end proc;
Div := proc( G)
DifferentialGeometry:-evalDG( - add( DifferentialGeometry:-evalDG( add( DifferentialGeometry:-
evalDG( ( diff( DifferentialGeometry:-GetComponents( DifferentialGeometry:-DGIm( G ), Basis ) [j], x
|| i ) * DifferentialGeometry:-DGRe( e || i . e || j ) , i = 1 ..7 ) , j = 1 ..7 ) )

```

(4.3)

```

end proc

```

```

O > Grad := proc(G)
  DGzip([seq(diff(GetComponents(DGRe(G),extBasis)[1],x||i),i=1..7)],Basis,
  "plus");
end proc;

```

```

Grad := proc( G)

```

(4.4)

```

DifferentialGeometry:-DGzip( [ seq( diff( DifferentialGeometry:-GetComponents( DifferentialGeometry:-
DGRe( G ), extBasis ) [ 1 ], x || i ), i = 1 ..7 ) ], Basis, "plus")

```

```

end proc

```

```

O > VLaplacian := proc(G)
  DGzip([seq(add(seq(diff(GetComponents(DGIm(G),Basis)[i],x||j$2),i=1..7),
  j=1..7))],Basis,"plus");
end proc;

```

```

VLaplacian := proc( G)

```

(4.5)

```

DifferentialGeometry:-DGzip( [ seq( add( seq( diff( DifferentialGeometry:-
GetComponents( DifferentialGeometry:-DGIm( G ), Basis ) [ i ], x || j$2 ) , i = 1 ..7 ) , j = 1 ..7 ) ) ], Basis,
"plus")

```

```

end proc

```

```

O > evalDG(Curl(Curl(Z))+VLaplacian(Z)-Grad(Div(Z)));

```

$$0 \ e0$$

(4.6)

The following is the "generalized" operator. Caution: the sign of the divergence is now negative.

```

O > DivNcurl := proc(G)
  evalDG(add(evalDG(add(evalDG(diff(GetComponents(DGIm(G),Basis)[j],x||i)*
  evalDG( e || i . e || j ) ) , i = 1 ..7 ) , j = 1 ..7 ) );
end proc;

```

```

DivNcurl := proc( G)

```

(4.7)

```

DifferentialGeometry:-evalDG( add( DifferentialGeometry:-evalDG( add( DifferentialGeometry:-
evalDG( ( diff( DifferentialGeometry:-GetComponents( DifferentialGeometry:-DGIm( G ), Basis ) [j], x
|| i ) * DifferentialGeometry:-evalDG( e || i . e || j ) ) , i = 1 ..7 ) , j = 1 ..7 ) )

```

```

end proc

```

```

O > evalDG(evalDG(-Div(Z)+Curl(Z))-DivNcurl(Z))

```

$$0$$

(4.8)

Maxwell's Equations

This section is the bulk of the project. We first define Maxwell's equations using the vector procedures. We then check for plane wave solutions and constrain them using Maxwell's equations. We conclude by constructing an Octonionic Faraday tensor in tandem with the conventional faraday tensor and show that they can be equated.

Define the equations

```

O > E := DGzip([seq(Epsilon[i](t,x1,x2,x3,x4,x5,x6,x7),i=1..7)],Basis,
  "plus");
B := DGzip([seq(beta[i](t,x1,x2,x3,x4,x5,x6,x7),i=1..7)],Basis,"plus");
E := E1(t,x1,x2,x3,x4,x5,x6,x7) e1 + E2(t,x1,x2,x3,x4,x5,x6,x7) e2 + E3(t,x1,x2,x3,x4,x5,x6,

```


$$x7) e3 + E_4(t, x1, x2, x3, x4, x5, x6, x7) e4 + E_5(t, x1, x2, x3, x4, x5, x6, x7) e5 + E_6(t, x1, x2, x3, x4, x5, x6, x7) e6 + E_7(t, x1, x2, x3, x4, x5, x6, x7) e7$$

$$B := \beta_1(t, x1, x2, x3, x4, x5, x6, x7) e1 + \beta_2(t, x1, x2, x3, x4, x5, x6, x7) e2 + \beta_3(t, x1, x2, x3, x4, x5, x6, x7) e3 + \beta_4(t, x1, x2, x3, x4, x5, x6, x7) e4 + \beta_5(t, x1, x2, x3, x4, x5, x6, x7) e5 + \beta_6(t, x1, x2, x3, x4, x5, x6, x7) e6 + \beta_7(t, x1, x2, x3, x4, x5, x6, x7) e7 \quad (5.1.1)$$

$$\mathbf{O} > \mathbf{J} := \mathbf{DGzip}([\mathbf{seq}(j[i](t, x1, x2, x3, x4, x5, x6, x7), i=1..7)], \mathbf{Basis}, "plus");$$

$$J := j_1(t, x1, x2, x3, x4, x5, x6, x7) e1 + j_2(t, x1, x2, x3, x4, x5, x6, x7) e2 + j_3(t, x1, x2, x3, x4, x5, x6, x7) e3 + j_4(t, x1, x2, x3, x4, x5, x6, x7) e4 + j_5(t, x1, x2, x3, x4, x5, x6, x7) e5 + j_6(t, x1, x2, x3, x4, x5, x6, x7) e6 + j_7(t, x1, x2, x3, x4, x5, x6, x7) e7 \quad (5.1.2)$$

$$\mathbf{O} > \mathbf{dBdt} := \mathbf{DGzip}([\mathbf{seq}(\mathbf{diff}(\mathbf{GetComponents}(\mathbf{B}, \mathbf{Basis})[i], t), i=1..7)], \mathbf{Basis}, "plus");$$

$$\mathbf{dEdt} := \mathbf{DGzip}([\mathbf{seq}(\mathbf{diff}(\mathbf{GetComponents}(\mathbf{E}, \mathbf{Basis})[i], t), i=1..7)], \mathbf{Basis}, "plus");$$

$$dBdt := \frac{\partial}{\partial t} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) e1 + \frac{\partial}{\partial t} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) e2 + \frac{\partial}{\partial t} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) e3 + \frac{\partial}{\partial t} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) e4 + \frac{\partial}{\partial t} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) e5 + \frac{\partial}{\partial t} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) e6 + \frac{\partial}{\partial t} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) e7$$

$$dEdt := \frac{\partial}{\partial t} E_1(t, x1, x2, x3, x4, x5, x6, x7) e1 + \frac{\partial}{\partial t} E_2(t, x1, x2, x3, x4, x5, x6, x7) e2 + \frac{\partial}{\partial t} E_3(t, x1, x2, x3, x4, x5, x6, x7) e3 + \frac{\partial}{\partial t} E_4(t, x1, x2, x3, x4, x5, x6, x7) e4 + \frac{\partial}{\partial t} E_5(t, x1, x2, x3, x4, x5, x6, x7) e5 + \frac{\partial}{\partial t} E_6(t, x1, x2, x3, x4, x5, x6, x7) e6 + \frac{\partial}{\partial t} E_7(t, x1, x2, x3, x4, x5, x6, x7) e7 \quad (5.1.3)$$

$$\mathbf{O} > \mathbf{eq1} := \mathbf{Div}(\mathbf{E}) = \mathbf{evalDG}((\mathbf{rho}(t, \mathbf{seq}(x||i, i=1..7)))/\mathbf{epsilon}) * \mathbf{e0}:$$

$$\mathbf{eq2} := \mathbf{Curl}(\mathbf{E}) = \mathbf{evalDG}(-1 * \mathbf{dBdt}):$$

$$\mathbf{eq3} := \mathbf{Div}(\mathbf{B}) = \mathbf{evalDG}(0 * \mathbf{e0}):$$

$$\mathbf{eq4} := \mathbf{Curl}(\mathbf{B}) = \mathbf{evalDG}(\mathbf{evalDG}(\mu * \mathbf{J}) + \mathbf{evalDG}(\mu * \mathbf{epsilon} * \mathbf{dEdt})):$$

$$\mathbf{O} > \mathbf{eqn1} := \mathbf{DivNcurl}(\mathbf{E}) = \mathbf{evalDG}(-1 * \mathbf{evalDG}((\mathbf{rho}(t, \mathbf{seq}(x||i, i=1..7)))/\mathbf{epsilon}) * \mathbf{e0}) + \mathbf{evalDG}(-1 * \mathbf{dBdt}));$$

$$\mathbf{eqn1} := - \left(\frac{\partial}{\partial x1} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x3} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x4} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x5} E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x6} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x7} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) e0 - \left(\frac{\partial}{\partial x3} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x2} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x5} E_4(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x4} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x7} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x6} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) e1 + \left(\frac{\partial}{\partial x3} E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x1} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x6} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x7} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x4} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x5} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) e2 - \left(\frac{\partial}{\partial x2} E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x1} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x7} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right) e3$$

$$\begin{aligned}
& x5, x6, x7) - \left(\frac{\partial}{\partial x6} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x5} E_6(t, x1, x2, x3, x4, x5, x6, x7) \\
& - \left(\frac{\partial}{\partial x4} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) e3 + \left(\frac{\partial}{\partial x5} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x6} \right. \\
& E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x7} E_3(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x1} E_5(t, x1, x2, x3, x4, \right. \\
& x5, x6, x7) \left. \right) - \left(\frac{\partial}{\partial x2} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x3} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) \\
& \left. \right) e4 - \left(\frac{\partial}{\partial x4} E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x7} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x6} \right. \\
& E_3(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x1} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x3} E_6(t, x1, x2, x3, \right. \\
& x4, x5, x6, x7) \left. \right) + \frac{\partial}{\partial x2} E_7(t, x1, x2, x3, x4, x5, x6, x7) \left. \right) e5 - \left(\frac{\partial}{\partial x7} E_1(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& + \frac{\partial}{\partial x4} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x5} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x2} E_4(t, x1, \right. \\
& x2, x3, x4, x5, x6, x7) \left. \right) + \frac{\partial}{\partial x3} E_5(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x1} E_7(t, x1, x2, x3, x4, x5, x6, \right. \\
& x7) \left. \right) \left. \right) e6 + \left(\frac{\partial}{\partial x6} E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x5} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right) \right. \\
& - \left(\frac{\partial}{\partial x4} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x3} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x2} E_5(t, x1, \\
& x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x1} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) \left. \right) e7 = \\
& - \frac{\rho(t, x1, x2, x3, x4, x5, x6, x7)}{\epsilon} e0 - \frac{\partial}{\partial t} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) e1 - \frac{\partial}{\partial t} \beta_2(t, x1, x2, x3, \\
& x4, x5, x6, x7) e2 - \frac{\partial}{\partial t} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) e3 - \frac{\partial}{\partial t} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) e4 \\
& - \frac{\partial}{\partial t} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) e5 - \frac{\partial}{\partial t} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) e6 - \frac{\partial}{\partial t} \beta_7(t, x1, x2, \\
& x3, x4, x5, x6, x7) e7
\end{aligned}$$

O > eqn2:=DivNcurl(B)=evalDG(-1*evalDG(0*e0)+evalDG(evalDG(mu*J)+evalDG(mu*epsilon*dEdt)));

$$\begin{aligned}
eqn2 := & - \left(\frac{\partial}{\partial x1} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x2} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x3} \beta_3(t, x1, \right. & \mathbf{(5.1.5)} \\
& x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x4} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x5} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) \\
& + \frac{\partial}{\partial x6} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x7} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) \left. \right) e0 - \left(\frac{\partial}{\partial x3} \beta_2(t, x1, \right. \\
& x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x2} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x5} \beta_4(t, x1, x2, x3, x4, x5, x6, \\
& x7) - \left(\frac{\partial}{\partial x4} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x7} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial}{\partial x6} \\
& \beta_7(t, x1, x2, x3, x4, x5, x6, x7) \left. \right) e1 + \left(\frac{\partial}{\partial x3} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial}{\partial x1} \beta_3(t, x1, x2, x3, \right. \\
& x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x6} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial}{\partial x7} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) \right) \\
& + \frac{\partial}{\partial x4} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial}{\partial x5} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) \left. \right) e2 - \left(\frac{\partial}{\partial x2} \beta_1(t, x1, \right.
\end{aligned}$$

$$\begin{aligned}
& x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_7} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) - \left(\frac{\partial}{\partial x_6} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_5} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_4} \beta_7(t, \right. \\
& \left. x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \Big) e_3 + \left(\frac{\partial}{\partial x_5} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_6} \beta_2(t, x_1, x_2, x_3, x_4, \right. \\
& \left. x_5, x_6, x_7) + \frac{\partial}{\partial x_7} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_2} \right. \right. \\
& \left. \left. \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_3} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_4 - \left(\frac{\partial}{\partial x_4} \beta_1(t, x_1, x_2, \right. \\
& \left. x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_7} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_6} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& \left. - \left(\frac{\partial}{\partial x_1} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_3} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_2} \beta_7(t, \right. \\
& \left. x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_5 - \left(\frac{\partial}{\partial x_7} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_4} \beta_2(t, x_1, x_2, x_3, x_4, x_5, \right. \\
& \left. x_6, x_7) - \left(\frac{\partial}{\partial x_5} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_3} \right. \\
& \left. \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_6 + \left(\frac{\partial}{\partial x_6} \beta_1(t, x_1, x_2, \right. \\
& \left. x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_5} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_4} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \right. \\
& \left. \left. x_7) \right) + \frac{\partial}{\partial x_3} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \beta_6(t, \right. \right. \\
& \left. \left. x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_7 = \mu \left(\left(\frac{\partial}{\partial t} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \epsilon + j_1(t, x_1, x_2, x_3, x_4, x_5, \right. \\
& \left. x_6, x_7) \right) e_1 + \mu \left(\left(\frac{\partial}{\partial t} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \epsilon + j_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_2 \\
& + \mu \left(\left(\frac{\partial}{\partial t} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \epsilon + j_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_3 + \mu \left(\left(\frac{\partial}{\partial t} E_4(t, \right. \right. \\
& \left. \left. x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \epsilon + j_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_4 + \mu \left(\left(\frac{\partial}{\partial t} E_5(t, x_1, x_2, x_3, x_4, x_5, \right. \right. \\
& \left. \left. x_6, x_7) \right) \epsilon + j_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_5 + \mu \left(\left(\frac{\partial}{\partial t} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \epsilon + j_6(t, \right. \\
& \left. x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_6 + \mu \left(\left(\frac{\partial}{\partial t} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \epsilon + j_7(t, x_1, x_2, x_3, x_4, x_5, \right. \\
& \left. x_6, x_7) \right) e_7
\end{aligned}$$

$$\begin{aligned}
\mathbf{O} &> \text{evalDG}(\text{eqn1}+\text{eq1}-\text{eq2}); \\
&\text{evalDG}(\text{rhs}(\text{eqn1})+\text{rhs}(\text{eq1})-\text{rhs}(\text{eq2})); \\
&\text{evalDG}(\text{lhs}(\text{eqn1})+\text{lhs}(\text{eq1})-\text{lhs}(\text{eq2})); \\
&0 \text{ e0} = 0 \text{ e0} \\
&0 \text{ e0} \\
&0 \text{ e0}
\end{aligned}$$

(5.1.6)

$$\begin{aligned}
\mathbf{O} &> \text{DivNcurl}(\text{lhs}(\text{eq1})); \\
&\text{DivNcurl}(\text{rhs}(\text{eq1})); \\
&\text{DivNcurl}(\text{lhs}(\text{eq3})); \\
&\text{DivNcurl}(\text{rhs}(\text{eq3})); \\
&0 \\
&0 \\
&0
\end{aligned}$$

Wave Equation in Vacuum?

This section explores whether the wave equation can be recovered for the Electric and Magnetic fields. This discussion relies heavily on what vector identities hold for octonions.

Electric Field

O > s1:=DivNcurl(lhs(eqn1))=DivNcurl(rhs(eqn1));

$$\begin{aligned}
 s1 := & - \left(\frac{\partial^2}{\partial x^3^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x3} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) \right. & (5.2.1.1) \\
 & + \frac{\partial^2}{\partial x^2^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x2} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^5^2} \\
 & E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x5} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^4^2} E_1(t, x1, \\
 & x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x4} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^7^2} E_1(t, x1, x2, x3, \\
 & x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x7} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^6^2} E_1(t, x1, x2, x3, x4, x5, \\
 & x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x6} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) \Big) e1 - \left(\frac{\partial^2}{\partial x^3^2} E_2(t, x1, x2, x3, x4, x5, x6, \right. \\
 & x7) - \left(\frac{\partial^2}{\partial x2 \partial x3} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial^2}{\partial x1 \partial x2} E_1(t, x1, x2, x3, x4, x5, x6, x7) \right) \\
 & + \frac{\partial^2}{\partial x^1^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x6} \right. \\
 & E_6(t, x1, x2, x3, x4, x5, x6, x7) \Big) + \frac{\partial^2}{\partial x^7^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x7} E_7(t, x1, \right. \\
 & x2, x3, x4, x5, x6, x7) \Big) + \frac{\partial^2}{\partial x^4^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x4} E_4(t, x1, x2, x3, \right. \\
 & x4, x5, x6, x7) \Big) + \frac{\partial^2}{\partial x^5^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x5} E_5(t, x1, x2, x3, x4, x5, \right. \\
 & x6, x7) \Big) \Big) e2 - \left(- \left(\frac{\partial^2}{\partial x2 \partial x3} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^2^2} E_3(t, x1, x2, x3, x4, x5, \right. \\
 & x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x3} E_1(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^1^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) \\
 & + \frac{\partial^2}{\partial x^7^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x7} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^6^2} \\
 & E_3(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x6} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^5^2} E_3(t, x1, \\
 & x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x5} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^4^2} E_3(t, x1, x2, x3, \\
 & x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x4} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right) \Big) e3 - \left(\frac{\partial^2}{\partial x^5^2} E_4(t, x1, x2, x3, x4, \right. \\
 & x5, x6, x7) - \left(\frac{\partial^2}{\partial x4 \partial x5} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^6^2} E_4(t, x1, x2, x3, x4, x5, x6, \\
 & x7) - \left(\frac{\partial^2}{\partial x4 \partial x6} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^7^2} E_4(t, x1, x2, x3, x4, x5, x6, x7)
 \end{aligned}$$

$$\begin{aligned}
& x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_6} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_0 \\
& + \left(\frac{\partial^2}{\partial t \partial x_3} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_5} \right. \\
& \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_7} \beta_6(t, x_1, \right. \\
& x_2, x_3, x_4, x_5, x_6, x_7) \Big) + \frac{\partial^2}{\partial t \partial x_6} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_1 - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_1(t, x_1, x_2, x_3, \right. \\
& x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_6} \beta_4(t, x_1, x_2, x_3, x_4, x_5, \right. \\
& x_6, x_7) \Big) - \left(\frac{\partial^2}{\partial t \partial x_7} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_4} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\
& + \frac{\partial^2}{\partial t \partial x_5} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_2 + \left(\frac{\partial^2}{\partial t \partial x_2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_7} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_6} \right. \\
& \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) + \frac{\partial^2}{\partial t \partial x_5} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_7(t, x_1, \right. \\
& x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) e_3 - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_6} \beta_2(t, x_1, x_2, x_3, \right. \\
& x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& x_7) \Big) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \\
& \Big) e_4 + \left(\frac{\partial^2}{\partial t \partial x_4} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_7} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. \\
& + \frac{\partial^2}{\partial t \partial x_6} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_3} \right. \\
& \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) + \frac{\partial^2}{\partial t \partial x_2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_5 + \left(\frac{\partial^2}{\partial t \partial x_7} \beta_1(t, \right. \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_4} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_3(t, x_1, x_2, x_3, \right. \\
& x_4, x_5, x_6, x_7) \Big) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_3} \beta_5(t, x_1, x_2, x_3, x_4, x_5, \\
& x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \Big) e_6 - \left(\frac{\partial^2}{\partial t \partial x_6} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& x_7) - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \\
& + \frac{\partial^2}{\partial t \partial x_3} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \right. \\
& \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) e_7
\end{aligned}$$

O > GetComponent(rhs(s1),extBasis)[1]

$$\begin{aligned}
& \frac{\partial^2}{\partial x_1 \partial t} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2 \partial t} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3 \partial t} \beta_3(t, x_1, x_2, \quad (5.2.1.2) \\
& x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4 \partial t} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5 \partial t} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) + \frac{\partial^2}{\partial x_6 \partial t} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7 \partial t} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7)
\end{aligned}$$

Because the Divergence of B is zero:

O > s2:=evalDG(subs(GetComponents(rhs(s1),extBasis)[1]=0,s1));

$$\begin{aligned}
 s2 := & - \left(\frac{\partial^2}{\partial x^3} E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x3} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) \right. \\
 & + \frac{\partial^2}{\partial x^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x2} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^5} \\
 & E_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x5} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^4} E_1(t, x1, \\
 & x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x4} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^7} E_1(t, x1, x2, x3, \\
 & x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x7} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^6} E_1(t, x1, x2, x3, x4, x5, \\
 & x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x6} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) \left. \right) e1 - \left(\frac{\partial^2}{\partial x^3} E_2(t, x1, x2, x3, x4, x5, x6, \right. \\
 & x7) - \left(\frac{\partial^2}{\partial x2 \partial x3} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial^2}{\partial x1 \partial x2} E_1(t, x1, x2, x3, x4, x5, x6, x7) \right) \\
 & + \frac{\partial^2}{\partial x^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x6} \right. \\
 & E_6(t, x1, x2, x3, x4, x5, x6, x7) \left. \right) + \frac{\partial^2}{\partial x^7} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x7} E_7(t, x1, \right. \\
 & x2, x3, x4, x5, x6, x7) \left. \right) + \frac{\partial^2}{\partial x^4} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x4} E_4(t, x1, x2, x3, \right. \\
 & x4, x5, x6, x7) \left. \right) + \frac{\partial^2}{\partial x^5} E_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x5} E_5(t, x1, x2, x3, x4, x5, \right. \\
 & x6, x7) \left. \right) \left. \right) e2 - \left(- \left(\frac{\partial^2}{\partial x2 \partial x3} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^2} E_3(t, x1, x2, x3, x4, x5, \right. \\
 & x6, x7) - \left(\frac{\partial^2}{\partial x1 \partial x3} E_1(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) \\
 & + \frac{\partial^2}{\partial x^7} E_3(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x7} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^6} \\
 & E_3(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x6} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^5} E_3(t, x1, \\
 & x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x5} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^4} E_3(t, x1, x2, x3, \\
 & x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x4} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right) \left. \right) e3 - \left(\frac{\partial^2}{\partial x^5} E_4(t, x1, x2, x3, x4, \right. \\
 & x5, x6, x7) - \left(\frac{\partial^2}{\partial x4 \partial x5} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^6} E_4(t, x1, x2, x3, x4, x5, x6, \\
 & x7) - \left(\frac{\partial^2}{\partial x4 \partial x6} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^7} E_4(t, x1, x2, x3, x4, x5, x6, x7) \\
 & - \left(\frac{\partial^2}{\partial x4 \partial x7} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial^2}{\partial x1 \partial x4} E_1(t, x1, x2, x3, x4, x5, x6, x7) \right) \\
 & + \frac{\partial^2}{\partial x^2} E_4(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x2 \partial x4} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^2} \\
 & E_4(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial x3 \partial x4} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^3} E_4(t, x1, \\
 & x2, x3, x4, x5, x6, x7) \left. \right) e4 - \left(- \left(\frac{\partial^2}{\partial x4 \partial x5} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial x^4} E_5(t, x1, \right.
 \end{aligned}$$

$$\begin{aligned}
& - \left(\frac{\partial^2}{\partial t \partial x_6} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_5} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} \right. \\
& \left. \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_3 - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_6} \beta_2(t, \right. \\
& \left. x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_5(t, x_1, x_2, x_3, \right. \right. \\
& \left. \left. x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_7(t, x_1, x_2, x_3, x_4, x_5, \right. \right. \\
& \left. \left. x_6, x_7) \right) \right) e_4 + \left(\frac{\partial^2}{\partial t \partial x_4} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_7} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \right. \\
& \left. \left. x_7) \right) + \frac{\partial^2}{\partial t \partial x_6} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. \\
& \left. - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_5 + \\
& \left(\frac{\partial^2}{\partial t \partial x_7} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_4} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_5} \right. \right. \\
& \left. \left. \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_3} \beta_5(t, x_1, \right. \\
& \left. x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_6 - \left(\frac{\partial^2}{\partial t \partial x_6} \beta_1(t, x_1, x_2, \right. \\
& \left. x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_3(t, x_1, x_2, x_3, x_4, \right. \right. \\
& \left. \left. x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_3} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& \left. - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_7
\end{aligned}$$

evalDG(Curl(Curl(F))+VLaplacian(F)-Grad(Div(F)));

O > evalDG(Ihs(s2)+VLaplacian(E)-Grad(Div(E)))

0 e0

(5.2.1.4)

O > Grad(rhs(eq1));

$$\begin{aligned}
& \frac{\partial}{\partial x_1} \rho(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_1 + \frac{\partial}{\partial x_2} \rho(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_2 \\
& + \frac{\partial}{\partial x_3} \rho(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_3 + \frac{\partial}{\partial x_4} \rho(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_4 \\
& + \frac{\partial}{\partial x_5} \rho(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_5 + \frac{\partial}{\partial x_6} \rho(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_6 \\
& + \frac{\partial}{\partial x_7} \rho(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_7
\end{aligned}$$

(5.2.1.5)

In a Vacuum, rho is zero, and so the divergence of E is zero:

O > s3:=evalDG(subs(Ihs(s2)=evalDG(-1*VLaplacian(E)),s2));

$$\begin{aligned}
s3 := & - \left(\frac{\partial^2}{\partial x_1^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \right. \\
& E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} E_1(t, x_1, x_2, \\
& \left. x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& \left. x_7) \right)
\end{aligned}$$

(5.2.1.6)

$$\begin{aligned}
& x7) \Big) e1 - \left(\frac{\partial^2}{\partial x1^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& + \frac{\partial^2}{\partial x3^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x5^2} E_2(t, \\
& x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x6^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} E_2(t, x1, x2, x3, x4, \\
& x5, x6, x7) \Big) e2 - \left(\frac{\partial^2}{\partial x1^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} E_3(t, x1, x2, x3, x4, x5, x6, \\
& x7) + \frac{\partial^2}{\partial x3^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x5^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& + \frac{\partial^2}{\partial x6^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) \Big) e3 - \left(\frac{\partial^2}{\partial x1^2} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} E_4(t, x1, x2, x3, x4, \\
& x5, x6, x7) + \frac{\partial^2}{\partial x3^2} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} E_4(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& + \frac{\partial^2}{\partial x5^2} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x6^2} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} E_4(t, \\
& x1, x2, x3, x4, x5, x6, x7) \Big) e4 - \left(\frac{\partial^2}{\partial x1^2} E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} E_5(t, x1, x2, \\
& x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x3^2} E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} E_5(t, x1, x2, x3, x4, x5, x6, \\
& x7) + \frac{\partial^2}{\partial x5^2} E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x6^2} E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& \Big) e5 - \left(\frac{\partial^2}{\partial x1^2} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} E_6(t, x1, \\
& x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x3^2} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} E_6(t, x1, x2, x3, x4, x5, \\
& x6, x7) + \frac{\partial^2}{\partial x5^2} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x6^2} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& \Big) e6 - \left(\frac{\partial^2}{\partial x1^2} E_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} E_7(t, x1, \\
& x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x3^2} E_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} E_7(t, x1, x2, x3, x4, x5, \\
& x6, x7) + \frac{\partial^2}{\partial x5^2} E_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x6^2} E_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& \Big) e7 = 0 e0 + \left(\frac{\partial^2}{\partial t \partial x3} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial t \partial x2} \right. \right. \\
& \beta_3(t, x1, x2, x3, x4, x5, x6, x7) \Big) + \frac{\partial^2}{\partial t \partial x5} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial t \partial x4} \beta_5(t, x1, \\
& x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial^2}{\partial t \partial x7} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial t \partial x6} \beta_7(t, x1, x2, x3, \\
& x4, x5, x6, x7) \Big) e1 - \left(\frac{\partial^2}{\partial t \partial x3} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial t \partial x1} \beta_3(t, x1, x2, x3, x4, x5, \\
& x6, x7) \right) - \left(\frac{\partial^2}{\partial t \partial x6} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial^2}{\partial t \partial x7} \beta_5(t, x1, x2, x3, x4, x5, x6, \\
& x7) \right) + \frac{\partial^2}{\partial t \partial x4} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial t \partial x5} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) \Big) e2 +
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\partial^2}{\partial t \partial x_2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_7} \right. \\
& \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_6} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_5} \beta_6(t, x_1, \\
& x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \left. \right) e_3 - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_1(t, x_1, x_2, \right. \\
& x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_6} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \\
& - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \left. \right) e_4 + \left(\frac{\partial^2}{\partial t \partial x_4} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& - \left(\frac{\partial^2}{\partial t \partial x_7} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_6} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \right. \\
& \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_2} \beta_7(t, x_1, \\
& x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) e_5 + \left(\frac{\partial^2}{\partial t \partial x_7} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_4} \beta_2(t, x_1, x_2, x_3, \\
& x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, \\
& x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_3} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. \\
& \left. \right) e_6 - \left(\frac{\partial^2}{\partial t \partial x_6} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. \\
& - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_3} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} \\
& \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \left. \right) e_7
\end{aligned}$$

O > bigsub:=DGzip([seq(diff(GetComponents(Curl(B),Basis)[i],t),i=1..7),Basis,"plus");

$$\begin{aligned}
\text{bigsub} := & - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) \quad (5.2.1.7) \\
& + \frac{\partial^2}{\partial t \partial x_5} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_7} \right. \\
& \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) + \frac{\partial^2}{\partial t \partial x_6} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) e_1 + \left(\frac{\partial^2}{\partial t \partial x_3} \beta_1(t, \right. \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_6} \beta_4(t, x_1, x_2, \right. \\
& x_3, x_4, x_5, x_6, x_7) \left. \right) - \left(\frac{\partial^2}{\partial t \partial x_7} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_4} \beta_6(t, x_1, x_2, x_3, x_4, \\
& x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_5} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) e_2 - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_7} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\
& - \left(\frac{\partial^2}{\partial t \partial x_6} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_5} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} \right. \\
& \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) \left. \right) e_3 + \left(\frac{\partial^2}{\partial t \partial x_5} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_6} \beta_2(t, \right. \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_5(t, x_1, x_2, x_3, \right.
\end{aligned}$$

$$\begin{aligned}
& x4, x5, x6, x7) \Big) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_7(t, x1, x2, x3, x4, x5, \right. \\
& \left. x6, x7) \right) \Big) e4 - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial t \partial x_7} \beta_2(t, x1, x2, x3, x4, x5, x6, \right. \right. \\
& \left. \left. x7) \right) + \frac{\partial^2}{\partial t \partial x_6} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) \right) \right) \\
& - \left(\frac{\partial^2}{\partial t \partial x_3} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial t \partial x_2} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) \Big) e5 - \\
& \left(\frac{\partial^2}{\partial t \partial x_7} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial t \partial x_4} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial t \partial x_5} \right. \right. \\
& \left. \left. \beta_3(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial^2}{\partial t \partial x_2} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) \right) + \frac{\partial^2}{\partial t \partial x_3} \beta_5(t, x1, \right. \\
& \left. x2, x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) \right) \right) e6 + \left(\frac{\partial^2}{\partial t \partial x_6} \beta_1(t, x1, x2, \right. \\
& \left. x3, x4, x5, x6, x7) - \left(\frac{\partial^2}{\partial t \partial x_5} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) \right) - \left(\frac{\partial^2}{\partial t \partial x_4} \beta_3(t, x1, x2, x3, x4, \right. \right. \\
& \left. \left. x5, x6, x7) \right) + \frac{\partial^2}{\partial t \partial x_3} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial t \partial x_2} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& \left. - \left(\frac{\partial^2}{\partial t \partial x_1} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) \right) \right) e7
\end{aligned}$$

O > evalDG(rhs(s3)+bigsub);

0 e0

(5.2.1.8)

Therefore, the rhs of s3 is the time-derivative of the curl of B. The Curl of B is equal to the time derivative of E, assuming J is zero.

O > dE2dt2:=DGzip([seq(diff(GetComponents(dEdt,Basis)[i],t),i=1..7)], Basis,"plus");

$$dE2dt2 := \frac{\partial^2}{\partial t^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) e1 + \frac{\partial^2}{\partial t^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) e2 + \frac{\partial^2}{\partial t^2} \quad (5.2.1.9)$$

$$\begin{aligned}
& E_3(t, x1, x2, x3, x4, x5, x6, x7) e3 + \frac{\partial^2}{\partial t^2} E_4(t, x1, x2, x3, x4, x5, x6, x7) e4 + \frac{\partial^2}{\partial t^2} E_5(t, x1, x2, \\
& x3, x4, x5, x6, x7) e5 + \frac{\partial^2}{\partial t^2} E_6(t, x1, x2, x3, x4, x5, x6, x7) e6 + \frac{\partial^2}{\partial t^2} E_7(t, x1, x2, x3, x4, x5, x6, \\
& x7) e7
\end{aligned}$$

O > s4:=evalDG(subs(rhs(s3)=-mu*epsilon*dE2dt2,s3));

$$s4 := - \left(\frac{\partial^2}{\partial x_1^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_2^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_3^2} \quad (5.2.1.10)$$

$$\begin{aligned}
& E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_4^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_5^2} E_1(t, x1, x2, \\
& x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_6^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_7^2} E_1(t, x1, x2, x3, x4, x5, x6, \\
& x7) \Big) e1 - \left(\frac{\partial^2}{\partial x_1^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_2^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& \left. + \frac{\partial^2}{\partial x_3^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_4^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_5^2} \right. \\
& E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_6^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_7^2} E_2(t, x1, x2, \\
& x3, x4, x5, x6, x7) \Big) e2 - \left(\frac{\partial^2}{\partial x_1^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x_2^2} E_3(t, x1, x2, x3, x4, \right.
\end{aligned}$$

$$\begin{aligned}
& x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} E_3(t, x1, x2, x3, x4, x5, x6, x7) \\
& + \frac{\partial^2}{\partial x^5} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} \\
& E_3(t, x1, x2, x3, x4, x5, x6, x7) \Big) e3 - \left(\frac{\partial^2}{\partial x^1} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} E_4(t, \right. \\
& x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} E_4(t, x1, x2, x3, x4, \\
& x5, x6, x7) + \frac{\partial^2}{\partial x^5} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} E_4(t, x1, x2, x3, x4, x5, x6, x7) \\
& + \frac{\partial^2}{\partial x^7} E_4(t, x1, x2, x3, x4, x5, x6, x7) \Big) e4 - \left(\frac{\partial^2}{\partial x^1} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& + \frac{\partial^2}{\partial x^2} E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} \\
& E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^5} E_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} E_5(t, x1, x2, \\
& x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} E_5(t, x1, x2, x3, x4, x5, x6, x7) \Big) e5 - \left(\frac{\partial^2}{\partial x^1} E_6(t, x1, x2, x3, x4, \\
& x5, x6, x7) + \frac{\partial^2}{\partial x^2} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_6(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& + \frac{\partial^2}{\partial x^4} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^5} E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} \\
& E_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} E_6(t, x1, x2, x3, x4, x5, x6, x7) \Big) e6 - \left(\frac{\partial^2}{\partial x^1} E_7(t, \right. \\
& x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} E_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_7(t, x1, x2, x3, x4, \\
& x5, x6, x7) + \frac{\partial^2}{\partial x^4} E_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^5} E_7(t, x1, x2, x3, x4, x5, x6, x7) \\
& + \frac{\partial^2}{\partial x^6} E_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} E_7(t, x1, x2, x3, x4, x5, x6, x7) \Big) e7 = \\
& - \mu \in \left(\frac{\partial^2}{\partial t^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) \right) e1 - \mu \in \left(\frac{\partial^2}{\partial t^2} E_2(t, x1, x2, x3, x4, x5, x6, \right. \\
& x7) \Big) e2 - \mu \in \left(\frac{\partial^2}{\partial t^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right) e3 - \mu \in \left(\frac{\partial^2}{\partial t^2} E_4(t, x1, x2, x3, x4, x5, \right. \\
& x6, x7) \Big) e4 - \mu \in \left(\frac{\partial^2}{\partial t^2} E_5(t, x1, x2, x3, x4, x5, x6, x7) \right) e5 - \mu \in \left(\frac{\partial^2}{\partial t^2} E_6(t, x1, x2, x3, x4, \right. \\
& x5, x6, x7) \Big) e6 - \mu \in \left(\frac{\partial^2}{\partial t^2} E_7(t, x1, x2, x3, x4, x5, x6, x7) \right) e7
\end{aligned}$$

O > s5:=evalDG(-s4)

$$\begin{aligned}
s5 := & \left(\frac{\partial^2}{\partial x^1} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} \right. \\
& E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^5} E_1(t, x1, x2, \\
& x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} E_1(t, x1, x2, x3, x4, x5, x6, \\
& x7) \Big) e1 + \left(\frac{\partial^2}{\partial x^1} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) \right.
\end{aligned} \tag{5.2.1.11}$$

$$\begin{aligned}
& \frac{\partial^2}{\partial x^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x^3} E_1(t, x_1, x_2, \\
& x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x^4} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x^5} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) + \frac{\partial^2}{\partial x^6} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x^7} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\
& \mu \in \left(\frac{\partial^2}{\partial t^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \tag{5.2.1.12}
\end{aligned}$$

Magnetic Field

O > Eqn2:=DivNcurl(B)=evalDG(-1*evalDG(0*e0)+evalDG(mu*epsilon*dEdt));

$$\begin{aligned}
Eqn2 := & - \left(\frac{\partial}{\partial x_1} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_3} \right. \tag{5.2.2.1} \\
& \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_4} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_5} \beta_5(t, x_1, x_2, x_3, \\
& x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_6} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \left. \frac{\partial}{\partial x_7} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_0 \\
& - \left(\frac{\partial}{\partial x_3} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_5} \right. \\
& \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_4} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_7} \beta_6(t, x_1, x_2, \right. \\
& x_3, x_4, x_5, x_6, x_7) \left. \right) + \frac{\partial}{\partial x_6} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) e_1 + \left(\frac{\partial}{\partial x_3} \beta_1(t, x_1, x_2, x_3, x_4, x_5, \right. \\
& x_6, x_7) - \left(\frac{\partial}{\partial x_1} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_6} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \\
& - \left(\frac{\partial}{\partial x_7} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_4} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_5} \beta_7(t, \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) e_2 - \left(\frac{\partial}{\partial x_2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \beta_2(t, x_1, x_2, x_3, \right. \right. \\
& x_4, x_5, x_6, x_7) \left. \right) + \frac{\partial}{\partial x_7} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_6} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& x_7) \left. \right) + \frac{\partial}{\partial x_5} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_4} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \left. \right) e_3 + \\
& \left(\frac{\partial}{\partial x_5} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_6} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_7} \beta_3(t, x_1, \right. \\
& x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_2} \beta_6(t, x_1, x_2, x_3, x_4, \right. \\
& x_5, x_6, x_7) \left. \right) - \left(\frac{\partial}{\partial x_3} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \left. \right) e_4 - \left(\frac{\partial}{\partial x_4} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& x_7) - \left(\frac{\partial}{\partial x_7} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_6} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \right. \\
& \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) - \left(\frac{\partial}{\partial x_3} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_2} \beta_7(t, x_1, x_2, \\
& x_3, x_4, x_5, x_6, x_7) \left. \right) e_5 - \left(\frac{\partial}{\partial x_7} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_4} \beta_2(t, x_1, x_2, x_3, x_4, x_5, \right. \\
& x_6, x_7) - \left(\frac{\partial}{\partial x_5} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \\
& + \frac{\partial}{\partial x_3} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \left. \right) e_6 + \left(\frac{\partial}{\partial x_6} \right.
\end{aligned}$$

$$\begin{aligned}
& \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_5} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial}{\partial x_4} \beta_3(t, x_1, x_2, \right. \\
& \left. x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial}{\partial x_3} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial}{\partial x_2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& \left. x_7) - \left(\frac{\partial}{\partial x_1} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_7 = \mu \in \left(\frac{\partial}{\partial t} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& \left. x_7) \right) e_1 + \mu \in \left(\frac{\partial}{\partial t} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_2 + \mu \in \left(\frac{\partial}{\partial t} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& \left. x_7) \right) e_3 + \mu \in \left(\frac{\partial}{\partial t} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_4 + \mu \in \left(\frac{\partial}{\partial t} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& \left. x_7) \right) e_5 + \mu \in \left(\frac{\partial}{\partial t} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_6 + \mu \in \left(\frac{\partial}{\partial t} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& \left. x_7) \right) e_7
\end{aligned}$$

O > r1:=DivNcurl(lhs(Eqn2))=DivNcurl(rhs(Eqn2));

$$\begin{aligned}
r1 := & - \left(\frac{\partial^2}{\partial x_3^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_3} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. & \text{(5.2.2.2)} \\
& + \frac{\partial^2}{\partial x_2^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_5^2} \\
& \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_5} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_4^2} \beta_1(t, x_1, \\
& \left. x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_4} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_7^2} \beta_1(t, x_1, x_2, x_3, x_4, \right. \\
& \left. x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_7} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_6^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& \left. - \left(\frac{\partial^2}{\partial x_1 \partial x_6} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_1 - \left(\frac{\partial^2}{\partial x_3^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& \left. - \left(\frac{\partial^2}{\partial x_2 \partial x_3} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial x_1 \partial x_2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. \\
& \left. + \frac{\partial^2}{\partial x_1^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_6} \right. \right. \\
& \left. \left. \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_7^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_7} \beta_7(t, x_1, \right. \right. \\
& \left. \left. x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_4^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_4} \beta_4(t, x_1, x_2, x_3, x_4, \right. \right. \\
& \left. \left. x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_5^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_5} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \right. \\
& \left. \left. x_7) \right) \right) e_2 - \left(- \left(\frac{\partial^2}{\partial x_2 \partial x_3} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_2^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \right. \\
& \left. \left. x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_3} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_1^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \right. \\
& \left. \left. + \frac{\partial^2}{\partial x_7^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_3 \partial x_7} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_6^2} \right. \right. \\
& \left. \left. \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_3 \partial x_6} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_5^2} \beta_3(t, x_1, \right. \right. \\
& \left. \left. x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_3 \partial x_5} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_4^2} \beta_3(t, x_1, x_2, x_3, x_4, \right.
\end{aligned}$$

$$\begin{aligned}
& - \left(\frac{\partial^2}{\partial x_1 \partial x_7} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_1^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_7 = \\
& - \mu \in \left(\frac{\partial^2}{\partial t \partial x_1} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_3} \right. \\
& E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_4} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_5} E_5(t, x_1, x_2, \\
& x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_6} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) \Big) e_0 - \mu \in \left(\frac{\partial^2}{\partial t \partial x_3} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_2} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \right. \\
& x_7) \Big) + \frac{\partial^2}{\partial t \partial x_5} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \\
& - \left(\frac{\partial^2}{\partial t \partial x_7} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_6} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_1 \\
& - \mu \in \left(- \left(\frac{\partial^2}{\partial t \partial x_3} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_1} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& + \frac{\partial^2}{\partial t \partial x_6} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} \right. \\
& E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) - \left(\frac{\partial^2}{\partial t \partial x_5} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \Big) e_2 - \mu \in \left(\frac{\partial^2}{\partial t \partial x_2} \right. \\
& E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_7} E_4(t, x_1, \\
& x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_6} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_5} E_6(t, x_1, x_2, x_3, x_4, \\
& x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_4} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \Big) e_3 - \mu \in \left(- \left(\frac{\partial^2}{\partial t \partial x_5} E_1(t, x_1, x_2, x_3, \right. \right. \\
& x_4, x_5, x_6, x_7) \Big) - \left(\frac{\partial^2}{\partial t \partial x_6} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_7} E_3(t, x_1, x_2, x_3, x_4, x_5, \right. \\
& x_6, x_7) \Big) + \frac{\partial^2}{\partial t \partial x_1} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\
& + \frac{\partial^2}{\partial t \partial x_3} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_4 - \mu \in \left(\frac{\partial^2}{\partial t \partial x_4} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& - \left(\frac{\partial^2}{\partial t \partial x_7} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_6} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \right. \\
& E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) - \left(\frac{\partial^2}{\partial t \partial x_3} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_2} E_7(t, x_1, \\
& x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_5 + \mu \in \left(- \left(\frac{\partial^2}{\partial t \partial x_7} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_4} E_2(t, \right. \right. \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) + \frac{\partial^2}{\partial t \partial x_5} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} E_4(t, x_1, x_2, x_3, \\
& x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_3} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_1} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) \Big) e_6 - \mu \in \left(- \left(\frac{\partial^2}{\partial t \partial x_6} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_5} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& x_7) + \frac{\partial^2}{\partial t \partial x_4} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_3} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \\
& - \left(\frac{\partial^2}{\partial t \partial x_2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_1} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_7
\end{aligned}$$

O > GetComponent(rhs(r1),extBasis)[1]

(5.2.2.3)

$$-\mu \in \left(\frac{\partial^2}{\partial x_1 \partial t} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2 \partial t} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3 \partial t} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4 \partial t} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5 \partial t} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6 \partial t} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7 \partial t} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \quad (5.2.2.3)$$

O > evalDG(Div(dEdt))

$$\left(\frac{\partial^2}{\partial t \partial x_1} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_3} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_4} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_5} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_6} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_0 \quad (5.2.2.4)$$

O > evalDG(evalDG(GetComponents(rhs(r1),extBasis)[1])*e0+evalDG(mu*epsilon*(Div(dEdt))))

$$0 e_0 \quad (5.2.2.5)$$

Because the divergence of E is zero (and because partial derivatives commute), we can replace the first component with zero.

O > r2:=evalDG(subs(GetComponents(rhs(r1),extBasis)[1]=0,r1));

$$r_2 := - \left(\frac{\partial^2}{\partial x_3^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_3} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_2^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_5^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_5} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_4^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_4} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_7^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_7} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_6^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_6} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_1 - \left(\frac{\partial^2}{\partial x_3^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_3} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial x_1 \partial x_2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_1^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_6} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_7^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_7} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_4^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_4} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_5^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_2 \partial x_5} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_2 - \left(- \left(\frac{\partial^2}{\partial x_2 \partial x_3} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_2^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_1 \partial x_3} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_1^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_3 \partial x_7} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_7^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_3 \partial x_6} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_5^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial x_3 \partial x_5} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) \quad (5.2.2.6)$$

$$\begin{aligned}
& - \left(\frac{\partial^2}{\partial x_2 \partial x_7} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_2^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\
& - \left(\frac{\partial^2}{\partial x_1 \partial x_7} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial x_1^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_7 \\
& = 0 e_0 - \mu \in \left(\frac{\partial^2}{\partial \bar{t} \partial x_3} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_2} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \right. \\
& \left. \left. x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_5} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_4} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. \\
& \left. - \left(\frac{\partial^2}{\partial \bar{t} \partial x_7} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_6} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_1 \\
& - \mu \in \left(- \left(\frac{\partial^2}{\partial \bar{t} \partial x_3} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_1} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& \left. + \frac{\partial^2}{\partial \bar{t} \partial x_6} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial \bar{t} \partial x_7} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_4} \right. \right. \\
& \left. \left. E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_5} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_2 - \mu \in \left(\frac{\partial^2}{\partial \bar{t} \partial x_2} \right. \\
& \left. E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_1} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_7} E_4(t, x_1, \right. \\
& \left. x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_6} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_5} E_6(t, x_1, x_2, x_3, x_4, \right. \\
& \left. x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_4} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_3 - \mu \in \left(- \left(\frac{\partial^2}{\partial \bar{t} \partial x_5} E_1(t, x_1, x_2, x_3, \right. \right. \\
& \left. \left. x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_6} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_7} E_3(t, x_1, x_2, x_3, x_4, x_5, \right. \right. \\
& \left. \left. x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_1} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial \bar{t} \partial x_2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& \left. + \frac{\partial^2}{\partial \bar{t} \partial x_3} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_4 - \mu \in \left(\frac{\partial^2}{\partial \bar{t} \partial x_4} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& \left. - \left(\frac{\partial^2}{\partial \bar{t} \partial x_7} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_6} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_1} \right. \right. \\
& \left. \left. E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_3} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_2} E_7(t, x_1, \right. \\
& \left. x_2, x_3, x_4, x_5, x_6, x_7) \right) e_5 + \mu \in \left(- \left(\frac{\partial^2}{\partial \bar{t} \partial x_7} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_4} E_2(t, \right. \right. \\
& \left. \left. x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_5} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial \bar{t} \partial x_2} E_4(t, x_1, x_2, x_3, \right. \\
& \left. x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_3} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_1} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& \left. x_7) \right) e_6 - \mu \in \left(- \left(\frac{\partial^2}{\partial \bar{t} \partial x_6} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_5} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& \left. x_7) + \frac{\partial^2}{\partial \bar{t} \partial x_4} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial \bar{t} \partial x_3} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. \\
& \left. - \left(\frac{\partial^2}{\partial \bar{t} \partial x_2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial \bar{t} \partial x_1} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_7
\end{aligned}$$

O > evalDG(Curl(Curl(B))-lhs(r2));

0

(5.2.2.7)

evalDG(Curl(Curl(F))+VLaplacian(F)-Grad(Div(F))); the Divergence of B is zero.

O > r3:=evalDG(subs(lhs(r2)=-1*VLaplacian(B),r2));

$$r3 := - \left(\frac{\partial^2}{\partial x_1^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \right)$$

(5.2.2.8)

$$\begin{aligned}
& -\mu \in \left(-\left(\frac{\partial^2}{\partial t \partial x_3} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_1} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& + \frac{\partial^2}{\partial t \partial x_6} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_7} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left. \left(\frac{\partial^2}{\partial t \partial x_4} \right. \right. \\
& E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \left. - \left(\frac{\partial^2}{\partial t \partial x_5} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) \right) e_2 - \mu \in \left(\frac{\partial^2}{\partial t \partial x_2} \right. \\
& E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left. \left(\frac{\partial^2}{\partial t \partial x_1} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_7} E_4(t, x_1, \right. \\
& x_2, x_3, x_4, x_5, x_6, x_7) - \left. \left(\frac{\partial^2}{\partial t \partial x_6} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_5} E_6(t, x_1, x_2, x_3, x_4, \right. \\
& x_5, x_6, x_7) - \left. \left(\frac{\partial^2}{\partial t \partial x_4} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) e_3 - \mu \in \left(-\left(\frac{\partial^2}{\partial t \partial x_5} E_1(t, x_1, x_2, x_3, \right. \right. \\
& x_4, x_5, x_6, x_7) \right) - \left. \left(\frac{\partial^2}{\partial t \partial x_6} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_7} E_3(t, x_1, x_2, x_3, x_4, x_5, \right. \right. \\
& x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_1} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\
& + \frac{\partial^2}{\partial t \partial x_3} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) e_4 - \mu \in \left(\frac{\partial^2}{\partial t \partial x_4} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& - \left. \left(\frac{\partial^2}{\partial t \partial x_7} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_6} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial^2}{\partial t \partial x_1} \right. \right. \\
& E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \left. - \left(\frac{\partial^2}{\partial t \partial x_3} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_2} E_7(t, x_1, \right. \right. \\
& x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) e_5 + \mu \in \left(-\left(\frac{\partial^2}{\partial t \partial x_7} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) - \left(\frac{\partial^2}{\partial t \partial x_4} E_2(t, \right. \right. \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) \left. \right) + \frac{\partial^2}{\partial t \partial x_5} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial t \partial x_2} E_4(t, x_1, x_2, x_3, \\
& x_4, x_5, x_6, x_7) - \left. \left(\frac{\partial^2}{\partial t \partial x_3} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_1} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& x_7) \left. \right) e_6 - \mu \in \left(-\left(\frac{\partial^2}{\partial t \partial x_6} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_5} E_2(t, x_1, x_2, x_3, x_4, x_5, x_6, \right. \\
& x_7) + \frac{\partial^2}{\partial t \partial x_4} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left. \left(\frac{\partial^2}{\partial t \partial x_3} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right. \\
& - \left. \left(\frac{\partial^2}{\partial t \partial x_2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) + \frac{\partial^2}{\partial t \partial x_1} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_7
\end{aligned}$$

the rhs(r3) is the time-derivative of the curl of E. The curl of E is the negative time-derivative of B, so:

O > dB2dt2:=DGzip([seq(diff(GetComponents(dBdt,Basis)[i],t),i=1..7)], Basis,"plus");

$$dB2dt2 := \frac{\partial^2}{\partial t^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_1 + \frac{\partial^2}{\partial t^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_2 + \frac{\partial^2}{\partial t^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_3 + \frac{\partial^2}{\partial t^2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_4 + \frac{\partial^2}{\partial t^2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_5 + \frac{\partial^2}{\partial t^2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_6 + \frac{\partial^2}{\partial t^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_7 \quad (5.2.2.9)$$

$$\begin{aligned}
& \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_3 + \frac{\partial^2}{\partial t^2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_4 + \frac{\partial^2}{\partial t^2} \beta_5(t, x_1, x_2, \\
& x_3, x_4, x_5, x_6, x_7) e_5 + \frac{\partial^2}{\partial t^2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) e_6 + \frac{\partial^2}{\partial t^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) e_7
\end{aligned}$$

O > r4:=evalDG(subs(rhs(r3)=-1*epsilon*mu*dB2dt2,r3));

$$r4 := -\left(\frac{\partial^2}{\partial x_1^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \quad (5.2.2.10)$$

$$\beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7)$$

$$\beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_7$$

O > r5:=evalDG(-1*r4);

$$\begin{aligned}
 r5 := & \left(\frac{\partial^2}{\partial x_1^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \right. \\
 & \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} \beta_1(t, x_1, x_2, \\
 & x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} \beta_1(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
 & \left. x_7) \right) e_1 + \left(\frac{\partial^2}{\partial x_1^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
 & + \frac{\partial^2}{\partial x_3^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} \\
 & \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_2(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} \beta_2(t, x_1, x_2, \\
 & x_3, x_4, x_5, x_6, x_7) \Big) e_2 + \left(\frac{\partial^2}{\partial x_1^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} \beta_3(t, x_1, x_2, x_3, x_4, \\
 & x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
 & + \frac{\partial^2}{\partial x_5^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} \\
 & \beta_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_3 + \left(\frac{\partial^2}{\partial x_1^2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} \beta_4(t, x_1, \\
 & x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, \\
 & x_6, x_7) + \frac{\partial^2}{\partial x_5^2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
 & + \frac{\partial^2}{\partial x_7^2} \beta_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_4 + \left(\frac{\partial^2}{\partial x_1^2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} \beta_5(t, x_1, x_2, \\
 & \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} \beta_5(t, x_1, x_2, \\
 & x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
 & x_7) + \frac{\partial^2}{\partial x_7^2} \beta_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_5 + \left(\frac{\partial^2}{\partial x_1^2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
 & + \frac{\partial^2}{\partial x_2^2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} \\
 & \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \beta_6(t, x_1, x_2, \\
 & x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} \beta_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_6 + \left(\frac{\partial^2}{\partial x_1^2} \beta_7(t, x_1, x_2, x_3, x_4, \\
 & x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
 & + \frac{\partial^2}{\partial x_4^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} \\
 & \left. \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} \beta_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_7 = \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_1(t,
 \end{aligned}
 \tag{5.2.2.11}$$

$$\begin{aligned} & x1, x2, x3, x4, x5, x6, x7) \Big) e1 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) \right) e2 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \right. \\ & \beta_3(t, x1, x2, x3, x4, x5, x6, x7) \Big) e3 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) \right) e4 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \right. \\ & \beta_5(t, x1, x2, x3, x4, x5, x6, x7) \Big) e5 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) \right) e6 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \right. \\ & \left. \beta_7(t, x1, x2, x3, x4, x5, x6, x7) \right) e7 \end{aligned}$$

**O > GetComponents(lhs(r5),Basis)[1];
GetComponents(rhs(r5),Basis)[1];**

$$\begin{aligned} & \frac{\partial^2}{\partial x1^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x3^2} \beta_1(t, x1, x2, \\ & x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x5^2} \beta_1(t, x1, x2, x3, x4, x5, x6, \\ & x7) + \frac{\partial^2}{\partial x6^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) \\ & \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) \right) \end{aligned} \quad (5.2.2.12)$$

Wave equation solution to ME solution

We now impose additional constraints to the wave equation solutions.

O > bwave:=VLaplacian(B)=evalDG(epsilon*mu*dB2dt2);

$$\begin{aligned} bwave := & \left(\frac{\partial^2}{\partial x1^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x3^2} \beta_1(t, \right. & (5.3.1) \\ & x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x5^2} \beta_1(t, x1, x2, x3, x4, x5, x6, \\ & x7) + \frac{\partial^2}{\partial x6^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) \Big) e1 + \left(\frac{\partial^2}{\partial x1^2} \right. \\ & \beta_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x3^2} \beta_2(t, x1, x2, x3, x4, \\ & x5, x6, x7) + \frac{\partial^2}{\partial x4^2} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x5^2} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x6^2} \\ & \beta_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} \beta_2(t, x1, x2, x3, x4, x5, x6, x7) \Big) e2 + \left(\frac{\partial^2}{\partial x1^2} \beta_3(t, x1, x2, x3, \right. \\ & x4, x5, x6, x7) + \frac{\partial^2}{\partial x2^2} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x3^2} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) \\ & + \frac{\partial^2}{\partial x4^2} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x5^2} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x6^2} \beta_3(t, x1, x2, \\ & x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x7^2} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) \Big) e3 + \left(\frac{\partial^2}{\partial x1^2} \beta_4(t, x1, x2, x3, x4, x5, x6, \right. \\ & x7) + \frac{\partial^2}{\partial x2^2} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x3^2} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x4^2} \beta_4(t, \\ & x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x5^2} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x6^2} \beta_4(t, x1, x2, x3, x4, x5, x6, \end{aligned}$$

$$\begin{aligned}
& x7) + \frac{\partial^2}{\partial x^7} \beta_4(t, x1, x2, x3, x4, x5, x6, x7) \Big) e4 + \left(\frac{\partial^2}{\partial x^1} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} \right. \\
& \beta_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} \beta_5(t, x1, x2, x3, x4, \\
& x5, x6, x7) + \frac{\partial^2}{\partial x^5} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} \\
& \beta_5(t, x1, x2, x3, x4, x5, x6, x7) \Big) e5 + \left(\frac{\partial^2}{\partial x^1} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} \beta_6(t, x1, x2, x3, \\
& x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& + \frac{\partial^2}{\partial x^5} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} \beta_6(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} \beta_6(t, x1, x2, \\
& x3, x4, x5, x6, x7) \Big) e6 + \left(\frac{\partial^2}{\partial x^1} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} \beta_7(t, x1, x2, x3, x4, x5, x6, \\
& x7) + \frac{\partial^2}{\partial x^3} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^5} \beta_7(t, \\
& x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} \beta_7(t, x1, x2, x3, x4, x5, x6, \\
& x7) \Big) e7 = \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_1(t, x1, x2, x3, x4, x5, x6, x7) \right) e1 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_2(t, x1, x2, x3, x4, x5, x6, \\
& x7) \right) e2 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_3(t, x1, x2, x3, x4, x5, x6, x7) \right) e3 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_4(t, x1, x2, x3, x4, x5, x6, \\
& x7) \right) e4 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_5(t, x1, x2, x3, x4, x5, x6, x7) \right) e5 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_6(t, x1, x2, x3, x4, x5, x6, \\
& x7) \right) e6 + \epsilon \mu \left(\frac{\partial^2}{\partial t^2} \beta_7(t, x1, x2, x3, x4, x5, x6, x7) \right) e7
\end{aligned}$$

O > ewave:=VLaplacian(E)=evalDG(epsilon*mu*dE2dt2);

$$\begin{aligned}
\text{ewave} := & \left(\frac{\partial^2}{\partial x^1} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_1(t, \right. & \mathbf{(5.3.2)} \\
& x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^5} E_1(t, x1, x2, x3, x4, x5, x6, \\
& x7) + \frac{\partial^2}{\partial x^6} E_1(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} E_1(t, x1, x2, x3, x4, x5, x6, x7) \Big) e1 + \left(\frac{\partial^2}{\partial x^1} \right. \\
& E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_2(t, x1, x2, x3, x4, \\
& x5, x6, x7) + \frac{\partial^2}{\partial x^4} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^5} E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} \\
& E_2(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} E_2(t, x1, x2, x3, x4, x5, x6, x7) \Big) e2 + \left(\frac{\partial^2}{\partial x^1} E_3(t, x1, x2, \\
& x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^2} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_3(t, x1, x2, x3, x4, x5, x6, x7) \right. \\
& + \frac{\partial^2}{\partial x^4} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^5} E_3(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^6} E_3(t, x1, \\
& x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^7} E_3(t, x1, x2, x3, x4, x5, x6, x7) \Big) e3 + \left(\frac{\partial^2}{\partial x^1} E_4(t, x1, x2, x3, x4, x5, \\
& x6, x7) + \frac{\partial^2}{\partial x^2} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^3} E_4(t, x1, x2, x3, x4, x5, x6, x7) + \frac{\partial^2}{\partial x^4} \right.
\end{aligned}$$

$$\begin{aligned}
& E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} E_4(t, x_1, x_2, x_3, x_4, \\
& x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_4 + \left(\frac{\partial^2}{\partial x_1^2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right. \\
& + \frac{\partial^2}{\partial x_2^2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} E_5(t, x_1, \\
& x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\
& + \frac{\partial^2}{\partial x_7^2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_5 + \left(\frac{\partial^2}{\partial x_1^2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} E_6(t, \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) + \frac{\partial^2}{\partial x_5^2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} E_6(t, \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) e_6 + \left(\frac{\partial^2}{\partial x_1^2} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_2^2} E_7(t, x_1, x_2, x_3, x_4, \\
& x_5, x_6, x_7) + \frac{\partial^2}{\partial x_3^2} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_4^2} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_5^2} E_7(t, x_1, x_2, x_3, x_4, \\
& x_5, x_6, x_7) + \frac{\partial^2}{\partial x_6^2} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) + \frac{\partial^2}{\partial x_7^2} E_7(t, x_1, x_2, x_3, x_4, \\
& x_5, x_6, x_7) \Big) e_7 = \mu \in \left(\frac{\partial^2}{\partial t^2} E_1(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_1 + \mu \in \left(\frac{\partial^2}{\partial t^2} E_2(t, x_1, x_2, x_3, x_4, x_5, \\
& x_6, x_7) \right) e_2 + \mu \in \left(\frac{\partial^2}{\partial t^2} E_3(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_3 + \mu \in \left(\frac{\partial^2}{\partial t^2} E_4(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) \right) e_4 + \mu \in \left(\frac{\partial^2}{\partial t^2} E_5(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_5 + \mu \in \left(\frac{\partial^2}{\partial t^2} E_6(t, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) \right) e_6 + \mu \in \left(\frac{\partial^2}{\partial t^2} E_7(t, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) e_7
\end{aligned}$$

$$\begin{aligned}
\mathbf{O} > \mathbf{kvec} := \mathbf{DGzip}([\mathbf{seq}(\mathbf{k}[i], i=1..7)], \mathbf{Basis}, \mathbf{"plus"}); \\
\mathbf{kvec} &:= k_1 e_1 + k_2 e_2 + k_3 e_3 + k_4 e_4 + k_5 e_5 + k_6 e_6 + k_7 e_7
\end{aligned} \tag{5.3.3}$$

$$\begin{aligned}
\mathbf{O} > \mathbf{xvec} := \mathbf{DGzip}([\mathbf{seq}(\mathbf{x}[i], i=1..7)], \mathbf{Basis}, \mathbf{"plus"}); \\
\mathbf{xvec} &:= x_1 e_1 + x_2 e_2 + x_3 e_3 + x_4 e_4 + x_5 e_5 + x_6 e_6 + x_7 e_7
\end{aligned} \tag{5.3.4}$$

$$\begin{aligned}
\mathbf{O} > \mathbf{E0} := \mathbf{DGzip}([\mathbf{seq}(\mathbf{E0}[i], i=1..7)], \mathbf{Basis}, \mathbf{"plus"}); \\
\mathbf{E0} &:= E0_1 e_1 + E0_2 e_2 + E0_3 e_3 + E0_4 e_4 + E0_5 e_5 + E0_6 e_6 + E0_7 e_7
\end{aligned} \tag{5.3.5}$$

$$\begin{aligned}
\mathbf{O} > \mathbf{B0} := \mathbf{DGzip}([\mathbf{seq}(\mathbf{B0}[i], i=1..7)], \mathbf{Basis}, \mathbf{"plus"}); \\
\mathbf{B0} &:= B0_1 e_1 + B0_2 e_2 + B0_3 e_3 + B0_4 e_4 + B0_5 e_5 + B0_6 e_6 + B0_7 e_7
\end{aligned} \tag{5.3.6}$$

$$\begin{aligned}
\mathbf{O} > \mathbf{kdotx} := \mathbf{GetComponents}(\mathbf{DGRe}(-1 * \mathbf{kvec} \cdot \mathbf{xvec}), \mathbf{extBasis})[1]; \\
\mathbf{kdotx} &:= x_1 k_1 + x_2 k_2 + x_3 k_3 + x_4 k_4 + x_5 k_5 + x_6 k_6 + x_7 k_7
\end{aligned} \tag{5.3.7}$$

$$\begin{aligned}
\mathbf{O} > \mathbf{ewavesol} := \mathbf{evalDG}(\mathbf{E0} * \mathbf{exp}(\mathbf{I} * (\mathbf{kdotx} - \mathbf{\omega} * \mathbf{t}))); \\
\mathbf{ewavesol} &:= e^{-\mathbf{I}(\mathbf{\omega} * \mathbf{t} - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} E0_1 e_1 \\
&+ e^{-\mathbf{I}(\mathbf{\omega} * \mathbf{t} - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} E0_2 e_2 \\
&+ e^{-\mathbf{I}(\mathbf{\omega} * \mathbf{t} - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} E0_3 e_3 \\
&+ e^{-\mathbf{I}(\mathbf{\omega} * \mathbf{t} - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} E0_4 e_4 \\
&+ e^{-\mathbf{I}(\mathbf{\omega} * \mathbf{t} - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} E0_5 e_5
\end{aligned} \tag{5.3.8}$$

$$+ e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} E06 e6$$

$$+ e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} E07 e7$$

M > seq(diff(GetComponents(ewavesol,Basis)[1],x[[i]],i=1..7)

$$I k_1 e^{-1(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7)} E01, \quad (5.3.9)$$

$$I k_2 e^{-1(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7)} E01,$$

$$I k_3 e^{-1(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7)} E01,$$

$$I k_4 e^{-1(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7)} E01,$$

$$I k_5 e^{-1(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7)} E01,$$

$$I k_6 e^{-1(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7)} E01,$$

$$I k_7 e^{-1(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7)} E01$$

O > bwavesol:=evalDG(B0*exp(I*(kdotx-omega*t)));

$$bwavesol := e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B01 e1 \quad (5.3.10)$$

$$+ e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B02 e2$$

$$+ e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B03 e3$$

$$+ e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B04 e4$$

$$+ e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B05 e5$$

$$+ e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B06 e6$$

$$+ e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B07 e7$$

Check solutions are solutions of the wave equation:

O > GetComponents(DGRe(kvec.kvec),extBasis)[1]

$$-k_1^2 - k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 - k_7^2 \quad (5.3.11)$$

O > DGzip([seq(diff(GetComponents(DGzip([seq(diff(GetComponents(bwavesol,Basis)[i],t),i=1..7)],Basis,"plus"),Basis)[i],t),i=1..7)],Basis,"plus");

$$-\omega^2 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B01 e1 \quad (5.3.12)$$

$$-\omega^2 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B02 e2$$

$$-\omega^2 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B03 e3$$

$$-\omega^2 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B04 e4$$

$$-\omega^2 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B05 e5$$

$$-\omega^2 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B06 e6$$

$$-\omega^2 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B07 e7$$

O > bwch:=evalDG(VLaplacian(bwavesol)-evalDG(epsilon*mu*DGzip([seq(diff(GetComponents(DGzip([seq(diff(GetComponents(bwavesol,Basis)[i],t),i=

1..7)],Basis,"plus"),Basis)[i],t),i=1..7)],Basis,"plus")));
ewch:=evalDG(VLaplacian(ewavesol)-evalDG(epsilon*mu*DGzip([seq(diff
(GetComponents(DGzip([seq(diff(GetComponents(ewavesol,Basis)[i],t),i=
1..7)],Basis,"plus"),Basis)[i],t),i=1..7)],Basis,"plus")));

$$\begin{aligned}
bwch := & B01 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 \\
& - k_7^2) e1 + B02 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 - k_4^2 - \\
& k_5^2 - k_6^2 - k_7^2) e2 + B03 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 \\
& - k_4^2 - k_5^2 - k_6^2 - k_7^2) e3 + B04 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - \\
& k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 - k_7^2) e4 + B05 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 \\
& - k_1^2 - k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 - k_7^2) e5 \\
& + B06 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 \\
& - k_7^2) e6 + B07 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 - k_4^2 - \\
& k_5^2 - k_6^2 - k_7^2) e7
\end{aligned}$$

$$\begin{aligned}
ewch := & E01 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 \quad (5.3.13) \\
& - k_7^2) e1 + E02 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 - k_4^2 - \\
& k_5^2 - k_6^2 - k_7^2) e2 + E03 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 \\
& - k_4^2 - k_5^2 - k_6^2 - k_7^2) e3 + E04 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - \\
& k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 - k_7^2) e4 + E05 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 \\
& - k_1^2 - k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 - k_7^2) e5 \\
& + E06 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 - k_4^2 - k_5^2 - k_6^2 \\
& - k_7^2) e6 + E07 e^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (\epsilon \mu \omega^2 - k_1^2 - k_2^2 - k_3^2 - k_4^2 - \\
& k_5^2 - k_6^2 - k_7^2) e7
\end{aligned}$$

**O > eval(subs(epsilon*mu*omega^2-k[1]^2-k[2]^2-k[3]^2-k[4]^2-k[5]^2-k[6]^2-k[7]^2=epsilon*mu*omega^2,ewch),[k^2=epsilon*mu*omega^2]);
eval(subs(epsilon*mu*omega^2-k[1]^2-k[2]^2-k[3]^2-k[4]^2-k[5]^2-k[6]^2-k[7]^2=epsilon*mu*omega^2,bwch),[k^2=epsilon*mu*omega^2]);**

$$\begin{aligned}
& 0 e1 + 0 e2 + 0 e3 + 0 e4 + 0 e5 + 0 e6 + 0 e7 \\
& 0 e1 + 0 e2 + 0 e3 + 0 e4 + 0 e5 + 0 e6 + 0 e7
\end{aligned}$$

(5.3.14)

Impose extra conditions to make them solutions to ME:

O > Div(ewavesol);

$$\begin{aligned}
Ie^{-1(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E01 k_1 + E02 k_2 + E03 k_3 + E04 k_4 + E05 k_5 \quad (5.3.15) \\
+ E06 k_6 + E07 k_7) e0
\end{aligned}$$

O > DGRe(E0.kvec);

$$- (E01 k_1 + E02 k_2 + E03 k_3 + E04 k_4 + E05 k_5 + E06 k_6 + E07 k_7) e0 \quad (5.3.16)$$

O > Div(bwavesol);

$$\text{Ie}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (B01 k_1 + B02 k_2 + B03 k_3 + B04 k_4 + B05 k_5 + B06 k_6 + B07 k_7) e0 \quad (5.3.17)$$

$$\mathbf{O} > \text{DGRe}(\mathbf{B0.kvec}); \\ - (B01 k_1 + B02 k_2 + B03 k_3 + B04 k_4 + B05 k_5 + B06 k_6 + B07 k_7) e0 \quad (5.3.18)$$

This implies that the wave vector is orthogonal to both E and B.

$$\mathbf{O} > \text{eqF:=Curl(ewavesol)=DGzip([seq(diff(-1*GetComponents(bwavesol,Basis)[i],t),i=1..7)],Basis,"plus");} \\ \text{eqF:= -Ie}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E02 k_3 - E03 k_2 + E04 k_5 - E05 k_4 - E06 k_7 + E07 k_6) e1 + \text{Ie}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E01 k_3 - E03 k_1 - E04 k_6 - E05 k_7 + E06 k_4 + E07 k_5) e2 \quad (5.3.19)$$

$$- \text{Ie}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E01 k_2 - E02 k_1 + E04 k_7 - E05 k_6 + E06 k_5 - E07 k_4) e3 + \text{Ie}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E01 k_5 + E02 k_6 + E03 k_7 - E05 k_1 - E06 k_2 - E07 k_3) e4 \\ - \text{Ie}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E01 k_4 - E02 k_7 + E03 k_6 - E04 k_1 - E06 k_3 + E07 k_2) e5 - \text{Ie}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E01 k_7 + E02 k_4 - E03 k_5 - E04 k_2 + E05 k_3 - E07 k_1) e6 \\ + \text{Ie}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E01 k_6 - E02 k_5 - E03 k_4 + E04 k_3 + E05 k_2 - E06 k_1) e7 = \text{I}\omega \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B01 e1 \\ + \text{I}\omega \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B02 e2 \\ + \text{I}\omega \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B03 e3 \\ + \text{I}\omega \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B04 e4 \\ + \text{I}\omega \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B05 e5 \\ + \text{I}\omega \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B06 e6 \\ + \text{I}\omega \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} B07 e7$$

$$\mathbf{O} > \text{GetComponents(lhs(eqF),Basis)[1];} \\ \text{GetComponents(rhs(eqF),Basis)[1];} \\ - \text{Ie}^{-i(\omega t - x1 k_1 - x2 k_2 - x3 k_3 - x4 k_4 - x5 k_5 - x6 k_6 - x7 k_7)} (E02 k_3 - E03 k_2 + E04 k_5 - E05 k_4 - E06 k_7 + E07 k_6) \\ + \text{I}\omega \text{e}^{-i(\omega t - x1 k_1 - x2 k_2 - x3 k_3 - x4 k_4 - x5 k_5 - x6 k_6 - x7 k_7)} B01 \quad (5.3.20)$$

$$\mathbf{O} > \text{DGIm(kvec.ewavesol);} \\ - \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E02 k_3 - E03 k_2 + E04 k_5 - E05 k_4 - E06 k_7 + E07 k_6) e1 + \text{e}^{-i(\omega t - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5 - k_6 x_6 - k_7 x_7)} (E01 k_3 - E03 k_1 - E04 k_6$$

$$\begin{aligned}
& -E05 k_7 + E06 k_4 + E07 k_5) e2 - e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_2 \\
& -E02 k_1 + E04 k_7 - E05 k_6 + E06 k_5 - E07 k_4) e3 \\
& + e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_5 + E02 k_6 + E03 k_7 - E05 k_1 \\
& -E06 k_2 - E07 k_3) e4 - e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_4 - E02 k_7 \\
& + E03 k_6 - E04 k_1 - E06 k_3 + E07 k_2) e5 \\
& - e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_7 + E02 k_4 - E03 k_5 - E04 k_2 \\
& + E05 k_3 - E07 k_1) e6 + e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_6 - E02 k_5 \\
& - E03 k_4 + E04 k_3 + E05 k_2 - E06 k_1) e7
\end{aligned}$$

This implies that $B=1/\omega$ * k_{CrossE} :

O > bwavesolE:=evalDG(1/omega*DGlm(kvec.ewavesol));

$$bwavesolE := -\frac{1}{\omega} \left(e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E02 k_3 - E03 k_2 + E04 k_5 \quad (5.3.22)
\right.$$

$$\begin{aligned}
& \left. -E05 k_4 - E06 k_7 + E07 k_6) \right) e1 \\
& + \frac{1}{\omega} \left(e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_3 - E03 k_1 - E04 k_6 - E05 k_7 \right. \\
& \left. + E06 k_4 + E07 k_5) \right) e2 - \frac{1}{\omega} \left(e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_2 \right. \\
& \left. - E02 k_1 + E04 k_7 - E05 k_6 + E06 k_5 - E07 k_4) \right) e3 \\
& + \frac{1}{\omega} \left(e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_5 + E02 k_6 + E03 k_7 - E05 k_1 \right. \\
& \left. - E06 k_2 - E07 k_3) \right) e4 - \frac{1}{\omega} \left(e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_4 \right. \\
& \left. - E02 k_7 + E03 k_6 - E04 k_1 - E06 k_3 + E07 k_2) \right) e5 \\
& - \frac{1}{\omega} \left(e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_7 + E02 k_4 - E03 k_5 - E04 k_2 \right. \\
& \left. + E05 k_3 - E07 k_1) \right) e6 + \frac{1}{\omega} \left(e^{-1(\omega t - k_1 x1 - k_2 x2 - k_3 x3 - k_4 x4 - k_5 x5 - k_6 x6 - k_7 x7)} (E01 k_6 \right. \\
& \left. - E02 k_5 - E03 k_4 + E04 k_3 + E05 k_2 - E06 k_1) \right) e7
\end{aligned}$$

O > GetComponents(ewavesol,Basis)[1];

$$e^{-1(\omega t - x1 k_1 - x2 k_2 - x3 k_3 - x4 k_4 - x5 k_5 - x6 k_6 - x7 k_7)} E01 \quad (5.3.23)$$

O > convert(exp(I*(kdotx-omega*t)),trig)

$$\cos(\omega t - x1 k_1 - x2 k_2 - x3 k_3 - x4 k_4 - x5 k_5 - x6 k_6 - x7 k_7) - I \sin(\omega t - x1 k_1 - x2 k_2 - x3 k_3 \quad (5.3.24)$$

$$- x4 k_4 - x5 k_5 - x6 k_6 - x7 k_7)$$

O > convert(GetComponents(ewavesol,Basis)[1],trig)

$$\left(\cos(\omega t - x1 k_1 - x2 k_2 - x3 k_3 - x4 k_4 - x5 k_5 - x6 k_6 - x7 k_7) - I \sin(\omega t - x1 k_1 - x2 k_2 - x3 k_3 \quad (5.3.25)
\right.$$

$$\left. - x4 k_4 - x5 k_5 - x6 k_6 - x7 k_7) \right) E01$$

O > convert(GetComponents(bwavesolE,Basis)[1],trig)

$$-\frac{1}{\omega} \left(\cos(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7) - I \sin(\omega t - x_1 k_1 - x_2 k_2 - x_3 k_3 - x_4 k_4 - x_5 k_5 - x_6 k_6 - x_7 k_7) \right) (E02 k_3 - E03 k_2 + E04 k_5 - E05 k_4 - E06 k_7 + E07 k_6) \quad (5.3.26)$$

Test solutions in ME's

We now test that these are solutions to Maxwell's equations.

O > tkvec:=evalDG((1/sqrt(2))*k*e1+(1/sqrt(2))*k*e2);

$$tkvec := \frac{\sqrt{2} k}{2} e1 + \frac{\sqrt{2} k}{2} e2 \quad (5.4.1)$$

The following works as long as the magnitude of the wave vector is k.

O > DGRe(-1*tkvec.tkvec);

$$k^2 e0 \quad (5.4.2)$$

O > tkdotx:=GetComponents(DGRe(-1*tkvec.xvec),extBasis)[1]

$$tkdotx := \frac{1}{2} \sqrt{2} kx1 + \frac{1}{2} \sqrt{2} kx2 \quad (5.4.3)$$

O > kdotE0:=GetComponents(DGRe(-1*tkvec.E0),extBasis)[1];

$$kdotE0 := \frac{1}{2} \sqrt{2} kE01 + \frac{1}{2} \sqrt{2} kE02 \quad (5.4.4)$$

O > indets(kdotE0,name);

$$\{E01, E02, k\} \quad (5.4.5)$$

O > orthcond:=solve(GetComponents(DGRe(-1*tkvec.E0),extBasis)[1]=0,indets(kdotE0,name),useassumptions) assuming k::positive;

$$orthcond := \{E01 = -E02, E02 = E02, k = k\} \quad (5.4.6)$$

O > tEwavesol:=evalDG(eval(evalDG(E0*exp(I*(tkdotx-omega*t+phi))), orthcond));

$$tEwavesol := -e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)} E02 e1 + e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)} E02 e2 \quad (5.4.7)$$

$$+ e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)} E03 e3 + e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)} E04 e4$$

$$+ e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)} E05 e5 + e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)} E06 e6$$

$$+ e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)} E07 e7$$

O > tBwavesol:=evalDG(1/omega*DGIm(tkvec.tEwavesol));

$$tBwavesol := \frac{\sqrt{2} k e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)}}{2 \omega} E03 e1 \quad (5.4.8)$$

$$- \frac{\sqrt{2} k e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)}}{2 \omega} E03 e2 + \frac{\sqrt{2} k e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)}}{\omega} E02 e3$$

$$- \frac{\sqrt{2} k e^{\frac{1}{2}(\sqrt{2} kx1 + \sqrt{2} kx2 - 2\omega t + 2\phi)}}{2 \omega} (E05 + E06) e4$$

$$\begin{aligned}
& + \frac{\sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi)}{2 \omega} (E04 - E07) e5 \\
& + \frac{\sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi)}{2 \omega} (E07 + E04) e6 \\
& + \frac{\sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi)}{2 \omega} (-E06 + E05) e7
\end{aligned}$$

O > dBWdt:=DGzip([seq(diff(GetComponents(tBwavesol,Basis)[i],t),i=1..7)], Basis,"plus");

$$\begin{aligned}
dBWdt := & -\frac{1}{2} \sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E03 e1 & (5.4.9) \\
& + \frac{1}{2} \sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E03 e2 - I \sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E02 e3 \\
& + \frac{1}{2} \sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) (E05 + E06) e4 \\
& - \frac{1}{2} \sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) (E04 - E07) e5 \\
& - \frac{1}{2} \sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) (E07 + E04) e6 \\
& - \frac{1}{2} \sqrt{2} k e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) (-E06 + E05) e7
\end{aligned}$$

O > dEWdt:=DGzip([seq(diff(GetComponents(tEwavesol,Basis)[i],t),i=1..7)], Basis,"plus");

$$\begin{aligned}
dEWdt := & I \omega e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E02 e1 - I \omega e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E02 e2 & (5.4.10) \\
& - I \omega e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E03 e3 - I \omega e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E04 e4 \\
& - I \omega e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E05 e5 - I \omega e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E06 e6 \\
& - I \omega e^{\frac{1}{2}} (\sqrt{2} k x_1 + \sqrt{2} k x_2 - 2 \omega t + 2 \phi) E07 e7
\end{aligned}$$

O > Div(tEwavesol);

$$0 e0 \quad (5.4.11)$$

O > evalDG(Curl(tEwavesol)-evalDG(-1*dBWdt));

$$0 \quad (5.4.12)$$

O > Div(tBwavesol);

$$0 e0 \quad (5.4.13)$$

O > evalDG(eval(evalDG(Curl(tBwavesol)-evalDG(mu*epsilon*dEWdt)),[omega=c*k,epsilon=1/(c^2*mu)]));

$$0 e1 + 0 e2 + 0 e3 + 0 e4 + 0 e5 + 0 e6 + 0 e7 \quad (5.4.14)$$

Construct the Faraday "Tensor"

O > newE:=subs([seq(GetComponents(tEwavesol,Basis)[i]=Epsilon[i],i=1..7)],tEwavesol);

$$newE := E_1 e1 + E_2 e2 + E_3 e3 + E_4 e4 + E_5 e5 + E_6 e6 + E_7 e7 \quad (5.4.15)$$

O > newB:=subs([seq(GetComponents(tBwavesol,Basis)[i]=beta[i],i=1..7)],

tBwavesol);

$$\text{newB} := \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + \beta_4 e_4 + \beta_5 e_5 + \beta_6 e_6 + \beta_7 e_7 \quad (5.4.16)$$

**O > emat:=Matrix([[0,seq(GetComponents(newE,Basis)[i],i=1..7)],[],[],[],
[],[],[],[]])**

$$\text{emat} := \begin{bmatrix} 0 & E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.4.17)$$

O > Emat:=emat-LinearAlgebra:-Transpose(emat);

$$\text{Emat} := \begin{bmatrix} 0 & E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 \\ -E_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.4.18)$$

O > seq(DGIm(e||1.e||j),j=1..7);

$$0 e_0, e_3, -e_2, e_5, -e_4, -e_7, e_6 \quad (5.4.19)$$

**O > subs([seq(e||i=beta[i],i=1..7),seq(evalDG(-e||i)=-beta[i],i=1..7),
evalDG(0*e0)=0],[0,seq(DGIm(e||1.e||j),j=1..7)]);**

$$[0, 0 e_0, \beta_3, -\beta_2, \beta_5, -\beta_4, -\beta_7, \beta_6] \quad (5.4.20)$$

**O > subs([seq(e||i=beta[i],i=1..7),seq(evalDG(-e||i)=-beta[i],i=1..7),
evalDG(0*e0)=0],[0,seq(DGIm(e||2.e||j),j=1..7)]);**

$$[0, -\beta_3, 0 e_0, \beta_1, \beta_6, \beta_7, -\beta_4, -\beta_5] \quad (5.4.21)$$

**O > seq(subs([seq(e||i=beta[i],i=1..7),seq(evalDG(-e||i)=-beta[i],i=1..7),
evalDG(0*e0)=0],[0,seq(DGIm(e||k.e||j),j=1..7)]),k=1..7)**

$$[0, 0 e_0, \beta_3, -\beta_2, \beta_5, -\beta_4, -\beta_7, \beta_6], [0, -\beta_3, 0 e_0, \beta_1, \beta_6, \beta_7, -\beta_4, -\beta_5], [0, \beta_2, -\beta_1, 0 e_0, \beta_7, -\beta_6, \beta_5, -\beta_4], [0, -\beta_5, -\beta_6, -\beta_7, 0 e_0, \beta_1, \beta_2, \beta_3], [0, \beta_4, -\beta_7, \beta_6, -\beta_1, 0 e_0, -\beta_3, \beta_2], [0, \beta_7, \beta_4, -\beta_5, -\beta_2, \beta_3, 0 e_0, -\beta_1], [0, -\beta_6, \beta_5, \beta_4, -\beta_3, -\beta_2, \beta_1, 0 e_0] \quad (5.4.22)$$

O > Bmat:=Matrix([[],seq(subs([seq(e||i=beta[i],i=1..7),seq(evalDG(-e||i)=-beta[i],i=1..7),evalDG(0*e0)=0],[0,seq(DGIm(e||k.e||j),j=1..7)]),k=1..7)]);

$$Bmat := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 e0 & \beta_3 & -\beta_2 & \beta_5 & -\beta_4 & -\beta_7 & \beta_6 \\ 0 & -\beta_3 & 0 e0 & \beta_1 & \beta_6 & \beta_7 & -\beta_4 & -\beta_5 \\ 0 & \beta_2 & -\beta_1 & 0 e0 & \beta_7 & -\beta_6 & \beta_5 & -\beta_4 \\ 0 & -\beta_5 & -\beta_6 & -\beta_7 & 0 e0 & \beta_1 & \beta_2 & \beta_3 \\ 0 & \beta_4 & -\beta_7 & \beta_6 & -\beta_1 & 0 e0 & -\beta_3 & \beta_2 \\ 0 & \beta_7 & \beta_4 & -\beta_5 & -\beta_2 & \beta_3 & 0 e0 & -\beta_1 \\ 0 & -\beta_6 & \beta_5 & \beta_4 & -\beta_3 & -\beta_2 & \beta_1 & 0 e0 \end{bmatrix} \quad (5.4.23)$$

O > Fmat:=Emat+Bmat;

$$Fmat := \begin{bmatrix} 0 & E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 \\ -E_1 & 0 e0 & \beta_3 & -\beta_2 & \beta_5 & -\beta_4 & -\beta_7 & \beta_6 \\ -E_2 & -\beta_3 & 0 e0 & \beta_1 & \beta_6 & \beta_7 & -\beta_4 & -\beta_5 \\ -E_3 & \beta_2 & -\beta_1 & 0 e0 & \beta_7 & -\beta_6 & \beta_5 & -\beta_4 \\ -E_4 & -\beta_5 & -\beta_6 & -\beta_7 & 0 e0 & \beta_1 & \beta_2 & \beta_3 \\ -E_5 & \beta_4 & -\beta_7 & \beta_6 & -\beta_1 & 0 e0 & -\beta_3 & \beta_2 \\ -E_6 & \beta_7 & \beta_4 & -\beta_5 & -\beta_2 & \beta_3 & 0 e0 & -\beta_1 \\ -E_7 & -\beta_6 & \beta_5 & \beta_4 & -\beta_3 & -\beta_2 & \beta_1 & 0 e0 \end{bmatrix} \quad (5.4.24)$$

O > DGI(m(e3.e6);

e5

(5.4.25)

Octonionic and conventional Faraday Tensors: equality of Tensors.

In this section, we first introduce the Octonionic Faraday Tensor. We then construct the conventional tensor in terms of the vector potential and show that these two tensors can be related.

O > DGEnvironment[Coordinate]([seq(x||i,i=0..7)],M);

Manifold: M

(5.5.1)

M > testF:=convert(Fmat,DGtensor,[["con_bas","con_bas"],[]]);

$$\begin{aligned} testF := & E_1 \partial_{x0} \otimes \partial_{x1} + E_2 \partial_{x0} \otimes \partial_{x2} + E_3 \partial_{x0} \otimes \partial_{x3} + E_4 \partial_{x0} \otimes \partial_{x4} + E_5 \partial_{x0} \otimes \partial_{x5} + E_6 \partial_{x0} \otimes \partial_{x6} + E_7 \partial_{x0} \otimes \partial_{x7} \\ & - E_1 \partial_{x1} \otimes \partial_{x0} + 0 e0 \partial_{x1} \otimes \partial_{x1} + \beta_3 \partial_{x1} \otimes \partial_{x2} - \beta_2 \partial_{x1} \otimes \partial_{x3} + \beta_5 \partial_{x1} \otimes \partial_{x4} - \beta_4 \partial_{x1} \otimes \partial_{x5} - \beta_7 \partial_{x1} \otimes \partial_{x6} \\ & + \beta_6 \partial_{x1} \otimes \partial_{x7} - E_2 \partial_{x2} \otimes \partial_{x0} - \beta_3 \partial_{x2} \otimes \partial_{x1} + 0 e0 \partial_{x2} \otimes \partial_{x2} + \beta_1 \partial_{x2} \otimes \partial_{x3} + \beta_6 \partial_{x2} \otimes \partial_{x4} + \beta_7 \partial_{x2} \otimes \partial_{x5} \\ & - \beta_4 \partial_{x2} \otimes \partial_{x6} - \beta_5 \partial_{x2} \otimes \partial_{x7} - E_3 \partial_{x3} \otimes \partial_{x0} + \beta_2 \partial_{x3} \otimes \partial_{x1} - \beta_1 \partial_{x3} \otimes \partial_{x2} + 0 e0 \partial_{x3} \otimes \partial_{x3} + \beta_7 \partial_{x3} \otimes \partial_{x4} \\ & - \beta_6 \partial_{x3} \otimes \partial_{x5} + \beta_5 \partial_{x3} \otimes \partial_{x6} - \beta_4 \partial_{x3} \otimes \partial_{x7} - E_4 \partial_{x4} \otimes \partial_{x0} - \beta_5 \partial_{x4} \otimes \partial_{x1} - \beta_6 \partial_{x4} \otimes \partial_{x2} - \beta_7 \partial_{x4} \otimes \partial_{x3} \\ & + 0 e0 \partial_{x4} \otimes \partial_{x4} + \beta_1 \partial_{x4} \otimes \partial_{x5} + \beta_2 \partial_{x4} \otimes \partial_{x6} + \beta_3 \partial_{x4} \otimes \partial_{x7} - E_5 \partial_{x5} \otimes \partial_{x0} + \beta_4 \partial_{x5} \otimes \partial_{x1} - \beta_7 \partial_{x5} \otimes \partial_{x2} \\ & + \beta_6 \partial_{x5} \otimes \partial_{x3} - \beta_1 \partial_{x5} \otimes \partial_{x4} + 0 e0 \partial_{x5} \otimes \partial_{x5} - \beta_3 \partial_{x5} \otimes \partial_{x6} + \beta_2 \partial_{x5} \otimes \partial_{x7} - E_6 \partial_{x6} \otimes \partial_{x0} + \beta_7 \partial_{x6} \otimes \partial_{x1} \\ & + \beta_4 \partial_{x6} \otimes \partial_{x2} - \beta_5 \partial_{x6} \otimes \partial_{x3} - \beta_2 \partial_{x6} \otimes \partial_{x4} + \beta_3 \partial_{x6} \otimes \partial_{x5} + 0 e0 \partial_{x6} \otimes \partial_{x6} - \beta_1 \partial_{x6} \otimes \partial_{x7} - E_7 \partial_{x7} \otimes \partial_{x0} \end{aligned} \quad (5.5.2)$$

$$-\beta_6 \partial_{x7} \otimes \partial_{x1} + \beta_5 \partial_{x7} \otimes \partial_{x2} + \beta_4 \partial_{x7} \otimes \partial_{x3} - \beta_3 \partial_{x7} \otimes \partial_{x4} - \beta_2 \partial_{x7} \otimes \partial_{x5} + \beta_1 \partial_{x7} \otimes \partial_{x6} + 0 e0 \partial_{x7} \otimes \partial_{x7}$$

M > testB:=convert(Bmat,DGtensor,[["con_bas"],"con_bas"],[]);

$$testB := 0 e0 \partial_{x1} \otimes \partial_{x1} + \beta_3 \partial_{x1} \otimes \partial_{x2} - \beta_2 \partial_{x1} \otimes \partial_{x3} + \beta_5 \partial_{x1} \otimes \partial_{x4} - \beta_4 \partial_{x1} \otimes \partial_{x5} - \beta_7 \partial_{x1} \otimes \partial_{x6} + \beta_6 \partial_{x1} \otimes \partial_{x7} \quad (5.5.3)$$

$$-\beta_3 \partial_{x2} \otimes \partial_{x1} + 0 e0 \partial_{x2} \otimes \partial_{x2} + \beta_1 \partial_{x2} \otimes \partial_{x3} + \beta_6 \partial_{x2} \otimes \partial_{x4} + \beta_7 \partial_{x2} \otimes \partial_{x5} - \beta_4 \partial_{x2} \otimes \partial_{x6} - \beta_5 \partial_{x2} \otimes \partial_{x7}$$

$$+ \beta_2 \partial_{x3} \otimes \partial_{x1} - \beta_1 \partial_{x3} \otimes \partial_{x2} + 0 e0 \partial_{x3} \otimes \partial_{x3} + \beta_7 \partial_{x3} \otimes \partial_{x4} - \beta_6 \partial_{x3} \otimes \partial_{x5} + \beta_5 \partial_{x3} \otimes \partial_{x6} - \beta_4 \partial_{x3} \otimes \partial_{x7}$$

$$-\beta_5 \partial_{x4} \otimes \partial_{x1} - \beta_6 \partial_{x4} \otimes \partial_{x2} - \beta_7 \partial_{x4} \otimes \partial_{x3} + 0 e0 \partial_{x4} \otimes \partial_{x4} + \beta_1 \partial_{x4} \otimes \partial_{x5} + \beta_2 \partial_{x4} \otimes \partial_{x6} + \beta_3 \partial_{x4} \otimes \partial_{x7}$$

$$+ \beta_4 \partial_{x5} \otimes \partial_{x1} - \beta_7 \partial_{x5} \otimes \partial_{x2} + \beta_6 \partial_{x5} \otimes \partial_{x3} - \beta_1 \partial_{x5} \otimes \partial_{x4} + 0 e0 \partial_{x5} \otimes \partial_{x5} - \beta_3 \partial_{x5} \otimes \partial_{x6} + \beta_2 \partial_{x5} \otimes \partial_{x7}$$

$$+ \beta_7 \partial_{x6} \otimes \partial_{x1} + \beta_4 \partial_{x6} \otimes \partial_{x2} - \beta_5 \partial_{x6} \otimes \partial_{x3} - \beta_2 \partial_{x6} \otimes \partial_{x4} + \beta_3 \partial_{x6} \otimes \partial_{x5} + 0 e0 \partial_{x6} \otimes \partial_{x6} - \beta_1 \partial_{x6} \otimes \partial_{x7}$$

$$-\beta_6 \partial_{x7} \otimes \partial_{x1} + \beta_5 \partial_{x7} \otimes \partial_{x2} + \beta_4 \partial_{x7} \otimes \partial_{x3} - \beta_3 \partial_{x7} \otimes \partial_{x4} - \beta_2 \partial_{x7} \otimes \partial_{x5} + \beta_1 \partial_{x7} \otimes \partial_{x6} + 0 e0 \partial_{x7} \otimes \partial_{x7}$$

O > newb:=DGIm(1/omega*kvec.newE);

$$newb := -\frac{k_3 E_2 - k_2 E_3 + k_5 E_4 - k_4 E_5 - k_7 E_6 + k_6 E_7}{\omega} e1 \quad (5.5.4)$$

$$+ \frac{k_3 E_1 - k_1 E_3 - k_6 E_4 - k_7 E_5 + k_4 E_6 + k_5 E_7}{\omega} e2$$

$$- \frac{k_2 E_1 - k_1 E_2 + k_7 E_4 - k_6 E_5 + k_5 E_6 - k_4 E_7}{\omega} e3$$

$$+ \frac{k_5 E_1 + k_6 E_2 + k_7 E_3 - k_1 E_5 - k_2 E_6 - k_3 E_7}{\omega} e4$$

$$- \frac{k_4 E_1 - k_7 E_2 + k_6 E_3 - k_1 E_4 - k_3 E_6 + k_2 E_7}{\omega} e5$$

$$- \frac{k_7 E_1 + k_4 E_2 - k_5 E_3 - k_2 E_4 + k_3 E_5 - k_1 E_7}{\omega} e6$$

$$+ \frac{k_6 E_1 - k_5 E_2 - k_4 E_3 + k_3 E_4 + k_2 E_5 - k_1 E_6}{\omega} e7$$

O > GetComponents(newb,Basis)[1];

$$-\frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} \quad (5.5.5)$$

O > testF3:=subs([seq(beta[i]=GetComponents(newb,Basis)[i],i=1..7)],testF);

$$testF3 := E_1 \partial_{x0} \otimes \partial_{x1} + E_2 \partial_{x0} \otimes \partial_{x2} + E_3 \partial_{x0} \otimes \partial_{x3} + E_4 \partial_{x0} \otimes \partial_{x4} + E_5 \partial_{x0} \otimes \partial_{x5} + E_6 \partial_{x0} \otimes \partial_{x6} + E_7 \partial_{x0} \otimes \partial_{x7} \quad (5.5.6)$$

$$- E_1 \partial_{x1} \otimes \partial_{x0} + 0 e0 \partial_{x1} \otimes \partial_{x1} - \frac{k_2 E_1 - k_1 E_2 + k_7 E_4 - k_6 E_5 + k_5 E_6 - k_4 E_7}{\omega} \partial_{x1} \otimes \partial_{x2}$$

$$- \frac{k_3 E_1 - k_1 E_3 - k_6 E_4 - k_7 E_5 + k_4 E_6 + k_5 E_7}{\omega} \partial_{x1} \otimes \partial_{x3}$$

$$- \frac{k_4 E_1 - k_7 E_2 + k_6 E_3 - k_1 E_4 - k_3 E_6 + k_2 E_7}{\omega} \partial_{x1} \otimes \partial_{x4}$$

$$- \frac{k_5 E_1 + k_6 E_2 + k_7 E_3 - k_1 E_5 - k_2 E_6 - k_3 E_7}{\omega} \partial_{x1} \otimes \partial_{x5}$$

$$\begin{aligned}
& - \frac{k_6 E_1 - k_5 E_2 - k_4 E_3 + k_3 E_4 + k_2 E_5 - k_1 E_6}{\omega} \partial_{x1} \otimes \partial_{x6} \\
& - \frac{k_7 E_1 + k_4 E_2 - k_5 E_3 - k_2 E_4 + k_3 E_5 - k_1 E_7}{\omega} \partial_{x1} \otimes \partial_{x7} - E_2 \partial_{x2} \otimes \partial_{x0} \\
& + \frac{k_2 E_1 - k_1 E_2 + k_7 E_4 - k_6 E_5 + k_5 E_6 - k_4 E_7}{\omega} \partial_{x2} \otimes \partial_{x1} + 0 e \theta \partial_{x2} \otimes \partial_{x2} \\
& - \frac{k_3 E_2 - k_2 E_3 + k_5 E_4 - k_4 E_5 - k_7 E_6 + k_6 E_7}{\omega} \partial_{x2} \otimes \partial_{x3} \\
& - \frac{k_7 E_1 + k_4 E_2 - k_5 E_3 - k_2 E_4 + k_3 E_5 - k_1 E_7}{\omega} \partial_{x2} \otimes \partial_{x4} \\
& + \frac{k_6 E_1 - k_5 E_2 - k_4 E_3 + k_3 E_4 + k_2 E_5 - k_1 E_6}{\omega} \partial_{x2} \otimes \partial_{x5} \\
& - \frac{k_5 E_1 + k_6 E_2 + k_7 E_3 - k_1 E_5 - k_2 E_6 - k_3 E_7}{\omega} \partial_{x2} \otimes \partial_{x6} \\
& + \frac{k_4 E_1 - k_7 E_2 + k_6 E_3 - k_1 E_4 - k_3 E_6 + k_2 E_7}{\omega} \partial_{x2} \otimes \partial_{x7} - E_3 \partial_{x3} \otimes \partial_{x0} \\
& + \frac{k_3 E_1 - k_1 E_3 - k_6 E_4 - k_7 E_5 + k_4 E_6 + k_5 E_7}{\omega} \partial_{x3} \otimes \partial_{x1} \\
& + \frac{k_3 E_2 - k_2 E_3 + k_5 E_4 - k_4 E_5 - k_7 E_6 + k_6 E_7}{\omega} \partial_{x3} \otimes \partial_{x2} + 0 e \theta \partial_{x3} \otimes \partial_{x3} \\
& + \frac{k_6 E_1 - k_5 E_2 - k_4 E_3 + k_3 E_4 + k_2 E_5 - k_1 E_6}{\omega} \partial_{x3} \otimes \partial_{x4} \\
& + \frac{k_7 E_1 + k_4 E_2 - k_5 E_3 - k_2 E_4 + k_3 E_5 - k_1 E_7}{\omega} \partial_{x3} \otimes \partial_{x5} \\
& - \frac{k_4 E_1 - k_7 E_2 + k_6 E_3 - k_1 E_4 - k_3 E_6 + k_2 E_7}{\omega} \partial_{x3} \otimes \partial_{x6} \\
& - \frac{k_5 E_1 + k_6 E_2 + k_7 E_3 - k_1 E_5 - k_2 E_6 - k_3 E_7}{\omega} \partial_{x3} \otimes \partial_{x7} - E_4 \partial_{x4} \otimes \partial_{x0} \\
& + \frac{k_4 E_1 - k_7 E_2 + k_6 E_3 - k_1 E_4 - k_3 E_6 + k_2 E_7}{\omega} \partial_{x4} \otimes \partial_{x1} \\
& + \frac{k_7 E_1 + k_4 E_2 - k_5 E_3 - k_2 E_4 + k_3 E_5 - k_1 E_7}{\omega} \partial_{x4} \otimes \partial_{x2} \\
& - \frac{k_6 E_1 - k_5 E_2 - k_4 E_3 + k_3 E_4 + k_2 E_5 - k_1 E_6}{\omega} \partial_{x4} \otimes \partial_{x3} + 0 e \theta \partial_{x4} \otimes \partial_{x4} \\
& - \frac{k_3 E_2 - k_2 E_3 + k_5 E_4 - k_4 E_5 - k_7 E_6 + k_6 E_7}{\omega} \partial_{x4} \otimes \partial_{x5} \\
& + \frac{k_3 E_1 - k_1 E_3 - k_6 E_4 - k_7 E_5 + k_4 E_6 + k_5 E_7}{\omega} \partial_{x4} \otimes \partial_{x6} \\
& - \frac{k_2 E_1 - k_1 E_2 + k_7 E_4 - k_6 E_5 + k_5 E_6 - k_4 E_7}{\omega} \partial_{x4} \otimes \partial_{x7} - E_5 \partial_{x5} \otimes \partial_{x0}
\end{aligned}$$

$$\begin{aligned}
& + \frac{k_5 E_1 + k_6 E_2 + k_7 E_3 - k_1 E_5 - k_2 E_6 - k_3 E_7}{\omega} \partial_{x5} \otimes \partial_{x1} \\
& - \frac{k_6 E_1 - k_5 E_2 - k_4 E_3 + k_3 E_4 + k_2 E_5 - k_1 E_6}{\omega} \partial_{x5} \otimes \partial_{x2} \\
& - \frac{k_7 E_1 + k_4 E_2 - k_5 E_3 - k_2 E_4 + k_3 E_5 - k_1 E_7}{\omega} \partial_{x5} \otimes \partial_{x3} \\
& + \frac{k_3 E_2 - k_2 E_3 + k_5 E_4 - k_4 E_5 - k_7 E_6 + k_6 E_7}{\omega} \partial_{x5} \otimes \partial_{x4} + 0 e\theta \partial_{x5} \otimes \partial_{x5} \\
& + \frac{k_2 E_1 - k_1 E_2 + k_7 E_4 - k_6 E_5 + k_5 E_6 - k_4 E_7}{\omega} \partial_{x5} \otimes \partial_{x6} \\
& + \frac{k_3 E_1 - k_1 E_3 - k_6 E_4 - k_7 E_5 + k_4 E_6 + k_5 E_7}{\omega} \partial_{x5} \otimes \partial_{x7} - E_6 \partial_{x6} \otimes \partial_{x0} \\
& + \frac{k_6 E_1 - k_5 E_2 - k_4 E_3 + k_3 E_4 + k_2 E_5 - k_1 E_6}{\omega} \partial_{x6} \otimes \partial_{x1} \\
& + \frac{k_5 E_1 + k_6 E_2 + k_7 E_3 - k_1 E_5 - k_2 E_6 - k_3 E_7}{\omega} \partial_{x6} \otimes \partial_{x2} \\
& + \frac{k_4 E_1 - k_7 E_2 + k_6 E_3 - k_1 E_4 - k_3 E_6 + k_2 E_7}{\omega} \partial_{x6} \otimes \partial_{x3} \\
& - \frac{k_3 E_1 - k_1 E_3 - k_6 E_4 - k_7 E_5 + k_4 E_6 + k_5 E_7}{\omega} \partial_{x6} \otimes \partial_{x4} \\
& - \frac{k_2 E_1 - k_1 E_2 + k_7 E_4 - k_6 E_5 + k_5 E_6 - k_4 E_7}{\omega} \partial_{x6} \otimes \partial_{x5} + 0 e\theta \partial_{x6} \otimes \partial_{x6} \\
& + \frac{k_3 E_2 - k_2 E_3 + k_5 E_4 - k_4 E_5 - k_7 E_6 + k_6 E_7}{\omega} \partial_{x6} \otimes \partial_{x7} - E_7 \partial_{x7} \otimes \partial_{x0} \\
& + \frac{k_7 E_1 + k_4 E_2 - k_5 E_3 - k_2 E_4 + k_3 E_5 - k_1 E_7}{\omega} \partial_{x7} \otimes \partial_{x1} \\
& - \frac{k_4 E_1 - k_7 E_2 + k_6 E_3 - k_1 E_4 - k_3 E_6 + k_2 E_7}{\omega} \partial_{x7} \otimes \partial_{x2} \\
& + \frac{k_5 E_1 + k_6 E_2 + k_7 E_3 - k_1 E_5 - k_2 E_6 - k_3 E_7}{\omega} \partial_{x7} \otimes \partial_{x3} \\
& + \frac{k_2 E_1 - k_1 E_2 + k_7 E_4 - k_6 E_5 + k_5 E_6 - k_4 E_7}{\omega} \partial_{x7} \otimes \partial_{x4} \\
& - \frac{k_3 E_1 - k_1 E_3 - k_6 E_4 - k_7 E_5 + k_4 E_6 + k_5 E_7}{\omega} \partial_{x7} \otimes \partial_{x5} \\
& - \frac{k_3 E_2 - k_2 E_3 + k_5 E_4 - k_4 E_5 - k_7 E_6 + k_6 E_7}{\omega} \partial_{x7} \otimes \partial_{x6} + 0 e\theta \partial_{x7} \otimes \partial_{x7}
\end{aligned}$$

Compare with conventional

$$\begin{aligned}
\mathbf{M} & > \text{eta} := \text{evalDG}(-dx||0 \ \&t \ dx||0 \ + \ \text{add}(dx||i \ \&t \ dx||i, i=1..7)); \\
\eta & := -dx0 \otimes dx0 + dx1 \otimes dx1 + dx2 \otimes dx2 + dx3 \otimes dx3 + dx4 \otimes dx4 + dx5 \otimes dx5 + dx6 \otimes dx6 \quad (5.5.1.1) \\
& \quad + dx7 \otimes dx7
\end{aligned}$$

M > Gamma:=Christoffel(eta);

$$\Gamma := \nabla_{\frac{\partial}{\partial x^0}} \frac{\partial}{\partial x^0} = 0 \frac{\partial}{\partial x^0} \quad (5.5.1.2)$$

M > formbasis:=DGinformation("FrameBaseForms");

$$\text{formbasis} := [dx0, dx1, dx2, dx3, dx4, dx5, dx6, dx7] \quad (5.5.1.3)$$

M > vectorbasis:=DGinformation("FrameBaseVectors");

$$\text{vectorbasis} := \left[\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}, \frac{\partial}{\partial x^4}, \frac{\partial}{\partial x^5}, \frac{\partial}{\partial x^6}, \frac{\partial}{\partial x^7} \right] \quad (5.5.1.4)$$

M > A:=DGzip([seq(a[i](seq(x[|j|],j=0..7)),i=0..7)],formbasis,"plus");

$$\begin{aligned} A := & a_0(x0, x1, x2, x3, x4, x5, x6, x7) dx0 + a_1(x0, x1, x2, x3, x4, x5, x6, x7) dx1 + a_2(x0, x1, x2, \\ & x3, x4, x5, x6, x7) dx2 + a_3(x0, x1, x2, x3, x4, x5, x6, x7) dx3 + a_4(x0, x1, x2, x3, x4, x5, x6, \\ & x7) dx4 + a_5(x0, x1, x2, x3, x4, x5, x6, x7) dx5 + a_6(x0, x1, x2, x3, x4, x5, x6, x7) dx6 \\ & + a_7(x0, x1, x2, x3, x4, x5, x6, x7) dx7 \end{aligned} \quad (5.5.1.5)$$

M > dA:=CovariantDerivative(A,Gamma);

$$\begin{aligned} dA := & \frac{\partial}{\partial x^0} a_0(x0, x1, x2, x3, x4, x5, x6, x7) dx0 \otimes dx0 + \frac{\partial}{\partial x^1} a_0(x0, x1, x2, x3, x4, x5, x6, x7) dx0 \\ & \otimes dx1 + \frac{\partial}{\partial x^2} a_0(x0, x1, x2, x3, x4, x5, x6, x7) dx0 \otimes dx2 + \frac{\partial}{\partial x^3} a_0(x0, x1, x2, x3, x4, x5, x6, \\ & x7) dx0 \otimes dx3 + \frac{\partial}{\partial x^4} a_0(x0, x1, x2, x3, x4, x5, x6, x7) dx0 \otimes dx4 + \frac{\partial}{\partial x^5} a_0(x0, x1, x2, x3, x4, \\ & x5, x6, x7) dx0 \otimes dx5 + \frac{\partial}{\partial x^6} a_0(x0, x1, x2, x3, x4, x5, x6, x7) dx0 \otimes dx6 + \frac{\partial}{\partial x^7} a_0(x0, x1, x2, \\ & x3, x4, x5, x6, x7) dx0 \otimes dx7 + \frac{\partial}{\partial x^0} a_1(x0, x1, x2, x3, x4, x5, x6, x7) dx1 \otimes dx0 + \frac{\partial}{\partial x^1} a_1(x0, \\ & x1, x2, x3, x4, x5, x6, x7) dx1 \otimes dx1 + \frac{\partial}{\partial x^2} a_1(x0, x1, x2, x3, x4, x5, x6, x7) dx1 \otimes dx2 + \frac{\partial}{\partial x^3} \\ & a_1(x0, x1, x2, x3, x4, x5, x6, x7) dx1 \otimes dx3 + \frac{\partial}{\partial x^4} a_1(x0, x1, x2, x3, x4, x5, x6, x7) dx1 \otimes dx4 \\ & + \frac{\partial}{\partial x^5} a_1(x0, x1, x2, x3, x4, x5, x6, x7) dx1 \otimes dx5 + \frac{\partial}{\partial x^6} a_1(x0, x1, x2, x3, x4, x5, x6, x7) dx1 \\ & \otimes dx6 + \frac{\partial}{\partial x^7} a_1(x0, x1, x2, x3, x4, x5, x6, x7) dx1 \otimes dx7 + \frac{\partial}{\partial x^0} a_2(x0, x1, x2, x3, x4, x5, x6, \\ & x7) dx2 \otimes dx0 + \frac{\partial}{\partial x^1} a_2(x0, x1, x2, x3, x4, x5, x6, x7) dx2 \otimes dx1 + \frac{\partial}{\partial x^2} a_2(x0, x1, x2, x3, x4, \\ & x5, x6, x7) dx2 \otimes dx2 + \frac{\partial}{\partial x^3} a_2(x0, x1, x2, x3, x4, x5, x6, x7) dx2 \otimes dx3 + \frac{\partial}{\partial x^4} a_2(x0, x1, x2, \\ & x3, x4, x5, x6, x7) dx2 \otimes dx4 + \frac{\partial}{\partial x^5} a_2(x0, x1, x2, x3, x4, x5, x6, x7) dx2 \otimes dx5 + \frac{\partial}{\partial x^6} a_2(x0, \\ & x1, x2, x3, x4, x5, x6, x7) dx2 \otimes dx6 + \frac{\partial}{\partial x^7} a_2(x0, x1, x2, x3, x4, x5, x6, x7) dx2 \otimes dx7 + \frac{\partial}{\partial x^0} \\ & a_3(x0, x1, x2, x3, x4, x5, x6, x7) dx3 \otimes dx0 + \frac{\partial}{\partial x^1} a_3(x0, x1, x2, x3, x4, x5, x6, x7) dx3 \otimes dx1 \\ & + \frac{\partial}{\partial x^2} a_3(x0, x1, x2, x3, x4, x5, x6, x7) dx3 \otimes dx2 + \frac{\partial}{\partial x^3} a_3(x0, x1, x2, x3, x4, x5, x6, x7) dx3 \\ & \otimes dx3 + \frac{\partial}{\partial x^4} a_3(x0, x1, x2, x3, x4, x5, x6, x7) dx3 \otimes dx4 + \frac{\partial}{\partial x^5} a_3(x0, x1, x2, x3, x4, x5, x6, \\ & x7) dx3 \otimes dx5 + \frac{\partial}{\partial x^6} a_3(x0, x1, x2, x3, x4, x5, x6, x7) dx3 \otimes dx6 + \frac{\partial}{\partial x^7} a_3(x0, x1, x2, x3, x4, \\ & x5, x6, x7) dx3 \otimes dx7 + \frac{\partial}{\partial x^0} a_4(x0, x1, x2, x3, x4, x5, x6, x7) dx4 \otimes dx0 + \frac{\partial}{\partial x^1} a_4(x0, x1, x2, \end{aligned}$$

$$\begin{aligned}
& x_3, x_4, x_5, x_6, x_7) dx_4 \otimes dx_1 + \frac{\partial}{\partial x_2} a_4(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_4 \otimes dx_2 + \frac{\partial}{\partial x_3} a_4(x_0, \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_4 \otimes dx_3 + \frac{\partial}{\partial x_4} a_4(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_4 \otimes dx_4 + \frac{\partial}{\partial x_5} \\
& a_4(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_4 \otimes dx_5 + \frac{\partial}{\partial x_6} a_4(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_4 \otimes dx_6 \\
& + \frac{\partial}{\partial x_7} a_4(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_4 \otimes dx_7 + \frac{\partial}{\partial x_0} a_5(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_5 \\
& \otimes dx_0 + \frac{\partial}{\partial x_1} a_5(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_5 \otimes dx_1 + \frac{\partial}{\partial x_2} a_5(x_0, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) dx_5 \otimes dx_2 + \frac{\partial}{\partial x_3} a_5(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_5 \otimes dx_3 + \frac{\partial}{\partial x_4} a_5(x_0, x_1, x_2, x_3, x_4, \\
& x_5, x_6, x_7) dx_5 \otimes dx_4 + \frac{\partial}{\partial x_5} a_5(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_5 \otimes dx_5 + \frac{\partial}{\partial x_6} a_5(x_0, x_1, x_2, \\
& x_3, x_4, x_5, x_6, x_7) dx_5 \otimes dx_6 + \frac{\partial}{\partial x_7} a_5(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_5 \otimes dx_7 + \frac{\partial}{\partial x_0} a_6(x_0, \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_6 \otimes dx_0 + \frac{\partial}{\partial x_1} a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_6 \otimes dx_1 + \frac{\partial}{\partial x_2} \\
& a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_6 \otimes dx_2 + \frac{\partial}{\partial x_3} a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_6 \otimes dx_3 \\
& + \frac{\partial}{\partial x_4} a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_6 \otimes dx_4 + \frac{\partial}{\partial x_5} a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_6 \\
& \otimes dx_5 + \frac{\partial}{\partial x_6} a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_6 \otimes dx_6 + \frac{\partial}{\partial x_7} a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, \\
& x_7) dx_6 \otimes dx_7 + \frac{\partial}{\partial x_0} a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_7 \otimes dx_0 + \frac{\partial}{\partial x_1} a_7(x_0, x_1, x_2, x_3, x_4, \\
& x_5, x_6, x_7) dx_7 \otimes dx_1 + \frac{\partial}{\partial x_2} a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_7 \otimes dx_2 + \frac{\partial}{\partial x_3} a_7(x_0, x_1, x_2, \\
& x_3, x_4, x_5, x_6, x_7) dx_7 \otimes dx_3 + \frac{\partial}{\partial x_4} a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_7 \otimes dx_4 + \frac{\partial}{\partial x_5} a_7(x_0, \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_7 \otimes dx_5 + \frac{\partial}{\partial x_6} a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_7 \otimes dx_6 + \frac{\partial}{\partial x_7} \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) dx_7 \otimes dx_7
\end{aligned}$$

M > F1:=SymmetrizeIndices(2*dA,[1,2],"SkewSymmetric");

$$\begin{aligned}
F1 := & \left(\frac{\partial}{\partial x_1} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} a_1(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) dx_0 \otimes d \quad (5.5.1.7) \\
& x_1 - \left(\frac{\partial}{\partial x_1} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} a_1(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) dx_1 \\
& \otimes dx_0 + \left(\frac{\partial}{\partial x_2} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} a_2(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) d \\
& x_0 \otimes dx_2 - \left(\frac{\partial}{\partial x_2} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} a_2(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \right) \right) \\
& \left. \right) dx_2 \otimes dx_0 + \left(\frac{\partial}{\partial x_3} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} a_3(x_0, x_1, x_2, x_3, x_4, x_5, x_6, \right. \right. \\
& \left. \left. x_7) \right) \right) dx_0 \otimes dx_3 - \left(\frac{\partial}{\partial x_3} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} a_3(x_0, x_1, x_2, x_3, x_4, x_5, \right. \right. \\
& \left. \left. x_6, x_7) \right) \right) dx_3 \otimes dx_0 + \left(\frac{\partial}{\partial x_4} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} a_4(x_0, x_1, x_2, x_3, x_4, \right. \right. \\
& \left. \left. x_5, x_6, x_7) \right) \right) dx_0 \otimes dx_4 - \left(\frac{\partial}{\partial x_4} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} a_4(x_0, x_1, x_2, x_3, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_6} \otimes \partial_{x_5} + \left(\frac{\partial}{\partial x_7} a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_6} \right. \right. \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_6} \otimes \partial_{x_7} + \left(\frac{\partial}{\partial x_7} a_0(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_0} \right. \right. \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_7} \otimes \partial_{x_0} - \left(\frac{\partial}{\partial x_7} a_1(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_1} \right. \right. \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_7} \otimes \partial_{x_1} - \left(\frac{\partial}{\partial x_7} a_2(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_2} \right. \right. \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_7} \otimes \partial_{x_2} - \left(\frac{\partial}{\partial x_7} a_3(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_3} \right. \right. \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_7} \otimes \partial_{x_3} - \left(\frac{\partial}{\partial x_7} a_4(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_4} \right. \right. \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_7} \otimes \partial_{x_4} - \left(\frac{\partial}{\partial x_7} a_5(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_5} \right. \right. \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_7} \otimes \partial_{x_5} - \left(\frac{\partial}{\partial x_7} a_6(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) - \left(\frac{\partial}{\partial x_6} \right. \right. \\
& a_7(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) \Big) \Big) \partial_{x_7} \otimes \partial_{x_6}
\end{aligned}$$

M > f1:=simplify(subs([seq(a[i](seq(x||j,j=0..7)) = Alpha[i]*exp(I*(add(seq(k[o]*x||o,o=1..7))-omega*x||0)),i=0..7)],f));

$$\begin{aligned}
fl := & -I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_1 + A_0 k_1) \partial_{x_0} \otimes \partial_{x_1} & (5.5.1.9) \\
& -I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_2 + A_0 k_2) \partial_{x_0} \otimes \partial_{x_2} \\
& -I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_3 + A_0 k_3) \partial_{x_0} \otimes \partial_{x_3} \\
& -I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_4 + A_0 k_4) \partial_{x_0} \otimes \partial_{x_4} \\
& -I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_5 + A_0 k_5) \partial_{x_0} \otimes \partial_{x_5} \\
& -I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_6 + A_0 k_6) \partial_{x_0} \otimes \partial_{x_6} \\
& -I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_7 + A_0 k_7) \partial_{x_0} \otimes \partial_{x_7} \\
& +I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_1 + A_0 k_1) \partial_{x_1} \otimes \partial_{x_0} \\
& +I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (A_1 k_2 - A_2 k_1) \partial_{x_1} \otimes \partial_{x_2} \\
& +I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (A_1 k_3 - A_3 k_1) \partial_{x_1} \otimes \partial_{x_3} \\
& +I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (A_1 k_4 - A_4 k_1) \partial_{x_1} \otimes \partial_{x_4} \\
& +I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (A_1 k_5 - A_5 k_1) \partial_{x_1} \otimes \partial_{x_5} \\
& +I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (A_1 k_6 - A_6 k_1) \partial_{x_1} \otimes \partial_{x_6} \\
& +I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (A_1 k_7 - A_7 k_1) \partial_{x_1} \otimes \partial_{x_7} \\
& +I e^{I x_6 k_6 + I x_2 k_2 + I x_3 k_3 + I x_4 k_4 + I x_5 k_5 + I x_1 k_1 + I x_7 k_7 - I x_0 \omega} (\omega A_2 + A_0 k_2) \partial_{x_2} \otimes \partial_{x_0}
\end{aligned}$$

$$\begin{aligned}
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_4 k_5 + A_5 k_4) \partial_{x_5} \otimes \partial_{x_4} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (A_5 k_6 - A_6 k_5) \partial_{x_5} \otimes \partial_{x_6} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (A_5 k_7 - A_7 k_5) \partial_{x_5} \otimes \partial_{x_7} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (\omega A_6 + A_0 k_6) \partial_{x_6} \otimes \partial_{x_0} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_1 k_6 + A_6 k_1) \partial_{x_6} \otimes \partial_{x_1} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_2 k_6 + A_6 k_2) \partial_{x_6} \otimes \partial_{x_2} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_3 k_6 + A_6 k_3) \partial_{x_6} \otimes \partial_{x_3} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_4 k_6 + A_6 k_4) \partial_{x_6} \otimes \partial_{x_4} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_5 k_6 + A_6 k_5) \partial_{x_6} \otimes \partial_{x_5} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (A_6 k_7 - A_7 k_6) \partial_{x_6} \otimes \partial_{x_7} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (\omega A_7 + A_0 k_7) \partial_{x_7} \otimes \partial_{x_0} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_1 k_7 + A_7 k_1) \partial_{x_7} \otimes \partial_{x_1} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_2 k_7 + A_7 k_2) \partial_{x_7} \otimes \partial_{x_2} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_3 k_7 + A_7 k_3) \partial_{x_7} \otimes \partial_{x_3} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_4 k_7 + A_7 k_4) \partial_{x_7} \otimes \partial_{x_4} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_5 k_7 + A_7 k_5) \partial_{x_7} \otimes \partial_{x_5} \\
& + Ie^{Ix_6 k_6 + Ix_2 k_2 + Ix_3 k_3 + Ix_4 k_4 + Ix_5 k_5 + Ix_1 k_1 + Ix_7 k_7 - Ix_0 \omega} (-A_6 k_7 + A_7 k_6) \partial_{x_7} \otimes \partial_{x_6}
\end{aligned}$$

Now, we will attempt to equate the conventional and octonionic faraday tensors.

M > f2:=simplify(eval(f1,[seq(Alpha[i]=alpha[i]*exp(-I*(add(seq(k[o]*x||o,o=1..7))-omega*x||0)),i=0..7]));

$$\begin{aligned}
f2 := & - (I\omega\alpha_1 + Ik_1\alpha_0) \partial_{x_0} \otimes \partial_{x_1} - (I\omega\alpha_2 + Ik_2\alpha_0) \partial_{x_0} \otimes \partial_{x_2} - (I\omega\alpha_3 + Ik_3\alpha_0) \partial_{x_0} \otimes \partial_{x_3} - \quad (5.5.1.10) \\
& (I\omega\alpha_4 + Ik_4\alpha_0) \partial_{x_0} \otimes \partial_{x_4} - (I\omega\alpha_5 + Ik_5\alpha_0) \partial_{x_0} \otimes \partial_{x_5} - (I\omega\alpha_6 + Ik_6\alpha_0) \partial_{x_0} \otimes \partial_{x_6} - \\
& (I\omega\alpha_7 + Ik_7\alpha_0) \partial_{x_0} \otimes \partial_{x_7} + (I\omega\alpha_1 + Ik_1\alpha_0) \partial_{x_1} \otimes \partial_{x_0} + (Ik_2\alpha_1 - Ik_1\alpha_2) \partial_{x_1} \otimes \partial_{x_2} + \\
& (Ik_3\alpha_1 - Ik_1\alpha_3) \partial_{x_1} \otimes \partial_{x_3} + (Ik_4\alpha_1 - Ik_1\alpha_4) \partial_{x_1} \otimes \partial_{x_4} + (Ik_5\alpha_1 - Ik_1\alpha_5) \partial_{x_1} \otimes \partial_{x_5} + \\
& (Ik_6\alpha_1 - Ik_1\alpha_6) \partial_{x_1} \otimes \partial_{x_6} + (Ik_7\alpha_1 - Ik_1\alpha_7) \partial_{x_1} \otimes \partial_{x_7} + (I\omega\alpha_2 + Ik_2\alpha_0) \partial_{x_2} \otimes \partial_{x_0} - \\
& (Ik_2\alpha_1 - Ik_1\alpha_2) \partial_{x_2} \otimes \partial_{x_1} + (Ik_3\alpha_2 - Ik_2\alpha_3) \partial_{x_2} \otimes \partial_{x_3} + (Ik_4\alpha_2 - Ik_2\alpha_4) \partial_{x_2} \otimes \partial_{x_4} + \\
& (Ik_5\alpha_2 - Ik_2\alpha_5) \partial_{x_2} \otimes \partial_{x_5} + (Ik_6\alpha_2 - Ik_2\alpha_6) \partial_{x_2} \otimes \partial_{x_6} + (Ik_7\alpha_2 - Ik_2\alpha_7) \partial_{x_2} \otimes \partial_{x_7} + \\
& (I\omega\alpha_3 + Ik_3\alpha_0) \partial_{x_3} \otimes \partial_{x_0} - (Ik_3\alpha_1 - Ik_1\alpha_3) \partial_{x_3} \otimes \partial_{x_1} - (Ik_3\alpha_2 - Ik_2\alpha_3) \partial_{x_3} \otimes \partial_{x_2} +
\end{aligned}$$

$$\begin{aligned}
& (Ik_4 \alpha_3 - Ik_3 \alpha_4) \partial_{x_3} \otimes \partial_{x_4} + (Ik_5 \alpha_3 - Ik_3 \alpha_5) \partial_{x_3} \otimes \partial_{x_5} + (Ik_6 \alpha_3 - Ik_3 \alpha_6) \partial_{x_3} \otimes \partial_{x_6} + \\
& (Ik_7 \alpha_3 - Ik_3 \alpha_7) \partial_{x_3} \otimes \partial_{x_7} + (I\omega \alpha_4 + Ik_4 \alpha_0) \partial_{x_4} \otimes \partial_{x_0} - (Ik_4 \alpha_1 - Ik_1 \alpha_4) \partial_{x_4} \otimes \partial_{x_1} - \\
& (Ik_4 \alpha_2 - Ik_2 \alpha_4) \partial_{x_4} \otimes \partial_{x_2} - (Ik_4 \alpha_3 - Ik_3 \alpha_4) \partial_{x_4} \otimes \partial_{x_3} + (Ik_5 \alpha_4 - Ik_4 \alpha_5) \partial_{x_4} \otimes \partial_{x_5} + \\
& (Ik_6 \alpha_4 - Ik_4 \alpha_6) \partial_{x_4} \otimes \partial_{x_6} + (Ik_7 \alpha_4 - Ik_4 \alpha_7) \partial_{x_4} \otimes \partial_{x_7} + (I\omega \alpha_5 + Ik_5 \alpha_0) \partial_{x_5} \otimes \partial_{x_0} - \\
& (Ik_5 \alpha_1 - Ik_1 \alpha_5) \partial_{x_5} \otimes \partial_{x_1} - (Ik_5 \alpha_2 - Ik_2 \alpha_5) \partial_{x_5} \otimes \partial_{x_2} - (Ik_5 \alpha_3 - Ik_3 \alpha_5) \partial_{x_5} \otimes \partial_{x_3} - \\
& (Ik_5 \alpha_4 - Ik_4 \alpha_5) \partial_{x_5} \otimes \partial_{x_4} + (Ik_6 \alpha_5 - Ik_5 \alpha_6) \partial_{x_5} \otimes \partial_{x_6} + (Ik_7 \alpha_5 - Ik_5 \alpha_7) \partial_{x_5} \otimes \partial_{x_7} + \\
& (I\omega \alpha_6 + Ik_6 \alpha_0) \partial_{x_6} \otimes \partial_{x_0} - (Ik_6 \alpha_1 - Ik_1 \alpha_6) \partial_{x_6} \otimes \partial_{x_1} - (Ik_6 \alpha_2 - Ik_2 \alpha_6) \partial_{x_6} \otimes \partial_{x_2} - \\
& (Ik_6 \alpha_3 - Ik_3 \alpha_6) \partial_{x_6} \otimes \partial_{x_3} - (Ik_6 \alpha_4 - Ik_4 \alpha_6) \partial_{x_6} \otimes \partial_{x_4} - (Ik_6 \alpha_5 - Ik_5 \alpha_6) \partial_{x_6} \otimes \partial_{x_5} + \\
& (Ik_7 \alpha_6 - Ik_6 \alpha_7) \partial_{x_6} \otimes \partial_{x_7} + (I\omega \alpha_7 + Ik_7 \alpha_0) \partial_{x_7} \otimes \partial_{x_0} - (Ik_7 \alpha_1 - Ik_1 \alpha_7) \partial_{x_7} \otimes \partial_{x_1} - \\
& (Ik_7 \alpha_2 - Ik_2 \alpha_7) \partial_{x_7} \otimes \partial_{x_2} - (Ik_7 \alpha_3 - Ik_3 \alpha_7) \partial_{x_7} \otimes \partial_{x_3} - (Ik_7 \alpha_4 - Ik_4 \alpha_7) \partial_{x_7} \otimes \partial_{x_4} - \\
& (Ik_7 \alpha_5 - Ik_5 \alpha_7) \partial_{x_7} \otimes \partial_{x_5} - (Ik_7 \alpha_6 - Ik_6 \alpha_7) \partial_{x_7} \otimes \partial_{x_6}
\end{aligned}$$

M > GetComponents(f2,[seq(seq(evalDG(D_x||i &t D_x||j),i=0..7),j=0..7)]);

$$\begin{aligned}
& [0, I\omega \alpha_1 + Ik_1 \alpha_0, I\omega \alpha_2 + Ik_2 \alpha_0, I\omega \alpha_3 + Ik_3 \alpha_0, I\omega \alpha_4 + Ik_4 \alpha_0, I\omega \alpha_5 + Ik_5 \alpha_0, I\omega \alpha_6 \\
& + Ik_6 \alpha_0, I\omega \alpha_7 + Ik_7 \alpha_0, -I\omega \alpha_1 - Ik_1 \alpha_0, 0, -Ik_2 \alpha_1 + Ik_1 \alpha_2, -Ik_3 \alpha_1 + Ik_1 \alpha_3, -Ik_4 \alpha_1 \\
& + Ik_1 \alpha_4, -Ik_5 \alpha_1 + Ik_1 \alpha_5, -Ik_6 \alpha_1 + Ik_1 \alpha_6, -Ik_7 \alpha_1 + Ik_1 \alpha_7, -I\omega \alpha_2 - Ik_2 \alpha_0, Ik_2 \alpha_1 \\
& - Ik_1 \alpha_2, 0, -Ik_3 \alpha_2 + Ik_2 \alpha_3, -Ik_4 \alpha_2 + Ik_2 \alpha_4, -Ik_5 \alpha_2 + Ik_2 \alpha_5, -Ik_6 \alpha_2 + Ik_2 \alpha_6, -Ik_7 \alpha_2 \\
& + Ik_2 \alpha_7, -I\omega \alpha_3 - Ik_3 \alpha_0, Ik_3 \alpha_1 - Ik_1 \alpha_3, Ik_3 \alpha_2 - Ik_2 \alpha_3, 0, -Ik_4 \alpha_3 + Ik_3 \alpha_4, -Ik_5 \alpha_3 \\
& + Ik_3 \alpha_5, -Ik_6 \alpha_3 + Ik_3 \alpha_6, -Ik_7 \alpha_3 + Ik_3 \alpha_7, -I\omega \alpha_4 - Ik_4 \alpha_0, Ik_4 \alpha_1 - Ik_1 \alpha_4, Ik_4 \alpha_2 \\
& - Ik_2 \alpha_4, Ik_4 \alpha_3 - Ik_3 \alpha_4, 0, -Ik_5 \alpha_4 + Ik_4 \alpha_5, -Ik_6 \alpha_4 + Ik_4 \alpha_6, -Ik_7 \alpha_4 + Ik_4 \alpha_7, -I\omega \alpha_5 \\
& - Ik_5 \alpha_0, Ik_5 \alpha_1 - Ik_1 \alpha_5, Ik_5 \alpha_2 - Ik_2 \alpha_5, Ik_5 \alpha_3 - Ik_3 \alpha_5, Ik_5 \alpha_4 - Ik_4 \alpha_5, 0, -Ik_6 \alpha_5 \\
& + Ik_5 \alpha_6, -Ik_7 \alpha_5 + Ik_5 \alpha_7, -I\omega \alpha_6 - Ik_6 \alpha_0, Ik_6 \alpha_1 - Ik_1 \alpha_6, Ik_6 \alpha_2 - Ik_2 \alpha_6, Ik_6 \alpha_3 \\
& - Ik_3 \alpha_6, Ik_6 \alpha_4 - Ik_4 \alpha_6, Ik_6 \alpha_5 - Ik_5 \alpha_6, 0, -Ik_7 \alpha_6 + Ik_6 \alpha_7, -I\omega \alpha_7 - Ik_7 \alpha_0, Ik_7 \alpha_1 \\
& - Ik_1 \alpha_7, Ik_7 \alpha_2 - Ik_2 \alpha_7, Ik_7 \alpha_3 - Ik_3 \alpha_7, Ik_7 \alpha_4 - Ik_4 \alpha_7, Ik_7 \alpha_5 - Ik_5 \alpha_7, Ik_7 \alpha_6 \\
& - Ik_6 \alpha_7, 0]
\end{aligned} \tag{5.5.1.11}$$

M > nops(GetComponents(f2,[seq(seq(evalDG(D_x||i &t D_x||j),i=0..7),j=0..7)]));

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(5.5.1.12)

M > eqns:= [seq(GetComponents(f2,[seq(seq(evalDG(D_x||i &t D_x||j),i=0..7),j=0..7)])[h]-GetComponents(testF3,[seq(seq(evalDG(D_x||i &t D_x||j),i=0..7),j=0..7)])[h]=0,h=1..64)];

$$eqns := \left[0 = 0, I\omega \alpha_1 + Ik_1 \alpha_0 + E_1 = 0, I\omega \alpha_2 + Ik_2 \alpha_0 + E_2 = 0, I\omega \alpha_3 + Ik_3 \alpha_0 + E_3 = 0, \right. \tag{5.5.1.13}$$

$$I\omega \alpha_4 + Ik_4 \alpha_0 + E_4 = 0, I\omega \alpha_5 + Ik_5 \alpha_0 + E_5 = 0, I\omega \alpha_6 + Ik_6 \alpha_0 + E_6 = 0, I\omega \alpha_7 + Ik_7 \alpha_0$$

$$+ E_7 = 0, -I\omega \alpha_1 - Ik_1 \alpha_0 - E_1 = 0, -0 e \theta = 0, -Ik_2 \alpha_1 + Ik_1 \alpha_2$$

$$\begin{aligned}
& - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I k_3 \alpha_1 + I k_1 \alpha_3 \\
& - \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_4 \alpha_1 + I k_1 \alpha_4 \\
& - \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_5 \alpha_1 + I k_1 \alpha_5 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_6 \alpha_1 + I k_1 \alpha_6 \\
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_7 \alpha_1 + I k_1 \alpha_7 \\
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I \omega \alpha_2 - I k_2 \alpha_0 - E_2 = 0, I k_2 \alpha_1 \\
& - I k_1 \alpha_2 + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -0 e \theta = 0, -I k_3 \alpha_2 + I k_2 \alpha_3 \\
& - \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -I k_4 \alpha_2 + I k_2 \alpha_4 \\
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_5 \alpha_2 + I k_2 \alpha_5 \\
& + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_6 \alpha_2 + I k_2 \alpha_6 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_7 \alpha_2 + I k_2 \alpha_7 \\
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I \omega \alpha_3 - I k_3 \alpha_0 - E_3 = 0, I k_3 \alpha_1 \\
& - I k_1 \alpha_3 + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, I k_3 \alpha_2 - I k_2 \alpha_3 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -0 e \theta = 0, -I k_4 \alpha_3 + I k_3 \alpha_4 \\
& + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_5 \alpha_3 + I k_3 \alpha_5 \\
& + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_6 \alpha_3 + I k_3 \alpha_6 \\
& - \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_7 \alpha_3 + I k_3 \alpha_7 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I \omega \alpha_4 - I k_4 \alpha_0 - E_4 = 0, I k_4 \alpha_1 \\
& - I k_1 \alpha_4 + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_4 \alpha_2 - I k_2 \alpha_4 \\
& + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_4 \alpha_3 - I k_3 \alpha_4
\end{aligned}$$

$$\begin{aligned}
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -0 \varrho 0 = 0, -I k_5 \alpha_4 + I k_4 \alpha_5 \\
& - \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -I k_6 \alpha_4 + I k_4 \alpha_6 \\
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_7 \alpha_4 + I k_4 \alpha_7 \\
& - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I \omega \alpha_5 - I k_5 \alpha_0 - E_5 = 0, I k_5 \alpha_1 \\
& - I k_1 \alpha_5 + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_5 \alpha_2 - I k_2 \alpha_5 \\
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, I k_5 \alpha_3 - I k_3 \alpha_5 \\
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_5 \alpha_4 - I k_4 \alpha_5 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -0 \varrho 0 = 0, -I k_6 \alpha_5 + I k_5 \alpha_6 \\
& + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I k_7 \alpha_5 + I k_5 \alpha_7 \\
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I \omega \alpha_6 - I k_6 \alpha_0 - E_6 = 0, I k_6 \alpha_1 \\
& - I k_1 \alpha_6 + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, I k_6 \alpha_2 - I k_2 \alpha_6 \\
& + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_6 \alpha_3 - I k_3 \alpha_6 \\
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_6 \alpha_4 - I k_4 \alpha_6 \\
& - \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, I k_6 \alpha_5 - I k_5 \alpha_6 \\
& - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -0 \varrho 0 = 0, -I k_7 \alpha_6 + I k_6 \alpha_7 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -I \omega \alpha_7 - I k_7 \alpha_0 - E_7 = 0, I k_7 \alpha_1 \\
& - I k_1 \alpha_7 + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_7 \alpha_2 - I k_2 \alpha_7 \\
& - \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_7 \alpha_3 - I k_3 \alpha_7 \\
& + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_7 \alpha_4 - I k_4 \alpha_7 \\
& + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, I k_7 \alpha_5 - I k_5 \alpha_7
\end{aligned}$$

$$\left. \begin{aligned} -\frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} &= 0, I k_7 \alpha_6 - I k_6 \alpha_7 \\ -\frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} &= 0, -0 \text{ e } 0 = 0 \end{aligned} \right\}$$

$$\mathbf{M} > \text{solve}(\text{eqns}, [\text{seq}(\text{Epsilon}[i], i=1..7)]); \quad (5.5.1.14)$$

$$\mathbf{O} > \text{solve}(\text{eqns}, [\text{seq}(\text{alpha}[i], i=0..7)]); \quad (5.5.1.15)$$

$$\mathbf{M} > \text{solve}(\text{eqns}, [\text{omega}]); \quad (5.5.1.16)$$

Solve the system of equations: solution to equality of tensors.

$$\begin{aligned} > \text{seq}(\text{eqns}[i], i=1..8); \\ 0 = 0, I \omega \alpha_1 + I k_1 \alpha_0 + E_1 = 0, I \omega \alpha_2 + I k_2 \alpha_0 + E_2 = 0, I \omega \alpha_3 + I k_3 \alpha_0 + E_3 = 0, I \omega \alpha_4 + I k_4 \alpha_0 \\ + E_4 = 0, I \omega \alpha_5 + I k_5 \alpha_0 + E_5 = 0, I \omega \alpha_6 + I k_6 \alpha_0 + E_6 = 0, I \omega \alpha_7 + I k_7 \alpha_0 + E_7 = 0 \end{aligned} \quad (5.5.2.1)$$

$$\begin{aligned} > \text{seq}(\text{eqns}[i], i=9..16); \\ -I \omega \alpha_1 - I k_1 \alpha_0 - E_1 = 0, 0 = 0, -I k_2 \alpha_1 + I k_1 \alpha_2 \end{aligned} \quad (5.5.2.2)$$

$$-\frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I k_3 \alpha_1 + I k_1 \alpha_3$$

$$-\frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_4 \alpha_1 + I k_1 \alpha_4$$

$$-\frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_5 \alpha_1 + I k_1 \alpha_5$$

$$-\frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_6 \alpha_1 + I k_1 \alpha_6$$

$$-\frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_7 \alpha_1 + I k_1 \alpha_7$$

$$-\frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0$$

$$> \text{eqns2} := \{\text{seq}(\text{eqns}[i], i=1..nops(\text{eqns}))\};$$

$$\text{eqns2} := \left\{ 0 = 0, -I \omega \alpha_1 - I k_1 \alpha_0 - E_1 = 0, -I \omega \alpha_2 - I k_2 \alpha_0 - E_2 = 0, -I \omega \alpha_3 - I k_3 \alpha_0 - E_3 = 0, \right. \quad (5.5.2.3)$$

$$\left. -I \omega \alpha_4 - I k_4 \alpha_0 - E_4 = 0, -I \omega \alpha_5 - I k_5 \alpha_0 - E_5 = 0, -I \omega \alpha_6 - I k_6 \alpha_0 - E_6 = 0, -I \omega \alpha_7$$

$$-I k_7 \alpha_0 - E_7 = 0, -I k_2 \alpha_1 + I k_1 \alpha_2 - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0,$$

$$-I k_3 \alpha_1 + I k_1 \alpha_3 - \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_3 \alpha_2 + I k_2 \alpha_3$$

$$-\frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -I k_4 \alpha_1 + I k_1 \alpha_4$$

$$-\frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_4 \alpha_2 + I k_2 \alpha_4$$

$$\begin{aligned}
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_4 \alpha_3 + I k_3 \alpha_4 \\
& + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_5 \alpha_1 + I k_1 \alpha_5 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_5 \alpha_2 + I k_2 \alpha_5 \\
& + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_5 \alpha_3 + I k_3 \alpha_5 \\
& + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_5 \alpha_4 + I k_4 \alpha_5 \\
& - \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -I k_6 \alpha_1 + I k_1 \alpha_6 \\
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_6 \alpha_2 + I k_2 \alpha_6 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_6 \alpha_3 + I k_3 \alpha_6 \\
& - \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_6 \alpha_4 + I k_4 \alpha_6 \\
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_6 \alpha_5 + I k_5 \alpha_6 \\
& + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I k_7 \alpha_1 + I k_1 \alpha_7 \\
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_7 \alpha_2 + I k_2 \alpha_7 \\
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_7 \alpha_3 + I k_3 \alpha_7 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_7 \alpha_4 + I k_4 \alpha_7 \\
& - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I k_7 \alpha_5 + I k_5 \alpha_7 \\
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_7 \alpha_6 + I k_6 \alpha_7 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, I \omega \alpha_1 + I k_1 \alpha_0 + E_1 = 0, I \omega \alpha_2 \\
& + I k_2 \alpha_0 + E_2 = 0, I \omega \alpha_3 + I k_3 \alpha_0 + E_3 = 0, I \omega \alpha_4 + I k_4 \alpha_0 + E_4 = 0, I \omega \alpha_5 + I k_5 \alpha_0 + E_5 \\
& = 0, I \omega \alpha_6 + I k_6 \alpha_0 + E_6 = 0, I \omega \alpha_7 + I k_7 \alpha_0 + E_7 = 0, I k_2 \alpha_1 - I k_1 \alpha_2 \\
& + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, I k_3 \alpha_1 - I k_1 \alpha_3
\end{aligned}$$

$$\begin{aligned}
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, I k_3 \alpha_2 - I k_2 \alpha_3 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, I k_4 \alpha_1 - I k_1 \alpha_4 \\
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_4 \alpha_2 - I k_2 \alpha_4 \\
& + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_4 \alpha_3 - I k_3 \alpha_4 \\
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, I k_5 \alpha_1 - I k_1 \alpha_5 \\
& + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_5 \alpha_2 - I k_2 \alpha_5 \\
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, I k_5 \alpha_3 - I k_3 \alpha_5 \\
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_5 \alpha_4 - I k_4 \alpha_5 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, I k_6 \alpha_1 - I k_1 \alpha_6 \\
& + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, I k_6 \alpha_2 - I k_2 \alpha_6 \\
& + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_6 \alpha_3 - I k_3 \alpha_6 \\
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_6 \alpha_4 - I k_4 \alpha_6 \\
& - \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, I k_6 \alpha_5 - I k_5 \alpha_6 \\
& - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, I k_7 \alpha_1 - I k_1 \alpha_7 \\
& + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_7 \alpha_2 - I k_2 \alpha_7 \\
& - \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_7 \alpha_3 - I k_3 \alpha_7 \\
& + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_7 \alpha_4 - I k_4 \alpha_7 \\
& + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, I k_7 \alpha_5 - I k_5 \alpha_7 \\
& - \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, I k_7 \alpha_6 - I k_6 \alpha_7 \\
& - \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0 \}
\end{aligned}$$

> nops(eqns2);
nops(eqns);

57

64

(5.5.2.4)

> eqns2[1];

0 = 0

(5.5.2.5)

> eqns3:={seq(eqns2[i],i=2..nops(eqns2))};

eqns3 := { -Iωα₁ - Ik₁α₀ - E₁ = 0, -Iωα₂ - Ik₂α₀ - E₂ = 0, -Iωα₃ - Ik₃α₀ - E₃ = 0, -Iωα₄ (5.5.2.6)

$$-Ik_4\alpha_0 - E_4 = 0, -I\omega\alpha_5 - Ik_5\alpha_0 - E_5 = 0, -I\omega\alpha_6 - Ik_6\alpha_0 - E_6 = 0, -I\omega\alpha_7 - Ik_7\alpha_0$$

$$-E_7 = 0, -Ik_2\alpha_1 + Ik_1\alpha_2 - \frac{E_1k_2 - E_2k_1 + E_4k_7 - E_5k_6 + E_6k_5 - E_7k_4}{\omega} = 0, -Ik_3\alpha_1$$

$$+ Ik_1\alpha_3 - \frac{E_1k_3 - E_3k_1 - E_4k_6 - E_5k_7 + E_6k_4 + E_7k_5}{\omega} = 0, -Ik_3\alpha_2 + Ik_2\alpha_3$$

$$- \frac{E_2k_3 - E_3k_2 + E_4k_5 - E_5k_4 - E_6k_7 + E_7k_6}{\omega} = 0, -Ik_4\alpha_1 + Ik_1\alpha_4$$

$$- \frac{E_1k_4 - E_2k_7 + E_3k_6 - E_4k_1 - E_6k_3 + E_7k_2}{\omega} = 0, -Ik_4\alpha_2 + Ik_2\alpha_4$$

$$- \frac{E_1k_7 + E_2k_4 - E_3k_5 - E_4k_2 + E_5k_3 - E_7k_1}{\omega} = 0, -Ik_4\alpha_3 + Ik_3\alpha_4$$

$$+ \frac{E_1k_6 - E_2k_5 - E_3k_4 + E_4k_3 + E_5k_2 - E_6k_1}{\omega} = 0, -Ik_5\alpha_1 + Ik_1\alpha_5$$

$$- \frac{E_1k_5 + E_2k_6 + E_3k_7 - E_5k_1 - E_6k_2 - E_7k_3}{\omega} = 0, -Ik_5\alpha_2 + Ik_2\alpha_5$$

$$+ \frac{E_1k_6 - E_2k_5 - E_3k_4 + E_4k_3 + E_5k_2 - E_6k_1}{\omega} = 0, -Ik_5\alpha_3 + Ik_3\alpha_5$$

$$+ \frac{E_1k_7 + E_2k_4 - E_3k_5 - E_4k_2 + E_5k_3 - E_7k_1}{\omega} = 0, -Ik_5\alpha_4 + Ik_4\alpha_5$$

$$- \frac{E_2k_3 - E_3k_2 + E_4k_5 - E_5k_4 - E_6k_7 + E_7k_6}{\omega} = 0, -Ik_6\alpha_1 + Ik_1\alpha_6$$

$$- \frac{E_1k_6 - E_2k_5 - E_3k_4 + E_4k_3 + E_5k_2 - E_6k_1}{\omega} = 0, -Ik_6\alpha_2 + Ik_2\alpha_6$$

$$- \frac{E_1k_5 + E_2k_6 + E_3k_7 - E_5k_1 - E_6k_2 - E_7k_3}{\omega} = 0, -Ik_6\alpha_3 + Ik_3\alpha_6$$

$$- \frac{E_1k_4 - E_2k_7 + E_3k_6 - E_4k_1 - E_6k_3 + E_7k_2}{\omega} = 0, -Ik_6\alpha_4 + Ik_4\alpha_6$$

$$+ \frac{E_1k_3 - E_3k_1 - E_4k_6 - E_5k_7 + E_6k_4 + E_7k_5}{\omega} = 0, -Ik_6\alpha_5 + Ik_5\alpha_6$$

$$+ \frac{E_1k_2 - E_2k_1 + E_4k_7 - E_5k_6 + E_6k_5 - E_7k_4}{\omega} = 0, -Ik_7\alpha_1 + Ik_1\alpha_7$$

$$\begin{aligned}
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_7 \alpha_2 + I k_2 \alpha_7 \\
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_7 \alpha_3 + I k_3 \alpha_7 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_7 \alpha_4 + I k_4 \alpha_7 \\
& - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I k_7 \alpha_5 + I k_5 \alpha_7 \\
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_7 \alpha_6 + I k_6 \alpha_7 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, I \omega \alpha_1 + I k_1 \alpha_0 + E_1 = 0, I \omega \alpha_2 \\
& + I k_2 \alpha_0 + E_2 = 0, I \omega \alpha_3 + I k_3 \alpha_0 + E_3 = 0, I \omega \alpha_4 + I k_4 \alpha_0 + E_4 = 0, I \omega \alpha_5 + I k_5 \alpha_0 + E_5 \\
& = 0, I \omega \alpha_6 + I k_6 \alpha_0 + E_6 = 0, I \omega \alpha_7 + I k_7 \alpha_0 + E_7 = 0, I k_2 \alpha_1 - I k_1 \alpha_2 \\
& + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, I k_3 \alpha_1 - I k_1 \alpha_3 \\
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, I k_3 \alpha_2 - I k_2 \alpha_3 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, I k_4 \alpha_1 - I k_1 \alpha_4 \\
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_4 \alpha_2 - I k_2 \alpha_4 \\
& + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_4 \alpha_3 - I k_3 \alpha_4 \\
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, I k_5 \alpha_1 - I k_1 \alpha_5 \\
& + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_5 \alpha_2 - I k_2 \alpha_5 \\
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, I k_5 \alpha_3 - I k_3 \alpha_5 \\
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_5 \alpha_4 - I k_4 \alpha_5 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, I k_6 \alpha_1 - I k_1 \alpha_6 \\
& + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, I k_6 \alpha_2 - I k_2 \alpha_6 \\
& + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_6 \alpha_3 - I k_3 \alpha_6
\end{aligned}$$

$$\begin{aligned}
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_6 \alpha_4 - I k_4 \alpha_6 \\
& - \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, I k_6 \alpha_5 - I k_5 \alpha_6 \\
& - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, I k_7 \alpha_1 - I k_1 \alpha_7 \\
& + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, I k_7 \alpha_2 - I k_2 \alpha_7 \\
& - \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, I k_7 \alpha_3 - I k_3 \alpha_7 \\
& + \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, I k_7 \alpha_4 - I k_4 \alpha_7 \\
& + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, I k_7 \alpha_5 - I k_5 \alpha_7 \\
& - \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, I k_7 \alpha_6 - I k_6 \alpha_7 \\
& - \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0 \}
\end{aligned}$$

> nops(eqns3);

56

(5.5.2.7)

> 56/2;

28

(5.5.2.8)

> eqns3[28];
> eqns3[56];

$$-I k_7 \alpha_6 + I k_6 \alpha_7 + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0$$

$$I k_7 \alpha_6 - I k_6 \alpha_7 - \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0$$

(5.5.2.9)

> lhs(eqns3[28])+lhs(eqns3[56]);

0

(5.5.2.10)

> seq(lhs(eqns3[i])+lhs(eqns3[i+28]),i=1..28);

0, 0

(5.5.2.11)

> eqns4:={seq(eqns3[i],i=1..28)};

$$eqns4 := \left\{ -I \omega \alpha_1 - I k_1 \alpha_0 - E_1 = 0, -I \omega \alpha_2 - I k_2 \alpha_0 - E_2 = 0, -I \omega \alpha_3 - I k_3 \alpha_0 - E_3 = 0, \right.$$

(5.5.2.12)

$$-I \omega \alpha_4 - I k_4 \alpha_0 - E_4 = 0, -I \omega \alpha_5 - I k_5 \alpha_0 - E_5 = 0, -I \omega \alpha_6 - I k_6 \alpha_0 - E_6 = 0, -I \omega \alpha_7$$

$$- I k_7 \alpha_0 - E_7 = 0, -I k_2 \alpha_1 + I k_1 \alpha_2 - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0,$$

$$-I k_3 \alpha_1 + I k_1 \alpha_3 - \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_3 \alpha_2 + I k_2 \alpha_3$$

$$\begin{aligned}
& - \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -I k_4 \alpha_1 + I k_1 \alpha_4 \\
& - \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_4 \alpha_2 + I k_2 \alpha_4 \\
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_4 \alpha_3 + I k_3 \alpha_4 \\
& + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_5 \alpha_1 + I k_1 \alpha_5 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_5 \alpha_2 + I k_2 \alpha_5 \\
& + \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_5 \alpha_3 + I k_3 \alpha_5 \\
& + \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_5 \alpha_4 + I k_4 \alpha_5 \\
& - \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0, -I k_6 \alpha_1 + I k_1 \alpha_6 \\
& - \frac{E_1 k_6 - E_2 k_5 - E_3 k_4 + E_4 k_3 + E_5 k_2 - E_6 k_1}{\omega} = 0, -I k_6 \alpha_2 + I k_2 \alpha_6 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_6 \alpha_3 + I k_3 \alpha_6 \\
& - \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_6 \alpha_4 + I k_4 \alpha_6 \\
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_6 \alpha_5 + I k_5 \alpha_6 \\
& + \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I k_7 \alpha_1 + I k_1 \alpha_7 \\
& - \frac{E_1 k_7 + E_2 k_4 - E_3 k_5 - E_4 k_2 + E_5 k_3 - E_7 k_1}{\omega} = 0, -I k_7 \alpha_2 + I k_2 \alpha_7 \\
& + \frac{E_1 k_4 - E_2 k_7 + E_3 k_6 - E_4 k_1 - E_6 k_3 + E_7 k_2}{\omega} = 0, -I k_7 \alpha_3 + I k_3 \alpha_7 \\
& - \frac{E_1 k_5 + E_2 k_6 + E_3 k_7 - E_5 k_1 - E_6 k_2 - E_7 k_3}{\omega} = 0, -I k_7 \alpha_4 + I k_4 \alpha_7 \\
& - \frac{E_1 k_2 - E_2 k_1 + E_4 k_7 - E_5 k_6 + E_6 k_5 - E_7 k_4}{\omega} = 0, -I k_7 \alpha_5 + I k_5 \alpha_7 \\
& + \frac{E_1 k_3 - E_3 k_1 - E_4 k_6 - E_5 k_7 + E_6 k_4 + E_7 k_5}{\omega} = 0, -I k_7 \alpha_6 + I k_6 \alpha_7 \\
& + \frac{E_2 k_3 - E_3 k_2 + E_4 k_5 - E_5 k_4 - E_6 k_7 + E_7 k_6}{\omega} = 0 \}
\end{aligned}$$

> eq1:={seq(eqns4[i],i=1..7)}

eq1 := { -I\omega\alpha_1 - I k_1 \alpha_0 - E_1 = 0, -I\omega\alpha_2 - I k_2 \alpha_0 - E_2 = 0, -I\omega\alpha_3 - I k_3 \alpha_0 - E_3 = 0, -I\omega\alpha_4 (5.5.2.13)

$$\begin{aligned} & -Ik_4\alpha_0 - E_4 = 0, -I\omega\alpha_5 - Ik_5\alpha_0 - E_5 = 0, -I\omega\alpha_6 - Ik_6\alpha_0 - E_6 = 0, -I\omega\alpha_7 - Ik_7\alpha_0 \\ & - E_7 = 0 \} \end{aligned}$$

The following is the proposed solution for the octonionic electric field:

$$\text{> esol:=solve(eq1,[seq(Epsilon[i],i=1..7)]);} \quad (5.5.2.14)$$

$$\text{esol := } \left[\left[E_1 = -I\omega\alpha_1 - Ik_1\alpha_0, E_2 = -I\omega\alpha_2 - Ik_2\alpha_0, E_3 = -I\omega\alpha_3 - Ik_3\alpha_0, E_4 = -I\omega\alpha_4 \right. \right. \\ \left. \left. - Ik_4\alpha_0, E_5 = -I\omega\alpha_5 - Ik_5\alpha_0, E_6 = -I\omega\alpha_6 - Ik_6\alpha_0, E_7 = -I\omega\alpha_7 - Ik_7\alpha_0 \right] \right]$$

The remainder of this section will be devoted to checking whether this is consistent with the system of equations.

$$\text{> ees:=seq(rhs(esol[1][i]),i=1..7)} \quad (5.5.2.15)$$

$$\text{ees := } -I\omega\alpha_1 - Ik_1\alpha_0, -I\omega\alpha_2 - Ik_2\alpha_0, -I\omega\alpha_3 - Ik_3\alpha_0, -I\omega\alpha_4 - Ik_4\alpha_0, -I\omega\alpha_5 - Ik_5\alpha_0, \\ -I\omega\alpha_6 - Ik_6\alpha_0, -I\omega\alpha_7 - Ik_7\alpha_0$$

$$\text{> ees[1];} \quad (5.5.2.16)$$

$$-I\omega\alpha_1 - Ik_1\alpha_0$$

$$\text{> test:=subs([seq(Epsilon[i]=Epsilon||i,i=1..7)],eqns4[8]);} \quad (5.5.2.17)$$

$$\text{test := } -Ik_2\alpha_1 + Ik_1\alpha_2 - \frac{E1k_2 - E2k_1 + E4k_7 - E5k_6 + E6k_5 - E7k_4}{\omega} = 0$$

$$\text{> simplify(subs([seq(Epsilon||i=ees[i],i=1..7)],test));} \quad (5.5.2.18)$$

$$I(\alpha_4k_7 - \alpha_5k_6 + \alpha_6k_5 - \alpha_7k_4) = 0$$

$$\text{> switcheqns:={seq(subs([seq(Epsilon[i]=Epsilon||i,i=1..7)],eqns4[j]),j=1..nops(eqns4))};}$$

$$\text{switcheqns := } \left\{ -I\omega\alpha_1 - Ik_1\alpha_0 - E1 = 0, -I\omega\alpha_2 - Ik_2\alpha_0 - E2 = 0, -I\omega\alpha_3 - Ik_3\alpha_0 - E3 = 0, \right. \quad (5.5.2.19)$$

$$-I\omega\alpha_4 - Ik_4\alpha_0 - E4 = 0, -I\omega\alpha_5 - Ik_5\alpha_0 - E5 = 0, -I\omega\alpha_6 - Ik_6\alpha_0 - E6 = 0, -I\omega\alpha_7$$

$$-Ik_7\alpha_0 - E7 = 0, -Ik_2\alpha_1 + Ik_1\alpha_2 - \frac{E1k_2 - E2k_1 + E4k_7 - E5k_6 + E6k_5 - E7k_4}{\omega}$$

$$= 0, -Ik_3\alpha_1 + Ik_1\alpha_3 - \frac{E1k_3 - E3k_1 - E4k_6 - E5k_7 + E6k_4 + E7k_5}{\omega} = 0, -Ik_3\alpha_2$$

$$+ Ik_2\alpha_3 - \frac{E2k_3 - E3k_2 + E4k_5 - E5k_4 - E6k_7 + E7k_6}{\omega} = 0, -Ik_4\alpha_1 + Ik_1\alpha_4$$

$$- \frac{E1k_4 - E2k_7 + E3k_6 - E4k_1 - E6k_3 + E7k_2}{\omega} = 0, -Ik_4\alpha_2 + Ik_2\alpha_4$$

$$- \frac{E1k_7 + E2k_4 - E3k_5 - E4k_2 + E5k_3 - E7k_1}{\omega} = 0, -Ik_4\alpha_3 + Ik_3\alpha_4$$

$$+ \frac{E1k_6 - E2k_5 - E3k_4 + E4k_3 + E5k_2 - E6k_1}{\omega} = 0, -Ik_5\alpha_1 + Ik_1\alpha_5$$

$$- \frac{E1k_5 + E2k_6 + E3k_7 - E5k_1 - E6k_2 - E7k_3}{\omega} = 0, -Ik_5\alpha_2 + Ik_2\alpha_5$$

$$+ \frac{E1k_6 - E2k_5 - E3k_4 + E4k_3 + E5k_2 - E6k_1}{\omega} = 0, -Ik_5\alpha_3 + Ik_3\alpha_5$$

$$\begin{aligned}
& + \frac{E1 k_7 + E2 k_4 - E3 k_5 - E4 k_2 + E5 k_3 - E7 k_1}{\omega} = 0, -1 k_5 \alpha_4 + 1 k_4 \alpha_5 \\
& - \frac{E2 k_3 - E3 k_2 + E4 k_5 - E5 k_4 - E6 k_7 + E7 k_6}{\omega} = 0, -1 k_6 \alpha_1 + 1 k_1 \alpha_6 \\
& - \frac{E1 k_6 - E2 k_5 - E3 k_4 + E4 k_3 + E5 k_2 - E6 k_1}{\omega} = 0, -1 k_6 \alpha_2 + 1 k_2 \alpha_6 \\
& - \frac{E1 k_5 + E2 k_6 + E3 k_7 - E5 k_1 - E6 k_2 - E7 k_3}{\omega} = 0, -1 k_6 \alpha_3 + 1 k_3 \alpha_6 \\
& - \frac{E1 k_4 - E2 k_7 + E3 k_6 - E4 k_1 - E6 k_3 + E7 k_2}{\omega} = 0, -1 k_6 \alpha_4 + 1 k_4 \alpha_6 \\
& + \frac{E1 k_3 - E3 k_1 - E4 k_6 - E5 k_7 + E6 k_4 + E7 k_5}{\omega} = 0, -1 k_6 \alpha_5 + 1 k_5 \alpha_6 \\
& + \frac{E1 k_2 - E2 k_1 + E4 k_7 - E5 k_6 + E6 k_5 - E7 k_4}{\omega} = 0, -1 k_7 \alpha_1 + 1 k_1 \alpha_7 \\
& - \frac{E1 k_7 + E2 k_4 - E3 k_5 - E4 k_2 + E5 k_3 - E7 k_1}{\omega} = 0, -1 k_7 \alpha_2 + 1 k_2 \alpha_7 \\
& + \frac{E1 k_4 - E2 k_7 + E3 k_6 - E4 k_1 - E6 k_3 + E7 k_2}{\omega} = 0, -1 k_7 \alpha_3 + 1 k_3 \alpha_7 \\
& - \frac{E1 k_5 + E2 k_6 + E3 k_7 - E5 k_1 - E6 k_2 - E7 k_3}{\omega} = 0, -1 k_7 \alpha_4 + 1 k_4 \alpha_7 \\
& - \frac{E1 k_2 - E2 k_1 + E4 k_7 - E5 k_6 + E6 k_5 - E7 k_4}{\omega} = 0, -1 k_7 \alpha_5 + 1 k_5 \alpha_7 \\
& + \frac{E1 k_3 - E3 k_1 - E4 k_6 - E5 k_7 + E6 k_4 + E7 k_5}{\omega} = 0, -1 k_7 \alpha_6 + 1 k_6 \alpha_7 \\
& + \frac{E2 k_3 - E3 k_2 + E4 k_5 - E5 k_4 - E6 k_7 + E7 k_6}{\omega} = 0 \}
\end{aligned}$$

> t1:={seq(simplify(subs([seq(Epsilon||i=ees[i],i=1..7)],switcheqns[j])), j=1..nops(switcheqns))};

$$\begin{aligned}
t1 := \{ & 0 = 0, I(-\alpha_1 k_7 - \alpha_2 k_4 + \alpha_4 k_2 + \alpha_7 k_1) = 0, I(\alpha_1 k_4 - \alpha_2 k_7 - \alpha_4 k_1 + \alpha_7 k_2) = 0, I(\alpha_1 k_5 + \alpha_2 k_6 \\
& - \alpha_5 k_1 - \alpha_6 k_2) = 0, I(\alpha_1 k_2 - \alpha_2 k_1 - \alpha_5 k_6 + \alpha_6 k_5) = 0, I(-\alpha_1 k_6 + \alpha_2 k_5 - \alpha_5 k_2 + \alpha_6 k_1) = 0, I(\alpha_1 k_3 - \alpha_2 k_7 \\
& + \alpha_6 k_1) = 0, I(-\alpha_1 k_4 - \alpha_3 k_6 + \alpha_4 k_1 + \alpha_6 k_3) = 0, I(-\alpha_1 k_3 + \alpha_3 k_1 + \alpha_4 k_6 - \alpha_6 k_4) \\
& = 0, I(\alpha_1 k_7 - \alpha_3 k_5 + \alpha_5 k_3 - \alpha_7 k_1) = 0, I(\alpha_1 k_5 + \alpha_3 k_7 - \alpha_5 k_1 - \alpha_7 k_3) = 0, I(-\alpha_1 k_3 \\
& + \alpha_3 k_1 + \alpha_5 k_7 - \alpha_7 k_5) = 0, I(-\alpha_2 k_5 - \alpha_3 k_4 + \alpha_4 k_3 + \alpha_5 k_2) = 0, I(\alpha_2 k_4 - \alpha_3 k_5 \\
& - \alpha_4 k_2 + \alpha_5 k_3) = 0, I(-\alpha_2 k_3 + \alpha_3 k_2 - \alpha_4 k_5 + \alpha_5 k_4) = 0, I(-\alpha_2 k_7 + \alpha_3 k_6 - \alpha_6 k_3 \\
& + \alpha_7 k_2) = 0, I(\alpha_2 k_6 + \alpha_3 k_7 - \alpha_6 k_2 - \alpha_7 k_3) = 0, I(\alpha_2 k_3 - \alpha_3 k_2 - \alpha_6 k_7 + \alpha_7 k_6) = 0, \\
& I(\alpha_4 k_7 - \alpha_5 k_6 + \alpha_6 k_5 - \alpha_7 k_4) = 0, I(-\alpha_4 k_6 - \alpha_5 k_7 + \alpha_6 k_4 + \alpha_7 k_5) = 0, I(\alpha_4 k_5 \\
& - \alpha_5 k_4 - \alpha_6 k_7 + \alpha_7 k_6) = 0 \}
\end{aligned}$$

> t2:={seq(t1[i],i=2..nops(t1))};

(5.5.21)

$$\begin{aligned}
t2 := & \{ I(-\alpha_1 k_7 - \alpha_2 k_4 + \alpha_4 k_2 + \alpha_7 k_1) = 0, I(\alpha_1 k_4 - \alpha_2 k_7 - \alpha_4 k_1 + \alpha_7 k_2) = 0, I(-\alpha_1 k_2 \\
& + \alpha_2 k_1 - \alpha_4 k_7 + \alpha_7 k_4) = 0, I(-\alpha_1 k_6 + \alpha_2 k_5 - \alpha_5 k_2 + \alpha_6 k_1) = 0, I(\alpha_1 k_5 + \alpha_2 k_6 \\
& - \alpha_5 k_1 - \alpha_6 k_2) = 0, I(\alpha_1 k_2 - \alpha_2 k_1 - \alpha_5 k_6 + \alpha_6 k_5) = 0, I(-\alpha_1 k_6 + \alpha_3 k_4 - \alpha_4 k_3 \\
& + \alpha_6 k_1) = 0, I(-\alpha_1 k_4 - \alpha_3 k_6 + \alpha_4 k_1 + \alpha_6 k_3) = 0, I(-\alpha_1 k_3 + \alpha_3 k_1 + \alpha_4 k_6 - \alpha_6 k_4) \\
& = 0, I(\alpha_1 k_7 - \alpha_3 k_5 + \alpha_5 k_3 - \alpha_7 k_1) = 0, I(\alpha_1 k_5 + \alpha_3 k_7 - \alpha_5 k_1 - \alpha_7 k_3) = 0, I(-\alpha_1 k_3 \\
& + \alpha_3 k_1 + \alpha_5 k_7 - \alpha_7 k_5) = 0, I(-\alpha_2 k_5 - \alpha_3 k_4 + \alpha_4 k_3 + \alpha_5 k_2) = 0, I(\alpha_2 k_4 - \alpha_3 k_5 \\
& - \alpha_4 k_2 + \alpha_5 k_3) = 0, I(-\alpha_2 k_3 + \alpha_3 k_2 - \alpha_4 k_5 + \alpha_5 k_4) = 0, I(-\alpha_2 k_7 + \alpha_3 k_6 - \alpha_6 k_3 \\
& + \alpha_7 k_2) = 0, I(\alpha_2 k_6 + \alpha_3 k_7 - \alpha_6 k_2 - \alpha_7 k_3) = 0, I(\alpha_2 k_3 - \alpha_3 k_2 - \alpha_6 k_7 + \alpha_7 k_6) = 0, \\
& I(\alpha_4 k_7 - \alpha_5 k_6 + \alpha_6 k_5 - \alpha_7 k_4) = 0, I(-\alpha_4 k_6 - \alpha_5 k_7 + \alpha_6 k_4 + \alpha_7 k_5) = 0, I(\alpha_4 k_5 \\
& - \alpha_5 k_4 - \alpha_6 k_7 + \alpha_7 k_6) = 0 \}
\end{aligned}
\tag{5.5.2.21}$$

Because the alphas and k's commute, each of these equations is true, and so the proposed solution for the octonionic electric field is valid. Note that there are only 7 independent components of the conventional vector potential, due to our choosing a divergenceless vector.