1-8-2018

Structure of matter, 4

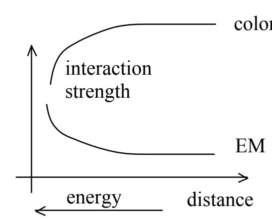
David Peak
Utah State University, david.peak@usu.edu

Structure of matter, 4

Antiscreening: The triumph of lattice QCD

QED is a phenomenally accurate theory of the interactions of electrically charged particles with photons. The way interactions are described in QED—by adding electromagnetic potential fields to the energy and momentum operators in the charged particle field equations—is essentially exactly correct given that the detailed calculations that can be made in QED agree so well with observation. These calculations are possible because simple processes (involving small numbers of interaction vertices) are significantly more important than complicated processes. That is, QED is a “perturbative” theory. Higher order QED effects, therefore, invariably consist of small corrections. QCD is different. The strength of the color interaction is greater than that of the electromagnetic interaction and, because gluons carry color, the processes that contribute importantly are more complex. In general, QCD is not a perturbative theory. Higher order QCD interactions are essential. While QED calculations typically involve only a few Feynman diagrams, QCD calculations of similar accuracy might involve hundreds of thousands!

Until recently, quantitatively accurate QCD results have been hard to come by. On the other hand, some QCD generalities have been known for about 40 years. For example, QCD, *like* QED, is renormalizable (that is, infinities can be removed by assigning finite measured values to a small number of quantities—such as masses and charges—that the theory predicts are infinite). In other words, QCD is a good possible quantum field theory; in principle, it can produce sensible finite results. Though free quarks have never been observed, their existence has been inferred from experiments in which high-energy electrons are used to bombard protons; these electrons emerge with lower kinetic energy–suggesting something in the protons has gained some–and scatter in directions as if the protons contain lumps as opposed to being uniform spheres. To account for not observing free quarks, the strength of the color interaction, *unlike* the electromagnetic interaction, is hypothesized to *increase* at long distances (and low energies) and *decrease* at short distances (and high energies). The latter suggests a perturbation approach might work, but only if the interaction energy is very high (as, for example, in a high energy quark-quark collision). The reason for this difference, is sometime attributed to the “screening” of electric charge at long distances by “clouds” of virtual electron-positron pairs around the charge, whereas virtual gluons supposedly “antiscreen” quarks, whatever that means.



Within the last few years numerical calculations using QCD for quark-gluon bound states have dramatically improved. One method, “QCD Amplitudes,” is a new way of efficiently summing Feynman diagrams under some restricted conditions. Its results suggest that QCD Amplitudes calculations at low energies (and large interaction strengths) can help distinguish observation of “new physics” from complicated QCD “molecule formation.” To date, there is no great departure from “ordinary” QCD in the experimental data at the LHC.

A second method is “Lattice QCD.” These calculations treat spacetime as a four-dimensional rectangular grid, in which the lattice points carry the quark fields and the links between points carry the gluon fields. Lattice QCD is a kind of 4D statistical mechanics picture of quarks and gluons. It employs an alternative to the **S** matrix first proposed by Feynman in his PhD dissertation for calculating quantum mechanical probabilities called “summation over paths” through spacetime. The method assumes that the probability a particle will get to a spacetime point B starting from A is obtained by summing "amplitudes" associated with every possible path connecting A and B, where each amplitude carries a dynamically determined weight. In the end, the desired probability is obtained by squaring the amplitude sum. The required infinite sum can be approximated by a finite sum by selecting paths drawn at random from a dynamically weighted distribution (a process called "Monte Carlo"). The spacetime grid is initialized with some arbitrary colorless distribution of quark and gluon field values, then at each calculation time step the probabilities of new values are calculated by doing a Monte Carlo path estimation of the field value amplitudes. Eventually, the field value distributions will all have the same statistical characteristics; that situation is a kind of "thermal equilibrium." Once the thermal equilibrium is determined a color perturbation can be "injected" into the lattice and its "relaxation" tracked.

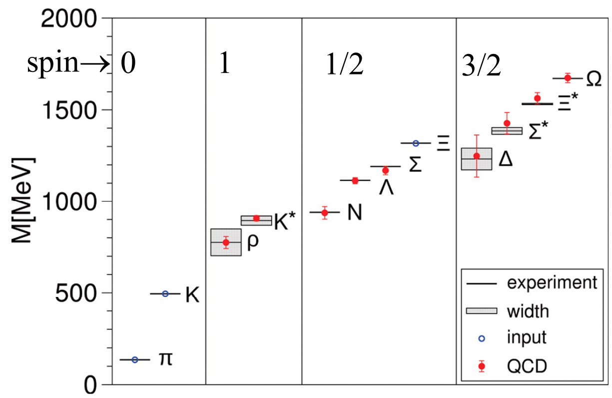
As an example, suppose the quark field amplitude is suddenly increased at one lattice point, representing the introduction of a single real quark into the equilibrium background. Lattice QCD shows that this excitation of the quark field rapidly produces excitations in the surrounding gluon fields and a subsequent enhanced appearance of quark-antiquark virtual pairs. This bubbling frenzy of activity does not die off as it would if the perturbation under study was an electron introduced into an analogous background for electromagnetic interactions. Rather it continues to grow, with the color of the source quark preferentially increasing in the cascade of virtual gluon excitations—numerically corroborating the qualitative expectation of antiscreening.

In electrodynamics the strength of an electric charge is greater the closer it is probed. The opposite is true for color. The color charge surrounding a single quark is greater at greater distances. A profound consequence of the growing excitation produced by injecting a quark into the lattice is that the multiplication of virtual particles never comes back to equilibrium after the perturbation. The existence of an isolated quark produces an untamed energy explosion, so we don't think such a thing exists. This inability to isolate a quark is called "quark confinement."

Fortunately for us (as we seem to be made of them), there are more quarks than a single isolated one. If the initial perturbation is equivalent to the injection of a quark of a given color and an antiquark of the corresponding anti-color, then the exploding excitations tend to cancel one another. The closer the quark and antiquark are, the more complete is the cancellation; there is less field energy. Thus, a quark and an antiquark of canceling color (a "white" combination) attract. Cancellation of the otherwise exploding color-excitations produces an effective attractive potential energy. The same is true for a white combination of three real quarks, one red, one blue, and one green (or three antiquarks, one anti-red, one anti-blue, and one anti-green). The three-color explosions tend to cancel, resulting in attraction.

In either circumstance, exact cancellation would require that the associated source quarks be at the same place in space at the same time. The Heisenberg Uncertainty Principle tells us that the *motions* of such highly localized quarks would be totally unconstrained. In other words, *perfect cancellation of exploding fields implies infinite kinetic energy*—an equally implausible physical situation. What happens after injection of the multi-quark perturbations described here, is that fields on the QCD lattice undergo a transient period during which color potential energy and quark kinetic energy are traded back and forth. Eventually, in the calculation, things settle down into a low energy state and an at least quasi-equilibrium is established. This more-or-less equilibrium configuration corresponds to a meson (for the quark, antiquark case) or a baryon (for the three-quark case).

Now, the total energy of the quieted-down configuration is the observed mass (times c^2) of the associated strongly interacting particle. So if QCD is to be a believable theory of matter it better be able to account for the masses of the observed mesons and baryons. In fact, state-of-the-art lattice QCD yields amazingly good values—±4%, or so—for all of the lowest mass mesons and baryons (i.e., the ones that motivated Gell-Mann's quark hypothesis in the first place). See the figure to the right (from S. Durr, et al., *Science*, **322**, 1224-1227 (2008)). To produce the values shown, only three free parameters are involved: the (assumed same) mass of the lightest quarks (the u and d), the mass of the strange quark (s), and the intrinsic strength of the color interaction. That's it; no other inputs allowed. It's a pretty impressive numerical accomplishment. Lattice QCD also does well for the masses of heavier, more exotic particles, but with larger uncertainties.



The excellent mass calculations of lattice QCD require that the mass of the u and d quarks be only a few MeV. The neutrons and protons from which all atomic nuclei are constructed consist of three u and d quarks. Neutrons and protons have a mass of about 1000 MeV and account for essentially all of the mass of atoms. Consequently, the constituent quarks account for only a few percent of the mass of the atoms in the universe. *The vast majority of atomic mass is due to quark kinetic energy and color potential energy*—that is, nothing massively tangible. What a surprise: we're made of (almost) nothing! There's a wonderfully poetic way to think of this. Very early in the hot universe—before there were nuclei, indeed, before there were even neutrons and protons—there presumably was a soup of highly energetic quarks and gluons (and electrons and neutrinos). As the universe expanded and cooled, quarks coalesced into neutrons and protons, trapping within them the densities of kinetic and color potential energy then prevalent in the universe. So, it's not precisely true that we're made of (almost) nothing. We are actually made of little droplets of the primordial cosmic fireball.

Though lattice QCD has not yet calculated the properties of even the simplest nucleus, the deuteron, we know what will happen. Cancellation of the color fields of the quarks and gluons in both neutron and proton is not exact, though it gets more so at larger distances. The color cancellation gets better as the neutron and proton are brought closer; thus there will be a short-range attraction between them. This short-range interaction is the "strong nuclear force." It results from the interactions of all of the quarks—virtual and real—and all of the gluons that the nucleons are made of. It is not directly the color force but certainly related to it. This situation is exactly analogous to electrical forces in atoms. Separated, the proton and electron of a hydrogen atom have strong electric fields. When brought together, their fields tend to cancel. But not exactly, since quantum mechanics forbids the electron to sit exactly on top of the proton. A little electric field leaks out. (Technically, it's a dipole field that falls off like 1/distance^3.) When a second hydrogen atom is brought close to the first, their interaction tends to make the cancellation more exact: the atoms are attracted to one another. This attraction is the electric "van der Waals force." Thus, the strong nuclear force is the color van der Waals force.

Summary comparison of QED and QCD

In modern language, both QED and QCD are *local gauge theories*. QED arises from the insensitivity of any physical measurement to changes in the phase of the electron's (or other charged particles') wavefunction. Such phase changes can be thought of as "rotations in one complex dimension," or, more conventionally (and also more obscurely), **"U(1) phase transformations."** *The symmetry of the dynamics describing the electron under U(1) phase transformations has an associated conservation law: conservation of electric charge*. This conservation law is equally valid when the phase transformations are applied point-by-point in spacetime, i.e., when they are "local" transformations. The electron's dynamical equations pick up extra derivative terms, however, under such local transformations. The electric and magnetic potentials save the day. When added to the energy and momentum operators of the electron field equation, they cancel the offending derivatives if, when the electron phase is transformed, the potentials transform also. Because the physical electric and magnetic fields are related to the potentials by differentiation, the transformed potentials can yield the same physical fields provided their derivatives vanish ("gauge freedom"). This will be true when the potential fields obey the Maxwell field equations. The particles of the potential fields are spin-1 (boson), massless, and electrically neutral photons. Photons "carry the electric force." Thus, the requirement that the electron field be invariant under local U(1) phase transformations automatically produces a complete theory of the interactions of electrons with electromagnetic fields.

In precisely the same way, QCD arises from the insensitivity of any physical measurement to changes in the color of a quark field. Quarks have three possible color values, so color transformations are "rotations in three complex dimensions," or, more conventionally (and also more obscurely), **"SU(3) color transformations."** *SU(3) color symmetry implies conservation of color*. Local SU(3) color symmetry has the same conservation law and to make such transformations consistent with dynamical invariance requires adding "color potentials" to the energy and momentum operators. In QED, the potentials are just a set of functions. In QCD, the potentials are a set of 3x3 matrix-valued functions. This extra piece of complexity implies the color potentials carry color, so the field equations they obey are more complicated than the Maxwell equations. The particles of the color potentials are spin-1 (bosons), massless, and electrically neutral *but* color-charged gluons. Gluons "carry the color force." The electric neutrality of photons implies the intrinsic strength of the electric force *increases as distance decreases*. The color charge of the gluons implies that gluon virtual pair density surrounding a bare color charge increases with distance. The intrinsic strength of the color force *decreases as distance decreases*.