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EVALUATION OF AN EXPERIMENT AFTER
ANALYSIS OF VARIANCE

by

Abdullah Sulaiman Atheem

A report submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Statistics

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CHAPTER I
INTRODUCTION AND REVIEW OF LITERATURE

Introduction

If experimental data are obtained according to an experimental design for which analysis of variance (AOV) is appropriate, the result of the analysis is one of the following decisions:

1. A significant difference exists among treatment effects.
2. No significant difference exists.

Interpretation in the first case has been studied very thoroughly by many statisticians resulting, for example, in the multiple comparison procedure. Too often, in the second case, the AOV analysis is thought to be completed with no further attempt to interpret a non-significant outcome.

Of the many possible explanations for significance in the first case, the multiple comparison procedure seeks to find the most plausible explanation. Similarly, there are two possible explanations for the non-significant result:

1. There is truly no meaningful difference among the treatments.
2. The experimental design did not offer sufficient precision to detect differences which may be of interest.

In this paper a method is sought which gives some indication of the most plausible explanation for a non-significant result.

There are two main objectives for developing this report.

1. To develop an analysis following a non-significant AOV result that provides a plausible interpretation of the reason for non-significance.
2. To present the analysis procedure in such a way that experimenters not highly trained in statistics can easily perform the calculation.

In order to attain these objectives, the power calculation presented by Henry Scheffé (1959) provides a good starting point for the main part of this work. His procedure will be modified to accomplish the first objective. Since he described in detail the power calculation for the one-way layout, it is necessary in order to accomplish the second objective that his work be extended in detail to a general balanced fixed effect factorial experiment. This extension will be modified as needed for the purpose of this report.

The entire balanced analysis of variance is reviewed including the expected mean square and the multiple comparison procedure. This review is intended to place the problem of a non-significant result from an analysis of variance in its proper context.

Some notation and definitions is presented in order to make the reader familiar with the terms that are used in the procedure. Then an interpretation of the meaningful difference between effects is presented, and also the nature and use of the available tables and charts for determining the power of the F distribution. The power calculation will be presented using the non-centrality parameter.

A detailed description of the procedure will then be given in a non-statistical term to describe the analysis by summarizing the

procedure in steps to make it easier to follow. Several examples using difference experimental designs will be worked out for illustration of the procedure.

Definitions and Notations

Let: H_0 be the null hypothesis

H_1 be the alternative hypothesis

α be the type I error. It is called the significant level.

$\alpha = P_r(\text{reject } H_0/H_1 \text{ true}).$

Thus α is the probability of rejecting the null hypothesis, when it is true.

β be the type II error.

$\beta = P_r(\text{accept } H_0/H_1 \text{ true}).$

Thus β is the probability of accepting the null hypothesis when the alternative hypothesis should be taken.

The power of the test is $(1-\beta)$ and it is the probability of rejecting the null hypothesis when it should be rejected.

f_1 is the degrees of freedom for the numerator of the F-test ratio.

f_2 is the degrees of freedom for the denominator of the F-test ratio.

n is the number of observations on each treatment in one-way layout, or the number of observations in each cell for higher-way layout in the balanced case.

δ is the non-centrality parameter for the non-central F distribution. For convenience in later work, define ϕ by

$$\phi = \frac{\delta}{\sqrt{f_1+1}}$$

Review of Literature

There is very little written that specifically addresses the problem described previously. However, the computations to be used in the solution of the problem are those associated with the standard power calculation.

On this subject there is a great deal written. Henry Scheffé (1959) wrote about the power calculation of the F-test in the analysis of variance in his book, The Analysis of Variance, and he did that for just one-way layout and stated "calculation for other experimental designs are similar". An example in Section 3.3 (example on power calculation) is given in his book. He uses both Pearson and Hartley charts and Fox's charts.

An early paper on this subject by Tang (1938), discussed the linear hypothesis for the analysis of variance including type I and type II error, and the power. He wrote his paper in a theoretical manner and finished it with applications in which he summarized his procedure into six steps and he gave two examples. Also, he tabulated the second type error β which is $(1-\text{power})$.

Pearson and Hartley (1951), give charts for the power function of the analysis of variance tests derived from the non-central F-distribution.

In this paper they started with the distribution of non-central chi-square and F-distributions, then they discussed applying the power function on the analysis of variance and derived the non-centrality parameter for special cases such as one-way classification, two-way classification with one observation in each cell, two-way classification

n observations per cell and latin square. They used the true effect in deriving the non-centrality parameter, then they interpreted the power function through examples. They constructed charts for power function of the F-distribution through examples. They constructed charts for power function of the F-distribution using ν_1 , ν_2 as degrees of freedom for the numerator and denominator, ϕ which is a function of the non-centrality parameter and the significance level, α . These charts were used to calculate the power.

As an extension of Tang's tables, Tiku (1967) has given tables for type II error which is (1-power). The advantage of these tables compared with Tang's tables is that these tables have smaller increments for the value of ϕ than Tang's tables have, and more values for the degrees of freedom for the numerator. He did the tables for two values of α , .01 and .05, but in 1972 he added another set for $\alpha = .1$. He discussed the computation of the tables.

Fox (1956) has constructed eight different charts for different power values and two different α 's, .01, .05, and two other charts for interpolation of the power and states an example to show how to use them.

Fox and Odeh (1975) extended the previous charts by using a combination of many levels of significance α and more levels for the power. They were trying to find the appropriate sample size for an experiment using specific power value. This procedure was applied on one and two-way layouts.

There are many books which include a discussion of hypothesis testing, type I and type II error and experimental design, such as

Lindgren (1962), Anderson and McLean (1974), Winner (1971) and Hicks (1974).

Tables and charts are available for calculating the power. The first tables for the power of the F-test were constructed by Tang (1938). He tabled type II error which is (1-power) for a given α , f_1 , f_2 , and ϕ , such that:

$$\alpha = .01, .05$$

$$f_1 = 1(1)8 \text{ (} f_1 \text{ value starting at one and increasing by one until 8)}$$

$$f_2 = 2, 4, 6(1) 30, 60, \infty$$

$$\phi = 1(.5)3(1)8 \text{ (} \phi \text{ value starting at one and increasing by .5 until three then increasing by one until 8)}$$

An extension of Tang's tables have been done by Tiku (1967). The parameters for Tiku's tables are:

$$\alpha = .005, .01, .025, .05$$

$$f_1 = 1(1)12$$

$$f_2 = 2(2)30, 40, 60, 120, \infty$$

$$\phi = .5, 1(.2)2.2(.4)3$$

In 1972 he added another set of tables with $\alpha = .1$ and the same values for the other parameters to use it for bigger α . In these tables, as Tang did, he tabulated type II error β which is equal to (1-power).

Pearson and Hartley (1951) constructed charts to calculate the power function for the analysis of variance test using the non-central F-distribution with these parameters:

$$\alpha = .01, .05$$

$$f_1 = \nu_1 \text{ (in his charts) } = 1 \text{ (1) } 8$$

$$f_2 = \nu_2 \text{ (in his charts) } = 6 \text{ (1) } 10, 12, 15, 20, 30, 60, \infty.$$

The parameter ϕ differs from one chart to another.

✓ Martin Fox (1956) constructed charts for the power of the F-test. Each chart has several curves indexed by the parameter ϕ for a given level of significance and power.

$$\alpha = .01, .05$$

$$\text{power} = .5, .7, .8, .9$$

$$f_1 = 3 \text{ (1) } 10 \text{ (2) } 20 \text{ (20) } 100, 200, \infty.$$

$$f_2 = 4 \text{ (1) } 10 \text{ (2) } 20 \text{ (20) } 100, 200, \infty.$$

There are eight charts for different combinations of α and power. In addition to these, two nomograms for $\alpha = .01, .05$ are provided to allow for interpolation in the power.

In 1975 Martin Fox and Robert E. Odeh again prepared the previous charts with more levels for α and power.

To discuss how to use the tables and the charts for calculating the power, an example is presented. Assume in an experiment $f_1 = 5$, $f_2 = 42$, $\phi = 2.6$, and the significant level $\alpha = .01$. From Tiku's tables go to the table which has $f_1 = 5$ and $\alpha = .01$, column 2.6 for ϕ and row 40 for f_2 the $\beta = .01$; the power = $1 - \beta = 1 - .01 = .99$.

✓ In Pearson and Hartley charts, enter the chart which is for $f_1 = 5$, for $\phi = 2.6$ and for $\alpha = .01$. Find the curve with $f_1 = 42$ (between 30,60) and note the power line you hit. The power is seen to be .99.

The charts are easier than the tables to use and do not require linear interpolation for the values of f_1, f_2 . However, Fox's charts

are more convenient to use if you want to determine the sample size for a given power. Pearson and Hartley charts are better for finding the power.

CHAPTER II

REVIEW OF THE ANALYSIS OF VARIANCE

Underlying Assumption

When the analysis of variance is used to summarize the properties of the data, no assumption is needed to validate them. On the other hand, when the analysis of variance is used as a tool of statistical inference to make inferences about the population parameters from which the data was drawn, a certain assumption about the population and the sampling procedures must be fulfilled if the inferences are to be valid. The analysis of variance can be used with two types of problems:

1. To investigate the existence of and to estimate the parameters of a fixed relation among the population means.
2. To investigate the existence of and to estimate the components of variance.

There are assumptions associated with each of these types of problems. Since the first type is the most important in this report, the assumptions associated with this type of problem will be considered below.

Assumption 1: (Random variable or random sampling procedure)

The sampling of individuals should be completely randomized, i.e., in an experiment for study in the effects of four doses of drugs on six rats allocated to each dose, the rats used within each dose must be selected at random.

The purpose of randomization is to avoid introducing a systematic bias, e.g., if one were to select rats in the order they are caught, the more active energetic ones may be in the last group.

Assumption 2: (Additivity)

If there is one observation per cell in a multi-way layout, some assumption of no interaction is required. The cell mean must be explained by μ , the grand mean and a_i , the effect of A treatment; b_j , the effect of B treatment, etc. If the response is not explained by these, there must be another term which is the interaction between the treatments. In this case, we can say that the treatments are not additive.

Assumption 3: (Equal variance and zero correlation)

The Y_{ij} 's are homoscedastic and mutually uncorrelated, they have a common variance, σ^2 , and all covariances among them are zero.

Assumption 4: (Normality)

The Y_{ij} 's are jointly distributed in a multivariate normal distribution. This assumption is used to conduct exact test of significance.

The error terms E_{ij} 's are independent and identically distributed as normal with mean zero and common variance, σ^2 . If the assumptions two through four are all satisfied, then the analysis of variance is a good device for analyzing and making inferences about the population parameters, if not the data might be transformed in order to meet assumptions by using the appropriate transformation.

Analysis of Variance Tables for Some Fixed Effect Model Designs

In this section some tables will be presented to summarize the analysis of variance degrees of freedom, sums of squares, means of squares, expected mean square (EMS) for fixed effect model and F-ratio for the one-way layout, two-way layout, three-way layout and latin square.

Multiple Comparison Procedures

Interpretation of a significant F-test in the analysis of variance usually leads to discussion about the means and the multiple comparison procedures. There are several multiple comparison procedures, i.e., least significance difference (LSD), Student-Newman Keuls procedure, Duncan Multiple Range test, Tukey Significant Difference, Scheffé, etc.

These multiple comparison procedures are prepared to determine which treatment(s) led to the significant difference.

In the multiple comparison procedures, one or more critical value for each procedure is calculated and the difference between any pair of means is compared to the corresponding critical value. If the difference between a pair of means exceeds the appropriate critical value, the means are declared significantly different, otherwise the difference is considered non-significant.

TABLE 1.--Analysis of variance table for one-way layout with equal number of observations per cell.

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square (Fixed)	F-Ratio
Treatment	I-1	$\sum_{i=1}^I \frac{Y_{i\cdot}^2}{n} - \frac{\left[\sum_{i=1}^I \sum_{j=1}^n Y_{ij} \right]^2}{In}$	$\frac{SS \text{ treatment}}{I-1}$	$\sigma_e^2 + \frac{n}{I-1} \sum_{i=1}^I a_i^2$	$\frac{MS \text{ Trt}}{MSE}$
Error	nI-I	SS total - SS treatment	$\frac{SS \text{ error}}{nI-I}$	σ_e^2	
Total Corrected	nI-1	$\sum_{i=1}^I \sum_{j=1}^n Y_{ij}^2 - \frac{\left[\sum_{i=1}^I \sum_{j=1}^n Y_{ij} \right]^2}{In}$			

I = number of treatments

TABLE 2.-- Analysis of variance table for two-way layout with one observation per cell

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Square	Expected Mean Square (EMS)	F-Ratio
A_i	$I-1$	$\sum_{i=1}^I Y_{i.}^2 / J - \frac{\left[\sum_{i=1}^I \sum_{j=1}^J Y_{ij} \right]^2}{IJ}$	$\frac{SSA}{I-1}$	$\sigma_e^2 + \frac{J}{I-1} \sum_{i=1}^I a_i^2$	$\frac{MSA}{MSE}$
B_j	$J-1$	$\sum_{j=1}^J Y_{.j}^2 / I - \frac{\left[\sum_{i=1}^I \sum_{j=1}^J Y_{ij} \right]^2}{IJ}$	$\frac{SSB}{J-1}$	$\sigma_e^2 + \frac{I}{J-1} \sum_{j=1}^J b_j^2$	$\frac{MSB}{MSE}$
Error	$(I-1)(J-1)$	SS total corrected-SSA-SSB	$\frac{SS \text{ error}}{(I-1)(J-1)}$	σ_e^2	
Total Corrected	$IJ-1$	$\sum_{i=1}^I \sum_{j=1}^J Y_{ij}^2 - \frac{\left[\sum_{i=1}^I \sum_{j=1}^J Y_{ij} \right]^2}{IJ}$	$\frac{SS \text{ total}}{IJ-1}$		

I = number of levels of factor A
 J = number of levels of factor B

TABLE 3.--Analysis of variance for two-way layout with n observations per cell

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Square	Expected Mean Square (EMS)	F-Ratio
A_i	$I-1$	$\sum_{i=1}^I \frac{Y_{i..}^2}{JN} - \frac{\left[\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^n Y_{ijk} \right]^2}{IJn}$	$\frac{SSA}{I-1}$	$\sigma_e^2 + \frac{Jn}{I-1} \sum_{i=1}^I a_i^2$	$\frac{MSA}{MSE}$
B_j	$J-1$	$\sum_{j=1}^J \frac{Y_{.j.}^2}{In} - \frac{\left[\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^n Y_{ijk} \right]^2}{IJn}$	$\frac{SSB}{J-1}$	$\sigma_e^2 + \frac{In}{J-1} \sum_{j=1}^J b_j^2$	$\frac{MSB}{MSE}$
AB_{ij}	$(I-1)(J-1)$	$\sum_{i=1}^I \sum_{j=1}^J \frac{Y_{ij.}^2}{n} - \sum_{i=1}^I \frac{Y_{i..}^2}{Jn} - \sum_{j=1}^J \frac{Y_{.j.}^2}{In} + \frac{\left[\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^n Y_{ijk} \right]^2}{IJn}$	$\frac{SSAB}{(I-1)(J-1)}$	$\sigma_e^2 + \frac{n}{(I-1)(J-1)} \sum_{i=1}^I \sum_{j=1}^J ab_{ij}^2$	$\frac{MSAB}{MSE}$
Error	$IJ(n-1)$	$SS_{total} - SS_A - SS_B - SS_{AB}$	$\frac{SS_{error}}{IJ(n-1)}$	σ_e^2	
Total Corrected	$IJ n-1$	$\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^n Y_{ijk}^2 - \frac{\left[\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^n Y_{ijk} \right]^2}{IJK}$	$\frac{SS_{total}}{IJn - 1}$		

TABLE 4.--Analysis of variance table for three-way layout

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Squares (EMS)	F-Ratio
A	I-1	$\sum_{i=1}^I \frac{Y_{i...}^2}{JKn} - \frac{\left(\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl} \right)^2}{IJKn}$	$\frac{SS_A}{I-1}$	$\sigma_e^2 + \frac{JKn}{I-1} \sum_{i=1}^I a_i^2$	$\frac{MSA}{MSE}$
B	J-1	$\sum_{j=1}^J \frac{Y_{.j..}^2}{IKn} - \frac{\left(\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl} \right)^2}{IJKn}$	$\frac{SS_B}{J-1}$	$\sigma_e^2 + \frac{IKn}{J-1} \sum_{j=1}^J b_j^2$	$\frac{MSB}{MSE}$
C	K-1	$\sum_{k=1}^K \frac{Y_{..k.}^2}{IJn} - \frac{\left(\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl} \right)^2}{IJKn}$	$\frac{SS_C}{K-1}$	$\sigma_e^2 + \frac{IJn}{K-1} \sum_{k=1}^K c_k^2$	$\frac{MSC}{MSE}$
AB	(I-1)(J-1)	$\sum_{i=1}^I \sum_{j=1}^J \frac{Y_{ij..}^2}{Kn} - \sum_{i=1}^I \frac{Y_{i...}^2}{JKn} - \sum_{j=1}^J \frac{Y_{.j..}^2}{IKn} + \frac{\left(\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl} \right)^2}{IJKn}$	$\frac{SS_{AB}}{(I-1)(J-1)}$	$\sigma_e^2 + \frac{Kn}{(I-1)(J-1)} \sum_{i=1}^I \sum_{j=1}^J ab_{ij}^2$	$\frac{MSAB}{MSE}$

TABLE 4.--Continued

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Squares (EMS)	F-Ratio
AC	(I-1)(K-1)	$\frac{\sum_{i=1}^I \sum_{k=1}^K Y_{i.k}^2}{Jn} - \frac{\sum_{i=1}^I \frac{Y_{i...}^2}{JKn}}{I} - \frac{\sum_{k=1}^K \frac{Y_{..k}^2}{IJn}}{K} + \frac{\left(\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl} \right)^2}{IJKn}$	$\frac{SS_{AC}}{(I-1)(K-1)}$	$\sigma_e^2 + \frac{Jn}{(I-1)(K-1)}$	$\frac{MS_{AC}}{MSE}$
				$\sum_{i=1}^I \sum_{k=1}^K ac_{ik}^2$	
BC	(J-1)(K-1)	$\sum_{j=1}^J \sum_{k=1}^K \frac{Y_{.jk}^2}{In} - \sum_{j=1}^J \frac{Y_{.j..}^2}{IJn} - \sum_{k=1}^K \frac{Y_{..k}^2}{IJn} + \frac{\left(\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl} \right)^2}{IJKn}$	$\frac{SS_{BC}}{(J-1)(K-1)}$	$\sigma_e^2 + \frac{In}{(J-1)(K-1)}$	$\frac{MS_{BC}}{MSE}$
				$\sum_{j=1}^J \sum_{k=1}^K bc_{jk}^2$	
ABC	(I-1)(J-1)(K-1)	$\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \frac{Y_{ijk}^2}{n} - SSA - SSB - SSC - SSAB - SSAC - SSBC - \frac{\left(\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl} \right)^2}{IJKn}$	$\frac{SS_{ABC}}{(I-1)(J-1)(K-1)}$	$\sigma_e^2 + \frac{n}{(I-1)(J-1)(K-1)}$	$\frac{MS_{ABC}}{MSE}$
				$\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K abc_{ijk}^2$	

TABLE 4.--Continued

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Squares (EMS)	F-Ratio
Error	IJK (n-1)	$SS_{total} - SS_A - SS_B - SS_C - SS_{AB} - SS_{AC} - SS_{BC} - SS_{ABC}$	$\frac{SSE}{IJK(n-1)}$	σ_e^2	
Total Corrected	IJKn-1	$\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl}^2 - \frac{\left[\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{l=1}^n Y_{ijkl} \right]^2}{IJKn}$	$\frac{SS_{total}}{IJKn-1}$		

I = number of levels of factor A

J = number of levels of factor B

K = number of levels of factor C

TABLE 5.--Analysis of variance table for latin square

Source of Variation	Degrees of Variation	Sum of Squares	Mean Square	Expected Mean Squares (EMS)	F-Ratio
Rows	K-1	$\sum_{i=1}^K \frac{Y_{i..}^2}{K} - \frac{\left(\sum_{i=1}^K \sum_{j=1}^K Y_{ijt} \right)^2}{K^2}$	$\frac{SS_R}{K-1}$	$\sigma_e^2 + \frac{K}{K-1} \sum_{i=1}^K R_i^2$	$\frac{MSR}{MSE}$
Columns	K-1	$\sum_{j=1}^K \frac{Y_{.j.}^2}{K} - \frac{\left(\sum_{i=1}^K \sum_{j=1}^K Y_{ijt} \right)^2}{K^2}$	$\frac{SS_C}{K-1}$	$\sigma_e^2 + \frac{K}{K-1} \sum_{j=1}^K C_j^2$	$\frac{MSC}{MSE}$
Treatment	K-1	$\sum_{t=1}^K \frac{Y_{..t}^2}{K} - \frac{\left(\sum_{i=1}^K \sum_{j=1}^K Y_{ijt} \right)^2}{K^2}$	$\frac{SS_{treat}}{K-1}$	$\sigma_e^2 + \frac{K}{K-1} \sum_{j=1}^K T_t^2$	$\frac{MST}{MSE}$
Error	(K-1)(K-2)	$SS_{total} - SS_{columns} - SS_{row} - SS_{treat}$	$\frac{SS_{error}}{(K-1)(K-2)}$	σ_e^2	
Total	K ² -1	$\sum_{i=1}^K \sum_{j=1}^K Y_{ijt}^2 - \frac{\left(\sum_{i=1}^K \sum_{j=1}^K Y_{ijt} \right)^2}{K^2}$	$\frac{SS_{total}}{K^2-1}$		

K = number of rows, columns and treatments

CHAPTER III

EVALUATION OF A NON-SIGNIFICANT ANALYSIS
USING THE POWER FUNCTIONMethod of Deriving the Non-Centrality Parameter

In order to find the non-centrality parameter, the appropriate model for the design, the hypothesis to be tested, and the sums of squares should be presented. The non-centrality parameter is a function of both the true effect and σ^2 which is estimated by the mean square error used in the denominator of the F-ratio.

To discuss the procedure for deriving the non-centrality parameter, a simple one-way layout example will be presented.

In an experiment six different teaching methods were used on a sample of 48 students (eight students for each method).

$$\text{Model: } y_{ij} = \mu + a_i + \epsilon_{ij}$$

$$\text{where } \begin{cases} i = 1, 2, \dots, I \\ j = 1, 2, \dots, n \end{cases}$$

y_{ij} is the score for j th student in the i th method.

$H_0: a_1 = a_2 = \dots a_I$ (testing whether the methods are the same or not?)

$$SS_A = n \sum_{i=1}^I (\bar{Y}_{i\cdot} - \bar{Y}_{\cdot\cdot})^2$$

where $\begin{cases} \bar{Y}_{i.} & \text{is the mean score for the } i\text{th method} \\ \bar{Y}_{..} & \text{is the overall mean score (over all groups)} \end{cases}$

If in the sum of squares used in the numerator of the F-ratio each value of y is replaced by its expected value, then this value would be equal to the non-centrality parameter $\sigma^2\delta^2$.

$$\text{Hence, } \sigma^2\delta^2 = n \sum_{i=1}^I (a_i - \bar{a})^2$$

where a_i is the true effect of the i th method and

$$\bar{a} = \frac{1}{I} \sum_{i=1}^I a_i.$$

It can be seen that given σ^2 , the δ depends upon the effect values a_i , $i = 1, \dots, I$.

Let Δ represent the maximum meaningful difference between any pair of effects, i.e.,

$$\Delta = \max |a_i - a_{i'}|.$$

By meaningful, it is meant that differences which are less than Δ are not worth detecting in a practical sense. Differences greater than Δ are important and worth detecting. Since the power of the F-test is monotone increasing with δ , we shall use the following criteria for determining δ . There are many possible values of δ given a value of Δ .

$$\text{If } \delta = \min_{a_i} \left[n \sum_{i=1}^I (a_i - \bar{a})^2 / \sigma^2 \right]$$

The constraint $\max |a_i - a_{i'}| = \Delta$, then δ represents a conservative choice for the non-centrality parameter (that is, given Δ the power of the test is at least as great as that computed using δ).

It will now be shown that $\epsilon^2\sigma^2$ will be minimum if two of the a_i 's differ by Δ or more and the remaining $(I - 2)$ a_i 's are equal to the average of the two.

This can be shown analytically as follows,

$$\sigma^2\delta^2 = n \sum_{i=1}^I (a_i - \bar{a})^2$$

$$\min \sigma^2\delta^2 = \min n \sum_{i=1}^I (a_i - \bar{a})^2$$

Conditions on minimizing $n \sum_{i=1}^I (a_i - \bar{a})^2$ are

$$a_1 = \min (a_i)$$

$$a_2 = \max (a_i)$$

$$a_2 - a_1 = \Delta$$

$$a_2 = a_1 + \Delta$$

Now we can write

$$\sum_{i=1}^I (a_i - \bar{a})^2 = (a_1 - \bar{a})^2 + (a_1 + \Delta - \bar{a})^2 + \sum_{i=3}^I (a_i - \bar{a})^2 \quad (1)$$

To minimize this value, it should be differentiated with respect to a_i and the derivative set equal to zero.

The partial derivative taken with respect to a_i is

$$\frac{\partial \sum_{i=1}^I (a_i - \bar{a})^2}{\partial a_i} = \begin{cases} 2(a_1 - \bar{a})(1 - \frac{2}{I}) + 2(a_1 + \Delta - \bar{a})(1 - \frac{2}{I}) \\ \text{if } i = 1 \\ \\ 2 \sum_{i=3}^I (a_i - \bar{a})(1 - \frac{1}{I}) \\ \text{if } i \neq 1 \end{cases}$$

If $i = 1$

$$2(a_1 - \bar{a})(1 - \frac{2}{I}) + 2(a_1 + \Delta - \bar{a})(1 - \frac{2}{I}) = 0$$

$$(a_1 - \bar{a}) + (a_1 + \Delta - \bar{a}) = 0$$

$$2a_1 + \Delta - 2\bar{a} = 0 \rightarrow \bar{a} = a_1 + \frac{\Delta}{2}$$

If $i \neq 1$

$$2 \sum_{i=3}^I (a_i - \bar{a})(1 - \frac{1}{I}) = 0$$

$$\sum_{i=3}^I (a_i - \bar{a}) = 0$$

There are many solutions for this equation, one of the solutions is obtained when

$$a_i = a_1 + \frac{\Delta}{2} = \bar{a} \quad i = 3, \dots, I$$

Going back to equation (1)

$$\sum_{i=1}^I (a_i - \bar{a})^2 = (a_1 - \bar{a})^2 + (a_1 + \Delta - \bar{a})^2 + \sum_{i=3}^I (a_i - \bar{a})^2$$

In the right hand side of this equation, plug in \bar{a} for each a_i in the term

$$\sum_{i=3}^I (a_i - \bar{a})^2$$

So this quantity will go to zero and

$$\sum_{i=1}^I (a_i - \bar{a})^2 = (a_1 - \bar{a})^2 + (a_1 + \Delta - \bar{a})^2$$

will be minimum if the usual assumption that $\sum_i a_i = 0$ is used.

This implies that $\bar{a} = 0$, and

$$\sum_{i=1}^I (a_i - \bar{a})^2 = \sum_{i=1}^I a_i^2,$$

$$|a_i| = \begin{cases} 1/2 \Delta & \text{for the two extremes} \\ 0 & \text{for the others} \end{cases}$$

$$\begin{aligned} \sigma^2 \delta^2 &= n \left(\frac{\Delta^2}{4} + \frac{\Delta^2}{4} \right) , \\ &= n \frac{\Delta^2}{2} , \end{aligned}$$

$$\delta^2 = \frac{n\Delta^2}{2\sigma^2} ,$$

Therefore,

$$\delta = \sqrt{\frac{n\Delta^2}{2}}$$

To show how to calculate δ , assume in our previous example we tested the hypothesis that there is no difference between the six methods and we fail to reject. Assume also that we found the mean square error to be 100 ($\hat{\sigma}^2 = 100$) and the meaningful difference for us is ($\Delta = 10$). This gives

$$\delta = \sqrt{\frac{(8)(10)^2}{2(100)}} = 2$$

Further derivation for the non-centrality parameter for different balanced factorial designs will be presented in the power calculation section.

Power Calculation for Specific Balanced Factorial Designs

In calculating the power function for the F-test in the analysis of variance, we need these parameters: α , f_1 , f_2 , σ^2 , ϕ .

If an AOV is done, we will get all these parameters except ϕ . But ϕ will be found by deriving the non-centrality parameter, since ϕ is a function of the non-centrality parameter

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}}$$

In this section the non-centrality parameter will be derived for the main effects and interactions in different balanced experimental designs and from this non-centrality parameter we will obtain the value of ϕ needed to enter the tables or the charts to calculate the power. The first derivation will be for the simplest design, a one-way layout, then the general balanced case will be considered. The derivation method discussed before will be used throughout this section.

One-way layout:

This case has been discussed before and the result was

$$\delta = \sqrt{\frac{n\Delta^2}{2\sigma^2}}$$

Since
$$\phi = \frac{\delta}{\sqrt{f_1 + 1}}$$

Thus,
$$\phi = \sqrt{\frac{n\Delta^2}{2(f_1 + 1)\sigma^2}}$$

The power of the test will be found in the tables or chart using the parameter ϕ .

Two-way layout with one observation per cell:

The model:
$$Y_{ij} = \mu + a_i + b_j + \epsilon_{ij}$$

where
$$\begin{cases} i = 1, 2, \dots, I \\ j = 1, 2, \dots, J \end{cases}$$

and

$$\sum_{i=1}^I a_i = \sum_{j=1}^J b_j = 0 \quad \epsilon_{ij} \text{ NID } (0, \sigma^2)$$

In this case we assume no interaction between the two factors and we wish to test the following two hypotheses:

$$H_0: a_1 = a_2 = \dots = a_I$$

and

$$H_0: b_1 = b_2 = \dots = b_J$$

This case is similar to the one-way layout. For this case we just change n , the number of observations in the one-way layout, to the number of levels in the other factor for finding the non-centrality parameter. So the non-centrality parameter for factor A is:

$$\delta_A = \sqrt{\frac{J\Delta^2}{2\sigma^2}}$$

and

$$\phi = \sqrt{\frac{J\Delta^2}{2(f_1 + 1)\sigma^2}} .$$

For factor B the necessary parameters are:

$$\delta_B = \sqrt{\frac{I\Delta^2}{2\sigma^2}}$$

and

$$\phi = \sqrt{\frac{I\Delta^2}{2(f_1 + 1)\sigma^2}}$$

Two-way layout with more than one observation per cell:

$$\text{Model: } Y_{ij} = \mu + a_i + b_j + ab_{ij} + \epsilon_{ijk}$$

where

$$\left\{ \begin{array}{l} i = 1, 2, \dots, I \\ j = 1, 2, \dots, J \\ k = 1, 2, \dots, n \end{array} \right.$$

and

$$\sum_{i=1}^I a_i = 0, \quad \sum_{j=1}^J b_j = 0, \quad \sum_{i=1}^I ab_{ij} = \sum_{j=1}^J ab_{ij} = 0$$

ϵ_{ijk} is NID $(0, \sigma^2)$

In the two-way and higher order layout, there are three different situations to consider in deriving the non-centrality parameters.

Situation A:

In this situation we test for the main effects with no additional assumptions about interactions.

To test the hypothesis that there is no treatment effect.

For factor A

$$H_0: a_1 = a_2 = \dots = a_I$$

To calculate the power, the non-centrality parameter should be derived.

$$SS_A = nJ \sum_{i=1}^I (\bar{Y}_i \dots - \bar{Y} \dots)^2$$

and

$$\sigma^2 \delta^2 = nJ \sum_{i=1}^I a_i^2$$

where a_i is the true effect of the i th level of factor A.

To find the minimum δ , as it was shown before, the two extreme a_i differ by Δ and the remaining $(I-2)$ are equal to the average of these two.

$$|a_i| = \begin{cases} 1/2 \Delta & \text{for the two extremes} \\ 0 & \text{for the others} \end{cases}$$

$$\sigma^2 \delta^2 = nJ \left(\frac{\Delta^2}{4} + \frac{\Delta^2}{4} \right) = nJ \frac{\Delta^2}{2} ,$$

$$\delta^2 = \frac{nJ \Delta^2}{2 \sigma^2} ,$$

$$\delta = \sqrt{\frac{nJ \Delta^2}{2 \sigma^2}} ,$$

Therefore,
$$\phi = \sqrt{\frac{nJ \Delta^2}{2 \sigma^2 (f_1 + 1)}}$$

where σ^2 is estimated by the mean square used in the denominator of the F-ratio.

For the other factor (B), ϕ would be the same except that J should be changed to I since

$$SS_B = In \sum_{j=1}^J (\bar{Y}_{.j.} - \bar{Y}_{...})^2$$

Therefore,
$$\phi = \sqrt{\frac{nI \Delta^2}{2 \sigma^2 (f_1 + 1)}}$$

where Δ is that for factor B.

Now the power is easy to find using either the tables or the charts.

Situation B:

In this situation we test for the main effects assuming that the interactions are zero (no interaction effects). That means that we throw the interactions in the error. So this situation will be the same as situation A except that f_2 would be $IJn - I - J + 1$.

Situation C:

In this situation, testing the hypothesis that the interactions are zeros (no interaction effect).

$$H_0: ab_{ij} = 0$$

$$f_1 = (J-1)(J-1),$$

$$f_2 = IJ(n-1)$$

$$SS_{AB} = n \sum_{i=1}^I \sum_{j=1}^J (\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2$$

where

$\bar{Y}_{ij.}$ is the mean of the ij th cell

$\bar{Y}_{i..}$ is the mean of the i th level in factor A

$\bar{Y}_{.j.}$ is the mean of the j th level in factor B

$\bar{Y}_{...}$ is the grand mean.

If (ab_{ij}) is the true effect of the ij th cell, then

$$\sigma^2 \delta^2 = n \sum_{i=1}^I \sum_{j=1}^J (ab_{ij})^2$$

To find $\min \delta^2$ the two extreme (ab_{ij}) will differ by Δ and the remaining $(IJ-2)$ will be equal to the average of the two

$$|ab_{ij}| = \begin{cases} 1/2 \Delta & \text{for the two extremes} \\ 0 & \text{for the other} \end{cases}$$

$$\sigma^2 \delta^2 = n \left(\frac{\Delta^2}{4} + \frac{\Delta^2}{4} \right) = n \left(\frac{\Delta^2}{2} \right),$$

$$\delta^2 = \frac{n\Delta^2}{2\sigma^2} \rightarrow \delta = \frac{\sqrt{n\Delta^2}}{2\sigma^2},$$

Therefore,
$$\phi = \frac{\sqrt{n\Delta^2}}{2(f_1 + 1)\sigma^2}$$

Then the power can be easily found from the tables or the charts.

Three and higher-way layout:

For three-way layout, the model is:

$$Y_{ijkl} = \mu + a_i + b_j + c_k + ab_{ij} + bc_{jk} + ac_{ik} + abc_{ijk} + \epsilon_{ijkl}$$

where
$$\begin{cases} i = 1, 2, \dots, I \\ j = 1, 2, \dots, J \\ k = 1, 2, \dots, K \\ l = 1, 2, \dots, n \end{cases}$$

and

$$\sum_{i=1}^I a_i = 0, \quad \sum_{j=1}^J b_j = 0, \quad \sum_{k=1}^K c_k = 0, \quad \sum_{i=1}^I ab_{ij} = \sum_{j=1}^J ab_{ij} = 0,$$

$$\sum_{j=1}^J bc_{jk} = \sum_{k=1}^K bc_{jk} = 0, \quad \sum_{i=1}^I ac_{ik} = \sum_{k=1}^K ac_{ik} = 0,$$

$$\sum_{i=1}^I abc_{ijk} = \sum_{j=1}^J abc_{ijk} = \sum_{k=1}^K abc_{ijk} = 0$$

$$\epsilon_{ijkl} \text{ NID } (0, \sigma^2).$$

In order to calculate the power, an analysis of variance table should be produced including the expected mean squares (EMS).

In deriving the non-centrality parameter, as in the two-way AOV, there are three situations with which we must be concerned.

Situation A:

In this situation we test for the main effects with no additional assumption about the interactions.

For factor A :

$$H_0: a_1 = a_2 = \dots = a_I$$

$$SS_A = nJK \sum_{i=1}^I (\bar{Y}_{i\dots} - \bar{Y}_{\dots})^2$$

$$\text{and } \sigma^2\delta^2 = nJK \sum_{i=1}^I a_i^2$$

where a_i is the true effect of the i th level of factor A.

To find min δ the two extreme a_i 's should differ by Δ and the remaining $(I-2)$ should be equal to the average of the two. So

$$|a_i| = \begin{cases} 1/2 \Delta & \text{for the two extremes} \\ 0 & \text{for the rest} \end{cases}$$

$$\sigma^2\delta^2 = nJK \left(\frac{\Delta^2}{4} + \frac{\Delta^2}{4} \right) ,$$

$$= nJK \frac{\Delta^2}{2}$$

$$\delta^2 = \frac{nJK\Delta^2}{2\sigma^2} ,$$

$$\delta = \sqrt{\frac{nJK\Delta^2}{2\sigma^2}},$$

Therefore,
$$\phi = \sqrt{\frac{nJK\Delta^2}{2\sigma^2(f_1 + 1)}}$$

For factor B substitute I for J and specify Δ , which is considered as a meaningful difference for the factor.

For factor C substitute I for K and put Δ associated with factor C. σ^2 will be estimated by the mean square in the denominator of the F-ratio for each factor.

In general, if we want to find the non-centrality parameter for any factor of a higher order layout, just multiply Δ^2 (the square of a difference that is meaningful to detect a significant difference in that factor) by the levels of the other factors and the number of observations per cell (n), and divide by $[2\sigma^2]$ and take the square root of that quantity.

For example, suppose we have an experiment with five factors. A contains I level, B contains J level, C contains K level, D contains L level, and E contains M level. To find the non-centrality parameter δ for A

$$\delta_A = \sqrt{\frac{JKLMn \Delta^2}{2\sigma^2}}$$

Therefore,
$$\phi = \sqrt{\frac{JKLMn \Delta^2}{2\sigma^2(f_1 + 1)}}$$

and the other factor would be the same except changing some number, i.e. for B substitute I for J etc., and σ^2 would be estimated by mean square

for the denominator in F-ratio and f_1 is the degrees of freedom for the numerator. The power may then be easily computed.

Situation B:

In this situation, we are testing for the main effects assuming no interaction. In this case, we include the interaction sum of squares with error. The procedure here would be the same as in situation A except that the degrees of freedom for the denominator will change, and for a higher-way layout in general it would be the same as in situation A.

Situation C:

In this situation we are testing for the interaction. For a three-way layout we have two kinds of interactions: interaction between any two factors and interaction between all the three factors.

Two-way interaction. We have three-two way interactions each of which has two factors present and the third one absent. To derive the AB interaction non-centrality parameter, we follow the same procedure as before.

$$SS_{AB} = Kn \sum_{i=1}^I \sum_{j=1}^J (\bar{Y}_{ij..} - \bar{Y}_{i...} - \bar{Y}_{.j..} + \bar{Y}_{....})^2$$

$$\sigma^2 \delta^2 = Kn \sum_{i=1}^I \sum_{j=1}^J (ab_{ij})^2$$

To find $\min \delta$ the two extremes (ab_{ij}) will differ by Δ and the remaining $(IJ-2)$ will be equal to the average of these two.

$$|ab_{ij}| = \begin{cases} 1/2 \Delta & \text{for the two extremes} \\ 0 & \text{for the others} \end{cases}$$

$$\sigma^2 \delta^2 = Kn \left(\frac{\Delta^2}{4} + \frac{\Delta^2}{4} \right) ,$$

$$= Kn \frac{\Delta^2}{2} ,$$

$$\delta^2 = \frac{Kn \Delta^2}{2\sigma^2} ,$$

$$\delta = \sqrt{\frac{Kn \Delta^2}{2\sigma^2}} ,$$

Therefore

$$\phi = \sqrt{\frac{Kn \Delta^2}{2\sigma^2 (f_1 + 1)}}$$

To find the non-centrality parameter for (AC) interaction, replace K by J and use the same formula above for (BC) interaction and replace K by I and use the same formula as above. Now we can calculate the power for any two-way interaction.

Three-way interaction: Also in three-way layout, there is one three-way interaction (ABC) deriving the non-centrality parameter δ for this interaction is as follows:

$$SS_{ABC} = n \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K (\bar{Y}_{ijk} - \bar{Y}_{ij..} - \bar{Y}_{i.k.} - \bar{Y}_{.jk.} + \bar{Y}_{i...} + \bar{Y}_{.j..} + \bar{Y}_{..k.} - \bar{Y}_{...})^2$$

and

$$\sigma^2 \delta^2 = n \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K (abc_{ijk})^2$$

To find the min δ , the two extremes (abc_{ijk}) will differ by Δ and the remaining $(IJK-2)$ equal to the average of these two. So

$$|abc_{ijk}| = \begin{cases} 1/2 \Delta & \text{for the two extremes} \\ 0 & \text{for the others} \end{cases}$$

$$\sigma^2\delta^2 = n\left(\frac{\Delta^2}{4} + \frac{\Delta^2}{4}\right),$$

$$= n\frac{\Delta^2}{2},$$

$$\delta^2 = \frac{n\Delta^2}{2\sigma^2},$$

$$\delta = \sqrt{\frac{n\Delta^2}{2\sigma^2}},$$

Therefore,
$$\phi = \sqrt{\frac{n\Delta^2}{2\sigma^2(f_1 + 1)}}$$

For higher-way layout interaction, if we have K factors, there will be $\binom{K}{2}$ two-way interactions, $\binom{K}{3}$ three-way interactions, and one K-way interaction.

To derive the non-centrality parameter for any of these interactions, just multiply Δ^2 by the number of observations per cell (n) and by the levels for the absent factor. In the previous example with five factors, A, B, C, D, and E.

The non-centrality parameter for ABC interaction would be:

$$\delta_{ABC} = \sqrt{\frac{LMn\Delta^2}{2\sigma^2}},$$

Therefore,
$$\phi = \sqrt{\frac{LMn\Delta^2}{2\sigma^2(f_1 + 1)}}$$

where Δ is the meaningful differences for detecting a significant difference. In testing for this interaction, σ^2 can be estimated by the mean square used in the denominator of F-ratio. The value f_1 is the degrees of freedom for one numerator. It is now easy to find the power for the main effects and the interactions in factorial experiment.

Latin Square

Latin square is an experimental design in which the levels of each of three factors is combined only once with each level of two other factors.

Latin square is K by K square with three factors which are usually designated row, column and treatment.

The model: $Y_{ijt} = \mu + R_i + C_j + T_t + \epsilon_{ijt}$

where $\left\{ \begin{array}{l} R_i \text{ is the effect of the } i\text{th row} \\ C_j \text{ is the effect of the } j\text{th column} \\ T_t \text{ is the effect of the } t\text{th treatment} \end{array} \right.$

and

$$\sum_{i=1}^K R_i = \sum_{j=1}^K C_j = \sum_{t=1}^K T_t = 0$$

$$\epsilon_{ijt} \text{ NID } (0, \sigma^2)$$

and also $(I=J=t=1, 2, \dots, K)$,

if we test the null hypothesis $T_t = 0$.

To find the non-centrality parameter:

$$SS_T = K \sum_{t=1}^K (\bar{Y}_{..t} - \bar{Y}_{...})^2$$

$$\sigma^2 \delta^2 = K \sum_{t=1}^K T_t^2$$

To find the min δ let the two extremes of T_t differ by Δ and the remaining $(K-2)$ equal to the average of these two as before. So,

$$|T_t| = \begin{cases} 1/2 \Delta & \text{for the two extremes} \\ 0 & \text{for the others} \end{cases}$$

$$\sigma^2\delta^2 = K\left(\frac{\Delta^2}{4} + \frac{\Delta^2}{4}\right),$$

$$= K\frac{\Delta^2}{2},$$

$$\delta^2 = \frac{K\Delta^2}{2\sigma^2},$$

$$\delta = \sqrt{\frac{K\Delta^2}{2\sigma^2}},$$

Therefore,
$$\phi = \sqrt{\frac{K\Delta^2}{2\sigma^2(f_1 + 1)}}$$

Since in latin square $I=J=t=K$ and $f_1 + 1 = K$, thus K in the numerator of ϕ will cancel with $(f_1 + 1)$ in the denominator, and

$$\phi = \sqrt{\frac{\Delta^2}{2}}$$

The non-centrality parameter for the rows and columns would be the same as the one for treatment which has been given above, then it is easy to find the power from either the tables or the charts.

Presentation of Post AOV Analysis

After designing the experiment and doing the appropriate analysis of variance for the hypotheses tested, the outcome will be one with these two results.

1. A significant difference exists among the treatments tested.
2. No significant difference exists. The first result can be interpreted by finding which is the most different treatment

using multiple comparison. In the second result, there are one of these decisions:

- a. There is truly no meaningful difference, or
- b. The experimental design did not offer sufficient precision to detect difference.

For the second case, investigation of which is the primary goal in this report, we have to use the non-centrality parameter and ϕ as derived in this chapter. If the experiment has high power, the decision should be there is truly no meaningful difference. If the power is low, it is indicated that the experimental design does not permit sufficient precision to detect the difference.

In this section some steps will be presented.

Step 1: Design the experiment and state the appropriate model for the experiment according to the best information available.

Step 2: State the hypotheses that you are interested in and the alternative hypotheses also.

Step 3: State the significance level α that you are going to use in your test.

Step 4: State the difference to be detected in order to consider it as a meaningful difference Δ .

Step 5: Do the analysis of variance for your experiment.

Step 6: Compute δ from the previous equations, since you know Δ , σ^2 , f_1 , number of observations per cell and the levels of each factor.

Step 7: Compute ϕ using the equation

$$\phi = \frac{\delta}{\sqrt{(f_1 + 1)}}$$

Step 8: Now since you know α , f_1 , f_2 , and ϕ , enter the charts or the tables in the appendices at the end of this report and find the power. If the power is high enough, stop and conclude that there is truly no meaningful difference otherwise continue these steps to find the appropriate sample size.

Step 9: Specify the power you need and enter the tables or the charts with f_1 from the AOV table and $f_2 = \infty$ and α , find ϕ value.

Step 10: Solve for n from the value of ϕ you found in Step (9) and round it to the next largest integer.

Step 11: Using the latest value of n compute the value of f_2 and ϕ .

Step 12: Return to the tables or the charts with the value of ϕ in Step 11, f_1 and the power specified in Step 9 and ϕ to find the value of f_2 .

Step 13: If the value of f_2 in Step 11 is less than in Step 12, increase n by one and go back to Step 11, otherwise this is the desired value n for specified power.

CHAPTER IV

ILLUSTRATIVE EXAMPLES ON THE EVALUATION OF
A NON-SIGNIFICANT AOV RESULTLayoutsOne-way layout

Example 1. An experimenter wishes to investigate three methods (fixed) of teaching. Five students are assigned randomly to each method. The improvement score is recorded below for each student.

TABLE 6.-- Teaching method

	I	II	III
	185	182	179
	175	183	181
	171	184	186
	165	191	188
	<u>182</u>	<u>162</u>	<u>158</u>
$Y_{i.}$	878	902	892
$\bar{Y}_{i.}$	175.6	180.4	178.4
	$Y_{..} = 2672$		

$$\text{Model: } Y_{ij} = \mu + M_i + \epsilon_{ij}$$

H_0 : No difference between the three methods.

$$M_1 = M_2 = M_3$$

TABLE 7.-- AOV table-a

Source	d.f.	MS	EMS	F-ratio
Methods	2	29.06665	$\sigma_e^2 + \frac{5}{2} \sum_{i=1}^I M_i^2$.266
Error	12	109.1333	σ_e^2	
Total corr.	14			

The conclusion in this case is fail to reject the hypothesis, i.e., there is no significant difference among the three methods. To investigate the reason for a non-significant result, we compute the power of the test using the meaningful difference Δ . Assume that $\Delta = 15$ has been determined by the experimenter. Then for this data

$$f_1 = 2, f_2 = 12, \sigma^2 = 109.1333$$

$$\delta = \sqrt{\frac{n\Delta^2}{2\sigma^2}} = \sqrt{\frac{5(15)^2}{2(109.133)}} = 2.2703$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{2.2703}{\sqrt{3}} = 1.31$$

Enter the charts with $f_1 = 2, f_2 = 12, \alpha = .05, \phi = 1.31$.

The estimated power is seen to be .40 with this design using the estimated $\hat{\sigma}^2 = 109.1333$. Thus, the probability of detecting the meaningful difference Δ is only .4, indicating that the experiment had little chance of detecting the difference even if it exists.

Since the power is very small (.4), he may decide to see how many observations one needs in order to conduct a more powerful experiment (say power .9).

Now enter the charts with $f_1 = 2$, $f_2 = \infty$, and power = .9. It is found that $\phi = 2.05$ for $\alpha = .05$. To find the value of n

$$\phi = \sqrt{\frac{n(15)^2}{2(109.1333)^3}} = 2.05,$$

$$\frac{n(15)^2}{6(109.1333)} = 4.2025,$$

Therefore $n = 12.23$ or $n = 13$.

Thus $f_2 = 3(13-1) = 36$

$$\phi = \sqrt{\frac{13(15)^2}{2(109.1333)^3}} = 2.11$$

Enter the charts with $f_1 = 2$ power = .9, $\phi = 2.11$ and $\alpha = .05$ and find f_2 . It will be found that

$$f_2 = 60$$

Since $36 < 60$, increase n by one

$$n = 13 + 1 = 14$$

Therefore, $f_2 = 3(14-1) = 39$, and

$$\phi = \sqrt{\frac{14(15)^2}{2(109.1333)^3}} = 2.2$$

Enter the charts with $f_1 = 2$, power = .9, $\phi = 2.2$, and $\alpha = .05$ to find f_2 . It will be found that

$$f_2 = 20, \text{ since } 39 > 20.$$

Stop and the sample size should be $n = 14$ in order to make the power .9.

Two-way layout with one observation per cell.

Example 2. (From Design of Experiments, V, L. Anderson and R. A. McLean). In an experiment on stress-rupture life of material used to make turbine blades in fan jet aircraft engines, two factors: blade temperatures, which might be encountered at take-off "of the aircraft", and alloys or materials that are of interest to the research engineer, are to be investigated. The data for this experiment are given below.

TABLE 8.-- Temperatures

Material	1	2	3	Total
a	185	182	182	549
b	175	183	184	542
c	171	184	189	544
d	165	191	189	545
Total	696	740	744	2180

Let us assume that for part of the experiment 12 treatment combinations (4 material x 3 temperature) are the only ones of interest and that these combinations can be run completely at random in the experiment. It can be analyzed as a fixed two-way AOV with only one observation per cell.

$$H_0: M_i = 0, T_j = 0 \quad H_1: M_i \neq 0, T_j \neq 0$$

The model: $Y_{ij} = \mu + M_i + T_j + \epsilon_{ij}$

TABLE 9.-- AOV table-b

Source	d.f.	Mean Square	Expected Mean Square	F-Ratio
Material (M_i)	3	2.89	$\sigma_e^2 + \frac{3}{3} \sum_{i=1}^I M_i^2$	< 1
Temperature (T_j)	2	177.34	$\sigma_e^2 + \frac{4}{2} \sum_{j=1}^J T_j^2$	3.7
Error	6	48.56	σ_e^2	
Total Corrected	11			

$$F(3, 6, .05) = 4.76 \text{ and } F(2, 6, .05) = 5.14$$

The conclusion is that no effects are significant at the .05 level. To investigate the reason for a non-significant result, we compute the power of the test for each factor using the meaningful difference Δ .

Assume that values $\Delta = 25$ for temperatures and $\Delta = 20$ for material have been determined by the experimenter.

Then for temperatures:

$$f_1 = 2, f_2 = 6, \sigma^2 = 48.56 \text{ using } \alpha = .05$$

$$\delta = \sqrt{\frac{I\Delta^2}{2\sigma^2}} = \sqrt{\frac{4(25)^2}{2(48.56)}} = 5.074$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{5.074}{\sqrt{3}} = 2.93$$

Enter the charts with $f_1 = 2$, $f_2 = 6$, $\alpha = .05$, and $\phi = 2.93$.

The estimated power is seen to be .94 for temperature with the design using the estimated $\sigma^2 = 48.56$. Thus the probability of detecting the meaningful difference Δ among the temperatures is .94. Thus indicating that we have a good chance of detecting the difference if it exists, but it seems that there is no meaningful difference among the temperature levels.

For the material:

$$f_1 = 3, f_2 = 6, \sigma^2 = 48.56, \text{ using } \alpha = .05 \text{ and } \Delta = 20.$$

$$\delta = \sqrt{\frac{J\Delta^2}{2\sigma^2}} = \sqrt{\frac{3(20)^2}{2(48.56)}} = 3.5151$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{3.5151}{\sqrt{4}} = 1.7575$$

Now enter the charts with $f_1 = 3$, $f_2 = 6$, $\alpha = .05$, and $\phi = 1.7575$.

The estimated power is seen to be .55 for material with this design, using the estimated $\sigma^2 = 48.56$. Thus, the probability of detecting a meaningful difference as defined by Δ is only .55, indicating that the experiment might not detect the difference even if it exists.

Since the power is small, the experimenter might decide to see the number of observations one needs in order to conduct a more powerful experiment with power equal to say .95.

Now enter the charts with $f_1 = 3$, $f_2 = \infty$, and power = .95, ϕ would be 2.07 for $\alpha = .05$. To find the value of n

$$\phi = \sqrt{\frac{Jn(\Delta)^2}{2\sigma^4}} = 2.07$$

$$\sqrt{\frac{3n(20)^2}{2(48.56)4}} = 2.07$$

$$\frac{1200n}{8(48.56)} = 4.2849$$

$$n = 1.387 \text{ or } n = 2$$

$$f_2 = (3)(4)(2 - 1) = 12$$

$$\phi = \sqrt{\frac{3(2)(20)^2}{2(48.56)4}} = 2.49$$

Enter the chart with $f_1 = 3$, power = .95, and $\phi = 2.49$, and $\alpha = .05$, and find f_2

$$f_2 = 11$$

Since $12 > 11$, we stop and the sample size should be $n = 2$ to gain .95 power.

Two-way layout with n observations

Example 3. An experiment was conducted to see the effect of price and advertising on television sales. Two levels of prices were used. Sales were examined with and without advertising. Four stores were chosen at random for each combination. The data represent the total units sold in these four stores. The pooled variance was 100.

	Price		Total
	\$60	\$65	
Advertising	110	92	202
No Advertising	170	104	274
Total	280	196	476

$$\text{Model: } Y_{ijk} = \mu + a_i + b_j + ab_{ij} + \epsilon_{ijk}$$

$$H_0: a_i = 0 \quad H_1: a_i \neq 0$$

$$b_j = 0 \quad b_j \neq 0$$

$$ab_{ij} = 0 \quad ab_{ij} \neq 0$$

TABLE 10.-- AOV table-c

Source	d.f.	M.S.	f	EMS
Total	15	140.6		
A	1	441	4.41	$\sigma_e^2 + 8\sum a_i^2$
B	1	324	3.24	$\sigma_e^2 + 8\sum b_j^2$
A x B	1	144	1.44	$\sigma_e^2 + 4\sum ab_{ij}^2$
Error	12	100		σ_e^2

The conclusion is that no effects are significant at the .05 level. To investigate the reason for a non-significant results we compute the power for the main effects and the interaction using meaningful difference Δ .

The meaningful difference for factor A: Since factor A is two different prices, Δ should be the difference that will give at least the same profit before reducing the price in order to be a meaningful difference. Assume that the cost is \$55; therefore, the profit is \$10 per unit before reducing the prices and \$5 per unit after reducing the price.

The profit before reducing the price = $196 \times 10 = 1960$. To find the total units that will give the same profit after reducing the

price: $196(10) = 5X$. $X = \frac{1960}{5} = 392$

therefore, $\Delta = \frac{392}{8} - \frac{196}{8} = 24.5$

From that

$$\delta = \sqrt{\frac{nJ\Delta^2}{2\sigma^2}},$$

$$= \sqrt{\frac{4(24.5)^2}{2(100)}} = 3.46$$

Thus $\phi = \frac{3.46}{2} = 2.45$

Enter the charts with $f_1 = 1$, $f_2 = 12$, $\alpha = .05$ and $\phi = 2.45$.

The probability is seen to be .89 for price effect using $\hat{\sigma}^2 = 100$. Thus, the probability of detecting the meaningful difference Δ is .89, indicating that the experiment has a good chance of detecting the difference if it exists. But since a non-significant AOV resulted, we can conclude that there is probably no meaningful difference existing among the price levels.

For factor B it can be done as in factor A considering the fact that there is an additional cost which is the advertising cost.

The interaction: When reducing the price and advertising the demand should be higher, therefore, the meaningful difference Δ for the interaction should be that which at least gives the same profit as before. Assume the cost before advertising is \$55 and the cost of advertising is \$1 per unit, therefore, there is a \$10 profit before advertising and reducing the price, but after reducing the price and advertising it is \$4 per unit. To gain at least the same profit

$$92(10) = 4X$$

$$X = \frac{920}{4} = 230, \text{ therefore}$$

$$\Delta = \frac{230}{4} - \frac{92}{4} = 34.5$$

From that

$$\delta = \sqrt{\frac{n\Delta^2}{2\sigma^2}},$$

$$= \sqrt{\frac{4(34.5)^2}{2(100)}} = 4.88$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{4.88}{\sqrt{2}} = 3.45$$

Enter the charts with $f_1 = 1$, $f_2 = 12$, $\alpha = .05$ and $\phi = 3.45$. The power is seen to be 100% for the interaction effect using $\hat{\sigma}^2 = 100$, indicating that we have a good chance of detecting the meaningful difference if it exists, but probably it does not exist.

Example 4. In an experiment there is two factors A with 4 level and B with 5 level, each cell has five observations replaced by the sum within each cell. Assume that mean square error = 4.6

		B					Total	Mean
		1	2	3	4	5		
A	1	21.0	21.5	14.4	19.0	17.1	93	3.72
	2	15.2	24.3	23.2	21.9	16.2	100.8	4.032
	3	16.1	30.8	26.5	26.1	20.2	119.7	4.788
	4	14.2	21.1	28.1	16.7	19.8	99.9	3.996
Total		66.5	97.7	92.2	83.7	73.3	413.4	
Mean		3.325	4.885	4.61	4.185	3.665		4.134

$$\text{Model: } Y_{ijk} = \mu + a_i + b_j + ab_{ij} + \epsilon_{ijk}$$

$$\begin{array}{ll}
 H_0: a_i = 0 & H_1: a_i \neq 0 \\
 b_j = 0 & b_j \neq 0 \\
 ab_{ij} = 0 & ab_{ij} \neq 0
 \end{array}$$

TABLE 11.-- AOV table-d

Source	d.f.	MS	F	EMS
Total	99			
A	3	5.238	1.14	$\sigma_e^2 + \frac{25}{3} \Sigma a_i^2$
B	4	8.3381	1.81	$\sigma_e^2 + 5 \Sigma b_j^2$
A x B	12	3.0378	<1	$\sigma_e^2 + \frac{5}{12} \Sigma \Sigma ab_{ij}^2$
Error	80	4.6		σ_e^2

The procedure has failed to reject the above three hypotheses, i.e. there are no significant differences existing among the main effects or the interaction. To investigate the reason for a non-significant result, we compute the power of the test for the main effects and the interaction using the meaningful difference Δ .

Main effects: Assume that Δ for A is 2, Δ for B is 3 has been determined by the experimenter. Then for A

$$\delta = \sqrt{\frac{nJ\Delta^2}{2\sigma^2}} = \sqrt{\frac{(5)(5)(2)^2}{2(4.6)}} = 3.297$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{3.297}{\sqrt{4}} = \frac{3.247}{2} = 1.65$$

Now enter the charts with $f_1 = 3$, $f_2 = 80$, $\alpha = .05$, and $\phi = 1.65$.

The estimated power is seen to be .82 for factor A with this design using the estimated $\sigma^2 = 4.6$. Thus the probability of detecting the meaningful difference Δ is .82, indicating that the experiment has a good chance of detecting the difference if it exists. But it might not exist, so we can conclude in this case that there is probably no meaningful difference existing among the level of factor A.

For factor B:

$$\delta = \sqrt{\frac{nI\Delta^2}{2\sigma^2}} = \sqrt{\frac{(5)(4)(3)^2}{2(4.6)}} = 4.4233$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{4.4233}{\sqrt{4 + 1}} = 1.98$$

Enter the charts with $f_1 = 4$, $f_2 = 80$, $\alpha = .05$, and $\phi = 1.98$. The power is seen to be .96 for factor B with this design using $\hat{\sigma}^2 = 4.6$.

Thus, the probability of detecting the meaningful difference Δ is .96, indicating that the experiment has a very good chance of detecting the difference if it exists. But since a non-significant AOV resulted, we can conclude that there is probably no meaningful difference existing among the level of factor B.

Interaction: Assume that Δ for the interaction is 5.5.

$$\delta = \sqrt{\frac{n\Delta^2}{2\sigma^2}} = \sqrt{\frac{5(5.5)^2}{2(4.6)}} = 4.055$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{4.055}{\sqrt{13}} = 1.125$$

Enter Tiku's tables with $f_1 = 12$, $\phi = 1.125$, $f_2 = 80$ and $\alpha = .05$ will find that $\beta = .30$.

$$\text{Power} = 1 - \beta = 1 - .30 = .70$$

The estimated power is .70 for the interaction with this design using the estimated $\sigma^2 = 4.6$. Thus, the probability of detecting the meaningful difference Δ is .70 indicating that the experiment has a good chance of detecting the difference if it exists. Because this probability is high, we can conclude that there is no meaningful difference among the levels of the main effects and the interaction.

Three-way layout.

Example 5. In an experiment of the effect of three kinds of fertilizer and two soils and three types of irrigation, the data recorded below is the yield in tons per acre. The experimenter is interested in only these levels of each factor (fixed).

TABLE 12.-- Yield in tons per acre

		Soil 1			Soil 2			Total
		<u>Irrigation</u>			<u>Irrigation</u>			
		1	2	3	1	2	3	
Fertilizer	1	28	32	40	38	30	25	296
		16	15	25	20	15	12	
	2	15	16	28	40	20	47	343
		27	30	35	23	30	32	
	3	15	32	29	38	15	27	308
		31	18	40	21	29	13	
Total		132	143	197	180	139	156	947

$$\text{The model: } y_{ijkl} = \mu + F_i + S_j + I_k + FS_{ij} + FI_{ik} + SI_{jk} + FSI_{ijk} + \epsilon_{ijkl}$$

where $\left\{ \begin{array}{l} i = 1, 2, 3 \\ j = 1, 2 \\ k = 1, 2, 3 \\ l = 1, 2 \end{array} \right.$

$$H_0: F_1 = F_2 = F_3 = 0$$

$$S_1 = S_2 = 0$$

$$I_1 = I_2 = I_3 = 0$$

$$FS_{ij} = 0$$

$$FI_{jk} = 0$$

$$FI_{ik} = 0$$

$$SI_{jk} = 0$$

$$FSI_{ijk} = 0$$

Using STATPAC/LIBRARYA program, the analysis of variance table is listed on the following page.

In this problem none of the hypothesis associated with the above effects are significant. To find out what is the reason for non-significance of our results, we compute the power of the test using the meaningful difference Δ for each of the main effects and the interactions.

Main effects.

Assume that the meaningful differences were determined by the experimenter for the main effects as

$$\Delta \text{ for fertilizer} = 8$$

$$\Delta \text{ for soils} = 6$$

$$\Delta \text{ for irrigation} = 10$$

TABLE 13.-- AOV table-e

Source	d.f.	MS	EMS	F-Ratio
Fertilizer (F)	2	49.695	$\sigma_e^2 + 6 \sum_{i=1}^I F_i^2$.49
Soil (S)	1	.25	$\sigma_e^2 + 18 \sum_{j=1}^J S_j^2$.0025
Irrigation (I)	2	105.86	$\sigma_e^2 + 6 \sum_{k=1}^K I_k^2$	1.05
F x S	2	100.75	$\sigma_e^2 + 3 \sum_{i=1}^I \sum_{j=1}^J FS_{ij}^2$.998
F x I	4	33.07	$\sigma_e^2 + \sum_{i=1}^I \sum_{k=1}^K FI_{ik}^2$.328
S x I	2	166.584	$\sigma_e^2 + 3 \sum_{j=1}^J \sum_{k=1}^K SI_{jk}^2$	1.651
F x S x I	4	37.71	$\sigma_e^2 + \frac{2}{4} \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K FSI_{ijk}^2$.374
Error	18	100.9167	σ_e^2	
Total	35	84.1611		

Fertilizer.

$f_1 = 2, f_2 = 18, \hat{\sigma}^2 = 100.9167$ using $\alpha = 0.5$

$$\delta = \sqrt{\frac{JKn(\Delta)^2}{2\sigma^2}} = \sqrt{\frac{(2)(3)(2)(8)^2}{(2)100.9167}} = 1.951$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{1.951}{\sqrt{3}} = 1.13$$

Enter the chart with $f_1 = 2$, $f_2 = 18$, $\phi = 1.13$, and $\alpha = .05$.

The estimated power is .35 for fertilizer with this design, using the estimate $\hat{\sigma}^2 = 100.916$. Therefore the probability of detecting the meaningful difference Δ is only .35, indicating that the experiment had little chance of detecting the difference even if it exists.

Because the power looks very small, the experimenter might be interested to see how many observations one needs in order to conduct a more powerful experiment (say power .85). To find out about this, go back to the chart with $f_1 = 2$, $f_2 = \infty$, power = .85 and $\alpha = .05$, you will find that $\phi = 1.9$.

To find n

$$\phi = \sqrt{\frac{JKn\Delta^2}{2\sigma^2}} = 1.9$$

$$\frac{n(2)(3)(8)^2}{(2)(100.9167)(3)} = 3.61$$

$$\frac{(8)^2 n}{100.9167} = 3.61$$

$$64n = 364.31$$

$$n = \frac{364.31}{64} = 5.69 \rightarrow 6$$

$$f_2 = (3)(2)(3)(6-1) = 90$$

$$\phi = \sqrt{\frac{(2)(3)(6)(8)^2}{(2)(100.9167)(3)}} = 1.951$$

Now enter the chart with $f_1 = 2$, power = .85, $\phi = 1.951$ and $\alpha = .05$ to find f_2 from the charts. You will find that $f_2 = 60$. Since $90 > 60$,

stop and the sample size should be $n = 6$ to gain .85 power for the fertilizer factor.

Soil.

$$f_1 = 1, f_2 = 18, \hat{\sigma}^2 = 100.9167 \text{ using } \alpha = .05$$

$$\delta = \sqrt{\frac{IKn(\Delta)^2}{2\sigma^2}} = \sqrt{\frac{(3)(3)(6)^2}{2(100.9167)}} = 1.61$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{1.61}{\sqrt{2}} = 1.14$$

Enter the charts with $f_1 = 1$, $f_2 = 18$, $\phi = 1.14$ and $\alpha = .05$. The power estimated is .35 for soil with this design, using the estimated $\sigma^2 = 100.9167$. Therefore the probability of detecting the meaningful difference Δ is only .35, indicating that the experiment had little chance of detecting the difference even if there is any.

Since the power is very low, the experimenter might be interested to see how large the sample should have been in order to conduct a more powerful experiment (say power .9). To find out about this, enter the chart with $f_1 = 1$, $f_2 = \infty$, power = .9 and $\alpha = .05$. You will find that $\phi = 2.3$.

To calculate n

$$\phi = \sqrt{\frac{IKn\Delta^2}{2\sigma^2(f_1 + 1)}} = 2.3$$

$$\frac{n(3)(3)(6)^2}{(2)(100.9167)(2)} = 5.29$$

$$\frac{324n}{403.6668} = 5.29$$

Therefore, $n = 6.59$ or $n = 7$

$$f_2 = (3)(2)(3)(7-1) = 108$$

$$\phi = \sqrt{\frac{(3)(3)(7)(6)^2}{(2)(100.9167)(2)}} = 2.37$$

Enter the charts with $f_1 = 1$, power = .9, $\phi = 2.37$ and $\alpha = .05$ to find f_2 .

$$f_2 = 30$$

Since $108 > 30$, stop and the appropriate sample size to get .9 power is $n = 7$.

Irrigation.

$$f_1 = 2, f_2 = 18, \hat{\sigma}^2 = 100.9167 \text{ and } \alpha = .05$$

$$\delta = \sqrt{\frac{I J n (\Delta)^2}{2\sigma^2}} = \frac{(3)(2)(2)(10)^2}{(2)100.9167} = 2.44$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{2.44}{\sqrt{3}} = 1.41$$

Enter the charts with $f_1 = 2$, $f_2 = 18$, $\phi = 1.41$ and $\alpha = .05$. We can see that the estimated power is .53 for the irrigation with this design using the estimated $\sigma^2 = 100.9167$. Therefore the probability of detecting the meaningful difference Δ in irrigation is only .53, implies that the experiment had little chance of detecting the difference even if it exists.

Since the power is low, the experimenter might be interested to see the sample size that provides good power (say .80), to find out enter the charts with $f_1 = 2$, $f_2 = \infty$, power = .80 and $\alpha = .05$. You will find that $\phi = 1.8$.

To find n

$$\phi = \sqrt{\frac{I J n \Delta^2}{2\sigma^2(f_1 + 1)}} = 1.8$$

$$\frac{n(3)(2)(10)^2}{2(100.9167)(3)} = 3.24$$

$$\frac{100n}{100.9167} = 3.24$$

$$n = 3.27 \rightarrow 4$$

$$f_2 = (3)(2)(3)(4-1) = 54$$

$$\phi = \sqrt{\frac{(3)(2)(4)(10)^2}{(2)(100.9167)(3)}} = 1.991$$

Now enter the charts with $f_1 = 2$, power = .8, $\phi = 1.991$ and $\alpha = .05$, to find f_2 from the charts you will find that $f_2 = 20$. Since $54 > 20$, stop and the sample size would be $n = 4$ in order to gain 0.8 power.

Two-way interactions.

In this problem we have three two-way interactions. The power for one of these interactions will be calculated as an example.

Fertilizer x irrigation. Assume that the meaningful difference for fertilizer by irrigation $\Delta = 20$. Then we have

$$f_1 = 4, f_2 = 18, \hat{\sigma}^2 = 100.9167 \text{ and } \alpha = .05.$$

$$\delta = \sqrt{\frac{J n (\Delta^2)}{2\sigma^2}} = \sqrt{\frac{(2)(2)(20)^2}{2(100.9167)}} = 2.82$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{2.82}{\sqrt{5}} = 1.26$$

Enter the chart with $f_1 = 4$, and $f_2 = 18$, $\phi = 1.26$ and $\alpha = .05$.

The estimated power is .52 for Fertilizer x Irrigation with this design using the estimated $\sigma^2 = 100.9167$. Therefore, the probability of detecting the meaningful difference in Fertilizer x Irrigation interaction is only .52, indicating that the experiment had little chance of detecting the meaningful difference even if it exists.

If the experimenter is interested in seeing the number of observations needed to perform more powerful experiment (say .9) enter the charts with $f_1 = 4$, $f_2 = \infty$, power = .9 and $\alpha = .05$, you will find that $\phi = 1.75$. To calculate n

$$\phi = \sqrt{\frac{nJ\Delta^2}{2\sigma^2(f_1 + 1)}} = 1.75$$

$$\sqrt{\frac{n(2)(20)^2}{2(100.9167)(5)}} = 1.75$$

$$\frac{800n}{1009.167} = 3.0625$$

$$n = 3.86 \rightarrow 4$$

$$f_2 = (3)(2)(3)(4-1) = 54$$

$$\phi = \sqrt{\frac{(4)(2)(20)^2}{2(100.9167)5}} = 1.781$$

Now enter the charts with $f_1 = 4$, power = .9, $\phi = 1.781$ and $\alpha = .05$, to find f_2 from the charts

$$f_2 = 60$$

Since f_2 found from the chart $>$ f_2 calculated increase n by one.

Therefore, $n = 4 + 1 = 5$

$$f_2 = (3)(2)(3)(5-1) = 72$$

$$\phi = \sqrt{\frac{(5)(2)(20)^2}{2(100.9167)(5)}} = 1.99$$

Now enter the table with $f_1 = 4$, power = .9, $\phi = 1.99$ and $\alpha = .05$. It will be found that $f_2 = 18$. Since $18 < 72$, stop and the appropriate sample size would be $n = 5$ in order to gain .9 power for Fertilizer x Irrigation in this experiment.

Three-way interaction.

In three-way interaction, Fertilizer x Soil x Irrigation, assume that the experimenter considered any difference more than or equal to 40 is a meaningful difference. Therefore

$$\Delta = 40$$

Now we have $f_1 = 4$, $f_2 = 18$, $\hat{\sigma}^2 = 100.916$ and $\alpha = .05$.

$$\delta = \sqrt{\frac{n\Delta^2}{2\sigma^2}} = \sqrt{\frac{(2)(40)^2}{2(100.9167)}} = 3.982$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{3.982}{\sqrt{5}} = 1.781$$

The power is .82 for Fertilizer x Soil x Irrigation with this design using the estimated $\sigma^2 = 100.9167$.

Thus the probability of detecting the meaningful difference for Fertilizer x Soil x Irrigation Δ is .82, indicating that we have a good chance of detecting the meaningful difference Δ if it exists. As before we conclude no meaningful difference exists since the probability of detecting that difference is estimated to be high (.82) if it exists for the given design.

Latin Square

Example 6. In a latin square design experiment on explosive switches, there were five (fixed) levels treatments, which is five levels of packing pressure (A = 10,000, B = 15,000, C = 20,000, D = 25,000, E = 30,000 PSI), five fixed machines (rows), and five fixed men (columns). The data below represent the firing time.

TABLE 14.-- Firing time

Machine	1	2	3	4	5	Total
I	25 (C)	17 (B)	18 (A)	22 (D)	32 (E)	114
II	12 (B)	26 (D)	25 (E)	12 (C)	20 (A)	95
III	19 (A)	33 (E)	16 (C)	24 (B)	25 (D)	117
IV	27 (D)	25 (A)	27 (B)	21 (E)	19 (C)	119
V	21 (E)	22 (C)	21 (D)	17 (A)	23 (B)	104
Total	104	123	107	96	119	549

$$\text{Model: } Y_{ijt} = \mu = R_i + C_j + T_t + \epsilon_{ijt}$$

$$H_0: R_i = 0$$

$$C_j = 0$$

$$T_t = 0$$

TABLE 15.-- AOV table-f

Source	d.f.	Sum of Square	Mean Square	EMS	F-Ratio
Row (men)	4	81.36	20.34	$\sigma_e^2 + \frac{K}{k-1} \sum_{i=1}^K R_i^2$.91
Column (machine)	4	98.16	24.54	$\sigma_e^2 + \frac{K}{k-1} \sum_{j=1}^K C_j^2$	1.1
Treatment	4	206.16	51.54	$\sigma_e^2 + \frac{K}{k-1} \sum_{t=1}^K T_t^2$	2.3
Error	12	269.28	22.44	σ_e^2	
Total	24	654.96	27.29		

The conclusion in this analysis is that none of these effects (row, columns, and treatment) are significantly different at .05 significance level. To investigate the reason for a non-significant result, we compute the power of the test for each factor using the meaningful difference Δ for each factor.

Assume the meaningful difference was determined by the experimenter as follows:

$$\Delta \text{ for row} = 10, \Delta \text{ for column} = 12, \Delta \text{ for treatment} = 15.$$

Row.

$$f_1 = 4, f_2 = 12, \hat{\sigma}^2 = 22.44, \Delta = 10$$

$$\text{Therefore, } \delta = \sqrt{\frac{K\Delta^2}{2\sigma^2}} = \sqrt{\frac{5(10)^2}{2(22.44)}} = 3.34$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{3.34}{\sqrt{5}} = 1.49$$

Enter the charts with $f_1 = 4$, $f_2 = 12$, $\phi = 1.49$, and $\alpha = .05$.

The estimated power is 0.60 for the row with this design using the estimated $\sigma^2 = 22.44$, indicating that the experiment had little chance of detecting the difference even if it exists. The experimenter might be interested to find the sample size that will provide a good power for his experiment (say .9). Now enter the charts with $f_1 = 4$, $f_2 = \infty$, power = .9 and $\alpha = .05$. It is found that $\phi = 1.75$, to find the value of n

$$\phi = \sqrt{\frac{nK\Delta^2}{2\sigma^2(f_1 + 1)}} = \sqrt{\frac{n5(10)^2}{2(22.44)5}} = 1.75$$

$$\frac{500n}{224.4} = 3.0625$$

Therefore, $n = 1.37$ or $n = 2$

$$f_2 = 25$$

$$\phi = \sqrt{\frac{2(5)(10)^2}{2(22.44)5}} = 2.11$$

Now enter the charts with $f_1 = 4$, power = .9, $\phi = 2.11$ and $\alpha = .05$.

It is found that $f_2 = 14$. Since $25 > 14$, stop and the appropriate sample size is $n = 2$.

Columns.

$$f_1 = 4, f_2 = 12, \hat{\sigma}^2 = 22.44 \text{ and } \Delta = 12.$$

$$\delta = \sqrt{\frac{K\Delta^2}{2\sigma^2}} = \sqrt{\frac{5(12)^2}{2(22.44)}} = 4.00$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{4.00}{\sqrt{5}} = 1.79$$

Now enter the charts with $f_1 = 4$, $f_2 = 12$, $\phi = 1.79$ and $\alpha = .05$.

The estimated value of the estimated power is seen to be .76 for the columns using the estimated $\sigma^2 = 22.44$. Thus the probability of detecting the meaningful difference in columns Δ is .76, indicating that we have a good chance of detecting this difference if it exists but it might not exist.

Treatment.

$$f_1 = 4, f_2 = 12, \hat{\sigma}^2 = 22.44 \text{ and } \Delta = 15$$

Therefore

$$\delta = \sqrt{\frac{K\Delta^2}{2\sigma^2}} = \sqrt{\frac{5(15)^2}{2(22.44)}} = 5.01$$

$$\phi = \frac{\delta}{\sqrt{f_1 + 1}} = \frac{5.01}{\sqrt{5}} = 2.24$$

Now enter the charts with $f_1 = 4$, $f_2 = 12$, $\phi = 2.24$ and $\alpha = .05$. The estimated value of the power is seen to be .93 for the treatment using the estimated $\sigma^2 = 22.44$.

Thus, the probability of detecting the meaningful difference among the treatment levels (Δ) is .93, indicating that we have a very good chance of detecting the meaningful difference if it exists, but it might not exist. So we can conclude that there are probably no meaningful differences existing among the treatment levels.

All the power calculated in these examples are the estimated powers because the variance used here is the sample variance.

CHAPTER V

SUMMARY AND CONCLUSIONS

The result of the analysis of variance for an experiment is one of these two outcomes:

1. A significant difference exists among the treatments.
2. No significant difference exists among the treatments.

Interpretation of the second outcome is the major emphasis of this report. Most of the experimental design and analysis of variance books devote little or no attention to the interpretation of a non-significant result. If they do, no quantitative evaluation is suggested. Since a non-significant result may be due either to poor design or lack of true difference, it seems that any information which can be obtained to help explain the result should be useful.

Interpretation of the non-significant result has been developed here through the power calculation. This calculation is based upon the difference between treatment effects which is meaningful as determined by the investigator. The power is an estimated power because the variance σ^2 is estimated by the sample variance.

If the power is reasonably high, we can say that there is truly no significant difference among the treatments, otherwise the experimental design does not offer sufficient precision to detect the difference. This might be because we did not have enough observations in

our experiment, in order to find the difference or else additional sources of variation should be identified.

If it is determined that the design had low power, an increased sample size may be all that is needed to improve the design. The procedure for estimating sample size from prior data is reviewed.

A P P E N D I C E S

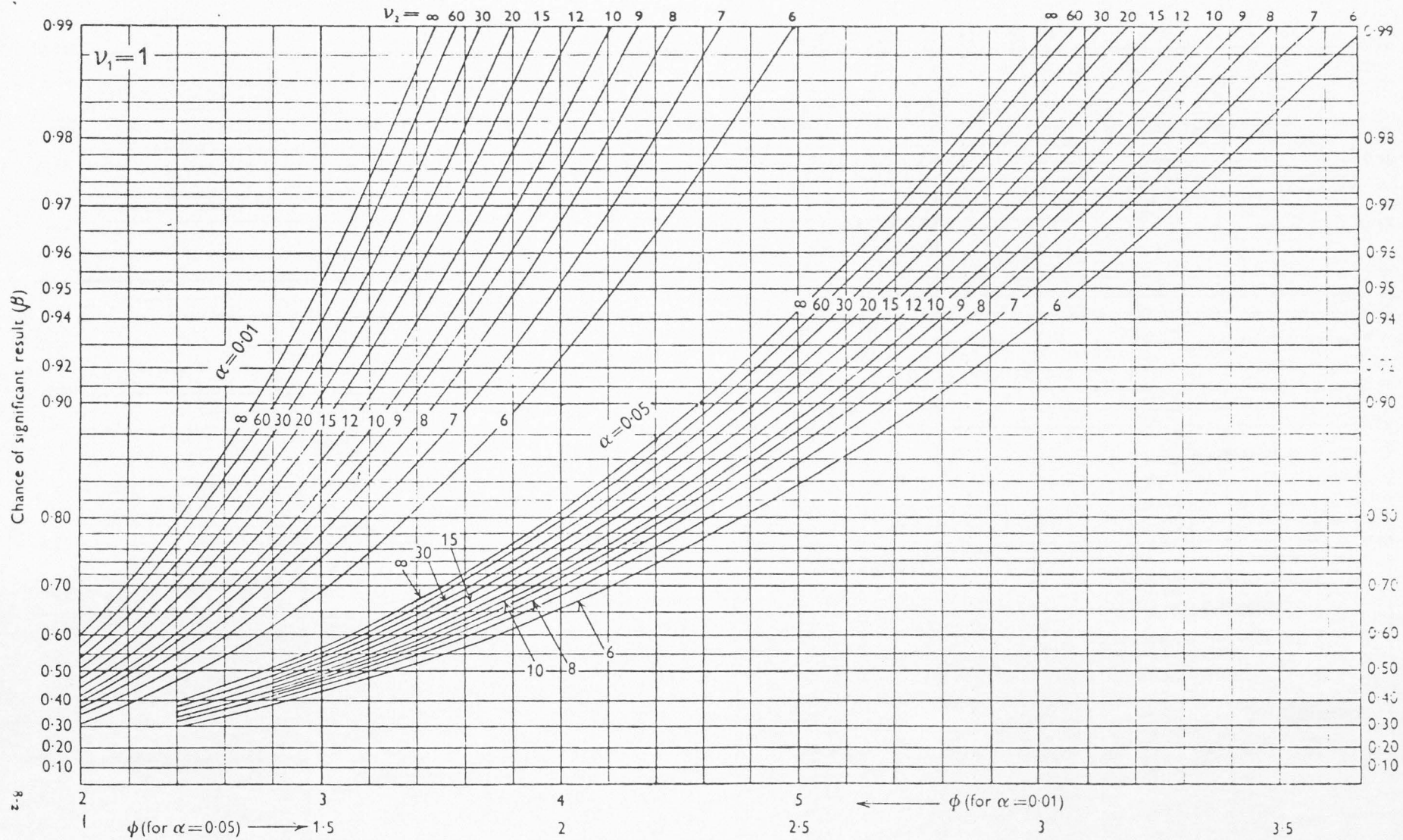
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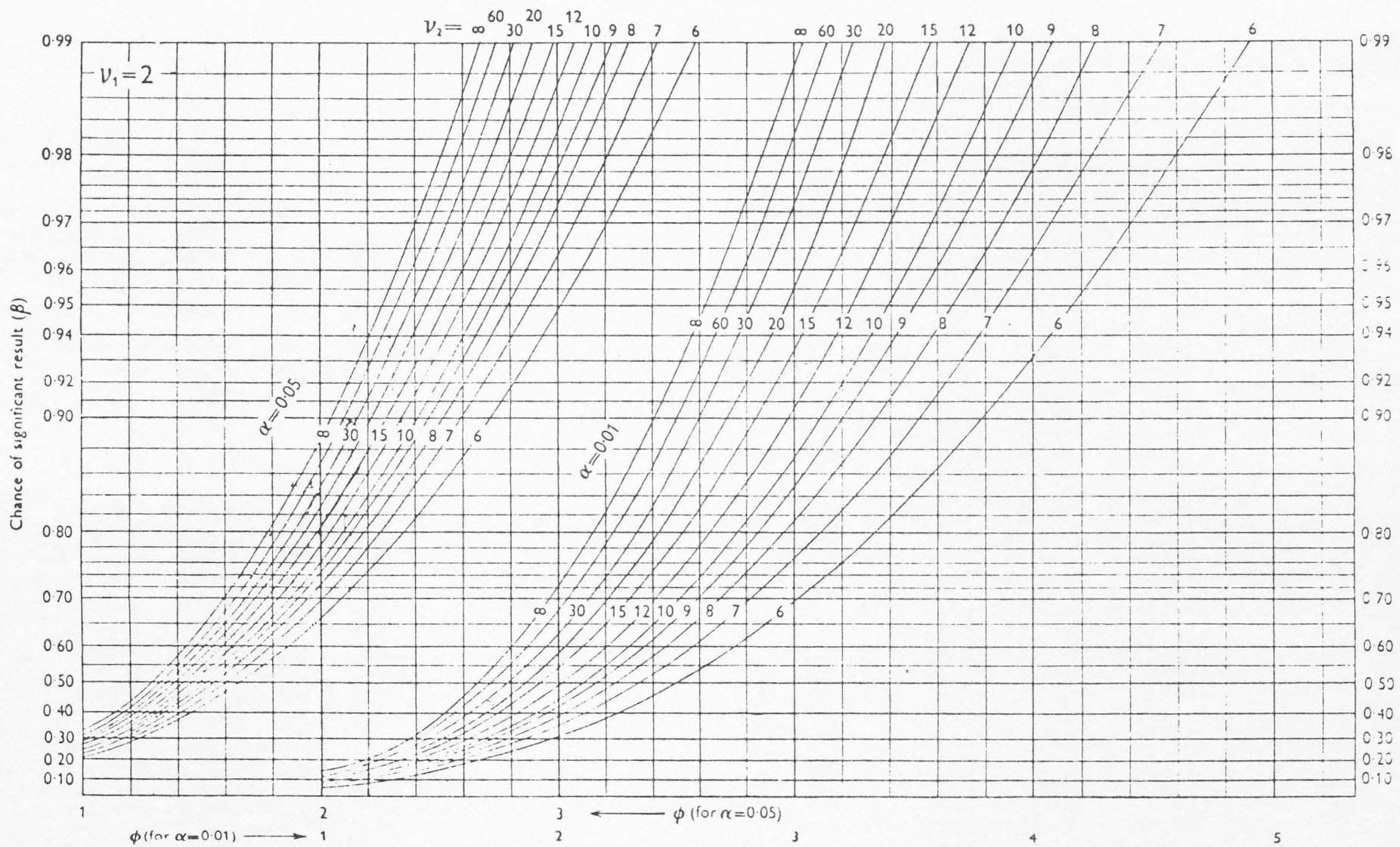
Charts to Find the Power

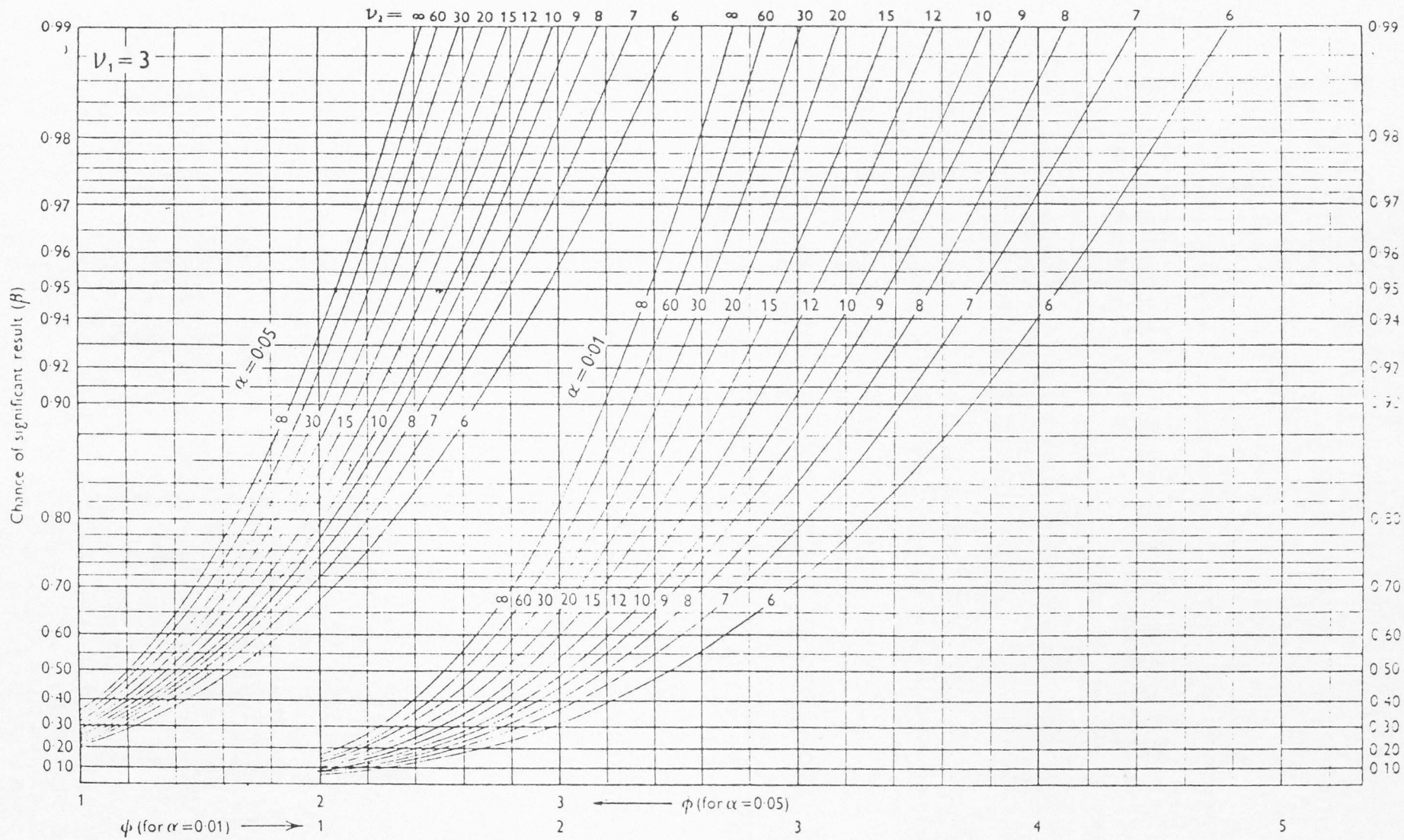
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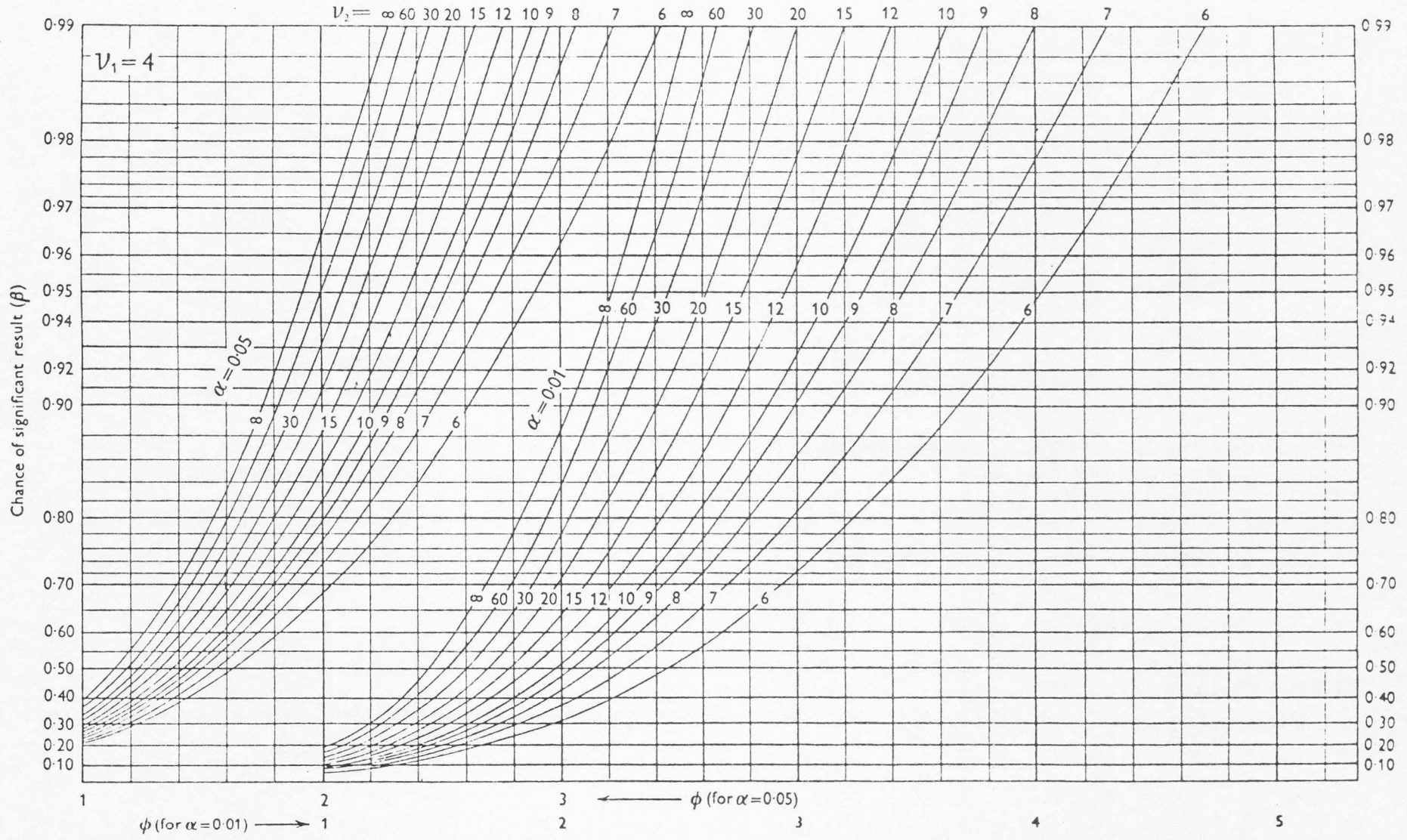
E. S. Pearson and H. O. Hartley

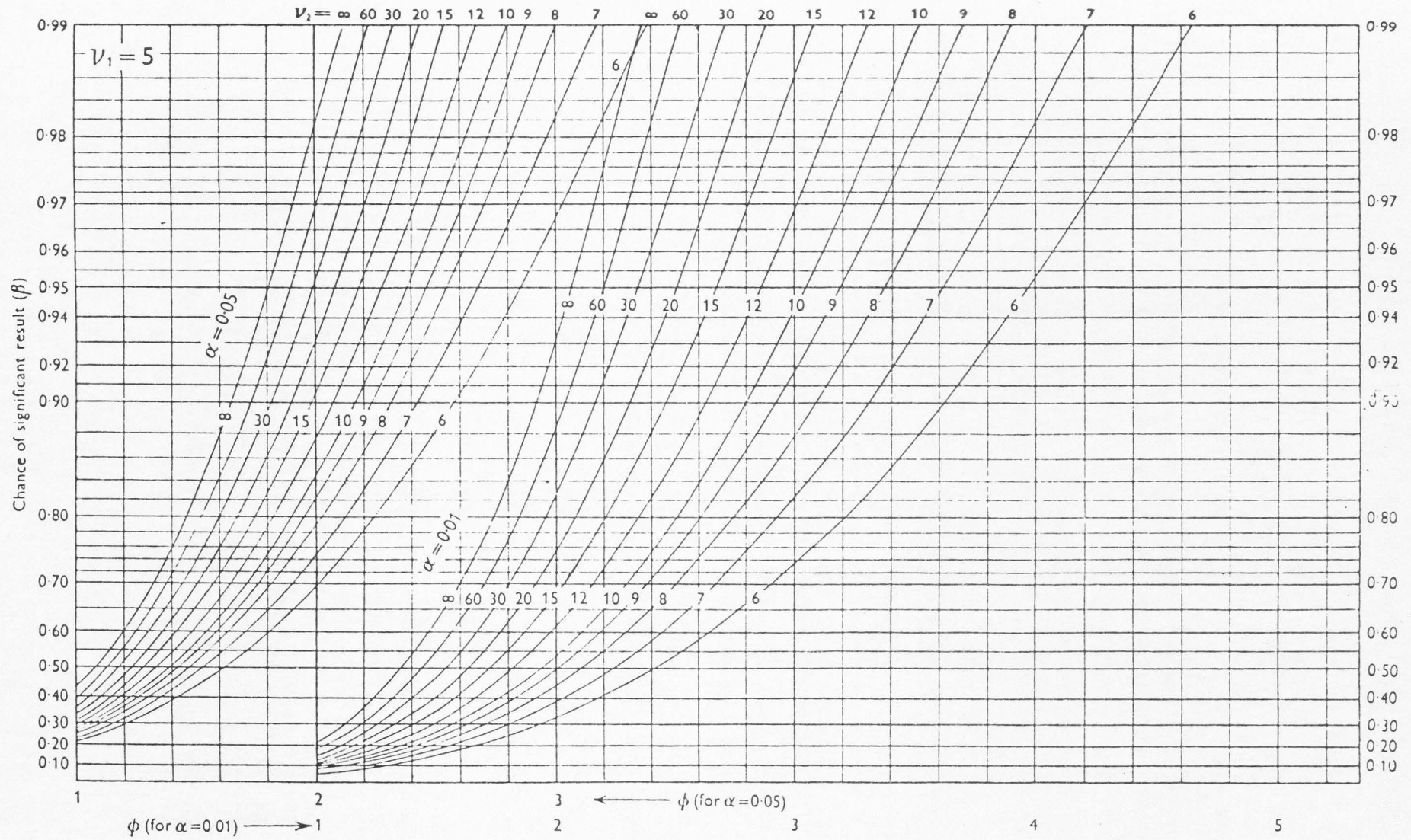
Biometrika, 38, (1951)

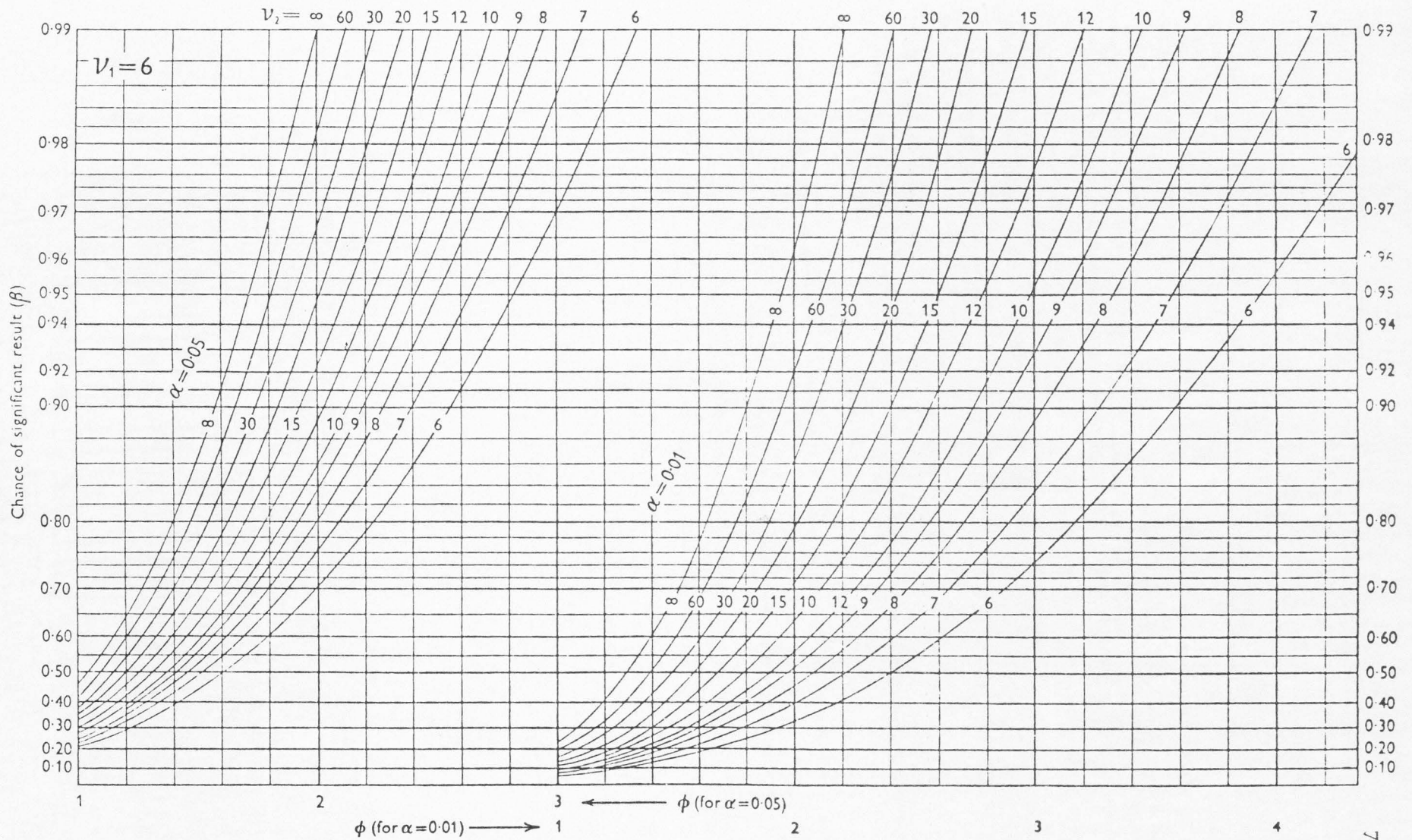


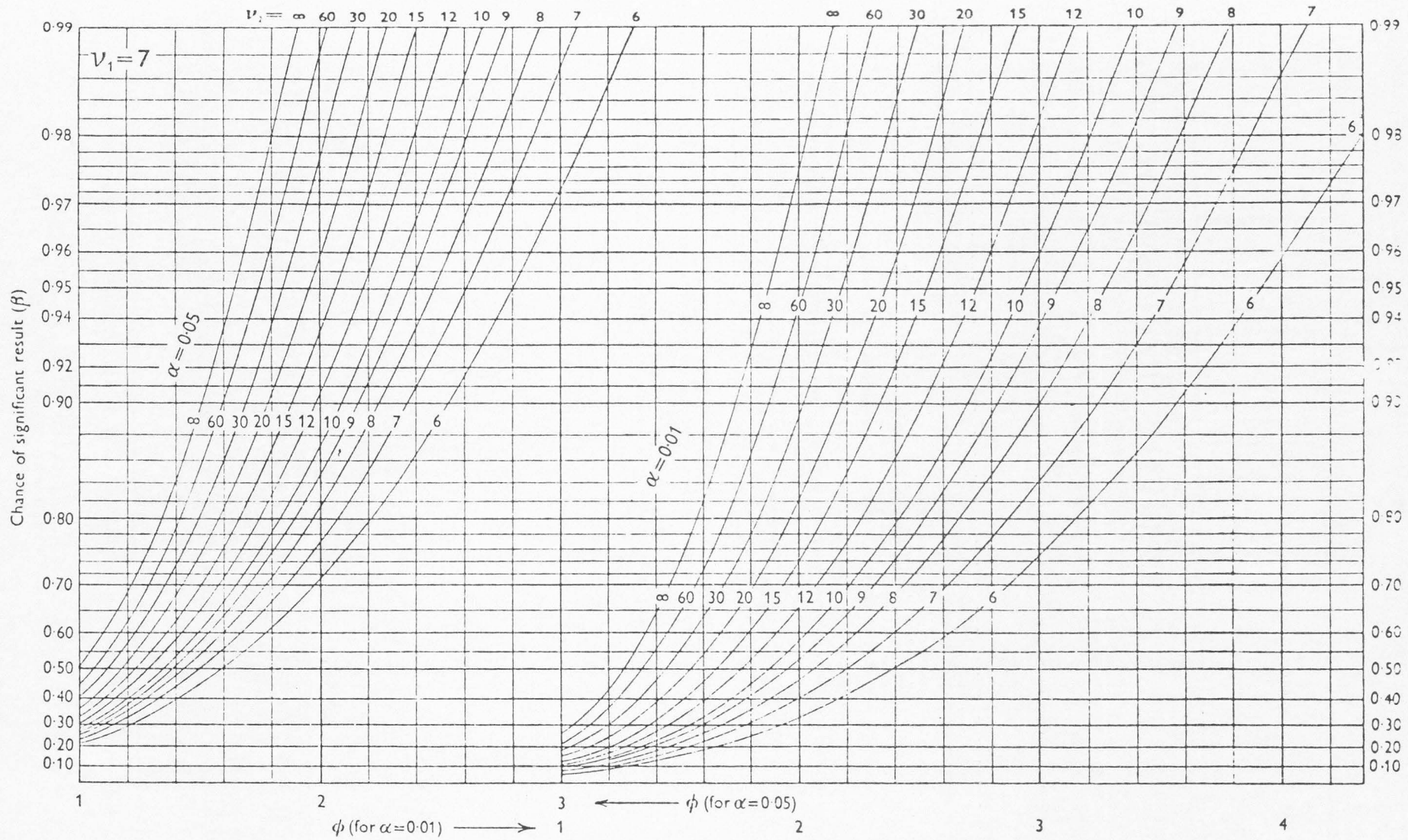


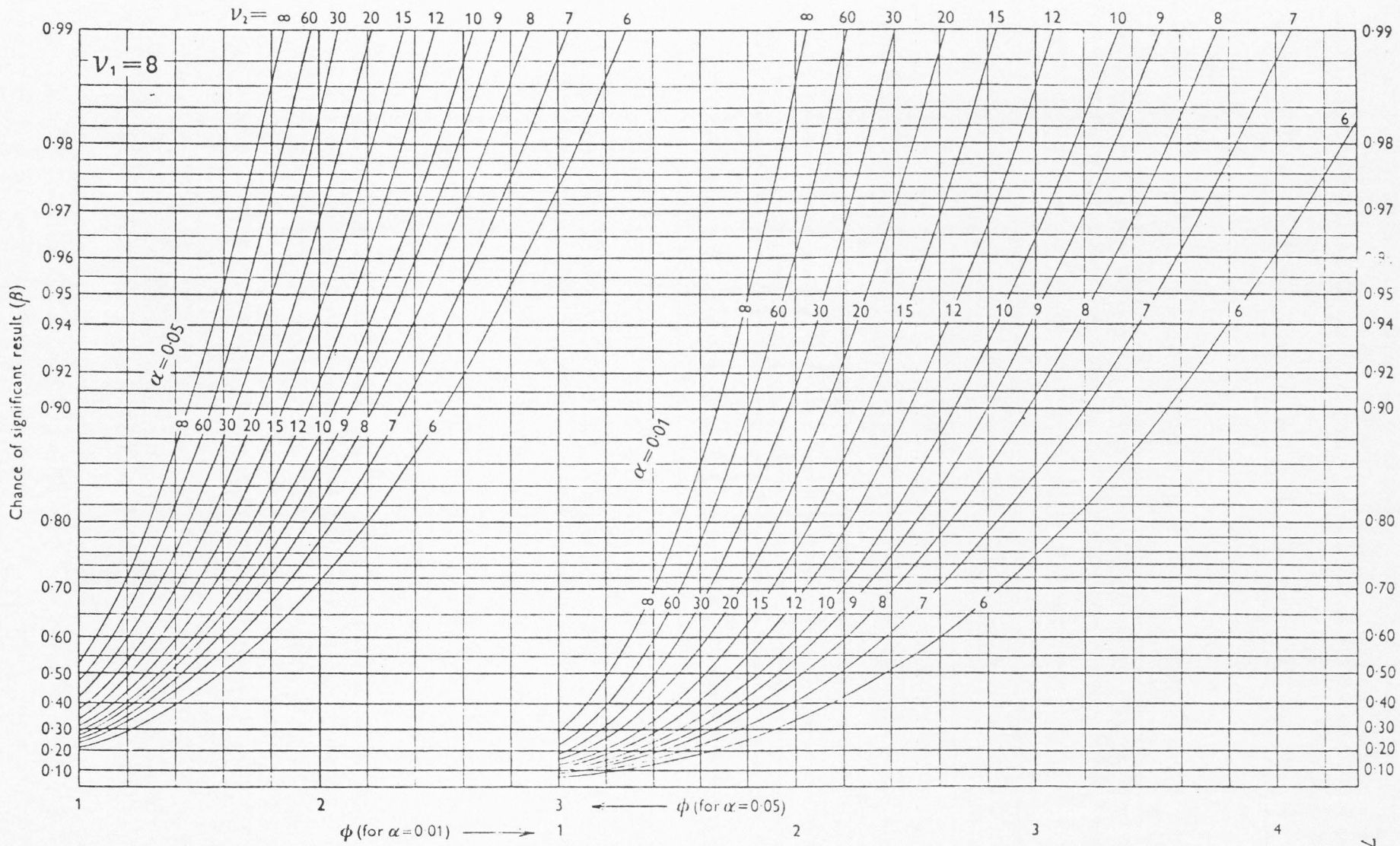












A P P E N D I X B

Table for Finding the Power

by

M. L. Tiku

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F_1	$P = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 1, A = 0.005$										
2	.9925	.9851	.9808	.9757	.9699	.9634	.9561	.9481	.9301	.9096
4	.9902	.9729	.9610	.9454	.9258	.9017	.8731	.8398	.7602	.6663
6	.9886	.9630	.9441	.9185	.8855	.8443	.7951	.7382	.6068	.4646
8	.9874	.9557	.9314	.8983	.8551	.8014	.7377	.6653	.5052	.3460
10	.9866	.9502	.9220	.8832	.8326	.7700	.6965	.6144	.4390	.2762
12	.9860	.9461	.9140	.8718	.8157	.7467	.6663	.5778	.3942	.2325
14	.9855	.9429	.9093	.8629	.8026	.7288	.6436	.5507	.3623	.2033
16	.9851	.9404	.9048	.8558	.7923	.7149	.6258	.5298	.3387	.1827
18	.9848	.9383	.9012	.8501	.7836	.7035	.6117	.5134	.3206	.1676
20	.9846	.9365	.8982	.8453	.7770	.6942	.6002	.5001	.3064	.1560
22	.9843	.9351	.8957	.8413	.7712	.6865	.5907	.4892	.2950	.1469
24	.9842	.9338	.8935	.8379	.7663	.6800	.5827	.4801	.2856	.1396
26	.9840	.9327	.8916	.8350	.7620	.6744	.5758	.4724	.2777	.1336
28	.9839	.9318	.8900	.8324	.7584	.6695	.5699	.4658	.2710	.1286
30	.9838	.9310	.8886	.8302	.7552	.6653	.5648	.4600	.2653	.1244
40	.9833	.9280	.8834	.8221	.7437	.6502	.5467	.4399	.2458	.1104
60	.9829	.9240	.8780	.8137	.7318	.6348	.5284	.4199	.2270	.0976
120	.9824	.9216	.8724	.8050	.7195	.6191	.5100	.4001	.2092	.0860
∞	.9819	.9182	.8665	.7959	.7069	.6031	.4915	.3805	.1922	.0756
$F_1 = 2, A = 0.005$										
2	.9931	.9876	.9843	.9805	.9761	.9711	.9656	.9595	.9458	.9201
4	.9913	.9772	.9673	.9541	.9371	.9161	.8907	.8608	.7881	.7004
6	.9898	.9674	.9500	.9259	.8938	.8530	.8033	.7452	.6093	.4615
8	.9886	.9593	.9354	.9015	.8562	.7956	.7293	.6503	.4784	.3089
10	.9878	.9528	.9235	.8818	.8258	.7553	.6720	.5793	.3866	.2199
12	.9871	.9475	.9139	.8658	.8015	.7214	.6250	.5265	.3256	.1664
14	.9865	.9432	.9061	.8529	.7820	.6945	.5939	.4867	.2828	.1326
16	.9861	.9397	.8996	.8423	.7662	.6728	.5670	.4560	.2517	.1099
18	.9857	.9368	.8942	.8334	.7530	.6552	.5453	.4317	.2284	.0941
20	.9854	.9343	.8897	.8259	.7421	.6405	.5276	.4121	.2104	.0825
22	.9852	.9321	.8858	.8196	.7327	.6282	.5128	.3961	.1961	.0738
24	.9849	.9302	.8824	.8141	.7247	.6177	.5003	.3827	.1846	.0670
26	.9847	.9286	.8794	.8093	.7178	.6086	.4897	.3714	.1751	.0617
28	.9846	.9272	.8768	.8051	.7117	.6008	.4805	.3617	.1672	.0573
30	.9844	.9259	.8745	.8014	.7064	.5939	.4725	.3534	.1605	.0537
40	.9839	.9212	.8661	.7878	.6870	.5692	.4443	.3244	.1383	.0425
60	.9833	.9162	.8570	.7734	.6667	.5438	.4159	.2961	.1181	.0332
120	.9828	.9108	.8472	.7581	.6455	.5177	.3876	.2686	.0999	.0256
∞	.9819	.9049	.8368	.7419	.6234	.4912	.3594	.2422	.0838	.0196
$F_1 = 3, A = 0.005$										
2	.9933	.9884	.9855	.9821	.9781	.9737	.9688	.9634	.9511	.9370
4	.9916	.9787	.9693	.9569	.9408	.9206	.8962	.8673	.7566	.7107
6	.9902	.9686	.9514	.9271	.8944	.8525	.8012	.7409	.6000	.4478
8	.9890	.9597	.9349	.8992	.8506	.7886	.7138	.6289	.4449	.2742
10	.9881	.9521	.9207	.8751	.8132	.7349	.6428	.5417	.3389	.1758
12	.9873	.9458	.9088	.8549	.7821	.6914	.5870	.4760	.2680	.1291
14	.9867	.9404	.8988	.8379	.7564	.6561	.5431	.4263	.2196	.0870
16	.9862	.9359	.8903	.8237	.7351	.6274	.5082	.3881	.1854	.0662
18	.9858	.9320	.8830	.8117	.7172	.6037	.4801	.3581	.1605	.0525
20	.9854	.9287	.8768	.8014	.7021	.5840	.4571	.3341	.1410	.0431
22	.9851	.9258	.8714	.7925	.6892	.5673	.4380	.3146	.1275	.0363
24	.9848	.9233	.8667	.7847	.6780	.5531	.4219	.2985	.1162	.0313
26	.9846	.9211	.8625	.7780	.6683	.5408	.4082	.2850	.1070	.0275
28	.9844	.9191	.8588	.7720	.6597	.5301	.3964	.2735	.0996	.0245
30	.9842	.9173	.8555	.7666	.6522	.5207	.3862	.2637	.0934	.0222
40	.9835	.9107	.8433	.7470	.6247	.4872	.3504	.2302	.0736	.0153
60	.9827	.9034	.8298	.7257	.5957	.4528	.3149	.1985	.0570	.0102
120	.9819	.8953	.8151	.7029	.5652	.4177	.2803	.1600	.0432	.0067
∞	.9809	.8863	.7990	.6783	.5333	.3824	.2469	.1420	.0322	.0012
$F_1 = 4, A = 0.005$										
2	.9934	.9888	.9861	.9829	.9792	.9750	.9704	.9653	.9538	.9405
4	.9918	.9794	.9703	.9582	.9425	.9228	.8988	.8703	.8004	.7153
6	.9903	.9691	.9518	.9271	.8937	.8506	.7978	.7357	.5907	.4354
8	.9892	.9595	.9347	.8991	.8448	.7788	.6995	.6099	.4189	.2477
10	.9882	.9510	.9175	.8683	.8011	.7183	.6171	.5097	.3016	.1448
12	.9874	.9436	.9034	.8441	.7638	.6642	.5512	.4338	.2248	.0897
14	.9867	.9372	.8912	.8234	.7323	.6215	.4990	.3765	.1739	.0592
16	.9861	.9318	.8807	.8057	.7059	.5863	.4576	.3328	.1322	.0414
18	.9856	.9270	.8716	.7904	.6834	.5572	.4242	.2989	.1148	.0304
20	.9852	.9228	.8637	.7773	.6643	.5329	.3970	.2722	.0971	.0232
22	.9848	.9192	.8568	.7658	.6479	.5123	.3745	.2507	.0839	.0184
24	.9845	.9159	.8506	.7558	.6336	.4948	.3557	.2331	.0737	.0150
26	.9842	.9131	.8452	.7470	.6212	.4797	.3398	.2186	.0658	.0125
28	.9840	.9105	.8404	.7391	.6103	.4665	.3261	.2064	.0594	.0106
30	.9838	.9082	.8360	.7321	.6006	.4550	.3144	.1960	.0543	.0092
40	.9829	.8994	.8197	.7061	.5653	.4140	.2737	.1615	.0387	.0054
60	.9820	.8960	.8014	.6776	.5278	.3723	.2345	.1303	.0287	.0030
120	.9809	.8784	.7810	.6467	.4885	.3306	.1973	.1027	.0177	.0016
∞	.9797	.8658	.7585	.6133	.4477	.2895	.1631	.0791	.0114	.0008

F_1	5	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 5, A = 0.005$										
2	.9935	.9890	.9861	.9834	.9798	.9758	.9714	.9665	.9554	.9426
4	.9919	.9798	.9709	.9590	.9435	.9240	.9002	.8720	.8025	.7177
6	.9905	.9693	.9518	.9267	.8927	.8486	.7944	.7308	.5826	.4251
8	.9892	.9591	.9325	.8933	.8394	.7703	.6873	.5940	.3981	.2275
10	.9882	.9498	.9145	.8621	.7903	.7000	.5952	.4832	.2726	.1225
12	.9873	.9415	.8984	.8343	.7474	.6404	.5207	.3991	.1923	.0695
14	.9866	.9342	.8842	.8100	.7107	.5911	.4617	.3361	.1407	.0420
16	.9860	.9278	.8718	.7890	.6794	.5503	.4148	.2886	.1069	.0289
18	.9854	.9221	.8609	.7707	.6527	.5164	.3773	.2522	.0839	.0183
20	.9850	.9172	.8513	.7547	.6299	.4881	.3469	.2239	.0678	.0130
22	.9846	.9127	.8428	.7408	.6102	.4642	.3220	.2015	.0562	.0097
24	.9842	.9088	.8353	.7285	.5931	.4438	.3013	.1835	.0476	.0074
26	.9839	.9053	.8285	.7176	.5781	.4263	.2838	.1687	.0410	.0059
28	.9836	.9021	.8225	.7079	.5649	.4111	.2690	.1565	.0360	.0047
30	.9833	.8992	.8170	.6992	.5552	.3979	.2563	.1462	.0319	.0039
40	.9823	.8882	.7963	.6667	.5106	.3510	.2132	.1130	.0203	.0019
60	.9812	.8755	.7729	.6309	.4655	.3041	.1730	.0845	.0123	.0009
120	.9798	.8609	.7461	.5918	.4185	.2581	.1364	.0609	.0070	.0004
∞	.9783	.8441	.7167	.5496	.3704	.2143	.1045	.0423	.0039	.0002
$F_1 = 6, A = 0.005$										
2	.9935	.9892	.9867	.9837	.9802	.9763	.9720	.9673	.9565	.9440
4	.9920	.9801	.9713	.9595	.9441	.9247	.9011	.8730	.8037	.7191
6	.9905	.9694	.9517	.9263	.8917	.8468	.7915	.7266	.5759	.4167
8	.9893	.9587	.9313	.8907	.8348	.7630	.6769	.5807	.3812	.2117
10	.9882	.9486	.9117	.8566	.7810	.6860	.5766	.4612	.2497	.1061
12	.9873	.9395	.8939	.8255	.7330	.6198	.4949	.3705	.1674	.0555
14	.9865	.9314	.8778	.7980	.6914	.5646	.4301	.3032	.1163	.0308
16	.9858	.9241	.8635	.7738	.6557	.5188	.3789	.2530	.0840	.0182
18	.9852	.9176	.8509	.7525	.6251	.4808	.3381	.2152	.0628	.0115
20	.9847	.9118	.8396	.7339	.5988	.4491	.3054	.1863	.0355	.0076
22	.9843	.9066	.8296	.7175	.5761	.4223	.2787	.1637	.0386	.0053
24	.9838	.9020	.8206	.7030	.5563	.3996	.2568	.1458	.0314	.0038
26	.9835	.8978	.8126	.6901	.5390	.3802	.2385	.1314	.0261	.0028
28	.9831	.8940	.8054	.6786	.5237	.3634	.2230	.1196	.0221	.0022
30	.9828	.8905	.7988	.6682	.5102	.3487	.2099	.1099	.0191	.0017
40	.9817	.8771	.7737	.6294	.4610	.2975	.1662	.0793	.0108	.0007
60	.9804	.8615	.7448	.5865	.4093	.2473	.1269	.0545	.0057	.0003
120	.9788	.8431	.7119	.5395	.3561	.1996	.0931	.0356	.0027	.0001
∞	.9770	.8216	.6745	.4588	.3026	.1558	.0653	.0219	.0012	.0000
$F_1 = 7, A = 0.005$										
2	.9936	.9893	.9868	.9839	.9805	.9767	.9725	.9678	.9572	.9450
4	.9920	.9802	.9716	.9598	.9445	.9252	.9017	.8737	.8046	.7200
6	.9906	.9694	.9516	.9259	.8907	.8451	.7889	.7230	.5703	.4098
8	.9893	.9587	.9302	.8885	.8308	.7567	.6681	.5695	.3674	.1992
10	.9882	.9476	.9093	.8517	.7727	.6738	.5608	.4428	.2314	.0937
12	.9872	.9378	.8898	.8177	.7202	.6019	.4729	.3467	.1481	.0455
14	.9864	.9288	.8720	.7871	.6743	.5415	.4033	.2761	.0980	.0234
16	.9857	.9207	.8560	.7599	.6346	.4914	.3485	.2242	.0673	.0128
18	.9850	.9134	.8416	.7359	.6004	.4497	.3053	.1857	.0189	.0075
20	.9845	.9068	.8288	.7148	.5709	.4151	.2708	.1567	.0355	.0046
22	.9840	.9009	.8173	.6960	.5454	.3860	.2430	.1345	.0270	.0030
24	.9835	.8955	.8069	.6794	.5231	.3614	.2204	.1171	.0212	.0020
26	.9831	.8906	.7976	.6645	.5037	.3404	.2017	.1034	.0170	.0014
28	.9827	.8862	.7891	.6513	.4866	.3223	.1860	.0923	.0139	.0010
30	.9824	.8822	.7814	.6394	.4715	.3067	.1729	.0833	.0116	.0008
40	.9811	.8654	.7518	.5944	.4166	.2526	.1300	.0559	.0058	.0003
60	.9796	.8476	.7175	.5446	.3594	.2007	.0930	.0351	.0026	.0001
120	.9778	.8252	.6778	.4901	.3013	.1531	.0629	.0205	.0010	.0000
∞	.9756	.7985	.6326	.4317	.2445	.1114	.0400	.0111	.0004	.0000
$F_1 = 8, A = 0.005$										
2	.9936	.9894	.9870	.9841	.9807	.9770	.9728	.9682	.9578	.9458
4	.9921	.9804	.9718	.9601	.9448	.9256	.9021	.8742	.8052	.7206
6	.9906	.9694	.9515	.9255	.8899	.8437	.7867	.7199	.5655	.4041
8	.9893	.9579	.9293	.8865	.8273	.7512	.6606	.5601	.3560	.1891
10	.9882	.9467	.9071	.8474	.7655	.6633	.5471	.4271	.2164	.0840
12	.9872	.9362	.8861	.8107	.7089	.5862	.4539	.3267	.1327	.0381
14	.9863	.9264	.8667	.7773	.6590	.5212	.3803	.2535	.0838	.0182
16	.9855	.9175	.8491	.7474	.6156	.4673	.3226	.2006	.0550	.0093
18	.9849	.9095	.8331	.7208	.5781	.4226	.2775	.1619	.0375	.0050
20	.9843	.9021	.8187	.6972	.5457	.3855	.2419	.1333	.0265	.0029
22	.9837	.8955	.8057	.6762	.5177	.3544	.2134	.1117	.0194	.0018
24	.9832	.8894	.7940	.6575	.4933	.3283	.1905	.0951	.0146	.0011
26	.9828	.8839	.7834	.6408	.4720	.3061	.1717	.0822	.0113	.0008
28	.9824	.8788	.7737	.6259	.4532	.2871	.1562	.0720	.0089	.0005
30	.9820	.8742	.7650	.6125	.4367	.2708	.1433	.0637	.0072	.0004
40	.9806	.8559	.7308	.5617	.3768	.2149	.1020	.0397	.0032	.0001
60	.9788	.8338	.6909	.5054	.3152	.1628	.0681	.0227	.0012	.0000
120	.9767	.8072	.6446	.4441	.2540	.1168	.0422	.0117	.0004	.0000
∞	.9742	.7750	.5913	.3790	.1956	.0786	.0240	.0055	.0001	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 9, A = 0.005$										
2	.9936	.9895	.9871	.9842	.9809	.9772	.9731	.9686	.9581	.9464
4	.9921	.9805	.9719	.9603	.9451	.9259	.9024	.8745	.8036	.7211
6	.9906	.9694	.9514	.9252	.8892	.8421	.7848	.7173	.5615	.3992
8	.9893	.9576	.9284	.8847	.8242	.7465	.6540	.5519	.3463	.1807
10	.9882	.9459	.9051	.8436	.7591	.6540	.5353	.4136	.2039	.0763
12	.9871	.9347	.8828	.8014	.6989	.5724	.4375	.3096	.1293	.0325
14	.9862	.9243	.8619	.7654	.6453	.5034	.3604	.2347	.0728	.0145
16	.9854	.9147	.8427	.7259	.5985	.4461	.3005	.1811	.0457	.0059
18	.9847	.9059	.8252	.7069	.5580	.3987	.2539	.1425	.0298	.0035
20	.9840	.8978	.8094	.6810	.5230	.3595	.2176	.1145	.0202	.0019
22	.9835	.8904	.7950	.6579	.4927	.3269	.1888	.0937	.0141	.0011
24	.9829	.8837	.7819	.6373	.4664	.2996	.1659	.0781	.0102	.0006
26	.9825	.8775	.7700	.6189	.4434	.2765	.1473	.0660	.0076	.0004
28	.9820	.8718	.7592	.6024	.4232	.2568	.1321	.0567	.0058	.0003
30	.9816	.8666	.7493	.5875	.4054	.2400	.1195	.0493	.0046	.0002
40	.9800	.8458	.7108	.5313	.3414	.1833	.0804	.0284	.0018	.0000
60	.9781	.8204	.6653	.4689	.2764	.1321	.0499	.0147	.0006	.0000
120	.9757	.7893	.6122	.4015	.2133	.0887	.0281	.0067	.0001	.0000
∞	.9727	.7512	.5510	.3309	.1552	.0548	.0142	.0027	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 10, A = 0.005$										
2	.9936	.9855	.9871	.9843	.9811	.9774	.9733	.9688	.9586	.9469
4	.9921	.9806	.9720	.9604	.9453	.9261	.9027	.8748	.8059	.7214
6	.9907	.9694	.9513	.9248	.8885	.8413	.7832	.7150	.5580	.3951
8	.9893	.9576	.9277	.8832	.8215	.7423	.6483	.5448	.3380	.1737
10	.9882	.9451	.9034	.8402	.7534	.6458	.5249	.4020	.1935	.0701
12	.9871	.9334	.8798	.7987	.6899	.5661	.4232	.2950	.1101	.0282
14	.9861	.9223	.8575	.7604	.6330	.4875	.3432	.2186	.0640	.0118
16	.9853	.9120	.8369	.7255	.5831	.4273	.2814	.1647	.0385	.0053
18	.9845	.9025	.8180	.6941	.5399	.3776	.2338	.1266	.0241	.0025
20	.9839	.8937	.8007	.6661	.5025	.3366	.1969	.0993	.0156	.0013
22	.9832	.8857	.7819	.6410	.4702	.3028	.1681	.0794	.0105	.0007
24	.9827	.8783	.7706	.6186	.4421	.2745	.1454	.0647	.0073	.0004
26	.9822	.8714	.7575	.5986	.4176	.2508	.1272	.0536	.0053	.0002
28	.9817	.8651	.7455	.5805	.3962	.2308	.1124	.0451	.0039	.0001
30	.9812	.8593	.7346	.5643	.3773	.2137	.1003	.0385	.0030	.0001
40	.9795	.8360	.6916	.5050	.3099	.1569	.0638	.0205	.0010	.0000
60	.9774	.8072	.6407	.4352	.2426	.1073	.0367	.0095	.0003	.0000
120	.9747	.7715	.5809	.3624	.1787	.0671	.0187	.0038	.0001	.0000
∞	.9713	.7273	.5120	.2874	.1221	.0378	.0083	.0015	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 12, A = 0.005$										
2	.9937	.9896	.9873	.9845	.9813	.9777	.9736	.9692	.9592	.9476
4	.9921	.9807	.9722	.9606	.9455	.9264	.9030	.8752	.8063	.7219
6	.9907	.9694	.9513	.9243	.8875	.8395	.7804	.7112	.5523	.3884
8	.9893	.9568	.9264	.8806	.8169	.7354	.6388	.5332	.3247	.1627
10	.9881	.9430	.9004	.8344	.7438	.6321	.5077	.3830	.1770	.0607
12	.9870	.9311	.8746	.7890	.6745	.5395	.3994	.2713	.0945	.0121
14	.9860	.9189	.8498	.7463	.6117	.4607	.3147	.1930	.0511	.0083
16	.9851	.9073	.8265	.7071	.5565	.3956	.2501	.1291	.0285	.0033
18	.9843	.8965	.8049	.6716	.5085	.3422	.2013	.1022	.0164	.0014
20	.9835	.8864	.7850	.6396	.4670	.2986	.1642	.0766	.0099	.0006
22	.9828	.8770	.7667	.6109	.4311	.2628	.1358	.0586	.0061	.0003
24	.9822	.8683	.7499	.5852	.4001	.2334	.1138	.0457	.0040	.0001
26	.9816	.8603	.7315	.5622	.3732	.2090	.0967	.0364	.0027	.0001
28	.9811	.8528	.7203	.5414	.3497	.1886	.0831	.0294	.0018	.0000
30	.9806	.8458	.7073	.5227	.3292	.1714	.0721	.0241	.0013	.0000
40	.9786	.8175	.6559	.4523	.2569	.1162	.0408	.0109	.0003	.0000
60	.9760	.7818	.5944	.3750	.1871	.0711	.0200	.0041	.0001	.0000
120	.9728	.7365	.5214	.2939	.1246	.0382	.0082	.0012	.0000	.0000
∞	.9684	.6792	.4386	.2137	.0740	.0174	.0031	.0003	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 1, A = 0.01$										
2	.9851	.9705	.9620	.9521	.9408	.9282	.9143	.8991	.8654	.8277
4	.9809	.9492	.9280	.9012	.8682	.8292	.7843	.7341	.6216	.5014
6	.9782	.9240	.9020	.8629	.8131	.7541	.6870	.6156	.4589	.3125
8	.9764	.9236	.8859	.8367	.7759	.7043	.6242	.5387	.3678	.2211
10	.9752	.9163	.8758	.8184	.7501	.6704	.5824	.4904	.3136	.1725
12	.9743	.9109	.8650	.8050	.7314	.6462	.5532	.4574	.2787	.1437
14	.9736	.9068	.8582	.7949	.7174	.6283	.5318	.4336	.2547	.1250
16	.9730	.9096	.8529	.7870	.7066	.6145	.5156	.4158	.2374	.1121
18	.9726	.9010	.8487	.7807	.6979	.6036	.5028	.4020	.2243	.1027
20	.9723	.8989	.8452	.7755	.6908	.5947	.4925	.3910	.2141	.0957
22	.9720	.8971	.8423	.7712	.6850	.5874	.4841	.3820	.2060	.0902
24	.9717	.8956	.8398	.7675	.6801	.5813	.4771	.3746	.1994	.0858
26	.9715	.8943	.8377	.7644	.6758	.5760	.4712	.3683	.1938	.0822
28	.9713	.8931	.8359	.7617	.6722	.5716	.4661	.3650	.1892	.0792
30	.9711	.8922	.8343	.7593	.6690	.5677	.4617	.3584	.1852	.0767
40	.9705	.8886	.8285	.7509	.6578	.5559	.4462	.3424	.1718	.0683
60	.9699	.8850	.8226	.7423	.6463	.5401	.4308	.3267	.1590	.0608
120	.9693	.8812	.8165	.7355	.6347	.5261	.4155	.3113	.1468	.0539
∞	.9687	.8773	.8102	.7244	.6229	.5120	.4003	.2962	.1354	.0478

F_1	$P = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 2, A = 0.01$										
2	.9863	.9753	.9688	.9613	.9527	.9430	.9323	.9207	.9045	.8850
4	.9828	.9547	.9386	.9153	.8862	.8511	.8100	.7635	.6571	.5401
6	.9803	.9409	.9118	.8730	.8237	.7640	.6951	.6191	.4576	.3052
8	.9784	.9288	.8910	.8401	.7754	.6982	.6110	.5182	.3358	.1869
10	.9770	.9196	.8751	.8150	.7393	.6500	.5515	.4498	.2626	.1268
12	.9760	.9124	.8627	.7957	.7118	.6142	.5085	.4022	.2163	.0934
14	.9752	.9067	.8529	.7806	.6905	.5839	.4765	.3678	.1854	.0733
16	.9745	.9021	.8450	.7684	.6736	.5653	.4519	.3420	.1636	.0603
18	.9740	.8983	.8386	.7585	.6600	.5485	.4326	.3221	.1476	.0513
20	.9735	.8951	.8331	.7502	.6486	.5345	.4170	.3063	.1354	.0449
22	.9731	.8924	.8285	.7433	.6392	.5229	.4042	.2936	.1260	.0401
24	.9728	.8901	.8246	.7373	.6312	.5132	.3936	.2830	.1184	.0364
26	.9725	.8881	.8212	.7322	.6243	.5048	.3845	.2742	.1122	.0335
28	.9723	.8863	.8182	.7277	.6182	.4976	.3768	.2667	.1070	.0312
30	.9721	.8848	.8156	.7238	.6130	.4914	.3701	.2603	.1027	.0293
40	.9713	.8791	.8060	.7096	.5943	.4693	.3468	.2382	.0885	.0233
60	.9704	.8731	.7960	.6948	.5749	.4469	.3237	.2170	.0757	.0183
120	.9695	.8668	.7854	.6794	.5551	.4244	.3011	.1968	.0643	.0143
∞	.9686	.8600	.7743	.6634	.5349	.4019	.2780	.1776	.0543	.0111
$F_1 = 3, A = 0.01$										
2	.9867	.9769	.9711	.9644	.9567	.9481	.9385	.9280	.9045	.8779
4	.9835	.9592	.9421	.9199	.8919	.8580	.8181	.7726	.6678	.5517
6	.9809	.9427	.9136	.8742	.8237	.7620	.6906	.6117	.4448	.2899
8	.9790	.9291	.8906	.8357	.7655	.6835	.5902	.4917	.3032	.1576
10	.9775	.9181	.8703	.8047	.7214	.6234	.5166	.4085	.2191	.0911
12	.9763	.9093	.8547	.7800	.6861	.5776	.4625	.3504	.1675	.0615
14	.9753	.9021	.8419	.7600	.6580	.5421	.4220	.3085	.1343	.0434
16	.9746	.8961	.8314	.7437	.6354	.5142	.3910	.2776	.1118	.0325
18	.9739	.8910	.8227	.7302	.6169	.4917	.3666	.2540	.0958	.0236
20	.9734	.8868	.8152	.7188	.6016	.4733	.3471	.2355	.0841	.0209
22	.9729	.8831	.8089	.7092	.5856	.4550	.3311	.2207	.0752	.0175
24	.9725	.8799	.8034	.7008	.5776	.4451	.3178	.2087	.0683	.0151
26	.9721	.8772	.7986	.6936	.5681	.4341	.3066	.1987	.0628	.0132
28	.9718	.8747	.7944	.6873	.5599	.4247	.2971	.1903	.0584	.0118
30	.9716	.8725	.7906	.6817	.5526	.4164	.2859	.1831	.0547	.0107
40	.9705	.8645	.7769	.6614	.5266	.3873	.2606	.1590	.0430	.0074
60	.9694	.8558	.7622	.6400	.4997	.3582	.2332	.1367	.0334	.0050
120	.9682	.8464	.7464	.6175	.4721	.3292	.2070	.1163	.0255	.0033
∞	.9669	.8361	.7295	.5938	.4439	.3005	.1821	.0978	.0192	.0022
$F_1 = 4, A = 0.01$										
2	.9869	.9777	.9723	.9660	.9587	.9506	.9416	.9317	.9096	.8844
4	.9838	.9604	.9438	.9221	.8946	.8612	.8217	.7767	.6725	.5566
6	.9812	.9433	.9139	.8738	.8219	.7585	.6848	.6036	.4330	.2767
8	.9792	.9284	.8873	.8306	.7575	.6897	.6116	.4691	.2776	.1363
10	.9776	.9180	.8650	.7944	.7047	.6196	.4865	.3745	.1867	.0726
12	.9763	.9056	.8464	.7647	.6622	.5452	.4236	.3087	.1330	.0424
14	.9752	.8989	.8309	.7403	.6281	.5027	.3765	.2620	.0998	.0270
16	.9743	.8896	.8178	.7200	.6003	.4691	.3405	.2279	.0783	.0184
18	.9736	.8834	.8068	.7030	.5774	.4420	.3124	.2023	.0636	.0133
20	.9730	.8780	.7974	.6886	.5583	.4199	.2901	.1825	.0533	.0101
22	.9724	.8734	.7892	.6763	.5421	.4015	.2719	.1670	.0457	.0079
24	.9719	.8693	.7821	.6656	.5283	.3861	.2570	.1545	.0400	.0064
26	.9715	.8657	.7759	.6563	.5164	.3730	.2445	.1442	.0355	.0054
28	.9711	.8625	.7704	.6482	.5060	.3617	.2340	.1357	.0320	.0046
30	.9703	.8597	.7655	.6409	.4969	.3519	.2249	.1286	.0292	.0040
40	.9695	.8491	.7473	.6145	.4643	.3176	.1942	.1051	.0207	.0023
60	.9682	.8374	.7275	.5844	.4306	.2836	.1653	.0844	.0143	.0013
120	.9666	.8245	.7060	.5566	.3962	.2504	.1386	.0665	.0096	.0007
∞	.9649	.8103	.6828	.5253	.3614	.2185	.1144	.0513	.0063	.0004
$F_1 = 5, A = 0.01$										
2	.9870	.9782	.9730	.9669	.9600	.9521	.9435	.9340	.9126	.8884
4	.9840	.9611	.9448	.9233	.8961	.8629	.8237	.7789	.6749	.5591
6	.9814	.9435	.9138	.8729	.8199	.7550	.6795	.5965	.4231	.2663
8	.9793	.9276	.8850	.8258	.7494	.6578	.5559	.4504	.2575	.1207
10	.9776	.9138	.8600	.7852	.6899	.5792	.4615	.3471	.1625	.0581
12	.9762	.9021	.8387	.7510	.6413	.5174	.3914	.2757	.1054	.0306
14	.9750	.8920	.8206	.7224	.6017	.4690	.3391	.2257	.0763	.0176
16	.9740	.8835	.8052	.6984	.5692	.4306	.2994	.1898	.0564	.0109
18	.9732	.8761	.7920	.6782	.5423	.3998	.2688	.1633	.0435	.0073
20	.9725	.8696	.7806	.6609	.5198	.3746	.2446	.1433	.0347	.0051
22	.9718	.8640	.7706	.6460	.5007	.3538	.2252	.1278	.0284	.0037
24	.9713	.8590	.7619	.6330	.4844	.3363	.2093	.1156	.0239	.0029
26	.9708	.8546	.7543	.6217	.4704	.3216	.1962	.1057	.0205	.0023
28	.9704	.8507	.7474	.6118	.4581	.3089	.1851	.0976	.0179	.0018
30	.9700	.8472	.7413	.6029	.4474	.2980	.1758	.0909	.0158	.0015
40	.9685	.8329	.7186	.5705	.4090	.2601	.1446	.0695	.0100	.0007
60	.9668	.8189	.6935	.5359	.3696	.2232	.1163	.0516	.0061	.0003
120	.9650	.8023	.6660	.4992	.3298	.1882	.0913	.0372	.0035	.0002
∞	.9628	.7835	.6360	.4606	.2901	.1556	.0699	.0250	.0020	.0000

F_1	$P = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 6, A = 0.01$										
2	.9871	.9785	.9735	.9675	.9608	.9532	.9447	.9355	.9147	.8910
4	.9841	.9616	.9454	.9241	.8971	.8640	.8250	.7802	.6764	.5605
6	.9815	.9435	.9135	.8720	.8180	.7518	.6749	.5905	.4149	.2578
8	.9794	.9267	.8829	.8216	.7424	.6477	.5428	.4351	.2417	.1090
10	.9776	.9118	.8556	.7770	.6772	.5618	.4400	.3248	.1442	.0480
12	.9761	.8988	.8318	.7388	.6230	.4938	.3647	.2492	.0905	.0230
14	.9748	.8870	.8113	.7064	.5785	.4402	.3083	.1971	.0600	.0120
16	.9737	.8778	.7936	.6790	.5418	.3978	.2659	.1604	.0419	.0068
18	.9728	.8692	.7783	.6556	.5113	.3639	.2335	.1339	.0306	.0042
20	.9720	.8617	.7650	.6356	.4857	.3363	.2082	.1142	.0232	.0027
22	.9713	.8551	.7533	.6183	.4641	.3136	.1881	.0992	.0182	.0019
24	.9707	.8493	.7430	.6032	.4456	.2946	.1719	.0876	.0147	.0013
26	.9701	.8441	.7339	.5900	.4297	.2787	.1586	.0784	.0121	.0010
28	.9696	.8394	.7258	.5783	.4158	.2651	.1476	.0710	.0102	.0008
30	.9692	.8351	.7185	.5679	.4037	.2533	.1383	.0649	.0088	.0006
40	.9675	.8191	.6911	.5299	.3605	.2133	.1090	.0462	.0049	.0002
60	.9655	.8008	.6607	.4891	.3166	.1753	.0816	.0315	.0026	.0001
120	.9633	.7800	.6271	.4459	.2728	.1402	.0595	.0205	.0013	.0000
∞	.9607	.7563	.5901	.4009	.2301	.1089	.0417	.0128	.0006	.0000

$F_1 = 7, A = 0.01$										
2	.9872	.9787	.9738	.9680	.9614	.9539	.9456	.9365	.9161	.8929
4	.9842	.9619	.9459	.9247	.8977	.8648	.8258	.7811	.6773	.5613
6	.9816	.9435	.9132	.8711	.8163	.7490	.6710	.5854	.4082	.2510
8	.9794	.9260	.8810	.8179	.7363	.6390	.5317	.4224	.2290	.1000
10	.9775	.9100	.8516	.7699	.6661	.5469	.4231	.3065	.1299	.0407
12	.9760	.8959	.8258	.7280	.6070	.4755	.3423	.2278	.0770	.0178
14	.9746	.8835	.8029	.6922	.5581	.4156	.2828	.1744	.0482	.0055
16	.9735	.8726	.7831	.6615	.5176	.3698	.2385	.1375	.0319	.0044
18	.9724	.8629	.7658	.6353	.4840	.3333	.2019	.1113	.0221	.0025
20	.9716	.8544	.7506	.6127	.4558	.3038	.1791	.0923	.0160	.0015
22	.9708	.8469	.7373	.5931	.4319	.2796	.1587	.0781	.0120	.0010
24	.9701	.8401	.7254	.5760	.4115	.2596	.1425	.0673	.0093	.0006
26	.9695	.8341	.7149	.5610	.3940	.2429	.1294	.0589	.0074	.0005
28	.9689	.8287	.7055	.5477	.3787	.2286	.1186	.0522	.0060	.0003
30	.9684	.8237	.6971	.5359	.3654	.2185	.1096	.0469	.0050	.0002
40	.9665	.8048	.6651	.4926	.3182	.1754	.0810	.0309	.0025	.0001
60	.9642	.7830	.6294	.4462	.2709	.1375	.0572	.0193	.0011	.0000
120	.9616	.7580	.5896	.3973	.2246	.1038	.0384	.0112	.0005	.0000
∞	.9585	.7290	.5456	.3467	.1807	.0751	.0244	.0062	.0002	.0000

$F_1 = 8, A = 0.01$										
2	.9872	.9789	.9740	.9683	.9618	.9544	.9463	.9373	.9172	.8943
4	.9843	.9621	.9462	.9251	.8982	.8653	.8264	.7817	.6779	.5619
6	.9817	.9435	.9128	.8703	.8148	.7467	.6676	.5811	.4026	.2454
8	.9794	.9252	.8793	.8147	.7311	.6315	.5223	.4117	.2186	.0928
10	.9775	.9084	.8481	.7635	.6564	.5341	.4082	.2912	.1185	.0352
12	.9758	.8933	.8201	.7184	.5930	.4559	.3234	.2101	.0668	.0142
14	.9744	.8798	.7953	.6794	.5401	.3943	.2614	.1560	.0396	.0063
16	.9732	.8678	.7735	.6458	.4963	.3458	.2157	.1193	.0248	.0030
18	.9721	.8571	.7543	.6169	.4598	.3072	.1815	.0938	.0163	.0016
20	.9711	.8477	.7374	.5919	.4292	.2762	.1554	.0756	.0113	.0009
22	.9703	.8392	.7224	.5702	.4034	.2510	.1352	.0624	.0081	.0005
24	.9695	.8316	.7091	.5512	.3814	.2302	.1193	.0524	.0060	.0003
26	.9689	.8247	.6972	.5345	.3625	.2129	.1065	.0449	.0046	.0002
28	.9682	.8185	.6866	.5198	.3461	.1953	.0962	.0389	.0036	.0001
30	.9677	.8129	.6770	.5067	.3318	.1859	.0876	.0343	.0029	.0001
40	.9655	.7911	.6406	.4584	.2815	.1447	.0611	.0209	.0012	.0000
60	.9630	.7658	.5995	.4070	.2318	.1079	.0402	.0118	.0005	.0000
120	.9600	.7362	.5536	.3531	.1843	.0765	.0247	.0061	.0002	.0000
∞	.9563	.7016	.5028	.2981	.1406	.0512	.0141	.0029	.0000	.0000

$F_1 = 9, A = 0.01$										
2	.9872	.9790	.9742	.9686	.9621	.9549	.9468	.9380	.9181	.8954
4	.9843	.9623	.9464	.9254	.8986	.8657	.8268	.7821	.6783	.5623
6	.9817	.9434	.9128	.8706	.8155	.7476	.6687	.5822	.4037	.2467
8	.9794	.9252	.8793	.8147	.7311	.6315	.5223	.4117	.2186	.0928
10	.9775	.9084	.8481	.7635	.6564	.5341	.4082	.2912	.1185	.0352
12	.9758	.8933	.8201	.7184	.5930	.4559	.3234	.2101	.0668	.0142
14	.9744	.8798	.7953	.6794	.5401	.3943	.2614	.1560	.0396	.0063
16	.9732	.8678	.7735	.6458	.4963	.3458	.2157	.1193	.0248	.0030
18	.9721	.8571	.7543	.6169	.4598	.3072	.1815	.0938	.0163	.0016
20	.9711	.8477	.7374	.5919	.4292	.2762	.1554	.0756	.0113	.0009
22	.9703	.8392	.7224	.5702	.4034	.2510	.1352	.0624	.0081	.0005
24	.9695	.8316	.7091	.5512	.3814	.2302	.1193	.0524	.0060	.0003
26	.9689	.8247	.6972	.5345	.3625	.2129	.1065	.0449	.0046	.0002
28	.9682	.8185	.6866	.5198	.3461	.1953	.0962	.0389	.0036	.0001
30	.9677	.8129	.6770	.5067	.3318	.1859	.0876	.0343	.0029	.0001
40	.9655	.7911	.6406	.4584	.2815	.1447	.0611	.0209	.0012	.0000
60	.9630	.7658	.5995	.4070	.2318	.1079	.0402	.0118	.0005	.0000
120	.9600	.7362	.5536	.3531	.1843	.0765	.0247	.0061	.0002	.0000
∞	.9542	.6745	.4620	.2549	.1085	.0345	.0080	.0014	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 10, A = 0.01$										
2	.9873	.9791	.9711	.9688	.9624	.9552	.9472	.9385	.9188	.8963
4	.9844	.9625	.9166	.9256	.8989	.8660	.8271	.7825	.6786	.5625
6	.9817	.9433	.9123	.8690	.8124	.7428	.6622	.5712	.3938	.2367
8	.9794	.9240	.8765	.8094	.7225	.6195	.5072	.3947	.2026	.0823
10	.9774	.9057	.8422	.7529	.6404	.5131	.3842	.2672	.1017	.0277
12	.9756	.8888	.8106	.7022	.5696	.4273	.2935	.1831	.0523	.0097
14	.9741	.8734	.7822	.6575	.5101	.3597	.2279	.1285	.0282	.0037
16	.9727	.8594	.7567	.6187	.4605	.3068	.1805	.0928	.0160	.0015
18	.9715	.8469	.7341	.5850	.4193	.2652	.1459	.0690	.0096	.0007
20	.9704	.8355	.7139	.5558	.3848	.2322	.1201	.0527	.0060	.0003
22	.9694	.8253	.6959	.5303	.3558	.2057	.1007	.0413	.0039	.0002
24	.9685	.8160	.6798	.5080	.3312	.1841	.0858	.0331	.0027	.0001
26	.9677	.8076	.6653	.4883	.3102	.1665	.0741	.0270	.0019	.0001
28	.9670	.7999	.6523	.4709	.2921	.1518	.0649	.0225	.0014	.0000
30	.9664	.7929	.6405	.4555	.2764	.1395	.0574	.0190	.0010	.0000
40	.9638	.7655	.5955	.3989	.2220	.0998	.0355	.0098	.0003	.0000
60	.9606	.7328	.5443	.3390	.1702	.0668	.0200	.0045	.0001	.0000
120	.9568	.6929	.4869	.2776	.1232	.0411	.0101	.0018	.0000	.0000
∞	.9520	.6475	.4234	.2170	.0832	.0230	.0045	.0007	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 12, A = 0.01$										
2	.9873	.9793	.9746	.9691	.9628	.9557	.9478	.9392	.9198	.8977
4	.9844	.9627	.9169	.9260	.8993	.8665	.8276	.7829	.6790	.5628
6	.9818	.9432	.9118	.8679	.8104	.7398	.6581	.5689	.3872	.2304
8	.9794	.9231	.8743	.8052	.7158	.6101	.4956	.3819	.1910	.0750
10	.9773	.9035	.8375	.7445	.6277	.4968	.3661	.2494	.0900	.0228
12	.9754	.8850	.8029	.6890	.5509	.4050	.2708	.1635	.0429	.0071
14	.9738	.8680	.7713	.6396	.4860	.3329	.2030	.1092	.0212	.0024
16	.9723	.8523	.7426	.5963	.4318	.2768	.1549	.0749	.0110	.0009
18	.9710	.8380	.7169	.5585	.3868	.2333	.1206	.0529	.0060	.0003
20	.9698	.8250	.6938	.5256	.3493	.1991	.0958	.0384	.0035	.0001
22	.9687	.8131	.6730	.4969	.3179	.1721	.0775	.0286	.0021	.0001
24	.9677	.8023	.6543	.4717	.2914	.1505	.0638	.0219	.0013	.0000
26	.9668	.7924	.6376	.4496	.2690	.1330	.0533	.0171	.0009	.0000
28	.9659	.7833	.6223	.4300	.2498	.1187	.0452	.0136	.0006	.0000
30	.9652	.7750	.6086	.4127	.2333	.1069	.0389	.0110	.0004	.0000
40	.9622	.7419	.5556	.3492	.1770	.0701	.0212	.0048	.0001	.0000
60	.9584	.7019	.4951	.2833	.1256	.0417	.0101	.0018	.0000	.0000
120	.9537	.6534	.4271	.2173	.0819	.0220	.0041	.0005	.0000	.0000
∞	.9476	.5948	.3530	.1552	.0479	.0100	.0015	.0001	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 1, A = 0.025$										
2	.9630	.9280	.9081	.8851	.8592	.8309	.8003	.7677	.6982	.6252
4	.9546	.8779	.8463	.7962	.7381	.6735	.6041	.5321	.3898	.2636
6	.9499	.8645	.8102	.7444	.6689	.5864	.5003	.4148	.2600	.1430
8	.9470	.8502	.7882	.7135	.6284	.5369	.4438	.3539	.2008	.0969
10	.9451	.8407	.7737	.6933	.6025	.5060	.4094	.3182	.1689	.0747
12	.9437	.8340	.7636	.6793	.5847	.4850	.3865	.2949	.1495	.0622
14	.9427	.8290	.7560	.6690	.5717	.4700	.3703	.2788	.1365	.0543
16	.9419	.8251	.7503	.6611	.5619	.4587	.3583	.2670	.1273	.0489
18	.9413	.8221	.7457	.6549	.5542	.4499	.3490	.2579	.1205	.0451
20	.9408	.8196	.7420	.6499	.5480	.4428	.3417	.2508	.1152	.0422
22	.9404	.8176	.7389	.6459	.5429	.4371	.3357	.2451	.1110	.0399
24	.9401	.8158	.7364	.6423	.5386	.4323	.3307	.2404	.1076	.0381
26	.9398	.8144	.7342	.6393	.5350	.4281	.3266	.2364	.1048	.0367
28	.9395	.8131	.7323	.6368	.5319	.4248	.3230	.2330	.1024	.0354
30	.9393	.8120	.7306	.6346	.5292	.4218	.3199	.2301	.1004	.0344
40	.9385	.8081	.7248	.6268	.5198	.4114	.3093	.2202	.0937	.0310
60	.9376	.8040	.7189	.6190	.5104	.4010	.2989	.2105	.0872	.0290
120	.9368	.8000	.7129	.6111	.5010	.3907	.2887	.2012	.0812	.0252
∞	.9359	.7958	.7069	.6031	.4915	.3805	.2786	.1922	.0756	.0227

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 2, A = 0.025$										
2	.9659	.9391	.9237	.9059	.8858	.8635	.8392	.8132	.7567	.6957
4	.9585	.9017	.8650	.8195	.7658	.7047	.6380	.5675	.4249	.2943
6	.9536	.8753	.8225	.7569	.6798	.5941	.5039	.4139	.2519	.1326
8	.9504	.8570	.7935	.7146	.6232	.5241	.4234	.3277	.1708	.0732
10	.9481	.8441	.7729	.6851	.5847	.4780	.3726	.2761	.1285	.0474
12	.9464	.8344	.7578	.6647	.5573	.4460	.3385	.2427	.1038	.0342
14	.9451	.8270	.7462	.6476	.5369	.4226	.3142	.2197	.0881	.0267
16	.9440	.8212	.7371	.6350	.5211	.4049	.2962	.2030	.0773	.0219
18	.9432	.8165	.7298	.6249	.5087	.3911	.2823	.1904	.0696	.0187
20	.9425	.8120	.7238	.6166	.4986	.3800	.2713	.1806	.0639	.0164
22	.9419	.8093	.7187	.6098	.4903	.3709	.2624	.1728	.0594	.0147
24	.9414	.8065	.7145	.6040	.4833	.3634	.2551	.1664	.0559	.0134
26	.9410	.8041	.7108	.5990	.4773	.3570	.2490	.1611	.0530	.0124
28	.9406	.8021	.7077	.5948	.4722	.3515	.2438	.1566	.0506	.0116
30	.9403	.8002	.7049	.5911	.4678	.3468	.2393	.1528	.0486	.0109
40	.9391	.7937	.6949	.5778	.4521	.3303	.2238	.1399	.0422	.0088
60	.9379	.7868	.6847	.5642	.4363	.3140	.2088	.1277	.0364	.0071
120	.9366	.7798	.6741	.5504	.4204	.2979	.1944	.1161	.0313	.0056
∞	.9353	.7724	.6632	.5364	.4045	.2820	.1804	.1053	.0268	.0045

F_2	$F_1 = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 3, A = 0.025$										
2	.9669	.9429	.9291	.9131	.8949	.8748	.8528	.8292	.7776	.7214
4	.9598	.9064	.8710	.8270	.7745	.7144	.6482	.5780	.4348	.3027
6	.9548	.8776	.8241	.7568	.6770	.5880	.4945	.4015	.2369	.1195
8	.9512	.8563	.7895	.7057	.6081	.5028	.3971	.2985	.1441	.0556
10	.9480	.8403	.7637	.6684	.5594	.4449	.3316	.2368	.0977	.0307
12	.9466	.8281	.7441	.6405	.5238	.4042	.2925	.1975	.0721	.0193
14	.9451	.8184	.7288	.6189	.4970	.3743	.2627	.1709	.0566	.0133
16	.9438	.8107	.7165	.6020	.4761	.3516	.2408	.1520	.0466	.0099
18	.9428	.8043	.7065	.5882	.4595	.3339	.2240	.1379	.0397	.0077
20	.9419	.7989	.6982	.5769	.4461	.3198	.2109	.1272	.0347	.0063
22	.9411	.7944	.6912	.5675	.4349	.3082	.2004	.1188	.0310	.0053
24	.9405	.7906	.6852	.5595	.4255	.2986	.1918	.1120	.0281	.0046
26	.9400	.7872	.6801	.5526	.4175	.2905	.1846	.1064	.0259	.0041
28	.9395	.7843	.6756	.5466	.4107	.2836	.1786	.1018	.0241	.0036
30	.9390	.7817	.6716	.5414	.4047	.2777	.1734	.0979	.0226	.0033
40	.9375	.7723	.6574	.5227	.3836	.2570	.1559	.0849	.0179	.0023
60	.9358	.7624	.6424	.5035	.3624	.2368	.1392	.0730	.0140	.0016
120	.9341	.7519	.6269	.4839	.3411	.2171	.1236	.0624	.0109	.0011
∞	.9322	.7408	.6107	.4638	.3199	.1981	.1090	.0529	.0083	.0008

F_2	$F_1 = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 4, A = 0.025$										
2	.9674	.9448	.9318	.9167	.8996	.8806	.8598	.8373	.7883	.7317
4	.9605	.9086	.8740	.8205	.7784	.7186	.6526	.5823	.4387	.3057
6	.9553	.8781	.8237	.7546	.6725	.5808	.4848	.3899	.2242	.1092
8	.9515	.8543	.7844	.6963	.5938	.4837	.3747	.2749	.1244	.0442
10	.9487	.8357	.7542	.6523	.5364	.4194	.3034	.2064	.0768	.0210
12	.9464	.8211	.7305	.6186	.4939	.3687	.2556	.1636	.0519	.0115
14	.9446	.8094	.7117	.5923	.4615	.3337	.2221	.1353	.0378	.0071
16	.9431	.7998	.6964	.5713	.4363	.3072	.1978	.1156	.0291	.0047
18	.9419	.7918	.6838	.5541	.4162	.2865	.1794	.1014	.0234	.0034
20	.9408	.7850	.6733	.5400	.3998	.2702	.1652	.0907	.0194	.0026
22	.9399	.7792	.6643	.5281	.3862	.2569	.1540	.0825	.0166	.0020
24	.9392	.7743	.6567	.5180	.3748	.2459	.1448	.0760	.0145	.0016
26	.9385	.7699	.6500	.5093	.3651	.2366	.1373	.0707	.0128	.0014
28	.9379	.7661	.6442	.5017	.3568	.2288	.1310	.0664	.0116	.0012
30	.9374	.7627	.6390	.4951	.3495	.2220	.1256	.0628	.0106	.0010
40	.9354	.7504	.6204	.4714	.3240	.1989	.1078	.0512	.0076	.0006
60	.9334	.7371	.6007	.4469	.2955	.1766	.0915	.0411	.0053	.0004
120	.9311	.7229	.5799	.4218	.2733	.1554	.0767	.0325	.0036	.0002
∞	.9286	.7077	.5581	.3961	.2483	.1355	.0636	.0254	.0024	.0001

F_2	$F_1 = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 5, A = 0.025$										
2	.9677	.9460	.9335	.9189	.9024	.8840	.8640	.8423	.7948	.7428
4	.9609	.9099	.8756	.8225	.7806	.7209	.6548	.5845	.4404	.3069
6	.9556	.8780	.8227	.7522	.6681	.5744	.4764	.3802	.2141	.1015
8	.9516	.8511	.7796	.6879	.5813	.4676	.3564	.2562	.1100	.0364
10	.9485	.8314	.7455	.6381	.5167	.3928	.2784	.1831	.0624	.0152
12	.9460	.8146	.7182	.5993	.4683	.3394	.2265	.1383	.0389	.0074
14	.9440	.8009	.6961	.5686	.4312	.3001	.1906	.1094	.0262	.0040
16	.9423	.7895	.6780	.5439	.4022	.2710	.1649	.0899	.0189	.0024
18	.9409	.7800	.6630	.5238	.3791	.2483	.1458	.0761	.0143	.0016
20	.9397	.7718	.6503	.5070	.3602	.2303	.1311	.0660	.0112	.0011
22	.9387	.7648	.6395	.4929	.3447	.2158	.1197	.0583	.0092	.0005
24	.9378	.7587	.6301	.4808	.3316	.2019	.1105	.0524	.0077	.0006
26	.9370	.7534	.6220	.4704	.3205	.1940	.1031	.0477	.0066	.0005
28	.9363	.7487	.6149	.4614	.3110	.1856	.0969	.0439	.0057	.0004
30	.9357	.7446	.6086	.4535	.3027	.1784	.0917	.0407	.0051	.0003
40	.9334	.7291	.5856	.4252	.2739	.1541	.0748	.0210	.0032	.0002
60	.9308	.7124	.5611	.3959	.2453	.1312	.0598	.0230	.0020	.0001
120	.9280	.6942	.5351	.3659	.2173	.1101	.0470	.0167	.0012	.0000
∞	.9249	.6745	.5077	.3355	.1903	.0910	.0561	.0118	.0007	.0000

F_2	$F_1 = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 6, A = 0.025$										
2	.9679	.9467	.9346	.9204	.9043	.8864	.8668	.8456	.7992	.7482
4	.9611	.9107	.8767	.8237	.7819	.7222	.6562	.5857	.4413	.3074
6	.9558	.8778	.8216	.7498	.6642	.5688	.4693	.3721	.2061	.0956
8	.9517	.8501	.7754	.6806	.5707	.4542	.3414	.2413	.0990	.0310
10	.9483	.8274	.7377	.6258	.4999	.3731	.2582	.1649	.0521	.0115
12	.9457	.8087	.7072	.5824	.4464	.3152	.2023	.1191	.0301	.0050
14	.9434	.7932	.6822	.5478	.4053	.2720	.1659	.0903	.0189	.0024
16	.9416	.7802	.6615	.5199	.3731	.2414	.1395	.0713	.0127	.0013
18	.9400	.7691	.6441	.4969	.3474	.2172	.1201	.0583	.0091	.0008
20	.9378	.7596	.6294	.4777	.3266	.1982	.1056	.0489	.0068	.0005
22	.9375	.7514	.6167	.4616	.3094	.1830	.0932	.0420	.0052	.0003
24	.9365	.7443	.6058	.4478	.2956	.1706	.0855	.0368	.0042	.0002
26	.9355	.7380	.5963	.4359	.2828	.1603	.0783	.0327	.0035	.0002
28	.9347	.7324	.5879	.4256	.2724	.1516	.0725	.0295	.0029	.0001
30	.9340	.7274	.5805	.4165	.2634	.1443	.0676	.0268	.0025	.0001
40	.9313	.7089	.5532	.3840	.2320	.1198	.0522	.0189	.0014	.0000
60	.9283	.6886	.5240	.3506	.2014	.0975	.0392	.0129	.0008	.0000
120	.9249	.6682	.4930	.3166	.1720	.0775	.0285	.0085	.0004	.0000
∞	.9211	.6418	.4601	.2823	.1443	.0602	.0022	.0053	.0002	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 7, A = 0.025$										
2	.9680	.9473	.9354	.9214	.9056	.8880	.8688	.8480	.8223	.7522
4	.9613	.9413	.8774	.8345	.7828	.7231	.6570	.5864	.4117	.3076
6	.9550	.8774	.8206	.7477	.6608	.5640	.4634	.3654	.1996	.0909
8	.9516	.8483	.7716	.6743	.5617	.4430	.3290	.2292	.0906	.0270
10	.9482	.8238	.7310	.6150	.4850	.3655	.2417	.1505	.0445	.0090
12	.9453	.8034	.6974	.5677	.4276	.2949	.1846	.1042	.0236	.0035
14	.9429	.7862	.6697	.5296	.3831	.2503	.1463	.0759	.0140	.0015
16	.9409	.7716	.6465	.4986	.3481	.2171	.1196	.0577	.0089	.0008
18	.9392	.7592	.6270	.4731	.3204	.1918	.1004	.0455	.0059	.0004
20	.9377	.7484	.6164	.4518	.2979	.1722	.0862	.0370	.0042	.0002
22	.9364	.7391	.5961	.4339	.2794	.1560	.0754	.0309	.0031	.0002
24	.9352	.7308	.5836	.4185	.2640	.1439	.0670	.0263	.0024	.0001
26	.9342	.7236	.5727	.4053	.2569	.1336	.0603	.0228	.0019	.0001
28	.9333	.7171	.5631	.3938	.2498	.1249	.0549	.0201	.0015	.0001
30	.9325	.7114	.5546	.3837	.2432	.1176	.0504	.0180	.0013	.0000
40	.9294	.6897	.5233	.3476	.1972	.0937	.0367	.0117	.0006	.0000
60	.9259	.6657	.4896	.3106	.1655	.0725	.0257	.0073	.0003	.0000
120	.9219	.6391	.4537	.2733	.1357	.0543	.0172	.0043	.0001	.0000
∞	.9173	.6098	.4157	.2301	.1084	.0393	.0111	.0024	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 8, A = 0.025$										
2	.9681	.9477	.9359	.9222	.9066	.8893	.8703	.8498	.8047	.7551
4	.9615	.9118	.8779	.8351	.7834	.7237	.6575	.5869	.4419	.3076
6	.9560	.8771	.8197	.7459	.6578	.5599	.4583	.3599	.1942	.0871
8	.9516	.8467	.7683	.6683	.5539	.4334	.3187	.2192	.0839	.0240
10	.9480	.8207	.7250	.6057	.4732	.3425	.2280	.1388	.0387	.0073
12	.9449	.7986	.6888	.5547	.4114	.2779	.1693	.0925	.0195	.0026
14	.9424	.7799	.6586	.5135	.3638	.2312	.1304	.0648	.0107	.0010
16	.9402	.7639	.6331	.4798	.3266	.1968	.1038	.0475	.0064	.0005
18	.9384	.7501	.6116	.4521	.2971	.1709	.0850	.0362	.0040	.0002
20	.9367	.7381	.5931	.4288	.2732	.1509	.0713	.0285	.0027	.0001
22	.9353	.7276	.5772	.4092	.2537	.1351	.0610	.0231	.0019	.0001
24	.9340	.7184	.5633	.3925	.2375	.1225	.0531	.0192	.0014	.0000
26	.9329	.7102	.5512	.3781	.2238	.1122	.0470	.0162	.0011	.0000
28	.9319	.7029	.5404	.3655	.2122	.1037	.0420	.0140	.0008	.0000
30	.9310	.6963	.5309	.3545	.2023	.0966	.0380	.0122	.0007	.0000
40	.9275	.6716	.4957	.3154	.1683	.0737	.0260	.0073	.0003	.0000
60	.9236	.6440	.4578	.2755	.1362	.0541	.0169	.0041	.0001	.0000
120	.9190	.6131	.4173	.2356	.1069	.0380	.0104	.0022	.0000	.0000
∞	.9136	.5786	.3745	.1956	.0809	.0254	.0060	.0010	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 9, A = 0.025$										
2	.9682	.9480	.9364	.9228	.9074	.8903	.8715	.8512	.8066	.7573
4	.9616	.9121	.8783	.8356	.7839	.7241	.6579	.5871	.4420	.3075
6	.9561	.8765	.8188	.7443	.6553	.5564	.4541	.3551	.1898	.0840
8	.9516	.8452	.7654	.6640	.5471	.4252	.3098	.2108	.0785	.0216
10	.9478	.8178	.7196	.5974	.4624	.3305	.2165	.1292	.0343	.0061
12	.9446	.7944	.6811	.5433	.3974	.2633	.1567	.0831	.0163	.0019
14	.9420	.7742	.6485	.4992	.3472	.2151	.1175	.0562	.0084	.0007
16	.9396	.7568	.6210	.4631	.3080	.1799	.0911	.0397	.0047	.0003
18	.9376	.7418	.5976	.4333	.2770	.1535	.0728	.0293	.0028	.0001
20	.9359	.7286	.5774	.4083	.2520	.1133	.0597	.0224	.0018	.0001
22	.9343	.7170	.5600	.3873	.2317	.1176	.0590	.0176	.0012	.0000
24	.9329	.7058	.5448	.3693	.2145	.1052	.0427	.0142	.0008	.0000
26	.9317	.6977	.5314	.3538	.2008	.0951	.0370	.0117	.0006	.0000
28	.9306	.6896	.5196	.3404	.1888	.0868	.0326	.0099	.0005	.0000
30	.9296	.6822	.5091	.3286	.1786	.0800	.0290	.0084	.0004	.0000
40	.9258	.6545	.4702	.2868	.1441	.0583	.0186	.0047	.0001	.0000
60	.9213	.6233	.4284	.2446	.1123	.0404	.0112	.0024	.0000	.0000
120	.9161	.5880	.3836	.2029	.0840	.0265	.0062	.0011	.0000	.0000
∞	.9099	.5485	.3366	.1630	.0600	.0163	.0032	.0004	.0000	.0000

F_1	$P = 5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	3.0
$F_1 = 10, A = 0.025$										
2	.9683	.9483	.9368	.9233	.9081	.8911	.8725	.8523	.8081	.7593
4	.9617	.9124	.8787	.8359	.7842	.7244	.6581	.5873	.4420	.3074
6	.9561	.8765	.8181	.7429	.6531	.5534	.4504	.3511	.1860	.0815
8	.9515	.8439	.7628	.6593	.5413	.4181	.3023	.2038	.0741	.0198
10	.9476	.8153	.7149	.5901	.4530	.3202	.2067	.1212	.0308	.0051
12	.9444	.7905	.6741	.5331	.3850	.2508	.1460	.0754	.0138	.0015
14	.9415	.7691	.6395	.4865	.3326	.2014	.1068	.0492	.0067	.0005
16	.9391	.7504	.6100	.4482	.2917	.1654	.0808	.0337	.0036	.0002
18	.9370	.7341	.5848	.4165	.2594	.1389	.0630	.0240	.0020	.0001
20	.9351	.7199	.5631	.3900	.2336	.1188	.0505	.0178	.0012	.0000
22	.9334	.7072	.5442	.3676	.2126	.1032	.0414	.0136	.0008	.0000
24	.9319	.6961	.5277	.3486	.1953	.0910	.0347	.0107	.0005	.0000
26	.9306	.6861	.5132	.3322	.1809	.0812	.0295	.0086	.0004	.0000
28	.9294	.6771	.5004	.3179	.1688	.0732	.0255	.0071	.0003	.0000
30	.9283	.6690	.4800	.3055	.1585	.0667	.0224	.0059	.0002	.0000
40	.9241	.6384	.4467	.2615	.1240	.0464	.0135	.0030	.0001	.0000
60	.9192	.6035	.4011	.2175	.0928	.0304	.0075	.0014	.0000	.0000
120	.9133	.5639	.3528	.1747	.0660	.0185	.0038	.0005	.0000	.0000
∞	.9062	.5193	.3018	.1345	.0442	.0104	.0017	.0002	.0000	.0000

F_1	$P = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 12, A = 0.025$										
2	.9084	.9487	.9373	.9241	.9090	.8923	.8730	.8540	.8103	.7821
4	.9018	.9127	.8791	.8364	.7847	.7249	.6581	.5875	.4420	.3071
6	.9502	.8760	.8108	.7405	.6494	.5184	.4414	.3446	.1801	.0775
8	.9514	.8417	.7585	.6527	.5315	.4064	.2901	.1924	.0672	.0170
10	.9474	.8110	.7069	.5778	.4373	.3032	.1910	.1087	.0250	.0039
12	.9439	.7839	.6622	.5158	.3644	.2305	.1292	.0637	.0104	.0010
14	.9408	.7660	.6238	.4647	.3082	.1792	.0903	.0391	.0046	.0003
16	.9331	.7391	.5909	.4227	.2647	.1426	.0652	.0251	.0022	.0001
18	.9358	.7206	.5625	.3878	.2305	.1159	.0486	.0168	.0011	.0000
20	.9337	.7042	.5379	.3587	.2034	.0961	.0373	.0118	.0006	.0000
22	.9318	.6897	.5165	.3341	.1816	.0812	.0293	.0085	.0004	.0000
24	.9301	.6767	.4977	.3132	.1638	.0696	.0236	.0063	.0002	.0000
26	.9258	.6651	.4811	.2953	.1491	.0605	.0194	.0048	.0001	.0000
28	.9272	.6546	.4665	.2798	.1369	.0533	.0162	.0038	.0001	.0000
30	.9260	.6451	.4534	.2664	.1266	.0474	.0138	.0031	.0001	.0000
40	.9210	.6087	.4050	.2192	.0929	.0360	.0072	.0013	.0000	.0000
60	.9151	.5668	.3528	.1728	.0640	.0173	.0034	.0005	.0000	.0000
120	.9079	.5188	.2977	.1294	.0407	.0090	.0014	.0001	.0000	.0000
∞	.8989	.4642	.2410	.0907	.0236	.0041	.0005	.0000	.0000	.0000

F_1	$P = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 1, A = 0.05$										
2	.9271	.8617	.8256	.7847	.7402	.6927	.6432	.5926	.4915	.3950
4	.9141	.8048	.7415	.6694	.5910	.5095	.4284	.3509	.2160	.1198
6	.9077	.7763	.7010	.6153	.5238	.4315	.3431	.2629	.1374	.0611
8	.9040	.7610	.6784	.5858	.4883	.3916	.3015	.2223	.1054	.0413
10	.9017	.7510	.6642	.5675	.4666	.3680	.2775	.1997	.0890	.0322
12	.9000	.7440	.6544	.5551	.4521	.3524	.2620	.1854	.0793	.0272
14	.8988	.7390	.6474	.5462	.4418	.3414	.2513	.1756	.0728	.0240
16	.8979	.7351	.6420	.5394	.4341	.3333	.2433	.1685	.0683	.0219
18	.8972	.7321	.6379	.5342	.4281	.3270	.2373	.1631	.0649	.0203
20	.8966	.7297	.6345	.5300	.4233	.3220	.2325	.1589	.0623	.0192
22	.8961	.7277	.6317	.5265	.4194	.3180	.2287	.1555	.0603	.0183
24	.8957	.7260	.6294	.5236	.4161	.3146	.2255	.1527	.0586	.0175
26	.8954	.7246	.6274	.5212	.4134	.3118	.2223	.1504	.0573	.0169
28	.8951	.7233	.6258	.5192	.4111	.3094	.2206	.1485	.0561	.0165
30	.8948	.7223	.6243	.5173	.4090	.3073	.2186	.1468	.0551	.0160
40	.8939	.7185	.6192	.5110	.4020	.3001	.2119	.1410	.0518	.0147
60	.8930	.7147	.6140	.5047	.3949	.2930	.2053	.1354	.0487	.0134
120	.8920	.7108	.6087	.4983	.3879	.2859	.1988	.1300	.0457	.0123
∞	.8910	.7070	.6036	.4920	.3810	.2791	.1926	.1248	.0430	.0112

F_1	$P = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 2, A = 0.05$										
2	.9324	.8814	.8527	.8201	.7840	.7451	.7038	.6608	.5722	.4537
4	.9201	.8239	.7657	.6976	.6219	.5414	.4598	.3804	.2400	.1353
6	.9129	.7891	.7135	.6257	.5303	.4330	.3396	.2554	.1264	.0520
8	.9083	.7672	.6810	.5821	.4769	.3729	.2773	.1955	.0821	.0273
10	.9052	.7523	.6592	.5538	.4430	.3361	.2408	.1624	.0609	.0175
12	.9030	.7417	.6438	.5336	.4197	.3115	.2173	.1419	.0490	.0126
14	.9013	.7337	.6323	.5189	.4029	.2941	.2010	.1281	.0416	.0099
16	.9000	.7274	.6234	.5077	.3901	.2812	.1892	.1183	.0367	.0082
18	.8989	.7225	.6164	.4988	.3802	.2713	.1802	.1110	.0331	.0071
20	.8980	.7184	.6107	.4917	.3723	.2634	.1732	.1054	.0305	.0063
22	.8973	.7150	.6059	.4858	.3658	.2570	.1675	.1009	.0285	.0057
24	.8967	.7122	.6019	.4808	.3603	.2517	.1629	.0973	.0269	.0052
26	.8961	.7097	.5985	.4767	.3558	.2472	.1590	.0943	.0256	.0048
28	.8957	.7076	.5956	.4730	.3518	.2434	.1558	.0918	.0245	.0045
30	.8953	.7058	.5930	.4699	.3484	.2401	.1530	.0896	.0236	.0043
40	.8938	.6992	.5830	.4538	.3265	.2288	.1434	.0824	.0207	.0035
60	.8923	.6924	.5746	.4478	.3247	.2177	.1341	.0756	.0181	.0029
120	.8908	.6855	.5651	.4364	.3129	.2069	.1253	.0692	.0157	.0024
∞	.8892	.6785	.5556	.4251	.3013	.1963	.1168	.0632	.0137	.0019

F_1	$P = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 3, A = 0.05$										
2	.9342	.8882	.8623	.8327	.7998	.7640	.7260	.6861	.6030	.5187
4	.9221	.8302	.7735	.7064	.6311	.5505	.4653	.3880	.2453	.1384
6	.9144	.7909	.7134	.6226	.5235	.4225	.3264	.2407	.1132	.0435
8	.9092	.7643	.6733	.5683	.4570	.3482	.2504	.1694	.0639	.0184
10	.9056	.7454	.6453	.5314	.4134	.3019	.2059	.1307	.0419	.0098
12	.9028	.7314	.6249	.5050	.3831	.2709	.1776	.1074	.0205	.0061
14	.9007	.7207	.6093	.4853	.3611	.2490	.1583	.0922	.0238	.0042
16	.8990	.7122	.5972	.4701	.3443	.2328	.1444	.0817	.0196	.0031
18	.8976	.7054	.5874	.4581	.3313	.2204	.1340	.0740	.0167	.0025
20	.8965	.6997	.5794	.4483	.3208	.2106	.1259	.0682	.0146	.0020
22	.8955	.6950	.5729	.4402	.3122	.2038	.1195	.0637	.0131	.0017
24	.8947	.6909	.5671	.4333	.3051	.1961	.1143	.0601	.0119	.0015
26	.8940	.6875	.5623	.4275	.2990	.1907	.1100	.0571	.0110	.0013
28	.8934	.6845	.5581	.4225	.2938	.1860	.1064	.0547	.0103	.0012
30	.8928	.6818	.5544	.4182	.2894	.1820	.1033	.0526	.0097	.0011
40	.8909	.6723	.5414	.4028	.2738	.1684	.0930	.0458	.0078	.0008
60	.8888	.6624	.5279	.3872	.2583	.1552	.0833	.0397	.0062	.0006
120	.8866	.6522	.5142	.3716	.2431	.1425	.0743	.0342	.0049	.0004
∞	.8843	.6415	.5000	.3557	.2280	.1304	.0659	.0293	.0038	.0003

F_1	$F_2 = 0.5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 4, A = 0.05$										
2	.9351	.8917	.8672	.8391	.8079	.7738	.7375	.6993	.6193	.5374
4	.9232	.8332	.7771	.7103	.6350	.5542	.4714	.3905	.2466	.1389
6	.9151	.7908	.7112	.6178	.5158	.4122	.3143	.2282	.1030	.0375
8	.9094	.7602	.6649	.5549	.4389	.3271	.2286	.1493	.0515	.0132
10	.9052	.7378	.6315	.5110	.3876	.2736	.1788	.1076	.0301	.0059
12	.9020	.7208	.6066	.4791	.3516	.2389	.1475	.0833	.0199	.0032
14	.8995	.7076	.5875	.4550	.3253	.2129	.1266	.0680	.0143	.0019
16	.8975	.6970	.5723	.4363	.3054	.1945	.1118	.0577	.0109	.0013
18	.8958	.6883	.5600	.4214	.2898	.1804	.1009	.0503	.0087	.0009
20	.8945	.6811	.5498	.4092	.2774	.1695	.0926	.0449	.0073	.0007
22	.8933	.6750	.5413	.3991	.2672	.1607	.0861	.0408	.0062	.0006
24	.8923	.6698	.5341	.3907	.2587	.1535	.0808	.0376	.0054	.0005
26	.8914	.6653	.5279	.3834	.2516	.1475	.0765	.0349	.0049	.0004
28	.8906	.6614	.5225	.3772	.2455	.1424	.0730	.0325	.0044	.0003
30	.8899	.6579	.5178	.3718	.2402	.1381	.0700	.0311	.0040	.0003
40	.8874	.6454	.5009	.3526	.2219	.1234	.0601	.0254	.0029	.0002
60	.8848	.6322	.4833	.3332	.2040	.1095	.0511	.0206	.0021	.0001
120	.8819	.6183	.4652	.3136	.1865	.0965	.0431	.0164	.0015	.0001
∞	.8789	.6038	.4466	.2940	.1695	.0844	.0360	.0130	.0011	.0000

F_1	$F_2 = 0.5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 5, A = 0.05$										
2	.9356	.8939	.8702	.8431	.8128	.7798	.7445	.7074	.6293	.5490
4	.9238	.8349	.7791	.7124	.6369	.5558	.4727	.3914	.2467	.1386
6	.9154	.7897	.7087	.6131	.5088	.4033	.3044	.2181	.0952	.0333
8	.9093	.7561	.6573	.5432	.4237	.3099	.2115	.1342	.0430	.0100
10	.9048	.7308	.6193	.4933	.3660	.2509	.1579	.0909	.0227	.0038
12	.9012	.7111	.5904	.4566	.3254	.2118	.1249	.0665	.0136	.0018
14	.8983	.6956	.5679	.4287	.2957	.1845	.1033	.0516	.0090	.0010
16	.8960	.6829	.5499	.4069	.2732	.1646	.0883	.0418	.0064	.0006
18	.8941	.6725	.5352	.3895	.2557	.1496	.0774	.0351	.0048	.0004
20	.8924	.6638	.5231	.3753	.2417	.1350	.0693	.0303	.0038	.0003
22	.8910	.6564	.5129	.3635	.2303	.1288	.0630	.0267	.0031	.0002
24	.8898	.6501	.5042	.3536	.2209	.1213	.0580	.0240	.0026	.0001
26	.8888	.6445	.4967	.3451	.2129	.1151	.0540	.0218	.0022	.0001
28	.8878	.6397	.4901	.3379	.2062	.1099	.0507	.0201	.0019	.0001
30	.8870	.6354	.4844	.3315	.2003	.1055	.0479	.0186	.0017	.0001
40	.8840	.6198	.4638	.3091	.1803	.0908	.0390	.0142	.0011	.0000
60	.8807	.6033	.4423	.2864	.1609	.0772	.0313	.0106	.0007	.0000
120	.8771	.5857	.4201	.2638	.1422	.0649	.0247	.0073	.0004	.0000
∞	.8733	.5671	.3971	.2412	.1245	.0538	.0192	.0056	.0003	.0000

F_1	$F_2 = 0.5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 6, A = 0.05$										
2	.9360	.8953	.8722	.8457	.8161	.7839	.7493	.7129	.6361	.5569
4	.9242	.8361	.7803	.7136	.6380	.5567	.4733	.3916	.2464	.1381
6	.9156	.7887	.7083	.6090	.5028	.3959	.2962	.2100	.0893	.0301
8	.9092	.7525	.6506	.5332	.4109	.2958	.1978	.1225	.0369	.0080
10	.9042	.7245	.6086	.4782	.3480	.2325	.1417	.0784	.0177	.0026
12	.9003	.7024	.5761	.4373	.3036	.1908	.1077	.0544	.0097	.0011
14	.8972	.6847	.5506	.4061	.2711	.1619	.0859	.0401	.0059	.0005
16	.8946	.6702	.5301	.3816	.2465	.1412	.0710	.0312	.0039	.0003
18	.8924	.6582	.5132	.3621	.2275	.1257	.0605	.0252	.0028	.0002
20	.8905	.6480	.4992	.3461	.2124	.1139	.0528	.0210	.0021	.0001
22	.8889	.6394	.4874	.3328	.2001	.1045	.0469	.0179	.0016	.0001
24	.8875	.6319	.4773	.3216	.1900	.0970	.0423	.0157	.0013	.0001
26	.8862	.6253	.4686	.3121	.1815	.0908	.0387	.0139	.0011	.0000
28	.8852	.6196	.4610	.3039	.1744	.0857	.0357	.0125	.0009	.0000
30	.8843	.6145	.4543	.2968	.1682	.0814	.0333	.0114	.0008	.0000
40	.8807	.5960	.4302	.2717	.1471	.0672	.0256	.0081	.0004	.0000
60	.8768	.5760	.4050	.2464	.1270	.0545	.0193	.0055	.0002	.0000
120	.8724	.5547	.3789	.2214	.1082	.0434	.0141	.0037	.0001	.0000
∞	.8677	.5319	.3520	.1967	.0907	.0339	.0101	.0024	.0000	.0000

F_1	$F_2 = 0.5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 7, A = 0.05$										
2	.9363	.8963	.8736	.8476	.8185	.7868	.7527	.7168	.6410	.5627
4	.9245	.8368	.7811	.7144	.6387	.5571	.4735	.3916	.2460	.1376
6	.9157	.7887	.7042	.6054	.4978	.3897	.2905	.2035	.0846	.0278
8	.9090	.7492	.6449	.5247	.4002	.2841	.1868	.1133	.0323	.0065
10	.9038	.7189	.5992	.4652	.3328	.2174	.1288	.0689	.0143	.0019
12	.8996	.6947	.5636	.4207	.2852	.1738	.0944	.0454	.0072	.0007
14	.8961	.6750	.5353	.3866	.2505	.1439	.0726	.0320	.0041	.0003
16	.8933	.6588	.5125	.3598	.2243	.1226	.0582	.0238	.0025	.0001
18	.8908	.6452	.4936	.3383	.2041	.1070	.0482	.0185	.0017	.0001
20	.8888	.6336	.4779	.3208	.1882	.0951	.0409	.0149	.0012	.0000
22	.8870	.6238	.4646	.3062	.1753	.0853	.0355	.0123	.0009	.0000
24	.8854	.6152	.4532	.2940	.1647	.0785	.0314	.0105	.0007	.0000
26	.8840	.6077	.4433	.2836	.1559	.0725	.0282	.0091	.0005	.0000
28	.8828	.6011	.4347	.2747	.1485	.0676	.0256	.0080	.0004	.0000
30	.8817	.5952	.4272	.2669	.1421	.0634	.0234	.0071	.0003	.0000
40	.8776	.5737	.3998	.2396	.1206	.0501	.0170	.0046	.0002	.0000
60	.8730	.5504	.3713	.2124	.1005	.0387	.0119	.0029	.0001	.0000
120	.8679	.5253	.3417	.1857	.0821	.0290	.0080	.0017	.0000	.0000
∞	.8622	.4983	.3112	.1597	.0656	.0211	.0052	.0010	.0000	.0000

F_2	$P = .5$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.6	3.0
$F_1 = 8, A = 0.05$										
2	.9305	.8971	.8747	.8490	.8203	.7889	.7553	.7198	.6118	.5671
4	.9274	.8874	.8617	.8149	.7891	.7574	.7235	.6914	.5456	.5371
6	.9158	.8789	.8504	.8023	.7805	.7445	.7139	.6839	.5009	.5259
8	.9088	.8763	.8498	.8017	.7800	.7444	.7177	.6899	.4889	.5055
10	.9053	.8740	.8490	.8010	.7800	.7449	.7184	.6915	.4818	.5014
12	.8989	.8678	.8426	.8003	.7800	.7457	.7198	.6937	.4755	.5005
14	.8951	.8663	.8418	.8000	.7800	.7462	.7202	.6940	.4759	.5002
16	.8920	.8648	.8408	.8000	.7800	.7466	.7206	.6943	.4760	.5001
18	.8894	.8633	.8397	.8000	.7800	.7470	.7210	.6946	.4761	.5000
20	.8871	.8620	.8388	.8000	.7800	.7474	.7214	.6947	.4762	.5000
22	.8851	.8609	.8382	.8000	.7800	.7478	.7218	.6948	.4763	.5000
24	.8834	.8599	.8375	.8000	.7800	.7482	.7222	.6949	.4764	.5000
26	.8819	.8591	.8370	.8000	.7800	.7486	.7226	.6950	.4765	.5000
28	.8805	.8584	.8361	.8000	.7800	.7490	.7230	.6951	.4766	.5000
30	.8793	.8577	.8357	.8000	.7800	.7494	.7234	.6952	.4767	.5000
40	.8746	.8530	.8325	.8000	.7800	.7509	.7249	.6957	.4772	.5000
60	.8694	.8504	.8308	.8000	.7800	.7524	.7264	.6962	.4777	.5000
120	.8635	.8475	.8301	.8000	.7800	.7539	.7279	.6967	.4782	.5000
∞	.8568	.8463	.8245	.8000	.7800	.7554	.7294	.6972	.4787	.5000

$F_1 = 9, A = 0.05$										
2	.9366	.8977	.8756	.8501	.8217	.7906	.7573	.7221	.6477	.5705
4	.9249	.8878	.8621	.8153	.7894	.7575	.7235	.6914	.5452	.5362
6	.9158	.8789	.8504	.8023	.7805	.7445	.7139	.6839	.5009	.5245
8	.9088	.8763	.8498	.8017	.7800	.7444	.7177	.6899	.4889	.5048
10	.9029	.8706	.8458	.8000	.7800	.7449	.7184	.6915	.4818	.5011
12	.8982	.8686	.8428	.8000	.7800	.7457	.7198	.6937	.4755	.5003
14	.8943	.8684	.8427	.8000	.7800	.7462	.7202	.6940	.4759	.5001
16	.8909	.8690	.8427	.8000	.7800	.7466	.7206	.6943	.4760	.5000
18	.8881	.8686	.8418	.8000	.7800	.7470	.7210	.6946	.4761	.5000
20	.8856	.8686	.8416	.8000	.7800	.7474	.7214	.6947	.4762	.5000
22	.8835	.8686	.8416	.8000	.7800	.7478	.7218	.6948	.4763	.5000
24	.8816	.8686	.8416	.8000	.7800	.7482	.7222	.6949	.4764	.5000
26	.8799	.8686	.8416	.8000	.7800	.7486	.7226	.6950	.4765	.5000
28	.8784	.8686	.8416	.8000	.7800	.7490	.7230	.6951	.4766	.5000
30	.8770	.8686	.8416	.8000	.7800	.7494	.7234	.6952	.4767	.5000
40	.8718	.8637	.8377	.8000	.7800	.7509	.7249	.6957	.4772	.5000
60	.8660	.8608	.8333	.8000	.7800	.7524	.7264	.6962	.4777	.5000
120	.8592	.8542	.8333	.8000	.7800	.7539	.7279	.6967	.4782	.5000
∞	.8514	.8461	.8247	.8000	.7800	.7554	.7294	.6972	.4787	.5000

$F_1 = 10, A = 0.05$										
2	.9368	.8981	.8762	.8510	.8228	.7920	.7589	.7240	.6500	.5732
4	.9250	.8881	.8625	.8156	.7895	.7575	.7235	.6914	.5448	.5362
6	.9158	.8784	.8504	.8023	.7805	.7445	.7139	.6839	.5009	.5233
8	.9085	.8747	.8498	.8017	.7800	.7444	.7177	.6899	.4889	.5042
10	.9026	.8707	.8457	.8000	.7800	.7449	.7184	.6915	.4818	.5009
12	.8976	.8686	.8428	.8000	.7800	.7457	.7198	.6937	.4755	.5002
14	.8935	.8684	.8427	.8000	.7800	.7462	.7202	.6940	.4759	.5001
16	.8900	.8690	.8427	.8000	.7800	.7466	.7206	.6943	.4760	.5000
18	.8871	.8686	.8418	.8000	.7800	.7470	.7210	.6946	.4761	.5000
20	.8843	.8686	.8416	.8000	.7800	.7474	.7214	.6947	.4762	.5000
22	.8819	.8686	.8416	.8000	.7800	.7478	.7218	.6948	.4763	.5000
24	.8799	.8686	.8416	.8000	.7800	.7482	.7222	.6949	.4764	.5000
26	.8780	.8686	.8416	.8000	.7800	.7486	.7226	.6950	.4765	.5000
28	.8764	.8686	.8416	.8000	.7800	.7490	.7230	.6951	.4766	.5000
30	.8749	.8686	.8416	.8000	.7800	.7494	.7234	.6952	.4767	.5000
40	.8692	.8637	.8353	.8000	.7800	.7509	.7249	.6957	.4772	.5000
60	.8627	.8587	.8333	.8000	.7800	.7524	.7264	.6962	.4777	.5000
120	.8551	.8547	.8333	.8000	.7800	.7539	.7279	.6967	.4782	.5000
∞	.8462	.8447	.8244	.8000	.7800	.7554	.7294	.6972	.4787	.5000

$F_1 = 12, A = 0.05$										
2	.9369	.8989	.8772	.8524	.8245	.7941	.7614	.7268	.6536	.5774
4	.9252	.8885	.8629	.8159	.7897	.7575	.7235	.6914	.5441	.5355
6	.9159	.8784	.8504	.8023	.7805	.7445	.7139	.6839	.5009	.5214
8	.9082	.8749	.8498	.8017	.7800	.7444	.7177	.6899	.4889	.5034
10	.9019	.8709	.8458	.8000	.7800	.7449	.7184	.6915	.4818	.5006
12	.8966	.8686	.8428	.8000	.7800	.7457	.7198	.6937	.4755	.5002
14	.8921	.8684	.8427	.8000	.7800	.7462	.7202	.6940	.4759	.5001
16	.8882	.8690	.8427	.8000	.7800	.7466	.7206	.6943	.4760	.5000
18	.8848	.8686	.8418	.8000	.7800	.7470	.7210	.6946	.4761	.5000
20	.8818	.8686	.8416	.8000	.7800	.7474	.7214	.6947	.4762	.5000
22	.8792	.8686	.8416	.8000	.7800	.7478	.7218	.6948	.4763	.5000
24	.8768	.8686	.8416	.8000	.7800	.7482	.7222	.6949	.4764	.5000
26	.8747	.8686	.8416	.8000	.7800	.7486	.7226	.6950	.4765	.5000
28	.8728	.8686	.8416	.8000	.7800	.7490	.7230	.6951	.4766	.5000
30	.8710	.8686	.8416	.8000	.7800	.7494	.7234	.6952	.4767	.5000
40	.8643	.8631	.8353	.8000	.7800	.7509	.7249	.6957	.4772	.5000
60	.8565	.8608	.8333	.8000	.7800	.7524	.7264	.6962	.4777	.5000
120	.8472	.8542	.8333	.8000	.7800	.7539	.7279	.6967	.4782	.5000
∞	.8359	.8548	.8333	.8000	.7800	.7554	.7294	.6972	.4787	.5000

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