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A Nonparametric Solution for Finding the Optimum Useful Life of Equipment

Barry T. Stoll
Utah State University

A NONPARAMETRIC SOLUTION FOR FINDING THE OPTIMUM USEFUL LIFE OF EQUIPMENT

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Barry T. Stoll

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CHAPTER I

INTRODUCTION

It is often the case that equipment used by industry must be replaced with new equipment from time to time either because frequent malfunctions make it too costly to repair, or because the equipment has simply worn out. The new equipment often has the nature of either malfunctioning soon after installation due to manufacturing defects, or functioning for an extended period of time because it is free of these defects. For this reason, equipment is often given a preliminary running called the burn-in which gives no useful output but merely tests for manufacturing defects. Also, after a given amount of time, equipment is often replaced so as to avoid the added cost of a breakdown while under use. The term useful life is here used to denote the time period starting after burn-in time is reached and ending when replacement time is reached (Shooman, 1968).

The amount of burn-in and/or replacement times can be controlled to minimize cost per unit of operating time or to maximize reliability for some specified operating time. Whether to minimize cost or to maximize reliability is dependent upon the ultimate goal of the equipment user. An example of a minimum cost goal would be that of a company manufacturing a commodity whose production line machinery must be replaced. This company would want to minimize their costs of production rather than maximize the reliability, because they are primarily interested in making a profit rather than insuring against production stops. On the other hand when the United States sends a man to the moon, they are not so much interested in minimizing costs as they are in maximizing the reliability of their equipment.

Given the various operating costs and previous operating data, this study proposes to develop equations and computer programs which could be used to determine the burn-in and/or replacement times for minimizing cost or maximizing reliability of equipment. The equations will be developed to include either burn-in time, replacement time or both.

CHAPTER II

FEASIBILITY OF BURN-IN AND/OR REPLACEMENT TIMES

Minimizing Cost

The total cost of operation of equipment can be considered to be composed of several different and contributing costs. There may be any number of these different costs but for practical purposes this study will consider six costs which will be either fixed or linear over time. Later, these six costs could easily be expanded to any number with small changes in the main computer programs. The costs will be as follows:

Cost number 1 - Fixed cost of purchase

Cost number 2 - Fixed cost of installation

Cost number 3 - Cost of operation per unit time

Cost number 4 - Fixed cost of burn-in installation

Cost number 5 - Cost of burn-in per unit time

Cost number 6 - Fixed cost of breakdown

The hazard function can be defined as the conditional probability that an equipment will fail in a unit time interval after t, given that it was working at time t (Sandler, 1963). The hazard function can therefore be given by the following equation:

$$\text{Hazard} = f(t)/(1 - F(t))$$

When considering the performance of large numbers of equipment it is often the case that when the hazard function is graphed, one of three general patterns emerges. These general forms occur because there is either large amounts of early failures or large amounts of late failures or both large amounts of early and late failures. Illustrations of these situations follow:

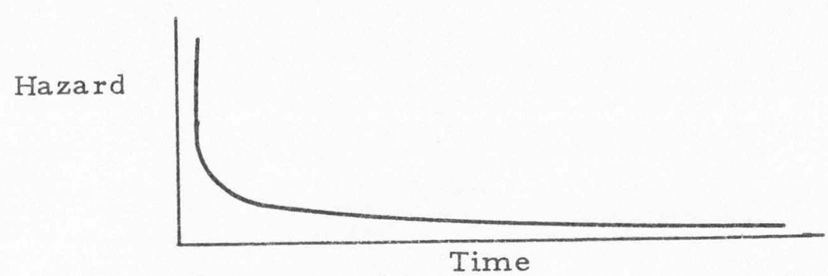

Figure 1. Hazard function - early failure.

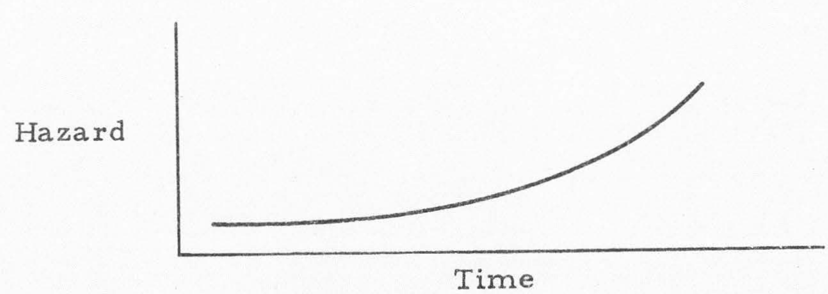

Figure 2. Hazard function - late failure.

Figure 3. Combined hazard function

If a burn-in time was imposed on a set of data which had a hazard function similar to Figure 1, it would eliminate many of the costs incurred by having to spend time and effort installing equipment which would run for only a short time. These costs would have to be paid if a burn-in time was not imposed.

If a replacement time was imposed on a set of data which had a hazard function similar to Figure 2 it would eliminate many of the breakdown costs incurred by having to suspend operations during the replacement, whereas, a replacement could have been made at a more opportune time.

A burn-in and replacement time could be imposed on a set of data which had a hazard function similar to Figure 3, to avoid many installation and breakdown costs.

By choosing an appropriate burn-in and/or replacement time the total costs can therefore be minimized.

Maximizing Reliability

Reliability can be defined as the probability of performing successfully for a specified time. Thus, as the probability of performing successfully increases, the reliability increases. By imposing a burn-in time to data with a hazard function similar to Figure 1, the probability of performing successfully increases as the burn-in time increases until the last data point is reached, since the hazard function is asymptotic to the time axis. This leads to a meaningless burn-in time solution for maximizing the reliability with data similar to Figure 1. However, by imposing a burn-in time to data with a hazard function similar to Figure 3, the probability of performing successfully increases as the burn-in time increases up to a point and then begins to decrease, since the hazard function is not asymptotic to the time axis. This leads to a meaningful burn-in time solution for maximizing the reliability with data similar to Figure 3. By choosing an appropriate burn-in time for a given time interval the reliability can be maximized.

CHAPTER III

OPTIMIZING USEFUL LIFE

Nonparametric Solution

A statistic is a term used to describe a measure computed from the observations in a sample. In computing a statistic, it is not necessary to have a knowledge of any unknown population. The observations in a sample determine the statistic and therefore the statistic can be thought of as a function of the observations in a sample. The random variable defined by this functional relationship can be defined to be a statistic. If $(x_1, \ldots x_n)$ is a possible sample point, then the functional relationship

$$y = t(x_1, \ldots x_n)$$

transforms from the space that contains all the values of the sample points to the space that contains the values of the function. A probability distribution is induced in the space that contains the values of the function by this transformation and thus defines a random variable:

$$Y = t(X_1, \ldots X_n) = t(X)$$

It is often the case that creating order out of a mass of data requires that the observations be put in numerical order. The result is a vector of ordered observations, from the smallest to largest and is sometimes referred to as the order statistic.

For a given sample, there can be defined a sample distribution function. A "mass" of amount $1/n$ can be placed at each observed value. This mass distribution then has a distribution function of

$$F_n(x) = 1/n \cdot \text{(number of observation} \leq x)$$

This is the sample distribution function. It can be computed from the order statistic. It is known that the sample distribution function converges to the population distribution function with probability one, uniformly in x. Therefore, it is a natural estimate of the population distribution function.

The sample distribution function is mathematically the same as a probability distribution function for a discrete distribution as it has the same mathematical properties as this type of function.

Since the sample central moments can be shown to be expressible as polynomial functions of sample moments about zero, it can be shown that the sample central moments tend in probability to the corresponding population moments (Lindgren, 1968).

If the population distribution function is not known but a sample distribution function is, it can be assumed that the sample distribution function will converge to the population distribution function as the sample size gets large. Therefore, for large sample sizes the population distribution function can be considered to be the sample distribution function and vise versa. This makes a nonparametric technique, for determining the minimum cost, possible. Since the sample distribution function is discrete this makes the burn-in and/or replacement times discrete because the time between data points need not be considered in finding the minimum cost. They need not be considered because, as can be seen in Figure 4, the discrete sample data gives a step function which, as previously stated, closely approximates the continuous population data's smooth curve function because of the large sample size. It can also be seen from Figure 4 that time values between data points of the step function give the same value for the number of breakdowns as the data point preceding the between data value. This of course, is what might be expected with a step function. Therefore, a burn-in or replacement time value between the discrete data points serves only to increase the burn-in or replacement time and their accompanying costs while not increasing the number of installation or replacement costs saved because the number of breakdowns have not increased. Since the number of data points is large but finite it is possible to compute the total cost of operation by using all the data points as burn-in and/or replacement times and choosing the times which minimize the cost.

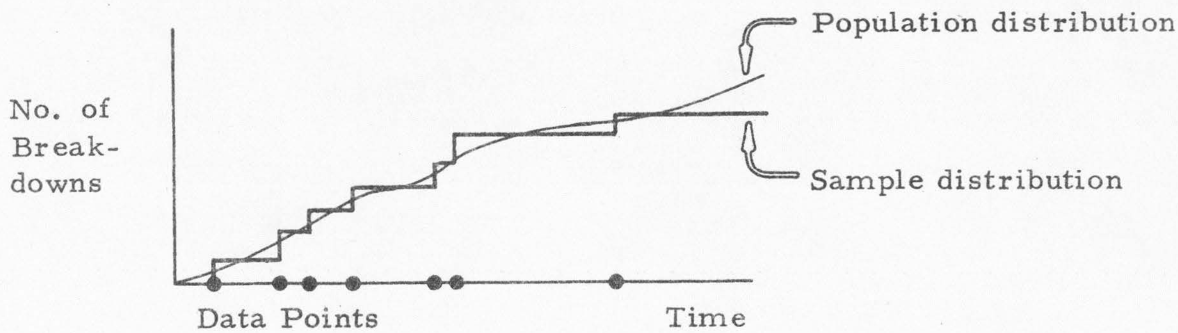

Figure 4. Step function.

Calculating the cost for each appropriate burn-in and/or replacement time rather than finding the burn-in and/or replacement time which minimizes the cost by more direct means has the advantage of showing the relative size of the costs before it reaches its minimum value. In some cases it might be more advantageous to use a cost which is not quite minimal but has a burn-in and/or replacement time which is more compatable to the equipment users time schedule. For example, it may be more desirable to burn-in for eight hours rather than for eight and one half hours, even though the cost might not quite be minimal because the burn-in could be done in one eight hour shift rather than be carried over into another shift of workers and possibly forgotten about.

Minimizing Cost

In a situation such as depicted by Figure 1, 2, or 3, where some burn-in and/or replacement time is going to be imposed, it is often not immediately evident what burn-in and/or replacement time will minimize the cost. One way of deciding the time or times to choose is to calculate the cost for each appropriate burn-in and/or replacement time in a large but finite sample which converges to the population and choose that particular time or times for which the cost is minimized.

Burn-in only

When it seems appropriate to impose only a burn-in time, costs one through five (as stated in Chapter II) may be considered. First, there is the cost of purchase. Assuming 100 pieces of equipment, this component of the total cost is 100 times the cost of purchase. Second, there is the cost of installation. There are 100 pieces of equipment but these will not all be installed due to malfunctions during the burn-in period. Therefore, there will be the number of equipment installed times the cost of installation for this component of total cost. The third cost consists of the cost of operation per unit time. This component of the total cost will be the total operation time, times the cost of operation per unit time. Fourth is the fixed cost of burn-in. This will apply to all the equipment, therefore, this component of the cost will be the number of equipment times the fixed cost of burn-in. Fifth is the cost per unit time of burn-in. This cost may be thought of as including within it, the cost per unit time of not making the profit which would be made if the equipment was being used instead of being burned-in. This component of the cost will be the total of the individual burn-in times, times the cost of burn-in per unit time. By imposing a burn-in time, some installation and breakdown costs can be saved. Where an installation cost is saved there will always be a cost of breakdown saved and vise versa, since, if equipment is not installed it can not breakdown. Therefore, in this part of the study breakdown cost as such is not considered seperately but is assumed to be includible in the installation cost.

For convenience the following notation will be used.

bt = the burn-in time

$n_1(bt)$ = the number of data points less than or equal to the burn-in time

n_2 = the number of data points

I_1 = the set of data points $X_{(i)}$ such that $i = [1, 2, \ldots n_1(bt)]$

so that for $i \epsilon I_1$ $X_{(i)} \leq bt$

I_2 = the set of data points $X_{(i)}$ such that $i = [n_1(bt)+1, \ldots n_2]$

so that for $i \epsilon I_2$ $X_{(i)} > bt$

E = total cost per unit time

The total of the contributing costs can be found by totaling the purchase cost times the number of data points, the installation cost times the total of the number of data points minus the number that failed during burn-in, the operation cost times the sum of the time of the data greater than the burn-in time, the fixed cost of burn-in times the number of data points, and the cost per unit time of burn-in times the total of all the burn-in time of those that failed and those that did not.

The total cost per unit time will be the total of the contributing costs divided by the total operating time and is given in the following equation:

$$E = \frac{C_1(n_2)+C_2(n_2-n_i(bt)+C_3(\sum_{i \in I_2} X_{(i)})+C_4(n_2)+C_5(\sum_{i \in I_1} X_{(i)} +bt(n_2-n_1(bt))}{\sum_{i \in I_2} X_{(i)}}$$

Without a burn-in time cost number four and cost number five would be equal to zero and the total operation time would increase but the number of equipment installed would also be increased, presumably offsetting any gains acquired by not using a burn-in time.

Replacement time only

When it seems appropriate to use only a replacement time, costs one, two, three and six (as stated in Chapter II) may be considered to apply. The first three components of the total costs, namely, costs one, two and three can be computed as in the case of burn-in time only. By imposing a replacement time some of the breakdown costs can be avoided because equipment that reach the replacement time successfully do not induce breakdown costs. The breakdown component of the total cost will be computed by multiplying the number of replacements needed times the cost of breakdown.

For convenience the following notation will be used.

$n_1(rt)$ = the number of data points less than or equal to the replacement time.

n_2 = the number of data points

I_1 = the set of data points $X_{(i)}$ such that $i = [1, 2, .. n_1(rt)]$ so that for $i \in I_1$ $X_{(i)} \leq rt$

I_2 = the set of data points $X_{(i)}$ such that $i = [n_1(rt)+1, ... n_2]$ so that for $i \in I_2$ $X_{(i)} > rt$

E = total cost per unit time

The total of the contributing costs can be found by totaling the purchase cost times the number of data points, the installation cost times the number of data points, the operation costs times the total of the operating time, and the breakdown cost times the number of data less than the replacement time.

The total cost per unit time will be the total of the contributing costs divided by the total operating time and is given in the following equation:

$$E = \frac{C_1(n_2)+C_2(n_2)+C_3(\sum_{i\in I_1} X_{(i)})+C_6(n_1(rt))}{\sum_{i\in I_1} X_{(i)}}$$

Without a replacement time the total operating time would be increased but the increased number of breakdown costs would presumably offset any gains acquired by not using a replacement time.

Burn-in and replacement time

When it seems appropriate to use burn-in and replacement times, costs one through six may be considered.

For convenience the following notation will be used.

bt = burn-in time

rt = replacement time

$n_1(bt)$ = the number of data points less than or equal to burn-in time

$n_2(rt)$ = the number of data points less than or equal to the replacement time

n_3 = the number of data points

I_1 = the set of data points $X_{(i)}$ such that $i = [1, 2, \ldots n_1(bt)]$ so that for $i \in I_1$ $X_{(i)} < bt$

I_2 = the set of data points $X_{(i)}$ such that $i = [n_1(bt) + 1, \ldots n_2(rt)]$

so that for $i \in I_2$ $X_{(i)} \leq rt$

I_3 = the set of data points $X_{(i)}$ such that $i = [n_2(rt)+1, \ldots n_3]$

so that for $i \in I_3$ $X_{(i)} > rt$

E = total cost per unit time

The total of the contributing costs can be found by totaling the cost of purchase times the number of points greater than the burn-in time, the cost of installation times the number of data points greater than the burn-in time, the operation cost times the sum of the time between burn-in and replacement times, the fixed cost of burn-in times the number of data points, the cost per unit time of burn-in times the total of the times of those that failed before burn-in time and those that did not, and the breakdown cost times the number of data points that fall between the burn-in time and the replacement times.

The total cost per unit time will be the total of the contributing costs divided by the total operating time and is given by the following equation:

$$E = \frac{C_1(n_3) + C_2(n_3 - n_1(bt)) + C_3\left(\sum_{i \in I_2} X_{(i)}\right) + C_4(n_3) + C_5\left(\sum_{i \in I_1} X_{(i)} + bt(n_3 - n(bt))\right) + C_6(n_2(nt) - n_1(bt))}{\sum_{i \in I_2} X_{(i)}}$$

Maximizing Reliability

In a situation such as depicted by Figure 3 where some burn-in time is going to be imposed it is often not immediately evident what burn-in time to use to maximize the reliability. One way of deciding the time to use is to calculate the reliability for each appropriate burn-in time noting that particular time for which the reliability is maximized.

Since the reliability can be considered to be the probability of performing successfully for a specified time, then it can be calculated by the following formula:

$$R = \frac{G(bt)}{H(bt)}$$

Where $G(bt)$ = number of successes as a function of the burn-in time

$H(bt)$ = number of events as a function of the burn-in time

When the situation is such as depicted by Figure 3, imposing an appropriate burn-in time reduces the total number of events without reducing the number of successes, thus, increasing reliability. The total number of events (H) can be computed by counting the number of times until failure greater than the burn-in time. The number of successes (G) can be computed by counting the number of times until failure greater than the burn-in and the specified (mission) times combined.

CHAPTER IV

DISCUSSION AND RESULTS

This study used the Weibull distribution to generate data. The Weibull distribution function is given by the following:

$$F(t) = 1 - \exp((-t/\Theta)^{V}) \tag{1}$$

where t is a time to failure and Θ and V are parameters of the distribution. A t with a Weibull distribution can be found from the following equation:

$$t = \Theta(-\ln x)^{1/V} \tag{2}$$

when x has a uniform distribution. The mean of the Weibull distribution is given by the following equation:

$$\text{Mean} = \frac{\Theta}{V}\Gamma\left(\frac{1}{V}\right) \tag{3}$$

The hazard function is given by the following:

$$H(t) = -\frac{R'(t)}{R(t)} = \frac{Vt^{V-1}}{\Theta^{V}}$$

where $R(t) = (1-F(t))$. When $V < 1$ a curve similar to Figure 5 occurs.

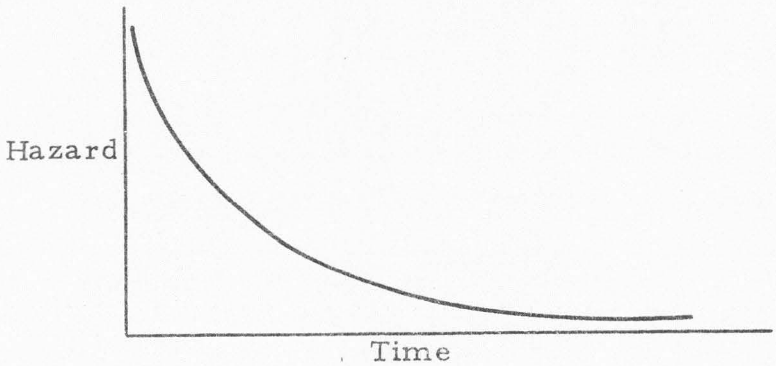

Figure 5. Hazard function V < 1

When V > 1 a curve similar to Figure 6 occurs.

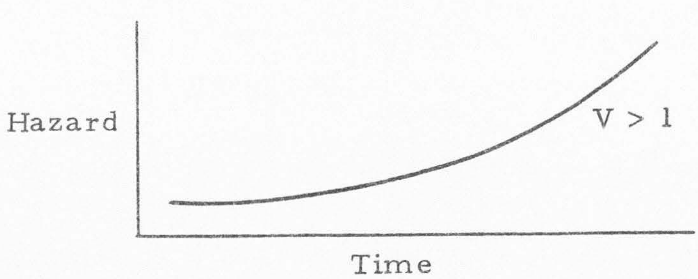

Figure 6. Hazard function V > 1

The curves of Figures 5 and 6 can be combined with an additional chance failure source for which the hazard function is constant, to form a combined hazard function by use of the following equation:

$$H(t) = \frac{V_1 t^{V_1 - 1}}{\theta_1^{V_1}} + \frac{1}{\theta_3} + \frac{V_2 t^{V_2 - 1}}{\theta_2^{V_2}} \tag{5}$$

The $\frac{1}{\theta_3}$ pertains to the chance failure. The central part of the curve represents a constant hazard function where chance failures are predominant. This represents the exponential distribution of failures which is a special case of the Weibull with V = 1. This is illustrated in Figure 7 (Shooman, 1968).

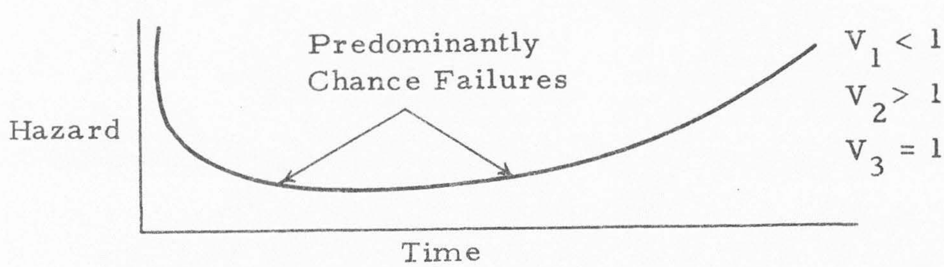

Figure 7. Combined hazard function.

To get time until failure data with a hazard function similar to Figure 5, a mean of 25 and V equal to 0.5 was selected. This makes θ approximately equal to 12 from equation 3. The t's can then be found by letting x be uniform random numbers between zero and one and solving equation two. To get time until failure data with a hazard function similar to Figure 6, again a mean of 25, but now a V equal to 5. was selected. This makes Θ approximately equal to 125 from equation three. Again the t's can be found by letting x be uniform random numbers between zero and one and solving equation two. To get time until failure data with a hazard function similar to Figure 7, Θ_1, Θ_2, Θ_3, V_1, V_2, and V_3 were assigned the values of 12, 5, 100, 0.5, 5 and 1, respectively. Now, however, the minimum value of t was choosen from equation two for each Θ, V set because a failure would occur at the shorter time.

Computer programs for generating the data as well as arranging it in assending order can be found in Appendix A.

Appendix B contains four computer programs which can be used for minimizing costs using only a burn-in time, only a replacement time, both a burn-in and replacement time or for maximizing reliability using a burn-in time, in that order. The first two programs compute all the total costs per unit of time as outlined in Chapter III using each data point for a burn-in or replacement time. The third program does not use all possible combinations of burn-in and replacement time but rather uses combinations of every n^{th} burn-in time with every n^{th} replacement time where n is determined by the user, to get an initial idea of where the best combination is. The user then looks at the costs as displayed in Figure 8, for the combinations chosen and decides in what general area the minimum lies. Then, after running the program again in this smaller area and with smaller jumps, he can narrow down the search area even more. This itervative procedure can be continued, using smaller and smaller search areas and jumps until the minimum is reached. This program has the advantage of showing the general trend of how much the cost is influenced by prescribed changes in burn-in and replacement times and may aid in choosing a time which may be better suited to a particular situation even though it does not quite minimize the cost. Other more sophisticated programing techniques such as the search methods which keep to rising paths or steepest gradients depend on the assumption that the function is unimodal as described by Wilde (1964). The cost function as seen in Figure 8 can be bimodal (doubly peaked). When the assumption of unimodel is not met, success can not be sure with such methods because the peak that is reached is dependent upon where the search starts (Wilde, 1964).

Figure 8 shows the calculated costs for combinations of every tenth burn-in time with every tenth replacement time where the burn-in time is less than the replacement time. The component costs one through six were 10, 600, 4, 1, 0.25, 250.

For example, the cell marked with "A" represents the total cost per unit time for the 20th ordered data point as the burn-in time and the 80th ordered data point as the replacement time.

The cost at point B is surrounded by combinations of times which give greater values for the cost. A similar situation exists at point A, thus showing that the cost function can be bimodal as described by Wilde (1964).

Data Point Used for Burn-in Time

Data point used for replacement time

	10th	20th	30th	40th	50th	60th	70th	80th
100th	669.9	658.1	671.2	861.7	1282.2	1458.2	1481.6	1872.1
90th	664.6	652.2	664.4	854.4	1297.9	1503.9	1549.4	1290.0
80th	664.3	651.4 A	663.9	870.5	1449.2	1861.9	2187.5	
70th	667.3	654.0	667.9	980.2	1872.6	3691.7		
60th	666.5	652.7 B	666.8	931.4	2492.2			
50th	674.3	660.1	677.5	1028.3				
40th	760.2	751.0	814.4					
30th	1348.9	1577.1						
20th	3367.0							
10th								

Burn-in time > replacement time in this lower region

Figure 8. Costs for 100 data in jumps of ten

It seems that the program used in this study and any other program which does not compute the cost for all the possible combinations of burn-in and replacement times, runs the risk of missing the optimum combination because the cost may have several local minimums before reaching the true minimum. Checking all possible combinations may be the only sure solution for finding the optimum combination, but this may prove too costly to the user. With the program given here, the user can come as close to checking all possible combinations as his resources allow by choosing the size of the "jumps".

The fourth program computes the reliability by using each data point as a burn-in time, as outlined in Chapter III, for any specified time interval. The program prints out all the reliabilities so that a burn-in time which does not quite maximize the reliability may be chosen if desired.

When using the first program, the user must read in the values for the component costs one, two, three, four and five, in addition to the number of data points, all on one card. The data must be read in one per card. Formats for reading in can be found from the comment card in the program and can be changed as needed. The program will then compute all the costs per unit time using each of the data points as a burn-in time and note the particular time for which the cost was minimized and the minimum cost. If the user wants to delete any of the component costs he can put a zero in its place when reading it in. If he wants to add a cost, then following the comment cards in the program should point the way.

When using the second program, the user must read in the values for the component costs one, two, three and six, in addition to the number of data points, all on one card. The data must be read in one per card. Formats for reading in can be found from the comment cards in the program and can be changed as needed. The program will then compute all the costs per unit time using each of the data points as a replacement time and note the particular time for which the cost was minimized and the minimum cost. If the user wants to delete any of the component costs he can put a zero in its place when reading it in. If he wants to add a cost, then following the comment cards in the program should point the way.

When using the third program, the user must read in the values for the component costs one, two, three, four, five and six, in addition to the number of data points, the size of the jumps and the starting and ending points in the data for the search, all on one card. The data must be read in one per card. Formats for reading in can be found from the comment cards in the program and can be changed as needed. If the user wants to delete any of the component costs he can put a zero in its place when reading it in. If he wants to add a cost, then following the comment cards in the program should point the way.

When using the fourth program the user must specify on one card the mission time and the total number of data points. The data itself is read in one per card. Formats for reading in can be found from the comment cards in the program and can be changed as needed. The program will then compute all the reliabilities using each of the data points as a burn-in time and note the particular time for which the reliability was maximized and the maximum reliability.

In testing the programs, component costs had to be chosen which would give practical results. These costs have the property that if one or more of them is too large or too small it may not be advantageous to use either a burn-in or a replacement time. The values of the costs as illustrated in Table 1 were chosen to give proctical results. Because other costs may produce the situation where a burn-in and/or replacement time was not advantageous, the programs first compute the total cost using no burn-in or replacement time.

Table 1. Costs used for each of the three minimum costs programs

	Program 1	Program 2	Program 3
Cost 1--cost of purchase	200	200	10
Cost 2--cost of installation	150	150	600
Cost 3--cost of operation	400	400	400
Cost 4--cost of burn-in (fixed)	1	--	1
Cost 5--cost of burn-in (per unit time)	0.25	--	0.25
Cost 6--cost of breakdown	--	50	250

The mission time for testing the reliability program was 4.1872667.

Each of the programs were run using 100 and 200 data points. The burn-in and/or replacement time can perhaps be illustrated best by showing their relative position in the data which was arranged from the smallest to the largest. This is illustrated in Table 2.

Table 2. Results using 100 and 200 data points

	Relative position in 100 data points	Relative position in 200 data points
Minimum cost Burn-in only program	28th	73th
Minimum cost Replacement only program	89th	179th
Minimum cost Burn-in & replacement program	22nd, 83rd	43rd, 168th
Maximum reliability Burn-in program	8th	18th

The relative size of each of the component costs taken together determine the optimum burn-in and/or replacement time. When the size of one of the component costs changes and the others remain constant, the burn-in and/or replacement time may or may not change. If they change, they change as illustrated in Table 3.

Each of the four programs were tested for errors by using ten data points except the third program which was tested by using thirty data points. Hand calculations were found to produce the same results.

In conclusion it was found that the size of the component costs relative to each other, determine what the optimum burn-in and/or replacement times should be when minimizing costs and that no burn-in and/or replacement time should be used when some of the component

Table 3. Results found when the component costs were changed

Component cost increased	Burn-in time	Replacement time
C#1 cost of purchase	decreased	increased
C#2 cost of installation	increased	increased
C#3 cost of operation	no effect	no effect
C#4 cost of burn-in (fixed)	decreased	increased
C#5 cost of burn-in (per unit time)	decreased	increased
C#6 cost of breakdown	increased	decreased
Component cost decreased	**Burn-in time**	**Replacement time**
C#1	increased	decreased
C#2	decreased	decreased
C#3	no effect	no effect
C#4	increased	decreased
C#5	increased	decreased
C#6	decreased	increased

costs have a relative size that is much greater or smaller than the others.

As can be seen in Table 3, the operation cost need not be considered in finding the optimum useful life.

With the maximum reliability program it was found that as the mission time decreased the optimum burn-in time decreased, and that a burn-in time only produced practical results when the hazard function took the form of Figure 3.

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APPENDIXES

Appendix A

```
      DIMENSION D(100),T(100)
      S(G,TH,X)=TH*((ABS(-ALOG(X)))**(1/G))
      G1=.5
      TH1=12.
      LARG=5139921
      DO 101 I=1,100
      X=RANDOM(LARG)
      D1=S(G1,TH1,X)
      D(I)=D1
  101 CONTINUE
```

```
      DIMENSION D(100),T(100)
      S(G,TH,X)=TH*((ABS(-ALOG(X)))**(1/G))
      G2=5.
      TH2=125.
      LARG=5139921
      DO 101 I=1,100
      X=RANDOM(LARG)
      D2=S(G2,TH2,X)
      D(I)=D2
  101 CONTINUE
```

```
      DIMENSION D(100),T(100)
      S(G,TH,X)=TH*((ABS(-ALOG(X)))**(1/G))
      G1=.5
      G2=5.
      G3=1.
      TH1=12.
      TH2=5.
      TH3=100.
      LARG=5139921
      DO 101 I=1,100
      X=RANDOM(LARG)
      D1=S(G1,TH1,X)
      X=RANDOM(LARG)
      D2=S(G2,TH2,X)
      X=RANDOM(LARG)
      D3=S(G3,TH3,X)
      IF(D1.GT.D2) GO TO 5
      D(I)=D1
      GO TO 100
    5 D(I)=D2
  100 CONTINUE
      IF(D3.GT.D(I)) GO TO 101
      D(I)=D3
  101 CONTINUE
```

```
      DO 500 J=2,100
      X=D(J)
      R=.5
      DO 600 I=1,J
      IF(R.GT.1.) GO TO 600
      IF(X.LT.D(I)) GO TO 700
      GO TO 600
  700 K=J-I
      R=2.
      DO 4 II=1,K
    4 D(J+1-II)=D(J-II)
      D(I)=X
  600 CONTINUE
  500 CONTINUE
```

Appendix B

```
      DIMENSION T(1000)
C     THIS PROGRAM COMPUTES THE BURN-IN TIME WHICH MINIMIZES COST
C     THE NEXT CARD READS IN THE COSTS AND NUMBER OF DATA,ND
C     COLUMNS 1-5 ARE COST NUMBER 1
C     COLUMNS 6-10 ARE COST NUMBER 2
C     COLUMNS 11-15 ARE COST NUMBER 3
C     COLUMNS 16-20 ARE COST NUMBER 4
C     COLUMNS 21-25 ARE COST NUMBER 5
C     COLUMNS 26-29 ARE THE NUMBER OF DATA
      READ(5,10)C1,C2,C3,C4,C5,ND
   10 FORMAT(5F5.2,I4)
C     THE NEXT FOUR CARDS READ IN THE DATA
      DO 15 I=1,ND
      READ(5,14) T(I)
   14 FORMAT(E15.8)
   15 CONTINUE
C     CSUM IS THE TOTAL OF ALL THE DATA TIMES
C     C9 IS THE COST WITHOUT A BURN-IN TIME
      CSUM=0.
      DO 97 I=1,ND
      CSUM=CSUM+T(I)
   97 CONTINUE
      XD=ND
      WRITE(6,98)
      C9=(ND*C1+ND*C2+CSUM*C3)/CSUM
      WRITE(6,99)C9
   98 FORMAT(' ','WITH NO BURN IN TIME')
   99 FORMAT(1X,F15.4)
      NND=ND-1
   30 FORMAT(' ','BURN-IN TIME           COST')
      WRITE(6,30)
      SMIN=1000000.
      DO 25 I=1,NND
C     BT IS THE BURN-IN TIME
C     XSUM IS THE NUMBER OF DATA POINTS GREATER THAN THE BURN-IN TIME
C     XQSUM IS THE SUM OF THE TIME GREATER THAN THE BURN-IN TIME
C     SUM IS THE NUMBER OF DATA POINTS LESS THAN THE BURN-IN TIME
C     XBSUM IS THE SUM OF THE TIME LESS THAN THE BURN-IN TIME
C     XD IS THE NUMBER OF DATA POINTS
      SUM=0.
      XSUM=0.
      XQSUM=0.
      BT=T(I)
      DO 35 J=1,ND
      IF(T(J).LT.BT)GO TO 45
      XSUM=XSUM+1.
      XQSUM=XQSUM+(T(J)-BT)
      GO TO 35
   45 SUM=SUM+T(I)
   35 CONTINUE
```

```
      XBSUM=SUM+(BT*XSUM)
      XD=ND
C     C6 IS THE COST WITH A BURN-IN TIME
      C6=((XD*C1)+(XD*C4)+(XBSUM*C5)+(XSUM*C2)+(XQSUM*C3))/XQSUM
   40 FORMAT(3X,E15.8,F15.4)
      WRITE(6,40)BT,C6
      IF(C6.GT.SMIN) GO TO 1
C     SMIN IS THE MINIMUM COST OF ALL THE COMPUTED COSTS
      SMIN=C6
      SSMIN=T(I)
C     SSMIN IS THE BURN-IN TIME THAT CORRESPONDS TO THE MINIMUM COST
    1 CONTINUE
   25 CONTINUE
      WRITE(6,6)
    6 FORMAT(' ','OPTIMUM TIME    MINIMUM COST')
      WRITE(6,7)SSMIN,SMIN
    7 FORMAT(1X,E15.8,F15.4)
      STOP
      END
```

```
      DIMENSION T(1000)
C     THIS PROGRAM COMPUTES THE REPLACEMENT TIME WHICH MINIMIZES COST
C     THE NEXT CARD READS IN THE COSTS AND NUMBER OF DATA,ND
C     COLUMNS 1-5 ARE COST NUMBER 1
C     COLUMNS 6-10 ARE COST NUMBER 2
C     COLUMNS 11-15 ARE COST NUMBER 3
C     COLUMNS 16-20 ARE COST NUMBER 6
C     COLUMNS 21-24 ARE THE NUMBER OF DATA
      READ(5,50)C1,C2,C3,C6,ND
   50 FORMAT(4F5.2,I4)
C     THE NEXT FOUR CARDS READ IN THE DATA
      DO 55 I=1,ND
      READ(5,14) T(I)
   14 FORMAT(E15.8)
   55 CONTINUE
C     CSUM IS THE TOTAL OF ALL THE DATA TIMES
C     C9 IS THE COST WITHOUT A BURN-IN OR REPLACEMENT TIME
      CSUM=0.
      DO 97 I=1,ND
      CSUM=CSUM+T(I)
   97 CONTINUE
      XD=ND
      WRITE(6,98)
      C9=(ND*C1+ND*C2+CSUM*C3+ND*C6)/CSUM
      WRITE(6,99)C9
   98 FORMAT(' ','WITH NO REPLACEMENT TIME')
   99 FORMAT(1X,F15.4)
   70 FORMAT(' ','REPLACEMENT TIME       COST')
      WRITE(6,70)
      SMIN=1000000.
      NND=ND-1
      DO 65 I=1,NND
C     RT IS THE REPLACEMENT TIME
C     YXQSUM IS THE SUM OF THE TIME LESS THAN THE REPLACEMENT TIME OF
C     OF THOSE DATA POINTS THAT ARE GREATER THAN THE REPLACEMENT TIME
C     YXSUM IS THE SUM OF THE TIME LESS THAN THE REPLACEMENT TIME OF
C     THOSE DATA POINTS THAT ARE LESS THAN THE REPLACEMENT TIME
C     YSUM IS THE NUMBER OF DATA POINTS LESS THAN THE REPLACEMENT TIME
C     YD IS THE NUMBER OF DATA POINTS
      RT=T(I)
      YXQSUM=0.
      YXSUM=0.
      YSUM=0.
      DO 75 J=1,ND
      IF(T(J).LT.RT)GO TO 85
      YXQSUM=YXQSUM+T(J)-RT
      GO TO 75
   85 YXSUM=YXSUM+T(J)
      YSUM=YSUM+1.
   75 CONTINUE
      YD=ND
```

```
C       C7 IS THE COST WITH A REPLACEMENT TIME
        C7=(YD*C1+YD*C2+YSUM*C6+((RT)*(YD-YSUM)+YXSUM)*C3)/((RT)
       1*(YD-YSUM)+YXSUM)
     80 FORMAT(2X,E15.8,6X,F15.4)
        WRITE(6,80)RT,C7
        IF(C7.GT.SMIN) GO TO 1
C       SMIN IS THE MINIMUM COST OF ALL THE COMPUTED COSTS
C       SSMIN IS THE REPLACEMENT TIME THAT CORRESPONDS TO THE MINIMUM COST
        SMIN=C7
        SSMIN=T(I)
      1 CONTINUE
     65 CONTINUE
        WRITE(6,6)
      6 FORMAT(' ','OPTIMUM TIME     MINIMUM COST')
        WRITE(6,7)SSMIN,SMIN
      7 FORMAT(1X,E15.8,F15.4)
        STOP
        END
```

```
C     THIS PROGRAM COMPUTES THE BURN-IN AND REPLACEMENT TIMES WHICH
C     MINIMIZE THE COST
C     THE NEXT CARD READS IN THE COSTS, THE NUMBER OF DATA,THE SIZE OF
C     THE JUMPS,THE STARTING POINT, AND THE ENDING POINT
C     COLUMNS 1-5 ARE COST NUMBER 1
C     COLUMNS 6-10 ARE COST NUMBER 2
C     COLUMNS 11-15 ARE COST NUMBER 3
C     COLUMNS 16-20 ARE COST NUMBER 4
C     COLUMNS 21-25 ARE COST NUMBER 5
C     COLUMNS 26-30 ARE COST NUMBER 6
C     COLUMNS 31-34 ARE THE NUMBER OF DATA
C     COLUMNS 35-37 ARE THE SIZE OF THE JUMPS
C     COLUMNS 38-41 ARE THE STARTING POINT
C     COLUMNS 42-45 ARE THE ENDING POINT
      READ(5,90)C1,C2,C3,C4,C5,C6,ND,NJ,ISP,IEP
   90 FORMAT(6F5.2,I4,I3,I4,I4)
C     THE NEXT CARD READS IN THE DATA
      DO 95 I=1,ND
      T(I)=D(I)
   95 CONTINUE
      CSUM=0.
      DO 97 I=1,ND
      CSUM=CSUM+T(I)
   97 CONTINUE
      XD=ND
      WRITE(6,98)
      C9=(XD*C1+XD*C2+CSUM*C3+XD*C6)/CSUM
      WRITE(6,99)C9
   98 FORMAT(' ','WITH NO BURN IN OR REPLACEMENT TIMES')
   99 FORMAT(1X,F15.4)
      WRITE(6,110)
  110 FORMAT(' ','   BURN-IN TIME   REPLACEMENT TIME      COST')
      C8=1000000.
      DO 105 I=ISP,IEP,NJ
      BT=T(I)
      DO 115 K=ISP,IEP,NJ
      IF (I.LT.K) GO TO 116
      GO TO 115
  116 CONTINUE
      XSUM=0.
      XQSUM=0.
      XBSUM=0.
      SUM=0.
      YXQSUM=0.
      YSUM=0.
      YXSUM=0.
      RT=T(K)
```

```
      DO 125 L=1,ND
      IF(T(L).LT.BT)GO TO 135
      XSUM=XSUM+1.
      XQSUM=XQSUM+(T(L)-BT)
      GO TO 125
  135 SUM=SUM+T(L)
  125 CONTINUE
      XBSUM=SUM+(BT*XSUM)
      DO 145 M=1,ND
      IF(T(M).LT.RT)GO TO 155
      YXQSUM=YXQSUM+T(M)-RT
      GO TO 145
  155 YXSUM=YXSUM+T(M)
      YSUM=YSUM+1.
  145 CONTINUE
      XD=ND
C     C7 IS THE COST WITH A BURN-IN AND REPLACEMENT TIME
      C7=((XD*C1)+(XSUM*C2)+(XD*C4)+(XBSUM*C5)+((YSUM-(XD-XSUM))*C6)+
     1((CSUM-XBSUM-YXQSUM)*C3))/((CSUM-XBSUM-YXQSUM))
      WRITE(6,120)BT,RT,C7
  120 FORMAT(3X,E15.8,E15.8,F15.4)
      IF(C7.LT.C8)GO TO 165
      GO TO 115
C     C8 IS THE FIRST APPROXIMATION OF THE MINIMUM COST
C     II IS THE FIRST APPROXIMATION OF THE BURN-IN TIME
C     KK IS THE FIRST APPROXIMATION OF THE REPLACEMENT TIME
  165 C8=C7
      II=I
      KK=K
  115 CONTINUE
  105 CONTINUE
      IIK=II-10
      IIKK=II+10
      WRITE(6,18)
   18 FORMAT(' ','FIRST APPROX')
      WRITE(6,150)T(II),T(KK),C8
  150 FORMAT(3X,E15.8,E15.8,F15.4)
      STOP
      END
```

```
      DIMENSION T(1000)
C     THIS PROGRAM COMPUTES THE BURN-IN TIME WHICH MAXIMIZES THE
C     RELIABILITY
C     THE NEXT CARD READS IN THE MISSION TIME AND THE NUMBER OF DATA
C     COLUMNS 1-15 ARE THE MISSION TIME
C     COLUMNS 16-19 ARE THE NUMBER OF DATA
      READ(5,160)XMT,ND
  160 FORMAT(E15.8,I4)
C     THE NEXT FOUR CARDS READ IN THE DATA
      DO 245 I=1,ND
      T(I)=D(I)
  245 CONTINUE
      RMIN=0.
      WRITE(6,180)
  180 FORMAT(' ','MISSION TIME        BURN-IN TIME     RELIABILITY')
      NND=ND-1
      DO 255 I=1,NND
C     XSUM IS THE NUMBER OF DATA POINTS GREATER THAN THE BURN-IN AND
C     MISSION TIMES COMBINED
C     XBSUM IS THE NUMBER OF DATA POINTS GREATER THAN THE BURN-IN TIME
C     XTO IS THE BURN-IN TIME
C     XMT IS THE MISSION TIME
C     XT IS A DATA POINT
      XSUM=0.
      XBSUM=0.
      XTO=T(I)
      XAT=XTO+XMT
      DO 265 J=1,ND
      XT=T(J)
      IF(XT.GT.XAT)GO TO 275
      GO TO 285
  275 XSUM=XSUM+1.
  285 IF(XT.GT.XTO)GO TO 295
      GO TO 265
  295 XBSUM=XBSUM+1.
  265 CONTINUE
      R=XSUM/XBSUM
      WRITE(6,190)XMT,XTO,R
  190 FORMAT(3X,E15.8,E15.8,F8.5)
      IF(R.LT.RMIN) GO TO 255
C     RMIN IS THE MAXIMUM RELIABILITY
C     XMIN IS THE  CORRESPONDING BURN-IN TIME
      RMIN=R
      XMIN=XTO
  255 CONTINUE
      WRITE(6,16)
      WRITE(6,17) XMT,XMIN,RMIN
   16 FORMAT(' ','MISSION TIME  BURN IN TIME  BEST RELIABILITY')
   17 FORMAT(3X,E15.8,E15.8,F8.5)
      STOP
      END
```