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BLOOD SUPPLY CHAIN NETWORKS
IN
HEALTHCARE:
GAME THEORY MODELS
AND
NUMERICAL CASE STUDIES

A Dissertation Presented

by

PRITHA DUTTA

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

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May 2019

Isenberg School of Management

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DEDICATION

To my parents, Saswati Dutta and Gopal Dutta

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ABSTRACT

BLOOD SUPPLY CHAIN NETWORKS
IN
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GAME THEORY MODELS
AND
NUMERICAL CASE STUDIES

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A crucial component of every healthcare system is the safe and steady supply of the life-saving product, blood. In order to meet the demand for blood consistently, it is imperative to maintain a robust supply chain. The blood banking industry in the United States, faced with emerging challenges, which include, an increase in operating costs, rise in competition among blood centers, insufficient reimbursement from payers such as insurance companies and government programs, in addition to inherent challenges such as donor motivation, seasonal shortages, perishability, is trying to adapt to the changing dynamics to sustain itself economically. The altruistic nature of this industry and the financial implications for its various stakeholders,

makes the efficient management of blood supply chains an important and interesting area of study.

In this dissertation, I contribute to the existing literature on blood banking by modeling the operational and economic challenges throughout the blood supply chain in the context of competition using game theory. I develop a model for blood service organizations competing for donations where they use service quality levels at collection sites as their strategic variables to increase their collection of blood from voluntary donors. I further construct a competitive blood supply chain network model that captures all major activities as well as perishability along the supply chain from collection of blood to distribution to hospitals.

As a crucial extension to the study on blood supply chains, I develop a network framework with multiple tiers of decision-makers including payers, that captures the decentralized nature of the blood banking system in the United States. The solutions from this multi-objective decision-making problem include quantities of blood to be supplied and transfused, given demand is known, as well as the financial transactions between the different tiers of stakeholders. For each model the governing equilibrium conditions are derived, and equivalent variational inequality formulations presented. The models and their relevance are further illustrated through simulated case studies. The results obtained provide valuable insights that can inform healthcare policy makers and regulators.

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CHAPTER 1

INTRODUCTION AND RESEARCH MOTIVATION

Blood supply chains constitute a critical part of the healthcare industry in any country. Blood transfusions are integral parts of any major surgeries. The number of units transfused can vary from one pint to dozens of pints for critical surgeries such as organ transplants. In addition, blood transfusions are used to treat certain diseases, including, for example, malaria-related anemia (see Obonyo et al. (1998)). Moreover, blood transfusions are often needed in the case of accidents as well as natural disasters because of injuries sustained by the victims.

The statistics provided by the American Red Cross on their website give an idea of the supply of, and demand for blood products in the United States (American Red Cross (2017)). Every day in the United States approximately 36,000 units of red blood cells are needed. Nearly 21 million blood components are transfused each year in the country while on average, 13.6 million whole blood and red blood cells are collected per year. While supply shortages in other industries imply financial losses for the organizations, in case of blood supply chains there are greater consequences as it can lead to societal loss through deaths.

Blood supply chains in the United States are decentralized and form complex networks with several interconnected stakeholders including the blood service organizations who collect, store and distribute blood, hospitals and trauma centers that use blood for transfusions, insurance companies and government payer programs that reimburse hospitals for their cost, and patients. The nonprofit and altruistic nature of the blood banking industry coupled with the financial ramifications for its var-

ious stakeholders in blood supply chains present interesting areas for research. At every stage in the supply chain stakeholders face numerous operational and strategic challenges. In the subsequent sections of this dissertation I provide details of these inherent issues in managing blood supply chains as well as emerging ones which provide the background and motivation for the research presented in Chapters 3, 4, 5 and suggested in Chapter 6 in this dissertation.

1.1. Supply Side Challenges

A unique feature of the blood banking industry is that the supply of the product is solely dependent on donations by individuals to the blood banks and blood service organizations collecting blood, which, for the most part, are nonprofits. Blood can neither be manufactured nor substituted by any other product. The American Red Cross (2017) reports that the number of donors in the United States in a year is approximately 6.8 million. Interestingly, according to the World Health Organization (2017), globally, 74 countries obtain more than 90% of their blood supply from voluntary unpaid blood donors, whereas 71 countries collect more than 50% of their blood supply from family/replacement or paid donors. In Britain, according to Gregory (2015), just 4% of the residents regularly donate blood with the National Health Service stating that it can no longer guarantee sufficient supplies. In New Zealand, about 4% of the population donates blood (see New Zealand Blood (2016)). According to the World Health Organization (2010), blood donation by 1% of the population is generally the minimum needed to meet a nation's most basic requirements for blood; the requirements are higher in countries with more advanced health care systems. However, the average donation rate is 15 times lower in developing countries than in developed countries. Globally, more than 70 countries had a blood donation rate of less than 1% in 2006.

While it is difficult to invoke altruism in people and to motivate them to donate blood, in some cases individuals who are motivated to donate blood might not qualify to do so. According to the Food and Drug Administration regulations (FDA (2018)), in the United States, blood donors have to go through a strict screening procedure. In addition to meeting the age, weight, and hemoglobin level requirements, as well as the time period between donations, donors are also screened for disease risk factors and may get deferred for reasons such as exhibiting signs and symptoms of colds or the flu, and/or relevant transfusion-transmitted infections, i.e., HIV, viral hepatitis, etc. (cf. American Association of Blood Banks (2017)). As a result, 38% of the country's population is eligible to donate blood at any given time. However, less than 10% actually donates blood in a year (cf. American Red Cross (2017)). Issues of seasonality place additional pressures on obtaining blood donations since donors may be preoccupied with holidays and/or weather-related issues. Hence, since the blood banking industry has to rely on voluntary donations from altruistic donors, it faces major challenges in terms of maintaining a sufficient supply of blood and will continue to do so, given the aging population. This fundamental supply side issue motivated the research in Chapter 3 and is also taken into consideration in Chapter 4 in this dissertation.

In conjecture to consumable product supply chains one might think of keeping excess supply as a solution to seasonal shortages. However, blood is a perishable product, with platelets lasting only 5 days and red blood cells having a lifetime of 42 days (cf. American Red Cross (2017), Blood Centers of America (2018). Hence, excess supply can also lead to loss through wastage. Moreover, at various stages in the blood supply chain such as testing, storage, a percentage of the collected blood is lost. This needs to be taken into account by the blood service organizations while targeting the collection amount and agreeing to the supply quantity with the hospitals.

1.2. Role of Competition

The blood banking industry in the United States is at a crossroads and is faced with several changes in the market dynamics. It is trying to adapt to these changes to sustain itself economically (cf. Nagurney (2017)). According to the American Red Cross, the leading supplier of blood in the United States, with about 40% of the market, there was a 33% decrease in blood transfusions in the period 2010-2014 (Wald (2014)). Further, Ellingson et al. (2017) report that in 2015 the number of red blood cell (RBC) units transfused in the United States was 11,349,000 marking a 13.9 percent decrease since 2013. The decrease in demand, resulting, for example, from medical advances associated with minimally invasive technologies, in addition to the scarcity of donors, has given rise to stiff competition among blood service organizations.

Industry experts are noting that the blood supply chain is becoming more and more similar to traditional commercial supply chains. Consequently, although the blood banking industry is characteristically not for profit, it is not surprising to find the prevalence of competition among blood service organizations in the United States. Competition exists for blood donations as well as for supply contracts with hospitals and other medical facilities (Snyder (2001), Hart (2011)). Pierskalla (2005), on page 141, emphasized in his overview of the supply chain management of blood banks that "To some extent there is a war out there. Many of the suppliers are in heavy, mostly negative competition among themselves and with many of the HBBs (hospital blood banks)" (see also Cohen, Pierskalla, and Sassetti (1987)).

Competition in the United States blood banking industry has been recognized in the popular press. As early as 1989, Gorman, reporting on competition among blood banks in California, remarked that the Palomar-Pomerado Hospital District established its own laboratory, staff, and recruiting drive to collect blood for use primarily at its two hospitals striking out against the decades old San Diego Blood

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Bank. Barton (2002) reported that in Florida the competition between Community Blood Centers and South Florida Blood Banks had become "ferocious." Since 2011, a small Sarasota-based blood bank, SunCoast Communities Blood Bank, had been competing for blood donations with a much larger organization, Florida Blood Services, that served hospitals in Tampa and neighboring areas (Smith (2011)). In 2012 the plight of SunCoast Communities Blood Bank in conducting its operations became more evident when it urged the state of Florida to stop the merger between three blood service organizations; namely, Orlando-based Florida's Blood Centers, the Community Blood Centers of Lauderhill, and Florida Blood Services of St. Petersburg on grounds of antitrust issues (Smith (2012)). More recently, in Tennessee (cf. Potts (2015)), a new blood center was opened by Blood Assurance in the town of Athens, where another blood center, Medic, currently uses six mobile blood centers with Medic officials noting their concern about the competition and whether they could honor their existing contracts with hospitals.

In 2013 the Eastern Maine Medical Center ended its contract with the American Red Cross to do business with Puget Sound Blood Center, a Seattle-based community blood bank (Barber (2013)). This trend is visible all across the country. Stone (2015) notes that "loyal blood donors will no longer see the iconic red cross on the side of the blood mobile next time they give blood at one of Mission Health's 17 facilities in Western North Carolina" because of a switch of supplier to a regional nonprofit blood bank of South Carolina, The Blood Connection, from the American Red Cross, ending a 30 year old contractual relationship. Prior to this new three year contract during which The Blood Connection will be the sole blood provider to Mission Health, it had been providing only a supplemental blood supply to the Mission hospitals. More recently, at the end of 2016, the American Red Cross lost its business in Central Arkansas to Arkansas Blood Institute, an affiliate of the Oklahoma Blood Institute.

This resulted in a layoff of 44 Red Cross employees at two blood centers (Brantley (2017)).

It is evident from the above examples that hospitals and medical centers may have several options for suppliers not only within the community and region, but also from out of state blood banks that may offer lower prices. In order to mitigate the risk of shortages due to possible supply shortfalls, hospitals and trauma medical centers may try to diversify their supplier base and contract with multiple blood service organizations. The contracts vary in terms of price, quantity, product mix and duration of partnerships] (Merola (2017)). There have been cases where reduced prices have led hospitals to switch blood suppliers (Schwartz (2012)). This competitive environment in the blood banking industry forms the basis of the research presented in this dissertation.

1.3. Economic Relationships

Blood banking is a multi-billion dollar industry with the global market for blood products projected to reach \$41.9 billion by 2020 (Global Industry Analysts, Inc. (2015)). The blood banking industry world-wide is a capital-intensive industry in which, however, the major stakeholders such as the blood suppliers and hospitals / medical centers in many countries are nonprofit organizations. At every stage in the blood supply chain the blood suppliers (blood service organizations) incur high costs associated with the collection of whole blood, the processing of the collected blood and the segregation of the components, the testing for disease markers, storage of blood bags at the appropriate temperature, and, finally, distribution to hospitals and other medical facilities. With strict regulations enforced in the United States by the Food and Drug Administration and the introduction of new disease markers, such as the one for the Zika virus to ensure safety of the transfused blood, there is, in

addition, a recognition of the importance of research and development, which also requires capital investment (Mulcahy et al. (2016)).

The United States constitutes the largest market for blood products in the world. However, the annual blood bank revenue is experiencing a decline, falling to 1.5 billion USD in 2014 from as high as 5 billion USD in 2008 (Wald (2014)) with the industry faced with, on the average, a decreasing demand and a rise in stiff competition (Nagurney (2017)). It is of utmost importance to ensure the economic sustainability of the blood service organizations in order to maintain a safe and steady supply of blood in the country.

The new economic landscape for this industry has been accompanied by an increasing number of mergers and acquisitions (cf. Toland (2014), Masoumi, Yu, and Nagurney (2017)) with the goal of identifying and exploiting various synergies, including cost-based ones. The growing trend of mergers is evident, for example, from the fact that the number of members in Americas' Blood Centers, the largest network of nonprofit community blood centers in North America, has dropped from 87 to 68 members due to 19 partnerships and 6 mergers formed in the five years between 2010-2015 among their member blood banks (Masoumi, Yu, and Nagurney (2017)).

Blood service organizations are not the only stakeholders in the supply chain facing issues in term of cost-effectiveness and revenue margins. There exists a lot of pressure on hospitals and, similarly, medical centers, in turn, to reduce the number of transfusions to check the overutilization and wastage of blood while managing their blood inventory efficiently to avoid shortages. Under financial stress, mergers and consolidations have been a characteristic of not only blood service organizations but also of hospitals, lately, and competition among hospitals has decreased (Kacik (2017), Gaynor, Mostashari, and Ginsburg (2017)). Some policy makers are emphasizing the need for more competition among health care providers for better service to patients at lower prices (Hyde (2016)). On another spectrum, industry executives argue that they

face sufficient competition from neighboring hospitals, new entrants, and alternative sources of healthcare. The cost of providing healthcare is continuing to increase in the United States and it is becoming more difficult to negotiate better reimbursement rates from hard-bargaining insurers (Dafny and Lee (2016)).

Hospitals in the United States get reimbursed for their patients' procedures, including blood transfusions, by different types of payers such as private insurers and government programs such as Medicare. The process of billing for blood products and receiving reimbursements for transfused units from different payer groups is a complicated process and has been a topic of concern among industry professionals (America's Blood Centers (2017a)). In 2018 the proposed reimbursement rates for blood products by the Centers for Medicare and Medicaid Services showed reduction in prices for several products. These payment policies have been criticized by organizations, including the America's Blood Centers, who continue to push the Centers for Medicare and Medicaid Services for fairer payment strategies (America's Blood Centers (2017a)). Inadequate reimbursements may affect the transactions between hospitals and blood service organizations who struggle to cover the rising cost of blood collection and testing, and, thereby, impact the economic sustainability of the entire blood supply chain (Mulcahy et al. (2016)). Although, currently, the payment methods used by Medicare and other private insurance companies following their suit succeed in checking for the overutilization of blood, the blood banking industry faces serious challenges due to a clear disconnect between the payments received and the actual cost of blood.

Mulcahy et al. (2016) propose alternative payment methods that might be beneficial for all stakeholders in the industry. One of the alternatives is a cost-based reimbursement policy which would take into account the number of units of blood transfused, in contrast to the current inpatient payment policy. The Medicare reimbursement for blood in inpatient setting falls under MS-DRG (Medicare Severity

Diagnosis Related Groups) which makes it difficult to separate the exact costs for the blood products (Toner et al. (2011)), while the outpatient reimbursement rates are determined using a cost-to-charge ratio methodology which uses data submitted by the hospitals and is susceptible to errors due to use of improper billing codes. The majority of blood transfusions occurs in an inpatient setting while around only 10-20 percent occurs in an outpatient setting. Hence, the former accounts for a large portion of revenue generated from blood transfusion (Mulcahy et al. (2016)).

The changing dynamics in the blood banking industry is forcing blood banks (also referred to as blood service organizations) to be more innovative in conducting their business. Blood banks need to price their products competitively based on the supplied quantity in order to recover costs of their operations and to generate revenue for activities such as research and development required for providing a steady supply of safe blood. Hence, there is a need for a change in the approach towards blood supply chain management, which should take into consideration not only the well-defined problems of perishability, outdating, shortage, and wastage (see, e.g., Nagurney, Masoumi, and Yu (2012)), but also limits on supply capacity and competition among blood banks. In light of the recent changes in the industry there also needs to be an intensified focus on the economics of blood supply chain networks to identify ways to make it more efficient and cost-effective and, hence, sustainable.

In this dissertation, I contribute to the study of blood supply chains by taking a holistic approach to developing game theory models that capture the current competitive landscape of the industry in the United States and include all major activities along the supply chain as well as the objectives of multiple stakeholders. The results obtained from analyzing the mathematical models through simulated case studies provide valuable insights and can inform policy makers. In particular, this dissertation attempts to find answers to the following research questions:

- In a competing scenario, what should be the service quality levels at the blood collection sites run by different blood service organizations?

- How do the varying service quality levels affect the amount of blood collected from different collection sites?

- What should be the optimal blood flows along various paths, taking perishability and capacity constraints into consideration, in a competitive blood supply chain network framework to ensure the supply of the minimum amount of blood required by hospitals?

- How do disruptions in various activities and reduction in capacities along the supply chain affect the revenue margin for blood service organizations?

- What should be the price per pint of blood charged by the blood service organizations in order to cover their costs?

- What should be the appropriate reimbursement rates received by the hospitals, and medical centers from the insurance companies and programs for blood transfusion to ensure economic sustainability of the entire supply chain?

This research adds to the literature on game theory and healthcare and, specifically, to game theory and blood supply chains, which has been very limited, to-date.

1.4. Literature Review

This section discusses the relevant literature on several topics covered in this dissertation. I provide a review of the extant literature and identify the gaps that I have tried to bridge.

1.4.1 Donor Motivation

As mentioned earlier, due to its reliance on voluntary donations one of the major challenges in the blood banking industry is donor motivation. There have been several studies on the factors that motivate people to become blood donors. While in some cases blood donations are incentivized even with monetary compensations, some economists and industry practitioners believe that it is altruism that mainly drives donors to donate their blood with the idea that it is going to help save the lives of people, at times, even in their communities, and, sometimes, even themselves (see, e.g., Lacetera, Macis, and Slonim (2012) and the references therein). Much of the theoretical works on blood donor motivation found in existing literature focus on altruism (Andreoni (1990), Mellström and Johanesson (2008), Evans and Ferguson (2013)). Others speculate that blood donors may be motivated by a notion of duty rather than unselfishness (see Wildman and Hollingsworth (2009)).

In their paper, Gillespie and Hillyer (2002) look at the studies conducted on the topic of blood donation decisions over the preceding three decades, focusing on both first time and return donors. They identify blood donation process measures such as the general donation experience for first time donors in terms of comfort, convenience of the process, and treatment by the staff in charge of technical and administrative activities as factors affecting donation decisions. It is further mentioned that negative donation experiences account for 6-19 percent attrition for all donors and 20 to as high as 41 percent of the drop-out rate for first time donors. In Charbonneau et al. (2015) the authors report deterrents among regular whole blood donors, lapsed whole blood donors and plasma/platelet donors. Based on their survey they found that for a significant percentage of donors in all three categories too much waiting time is a deterrent. These claims are also supported by the empirical evidence provided in Yuan et al. (2011), Finck et al. (2016) and Schreiber et al. (2006). These papers suggest that, while motivators are mainly altruistic, the deterrents are all factors

controlled by the blood service organizations. Hence, there is reason to believe that improving such aspects can have a positive impact on blood donations.

Personal experience and donor satisfaction from the blood donation process, along with the image or awareness of the impact of the organization collecting blood, have been pointed out as significant factors in donor motivation and retention in several studies (Nguyen et al. (2008), Aldamiz-echevarria and Aguirre-Garcia (2014)). In other words, it can be said that the quality of services provided by the blood banks during the collection or donation process plays a significant role in the decision-making process of first time as well as return donors.

There also exists literature on the assessment of blood banks in terms of service quality as perceived by donors, which, again, suggests a direct relationship between donor satisfaction and donation decisions (Al-Zubaidi and Al-Asousi (2012), Jain, Doshit, and Joshi (2015)). For example, Schlumpf et al. (2008), in a survey of over 7,900 blood donors, determined that prior donation frequency, intention to return, donation experience, and having a convenient location appear to significantly predict donor return. Craig et al. (2016), in turn, estimated the effect of wait time at the blood donation center on the satisfaction, intention to donate, and actual return behavior of blood donors and found that for a 38% increase in wait time there is a 14% decrease in whole blood donations. Thus, longer wait times entail substantial social costs and also attest to the importance of the quality of service for blood organizations in terms of donations. Convenience is also identified in the literature as a factor that can influence donor behavior (Schreiber et al. (2006)) in terms of clinic accessibility, which also attests to the relevance of quality of service. Cimarolli (2012) in her thesis, which focused on blood donations in Canada, emphasized that it is of utmost importance to improve the experiences of those donating blood, especially first-timers, by optimizing clinic locations and resources, minimizing negative reactions, lowering wait time, and increasing donor comfort. Perera et al. (2015) specifically noted, in

their study of blood donor programs in Sri Lanka, that blood donor programs could be improved there by addressing the provision of quality service.

While there exists a rich body of literature on the optimization of blood supply chains, as discussed in the following section, to ensure that demand is met as closely as possible and that shortages are minimized, with a view towards the perishability of this life-saving product (see, e.g., Nagurney et al. (2013), Duan and Liao (2014)), a thorough search of the published journal literature fails to return any significant work on the modeling of competition for donors among blood service organizations. Blood donations comprise the very basis of blood supply chains and play a crucial role in the stability of the entire blood supply system. Hence, it is of academic as well as practical significance to develop a model capturing the competition among organizations collecting blood.

1.4.2   Blood Supply Chain

The body of literature on blood supply optimization has been growing steadily over the years with some of the fundamental literature including that of Nahmias (1982) on perishable product inventory and that of Cohen and Pierskalla (1979) targeted at hospital and regional blood banks. Beliën and Forcé (2012) provide a comprehensive review of the supply chain management of blood products. The authors classify the works according to various categories such as the type of blood product, the solution method utilized, etc. Given the uncertainty in demand and supply of blood and the complex nature of blood supply chains, some authors have used simulation techniques to optimize the inventory levels (see, e.g., Rytilä and Spens (2006), Kopach (2008)) while others (see, e.g., Pierskalla (2005), Hemmelmayr et al. (2009)) have used mathematical programming to solve associated facility location and routing problems. Sarhangian et al. (2017) studied the performance of threshold-based allocation policies for optimizing blood inventory taking into consideration the trade-

off between age of the blood and availability. Ramezanian and Behboodi (2017) used mixed integer linear programming (MILP) to solve a deterministic location/allocation problem for blood collection facilities that takes into consideration the utility of blood donors in order to motivate them. They further utilized a robust optimization method to account for the issue of uncertainty in demand for blood. Osorio, Brailsford, and Smith (2018) distinguish between blood collection methods and present a multiobjective optimization problem that deals with the issue of assigning donors to a certain collection method while considering stochastic demand

Fortsch and Khapalova (2016) tackle some of the issues of blood supply chain management by using various forecasting techniques to better predict the demand for blood at the blood centers; thereby, reducing the uncertainty regarding the demand for blood. Nagurney, Masoumi, and Yu (2012) take into account uncertainty in the demand for blood and construct the full associated supply chain network of a blood service organization. Dillon, Oliveira, and Abbasi (2017) uses stochastic programming to deal with the uncertainty in the demand for blood. Another paper that handles the stochastic demand for blood is that of Zahiri and Pishvaee (2017).

El-Amine, Bish, and Bish (2017) focus on blood screening and consider that the budget-constrained blood center's goal is to construct a robust postdonation blood-screening scheme that minimizes the risk of an infectious donation being released into the blood supply. Ayer et al. (2017), in turn, consider when and from which mobile collection sites to collect blood for cryo production, so that the weekly collection target is met while the collection costs are minimized. Further, in Ayer et al. (2018), the authors develop a dynamic programming approach to the problem and create a decision support tool to help the American Red Cross select cryo collection sites. Hosseinifard and Abbasi (2016) consider the effect of centralized inventory in a two echelon supply chain model and show how centralization of the hospital level inventory increases the sustainability and resilience of the blood supply chain. A

recent interesting work has also considered the design of a blood supply chain network in a crisis via a robust stochastic model (see Salehi, Mahootchi, and Husseini (2017). Overall, there are few studies that address the supply side in blood banking (Fahimnia et al. (2017), Ramezanian and Behboodi (2017)) and especially in the context of competition.

Perishability of blood and wastage due to outdating is another common issue in blood supply chains and, hence, has been incorporated into some studies (Chazan and Gal (1977), Nagurney and Masoumi (2012), Nagurney, Masoumi, and Yu (2012), Duan and Liao (2014), Wang and Ma (2015)). In addition to optimizing the inventory of Red Blood Cells, Duan and Liao (2014) tackle the issue of blood group compatibility and substitution, while Dillon, Oliveira, and Abbasi (2017) use stochastic programming to deal with the uncertainty in the demand for blood. Much of the recent work on blood supply chains focuses on the optimization of the inventory at the hospital and blood bank levels as well as on the optimization of the shipment of blood from the blood banks to the hospitals (Gunpinar and Centeno (2015), Wang and Ma (2015)). However, there is a dearth of research on optimization of the entire blood supply chain network due to its complexity. In their literature review paper, focusing on quantitative models in blood supply chain management, Osorio, Brailsford, and Smith (2015) mention a few studies on integrated models or models that include all stages of the blood supply chain (see also, e.g., Katsaliaki and Brailsford (2007), Delen, Erraguntla, and Mayer (2011), Nagurney, Masoumi, and Yu (2012)). For example, Nagurney, Masoumi, and Yu (2012) developed a multicriteria optimization model for a regional blood banking system while capturing the myriad associated supply chain network activities. Inspired by this work I develop a blood supply chain network in a noncooperative competitive environment in Chapter 4 which adds to our understanding of how competition among blood supply chains affects the supply of the product and the revenue generated by the blood banks.

1.4.3 Competitive Supply Chain Networks

There exists a body of scientific literature that uses the concept of Nash equilibrium (cf. Nash (1950, 1951)) in decentralized supply chains, although, overall, the work is fairly recent (see, e.g., Nagurney, Dong, and Zhang (2002), Ha, Li, and Ng (2003), Bernstein and Federgruen (2005), Dong et al. (2005), Xiao and Yang (2008), Anderson and Bao (2010), Toyasaki, Daniele, and Wakolbinger (2014)), with the books by Nagurney (2006) and Nagurney and Li (2016) providing extensive references. In particular, Nagurney, Dong, and Zhang (2002) developed an equilibrium model for a competitive supply chain network with separate tiers for multiple manufacturers, multiple retailers, and multiple demand markets. They formulated and solved the multitiered supply chain network equilibrium problem as a variational inequality problem in order to obtain the equilibrium product flows and prices. Dong et al. (2005) conceptualized the three tiers in their supply chain network to denote manufacturers who can use one of several shipment alternatives to send the products to the distributors who comprise the second tier, and, finally, to retailers who are faced with stochastic demand. Other papers dealing with competition among supply chain stakeholders and demand uncertainty include those by Tsiakis et al. (2001), Bernstein and Federgruen (2005), Xiao and Yang (2008), and Mahmoodi and Eshghi (2014).

Bernstein and Federgruen (2005) studied the equilibrium conditions in a two-echelon supply chain where a single supplier supplies materials to multiple competing retailers who face uncertain demand. The authors also explored the impacts of coordination between the two echelons through contracts. Mahmoodi and Eshghi (2014) considered price competition between two-tiered supply chains consisting of manufacturers and retailers. The authors proposed three different algorithms to obtain the equilibrium solutions in three possible industry structures and examined the effects of competition and demand uncertainty intensity on the solutions and supply chain

profits in a numerical example. Farahani et al. (2014) provided a comprehensive literature review of competitive supply chain design models in which they classified the papers based on several major features of the models such as the number of tiers considered, the type of demand, the type of competition, etc. While the majority of the papers discussed above deal with supply chain structures with two tiers, in reality, supply chain networks may be more complex and involve multiple network economic activities as well as several stakeholders competing. Nagurney (2010) proposed a supply chain competition model with activities such as manufacturing, storage, and distribution for profit-maximizing firms. Masoumi, Yu, and Nagurney (2012), in turn, constructed a supply chain network model for oligopolistic competition among pharmaceutical companies while taking into account the perishable nature of drugs, whereas Yu and Nagurney (2013) developed a competitive food supply chain network model, which also included perishability and price differentiation. However, as pointed out in the previous section as well, there does not exist any work on the modeling of blood supply chain networks in the context of competition.

1.4.4 Capacity Constraints

Capacity constraints in supply chains have been studied extensively as evidenced in the papers by Gavirneni (2002), Lee and Kim (2002), Choi, Dai, and Song (2004), Goh, Lim, and Meng (2007), Jung et al. (2008), Nagurney and Li (2016), Nagurney, Yu, and Besik (2017), and Nagurney (2018). The capacities considered in these works pertain to physical capacities of production plants, distribution channels, freight service providers, etc., which vary from one firm to another. Similar to commercial firms, blood service organizations have limited resources in terms of space for collection, processing, and storage, and access to transportation vehicles, etc. Masoumi, Yu, and Nagurney (2017) introduce upper bounds on the capacity volume of various activities in the blood supply chain network consisting of collection, processing,

shipment, storage, and distribution along with frequencies of supply chain activities. However, blood service organizations face an additional constraint on the supply side due to limited number of eligible and motivated donors in any region.

Nagurney, Yu, and Besik (2017) utilize the concept of a Variational Equilibrium to formulate and solve the Generalized Nash Equilibrium (GNE) problem in the case of commercial supply chain network capacity competition with outsourcing with capacities associated with shared links of storage and freight service provision. A detailed discussion of GNE, in the context of a nonprofit, disaster relief framework, and the associated challenges of formulation and solution, is available in Nagurney, Alvarez Flores, and Soylu (2016) and the references therein.

As mentioned earlier, supply side capacity is a major issue in blood supply chains. While the demand for blood is highly volatile, through supply contracts with blood banks hospitals try to ensure that they have a minimum amount of blood available to avoid shortages, and at the same time set an upper bound on the demand to minimize wastage. Such supply and demand constraints have not been included in the studies present in the existing literature on blood supply chains.

1.4.5 Game Theory and Nonprofits

Noncooperative game theory is a powerful tool that is used extensively for formulating and solving problems where there is competition. While it is primarily used in the case of profit-making entities, several studies have used game theory to model competition among nonprofit organizations; see, e.g., Ortmann (1996), Tuckman (1998), Castaneda, Garen, and Thornton (2008), Bose (2015), and Nagurney and Li (2017). There is, however, a limited number of studies applying game theory in the realm of nonprofit supply chains and even fewer in the context of blood services. Saxton and Zhuang (2013) argued for the relevance of game theory in markets for charitable contributions and presented a model consisting of an organization and

a donor. Zhuang, Saxton, and Wu (2014) provided a sequential game theoretical model of disclosure-donation interactions with one nonprofit organization and multiple donors. Nagurney, Alvarez Flores, and Soylu (2016) developed a Generalized Nash Equilibrium network model in which nonprofit organizations are competing for financial funds for post-disaster relief operations, while minimizing costs associated with relief item distribution.

While there are some studies on the effects of competition among nonprofit organizations (see Muggy and Heier Stamm (2014), Nagurney, Alvarez Flores, and Soylu (2016), Nagurney and Li (2017), Nagurney et al. (2018)), and the references therein), to the best of my knowledge there is no prior work modeling the competition among blood service organizations.

1.5. Dissertation Overview

The dissertation consists of six chapters with the first chapter dealing with the research motivation and literature review. In Chapter 2, I recall the methodologies that are utilized in this dissertation, mainly variational inequality theory (Nagurney (1999)) and its relation to game theory (Nash (1950, 1951)). Below I present the contributions in Chapters 3 through 5 and outline a possible extension and future directions for the blood supply chain research in Chapter 6 of this dissertation.

1.5.1 Contributions in Chapter 3

It is of great value and importance to blood service organizations to understand the reasons behind donation decisions in order to ensure a sufficient supply of this life-saving product. I conceptualize the operational factors affecting donor decisions reported in empirical studies combined together as the service quality of the blood collection sites and develop a game theory model, whose solution yields the desired service quality levels at the blood collection sites and the corresponding quantities of

blood received. I provide a game theory Nash Equilibrium network framework since changes in the blood industry have resulted in greater competition among blood collection organizations.

Indeed, as noted by Meckler and Neergaard (2002), the blood banking industry has seen a rise in competition among blood banks in terms of recruiting and retaining donors as the number of organizations providing blood services has grown. Donors in parts of the United States, for example, may have the option of giving to the American Red Cross, to a local community blood center or hospital, or to America's Blood Centers' member organizations which make up North America's largest network of nonprofit community blood centers, and operate more than 600 blood donation collection sites, or to the United Blood Services, if in proximity.

It is also important to emphasize that blood organizations incur huge costs for organizing blood donation drives and other operations such as testing, processing of collected blood, transportation, and storage which have to be met through revenue generation. This is achieved by charging money per pint to hospitals, healthcare clinics, and trauma centers, etc. that require blood (cf. Nagurney, Masoumi, and Yu (2012)). The revenue generated covers costs and can be invested back into the process to improve the services provided by the blood banks and collection agencies. Moreover, organizations involved in blood banking and supply derive a utility in terms of satisfaction from providing quality service. Hence, it is beneficial for them to increase the number of donors and the amount of blood donated. However, improving the service quality to attract more donors implies, in turn, an increase in costs. The trade-off between the two can be easily observed by analyzing the model presented in this chapter. Chapter 3 of this dissertation, is based on the paper by Nagurney and Dutta (2019a).

1.5.2 Contributions in Chapter 4

The supply chain network competition model for blood service organizations (BSOs) in Chapter 4 makes several contributions to the existing literature on blood supply chains. The blood supply chain network structure developed here is the first of its kind that can capture competition on a regional as well as national level and traces all major blood supply chain activities starting from collection, testing and processing to storage, and distribution. I include multiple, competing blood service organizations in which the link cost functions are not assumed to be separable. The cost on a link may, in general, depend not only on the flow on that link but also on flows on other links of the specific BSO's supply chain network as well as on the flows on links of other BSOs' supply chain networks.

Common/shared capacities are incorporated on the supply side in terms of blood donations, and common/shared constraints on the demand side due to demand point constraints consisting of lower and upper bounds on the blood needed. No model, to-date, considers such features with the former uniquely relevant to blood supply chain network competition, not considered until this study, and the latter also very relevant due to the need to meet the demand for blood while also minimizing wastage. The utility functions of the blood service organizations contain revenue as well as altruism/ benefit components with the latter being weighted. Nagurney and Li (2017) also considered nonprofit competition with revenue and altruism features but in the case of hospital competition on a simpler, bipartite network and with the altruism component of an entirely different construct than herein. This model enables the incorporation of different factors that would affect prices that distinct hospitals and medical centers would be willing to pay for RBCs and that different blood service organizations would, therefore, be able to charge them. This chapter is based on the paper by Nagurney and Dutta (2019b).

1.5.3   Contributions in Chapter 5

In this chapter, I extend the work done in Chapter 4 by adding another tier of decision-makers comprised of insurance companies and programs or patient payer groups that reimburse hospitals for providing transfusion services, and the last activity in the blood supply chain which is the transfusion of blood. Hence, the integrated supply chain network model that I develop here includes all major stakeholders in the blood supply chain; namely, the blood banks or blood service organizations, the hospitals, and the patient payer groups. The network structure captures the decentralized nature of the blood supply chains in the United States and is the first competitive perishable product supply chain network model in healthcare with multiple tiers, multiple paths, and multiple associated distinct types of stakeholders.

While equilibrium optimal flow of blood from blood service organizations to hospitals along different paths are computed, this model also determines the amount of blood transfused by different hospitals to patients in different payer groups. A main focus of this chapter is to find a way to bridge the gap between payments received by hospitals and their cost of procuring blood from blood service organizations, that threatens the economic sustainability of blood supply chains in the United States. Deriving inspiration from an alternative payment policy suggested in Mulcahy et al. (2016), I developed the mathematical model here to also determine the optimal reimbursement rates received by hospitals from payers and the optimal prices that hospitals agree to pay to blood service organizations. This chapter is based on the paper by Dutta and Nagurney (2018).

1.5.4   Concluding Comments

The main contributions of the research done in this dissertation are summarized below.

1. The mathematical models constructed in Chapters 3, 4, and 5 are the first competitive network models in the context of blood banking that capture the economics of the blood supply chains. While in Chapter 3 the focus is only on the collection operation from voluntary blood donors, in the subsequent chapters the entire supply chain from blood service organizations to the end users, that is, the hospitals and patients is modeled. The contributions are in terms of the progression of each model with added layers of complexity.

2. It is a well-established fact that quality and customer satisfaction in service industries can provide competitive advantage to firms (Ghobadian, Speller and Jones (1994). There have been studies on the effect of quality in the healthcare industry (Brook and Kosecoff (1988), Rivers and Glover (2008), Nagurney and Li (2017)). However, the model presented in Chapter 3 is the first model that captures the effect of service quality on competition in the blood banking industry which makes it unique.

3. In Chapters 4 and 5, I assume multiple, competing blood service organizations in which the link cost functions are not assumed to be separable. In the case of multiple blood supply chain networks, in contrast, Masoumi, Yu, and Nagurney (2017) considered link cost functions that were separable. The generality of the cost functions here enables the modeling of supply chain network competition for resources among the BSOs.

4. The inclusion of perishability in the models in Chapters 4 and 5 make them pragmatic. When dealing with a highly perishable product such as blood it is imperative to account for it.

5. The objective functions of the blood service organizations and hospitals include an altruisim component which helps in capturing their nonprofit behavior.

6. Lagrange analysis presented in the different chapters provide nice economic interpretations for the analytical results. In Chapter 3, the analysis provides economic trade-offs associated with the upper and lower bounds of service quality levels. In

Chapter 4, the Lagrange analysis again sheds light on the marginal loss and gain associated with the lower and upper bounds on supply, link capacities as well as demand, while in Chapter 5, the equilibrium Lagrange multipliers reveal the prices charged by the BSOs to hospitals.

7. The methodologies used in this dissertation are game theory and variational inequality theory. In Chapters 3 and 5 the governing equilibrium concept is that of Nash equilibrium, and in Chapter 4 the problem is formulated as a Generalized Nash Equilibrium one. Qualitative results are provided in this dissertation along with discussion of computational procedures and results obtained from numerical cases.

8. In the final chapter, I present my conclusions and provide further directions for research in this area.

CHAPTER 2

METHODOLOGIES

In this chapter, I provide an overview of the fundamental theories and methodologies that are utilized in this dissertation. I first review variational inequality theory which has been utilized throughout this dissertation as the essential methodology to analyze the equilibria of blood supply chain networks with supply side and demand side competition, and multiple decision-makers. Variational inequality theory is a powerful methodology with widespread use in solving network economic equilibrium models. Some of the relationships between variational inequality and game theory to model the competition among blood service organizations as well as other stakeholders in the blood banking industry are also presented in this chapter.

In addition, I recall the theory of Generalized Nash Equilibrium which is utilized in Chapter 4 of this dissertation. Additional theorems and proofs associated with finite-dimensional variational inequality theory can be found in Nagurney (1999).

Finally, I review the algorithms: the Euler method and the modified projection method. The Euler method is employed to solve the variational inequality problems in Chapters 3 and 4. The modified projection method is applied to solve the variational inequality problem in Chapter 5.

2.1. Variational Inequality Theory

In this section, I provide a brief overview of the theory of variational inequalities followed by qualitative results, specifically concerning the existence and uniqueness of

solutions. All definitions and theorems are taken from Nagurney (1999). All vectors are assumed to be column vectors.

Definition 2.1 (Finite-Dimensional Variational Inequality Problem)

The finite-dimensional variational inequality problem, $VI(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset \mathcal{R}^n$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.1a}$$

where F is a given continuous function from $\mathcal{K}$ to $\mathcal{R}^n$, $\mathcal{K}$ is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space.

In (2.1a), $F(X) \equiv (F_1(X), F_2(X), \ldots, F_n(X))^T$, and $X \equiv (X_1, X_2, \ldots, X_n)^T$. Recall that for two vectors $u, v \in \mathcal{R}^n$, the inner product $\langle u, v \rangle = \|u\| \|v\| cos\theta$, where θ is the angle between the vectors u and v, and (2.1a) is equivalent to

$$\sum_{i=1}^{n} F_i(X^*) \cdot (X_i - X_i^*) \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.1b}$$

The variational inequality problem is a general problem that encompasses a wide spectrum of mathematical problems, including; optimization problems, complementarity problems, and fixed point problems (see Nagurney (1999)). It has been shown that optimization problems, both constrained and unconstrained, can be reformulated as variational inequality problems. The relationship between variational inequalities and optimization problems is now briefly reviewed.

Proposition 2.1 (Formulation of a Constrained Optimization Problem as a Variational Inequality)

Let X^* be a solution to the optimization problem:

$$Minimize \quad f(X) \tag{2.2}$$

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subject to:

$$X \in \mathcal{K},$$

where f is continuously differentiable and $\mathcal{K}$ is closed and convex. Then X^* is a solution of the variational inequality problem:

$$\langle \nabla f(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.3}$$

where $\nabla f(X)$ is the gradient vector of f with respect to X, where $\nabla f(X) \equiv (\frac{\partial f(X)}{\partial X_1}, \dots, \frac{\partial f(X)}{\partial X_n})^T$.

Proposition 2.2 (Formulation of an Unconstrained Optimization Problem as a Variational Inequality)

If $f(X)$ is a convex function and X^* is a solution to $\mathrm{VI}(\nabla f, \mathcal{K})$, then X^* is a solution to the optimization problem (2.2). In the case that the feasible set $\mathcal{K} = \mathcal{R}^n$, then the unconstrained optimization problem is also a variational inequality problem.

The variational inequality problem can be reformulated as an optimization problem under certain symmetry conditions. The definitions of positive-semidefiniteness, positive-definiteness, and strong positive-definiteness are recalled next, followed by a theorem presenting the above relationship.

Definition 2.2 (Positive Semi-Definiteness and Definiteness)

An $n \times n$ matrix $M(X)$, whose elements $m_{ij}(X); i, j = 1, \dots, n$, are functions defined on the set $\mathcal{S} \subset \mathcal{R}^n$, is said to be positive-semidefinite on $\mathcal{S}$ if

$$v^T M(X) v \geq 0, \quad \forall v \in \mathcal{R}^n, X \in \mathcal{S}. \tag{2.4}$$

It is said to be positive-definite on $\mathcal{S}$ if

$$v^T M(X) v > 0, \quad \forall v \neq 0, v \in \mathcal{R}^n, X \in \mathcal{S}. \tag{2.5}$$

It is said to be strongly positive-definite on $\mathcal{S}$ if

$$v^T M(X) v \geq \alpha \|v\|^2, \, for \, some \; \alpha > 0, \quad \forall v \in \mathcal{R}^n, \, X \in \mathcal{S}. \tag{2.6}$$

Theorem 2.1 (Formulation of an Optimization Problem from a Variational Inequality Problem Under Symmetry Assumption)

Assume that $F(X)$ is continuously differentiable on $\mathcal{K}$ and that the Jacobian matrix

$$\nabla F(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_1} & \cdots & \frac{\partial F_1}{\partial X_n} \\ \vdots & \cdots & \vdots \\ \frac{\partial F_n}{\partial X_1} & \cdots & \frac{\partial F_n}{\partial X_n} \end{bmatrix} \tag{2.7}$$

is symmetric and positive-semidefinite. Then there is a real-valued convex function $f : \mathcal{K} \longmapsto \mathcal{R}^1$ satisfying

$$\nabla f(X) = F(X) \tag{2.8}$$

with X^* the solution of $\mathrm{VI}(F, \mathcal{K})$ also being the solution of the mathematical programming problem:

$$Minimize \quad f(X)$$

subject to:

$$X \in \mathcal{K},$$

where $f(X) = \int F(X)^T dx$, and $\int$ is a line integral.

Thus, variational inequality is a more general problem formulation than an optimization problem formulation, since it can also handle a function $F(X)$ with an asymmetric Jacobian (see Nagurney (1999)). Next, I recall the qualitative properties of variational inequality problems, especially, the conditions for existence and uniqueness of a solution.

Existence of a solution to a variational inequality problem follows from continuity of the function $F(X)$ entering the variational inequality, provided the feasible set $\mathcal{K}$ is compact as stated in Theorem 2.2.

Theorem 2.2 (Existence of a Solution)

If $\mathcal{K}$ is a compact convex set and $F(X)$ is continuous on $\mathcal{K}$, then the variational inequality problem admits at least one solution X^*.

Theorem 2.3 (Condition for Existence if Feasible Set is Unbounded)

If the feasible set $\mathcal{K}$ is unbounded, then $\mathrm{VI}(F, \mathcal{K})$ admits a solution if and only if there exists an $\mathcal{R} > 0$ and a solution of $\mathrm{VI}(F, \mathcal{S})$, X_R^*, such that $\|X_R^*\| < \mathcal{R}$, where $\mathcal{S} = \{X : \|X\| \leq \mathcal{R}\}$.

Theorem 2.4 (Existence Following a Coercivity Condition)

Suppose that $F(X)$ satisfies the coercivity condition

$$\frac{\langle F(X) - F(X_0), X - X_0 \rangle}{\|X - X_0\|} \to \infty \tag{2.9}$$

as $\|X\| \to \infty$ for $X \in \mathcal{K}$ and for some $X_0 \in \mathcal{K}$. Then $\mathrm{VI}(F, \mathcal{K})$ always has a solution.

According to Theorem 2.4, the existence condition of a solution to a variational inequality problem is guaranteed if the coercivity condition holds. Next, certain monotonicity conditions are utilized to discuss the qualitative properties of existence and uniqueness. Some basic definitions of monotonicity are provided first.

Definition 2.3 (Monotonicity)

$F(X)$ is monotone on $\mathcal{K}$ if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{2.10}$$

Definition 2.4 (Strict Monotonicity)

$F(X)$ *is strictly monotone on* $\mathcal{K}$ *if*

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, X^1 \neq X^2. \qquad (2.11)$$

Definition 2.5 (Strong Monotonicity)

$F(X)$ *is strongly monotone on* $\mathcal{K}$ *if*

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq \alpha \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}, \qquad (2.12)$$

where $\alpha > 0$.

Definition 2.6 (Lipschitz Continuity)

$F(X)$ *is Lipschitz continuous on* $\mathcal{K}$ *if there exists an* $L > 0$, *such that*

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \leq L \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}. \qquad (2.13)$$

L *is called the Lipschitz constant.*

Theorem 2.5 (Uniqueness Under Strict Monotonicity)

Suppose that $F(X)$ *is strictly monotone on* $\mathcal{K}$. *Then the solution to the* VI$(F, \mathcal{K})$ *problem is unique, if one exists.*

Theorem 2.6 (Uniqueness Under Strong Monotonicity)

Suppose that $F(X)$ *is strongly monotone on* $\mathcal{K}$. *Then there exists precisely one solution* X^* *to* VI$(F, \mathcal{K})$.

In summary of Theorems 2.2, 2.5, and 2.6, strong monotonicity of the function F guarantees both existence and uniqueness, in the case of an unbounded feasible set $\mathcal{K}$. If the feasible set $\mathcal{K}$ is compact, that is, closed and bounded, the continuity of F guarantees the existence of a solution. The strict monotonicity of F is then sufficient to guarantee its uniqueness provided its existence.

2.2. The Relationships between Variational Inequalities and Game Theory

In this section, some of the relationships between variational inequalities and game theory are briefly discussed.

Nash (1950, 1951) developed noncooperative game theory, involving multiple players, each of whom acts in his/her own interest. In particular, consider a game with m players, each player i having a strategy vector $X_i = \{X_{i1}, ..., X_{in}\}$ selected from a closed, convex set $\mathcal{K}^i \subset R^n$. Each player i seeks to maximize his/her own utility function, $U_i \colon \mathcal{K} \to R$, where $\mathcal{K} = \mathcal{K}^1 \times \mathcal{K}^2 \times \ldots \times \mathcal{K}^m \subset R^{mn}$. The utility of player i, U_i, depends not only on his/her own strategy vector, X_i, but also on the strategy vectors of all the other players, $(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_m)$. An equilibrium is achieved if no one can increase his/her utility by unilaterally altering the value of its strategy vector. The formal definition of the Nash equilibrium is recalled as follows.

Definition 2.7 (Nash Equilibrium)

A Nash equilibrium is a strategy vector

$$X^* = (X_1^*, \ldots, X_m^*) \in \mathcal{K}, \tag{2.14}$$

such that

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in \mathcal{K}^i, \forall i, \tag{2.15}$$

where $\hat{X}_i^* = (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_m^*).$

It has been shown by Hartman and Stampacchia (1966) and Gabay and Moulin (1980) that given continuously differentiable and concave utility functions, U_i, $\forall i$, the Nash equilibrium problem can be formulated as a variational inequality problem defined on $\mathcal{K}$.

Theorem 2.7 (Variational Inequality Formulation of Nash Equilibrium)

Under the assumption that each utility function U_i is continuously differentiable and concave, X^* is a Nash equilibrium if and only if $X^* \in \mathcal{K}$ is a solution of the variational inequality

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad X \in \mathcal{K}, \tag{2.16}$$

where $F(X) \equiv (-\nabla_{X_1} U_1(X), \ldots, -\nabla_{X_m} U_m(X))^\mathsf{T}$, and $\nabla_{X_i} U_i(X) = (\frac{\partial U_i(X)}{\partial X_{i1}}, \ldots, \frac{\partial U_i(X)}{\partial X_{in}})$.

The conditions for existence and uniqueness of a Nash equilibrium are now introduced. As stated in the following theorem, Rosen (1965) presented existence under the assumptions that $\mathcal{K}$ is compact and each U_i is continuously differentiable.

Theorem 2.8 (Existence Under Compactness and Continuous Differentiability)

Suppose that the feasible set $\mathcal{K}$ is compact and each U_i, is continuously differentiable $\forall i$. Then existence of a Nash equilibrium is guaranteed.

On the other hand, Gabay and Moulin (1980) relaxed the assumption of the compactness of $\mathcal{K}$ and proved existence of a Nash equilibrium after imposing a coercivity condition on $F(X)$.

Theorem 2.9 (Existence Under Coercivity)

Suppose that $F(X)$, as given in Theorem 2.7, satisfies the coercivity condition (2.9). Then there always exists a Nash equilibrium.

Furthermore, Karamardian (1969) demonstrated existence and uniqueness of a Nash equilibrium under the strong monotonicity assumption.

Theorem 2.10 (Existence and Uniqueness Under Strong Monotonicity)

Assume that $F(X)$, as given in Theorem 2.7, is strongly monotone on $\mathcal{K}$. Then there exists precisely one Nash equilibrium X^*.

Additionally, based on Theorem 2.5, uniqueness of a Nash equilibrium can be guaranteed under the assumptions that $F(X)$ is strictly monotone and an equilibrium exists.

Theorem 2.11 (Uniqueness Under Strict Monotonicity)

Suppose that $F(X)$, as given in Theorem 2.7, is strictly monotone on $\mathcal{K}$. Then the Nash equilibrium, X^*, is unique, if it exists.

2.3. Generalized Nash Equilibrium (GNE)

In this section, I present a brief discussion on Generalized Nash Equilibrium (GNE) in which the strategies of the players, defined by the underlying constraints, depend also on the strategies of their rivals. A frequently encountered class of Generalized Nash games deals with a common coupling constraint that the players' strategies are required to satisfy (Kulkarni and Shanbhag (2012)). These games are also known as Generalized Nash games with shared constraints (Facchinei and Kanow (2007), Rosen (1965)).

Definition 2.10 (Generalized Nash Equilibrium)

A strategy vector $X^* \in K \equiv \prod_{i=1}^{I} K^i, X^* \in \mathcal{S}$, constitutes a Generalized Nash Equilibrium if for each player i; $i = 1, ..., I$:

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K^i, \forall X \in \mathcal{S}, \tag{2.17}$$

where

$$\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*),$$

K^i is the feasible set of individual player and $\mathcal{S}$ is the feasible set consisting of the shared constraints.

Bensoussan (1974) formulated the GNE problem as a quasivariational inequality. However, GNE problems are challenging to solve when formulated as quasivariational

inequality problems for which the state-of-the-art in terms of algorithmic procedures is not as advanced as that for variational inequality problems. In Kulkarni and Shanbhag (2012), the authors provide sufficient conditions to establish the theory of Variational Equilibrium as a refinement of the GNE which is utilized in Chapter 4 of this dissertation.

Definition 2.11 (Variational Equilibrium)

A strategy vector X^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $X^* \in K, X^* \in \mathcal{S}$ is a solution of the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \forall X \in \mathcal{S}. \qquad (2.18)$$

A notable feature of a variational equilibrium that I discuss in Chapter 4 in this dissertation is that the Lagrange multipliers associated with the shared or coupling constraints are the same for all players in the game which provides an elegant economic interpretation in terms of fairness.

2.4. Algorithms

In this section, I review the algorithms that are used in this dissertation. The Euler method, which is based on the general iterative scheme of Dupuis and Nagurney (1993), and the modified projection method of Korpelevich (1977) are presented.

2.4.1 The Euler Method

The Euler-type method algorithm and it's convergence conditions are given below. At an iteration $\tau+1$ of the Euler method (see also Nagurney and Zhang (1996)), where τ denotes an iteration counter, one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - \alpha_\tau F(X^\tau)), \qquad (2.19)$$

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where F is the function in (2.1a), and $P_\mathcal{K}$ is the projection on the feasible set $\mathcal{K}$, defined by

$$P_\mathcal{K}(X) = \text{argmin}_{X' \in \mathcal{K}} \|X' - X\|. \tag{2.20}$$

I now provide the complete statement of the Euler method.

Step 0: Initialization

Set $X^0 \in \mathcal{K}$.

Let $\tau = 0$ and set the sequence $\{\alpha_\tau\}$ so that $\sum_{\tau=0}^{\infty} \alpha_\tau = \infty$, $\alpha_\tau > 0$ for all τ, and $\alpha_\tau \to 0$ as $\tau \to \infty$.

Step 1: Computation

Compute $X^{\tau+1} \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^{\tau+1} + \alpha_\tau F(X^\tau) - X^\tau, X - X^{\tau+1} \rangle \geq 0, \qquad \forall X \in \mathcal{K}. \tag{2.21}$$

Step 2: Convergence Verification

If $\max |X_l^{\tau+1} - X_l^\tau| \leq \epsilon$, for all l, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1.

Assumption 2.1

Suppose we fix an initial condition $X_0 \in \mathcal{K}$ and define the sequence $\{X_\tau, \tau \in N\}$ by (2.19). I assume the following conditions:

1. *$\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$ as $\tau \to \infty$.*
2. *$d(F_\tau(x), \overline{F}(x)) \to 0$ uniformly on compact subsets of $\mathcal{K}$ as $\tau \to \infty$.*
3. *The sequence $\{X_\tau, \tau \in N\}$ is bounded.*

Theorem 2.12 (Convergence of the General Iterative Scheme)

Let S denote the set of stationary point of the projected dynamical system (2.19), equivalently, the set of solutions to the variational inequality problem (2.1a). Assume Assuption 2.1. Suppose $\{X_\tau, \tau \in N\}$ is the scheme generated by (2.19). Then $d(X_\tau, S) \to 0$ as $\tau \to \infty$, where $d(X_\tau, S) \to 0 = inf_{X \in S}\|X_\tau - X\|$.

Corollary 2.1 (Existence of a Solution Under the General Iterative Scheme)

Assume the conditions of Theorem 2.12, and also that S consists of a finite set of points. Then $\lim_{\tau \to \infty} X_\tau$ exists and equals to a solution to the variational inequality.

In the subsequent chapters, for each model, I derive the explicit formulae for all the strategy vectors in the respective variational inequalities formulated.

2.4.2 The Modified Projection Method

The modified projection method of Korpelevich (1977) can be utilized to solve a variational inequality problem in standard form (cf. (2.1a)). This method is guaranteed to converge if the monotonicity (cf. (2.10)) and Lipschitz continuity (cf. (2.13)) of the function F that enters the variational inequality (cf. (2.1a)) hold, and a solution to the variational inequality exists.

I now recall the modified projection method, and let τ denote an iteration counter.

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\tau = 1$ and let α be a scalar such that $0 < \alpha \leq \frac{1}{L}$, where L is the Lipschitz continuity constant (cf. (2.13)).

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \alpha F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.22}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \alpha F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.23}$$

Step 3: Convergence Verification

If max $|X_l^\tau - X_l^{\tau-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\tau := \tau + 1$, and go to Step 1.

Theorem 2.13 (Convergence of the Modified Projection Method)

If $F(X)$ is monotone and Lipschitz continuous (and a solution exists), the modified projection algorithm converges to a solution of variational inequality (2.1a).

In the following chapters, I derive the variational inequality formulations of the competitive blood supply chain network models. The computational algorithms reviewed in this chapter, which are the Euler method and the modified projection method, are also adapted accordingly to solve a number of simulated case studies.

CHAPTER 3

COMPETITION FOR BLOOD DONATIONS

In this chapter, I develop a bipartite supply chain network consisting of two tiers representing blood service organizations (BSOs) and blood collection regions. Each BSO collects blood from multiple sites. The blood service organizations have, as their strategic variables, the quality of service that they provide donors at their collection sites in the regions. It is well-recognized that and, as highlighted in Chapter 1, the quality of service at the blood collection sites plays a big role in repeat donations. Moreover, donors receive no financial remuneration in this model and this is quite reasonable since, in many countries, payments for blood donations are not permitted. I assume that the voluntary donors react to the service quality levels and, hence, the amount of blood received from each collection site varies depending on the level of service quality provided.

The formal definition of the word "quality" implies the standard of something as measured against other things of similar kind or, in other words, the degree of excellence of something. Service quality pertaining to the blood collection process at different facilities operated by blood service organizations would include operational characteristics such as cleanliness, wait time, hours of donation (convenience), location of the facilities, and treatment by staff that affect donation decisions as supported by empirical findings in the existing literature (Gillespie and Hillyer (2002), Nguyen et al. (2008), Aldamiz-echevarria and Aguirre-Garcia (2014), Al-Zubaidi and Al-Asousi (2012), Yuan et al. (2011), Finck et al. (2016), Schreiber et al. (2006)). Different facilities, even operated by the same blood service organization, may have different

associated service quality, depending on the size, the resources available there, and location of the service. For example, a permanent unit may be more comfortable to some donors, and be viewed as providing a higher quality of service than a mobile unit. Moreover, the service quality may vary from one organization to another for similar reasons and even the experience of personnel can be a factor.

The blood service organizations in the model developed in this chapter compete noncooperatively with the objective of maximizing their transaction utilities until a Nash Equilibrium is achieved. Since in the United States, blood service organizations are, for the most part, not-for-profit, the utility function consists of a service utility or altruism term in addition to revenue generated from selling the collected blood to healthcare facilities such as hospitals, trauma centers, and the cost of collection. The solution of the model yields the desired service quality levels at the blood collection sites and the corresponding quantities of blood received. Results obtained from such a model can provide managerial insights to the blood service organizations in terms of decisions about improving the service quality levels at their various blood collection sites to increase the amount of blood donations that they receive as well as the associated financial implications.

This chapter is based on the paper Nagurney and Dutta (2019a), and is organized as follows. In Section 3.1, I present the competitive network model for blood donations, identify the Nash Equilibrium conditions, and provide the variational inequality formulation. I also establish that the equilibrium quality level pattern of the blood service organizations is guaranteed to exist and provide conditions for uniqueness of the solution. Further, I derive an equivalent formulation to the variational inequality problem for competition among the blood service organizations which utilizes Lagrange multipliers associated with the lower and upper bounds of the quality levels that the blood service organizations can provide in the different regions in which they have or may desire to have blood collection services and provide a richer interpre-

tation of the underlying economic behavior of the blood service organizations. An illustrative example is also presented. In Section 3.2, I outline the algorithm, which yields closed form expressions at each iteration for the service quality levels, until convergence is achieved and then demonstrate the generality of the modeling and algorithmic framework through a series of numerical examples with accompanying insights in Section 3.3. Lastly, I summarize the results and present the conclusions in Section 3.4.

3.1. The Competitive Network Model for Blood Donations

Assume that there are m blood service organizations responsible for collecting the blood donations, which are then tested, processed, and distributed to hospitals and other medical facilities. A typical blood service organization is denoted by i. There are n regions in which the blood collections can take place. The collection sites may be fixed or mobile. A typical region is denoted by j. The time horizon for this model is flexible but I have in mind a week or a month. The illustrative example in Section 3 provides an actual context for an application of the model, which is then further expanded in the numerical examples in Section 3.3. The network structure of the problem is depicted in Figure 3.1.
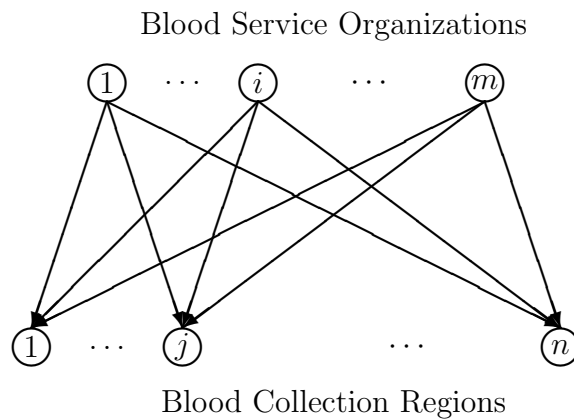
Figure 3.1. The Network Structure of the Game Theory Model for Blood Donations

Let Q_{ij} denote the quality of service that BSO i provides in region j. In order to allow flexibility I introduce upper and lower bounds on the service quality levels. Lower bounds could be employed as a target for a blood service organization or correspond to a minimum set by a regulatory body. An upper bound would represent the maximum achievable quality level by an organization in a region.

Group the strategic variables for each blood service organization into the vector $Q_i \in R_+^n$, and then group the quality of service levels for all blood service organizations into the vector $Q \in R_+^{mn}$. So Q is essentially an $m \times n$ matrix. There is a nonnegative lower bound and a positive upper bound imposed on each strategic variable, such that

$$\underline{Q}_{ij} \leq Q_{ij} \leq \bar{Q}_{ij}, \quad j = 1, \ldots, n. \tag{3.1}$$

The feasible set K^i for blood service organization i; $i = 1, \ldots, m$, is defined as $K^i \equiv \{Q_i | (3.1) \text{ holds}\}$. The feasible set underlying all players in the game, that is, the blood service organizations, is denoted by K where $K \equiv \prod_{i=1}^m K^i$.

I now describe the components of the transaction utility faced by each blood service organization that capture the total cost associated with the blood collection in the different regions, the utility corresponding to providing the quality levels, and the revenue generated from the blood donations. Each blood service organization seeks to maximize its transaction utility, which depends on the quality levels not only that it controls but also on those determined by the competing blood service organizations in the various regions.

Each blood service organization i encumbers a total cost $\hat{c}_{ij}$ associated with collecting blood in region j, such that

$$\hat{c}_{ij} = \hat{c}_{ij}(Q), \quad j = 1, \ldots, n, \tag{3.2}$$

where $\hat{c}_{ij}$ is assumed to be convex and continuously differentiable for all i, j. Note that the total cost depends, in general, on the quality of service that the blood service

organization i provides at its facility in each region j, as well as on the quality levels of the other blood service organizations. Some facilities may be more spacious, have more staff, provide greater comfort, have shorter waiting times, and also be cleaner, all factors that enter into the quality of the service and experience of the blood donors. Also, the blood service organizations may be competing for blood service professionals as well as other resources so that the generality of the functions in (3.2) allows for greater modeling flexibility. The total cost functions in (3.2) also include the cost of supplies for collecting the blood.

Each blood service organization i, since it values the service that it provides to donors, enjoys a utility associated with the service given by: $\omega_i \sum_{j=1}^{n} \gamma_{ij} Q_{ij}$, where the ω_i and the γ_{ij}s; $j = 1, \ldots, n$, take on positive values (cf. Nagurney, Alvarez Flores, and Soylu (2016) and the references therein). This component of the transaction utility represents a monetized utility reflecting the value that the blood service organization places on providing collection services at quality levels in the regions under study. If the blood service organization does not wish to consider this component, then ω_i can be set equal to zero. However, for organizations such as the American Red Cross, these operations might give more visibility and create a goodwill among donors that can eventually aid in fundraising for their other humanitarian operations.

In addition, each blood service organization i receives a volume of blood donations in region j, denoted by P_{ij}; $j = 1, \ldots, n$, where

$$P_{ij} = P_{ij}(Q), \tag{3.3}$$

where each P_{ij} is assumed to be concave and continuously differentiable. These blood donation functions capture competition for blood donations among the blood service organizations based on the levels of quality of service that they provide. If need be, these functions can include parameters associated with the level of organizational effectiveness as well as the impact preference of donors. Donors can be expected to be

more willing to give to reputable blood collection service organizations. Also, donors often prefer that the blood that they donate be used in a region in proximity to them. For example, Mews and Boenigk (2013), in an online experiment with 144 potential donors, found that organizational reputation is easily damaged by negative news in the press and that this leads to a significantly lower willingness to donate blood for such an organization among potential donors. They also note that, in highly competitive markets, as is the case in Germany (and other countries), intangible assets such as organizational reputation and nonprofit brands have been proven to be of critical importance.

Since blood service organizations charge for the blood that they provide and different organizations can and do price differently, I associate an average price π_i for blood (typically, measured in pints) for blood service organization i; $i = 1, \ldots, m$. These prices correspond to the price associated with the blood collection activity and, hence, would be fraction of the price charged for a pint of blood to hospitals and other medical facilities. For example, the price for a pint of blood can range in the United States from about \$150 to as much as \$300 and this price would also cover testing and delivery. The revenue that blood service organization i achieves that is associated with its blood collection activities over the time horizon is, hence, given by $\pi_i \sum_{j=1}^{n} P_{ij}(Q)$.

I now construct the optimization problem faced by blood service organization i; $i = 1, \ldots, m$. Each blood service organization i seeks to maximize its transaction utility, U_i, with the transaction utility capturing income due to contractual payments for the blood that it will distribute as well as the monetized utility that the organization gains from providing the collection services to various regions and the costs associated with collection. In particular, the optimization problem is as follows:

$$\text{Maximize} \quad U_i = \pi_i \sum_{j=1}^{n} P_{ij}(Q) + \omega_i \sum_{j=1}^{n} \gamma_{ij} Q_{ij} - \sum_{j=1}^{n} \hat{c}_{ij}(Q) \qquad (3.4)$$

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subject to (3.1).

For additional background on utility functions for nonprofit and charitable organizations, see Rose-Ackerman (1982) and Malani, Philipson, and David (2003).

The Nash Equilibrium conditions for the noncooperative game (cf. Nash (1950, 1951)) are stated below.

Definition 3.1: Nash Equilibrium for Blood Donations

A service quality level pattern $Q^* \in K$ is said to constitute a Nash Equilibrium in blood donations if for each blood service organization $i; i = 1, \ldots, m,$

$$U_i(Q_i^*, \hat{Q}_i^*) \geq U_i(Q_i, \hat{Q}_i^*), \quad \forall Q_i \in K^i, \tag{3.5}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_m^*). \tag{3.6}$$

According to (3.5), a Nash Equilibrium is established if no blood service organization can improve upon its transaction utility by altering its quality service levels, given that the other organizations have decided on their quality service levels.

3.1.1 Variational Inequality Formulation

I now present the variational inequality formulation of the above supply chain Nash Equilibrium in quality of service levels.

Theorem 3.1: Variational Inequality Formulation of the Nash Equilibrium for Blood Donations

A service quality level pattern $Q^* \in K$ is a Nash Equilibrium according to Definition 3.1 if and only if it satisfies the variational inequality problem:

$$-\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) \geq 0, \quad \forall Q \in K, \tag{3.7}$$

or, equivalently, the variational inequality:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\sum_{k=1}^{n}\frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i\gamma_{ij} - \pi_i\sum_{k=1}^{n}\frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}\right] \times \left[Q_{ij} - Q_{ij}^*\right] \geq 0, \quad \forall Q \in K.$$

(3.8)

Proof: We know that each feasible set K^i; $i = 1,\ldots,m$, is convex since it consists of simple box constraints. Hence, it follows that the Cartesian product K of these sets is also convex. Under the imposed conditions on the blood donation functions $P_{ij}(Q)$, and the total cost functions $\hat{c}_{ij}(Q)$, for all i, j, we also know that the utility functions U_i; $i = 1,\ldots,m$, are concave and continuously differentiable, since the utility functions consist of such functions, and a linear expression. Therefore, according to Proposition 2.2 in Gabay and Moulin (1980), which established the equivalence between the solution to a Nash equilibrium problem and the solution to the corresponding variational inequality problem, we know that each blood service organization i; $i = 1,\ldots,m$, maximizes its utility according to Definition 3.1 if and only if $Q^* \in K$ solves:

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) \geq 0, \quad \forall Q \in K,$$

which is precisely variational inequality (3.7).

In order to obtain variational inequality (3.8) from variational inequality (3.7), note that:

$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} = \sum_{k=1}^{n}\frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i\gamma_{ij} - \pi_i\sum_{k=1}^{n}\frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}, \quad \forall i, j,$$

and, therefore, variational inequality (3.8) also holds. □

The above variational inequality formulations of the Nash Equilibrium problem can be put into standard variational inequality form given by (2.1a) presented in

Chapter 2. The mn-dimensional column vector $X \equiv Q$ and the mn-dimensional column vector $F(X)$ has as its (i,j)-th component, F_{ij}:

$$F_{ij}(X) \equiv -\frac{\partial U_i(Q))}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q)}{\partial Q_{ij}} \right], \qquad (3.9)$$

with the feasible set $\mathcal{K} \equiv K$. Then, clearly, variational inequality (3.7) and (3.8) can be put into standard form (2.1a).

Existence of a solution Q^* to variational inequality (3.7) and also (3.8) is guaranteed from the standard theory of variational inequalities (cf. Chapter 2) since the function $F(X)$ that enters the variational inequality is continuous and the feasible set K is compact. Moreover, from the strict monotonicity condition described in Definition 2.4 (cf. Chapter 2) we have that the equilibrium solution Q^* is unique.

3.1.2 An Equivalent Formulation of Variational Inequality with Lagrange Multipliers

I now describe and analyze an equivalent formulation of variational inequality (3.7) which provides a deeper analysis of the Lagrange multipliers that are associated with the constraints (3.1), in the form of lower and upper bounds, on the levels of quality.

Observe that the feasible set K can be rewritten as

$$K = \{ Q \in R^{mn} : \underline{Q}_{ij} - Q_{ij} \le 0, Q_{ij} - \bar{Q}_{ij} \le 0, i = 1, \ldots, m; j = 1, \ldots, n \}. \quad (3.10)$$

Also, variational inequality (3.7) can be rewritten as a minimization problem. For example, by setting

$$V(Q) = -\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*), \qquad (3.11)$$

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we have that

$$V(Q) \geq 0 \text{ in } K \text{ and } \min_K V(Q) = V(Q^*) = 0. \tag{3.12}$$

Now consider the Lagrange function:

$$\mathcal{L}(Q, \lambda^1, \lambda^2) = -\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*)$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}^1 (\underline{Q}_{ij} - Q_{ij})$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}^2 (Q_{ij} - \bar{Q}_{ij}), \tag{3.13}$$

where $Q \in R^{mn}$, $\lambda^1, \lambda^2 \in R_+^{mn}$, $\lambda^1 = \{\lambda_{11}, \ldots, \lambda_{mn}\}$, and $\lambda^2 = \{\lambda_{11}^2, \ldots, \lambda_{mn}^2\}$.

Since for the convex set K the Slater condition is verified and Q^* is a minimal solution to problem (3.14), according to well-known theorems (see Jahn (1994)), there exist Lagrange multiplier vectors $\bar{\lambda}^1$, $\bar{\lambda}^2 \in R_+^{mn}$ such that the vector $(Q^*, \bar{\lambda}^1, \bar{\lambda}^2)$ is a saddle point of the Lagrange function (15), that is,

$$\mathcal{L}(Q^*, \lambda^1, \lambda^2) \leq \mathcal{L}(Q^*, \bar{\lambda}^1, \bar{\lambda}^2) \leq \mathcal{L}(Q, \bar{\lambda}^1, \bar{\lambda}^2), \quad \forall Q \in K, \forall \lambda^1, \lambda^2 \in R_+^{mn}, \tag{3.14}$$

and

$$\bar{\lambda}_{ij}^1 (\underline{Q}_{ij} - Q_{ij}^*) = 0, \quad \bar{\lambda}_{ij}^2 (Q_{ij}^* - \bar{Q}_{ij}) = 0, \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{3.15}$$

From the right-hand side of (3.16) it follows that $Q^* \in R_+^{mn}$ is a minimal point of $\mathcal{L}(Q, \bar{\lambda}^1, \bar{\lambda}^2)$ in the whole space R^{mn} and, hence, for all i and j:

$$\frac{\partial \mathcal{L}(Q^*, \bar{\lambda}^1, \bar{\lambda}^2)}{\partial Q_{ij}} = -\frac{\partial U_i(Q^*)}{\partial Q_{ij}} - \bar{\lambda}_{ij}^1 + \bar{\lambda}_{ij}^2 = 0 \tag{3.16}$$

together with conditions (3.15).

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Conditions (3.15) and (3.16), which hold for all i and j, represent an equivalent formulation of variational inequality (3.7).

Indeed, it is easy to see that from (3.15) and (3.16), variational inequality (3.7) follows. For example, multiplication of (3.16) by $(Q_{ij} - Q_{ij}^*)$ yields

$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) - \bar{\lambda}_{ij}^1(Q_{ij} - Q_{ij}^*) + \bar{\lambda}_{ij}^2(Q_{ij} - Q_{ij}^*) = 0$$

and, by taking into account (3.15):

$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) = \bar{\lambda}_{ij}^1(Q_{ij} - Q_{ij}^*) - \bar{\lambda}_{ij}^2(Q_{ij} - Q_{ij}^*)$$

$$= \bar{\lambda}_{ij}^1(Q_{ij} - \underline{Q}_{ij}) + \bar{\lambda}_{ij}^2(\bar{Q}_{ij} - Q_{ij}) \geq 0. \tag{3.17}$$

Summation over all i and j of (3.17) yields variational inequality (3.7).

I now proceed to analyze the marginal transaction utilities of the blood service organizations.

From (3.16) we have that

$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} - \bar{\lambda}_{ij}^1 + \bar{\lambda}_{ij}^2 = 0, \quad i = 1, \ldots, m; j = 1, \ldots, n.$$

Therefore, if $\underline{Q}_{ij} < Q_{ij}^* < \bar{Q}_{ij}$, then we get, using also (3.8):

$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}} \right] = 0,$$

$$i = 1, \ldots, m; j = 1, \ldots, n. \tag{3.18}$$

On the other hand, if $\bar{\lambda}_{ij}^1 > 0$ and, hence, $Q_{ij}^* = \underline{Q}_{ij}$ and $\bar{\lambda}_{ij}^2 = 0$, then we get that

$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}} \right] = \bar{\lambda}_{ij}^1,$$

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$$i = 1, \ldots, m; j = 1, \ldots, n, \tag{3.19}$$

and if $\bar{\lambda}_{ij}^2 > 0$ and, hence, $Q_{ij}^* = \bar{Q}_{ij}$ and $\bar{\lambda}_{ij}^1 = 0$, we have that

$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} = \left[\sum_{k=1}^n \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^n \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}} \right] = -\bar{\lambda}_{ij}^2,$$

$$i = 1, \ldots, m; j = 1, \ldots, n. \tag{3.20}$$

I now proceed to analyze (3.18) through (3.20). From equality (3.18), which holds when $\underline{Q}_{ij} < Q_{ij}^* < \bar{Q}_{ij}$, observe that for blood service organization i, which provides a quality level of service of Q_{ij}^* to blood donors in region j, the marginal transaction utility is zero; that is, the marginal total cost: $\sum_{k=1}^n \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}}$ is equal to the marginal utility associated with providing the service and the marginal revenue associated with the acquired blood donations in region j: $\omega_i \gamma_{ij} + \pi_i \sum_{k=1}^n \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}$.

According to equality (3.19), if $Q_{ij}^* = \underline{Q}_{ij}$, then the minus marginal transaction utility is equal to $\bar{\lambda}_{ij}^1$. In other words, the marginal total cost exceeds the marginal utility associated with providing the service and the marginal revenue. The blood service organization i then suffers a marginal loss given by $\bar{\lambda}_{ij}^1$.

Finally, according to (3.20), in which case $Q_{ij}^* = \bar{Q}_{ij}$ and $\bar{\lambda}_{ij}^2 > 0$, minus the marginal transaction utility is equal to $-\bar{\lambda}_{ij}^2$. In this case the marginal utility plus the marginal revenue exceeds the marginal total cost. Blood service organization i experiences a marginal gain given by $\bar{\lambda}_{ij}^2$.

From the above analysis, the equilibrium/optimal Lagrange multiplier vectors of variables $\bar{\lambda}^1$ and $\bar{\lambda}^2$ provide a rigorous interpretation of the behavior of the competition with respect to the provision of blood services, under the quality service level bounds. I remark that Langrangean analysis has also yielded useful insights into equilibrium problems in cybersecurity (cf. Daniele, Maugeri, and Nagurney (2017)) and in finance (see Daniele, Giuffre, and Lorino (2016)).

3.1.3 An Illustrative Example

An example is now presented to illustrate some of the above concepts. I will expand it in Section 3.3 to construct a series of numerical examples.

The example is inspired, in part, by the American Red Cross (cf. Arizona Blood Services Region (2016)) issuing a recent call for donations since its supplies of blood are low due to seasonal colds and flu and the devastating impact of Hurricane Matthew, which made landfall in the United States on October 8, 2016, affected such states as Florida, Georgia, and the Carolinas, and disrupted blood donations in many locations in the Southeast of the United States. Specifically, I focus on Tucson, Arizona, where the American Red Cross held recent blood drives at multiple locations and where there are also competitors for blood, including United Blood Services.

Example 3.1: Two BSOs and Two Blood Collection Regions

Example 3.1 serves as the baseline. It consists of two blood service organizations (BSOs), the American Red Cross and the United Blood Services, corresponding to uppermost nodes 1 and 2 in Figure 3.2. There are also two blood collection regions in Tucson, denoted by the lowermost nodes in Figure 3.2. For example, in its recent call for blood donations (cf. Arizona Blood Services Region (2016)), the America Red Cross had two locations in Tucson for collection. United Blood Services is a nonprofit organization that was founded in 1943 in Arizona and provides blood and services to more than 500 hospitals in 18 states (cf. United Blood Services (2016)). United Blood Services also collects blood in Tucson. The collection site nodes 1 and 2 are fixed, rather than mobile, for both organizations.

The components of the transaction utility functions (3.4) of the blood service organizations are as follows.

I consider a month of collection of whole blood cells. According to Meyer (2017), Executive Vice President of the American Red Cross (in a private communication), productive Red Cross sites collect, on the average, 700-840 whole blood units a month.

Blood Service Organizations

Blood Collection Regions
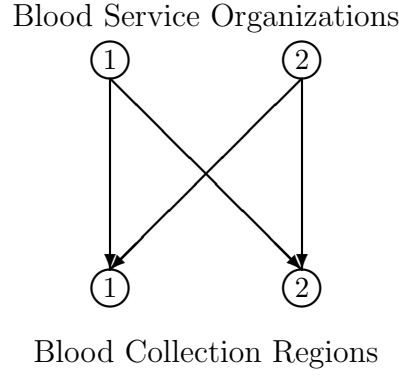
Figure 3.2. Example 3.1

The blood donation functions for the American Red Cross are:

$$P_{11}(Q) = 10Q_{11} - Q_{21} - Q_{22} + 130, \quad P_{12}(Q) = 12Q_{12} - Q_{21} - 2Q_{22} + 135.$$

Note that the fixed terms of 130 and 135 reflect the baseline of repeat donors that the American Red Cross expects over the month in the two regions, respectively.

The blood donation functions for the United Blood Services are:

$$P_{21}(Q) = 11Q_{21} - Q_{11} - Q_{12} + 123, \quad P_{22}(Q) = 12Q_{22} - Q_{11} - Q_{12} + 135.$$

The United Blood Services has a lower baseline population of donors in these blood collection regions than the America Red Cross. Its attention to quality is as good or higher than that of the American Red Cross, according to the respective functions.

The utility function components of the transaction utilities of these blood service organizations are:

$$\omega_1 = 9, \quad \gamma_{11} = 8, \quad \gamma_{12} = 9,$$

$$\omega_2 = 10, \quad \gamma_{21} = 9, \quad \gamma_{22} = 10.$$

The above parameters reflect that Organization 2 derives greater utility from providing quality blood collection services than Organization 1.

The total costs of operating the blood collection sites over the time horizon, which must cover costs of employees, supplies, and energy, and providing the level of quality service, are:

$$\hat{c}_{11}(Q) = 5Q_{11}^2 + 10,000, \quad \hat{c}_{12}(Q) = 18Q_{12}^2 + 12,000.$$

$$\hat{c}_{21}(Q) = 4.5Q_{21}^2 + 12,000, \quad \hat{c}_{22}(Q) = 5Q_{22}^2 + 14,000.$$

The bounds on the quality levels are:

$$\underline{Q}_{11} = 50, \quad \bar{Q}_{11} = 80, \quad \underline{Q}_{12} = 40, \bar{Q}_{12} = 70,$$

$$\underline{Q}_{21} = 60, \quad \bar{Q}_{21} = 90, \quad \underline{Q}_{22} = 70, \bar{Q}_{22} = 90.$$

The prices, which correspond to the collection component of the blood supply chain, are: $\pi_1 = 70$ and $\pi_2 = 60$.

First, observe that the objective function (3.4) of each blood service organization is, given the above functional forms, concave and continuously differentiable. In order to solve Example 3.1, I utilize variational inequality (3.8), and, because of the simplicity of the functions above (which I generalize for additional numerical examples), equilibrium quality levels can easily be obtained.

Specifically, using formula (3.10) for each F_{ij}, I obtain the following equations:

$$F_{11}(Q^*) = 10Q_{11}^* - 772 = 0,$$

$$F_{12}(Q^*) = 36Q_{12}^* - 921 = 0,$$

$$F_{21}(Q^*) = 9Q_{21}^* - 750 = 0,$$

$$F_{22}(Q^*) = 10Q_{22}^* - 820 = 0.$$

Note that we also have to be cognizant of the lower and upper bounds on the quality levels. Solving the above equations yields : $Q_{11}^* = 77.2$, $Q_{12}^* = 25.5$, $Q_{21}^* = 83.3$, and

$Q_{22}^* = 82$. Checking whether the values lie within the respective bounds I observe that they all do, except for Q_{12}^*, which, hence, is set to its lower bound so that: $Q_{12}^* = 40$.

According to this solution, the Red Cross stands to collect 736.7 units of blood at region 1, since $P_{11}(Q^*) = 736.7$ and 367.7 units of whole blood at region 2. United Blood service, on the other hand, stands to collect, since $P_{21}(Q^*) = 922.1$, that number of units per month at region 1, and 1001.80 units in region 2 (since $P_{22}(Q^*) = 1001.8$). Hence, United Blood Services collects a larger number of units of blood in the two regions.

It is also known, according to the Lagrangean analysis above, that, since only Q_{12}^* is at its lower bound and no quality service levels are at their upper bounds: $\bar{\lambda}_{11}^1 = 0$, $\bar{\lambda}_{21}^1 = 0$, $\bar{\lambda}_{22}^1 = 0$, and $\bar{\lambda}_{11}^2 = 0$, $\bar{\lambda}_{12}^2 = 0$, $\bar{\lambda}_{21}^2 = 0$, $\bar{\lambda}_{22}^2 = 0$. Also, since $Q_{12}^* = \underline{Q}_{12}$, I get $\bar{\lambda}_{12}^1 = \sum_{k=1}^2 \frac{\partial \hat{c}_{1k}(Q^*)}{\partial Q_{12}} - \omega_1 \gamma_{12} - \pi_1 \sum_{k=1}^2 \frac{\partial P_{1k}(Q^*)}{\partial Q_{12}} = 1359$. The American Red Cross suffers a marginal loss given by $\bar{\lambda}_{12}^1$. The transaction utilities at the equilibrium quality levels are: $U_1(Q^*) = 5,507.20$ and $U_2(Q^*) = 40,285.99$. In this illustrative example, the United Blood Services organization provides a higher level of quality services at each of its locations in Tucson and garners a higher transaction utility than the American Red Cross.

In the numerical examples in Section 3.3, I consider more general blood donation functions and also add both a competitor and a new region and analyze the resulting blood service organization quality levels and the incurred transaction utilities.

3.2.   The Algorithm

Here I outline the computational procedure used to solve the examples in Section 3.3. Specifically, the variational inequalities (3.7) and (3.8) are amenable to solution via the Euler method of Dupuis and Nagurney (1993) which is described in Section 2.4. Below I present the closed form expression for the service quality levels of blood service organizations.

Explicit Formulae for the Euler Method Applied to the Blood Donation Service Organization Game Theory Model

The elegance of this algorithm for the variational inequality (3.8) for the computation of solutions to the model is clear from the following explicit formula (cf. also (2.21)). In particular, I have the following closed form expression for the quality service levels $i = 1, \ldots, m; j = 1, \ldots, n$, at iteration $\tau + 1$:

$$Q_{ij}^{\tau+1} = \max\{\underline{Q}_{ij}, \min\{\bar{Q}_{ij}, Q_{ij}^\tau + a_\tau(\pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^\tau)}{\partial Q_{ij}} + \omega_i \gamma_{ij} - \sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^\tau)}{\partial Q_{ij}})\}\}. \quad (3.21)$$

3.3. Numerical Examples

I now present numerical examples focused on an area of Arizona. These build on the illustrative example in Section 3.1. In Nagurney and Dutta (2019a) the Euler method was implemented using FORTRAN on a Linux system. The convergence criterion was $\epsilon = 10^{-5}$, that is, the Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each quality service level (see (3.21)) differed from its respective value at the preceding iteration by no more than the ϵ. The Euler method was initialized by setting all the quality service levels to their lower bounds.

In the subsequent examples, Examples 3.2 through 3.6, different features are added. In Example 3.2, more general blood donation functions than those used in Example 3.1 are introduced and this changes the solution significantly. In Example 3.3, a collection region is added to show how that can benefit the competing blood service organizations. Example 3.4 incorporates an additional blood service organization to increase the competition. With Example 3.5, it is shown how over time increased competition can affect the outcomes. Lastly, in Example 3.6, there are three blood service organizations competing for donations in three collection regions. Hence, with each example a layer of complexity is added, which demonstrates the generality of the modeling and computational framework.

Below I provide additional information for each numerical example.

Example 3.2: Two BSOs and Two Blood Collection Regions with Different Form of Blood Donation Functions

Example 3.2 has the same network topology as Example 3.1, that is, the one depicted in Figure 3.2. The data are identical to those in Example 3.1 except here, significantly more general blood donation functions are used to observe how that changes the solutions. In particular, the new P_{ij} functions are constructed from the ones in Example 3.1 thus: $\alpha_{ij}\sqrt{P_{ij}}$ for $i = 1, 2$; $j = 1, 2$ with $\alpha_{11} = 50$, $\alpha_{12} = 30$, $\alpha_{21} = 40$, and $\alpha_{22} = 20$.

The computed equilibrium quality levels are:

$$Q_{11}^* = 72.43, \quad Q_{12}^* = 40.00, \quad Q_{21}^* = 64.61, \quad Q_{22}^* = 70.00.$$

The Euler method requires 34 iterations to converge to this solution. BSO 1 provides a higher level of quality of service in Region 1 whereas BSO 2 does in Region 2. BSO 1 collects $P_{11} = 1341.37$ units of blood in Region 1 and $P_{12} = 607.74$ units of blood in Region 2, whereas BSO 2 collects $P_{21} = 1074.27$ units in Region 1 and $P_{22} = 587.39$ units of blood in Region 2. The revenue of BSO 1 is 136,437.78 and that of BSO 2 is: 99,699.67. The monetized service utility component of the transaction utility of BSO 1 is 8,455.10 and that of BSO 2 is 12,814.97. BSO 1 incurs costs of 77,031.92 and BSO 2 incurs costs of 69,285.48. The revenue minus the cost (net revenue) for BSO 1 is: 59,405.86, whereas the revenue minus the cost for BSO 2 is: 30,414.19.

The values of the transaction utilities of the blood service organizations at the equilibrium values are, hence: $U_1 = 67,860.96$ and $U_2 = 43,229.16$. Note that both organizations have over the time period revenues that exceed their costs, which is important for the sustainability of their operations. Also, BSO 1 garners a larger number of units of blood donated in each region than does BSO 2. In Region 1 BSO 1 has a higher level of quality service than does BSO 2 but in Region 2 BSO

2 provides a higher level of quality service. However, the "brand" of BSO 1, which is also reflected in its P_{12} function, dominates, although not significantly in terms of units collected in Region 2.

Observe that both Q_{12}^* and Q_{22}^* are at their lower bounds. Hence, according to the Lagrange analysis theoretical results presented in Section 3.1 we know that: $\bar{\lambda}_{11}^1 = \bar{\lambda}_{11}^2 = 0$ and $\bar{\lambda}_{21}^1 = \bar{\lambda}_{21}^2 = 0$. Also, we have that $\bar{\lambda}_{12}^1 = 737.03, \bar{\lambda}_{12}^2 = 0$, and $\bar{\lambda}_{22}^1 = 354.85, \bar{\lambda}_{22}^2 = 0$. Therefore, BSO 1 suffers a marginal loss of 737.03 associated with its services in Region 2 and BSO 2 suffers a marginal loss of 354.85 associated with its services in Region 2.

Example 3.3: Two BSOs and Three Blood Collection Regions

Example 3.3 has the network topology in Figure 3.3. The data are as in Example 3.2 but with the addition of a possible new blood collection point in Region 3. In this example, the quality of service level lower bounds associated with the blood service organizations servicing Region 3 in terms of collections are set to 0.

Blood Service Organizations

Blood Collection Regions
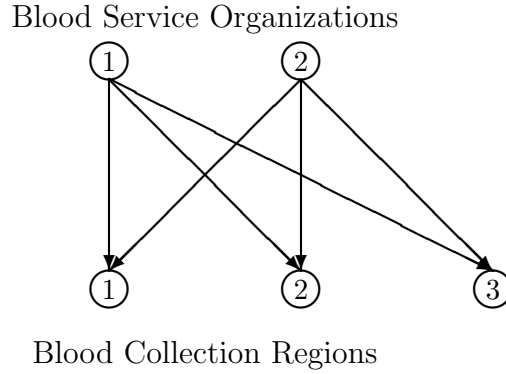
Figure 3.3. Example 3.3

The data are as follows: $\alpha_{13} = 40$, $\alpha_{23} = 30$, and

$$P_{13}(Q) = 40\sqrt{10Q_{13} - Q_{23} + 50}, \quad P_{23}(Q) = 30\sqrt{11Q_{23} - Q_{13} + 50},$$

$$\gamma_{13} = 9, \quad \gamma_{23} = 10,$$

and

$$\hat{c}_{13}(Q) = 10Q_{13}^2 + 15,000, \quad \hat{c}_{23}(Q) = 9Q_{23}^2 + 13,000.$$

The lower and upper bounds on the new links, in turn, are:

$$\underline{Q}_{13} = 0, \quad \underline{Q}_{23} = 0,$$

$$\bar{Q}_{13} = 60, \quad \bar{Q}_{23} = 70.$$

The Euler method converges in 34 iterations to the following equilibrium quality level pattern:

$$Q_{11}^* = 72.43, \quad Q_{12}^* = 40.00, \quad Q_{13}^* = 38.84,$$

$$Q_{21}^* = 64.61, \quad Q_{22}^* = 70.00, \quad Q_{23}^* = 33.70.$$

Note that the equilibrium quality levels for Q_{11}^*, Q_{12}^*, and Q_{21}^*, Q_{22}^* remain as in Example 3.2 since the underlying functions for these blood service organization and blood region pairs remain as in Example 3.2. Also, it is known from the theoretical analysis in Section 3.1 that since both Q_{13}^* and Q_{23}^* are neither at their lower or at their upper bounds, we have that: $\bar{\lambda}_{13}^1 = \bar{\lambda}_{13}^2 = \bar{\lambda}_{23}^1 = \bar{\lambda}_{23}^2 = 0.00$.

BSO 1 receives 804.73 units of blood in Region 3 whereas BSO 2 receives 586.24 units of blood in Region 3. The revenue of BSO 1 is now 196,739.25 and that of BSO 2 is: 134,874.17. The monetized service utility component of the transaction utility is 11,601.49 for BSO 1 and 16,185.06 for BSO 2. Since now both organizations operate a facility in an additional region the costs for BSO 1 are equal to 167,283.03 and for BSO 2 the costs are: 127,589.64. The transaction utility for BSO 1, $U_1 = 41,057.70$, and the transaction utility for BSO 2, U_2, is now $23,469.59$.

The revenue for each organization is higher in this example, with a new blood collection facility in a new region, than that obtained in Example 3.2. However, the cost is also higher. By collecting blood donations from three regions, rather than

two, the organizations achieve a higher monetized service utility by serving more regions. The net revenue for BSO 1 is now 29,456.22 and it is 7,284.53 for BSO 2. For each organization the revenue still exceeds the costs, which means that collecting blood in Region 3 does not hurt them financially. However, the net revenue for each organization is lower in this example than in Example 3.2.

Example 3.4: Three BSOs and Two Blood Collection Regions

Example 3.4 is constructed from Example 3.2, and has the same data, except for the new data corresponding to the addition of a new competitor, BSO 3, as depicted in Figure 3.4.

Blood Service Organizations

Blood Collection Regions
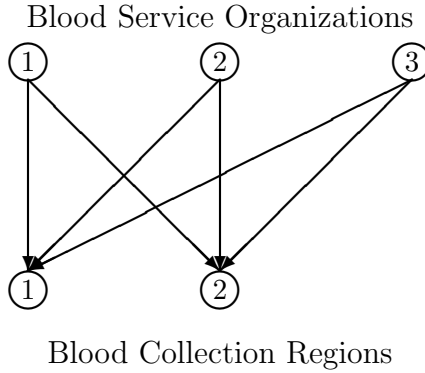
Figure 3.4. Example 3.4

The data for BSO 3 are:

$$P_{31}(Q) = 50\sqrt{11Q_{31} - Q_{21} + 50}, \quad P_{32}(Q) = 40\sqrt{10Q_{32} - Q_{12} + 2000},$$

$$\omega_3 = 10, \quad \gamma_{31} = 10, \quad \gamma_{32} = 11,$$

with the total cost functions given by:

$$\hat{c}_{31}(Q) = 6q_{31}^2 + 10,000, \quad \hat{c}_{32}(Q) = 5Q_{32}^2 + 12,000,$$

and with the lower and upper bounds as follows:

$$\underline{Q}_{31} = 50, \quad \bar{Q}_{31} = 90,$$

$$\underline{Q}_{32} = 40, \quad \bar{Q}_{32} = 80.$$

The price $\pi_3 = 80$. Recall that here I just consider the collection component of the blood supply chains.

The Euler method requires 40 iterations for convergence and yields the following equilibrium quality service level pattern:

$$Q_{11}^* = 72.43, \quad Q_{12}^* = 40, \quad Q_{21}^* = 64.61, \quad Q_{22}^* = 70, \quad Q_{31}^* = 70.73, \quad Q_{32}^* = 66.65.$$

Observe that, since the blood donation functions of the original blood service organizations have not changed, their quality service levels and, hence, their transaction utilities remain as in Example 3.2; the same holds for the donations, revenue amounts, costs, as well as the monetized service utility component of the transaction utilities.

BSO 3 has a transaction utility $U_3 = 104,706.44$. The amounts of its blood donations received are: $P_{31} = 1,381.47$ and $P_{32} = 2,049.99$. Its revenue is: 274,516.72 and its monetized service utility component of its transaction utility is: 14,111.81, with its cost equal to 184,922.09. This blood service organization has a net revenue equal to 89,594.63. BSO 3 has the highest net revenue of all the organizations, in this example, since the net revenue for BSO 1 in Example 3.2 was 59,405.86 and that for BSO 2: 30,414.19. This is due, in part to BSO 3 being able to achieve the highest volume of donations. Its quality levels do not lie at the bounds so that: $\bar{\lambda}_{31}^1 = \bar{\lambda}_{31}^2 = \bar{\lambda}_{32}^1 = \bar{\lambda}_{32}^2 = 0$.

Example 3.5: Competition from BSO 3 Intensifies

Example 3.5 is constructed from Example 3.4. It is assumed that some time has transpired and now both BSOs 1 and 2 realize that there is more competition from BSO 3.

Hence, their blood donation functions are now modified to capture the impact of competition from BSO 3 as follows:

For BSO 1:

$$P_{11}(Q) = 50\sqrt{10Q_{11} - Q_{21} - Q_{22} - .5Q_{31} + 130},$$

$$P_{12}(Q) = 30\sqrt{12Q_{12} - Q_{21} - 2Q_{22} - .3Q_{32} + 135},$$

and BSO 2:

$$P_{21}(Q) = 40\sqrt{11Q_{21} - Q_{11} - Q_{12} - .2Q_{21} + 113},$$

$$P_{22}(Q) = 20\sqrt{12Q_{22} - Q_{11} - Q_{12} - .3Q_{32} + 135}.$$

The remainder of the data is identical to that in Example 3.4.

The Euler method converges in 40 iterations and yields the following quality service level pattern:

$$Q_{11}^* = 73.57, \quad Q_{12}^* = 40, \quad Q_{21}^* = 64.99, \quad Q_{22}^* = 70, \quad Q_{31}^* = 70.73, \quad Q_{32}^* = 66.65.$$

The transaction utilities are now: $U_1 = 64,439.25$, $U_2 = 42,572.30$, and $U_3 = 104,222.39$. The volumes of blood donations are now as follows: for BSO 1: $P_{11} = 1,318.43$, $P_{12} = 592.46$; for BSO 2: $P_{21} = 1,059.31$, $P_{22} = 580.15$, and for BSO 3: $P_{31} = 1,381.22$ and $P_{32} = 2,049.99$. BSO 1 has a revenue of 133,762.72, costs equal to 77,860.27, and a monetized service quality component of the transaction utility equal to 8,536.80. BSO 2, in turn, enjoys a revenue of 98,367.77, encumbers costs equal to 68,644.69, and a monetized service quality component of its transaction utility equal to 12,849.21. BSO 3 obtains a revenue of 274,497.03, incurs costs of 185,38.53, and a monetized service quality component of its transaction utility equal to 15,112.89.

Observe that only Q_{12}^* and Q_{22}^* are, again, at their lower bounds. Hence, according to the Lagrange analysis theoretical results presented in Section 3.1 we know that: $\bar{\lambda}_{11}^1 = \bar{\lambda}_{11}^2 = 0$, $\bar{\lambda}_{21}^1 = \bar{\lambda}_{21}^2 = 0$, $\bar{\lambda}_{31}^1 = \bar{\lambda}_{31}^2 = 0$ and also $\bar{\lambda}_{32}^1 = \bar{\lambda}_{32}^2 = 0$. Also, we now have that $\bar{\lambda}_{12}^1 = 720.98, \bar{\lambda}_{12}^2 = 0$, and $\bar{\lambda}_{22}^1 = 351.79, \bar{\lambda}_{22}^2 = 0$. Therefore, BSO 1 suffers a marginal loss of 720.98 associated with its services in Region 2 and BSO 2 suffers

a marginal loss of 351.79 associated with its services in Region 2. These marginal losses are lower than those they suffered in Example 3.2.

Interestingly, with increased competition, blood donors benefit in that the quality service levels provided are now as high or higher than in Example 3.4 and both blood service organizations (BSOs) 1 and 2 provide a higher quality service in Region 1 than in Example 3.4. But, of course, this comes at a higher cost so their transaction utilities are lower now than in Example 3.4. Also, in terms of financial sustainability, note that for BSO 1, its net revenue is now: 55,902.45; for BSO 2, this value is: 29,723.08, and for BSO 3: 89,109.50. With increased competition, the net revenues decrease for all blood service organizations but these are, nevertheless, still significant and would allow for investment, whether to enhance their operations or, if feasible, to engage in R&D and further innovation for blood services.

In addition, comparing the amounts of blood collected by the two organizations in the two regions in Example 3.2 with the results obtained here, it is seen that the blood collections from both regions decrease for BSO 1 and BSO 2. However, due to the presence of a competing organization the overall blood collection increases. This finding is consistent with the empirical findings in Bose (2015).

Example 3.6: Three BSOs and Three Blood Collection Regions

Example 3.6 is constructed from Example 3.5 but it has a new blood collection region. Hence, as depicted in Figure 3.5, there are now three regions for blood collection, as well as three blood service organizations.

The data remain as in Example 3.5 with the addition of the new data below:

$$\alpha_{13} = 40, \quad \alpha_{23} = 30, \quad \alpha_{33} = 50,$$

$$P_{13}(Q) = 40\sqrt{10Q_{13} - Q_{23} - .2Q_{33} + 150},$$

$$P_{23}(Q) = 30\sqrt{11Q_{23} - Q_{13} - .2Q_{33} + 150},$$

Blood Service Organizations

Blood Collection Regions
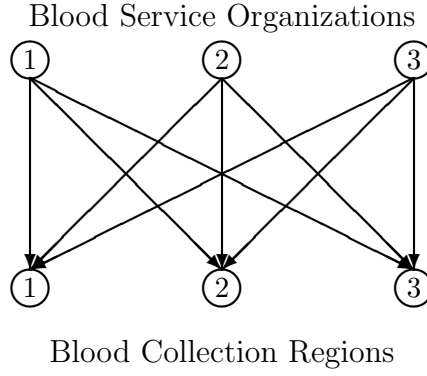
Figure 3.5. Example 3.6

$$P_{33}(Q) = 50\sqrt{10Q_{33} - Q_{23} - .3Q_{13} + 100},$$

$$\hat{c}_{13}(Q) = 100Q_{13}^2 + 15,000 \quad \hat{c}_{23}(Q) = 9Q_{23}^2 + 13000, \quad \hat{c}_{33}(Q) = 8Q_{33}^2 + 10000,$$

and with lower and upper bounds on the new links to Region 3 given by:

$$\underline{Q}_{13} = 0, \quad \underline{Q}_{23} = 0, \quad \underline{Q}_{33} = 40,$$

$$\bar{Q}_{13} = 60, \quad \bar{Q}_{23} = 70, \quad \bar{Q}_{33} = 90.$$

Also, we have that

$$\gamma_{13} = 9, \quad \gamma_{23} = 10, \gamma_{33} = 10.$$

The Euler method, again, converges in 40 iterations to the following equilibrium pattern:

$$Q_{11}^* = 73.57, \quad Q_{12}^* = 40, \quad Q_{13}^* = 36.32,$$

$$Q_{21}^* = 64.99, \quad Q_{22}^* = 70, \quad Q_{23}^* = 31.51,$$

$$Q_{31}^* = 70.73, \quad Q_{32}^* = 66.65, \quad Q_{33}^* = 56.39.$$

The transaction utilities are now: $U_1 = 129,918.82$, $U_2 = 58,877.95$, and $U_3 = 168,602.63$.

All of the Lagrange multipliers are equal to 0 except for the following: $\bar{\lambda}_{12}^1 = 720.98$, $\bar{\lambda}_{22}^1 = 351.79$.

All three blood service organizations enjoy a higher transaction utility by collecting blood in all three regions, rather than just two regions.

The volumes of blood donations are now: for BSO 1: $P_{11} = 1,318.43$, $P_{12} = 592.46$, $P_{13} = 867.59$; for BSO 2: $P_{21} = 1,059.31$, $P_{22} = 580.15$, $P_{23} = 635.70$, and for BSO 3: $P_{31} = 1,381.22$, $P_{32} = 2,049.99$, and $P_{33} = 1,246.49$. BSO 1 has a revenue of 194,493.95, a cost equal to 76,054.13, and a monetized service quality component of the transaction utility equal to 11,478.99. BSO 2, in turn, enjoys a revenue of 136,509.97, a cost 93,632.34, and a monetized service quality component of its transaction utility equal to 16,000.32. BSO 3 obtains a revenue of 374,216.53, incurs a cost 226,36.88, and a monetized service quality component of its transaction utility equal to 20,751.96.

The net revenue of BSO 1 is now equal to 118,439.83; that of BSO 2 is: 42,877.63, and that of BSO 3: 147,850.66. All blood service organizations gain by servicing another region even in the case of competition.

3.4. Summary and Conclusions

In this chapter, I developed a game theory model for blood donations that focuses on blood service organizations, which are increasingly challenged by competition in this unique industry. Indeed, blood is a product that is life-saving, but, at the same time, it cannot be manufactured, but must be donated by individuals. The model is network-based and the governing concept is that of Nash Equilibrium. The blood service organizations compete for blood donations in different regions and donors respond to the quality of service that the blood service organizations provide in blood collection. I formulated the governing equilibrium conditions as a variational inequality problem and prove that the solution is guaranteed to exist. Conditions for uniqueness are also provided and additional theoretical results based on Lagrange

theory associated with the lower and upper bounds on the service quality levels are established.

The modeling and algorithmic framework is illustrated through a series of examples for which the computed equilibrium quality service levels as well as the Lagrange multipliers, along with analysis are provided. The results obtained for the volume of donations, the revenue, and the costs of each blood service organization, and the monetized component of the transaction utility, which corresponds to the utility associated with providing quality service from the different examples, show how these components vary depending on the intensity of competition. The results also demonstrate how increased competition can yield benefits for blood donors in terms of quality level of service. In addition, the examples show that the blood service organizations can benefit, from enhanced transaction utility, by providing additional blood collection sites. Importantly, the results also reveal that blood service organizations that do "good," can also be financially sustainable.

The results obtained from this model can also provide important managerial insights. For blood collection regions that have lower quality levels, internal assessments can be made by the blood service organizations to figure out the individual factors responsible for such low levels such as longer wait time, unfriendly staff, etc. Accordingly, intervention techniques such as better scheduling, and improved training of staff can be implemented. However, improving the service quality implies increase in cost. The blood service organizations can easily observe the financial implications of their decisions from the analysis.

In addition, it might be of value to blood service organizations to conduct surveys at the blood collection sites and come up with service quality measures that are weighted averages of the scores on the different operational facets mentioned in extant literature that affect donor retention. The blood banking industry is highly regulated by the Food and Drug Administration (FDA). Blood collection procedures have to

follow strict protocols stated by the FDA in order to ensure that the collected blood is safe. This research suggests that, in addition to emphasizing safety measures for blood collection such as proper donor screening, attention should be given to the service quality aspects of the blood collection sites to maintain a steady supply of blood from motivated donors.

CHAPTER 4

SUPPLY CHAIN NETWORK COMPETITION AMONG BLOOD SERVICE ORGANIZATIONS

This chapter is based on the paper by Nagurney and Dutta (2019b). In the competitive supply chain network model developed in this chapter I include multiple, competing blood service organizations which are supplying blood to different hospitals and trauma centers through multiple paths that include all major supply activities such collection, processing and testing, shipment, storage and distribution. There is a cost associated with each link representing the cost for that particular operation.

Similar to the utility function in Chapter 3, the utility function of the blood service organizations here contains revenue as well as altruism/benefit components with the latter being weighted. Further, in this model common/shared capacities on the blood donations are imposed to incorporate supply side competition while upper and lower bounds on the demand are included to address issues of shortage and wastage.

The equilibrium blood product flows in terms of RBCs are determined for each blood service organization, given the competition and the constraints. Perishability along the supply chain is captured and also differentiated prices are revealed. The governing equilibrium conditions are that of Generalized Nash Equilibrium due to the presence of shared constraints. I provide alternative variational inequality formulations of the Generalized Nash Equilibrium problem, along with economic analysis utilizing Lagrange theory associated with the various capacity constraints as well as the demand constraints. Finally, an effective computational scheme is applied to compute the equilibrium solutions in numerical examples comprising the case study.

The remainder of this chapter is organized as follows. In Section 4.1, the model is constructed, and alternative variational inequality formulations provided. In addition, Lagrange analysis is conducted to gain insights into the economic meaning associated with the supply and demand constraints. In Section 4.2, the computational procedure is described, along with explicit formulae, at each iteration, for the RBC path flows, and the Lagrange multipliers associated with the blood collection links, the physical capacity link bounds, and the demand point upper and lower bounds for RBCs. I then demonstrate the applicability of the framework through a case study consisting of a series of numerical examples in Section 4.3. The summary of the results, and conclusions are provided in Section 4.4.

4.1. The Multiple Blood Service Organizations Supply Chain Network Competition Model

Blood service organizations (blood banks) collect blood periodically through blood drives at collection facilities and/or through blood mobile units. Once whole blood is collected at the collection sites it is sent to component laboratories for processing and testing for disease markers. The processing involves separation of the whole blood into components such as red blood cells, plasma, and platelets. Different blood products have different shelf lives and each type of product also needs to be stored at specific temperatures. Hence, supply chain management strategies for blood need to be component-specific. I focus on RBCs in this model since these are the most common type of blood product and are used for transfusions in surgeries, treatments for cancer and other diseases, etc.

As depicted in Figure 4.1, there are I blood service organizations competing with each other. Each blood service organization i can collect blood at n_C^i collection sites. I assume that there are J regions in which blood banks can set up collection sites or send blood mobiles to. Each of the $n_C^i; i = 1, .., I$, collection sites belongs to a region

67
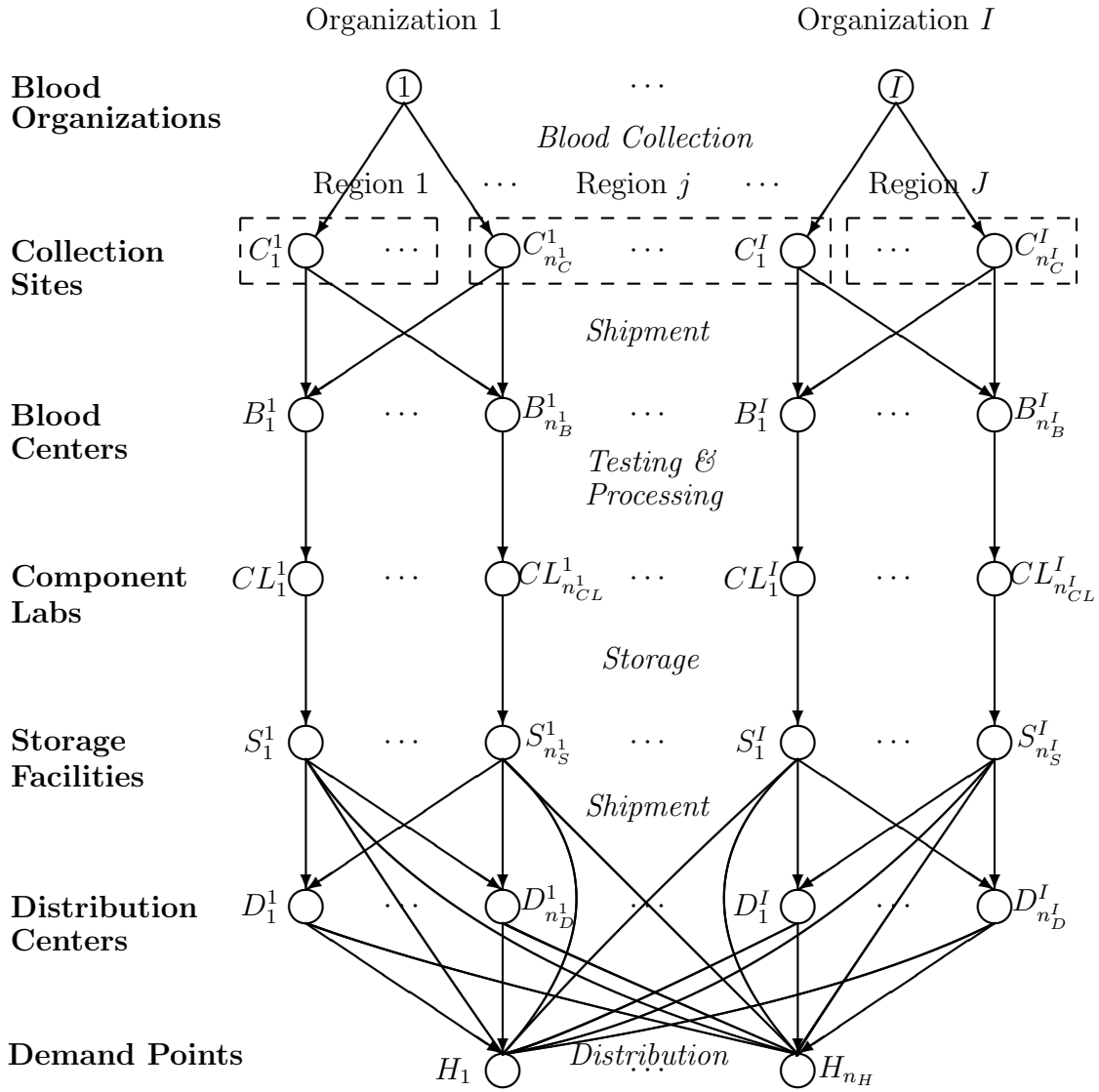
Figure 4.1. Supply Chain Network Topology for I Blood Organizations

$j; j = 1, \ldots, J$. Collected blood by i is then shipped to n_B^i blood centers. From there, blood is sent to n_{CL}^i component laboratories for testing and processing and, subsequently, shipped to n_S^i storage facilities. The component laboratories may not

be separate physical entities but may exist within the blood centers (cf. Nagurney, Masoumi, and Yu (2012)). The subsequent tier of the supply chain network for i; $i = 1, \ldots, I$, in Figure 4.1, is comprised of n_D^i distribution centers. The blood banks may serve the same n_H demand points consisting of hospitals, medical centers, etc., and denoted by the bottom nodes: $H_1, \ldots, H_{n_H}$ in Figure 4.1. These "demand markets" may be served by multiple blood banks since this is the case in reality. For example, Baystate Health in Massachusetts procures blood from the American Red Cross and from the Rhode Island Blood Bank in addition to having in-house blood collection (Merola (2017)).

Each link between a pair of nodes denotes an activity along the supply chain. The links from the blood service organizations to the collection sites represent the collection procedure. The next set of links to the component labs represent the processing and testing of blood. The successive sets of links denote, respectively, storage, shipment, and distribution to demand points. There are also some direct links from storage facilities to demand points since in some cases blood banks work closely with the hospitals and monitor their inventory levels and ship the required amount of blood directly to reduce cost (Wellis (2017)). Hence, the network topology corresponding to even a single blood service organization, as depicted in Figure 4.1, is more general than those constructed in Nagurney, Masoumi, and Yu (2012) and in Masoumi, Yu, and Nagurney (2017).

This model can be used to capture regional as well as nationwide competition. Moreover, large blood service organizations such as the American Red Cross and the New York Blood Center have multiple component labs, storage, and distribution centers, whereas smaller community ones might have one each. The time horizon in which all the activities are occurring is assumed to be one week in this model.

The network topology of the blood service organizations' supply chains is represented by $G = [N, L]$ where N and L denote the sets of nodes and links, respectively.

Table 4.1. Multiplier Notation for Perishability

Notation	Definition
α_a	The arc multiplier associated with link a, which represents the percentage of throughput on link a. $\alpha_a \in (0, 1]$; $a \in L$.
α_{ap}	The arc-path multiplier, which is the product of the multipliers of the links on path p that precede link a; $a \in L$ and $p \in P$; that is, $$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \displaystyle\prod_{b \in \{a' < a\}_p} \alpha_b, & \text{if } \{a' < a\}_p \neq \varnothing, \\ \delta_{ap}, & \text{if } \{a' < a\}_p = \varnothing, \end{cases}$$ where $\{a' < a\}_p$ denotes the set of the links preceding link a in path p and $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.
μ_p	The multiplier corresponding to the percentage of throughput on path p; that is, $\mu_p \equiv \displaystyle\prod_{a \in p} \alpha_a$; $p \in P$.

L^i is defined as the set of all the directed links corresponding to the sequence of activities pertaining to the supply chain network of blood service organization i; $i = 1, \ldots, I$. Associated with each link a is a total operational cost function, denoted by $\hat{c}_a \ \forall a \in L$, representing the cost for each activity corresponding to collection, processing and testing, storage and distribution. A path p consists of a sequence of links originating at one of the top origin nodes in Figure 4.1, ranging from node 1 through node I, and ending at a destination node, corresponding to one of the demand points: $H_1, \ldots, H_{n_H}$.

In order to capture perishability, I utilize a generalized network approach with appropriate arc and path multipliers (see also, e.g., Nagurney et al. (2013)) as defined in Table 4.1. Moreover, since the blood product under consideration is RBCs those paths that would have a time length greater than 42 days are explicitly removed from the network(s) in Figure 4.1 since they would, in effect, be infeasible (and against Food and Drug Administration regulations).

Let x_p denote the nonnegative flow of blood on path p. Let the weekly demand for blood from blood service organization i at demand point k be denoted by d_{ik}; $i = 1, ..., I$; $k = H_1, ..., H_{n_H}$. Let P_k^i denote the set of all paths joining blood service organization node i with destination node H_k. The demands are grouped into the vector $d \in R_+^{I n_H}$.

The conservation of flow equation that has to hold for each blood service organization i; $i = 1, ..., I$, at each demand point k; $k = H_1, ..., H_{n_H}$, is

$$\sum_{p \in P_k^i} \mu_p x_p = d_{ik}, \tag{4.1}$$

that is, the demand for blood at each demand point from each blood service organization has to be satisfied. Observe that, according to (4.1) the amount of blood product flow along a path that arrives at a destination node is equal to the path multiplier times the initial flow on the path since there may be losses due to testing, etc.

Moreover, the path flows must be nonnegative, that is:

$$x_p \geq 0, \quad \forall p \in P, \tag{4.2}$$

where P denotes the set of all paths in the network in Figure 4.1 from origin nodes corresponding to the organizations to the destination nodes corresponding to the demand points.

Let f_a denote the flow of blood on link a. Then, the following conservation of flow equations must also hold:

$$f_a = \sum_{p \in P} x_p \alpha_{ap}, \quad \forall a \in L. \tag{4.3}$$

Note that, according to (4.3), the initial product flow on link a is the sum of the product flows along paths that contain that link, taking into account possible losses in the preceding activities.

As mentioned earlier, the raw material in the supply chain for blood products cannot be manufactured but has to be collected from voluntary donors. Hence, the total amount of blood that can be collected is restricted in terms of the percentage of population that is eligible to donate blood in a particular region in a given week. An eligible donor, say, in Sarasota County in Florida, is unlikely to travel to a distant region in the state or to another state, unless it is in proximity, to donate blood. Therefore, I specify region-based populations and recall that, typically, a donor donates one pint of blood at a time. Let L_1^j denote the set of top-tier links in the network in Figure 4.1 representing blood collection in region j. Then we have the following constraint for each region j; $j = 1, \ldots, J$:

$$\sum_{a \in L_1^j} f_a \leq S^j, \qquad (4.4)$$

where S^j represents the total population eligible to donate blood in a given week in region j; $j = 1, \ldots, J$. Unlike commercial product supply chains with capacity constraints, in this case, the constraint is not on the physical capacity of the production or collection facilities but on the actual supply of the raw material. Observe that (4.4) is a common, that is, a shared constraint among the blood service organizations if a given region includes collection links of multiple blood service organizations.

In addition, explicit link capacities are incorporated on all the network links in Figure 4.1, which represent the actual physical capacities. Hence, for each blood service organization i; $i = 1, \ldots, I$, each link $a \in L^i$ has a positive associated capacity denoted by u_a. Then, the following constraints must also be satisfied:

$$f_a \leq u_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \qquad (4.5)$$

All the link flows in the network are grouped into the vector $f \in R^{n_L}$ where n_L is the total number of elements in L, the set of all links.

Finally, hospitals and medical centers, that is, the demand points, have constraints, which may be included in the contracts with the blood service organizations. In particular, they contract for a lower bound for the weekly deliveries of blood, while also dealing with upper bounds on the amounts that they can safely store in order to also reduce wastage and associated costs. These constraints are as follows:

$$\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p \geq \underline{d}_k, \quad k = H_1, \ldots, H_{n_H}, \tag{4.6}$$

$$\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p \leq \bar{d}_k, \quad k = H_1, \ldots, H_{n_H}, \tag{4.7}$$

where $\underline{d}_k$ denotes the lower bound for units of RBCs at demand point k and $\bar{d}_k$ denotes the upper bound at k. Observe that these are common/shared constraints for the blood service organizations and, hence, will affect their feasible sets, as they compete to serve the hospitals and medical centers with blood.

The total link cost on link a, denoted by $\hat{c}_a$, $\forall a \in L$, may, in general, be a function of all the link flows in the network. This is to enable the modeling of competition for resources across the blood service organizations' supply chain networks. Hence, we have that

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L. \tag{4.8}$$

For example, blood service organizations may compete for staff to conduct the various supply chain network activities; moreover, they may compete for freight services for distribution purposes, etc.

The price that demand point k is willing to pay for a unit of RBCs from blood service organization i is denoted by ρ_{ik} for $i = 1, \ldots, I$; $k = H_1, \ldots H_{n_H}$ and is given by the function:

$$\rho_{ik} = \rho_{ik}(d), \quad i = 1, \ldots, I; \quad k = H_1 \ldots, H_{n_H}. \tag{4.9}$$

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Hence, the price charged per unit of RBCs may, in general, depend on the vector of demands, due to the competition among the hospitals and medical centers for blood. The prices represent the value that a hospital or medical center places on a unit of RBC from a specific blood service organization and that it is willing to pay. These price functions may incorporate parameters reflecting the duration of the contract, if the BSO is selected by the particular hospital, as well as historical information as to the reliability of the former.

In addition, since the majority of blood banks in the United States are nonprofits, there is a utility associated with the service that they provide (cf. Nagurney, Alvarez Flores, and Soylu (2016)). Let γ_{ik} correspond to a measurement of the satisfaction that blood service organization i derives from supplying blood to demand point k. The overall such "service" utility of blood service organization i associated with all the demand points is then given by $\sum_{k=H_1}^{H_{n_H}} \gamma_{ik} d_{ik}$. This service utility also represents altruism (cf. Nagurney, Alvarez Flores, and Soylu (2016)). In addition, each blood service organization i associates a weight ω_i with its service utility, which monetizes it. According to the function $\omega_i \sum_{k=H_1}^{H_{n_H}} \gamma_{ik} d_{ik}$, the greater the amount made available, the more patients that can benefit and, therefore, the greater the good that can be accomplished.

By synthesizing the above revenue and cost terms as well as what may be considered to be a weighted altruism function, the utility function of blood service organization i; $i = 1, \ldots, I$, denoted by U_i, can be expressed as:

$$U_i = \sum_{k=H_1}^{H_{n_H}} \rho_{ik}(d) d_{ik} + \omega_i \sum_{k=H_1}^{H_{n_H}} \gamma_{ik} d_{ik} - \sum_{a \in L^i} \hat{c}_a(f). \tag{4.10}$$

The utility function (4.10) is assumed to be concave and continuously differentiable. It is to be noted that this is the utility of each blood service organization over a time horizon of a week.

In this model, the blood service organizations are trying to maximize their utility, subject to constraints (4.1)-(4.7), while competing for the quantity of blood to be obtained and to be supplied to the hospitals and medical centers. Hence, each blood service organization has, as its strategies, its vector of path flows, X_i, such that

$$X_i \equiv \{\{x_p\}|p \in P^i\} \in R_+^{n_{P^i}}, \tag{4.11}$$

where P^i denotes the set of all paths associated with i and n_{P^i} denotes the number of paths from i to the demand points. Then, X is the vector of all the blood banks' path flows, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$. I, also, for simplicity of notation, use $x \equiv X$.

Using the conservation of flow equations (4.3), shared constraint (4.4) can be rewritten, for each region $j = 1, \ldots, J$, in terms of the strategic variables, i.e., the path flows, as:

$$\sum_{a \in L_1^j} \sum_{p \in P} x_p \delta_{ap} \leq S^j, \quad j = 1, \ldots, J. \tag{4.12}$$

Since collection of blood is the first activity in the network and there are no preceding links, from the definition of the arc-path multiplier we have $\alpha_{ap} = \delta_{ap}$.

Similarly, the individual blood bank's capacity constraints for all activities can be rewritten as follows:

$$\sum_{p \in P} x_p \alpha_{ap} \leq u_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{4.13}$$

The i-th blood bank's individual feasible set is defined as, K_i, given by

$$K_i \equiv \{X_i|(4.2) \text{ and } (4.13) \text{ hold for } i\}. \tag{4.14}$$

Further, I define the feasible set consisting of the shared constraints, $\mathcal{S}$, as:

$$\mathcal{S} \equiv \{X|(4.12), (4.6), \text{ and } (4.7) \text{ hold}\}. \tag{4.15}$$

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Also, in view of (4.1), the demand price functions (4.9) can be reexpressed as:

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(x) \equiv \rho_{ik}(d), \quad i = 1, \ldots, I; k = H_1, \ldots, H_{n_H}. \tag{4.16}$$

Using the conservation of flow equations (4.1) through (4.3), and, given the form of the total link cost functions, the demand price functions, and the weighted altruism functions, I can define each blood service organization utility function in terms of path flows only, that is, $\hat{U}_i(X) \equiv U_i; i = 1, \ldots I$. These utilities are then grouped into an I-dimensional vector $\hat{U}$, where

$$\hat{U} = \hat{U}(X). \tag{4.17}$$

In this model it is assumed that the blood service organizations compete noncooperatively in an oligopolistic market framework in which each blood service organization selects its blood product flows to maximize its utility, until an equilibrium is achieved, according to the following definition.

Definition 4.1: Blood Supply Chain Network Generalized Nash Equilibrium

A blood product path flow pattern $X^* \in K \equiv \prod_{i=1}^{I} K^i, X^* \in \mathcal{S}$, constitutes a blood supply chain network Generalized Nash Equilibrium if for each blood service organization $i; i = 1, ..., I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K^i, \forall X \in \mathcal{S}, \tag{4.18}$$

where

$$\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*).$$

According to (4.18) an equilibrium is established if no blood service organization can unilaterally improve upon its utility by selecting an alternative vector of blood

product flows, given the blood product flows of the other blood service organizations, and subject to the capacity constraints, both individual and shared ones, the shared demand constraints, and the nonnegativity constraints. It is to be noted that K and $\mathcal{S}$ are both convex sets.

If there are no coupling, that is, shared, constraints in this problem then X and X^* in Definition 4.1 need only lie in the feasible set K, and, under the assumption of concavity of the utility functions and that they are continuously differentiable, we know that (cf. Gabay and Moulin (1980) and Nagurney (1999)) the solution to what would then be a Nash Equilibrium problem (see Nash (1950, 1951)) would coincide with the solution to the following variational inequality problem: determine $X^* \in K$, such that

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{4.19}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i.

However, as mentioned earlier, since here the blood service organizations have common constraints on the amount of blood that can be collected, and on the amounts to be delivered, the strategies of each BSO affect both the objective functions as well as the feasible sets of the other BSOs. Consequently, this is a Generalized Nash Equilibrium (GNE) which cannot be directly formulated as variational inequality problem, but may be formulated as a quasi-variational inequality.

4.1.1 Variational Equilibrium and Variational Inequality Formulation

I now define the variational equilibrium which, as emphasized in Chapter 2, Section 2.3 is a refinement of the Generalized Nash Equilibrium and is a specific type of GNE (see Kulkarni and Shahbhang (2012)). In a GNE defined by a variational equilibrium, the Lagrange multipliers associated with the shared constraints are all

the same which provides a fairness interpretation and makes sense from an economic standpoint. Specifically, we have:

Definition 4.2: Variational Equilibrium

A strategy vector X^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $X^* \in K, X^* \in \mathcal{S}$ is a solution of the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \forall X \in \mathcal{S}. \tag{4.20}$$

I now expand the terms in the variational inequality (4.20). From the definition of a gradient, it is known that

$$-\nabla_{X_i} \hat{U}_i(X) = \left[-\frac{\partial \hat{U}_i}{\partial x_p}; p \in P_k^i; k = H_1, \ldots, H_{n_H} \right]. \tag{4.21}$$

We also know that, in view of (4.1) and (4.10), that for paths $p \in P_k^i$:

$$-\frac{\partial \hat{U}_i}{\partial x_p} = -\frac{\partial(\sum_{l=H_1}^{H_{n_H}} \rho_{il}(d) \sum_{q \in P_l^i} \mu_q x_q + \omega_i \sum_{l=H_1}^{H_{n_H}} \gamma_{il} \sum_{q \in P_l^i} \mu_q x_q - \sum_{b \in L^i} \hat{c}_b(f))}{\partial x_p}. \tag{4.22}$$

Then, making use of (4.1) and (4.3) and the expression (4.16), we have that for $p \in P_k^i$:

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \alpha_{ap}, \tag{4.23a}$$

$$\frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \equiv \frac{\partial \rho_{il}(d)}{\partial d_{ik}} \mu_p, \tag{4.23b}$$

and obtain for $p \in P_k^i$:

$$-\frac{\partial \hat{U}_i}{\partial x_p} = \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x)\mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q \right]. \tag{4.24}$$

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Hence, (4.20) is equivalent to the variational inequality: determine $x^* \in K, x^* \in \mathcal{S}$ such that:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x^*)\mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \right] \times [x_p - x_p^*] \geq 0,$$

$$\forall x \in K, x \in \mathcal{S}. \tag{4.25}$$

For simplicity, I refer to $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ as the *marginal total cost of path p*.

Variational inequality (4.25) can now be put into the standard form given by (2.1a) in Chapter 2. Let the p-th component of $F(X)$ for a given i, k, $p \in P_k^i$, $\forall i, k$, be

$$\left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x)\mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q \right], \tag{4.26}$$

with $\mathcal{K} \equiv \mathcal{K}^1 \equiv K \cap \mathcal{S}$, then variational inequality (4.25) can be put into the standard form.

Remark: Existence of an Equilibrium Solution

It is assumed that the feasible set $\mathcal{K}$ is nonempty, which will be the case if the capacities on the links and blood donor regions are sufficient to satisfy the sum of the demands for blood at the demand points. An equilibrium blood flow pattern $X^* = x^* \in \mathcal{K}$ satisfying variational inequality (4.26); equivalently, variational inequality (4.25), is guaranteed to exist since the function $F(X)$ is continuous under the imposed assumptions and the feasible set $\mathcal{K}$ is compact, due to the nonnegative assumption on the blood path flows and the link and blood donor regional upper bound capacities.

4.1.2 Alternative Variational Inequality Formulations and Lagrange Analysis with Economic Interpretation

In this part, I first present an alternative variational inequality formulation to the one in (4.25), again, in path flows, but using Lagrange multipliers. I then conduct an

economic analysis using Lagrange theory and conclude with, yet, another variational inequality, which I utilize for computational purposes in Section 4.2.

Let η_j, $\forall j$, and $\theta_a; a \in L$, denote the Lagrange multipliers associated with constraints (4.12) and (4.13), respectively. In addition, let σ_k; $\forall k$, denote the Lagrange multiplier associated with the k-th lower bound demand constraint (4.6) and let ϵ_k; $\forall k$, denote the Lagrange multiplier associated with the k-th upper bound demand constraint (4.7). The above Lagrange multipliers are grouped into the respective vectors: $\eta \in R_+^J$, $\theta \in R_+^{n_L}$, $\sigma \in R_+^{n_H}$, and $\epsilon \in R_+^{n_H}$. Also, let β_p; $\forall p \in P$, denote the Lagrange multiplier associated with each path p nonnegativity constraint (4.2) and I group these Lagrange multipliers into the vector $\beta \in R_+^{n_P}$. I define the feasible set $\mathcal{K}^2 \equiv \{(x, \beta, \eta, \theta, \sigma, \epsilon) | x \in R_+^{n_P}, \beta \in R_+^{n_P}, \eta \in R_+^J, \theta \in R_+^{n_L}, \sigma \in R_+^{n_H}, \epsilon \in R_+^{n_H}\}$. Then, we have the following result:

Theorem 4.1: Alternative Variational Inequality Formulation of the Variational Equilibrium in Path Flows

The variational inequlity (4.25) is equivalent to the variational inequality: determine the vector of equilibrium path flows and Lagrange multipliers, $(x^*, \beta^*, \eta^*, \theta^*, \sigma^*, \epsilon^*) \in \mathcal{K}^2$, such that:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \beta_p^* + \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* \delta_{ap} + \sum_{a \in L^i} \theta_a^* \alpha_{ap} - \omega_i \gamma_{ik} \mu_p - \sigma_k^* \mu_p + \epsilon_k^* \mu_p - \hat{\rho}_{ik}(x^*) \mu_p \right.$$

$$\left. - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \right] \times [x_p - x_p^*]$$

$$+ \sum_{p \in P} x_p^* \times [\beta_p - \beta_p^*] + \sum_{j=1}^{J} \left[S^j - \sum_{a \in L_1^j} \sum_{p \in P} x_p^* \delta_{ap} \right] \times [\eta_j - \eta_j^*] + \sum_{i=1}^{I} \sum_{a \in L^i} \left[u_a - \sum_{p \in P} x_p^* \alpha_{ap} \right] \times [\theta_a - \theta_a^*]$$

$$+ \sum_{k=H_1}^{H_{n_H}} (\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^* - \underline{d}_k) \times (\sigma_k - \sigma_k^*) + \sum_{k=H_1}^{H_{n_H}} (\bar{d}_k - \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^*) \times (\epsilon_k - \epsilon_k^*) \geq 0,$$

$$\forall (x, \beta, \eta, \theta, \sigma, \epsilon) \in \mathcal{K}^2. \tag{4.27}$$

Proof: By setting:

$$V(x) = \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x^*) \mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \right] \times [x_p - x_p^*], \tag{4.28}$$

variational inequality (4.25) can be rewritten as:

$$\text{Min}_{\mathcal{K}} V(x) = V(x^*) = 0. \tag{4.29}$$

Under the previously imposed assumptions we know that all the underlying functions in (4.29) are continuously differentiable and convex.

Then let:

$$
\begin{aligned}
b_p &= -x_p \leq 0, & \forall p, \\
e_j &= \sum_{a \in L_1^j} \sum_{p \in P} x_p \delta_{ap} - S^j \leq 0, & \forall j, \\
g_a &= \sum_{p \in P} x_p \alpha_{ap} - u_a \leq 0, & \forall a, \\
h_k &= \underline{d}_k - \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p \leq 0, & \forall k, \\
r_k &= \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p - \bar{d}_k \leq 0, & \forall k,
\end{aligned}
\tag{4.30}
$$

and

$$\Gamma(x) = (b_p, e_j, g_a, h_k, r_k)_{p \in P; j=1,\dots,J; a \in L; k=H_1,\dots,H_{n_H}}. \tag{4.31}$$

Hence, the feasible set $\mathcal{K}$ can be rewritten as

$$\mathcal{K} = \{x \in R_+^{n_P} : \Gamma(x) \leq 0\}. \tag{4.32}$$

I now construct the Lagrange function:

$$\mathcal{L}(x, \beta, \eta, \theta, \sigma, \epsilon) = \sum_{i=1}^{I} \left(-\sum_{k=H_1}^{H_{n_H}} \hat{\rho}_{ik}(x) \sum_{p \in P_k^i} \mu_p x_p - \omega_i \sum_{k=H_1}^{H_{n_H}} \gamma_{ik} \sum_{p \in P_k^i} \mu_p x_p + \sum_{a \in L^i} \hat{c}_a(Ax) \right)$$

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$$+\sum_{p\in P}\beta_p b_p + \sum_{j=1}^{J}\eta_j e_j + \sum_{a\in L}\theta_a g_a + \sum_{k=H_1}^{H_{n_H}}\sigma_k h_k + \sum_{k=H_1}^{H_{n_H}}\epsilon_k r_k, \qquad (4.33)$$

$$\forall x \in R_+^{n_P}, \forall \beta \in R_+^{n_P}, \forall \eta \in R_+^{J}, \forall \theta \in R_+^{n_L}, \forall \sigma \in R_+^{n_H}, \forall \epsilon \in R_+^{n_H},$$

where A is the arc-path incidence matrix with component $ap = 1$, if link a is contained in path p and 0, otherwise; β is the vector with components: $\{\beta_p, \forall p \in P\}$, with η and the other vectors of Lagrange multipliers as defined above.

It is straightforward to establish that the feasible set $\mathcal{K}$ is convex and that the Slater condition holds. Then, if x^* is the minimal solution to problem (4.29), there exist $\beta^* \in R_+^{n_P}$, $\eta^* \in R_+^{J}$, $\theta^* \in R_+^{n_L}$, $\sigma^* \in R_+^{n_H}$, and $\epsilon^* \in R_+^{n_H}$ such that the vector $(x^*, \beta^*, \eta^*, \theta^*, \sigma^*, \epsilon^*)$ is a saddle point of the Lagrange function (4.33), that is:

$$\mathcal{L}(x^*, \beta, \eta, \theta, \sigma, \epsilon) \le \mathcal{L}(x^*, \beta^*, \eta^*, \theta^*, \sigma^*, \epsilon^*) \le \mathcal{L}(x, \beta^*, \eta^*, \theta^*, \sigma^*, \epsilon^*), \qquad (4.34)$$

$$\forall x \in R_+^{n_P}, \forall \beta \in R_+^{n_P}, \forall \eta \in R_+^{J}, \forall \theta \in R_+^{n_L}, \forall \sigma \in R_+^{n_H}, \forall \epsilon \in R_+^{n_H},$$

and

$$\beta_p^* b_p^* = 0, \quad \forall p \in P,$$

$$\eta_j^* e_j^* = 0, \quad \forall j,$$

$$\theta_a^* g_a^* = 0, \quad \forall a \in L,$$

$$\sigma_k^* h_k^* = 0, \quad \epsilon_k^* r_k^* = 0, \quad \forall k. \qquad (4.35)$$

From the right-hand side of (4.35) it follows that $x^* \in R_+^{n_P}$ is a minimal point of $\mathcal{L}(x, \beta^*, \eta^*, \theta^*, \sigma^*, \epsilon^*)$ in the entire space R^{n_P} and, therefore, we have that for all $p \in P_k^i, \forall i, \forall k$:

$$\frac{\partial \mathcal{L}(x^*, \beta^*, \eta^*, \theta^*, \sigma^*, \epsilon^*)}{\partial x_p}$$

$$= \frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ik}\mu_p - \hat{\rho}_{ik}(x^*)\mu_p - \sum_{l=H_1}^{H_{n_H}}\frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}\mu_q x_q^* - \beta_p^* + \sum_{j=1}^{J}\sum_{a\in L_1^j}\eta_j^*\delta_{ap}$$

$$+ \sum_{a \in L^i} \theta_a^* \alpha_{ap} - \sigma_k^* \mu_p + \epsilon_k^* \mu_p = 0, \tag{4.36}$$

together with conditions (4.35). Conditions (4.35) and (4.36) correspond to an equivalent variational inequality to that in (4.25). For example, if we multiply (4.36) by $(x_p - x_p^*)$ and sum with respect to $p \in P_k^i$, $\forall i$, $\forall k$, we obtain:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x^*) \mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \right] \times (x_p - x_p^*)$$

$$= \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \beta_p^* x_p - \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \beta_p^* x_p^*$$

$$- \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{j=1}^{J} \sum_{a \in L_j^i} \eta_j^* x_p \delta_{ap} + \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* x_p^* \delta_{ap}$$

$$- \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{a \in L^i} \theta_a^* \alpha_{ap} x_p + \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{a \in L^i} \theta_a^* \alpha_{ap} x_p^*$$

$$+ \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sigma_k^* \mu_p x_p - \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sigma_k^* \mu_p x_p^* - \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \epsilon_k^* \mu_p x_p + \sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \epsilon_k^* \mu_p x_p^*. \tag{4.37}$$

Examining the expressions on the right-hand side of the equal sign in (4.37) it is known that for $j = 1, \ldots, J$, $\forall a \in L$, and for $k; k = H_1, \ldots, H_{n_H}$:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \beta_p x_p^* = 0,$$

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \sum_{a \in L_1^j} \eta_j^* x_p^* \delta_{ap} = \eta_j^* S_j,$$

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \theta_a^* \alpha_{ap} x_p^* = \theta_a^* u_a,$$

$$\sum_{i=1}^{I}\sum_{p\in P_k^i}\sigma_k^*\mu_p x_p^* = \sigma_k^*\underline{d}_k, \quad \sum_{i=1}^{I}\sum_{p\in P_k^i}\epsilon_k^*\mu_p x_p^* = \epsilon_k^*\bar{d}_k. \tag{4.38}$$

Hence, the right-hand side of (4.37) simplifies to:

$$\sum_{i=1}^{I}\sum_{k=H_1}^{H_{n_H}}\sum_{p\in P_k^i}\beta_p^* x_p - \sum_{j=1}^{J}\eta_j^*\left(\sum_{i=1}^{I}\sum_{a\in L_j^i}\sum_{k=H_1}^{H_{n_H}}\sum_{p\in P_k^i} x_p\delta_{ap} - S^j\right) - \sum_{a\in L}\theta_a^*(f_a - u_a)$$

$$+ \sum_{k=H_1}^{H_{n_H}}\sigma_k^*\left(\sum_{i=1}^{I}\sum_{p\in P_k^i}\mu_p x_p - \underline{d}_k\right) - \sum_{k=H_1}^{H_{n_H}}\epsilon_k^*\left(\sum_{i=1}^{I}\sum_{p\in P_k^i}\mu_p x_p - \bar{d}_k\right) \geq 0, \tag{4.39}$$

and the conclusion follows. $\square$

I now provide an economic interpretation of the Lagrange multipliers. I consider a path $p \in P_k^i$ for a fixed i and k where $x_p^* > 0$, that is, the equilibrium blood flow on the path is positive. Then, from the first line of (4.35) it is known that $\beta_p^* = 0$. In particular, I consider multiple distinct cases which are given below.

Case I: None of the Associated Constraints are Active

Let us first consider the case when the associated path capacity and demand constraints are not active, that is, in equality (4.36) we have that, in addition to $\beta_p^* = 0$, the corresponding $\eta_j^* = 0$, as well as the corresponding $\theta_a^* = 0$, with also $\sigma_k^* = 0$ and $\epsilon_k^* = 0$. Hence, we then have that (4.36) satisfies

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i\gamma_{ik}\mu_p - \hat{\rho}_{ik}(x^*)\mu_p - \sum_{l=H_1}^{H_{n_H}}\frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}\mu_q x_q^* - \beta_p^* + \sum_{j=1}^{J}\sum_{a\in L_1^j}\eta_j^*\delta_{ap}$$

$$+ \sum_{a\in L^i}\theta_a^*\alpha_{ap} - \mu_p\sigma_k^* + \mu_p\epsilon_k^* = 0$$

$$\iff \frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i\gamma_{ik}\mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}}\frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}\mu_q x_q^*, \tag{4.40}$$

which means that, in this case, the marginal total cost on path p is equal to the marginal utility associated with weighted altruism of the pair (i, k) plus the marginal revenue associated with the path p.

Case II: The Associated Donor Supply Constraints Are Active but Other Capacity and Demand Constraints Associated with the Path Are Not

I now consider the situation in which the blood collected in the regions that link a of path p is contained in is equal to the available supply, in which case the corresponding η_j^* of those regions j will be positive. Also, the other capacity and demand constraints relevant to path p are not at their bounds. Hence, we then get from (4.36) that

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* - \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* \delta_{ap}, \quad (4.41)$$

and, therefore,

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \quad (4.42)$$

This result is quite intuitive, since it implies that the marginal total cost on path p is less than the marginal utility associated with the weighted altruism plus the marginal revenue associated with the path p. This situation is beneficial for BSO i.

Case III: One or More Links on the Path Are at Their Capacities But No Other Associated Capacity or Demand Constraints Are Active

In this case it is known that (4.36) yields:

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* - \sum_{a \in L^i} \theta_a^* \alpha_{ap}, \quad (4.43)$$

and, therefore,

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \quad (4.44)$$

This is also reasonable, since if the path p has one or more links at their capacities, then one would expect that the marginal total cost of that path would be less that

the marginal utility associated with the weighted altruism/benefit function plus the marginal revenue associated with the path p.

Case IV: The Demand Point That the Path Is Destined to Has Its Demand at the Lower Bound Whereas No Other Associated Constraints Are Active

In this case it is known that $\sigma_k^* > 0$ and all other relevant Lagrange multipliers are zero so that expression (4.36) now yields:

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* + \mu_p \sigma_k^*, \qquad (4.45)$$

and, hence,

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} > \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \qquad (4.46)$$

This is not a desirable situation since the marginal total cost on the path p now exceeds the marginal utility associated with the weighted altruism/benefit function plus the marginal revenue associated with the path p.

Case V: The Demand Point That the Path Is Destined to Has Its Demand at the Upper Bound Whereas No Other Associated Constraints Are Active

I now consider the case when the demand at point k is at its upper bound and no other associated constraints are active (and, therefore, all other associated Lagrange multipliers are equal to zero). We know that then $\epsilon_k^* > 0$ and we have that, according to (4.36):

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* - \mu_p \epsilon_k^*, \qquad (4.47)$$

and, consequently,

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \tag{4.48}$$

According to (4.48), the marginal total cost on path p, in this case, is less than the marginal utility associated with the weighted altruism/benefit function plus the marginal revenue of the BSO and demand point pair (i, k). This is clearly another desirable situation.

Taking into account the Lagrange multipliers, an equivalent variational formulation to variational inequality (4.27) is the following: determine the vector of equilibrium path flows and Lagrange multipliers, $(x^*, \eta^*, \theta^*, \sigma^*, \epsilon^*) \in \mathcal{K}^3$, such that:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* \delta_{ap} + \sum_{a \in L^i} \theta_a^* \alpha_{ap} - \omega_i \gamma_{ik} \mu_p - \sigma_k^* \mu_p + \epsilon_k^* \mu_p - \hat{\rho}_{ik}(x^*)\mu_p \right.$$

$$\left. - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \right] \times [x_p - x_p^*]$$

$$+ \sum_{j=1}^{J} \left[S^j - \sum_{a \in L_1^j} \sum_{p \in P} x_p^* \delta_{ap} \right] \times [\eta_j - \eta_j^*] + \sum_{i=1}^{I} \sum_{a \in L^i} \left[u_a - \sum_{p \in P} x_p^* \alpha_{ap} \right] \times [\theta_a - \theta_a^*]$$

$$+ \sum_{k=H_1}^{H_{n_H}} (\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^* - \underline{d}_k) \times (\sigma_k - \sigma_k^*) + \sum_{k=H_1}^{H_{n_H}} (\bar{d}_k - \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^*) \times (\epsilon_k - \epsilon_k^*) \geq 0,$$

$$\forall (x, \eta, \theta, \sigma, \epsilon) \in \mathcal{K}^3, \tag{4.49}$$

where $\mathcal{K}^3 \equiv \{(x, \eta, \theta, \sigma, \epsilon) | x \in R_+^{n_P}, \eta \in R_+^J, \theta \in R_+^{n_L}, \sigma \in R_+^{n_H}, \epsilon \in R_+^{n_H}\}$.

For the case study in Section 4.3, I utilize variational inequality (4.49). It is to be noted that variational inequality (4.49) can also be put into standard form (2.1a) (cf. Chapter 2).

4.2. The Computational Procedure

In this section, I describe the realization of the Euler method (cf. Chapter 2) which is discussed in details in Section 2.4. As shown in Dupuis and Nagurney (1993)

and Nagurney and Zhang (1996), for convergence of the general iterative scheme, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} = \infty, a_\tau > 0, a_\tau \to 0$, as $\tau \to \infty$.

Specifically, the notable feature of this algorithm, when applied to the blood supply chain network competition model, is that it yields closed form expressions for the variables at each iteration, resulting in an elegant procedure for computations and solution.

Explicit Formulae for the Euler Method Applied to the Alternative Variational Inequality Formulation (4.49)

In particular, for this problem, we have the following closed form expressions for the path flows at iteration $\tau + 1$. For each path $p \in P_k^i, \forall i, k$, we have:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ik}(x^\tau)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^\tau)}{\partial x_p} \sum_{q \in P_l^i} x_q^\tau \mu_q + \omega_i \gamma_{ik}\mu_p - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p}$$

$$- \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^\tau \delta_{ap} - \sum_{a \in L^i} \theta_a^\tau \alpha_{ap} + \sigma_k^\tau \mu_p - \epsilon_k^\tau \mu_p)\}. \tag{4.50}$$

The Lagrange multipliers associated with blood collection links $a \in L_1^j$; $j = 1, \ldots, J$, are computed according to:

$$\eta_j^{\tau+1} = \max\{0, \eta_j^\tau + a_\tau(\sum_{a \in L_1^j} \sum_{p \in P} x_p^\tau \delta_{ap} - S^j)\}. \tag{4.51}$$

The closed form expression for the Lagrange multipliers for the capacity constraint on link $a \in L^i$; $i = 1, \ldots, I$ is:

$$\theta_a^{\tau+1} = \max\{0, \theta_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \alpha_{ap} - u_a)\}. \tag{4.52}$$

Next, I provide the closed form expressions for the Lagrange multipliers associated with the upper and lower bounds on the demands. The explicit formulae for the

Lagrange multipliers associated with the lower bounds on the demands at demand points: $k = H_1, \ldots, H_{n_H}$, are:

$$\sigma_k^{\tau+1} = \max\{0, \sigma_k^\tau + a_\tau(\underline{d}_k - \sum_{i=1}^I \sum_{p \in P_k^i} \mu_p x_p^\tau)\}. \tag{4.53}$$

The Lagrange multipliers associated with the upper bounds on the demands at the demand points: $k = H_1, \ldots, H_{n_H}$, in turn, are computed according to:

$$\epsilon_k^{\tau+1} = \max\{0, \epsilon_k^\tau + a_\tau(\sum_{i=1}^I \sum_{p \in P_k^i} \mu_p x_p^\tau - \overline{d}_k)\}. \tag{4.54}$$

The algorithm is assumed to have converged when the absolute value of successive iterates is less than or equal to the imposed convergence tolerance ϵ.

4.3. Numerical Examples

The numerical examples are inspired by a particular region of New England in which there is growing competition between blood service organizations. The examples are stylized but capture the features of the game theory model and demonstrate the types of insights that can be revealed.

In the paper Nagurney and Dutta (2019b) the Euler method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst was used for the computations. The Euler method was initialized with all variables identically equal to 0.00. The $\{a_\tau\}$ sequence utilized was: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$. The convergence tolerance utilized was 10^{-6}; in other words, the algorithm was terminated when the absolute value of successive computed variable iterates was less than or equal to this value. The numerical examples below contain explicit input and output data.

Example 4.1: Baseline Example with Two BSOs and Four Hospitals

I consider two blood service organizations (BSOs), with BSO 1 being a local one, and BSO 2 being an iconic national one. Please refer to Figure 4.2 for the supply chain network topology for all the numerical examples. BOS 1 has two collection sites, a single blood center for processing and testing as well as a single component lab and storage facility, similar to, for example, the Rhode Island Blood Center, which is based in Providence, Rhode Island. BSO 2, in turn, has three collection sites, two blood centers for testing and processing, two component labs and storage facilities, as well as distribution centers.

There are four demand points with the first and third, denoted, respectively, by H_1 and H_3 denoting major trauma hospitals and the other two: H_2 and H_4 corresponding to smaller hospitals.

Also, there are three regions, as depicted in Figure 4.2, with Region 2 being common (that is, in proximity) to a collection site of each organization. Here regions correspond to counties.

I now provide the data for this example. Example 4.1 serves as the baseline from which other examples are constructed.

The number of people eligible to donate blood in each of the regions are:

$$S^1 = 6000, \quad S^2 = 2400, \quad S^3 = 3000.$$

Such values are reasonable since the Northeast has a higher percentage of senior citizens (65 years and above) (Wilson (2013)) and older people are found to be less likely to donate or retire from donating after a certain age (Shaz et al. (2011), Aleccia (2017)).

The weekly upper and lower bounds on the demand at each hospital are given below:

$$\underline{d}_{H_1} = 200, \quad \overline{d}_{H_1} = 350,$$
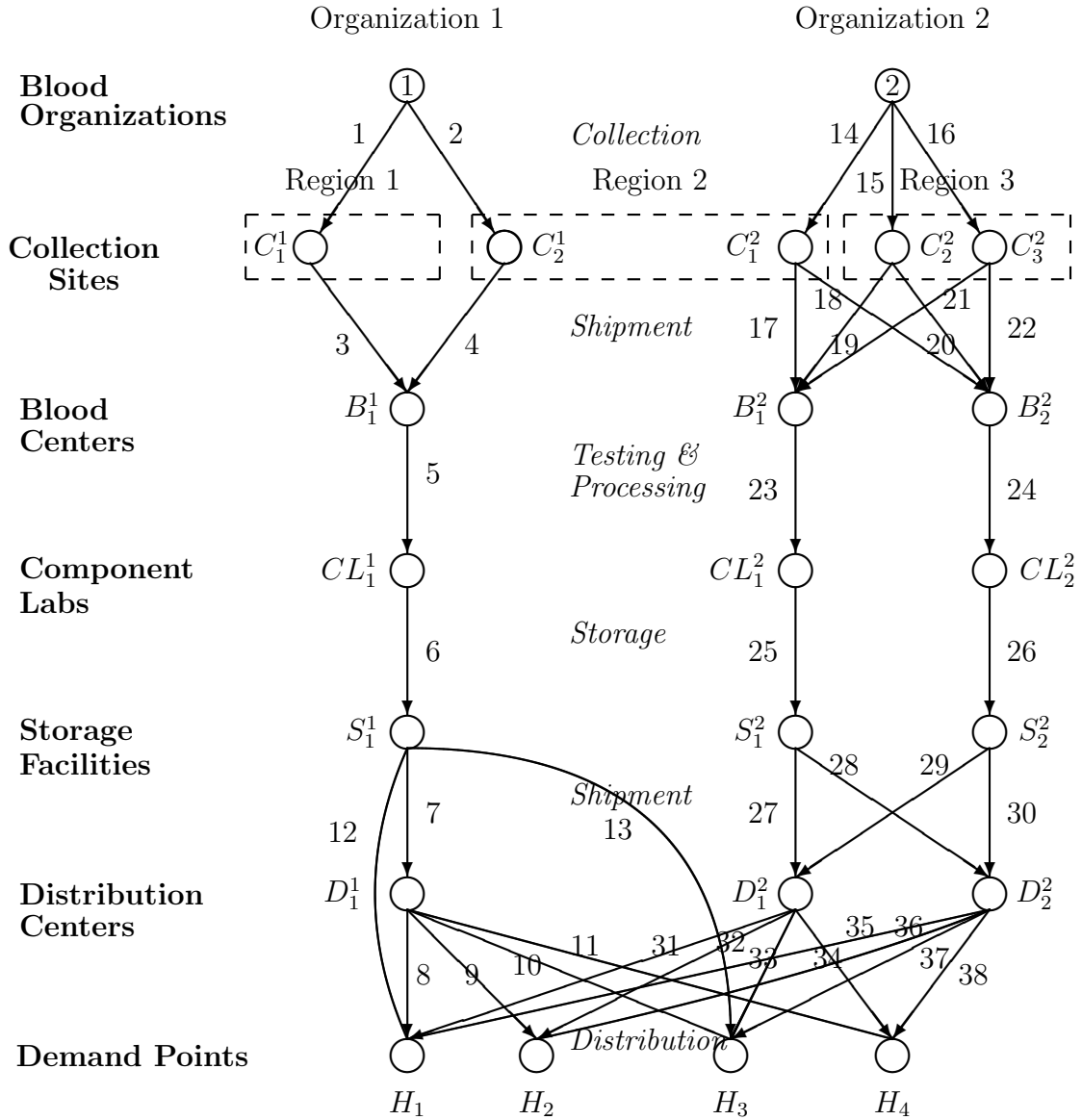
Figure 4.2. The Supply Chain Network Topology for Examples 4.1-4.4

$$\underline{d}_{H_2} = 60, \quad \overline{d}_{H_2} = 150,$$

$$\underline{d}_{H_3} = 200, \quad \overline{d}_{H_3} = 300,$$

$$\underline{d}_{H_4} = 100, \quad \overline{d}_{H_4} = 120.$$

The link definitions, associated link capacities, arc multipliers, total cost functions, and computed equilibrium link flows and associated link Lagrange multipliers

are provided in Table 4.2. Since BSO 2 operates on a national level, it has more resources than BSO 1 which is reflected in many of the link capacities. The cost functions and demand price functions are constructed using information obtained from Tracy (2010), Carlyle (2012), Gunpinar and Centeno (2015), and Masoumi, Yu and Nagurney (2017). Also, there are losses on links associated with testing and processing, and, hence, those arc multipliers are less than 1.

The demand price functions are as follows:

BSO 1:

$$\rho_{1H_1}(d) = -0.07d_{1H_1} - 0.02d_{2H_1} + 300, \quad \rho_{1H_2}(d) = -0.08d_{1H_2} - 0.03d_{2H_2} + 310,$$

$$\rho_{1H_3}(d) = -0.05d_{1H_3} - 0.01d_{2H_3} + 300, \quad \rho_{1H_4}(d) = -0.04d_{1H_4} - 0.02d_{2H_4} + 280.$$

BSO 2:

$$\rho_{2H_1}(d) = -0.05d_{2H_1} - 0.01d_{1H_1} + 280, \quad \rho_{2H_2}(d) = -0.07d_{2H_2} - 0.04d_{1H_2} + 290,$$

$$\rho_{2H_3}(d) = -0.03d_{2H_3} - 0.01d_{1H_3} + 280, \quad \rho_{2H_4}(d) = -0.05d_{2H_4} - 0.02d_{1H_4} + 270.$$

The equilibrium link solution are reported in Table 4.2 since the number of paths is quite large - equal to 60, whereas the number of links is 38.

In addition, it is assumed that the weights associated with the altruism component of the BSOs' objective functions are both equal to 1 so that $\omega_1 = \omega_2 = 1$. Furthermore, we have that $\gamma_{1H_1} = 2$, $\gamma_{1H_2} = 1$, $\gamma_{1H_3} = 2$, and $\gamma_{1H_4} = 1$, whereas $\gamma_{2H_1} = 2$, $\gamma_{2H_2} = 1$, $\gamma_{2H_3} = 2$, and $\gamma_{2H_4} = 1$. Hence, both BSOs assign a higher value to servicing the larger hospitals.

It can be seen from Table 4.2 that four of the links are at their capacities and these are links: 13, 34, 36, and 38. All these links are shipment links. Link 13 is associated with BSO 1, whereas the other links are in BSO 2's supply chain network. The BSOs are advised to invest in enhancing the capacities in these links.

Table 4.2. Definition of Links, Associated Weekly Link Capacities, Arc Multipliers, Total Operational Link Cost Functions, Equilibrium Link Solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 4.1

Link a	From Node	To Node	u_a	α_a	$\hat{c}_a(f)$	f_a^*	θ_a^*
1	1	C_1^1	250	1.00	$0.24f_1^2 + 0.6f_1$	139.33	0.00
2	1	C_2^1	200	1.00	$0.4f_2^2 + 0.9f_2$	87.59	0.00
3	C_1^1	B_1^1	300	1.00	$0.06f_3^2 + 0.1f_3$	139.33	0.00
4	C_2^1	B_1^1	250	1.00	$0.07f_4^2 + 0.16f_4$	87.59	0.00
5	B_1^1	CL_1^1	500	0.97	$0.36f_5^2 + 0.45f_5$	226.92	0.00
6	CL_1^1	S_1^1	500	1.00	$0.02f_6^2 + 0.04f_6$	220.11	0.00
7	S_1^1	D_1^1	500	1.00	$0.03f_7^2 + 0.09f_7$	166.40	0.00
8	D_1^1	H_1	50	1.00	$0.4f_8^2 + 0.7f_8$	21.37	0.00
9	D_1^1	H_2	50	1.00	$0.5f_9^2 + 0.9f_9$	28.38	0.00
10	D_1^1	H_3	100	1.00	$0.15f_{10}^2 + 0.8f_{10}$	76.64	0.00
11	D_1^1	H_4	60	1.00	$0.35f_{11}^2 + 0.6f_{11}$	40.00	0.00
12	S_1^1	H_1	50	1.00	$0.4f_{12}^2 + 0.9f_{12}$	33.71	0.00
13	S_1^1	H_3	20	1.00	$0.7f_{13}^2 + 1f_{13}$	20.00	5.02
14	2	C_1^2	250	1.00	$0.25f_{14}^2 + 0.7f_{14}$	130.81	0.00
15	2	C_2^2	300	1.00	$0.2f_{15}^2 + 0.8f_{15}$	148.27	0.00
16	2	C_3^2	200	1.00	$0.3f_{16}^2 + 0.5f_{16}$	112.99	0.00
17	C_1^2	B_1^2	100	1.00	$0.12f_{17}^2 + 0.3f_{17}$	70.11	0.00
18	C_1^2	B_2^2	150	1.00	$0.08f_{18}^2 + 0.27f_{18}$	60.71	0.00
19	C_2^2	B_1^2	100	1.00	$0.16f_{19}^2 + 0.45f_{19}$	70.86	0.00
20	C_2^2	B_2^2	200	1.00	$0.1f_{20}^2 + 0.5f_{20}$	77.41	0.00
21	C_3^2	B_1^2	100	1.00	$0.2f_{21}^2 + 0.6f_{21}$	35.85	0.00
22	C_3^2	B_2^2	100	1.00	$0.05f_{22}^2 + 0.08f_{22}$	77.14	0.00
23	B_1^2	CL_1^2	600	0.98	$0.36f_{23}^2 + 0.8f_{23}$	176.81	0.00
24	B_2^2	CL_2^2	500	0.96	$0.3f_{24}^2 + 0.7f_{24}$	215.25	0.00
25	CL_1^2	S_1^2	500	1.00	$0.02f_{25}^2 + 0.05f_{25}$	173.28	0.00
26	CL_2^2	S_2^2	500	1.00	$0.03f_{26}^2 + 0.04f_{26}$	206.64	0.00
27	S_1^2	D_1^2	150	1.00	$0.15f_{27}^2 + 0.4f_{27}$	88.02	0.00
28	S_1^2	D_2^2	150	1.00	$0.18f_{28}^2 + 0.65f_{28}$	85.25	0.00
29	S_2^2	D_1^2	200	1.00	$0.09f_{29}^2 + 0.12f_{29}$	116.35	0.00
30	S_2^2	D_2^2	150	1.00	$0.14f_{30}^2 + 0.5f_{30}$	90.30	0.00
31	D_1^2	H_1	100	1.00	$0.24f_{31}^2 + 0.8f_{31}$	48.90	0.00
32	D_1^2	H_2	80	1.00	$0.32f_{32}^2 + 0.9f_{32}$	51.65	0.00
33	D_1^2	H_3	100	1.00	$0.25f_{33}^2 + f_{33}$	63.82	0.00
34	D_1^2	H_4	40	1.00	$0.5f_{34}^2 + 0.8f_{34}$	40.00	3.02
35	D_2^2	H_1	150	1.00	$0.1f_{35}^2 + 0.35f_{35}$	96.01	0.00
36	D_2^2	H_2	20	1.00	$0.5f_{36}^2 + 0.8f_{36}$	20.00	8.80
37	D_2^2	H_3	80	1.00	$0.35f_{37}^2 + 0.7f_{37}$	39.53	0.00
38	D_2^2	H_4	20	1.00	$0.4f_{38}^2 + 0.9f_{38}$	20.00	22.84

I also report the additional equilibrium Lagrange multipliers for this example. In particular, we have that: $\eta_1^* = \eta_2^* = \eta_3^* = 0.00$, since none of the supply/donor upper bound constraints in the three regions are binding.

The equilibrium demands for the RBCs at the demand points from the BSOs are:

$$d_{1H_1}^* = 55.09, \quad d_{1H_2}^* = 28.39, \quad d_{1H_3}^* = 96.64, \quad d_{1H_4}^* = 40.00,$$

$$d_{2H_1}^* = 144.91, \quad d_{2H_2}^* = 71.65, \quad d_{2H_3}^* = 103.36, \quad d_{2H_4}^* = 60.00,$$

and the associated demand prices for the RBCs at the equilibrium demand solution are:

$$\rho_{1H_1}(d^*) = 293.25, \quad \rho_{1H_2}(d^*) = 305.58, \quad \rho_{1H_3}(d^*) = 294.13, \quad \rho_{1H_4}(d^*) = 277.20,$$

$$\rho_{2H_1}(d^*) = 272.20, \quad \rho_{2H_2}(d^*) = 283.85, \quad \rho_{2H_3}(d^*) = 275.93, \quad \rho_{2H_4}(d^*) = 266.20.$$

Hence, none of the demands are at the imposed upper bounds and, consequently, all the associated Lagrange multipliers ϵ_k^*; $k = 1, \ldots, 4$, are equal to 0.00. On the other hand, three of the demands are at the imposed lower bounds, at demand points: H_1, H_3, and H_4, and, therefore, the associated Lagrange multipliers: $\sigma_{H_1}^*$, $\sigma_{H_3}^*$, and $\sigma_{H_4}^*$, are all positive. In particular, these Lagrange multipliers, at equilibrium, have the following computed values:

$$\sigma_{H_1}^* = 0.55, \quad \sigma_{H_2}^* = 0.00, \quad \sigma_{H_3}^* = 5.80, \quad \sigma_{H_4}^* = 27.63.$$

I now report, for completeness, components of the objective function (cf. (4.10)) for BSO 1 and for BSO 2. For BSO 1, at the equilibrium solution, the revenue is equal to: 64,341.70; the altruism component of the utility function is: 371.84, and the total cost associated with its supply chain is: 33,099.85, resulting in a net revenue of:

31,241.85 and a utility of: 31,613.70. As for BSO 2 its revenue is equal to: 104,275.07; its altruism component of its utility function is: 628.19, and the total cost associated with its supply chain network is: 86,525.06, yielding a net revenue of: 17,750.01 and a utility of: 18,378.20.

Example 4.2: Upper and Lower Bounds on Demands Removed

Example 4.2 is constructed from Example 4.1 and has the identical data except that the demand lower and upper bound constraints at the four hospital demand points are removed. In Example 4.2 the potential impacts of removing such constraints in terms of the RBC deliveries and the associated prices as well as the BSOs' net revenues and utilities are explored. The variational inequality (4.49) was adapted accordingly to remove the terms and variables associated with the demand lower and upper bounds and the Euler method was, as well. The computed equilibrium link flow pattern and link capacity Lagrange multipliers are reported in Table 4.3.

The Lagrange multipliers associated with the three regions remain as in Example 4.1, that is, $\eta_1^* = \eta_2^* = \eta_3^* = 0.00$, since none of the supply/donor upper bound constraints in the three regions are binding. The link capacities at links 13 and 38 are now at their upper bounds. These were also at their upper bounds in Example 4.1 but links 34 and 36 are no longer at their upper bounds.

The equilibrium demands for the RBCs at the demand points from the BSOs are now:

$$d_{1H_1}^* = 67.08, \quad d_{1H_2}^* = 34.48, \quad d_{1H_3}^* = 99.99, \quad d_{1H_4}^* = 13.32,$$

$$d_{2H_1}^* = 158.41, \quad d_{2H_2}^* = 77.42, \quad d_{2H_3}^* = 97.0, \quad d_{2H_4}^* = 41.32,$$

and the associated demand prices for the RBCs at the equilibrium demand solution are:

$$\rho_{1H_1}(d^*) = 292.14, \quad \rho_{1H_2}(d^*) = 304.92, \quad \rho_{1H_3}(d^*) = 294.03, \quad \rho_{1H_4}(d^*) = 278.64,$$

Table 4.3. Link, Equilibrium Link Solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 4.2

Link a	f_a^*	θ_a^*
1	136.03	0.00
2	85.49	0.00
3	136.03	0.00
4	85.49	0.00
5	221.51	0.00
6	214.87	0.00
7	155.56	0.00
8	27.78	0.00
9	34.48	0.00
10	79.99	0.00
11	13.32	0.00
12	39.30	0.00
13	20.00	13.10
14	128.84	0.00
15	146.03	0.00
16	111.29	0.00
17	69.08	0.00
18	59.76	0.00
19	69.81	0.00
20	76.22	0.00
21	35.31	0.00
22	75.98	0.00
23	174.20	0.00
24	211.96	0.00
25	170.71	0.00
26	203.48	0.00
27	84.32	0.00
28	86.39	0.00
29	111.31	0.00
30	92.17	0.00
31	54.97	0.00
32	57.42	0.00
33	61.40	0.00
34	21.84	0.00
35	103.44	0.00
36	20.00	0.00
37	35.64	0.00
38	19.84	13.60

$$\rho_{2H_1}(d^*) = 271.41, \quad \rho_{2H_2}(d^*) = 283.20, \quad \rho_{2H_3}(d^*) = 276.09, \quad \rho_{2H_4}(d^*) = 267.67.$$

Without the imposition of demand bounds at the hospital demand points, the total equilibrium demand at $H_1 = 225.49$; the total demand at $H_2 = 111.90$; the total demand at $H_3 = 197.03$, and that at $H_4 = 54.64$.

For BSO 1 we have that, at the equilibrium solution, the revenue is equal to: 63,221.34; the altruism component of the utility function is: 381.94, and the total cost associated with its supply chain is: 31,685.55, leading to a net revenue of: 31,535.79 and a utility of: 31,917.73. As for BSO 2, its revenue is now equal to: 102,772.48; its altruism component of its utility function is: 629.65, and the total cost associated with its supply chain network is now: 83,461.70, yielding a net revenue of: 19,310.78 and a utility of: 19,940.43.

Both blood service organizations now enjoy higher net revenues, as well as higher utilities, without the demand constraints. However, observe that, without those constraints, both hospitals H_3 and H_4 may suffer serious shortfalls in terms on needed RBCs since $\underline{d}_{H_3} = 200$ and $\underline{d}_{H_4} = 100$ and the total deliveries are only, respectively 197.03 and 54.64. This example illustrates the merits of imposing lower demand bounds, which can be part of the contracts between the hospital(s) and the BSO(s).

Also, another interesting result is regarding the altruism component of the BSO utility functions. In Example 4.1, BSO 1 enjoyed an altruism component value of 371.84, whereas now, in Example 4.2, it enjoys an altruism component value of 381.94. BSO 2 enjoyed an altruism component value of 628.19, whereas, in Example 4.2, the corresponding value is 629.65. Hence, the respective BSO altruism component values have increased.

Example 4.3: Decrease in Supply Capacity

Example 4.3 is also constructed from Example 4.1 and has the same data except for the following. However, a major disruption is introduced in the form of a disease so that the number of those eligible to donate blood drops considerably. In particular,

let us now have that:

$$S^1 = 500, \quad S^2 = 220, \quad S^3 = 120.$$

The new computed equilibrium link flow solution and corresponding Lagrange multipliers associated with the link capacity constraints are reported in Table 4.4.

Observe that now, unlike in Example 4.1, and due to a much decreased volume of possible donations, the constraints for both Regions 2 and 3 are tight and the associated Lagrange multipliers are now: $\eta_1^* = 0.00$, $\eta_2^* = 109.82$, and $\eta_3^* = 85.00$.

The equilibrium demands for the RBCs at the demand points in Example 4.3 are:

$$d_{1H_1}^* = 77.39, \quad d_{1H_2}^* = 23.21, \quad d_{1H_3}^* = 116.17, \quad d_{1H_4}^* = 45.68,$$

$$d_{2H_1}^* = 122.61, \quad d_{2H_2}^* = 36.79, \quad d_{2H_3}^* = 83.33, \quad d_{2H_4}^* = 54.33.$$

The associated demand prices for the RBCs at the equilibrium demand solution are:

$$\rho_{1H_1}(d^*) = 292.13, \quad \rho_{1H_2}(d^*) = 307.04, \quad \rho_{1H_3}(d^*) = 293.33, \quad \rho_{1H_4}(d^*) = 277.09,$$

$$\rho_{2H_1}(d^*) = 273.10, \quad \rho_{2H_2}(d^*) = 286.50, \quad \rho_{2H_3}(d^*) = 276.33, \quad \rho_{2H_4}(d^*) = 266.37.$$

The equilibrium total demands at the four hospital demand points are at their respective lower bounds. The Lagrange multipliers, at the equilibrium, associated with the lower and upper bounds on the demands at the four demand points are now:

$$\sigma_{H_1}^* = 107.14, \quad \sigma_{H_2}^* = 90.43, \quad \sigma_{H_3}^* = 110.02, \quad \sigma_{H_4}^* = 129.07,$$

and

$$\epsilon_{H_1}^* = 0.00, \quad \epsilon_{H_2}^* = 0.00, \quad \epsilon_{H_3}^* = 0.00, \quad \epsilon_{H_4}^* = 0.00.$$

I now report the values, at the computed equilibrium, of the components of the objective function (cf. (4.10)) for BSO 1 and for BSO 2. For BSO 1 the revenue

Table 4.4. Link, Equilibrium Link Solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 4.3

Link a	f_a^*	θ_a^*
1	237.60	0.00
2	33.49	0.00
3	237.60	0.00
4	33.49	0.00
5	271.09	0.00
6	262.96	0.00
7	196.94	0.00
8	3.38	0.00
9	23.21	0.00
10	96.67	0.00
11	45.68	0.00
12	46.01	0.00
13	20.00	13.10
14	186.51	0.00
15	68.06	0.00
16	51.94	0.00
17	86.42	0.00
18	100.09	0.00
19	35.42	0.00
20	32.64	0.00
21	18.86	0.00
22	33.07	0.00
23	140.70	0.00
24	165.80	0.00
25	137.89	0.00
26	159.17	0.00
27	68.17	0.00
28	9.72	0.00
29	86.81	0.00
30	72.36	0.00
31	42.78	0.00
32	25.43	0.00
33	52.44	0.00
34	34.33	0.00
35	79.83	0.00
36	11.36	0.00
37	30.89	0.00
38	20.00	13.60

obtained is now equal to: 76,616.49; the altruism component of the utility function is: 574.02, and the total costs associated with its supply chain is: 50,978.78 resulting in a net revenue of: 25,637.71 and a utility of: 26,094.73. As for BSO 2, its revenue is now equal to: 81,522.08; its altruism component of its utility function is: 503.00, and the total cost associated with its supply chain network is: 87,042.11, yielding a net revenue of: -5,520.03 and a utility of: -5,017,03.

With a much reduced donor base, the two BSOs still manage to meet their delivery obligations. However, BSO 1 suffers a reduction in net revenue and utility of approximately 20%, as compared to the corresponding values in Example 4.1. BSO 2, on the other hand, experiences not only a significant reduction in net revenue and utility, but these attain negative values and, hence, BSO 2 incurs a financial loss. This example illustrates that blood service organizations need to maintain a sufficiently large donor base for the life-saving product that is blood. This is especially essential in times such as disease outbreaks as well as during different times of various seasons when donors may not be available due to holidays or inclement weather.

Example 4.4: Decrease in Capacities of Critical Links

Example 4.4 is also constructed from Example 4.1 but in Example 4.4 the impacts of decreased capacity associated with BSO 2's testing and processing and storage links 24 and 26 due to a natural disaster are explored. The data is the same as in Example 4.1 except that now the link upper bounds $u_{24} = 200$ and $u_{26} = 200$. The computed equilibrium link flow pattern and associated Lagrange multiplier pattern are reported in Table 4.5.

Observe that links: 13, 24, 34, 36, and 38 are now at their capacities.

The equilibrium Lagrange multipliers associated with the bounds on donors in the three regions are: $\eta_1^* = \eta_2^* = \eta_3^* = 0.00$ since these constraints are not binding.

The equilibrium demands for the RBCs at the demand points in Example 4.4 are:

100

Table 4.5. Link, Equilibrium Link Solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 4.4

Link a	f_a^*	θ_a^*
1	140.65	0.00
2	88.43	0.00
3	140.65	0.00
4	88.43	0.00
5	229.08	0.00
6	222.21	0.00
7	167.35	0.00
8	22.45	0.00
9	26.22	0.00
10	78.68	0.00
11	40.00	0.00
12	34.86	0.00
13	20.00	5.71
14	127.67	0.00
15	144.65	0.00
16	109.84	0.00
17	72.21	0.00
18	55.46	0.00
19	72.05	0.00
20	72.60	0.00
21	37.80	0.00
22	71.94	0.00
23	182.15	0.00
24	200.00	17.81
25	178.51	0.00
26	192.00	0.00
27	90.63	0.00
28	87.88	0.00
29	107.11	0.00
30	84.89	0.00
31	48.47	0.00
32	46.50	0.00
33	62.77	0.00
34	40.00	1.74
35	94.22	0.00
36	20.00	5.23
37	38.55	0.00
38	20.00	21.38

$$d^*_{1H_1} = 57.31, \quad d^*_{1H_2} = 26.22, \quad d^*_{1H_3} = 98.68, \quad d^*_{1H_4} = 40.00,$$

$$d^*_{2H_1} = 142.69, \quad d^*_{2H_2} = 66.50, \quad d^*_{2H_3} = 101.32, \quad d^*_{2H_4} = 60.00.$$

The associated demand prices for the RBCs, in turn, at the equilibrium demand solution are:

$$\rho_{1H_1}(d^*) = 293.13, \quad \rho_{1H_2}(d^*) = 305.91, \quad \rho_{1H_3}(d^*) = 294.05, \quad \rho_{1H_4}(d^*) = 277.20,$$

$$\rho_{2H_1}(d^*) = 272.29, \quad \rho_{2H_2}(d^*) = 284.30, \quad \rho_{2H_3}(d^*) = 275.97, \quad \rho_{2H_4}(d^*) = 266.20.$$

The equilibrium total demands at the hospital demand points H_1, H_3, and H_4 are at their respective lower bounds. The Lagrange multipliers, at the equilibrium, associated with the lower and upper bounds on the demands at the four demand points are now:

$$\sigma^*_{H_1} = 4.11, \quad \sigma^*_{H_2} = 0.00, \quad \sigma^*_{H_3} = 9.08, \quad \sigma^*_{H_4} = 30.20,$$

and

$$\epsilon^*_{H_1} = 0.00, \quad \epsilon^*_{H_2} = 0.00, \quad \epsilon^*_{H_3} = 0.00, \quad \epsilon^*_{H_4} = 0.00.$$

BSO 1's revenue is now equal to: 64,925.04; the altruism component of the utility function is: 78.20, and the total costs associated with its supply chain is: 33,706.32. Hence, the net revenue is: 31,218.71 and the utility is: 31,596.91. BSO 2's revenue is now equal to: 101,693.17; its altruism component of its utility function is: 614.52, and the total cost associated with its supply chain network is: 84,635.16, resulting in a net revenue of: 17,058.01 and a utility of: 17,672.53.

Note that, with decreased capacity on critical links, BSO 2's net revenue as well as utility decrease relative to their respective values in Example 4.1. Interestingly, the

reduced capacity of BSO 2 also affects BSO 1 and, although it now has higher revenues, it also incurs higher costs, resulting in a reduced value of net revenue (31,218.71 versus 31,241.85). This suggests that the blood service organizations may gain by cooperating rather than competing.

4.4.   Summary and Conclusions

The blood services industry in the United States is undergoing major changes, which include increasing competition among blood banks, that is, blood service organizations. In this chapter, I presented the first game theory model for competitive supply chain networks associated with blood service organizations that includes not only perishability but also an altruism component in their objective functions since they are nonprofit organizations. In addition to capacities on the links representing the network economic activities associated with such supply chain networks, I also incorporated upper bounds reflecting donations in different regions as well as lower bounds and upper bounds associated with the demand for RBCs at the various demand points, which correspond to hospitals and medical centers. Such demand constraints ensure that each hospital or medical center has the minimum amount needed for a given week while also guaranteeing that waste will be reduced because of the upper bounds.

The novel features of the competitive supply chain network game theory model resulted in a Generalized Nash Equilibrium (GNE), rather than just a Nash Equilibrium, since the utility function of each blood service organization depends on its own strategies in the form of blood path flows, as well as those of the other BSOs, and the feasible sets do as well. The concept of a variational equilibrium is utilized to transform the problem into a variational inequality problem in which the Lagrange multipliers corresponding to the shared / common constraints are equal among the competitors. This provides a nice economic fairness interpretation.

Alternative variational inequality formulations are also provided and a Lagrange analysis with economic interpretations are presented. An algorithm was outlined which resolves the problem into closed form expressions at each iteration in terms of path flows and the various Lagrange multipliers. The algorithm was then applied to compute solutions to a series of numerical examples for which full input and output data are reported. The examples illustrated the impacts of disruptions as in a reduction in the number of donors as well as that of decreases in capacities of critical links such as testing and processing on RBC prices, demands, net revenues of the blood service organizations, and their overall utilities. The framework here focused on competition among blood service organizations not only in terms of blood donations but also for business with hospitals as well as along their supply chain networks through the generality of the link total cost functions.

CHAPTER 5

MULTITIERED BLOOD SUPPLY CHAIN NETWORK COMPETITION: LINKING BLOOD SERVICE ORGANIZATIONS, HOSPITALS, AND PAYERS

This chapter is based on the paper by Dutta and Nagurney (2018). In this chapter, I develop a multitiered blood supply chain network model. Extending the work in Chapter 4, here I move further down the blood supply chain to include another tier of stakeholders to the competitive blood supply chain network framework. In the network structure presented here there are three distinct tiers of decision-makers namely blood service organizations, hospitals and medical centers, and payers such as government programs and private insurance companies.

While in the Chapter 4 I focused only on the economic transactions between the blood service organizations and hospitals, in this chapter both the key economic relationships in the blood banking industry are considered through the reimbursements received by hospitals for transfusing blood to patients from the payer groups as well as the price per unit of blood charged by the blood service organizations to the hospitals.

The behavior of the individual decision-makers is discussed and their optimality conditions are presented. The variational inequality formulation of the equilibrium conditions for the entire blood supply chain is also provided. The objective here is to determine the equilibrium flows along various paths from the blood service organizations to the hospitals, the amount of blood transfused by different hospitals to patients belonging to different payer groups in order to meet the demand, the price per unit that hospitals agree to pay to the different blood suppliers, and the reimbursements received by different hospitals from different payer groups.

The rest of this chapter is organized as follows. In Section 5.1, I present the multitiered blood supply chain network competition model, consisting of blood service organizations, hospitals, and the payers. The behavior of each class of decision-makers is described, and a unified variational inequality derived, whose solution yields equilibrium blood logistical flows and prices. Illustrative examples are presented for clarification and exposition purposes. In Section 5.2, qualitative properties of the equilibrium pattern are given, and existence established under reasonable conditions. The algorithm that is applied is outlined in Section 5.3, and explicit formulae at a given iteration presented, along with conditions for convergence. Lastly, I present a case study in Section 5.4 and summarize the results, and present my conclusions in Section 5.5.

5.1. The Multitiered Blood Supply Chain Network Competition Model

The blood supply chain network (cf. Figure 5.1) consists of I competing blood service organizations (BSOs), with a typical BSO denoted by i, H_{n_H} hospitals, with a typical hospital denoted by j, and T_{n_T} payers, with a typical payer denoted by k. The BSOs are depicted by the top-most nodes in Figure 5.1 and the payers by the bottom nodes. Examples of patient payer types include: Medicare, Medicaid, other public health insurance programs, private health insurance such as UnitedHealthcare, the uninsured, etc. Each blood service organization i collects blood from n_C^i collection sites that include fixed and mobile sites. Once blood is collected by BSO i; $i = 1, \ldots, I$, it is sent to n_P^i component laboratories for testing and processing where whole blood is separated into components such as Red Blood Cells (RBCs), platelets, and plasma.

Blood is shipped from the component laboratories of each BSO i to n_S^i storage facilities that constitute the fifth tier of the supply chain network. The next level
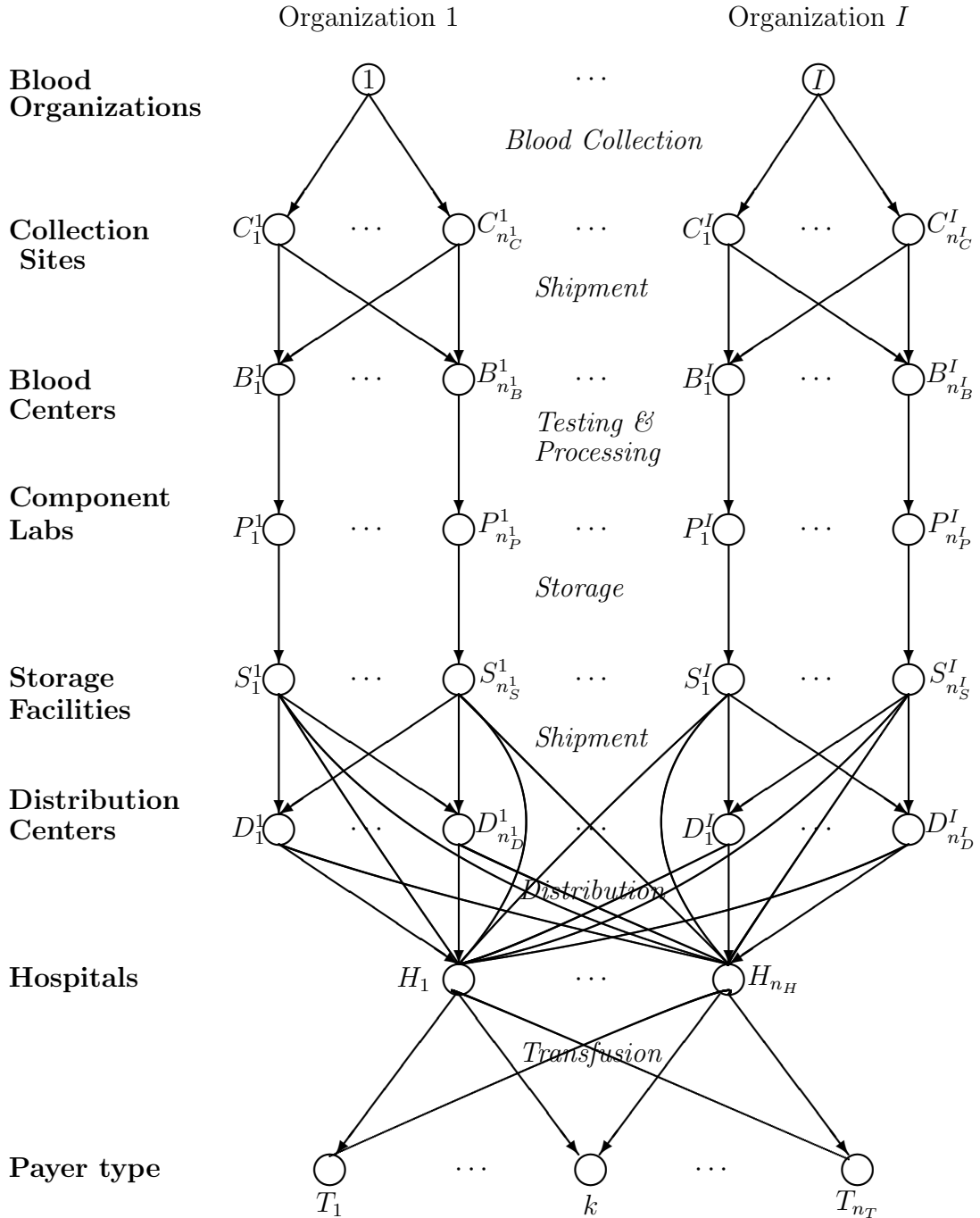
Figure 5.1. Blood Supply Chain Network Topology

of nodes represents the n_D^i; $i = 1, \ldots, I$, distribution centers. At times, the component laboratories, storage facilities, and distribution centers are not separate physical entities but exist within the blood centers. At the seventh tier blood reaches hos-

pitals from multiple suppliers with whom they have contracts (Merola (2017)). Up till the seventh tier this network structure is identical to Figure 4.1 (cf. Chapter 4). However, in contrast to Chapter 4, here the hospitals also compete with one another for patients belonging to different payer groups denoted by the last tier of nodes as mentioned earlier. Each set of links between a pair of nodes denotes an activity along the supply chain such as, for example, the collection of whole blood from donors, shipment, testing and processing, storage, distribution, and, finally, transfusion.

In the subsequent sections the behavior of the blood service organizations is discussed and then that of the hospitals and, finally, the payers.

5.1.1 Behavior of the Blood Service Organizations and Their Optimality Conditions

A path is a sequence of links, which are directed, and originates at a top origin node representing a blood service organization and ending at a hospital node (cf. Figure 5.1). N and L are defined as the sets of nodes and links, respectively, up to the seventh tier representing the hospitals with L^i denoting the set of links in BSO i's supply chain for $i = 1, \ldots, I$. Associated with each link a, $\forall a \in L$, is a total cost function $\hat{c}_a$ representing the cost for the activity.

A significant challenge in managing blood supply chain is the perishable nature of blood. Every component of blood is perishable with different expiration rates and shelf lives. In order to capture perishability, the generalized network approach with appropriate arc and path multipliers provided in Table 4.1 (cf. Chapter 4) is utilized here. Since, again, we are dealing with RBCs those paths that would have a time length greater than 42 days are explicitly removed from the network(s) in Figure 5.1 since they would, in effect, be infeasible (and against Food and Drug Administration regulations).

Some of the functions defined below were used in Chapter 4. x_p denotes the nonnegative flow of blood on a path p sent from a BSO to a hospital. Let the contracted amount of blood supplied by BSO i to hospital j be denoted by q_{ij}; $i = \ldots, I; j = H_1, \ldots, H_{n_H}$. This is assumed to be the projected demand for a week. Let P_j^i denote the set of all paths joining BSO i with hospital j and P is the set of all paths.

The conservation of flow equation that has to hold for each BSO i; $i = 1, \ldots, I$, at hospital j; $j = H_1, \ldots, H_{n_H}$, is:

$$\sum_{p \in P_j^i} x_p \mu_p = q_{ij}, \tag{5.1}$$

where $\mu_p \equiv \prod_{a \in p} \alpha_a$; $p \in P$, denotes the multiplier corresponding to the percentage of throughput on path p (cf. Table 4.1). Hence, (5.1) implies that the sum of all the actual, after perishability is factored in, path flows from a particular BSO to a particular hospital should be equal to their contracted supply amount. The total amount of blood supplied by a blood service organization i, q_i, can be written as $\sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} x_p \mu_p = q_i$.

Since the path flows must be nonnegative, we have that:

$$x_p \geq 0, \quad \forall p \in P. \tag{5.2}$$

Let f_a denote the flow of blood on link a. Then, the following conservation of flow equations must hold:

$$f_a = \sum_{p \in P} x_p \alpha_{ap}, \quad \forall a \in L. \tag{5.3}$$

According to (5.3), the initial blood product flow on link a is the sum of the product flows along paths that contain that link, taking into account possible losses in the preceding activities. All the flows corresponding to links in L are grouped into

the vector $f \in R^{n_L}$ where n_L is the total number of elements in L. The total link cost

on a link a is assumed to be, in general, a function of all the flows in the network.

Therefore, we have that

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L. \tag{5.4}$$

The total cost on each link is assumed to be convex and continuously differentiable.

The total cost incurred by a blood service organization will be the sum of all the total

costs on links operated by the blood service organization. The price per unit charged

by BSO i to hospital j is denoted by ρ_{ij}^{1*}. I discuss how the equilibrium prices are

recovered, once the model is solved, later in this section. The revenue generated by

each BSO is the product of the unit price and the amount of blood supplied.

As noted earlier, blood service organizations in the United States are predomi-

nantly nonprofits. Therefore, there is a utility associated with the service that they

provide (cf. Nagurney, Alvarez Flores, and Soylu (2016) and Nagurney and Li (2017)).

Let γ_{ij} correspond to a measurement of the satisfaction that blood service organiza-

tion i derives from supplying blood to hospital j. The overall such "service" utility

of blood service organization i associated with all the demand points is then given by

$\sum_{j=H_1}^{H_{n_H}} \gamma_{ij} q_{ij}$, similar to the service utility function described in Chapter 4. In addi-

tion, each blood service organization i associates a weight ω_i with its service utility,

which monetizes it.

The utility function of blood service organization i; $i = 1, \ldots, I$, denoted by U_i,

can be expressed as:

$$U_i = \sum_{j=H_1}^{H_{n_H}} \rho_{ij}^{1*} q_{ij} + \omega_i \sum_{j=H_1}^{H_{n_H}} \gamma_{ij} q_{ij} - \sum_{a \in L^i} \hat{c}_a(f), \tag{5.5a}$$

or, equivalently, in terms of path flows, through the use of (5.1) and (5.3):

$$\hat{U}_i = \sum_{j=H_1}^{H_{n_H}} \rho_{ij}^{1*} \sum_{p \in P_j^i} x_p \mu_p + \omega_i \sum_{j=H_1}^{H_{n_H}} \gamma_{ij} \sum_{p \in P_j^i} x_p \mu_p - \sum_{a \in L^i} \tilde{\hat{c}}_a(x), \tag{5.5b}$$

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with $\tilde{\hat{c}}_a(x) \equiv \hat{c}_a(f)$, $\forall a \in L$.

It is to be noted that the utility of each blood service organization is over a time horizon of a week.

The blood service organizations seek to maximize their utility, while competing for the quantity of blood supplied. Hence, each BSO has as its strategic variables, its path flows, with X_i denoting the vector of path flows corresponding to blood service organization i; $i = 1, \ldots, I$:

$$X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R_+^{n_{Pi}}, \tag{5.6}$$

where P^i denotes the set of all paths associated with BSO i and n_{Pi} denotes the number of paths from BSO i to the hospitals. X is the vector of all path flows, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$. Further, the feasible set for blood service organization i is defined as $K_i \equiv \{X_i|x_i \in R_+^{n_{Pi}}\}$. All vectors, as before, are column vectors.

The blood service organizations compete noncooperatively in an oligopolistic market framework in which each blood service organization selects its own optimal blood product flows to maximize its utility, given the optimal ones of its competitors. The governing equilibrium concept underlying the behavior of the blood service organizations is, therefor, that of Nash (1950, 1951) equilibrium. The optimality conditions for all the blood service organizations simultaneously can be expressed as the following variational inequality (cf. Gabay and Moulin (1980), Nagurney (1999)): determine $x^* \in K^1, K^1 \equiv \prod_{i=1}^{I} K_i$, such that:

$$\sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ij} \mu_p - \rho_{ij}^{1*} \mu_p \right] \times [x_p - x_p^*] \geq 0 \quad \forall x \in K^1, \tag{5.7}$$

where $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ is for path $p \in P_j^i$ given by

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \alpha_{ap}. \tag{5.8}$$

The optimality conditions as expressed by (5.7) provide a nice economic interpretation that a blood service organization will supply blood to a hospital by a path p (flow on the path will be positive) if the "marginal total cost" on the path is exactly equal to marginal utility associated with weighted altruism of the pair (i, j) plus the marginal revenue associated with the path p, with perishability accounted for.

5.1.2 Behavior of the Hospitals and Their Optimality Conditions

I now discuss the competition among the hospitals. Hospitals are the stakeholders in the blood supply chain network who are involved in transactions with both blood suppliers, the BSOs, and the patient payer groups.

Each hospital j decides to transfuse an amount q_{jk} of RBCs to patient group k. The total amount of blood transfused by hospital j; $j = H_1, \ldots, H_{n_H}$, cannot exceed the total amount it receives from its contracted suppliers. Therefore, the following condition must be satisfied

$$\sum_{k=T_1}^{T_{n_T}} q_{jk} \leq \sum_{i=1}^{I} q_{ij}, \tag{5.9a}$$

or, equivalently,

$$\sum_{k=T_1}^{T_{n_T}} q_{jk} \leq \sum_{i=1}^{I} \sum_{p \in P_j^i} x_p \mu_p. \tag{5.9b}$$

The price charged by hospital j; $j = H_1, \ldots, H_{n_H}$, for per unit of RBC transfused is denoted by ρ_j^{2*}. Similar to blood service organizations, many hospitals are not-for-profits and, hence, will have a weighted altruism factor in their utility function which is given as $\beta_j \sum_{k=T_1}^{T_{n_T}} \theta_{jk} q_{jk}$. In the case of a for profit hospital the weight β_j will simply be zero.

In addition to the cost of acquiring blood from the suppliers, hospitals incur a holding cost for maintaining a proper inventory of blood. This cost is denoted by h_j

for hospital j; $j = H_1, \ldots, H_{n_H}$, and is a function of $\sum_{k=T_1}^{T_{n_T}} q_{jk}$, the total amount of blood transfused at hospital j.

The optimization problem for hospital j; $j = H_1, \ldots, H_{n_H}$, then becomes

$$\text{Maximize} \quad \rho_j^{2*} \sum_{k=T_1}^{T_{n_T}} q_{jk} + \beta_j \sum_{k=T_1}^{T_{n_T}} \theta_{jk} q_{jk} - h_j\left(\sum_{k=T_1}^{T_{n_T}} q_{jk}\right) - \sum_{i=1}^{I} \rho_{ij}^{1*} \sum_{p \in P_j^i} x_p \mu_p, \quad (5.10)$$

subject to constraint (5.9b) and the nonnegativity constraints: $x_p \geq 0$, $\forall p \in P_j$, where P_j is the set of all paths terminating in j, and $q_{jk} \geq 0$ for all j and k.

Now the optimality conditions of the hospitals are obtained, assuming that each hospital is faced with the above optimization problem, and that the hospitals compete in a noncooperative manner to maximize their utilities, given the actions of the other hospitals. It is to be noted that the hospitals seek to determine the optimal quantities to be supplied to the patient groups as well as the amount to be received from different suppliers. Assuming that the holding cost for each hospital is convex and continuous, the optimality conditions for all hospitals, simultaneously, coincide with the solution of the variational inequality: determine $(x^*, q^*, \eta^*) \in K^2$ satisfying

$$\sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\rho_{ij}^{1*} \mu_p - \eta_j^* \mu_p \right] \times [x_p - x_p^*] + \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[-\rho_j^{2*} - \beta_j \theta_{jk} + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^*)}{\partial q_{jk}} + \eta_j^* \right]$$

$$\times [q_{jk} - q_{jk}^*] + \sum_{j=H_1}^{H_{n_H}} \left[\sum_{i=1}^{I} \sum_{p \in P_j^i} x_p^* \mu_p - \sum_{k=T_1}^{T_{n_T}} q_{jk}^* \right] \times [\eta_j - \eta_j^*] \geq 0$$

$$\forall (x, q, \eta) \in K^2, \quad (5.11)$$

with the feasible set K^2 defined as:

$$K^2 \equiv \{(x, q, \eta) | x \in R_+^{n_P}, q \in R_+^{n_H n_T}, \eta \in R_+^{n_H}\}. \quad (5.12)$$

Here η_j is the Lagrange multiplier associated with constraint (5.9) for hospital j, η is the n_H-dimensional vector of all the multipliers, and q denotes the $n_H n_T$-dimensional

vector of blood flows between the hospitals and patient groups. For further background on such a derivation, see Nagurney, Dong, and Zhang (2002) and the references therein. Similar to (5.7), in this derivation of the variational inequality the prices charged are not considered to be variables. They become endogenous variables in the complete equilibrium model.

The economic interpretation of the hospitals' optimality conditions are discussed and the justification of the h_j functions is provided. From the first term in (5.11) it can be inferred that if there is positive flow of blood products between a blood service organization and hospital, then η_j^* is precisely equal to the hospital's payment to the supplier, ρ_{ij}^{1*}. From the second term of (5.11) we see that if q_{jk}^* is positive, that is, if patients from payer group k get transfusions from hospital j, then the unit price charged by hospital j, ρ_j^{2*}, plus its marginal service utility, $\beta_j\theta_{jk}$, is exactly equal to its marginal cost of holding inventory plus its unit cost of procuring blood (since $\eta_j^* = \rho_{ij}^{1*}$). Further, from the third term in (5.11) it can be inferred that if η_j^* is positive, then the amount of blood received by hospital j is exactly equal to the amount of blood transfused at hospital j. Hence, it can be said that the inventory holding cost of a hospital is a function of the total amount of blood transfused at a hospital.

5.1.3 Behavior of the Payer Groups and Equilibrium Conditions

As mentioned earlier, the payers are conceptualized as patients belonging to different payer groups. Since most of the surgeries requiring blood transfusion such as knee replacements, cardiovascular surgery, organ transplants, etc., are planned ahead of time, it is assumed that the demand for blood at each payer node depends on the reimbursement that the payers (insurers) are willing to give. The type of insurance is also known before blood transfusion takes place. Since for a patient getting treatment at a hospital, the blood required for transfusion will be provided by that particular

hospital, it is expected that the demand at each hospital from each payer group may be different. Demand at patient payer group k for transfusions at hospital j is denoted by d_{jk}. The amount that payer type k is willing to reimburse to hospital j is given as ρ_{jk}^3. Thus, we have that

$$d_{jk} = d_{jk}(\rho^3), \quad \forall j, \forall k, \tag{5.13}$$

where ρ^3 is the $n_H n_T$-dimensional vector of payer prices. The demand is assumed to be monotonically decreasing in the reimbursement for j, but increasing in the reimbursements for other hospitals. However, it is to be noted that if the demand is quite price inelastic then the coefficients should be set accordingly.

In reality, healthcare payments received by different hospitals from the same payer might vary significantly (Luhby (2013)). The reimbursement or payer price can depend on several factors such as the payer mix, whether it is a teaching or non-teaching hospital, the hospital's location, its healthcare network, etc. For example, Medicare pays a higher rate to teaching hospitals while private insurance companies negotiate better rates for hospitals in their network.

The payers take into account not only the price charged by the hospitals in determining which hospital to choose, but also the transaction cost. Let c_{jk} denote the transaction cost between hospital j and payer group k. It is assumed that the transaction cost is continuous, positive, and of the general form

$$c_{jk} = c_{jk}(q), \quad \forall j, \forall k. \tag{5.14}$$

Following the work of Nagurney, Dong and Zhang (2002) (see also Nagurney (2006)), we have that the equilibrium conditions are: For all hospitals j; $j = H_1, \ldots, H_{n_H}$, and payers $k = T_1, \ldots, T_{n_T}$:

$$\rho_j^{2*} + c_{jk}(q^*) \begin{cases} = \rho_{jk}^{3*} & \text{if} \quad q_{jk}^* > 0, \\[2mm] \geq \rho_{jk}^{3*} & \text{if} \quad q_{jk}^* = 0. \end{cases} \tag{5.15}$$

and

$$d_{jk}(\rho^{3*}) \begin{cases} = q_{jk}^* & \text{if} \quad \rho_{jk}^{3*} > 0, \\[2mm] \leq q_{jk}^* & \text{if} \quad \rho_{jk}^{3*} = 0. \end{cases} \tag{5.16}$$

Conditions (5.15) imply that, in equilibrium, if q_{jk}^* is positive, that is, there are patients at demand market k that get blood transfusions from hospital j, then the price charged by the hospital plus the transaction cost does not exceed the price that the payer is willing to reimburse. Conditions (5.16) state that, if the equilibrium price that a particular payer group is willing to pay for the blood product from a particular hospital is positive, then the quantity of blood obtained from a hospital is precisely equal to the demand of blood for that payer group. These conditions correspond to the well-known spatial price equilibrium conditions but applied to an entirely novel context of multitiered blood supply chain networks (cf. Takayama and Judge (1971), Nagurney (1999), and the references therein).

In equilibrium, conditions (5.15) and (5.16) will have to hold for all k, and can, in turn, be expressed as the variational inequality problem (see, e.g., Nagurney (1999)): determine $(q^*, \rho^{3*}) \in K^3$, such that

$$\sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [\rho_j^{2*} + c_{jk}(q^*) - \rho_{jk}^{3*}] \times [q_{jk} - q_{jk}^*] + \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [q_{jk}^* - d_{jk}(\rho^{3*})] \times [\rho_{jk}^3 - \rho_{jk}^{3*}] \geq 0$$

$$\forall (q, \rho^3) \in K^3, \tag{5.17}$$

where the feasible set $K^3 \equiv \{(q, \rho^3) \in R_+^{2n_H n_T}\}$.

5.1.4   The Equilibrium Conditions of the Blood Supply Chain

In equilibrium, the amount of blood supplied by the blood service organizations must be equal to the amount of blood received by the hospitals. In addition, the

amount of blood transfused by the hospitals must be equal to the amount needed by the patients. Furthermore, the equilibrium quantities and price pattern in the blood supply chain must satisfy the sum of the inequalities (5.7), (5.11), and (5.17), in order to formalize the agreements between the tiers. Hence, although there is competition across a tier of decision-makers, whether BSOs or hospitals, there is cooperation between tiers and the prices assist in this. I now state this explicitly in the following definition.

Definition 5.1: Multitiered Blood Supply Chain Network Equilibrium

The equilibrium state of the supply chain is one where the blood product (RBC) flows between the three distinct tiers of decision-makers coincide and the blood flows and prices satisfy the sum of the optimality conditions (5.7), (5.11), and (5.17).

The following theorem is now established.

Theorem 5.1: Variational Inequality Formulation

The equilibrium conditions governing the supply chain model with competition are equivalent to the solution of the variational inequality problem given by: determine $(x^*, q^*, \eta^*, \rho^{3*}) \in K^4$ satisfying:

$$\sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ij} \mu_p - \eta_j^* \mu_p \right] \times [x_p - x_p^*]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [c_{jk}(q^*) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^*)}{\partial q_{jk}} + \eta_j^* - \beta_j \theta_{jk} - \rho_{jk}^{3*}] \times [q_{jk} - q_{jk}^*]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \left[\sum_{i=1}^{I} \sum_{p \in P_j^i} x_p^* \mu_p - \sum_{k=T_1}^{T_{n_T}} q_{jk}^* \right] \times [\eta_j - \eta_j^*] + \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [q_{jk}^* - d_{jk}(\rho^{3*})] \times [\rho_{jk}^3 - \rho_{jk}^{3*}] \geq 0$$

$$\forall (x, q, \eta, \rho^3) \in K^4, \tag{5.18}$$

where $K^4 \equiv \{(x, q, \eta, \rho^3) \in R_+^{n_P + 2 n_H n_T + n_H}\}$.

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Proof: At first the necessity condition is established, that the equilibrium conditions imply variational inequality (5.18). Observe that, indeed, the summation of (5.7), (5.11), and (5.17), yields variational inequality (5.18), after algebraic simplification.

For sufficiency the converse is now established, that is, that a solution to variational inequality (5.18) satisfies the sum of inequalities (5.7), (5.11), and (5.17), and is, therefore, an equilibrium according to Definition 5.1. To inequality (5.18) add the term $-\rho_{ij}^{1*}\mu_p + \rho_{ij}^{1*}\mu_p$ to the term in the first set of brackets preceding the multiplication sign and add the term $-\rho_j^{2*} + \rho_j^{2*}$ to the term in brackets preceding the second multiplication sign. Such "terms" do not change the value of the inequality since their value is equal to zero, with the resulting inequality of the form

$$\sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\sum_{p\in P_j^i}\left[\frac{\partial\hat{C}_p(x^*)}{\partial x_p} - \omega_i\gamma_{ij}\mu_p - \eta_j^*\mu_p - \rho_{ij}^{1*}\mu_p + \rho_{ij}^{1*}\mu_p\right]\times[x_p - x_p^*]$$

$$+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}[c_{jk}(q^*) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}}q_{jk}^*)}{\partial q_{jk}} + \eta_j^* - \beta_j\theta_{jk} - \rho_{jk}^{3*} - \rho_j^{2*} + \rho_j^{2*}]\times[q_{jk} - q_{jk}^*]$$

$$+\sum_{j=H_1}^{H_{n_H}}\left[\sum_{i=1}^{I}\sum_{p\in P_j^i}x_p^*\mu_p - \sum_{k=T_1}^{T_{n_T}}q_{jk}^*\right]\times[\eta_j - \eta_j^*] + \sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}[q_{jk}^* - d_{jk}(\rho^{3*})]\times[\rho_{jk}^3 - \rho_{jk}^{3*}] \geq 0,$$

$$\forall(x, q, \eta, \rho^3) \in K^4, \tag{5.19}$$

which, in turn, can be rewritten as

$$\sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\sum_{p\in P_j^i}\left[\frac{\partial\hat{C}_p(x^*)}{\partial x_p} - \omega_i\gamma_{ij}\mu_p - \rho_{ij}^{1*}\mu_p\right]\times[x_p - x_p^*] + \sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\left[\rho_{ij}^{1*}\mu_p - \eta_j^*\mu_p\right]\times[x_p - x_p^*]$$

$$+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}\left[-\rho_j^{2*} - \beta_j\theta_{jk} + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}}q_{jk}^*)}{\partial q_{jk}} + \eta_j^*\right]\times[q_{jk} - q_{jk}^*] + \sum_{j=H_1}^{H_{n_H}}\left[\sum_{i=1}^{I}\sum_{p\in P_j^i}x_p^*\mu_p\right.$$

$$\left. - \sum_{k=T_1}^{T_{n_T}}q_{jk}^*\right]\times[\eta_j - \eta_j^*] + \sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}[\rho_j^{2*} + c_{jk}(q^*) - \rho_{jk}^{3*}]\times[q_{jk} - q_{jk}^*]$$

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$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [q_{jk}^* - d_{jk}(\rho^{3*})] \times [\rho_{jk}^3 - \rho_{jk}^{3*}] \geq 0, \quad \forall (x, q, \eta, \rho^3) \in K^4. \tag{5.20}$$

But inequality (5.20) is equivalent to the price and product flow pattern satisfying the sum of (5.7), (5.11), and (5.17). The proof is complete. □

The variational inequality (5.18) can be rewritten in standard variational inequality form (cf. 2.1a) that is: determine $Y^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(Y^*), Y - Y^* \rangle \geq 0, \quad \forall Y \in \mathcal{K}, \tag{5.21}$$

where $Y \equiv (x, q, \eta, \rho^3)$, $F(Y) \equiv (F_p, F_{jk}^1, F_j, F_{jk}^2)_{p \in P_j^i; i=1,\ldots,I; j=H_1,\ldots,H_{n_H}; k=T_1,\ldots,T_{n_T}}$, and the specific components of the function F are given by the functional terms preceding the multiplication signs in (5.18). The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space, where N here is $n_P + 2n_H n_T + n_H$ and $\mathcal{K} \equiv K^4$.

The variables in the variational inequality problem are: the product (RBC) flows from the blood service organizations to the hospitals, x, the quantities of blood transfused by the hospitals to the patient groups, q, the prices associated with transfusing and storing blood by the hospitals, η, and the demand market prices or reimbursement rates, ρ^3.

I now discuss how to recover the blood service organizations' equilibrium prices, ρ_{ij}^{1*}, for all i, j, and the hospitals' equilibrium prices, ρ_j^{2*}, for all j, from the solution of the variational inequality (5.18). In the previous discussion in Section 5.1.2, it is mentioned that if there is positive flow of blood products between a blood service organization and hospital, then η_j^* is precisely equal to the hospital's payment to the supplier, ρ_{ij}^{1*}. η_j^* is obtained from the solution of the inequality (5.18). On the other hand, prices charged by the hospitals, ρ_j^{2*}s, can be obtained by finding a $q_{jk}^* > 0$, and then from (5.15) setting

$$\rho_j^{2*} = \rho_{jk}^{3*} - c_{jk}(q^*),$$

where ρ_{jk}^{3*} is obtained from the solution of variational inequality (5.18).

The result that, in equilibrium, the sum of the amounts of blood supplied to each hospital is equal to the sum of the amounts of blood transfused at that hospital is now established. This implies that each hospital, assuming utility maximization, purchases from the blood service organizations only the amount of blood that is actually transfused to the patients. Variational inequality (5.18) is utilized to establish the above mentioned result. From the third term in (5.18) we can see that if $\eta_j^* > 0$, then we have $\sum_{i=1}^{I} \sum_{p \in P_j^i} x_p^* \mu_p = \sum_{k=T_1}^{T_{n_T}} q_{jk}^*$. In other words, the "market clears" for hospital j. Let us now consider the case where $\eta_j^* = 0$. From (5.7) it is evident that, if $x_p^* > 0$, then we have that

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ij} \mu_p + \rho_{ij}^{1*} \mu_p,$$

and, if $x_p^* = 0$, then we have that

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} \geq \omega_i \gamma_{ij} \mu_p + \rho_{ij}^{1*} \mu_p.$$

Hence, from the first term in inequality (5.18), we can say that, if $\eta_j^* = 0$, then

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ij} \mu_p \geq 0,$$

which implies that $x_p^* = 0, p \in P_j^i, \quad \forall i, j$. It follows then from the third term of (5.18) that $\sum_{k=T_1}^{T_{n_T}} q_{jk}^* = 0$, and, hence, the market clears also in this case since the flow into a hospital is equal to the flow out and equal to zero. Thus, established the following is established:

Corollary 5.1

The market for the blood product clears for each hospital at the supply chain network equilibrium.

5.1.5 Illustrative Examples

In this section I present two examples to illustrate some of the above mentioned concepts. The blood supply chain topology is depicted in Figure 5.2. A case study with a more elaborate blood supply chain network topology is presented in Section 5.4.

Organization 1   Organization 2
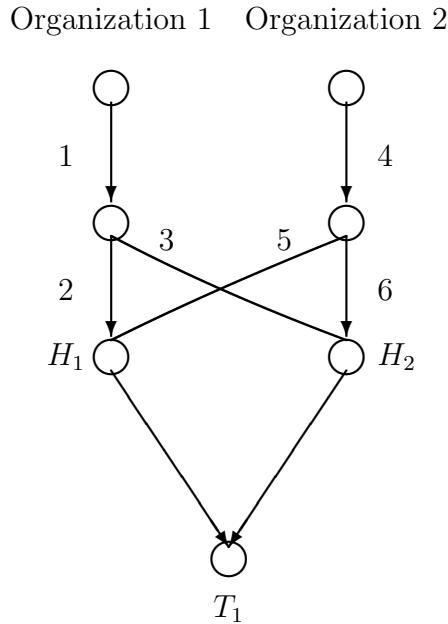
Figure 5.2. The Blood Supply Chain Network Topology for the Illustrative Examples

Example 5.1: Network with Two BSOs, Two Hospitals and One Payer

There are two blood service organizations supplying blood to two hospitals. The hospitals in turn treat patients who belong to the group Payer Type T_1. From each BSO there are two paths reaching each hospital. For simplicity, each path consists of two links. The paths are defined as follows: $p_1 = (1, 2)$, $p_2 = (1, 3)$, $p_3 = (4, 5)$, and $p_6 = (4, 6)$. The time horizon is assumed to be a week.

The total link cost functions are:

$$\hat{c}_1(f_1) = f_1^2 + 1.5f_1, \quad \hat{c}_2(f_2) = f_2^2 + 2f_2, \quad \hat{c}_3(f_3) = f_3^2 + 2.5f_3,$$

$$\hat{c}_4(f_4) = f_4^2 + 2f_4, \quad \hat{c}_5(f_5) = f_5^2 + 2f_5, \quad \hat{c}_6(f_6) = f_6^2 + 2.5f_6.$$

In this example it is assumed that no amount blood is lost in the supply chain; hence, the arc-path multipliers, μ_ps are all equal to 1, $\forall p$. Again, for ease of calculation, the parameters associated with the altruism components of the utility functions are all zero, i.e, $\omega_1 = \omega_2 = \beta_{H_1} = \beta_{H_2} = 0$. The holding cost functions for the two hospitals are:

$$h_{H_1}(q_{H_1 T_1}) = 1.5 \times q_{H_1 T_1}, \quad h_{H_2}(q_{H_2 T_1}) = 1.5 \times q_{H_2 T_1}.$$

The transaction cost functions between the hospitals and payers are:

$$c_{H_1 T_1}(q_{H_1 T_1}) = q_{H_1 T_1} + 100, \quad c_{H_2 T_1}(q_{H_2 T_1}) = q_{H_2 T_1} + 100.$$

The demand price functions are:

$$d_{H_1 T_1} = -0.005\rho_{H_1 T_1}^3 + 0.002\rho_{H_2 T_1}^3 + 100, \quad d_{H_2 T_1} = -0.005\rho_{H_2 T_1}^3 + 0.002\rho_{H_1 T_1}^3 + 100.$$

Using inequality (5.18) ten linear equations are obtained which are as follows:

$$4x_{p1}^* + 2x_{p2}^* + 3.5 - \eta_{H_1}^* = 0,$$

$$4x_{p_2}^* + 2x_{p_1}^* + 4 - \eta_{H_2}^* = 0,$$

$$4x_{p_3}^* + 2x_{p_4}^* + 4 - \eta_{H_1}^* = 0,$$

$$4x_{p_4}^* + 2x_{p_3}^* + 4.5 - \eta_{H_2}^* = 0,$$

$$q_{H_1 T_1}^* + \eta_{H_1}^* - \rho_{H_1 T_1}^{3*} + 101.5 = 0,$$

$$q_{H_2 T_1}^* + \eta_{H_2}^* - \rho_{H_2 T_1}^{3*} + 101.5 = 0,$$

$$x_{p_1}^* + x_{p_3}^* - q_{H_1 T_1}^* = 0,$$

$$x_{p_2}^* + x_{p_4}^* - q_{H_2 T_1}^* = 0,$$

$$q_{H_1 T_1}^* + 0.005\rho_{H_1 T_1}^{3*} - 0.002\rho_{H_2 T_1}^{3*} - 100 = 0,$$

$$q_{H_2 T_1}^* + 0.005\rho_{H_2 T_1}^{3*} - 0.002\rho_{H_1 T_1}^{3*} - 100 = 0.$$

The equilibrium blood path flows from the blood service organizations to the hospitals obtained by solving the above equations are: $x_{p_1}^* = x_{p_2}^* = 49.29, x_{p_3}^* = x_{p_4}^* = 49.21$.

In the absence of the altruism factors, we have that the η_j^*s are precisely equal to the prices charged by the blood service organizations, ρ_{ij}^{1*}s. The equilibrium prices charged by the BSOs are: $\eta_{H_1}^* = 299.25, \eta_{H_2}^* = 299.75$, respectively. This means that Hospital H_1 agrees to pay \$299.25 per unit of blood and Hospital H_2 agrees to pay \$299.75 per unit of blood. The price per unit charged by the hospitals are: $\rho_{H_1}^{2*} = 300.75$ and $\rho_{H_2}^{2*} = 301.25$. This makes sense and is fair since Hospital H_2 pays a higher price for acquiring the blood, the price charged is also slightly higher than that of Hospital H_1.

At equilibrium, the quantities of blood transfused at each hospital are: $q_{H_1 T_1}^* = q_{H_2 T_1}^* = 98.50$. Lastly, the reimbursements that Payer type T_1 is willing to pay to Hospital H_1 and Hospital H_2 are: $\rho_{H_1 T_1}^{3*} = 499.25$ and $\rho_{H_2 T_1}^{3*} = 499.75$, respectively.

Example 5.2: Introducing Fractional Arc Multipliers

In this example, the data remain as in Example 5.1, except that the arc multipliers are modified so that not all are equal to 1:

$$\alpha_1 = 1, \quad \alpha_2 = 0.95, \quad \alpha_3 = 1, \quad \alpha_4 = 1, \quad \alpha_5 = 1, \quad \alpha_6 = 0.98.$$

Hence, the path multipliers are:

$$\mu_{p_1} = 1 \times 0.95 = 0.95, \quad \mu_{p_2} = 1, \quad \mu_{p_3} = 1, \quad \mu_{p_4} = 1 \times 0.98 = 0.98.$$

Again, using inequality (5.18), we obtain the following set of equations:

$$3.9x_{p_1}^* + 2x_{p_2}^* + 3.5 - 0.95\eta_{H_1}^* = 0,$$

$$4x_{p_2}^* + 2x_{p_1}^* + 4 - \eta_{H_2}^* = 0,$$

$$4x_{p_3}^* + 2x_{p_4}^* + 4 - \eta_{H_1}^* = 0,$$

$$3.96x_{p_4}^* + 2x_{p_3}^* + 4.5 - 0.98\eta_{H_2}^* = 0,$$

$$q_{H_1 T_1}^* + \eta_{H_1}^* - \rho_{H_1 T_1}^{3*} + 101.5 = 0,$$

$$q_{H_2 T_1}^* + \eta_{H_2}^* - \rho_{H_2 T_1}^{3*} + 101.5 = 0,$$

$$0.95x_{p_1}^* + x_{p_3}^* - q_{H_1 T_1}^* = 0,$$

$$x_{p_2}^* + 0.98x_{p_4}^* - q_{H_2 T_1}^* = 0,$$

$$q_{11}^* + 0.005\rho_{H_1 T_1}^{3*} - 0.002\rho_{H_2 T_1}^{3*} - 100 = 0,$$

$$q_{H_2 T_1}^* + 0.005\rho_{H_2 T_1}^{3*} - 0.002\rho_{H_1 T_1}^{3*} - 100 = 0.$$

The equilibrium path flows are now: $x_{p_1}^* = 48.35, x_{p_2}^* = 51.36, x_{p_3}^* = 52.53, x_{p_4}^* = 48.10$.

The equilibrium prices charged by the BSOs are: $\eta^*_{H_1} = 310.29, \eta^*_{H_2} = 307.63$, respectively. The price per unit charged by Hospital H_1 and Hospital H_2 are $\rho^{2*}_{H_1} = 311.75$ and $\rho^{2*}_{H_2} = 307.63$, respectively.

At equilibrium, the quantities of blood transfused at each hospital are: $q^*_{H_1 T_1} = 98.46$, and $q^*_{H_2 T_1} = 98.49$. The reimbursements that Payer Type T_1 is willing to pay to Hospital H_1 and Hospital H_2 are: $\rho^{3*}_{H_1 T_1} = 510.26$, and $\rho^{3*}_{H_2 T_1} = 506.12$, respectively.

If we compare the results in the two examples, we can see that even under the consideration that a fraction of the collected blood perishes or is wasted along the supply chain, the amount of blood transfused at each hospital remains almost same. However, while in Example 5.1 all the paths had similar flows, in Example 5.2, the paths without perishability have higher flows than those with perishability. Due to wastage of some amount of collected blood the cost is likely to increase, and hence, the equilibrium prices obtained in Example 5.2 are higher. The payer agrees to pay higher rates in Example 5.2 which enables the hospitals to pay higher prices to the blood suppliers in order to cover the cost of the wasted blood and still meet the demand. Hence, the blood supply chain functions efficiently even in the face of perishability of blood products.

5.2.   Qualitative Properties

In this section, some qualitative properties of the solution to the variational inequality (5.18) are provided. I first present the existence results. Since the feasible set underlying the variational inequality problem (5.18), K^4, is not compact it is not possible to derive existence of a solution from the sole assumption of continuity of the function $F(Y)$ (cf. Kinderlehrer and Stampacchia (1980)). However, we can impose a rather weak condition to ensure the existence of a solution pattern. Let

$$K_b \equiv \{(x, q, \eta, \rho^3) | 0 \le x \le b_1; 0 \le q \le b_2; 0 \le \eta \le b_3; 0 \le \rho^3 \le b_4\}, \qquad (5.22)$$

where $b = (b_1, b_2, b_3, b_4)$ and $x \leq b_1; q \leq b_2; \eta \leq b_3; \rho^3 \leq b_4$ means that $x_p \leq b_1; q_{jk} \leq b_2; \eta_j \leq b_3; \rho_{jk}^3 \leq b_4$ for all $p \in P_j^i, \forall i, j, k$. Then K_b is a bounded, closed convex subset of $R_+^{n_P + 2n_H n_T + n_H}$. Thus, the following variational inequality:

$$\langle F(Y^b), Y - Y^b \rangle \geq 0, \quad \forall Y^b \in K_b, \tag{5.23}$$

admits at least one solution $Y^b \in K_b$, from the standard theory of variational inequalities, since K_b is compact and F is continuous. Following Kinderlehrer and Stampacchia (1980)(see also Theorem 1.5 in Nagurney (1999)), we have:

Lemma 5.1

Variational inequality 5.21 admits a solution if and only if there exists a $b > 0$ such that variational inequality (5.23) admits a solution in K_b with

$$x^b < b_1, \quad q^b < b_2, \quad \eta^b < b_3, \quad \rho^{3b} < b_4. \tag{5.24}$$

Under the conditions in Theorem 5.2 below it is possible to construct the upper bounds b_1, b_2, b_3, and b_4 large enough so that the restricted variational inequality (5.23) will satisfy the boundedness condition (5.24) and, thus, existence of a solution to the original variational inequality problem according to Lemma 5.1 will hold.

Theorem 5.2: Existence of a Solution

Suppose that there exist positive constants M, N, and R with $R > 0$ such that:

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} - \omega_i \gamma_{ij} \mu_p \geq M, \quad \forall x \quad with \quad x_p \geq N, p \in P_j^i, \forall i, j, \tag{5.25}$$

$$c_{jk}(q) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk})}{\partial q_{jk}} \geq M, \quad \forall q \quad with \quad q_{jk} \geq N, \forall j, k,$$

$$d_{jk}(\rho^3) \leq N, \quad \forall \rho^3 \quad with \quad \rho_{jk}^3 > R, \forall j, k. \tag{5.26}$$

Then variational inequality (5.21); equivalently, variational inequality (5.18), admits at least one solution.

Proof: Follows from Lemma 5.1. See also the proof of existence for Proposition 1 in Nagurney and Zhao (1993) and the existence proof in Nagurney, Dong, and Zhang (2003). □

It can be argued that, from an economics perspective, assumptions (5.25) and (5.26) are reasonable, since, when flow of RBCs on a path between a blood service organization and a hospital pair is large, it can be expected the "marginal" cost on the path minus the marginal service utility associated with the weighted altruism to exceed a positive lower bound. Similarly, when the amount of blood transfused by a hospital to a patient group is positive, the transaction cost between the pair and marginal cost of holding the blood in inventory by the hospital will exceed a lower bound. Lastly, in case the demand market price is very high, the demand for the product can be expected to be low (even if slightly).

Lemma 5.2: Monotonicity

Assume that the link total cost functions and the inventory holding cost functions are convex, the transaction cost functions are monotone increasing, and the demand functions are monotone decreasing functions. Then the vector function F that enters the variational inequality (5.21) is monotone, that is,

$$\langle F(Y') - F(Y''), Y' - Y'' \rangle \geq 0, \quad \forall Y', Y'' \in \mathcal{K}. \tag{5.27}$$

Proof: Let $Y' = (x', q', \eta', \rho^{3\prime}), Y'' = (x'', q'', \eta'', \rho^{3\prime\prime})$ with $Y' \in \mathcal{K}$ and $Y'' \in \mathcal{K}$. Then inequality (5.26) can be seen in the following deduction:

$$\langle F(Y') - F(Y''), Y' - Y'' \rangle = \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\frac{\partial \hat{C}_p(x')}{\partial x_p} - \frac{\partial \hat{C}_p(x'')}{\partial x_p} \right] \times [x'_p - x''_p]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[\frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q'_{jk})}{\partial q_{jk}} - \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q''_{jk})}{\partial q_{jk}} \right] \times [q'_{jk} - q''_{jk}]$$

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$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[c_{jk}(q') - c_{jk}(q'') \right] \times [q'_{jk} - q''_{jk}]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [-d_{jk}(\rho^{3\prime}) + d_{jk}(\rho^{3\prime\prime})] \times [\rho^{3\prime}_{jk} - \rho^{3\prime\prime}_{jk}]$$

$$= (I) + (II) + (III) + (IV). \tag{5.28}$$

Now, we can write (I) as:

$$(I) = \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\frac{\partial \hat{C}_p(x')}{\partial x_p} - \frac{\partial \hat{C}_p(x'')}{\partial x_p} \right] \times [x'_p - x''_p]$$

$$= \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} \alpha_{ap} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \alpha_{ap} \right] \times [x'_p - x''_p]$$

$$= \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \right] \times \sum_{p \in P_j^i} [x'_p \alpha_{ap} - x''_p \alpha_{ap}]$$

$$= \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \right] \times \left[\sum_{p \in P_j^i} x'_p \alpha_{ap} - \sum_{p \in P_j^i} x''_p \alpha_{ap} \right]$$

$$= \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \right] \times [f'_a - f''_a]. \tag{5.29}$$

Since the total link cost functions are convex, we have:

$$(I) = \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \right] \times [f'_a - f''_a] \geq 0. \tag{5.30}$$

The convexity of the holding cost functions, $h_j(q_j)$ for all j yields

$$(II) = \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[\frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q'_{jk})}{\partial q_{jk}} - \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q''_{jk})}{\partial q_{jk}} \right] \times [q'_{jk} - q''_{jk}] \geq 0. \tag{5.31}$$

Since c_{jk}, for all j, k, are assumed to be monotone increasing, and d_{jk}, for all j, k, are assumed to be monotone decreasing, we have that

$$(III) = \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[c_{jk}(q') - c_{jk}(q'') \right] \times [q'_{jk} - q''_{jk}] \geq 0, \qquad (5.32)$$

and

$$(IV) = \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [-d_{jk}(\rho^{3\prime}) + d_{jk}(\rho^{3\prime\prime})] \times [\rho^{3\prime}_{jk} - \rho^{3\prime\prime}_{jk}] \geq 0. \qquad (5.33)$$

Substituting (5.30) – (5.33) into the right-hand side of (5.28), it can be concluded that (5.28) is nonnegative. The proof is complete. $\square$

Definition 5.2: Lipschitz Continuity

The function that enters the variational inequality problem (5.21) is Lipschitz continuous if

$$\|F(Y') - F(Y'')\| \leq L \|Y' - Y''\| \quad \forall Y', Y'' \in \mathcal{K}, \qquad (5.34)$$

where $L > 0$ is known as the Lipschitz constant.

The properties of monotonicity and Lipschitz continuity are utilized to establish the convergence of the algorithm in the following section.

5.3. Algorithm: The Modified Projection Method

In this section, the realization of the modified projected method (cf. Section 2.4.2) for the computation of the variational inequality (5.18) is described. Below I present each step of the computation with respect to this model and provide the explicit formulae.

The statement of the modified projection method for the solution of variational inequality 5.21, applied to the blood supply chain network model, is as follows, where τ is the iteration counter:

Step 0. Initialization

Initialize with $Y^0 \in \mathcal{K}$. Set $\tau = 1$ and select ψ, such that $0 < \psi \leq 1/L$, where L is the Lipschitz constant (see (5.34)).

Step 1: Computation

Compute $\bar{Y}^\tau$ by solving the variational inequality subproblem (cf. (2.22)):

$$\langle \bar{Y}^\tau + \psi F(Y^{\tau-1}) - Y^{\tau-1}, Y - \bar{Y}^\tau \rangle \geq 0, \quad \forall Y \in \mathcal{K}. \tag{5.35a}$$

(5.35a) is now expanded, according to the details of (5.18), for this model. Compute $(\bar{x}^\tau, \bar{q}^\tau, \bar{\eta}^\tau, \bar{\rho}^{3\tau}) \in K^4$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\bar{x}_p^\tau + \psi \left(\frac{\partial \hat{C}_p(x^{\tau-1})}{\partial x_p} - \omega_i \gamma_{ij} \mu_p - \eta_j^{\tau-1} \mu_p \right) - x_p^{\tau-1} \right] \times [x_p - \bar{x}_p^\tau]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[\bar{q}_{jk}^\tau + \psi \left(c_{jk}(q^{\tau-1}) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^{\tau-1})}{\partial q_{jk}} + \eta_j^{\tau-1} - \beta_j \theta_{jk} - \rho_{jk}^{3\,\tau-1} \right) - q_{jk}^{\tau-1} \right]$$

$$\times [q_{jk} - \bar{q}_{jk}^\tau + \sum_{j=H_1}^{H_{n_H}} \left[\bar{\eta}_j^\tau + \psi \left(\sum_{i=1}^{I} \sum_{p \in P_j^i} x_p^{\tau-1} \mu_p - \sum_{k=T_1}^{T_{n_T}} q_{jk}^{\tau-1} \right) - \eta_j^{\tau-1} \right] \times [\eta_j - \bar{\eta}_j^\tau]$$

$$+ \sum_{k=T_1}^{T_{n_T}} \left[\bar{\rho}_{jk}^{3\tau} + \psi \left(q_{jk}^{\tau-1} - d_{jk}(\rho^{3\,\tau-1}) \right) - \rho_{jk}^{3\,\tau-1} \right] \times [\rho_{jk}^3 - \bar{\rho}_{jk}^{3\tau}] \geq 0, \quad \forall (x, q, \eta, \rho^3) \in K^4. \tag{5.35b}$$

Step 2: Adaptation

Compute Y^τ by solving the variational inequality subproblem (cf. (2.23)):

$$\langle Y^\tau + \psi F(\bar{Y}^\tau) - Y^{\tau-1}, Y - Y^\tau \rangle \geq 0, \quad \forall Y \in \mathcal{K}. \tag{5.36a}$$

(5.36a) is now expanded according to (5.18). Compute $(x^\tau, q^\tau, \eta^\tau, \rho^{3\tau}) \in K^4$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[x_p^\tau + \psi \left(\frac{\partial \hat{C}_p(\bar{x}^\tau)}{\partial x_p} - \omega_i \gamma_{ij} \mu_p - \bar{\eta}_j^\tau \mu_p \right) - x_p^{\tau-1} \right] \times [x_p - x_p^\tau]$$

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$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[q_{jk}^\tau + \psi \left(c_{jk}(\bar{q}^\tau) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} \bar{q}_{jk}^\tau)}{\partial q_{jk}} + \bar{\eta}_j^\tau - \beta_j \theta_{jk} - \bar{\rho}_{jk}^{3\tau} \right) - q_{jk}^{\tau-1} \right] \times [q_{jk} - q_{jk}^\tau]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \left[\eta^\tau + \psi \left(\sum_{i=1}^{I} \sum_{p \in P_j^i} \bar{x}_p^\tau \mu_p - \sum_{k=T_1}^{T_{n_T}} \bar{q}_{jk}^\tau \right) - \eta^{\tau-1} \right] \times [\eta_j - \eta_j^\tau]$$

$$+ \sum_{k=T_1}^{T_{n_T}} \left[\rho_{jk}^{3\tau} + \psi \left(\bar{q}_{jk}^\tau - d_{jk}(\bar{\rho}^{3\tau}) \right) - \rho_{jk}^{3\ \tau-1} \right] \times [\rho_{jk}^3 - \rho_{jk}^{3\tau}] \geq 0, \quad \forall (x, q, \eta, \rho^3) \in K^4.$$

$$(5.36b)$$

Step 3: Convergence Verification

If $|Y^\tau - Y^{\tau-1}| \leq \epsilon$, for $\epsilon > 0$, a pre-specified tolerance level, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1 (cf. Subsection 2.4.2).

Specifically, for this model, if $|x_p^\tau - x_p^{\tau-1}| \leq \epsilon, |q_{jk}^\tau - q_{jk}^{\tau-1}| \leq \epsilon, |\eta_j^\tau - \eta_j^{\tau-1}| \leq \epsilon, |\rho^{3\tau} - \rho^{3\tau-1}| \leq \epsilon \quad \forall p \in P_j^i, i = 1, ..., I, j = H_1, ..., H_{n_H}, k = T_1, ..., T_{n_T}$ for $\epsilon > 0$, a pre-specified tolerance level, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1.

Explicit Formulae for the Modified Projection Method

The elegance of this algorithm applied to the blood supply chain network competition model in that at each iteration, closed form expressions are obtained for the variables, resulting in an easy to implement computational procedure. Below the closed form expressions for the solutions of (5.35b) are provided .

The closed form expression for the blood path flows at iteration τ is: For each path $p \in P_j^i, \forall i, j$, compute:

$$\bar{x}_p^\tau = \max \left\{ 0, x_p^{\tau-1} - \psi \left(\frac{\partial \hat{C}_p(x^{\tau-1})}{\partial x_p} - \omega_i \gamma_{ij} \mu_p - \eta_j^{\tau-1} \mu_p \right) \right\}. \tag{5.37}$$

The amount of blood transfused, $q_{jk}, \forall j, k$, at iteration τ, is computed according to:

$$\bar{q}_{jk}^\tau = \max \left\{ 0, q_{jk}^{\tau-1} - \psi \left(c_{jk}(q^{\tau-1}) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^{\tau-1})}{\partial q_{jk}} + \eta_j^{\tau-1} - \beta_j \theta_{jk} - \rho_{jk}^{3\ \tau-1} \right) \right\}. \tag{5.38}$$

The Lagrange multipliers, η_j, $j = H_1, \ldots, H_{n_H}$, are computed at iteration τ using the formula:

$$\bar{\eta}_j^\tau = \max\left\{0, \eta^{\tau-1} - \psi\left(\sum_{i=1}^I \sum_{p \in P_j^i} x_p^{\tau-1} \mu_p - \sum_{k=T_1}^{T_{n_T}} q_{jk}^{\tau-1}\right)\right\}. \tag{5.39}$$

Lastly, at iteration τ, the closed form expression for the demand prices, ρ_{jk}^3, $j = H_1, \ldots, H_{n_H}$; $k = T_1, \ldots, T_{n_T}$, is:

$$\bar{\rho}_{jk}^{3\tau} = \max\left\{0, \rho^{3\tau-1}_{jk} - \psi\left(q_{jk}^{\tau-1} - d_{jk}(\rho^{3\tau-1})\right)\right\}. \tag{5.40}$$

Analogous closed form expressions to those above can be easily obtained also for (5.36b).

5.4. The Case Study

In this section, I present a case study that is solved using the modified Projection Method. Although the network structure is stylized the aim is to capture the complex landscape of blood banking in Southern California. There are three major blood service organizations; namely, the American Red Cross, the San Diego Blood Bank, and LifeStream, that collect and supply blood to hospitals in Southern California that have been facing shortages in recent times (Austin (2018), Hayden (2018)). LifeStream, a community based nonprofit blood bank, supplies blood products and services to more than 80 Southern California hospitals in five counties: San Bernardino, Riverside, Los Angeles, Orange, San Diego, and Imperial, while San Diego Blood Bank, another a regional nonprofit blood service organization, serves San Diego, Imperial, Los Angeles, and Orange Counties. The American Red Cross also has a strong presence in this region with multiple donor centers and supply contracts with UC San Diego and Scripps Health facilities (Sisson (2017)).

In the network structure shown in Figure 5.3 there are two blood service organizations, one a smaller regional blood bank such as LifeStream or the San Diego

Blood Bank, and, the other – a larger one such as the American Red Cross. Both of these blood service organizations supply blood to two hospitals which treat patients belonging to three payer groups: two private ones such as those covered by the Kaiser Foundation Health Plan, which has one of the highest customer satisfaction ratings according to a NCQA 2015-2016 report, and Blue Shield of California, which has one of the lowest ratings according to the same report, and the other – a government payer program such as Medicare or Medicaid. Given that hospitals and blood centers are facing shortages, it is not unreasonable to assume that each hospital has more than one supplier to mitigate the risk of running out of blood as observed in the Northeastern part of the United States. I now provide the data for this problem.

Baseline Example: Two BSOs, Two Hospitals and Three Payers

In Lagerquist et al. (2017), the authors provide an analysis of the cost of transfusing one unit of RBC in a Canadian hospital. Based on their data, the per unit cost of inventory and storage at the hospital was obtained as 30.80 CAD.

Using this information and converting it to USD the inventory holding costs for the two hospitals are constructed as:

$$h_{H_1}(\sum_{k=T_1}^{T_3} q_{H_1k}) = 23.6 \times (q_{H_1T_1} + q_{H_1T_2} + q_{H_1T_3}),$$

and

$$h_{H_2}(\sum_{k=T_1}^{T_3} q_{H_2k}) = 24 \times (q_{H_2T_1} + q_{H_2T_2} + q_{H_2T_3}).$$

Below, I present the transaction cost functions for this problem. It is to be noted that this cost might include various costs that are not directly associated with the procurement of blood, and maintaining it's inventory such as cost of cross matching, transfusion, and administrative costs for billing, etc. Linear cost functions are used which are given as follows:

$$c_{H_1T_1}(q_{H_1T_1}) = 0.5q_{H_1T_1}+10, \quad c_{H_1T_2}(q_{H_1T_2}) = 0.5q_{H_1T_2}+9, \quad c_{H_1T_3}(q_{H_1T_3}) = 0.5q_{H_1T_3}+8,$$
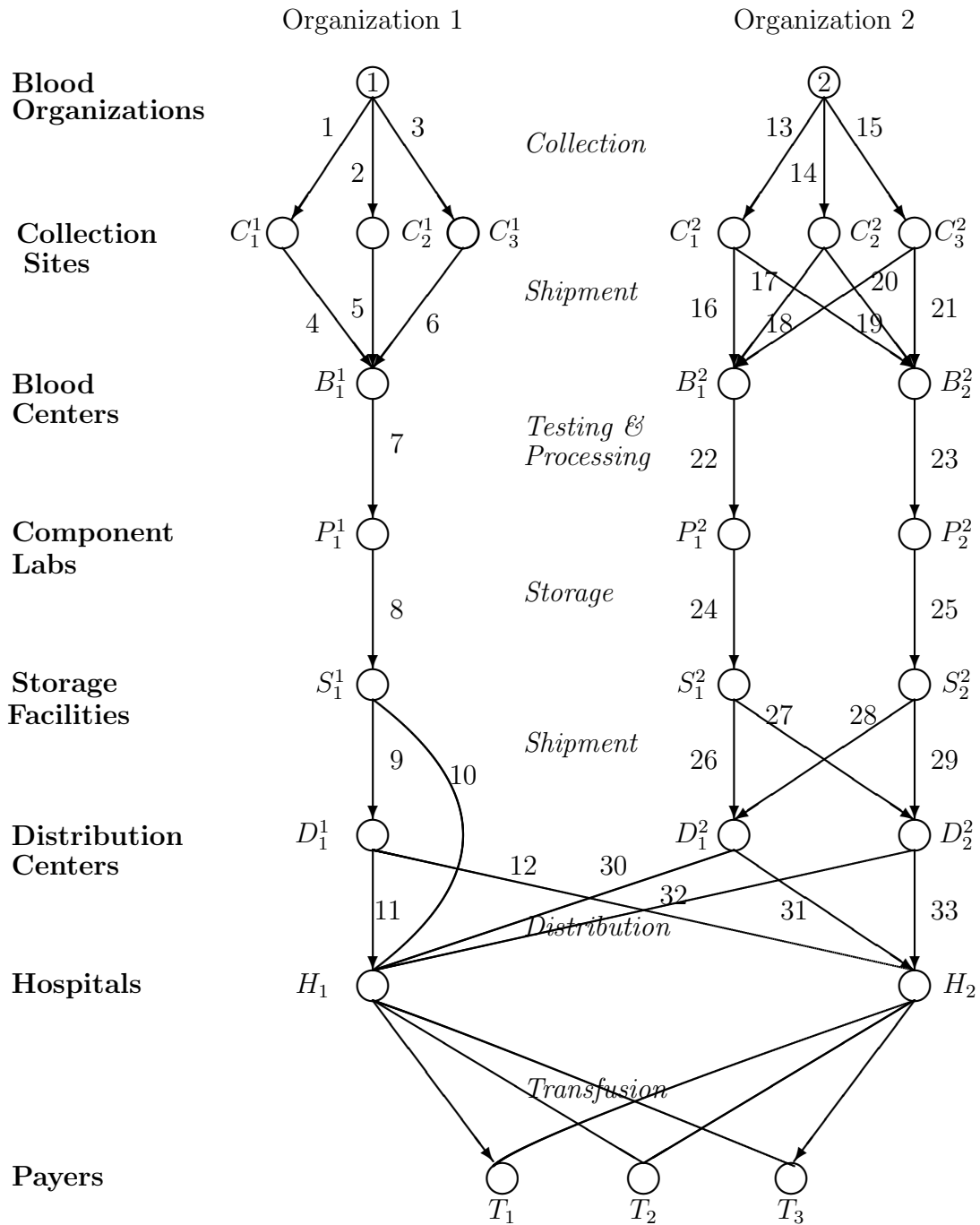
Figure 5.3. The Supply Chain Network Topology for Case Study

$$c_{H_2T_1}(q_{H_2T_1}) = 0.5q_{H_2T_1} + 10, \quad c_{H_2T_2}(q_{H_2T_2}) = 0.5q_{H_2T_2} + 10, \quad c_{H_2T_3}(q_{H_2T_3}) = 0.5q_{H_2T_3} + 8.$$

While it is mentioned in Section 5.1 that reimbursements received by a hospital from different payers might vary, the transaction costs might also vary depending on the type of payer. According to Ho and Lee (2017) average cost per patient for a hospital varies from one payer to another due to long-term relationships with particular insurance companies or due to " complementarities in information systems with some insurers."

Assuming that the overall base weekly demand for RBCs across all payer types at each hospital is 250 units the demand price functions are constructed as follows:

$$d_{H_1 T_1} = -0.007\rho^3_{H_1 T_1} + 0.001\rho^3_{H_2 T_1} + 100, \quad d_{H_2 T_1} = -0.005\rho^3_{H_2 T_1} + 0.003\rho^3_{H_1 T_1} + 100,$$

$$d_{H_1 T_2} = -0.007\rho^3_{H_1 T_2} + 0.001\rho^3_{H_2 T_2} + 50, \quad d_{H_2 T_2} = -0.005\rho^3_{H_2 T_2} + 0.003\rho^3_{H_1 T_2} + 50,$$

$$d_{H_1 T_3} = -0.007\rho^3_{H_1 T_3} + 0.001\rho^3_{H_3 T_1} + 100, \quad d_{H_2 T_3} = -0.005\rho^3_{H_2 T_3} + 0.003\rho^3_{H_1 T_3} + 100.$$

The weights associated with the altruism components of the blood service organizations' objective function are $\omega_1 = \omega_2 = 1$. The coefficients of the altruism function are assumed to be $\gamma_{1H_1} = 1, \gamma_{1H_2} = 1, \gamma_{2H_1} = 1, \gamma_{2H_2} = 1$. The hospitals also have an altruism component in their objective functions and the associated weights are assumed to be $\beta_{H_1} = \beta_{H_2} = 1$, while the coefficients are $\theta_{H_1 T_1} = 1, \theta_{H_1 T_2} = 1, \theta_{H_1 T_3} = 2, \theta_{H_2 T_1} = 1, \theta_{H_2 T_2} = 1, \theta_{H_2 T_3} = 2$. Both the hospitals associate greater service utility in treating patients belonging to government payer program.

In the paper by Dutta and Nagurney (2018) the modified projection method was implemented in FORTRAN and a Linux systems at the University of Massachusetts Amherst was used for the computations. The algorithm was initialized by setting all the variables equal to 0.00 and with ψ set to .05. The algorithm was considered to have converged when the absolute difference of the difference of each successive iterate was less than or equal to 10^{-4}. The equilibrium conditions held with an excellent accuracy.

In Table 5.1 I provide the total link cost functions, the arc multipliers associated with each link as well as the computed equilibrium link flows. The total link cost functions capture the fact that the two most expensive operations for the blood services organizations are collection of blood from donors, and testing and processing of the collected units.

In addition to the computed equilibrium link flow values in Table 5.1, which are obtained from the equilibrium path flows, the other computed equilibrium values of the variables are:

$$\eta^*_{H_1} = 184.92, \quad \eta^*_{H_2} = 194.67,$$

$$q^*_{H_1 T_1} = 98.46, \quad q^*_{H_1 T_2} = 48.61, \quad q^*_{H_1 T_3} = 98.55,$$

$$q^*_{H_2 T_1} = 99.43, \quad q^*_{H_2 T_2} = 49.53, \quad q^*_{H_2 T_3} = 99.41.$$

and

$$\rho^{3*}_{H_1 T_1} = 257.76, \quad \rho^{3*}_{H_1 T_2} = 231.83, \rho^{3*}_{H_1 T_3} = 245.80$$

$$\rho^{3*}_{H_2 T_1} = 268.39, \quad \rho^{3*}_{H_2 T_2} = 233.43, \quad \rho^{3*}_{H_2 T_3} = 266.37.$$

Using the procedure described in Section 5.1.4, the equilibrium prices of the BSOs and that of the hospitals are recovered: $\rho^{1*}_{1H_1} = \rho^{1*}_{2H_1} = 184.92$ and $\rho^{1*}_{1H_2} = \rho^{1*}_{2H_2} = 194.67$. Also, $\rho^{2*}_{H_1} = 198.52$ and $\rho^{2*}_{H_2} = 208.67$.

Also, for completeness, I provide the incurred demands at the equilibrium prices at the different payers and these are:

$$d_{H_1 T_1} = 98.46, \quad d_{H_1 T_2} = 48.61, \quad d_{H_1 T_3} = 98.55,$$

$$d_{H_2 T_1} = 99.43, \quad d_{H_2 T_2} = 49.53, \quad d_{H_2 T_3} = 99.41,$$

Finally, the incurred utilities of the blood service organizations and the hospitals at the equilibrium pattern are presented. The utility of BSO 1 is: 25,187.59 and that

Table 5.1. Definition of Links, Associated activity, Arc Multipliers, Total Operational Link Cost Functions, and Equilibrium Link Solution

Link a	From Node	To Node	α_a	$\hat{c}_a(f)$	f_a^*
1	1	C_1^1	1.00	$0.45_1^2 + 0.6f_1$	49.96
2	1	C_2^1	1.00	$0.35f_2^2 + 0.5f_2$	57.35
3	1	C_3^1	1.00	$0.32f_3^2 + 0.6f_3$	64.24
4	C_1^1	B_1^1	1.00	$0.09f_4^2 + 0.36f_4$	49.96
5	C_2^1	B_1^1	1.00	$0.12f_5^2 + 0.5f_5$	57.35
6	C_3^1	B_1^1	1.00	$0.1f_6^2 + 0.35f_6$	64.24
7	B_1^1	P_1^1	0.98	$0.5f_7^2 + 0.86f_7$	171.55
8	P_1^1	S_1^1	1.00	$0.12f_8^2 + 0.5f_8$	168.12
9	S_1^1	D_1^1	1.00	$0.09f_9^2 + 0.5f_9$	64.36
10	S_1^1	H_1	1.00	$0.05f_{10}^2 + 0.68f_{10}$	103.76
11	D_1^1	H_1	1.00	$0.04f_{11}^2 + 0.8f_{11}$	0.00
12	D_1^1	H_2	1.00	$0.06f_{12}^2 + 0.8f_{12}$	64.36
13	2	C_1^2	1.00	$0.3f_{13}^2 + 0.8f_{13}$	104.72
14	2	C_2^2	1.00	$0.25f_{14}^2 + 0.65f_{14}$	138.95
15	2	C_3^2	1.00	$0.32f_{15}^2 + 0.6f_{15}$	97.36
16	C_1^2	B_1^2	1.00	$0.1f_{16}^2 + 0.28f_{16}$	82.57
17	C_1^2	B_2^2	1.00	$0.15f_{17}^2 + 0.3f_{17}$	22.15
18	C_2^2	B_1^2	1.00	$0.15f_{18}^2 + 0.35f_{18}$	33.17
19	C_2^2	B_2^2	1.00	$0.12f_{19}^2 + 0.45f_{19}$	105.78
20	C_3^2	B_1^2	1.00	$0.16f_{20}^2 + 0.5f_{20}$	53.18
21	C_3^2	B_2^2	1.00	$0.08f_{21}^2 + 0.6f_{21}$	44.17
22	B_1^2	P_1^2	0.98	$0.4f_{22}^2 + 0.65f_{22}$	168.93
23	B_2^2	P_2^2	0.97	$0.45f_{23}^2 + 0.8f_{23}$	172.11
24	P_1^2	S_1^2	0.96	$0.02f_{24}^2 + 0.05f_{24}$	165.55
25	P_2^2	S_2^2	1.00	$0.04f_{25}^2 + 0.07f_{25}$	166.94
26	S_1^2	D_1^2	1.00	$0.2f_{26}^2 + 0.4f_{26}$	80.61
27	S_1^2	D_2^2	1.00	$0.18f_{27}^2 + 0.6f_{27}$	78.32
28	S_2^2	D_1^2	1.00	$0.12f_{28}^2 + 0.45f_{28}$	99.96
29	S_2^2	D_2^2	1.00	$0.15f_{29}^2 + 0.5f_{29}$	66.98
30	D_1^2	H_1	1.00	$0.08f_{30}^2 + 0.5f_{30}$	75.12
31	D_1^2	H_2	1.00	$0.1f_{31}^2 + 0.6f_{31}$	105.45
32	D_2^2	H_1	1.00	$0.12f_{32}^2 + 0.35f_{32}$	66.75
33	D_2^2	H_2	1.00	$0.16f_{33}^2 + 0.4f_{33}$	78.55

of BSO 2: 47,806.95. The utility of Hospital H_1 is: 985.40 and that of Hospital H_2: 495.25.

I now consider three variants of the baseline example, in which the weights associated with altruism are modified. Specific changes made to the Basline Example are reported below. The remainder of the data remains as in the Baseline Example. The computed equilibrium flows are reported in Table 5.2 for all the Variants.

Variant 1: Altruism Weights for BSOs Set to Zero

In Variant 1 the weights of the BSOs are all set to zero, that is, $\omega_1 = \omega_2 = 0$.

Besides the reported link equilibrium values in Table 5.2, the modified projection method also yielded the following equilibrium values for the other variables:

$$\eta_{H_1}^* = 283.54, \quad \eta_{H_2}^* = 293.33,$$

$$q_{H_1 T_1}^* = 97.87, \quad q_{H_1 T_2}^* = 48.02, \quad q_{H_1 T_3}^* = 97.96,$$

$$q_{H_2 T_1}^* = 99.23, \quad q_{H_2 T_2}^* = 49.33, \quad q_{H_2 T_3}^* = 99.21,$$

and

$$\rho_{H_1 T_1}^{3*} = 356.08, \quad \rho_{H_1 T_2}^{3*} = 330.15, \quad \rho_{H_1 T_3}^{3*} = 344.12$$

$$\rho_{H_2 T_1}^{3*} = 366.95, \quad \rho_{H_2 T_2}^{3*} = 332.00, \quad \rho_{H_2 T_3}^{3*} = 364.93$$

For completeness, I also report the recovered prices at the top tier and the middle tier. Specifically, we have: $\rho_{1H_1}^{1*} = \rho_{2H_1}^{1*} = 283.54$ and $\rho_{1H_2}^{1*} = \rho_{2H_2}^{1*} = 293.33$. In addition, we have that: $\rho_{H_1}^{2*} = 297.14$ and $\rho_{H_2}^{2*} = 307.33$.

Also, for completeness, I would like to mention that the incurred demands at the equilibrium prices at the different payers are exactly equal to the corresponding q_{jk}^* value, which conforms well with the equilibrium conditions.

The utility of BSO 1 is now: 24,952.65 and that of BSO 2 is: 47,365.72, whereas the utility of Hospital H_1 is: 979.48 and that of Hospital H_2 is: 493.29.

Table 5.2. Links and Link Equilibrium Solution for Variant Examples

Link a	From Node	To Node	Variant 1 f_a^*	Variant 2 f_a^*	Variant 3 f_a^*
1	1	C_1^1	49.71	49.94	49.68
2	1	C_2^1	57.08	57.32	57.04
3	1	C_3^1	63.93	64.20	63.89
4	C_1^1	B_1^1	49.71	49.92	49.68
5	C_2^1	B_1^1	57.08	57.32	57.04
6	C_3^1	B_1^1	63.93	64.20	63.89
7	B_1^1	P_1^1	170.72	171.44	170.61
8	P_1^1	S_1^1	167.31	168.01	167.20
9	S_1^1	D_1^1	64.26	64.34	64.25
10	S_1^1	H_1	103.05	103.67	102.95
11	D_1^1	H_1	0.00	0.00	0.00
12	D_1^1	H_2	64.26	64.34	64.25
13	2	C_1^2	104.22	104.66	104.16
14	2	C_2^2	138.29	138.27	138.21
15	2	C_3^2	96.89	97.30	96.83
16	C_1^2	B_1^2	82.18	82.52	82.13
17	C_1^2	B_2^2	22.04	22.13	22.03
18	C_2^2	B_1^2	33.01	33.15	32.99
19	C_2^2	B_2^2	105.28	105.72	105.21
20	C_3^2	B_1^2	52.93	53.15	52.90
21	C_3^2	B_2^2	43.96	44.15	43.94
22	B_1^2	P_1^2	168.12	168.82	168.02
23	B_2^2	P_2^2	171.29	172.00	171.18
24	P_1^2	S_1^2	164.76	165.45	164.66
25	P_2^2	S_2^2	166.15	166.84	166.04
26	S_1^2	D_1^2	80.23	80.56	80.18
27	S_1^2	D_2^2	77.94	78.27	77.89
28	S_2^2	D_1^2	99.49	99.90	99.42
29	S_2^2	D_2^2	66.66	66.94	66.62
30	D_1^2	H_1	74.53	75.04	74.45
31	D_1^2	H_2	105.18	105.42	105.15
32	D_2^2	H_1	66.28	66.68	66.22
33	D_2^2	H_2	78.33	78.52	78.30

Variant 2: Altruism Weights for Hospitals Set to Zero

In Variant 2, only the weights associated with the hospitals are set to zero, that is, we have $\beta_{H_1} = \beta_{H_2} = 0$ with all the rest of the data as in the Baseline Example. Please refer to Table 5.2 for the computed equilibrium link flows.

As can be seen from Table 5.2 all the equilibrium link flows are higher than the corresponding values for Variant 1.

The other computed equilibrium values of the variables are:

$$\eta_{H_1}^* = 184.74, \quad \eta_{H_2}^* = 194.49,$$

$$q_{H_1 T_1}^* = 98.41, \quad q_{H_1 T_2}^* = 48.56, \quad q_{H_1 T_3}^* = 98.42,$$

$$q_{H_2 T_1}^* = 99.41, \quad q_{H_2 T_2}^* = 49.46, \quad q_{H_2 T_3}^* = 99.42,$$

and

$$\rho_{H_1 T_1}^{3*} = 267.54, \quad \rho_{H_1 T_2}^{3*} = 241.62, \quad \rho_{H_1 T_3}^{3*} = 265.55$$

$$\rho_{H_2 T_1}^{3*} = 278.20, \quad \rho_{H_2 T_2}^{3*} = 253.33, \quad \rho_{H_2 T_3}^{3*} = 276.20$$

As for the equilibrium prices at the top and middle tiers, these are now: $\rho_{1 H_1}^{1*} = \rho_{2 H_1}^{1*} = 184.74$ and $\rho_{1 H_2}^{1*} = \rho_{2 H_2}^{1*} = 194.49$. In addition, we have: $\rho_{H_1}^{2*} = 278.20$ and $\rho_{H_2}^{2*} = 218.49$

As in the above examples, the q_{jk}^* value coincides with the incurred equilibrium demand $d_{jk}(\rho^{3*}), \forall j, k$.

The utility of BSO 1 is now: 25,156.20 and that of BSO 2: 47,747.99, whereas the utility of Hospital H_1 is: -0.02 and that of Hospital H_2 is: 0.01.

This result is quite interesting. First, note that the entire supply chain of each BSO is captured in the model. As for the hospitals, the focus is on its blood supply operations, but each hospital engages in numerous other activities. The utilities of both hospitals without the altruism component of the objective functions are essentially zero, which implies economic sustainability on the part of the blood operations.

It is important to emphasize that the hospitals are nonprofits and, were they for profit organizations, then their respective objective functions would be modified from those in (5.10). Moreover, it is to be noted that in the United States, blood transfusion costs account for 1% of a hospitals budget, typically, which is considered to be high (Hemez (2016)).

Variant 3: All Altruism Weights Set to Zero

In Variant 3, all the weights associated with altruism for all the BSOs and all the hospitals were identically equal to zero, with the rest of the data as in the Baseline Example. This would correspond, in effect, to the stakeholders in terms of the BSOs and the hospitals being non altruistic and operating, more or less, in a profit-like manner. Please refer to Table 5.2 for the computed equilibrium link flow pattern. Observe that, of the three Variant examples, the equilibrium link flows are the lowest for Variant 3. Also, note that the highest equilibrium link flows occur in the Baseline Example. The computed equilibrium values for the other variables are:

$$\eta^*_{H_1} = 283.36, \quad \eta^*_{H_2} = 293.15,$$

$$q^*_{H_1 T_1} = 97.82, \quad q^*_{H_1 T_2} = 47.94, \quad q^*_{H_1 T_3} = 97.83,$$

$$q^*_{H_2 T_1} = 99.21, \quad q^*_{H_2 T_2} = 49.26, \quad q^*_{H_2 T_3} = 99.22,$$

and

$$\rho^{3*}_{H_1 T_1} = 365.87, \quad \rho^{3*}_{H_1 T_2} = 339.94, \quad \rho^{3*}_{H_1 T_3} = 363.87$$

$$\rho^{3*}_{H_2 T_1} = 376.76, \quad \rho^{3*}_{H_2 T_2} = 351.78, \quad \rho^{3*}_{H_2 T_3} = 374.76$$

The recovered equilibrium prices charged by the hospitals to the BSOs are: $\rho^{1*}_{1 H_1} = \rho^{1*}_{2 H_1} = 283.36$ and $\rho^{1*}_{1 H_2} = \rho^{1*}_{2 H_2} = 293.15$. The recovered prices at the hospitals are: $\rho^{2*}_{H_1} = 306.96$ and $\rho^{2*}_{H_2} = 317.15$. The utility of BSO 1 is now: 24,921.40 and that of BSO 2: 47,307.50, whereas the utility of Hospital H_1 is now: -.12 and that of

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Hospital H_2: .06. Again, the utilities of both hospitals are essentially zero in this variant, which represents that none of the stakeholders assign a positive value to the altruism component in their respective objective functions.

5.4.1 Additional Discussion of the Numerical Results for Baseline Example and Its Variants

As can be seen from the numerical results in the case study the equilibrium prices increase down the tiers, which is very reasonable economic behavior. Furthermore, as can be seen from Variant 3, the equilibrium prices charged to the various payers are the highest of all the examples comprising the case study. This is intuitive since in Variant 3 without the altruism weights, the blood service organizations and hospitals act like profit-maximizing entities. The prices paid by the hospitals to the blood suppliers that are obtained in the Baseline Example and its Variants more or less tally with the prices reported in empirical studies of Toner et al. (2011) and Ellingson et al. (2017). In Toner at al. (2011), the authors report that the average cost of acquisition of one unit of RBCs in the West is 228.31 with a standard deviation of 42. According to Ellingson et al. (2017), the interquartile range for the price paid by hospitals in the United States for a unit of leukocyte-reduced RBCs in 2015 was 197 to 228, while that of non-leukocyte-reduced RBCs was 185 to 205.

Again, I emphasize that, although the case study is stylized it, nevertheless, illustrates important features of this unique supply chain in which the product cannot be produced but must be donated, while, at the same time, it then undergoes multiple activities of testing, processing, and distribution to hospitals, with subsequent dissemination to needy patients for the medical procedures. Moreover, the model captures, in a novel way, that payments for blood services can depend on the method of payment and reimbursement to hospitals.

5.5. Summary and Conclusions

In this chapter, I developed a mathematical model that integrates the behaviors of three major stakeholders in the blood supply chain: blood service organizations, hospitals and medical centers, and patient payer groups. The model captures the current competitive landscape of the blood banking industry in the United States, and explores a cost-based pricing scheme for blood products that is aimed at bridging the disconnect between actual costs of acquiring a unit of red blood cells for transfusion and the payments received by the hospitals and the blood suppliers. The model optimizes, under competition, the flow of blood through the paths joining the blood service organizations with the hospitals, the amount of blood transfused to each patient payer group at each hospital, and determines the equilibrium prices charged by the blood service organizations, and the reimbursements received by the hospitals. To the best of my knowledge, this is the first perishable product supply chain network model to include the complex economic interplays between the different tiers of decision-makers in the blood supply chain. I also quantify and incorporate the nonprofit or altruistic nature of blood centers and hospitals through a service utility component in their utility functions.

The theory of variational inequalities was utilized to formulate the equilibrium conditions for each stakeholder, and, subsequently, the entire integrated supply chain. Qualitative properties are also presented as well as examples for illustrative purposes. The algorithm is outlined and applied to solve numerical examples comprising a case study focusing on blood service organizations in California.

The equilibrium prices obtained reveal how the prices increase as the blood service organizations and hospitals act less altruistically. Under every scenario that is examined the prices obtained closely resemble those in practice. The results also show that the equilibrium prices increase in progression down the tiers, which ensures the economic stability of the blood supply chain. In terms of policy implications the

results show the benefit of having a pricing scheme for blood products based on the volume of blood transfused and the actual costs of all the supply chain operations, and how the reimbursements to hospitals vary by payer type.

CHAPTER 6

CONCLUSIONS AND FUTURE RESEARCH

6.1. Conclusions

The aim of this dissertation was to analyze how horizontal competition among blood centers, the key players in the blood banking industry, and their supply chains has affected the supply of the product and the economic stability of the industry. Rise in competition among blood centers is one of the several emerging issues in the blood banking system in the United States. The changes in the industry in response to these challenges provide interesting areas for research that have immense healthcare policy implications.

One of the most pressing issues faced by blood centers or blood service organizations is recruitment of donors since the blood banking industry is solely dependent on altruistic voluntary donors for the raw material. With the rise in the number of blood service organizations, the blood collection operation has become more and more competitive (Barton (2002), Smith (2011)). Competition among blood service organizations also exists for supply contracts with hospitals and trauma centers (Barber (2013), Brantley (2017)).

Due to the potential for fatal consequences caused by unavailability of blood products, to-date, most of the studies on blood banking have justifiably focused on problems such as inventory planning to minimize shortage and wastage, vehicle routing problems for transshipment of excess blood between hospitals, and so on. However, with phenomena such as increase in cost of testing donated blood and other operations, the price of blood varying considerably across the United States, it is essential

to scrutinize the economic aspects associated with each activity in the supply chain. Blood centers, which are nonprofit organizations, incur high costs in collecting, processing, testing, storing, and distributing blood to the demand markets. They charge the hospitals and other medical facilities a fee per pint of blood supplied to recover these costs. The hospitals in turn get reimbursed for their costs by payers such as private insurance companies, and government programs such as Medicare, Medicaid. However, there appears to be a disconnect between the reimbursements received and the actual costs of acquiring blood. Hence, several regional as well as national level blood service organizations have been operating at low margins (Mulcahy et al. (2016)).

In Chapter 3 of this dissertation, I developed a non-cooperative game theory model for blood service organizations competing on the quality of service provided at their collection sites with the objective of maximizing their utility which consists of revenue, cost and an altruism component. I imposed upper and lower bounds on the quality levels and derived the economic implications of each. In Chapter 4, I constructed and analyzed a competitive blood supply chain network model that captures the supply side as well as demand side competition among blood centers. Each important activity in the blood supply chain is mapped, associated costs considered and persihability of the product along the various paths in the network is also included. The novel features of the model resulted in a Generalized Nash Equilibrium. In Chapter 5, I extended the blood supply chain network framework by incorporating an additional tier of decision-makers; namely, the patient payer groups. The multitiered network captures the decentralized nature of the blood banking system in the United States and tests the merits of a proposed cost-based payment policy for blood products.

6.2. Future Research

This dissertation has demonstrated how competitive game theory techniques can be used effectively to test the status quo in blood banking as well as proposed policy changes that can make management of the blood supply chain more efficient. However, as the industry adapts itself to various changes and new policies are put forward, opportunities for future research emerge. In this section, I discuss some of those directions.

6.2.1 Outsourcing of Donor Testing

In recent times in response to the greater uncertainty in demand, and an increase in the cost of operations, blood service organizations have adopted various methods to adjust to the changes in the market and to sustain themselves economically. As mentioned in Chapter 1, there has been a growing trend of mergers and acquisitions among blood service organizations. However, another recent development in the blood banking industry is the consolidation of the donor testing facilities and outsourcing of the testing operation by smaller blood collection establishments to centralized laboratories that can afford to maintain this capital-intensive operation. According to Dr. Jorge Rios, Medical Director of the American Red Cross Blood Services, East Division, at present the cost of testing ranges from 40 to 60 USD per donor, and it varies with the volume of units tested. The cost also depends on the size of the independent testing laboratory; the bigger the lab, the lower is the cost (Rios (2018)). I intend to study the financial implications of outsourcing the blood testing operation by the regional blood banks to centralized testing laboratories on the various stakeholders in the industry.

Due to the nonprofit nature of blood service organizations they do not sell blood to hospitals to increase profits but charge them a fee to recover their costs (Engel (2007)) and a large part of that cost is attributed to testing the collected blood for

various infectious diseases. The Food and Drug Administration (FDA) of the United States is responsible for ensuring that the supply of blood in the country is safe. The FDA provides a list of tests to be carried out on the collected blood which include tests for HIV, Hepatitis B, C, Syphilis, Zika virus, etc. FDA recommended the screening of individual units of donated whole blood and blood components for Zika virus by blood centers in all states in the United States in 2016 (Nelson (2017)).

Ellingson et al. (2017) conducted a nation wide study to determine the projected cost of the Zika virus test on donated blood and found that it would cost 137 million USD (with 95% confidence interval) annually. Indeed, this test has added to the financial stress faced by blood service organizations in the times of decreasing revenue margins. While it is important to test collected blood for the virus before transfusion to minimize any risk, Saá et al. (2018) found that the screening of individual donations in the United States had a low yield while being extremely costly. According to Branswell (2018) the Zika testing operation ran by the American Red Cross costs roughly $137 million a year, and resulted in detection of only eight units that tested positive for the virus between June 2016 and September 2017. The article also mentions that "The high screening cost and low number of positive detections works out to about $5.3 million for each positive unit the Red Cross pulled from the system".

Given such conditions it is not surprising that in 2018 the American Red Cross consolidated its National Testing Laboratories and joined Creative Testing Solutions (CTS) to become the largest nonprofit blood donor testing laboratory organization in the United States (America's Blood Centers (2017b)). According to the website Creative Testing Solutions, owned by three major players in the industry, the American Red Cross, OneBlood, and Vitalant, will test approximately 9 million donor samples, which is about 75 percent of the country's blood supply, at six high volume laboratory facilities located in Charlotte, Dallas, Phoenix, Portland, St. Louis, and

Tampa in 2018 (CTS (2018)). In addition to CTS, the other players in the market include Qualtex Laboratories, ViroMed Laboratories, and blood service organizations such as the New York Blood Center, which conduct their own testing, and also for other regional blood banks.

While the size of CTS might allow it to offer lower prices than its competitors to blood centers due to economies of scale, it might also provide leverage with test builders and even with regulators. However, the creation of such an organization also raises concern with respect to the stability and the resilience of the blood supply chain in the United States in case of disruption leading to unavailbility of testing services for days, or even weeks (Katz (2017)). Hence, it would be of great value to study the effect of this new development in the blood banking industry and the trade-offs between outsourcing and in-house blood testing. The network structure in Chapter 4 can be extended to include outsourced testing. This can be achieved by introducing links denoting outsourced testing and nodes representing external testing laboratories. In addition, we can investigate and model the behavior of the donor testing facilities as a separate tier of decision makers in the multitiered blood supply chain network.

6.2.2 Inclusion of Brokers

The changing dynamics in the blood banking sector have also given rise to new business models such as the brokerage model. Mulcahy et al. (2016) explains the brokerage business model where organizations connect hospitals in need of blood with blood centers with excess blood. Revenue is generated through fees charged on each completed transaction. Two prominent examples of a brokerage model are the National Blood Exchange (NBE), a program run by the American Association of Blood Banks, and a software technology and services company called Bloodbuy. Bloodbuy uses a series of matching algorithms to pair a hospital with a blood bank

based on the amount of blood available at the blood bank and what the hospital is willing to pay.

The brokerage system has several merits. Hospitals contracting with blood centers from various parts of the country have already become prevalent. These new organizations make nationwide procurement of blood easier as they can ensure that hospitals and health centers anywhere in the country can get their demand fulfilled without having to pay unfair high prices. One of the advantages of this model reported in Mulcahy et. al. (2016) is that it provides price transparency and can result in cost savings. It can induce more competitive pricing for both blood centers and hospitals while minimizing shortage and wastage of blood. A game theory model can be developed and analyzed with the brokers constituting a separate tier of decision-makers. Such a model can capture the monetary transactions of the brokers and can also verify the perceived merits of the brokerage model.

The brokerage model also adds an intriguing layer of complexity to the modeling of blood supply chain networks. NBE claims to be a resource sharing program but Bloodbuy is a profit-maximizing company. The inclusion of profit-maximizing entities in the blood banking industry, which has traditionally operated in the nonprofit sector, has not been studied. At the same time we can study the operation of a program such as NBE from a system optimization perspective which can reveal the effect of centralization on the efficiency of the blood banking system.

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