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Elementary school teachers' beliefs about teaching and learning mathematics : selected case studies in Taiwan.

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ELEMENTARY SCHOOL TEACHERS' BELIEFS ABOUT TEACHING AND LEARNING MATHEMATICS: SELECTED CASE STUDIES IN TAIWAN

A Dissertation Presented

by

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ABSTRACT

ELEMENTARY SCHOOL TEACHERS' BELIEFS ABOUT TEACHING AND LEARNING MATHEMATICS: SELECTED CASE STUDIES IN TAIWAN

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Mathematics curriculum innovation has been launched in Taiwan recently in order to reflect the changing needs of the 21st century. The underlying assumptions of reform are: a learner-centered approach, emphasis on confluent education, and a problem-solving & reasoning approach. Research has revealed that teachers' beliefs can negatively interact with curriculum reform. On the other hand, some studies document that beliefs have little effect on instructional behavior. Therefore, this study attempts to investigate three questions: 1) what are the teachers' beliefs about the teaching and learning of mathematics in Taiwanese elementary schools and in what ways are teachers' beliefs congruent with the ongoing trend of reform; 2) what is the general picture of teachers' mathematical instructional practices in Taiwanese elementary schools and in what ways are these instructional practices congruent with the ongoing trend of reform; and 3) what is the relationship between teachers' beliefs and their instructional practices?

Basically, this study combines qualitative and quantitative methods in collecting and analyzing data. That is, teacher interviews and questionnaires were administered in order to understand teachers' beliefs about teaching and learning mathematics while observational checklists and naturalistic field observations were used to portray instructional behavior. The major findings of this study are:

1) Elementary school teachers' beliefs tend to hold with the traditional absorption learning theory and seem incongruent with the undergoing curriculum reform.

2) The instructional practices tend to reflect a traditional teacher-centered classroom and also seem incongruent with the launched reform.

3) Teachers' beliefs about teaching and learning play a vital role in shaping their instructional behavior; the situational constraints merely play a minor role.

In light of the above findings, some implications such as teacher education were drawn to broaden teachers' beliefs.

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CHAPTER I

INTRODUCTION

Background

Recently there has been an emerging interest in the study of teachers' beliefs. In their review of the small number of studies on the topic, Clark and Peterson (1986) concluded that "a teacher's cognitive and other behaviors are guided by and make sense in relation to a personally held system of beliefs." In their review of teachers' beliefs about their work activities, Eisenhart, Cuthbert, Shrum, and Harding (1988) attested that teachers' beliefs have significant bearing on the implementation of educational policy.

Like research on teachers' general educational beliefs, most studies on teachers' beliefs about mathematics and its teaching bear witness to the fact that teachers' beliefs affect the way in which they teach mathematics (Shirk, 1973; Shroyer, 1981; Thompson, 1982; McGalliard, 1983; Kesler, 1985). Studies also demonstrated that teachers' belief directly influence students' behavior (Harvey et. al., 1966) and achievement (Peterson et al., 1989).

Moreover, research supports the contention that educational policies or innovations that are not compatible with teachers' beliefs are implemented distortedly (Olson, 1981; Bussis, et al., 1976) or resistantly (Wolcott, 1977). As Brousseau, Book and Byers (1988) put it, "a first step toward understanding how to effect the process of schooling would be to understand the value and beliefs of those who drive those processes." Because a teacher's predispositions determine much of what the teacher "sees" and how the teacher defines daily teaching problems (Cooney, 1990).

On the other hand, Clark and Peterson (1986) pose constraints and opportunities on their review model. That is, there are some constraints which intercede between teachers' beliefs and actions. Teachers' actions are often constrained by the physical setting or by external influences such as the school, principal, community, or curriculum. This argument has been substantiated by empirical studies. For example, McNeil (1986, 1988) found that discipline problems and other administrative constraints made teachers adopt practices incongruent with their beliefs. Bawden, Buike, and Duffy (1979) reported that beliefs have only a minimal effect upon practice, and that other aspects of the teaching act -- the context of work, classroom management, activity flow etc., -- do mediate instructional behavior.

In light of the above studies, sociological research on "teachers work" lend support to the argument that teachers are often constrained by their work situation (Metz, 1978; Sarason, 1982; Gracey, 1972; Kounin, 1977; Jackson, 1968). In view of these studies and their three-year ethnographic study, Grant and Sleeter (1985) concluded that "teachers' work is determined as much by their conceptions as by factors in their work place." Indeed, teachers do have beliefs about teaching, and these beliefs do influence their practice. But to what degree that beliefs and situational factors influence the teaching behavior is needed to further examine.

Statement of Problem

Since teachers' beliefs interact significantly with curriculum, it is imperative to consider this concern in making any curricular innovation. As Romberg (1988a) put it, the most essential barriers to reform are strongly held beliefs and attitudes. In 1972, the Ministry of Education in the Republic of China gave the Taiwan Provincial Institute for Elementary School Teachers In-service Education (TPIESTIE) primary responsibility for research and development in all elementary school curricula. The new mathematics curriculum was thoroughly designed, evaluated and revised before its nationwide adoption in 1983 (Tsui, 1989). The overall goal of the new mathematics curriculum, according to the mathematics section of Curriculum Standards for Elementary School, is "to help children obtain relevant mathematical knowledge from daily-living experience and furthermore to foster the positive attitude and ability to apply mathematics in solving real life problems" (Ministry of Education, 1989).

Furthermore, according to the Teacher Handbook of Elementary School Mathematics In-service Training edited by TPIESTIE (1978), teachers should emphasize the following points in teaching the new mathematics curricula: 1) the procession from concrete to semi-concrete and finally to abstract thinking levels; 2) greater application of the "learning by discovery" method in fostering children's independent problem-solving ability and 3) greater stress on individualized instruction pedagogy to accommodate the differences in children's abilities.

It is clear that the new curriculum lays stress on both process and result in students' mathematics learning. It puts more emphasis on concept-fostering and thought training than ever before. Most importantly, it is the first time that manipulative materials and the discovery learning approach were introduced to elementary schools. This called for really big changes among teachers.

In order to assure successful implementation, some follow-up evaluations and in-service teacher training sessions were held. In examining these evaluations, it is easy to see that emphasis was put on measuring learning outcomes and on examining the curriculum content itself (TPIESTIE, 1988; Liu, 1985, 1988). Few studies addressed the problems of actually implementing the new curriculum, and these studies employed mainly questionnaires instead of direct classroom observations or teacher interviews (Wu, 1983; Kao, 1981). Very little attention was directed to studying teachers' beliefs and actual classroom practices. Little is know about teachers' beliefs toward the discovery learning method, reasoning, problem-solving, and the concrete-semiconcrete-abstract learning approach as addressed by the new curriculum. It is unknown how thoroughly teachers actually implement the new pedagogy as prescribed by the new curriculum.

As to in-service teacher training, there are still some concerns as to its effectiveness. They include: 1) the fact that it is difficult to implement the in-service training in a nationwide program, so the new objectives and the pedagogy of the curriculum might not be disseminated throughout the country, and 2) the two-week in-service training is too short to overcome the long-standing beliefs held by teachers who were trained and taught under the old curriculum (TPIESTIE, 1988).

The government recently set about further revision of Curriculum Standards for Elementary School, in response to the coming needs of the 21st century (Hung and Chuang, 1991; TPIESTIE, 1991, 1992). The potential mathematics curriculum will be an extension and enrichment of the present curriculum, and will much parallel the content of Curriculum and Evaluation Standards for School Mathematics edited by National Council of Teachers of Mathematics (NCTM, 1989). As Cooney (1988) claimed "whether teachers implement the full intent of the Standards depends on how the intended curriculum is filtered through the teachers' beliefs and conceptions of mathematics." Since teachers' beliefs govern their instructional practices, there is an urgent need to identify teachers' beliefs at this critical point.

On the other hand, some studies about curricular implementation revealed that environmental constraints kept teachers from implementing the prescribed pedagogy in the innovative curriculum. These constraints included a limited time schedule, classroom management problems, and an overload of students (Wu, 1983; Kao, 1981). It is true that over populated classrooms (average ratio: 50 students to 1 teacher) is a major teaching problem in Taiwan. In addition, the heavy load of teacher's work (Chao, 1990; Kao, et al., 1987) may have some bearing on teachers' instructional practices.

The main purpose of this study, therefore, is to investigate Taiwanese elementary school teachers' beliefs about mathematics teaching and learning by incorporating both classroom observation and teacher interviews with questionnaire. By so doing, teachers' instructional practices can be simultaneously portrayed along with the classroom observation. An examination of the relationship between teachers' beliefs and instructional practices with considerable openness to the emergence of any situational constraints in the inquiry process can also be achieved. And so, the specific research questions are:

1) What are the teachers' beliefs about the teaching and learning of mathematics in Taiwanese elementary schools and in what ways are teachers' beliefs congruent with the recent trend of curriculum reform?

2) What is the general picture of teachers' mathematical instructional practices in Taiwanese elementary schools and in what ways are these instructional practices congruent with the recent trend of curriculum reform?

3) What is the relationship between teachers' beliefs and their instructional practices?

Significance of Study

The significance of this study is three-fold. First of all, it will contribute to curricular innovations in Taiwan. Research has shown that teachers' beliefs can interact with curriculum reform negatively. For example, Olson (1981) reported that innovations caused teachers dilemmas, and that teachers dealt with the tension between their belief that their classroom influence should be high and the curriculum developers' belief that teachers' influence should be low by "domesticating" the curriculum project so that it became compatible with teachers' conceptions. In other words, the essential components of the innovation were either neglected or redefined in more traditional manner. Teachers translated new programs into their ways of understanding.

Bussis, Chittenden and Amarel (1976) discovered four types of define teachers in attempting to implement "open education," ranging from those who put an extreme emphasis on traditional "grade-level facts and skills" to those whose primary stress was on "broad developmental goals." A large number of teachers held beliefs incongruent with the new approach and resolved the conflict by behaving in their traditional way or changing only their surface curricular activities. In view of this information, teachers' beliefs have to be taken into account when initiating any curriculum innovation.

Since teachers' conceptions can be overlooked only at the innovation's own risk and since all research regarding the evaluation of curriculum reform in Taiwan failed to take teachers' beliefs into account, the present study will contribute to innovation in two ways: 1) it offers a different lens through which to evaluate the present curriculum reform and 2) it provides an overview of teachers' beliefs and classroom practices, which serves as referential base for enacting new curriculum standards and designing corresponding in-service and pre-service training.

Secondly, from the research standpoint, this study will also contribute to the ongoing dialogue concerning the relationship between teachers' beliefs and instructional practices. Do teachers' beliefs completely create their instructional practice, or do these beliefs have only a small effect upon practice, while other external teaching aspects greatly influence practices? Or do both beliefs and situational factors interactively account for instructional behavior? Basically, this study takes the position that teachers' beliefs do influence practices, but the degree that beliefs shape behavior remains open to investigate.

Thirdly, there are only two studies of teachers' beliefs to be found in Taiwan; one concerns general education (Lin, 1989) while the other is about mathematics and its teaching (Lin, 1990). The results of the latter study showed that teachers' beliefs about mathematics and its teaching are more or less traditionally oriented in Taiwan, and that heavy emphasis on computational skill still dominates the practices. This phenomenon was also reported in cross-cultural studies of mathematics learning (Stigler and Perry, 1988; Stevenson, et al., 1987). The present study connects to this line of inquiry, and hopefully it can update the information about teachers' beliefs in Taiwan.

Moreover, from a methodological viewpoint this study, combining quantitative and qualitative methods can be a model for future studies, since teachers' beliefs studies are still in their infancy throughout the world.

<u>Definition of Belief</u>

We repeat the important message that teachers' beliefs can significantly influence their teaching practices, however, what are beliefs? Before presenting this study, an understanding of the definition of "belief" is necessary.

<u>The Handbook of Psychological Terms</u> (1975) defines belief as "a proposition accepted with unquestioning confidence, often the result of a strong wish for credence in the belief and of a dislike to evaluate it." It defines attitude as "a readiness to respond in a certain way when the appropriate situation occurs; a mental set."

<u>A Comprehensive Dictionary of Psychological and Psychoanalytical Terms</u> (1961) defines belief as "an emotional acceptance of proposition or doctrine upon what one implicitly considers adequate ground. The grounds for belief, however, are often not examined, nor does the believer imply that other need have the same grounds. Beliefs have varying degrees of subjective certitude." This dictionary defines attitude as "an enduring, learned predisposition to behave in a consistent way toward a given class of objects."

Rokeach (1968) stated that "a belief is any simple proposition, conscious or unconscious, inferred from what a person says or does." According to him, there are three types of beliefs in content: descriptive beliefs, evaluative beliefs, and prescriptive beliefs. A descriptive belief describes the object of belief as true or false; for example, "I believe that the sun rises in the east." An evaluative belief evaluates the object of belief as good or bad; for instance, "I believe this guy is good." A prescriptive belief advocates a certain course of action or a certain state of existence as desirable or undesirable; for example, "I believe it is desirable that teachers should foster children's reasoning abilities in teaching mathematics." Rokeach (1968) sees beliefs as underlying the formation of attitude. He contends that "attitude is a relatively enduring organization of beliefs around an object or situation predisposing one to respond in some preferential manner."

Fishbein and Ajzen (1975) define belief as "representing the information a person has about the object." In other words, a belief links an object to some attribute. According to them, "the object of a belief may be a person, a group of people, an institution, a behavior, a policy, an event, etc., and the associated attribute may be any object, trait, property, quality, characteristic, outcome, or event;" for example, "America is a democratic country" links the object "America" to the attribute "democratic country." He also argued that beliefs are elements of attitude. He stated "attitude is effective or evaluative in nature which is determined by the person's beliefs about the attitude object."

Both Rokeach and Fishbein and Ajzen acknowledge that beliefs have strength. People may differ in concerning a specific object-attribute association, that is, in their perceived likelihood that the object has a specific attribute (Fishbein & Ajzen, 1975). Rokeach (1968) believed that all beliefs are not equally essential to the individual; beliefs may vary along a "central-peripheral" dimension. Hence, the more central a belief, the more it will resist change. Kerlinger (1967) used the term "Criteria Referents of Attitudes" to convey the notion of strength. If referents are criteria to one person, his attitudes will cluster around them.

To synthesize, belief is:

1. containing emotion and affection in nature
2. constituting the basic element of attitude
3. predisposition to action
4. having strength

CHAPTER II

REVIEW OF RELATED LITERATURE

Research into teachers' beliefs is very new, and each study seems to act as a pioneer work (Clark and Peterson, 1986). There is a great of diversity in research focus (e.g., beliefs about overall curriculum, beliefs about a specific subject matter, beliefs about education in general, etc.), in research methods (e.g., stimulated recall interview, repertory grid technique, questionnaire, classroom observation, etc.), and in research subjects (e.g., In-service elementary school teachers, In-service secondary school teachers, pre-service teachers, etc.). In addition, the terms used in these studies are also divergent. The variations include beliefs, views, conceptions, conceptual framework, implicit theory, etc.

This chapter, the review of literature, is organized into four sections. The first section deals with empirical studies of teachers' beliefs about education in general, curriculum/subject matter and its teaching, and other types of beliefs studies. The second section focuses solely on empirical studies of teachers' beliefs about mathematics and its teaching. A discussion of research findings on the relationship between beliefs and classroom practices is presented in the third session. Since the present study focuses on teachers' beliefs about the mathematics curricular innovation in Taiwan, the last section, reviews recent trends of mathematics reform.

Relevant Studies of Teachers' Beliefs About Education, Curriculum/Subject Matter and Its Teaching, and Others

The reviews in this section are divided into three categories of teachers' beliefs studies: 1) education in general; 2) a specific curriculum/subject matter and its teaching; 3) other variations with more narrow focus.

Education in General

In an effort to develop a teacher preparation program at Michigan State University, a series of studies were conducted primarily to investigate the general educational beliefs of different populations. These studies typically administered a questionnaire reflecting beliefs scales such as pedagogy, milieu, curriculum, and students' and teachers' roles to a large number of subjects and employed statistics to manage the data. Generally speaking, the finding of each study is comparison of different groups in nature. For instance, Brousseau, Freeman and Book (1984) compared 258 education majors with 146 non-education majors and found that the educational beliefs of these two groups were different. Book and Freeman (1985) then compared 174 elementary education majors with 178 secondary education majors and found that the educational beliefs of these two groups were remarkably similar. Brousseau, Book and Byers (1988) compared prospective and experienced teachers and found that experienced teachers were different from inexperienced teachers in their educational beliefs. Although these studies offer information for teacher education programs, they are limited in the sense that they provide only a superficial understanding of subjects' beliefs as is inherent in typical questionnaire studies. As Borg and Gall (1983) put it, "they fail to dig deeply enough to provide a true picture of opinions and feeling."

Bauch (1982, 1984) also investigated teachers' educational beliefs and the possible relationship between beliefs and practices. From the analysis of a belief questionnaire (182 elementary school teachers), she identified four types of teacher beliefs based on high/low scores on two dimensions of beliefs: teacher control and student participation. The four types of teachers include 1) "autocrats," with high discipline and low participation scores; 2) "strategist," with high discipline and participation scores; 3) "laissez-faire," with low discipline and participation scores; and 4) "democrats," with low discipline but high participation scores. Following this questionnaire, classroom observation and teacher interviews were conducted, and Bauch found that, generally speaking, teachers' instructional practices reflected their specific types of beliefs.

As noted, Bauch identifies 2 dimensions of educational beliefs, while the studies at Michigan State University identified 5 dimensions. The content of educational beliefs did have variations among studies. For example, Wehling and Charters (1969) identified 8 dimensions of teachers' beliefs about the teaching process in their investigation: subject-matter emphasis, personal adjustment ideology, student autonomy vs. teacher direction, emotional disengagement, consideration of student viewpoint, classroom order, student challenge, and integrative learning; whereas Bunting (1984) identified 4 dimensions: the effective factor, cognitive factor, directive factor, and interpretive factor. In a factor-analytic study of the Minnesota Teacher Attitude Inventory, Horn and Morrison (1965) found the existence of 5 dimensions. An important assumption of multi-dimensionality of beliefs is that individual teachers may simultaneously hold beliefs which are considered contradictory to each other under the traditional bipolar assumption. For instance, teachers who place weight on the emotional development of the students may, at the same time, support the more traditional values of content mastery and authority compliance (Bunting, 1984).

Curriculum/Subject Matter and Its Teaching

Some studies on teachers' beliefs about a specific curriculum bear testimony to the fact that beliefs significantly interact with curriculum implementation. In an in-depth interview study of 60 elementary teachers who were implementing open education, Bussis, Chittenden and Amarel (1976) identified four groups of teachers representing differences in their personally-held curriculum construct systems. Group 1 teachers (12%) were characterized by great concern for "grade-level facts and skills." They showed little evidence of change in the curricular activities. Group 2 teachers (22%) also heavily emphasized grade-level facts and skills, but they showed much evidence of change and experimentation with the curricular activities, however, there was no connection between their arranged activities and underlying rationales. They were struggling to understand the innovative programs. The third group of teachers (39%) were also concerned with grade level facts and skills, but the concerns of children's initiative and confidence were dominant. There is also evidence of rich curricular activities with connection to their organizing priorities/concerns. The fourth group of teachers (27%) emphasized children's initiative and reflectivity in cognitive concern or confidence and acceptance of self in personal/social concern. Furthermore, these teachers were very reflective about their curricular activities and organizing priorities/concerns.

Olson (1981, 1982) employed a repertory grid interview technique to elicit 8 British secondary school teachers' views about implementing a new science curriculum. He discovered that the dilemma of "teacher role" confronted these teachers. The new science curriculum called for "low influence teaching" which emphasized students' own discovery learning and discussion. In contrast, these teachers viewed their role as that of the traditional "high influence" teacher who exerted considerable authority in the classroom. The dilemma was resolved by "domestication" to favor more familiar and comfortable ways. For instance, discussions became lectures or recitations; intellectual skills development was translated as content memorization and examination rehearsal, etc. In short, the innovation was translated unrecognizably.

Munby (1983, 1984) also used a repertory grid technique to prove that curriculum change is doomed to different interpretations and implementations by teachers of diverse beliefs. The subjects, 14 junior high school teachers of different subject matters, were found to have wide individual differences in their beliefs and principles; and many of the principles and beliefs held by teachers were formulated as dichotomies. Each teacher had between three and six principles, scattered on five main categories and subcategories of all enunciated principles: goals, management, teacher needs, student needs, and the facilitation of learning. The five most frequently mentioned principles were curriculum goals, student involvement, teacher control, student needs and limitations, and motivation.

The above three studies (Bussis et al., Olson, Munby) all employed self-reported interview as the sole research method and concluded that beliefs influence practice. Although they achieved some in-depth understanding of teachers' beliefs, relying on a single research method results in bias. As Weiss (1975) pointed out, the interviewer and respondent are subject to bias from many sources: predispositions of respondent, predispositions of the interviewer, and procedures use. The biggest problem was that the self-reported data didn't embed on referential classroom instruction. If the purpose of study is to investigate the impact of beliefs upon practice, then self-reported interviews are not enough.

The disparate views among teachers who teach different subject matters, and even among those who teach the same subject matter first reported by Muni were also revealed in the findings of Nespor (1984, 1985). In addition to repertory grid interview techniques, Nestor observed classroom teaching and employed stimulated recall interviews to uncover the beliefs of eight teachers who taught different subjects and grades in three different school districts (the basic assumption he held is that context such as community, students taught, and task structures exerts influences on teachers' beliefs and actions). Nespor's multiple methods of research design increased the validity and credibility of his findings. As Patton (1990) put it, triangulation is a powerful solution to the problem of relying too much on any single data source or method.

Contrary to the findings of Olson, Munby and Bussis et al., a naturalistic field study of teachers' reading conceptions conducted by Bawden, Buike, and Duffy (1979) found that conceptions have little influence on teaching practice. The important findings are: 1) teachers do have reading and non-reading conceptions (such as activity flow, student level, management problem, etc.); 2) some teachers possess more complex conceptions than others; 3) teachers modify their conceptions and instruction over time; and 4) a substantial amount of teachers' non-reading conception seems to dominate the teachers' work and influence practices more than the reading conception.

Other Beliefs Studies

In addition to education in general, curriculum/subject matter and its teaching, the variety of studies of teacher beliefs also include beliefs about the teachers' role (Janesick, 1978), content emphasis of different subject matters (Schmidt and Buchman, 1983), and rationalization of classroom procedures and outcomes (Ignatovich, Cusick, and Ray, 1979).

Based on the theory of symbolic interaction, Janesick (1978) presented an ethnographic case study of a sixth-grade teachers' classroom perspective in terms of his role. A primary concern of this teacher was to establish a sense of "groupness" in classroom. The teacher defined his role as the leader of the group and struggled to achieve his goal by modeling and initiating activities which called for cooperation and respect throughout the whole school year. Janesick's single case study offers an in-depth and detailed understanding how the teacher defined his classroom world and constructed his actions, but this study is limited in terms of of generalization across cases.

Ignatovick et al., (1979) used a Q-sort methodology to identify the beliefs about classroom procedures and outcomes of three different groups. They found teachers and principals believed in humanistic approaches to instruction and viewed external administrative acts (such as standardized tests and administrative evaluation) as negatively, whereas administrators emphasized the abstract modeling of classroom learning and believed in the importance of external administrative acts on classroom practice.

Schmidt and Buchman (1983) studied six elementary school teachers' beliefs about the content emphasis of five subject areas and their sense of competence in teaching these areas. They found a connection between teachers' beliefs about school subjects and the amount of instructional time allocated to them.

In Taiwan, interest in the study of teachers' beliefs has begun to emerge. So far, only a single study regarding educational beliefs has been found. Lin (1989) investigated three populations' beliefs: elementary school teachers, pre-service teachers, and teaching faculties in nine teachers colleges. The research instrument consisted of three types of questionnaires: educational beliefs, democracy orientation, and authority conformity. The educational beliefs questionnaire includes four elements: knowledge and curriculum, community role, classroom control and relationships, and equal/differential treatment.

The findings are useful in providing a general picture of teachers' educational beliefs among different groups and subgroups in Taiwan, but it lacked an in-depth understanding of teachers' beliefs about specific subject matter and its teaching or about the innovative curriculum in order to make a substantial contribution to teaching. The findings are such that, female subjects are more progressive than their male counterparts; that teachers who graduated from four-year colleges are more progressive than those from junior teachers colleges; and that more "democracy-oriented" and less "authority-obedient" teachers or prospective teachers held more progressive views.

Relevant Studies of Teachers' Beliefs About Mathematics and Its Teaching

Few studies have taken subject matter into consideration. Take mathematics for an example. Among these belief studies, some focus on the broad context of mathematics and its teaching; others direct their attention focus solely on a specific topic and/or its teaching within mathematics. Despite variations in focus, these beliefs studies are all embedded on a specific subject matter instead of on broad educational context, and therefore they are appealing to me because they are more likely to prescribe some kinds of possible intervention in order to aid us as we attempt to improve teaching.

Mathematics and Its Teaching

Thompson (1982, 1984) conducted case studies in order to investigate three junior high school teachers' conceptions of mathematics and mathematics teaching, and the relationship between conceptions and practices. Classroom observation, stimulated recall interviews, and some paper and pencil instruments were employed to collect the data. Because of the advantage of the multiple methods design, the professed beliefs can be referenced to actual teaching contexts. The cross-data validity checks strengthen the study. The result shows that each teacher held prevailing views of mathematics and its teaching, and differences in teacher's beliefs were generally reflected in their instructional practices.

The first teacher regarded mathematics primarily as an organized and logical system of symbols and procedures; therefore, her views about teaching were basically that the teacher must stress the reasons and logic underlying mathematical rules and procedures. The content-oriented and conceptual approaches best characterize her views. The second teacher viewed mathematics primarily as a challenging subject which is discovery in nature. Her views on teaching held that teachers must encourage students to reason, question, and guess on their own. The process-oriented and discovery approaches best describe her view of teaching. The third teacher held very traditional beliefs about mathematics. She regarded mathematics as a collection of more or less arbitrary rules and procedures which are prescriptive in nature. Her views on teaching were that transferring information was the main task of teaching. The computational approach best portrays her teaching. Thompson studied in-service teachers' beliefs, while Collier (1972) and Shirk (1973) investigated pre-service teachers' beliefs. Shirk (1973) also triangulated the research design through collecting class assignments (papers describing subject's concept of teaching and the nature of mathematics), collecting materials about mini-teaching experience (lesson plans, lesson self-comment lesson cards, teaching materials, etc.), observing mini-teaching sessions, and conducting subject interviews in an attempt to identify the conceptual framework of four pre-service teachers' who were enrolled in a mathematics method course.

The result was that the four prospective teachers held many common elements among their conceptual frameworks, but the unique combination of elements in each case resulted in different teaching behaviors. The conceptual frameworks appeared to be activated in teaching situations. The first subject believed in the teacher's role of transmitting a well-ordered system of mathematical knowledge. The second subject regarded mathematics as primarily a vehicle to educate the "whole" person. The third subject believed that establishing respect for herself both as a person and a mathematician was essential to teaching. The fourth subject's framework was similar to the third subject's in the way of emphasizing building a relationship with the students, but was more relaxed in her teaching style.

Collier (1972) also examined pre-service teachers' beliefs about mathematics and mathematics teaching, but the method and technique he employed, a Likert-type scales questionnaire, is very different from that of Thompson and Shirk. The scales were constructed with half of the items describing mathematics as formal and the other half describing mathematics as informal. The formal view holds that mathematics is a rigid, organized body of knowledge and that teaching mathematics should focus on teacher demonstration, rote memory, and specific approach in solving problems. The informal view is that mathematics is reasoning and creative in nature and that teaching mathematics should focus on discovery, exploration and multiple-ways of problem solving.

Contrary to Shirk's finding, the prospective elementary school teachers who were in the last stage of preparation moved toward more informal beliefs about mathematics and mathematics instruction than those who had just entered the program. In addition, their beliefs about mathematics and mathematics instruction were less ambivalent. Shirk found no major discernable change within the conceptual framework of prospective teachers.

Schmidt and Kennedy (1990) also employed questionnaires to assess teachers' beliefs, but the research subjects included prospective, beginning, and experienced teachers. The nice thing about this study is that it identified each respondent's belief pattern in terms of representative numbers of characteristics of various beliefs. The most notable finding was the wide diversity of belief patterns. For example, in regard to beliefs about the nature of mathematics, 12 patterns of beliefs accounted for 90 percent of all respondents, with 10 percent holding the remaining 42 belief patterns. Even though experienced teachers held different patterns of beliefs from prospective teachers, they were noticeably more homogeneous in their beliefs.

The second essential result is that teacher's beliefs showed no polarity. Each of the belief patterns is an all-encompassing belief pattern. In other words, each belief pattern included elements from both poles of the educational dichotomy. This finding echoes the view of multi-dimensionality of educational beliefs. Fifty four percent of respondents belonged to the first or second belief pattern. The first belief pattern, for example, included the belief that being good at mathematics required rote memory and having basic understandings; the capability of thinking logically as well as flexibly; and ability, work and an interest in mathematics as well.

Problem Solving

Recently researchers turned their attention to teachers' beliefs about mathematical problem solving. Ford (1988) interviewed ten 5th-grade teachers and twenty students to discover what teachers beliefs about problem solving were and to determine to what extent their beliefs were reflected in the beliefs of students. The major findings were: 1) both teachers and students believed in problem solving as primarily an application of computational skills; 2) both teachers and students regarded successful problem solving as having achieved the right answer; 3) teachers attributed ability, whereas students attributed both ability and effort reason for success and failure in problem solving; 4) the reported teaching/learning method in classroom was computational activity and was textbook oriented; the use of calculators was discouraged; 5) teachers tended to overestimate students' ability to do problems involving computation and underestimate students' ability to do reasoning problems. In short, Ford found that elementary school teachers held very limited views about mathematical problem solving.

In his in-depth study of a beginning teacher's beliefs, Cooney (1985) also directed his attention to mathematical problem solving. In the process of interviews and classroom observation, the teacher revealed the belief that "solving problems is the essence of mathematics" and that "a central point of teaching problem solving is teaching heuristics." His problem solving approach was characterized by motivation, fun and casualness in his teaching practice. He "seemed to interpret problem solving as a technique of presenting interesting problems for the purpose of capturing the interest of students" (Cooney, 1983). However, this teacher experienced some difficulties in implementing a problem solving approach in the classroom. A chasm was found between his beliefs and actions.

Thompson (1988) documented changes in 16 elementary school teachers' conceptions of mathematical problem-solving and their instructional practices over a 3 week summer course and after a year of teaching. He approached the study by administering the questionnaire, conducting informal interviews, having teachers keep journals and write instructional report, and observing classrooms.

The initial data showed that some teachers had limited conceptions of the "right" method to solve "word problems," application of computational skills, knowing and remembering the procedure in order to be successful in solving problems. Moreover, most teachers lacked confidence in teaching problem solving. By the end of the course, all teachers reported that they felt more confident and knowledgeable, and many teachers saw problem solving as a way of teaching. As to the changes in instructional practices, a substantial number of teachers were observed teaching problem solving in a systematic and qualitative way.

Specific Topic and/or Its Teaching

Two studies analyzed teachers' beliefs within a specific topic area of mathematics: Peterson, Fennema, Carpenter and Loef (1989) on addition and subtraction; and Tiros and Grabber (1989) on multiplication and division. The difference is that the former study examined first-grade teachers' beliefs about teaching the topic and the latter investigated pre-service elementary teachers' misconceptions about the topic itself.

In an attempt to examine the relationship between teachers' beliefs and students' achievement, Peterson et al. identified two groups of teachers' pedagogical content beliefs through administering belief questionnaires and interviews. Teachers with a more cognitively based perspective (CB teacher) believed that : 1) children construct mathematical knowledge in light of their intuitive knowledge; 2) mathematical skills should be taught in relation to problem solving; 3) instruction should be sequenced to build on children's development of mathematical ideas, for example, counting strategy; and 4) instruction should be organized to facilitate children's construction. Teachers with a less cognitively based perspective (LC teacher) are on the opposite extreme from CB teachers. The result showed that there was a significantly positive relationship among teachers' beliefs, teaching practice, and students' problem-solving achievement.

On the other hand, Tiros and Grabber administered paper and pencil instruments and interviews to assess the extent to which the beliefs, "multiplication always makes bigger" and "division always makes smaller," were held by 136 prospect teachers enrolled in the mathematics content or methods course. The results indicated that a substantial percentage of the pre-service teachers held misconceptions about multiplication and division. Fifty-two percent of pre-service teachers believed that "in division problems, the quotient must be less than the dividend." Although the finding attracted attention to the teacher education program, it is limited because it didn't focus on teachers beliefs about teaching per se.

In Taiwan, the interest in the study of teachers' beliefs, especially on mathematics and its teaching, has just begun. Cooney has been invited to Taiwan to deliver lectures on the topic (TPIESTIE, 1990). So far systematic research on the subject in Taiwan is found only in Lin's work (TPIESTIE, 1990). The findings of his teacher belief interviews were that: 1) mathematics is very difficult to learn; 2) repeated drill is essential in learning and speedy calculation is a desired goal; 3) a pedagogical "recipe" to guide each step of teaching should be provided; 4) reward and punishment can improve learning; and 5) repeated explanation can help understanding (instead of a diagnosis of learning difficulty).

The results revealed that teachers' beliefs about mathematics and its teaching are more or less traditionally orientated in Taiwan. The present study will extend the investigation of whether teachers' beliefs are congruent with the recent trend of curriculum reform. The premise of curricular innovation will be used as a criteria to assess teachers' beliefs. Hopefully, by doing so, this study can contribute to the current reform. Furthermore, the present research will combine qualitative and quantitative methods to achieve both breadth and depth in understanding teachers' beliefs and to enhance the credibility of research findings.

The Relationship Between Beliefs and Behavior

Few studies directly or indirectly documente the relationship between teachers' beliefs and actions. Some find a consistent relationship, while others find an inconsistent relationship.

Consistency

In her educational beliefs study, Bauch (1982, 1984) found four types of teachers. In addition, teachers' instructional behavior generally reflected their types of educational beliefs. The "autocratic," "strategist," "laissez-faire," and "democrat" can be respectively characterized as being control-oriented, management-oriented, neutrally-oriented, and participation-oriented respectively in their instructional practices.

Earlier research by Harvey, White, Prather, Alter, and Hoffmeister (1966), which investigated how teachers representing different belief systems (Systems 1, 2, 3, 4) influence their teaching approaches and the classroom atmospheres in the preschool setting, also lends support to the notion of consistency between beliefs and behavior.

Studies on teachers' beliefs about curriculum also show that these beliefs affect teaching practices. For example, Bussis et al., (1976) provided evidence that differences in beliefs resulted in variations in surface curricular activities. The phenomenon of "domestication" reported by

Olson (1981, 1982) testified that instructional behaviors reflected personally-held beliefs. In addition, Schmidt and Buchman (1983) found a consistent relationship between beliefs about the emphasis of school subjects and the allocation of instructional time given to these subjects.

The consistent relationship is also found in the studies of teachers' beliefs about mathematics and its teaching. For example, in his three case studies of teachers' conceptions, Thompson (1982, 1984) concluded that "teachers' beliefs, views and preferences about mathematics and its teaching played a significant, albeit subtle, role in shaping their instructional behavior." Consistent with the findings of Thompson, the study of geometry teachers' conceptual systems by Mcgalliard (1983) and the study of high school mathematics teachers' instructional behavior by Kesler (1985) also found that teachers' conceptions of teaching are related to their own teaching behavior. In his study of four pre-service teachers' conceptual frameworks, Shirk (1973) found that these teachers' classroom behavior provided evidence that the conceptual frameworks were "activated" in teaching situations. In their study of teachers' pedagogical content beliefs, Peterson, Fennema, Carpenter and Loef (1989) identified two groups of teachers. Teachers with a more cognitively based perspective (CB teacher) reported in interviews that they made extensive use of word problems in teaching and paid closer attention to children's developmental levels in teaching compared to teachers with a less cognitively based perspective (LCB teachers).

In addition, Kuhs (1980) found that elementary school teachers' conceptions of mathematics content affected the selection of content for classroom instruction. Shroyer (1981) found that when teachers were confronted with "critical moments" in teaching mathematics, the way they handled the situation reflected their beliefs on teaching.

Inconsistency

It seems plausible to assume that teachers' beliefs significantly influence the way they teach in the classroom based on the above research. But classroom life is complex, some studies document a discrepency between teachers' beliefs and classroom practices. In a case study of a beginning mathematics teacher's belief about problem solving, Cooney (1983, 1985) found a chasm between a teacher's beliefs espoused prior to teaching and his actual teaching performance. The teacher's professed idealism was that problem-solving was the focal point of mathematical instruction, but classroom reality frustrated him in such ways that his students were not receptive to his problem-solving teaching strategies and the demands of teaching impeded his ability to create an episode of "real problems." Indeed, the use of a problem-solving approach demands not only extensive preparation, but also the development of ways to maintain classroom control as Cooney contended. Moreover, teachers might not feel confident and competent in teaching problem solving as reported by Thompson (1988). Cooney's study implies that we have to take classroom reality into consideration when we are trying to examine the relationship between teachers' beliefs and practices.

Bawden, Buike, and Duffy's (1979) study of teachers' reading conceptions fully demonstrated that other aspects of teaching do mediate a teacher's teaching behavior. These non-reading conceptions include classroom management and routine, mutual teacher-pupil respect, the amount of assistance needed by low or high ability pupils, etc. 15 out of 23 teachers studied possessed such non-reading conceptions, which modified decision making during the teaching of reading. Moreover, 7 out of 15 teachers who held non-reading conceptions seemed to be governed by these conceptions more than by the reading conception. This led Bawden, Buike, and Duffy to conclude that "a teacher's conception of reading is a free-floating element which has little meaning until it is filtered through the teacher's non-reading conceptions and applied to a specific teaching context." Furthermore, Duffy (1981) reviewed of four types of studies: teacher planning, teacher decision-making, classroom reading practices, and teachers' conceptions of reading, and the results supported the previous contention that there is a hiatus between the abstract theory and the reality of practice. Teachers may possess conceptions of reading, but these conceptions do not significantly affect their teaching because other aspects of teaching demand the teacher's immediate attention. In short, Duffy argued that teachers' beliefs have only a minimal effect upon teaching practices.

McNeil (1986, 1988) also observed discrepancies between teachers' personal beliefs as expressed in interviews and their classroom practices. In an extensive ethnographic study of four high schools, teachers articulated goals for active learning, inquiry and discussion for their students, but these goals were neglected so as to live up to the expectations of administrators whose priority was either the students' orderly progression towards their diplomas by way of good test scores or maintaining the students' discipline. As a result, teachers exhibited "defensive teaching strategies" in both presenting course contents and employing teaching methods. These strategies include fragmentation, mystification, omission and defensive simplification. Instead of allowing students to be actively involved in learning process, teachers lectured and reduced their presentations to lists of terms and unelaborated facts. By doing so, the course contents were easily transmitted, answered and graded; the behavior disorder was reduced. McNeil's findings of beliefs conflict between classroom teachers and administrators was also reported by Ignatovick, Cusick, and Ray (1979) in their study of teachers' beliefs.

The above studies strongly suggest that teaching practices are subservient to the classroom reality to a large degree. As noted by Jackson (1968), classroom life is complex. Some sociological research on teachers work also supports the notion that teachers are often constrained by their work situations. Sometimes these situational factors take precedence even over other educational concerns. These situational constraints include such as class size (Jackson, 1968; Metz, 1978; Sarason, 1982), parent expectations (Lortie, 1975; Gracey, 1972; Metz, 1978), student characteristics and levels (Sarason, 1982), outside pressures and testing systems (Porter, 1989), and management problems (Kounin, 1977).

The consistency-inconsistency argument has its origin in the field of psychology. Some attitudinal researchers had made attempts to offer conceptual frameworks in order to better account for the relationship between beliefs (attitude) and behaviors, which I found very useful in organizing the present study. Although they are different to some degree, they share the common premise that beliefs and situational factors interact to shape behaviors.

For example, Rokeach and Kliejunas (1972) proposed the formula that "behavior-with-respect-to-an-object-within-a-situation (Bos) is always a function of at least two interacting attitudes: attitude-toward-object (Ao) and attitude-toward-situation (As):"

Bos = f (Ao As)

Furthermore, whenever a person encounters an object within a situation, two attitudes, Ao and As, are activated, and the person will compare the two attitudes for their relative importance with respect to one another. Thus, AoAs = (w)Ao + (1-w)As, where w and 1-w refer to the perceived importance of Ao and As with respect to one another. Take students' "cutting class" behavior (Bos) for instance, it can be best predicted by the attitude the student holds toward the particular professor (Ao), the attitude the student holds toward the situation (As) the classroom conditions, the classmates, the general activity of attending class, etc., and the perceived relative importance of these two attitudes. If a subject rated in a 9-point scale Ao as 3, As as 7, and the perceived importance of these two attitude as 20% and 80% respectively, the weighted value of AoAs is:

$$6.2 = ((.20)3 + (.80)7)$$

The higher scores represent more favorable feeling toward object and situation. Rokeach assumes that this score should turn out to be the best predictors of behavior.

Fishbein and Ajzen (1975) also incorporate situational factors into their conceptual framework to account for behavior. According to them, intentions are viewed as the immediate antecedents over behavior, and beliefs are the basic building blocks. The two major determinants of intentions are attitudes toward the behavior (Ab) and subjective norms (Sn). Attitude towards the behavior (Ab) is a function of beliefs about the behavior's consequences and evaluations of those consequences. The subjective norm (Sn) is a function of normative beliefs and motivation to comply.

In order to understand the formation of Ab and Sn, one must examine the effects of stimulus on them. These stimulus conditions include situational variations, time, the characteristics of the target, etc. In other words, variables external to this model can influence behavior indirectly by affecting the determinants of behavior intentions.

Both the two models presented above suggest that beliefs and situational variables together account for behavior. In their ethnographic study of teachers' work, Grant and Sleeter's (1985) conclude that teachers' work is determined as much by their conceptions as by factors in their work place. In fact, some studies on teachers' beliefs have based on the conceptual framework that teachers' beliefs are continually modified by contextual variables in teaching (Janesick, 1978; Nespor, 1984; Elbaz, 1981). Parallel to this is the conclusion drawn from Shavelson and Stern (1981), etc.'s reviews of research on teachers' thought processes. The reviews summarized that teaching involved making ongoing decisions in solving instructional problems in the teaching context (Shavelson and Stern, 1981; Clark and Peterson, 1986).

It is true that the dualistic assumption of personally-held beliefs which assumes that a person labeled as being in one pole does not necessarily disagree the views of the other pole (Kerlinger, 1967), together with the existence of situational factors do make the relationship between beliefs and actions complex. Furthermore, teachers' knowledge may plays an important role in shaping teaching practices as claimed by Carpenter (1988). Teacher's knowledge includes content, curricular, and pedagogical knowledge according to Shulman (1986). Recall Cooney and Tompson's studies, where the subjects felt incompetent and unknowledgeable in teaching problem solving. The present study is therefore open to many potential factors in exploring the relationship between teachers' beliefs and instructional practices.

Recent Trends of Mathematics Curriculum Reform in Taiwan

As Romberg (1988a, 1988b, 1988c) indicated, the continued innovation in information technology accelerated the need for change in school mathematics, and so the government of Taiwan also recognized that need and set about making changes in its mathematics curriculum in order to equip its students to meet the needs of society in the twenty-first century. According to the working draft of Curriculum Standards for Elementary School Mathematics written by the Taiwan Provincial Institute for Elementary School Teachers Inservice Education (TPIESTIE, 1991, 1992), the potential curriculum extends the essentials of present curriculum while reflecting the spirit of Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989). The final document will be available sometimes at the middle of this year.

This study investigates whether teachers' beliefs and instructional practices are congruent with the recent trend of curriculum reform. At this critical moment, the underlying assumptions of recent trend reform must be provided as a framework for organizing instructional observation and teacher interview for the study. It includes the premise of following three sources: the present curriculum of Taiwan, the potential curriculum of Taiwan, and NCTM's Curriculum and Evaluation Standards for School Mathematics.

The Present Curriculum of Taiwan

Actually, the present mathematics curriculum in Taiwan is already far removed from the previous one. As discussed in the first chapter, this is the first time that manipulative materials and the discovery learning method are being introduced into Taiwanese schools. This puts more emphasis on conceptual understanding and reasoning than ever before. Generally speaking, the present mathematics curriculum differs from the previous curriculum in prescribing a pedagogy as follows:

Learning by Discovery. The present curriculum prescribes a "learning by discovery" pedagogy. For example, according to the mathematics section of Curriculum Standards for Elementary School (Ministry of Education, 1989), "instead of immediate demonstration or instruction, teacher should greatly give children opportunities for thinking, trying, discussing, hypothesizing, proving, discovering, and presenting along with cultivating children's independent problem-solving abilities." It is also clearly stated in the Teacher Handbook of Elementary School Mathematics In-service Training (TPIESTIE, 1978) that teachers should greatly use the "learning by discovery" pedagogy to cultivate independent problem-solving abilities.

Connecting Concrete with Abstract Thinking. Another major component paralleling the discovery method is the use of manipulatives and semi-concrete materials in learning process. For example, "instruction should begin with concrete, and/or semi-concrete levels and lead to abstract thinking" and "teachers should extensively adopt manipulative learning activities and fully apply concrete materials, audio-visual aids and social resources so that children can draw their own conclusions from their observations and actions" (Ministry of Education, 1989).

Individualized Instruction. Current curriculum prescribes individualized instruction. For example, "teachers ... should make reference to children's differences in abilities and learning experiences so as to design reasonable and effective learning activities," and "teachers should often apply diagnostic techniques to uncover differences among children and reasons for learning difficulties and low performance in order to give remedy. In the Teacher Handbook of Elementary School Mathematics In-service Training (TPIESTIC, 1978), Underhill suggested a model to accomplish individualized instruction which indicates the importance of diagnosing individual differences and adopting corresponding instructional and remedial activities in mathematics teaching. According to this model, children take a pre-learning test and participate in activities of different purposes according to individual test results. Following whole-class, group, or individualized instruction, a diagnostic test has to be administered to measure individual learning outcomes and difficulties. By the same token, children should participate in different instructional activities for practice, enrichment, or re-instruction accordingly, before moving on to the next topic.

From the description above, we can realize that cultivating reasoning and problem-solving skills becomes the emphatic goal and direction of effort in the present curriculum. It is clearly indicated in the mathematics session of Curriculum Standards for Elementary School that teaching is not confined to textbooks and classroom activities. Any activity involving numbers, quantity and shape could be included in lessons. Further examples might include "teachers should give children more opportunities to participate in designing and choosing learning activities ... to design and try out different solutions in order to choose appropriate and effective method to solve problems;" or "teachers should often encourage children to ask questions ... or use clues or provoking-questions to inspire children's reasoning, thinking and mental activity;" or "paper and pencil work should inspire students' thinking and work time should be short." Furthermore, "teachers should focus not only on answers but also on the reasoning process in assessing students' learning outcomes."

The Potential Curriculum of Taiwan

It is stated in the preface of working draft of Curriculum Standards for Elementary School Mathematics that teachers don't understand the process of learning and they deliver algorithm and rote rules by traditional transmission. Students learn without understanding and reasoning and spend lots of time on computational skill. As a result they lose their reasoning ability and interest in mathematics (TPIESTIE, 1991). In order to resolve the above problems and consider the three following needs, the government and TPIESTIE conceived the change of curriculum.

Reflecting Social Need. A democratic society requires communication and coordination. One can foster children's communication and coordination abilities through mathematical learning activities. Further, the modern techniques progress rapidly, and computers and calculators decrease the need for paper and pencil calculation. Finally, under the industrial revolution, human beings often confront non-routine problems. A problem-solving orientation of learning helps children to face problems.

Consolidating the Learner-Centered Approach. First of all, it is only when students autonomously participate in the learning activities that learning will occur. All curriculum should have the children at the main consideration. Secondly, meaningful learning must put children in an rich context, then connect their intuitive knowing to formal mathematics. Finally, any activities should individual differences into consideration.

Emphasizing Problem Solving. Mathematics is regarded as problem solving under the modern trend. Children must often confront non-routine problems in order to foster their reasoning ability.

NCTM Curriculum Standards

According to the Curriculum and Evaluation Standards for School Mathematics prepared by National Council of Teachers of Mathematics (NCTM), mathematics are regarded as follows:

Mathematics as Problem Solving. Standard 1 states that problem solving should be a primary goal of all mathematics instruction and an integral part of all mathematical activity. In other words, instruction should be based on problem situations in everyday experience instead of teaching a distinct topic as problem solving. In this problem solving approach to instruction, the classroom teacher should encourage thought-provoking questions, conjecture, investigations, discussion and discovery.

Mathematics as Communication. Communication enables children to clarify their thinking when they construct links between their informal notions and formal, symbolic and abstract mathematics; therefore, representing, talking, listening, writing, and reading are essential to instruction. In this case, the use of concrete physical manipulatives is indispensable because they offer the basis for conveying an idea.

Mathematics as Reasoning. Instruction should help children make sense of mathematics. They should be encouraged to think and conjecture in many ways and justify their solutions as opposed to being forced to do meaningless memorization. In a words, the process of solving a problem is as important as its answer.

<u>Mathematical Connections</u>. Mathematical connections refer to 1) connecting ideas both within and among areas of mathematics; 2) connecting procedural and conceptual knowledge; 3) connecting to everyday experiences. Again, the concrete materials play an important role in making the connections.

The core ideas of the NCTM Standards specify that instruction should be based on solving problem situations (Romberg, 1988a) through conjecture, representing (either through concrete manipulatives or by drawing diagrams and table), investigating, communicating, and finally verifying. Problem solving becomes an approach, not a topic to be taught.

Obviously, the present curriculum of Taiwan, the potential curriculum of Taiwan, and the NCTM Standards have their roots in Constructivist theory. According to Piaget's genetic epistemology (1970), knowledge is actively constructed, especially in the logico-mathematical realm; "to understand is to invent" (1973a). He also postulated that "we should emphasize the role of actions in mathematical education, particularly with young children: activity with objects is indispensable to the comprehension of arithmetic" (Piaget, 1973b).

This notion is supported by recent research findings that children actively construct meaning based on prior knowledge. For instance, without having been taught, young children can use special shortcuts or have their own inventions in solving problems (Groen & Resnick, 1977; Carpenter & Moser, 1982; Madell, 1985; Baroody, 1986, 1987; Ginsburg, 1989; Kamii, 1985, 1989).

The learner-centered approach, connection building approach, and problem-solving & reasoning approach are the three common focuses of current trend of curricular innovation. The research instruments and data analysis of the present study are based on these three emphases.

CHAPTER III

METHODOLOGY

The primary intent of this study was to investigate whether teachers' beliefs about teaching and learning mathematics in Taiwan are congruent with curricular innovation. The secondary interest was to take a look at instructional practices and further examine the relationship between beliefs and instructional behaviors. For these purposes, classroom observations, (which include one unfocused, qualitative-oriented observation and one focused, quantitative-oriented observation) were employed to collect data about teachers' instructional practices and also to serve as a complementary method of inferring teachers' beliefs by offering a referential context for understanding. Post-observational teacher interviews were conducted to elicit teachers' expressed beliefs about the teaching and learning of mathematics. In addition, a simple questionnaire was administered to collect more data about teachers' beliefs. Data gathered from these sources were cross-referenced in order to explore the relationship between teachers' beliefs and instructional behaviors. A pilot study was conducted to test the feasibility of the research design prior to the study.

Cooney (1990) suggested a "humanistic orientation" with which to study teachers' beliefs. Basically, he assumes that knowledge and meaning are constructed by the individual through interaction with his or her environment. Therefore, they are idiosyncratic and unique to individual. To understand teachers' beliefs, one must adopt the processes that promote intimate communication between the researcher and the informant. This study emulates his method of studying teachers' beliefs.

Generally speaking, the present study combines qualitative and quantitative methods in the research design. In order to make generalization possible within the area I investigated and at the same time fulfill deep understanding of beliefs, it seems plausible and reasonable to triangulate the research methods. There has been a tendency recently not to view the quantitative and qualitative research methods as dichotomy, but rather as being complementary to one another (Denzin, 1978; Madey, 1982; Patton, 1990). As Rossman and Wilson (1985) noted, "numbers and words can be used together in a variety of ways to produce richer and more insightful analyses of complex phenomena than can be achieved by either one alone."

On the other hand, although incorporating multiple methods can increase the quality and credibility of research, it does result in the complexity of data analysis. The complexity comes from not only the difficulties of putting data of different nature together (e.g. quantitative and qualitative), but also from the possibility of inconsistency and contradictory results among data (Mathison, 1988).

Sources of Data

There are 2,487 elementary schools with 56,120 classes and 82,583 teachers in Taiwan (Ministry of Taiwan, 1991). It seems overwhelming to investigate teachers' beliefs and instructional behaviors with such huge numbers. Hence, this study investigated only one administrative area in a northern Taiwanese city. Due to the nature of the study, combining qualitative and quantitative approaches, a random sampling technique which accomplishes generalization was not applied in this study because the cooperation of schools' and teachers' is the prerequisite of qualitative field research. Under these circumstances, a "maximum variation sampling" technique was chosen in order to maximize both the representativeness and the depth of the study. In this way, at least, one can be sure that the variation among schools is represented in the study.

There are 16 public schools (including 1 rural school), 3 private schools, and 1 laboratory school within this area. Twelve teachers from three of the public schools, four teachers from one of the private school, three teachers from the rural school, and three teachers from the laboratory school participated in this study, and so, the diverse characteristics of various types of schools were included in the samples. All told, 6 schools with 22 teachers (classes) took part in the field study. In order to protect the confidentiality of the participants, individual teachers' biographies are not presented. Table 1 presents the number of teachers according to their sex, age, and school types.

Table 1 The Sample of Study

Sex		Age			School Type			
Male	Female	21-30	31-40	41-50	Public	Private	Rural	Lab.
6	16	11	5	6	12	4	3	3

Collection of Data

Data collection was the central part of study. Basically, this research combined both the qualitative and quantitative approaches to collect data. It includes instructional observation, teacher interview, and a beliefs questionnaire.

Instructional Observation

In order to reduce the possibility of teachers acting out what they expressed in the previously conducted beliefs interview, classroom observation was arranged prior to the interview. This arrangement enabled me to make a more precise inference about teachers' beliefs and, simultaneously, to provide a more accurate picture of teachers' instructional practices in Taiwan.

Each teacher (classroom) was observed for 2 sessions of mathematics lessons. Triangulation is commonly accepted as a strategy for increasing the validity of evaluation and research findings; therefore, the first observation session was a focused observation using a pre-categorized observation checklist to code the presence or absence of teacher's and students' behaviors, whereas the second observation was unfocused and observational notes were taken to record as much as possible about what was transpiring during the class period so as to get a general sense of the setting and the teacher. Data coming from these two sources were brought together in order to get a holistic view of instructional practice. In more technical terms, the descriptive, qualitative-orientation of the second data source was the supplementary explication of the first source, which was of more or less quantitative-orientation.

The development of the classroom observational checklist (see appendix A, B) primarily made reference to The Behavior Checklist of Child-Environment Interaction developed by Day, Perkins and Weinthaler (1982) and a sourcebook of observational instrument --- Evaluating Classroom Instruction --- edited by Borich and Madden (1977). It also followed some suggestions about designing observational instrument in the articles or books (e.g. Herbert & Attridge, 1975; Siedentop, 1991; Borg & Gall, 1983). The important components of the current trend of curricular innovation constituted the observational variables. An interval recording method was adopted in the observation. The term refers to "observing behavior for short time periods (intervals) and deciding what behavior best characterizes that time period" (Siedentop, 1991). The present study employed 15 second intervals, or more specifically, 15 seconds was used for observation followed by 15 seconds of recording. In addition, a fixed schedule of time sampling was followed during the observation; that is, I observed the first 10 minutes, the middle 10 minutes, and last 10 minutes with 5 minutes breaks between them. The result is a total of 60 observational intervals in a mathematics lesson.

Teacher Interview

Following the observations, an interview was conducted to elicit teachers' beliefs and to explore the relationship between beliefs and behaviors. The "general interview guide approach" as described by Patton (1990), which keeps the interactions focused but allows flexibility in the wording and sequencing of questions to specific respondents in the context of the actual interview was adopted in the research. The interview guide (see Appendix C) served only as a basic checklist in the course of the interview to make sure that all pertinent issues were covered. Besides, some interview techniques suggested by seidman (1991) were applied in conducting teacher interview. Generally, the in-depth interview included the following categories of questions.

Teachers were asked to describe a "typical lesson" or "the routine activity of a lesson." This allowed teachers to begin on familiar ground and acquainted me with their general instructional approach and underlying rationale. Following this general information, the teacher was asked to offer a concrete example of how he or she taught a new topic (e.g. multiplication, division). This provided a referential context for understanding the teachers' expressed beliefs.

Next, an informal stimulated-recall technique was applied if it was necessary. By informal stimulated recall, I mean that some specific events in the observed lesson are mentioned for the purpose of helping teacher recall covert mental activities. The intention here was to test my inferred beliefs which were gained through the lesson observations. For example, "This morning, you arranged the students into groups and gave them some manipulatives, can you tell me what your thinking was there?" I also utilized this method to explore the relationship between beliefs and behaviors. The inquiry method usually is accompanied by the use and replay of audio-visual aids in order to stimulate thinking. In consideration of the specific cultural background of Taiwan, I did not use these facilities. I tried to arrange the proposed interview session to be as close as possible to the observed lessons so as to keep fresh memory while allowing me enough time to prepare inquiry questions for the interview.

Finally, some questions were posed as a way to extend the questionnaire in order to achieve a deeper level of understanding, or to fill in information missed during the observations and on-going interview. These questions were organized around the important components of the present mathematics curricular innovation; for example, "What do you think the teacher's role should be in teaching mathematics?" According to the three focuses of innovation, teachers should play a "low influence" role as opposed to the traditional authoritarian "high influence" role. In this way, I could infer his perceived role concerning his beliefs about the "learner-centered approach." Another example is like, "In your opinion, what is the best way for students to learn mathematics?", by which I could infer whether he/she perceived that reasoning and problem solving as important by the answer.

A probe into the relationship between beliefs and behaviors was also included. An example of this sort of questioning might be, "What are your difficulties in putting your beliefs about teaching into practice?"

Beliefs Questionnaire

Each teacher was asked to fill out a short beliefs questionnaire at the end of the interview (see Appendix D). The questionnaire was constructed with 10 question/ statements which reflected the underlying assumptions of the recent trend of curriculum reform. Teachers responded on a 4-point Likert scale ranging from strongly agree to strongly disagree. The results from two sources -- the teacher interview and the short questionnaire -- were brought together so as to maximize the understanding of teachers' beliefs.

Analysis of Data

Data analysis was an ongoing process throughout the investigation in order to make inferences about teachers' beliefs and to act as a "double-check" base in the post-observation interview. In other words, the observational data were used to generate relevant questions for inquiry in the interview session. As is inherent in a qualitative study, analytic insights and interpretations often emerged during the data collection stage.

Analysis of Instructional Behavior

The two classroom observations constituted the basic data for the analysis of instructional behavior. The data generated from the checklist observation computed the percentage of intervals at which each behavior occurred. For example, within the nature of instructional activity, what percentage of intervals (time) was engaged in whole-class direct instruction or in whole-class group activity? In this way, the whole picture of instructional practice could be portrayed.

On the other hand, the vivid, concrete, and descriptive information of field notes complemented the statistical skeletons. For example, the instructional episodes (e.g. teaching division) taken from the field notes provided for a better understanding of the common pattern of instruction. Together, these two types data provided a full view of classroom practices. For the sake of safety and credibility, the first observation was videotaped and the second observation was audiotaped.

In addition, a behavior score for each teacher was rated on a 4-point scale by the field observer and a side observer and mean behavior scores were given for each of the three curriculum focuses. The field observer is the person who actually went to classrooms and conducted two observations. The side observer was the person who checked the credibility of the classroom observational checklist through reviewing the video tapes of the first observation. He was also the reader of field notes. In addition to the checklist's statistic and the field notes, both the field observer and the side observer reviewed the tapes and rated the behavior score of each teacher. Considerable communication and discussion occurred between the two observers during the rating procedure.

Analysis of Teachers' Beliefs

For the analysis of teachers' beliefs, two types of data were brought together: the beliefs questionnaire and teacher interview. The questionnaire offered the percent distribution of teachers' views about underlying assumptions in ongoing trends of reform. The interviewer conducted the teacher interviews and obtained notes and audio tapes of the interviews. The teachers' responses to the beliefs interview were later transcribed by the interviewer and a coder. Content analysis was employed to generate patterns, themes, or categories of teachers' conceptions from the interview transcripts which supplemented the statistical data of beliefs questionnares. Both the questionnaire and interview protocols were read and rated on a 4-point scale for each teacher by the interviewer and rater and the mean beliefs scores on three curriculum focuses for all teachers were then computed.

Analysis of the Relationships Between Beliefs and Behavior

Both the classroom behavior and beliefs data were thoroughly examined to decide the strength (rating score) of the beliefs and behavior of each teacher and further to define the in-between relationship. The qualitative data then gave a factual description of the relationship. One of the issues explored in the interview -- what the difficulties were in putting his/her beliefs into practice -- was especially helpful in understanding the relationship between beliefs and behavior.

Procedures of Study

A pilot study was conducted prior to the investigation (June, 1991). It included 1) sending questionnaires to a school in an administrative area outside of the area studied but in the same city so that the wording of questionnaire might be in accord with teachers' language; 2) observing some classrooms in order to fix the observational checklist so that the categories of behavior might reflect the context of Taiwanese classrooms; 3) interviewing teachers to familiarize the interviewer with the interview technique and context and to fix the wording of interview guide.

A major change was made based on the results of the pilot study. In the pilot study, questionnaire was administered before the beliefs interview, but teachers discerned the orientation of the research from the wording in the questionnaire. They became conservative or expressed a very different view from the questionnaire. For example, when asked what is the best way for learning/teaching mathematics, they responded the use of manipulatives or understanding (the use of manipulatives is very emphasized in the teacher's manual), but this was not reflected in their description of a typical lesson and an instance of an example teaching. As a result, the beliefs interview was conducted before the questionnaire in the actual study.

In addition to making changes in the wording of research instruments and in the research precedures, building relationships with schools and teachers was a major occupation in the summer of 1991. The field research began in September of that year and it took three months to collect the data. Data management and analysis proceeded together with the data collection.

CHAPTER IV

RESULTS

This chapter presents an analysis of the data gathered in this study. Results are organized into three major sections. The first section reports teachers' beliefs about the teaching and learning of mathematics according to the questionnaires and interviews data. The second section documents teachers' instructional practices resulting from the observational data. The last section examines the relationship between teachers' beliefs and instructional behaviors.

Teachers' Beliefs: The Analysis of Questionnaires & Interviews Data

The learner-centered approach, the problem-solving & reasoning approach, and the connection building approach are the three focuses of recent trend of curriculum reform. The data presentation will be centered around these three themes.

Beliefs About the Learner-centered Approach

The working draft of Curriculum Standards for Elementary School Mathematics specifies that mathematics concepts and skills should be constructed by the children themselves rather than instilled by their teachers (TPIESTIE, 1991, 1992). The construction of new ideas occurs in an active way. The meaning that a new idea has is given to it by the learners they reflect on it and relate the new information to what is already known (VanDe Walle, 1990). Instruction, therefore, should reflect this constructive, active view of learning. Teachers need to create an environment that encourages children to explore, develop, test, discuss, and apply ideas (NCTM, 1989). Children should be both mentally and physically involved in the learning process.

This presentation begins by providing the statistical results of each question in the beliefs questionnaire and then supplements this with related information obtained from the beliefs interview. Three questions in the questionnaire are designed to assess teachers' beliefs about a "learner-centered approach." The first question is: "Children learn mathematics best by attending to the teachers' explanations and by more frequent drilling." Table 2 presents the degree of agreement with this belief.

Over eighty percent of teachers agreed that children learn mathematics best by attending to teachers' explanations and by more frequent drilling. The data strongly indicated that teachers tended to hold what Baroody called the "absorption theory" which views the learner as "a blank slate, a passive receptor of knowledge" (Baroody, 1987). In this view of learning, teaching is seen as imparting content and providing drills for students to stabilize the new skill.

Table 2 The Distribution of Teachers' Beliefs About the Learner-centered Approach (1)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	8	10	4	0
Percentage	36.36%	45.45%	18.18%	0%

Support for this perspective can be seen in answering the question "what is the best way to learn/teach mathematics?" or in describing "a typical lesson" during the interviews. Some examples that emerged from the data are as follows:

> Teacher #4: ... if some children they don't pay attention to the instruction, you should warn them, tell them: "I see you're not listening to what I said." Because if he misses out on some of the information in the lesson, he won't be able to understand what goes on later, and this causes him to lose interest later on. Therefore, you have to keep an eye out for student who can't concentrate in order to make sure that he doesn't miss out on any information.
>
> ... If they understand, the most important thing is to have them practice ... because practice will increase one's performance. Once they have achievement, it brings a willingness to learn ... then they will pay more attention to the lesson, because they seek teacher's praise.
>
> Teacher #18: ... In order to teach a new concept, you must use "lecture" method to explain it to students. After they understand, of course, they have to drill repeatedly. If they don't practice, then they will lose the computational ability.

By drilling more, one can reinforce what has been learned ... Actually, the real purpose of games or group tournaments is to repeatedly practice.

Teacher #5: ... After they attend to my instruction, the most important thing is to drill, frequently drill. Drill is very important... Children's most common problem is that they listen and then understand, but they don't know what to do when they practice later.

Teacher #7: (when being asked how she promotes understanding since she assumed that the best way to learn/teach mathematics is to "understand," she replied) By means of explaining. After explaining, I usually let them practice. I usually spend most of the time on explaining ...

... You have to demonstrate how to do this problem, how to find the relationship, because even among fifth and sixth grade students, some are still unable to find the relationship, so, the teacher has to do it for them.

... When I teach, I teach them a recitation rhyme (16 + 17, 6 + 7 = 13, write 3 regroup 1 ...), they repeat it after me, which I think makes it easier for them to remember.

Teacher #10: My thinking is that I tell my students the concepts and procedures first, then I let them try to do it. Some experts say, you have to let students discover by themselves, but I think some middle and low level students have difficulty doing this ... in order for them to discover, it takes a long time, therefore, I tell them first, then let them to do it.

Teacher #15: (In response to what is the best way to teach/learn mathematics.) Direct instruction ... to show and tell students the steps, then have them drill independently ...

Teacher #20: ... Right! like with division, teacher has to illustrate on the board, and explain the procedures first, then have the children practice ... tell them the method (referring the procedures of long division algorithm), then have them drill; drill becomes the vital part, very important.

If they don't use the method right after I teach it, they soon forget it. Repeated drill is very important in learning mathematics, as you see, the "mental calculation" ability is obtained by training, nothing else.

Although Some of the above-mentioned teachers and some of the other teachers did contend that using real life examples to explain is the best way of teaching and some teachers espoused the belief that employing concrete manipulatives is the most effective way to teach, the typical lessons they described were all teacher-centered, and in a show-and-tell style. This viewpoint will be illustrated in a discussion of our "building connection approach" later.

A few teachers did hold a different view from the most salient one. This is best represented by the following comment:

> Teacher #3: ... If you merely instruct them, it is very hard... You must have something to appeal to them, that is, to keep their hands, feet and minds continuously busy.

The second questionnaire item which is also designed to assess teachers' belief about the "learner-centered approach" is: "In teaching mathematics, the role of the teacher is to impart mathematics knowledge and correspondingly, the role of student is to attend to the instruction." The working draft of the new potential curriculum standards requires teachers to play a "low influence" role in the students' knowledge constructing process. The rate of agreement with this belief question is presented in Table 3.

Table 3 The Distribution of Teachers' Beliefs About the Learner-centered Approach (2)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	1	8	11	2
Percentage	4.55%	36.365	50.00%	9.09%

Over forty percent of the teachers agreed that the teachers' role is that of a knowledge dispenser and by contrast, the students' role is that of a passive knowledge receptor rather than a positive knowledge constructor. Approximately sixty percent of the teachers disagreed with this notion.

The inductive analysis of the interview data suggested that teachers' perceptions about their roles could be conceptualized along a continuum according to the extent to which teachers exert their authority during instruction. Hence, teachers' role varied from a "high influence" role at one end to a "low influence" role at the other. The high influence role is illustrated by the following teachers' responses:

> Teacher #18: When you are teaching, you are not only a teacher, but also a leader ... As "leader" you arrange all classroom activities, who has to go where, who has to do what. You are the introducer of the concept, but you can't say, my job is only introducing the concepts and whether students listen or not is up to them. You are the leader and students should be under your control ... Student should follow the teacher's directions.

> Teacher #5: When instructing, I hope my role is that of a director. In such manner, showing authority, so that students may attend to me.

Teacher #11: To be a main actor in order to attract student's attention ... I would like all students to listen carefully. In order for them not to be distracted, I ask them to have a clean desk and to give me their full attention.

Teacher #7: You have to play "Black Face" and "White Face," right? It is true, sometimes, you should be mean to them, you play black face if they don't pay attention to what you said ... Students should be a "good audience" (smile), right? They must listen, and pay close attention to what I say.

The low influence role located at the other end of the continuum might best be typified by the following responses:

Teacher #9: To guide them into a learning context ... Students are the main actors ... If they completely accept what teachers say, they have no chance to think, to solve the problem ... Because doing mathematics is to solve problems.

Teacher #3: Somewhat like a theatrical director, that is, to let them perform on the stage, then, I raise some (questions), to guide ... Because students are the main players in the learning process, you have to clearly know what they are thinking about, to let them speak out, to let them explain why they use a certain method ... Um ... He writes, he talks about the way he solves the problem, he manipulates concrete material, all of these things encompass his performance ...

Teacher #19: The ideal type is to help them discreetly, that is, to guide, then let them discover by themselves. It is better to have a group discussion ...

Some teachers conceived their roles as a guide and at the same time also as a dispenser, some examples are:

Teacher #22: Generally speaking, I guide them into the learning topic, then, I might have something specific to transmit to them.

Teacher #1: Teacher is a guide, and sometimes, an instructor. He might teach something.

The last question used to measure teachers' beliefs about "learner-centered approach" is: "teachers should teach students exact procedures for solving problems in order to avoid aimless groping." Table 4 presents the degree of agreement with this belief question.

Table 4 The Distribution of Teachers' Beliefs About the Learner-centered Approach (3)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	7	7	7	1
Percentage	31.82%	31.82%	31.82%	4.55%

Sixty-three percent of the teachers felt that teachers should teach students exact procedures for solving problems in order to avoid aimless groping; approximately one-third of the teachers disagreed with this. The data suggest that most of the teachers' conceptions deviates from the constructivist view that learners should be kept mentally and physically active by means of confronting problems, manipulating concrete materials, conjecturing, discussing, representing, and validating in the learning process.

Both the interviews data quoted above and the questionnaires data revealed the same information. This notion is further supported by the data from the typical lesson or by the example teaching teachers described. According to this data, the most prominent teaching style is explaining, illustrating, and demonstrating. This will be further discussed in the presentation of the result of the following approach.

Beliefs About the Building Connection Approach

Building connection between conceptual and procedural knowledge (or between informal understanding and formal mathematics) is a primary concern in the TPIESTIE's working draft. To make connections between conceptual and procedural knowledge refers to the idea that "the rules and processes of procedural knowledge have a conceptual basis or meaningful rationale and that the symbolism used represents the appropriate concepts" (VanDe Walle, 1990). Concrete or semi-concrete models therefore, are used as what Ginsburg called "intermediary schemata" for building the connections. As Ginsburg and Yamamoto (1986) put it, "genuine understanding must involve the creation of harmonious links among informal and formal procedures and concepts." Three questionnaire items are designed to assess teachers' beliefs about the "connections building approach."

The first belief question is: "Teachers should present new mathematical symbols immediately in teaching a new topic so that the students can have a clear idea of what they are about to learn." The constructivist view of learning indicates that instruction is embeded in children's intuitive knowing rather than directly presenting formal symbolism since children use their prior informal knowledge to interpret formal mathematics. Children must be able to make sense of their learning by relating the newly introduced symbolism to what they already know and are comfortable with. The statistical results of teachers' beliefs about embedding instruction in children's informal knowledge in teaching a new topic are presented in the following Table:

Table 5 The Distribution of Teachers' Beliefs About the Building Connection Approach (1)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	0	1	13	8
Percentage	0%	4.55%	59.09%	36.36%

All but one teacher held the belief that teachers shouldn't present new mathematical symbols immediately in teaching a new lesson. The data suggest that teachers in general hold considerably homogeneous view about embedding instruction in intuitive knowledge.

There are three ways to relate symbolism to children's informal knowledge in presenting a new topic: giving relevant life examples, employing informal procedures, and making use of concrete or semi-concrete models. The typical lessons and the example teaching of interview data reveal that all teachers give daily life examples and employ informal procedures in teaching a new topic. To begin the instruction of the concept of multiplication, teachers provide life-related examples and problems. Further, teachers acknowledged that the concept of multiplication may connect with children's already existing knowledge of addition --- "repeated addition." As one teacher commented: "To begin with, the symbol is very abstract, therefore, I must present in a more life-related way to pull them (students) over." The following episode best illustrates how teachers employ life relevant example and "repeated addition" to introduce the concept of multiplication:

> Teacher #3: ... "Each of us has two hands, so how many hands do five persons have?" I call on children to perform in front of the whole class. They all show their two hands. Children say 10 hands in total. I ask them how did you arrive at this answer, how did you think? Some children said they counted; some children replied, they use the method of 2 + 2 + 2 + 2 + 2, to add five times. Then I tell them we can use a more convenient method to save time.

As to making use of the concrete or semi-concrete models in teaching a new concept, only a few teachers adopt these models. This will be described in "the salient patterns of instruction" to follow.

The second question designed to measure teachers' beliefs about the "connections building approach" is: "The most effective way for students to learn concept and algorithm is to have them observe the teacher demonstrating by the use of manipulatives."

VanDe Walle put it well in saying that "First hand physical interaction with something is simply a better thinking tool than passively observing it." Manipulatives are indispensable connecting links in learning about the abstract formalism of mathematics for children. As Curriculum and Evaluation Standards for School Mathematics points out, "manipulatives and other physical models help children relate processes to their conceptual underpinnings and give them concrete objects to talk about in explaining and justifying their thinking" (NCTM, 1989). Hence, children should be provided more opportunities to actively manipulate concrete materials in constructing mathematical concepts and computational skills. Teachers' response to this belief are presented as follows:

Table 6 The Distribution of Teachers' Beliefs About the Building Connection Approach (2)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	1	7	11	3
Percentage	4.55%	31.82%	50.00%	13.64%

Approximately thirty-seven percent of the teachers agreed that observing the teacher demonstrating the use of manipulatives is the most effective way to learn concept and algorithm. The other sixty-three percent of teachers disagreed with this contention. The data implicitly suggest that teachers don't believe that students should be engaged in manipulative activities.

The typical lessons and example teaching teachers described in the interview transcript provided a clear picture of teachers' instructional styles and their application of concrete materials. Some patterns in the way teachers introduce concept or algorithm were identified:

<u>Illustrating on the Board</u>. This is the most common method of instruction. Teachers employed life relevant examples or further drew pictures on the chalk board while presenting the lesson contents. Teachers' verbal explanations are the main element of instruction. Students act as an audience as in a lecture. This is represented by the following teachers' descriptions:

> Teacher #2: ... Okay, there are six groups in our class, and each group has six students, how many students are there in our class? Then I approach the problem using addition, that is, "how many in a group, six, six plus six, one adds six times in total," from here I move on to multiplication.
>
> Teacher #6: (On being asked how will he teach multiplication since he was never taught this topic before, he replied) ... I will explain to them why two times two is equal four. Um, perhaps, I might illustrate by drawing a rectangle grid ...
>
> Teacher #5: When I taught multiplication, I drew picture on the board, that is, I have how many sets of things, and how many things in a set, and then to lead them to multiplication.
>
> Teacher #18: (On being asked how will he teach multiplication since he has never taught this topic before, he replied) I can think of two ways, the first one is to draw on the board. The second way might be to let them think: now the teacher distributes candies, one student gets two ...

Demonstrating with Semi-Abstract Tallies. Drawing semi-abstract tallies or marks on the board to demonstrate mathematics concepts or procedures is also a common pattern of teaching. Usually, teachers asks students to apply the same technique in solving problems. This is reflected in the following comments:

> Teacher #21: The "regrouping" concept is very hard to understand. Therefore, I demonstrate double-column addition and subtraction by drawing marks representing tens and ones in order to explain the trading process ... I also ask students to draw these tallies to help them understand the whole regrouping process.

> Teacher #7: ... like distributing twelve items, if four is a group, then you circle four things (referring to four marks drawn on the paper), this is the best way ... If I let the students manipulate blocks, it will cause classroom disorder. I think drawing circles is better than manipulating blocks.

> Teacher #14: ... Right! to draw circles, to draw circles around 10 marks, the leftover 1 mark is the answer of the first column ... At this moment I will tell them how to accomplish this without drawing marks and circles.

Demonstrating with Concrete Manipulatives. In addition to giving examples, drawing pictures, or making tallies on board, teachers sometimes further employ manipulatives in explaining mathematics concepts or procedures. Usually students observe the teacher's demonstrations without manipulating any tangible materials. The following exemplify the teachers' response:

> Teacher #8: Anyway, if "multiplication" is completely new to them (referring to students), you must show them concrete things ... for example, taking out six plastic fish and grouping them by two, then asking them how many fishes are in one pile and how many fish are in total ... What follows is that I tell them what 2

represents and what 3 represents in "two times three equals six".

> Teacher #10: Take division as an example, there is candy to be distributed to a certain people, you have to use division, you have to present real items or to draw a picture. For instance, if there are eight candies, then you show them eight candies. To distribute these to four people, then you divide them into four groups ... After concrete manipulation, you tell them the mathematical sentence is "eight divided by four equal two ...

<u>Teacher Demonstrating and Student Following</u>. This type of teaching is similar to the above-mentioned pattern of teaching. The difference lies in the fact that this category of teaching will provide the opportunities for students to manipulate physical materials. Some teachers call on a few students to work on manipulatives in front of whole class; still others teachers have whole class work on tangible materials. However, for the most part teachers demonstrate with manipulatives first, then have students follow the steps the teacher shows them. The following remarks might best typify:

> Teacher #11: After demonstrating, then I call on an individual student to manipulate materials in front of whole class ... we have lots of picture cards, for example, there are eight frogs in the pond, four frog are gone, how many are left? Then, I tell children to take away four frog card from the board.

> Teacher #13: ... The most important thing is to use concrete materials. I found if all of the students can manipulate concrete materials, they drill more accurately and rapidly (From observing the lesson she taught, students worked on tangible materials at the teacher's dictation).

> Teacher #9: ... They have the experience of distributing things, so suppose we are going to distribute something, right? If time allows, I will call on some children to manipulate tangible items in

> front of the whole class. If we don't have spare time, then I merely demonstrate with manipulatives throughout the whole lesson.

<u>Teacher Guiding and Student Acting</u>. This category of teaching presents lesson in a guided way as opposed to direct presenting or demonstrating. Concrete manipulatives are indispensable to both teachers and students. Teachers make extensive use of physical materials to develop a mathematical concept or procedure. The following interview protocols exemplify this guided teaching:

> Teacher #17: First, I ask them to solve some multiplication word problems. Then I begin to raise some real-life "division" problems, for example, in birthday party, you bring a box of candy to share with your classmates, how do you distribute these candies? I do my best to let them manipulate concrete materials such as plastic flower to solve the problems and discuss the methods they use.

It is obvious from the above description the belief that students should be provided opportunities to actively engage in manipulative activity is not too common among teachers. Most of the Teachers hold the beliefs that mathematics concepts and procedures should be presented by means of explanation, illustration and demonstration. For this sake, manipulatives are used more on the situation of the teachers' presentation than the circumstance of the students' exploration, construction of knowledge.

Even though some teachers allow students to use concrete models, students' manipulation is mostly at teacher's dictation. Teacher #9's accompanying remarks in answering questionnaire item best reflects this phenomenon:

> I generally agree (referring to the statement that "observing teacher demonstrating the use of manipulatives is the best way for students to learn concepts and algorithm"), that is, teachers have to demonstrate how to do it with the concrete objects first, to give student a model, then they follow ... Um, I might give them a logic first.

The third question to measure teachers' beliefs about the "connection building approach" is: "Teachers should let students work on concrete materials in the beginning of introducing a new concept or algorithm (e.g. single-digit multiplication or division); As to Approaching the complex algorithm (Multi-column multiplication or long division), teachers must rely on demonstrating each step on the board."

According to TPIESTIE's (1991, 1992) working draft, "the concrete manipulation and the symbolic manipulation should correspondingly appear and connect to each other in order that children may understand the meaning of abstract mathematical concepts and algorithm." In other words, mapping between the steps in written procedures and the performance with concrete materials is essential to children's understanding. Children should experience that writing procedures is simply a way to record their work with manipulatives (e.g. blocks). Teachers' responses to this belief question were as follows (Table 7):

Table 7 The Distribution of Teachers' Beliefs About the Building Connection Approach (3)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	13	9	0	0
Percentage	59.09%	40.91%	0%	0%

The Table indicates that all teachers believe that they should rely on demonstrating the steps on the board in introducing the complex concepts or algorithms; and that they should let students work on manipulatives in the beginning of introducing a new concept or simple algorithm. This belief is also reflected in their description of a typical lesson or an example teaching:

> Teacher #15: Division is taught in the third grade. To begin, one must use these concrete objects, to let students have the experience of concrete manipulation ... By fourth grade, they should have acquired enough concepts so that you don't need these concrete things (referring to the teaching of long division).

> Teacher #12: In the beginning of teaching division, I will let them distribute things ... take out some small objects, plastic flowers or other plastic materials, whatever ... to distribute into piles. Um, like this, to let them get this concept that division is distribution work. Then I lead them to do paper and pencil computation according to the contents of textbook.

"Manipulatives are of little use unless the bridge is made to the symbolic aspects of mathematics" (Lindquist, 1989). "Connecting concepts to symbols" seems not to be demonstrated in teaching mathematics. This is illustrated by the following fourth-grade teachers' remarks:

> Teacher #5: ... The other one is the multi-digit multiplication algorithm, the way of lining up of products might be easily mixed up. You must tell them that the product of the first column lines up here; the product of the tens column has a zero after it, you should put it in the second line; and the product of the hundreds column is put in the third line. Like this, following the order.
>
> Teacher #15: ... Division is approached from the left side of the dividend, you have to compare the left two numbers with the divisor. If it is not enough for distribution, then you go down to the third number ... then you put the fourth column down ... Teacher has to demonstrate each step on the chalk board and to have students watch carefully.
>
> Teacher #18: ... to compare and decide which column to start with, for example, if this side is 20 (referring to divisor) and this side is 10 (referring to dividend), you have to go down one column. I always tell them to cover the remaining column, for instance, "302 divided by 25, the number "2" is covered, then you record the quotient up, by doing so, you won't mix up. Proceed in a similar manner, going down column by column."

It is evident that students are taught by rule-example methods. The mechanical, step-by-step rules are mastered by rote rather than by building on conceptual understanding. Concrete or semi-concrete models are not employed either in teachers' demonstrations or students' manipulations.

Not using concrete or semi-concrete models is evident when teaching complex algorithm in fourth grade and also in second grade. For example, teacher #21 knew that the regrouping concept is very hard to understand for children, yet she still approached the concept by drawing semi-abstract tallies. As she commented:

> Many students get stuck here, so, I have to instruct them repeatedly, drawing tallies on the board many times, one time after another. Actually, even after I've already drawn it "N" times on the board, some students still can't get it. It's not until they repeatedly drill, that they understand it.

Teacher #8 is another example. The following protocol also demonstrates that concrete objects are not employed in teaching difficult concepts as regrouping:

> ... Put 1 above the second column; they often ask why they always carry 1, why not 2. Then I tell them because the sum is 10 more, like 15, you write 1 above, 25, then you write 2 above. In the beginning, some children, keep putting 1 or 2 above the second column regardless of whether or not they need to carry. After I teach two or three sessions, they understand.

The last question to evaluate beliefs about the "connection building approach" is: "Students discuss mathematical problems by groups is helpful in clarifying thinking and promoting understanding, therefore, it should be largely applied in the mathematical instruction." In addition to as problem solving, reasoning and connection, mathematics is also communication in NCTN Standards. Communication plays an important role in helping children construct links between their intuitive knowing and the abstract symbolism of mathematics (NCTM, 1989). Teachers' responses to this belief question is presented as Table 8:

Table 8 The Distribution of Teachers' Beliefs About the Building Connection Approach (4)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	4	12	6	0
Percentage	18.18%	54.55%	27.27%	0%

Over seventy percent of the teachers agreed that interactions between students may help in clarifying thinking and sharping understanding, and therefore, it should be largely applied in instruction. Less than thirty percent of the teachers disagreed with this.

Interaction between students is also prescribed in TPIESTIE's working draft of Standards. For instance, "A teacher may arrange students into cooperative learning groups in order that each child may fully have opportunities to discuss and present" (TPIESTIE, 1991, 1992). Another example from the proposed durriculum is: "A teacher should provide children enough time for observing, discussing, manipulating, thinking and presenting" (TPIESTIE, 1991, 1992).

The interviews data do not correspond with the questionnaires data. Drawing from the data of "best way for teaching/learning mathematics," there were no teachers who contended the importance of student interaction. There were no demonstrations of students' interchanges according to the typical lessons or examples teaching. Although there were not any signs of interaction, some classrooms were arranged into groups of seats during the observational segment. Teachers were asked to enunciate their thinking on grouping students. None of the teachers' reported reasons for grouping students involved promoting pupils interchange:

> Teacher #6: Generally speaking, there are three reasons. The first reason is, I can take care of everyone in a group when I go down from platform, because the seats have been put together. The second reason is for group competition of speed and achievement, to inspire group honor. Um, the last one is to have "student teachers" in the group to help the slow students (Usually, a class consisted of 40 - 60 students with teachers always appointing a few high-achivement students to help the slower students).

<u>Beliefs About the Problem-solving & Reasoning Approach</u>

A constructivist view of learning prescribes a "problem-solving & reasoning approach." The TPIESTIE's working draft of Standards reflects this contention; for example, "Teachers have to design problem-solving activities in order to have children experience the thinking process of non-routine problem" (TPIESTIE, 1991, 1992). Indeed, the concrete or semi-concrete materials are only effective under circumstances in which children are mentally active in constructing the underlying mathematical relationship. Activating children's mind is the most vital element of learning mathematics.

Three questions are designed to measure teachers' beliefs about this approach. The first is: "Problem-solving is an important topic, and should be incorporated in the textbook as a unit to be taught." According to the Curriculum and Evaluation Standards for School Mathematics, "Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned" (NCTM, 1989). This is what Schroeder and Lester (1989) calls "teaching via problem solving," which deviates from the most common view of "teaching for problem solving" or "teaching about problem solving." In this way, problem solving becomes the focus of the curriculum. Mathematics concepts and skills are better learned in a problem solving context so that children's inquiring minds and reasoning ability can be fostered. The rate of agreement with this belief question is presented in Table 9.

Table 9 The Distribution of Teachers' Beliefs About the Problem-solving & Reasoning Approach (1)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	13	5	4	0
Percentage	59.09%	22.73%	18.18%	0%

The majority of teachers expressed the feeling that problem-solving should become a lesson unit to be taught. Only about eighteen percent of the teachers disagreed with this viewpoint. Since, in answering the previous questionnaire item most of the teachers asserted that to teach exact procedures for solving problems avoided aimlessly groping, teachers' response that problem-solving should become a lesson unit to be taught can be reasonably inferred.

In expressing their views about how to apply the "problem solving approach" in teaching during the interviews, most of the teachers stated that they had no ideas about this approach. I rephrased it by saying that problem solving is the essential focus of mathematics and how would he or she apply it in teaching? Still about one-fourth of the teachers did not grasp the concept.

There appears to be four salient patterns in the application of a problem solving approach in teaching among the remaining three-fourths of the teachers:

<u>Pattern I</u>. The teachers who hold this pattern say that it is teacher's responsibility to teach the exact procedures of solving a problem or to teach the right path for problem solving. Teachers' comments reveal that they have no confidence in children's problem solving ability.

Teacher #11: I still don't understand the problem solving approach. Is it that students cannot solve a mathematics problem in practice, so how would the teacher do it?

Interviewer: How would you apply the problem solving approach in instruction?

Teacher #11: ... If a student couldn't do the problem, I would say: " Did you listen carefully during instruction? I already taught you this problem, why can't you do it?

Interviewer: If you didn't teach that problem before?

Teacher #11: If I didn't teach this problem before, I must to explain to the whole class ... If I didn't teach a problem before, I won't let them practice it. In the event of a problem I really didn't teach before, I have to illustrate it to them.

Interviewer: Explain to them how to do it?

Teacher #11: Right, how to do this problem ... The problem hasn't been taught, that is the teacher's responsibility, therefore, teacher must reinstruct the students.

- - - - - - - - - - - - - -

Teacher #18: (After the interviewer stated the meaning of the problem solving approach, the teacher said that he didn't know how to comment, he expressed as follows.) To talk about the reality, I will tell them the right procedures, let them follow my way because time constraints do not allowed to let them think ... I don't think all students can adapt to this style and students need some "foundation" ...

Pattern II. This pattern teachers regard the problem solving approach as when children encounter the problems and raise them to the class rather than having teachers teach mathematics contents in a problem-solving context; but teachers believe that they should encourage children to reason through the problem once the children raise the problem.

Teacher #1: If students confront a life problem related to the textbook, it should be presented ... So far, my students haven't asked me any daily-life problem which is related to textbook contents. Once the problem is presented, then ask them ways for solving problem ...

Teacher #14: For me, once children raise the problem, we may explore and conjecture together. I assume that problem solving means children may propose their own ways of solving a problem.

Pattern III. Teachers of this pattern hold the view that they must conceive a problem context and then "lead" children to the right path in order to solve the problem. They seem to "worry a little bit" about students' problem solving abilities.

> Teacher #8: The basic problem with the design must rely on teachers. Teacher has to lead children's thinking in the right direction, to give hints, and then to let them discuss among themselves and tell me the results. If they can't solve a problem by themselves, then I tell them how to ... To totally let children solve problems by themselves, it can't work, children have limited abilities.

Pattern IV. Teachers whose conceptions belong to this pattern expressed the belief that they must design a problem situation and then invite children to reason through the problem situation. This is the closest view to the idea behinds of the current curriculum reform. Very few teachers belong to this category.

> Teacher #17: I think that problem solving, the teacher has to present the problem, then I think I will allow the students to think out how to solve it, to try each method by groups. In the end, we discuss it together and evaluate, then the teacher synthesizes it and makes comments.

It is obvious from the descriptions above that most teachers' views on the problem solving approach are distant from the thrust of the ongoing curriculum reform. Some teachers even mentioned that this was the first time that they had ever heard of the problem solving approach. This limited view of problem solving corresponds with the typical lessons or the example teaching which teachers conducted.

As stated before, the prominent patterns of instructional styles were explaining, illustrating and demonstrating. Very few teachers demonstrated a kind of guided discovery approach in their reports of typical lessons or example teaching. Furthermore, these typical lessons and example teaching were made of almost the same invariant sequences of: 1) arousing interest or reviewing old material related to the topic; 2) instructing on the topic; 3) to providing seat work; 4) checking the seat work or reinstructing if the students needed more help. Teachers worked hard to make sure that all students learned from what he or she said.

In short, "teaching via problem solving" was not, for the most part, demonstrated in the teachers' reported lessons. As teacher #2 commented: "I have students drill repeatedly after instruction and if they make errors, I correct them. Therefore, I might teach the same problem many times, and explain it many times." This kind of teaching -- teaching the right procedures that later can be applied to computational or word problems -- only involves part of "teaching for problem solving" at the most.

The second question used to assess teachers' beliefs about the "problem-solving & reasoning approach" is: "The main objective of teaching mathematics is to equip students with speedy and accurate computational skills and relevant mathematics knowledge." Under the constructivist view, mathematics is full of "relationships." There is no means of passive absorption, hence, to free children to think, explore, and validate is the main goal of instruction. In examining the TPIESTIE's working draft, phrases such as "stimulating children's thinking" and "promoting deep-level thinking" saturate it. Table 10 presents the degree of agreement with this belief question.

Table 10 The Distribution of Teachers' Beliefs About the Problem-solving & Reasoning Approach (2)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	4	6	9	3
Percentage	18.18%	27.27%	40.91%	13.64

About half of the teachers disagreed with the notion that the main objective of teaching mathematics is to equip students with speedy and accurate computational skills and relevant mathematical knowledge. The data suggest that for almost half of the teachers reasoning is not the main goal of teaching mathematics; instead, teaching speedy and accurate computational skills and mathematics knowledge is the focus. The following quotations give a vivid description of this view:

Teacher #11: Learning mathematics requires speed in computation. If you calculate slowly, even though it may be accurate, it is too slow. Therefore, I always take five minutes to practice mental calculation in each lesson through the use of flash cards in order that children may answer as soon as they see the problem.

Teacher #20: The students in our school all understand, but they calculate very slowly. This is due to too little practice ... and you have to set a time limit, you give them more time in the beginning, then you reduce the time allowed.

This conception could also be reasonably drawn from the fact that sixty-three percent of the teachers agreed that they should teach exact procedures for solving problems so as to avoid aimless groping.

The four common goals of mathematical teaching which teachers enunciated in the interviews were 1) grade-level skills, 2) the application of what is learned in solving daily-life problems or fostering problem solving ability, 3) an interest in mathematics, and 4) real understanding. Fourteen out of twenty-two teachers (63.64%) included the application in daily-life problems or fostering problem solving ability in their statement of goals. This figure is a little higher than the statistical results of the questionnaire. The reasonable explanation is that the term "objective" makes for a distant target for which they may endeavor. Teachers recognize that they have to work toward this goal.

The last question designed to measure teachers' beliefs about "problem solving & reasoning approach" is:

"Mathematical problem solving is essentially the application of computational skills in order to get the right answer to word problems in a textbook or workbook."

The core of the problem solving approach is not only to provoke children's reasoning skills but also to embed instructional problems in daily-life experience. The process of solving a problem is more important than merely getting the right answer. Table 11 presents the degree of agreement with this belief question:

Table 11 The Distribution of Teachers' Beliefs About the Problem-solving & Reasoning Approach (3)

	Strongly Agree	Agree	Disagree	Strongly Disagree
Numbers of Teachers	1	6	11	4
Percentage	4.55%	27.27%	50.00%	18.18%

Approximately thirty-two percent of the teachers believed that mathematical problem solving means to apply computational skills in order to obtain the right answer to word problems in a textbook or workbook. Teachers who disagreed with this statement were further asked to express their feelings. The expressions contained two arguments: 1) comment on the application of computational skills; and 2) comment on the right answer of word problems in textbook or workbooks. Some teachers commented on both arguments.

Teacher #3: It is the application of problem solving ability (pointed to the words "computational skill") and it could have many ways in solving a problem.

Teacher #17: I think, the purpose is not only to get the right answer listed in the textbook or workbook, but also to become flexible enough to apply the acquired in real life.

Teacher #12: The right answer is not most important things, what is important is the thinking process, the reasons for solving the problem in a certain way.

Summary of Teachers' Beliefs

Table 12 presents the means of teachers' scores on the three curriculum focuses as measured by the beliefs questionnaire. All items except item 7 (belief about students interchanges) were worded so that agreement with the statement indicated less agreement with the themes of the ongoing trend of curriculum reform. All items except item 7 are given scores 1, 2, 3, and 4 according to whether they strongly agreed, agreed, disagreed or strongly disagreed respectively. Item 7 was given scores inversely.

Table 12 also presents the interviewer's and side rater's assessments of the interview protocols for each of the three curriculum focuses. Both the interviewer and the rater read the written protocols and scored each teacher on a 4-point scale for each of the three focuses. That is, they judged where the teacher's response fell on the continuum for each of three focuses. A mean score of each focus for all teachers was then calculated. A higher score indicated that the teachers' beliefs were closer to the themes of the ongoing trend of mathematics curriculum reform. The mid-point score was 2.50.

The teachers' overall mean score on beliefs was 2.12 which was lower than the mid-point score of 2.50. In summary, it suggests that teachers beliefs tended to be close to the extreme characterized as the traditional absorption view as opposed to the other extreme which is characterized as the constructivist trend.

Table 12 Means of Teachers' Scores on the Beliefs About Curriculum Focuses as Measured by the Beliefs Questionnaire and by Interviewer's and Rater's Ratings of Belief Interview

Curriculum Focuses	Beliefs Questionnaire	Beliefs Interview			Mean
		Interviewer	Rater	Mean	
Learner-centered Approach	2.18	1.77	1.86	1.82	**2.00**
Connection Building Approach	2.59	2.09	2.23	2.16	**2.38**
Problem-solving Approach	2.32	1.77	1.55	1.66	**1.99**
Mean	**2.36**	**1.88**	**1.88**	**1.88**	**2.12**

Teachers' Instructional Behavior: The Analysis of Observational Data

Data analysis was derived from two types of observation: the Classroom Observation Checklists and the field notes. The statistical results of the Classroom Observation Checklists present a profile of teaching practices, which will be further supplemented by the more vivid, descriptive information of the field notes data. As in the previous section on teachers' beliefs, this presentation will also be organized into the three themes permeated the working draft of Curriculum Standards for Elementary School Mathematics (TPIESTIE, 1991, 1992) which reflects the current trends of reform: the learner centered approach, connection building approach, and problem-solving & reasoning Approach.

Behavior Portraits of the Learner-centered Approach

Mathematics is full of relationships and the construction of mathematicaI relationships takes into consideration an active means rather than a passive means of absorption and accumulation. It is the students themselves who must be the central figures in the process of construction as opposed to the traditional phenomenon of teacher domination. The following three Tables -- The Distribution of Instructional Activities (Table 13), The Distribution of Teachers' Instructional Behavior (Table 14), and The Distribution of Students' Instructional Behavior (Table 15) -- together paint an overview pictures of classroom practices. We will go through each Table to examine the instructional behavior of both teachers and pupils.

Table 13 The Distribution of Instructional Activities

Instructional Activity	Number of Observation	Percent Distribution
Whole-Class Direct	669	50.68%
Activity	147	11.14%
Practice	263	19.92%
Feedback	189	14.32%
Transition	49	3.71%
Other	3	0.22%
Total Observations	1320	

More than fifty percent of the observed instructional segments were the "whole-class direct instruction." According to the operational definition (See Appendix B), whole class direct instruction is when the teacher presents and transmits academic information/textbook contents to whole class and usually students sit and listen to the teacher's lecture during instruction. From this definition, and statistical results, a picture of teacher-dominated classrooms and show-and-tell teaching approach emerges.

Together with the occurrence of practice (19.92%) and feedback segments (14.32%), the overall frequency is high to 84.92% of the instructional activities. The statistics figure portrays a traditional teaching and learning style --

the teacher imparts knowledge and then a drill is provided for consolidating the newly learned concepts or procedures. Obviously, the teacher is the main actor in the classroom, and it is far from a learner-centered approach. This finding are in accordance with the main results of the observational study of "the roles of students and teachers in 1989 first grade curricula" conducted by Ko (1990). This study showed that no matter what subject was taught, most teachers delivered lectures and made students recite.

The percent distribution of teachers' instructional behavior (including verbal behavior and material use behavior, see Table 14) further provides a strong evidence as to how teachers actually behave during instruction. The most frequently occurring teachers' verbal behavior was asking low cognitive questions. That is, the overall frequency at which teachers were observed to asks questions involving merely factual recall or mindless responding was 23.11% of the time. Together with the total percentage of more or less teacher-centered verbal behavior (45.01%) such as giving direction, imparting information, explanation, and asking recitation, the percentage was high at 68.12%. In contrast, there was little evidence of student-centered verbal behavior such as asking high cognitive questions, encouraging reasoning, and encouraging communication. The overall frequency of this behavior was only 10.30% of the time.

Table 14 The Distribution of Teachers' Instructional Behavior

Teacher's Behavior	Number of Observation	Percent Distribution
<u>Verbal Behavior</u>		
Giving Directions	140	10.61%
Imparting Information	199	15.08%
Explaination-Informal	73	5.53%
-Formal	75	5.68%
Asking Ques. - H. Cog.	63	4.77%
- L. Cog.	305	23.11%
Responding	62	4.70%
Asking Recitation	107	8.11%
Encouraging Reasoning	48	3.64%
Encouraging Disc./Commu	25	1.89%
Other Speech	101	7.65%
No Speech	122	9.24%
<u>Material Use Behavior</u>		
Chalk & Board	411	31.14%
Textbook	140	10.61%
Manipulatives	264	20.00%
Workbooks/Worksheets	32	2.42%
Other Materials	53	4.02%
No Material Use	420	31.82%
Total Observations	1320	

The findings correspond with the results of "Research on Teacher Effects in the Republic of China" conducted by Chen, Schaffer, Wu, Jaing and Hung (1981). This study also developed a classroom observation instrument with which to code teachers' behavior in forty sixth grade mathematics classrooms. One important finding was that the most common features of teachers' instructional behavior were that 45 percent of the segments were spent in delivering lectures and that the considerably high percentage of asking low-cognitive questions was found among the other categories of teaching behavior.

Among all the observed teachers' material use behavior, the percent distribution of "chalk & board" and "textbook" behavior is 41.75%. It appears that 41.75 percent of the time teachers were observed either explaining on the board or imparting knowledge. The "no material use" behavior (31.82%) could mean that teachers verbally instructed without the use of any material aids or that teachers watched students doing paper and pencil work without using any materials themselves. Hence, it is reasonably concluded that the possible percentage of time spent in directly transmitting mathematics contents was more than 41.75%. Nonetheless, it is still a picture of teacher-centered instructional style.

On the other hand, the percent distribution of students' behavior (see Table 15) offers a general view of how students actually behaved during the observed instruction. The "no speech" behavior is high at 52.20% of the total students' verbal behavior. This might suggest that students were quiet either while listening to teachers' or engaging in paper-and-pencil work for most of the instructional segments. The most uttered speech coded was supplying low cognitive answer (20.83%), other speech (10%), and recitation (9.24%) respectively. In contrast, the total percentage of answering high cognitive questions and discussion/communication accounted for only 6.75% of all verbal behavior.

Table 15 The Distribution of Students' Instructional Behavior

Student's Behavior	Number of Observation	Percent Distribution
Verbal Behavior		
Answering Ques.- H. Cog.	5	3.79%
- L. Cog.	275	20.83%
Recitation	122	9.24%
Asking Question	13	0.99%
Discussion/Communication	39	2.96%
Other Speech	132	10.00%
No Speech	689	52.20%
Material Use Behavior		
Chalk & Board	64	4.85%
Textbook	222	16.82%
Manipulatives	188	14.24%
Workbooks/Worksheets	86	6.52%
Other Materials	27	2.05%
No Material Use	733	55.53%
Total Observations	1320	

As to material use behavior, The "no material use" behavior is high at 55.53% of the total student's material use behavior. It is very probable that students either looked at the board or listened to the teacher during the observed instructional segments. Textbook (16.82%) was the most used materials. The overall frequency of textbook, workbooks/worksheets, and chalk & board use (students usually copied their procedures and answers of the problems drilled on the chalk board during feedback time) was 28.19%

during the observation. Using Manipulatives accounted for just 14.24% of the all observed segments.

All of these statistics provide a profile of classrooms which consist of leading knowledge dispensers and passive learners. Transmitting and Drilling were the prevailing instructional activities. The constructive, active learning approach was rarely exhibited in the classrooms. They tended to be content-oriented with teacher-dominated teaching.

It appears that the qualitative observation supports the statistical profiles of teaching. What transpires in the field notes is almost the same invariant sequences of instructional segments: reviewing old material related to the present topic (sometimes practicing mental calculation), presenting through illustrations and demonstrations, providing paper and pencil work, and lastly giving feedback on students' work. Sometimes, giving feedback on the previous night's homework assignment would be part of the opening sequence. Teachers' presentation were very textbook-defined. Usually, a problem would be put on the board or a problem in the textbook would be read aloud by the whole class. The teachers would then demonstrate procedures step by step for two or three problems. Generally speaking, teacher illustration and student listening or following (following the steps the teacher demonstrated such as in learning the use of protractor) were the main methods of instruction.

When students practiced newly learned procedures or skills, the teacher would always circulated around the room to provide individual help or to remind or impart repeatedly some important steps just taught. Teachers would reinstruct the children by illustrating on the board if they found a common error being made. What followed after paper and pencil work was the feedback or answer checking time. Generally, a few students would be called on to copy their procedures and answers on the board. Interestingly, during this period, the teachers would usually "reinstruct" or remind pupils as in the foregoing presentation. The following quotations of episodes best describe this senario:

> Teacher #9: First, write the total number of items in the first blank. Then, write the number of "the people to be distributed to" in the second blank, and lastly, put "the quantity each person gets" in the answer blank ...
>
> Teacher #20: Remember, the operations within the parentheses in a mathematical sentence must be calculated first ... Don't forget!
>
> Teacher #11: You must remember to proceed from the ones column, you can't do it from the tens column ... remember to line up the digits in the ones column and to line up the digits in the tens column!
>
> Teacher #16: ... The central point of the protractor must be placed on the vertex of the measured angle, then the side of the protractor must be placed over one side of the angle ... One more point to be remembered is ... You must remember ...

As revealed above, phrases like "remember!" or "don't forget!" seemed to extend the foregoing instruction. It makes the instructional segments of drill and feedback not much different from the whole-class instruction except that students are doing or checking the paper-and-pencil work in addition to listening. The teachers' behaviors make it appear as if they don't have much confidence in their students' independent work. More teaching scenarios will be supplemented in later discussions which more vividly describe teacher-centered classrooms. These teachers work very hard to make sure that all students have listened and absorbed. If students make errors, it is either that they they haven't absorbed the material or that they didn't follow the steps the teacher showed and therefore reinstruction is needed. Even at recess time, some teachers help individual students or correct their students' workbooks.

In summary, it appears from the field observation that repetitive instruction and practice constitute most scenarios of mathematics lessons and teachers exert as much influence as they can on students' learning. It is quite a distance from the learner-centered approach. The qualitative data pretty much reflect the statistical findings.

Behavior Portraits of the Building Connection Approach

In building the connections between conceptual and procedural knowledge, the concrete or semi-concrete models are considerably important bridging materials. These tools must be used not only by teachers but most importantly, by learners. They must be used not only in the introduction of a new concept or algorithm, but also in the process of teaching more complex concepts or algorithms.

Table 16 offers very detailed information about teachers' material use behavior. It is helpful in understanding each material use behavior in various instructional activities and the features of each activity. Take manipulative use behavior as an example, the occurrence of manipulative use behavior in whole-class direct instruction, activity, practice, feedback and transition were 70.45%, 18.56%, 2.65%, 5.30%, and 3.03% of the time respectively. 70.45% of manipulative use occured in whole-class direct instruction but at the same time the frequency of teachers' use of manipulatives accounted for 27.80% of all material use behavior in whole-class direct instruction. Manipulatives were mostly used in whole-class direct instruction. This data, together with the evidence in Table 19 (The Distribution of Teachers' Verbal Behavior by Instructional Activities) imply that concrete or semi-concrete materials were used for the purposes of demonstration and illustration (Table 19 shows little evidence of thought provoking behavior such as asking high cognitive questions, encouraging reasoning, and discussion in whole-class direct instruction).

Table 16 The Distribution of Teachers' Material Use Behavior by Instructional Activities

	Whole	Acti.	Pract.	Feedb.	Trans.	Other	Total
Chalk & Board	263	20	24	104	0	0	411
	63.99	4.87	5.84	25.30	0	0	100%
	39.31	13.61	9.13	55.03	0	0	
Textbook	73	3	28	31	5	0	140
	52.14	2.14	20.00	22.14	3.57	0	100%
	10.91	2.04	10.65	16.40	10.20	0	
Manipulatives	186	49	7	14	8	0	264
	70.45	18.56	2.65	5.30	3.03	0	100%
	27.80	33.33	2.66	7.41	16.33	0	
Workbooks/ Worksheets	3	0	22	7	0	0	32
	9.38	0	68.75	21.88	0	0	100%
	0.45	0	8.37	3.70	0	0	
Other Materials	21	2	27	2	1	0	53
	39.62	3.77	50.94	3.77	1.89	0	100%
	3.14	1.36	10.27	1.06	2.04	0	
No Material Use	123	73	155	31	35	3	420
	29.29	17.38	36.90	7.38	8.33	0.71	100%
	18.39	49.66	58.94	16.40	71.43	100.00	
Total	**669**	147	263	189	49	3	1320

As to students' material use behavior, Table 17 provides very detailed information about each material use behavior in various instructional activities and is helpful in understanding the features of each activity. Take manipulative use behavior as an example, tangible materials are mostly used by children in both whole-class instruction (38.83%) and activity (40.96). 15.96%, 1.60%, and 2.66% of manipulative use occured in practice, feedback, and transition respectively. The data imply the possibility that students' use of manipulatives was at the teacher's dictation since there was a high percentage of teacher-dominated verbal behavior such as giving directions, imparting information, and explaining in both activities as presented in the Table 19 (The Distribution of Teachers' Verbal Behavior by Instructional Activities). This point will be further demonstrated by looking into the 188 instructional segments of student's manipulative use behavior (Table 18) described later.

Table 17 The Distribution of Students' Material Use Behavior by Instructional Activities

	Whole	Acti.	Pract.	Feedb.	Trans.	Other	Total
Chalk & Board	22	0	29	12	1	0	64
	34.38	0	45.31	18.75	1.56	0	100%
	3.29	0	11.03	10.43	2.40		
Textbook	88	0	83	44	7	0	222
	39.64	0	37.39	19.82	3.15	0	100%
	13.15	0	31.56	23.28	14.29	0	
Manipulatives	73	77	30	3	5	0	188
	38.83	40.96	15.96	1.60	2.66	0	100%
	10.91	52.38	11.41	1.59	10.20	0	
Workbooks/ Worksheets	3	0	70	10	3	0	86
	3.49	0	81.40	11.63	3.49	0	100%
	0.45	0	26.62	5.29	6.12	0	
Other Materials	15	2	5	5	0	0	27
	55.56	7.41	18.52	18.52	0	0	100%
	2.24	1.36	1.90	2.65	0	0	
No Material Use	468	68	46	115	33	3	733
	63.85	9.28	6.28	15.69	4.27	4.5	100%
	69.96	46.26	17.49	60.85	67.35	100.0	
Total	**669**	147	263	189	49	3	1320

The coded 15.96% of manipulative use in practice activity was the use of physical materials for skill development (such as the protractor). No manipulative use in bridging concepts and algorithm (such as Base-Ten Blocks) was found. Actually, such bridging materials are also found not much in other instructional activities. For a better understanding all these statistical findings, the following discussion presents vivid instructional episodes taken from field notes.

Seven of the twenty two teachers were observed conducting "division" lessons. Out of these, three (third grade) teachers were teaching the beginning concepts of division and four (fourth grade) teachers were teaching long division. The inductive analysis of field notes and video-tape reveals that no teacher employed concrete or semi-concrete models while teaching long division. Mapping the steps between the written symbols and the manipulative actions was far outside students' learning experience. Carefully leading students through the mechanical steps of algorithm by demonstrating on the board was the main endeavor:

Teacher #5: (Writing $30\,/\overline{\,290\,}$ on the board and drawing a line under divisor 30 and a line under the digits 2 and 9 in the dividend separately) There are two digits here (divisor), so we look at two digits here (dividend).

Teacher #5: 30 and 29, which is bigger (writing 30 and 29 down separately)?

Class: 30

Teacher #5: (putting the symbol ">" between 30 and 29) 30 is bigger, it can't be divided. You can't beat him, you must seek help (erasing the line under 29 in the dividend and redrawing a line under 290).

Teacher #5: Okay! it becomes 290. Now, our quotient has to be written above this (pointing to 0 in the dividend and the position above 0, and putting a small mark on the position where he pointed).

Teacher #5: Which number will you pick to divide?

Class: (Silent)

Teacher #5: Watch this (drawing a circle around 29 of the dividend). Three (times) how much, is 29 (pointing to 3 of the divisor and 29 of the dividend)?

Class: Three nine twenty seven (3 x 9 = 27) (In Chinese, the word "times" is understood but not spoken in this situation)

Teacher #5: (Writing 9 at the position of the quotient) 9 (times) 0 ...
(waiting for class to supply product)

Class: 0

Teacher #5: (Writing 0 down) 9 (times) 3 ...

Class: 27

Teacher #5: (Writing 27 and drawing a line under 270) How much is the remainder?

Class: Zero, two (reading when teacher puts down the remainder from right column to left), twenty.

\- - - - - - - - - - - - - -

Teacher #6: Okay! Let's do one more problem (writing 30/$\overline{810}$ on the board and covering the digit 0 in 810 with magnet).

Teacher #6: (Drawing a dotted line between 81 and magnet) We cover it, should the quotient be put on the right side or left side of dotted line?

Class: Left side

Teacher #6: We'll try 2 (putting 2 in the quotient position), I already told you, this (pointing to the 2 just written) times that and that (pointing to the two digits -- 0 and 3 -- of the divisor) write here. 2 (times) 0 ...
(waiting for class to supply product)

Class: 0

Teacher #6: (Writing 0 down) Then, 2 (times) 3 ...

Class: 6

Teacher #6: (Writing 6 down) Then, 1 minus 0 ...

Class: 1

Teacher #6: (Writing 1 down) 8 minus 6 ...

Class: 2

Teacher #6: (Writing 2 down) Don't forget, we just covered this digit. Now, we return to it. We must bring it down. Bring it down. Do you see (pointing to the students who didn't pay attention to the instruction)? Bring it down (taking away the magnet). We find that we haven't written here yet (pointing to the empty position next to the first quotient 2).

Teacher #6: It is very simple, we cover these two digits again (covering the digit 0 of divisor and the digit 0 just brought down). 3 (times) how much is 21?

Class: 7

Teacher #6: (putting 7 next to the first quotient 2) 7 (times) 0 ...

Class: 0

Teacher #6: (putting 0 down) 7 (times) 3 ...

Class: 21

Teacher #6: The answer is 21.

It is apparent that manipulatives were not used as a bridge to connect to the symbolic aspects of mathematics. There is no rationale and conceptual basis for the symbolic procedures. Delivering rote rules is the main method. What about developing the beginning concept of division? Among the three observed lessons, Teacher #7 illustrated by drawing tallies and circling the tallies (as a group) on the board, while teacher #9 demonstrated with Semi-concrete manipulatives. The other teacher, teacher 13, was the best at supplying students with semi-concrete materials. But students' working on tangible materials was at the teacher's dictation or following demonstrative steps to work out similar problems. No critical thinking occurred in this learning episode. In none of the above cases had students playing with the models on their own to explore the beginning concept of division by testing an idea they conjecture or solving a simple word problem. Concrete or semi-concrete models became the teachers' presentational aids more than students' materials for active construction and exploration.

Teacher 13: Page 20 ... please read the first problem.

Class: (in chorus) A paper strip is 24 centimeters long, if we cut it into 8 centimeters, how many pieces can we get?

Teacher 13: Read that pieces, one more time.

Class: (in chorus) ...

Teacher 13: Pass the paper strips, everyone takes one and measures whether it is 24 centimeters.

Class: (Some noise ...)

Teacher 13: (lining up three green paper strips on the board and putting "24 cm" on the top of the strips and three "8 cm" down the strips on the board)

Teacher 13: Please look at the board after you have finished measuring ... Okay! Look at the board.

Teacher 13: The whole length is 24 centimeters (pointing to the paper strips she put on the board), each of your paper strip is 24 centimeters too. Please mark every 8 centimeters to get 3 pieces, like mine.

Class: (Some talking ...)

Teacher 13: Like mine on the board. Right! Mark it every 8 centimeters (watching a student make marks). Start from 0, draw a mark from 0 to 8, completely like mine on the board. Such students are most competent! Start from 0 ..., Okay, raise your hand if you have finished marking.

Class: (Most students raise their hands)

Teacher 13: Okay, take the scissors and cut it into 3 pieces.

Class: (Cutting ...)

Teacher 13: Look at the board and put your scissor down. Tell me, children, how would you write the mathematical sentence?

Class: 24 divided by 8 equals 3 (teacher reads out loud as she writes the sentence: $24 \div 8 = 3$).

Teacher 13: (explaining what 24, 8 and 3 represent respectively ...)

Even in teaching regrouping concept in the second grade, concrete or semi-concrete materials work mainly as teachers' presentational aids (Teacher #11 and #3). Although a few students would be called to work with the materials in front of the class, they all followed the teacher's demonstrative steps in the prior similar problems. One teacher (teacher #21) illustrated the concept of "trading" (borrowing) by breaking 1 "ten" tally into 10 "one" tallies on the board. Both the quantity (whole class manipulation) and quality (materials for constructing and reasoning) of the students' use of manipulative were not achieved.

The fact that teachers didn't make good use of physical materials in order to build connections between concepts and symbols is also revealed in conducting the lesson units such as the concept of an angle. Five teachers were observed developing the concept and measurement skills of "angle." Three of them followed the teacher's manual to allow students to use "circular boards" (two circular boards with different colors are crossed through the cut radius which could be turned to show different degree of angles). One teacher used the boards for presentation. The other teacher ignored and skipped the use of this material in both the presentation and students' exploration. With respect to the way in which students manipulated the circular boards, only one (Teacher #12) out of the three teachers gave children room to explore. Both of the other two teachers (#15, #16) had pupils work after their demonstration.

> Teacher #15: Can you do it after I show you, Okay? Now, you haven't seen clearly yet.
>
> Teacher #16: Now I'll demonstrate, you watch first.

It is prevalent that physical models become primarily teacher's instructional aids as revealed from the field notes. If whole-class manipulating is the case, it tends to occur under the teacher's dictation. In Table 18, the context of 188 instructional segments in which students were observed using concrete materials (including 24 segments in which individual children were called on to perform in front of the whole-class rather than whole class manipulation) further strongly supports this idea. It provides the instructional context -- the teachers' verbal behavior, teachers' material use behavior, as well as students' verbal behavior -- during students' use of manipulatives.

It is indisputable from the data that the quality of students' manipulative use behavior is not as high as the quantity of it (188 segments out of 1320 segments, 14.24% of the total observation). The most frequently observed teachers' verbal behaviors during students' manipulative use were asking low cognitive questions (24.47%), giving directions (20.21%), other speech (12.76%), and imparting information (11.17%). The overall frequency of this kind of teacher-dominated behavior was high at 68.61% of the time. In contrast, the overall frequency of provoking thought behaviors like asking high cognitive question (6.38%), encouraging reasoning (7.45%), and encouraging discussion (3.19%) accounted for merely 17.02% of the observation. Students' use of manipulatives was far from exploration and encouraging reflective minds.

Table 18 The Instructional Context of Students' Manipulative Use Behavior

	Number of Observations	Percent Distribution
<u>Instructional Activity</u>		
Whole-Class Direct	70	37.23%
Activity	85	45.21%
Practice	25	13.30%
Feedback	3	1.60%
Transition	5	2.70%
Other		
<u>Teacher's Verbal Behavior</u>		
Giving Directions	38	20.21%
Imparting Information	21	11.17%
Explaination-Informal	3	1.60%
-Formal	3	1.60%
Asking Ques. - H. Cog.	12	6.38%
- L. Cog.	46	24.47%
Responding	2	1.06%
Asking Recitation	1	0.53%
Encouraging Reasoning	14	7.45%
Encouraging Disc./Commu	6	3.19%
Other Speech	24	12.76%
No Speech	18	9.57%
<u>Teacher's Material Use Behavior</u>		
Chalk & Board	6	3.19%
Textbook	15	7.98%
Manipulatives	52	27.66%
Workbooks/Worksheets	3	1.60%
Other Materials	3	1.60%
No Material Use	109	57.98%
<u>Student's Verbal Behavior</u>		
Answering Ques.- H. Cog.	10	5.32%
- L. Cog.	25	13.30%
Recitation	2	1.06%
Asking Question	2	1.06%
Discussion/Communication	13	6.91%
Other Speech	35	18.62%
No Speech	101	53.72%

Total Observations of Student Using Manipulative: **188**

In addition, Table 18 also shows that the frequency of the students' use of concrete or semi-concrete model in whole-class direct instruction was 37.23% of the time. Accordingly, it is reasonable to conclude that manipulatives are more used for presentation than for students' construction.

Admittedly, not all teachers demonstrated the same type of teaching as such. A few teachers did provide pupils opportunities to explore physical materials and apply a more or less guided approach in developing concepts. For example, teacher #8 urged each child to measure the length of his desk with his own fingers and guided children to understand the need of a common measuring unit: the ruler. Teacher #22 and her students alternately used semi-concrete models to develop the whole number place value concept. The point here is that throughout the observation, most of the teachers didn't make the most of concrete or semi-concrete materials or even provide opportunities for manipulation. For the most part, concrete or semi-concrete models were solely used as a presentational aids. In addition, delivering rote rules was all too common. As a result, building the connections between symbolic procedures and conceptual understanding is far from being achieved. To sum up, both the qualitative and quantitative data mutually support the view that the quantity and quality of students' use of manipulative are less than satisfactory.

Behavior Portraits of the Problem-solving & Reasoning Approach

Manipulative use alone can't work; a reflective mind is more crucial than a mindless manipulation. Accordingly, in addition to supplying tangible materials, it is imperative that teachers activate children's mind by confronting them with problems, asking them thought-provoking questions and encouraging them to reason through problem situations. Problem-solving is an approach to teaching, not a separate unit to instill. In short, computational skills are not the vital goal of instruction, especially in computer age. The most significant objective of teaching mathematics is to foster problem solving and reasoning skills.

An important index of the problem-solving and reasoning approach is the occurrence of provoking student thought. As shown in the previous tables, behavior such as asking (answering) high cognitive questions, encouraging reasoning, and (encouraging) discussion/communication occurred much less frequently. Students attending to teachers was coded most frequently. Teachers as dominant speakers repeatedly occurred in the instructional scenarios. Furthermore, it is substantiated in Table 19 that the overall frequency of teacher-centered speech (such as giving directions, imparting information, explaining, asking low cognitive questions, and asking for recitation) in the whole-class instruction, practice and feedback activity is high at 78.62%, 51.34%, and 66.14% of all verbal behaviors respectively.

Table 19 The Distribution of Teachers' Verbal Behavior by Instructional Activities

	Whole	Acti.	Pract.	Feedb.	Trans.	Other	Total
Giving	45	12	39	13	30	1	140
Direction	32.14	8.57	27.86	9.29	21.43	0.71	100%
	6.73	8.16	14.83	6.88	61.22	33.33	
Imparting	126	11	35	26	1	0	199
Information	63.32	5.53	17.5	13.06	0.50	0	100%
	18.83	7.48	13.31	13.76	2.04	0	
Explaining	58	3	1	11	0	0	73
Informal	79.45	4.11	1.37	15.07	0	0	100%
	8.67	2.04	0.38	5.82	0	0	
Explaining	51	6	0	18	0	0	75
Formal	68.00	8.00	0	24.00	0	0	100%
	7.62	4.08	0	9.52	0	0	
Asking Quest.	39	17	2	5	0	0	63
High Cog.	**61.90**	26.98	3.17	7.94	0	0	100%
	5.83	11.56	0.76	2.65	0	0	
Asking Quest.	174	47	35	49	0	0	305
Low Cog.	**57.05**	15.46	11.48	16.07	0	0	100%
	26.01	31.97	13.31	25.93	0	0	
Responding/	19	1	4	38	0	0	62
Feedback	30.65	1.61	6.45	61.29	0	0	100%
	2.84	0.68	1.52	20.11	0	0	
Asking	72	2	25	8	0	0	107
Recitation	67.29	1.87	23.36	7.48	0	0	100%
	10.76	1.36	9.51	4.23	0	0	
Encouraging	22	17	6	3	0	0	48
Reasoning	45.83	35.42	12.50	6.25	0	0	100%
	3.29	1.56	2.28	1.59	0	0	
Encouraging	8	16	0	1	0	0	25
Dis./Commu.	32.00	64.00	0	4.00	0	0	100%
	1.2	10.88	0	5.29	0	0	
Other Speech	27	9	40	12	11	0	101
	26.73	8.91	39.60	11.88	10.89	0	100%
	4.04	6.12	15.21	6.35	22.45	0	
No Speech	28	6	76	5	7	2	122
	22.95	4.92	62.30	4.10	5.74	1.64	100%
	4.19	4.08	28.90	2.65	14.29	66.66	
Total	**669**	147	263	189	49	3	1320

It is self-evident from the figures that teachers keep recalling and explaining the important steps or information, asking low cognitive questions, and asking for recitation even during the practice and feedback period as if the instruction they just delivered was not sufficient or they had no any confidence in their students' abilities. Most teachers act as knowledge distributors, and independent student thinking seldom prevailed in the classrooms.

Table 19 also offers very rich information about the nature of each instructional activity and the distribution of each specific behavior in various activities. For example, asking low cognitive questions was the most occurring behavior (26.01%) among all verbal behavior in whole-class instruction. Simultaneously, this behavior occurred most in the whole-class direct instruction (57.05%) among all instructional activities.

On the other hand, the index of active construction of students' verbal behavior such as supplying high cognitive answers, asking questions and discussion/communication were much less frequently observed as shown in Table 20. The overall frequency of such behavior in the whole-class instruction, activity, practice, and feedback was 7.92%, 26.53%, 1.90%, 2.65% of all verbal behaviors respectively. Again, this Table supplies much useful information. Take no speech behavior as an example, students spent a high percentage of their time engaged in listening or kept silence (50.07%) during whole-class instruction. This behavior happened to have the highest occurrence (48.62%) in the same activity among all the instructional activities. Therefore, we can image a picture of a passive audience sitting in the classrooms.

Table 20 The Distribution of Students' Verbal Behavior by Instructional Activities

	Whole	Acti.	Pract.	Feedb.	Trans.	Other	Total
Answer Ques.	32	15	0	3	0	0	50
High Cog.	64.00	30.00	0	6.00	0	0	100%
	4.78	10.20	0	1.59	0	0	
Answer Ques.	159	36	28	52	0	0	275
Low Cog.	57.82	13.09	10.18	18.91	0	0	100%
	23.77	24.49	10.65	27.51	0	0	
Recitation	82	3	25	12	0	0	122
	67.21	2.46	20.49	9.84	0	0	100%
	12.26	2.04	9.51	6.35	0	0	
Asking Ques.	8	0	5	0	0	0	13
	61.54	0	38.46	0	0	0	100%
	1.20	0	1.90	0	0	0	
Discu./Commu.	13	24	0	2	0	0	39
	33.33	61.54	0	5.13	0	0	100%
	1.94	16.33	0	1.06	0	0	
Other Speech	40	15	27	17	33	0	132
	30.30	11.36	20.45	12.88	25.00	0	100%
	5.98	10.20	10.27	8.99	67.35	0	
No Speech	335	54	178	103	16	3	689
	48.62	7.84	25.83	14.95	2.32	0.44	100%
	50.07	36.73	67.68	54.50	32.65	100.00	
Total	**669**	147	263	189	49	3	1320

There were very few occasions recorded in the field notes in which pupils posed questions, supplied high cognitive answers, or make verbal exchanges. All communications tended to flow one way - from the top. All the observed verbal exchanges were between teacher and students. Interaction between students were not observed at all. In short, the teacher acted as a knowledge dispenser and pupils correspondingly acted as passive vessels. Thinking and reasoning were not the prime concerns in the classroom. This phenomenon is exhibited in the following episode. At the end of the paper and pencil work, Teacher #1 asked some of her students to copy their procedures and answers on the board. She proceeded problem by problem with careful articulation.

Teacher #1: 552 divided by 24, in the same manner, is 5 enough (referring to can 5 be divided by 24, and covering the last two digits -- 5 and 2 -- by hand)?

Class: Not enough!

Teacher #1: Is 55 enough (moving her hand one digit to the right to reveal the 5, with the number 2 is still covered)?

Class: Enough!

Teacher #1: Okay! 55 is enough, right? This it means that the first digit of the quotient has to be written above this digit 5 (still covering number 2 and pointing to the second digit 5 and the position above 5 with her other hand).

Teacher #1: Is it put at the wrong place? No, it's accurate (asking class whether the student put the first digit of the quotient in wrong place, but answering herself). 55 is enough, the first digit of the quotient has to be put above this number (pointing to the number 5 again).

Teacher #1: Now then, 55 and 24, what number is 24 closer to? What number (pointing to 24)?

Class: 20

Teacher #1: Okay, we'll think of 24 as 20; okay, I already taught this in the previous lesson. 55 and 20, we cover one digit each. 5 and 2, children, which number will you pick (only 5 and 2 being revealed, the other digits being covered by hands)

Class: 2 (not very loud)

Teacher #1: How much is 2 (times) 2 ...

Class: 4

Teacher #1: 4 is less than 5, 2 (times) 3 is 6, 6 is (blank) than 5? (leading class to answer).

Class: Bigger!

Teacher #1: So, of course I pick ...

Class: 2

Teacher #1: (Writing 2 at the position of quotient) Okay, 2 (times) 4 is 8, 2 (times) 2 is 4 (teacher and class recite in unison as the teacher points to 2, 4, 8, 2, 2, and 4, written by the student on the board).

Teacher #1: 7 is the remainder, right? 7? But, there is a number 2, what should we do? Bring it down (asking class the question and answering herself, and, when speaking, pointing to the number 2 which has been brought down by the student).

Teacher #1: Bring 2 down, so it becomes 72 divided by 24 (pointing to 72 and 24). The same thing, same as this (writing 72 divided by 24 in vertical way on the board). Children, look here, I just say, think of 24 as 20, then to cover one digit of each (covering 2 and 4). 7 and 2, children, which number will you pick?

Class: 3

Teacher #1: Because 3 (times) 2 is ...

Class: 6

Teacher #1: If 2 (times) 4, is over (estimate), right? 2 (times) 4 is ...

Class: 8

Teacher #1: So, we pick 3. 3 (times) 4 ...

Class: 12

Teacher #1: This can be ...

Class: divided.

The above teaching scenario reflects an important feature of teachers' instruction; that is, low cognitive questions are often employed by teachers to conduct the lesson. This is a very interesting phenomenon, regardless of the different types of teachers, since most teachers demonstrated this approach in teaching. The low cognitive questions they asked have a nature of "leading"; like a hole in a slope, the ball (the analogy of answer) must fall into the hole without hesitating (the analogy of thinking). Furthermore, some teachers didn't wait for students' answers and then supply answer by themselves right after they posed their low cognitive questions as if they were asking themselves. As presented before, asking low cognitive questions was the most frequently observed behavior (23.11%) among other categories of teacher' verbal behavior.

Another interesting phenomenon teachers demonstrated during the observation is that they asked students to recite including reading the problem to be taught, the title of the lesson unit, and the term just learned; and reciting the procedures or steps of solving a problem type, as if learning mathematics involved rote memory.

Teacher #11: Repeat after me! "34 plus 58"

Class: 34 plus 58

Teacher #11: 4 plus 8 is 12

Class: 4 plus 8 is 12

Teacher #11: Write the 2 and carry the 1

Class: Write the 2 and carry the 1

Teacher #11: 1 plus 3 plus 5 is 9

Class: 1 plus 3 plus 5 is 9

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Teacher #9: 12 divided by 3, "12 represents 12 candies," repeat after me.

Class: 12 represents 12 candies (the teacher pointing to the mathematical sentence written on the board).

Teacher #9: Distribute to 3 children (pointing to the 3 in the mathematical sentence).

Class: Distribute to 3 children

Teacher #9: Everyone gets 4 (pointing to the 4 in the mathematical sentence).

Class: Everyone gets 4.

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Teacher 13: The whole length is read "Chyuan charng", read it!

Class: Chyuan charng

Teacher 13: 24 divided by 8 is 3 (pointing to the mathematical sentence on the board)

Class: 24 divided by 8 is 3

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Teacher #20: Read the problem on the board!

Class: ... (in chorus)

Teacher #20: Wait (interrupting), not too loud.

Class: Shio-yin has 100 dollars, she spent 60 dollars to buy ...

It is also notable in the field notes that speed was the emphasis of doing mathematics instead of reason. Teachers often threatened students with scores. The recurring remarks is like this:

Teacher #2: I'll give you some problems to do, let's see which group is the fastest one. (writing 56 x 6, 500 ÷ 60,... on the board)

Teacher #20: Finished (looking at the students who still engaged in the paper and pencil work)? From now on if you write too slow, I will count the problems that you haven't finished yet as errors.

Teacher #19: Did you find January (referring to January on calender)? Check one more time, how many days in January? Let's see who is the fastest one to point out January?

Teacher #21: 30 seconds left (addressing to the class who was doing computational problems). Fane (calling a pupil who is talking)! I will give you a zero!

Teacher #15: Workbook, page 13! Use the protractor to measure. Do it quickly, hurry up!

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Teacher #11: (showing a flash card -- "4 + 2" --) One, two, three (implying give the answer right away)!

Class: 6 (in choral response)

Teacher #11: (showing a flash card -- "5 + 4" --) One, two, three (implying give the answer right away)!

Class: 9 (in choral response)

To sum up, problem-solving and reasoning was not the apparent concern of teachers' instructional practices. Generally speaking, the observed lessons didn't manifest this approach at all. Instead, the observed prevalent scene was one in which teachers spent a great percentage of the time in delivering rule-exampled procedures and provided paper and pencil work after demonstration. This type of teaching is far from "instruction embedding on problem-solving context". If we count doing paper-and-pencil work in applying the concept and skill just learned as solving problem, it is at the very most merely what Schroeder and Lester called "teaching for problem solving"; it is obviously not "teaching via problem solving" as prescribed in the potential curriculum outline.

Summary of Teachers' Instructional Behavior

Table 21 presents two observers' ratings of the field notes and observational checklists on a 4-point scale for each of the three curriculum focus.

The field observer judged where the teacher's behavior gathered from checklists and field notes fell on the continuum for each of three curriculum focus. The video tapes of the first observations and the audio tapes of the second observations were often reviewed during rating. The side observer devoted herself to reading the mutually agreed statistical results of the checklist (the first observation) and reading the field notes taken by the field observer (the second observation). As was the case with the field observer, the video tapes of the first observation and the audio tapes of the second observation were also often reviewed by the side observer during rating.

Table 21 Means of Teachers' Scores on the Behavior Concerning Curriculum Focuses as Measured by Both the Field Observer and the Side Observer Based on Ratings of Classroom Observational Checklists and Field Notes

Curriculum Focuses	Field Observer	Side Observer	Means
Learner-centered Approach	1.41	1.59	**1.50**
Connection Building Approach	2.18	2.09	**2.14**
Problem-solving & Reasoning Approach	1.36	1.45	**1.41**
(Means)	**1.65**	**1.71**	**1.68**

The two observers assessed the ratings one teacher at a time; that is, they gave each teacher a score on each of the three curriculum focus. The mean score for each of the curriculum focus for all teachers was then computed. A higher score means that teachers' instructional behaviors were closer to the themes of the ongoing trend of mathematics curricular innovation. The mid-point of score is 2.50.

Table 21 shows that the overall mean score of teachers' instructional behavior was low at 1.68. It is clear that these teachers' classroom teaching was quite distant from the ongoing trend of reform. In short, we still have a long way to go under the pressure of reform.

The Relationships Between Beliefs and Behavior

In the preceding sections, we have presented a lengthy discussion of both teachers' beliefs about the teaching and learning mathematics and the corresponding instructional practices derived from multiple sources. This section will focus on the relationships between teachers' conceptions and their teaching behavior; that is, whether teachers' professed views were manifested in their classroom teaching, and whether teachers' classroom behavior reflected their expressed beliefs.

To try to unravel the complexity of the multiple sources of data and further to examine the relationship of beliefs to behavior is a difficult job. The most troublesome problem is the dualistic nature of personally-held beliefs as expressed by Kerlinger (1967) that a person identified as being in one pole does not necessarily disapprove of the views of the opposite pole. According to Schmidt and Kennedy (1990) in their beliefs study, "any belief pattern is an all-encompassing beliefs pattern, one that includes both poles of the education dichotomy." The best example in the present study is that most teachers state that students attending to teacher's instruction and more frequent drill are the best ways for learning mathematics, nevertheless, they argue the importance of thinking and reasoning in learning.

However, the strength of a teacher's beliefs and behavior can still be discerned from the recurring regularities revealed in the data. Realizing this, a careful and thorough examination of various data sources to decide (rate) the strength of the beliefs and behavior of each teacher was conducted in defining the in-between relationships. Table 22 ranks all teachers' beliefs and behavior scores into three levels. Table 23 presents each teacher's scores on beliefs and behavior. These two Tables taken together provide valuable information about the relationship between beliefs and behavior.

Table 22 shows that most teachers' beliefs scores and behavior scores stay in the same level; for example, teacher #3 has a high beliefs score which ranked in the first level and her behavior score is also rated high in the first level. By contrast, teacher #18 has the lowest beliefs score, his behavior score is also low ranked in the bottom level. Although four teachers' beliefs and behavior scores are not ranked in the same level (Teacher #19, #13, #5, #10), they merely shift slightly to the next level. There were no jumps from the top to the lowest level or vise versa. The data strongly suggest that teachers' conceptions of teaching and learning affect their instructional behaviors. That is, teachers who hold more constructivist-oriented beliefs are more likely to behave as such in teaching and teachers who hold more absoption-oriented conceptions act more as such in the classroom.

Table 22 The Ranks of Teachers' Scores on Beliefs and Instructional Behavior

	Beliefs Teacher #	Beliefs Score	Behavior Teacher #	Behavior Score
Level 1:				
	* **#19**	**2.94**	#3	3.17
	#3	2.77	#12	2.83
	#9	2.75	#22	2.67
	#22	2.74	#8	2.50
	#12	2.72	#17	2.33
	#17	2.69	#9	2.00
	#14	2.59	#14	2.00
	#8	2.54	(#13	2.00)
Level 2:				
	(#13	2.29)	#7	1.50
	#4	2.00	#11	1.50
	#7	1.95	* **#19**	**1.33**
	#15	1.89	#4	1.33
	#21	1.85	#15	1.33
	#11	1.82	#21	1.33
	#5	1.80	**<#10**	**1.33>**
Level 3:				
	#1	1.72	#5	1.17
	<#10	**1.70>**	#1	1.17
	#20	1.70	#20	1.17
	#6	1.69	#6	1.17
	#16	1.67	#16	1.17
	#2	1.67	#2	1.00
	#18	1.50	#18	1.00

It is true that personal-held conceptions act as driving forces in shaping the patterns of behavior revealed in the qualitative data. Take the use of manipulatives as an example: teachers who held strong beliefs about the use of manipulatives (Teacher #3, #9, #13, #11, #12), used their own time to "make" semi-concrete materials (either by themselves or with the help of students) and employed these materials in teaching. Teacher #12 made almost thirty circular boards (two students shared each board) in teaching the concept of angles. In the interview he expressed the conception that "teachers should let students discover patterns from the real manipulation of materials." Teacher #3 enunciated the view of "concretizing abstract concepts." When teaching she made some semi-concrete materials like the flat Base-Ten Blocks to teach regrouping concepts. Unfortunately, due to the influence of traditional high-influence beliefs, the use of materials by some teachers were hard to seperate from teacher domination (e.g. teacher demonstrates or students follow direction).

In contrast, teacher #18 never uttered a word concerning the use of manipulatives and he envisioned a very authoritative role in the interview. Consistent with this view was his instructional practice in which he stood in the front of the classroom on a raised platform and pointed to the textbook (he never even wrote anything on board except the lesson title written in the beginning -- "Angle and Congruency") as if he was delivering a lecture or broadcasting. An authoritarian atmosphere was detected in his classroom. All of these demonstrate that beliefs influence behavior to a large degree.

On the other hand, Table 23 shows that all teachers' beliefs scores are somewhat higher than their behavior scores except for teachers #3 and #12. It can be inferred drawing from Table 22 that teachers' instructional behaviors pretty much reflects what teachers believe. One of the reasonable explanation for the slightly higher score between beliefs and behavior might be that some other factor

Table 23 Individual Teacher's Scores on Beliefs and Instructional Behavior

Teacher	Score on Beliefs: Ques.	Interv.	Mean	Score on Behavior	DIf. b/w Beliefs & Behavior
#1	2.1	1.33	1.72	1.17	0.55
#2	2.0	1.33	1.67	1.00	0.67
#3	2.7	2.83	2.77	3.17	- 0.40
#4	2.5	1.50	2.00	1.33	0.67
#5	2.6	1.00	1.80	1.17	0.63
#6	2.2	1.17	1.69	1.17	0.52
#7	2.4	1.50	1.95	1.50	0.45
#8	2.9	2.17	2.54	2.50	0.04
#9	2.5	3.00	2.75	2.00	0.75
#10	1.9	1.50	1.70	1.33	0.37
#11	1.8	1.83	1.82	1.50	0.32
#12	2.6	2.83	2.72	2.83	- 0.11
#13	2.4	2.17	2.29	2.00	0.29
#14	2.5	2.67	2.59	2.00	0.59
#15	2.1	1.67	1.89	1.33	0.56
#16	2.0	1.33	1.67	1.17	0.50
#17	2.2	3.17	2.69	2.33	0.36
#18	2.0	1.00	1.50	1.00	0.50
#19	3.2	2.67	2.94	1.33	1.61
#20	2.4	1.00	1.70	1.17	0.53
#21	2.2	1.50	1.85	1.33	0.52
#22	3.3	2.17	2.74	2.67	0.07

"intervenes" between conceptions and behavior and accordingly decreases the quality of teaching behavior. A plausible one is that situational constraints interferred as claimed by most sociological research on teachers work as discussed in Chapter 2.

Indeed, teachers complained a lot about the heavy workload during the interview. The common most complaint about the workload was that it influences the actualization of beliefs in teaching. This includes big class size, overloaded administrative work and non-academic activities, ceaselessly correcting workbook, heavy load of content materials (e.g. textbook, workbook), and bad management of teaching materials (manipulatives), etc. Such complaints are very often reflected in research on teachers' workloads or pressures (Chao, 1990; Kao, et al., 1987).

Class size is the primary problem. It brings about many relevant problems. First of all, how can a teacher implement whole-class manipulation under the discipline pressure of a class of 50 - 60 students? How can a teacher take care of individual students in a huge and mixed ability class? How can a teacher correct overwhelming piles of workbooks (every subject has workbooks) while preparing good lesson? According to Educational Statistic of the Republic of China (Ministry of Education, 1991), the most common class size is from 41 to 50 students and the second most common is 51 - 60 students. With over-population in the classroom, heavy administration and non-academic work, and other situational factors, it is inevitable that teachers' immediate attention will be distracted and as a result, their teaching performance will be weakened. The following interview protocols describe this situation:

Interviewer: What role should you play in teaching mathematics?

Teacher #8: Ideally, teacher should play a role of helping aside. That is, a guide ..., not to directly transmit. Let students think by themselves. But there are so many children in our class, if I let them think and discover... And we don't have much time (referring the heavy load of content materials). Like using concrete material to guide children to solve problems - it really takes lots of time.

Interviewer: Class size and time constraints ...

Teacher #8: So, sometimes, I teach them directly ...

Interviewer: So, you think, a teacher's role should be that of a guide ...

Teacher #8: But, to tell the truth, sometimes it is it is superceded by the classroom reality.

Interviewer: Reality ...

Teacher #8: One becomes a leader.

When being asked about what difficulties she encountered in realizing teaching ideas or beliefs, she replied as follows:

> The main problem is the over-population. I don't have any time to take care of individual students. If I insist in doing so, then some students will raise their voices which interferes with other students and finally the whole class is out of control.

The following interview protocols also reveal the difficultly of carrying beliefs over practice owing to the situational factors:

Teacher #19: I think the most important thing is to take care of individual differences ... children have different abilities ... But I can't handle it under the present conditions. Besides, I have to catch up the teaching schedule ...

Interviewer: Teaching schedule? How does it influence you?

Teacher #19: Right, catch up the schedule (didn't answer question). If I repeat the instruction, the students who already understand will get bored. Hence, I have to utilize recess. But I have lots of things to do during recess like correcting workbooks, etc. I don't have the extra time to make good use of manipulative materials (referring to making materials or finding materials), therefore, it is impossible to realize my ideas and hopes very well.

It appears that the situational factors are overwhelming and definitely interfere the realization of beliefs about teaching. A caution which should be placed here is that situational factors alone can't decide behavior. One can't completely attribute the low scores of instructional practice to the function of situational factors. Evidently, both teacher #3 and #12's behavior scores are not lower (even higher) than their beliefs scores. Both of them are ranked in the highest level of beliefs and behavior. The personally-held beliefs are crucial to the formation of behavior.

Take the use of manipulatives as an example, both teachers made their own semi-concrete materials in teaching as mentioned before, whereas some other teachers merely complained about the bad management of materials (e.g. not enough materials, broken pieces, etc.) and about the tightly scheduled school day (e.g. no time for finding the right materials in material room). Doyle and Ponder (1977) found that teachers were most receptive to proposals for change that fit with current classroom procedures and did not create major disruptions (cited from Feiman-Nemser & Floden, 1986). Contributing one's recess or extra personal time to make materials for teaching demonstrates the strength of beliefs and the willingness to put conceptions into effect of these two teachers.

Table 22 apparently shows that for the teachers who hold high beliefs score, the behavior score is also ranked at the highest level. It is also true that for the teachers who hold low beliefs score, have behavior scores in the lowest level. How can one deny that conceptions are not the driving forces of one's behavior? Moreover, if situational factors alone decide behavior, how can one account for the fact that teacher #18 ignored the use of circular boards since there are only around ten students in his class?

From the above analysis, it seems plausible to conclude that beliefs about teaching and learning do shape instructional practices. Furthermore, if we want to make predictions about teaching behavior, then the situational constraints have to be taken into consideration. These constraints "might" decrease the quality of teaching behavior, but they can't totally determine instructional practice. In other words, personally-held beliefs are the vital, decisive factors of teaching behavior and situational factors are minor ones.

CHAPTER V

CONCLUSIONS

The central concern of this study was to investigate whether elementary school teachers' beliefs about the teaching and learning of mathematics and their instructional practice parallel the underlying assumptions of the current trend of curriculum reform. Furthermore, what is the relationship between teachers' beliefs and their instructional behavior? The first section of this conclusion chapter will summarize and discuss the prime findings drawing from the multiple sources of data in the hope that this will shed some light on the undergoing reform and relevant policies. Accordingly, Section II will focus on the implications and recommendations based on these findings. Finally, some suggestions for further research are offered.

Summary of Results and Discussion

This study found that skill training and memorization receive many times the emphasis given to either conceptual understanding or problem-solving in our Taiwanese sample. This conclusion is also supported in a study that was done by Porter (1989) on a similar American sample. Table 24 presents the overall scores of teachers' conceptions and teaching behaviors. As the Table shows, the mean scores of teachers' beliefs and behavior are less than the mid-point score of 2.50 in the 4-point rating scale. It suggests that both teachers' conceptions and behaviors tend to be close to the extreme in the scale characterized as the traditional absorption theory as opposed to the other extreme which is characterized as the constructivist trend as shown in Figure 1.

Table 24 The Overall Scores of Teachers' Beliefs and Instructional Behaviors

	Beliefs			Behavior	Means
	Quest.	Interview	Means		
Learner-centered Approach	2.18	1.82	2.00	1.50	**1.75**
Connection Building Approach	2.59	2.16	2.38	2.14	**2.26**
Problem-solving & Reasoning	2.32	1.66	1.99	1.41	**1.70**
Means	**2.36**	**1.88**	**2.12**	**1.68**	**1.90**

```
Learner-centered      |----T----B----*----|---------|
  Approach

Connection Building   |---------|-T-B*----|---------|
  Approach

Problem-solving &     |---T----B|----*----|---------|
  Reasoning Appr.

Overall Mean Scores   |-----T---|-B--*----|---------|
                      1         2         3         4
          (Absorption Theory)                (Constructivism)
-------------------------------------------------------------
     "T" represents teaching behavior
     "B" represents beliefs
     "*" represents the mid-point score
```

Figure 1. The Overall Scores of Teachers' Beliefs and Instructional Behaviors

Summary of Results

The Learner-centered Approach. The mean scores of beliefs and behavior in the learner-center approach are 2.0 and 1.50 respectively. It is true that most teachers enunciated a belief in the high-influence role and their classroom behaviors reflected this view. Repeated instruction, paper-and-pencil work, and passively attending to teacher's lecture constituted the majority of classroom practices. Apparently, the teacher is the central figure in the instruction of children, and therefore it is not a learner-centered approach at all.

The Building Connection Approach. The mean scores of beliefs and behaviors in the connection building approach are 2.38 and 2.14 respectively. It was clear that most teachers didn't regard manipulatives as a crucial medium for building connections between procedural and conceptual knowledge and correspondingly didn't make the most of concrete or semi-concrete models in teaching. Manipulative materials were used mostly in the beginning of teaching concepts for the purpose of demonstration. There was absolutely no mapping between the steps of symbolic procedures and manipulative actions. If student manipulation of materials did take place, it fell under teacher's direction. Both the quality and the quantity of using manipulatives as connection building tools were not demonstrated.

The Problem-solving & Reasoning Approach. The mean scores of beliefs and behaviors on problem-solving & reasoning approach are 1.99 and 1.41 respectively. Teachers deeply believe that they should teach the exact procedures of solving a problem and that the main focus of mathematics is to teach computational skills. Not much thought was provoked in the classroom. Most teachers didn't place critical thinking at the heart of instruction. The phenomenons of exploring, conjecturing, reasoning, and communicating were minimally detected in students' learning. "Teaching via problem solving" seems far removed from actual practice.

Lastly, the overall mean scores of beliefs and behaviors are 2.12 and 1.68 respectively. Undoubtedly, the above summary demonstrates that teaching for what Skemp (1978) called instrumental understanding (as opposed to relational understanding) is the prevailing beliefs and behavior pattern among the teachers investigated. In other words, manipulating symbols without thinking is of concern among teachers and students. Most teachers' conceptions and instructional behaviors deviate from the constructivist view of learning which is the underlying assumption of the current trend of curriculum reform.

Discussion of Results

It is interesting that the beliefs and behavior scores on the connection building approach are higher than the scores of the other two approaches. This is probably due to the fact that teachers more or less capitalize on children's intuitive knowing or employ manipulative materials in teaching. But the fact that manipulatives became teachers' presentational aids rather than students' materials for exploring and constructing mathematical concepts and relationship taken together with the fact that students' manipulation followed teachers' direction kept the rating score lower than the midpoint score 2.50.

The above facts demonstrate the phenomenon of using manipulatives takes place only at a "surface" level.

Although teachers follow teachers' manuals to employ manipulatives in teaching, they interpret the manual in terms of their own conceptions. Since they envision their role as that of a high-influence knowledge dispenser and since they believe in learning by absorption and rote, it is inevitable that they respond to reform with superficial conformation. That is, one adopts the new materials but uses them in a traditional, authoritarian way. This finding echoes the results -- "domesticating" -- reported by Olson (1981) and "surface curriculum" documented by Bussis, Chittenden and Amarel (1976). Moreover, an instruction rooted in the beliefs of authoritarianism is contradictory to the assumptions of teaching mathematics via a problem solving approach. This is why low scores were obtained for the "problem-solving approach."

The fact that long-held personal beliefs about teaching and learning (e.g. the authoritarian role) strongly influence the ways in which curriculum are implemented (e.g. the way manipulatives are used) demonstrates that beliefs affect behaviors in a profound way. In short, the present study finds that beliefs are the driving forces behind behaviors and situational factors play only a minor role in shaping behavior.

Teachers' beliefs seem incongruent with the premise of the present trend of reform and moreover current teaching practices fail to capitalize on the assumption that children construct knowledge. It seems that we still have a long way to go under the reform trend. The point here is not to blame the low beliefs and behavior scores of teachers. Under the circumstances of the present heavy workload and large class sizes, our teachers work hard and try to conform to the reform implementations. The point here is instead to show the need to study how teachers' beliefs are constructed in their life experiences and correspondingly to "enrich" or "broaden" teachers' beliefs.

Implications and Recommendations

Beliefs about teaching and learning mathematics significantly dictate the way teachers teach. This is one of the findings of this study and a common finding of many other studies (Thompson, 1982, 1984; Kesler, 1985; McGalliard, 1983; Bauch, 1982, 1984; Shirk, 1973; Olson, 1981, 1982; Bussis et. al. 1976; Peterson et. al. 1989, etc.). Ethnographic research can help us form a better understanding of teachers' beliefs and their life experience so that one can take corresponding measures to "enrich" teachers' conceptions. From this study, I can draw the following five implications:

1) Preparing preservice teachers properly is an immediate need. The strongest implication from this study is that holding congruent beliefs is more essential than prescribing any pedagogy of practice. The traditional,

pedagogical skills development, such as teaching techniques of classroom management or techniques of applying manipulative materials, is important, but the primary concern here may be to educate teachers adequately with the philosophy of Curriculum Standards. As Thompson (1985) put it, "A skills development approach is unlikely to bring about significant changes in the teachers' views." More specifically, preservice teachers should be taught in a constructivist learning context rather than being told of the constructivist theory and then being expected to reflect this view in future teaching. For example, the Curriculum Standards prescribe that teachers present mathematical content in a problem solving context, then inservice teachers should be provided with the problem context in which they solve problems by reasoning, exploring, conjecturing, testing and discussing. In other words, they have to learn the mathematical content in the way in which their students will learn in future.

2) It also seems important to equip inservice teachers with appropriate philosophy because they are the ones who will be implementing the curriculum. In the same manner, rather than attempting to derive prescriptions for teaching, this study suggests proceeding from teachers' beliefs. Although long standing beliefs -- the most essential impediment to reform -- seem difficult to change, some research documents changes in teachers' conceptions through short-term training (Tompson, 1988; Carpenter et. al.,

1989). Carpenter, Fennema, Peterson, Chiang, and Loef (1989) conclude from their research that "giving teachers access to research-based knowledge about students' thinking and problem solving can affect teachers' beliefs about learning and instruction, their classroom practices ..." Therefore, to immerse teachers in a short-term research-based context which is filled with the underlying philosophy of reform seems to be needed in relevant policy.

3) Incorporating qualitative data such as the data gathered in this study -- interview protocols, field notes, audio, and video tapes -- into teacher education programs might be considered as a way to reflect on one's beliefs and teaching. An obvious phenomenon is that, when these data are applied in teacher training programs, some techniques must be adopted to avoid embarrassing teachers. If teachers can be trained (taught) in a constructivist-based way as described above, then the alternative effect of exposure to both the presentation of traditional-tended data and to the learning context of a constructivist atmosphere will make teachers reflect on what they do and believe.

4) The decreased amount of learning materials (e.g. decreased contents of textbooks, less drill in textbook and workbooks) will probably be a result of the newly enacted curriculum. With the present overloaded of materials, it is hard for teachers not to teach topics of mathematics by way of content exposure before delving into practice leaving no time for developing thinking. Since over emphasis on skills

and rote learning is a common phenomenon of teaching and since understanding, reasoning, and problem-solving are the direction of the curriculum reform endeavor, to decrease the amount of learning materials might be one way in which to lead teachers to focus on conceptual understanding and problem solving.

5) Decreasing situational constraints must be taken into account. Heavy workload and large class size are often used by teachers as reason to oppose the proposed change but this study finds that situational factors are more or less as a minor factor in the influence of instructional behaviors. Therefore, in addition to the primary concern of working on teachers' beliefs, it is necessary to remove these hindrances or to decrease of their influence to the least degree.

Limitations and Suggestions for Future Research

The major feature of present study is the combining of the qualitative and quantitative approaches. The in-depth nature of qualitative data provides a better understanding of the quantitative data. Nevertheless, even a well-designed research has its limitations, and the present study also has limitations as follows:

1) Although this study adopts a "maximum variation sampling" strategy which includes various natures of samples

commonly existing in Taiwan (teachers from four different types of schools), the sample consists of only twenty-two teachers in one administrative area of a city. Therefore, the findings only account for the beliefs and instructional behavior of these twenty-two teachers within that area. Over generalization has to be avoided.

2) In order to handle all multiple methods, to take care of both quality and quantity, and to consider other factors (e.g. personal labor, teachers' cooperation), a trade off is applied in the study -- only two classroom observations and one belief interview with each teacher were conducted in addition to the questionnaire. Hence, it is hard to say in general that teachers always perform the same way that they did in these two observations or speak the same way that they did in this interview.

Based on the above limitations and other considerations, here are some suggestions for future research:

1) A large scale of investigation of teachers' beliefs and instructional practices based on the present study has to be extended under the trend of reform, particularly in Taiwan, where the new curriculum will be implemented two years from now. The important finding of the present study is that beliefs affect teaching behavior to a large degree, therefore, the first priority for successfully implementing curriculum reform is to identify teachers' conceptions and to portray teachers' teaching behaviors in a nation-wide

basis. Although, it is labor, money, and time-consuming, the pay off is worth it. It goes without dispute that if a large scale study is held, then a team approach may be required: a team consisting interviewers, observers and raters must cooperate and accordingly a structured interview and observation must be administered.

2) Perhaps researchers or educators need to focus much more attention on the research question of how beliefs evolve in life experience. Teachers' behaviors in this study are deeply influenced by an authoritative view. Is this related to the whole cultural background? Does this view come from the learning experience they had before entering a teacher education programs? How does a teacher education program affect the development of teachers' beliefs? Do experienced teachers' beliefs become modified during their teaching? The more we understand, the more success we might have in taking appropriate measures to improve the situation. Further research will be required to answer these kinds of questions.

3) Further research based on long term observation and successive interviews is necessary to determine the relationship of beliefs to behaviors. A well-designed long term study and small scale of research will allow us to better understand how teachers' conceptions interact with contextual factors.

APPENDIX A

CLASSROOM OBSERVATIONAL CHECKLIST

Nature of Instructional Activities	1	2	3	4	5	6	7	8	9	10
	Observations									
Whole-Class Direct Instruction	O	O	O	O	O	O	O	O	O	O
Whole-Class/Group Activity	O	O	O	O	O	O	O	O	O	O
Practice Activity	O	O	O	O	O	O	O	O	O	O
Feedback To Practice Work	O	O	O	O	O	O	O	O	O	O
Transition	O	O	O	O	O	O	O	O	O	O
Other Activity	O	O	O	O	O	O	O	O	O	O

Teachers' Verbal Behavior

Giving Directions	O	O	O	O	O	O	O	O	O	O
Imparting Information	O	O	O	O	O	O	O	O	O	O
Explanation: informal	O	O	O	O	O	O	O	O	O	O
formal	O	O	O	O	O	O	O	O	O	O
Asking Question: high cognitive	O	O	O	O	O	O	O	O	O	O
low cognitive	O	O	O	O	O	O	O	O	O	O
Responding	O	O	O	O	O	O	O	O	O	O
Asking Recitation	O	O	O	O	O	O	O	O	O	O
Encouraging Reasoning	O	O	O	O	O	O	O	O	O	O
Encouraging Discussion/Communication	O	O	O	O	O	O	O	O	O	O
Other Speech	O	O	O	O	O	O	O	O	O	O
No Speech	O	O	O	O	O	O	O	O	O	O

Teachers' Material Use Behavior

Chalk & Board	O	O	O	O	O	O	O	O	O	O
Textbook	O	O	O	O	O	O	O	O	O	O
Manipulatives	O	O	O	O	O	O	O	O	O	O
Workbooks/Worksheets	O	O	O	O	O	O	O	O	O	O
Other Materials	O	O	O	O	O	O	O	O	O	O
No Materials Use	O	O	O	O	O	O	O	O	O	O

Students' Verbal Behavior

Answering Question: high cognitive	O	O	O	O	O	O	O	O	O	O
low cognitive	O	O	O	O	O	O	O	O	O	O
Recitation	O	O	O	O	O	O	O	O	O	O
Asking Question	O	O	O	O	O	O	O	O	O	O
Discussion/Communication	O	O	O	O	O	O	O	O	O	O
Other Speech	O	O	O	O	O	O	O	O	O	O
No Speech	O	O	O	O	O	O	O	O	O	O

Students' Material Use Behavior

Chalk & Board	O	O	O	O	O	O	O	O	O	O
Textbook	O	O	O	O	O	O	O	O	O	O
Manipulatives	O	O	O	O	O	O	O	O	O	O
Workbooks/Worksheets	O	O	O	O	O	O	O	O	O	O
Other Materials	O	O	O	O	O	O	O	O	O	O
No Materials Use	O	O	O	O	O	O	O	O	O	O

APPENDIX B

OPERATIONAL DEFINITION OF CLASSROOM OBSERVATIONAL CHECKLIST

Nature of Instructional Activities

Whole-Class Direct Instruction

The teacher presents and transmits academic information /textbook content to the whole class; students usually sit and listen to the teacher's lecture during instruction (students using physical materials at teacher's ditaction is also coded as "Whole-Class Direct Instruction").

Whole-Class/Group Activity

Instruction takes the form of "activities," including games, class/group discussion, class/group problem-solving activities, etc. Students Usually play a more active role than they do in whole class direct instruction.

Practice Activity

Any activity that involves skill practice, including individual seatwork, practice on the blackboard, and whole-class practice of mental calculation.

Feedback To Practice Work

When the teacher spents a period of time commenting on students' practice work, including homework assignments and classroom practice work.

Transition

The time at which a class is between activities; for example, the period between when the manipulative activity is announced and when it is actually engaged, when the teacher is passing out materials. Another example would the time between whole class instruction and individual seat work while the teacher is passing out materials.

Other Activity

This refers to any activity that involves non-mathematical academic learning; for example, when the teacher has students collect field trip money or announces an important school event.

Teachers' Verbal Behavior

Giving Directions

When the teacher gives commands,or directions about work to be done and "how" to go about an activity; for example, when the teacher says: "turn to page 15 ..." before she begins her instruction, or "first, exchange 1 long block for 10 small cubes, then take away 3 small cubes ..." when students engage in manipulative activity.

Imparting Information

When the teacher provides academic information, such as lesson content, algorithmic procedures, or rules. For example, "To add two three-digit numbers you first add the numbers in the right-hand column. If the answer is 10 or more, put the 1 above the second column. Proceed in a similar manner for the next two columns in order."

Explanation

When the teacher explains a concept, algorithmic procedure, or rule to be learned. The explanation depends on children's intuitive knowledge, such as their real life experience or manipulative models that are coded as "informal/real life." The explanation according to logical relationships is coded as "formal/logic."

Asking Question

When the teacher asks a question which requires critical thinking, it is coded as a "high cognitive question" because it provokes children's thought. It is high level in terms of reasoning. On the other hand, when the teacher asks a question which merely involves factual recall or mindless response such as "Is 3 bigger or smaller than 5?", it is coded as a "low level question" because it is low level in terms of reasoning.

Responding

When the teacher responds to students' questions. The teacher's response to the correctness of the answer that the student provides (oral or written) is also coded as responding.

Asking Recitation

The teacher asks students to recite from memory, or to read aloud textbook content or written messages from the blackboard, or to repeat what teacher said.

Encouraging Reasoning

Refers to the teacher's provoking students to reason, conjecture, or justify their thinking in the instructional activity by using either direct questioning or indirect hints. For example, "When you measure desk by your fingers, some says 6, some says 7, other says 8; why does the same table have different length? How can you get the same length?"

Encouraging Discussion/Communication

When the teacher encourages students to explain their thinking process or to exchange ideas either in small group activities or in whole-class instruction.

Other Speech

When the teacher's speech doesn't belong to any of the above categories or is not related to mathematical learning is coded as "other speech." For example, "Su! Don't fool around! This is the last warning." or "Two minutes left, hurry up!"

No Speech

No verbal expression at the moment of instruction.

Teachers' Material Use Behavior

Chalk & Board

When the teacher writes something on the board or points to the written message on the board during observed segments.

Textbook

Textbook refers to the national edition of learning materials. The teacher actually uses the textbook; for example, reading the instructions from the textbook, pointing to the instructions in the textbook, etc.

Manipulatives

Refers to physical materials which can be manipulated to enhance conceptual understanding or to obtain skills. Included are all the concrete and semi-concrete materials such as Base-Ten Blocks, paper cutted fruit, protractors, etc.. The teacher may uses manipulatives for demonstration or for stimulating children's thinking.

Workbooks/Worksheets

The teacher uses the national edition of the workbook or uses the worksheets; for example, checking individual student's workbook, giving direction about workbook to be done, etc..

Other Materials

Any material which is not included in above mentioned categories is coded as other materials, such as flash cards, number cards, hands, etc..

No Material Use

The teacher is not using any materials. She or he may or may not be engaged in verbal behavior without using any materials, for instance, when the teacher circulates in the classroom during seatwork.

Students' Verbal Behavior

Answering Question

When the student's (or the whole class) response to the teacher's question reflects critical thinking, it is coded as "Answering Question: High Cognitive." On the other hand, when the student (or the whole class) responds to a teacher's question which merely involves factual recall or low-level reasoning, it is coded as "Answering Question: Low Cognitive."

Recitation

When the student (or the whole class) speaks aloud from memory, or reads aloud textbook content or written messages from the blackboard, or repeats what teacher has just said.

Asking Question

When the student (or the whole class) asks an academic question which relates to the concept or skill taught.

Discussion/Communication

The students explain their thinking processes or exchange ideas in small group or whole class discussion.

Other Speech

When the student's (or the whole class) speech doesn't relate to the concept or skill beibg taught or doesn't belong to any of the above categories of speech. Examples of other speech include asking information about seatwork to be done and other non-academic question.

No Speech

The student (or the class) did not utter a word during the observed 15 second segment. The student (or the class) might listen to the instruction or do paper and pencil work silently.

Students' Material Use Behavior

Chalk & Board

When the students writes something on the board during the observed segments. Usually, this category of material use behavior occurs in the feedback activity when the teacher asks the students to copy their procedures and answers on the board.

Textbook

When the students use textbook; for example, reading the message in the textbook, drilling on the problems in the textbook, etc..

Manipulatives

Students manipulate concrete or semi-concrete materials (e.g. Base-Ten Blocks, paper cutted fruit, protractor, etc.) for the purpose of enhancing understanding or learning skills.

Workbooks/Worksheets

The students use the national edition workbook, or do worksheets given by the teacher, or paper and pencil work.

Other Materials

Any material which is not included in the above mentioned categories is coded as "Other Material," for example, flash cards, number cards, hands, etc..

No Material Use

The students did not use any materials. Usually, no material use behavior occured when they were listening to the instruction.

APPENDIX C

TEACHER INTERVIEW GUIDE

1. Interviewer asks questions about personal background information (e.g. age)

2. Please describe a typical mathematics lesson (the routine activity of a mathematics lesson).

3. Please provide the rationale for the above-arranged routine activities.

4. Please describe how you teach the beginning concept of multiplication or division?

5. (Optional question)
   Interviewer mentions specific events in the observed lesson and asks teacher what his or her thinking was there.

6. What is the best (or most effective) way for students to learn mathematics (or What is the best way to teach mathematics)? And why do you think that it is the best way?

7. What do you think the teacher's role should be in teaching mathematics? And why should teacher's role be like this.

8. Please describe your main objective for teaching mathematics?

9. What are your difficulties in putting your beliefs (about teaching mathematics) into practice?

10. In your opinion, how should you apply the problem solving approach in teaching mathematics?

APPENDIX D

BELIEFS QUESTIONAIRE

Strongly Agree	Agree	Disagree	Strongly Disagree
1	2	3	4

1. Children learn mathematics best by attending to the teacher's explanations and by more frequent drilling. 1 2 3 4

2. In teaching mathematics, the role of the teacher is to impart mathematics knowledge and correspondingly, the role of student is to attend to the instruction. 1 2 3 4

3. Teachers should teach students exact procedures for solving problems in order to avoid aimless groping. 1 2 3 4

4. Teachers should presnt new mathematical symbols immediately in teaching a new topic so that the students can have a clear idea of what they are about to learn. 1 2 3 4

5. The most effective way for students to learn concept and algorithm is to have them observe the teacher demonstrating the use of manipulatives. 1 2 3 4

6. Teachers should let students work on concrete materials in the beginning of introducing a new concept or algorithm (e.g. single-digit multiplication or division); as to approaching the complex algorithm (multi-column multiplication or long division), teachers must rely on demonstrating each step on the board. 1 2 3 4

7. Students discuss mathematical problems by groups is helpful in clarifying thinking and promoting understanding, therefore, it should be largely applied in the mathematical instruction. 1 2 3 4

8.	Problem-solving is an important topic, and should be incorporated in the textbook as a unit to be taught.	1	2	3	4
9.	The main objective of teaching mathematics is to equip students with speedy and accurate computational skills and relevant mathematics knowledge.	1	2	3	4
10.	Mathematical problem solving is essentially the application of computational skills in order to get the right answer to word problems in a textbook or workbook.	1	2	3	4

- Thank you -

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