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What is the relationship between what teachers believe about how children learn mathematics and how those teachers teach mathematics? : a case study of elementary school teachers' beliefs and behaviors.

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A CASE STUDY OF ELEMENTARY SCHOOL TEACHERS'

BELIEFS AND BEHAVIORS

A Dissertation Presented

by

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ABSTRACT

WHAT IS THE RELATIONSHIP BETWEEN WHAT TEACHERS BELIEVE ABOUT HOW CHILDREN LEARN MATHEMATICS

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MAY 1992

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In a qualitative study of the beliefs and behaviors of four third and fourth grade teachers as they taught mathematics in an industrial Vermont town, teachers were found to have four fundamental common beliefs about how children learn mathematics: (a) children learn mathematical concepts by manipulating or visualizing concrete materials; (b) children learn arithmetic through specific sequenced steps; (c) children learn mathematics through practice and repetition; and (d) children learn mathematics best when they feel good about themselves and experience success in

mathematics. Not all of their beliefs are in concert with the learning theories foundational to the 1989 National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics.*

Associated with each belief, the teachers had one or more factors they considered when planning mathematics lessons. They demonstrated a variety of behaviors including classroom activities and strategies concomitant with, although not necessarily congruent with each belief. There were discrepancies most commonly because of tendencies to acquiesce to the pressures of time and curricular expectations (including those expectations from the next year's teachers) and to rely upon the textbook rather than build upon the strength of their convictions and beliefs about how children learn.

While teachers believed that manipulating materials helps students grasp and develop concepts about the real world in mathematical terms, there was limited time devoted to the manipulation of materials. Although sequential learning was believed to be valuable, many mathematical concepts such as measurement and geometry were taught out of the context and sequence of similar concepts. Practice was typical in each classroom; repetition was prevalent in two classrooms. Many ways of boosting the confidence of students were demonstrated, although one of the teachers believed she was supportive to students when in fact supportive behaviors were not displayed.

Staff development implications include recommendations for teachers to increase their knowledge of constructivism as a way that children learn and of mathematics as a field of knowledge. There are suggested actions for teacher unions, school administrations, state departments of education, post secondary schools of education, and professional organizations.

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CHAPTER 1

INTRODUCTION

"All students need to learn more, and often different, mathematics and . . . instruction in mathematics must be significantly revised".

NCTM, *Curriculum and Evaluation Standards for School Mathematics*, p. 1

Statement of the Problem

As we approach the twenty-first century and technology dominates our lives, our schools underserve students in the area of mathematics education. A variety of mathematics curricula have been employed in our schools in the past fifty years, yet children's performance lags behind that of the youth of Europe and East Asia. Africa (NAEP, 1989; National Center on Education and the Economy, 1990; Goodlad, 1984). Report after report tells of the failure of teachers to teach math effectively, most notably *A Nation at Risk* (National Commission on Excellence in Education, 1983), *A Place Called School* (Goodlad, 1984), and *Educating Americans for the 21st Century* (National Science Board Commission on Precollege Education in Mathematics, Science and Technology, 1983). A look into many elementary classrooms shows short periods of mathematics instruction being taught using textbooks filled with pages of numerical problems, suggesting that mathematics education is a low priority and may be taught with significantly less student engagement than other subjects.

Mathematics curricula have been changed and modified in the past fifty years, reflecting several efforts to improve and enhance student learning. In the 1940's and 1950's, elementary school mathematics instruction included memorizing times tables, flash cards, and algorithms; the new math of the 1960's brought set theory and a heavy emphasis on the structures and principles of mathematics. In 1975, the National Assessment of Education Progress (NAEP) reported mathematical achievement as inadequate for the challenges students faced. Mathematics curricula refocused on competency and the basics of computational skills and skills for everyday living. Even so, the 1989 NAEP mathematics assessment reported that many students had "serious gaps in their knowledge of underlying concepts" (Carpenter & Lindquist, 1989, p. 169).

To address concerns about students' low levels of mathematical achievement and corresponding concerns about the teaching of mathematics, in 1989, the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards for School Mathematics*. The standards are designed to establish a coherent and common vision of mathematical literacy and to guide the revision of the teaching and learning of mathematics in schools. They outline what mathematics should be included in the curricula and examples of student activities appropriate for learning each area of mathematics. The standards are not a curriculum, but rather guidelines outlining expectations. The curricula used in classrooms must be developed by the teachers, schools, and districts in accordance with their missions and philosophies.

A basic premise of the NCTM standards is that children construct their knowledge from experience. To implement NCTM's standards, to develop effective curricula, and to improve students' understanding and uses of mathematics, teachers must think about and understand how children learn mathematics and they must use that knowledge when communicating mathematically and teaching mathematics. When asking teachers to make changes in line with the latest reform movement in mathematics, it is first necessary to understand what teachers believe about how children learn mathematics and how those teachers teach mathematics.

Frequently in the elementary classroom, mathematics is taught differently than other areas of the curriculum. A common focus of elementary school teachers is on reading and the language arts. Typically, classrooms are equipped with an abundance of reading and writing materials and the largest segment of the school day is involved with the production and interpretation of language: reading, grammar, spelling, penmanship, oral expression, creative writing, etc. Science, music, art, physical education, and to some extent social studies, usually involve active participation on the part of the students. Yet mathematics is often taught through the rote memorization of facts and the mechanistic application of algorithms.

When teachers guide children through learning experiences that engage them in the content through interesting and challenging activities which move progressively from concrete experiences to abstract ideas, they ground their teaching in a constructivist theory of how children learn. Teachers provide opportunities for students to interact with physical manipulatives and to develop strategies and personal theories about how things work in the world.

Virtually every teacher preparation program in the United States requires elementary education majors to study child development and psychology which focus on Western thinkers such as Dewey, Piaget, Vygotsky, Erikson, and others who espouse an essentially constructivist form of cognition. Constructivism explains children's knowledge as mental constructs built from multiple experiences with the world around them. As children play with, manipulate, organize, process, and internalize their experiences, they develop understandings and concepts which are then used as building blocks for and bridges to further learning.

Although teachers may incorporate these ideas of learning in many of their planning and teaching activities, the teaching of mathematics today does not appear to be well grounded in a constructivist philosophy.

There is a crisis in mathematics education in America. A reformation of mathematics education which is based on a coherent relationship between how children learn and how teachers teach mathematics is now dawning.

Purpose of the Study

The purpose of this qualitative study is to look at the intersection of elementary school teachers' beliefs about how children learn mathematics and how they teach mathematics. The study is designed to determine whether there is a gap between the beliefs and actions of elementary school mathematics teachers and to specify some of the relationships between what teachers believe and do. Outcomes of the study include a description of some teachers' beliefs and teaching behaviors and specific professional development directions and activities. These outcomes may also be useful in redesigning pre-service teacher education, in making rational curriculum changes, and in undertaking other reform efforts designed to address the crisis in mathematics education.

Questions guiding this study include

1. What do teachers believe about how children learn mathematics?

2. What factors do teachers consider when planning their mathematics lessons?

3. What behaviors, activities, and teaching strategies do teachers use in the mathematics classroom?

4. How do the teacher's beliefs about how children learn mathematics relate to the ways teachers behave as they teach mathematics?

Significance of the Study

The results of this study will contribute to the research base of information about what elementary teachers believe about how children learn and how that relates to how they teach mathematics. Principals, curriculum coordinators, and other administrators concerned with improving mathematics education will be interested in the findings of this study as they may be related to the school personnel with whom they work. As a result of this study, administrators and teachers may become more conscious of the need to teach mathematics in a manner consistent with what is known about how children learn.

At this time, as the nation undertakes new reform efforts in mathematics education, the results of the study will also inform members of the education community about some of the current beliefs and behaviors of elementary school teachers that should be considered when contemplating curricular change. If we can determine whether there are gaps between beliefs and practices and what those gaps are, educators can implement curricular changes drawn from the NCTM standards, and, based on professional development activities designed to build upon learning theory and teaching strategies, strengthen teachers and their teaching.

Teachers have final control over what is taught in their classrooms. By identifying connections between teacher beliefs and behaviors and how children learn, teachers will have information to use while developing their curricula. Because the NCTM standards urge significant manipulation of materials and rely upon substantial real life problem solving in the traditions of Piaget, Vygotsky, and others, implementation of the standards will engage teachers in developing mathematics curricula based on how students learn.

Textbooks can not identify nor anticipate the teachable moments found every day in an elementary school classroom. Teachers are the critical agents in linking mathematical learning such as reading a clock, figuring averages, measuring heart rates, generating graphs, and counting money collected to the physical educator's Jump Rope for Heart marathon for the American Heart Association. This study helps identify the extent to which teachers' beliefs and behaviors capitalize on using the surrounding educational environment to teach mathematics as an integral part of children's thinking, problem solving, and school life.

The NCTM standards are grounded in assumptions that children acquire mathematical knowledge by developing their own constructs from direct experiences. It follows that curricula developed from the standards will effect changes in student learning when teachers provide concrete experiences from which children can build number concepts, relationships, and computations. This pedagogy requires more than believing; it also requires acting on beliefs. This study will help identify how teachers' actions are coordinated with their beliefs.

This exploratory descriptive study may offer researchers and educators insights into teachers' thinking and also provoke further questions about what, how, and why teachers behave as they do in their mathematics classrooms, especially if their behaviors are in discord with their beliefs about how children learn. This study will provide the foundation for future research about the degree to which teachers' beliefs and behaviors are related.

Researcher's Assumptions and Definitions of Terms

The study is founded on a variety of assumptions about the roles of schools and teachers as those responsible for educating our children and preparing the next generation for a complex technological world. While recognizing the vast number of responsibilities elementary school teachers are asked to take on, ranging from teaching reading to identifying possible child abuse to counting milk money, there is a fundamental assumption that elementary school teachers teach most areas of the curriculum including mathematics.

This study rests on the assumption that mathematics is an essential field of knowledge which will be taught in all elementary schools to all students. Because our world is becoming increasingly infused with new information, stored and retrieved electronically and requiring analysis and reasoning in order for it to be useful, mathematical thinking is becoming a more valuable tool for all members of our society. Thus, the teaching of mathematics must result in higher levels of achievement than have been recorded on recent NAEP examinations.

Elementary teachers have been assumed to be committed to helping their students learn mathematics and to be able to identify and communicate their beliefs about how children learn mathematics. In order to conduct this study, it was necessary to work with teachers who could articulate their beliefs and were willing to have their mathematics classes observed. There were no individuals who self-selected out of the study because they were unclear, unsure, or unwilling to articulate their thoughts to the researcher. This study was founded on the assumption that teachers were willing and able to share their teaching with others.

This study is also founded on the notion that the 1989 NCTM standards are logical and rational guidelines for reforming the teaching of mathematics in the next few years. They outline fundamental concepts and skills for students to master: communicating, problem solving, reasoning, estimating, computing, organizing data, establishing patterns and relationships, and measuring through the use of mathematics. The standards are straightforward and reflect basic thinking skills.

Some terms and definitions are used consistently throughout the study. References to schools and teachers refer to elementary schools and elementary teachers who work with students between the ages of seven and eleven. The focus is on teachers of third and fourth grades, although it may be possible to extend the findings beyond that population.

Finally, as the literature review explicates, both the standards and the researcher assume that children construct their own mathematical knowledge and thinking structures based on their interactions with tasks and materials rather than simply by being provided information from an external source. While the NCTM curriculum standards encompass kindergarten through grade twelve, the specific standards referred to in this research undertaking are those recommended for the first tier of education outlined, kindergarten through grade four.

Limitations of the Study

There are a number of limitations to this study which should influence how the reader interprets and uses the findings. The research is a study of four third and fourth grade teachers in a rural Vermont school. Although the teachers reflect various views and styles, each is committed to teaching. They do not represent teachers as a whole; detailed descriptions of their beliefs and classroom behaviors should help the reader decide if or how much of the findings can be transferred to other teachers and settings.

The research data are drawn from an intensive study of a small number of individuals and consist of rich descriptions of teachers' thinking, beliefs, attitudes, fears, questions, concerns, thrills, and dreams. Observations and conclusions are drawn from interactions and communications with students, teachers, and administrators. Analyses of the complex planning, teaching, and learning relationships are set in context.

Economic and time constraints limited the extent of the study. No study of this duration can paint a complete picture. Nonetheless, the intensity of the study, the depth of the interviews, and the triangulation procedures helped to provide an accurate and trustworthy portrayal of these particular teachers.

A potential limitation of the study resides with the biases of the researcher. I have been a student and a teacher. I am an administrator. I have observed master teachers and student teachers. I have interviewed hundreds of people for jobs, information, and a variety of other reasons. I have my own prejudices and experiences which influence my responses to what I see and hear. I enjoy exploring ideas and concepts with people. In this study, I endeavored always to be open and supportive of others directing their own thoughts and expressions. I examined and checked continually my own reactions and interpretations of what I saw, heard, felt, and intuited. I used multiple data collection methods and triangulation in order to verify impressions. I explicated my winces and hunches as they occurred in order to identify and use them in the analyses of the data. I was a tool in this study in order to enrich the findings rather than to constrain them.

In response to the myriad of reports indicating students' mathematical achievement is inadequate to the challenges of the twenty-first century and suggesting that elementary teachers and elementary mathematics curriculum need reform, this qualitative dissertation study is about third and fourth grade teachers' beliefs about how children learn mathematics and how those teachers teach mathematics. The report assesses whether there are gaps between the beliefs and actions of teachers, identifies some of the relationships between what teachers believe and do, and suggests some professional development activities to remedy the status of mathematics education today.

CHAPTER 2

LITERATURE REVIEW

Before investigating the relationships between teachers' beliefs about how children learn mathematics and teachers' behaviors when they teach mathematics, it is useful to know what the research indicates about these two areas. There is much rich literature focusing on children's cognition and more specifically on how children learn mathematics. Rooted in the studies of Jean Piaget, including *The Child's Conception of the World* (1929), *The Child's Conception of Number* (1952), and *The Child's Construction of Quantities* (1974), research on children's acquisition of mathematical skills continues to grow.

There is a significant body of literature on the influences teachers have on students' mathematical learning because of their knowledge and their affective behaviors. Because influences result from teachers' actions in the classroom, it is important to look to that body of literature as a foundation for this research study.

The recent NCTM *Curriculum and Evaluation Standards for School Mathematics*, published in 1989, is of particular interest to this study. The purpose and foundational beliefs of the NCTM standards and its 1991 companion piece, *Professional Standards for Teaching Mathematics*, are also reviewed.

How Children Learn Mathematics

We must understand how children learn mathematics if we are to teach it effectively. In the 1920's educational philosophers abandoned the notion that children's minds were blank slates ready to be filled up with information, yet much teaching in elementary schools in the 1990's still relies upon rote memorization and practicing algorithms outside of students' contexts.

Over fifteen years ago, Lazarus (1974) warned that students who had problems with math often go undetected for several years because their memorization of mathematics could overlay understanding. In order to help students learn mathematics effectively, it is important that teachers understand how children acquire mathematics and that students be encouraged to demonstrate their knowledge as they see it, not as others have conveyed it.

Current theories about how children learn and develop cognitively build upon Jean Piaget's research that children construct their own understandings of logic and mathematics. That is, children construct logico-mathematical knowledge by acting on and manipulating the environment around them (Ginsburg and Opper, 1969; Piaget, 1928, 1952; Piaget & Inhelder, 1974). They engage in a variety of experiences, building with blocks, fitting small containers into larger ones, and dividing pies into pieces so that each member of the family has one. Children try things out, find patterns and repetitions, and pair and count items until they say "ahaa!" and begin to integrate their repeated discoveries into their intellectual frameworks.

Sinclair reiterates this constructivist theory of how children learn mathematics when she says, "From all we know about children as constructors of knowledge, mathematical meanings are constructed as action-patterns, first on real objects and later interiorized." (cited in Steen & Albers, 1981, p. 12).

Children do not gain new insights and understandings about their mathematical world because of being told by another person, but rather from cognitive activity and learning enacted within the child's head. By engaging in activities, new relationships and interrelationships are understood and new meanings are attached to observable concrete experiences and constructions.

The concept of number is not empirical in nature; the child constructs it and makes sense of it from within by using his/her own mental action of reflective abstraction to put things in some relationship or order. Kamii (1985) contends that numerical concepts are developed by each child from his/her natural ability to think. The concept of number is not teachable or learned solely from the external environment or from transmissions from people; it emerges within a child's mind when s/he has experienced, manipulated, and attached meaning to quantities. The child is the constructor of the concept through making use of the external environment; the material or social environment has provided the setting while it is the child who has imparted meaning to it.

Number concepts can be explained in concert with cognitive development theory. As children become comfortable and facile with concrete objects, they begin to internalize the relationships among the objects and thus develop mathematical and relational concepts. By establishing mental number lines, children construct their internal understandings of quantities, more and less, part/whole schemata, and the partitioning qualities of numbers. As children envision a numerical quantity on a mental number line, they can move up and down the scale to count, or to establish relative values (which is greater, which is lesser), or to see how one number may be segmented in various ways.

Children begin to visualize and establish relationships between two or more numerical quantities as a result of their experiences: two and three combine to equal five; a pizza can be divided into half and half again to create fourths; twenty-four hot dogs divided among eight campers allows for three hot dogs per camper. Repeated experiences help a child see patterns which, in turn, enable relationships to emerge in the child's mind (Battista, Wheatley & Talsma, 1989; Landis & Maher, 1989; Resnick, 1983).

As children grow and develop mathematically they move through various stages, combining existing information and processes into more complex ones. By using and breaking down more complex processes, problem solving skills develop. Solving a problem is a cognitive process a child goes through, not as a result of didactic instruction, but as a result of combining existing knowledge with a new task or challenge. To solve a problem, the child undertakes a series of steps:

1. S/He builds a representation in his/her working memory.

2. S/He searches long term memory for a stored problem-solving routine relevant to the problem as formulated.

3. If a routine that works under the present conditions is not found, then further features of the problem task are noted or the immediate goal of the problem solving activity is redefined so that routines not previously recognized as relevant or usable will become so.

The processes of redefining the goal or refining the tasks of the problem are repeated until a solution to the problem is generated. When the goal has been met, the child's working memory is modified to encompass the new problem solution and is available for his or her use with any future dilemmas (Resnick & Glaser, 1976).

The internal problem solving routine may be manifested in different ways depending upon the successful problem solving experiences of the individual. While one child may determine the number of marshmallows per camper by counting out piles for each camper, another may use a division fact which has been stored in long term memory.

In spite of this knowledge about how children develop their concepts of number and numerical relationships and how they solve problems, students frequently are asked to memorize facts rather than reason out mathematical relationships. Similarly, they are required to memorize procedures for solving problems rather than asked to analyze the situation, identify relationships, and apply logical steps to figure a solution.

As a result of procedures memorized and misapplied (what Resnick refers to as buggy algorithms), children engage in errorful calculations. Children's mathematical mistakes do not emanate from their cognitive understandings, reasoning, or understanding of mathematical principles, but rather from attempts to apply sometimes haphazardly and illogically memorized algorithms to number problems outside of meaningful contexts. While children must acquire, often through teaching, some level of procedural skill in order to become mathematically competent and proficient, it is their cognitive development that is the foundation for their reasoning and problem solving skills (Resnick & Omanson, 1987).

Teachers' understanding of and respect for children's informal problem solving methods and use of a wide variety of models is important when teaching arithmetic operations. With a broad range of experiences, children develop greater independence of thought. Schematic diagrams, verbal descriptions, and other strategies that children use while problem solving need to be heeded, modeled, and incorporated into effective teaching (Fuson & Willis; 1989; Greer, 1987; Siegler, 1987).

In addition to benefiting from a variety of models, children learn through multiple modes, including visual, oral, aural, and kinesthetic, and have different preferred learning styles. Many aspects of intelligence come into play and contribute to effective problem solving. Says Elliott: "All layers, indeed all regions, of the brain must be brought into synchronization if learners are to develop to their fullest problem solving potentials" (Elliott, 1987, p. 34).

In the realm of geometry, children must experience, touch and create, two- and three-dimensional shapes, angles, lines, and other geometric concepts in concrete ways including creating them with their bodies on the floor or on an electronic sketchboard, (i.e. computer screen) in order for the concepts of geometric configurations and relationships to be meaningful. Children's conceptualization of geometric shapes, objects, and figures has been enhanced through the use of computer models (Clements & Battista, 1989; Papert, 1980; Watt & Watt, 1986).

In addition to providing ways for children to create and interact with geometric concepts, computer experiences also can enhance and build the problem solving skills of children. The experience of being an active and self-directed programmer aids in the development of being an active and self-directed thinker. Seymour Papert, the inventor of the computer language Logo, argues that by "teaching the computer how to think children embark on an exploration about how they themselves think" (Papert, 1980, page 19).

Concern has been expressed over the years about whether math anxiety inhibits students' various mathematical learning and performance. Early studies indicated that math anxiety might interfere with some students' abilities to perform (Richardson & Suinn, 1972). More recent research suggest that positive attitudes of students have positive effects on math achievement regardless of their anxiety levels (Aiken, 1976; Daane & Post, 1988). These findings suggest that anxiety does not affect the cognitive development of children and that attitude, a manifestation of the affective domain, is important.

Teachers' Influences on Children's Mathematical Learning

There is ample evidence, as cited above, that number concepts, mathematical relationships, and problem solving activities are cognitive constructs made by the child. It is encouraging for those of us in education to know that teachers' knowledge and behaviors effect learning as do teachers' attitudes and beliefs.

Teachers have tremendous effects on the many opportunities available for students to construct their own knowledge. Teachers determine the content and how it is presented in the classroom. They determine the materials available, the time spent on content, the pace of activities, the skills taught, the sequence of events, the means of assessing student needs and knowledge, grouping practices, the standards of achievement, and most other things that occur in the classroom. Their judgments are often effected by their own knowledge base, attitudes, and personal preferences. Teachers' contributions to student learning are significant; all these items under the teacher's control effect student learning (Barr, 1988; Brophy, 1982; Freeman & Porter, 1989; Hunter, 1990).

By grounding learning experiences in the interests and backgrounds of the learners, teachers can develop effective curriculum. As indicated earlier in this chapter, Piagetian philosophy and developmental theory are important foundations for understanding the learning paths and needs of students and thus, for planning activities in the classroom. When teachers understand their students' cognitive development processes, they can predict the likelihood that a specific experience will bring about the desired learning on the part of the student. This requires teachers to be knowledgeable about how children learn and about the content to be learned, to be cognizant of children's abilities to perform certain tasks collaboratively even though they are not yet able to perform them on their own, and to be attuned to the environment and the learners' needs (Brophy, 1982; Shulman, 1986, 1987; Tyler, 1949; Vygotsky, 1986).

The better teachers know the content and the pedagogical implications of the content, the more effective the teaching will be (Lampert, 1988; Shulman, 1986, 1987). Through their own educational experiences teachers can learn to have a more complex view of what it means to teach and learn mathematics. The experiences teachers have as learners and knowers of the subject influence how they learn mathematics, and consequently how they teach it.

At the primary level, teachers' perceptions of mathematics are likely to be influenced by the amount of mathematics they know. Shulman's (1987) construct of pedagogical knowledge supports and expands Bishop and Nickson's (1983) findings that tell us that teachers "tend to follow through topics in a step-by-step approach, which possibly lacks breadth and depth and does not make use of appropriate concrete experience, because they are not confident enough in what they are doing to deviate from the narrow factual path." (p. 43). By knowing more mathematics and more about the pedagogical nature of the content, however, their teaching can become more vibrant.

Teachers who function at higher order levels of thinking are adept at diagnosing, exposing, eliminating, and correcting student misconceptions. Good arithmetic teaching includes the use of word problems which give meaning to the manipulation of numbers, respect for children's informal methods of problem solving, and the gradual and sensitive introduction of formal methods. Weaving lessons together to build upon earlier learning, ensuring consistency in structure, and balancing problem types all facilitate mathematical learning (Leinhardt, 1989; Greer, 1987; Vobejda, 1987).

In addition to the knowledge bases of learning theory and of mathematical content and processes which contribute to teachers' influences on children's mathematical learning, teachers' attitudes also influence student learning. Teachers' beliefs and attitudes determine much of what is taught, along with textbook characteristics and the composition of the class (Bell, Costello & Kucheman, 1983; Barr, 1988). Those with very positive attitudes towards math tend to have students who have high attitudes. Similarly, teachers with high math personal achievement engender high achievement among their students, although sometimes at the expense of making the attitudes of their students more negative (Schofield, 1981).

When teachers feel unsure of themselves in a subject area, they are inclined to stick to memory, convergent thinking, and simplistic questioning. Teachers with high self-concepts speak only 38% of the time, spend 24% of their time on housekeeping items, and elicit higher order thinking skills on the part of their students more than teachers who have low self-concepts. Those with low self-concepts talk more than their students (69% vs. 31% of the time for their students) and spend up to 45% of classroom time on routine and housekeeping items. These classroom behaviors are less likely to help students stretch their thinking capacities than those used by other teachers (Trowbridge, 1973).

In contrast to self-concept levels, mathematics anxiety among teachers does not necessarily inhibit their learning of effective mathematics pedagogy. On the other hand, it may effect mathematics learning in the classroom. Mathematically anxious teachers tend to be slightly more traditional in their teaching and take fewer risks; as a result their students ask slightly fewer questions (Bush, 1989). It is encouraging to note that teachers with more recent training in mathematics show less anxiety (Widmer & Chavez, 1982) and that anxiety can be reduced through mathematics methods courses (Battista, 1986; Sovchick, Meconi & Steiner, 1981). In addition, Aiken (1976) indicates that preservice teacher education courses can be used to improve teachers' attitudes toward mathematics.

Recognizing that students develop mathematical concepts by building their own constructs representative of the mathematical world as they experience it, it is important that teachers provide opportunities and guidance for students to explore mathematics in a variety of ways. In the process of teaching, teachers also must be cognizant of their own knowledge about mathematics, their behaviors, and their attitudes as they teach students. Teachers influence both the affective and the cognitive domains of students as a result of their own affects and cognition.

NCTM Curriculum Standards

As pointed out earlier, in 1989 NCTM established curriculum standards to guide the revision of school mathematics for grades kindergarten through 12. Intended to address the inadequacies of mathematics education as described in numerous reports in the early 1980's, the standards outline a vision of what it means to be mathematically literate - to be able to explore, conjecture, and reason logically as well as to use a variety of mathematical methods to solve problems effectively. The NCTM standards also create a set of guidelines for the revision of school mathematics curriculum. They are not a curriculum, per se. Curricula are expected to be developed by school personnel based on their specific needs and resources while simultaneously embracing the NCTM standards.

Undergirding the standards are goals that the educational system will develop: (a) mathematically literate workers, (b) lifelong learners, (c) opportunity for all, and (d) an informed electorate. To that end, all students should learn to value mathematics, become confident in their abilities to do mathematics, become mathematical problem solvers, learn to communicate mathematically, and learn to reason mathematically. In spite of a multitude of efforts, past reforms have not addressed these goals nor achieved significant advances in mathematical teaching and learning. The NCTM goals represent the most recent shift in the focus of mathematics education from memorizing rules and procedures in order to mechanistically find the correct answer to mathematical problems to using mathematical reasoning, conjecture, invention, and problem solving to address issues of our world.

There are 13 curriculum standards for grades kindergarten through four which identify what mathematics should be included within those grade levels: (a) mathematics as problem solving, (b) mathematics as communication, (c) mathematics as reasoning, (d) mathematical connections, (e) estimation, (f) number sense and numeration, (g) concepts of whole number operations, (h) whole number computation, (i) geometry and spatial sense, (j) measurement, (k) statistics and probability, (l) fractions and decimals, and (m) patterns and relationships. Attention to the processes of mathematics pervades the curriculum standards including the areas of computation and operations where there are expectations that students will investigate the meanings of various operations and engage in mental and calculator computations as well as paper and pencil computations.

The expectation is that schools in North America will change the teaching and learning of mathematics so that instead of focusing primarily upon computation and rote activities, there will be emphases on mathematical insight, reasoning, and problem solving. Significant attention is to be given to developing a number sense and relationships, collecting and organizing data, and applying problem solving strategies to everyday problems.

Foundational Beliefs of the NCTM Standards

NCTM's standards for mathematics curricula outline concepts and skills to be taught and learned. Their foundation is that each child constructs "meanings in the context of physical situations and allows mathematical abstractions to emerge from empirical experience . . . [which] provides anchoring for skill acquisition" (NCTM, 1989, p. 17).

In addition to the curriculum standards, in 1991, NCTM published *Professional Standards for Teaching Mathematics.* The professional standards are designed to "make explicit and expand the images of teaching and learning implicit in the *Curriculum and Evaluation Standards for School*

Mathematics, [and] to elaborate a vision of instruction that can light the path toward such change" (NCTM, 1991, p. 20). In the professional standards, four assumptions about the practice of teaching mathematics are underscored:

1. The goal of mathematics teaching is to help all students develop mathematical power.

2. What students learn is fundamentally connected with how they learn it.

3. All students can learn to think mathematically.

4. Teaching is a complex practice and hence not reducible to recipes or prescriptions.

The professional standards explicate types of tasks to be posed to students such as those that "engage students' intellect; develop students' mathematical understandings and skills. . . [and] stimulate students to make connections and develop a coherent framework for mathematical ideas" (NCTM, 1991, p.25). Teachers are invited to consider the content, the message about mathematical thinking, and the skill orientation of tasks they design for their students.

The professional standards also outline the roles of teachers, students, and various tools (such as calculators, graphs, and dramatizations) in the discourse of teaching mathematics. Annotated vignettes of classrooms using various kinds of discourse are included in the standards in order to stimulate teachers to think about the discourse in which they engage while teaching.

The other two areas the professional standards address are those of the learning environment and the analysis of teaching and learning. These two topics recommend that teachers provide the time, space, materials, and context that facilitate students' learning of mathematics and engage in ongoing analysis of the teaching and learning in order to assess the soundness and significance of the mathematical learning and the effects of the tasks, discourse, and environment.

While the curriculum standards and the professional standards state expectations and desirable outcomes, they also speak to teacher knowledge and beliefs about how children learn mathematics: "Teachers' understanding about how students learn mathematics should be informed by research as well as their own experience" (NCTM, 1991, p. 27). The professional standards recommend that professional development activities enhance the knowledge teachers have about how children learn mathematics as well as knowledge about mathematical pedagogy.

As cited earlier, it is important that children be immersed in their physical world with a variety of manipulative materials and experiences that enable them to feel, touch, explore, and replicate their surroundings. This is so in order for students to achieve the mathematical learning outlined in the curriculum standards. As a result of their experiences, students are able to develop an understanding of the physical and mathematical world and then develop ways to express their knowledge. The theoretical foundation upon which the NCTM standards are built is in accordance with Piaget, Sinclair, Vygotsky, Duckworth, Kamii, Resnick, and others.

The process is akin to young children learning to speak, read and write; by hearing others, experiencing language, and experimenting and manipulating language sounds and structures, their own language emerges. So, too, in their mathematical worlds, as children see how the physical world fits together; as they experience buildings, shapes, patterns and interconnections; as they experiment and manipulate the real world and models of it, then they can generate mathematical explanations of their world.

CHAPTER 3

METHODS: DESIGN OF THE STUDY

This study was designed to investigate how teachers' beliefs about how children learn mathematics are related to how those same teachers teach mathematics. The interrelatedness of teachers' beliefs and actions, their perspectives on their teaching, and the dynamics of practices in the classroom are conducive to a qualitative case study approach. The aim of this study is to describe how and why teachers conduct their business as they do. The goals are to provide thick descriptions, explanations, and explorations of situations and phenomena, to interpret the findings, and thus to provide a foundation for future planning and staff development.

Through a qualitative case study, it is possible to observe people at work, to delve into and query the thought processes and decisions of teachers, to engage in in-depth interviews and conversations, and to follow emerging ideas to their fruition. The qualitative researcher is able to explore the multiple realities of teachers behaving and believing in different ways (Merriam, 1988). She can "discover important questions, processes, and relationships" (Marshall & Rossman, p.43).

Ethical Considerations

Because of the nature of a qualitative study, ethical dilemmas could have arisen either during data collection or the dissemination of findings. Sensitivity to and respect for individual participant's needs and beliefs were essential to the entire enterprise both in terms of trustworthiness and propriety, and were maintained at all times. Several steps were taken to ensure confidence in the researcher, as well as confidentiality of materials and anonymity of the participants.

All participation was voluntary and all participants signed an informed consent form prior to engaging in the study. They could withdraw from the study at any time before June 15, 1991, a time when observations, interviews, and initial analyses were completed but final analyses were not completed. Anonymity of all participants, schools, and the town were protected through the use of pseudonyms in all written materials and oral reports. All University of Massachusetts regulations and guidelines for working with human subjects in dissertation research were followed.

Participants had the opportunity to review transcripts and summaries and add information and emphases. While their input was considered, it was the researcher who was responsible for final interpretations.

The Role and Background of the Researcher

For over twenty years I have been an educator working as an elementary school teacher, a curriculum developer, a teacher of adults, a college administrator, and a supervisor of student teachers. I have been a student of education both as an administrator in the workplace and as a student in the classroom. The issues of rural education, teacher education, and effective schools are of substantial interest to me. It has long been puzzling to me why many elementary school teachers and other adults in our society are uncomfortable with, if not deficient in, mathematics. And as I look at the economic and social futures of our youth, I see foundational mathematics as a critical element to survival in the next century.

As an elementary school teacher, I frequently worked with fellow teachers to encourage their students to engage in hands-on experiences. Once those teachers worked with the materials themselves they often changed their classroom teaching styles. It seemed so logical to me that children would benefit from and learn more effectively from engaging in experiments and real-life activities while learning new concepts. Like me, many teachers have some knowledge of Piagetian and constructivist philosophies, yet many had to be shown teaching techniques to match their beliefs about student learning.

After an absence from the elementary school classroom, I was perplexed to find the teaching of mathematics virtually unchanged from 1976 to 1990. Mechanistic manipulation of numbers prevailed in most classrooms I observed; real-life experiences and tasks with problem solving were rare.

Recent coursework, readings, and discussions of educational philosophy, Piaget, early learning and problem solving, curriculum development, and the teaching profession have convinced me that more information and research are necessary to understand better what relationships exist between teacher beliefs about how children learn mathematics and the ways teachers teach mathematics. I am confident that information about teachers' beliefs and behaviors can also enhance the current reform efforts of NCTM, a reform that I support.

Within the year, I have begun working as an elementary school principal; I am now working with teachers who are grappling with the design and implementation of new mathematics curricula based on the NCTM standards. The information from this dissertation is invaluable to those efforts.

My experiences in classrooms and schools and with supervision, interviewing, and curriculum development provided me with a solid foundation for gathering and working with data. I have been an administrator responsible for managing large amounts of confidential information and for writing numerous reports synthesizing information and ideas. I am sensitive to the nuances of resistance and conflict and have demonstrated skills appropriate to resolving issues between factions. The technical skills and knowledge necessary for undertaking this dissertation were in place from the outset.

As a woman working in elementary education, a field predominantly made up of women, it was relatively easy to establish rapport with the participants - students and teachers alike. For me, the research tasks are exciting and the outcomes valuable.

A Pilot Study

In the fall of 1990, I studied the teaching activities and beliefs of one sixth grade teacher and found great disparities between what the teacher believed about how children learn mathematics and how she taught mathematics in her classroom. Several significant themes emerged from these research data, partially explaining the differences between "Candy's" beliefs and actions.

Specifically, she believed

1. She needs to be accountable to the next grade for teaching all the content in the curriculum.

2. Mathematics is not as important as other areas of the curriculum.

3. During mathematics instruction she teaches in a different way than what she believes about how children learn.

4. Teachers can make curricular changes.

For this teacher, covering the content outlined in the textbook superseded mastering the content because she perceived that the next grade's teacher would judge her performance based on whether all the content was introduced to the students. This translated into the idea that the textbook was the curriculum and that it must be followed. As a result of this, the teacher read the instructional lessons from the teacher's manual and assigned practice problems from the book in the sequence provided.

By teaching math for 20 to 30 minutes in the afternoon, there was evidence that Candy valued math less than other areas of her sixth grade curriculum, each of which she spent more time teaching. In addition, the school had not reviewed, changed, or even scheduled a review of the math curriculum in several years; language arts and science had been overhauled within the past five years. Her performance evaluations did not cite any shortcomings in her mathematics teaching. This teacher had no reason to believe that mathematics was a particularly important content area that she should focus on any more than she already had.

Candy's teaching of other content areas was active and related to students' interests. She believed in providing a variety of activities to accommodate different learning styles and preferences; she gave children choices of activities and talked about the need for children to make their own connections when learning new concepts. But when teaching mathematics, she spent much time on drill, practice, and recitation. In her own words, "I teach [mathematics] in a different way than what I believe much of the time."

As a result of personal interest and a variety of professional development activities, Candy created a new science curriculum in her classroom and promoted a variety of new science activities with other teachers. She was active and creative in her planning and teaching. She also pushed to ensure that adequate materials were available to accomplish the tasks. She had firsthand evidence that teachers can make curricular changes in their own classrooms and in the school district.

For Candy, time, support, training, materials, and recognition for her efforts would help her change her teaching of mathematics. She had a strong desire to be a good teacher. She recognized the dichotomy within her work and believed she could alter her teaching to be more compatible with her beliefs about how children learn. She also would have liked some assistance from her colleagues and superiors to change quickly and effectively.

As learned from the pilot study, rich data can be obtained from numerous prolonged observations and lengthy, probing questioning. In order to get the best data, it was necessary to focus and refocus regularly and constantly on the teacher's messages and nuances, both verbal and non-verbal.

During the pilot study, an unpolished miniature of what the dissertation would entail, it was invaluable to request elaborations and expansions of responses and to revisit areas of interest again and again to eliminate and resolve multiple interpretations and to gain clarity on specific points. It was important to identify matching and mismatching actions and beliefs. And it was a detailed task to sort, resort, match, and make sense of the data.

The themes that emerged from the study of Candy - that she must be accountable to the next grade level, that mathematics is not as important as other areas of the curriculum, that she teaches mathematics in a different way than what she believes about how children learn, and that teachers can make curricular changes - were useful starting points for looking at data from the dissertation. It was equally important to be open to what the new data showed. While the pilot study served as a useful training tool for the dissertation, the research of the dissertation went far beyond.

Participant Selection

This study was an in-depth investigation with the goal of identifying commonalities as well as differences between teachers' beliefs and their behaviors while teaching mathematics. Four teachers were selected from the same rural school district in Vermont, so that they had common textbooks and manipulative materials, curriculum guides, and opportunities for staff development.

Only third and fourth grade teachers were selected, ensuring a limited range of mathematical topics within the curricula. Each of the teachers was experienced, having taught at least twelve years, and demonstrated a commitment to teaching as evidenced by enthusiasm and an investment of time in planning and preparing for their teaching activities. Selection was also based on the teacher's ability to articulate rationales, beliefs, and other thoughts related to the teaching processes. Participation was voluntary.

Students, teaching colleagues, and administrators affiliated with the teachers were asked additional questions in order to assist the researcher in corroborating perceptions and information. Their selection was based primarily on their proximity to the teachers being studied, although student selection was also based on their abilities to communicate and articulate their views.

Site Entry Procedures

The Superintendent of Schools in the selected town was contacted and approved the use of that school district for this study. The district was concurrently undergoing a review of the mathematics curricula from kindergarten through grade twelve with the expectation that the curricula would be revised within a year. The study committee was chaired by the Assistant Superintendent whose guidance was sought regarding curriculum expectations and some potential teachers for the study. Individual teachers were contacted to determine their interest in being involved in this research project and to determine whether they met the criteria of grade level, experience, commitment, and ability to articulate their ideas.

While the participant pool to be studied was identified by general characteristics and geography, the individuals were not contacted or selected until permission to undertake the study was granted. Teacher participants were identified in March 1991. Student participants were selected for the study later based on their being representative of the student population and their abilities to respond to pertinent questions.

After an informal contact by telephone or hallway conversation, each participant was formally contacted by letter with an explanation of the project and a request to sign a consent form. In the case of students, parents were contacted and they were asked to sign the consent form in concert with their child. No observations or interviews were conducted until the individual had consented to participate (see Appendix A).

Data Collection Procedures

Data were collected in a variety of ways; the dominant modes were the eighty-three direct observation of teachers while teaching mathematics and fourteen in-depth interviews with teachers, students, and administrators.

In order to unearth and understand how teachers teach mathematics including what activities and teaching strategies are used, each of the four teachers was observed teaching mathematics between thirteen and twenty-nine times over a three-month period. The observation field notes included notations of the classroom environments, equipment and materials used, attitudes expressed by the teachers with the children, the content and pedagogical techniques used, children's responses and actions, etc. Audiotape was used to supplement field notes. While recording some observations in the classroom, the researcher expanded and added to notes within the day (often within the hour) of each observation to ensure clarity and detail. The eighty-three class sessions observed totaled sixty-six hours.

Each participating teacher was also formally interviewed an hour to an hour and one-half both at the beginning of the study and at the end of the study. Twenty broad, open-ended questions were developed to guide the interviews to explore teachers' beliefs about how children learn mathematics and what factors teachers consider when planning their mathematics lessons (see Appendix B). Additional questions were drawn from the data collected during observation sessions; others evolved from emerging themes identified during ongoing data analyses. Each interview was conducted individually; total interviewing time with the teachers was approximately eight and one-half hours. Interviews were audiotaped and transcribed.

In addition, there were numerous informal conversations and chats with the teachers based on observations, prior conversations, and themes evolving from the data. Conversations with participants, recorded in the field notes, were used to check and verify information at various times throughout the study. A variety of documents also were collected and examined including school district curricula, textbooks, lesson plans, relevant school policies, and school district mission statements. Interviews and conversations with administrators also were scheduled and conducted as the study progressed in order to identify and clarify expectations for teachers regarding the instruction of mathematics.

In order to corroborate information and themes, students from each mathematics class were interviewed (with parental consent) during the study. Areas of inquiry included their attitudes, achievement levels, uses of mathematics, their preferred learning activities, and how they think they learn best.

One of the characteristics of qualitative research is the emergent design (Lincoln & Guba, 1985). This study changed and evolved based on the people and the information obtained from them at various stages of the inquiry. The researcher continually monitored the procedures being used as well as the content of the data and made adjustments as necessary to assure ethical access to maximum information.

The chair of the dissertation committee was consulted as questions arose.

Data Management

Data collected from this study were plentiful and rich. Managing the volume entailed significant organization and patience. The first step to controlling the information was to date, identify participants by pseudonym, and record as much information as possible as it was observed and collected. All original data were maintained throughout the project so that source materials were available for reference at any point. Both original data and copies were kept secure and away from the study site.

Parallel journals were maintained: a white one of field notes for data heard and observed and a yellow one for the researcher's responses, thoughts, and initial analysis. This one included what some researchers call theoretical memos. Each of these was maintained each day in the field. Notes taken during observations were expanded within the day in order to minimize blurring and distortions of perceptions and to maximize details.

All formal interviews and some observations were taped. The audiotapes were transcribed and significant information was highlighted on the written text. Both the tapes and hard copies were maintained for future reference. As a means of verifying the data, all teacher participants were asked to review the transcripts of their interviews and other printed summaries and to add comments and emphases as they saw fit. The comments and notes added by participants became part of the data. As data were culled and analyzed (see below) summaries were written and charted with reference to the primary source materials. The summaries were treated with the same safeguards as the raw data.

Data Analysis

Much has been written about analyzing qualitative data suggesting the need for continual analysis, review, and integration of new data into emerging categories and themes (Lincoln & Guba, 1985; Marshall & Rossman, 1989; Merriam, 1988). Beginning with the classroom observations, the data were reviewed for patterns, repetitive behaviors, and teacher beliefs. As general categories emerged and additional data were gathered, the data were reviewed periodically to verify, elaborate, modify, or negate the researcher's ongoing analyses.

Information gleaned from classroom observations was used to frame questions for interviews. Questions were used to draw out further information and test the researcher's initial hunches. There were modifications and revisions of tentative answers to the research questions as the research progressed.

To assist with sorting the data, broad categories of data about curriculum, children's learning, teaching beliefs, etc. and narrower themes identifying commonalities and differences among the teachers' beliefs and behaviors were identified and coded using colors for the content and symbols referring to the data sources. As tentative categories with unique characteristics were identified, they were described in the researcher's journal. Wall charts served to bring similar pieces of data together to enhance and refine the themes and to decipher sets of distinctive features which made them mutually exclusive.

The continuous spiraling process of analysis, review, and incorporation of new data occurred throughout the data gathering and afterwards in order to feed the inductive analysis process which led to developing categories and themes. References to themes in the literature and identified in the pilot research generated some initial categories and more refined themes; as data were collected, additional themes emerged leading to final assertions.

Tentative assertions were compared with the data. Alternative explanations were sought and tested. A peer de-briefer and the dissertation committee chair were used periodically to challenge and corroborate tentative and final categories, themes, and assertions. Only after the data were manipulated, analyzed, and tested in numerous ways were the most plausible assertions about the relationships between teachers' beliefs about how children learn mathematics and how teachers teach mathematics summarized and described.

Assurance of Trustworthiness

The data were triangulated throughout the study. The use of a variety of qualitative techniques including observations of classroom teaching; in-depth structured and free-form interviews with teachers; examinations of teachers' notes, textbooks, and other teaching materials; conversations and interviews with students and administrators; and a prolonged engagement with each teacher helped to ensure trustworthiness of the procedures and substantive findings of this study.

By spending three months observing and engaging in conversations in the school, there were ample opportunities for trust to be established between the researcher and the participants. This study was buttressed with a researcher's journal of the natural history of the study, participant member checks whereby participants reviewed and added to the study in progress, and periodic peer de-briefing before the final dissertation report was written.

CHAPTER 4

RESULTS: DATA DESCRIPTION

Participants and School Setting

The research for this study was undertaken during the spring of 1991 in one of the four elementary schools in a district which serves fewer than 2,000 students from kindergarten through grade 12. The Kirby School serves the 400 children in grades three, four, and five in the Vermont industrial town of Winston. Two third grade teachers, Susan and Mary, and two fourth grade teachers, Oliver and Faith, (all pseudonyms) were the participants in my investigation.

The first time I entered the Kirby School, originally built as the town's high school in 1895 and sitting partway up one of the hills overlooking some of the empty brick factories built alongside the river during the last century, I was struck by the massive brick structure with its two wings aside of the central face, each two and one-half stories tall. Walking up the granite steps to the double green doors, I had a strong feeling of entering an old-time school. The doors swung easily from years of use, the ceilings were high, and the brown linoleum floors worn through in several spots, most notably at the bottom of each staircase.

In the south wing, home to the fourth grades, the rippled hardwood floors creaked, the wide staircases easily accommodated four abreast, and in the classrooms, the floors showed oval scars from the screwed down desks of past generations. The black slate boards stretched across the walls of the large classrooms, leaving spaces only for bulletin boards and closets. Bookshelves added over the years in each room teemed with paperback books, games, encyclopedias, reading books and textbooks, writing paper, boxes of science equipment, and supplies for making projects. Fluorescent lights adorned the classroom ceilings, lowered to the top of the tall doorways and windows. Unpainted plywood tracks held windowshades, recently installed over the huge double hung windows to help save the heat spewed from the large radiators in each room.

The short flight of four stairs connecting the south wing to the central section of the building was half covered with a rubber-matted ramp, showing the adaptability of the old building to newer codes and student needs. In the hallway, an aide guided a blind student up the stairs to a resource room, a chubby boy skipped down the central staircase on an errand, and quiet murmurs were heard from within the classrooms as students tended to their academic chores.

Shortly after a bell rang, one class of fourth graders flowed into the hallway to get their coats and lunch bags, lined up quietly, and upon the "let's go" of their teacher, headed down the stairs to the basement cafeteria with a muffled chatter and brisk pace.

In the principal's office the secretary, trying to type a newsletter to parents, was interrupted by phone calls, a student being signed out by her mother in order to get her cast changed at the doctor's, a teacher coming in to say her intercom did not seem to work, another phone call, and the principal whisking by from the hallway to his inner office asking her to contact a child's grandmother. With a calm smile, she turned to ask how she could help me. I told her my business of observing teachers and indicated I was waiting to see the principal with whom I had an appointment. My introduction to the Kirby School was warm and comfortable; just like an old leather slipper, lost in the back of a closet for a few years and found to fit just as well as the day it disappeared.

The ambiance of the school was one of respectfulness and seriousness; students had a purpose to their activities and a focus on pride and cooperation. Teachers referred to the effort and time they had put into helping students work cooperatively and respect one another's strengths and values. Classroom rules such as "Don't interfere with another's thinking and learning" were posted in nearly every classroom and were regularly invoked whenever an infraction occurred. With great admiration, the teachers referred to the Planning Room as the place where errant students were effectively helped to establish plans to get them through their ill-behaved days.

I had been invited by Faith to join her in the teachers' lounge for lunch and was introduced to Barbara (a retiring fourth grade teacher), Oma (one of the teachers I interviewed briefly on the phone only to discover she did not teach math to her fourth graders), Connie (the Planning Room aide), and two or three other teachers as they came and went with their brown bag lunches. The lounge was equipped with a telephone, refrigerator, microwave oven, three side tables, one wooden chair, a sagging vinyl couch, and six vinyl chairs with varying states of sprung coils.

As teachers in the same school, Susan, Mary, Faith, and Oliver were accountable to the same several-years-old curriculum outline. In the school district, the Addison-Wesley *Mathematics In Our World* textbooks were recommended. Teachers determined when and if they needed to replace their textbooks with newer editions and ordered their own materials and supplies for their classrooms based on their needs as they saw them. There were no specific guidelines on how to spend the $300 to $500 available in each year's budget for each classroom.

Each of the teachers in this study had worked in the Winston School District for at least five years, Faith for seventeen years. All of them were in their mid-thirties to mid-forties and had elementary school-age children of their own. Three of them had children in the district; Mary lived in another town where her daughter was in the second grade. Although Oliver and Mary both worked with special education children in the past, all had chosen to teach in regular education classrooms and were so employed at the time of data collection.

Susan was a shy, quiet third grade teacher who at first was unsure about having someone spend time in her room. She liked the idea of being part of a study, but was torn by the fact that she felt nervous whenever anyone was in the room. After thinking about it overnight, she decided that it would be good for her and for her students to have another person in the room on a regular basis. In her view, her role as teacher was to make learning fun for students. She enjoyed doing art projects; one of her re quests when she moved from teaching first grade to teaching third grade was that she have a sink in her room. Her room was cluttered with various projects in process; her bulle-tin boards remained the same throughout the three months of observations; the students sat in rows facing the front of the room, and she spent time during nearly every observation reminding students to be quiet and pay attention.

In contrast, Mary appeared comfortable with herself and relaxed with her third grade students. She had a bounce in her voice and was able to cajole students into expected behavior. If she was pleased with their behavior she was quick to tell them; likewise if she was disappointed in her students, that was quickly communicated. The room had different student work displayed on a regular basis such as solar system mobiles, tangram solutions, stories, and water cycle charts. Student desks were rearranged every month or so depending on the nature of their studies and the social interactions of the students.

Oliver, tall and slim, team taught fourth grade with his wife, Oma. His two sections of math were usually lively and incorporated students' out-loud thinking in the teaching process. For him there was a little time for play and a lot of time for work. He used exaggeration, humor, and personal anecdotes to spark students' interests. For example, after spring vacation he had the class calculate the cost per hour of one child's tan based on approximate airfare, hotel, food, entertainment costs, and time spent on the beach while reviewing addition, division, and averaging throughout the problem solving exercise. Oliver had taken a course in cooperative learning techniques and had set up the room so that students sat in groups of four, enabling them to work together when appropriate.

Faith's fourth grade room also was arranged with groups of desks clustered together. School work was serious business and students were expected to do their own work quietly. Her broad and frequent smile showed a caring for each child just as her frown indicated when a child was out of line. The room was filled with books and boxes of work sheets. With seventeen years of teaching fourth and fifth grades, Faith had her eyes set on moving into an administrative position. In the past year, Faith, a single parent, had taken three courses for her master's degree and established a Parent Center at the school.

Beliefs, Associated Factors Considered When Planning, and Related Behaviors

The data reveal four beliefs common to the four teachers in this study:

1. Children learn mathematical concepts by manipulating or visualizing concrete material.

2. Children learn arithmetic through specific sequenced steps.

3. Children learn mathematics through practice and repetition.

4. Children learn mathematics best when they feel good about themselves and experience success in mathematics.

Associated with each belief, each of the teachers had one or more factors they considered when planning their mathematics lessons. They also demonstrated a variety of behaviors including classroom activities and strategies concomitant with, although not necessarily congruent with, each belief.

First Common Belief: Children Learn Mathematical Concepts by Manipulating or Visualizing Concrete Materials

Teachers' beliefs were evident in their materials selection as well as their teaching actions. One belief common to Susan, Mary, and Oliver was that **children learn mathematical concepts by manipulating concrete materials.**

When asked about how she thought children learn, Susan was quick to respond in terms of where her third grade children were.

> Some children are still functioning, actually a great majority of them are still functioning at the concrete level, so they really need to start out with the manipulatives and they need to do a lot of seeing how that works, how each concept works, why it is the way it is, rather than just going on to the pencil and paper activities. (interview, 5/10/91)

Susan believed manipulatives were very valuable for introducing concepts to children so they understood the concept before being asked to use the algorithm or solve mathematical problems. As she planned for teaching her students multiplication, Susan considered her students' needs for concrete manipulation.

> I use the manipulatives a lot for introductions to something. For instance, when we started multiplication, they didn't know that they were doing multiplication for the first week or so. What I did was I taught them through using the tiles and beans in cups and we worked with grouping and we worked with patterns in the grouping and we see how they all work together. Then when we finally did a chart I asked them to look at it to see if it reminded them of anything. That's when I was looking for the realization that they had been doing multiplication, Some of them got this big "uh huh, this is what we've been doing." (interview, 4/3/91)

In practice, however, Susan rarely employed learning activities in which children used manipulatives. Over the 20 observations I conducted between March and June in Susan's classroom, manipulatives were used only once. On April 22, when she was introducing the use of a division radical, or bracket, Susan used tiles on the overhead projector and each group of three students had a quantity of tiles. Susan showed the use of the division radical and remainders by showing 7 rows of 3 tiles under the radical sign. She then asked the students to try to make a perfect rectangle under their division brackets using 26 tiles. Various answers of 4 x 6, remainder 2, 3 x 8, remainder 2, and 5 x 5, remainder 1 were offered by the class. At the end of the lesson, Susan asked,

> Do you think using this bracket will help you when you have a problem when you have something left over? I'll

show you another way to do problems with remainders that's faster another time. (observation notes, 4/22/91)

During this lesson, students explored ways to divide a quantity into equal sets and that sometimes there are remainders. She did not explain the concept of division as splitting the full quantity into given sized quantities as determined by the divisor. Nor did they discuss the initial focus of the lesson, reasons for using a radical sign in division, i.e., that the radical permits the recording of remainders and that the divisor is the determinant of the size of the sets.

She followed up with a summary of the day's lesson:

> Now let's review what we did today. What did we talk about? (remainders) Did we talk about relating multiplication and division? What did we say about multiplication and division? What about multiplication? What shapes did they make? What about remainders? Did they make a rectangle? What does that tell you about problems with remainders? Right, they don't make a rectangle. (observation notes, 4/22/91)

Although Susan believed that concrete manipulative materials were useful to students' learning, she rarely planned ways for students to learn through that mode. It seemed difficult for her to help students make appropriate connections between concrete operations, the related mathematical concepts, and the symbolic notations.

Like Susan, Mary believed in the value of manipulatives.

> I use tiles so they can see. I really think that they really need to see what they are trying to figure out so it becomes more meaningful to them. . . . I feel that they really need to be able to see and move things around and then they can go into doing the pencil and paper. And that becomes meaningful to them because they can picture the piles of tiles, moving them around. . . . Whenever I introduce anything new they are either using beans and cups or tiles, because I believe they really need to see what they are doing and understand it so it becomes more meaningful to them, to know that they are really dividing, really breaking up a total number into sets. . . . I do believe that things need to be concrete and then you can expand. (interview, 5/1/91)

As she planned her math lessons Mary was somewhat ambivalent about the use of manipulatives and tried to balance what she believed about their value to children and the effective use of them with her third graders.

> I would really like to. . . have more manipulatives around. As a resource teacher I used a lot of them. I find it hard to use them with so many children at one time. (laughter) That is the one thing I guess coming from a special ed room to this room is that there are so many children at one time. I am still getting used to that. The noise level has taken me three years to get used to. I want to allow the children to be able to talk to each other and to support each other, but it has taken me a while to be able to do that and feel comfortable doing it.
>
> I hate beans in cups, counting with beans in cups. I do that at the beginning of the addition and subtraction when it is introduced in the textbook when there is something new. I bring out the beans and cups but I will use them for a day or two and then that is it. I could never use those all the time.
>
> Yeah, I know it's good for them. My children have not really needed it [manipulative activities] much, so I don't use them much but I do use it. I think they get a lot of that when they are younger. Now they are ready to . . . my feeling is they love using the text book. It makes them feel big. (interview, 3/21/91)

Nevertheless, in practice, Mary's children used manipulative materials a great deal. For example, when students were working on division problems involving one digit divisors, two of the children helped themselves to some tiles to assist them in finding the answers to the problems. Others were using their fingers to find the answers or the less tactile and more visual means of making hatch marks on paper. When I mentioned to Mary that I had noticed some children using tiles, she indicated,

> At this point if they feel they need them [tiles] they are going on their own. I don't think I have said, "go get some tiles and work it out," for the last couple of days. They are doing that on their own. (interview, 5/1/91)

She went on further to explain how she had found the tiles useful for one child (one of the more successful learners in the room) when he was working on simple division.

> Now with Joe, you know when they had to do the work sheets on the last page they had the remainders? I knew he could do it, but he couldn't see, he couldn't figure it out in his head that 20 divided by 9, he couldn't say 2 X 9 is 18 with 2 left over. It took him two days, really maybe even three. He struggled with that but I didn't want to tell him how. I let him struggle and he was building them. He got like through the first column and maybe part of the other column and he still hadn't caught on. So I showed him the pattern if you multiply these two numbers it comes to this and you add this and then he goes, "Oh, I get it!" But he had to go through that building, I think, first so he could picture those tiles and then see that you added on the remainder, what we call the leftovers. It was at least two days. (interview, 5/1/91)

Although Mary indicated she had difficulty with the noise level of manipulative materials in the classroom, she found ways to meet her beliefs about how children learn by accommodating individual student needs without the clatter of the whole class's use of them at once. As illustrated by Mary's work with Joe, she planned and implemented individual activities based on her belief that children learn through the use of manipulatives. Those who needed the concrete materials were encouraged to make use of them until they were ready to make the transition to symbolic notations.

Oliver, who taught fourth grade, theoretically supported the notion that children learn by manipulating concrete objects while he was perfectly open about the disparity between his beliefs and practice.

> What I do in theory is to try to bring in as many concrete things as possible to expand their [the students'] horizons. That is theory. Sometimes the press of trying to reach a certain point by mid-year and by the end of the year short circuits that theory. (interview, 5/9/91)

He acknowledged some of his frustrations when planning for mathematics classes. He believed he did not always meet the needs of some of his fourth graders

> . . . who seem to be very low and almost on the stage of concrete operations. . . they need time. Sometimes they need not to be on what they are on, but to go way back to very elementary levels and do hands on things over and over and over again.

> There's things that mitigate against that though. Sometimes it's the kids themselves. They don't want to be doing playtime things and they understand that their colleagues are doing things in division and multiplication and they want to be there - they don't want to be behind and yet they have very obvious - they look around and see that in fact they are behind. So that mitigates against going back and forming that good baseline. As do parents: "What do you mean my kid is doing addition and subtraction with blocks!" Finally there is my own need to sort of try to keep them up to pace as much as possible. (interview, 5/9/91)

Groups of three to eight children generally worked together on the same assignments after Oliver had introduced the topic or concept and had written the assignment on the board or in the children's folders. May 7th was a typical day in Oliver's mathematics class which showed how he incorporated the visualization of manipulatives in his teaching:

> Oliver requests the students to please get in their math groups which they do quickly. He works with the eight students sitting closest to the blackboard who are clustered at their desks in two groups of three and a group of two.
>
> After requesting the group to open their textbooks to p. 234, he asks them what mental math is. Following their response of "Do it in your head," he draws ten fruits on the board. He indicates he wants to put the fruits into five groups, and has a child go to the board and draw circles around five groups of two fruits.
>
> He asks, "If I had one group, I would have how many?" After a child responds, "two", he writes 1/5 = 2 on the blackboard.
> -"If I had two of the five groups, how many would I have?"
> - "Four," responds the group of children together, and he writes 2/5 = 4.
> - "If I had three of the five groups?"
> - "Six," and 3/5 = 6 is written.

- "Four groups?"
- "Eight," at which point Oliver writes 4/5 = 8.
- "I didn't see you do that with pencil and paper. How did you do it?"
- "I just doubled the numerators," said one child. "I just add 2 more and add 2 more and add 2 more," said another.

Oliver went on with the lesson increasing the whole to twelve fruits and finding 1/4, 2/4, and 3/4 of those fruits, at which point the first responder said, "My rule doesn't work."
- Another child offered, "Maybe it works for odd numbers, but not for even." Oliver urged students to check it out to see if it works for other odd denominators.
- A third child says, "I see a pattern," while a fourth tells everyone "If you triple the number, it'll work."

Oliver summed up the discussion saying, "For fifths, we doubled it, and for fourths, we tripled it. We may be onto something. Let's try twelve divided into three equal groups. That's how many in each group?" They draw their groups, talk about, record 1/3 = 4, 2/3 = 8, and again review what they have found with their three various groups of fruit. Oliver then exclaims, "I'll be darned, there is a pattern!" (observation notes, 5/7/91)

Over and over again, Oliver used visual assists with the children. During nine of the twenty-nine observations I made, at least some of Oliver's children were using concrete objects to help them make the connection between the physical world and the mathematical symbols. Most other class periods involved some illustrations on the board or on paper to help students make the transition from the concrete to the symbolic. Whether it was drawings on the board, cardboard fraction circles, beans and cups to help some children find groupings, tiles to make multiplication arrays, or cutting up egg cartons to show various ways to make fractional parts of the number twelve, he found ways to help the children see and understand the concrete foundation and concept which the mathematical symbols were designed to represent.

As one of Oliver's student's said, "When he explains, he doesn't just talk about it; he shows it and does it". Later the same student explained, "Having someone explain it to me and using chips and stuff like that helps me. Chips and tiles and beans and stuff." (interview, 6/10/91)

The other fourth teacher, Faith, held slightly different beliefs, indicating that manipulatives were not so useful to her and her students as she taught fourth graders:

> I am not a firm believer that you have to have "things" in order for kids to be successful in math. I don't know necessarily the connections are made with these things in their hands. (interview, 3/26/91)

But, according to Faith, children learn mathematics by **visualizing** it. She gave vivid details about her teaching as she explained the visualizations that helped her fourth graders understand fractions.

> Well, if it is word problems, we draw them out. They have to see it in order to understand it. Today, like in fractions, we spend time drawing these on the board and drawing it out and they suddenly - I knew they understood it because suddenly they were! I was putting pictures on the board and they were able to give me all the fractions. And then I was putting the fractions and leaving out numbers here and there and they could fill them all, see it in the pictures.

> I think that kids at this age need to see things. They have got to have a picture in their mind. I think this age level has to be able to picture it and they have to either act it out - if it is a behavior problem they act it out. And if they are acting it then it is real to them and I think that they can see it, it is real to them. Abstract thinking is not easy at this age for most children. . . . They have to see it and feel it and touch it. (interview, 5/23/91)

Since Faith believed they need to "see it and feel it and touch it", she planned and taught her lessons using the existing physical attributes and materials within the room.

> I don't have a lot of things in my room, I guess, but I do a lot of pointing out things that they can see in the room or we draw things. Like right angles - when we talk about right angles, you know they take a long yard stick and they walk around the room pointing, drawing out the right angles they can see in the doors or the windows. We talk about polygons; they draw them for me. We do perimeter. We can measure the room or we can measure the edges of our books or around our desks, things like that. Things that we have in the room. (interview, 5/23/91)

Although teachers varied in their intensity and commitment to using manipulative materials, or visual substitutes for them, each of the four teachers demonstrated belief in the value of and use of concrete materials to help students gain conceptual understandings of mathematics. Through beans and cups, tiles, tangrams, hatch marks, pictures, and examples within the classroom, teachers encouraged students to build from the concrete world in order to create their own symbolic algorithms and calculations during mathematics class.

As these teachers planned their mathematics lessons, some consideration was given to their beliefs about the value of manipulating or visualizing concrete materials. But time, noise, student attitudes, availability, and the strength of the belief controlled their uses in the classrooms.

Although Susan espoused a strong belief in the value of concrete experiences through the use of manipulative materials for third graders, she used them significantly less than either Mary or Oliver, both of whom indicated they used them less than they thought was best for their students. Faith's use of visualization of concepts was limited to geometry and fractions. For none of these four teachers were manipulatives a primary part of the teaching/learning process.

That manipulatives occupied only a supportive role in teaching mathematics was not a function of their scarcity. There was a variety of manipulative materials available in all these teachers' classroom: tiles, cups and beans, counters of one sort or another, tangrams, geoboards, and cuisenaire rods were evident to some degree in each room. It was also clear that teachers could borrow from each other if they so chose.

Teachers did not discourage the occasional student who initiated the use of manipulatives to help envision or solve a problem. Those few instances observed indicated that at least some of the students had had experience using manipulatives in their mathematical work. In the classrooms of the four teachers studied, however, student manipulation and visualization of concrete materials ranged from occasionally to nearly never, even though these teachers believed that manipulating and visualizing were effective ways for students to learn mathematics.

Second Common Belief: Children Learn Arithmetic Through Specific Sequenced Steps

A second belief common to the four teachers was that children learn arithmetic through specific sequenced steps, building new mathematical knowledge upon previous knowledge. Both planning and teaching behaviors related to sequential mathematical learning indicated a heavy reliance on the Addison-Wesley textbook for the sequence of arithmetic topics taught to students. Non-arithmetic mathematics, such as time, geometry, and measurement, had no apparent sequence or ties to other areas of mathematics.

Oliver depended extensively on the fourth grade mathematics textbook for the sequence of his mathematics lessons:

> I have a great deal of respect for the text. I have the Addison-Wesley texts that seem to be very well constructed. There is a lot of thought put into what goes with what and how to teach it. [There are] a couple of workshops I have attended this year with the publisher. They seem to know exactly what they are doing and why they are doing it and how things fit together. There seems to be a rhyme and reason to it. So I would say the text itself with all the ancillary activities and materials suggested that go with it could probably build a substantial program. (interview, 4/2/91)

> Later, Oliver reiterated, "The text drives a lot of the sequence. . . . The sequence is pretty much determined by the textbook. . . .I follow the sequence that is in the book. It has been well researched from my perspective. . . .I don't see any value in taking what I consider a well researched book and reinventing the wheel. (interview, 4/2/91)

During the three month period I observed Oliver teaching his math classes, I never saw him deviate from the textbook's sequence other than for review or to capitalize on a real-world problem that presented itself that day. He planned his lessons based on what came next in the book, sometimes using his own expansions of the material as it was presented.

> The text drives a lot of the sequence and depending on the success of the lesson, I would either dig deeper, if we were not successful with the lesson that day, or I would piggy back two lessons together. . . . They [my lessons] are planned in theory on the success of the previous lesson and the sequence is pretty much determined by the textbook. (interview, 5/9/91)

An example of Oliver's putting his own twist on the text was when teaching long division, he taught the students the steps as <u>D</u>ivide, <u>M</u>ultiply, <u>S</u>ubtract, <u>B</u>ring down the next number, otherwise remembered as <u>D</u>unk <u>M</u>y <u>S</u>illy

Brother. Although Oliver grouped students by the pace at which they went, each group proceeded through the book, doing assignments in the book in the same order.

Faith's mathematics curriculum followed the 1978 fourth grade Addison-Wesley textbook, supplemented with a few additional activities.

> I stick with the text except for like telling time isn't in there, measuring isn't in there, there is not a lot of measuring in there so when I do telling time - and making change isn't in there anymore. . . . When we get off on the concepts we expect kids to know [based on the curriculum guides] they are all not in there, so no I don't, I kind of use my own things. I do stuff together in a group on the board. (interview, 3/26/91)

During Faith's classes, there was a predictable routine. Each group's assignment (invariably the next page or two in the book) was on the board and the children worked quietly in their books doing the problems, raising their hands when they needed help.

Faith called groups together every three or four days to introduce the next concept from the book prior to their working all the problems in the chapter and usually half the review problems on the pages in the back of the book. When a student completed his or her assignments, there was a box of related worksheets in the back of the room to provide further practice.

When asked about planning, Faith responded in a way that suggested she did not spend much time thinking about it:

> Correcting [takes] more time than planning. . . . It depends on - you see I've got to juggle my time because I put in time with the Parent Center, I have my own kids. . . . Then I have my own course stuff to do. (interview, 3/26/91)

Faith followed the book for daily work, except when she decided to teach non-arithmetic concepts.

> I just take a comfortable break when I think that they are not involved in something really new, when I feel like maybe we've come to closure on a particular concept or, not a concept but a computational skills level. . . . I just kind of close down for a week and do something else and then I review a lot. (interview, 3/26/91)

Twice during the twenty-one observations I made, Faith worked with supplementary concepts, measuring using quarter inches and identifying symmetry. In these two instances, there was no apparent connection of the tasks to any arithmetic or previously taught concepts.

For Mary and her third graders, the mathematics curriculum and sequence were dictated by the book because she believed the textbook presented most of the mathematical topics in a logical order. It was a primary vehicle for determining the scope and sequence of teaching mathematics:

> We have a curriculum. I don't look at it at all. I follow what is in the math book. The school district has their own set of tests that the children need to pass in third grade. I know what those are so I am sure that I teach those skills. . . . I found [having a test] was helpful. It gave me some guidelines. The thing I use most is my textbook. . . .I used it [the textbook] a lot in the beginning [of the year]. I think maybe as I feel comfortable teaching I know I use it less and less. . . I wanted to be sure I was being responsible and covering what I was supposed to cover. (interview, 3/29/91)

Although Mary's tendency was to use the textbook for the sequence of mathematical concepts taught, she also believed in varying the children's experiences with games to reinforce concepts and with different enrichment activities such as using tangrams to build geometry concepts.

> [Tangrams] no, that is not out of the textbook. That's because I felt that they had enough of the multiplication. We did most of the work from the textbook. . . and I felt they needed to change and so we did that. And this week they will still be doing some multiplication but also reviewing a little of the subtraction and also working with telling time. Next week I have a little bit more measurement and then next week we will go back and really work on the multiplication. (interview, 3/29/91)

While Mary believed sequence was important, she also believed variety was important to keep children's interest from lagging and planned so that she taught other mathematical concepts interspersed throughout the year.

Susan's approach to the third grade mathematics curriculum was less beholden to the text than the other three teachers studied:

> When I started teaching I used to use a text all the time, and although it taught the children what they need to know, I don't think it was necessarily the best way to deliver what they needed. I don't think it gave me the freedom that I needed either. In looking at what I needed to cover for the year, if I just make sure I cover the skills and do them so that I feel the children for the most part are enjoying what they are learning, then that is probably the best method of delivery at least for me. (interview, 4/3/91)

Later Susan again stated her approach to planning and teaching mathematics:

> You don't have to teach math from a textbook. You take what you know, go in the direction you want to go, think about the goals you want to achieve, look for the outcomes, and you are all set. That is the whole nuts and bolts. (interview, 5/10/91)

Although Susan did not use the textbook, in the three months that I observed her teaching her third graders, she spent the major part of her time teaching two essential concepts in the general order presented in the third grade curriculum: multiplication facts from zero to ten and division, the inverse of multiplication, using factors from zero to ten.

As a secondary and unrelated mathematics activity, Susan's class spent one day a week with her in the computer lab using the program Logo to develop concepts of plane geometry including shapes and angles. This part of the mathematics curriculum was viewed by Susan and her students as supplementary, unstructured, and unrelated to the multiplication and division of their in-classroom work.

These four teachers generally believed sequence to be important. Their planning was usually drawn from the text book which they believed to be appropriately sequenced for their students and from the additional materials and opportunities available to them. Review and non-arithmetic topics were addressed in no particular sequence. For Susan, Thursday was computer day so Logo was used; Faith reviewed concepts on Fridays and did measurement and symmetry when one of the groups reached the end of a chapter in the book. Mary felt the students needed a change so they took a week's break to study geometry with tangrams. Only Oliver followed the book in its entirety without sidetracking.

While each of the teachers espoused the importance of learning mathematics through specific sequential steps, three of them demonstrated only partial commitment to this belief by their planning and teaching behaviors. Each of the teachers followed a sequence for the computational mathematical concepts they taught, yet none of them thought of the mathematical concepts of time, money, measurement, and geometry as being related concepts that could be taught within other segments of the mathematics curriculum. There is no evidence that any of the teachers related the conversion of hours to minutes or feet to inches to the similar concept of converting tens to ones as is done in subtraction when borrowing. Nor was plane geometry viewed as relational as are multiplication, division, and fractions. Thus, the teachers' development of the mathematics curriculum in their classrooms was based less on the belief that concepts should be logically sequenced with related concepts or with student development than on the belief that what is presented in the textbook, or has been traditionally presented in textbooks, is a sacred sequence appropriate for third and fourth graders.

There was no recent mathematics curriculum for the Kirby school. Teachers picked and chose items from the old curriculum for teaching ideas and materials, but did not regularly use it as a basis for their sequencing or planning. The calendar dictated when some of the "incidental" concepts such as geometry, time, money, or measurement were taught, either because there were a few days before or after vacation when variety would be an asset, or because it was nearing the end of the year and the teachers knew that certain concepts were expected to be taught before students go on to the next grade level.

The pressure of what the next grade level's teacher expected students to know also influenced what the teachers teach. This "academic press", as Oliver referred to it, did not influence sequence as much as content and pace. Seldom was time taken to appraise students' mathematical progress and prowess as a basis for the subsequently structured segments of the curriculum. The typical pattern of the teachers was to turn the page of the textbook to determine what was next.

Third Common Belief: Children Learn Mathematics Through Practice and Repetition

Practice, the performance of a skill to develop proficiency, and repetition, the review of a skill already developed, were methodologies all four teachers believed were important in order for students to learn mathematics.

Mary believed review benefited students:

> I still feel that children can go over their material because. . . they always pick up something new. . . . We all start out with a review. (interview, 5/1/91)

Mary planned her teaching so that her students learned a concept and followed it up by practice for one or more days. Repetition was limited to review at different intervals to ensure the students' retention of the concepts and skills. She interspersed review of multiplication facts and timed tests with the practice of one and two digit multiplication, addition and subtraction of money, tangram patterning, single digit division, use of calculators, units of measurement, and telling time. She reviewed previously taught concepts and skills at least four times (3/19, 3/21, 4/4, and 5/22). In doing so, she provided the students with a review of the strategies for doing the algorithms as well as additional practice:

> Now I have an end of the chapter review. I really want you to do this. It has a little subtraction and a little multiplication. Now, you'll have to think about

regrouping. If there's a zero in the middle, what do you have to do? (observation notes, 4/4/91)

Susan spoke a great deal about her belief that she needs to teach skills and that third grade children need to practice and repeat skills using a variety of modes in order to accommodate different learning styles of students.

> There are some children who are very visual and I could talk all day and explain all day, but if they can't see it then they wouldn't get anything out of it. Then there are the auditory learners who pick up everything very nicely because we as teachers have a tendency to talk quite loud. (laughter) And talk and talk and talk. What I have also found is that with some kids, even though we may explain it in one certain way, they're not seeing it or hearing in the same way that you are verbalizing it, so I found that it is important to restate and check to make sure they are getting what you are saying. . . . I try to combine all of those as much as possible, just to make sure I cover everybody's learning styles.
>
> I have also found that sometimes I will have another child explain simply because they might have another way of looking at it. As far as I am concerned regarding math there isn't really a wrong way to do it. There are many ways to do it. As long as they are coming up with the right answer and you have a logical explanation for the way you solved it and it follows with pattern, then it is a good way to solve. (interview, 5/10/91)

Susan also spoke of the need for practice and for students to know their facts.

> Practice is good; it's really important. . . . At the end of the year I would like them all to have multiplication facts solid. (interview, 5/10/91)

Later, she talked about how she planned most of her lessons:

> When I am planning math, I think of it in components. The introduction which I usually try to do is sort of a

> warm up. . . . If it is a new activity then it is more of a preteaching type thing and then also why we need to know this. Then there is a teaching or practice session depending on where we are, and then a closure. (interview, 5/10/91)

Nearly every day during this study, Susan's beliefs in a warm up, task variety, and multiple modes of practice and repetition were borne out in her teaching behaviors; the students practiced multiplication and division problems orally, on worksheets with games or answer searches, on the board with illustrations, or silently using fingers to communicate the factors and products. Students used several strategies to figure out the answers including hatch marks, counting on the fingers, skip counting, and cross-lines. Each day the students had two or three different activities, each focusing on the same concept - one digit multiplication and division.

Susan also spoke about her planning this way:

> I plan for the week. I look at the overall goal for the week. . . . As I'm going along, I'll tailor the plans and sometimes if it is one of those days where it looks as though they are not going to be able to handle the activities the way I had planned doing them, I'll go "Whoops, ok, let's rethink how we can do this." Then I will change my plans within a few minutes even, if necessary. (interview, 5/10/91)

Susan's classes reflected a significant amount of changing of plans within minutes as she mused over which game to play, how to revise the rules to make a game more interesting, or searched out a new ditto sheet. The changes, however, were within the topic of the week, ensuring practice and repetition.

Oliver distinguished among his students about how much practice and repetition was appropriate for students, based on their learning needs in mathematics.

> I would say that for the great middle and down that . . . repetition was very beneficial. However, from above the middle up that repetition could become old very quickly. It could become mundane even. (interview, 5/9/91)

Oliver also had a different approach regarding how students might gain additional practice. In his classes, students were subdivided into groups for their mathematics instruction. Each day Oliver taught one or more groups while the other students were practicing their skills. Oliver also believed that the fact that different concepts and skills were being taught to the different groups meant that the students were exposed to concepts that they were not necessarily working on themselves. For many students who eavesdropped on other groups, this meant that there was reteaching or review for them when they were also focusing on their assignments.

Faith believed very strongly that practice and review were the best way for fourth grade children to learn mathematics. In order to help students connect their current learning with previous foundational concept development, approximately once a month, Faith took a day to review concepts taught earlier in the year, using a game or team competitions.

> We do facts tests, the times tests a lot just to review the facts. I really feel that at this level that if you don't review and don't see it and use it, that they lose it. So that is what I do is constantly review and review. So if I had more time that is what I would do, I would use some for review and then maybe even have one block just for the computational skills and then another block just for introducing a new concept and then reviewing. (interview, 3/26/91)

> With a lot of practice. I really believe in a lot of practice. Children at this age don't learn something forever without a great deal of repeated, repeated practice. . . . It is a constant review of facts, of times tests. . . .Children need to do a lot of review in math, practice. (interview, 5/23/91)

One of Faith's students explained his mathematics' learning in an interview:

> Interviewer: What are the things that help you "get" things, help you learn math?
> Student: I just listen to what the teacher says to - how to do it, and try to remember all that.
> I: Is it hard to remember?
> S: Some of it is and some of it isn't.
> I: Are there tricks or are there things that help you to remember that, things that you do to help you remember?
> S: No.
> I: Just practice?
> S: Yes. (interview, 6/5/91)

Throughout the three months' observation period, Faith's teaching behaviors included reviews of previously studied mathematics concepts with the whole class at least five times (3/22, 4/26, 4/30, 5/9, and 5/29). Some reviews were in the form of tests, others were games.

> You people would never have remembered everything from September without review. Sometimes we do this with

> games. So today we'll have a game. (observation notes, 4/26/91)

As Faith taught her fourth grade groups two-digit division, averaging, fractions, and some geometry concepts, she followed the textbook carefully. Students usually were assigned every problem in the book, the practice problems in the back of the book, and extra worksheets for reinforcement. Students who completed their work more quickly were frequently offered challenge worksheets to help them practice their mathematics even more.

She also supplemented the textbook with worksheets:

> They have given me a couple of worksheets to go with the book, but it is not necessarily enough because they still need drill on it. (interview, 3/26/91)

Belief in the value of practice and repetition was common to all four teachers, although it was planned for and carried out in different manners. For Faith's students, practice and repetition occurred nearly every day as they did pages and pages of problems and then reviewed concepts every few weeks. Oliver's students received extra practice as they peered over other's shoulders as well as when the book had review pages in it. Susan's class engaged in a great deal of practice and repetition as they spent the bulk of the semester learning multiplication using a variety of modes. And Mary followed the sequence in the book as well as gauged her students' need for variety when determining when and how much to practice and repeat concepts.

Repetition was a teaching technique that Faith believed in and rigorously practiced in her classroom. Students were regularly assigned several pages of similar calculating problems and required to correct any errors. Susan, on the other hand, believed that games and different strategies were important and interesting to her students. The result of her various games, however, was that her students spent many many hours doing repetitive multiplication facts, albeit in the form of a cross-number puzzle one day and a coloring paper the next.

Practice and repetition were not as important to Oliver or Mary, and were less evident. For practice, often their students were assigned fewer than the full complement of problems in the book, or students shared an assignment. Repetition and review were more often based on the needs of an individual child or small group than on an overall assumption that repetition was necessary.

Both Mary and Oliver encouraged cooperation among students when they were learning new concepts. Through team efforts, students were less inclined to engage in repetitive activities and more likely to explore new ways to solve problems.

When word problems were part of an assignment in any of the classrooms studied, they usually required the same algorithms for their solutions, and thus became mechanical and repetitious. The students were seldom required to figure out how to set up a simple problem; rather they plucked the numbers from the problem and calculated the answer. Sometimes students challenged and prodded one another in their attempts to either reach a conclusion or teach one another different ways to approach a problem. Students occasionally challenged one another with a new twist to a problem. "What if you changed this. . ." was a refrain that could be heard from children-based problem-solving.

Fourth Common Belief: Children Learn Mathematics Best When They Feel Good About Themselves and Experience Success in Mathematics

Although this belief seemed to pervade all subject areas, the teachers in this study noted the importance of students' feeling good about themselves, feeling success, and feeling confidence as learners and as individuals in the mathematics classroom.

Faith was adamant in her belief that children would learn only when they feel good about themselves and experience success in their lives. This was a foundation for most of her teaching.

> Self-esteem and the person is your starting point and then you work into math. . . .I think it is a school-wide goal, a personal goal of mine too, is making children feel good about themselves and when they do then they can do anything. (interview, 3/26/91)

> Kids don't learn at all. . . unless they are really feeling good about themselves. . . . My top priorities are always trying to make kids feel positive about what is happening and not feel like they are a failure. . . . Academics don't come before you have the human being together and feeling like they are feeling positive about themselves and the people around them. (interview, 5/23/91)

Faith incorporated these beliefs as she talked about her overall role as a teacher.

> I think I have always felt that my job is really in two parts. I am a teacher of academics, but I am also a teacher of what you need - all the qualities and the things you need to survive in this world. As a teacher I am teaching them to be a good person and what they need to be successful. (interview, 5/23/91)

Faith planned small celebrations for accomplishments and reaching marker points. She was often a cheerleader for her fourth graders with comments such as "good job" or "We have gym to look forward to this afternoon so let's work our heads off" and "You're doing a nice job sticking to business."

Oliver believed that children learn effectively and feel good about themselves when they take charge of their own learning. Through cooperative learning activities and study buddies, students were able to get

> . . . a lot of one on one instruction; peer instruction basically. . . . They got together on projects that seemed to have some drive for those kids. . . . There was a lot of camaraderie and there was a lot of individual help that you don't get when you have one teacher serving 24 students. (interview, 4/2/91)

Cooperative learning activities were common in Oliver's mathematics classroom. Oliver frequently planned for students to work in pairs at the onset of a new assignment so they could check, correct, and/or teach one another. Sometimes students were paired so that one student did the even problems on a page while the partner would do the odd problems. The students often used each other as resources to figure out whether they were solving problems correctly.

Oliver also had a bulletin board of students' finest work, displaying perfect papers or tests with high scores. Additionally, he conducted many class discussions around mathematical problems, providing significant wait time for students to think and encouraging every student to have an answer before analyzing various responses.

Susan believed that students feel good about themselves and their learning when mathematics is fun and engaging. By providing them with a variety of games and fun activities, she believed students would be motivated to learn and enjoy the process.

> What I get mostly [in my classes] are kids that need a little bit of self-confidence because that is one of the things that I do well. . . . I cover the skills and do them so that I feel the children for the most part are enjoying what they are learning, then that is

> probably the best method of delivery. (interview, 4/3/91)

> Third graders still, I feel, need a lot of activity and games. (interview, 5/10/91)

Much of the work students did in Susan's room was in the form of games such as Bingo, Around the World with flash cards, or a worksheet with an element of detective work. One example was "The Swiss Vault Caper" where students had to determine the correct answers to division problems in order to find the correct combination to the safe drawn on the worksheet. However, there was little or no celebration or even recognition of student achievement. All students were assigned the same games and papers regardless of their levels of accomplishment. While the activities were intended to be fun and build good self-concept, there were seldom any cues from Susan differentiating students with meritorious learning from those who did not even complete the work.

Mary also believed students' learning was largely dependent on the students' self-esteem.

> They are very proud of themselves. I feel it is a very important part of teaching that children feel good about themselves and have a good self-esteem. Because from being a special educator [I know] when that is low, it really interferes with their learning. (interview, 5/1/91)

In her classroom, Mary had student work displayed on every wall and hanging from the ceiling as a way to show her pride in students' accomplishments. Students received praise and credit for their efforts and achievements. Mary was also quick to give children credit for knowing when they had forgotten something:

> That's a good thing to tell us you forgot so we can reteach it. (observation, 3/19/91)

> Don't worry, Anne, we'll get you through it. (observation, 3/21/91)

The individual attention and respect accorded each child in Mary's room reflected her belief that children must feel good about themselves in order to be good learners.

Embedded in the belief that children learn mathematics only when they feel good about themselves and experience success was the notion that part of each teacher's role is to engender confidence as learners and as individuals. Each teacher believed that he or she was establishing an environment conducive to good learning, accomplishments, and recognition for the same. Each teacher's personal style influenced the physical structure, planning, and activities in the classroom. Many messages about the importance of children's work were conveyed by the richness or absence of bulletin boards and other displays of students work as well as by verbal cues. These teacher behaviors were as varied as the mathematical concepts they taught; different from room to room and from day to day.

All of the teachers subscribed to the belief that students need to feel good about themselves and experience success in order to learn. Each teacher demonstrated ways to provide students with successful experiences through praise, grades, enthusiasm, or student self-appraisal. Susan, who believed learning should be fun and believed that enthusiasm engenders interest and ultimately success and self-confidence, gave few grades and little praise. Of the classes studied, her students seemed least enthusiastic about their studies and classroom.

Both Mary and Oliver held high expectations, and individual students received a great deal of attention and praise. Students in their classes also accepted significant responsibility for their own learning. They frequently initiated projects and activities, generated questions, and took on challenges. Oliver's students often engaged in cooperative learning activities which provided them with opportunities to teach and receive feedback from their peers as well as from him.

Faith was constant in her enthusiasm and belief in her students' abilities to put forth effort, and she demanded that students do and redo each paper until it was correct. Quiet was the norm in Faith's fourth grade as students attended to their papers and book work. Students had a clear understanding of what was expected of them and exhibited pride in their accomplishments.

In summary, although each of the teachers in this study had his or her own belief structures, similarities were evident. All of the teachers believed that children learn mathematical concepts by manipulating concrete materials or by visualizing them, that they learn arithmetic concepts through specific sequenced steps, that practice and repetition are necessary, and that children learn mathematics only when they feel good about themselves and experience success in mathematics. Their planning for mathematics instruction incorporated their beliefs to some degree, but was frequently dominated by other factors including time, the textbook, the need to cover content, available materials, and perceived expectations from the next grade's teachers. The behaviors that each exhibited while teaching usually supported their beliefs to some degree, although there were variances in intensity and quality. In some instances, behavior was not congruent with beliefs.

Relationships Between Beliefs and Behaviors

As stated earlier, four prevailing beliefs were articulated by the four teachers studied.

1. Children learn mathematical concepts by manipulating or visualizing concrete materials.

2. Children learn arithmetic through specific sequenced steps.

3. Children learn mathematics through practice and repetition.

4. Children learn mathematics best when they feel good about themselves and experience success in mathematics.

Based on these beliefs, some clear and specific teaching behaviors might be anticipated. Upon close examination, however, the teachers in this study did not live up to behaviors expected. There were many gaps between the teachers' beliefs and their actions.

First, if teachers believe that children learn mathematical concepts by manipulating concrete materials or visualizing them, then teachers might be expected to plan for children to use and see concrete examples of the mathematical concepts under discussion. Chips, interlocking cubes, attribute blocks, base ten blocks, fraction bars, tiles, rulers, geoboards, geometric models, graph paper, and much more might be available and in use by students as they explored mathematical relationships and concepts. Students might build models of buildings, towns and cities, establish scales, compare relative sizes, and create graphs to represent their findings. They might discover ways to count, tally quantities, and determine fractional parts using materials before using pencil and paper. Students could use abaci, calculators, and computers as tools to explore further ideas, patterns, and methods of problem solving.

As the research cited earlier indicated, students engage in and experience various mathematical relationships, patterns, and connections, and build their own mental pictures and understandings of those mathematical concepts (Kamii, 1985, Piaget, 1952; Resnick, 1983). They then process and build upon their knowledge to build more concepts (Battista, Wheatley & Talsma, 1989; Landis & Maher, 1989; Resnick, 1983). Through the use of manipulatives, teachers can help students express the many ideas they generate from their experiences and mental models.

Two of the teachers in this study made manipulatives accessible to students most of the time. Mary sometimes directed students to their use and planned for their incorporation in her teaching, and Oliver occasionally referred students to manipulative materials while more frequently relying on pictorials as he taught. Neither Mary nor Oliver thought they made use of manipulatives or visualizations as much as they would like, largely because of time. Both of these teachers wished to cover the content expected of their students for the academic year and found the use of manipulatives time consuming.

In addition, Mary did not like the noise that manipulatives made and found the management of materials with a class of 22 children difficult. Like Oliver, she knew that students had made use of them in the past and encouraged individual students to use them when they were having difficulty with a particular concept.

Susan declared her belief in the value of manipulatives, yet was observed using them only once in three months. Susan taught her students in one group, all working on the same mathematics activity. Since using manipulatives requires planning and thoughtful organization of both the materials and the students, Susan's tendencies to spend significant time on housekeeping and discipline issues and to readjust her plans frequently may have contributed to her failure to use manipulatives as often as she wished.

Faith acted on her belief in the value of visualizing geometric shapes and fractional pieces and made no use of manipulatives or visualizations while teaching arithmetic computation concepts. Her conviction that practice and repetition were the keys to learning fourth grade arithmetic overrode any of her thoughts about the need for students to see, feel, or touch objects.

If teachers believe that children learn arithmetic through specific sequenced steps, building new mathematical knowledge upon previous knowledge, then it might be expected that teachers would plan and follow conceptual trains of thinking. They would ensure an interrelatedness of concepts being introduced within the tasks of problem solving, computing, and procedures. There might be a webbing of concepts so that topics such as measurement would be introduced with geometric topics such as perimeter and shape or addition computation and not isolated to a few days before vacation or introduced independently. Division would be a short way of doing subtraction, not an entity of its own or merely the inverse of multiplication. Money might be an example of the decimal system at work rather than a different topic to be mastered.

If sequence were foundational to learning arithmetic, curriculum might be partially driven by student development and rely on students' knowledge bases for next steps in the sequence. Computation would be grounded in discoveries and activities related to the sequence of concepts experienced. Subtraction would follow addition, and so might negative numbers.

Each of these teachers indicated that children learn arithmetic through specific sequenced steps, building new mathematical knowledge upon previous knowledge. Each of them followed a specific sequence as outlined in the text-book. However, they limited their application of this belief to arithmetic, seeming not to be able to see further connections within the mathematical world such as how symmetry can be expressed in numbers as well as shapes. They also seemed not to be able to see sequencing and relationships among different topics. Their planning did not encompass sequencing outside of that offered through the textbook.

The textbook was the basis of each of these teachers' curriculum sequence. Oliver had decided that the research done by the publisher (Addison-Wesley) was thorough and well founded. Thus, he relied on the text to determine what topic to teach next.

Faith also followed the textbook and recognized that her 1978 edition did not cover all the topics expected in the fourth grade. She supplemented the text with other topics inserted into her curriculum when the time seemed right - before vacation, when one group had finished a chapter, or when it was time for a break. Faith's busy schedule precluded her from spending much time planning her mathematics lessons.

Mary's adherence to the sequence in the textbook seemed as much for convenience as any other factor. The text provided the basics necessary to accomplish the expected curriculum and to meet the testing standards for third grade. At the same time, Mary did not feel bound to the text and deviated from it when she felt the need for variety. Although Susan did not use the textbook as a teaching material with her students very often, she did follow the sequence outlined in the text and the curriculum guide.

Even though each of the teachers indicated that they believed children learn mathematics through specific sequenced steps, it appears that none of them put significant thought or planning into developing a sequence that was in concert with their beliefs. They relied on the curriculum for topics to be covered and the text for the sequence of arithmetical concepts, but failed to integrate other topics into a sequence, suggesting that they either did not know or understand the relationships of various mathematical topics to one another, or they did not devote the necessary time to planning their mathematics teaching so that they would match their behaviors to their beliefs.

These four teachers also believed that children learn mathematics through practice and repetition. Many practice and repetitive activities were in evidence in each of the four classrooms, although they varied according to the teacher.

For Faith, pages and worksheets of similar problems, timed tests, and word problems that correlated with the computation of the week became the hallmark of the students' activities. Mathematics was largely a solitary activity with students working on their own perfecting number facts, algorithms, and clue words for problem solving. Speedy answers to problems were important.

Time limits also were put on tests by Susan and Mary. Memorized procedures were stressed. In Oliver's classroom, mnemonics were used to help remember steps in a process (e.g., Dunk My Silly Brother for the sequence of steps in long division: Divide, Multiply, Subtract, Bring down). In Susan's room, "tricks" and unreasoned shortcuts were used such as using finger spaces to identify the answers to multiples of nines.

All students in Mary's, Faith's and Susan's classrooms completed the same number of problems without consideration of each student's level of mastery. There seemed to be little planning or organizing of the mathematics lessons to ensure understanding of various mathematical phenomena. Instead, procedures were followed, practiced, and repeated as outlined in the textbook until students had memorized computational techniques by rote. Again, the order and quantity of items in the textbook influenced teachers' actions.

The teachers had different means to ensure practice and repetition. Oliver provided different quantities of practice to groups of students based on their performance. He also saw eavesdropping and cooperative learning strategies as a way for students to review skills. Susan had students engage in numerous strategies and learning modes to practice and reinforce their work. And Mary helped students think through old strategies with new problems as students encountered them.

Regardless of their strategies, all of the teachers in this study acted on their belief about learning through practice and repetition. Even though time was viewed as a constraint for using manipulative materials more frequently, only Faith thought there was not enough time for the practice and repetition she desired for her students. While some practice allows children to confirm their learning, the extent of repetition observed in this study suggests a lack of a firm commitment to constructivism and the teachers' first belief - that mathematical concepts are learned by manipulating concrete materials.

Finally, if children learn mathematics only when they feel good about themselves and experience success in mathematics, then many successful experiences that build confidence would be at work in the classroom. Achievable and accessible goals would be set for each child in the classroom. Praise and feedback from both teachers and peers would be genuine and frequent. Teachers would have good understandings of the needs and cultural values of each student and provide appropriate comments and commendations.

Recognition and rewards such as displayed work, stickers, or certificates might be awarded to students who have achieved specified success levels. Various activities might be present allowing one student or a group of students to be recognized for their accomplishments.

These teachers demonstrated different ways of recognizing student achievement and success. Oliver's bulletin board of exemplary papers and good test scores was the source of pride for some of his students, yet there were many whose work never was displayed during the three months of observation. Some students felt good about themselves because of his respect for them and kind humor with them. However, there was no avenue evident for some of the lower achieving and/or more shy students to experience success in his room.

Susan believed she was effective with students who had low self-confidence. By the same token, she had almost no student work displayed in the room and was reserved in her deportment such that praise was slight and seldom. It was common for the entire class to hear "Good Job!" at the end of a lesson, however, students rarely received specific feedback on their work. Susan's beliefs were not displayed in her behaviors.

Faith's daily grading of papers provided students with information about their level of success with the subject matter. Since students worked until they got their papers correct, ultimately each experienced some success in mathematics. In addition, Faith challenged, cheered, and encouraged her students to work hard and meet goals. Faith's strong belief that children must feel good about themselves and experience success was a driving force behind Faith's enthusiasm and insistence that students complete all of their work.

Mary also supported and provided successful experiences for her students by evaluating each one based on his or her effort and progress. She recognized each student's work, displayed their work to demonstrate her pride in their accomplishments, and had frequent individual comments for students regarding their personal attributes as well as their academic achievements.

Susan, Faith, and Mary organized their mathematics classrooms as places where solitary learning activities usually occurred so that students seldom engaged in social interactions to build good feelings and self-esteem through peer relationships. The cooperative learning environments established in Oliver's room, however, were designed to foster recognition as well as non-competitive learning and led to students feeling comfortable with one another and feeling successful among their fellow students.

Each teacher's style and personality had some bearing on their work with their students, especially in the area of helping students feel good about themselves. Teacher attitudes, comments, levels of concern, and self-confidence were communicated to students every day. Recognition of progress towards their students' budding mathematical skills was one means of helping students feel good about themselves. Setting realistic and appropriate goals for each student and helping them attain them was one way of helping students experience success, whether in the mathematics classroom or any other aspect of school life. The teachers' effectiveness was enhanced by how much the teacher knew and understood developmentalism, effective pedagogy, and mathematics as a subject.

Each of these teachers demonstrated concern and interest in their students in their own ways. Oliver focused largely on the content area and thought about the pedagogical implications of it more than the other teachers studied. Mary's understanding of child development helped her focus on the individual needs of students and permitted her to tailor many activities. Faith's underlying belief in the goodness of each child helped her to encourage every student to achieve some success. And Susan's teaching was based largely on her caring for students and their various learning styles.

In these four classrooms, there was limited congruence between beliefs and actions. There were discrepancies, most commonly because of tendencies to acquiesce to the pressures of time and curricular expectations, and to rely upon the textbook rather than build upon the strength of their convictions and beliefs. While teachers believed that manipulating materials helps students grasp and develop concepts about the real world in mathematical terms, there was limited time devoted to the manipulation of materials. Although sequential learning was believed to be valuable, many mathematical concepts were taught out of the context and sequence of similar concepts. Practice and repetition were common, as were many ways of boosting the confidence of students.

CHAPTER 5

ANALYSIS AND DISCUSSION OF THE FINDINGS

In this chapter, there will be an analysis and discussion of the beliefs and behaviors of the teachers studied in the context of the National Council of Teachers of Mathematics (NCTM) curriculum standards for school mathematics and professional standards for teachers of mathematics and an exploration of professional development implications of this study.

Teachers' Beliefs and Behaviors in the Context of NCTM Standards

With their focus on mathematical thinking and problem solving, the NCTM standards require teachers to help students see mathematics as a language of expression and a means of solving problems. There are gaps between what the teachers in this study believed, how they behaved, and the behaviors necessary to implement the NCTM standards.

The beliefs of the teachers in this study drove many of the behaviors observed in the teaching process. These teachers also cited the external pressures of time constraints, the need to cover the content, reliance on the textbook, and expectations from the next year's teacher as reasons for behaving the ways they did.

These teachers' first belief that children learn mathematical concepts by manipulating concrete materials is supported by constructivist learning theory which is also a strong underpinning of the NCTM curriculum standards. In order to operationalize the standards, teachers must not only understand how children learn mathematics, but also understand how to provide good, rich opportunities for children to make sense of their worlds through self-discovery and the manipulation of materials.

The teachers in this study provided some time, materials, and relevant tasks so that children could explore, create, and recreate physical structures and relationships. In order to meet the NCTM standards, it is necessary to expand these opportunities, reinforce discoveries, and help students translate their concrete understandings into symbolic notations. Extensive use of manipulative materials and visualization is helpful to students; it is also important to help students translate their concrete understandings to the language of mathematics, the signs, symbols, and numbers of mathematics.

As language sounds and ideas can be transcribed into letters and words, so quantities, shapes, and physical relationships and can be transcribed into signs, numerals, and equations. Mathematics must be understood as a means of communication, make sense to students, and be useful to them. In order to reinforce the meanings students attach to their experiences, sequencing the tasks and structuring the learning environment become teaching tasks. It is essential that teachers be attuned to students' growing knowledge and relate one experience to another and to students' understandings and misconceptions. These teaching behaviors require a knowledge and understanding of the structure of mathematics as well as of the power of mathematics as a means of communication. When teaching and learning mathematics, one needs to go beyond the mechanics of arithmetic, see the connections of one area of mathematics to another, and build upon constructivist learning theory by providing manipulative activities so students can experience and gain understandings of the various concepts of mathematics.

While the teachers studied held a second belief that children learn arithmetic through specific sequenced steps, building new mathematical knowledge upon previous knowledge, they limited the application of this belief to arithmetical computations. Computation, however, is only valuable when applied to an experience or solving a problem. These teachers did not seem to know or understand how one mathematical task related to or built upon another, or how to set the stage to enable students to build relationships in sequence well enough to go beyond providing simple computational tasks for their students. They followed the textbook and covered the year's content without coordinating or fully understanding how the various topics within the curriculum fit together - such as that time is a measurement which can be calculated, like addition, by counting on. Further understanding of the discipline of mathematics would be helpful to these teachers so they can establish sequence in their teaching and ensure that students build new mathematical knowledge upon previous knowledge.

The third belief of the teachers studied, that children learn mathematics through practice and repetition, is disputed by research and literature. The NCTM Standards recommend that students master the basic facts of arithmetic, but only after they have had enough exploratory experiences to identify relationships among numbers and to develop efficient thinking strategies to derive answers from known facts. (NCTM, 1989, p. 47)

Classroom time currently devoted to practice and repetition might well be spent developing solid number concepts and number relationships through the use of manipulatives, applying concepts to real problems, focusing on geometry and spatial sense, and exploring relational constructs such as statistics, probability, and mathematical patterns. It is also important that students spend time becoming facile with various tools to help them do mathematics including the calculator, the abacus, and the computer.

The teachers' fourth belief that children learn mathematics only when they feel good about themselves and experience success can be questioned about its origins versus outcomes: Does one feel good about oneself because of success, or does one feel good about oneself as a learner as a precursor to success? It might well be unfair to students for teachers to assume that they cannot learn mathematics if they are not feeling good about themselves. If schools are to educate every student to his or her potential and meet the expectations outlined in the NCTM standards, it behooves teachers to set high expectations and good examples for all students.

As pointed out previously, teachers who have high self-concepts and good attitudes about mathematics generally engender good self-concepts, higher order thinking skills, and good attitudes among their students (Schofield, 1981; Trowbridge, 1973). Excitement about the process of learning and about content is contagious. Each of the teachers in this study expressed a desire to raise their students' self-concepts and successful experiences. Oliver and Mary exuded their own self-confidence and interest in mathematics. Faith was exuberant in her praise for students although her plodding through the mathematics textbook suggested she was not particularly invested in the subject. Susan believed she was good at helping students feel good about themselves, but her shyness and flat affect did not seem to engender student self-confidence or success. Thus, these teachers behaved in concert with their beliefs only to a limited extent.

Teachers serve as role models and frequently are emulated by students such that the enthusiasm and values attributed to mathematics as a field of study are transferred to students. Teachers both reflect and set standards within the communities in which they teach. They have opportunities to help students and parents envision a future with mathematically literate workers where mathematics is a common language used for expressing ideas and solving problems. By being knowledgeable about mathematics and the standards espoused by NCTM, teachers can enhance their own self-concepts and attitudes about mathematics and thereby foster students' learning of mathematics by helping them feel good about themselves and experience success.

Professional Development Implications

Underlying this study are issues concerning changes necessary to teach mathematics at the elementary level in accordance with NCTM's *Curriculum and Evaluation Standards for School Mathematics*. As this study shows, beliefs do not always control a teacher's actions, and behaviors do not necessarily reflect beliefs.

The 1989 and 1991 NCTM curriculum and professional standards for mathematics set a course for significant change in schools. The expectation that mathematics education will encompass broader mathematical thinking and problem solving such that mathematics becomes a common means of communicating ideas, dictates that there be changes in elementary school classrooms.

This study shows that even though teachers believed that children learn mathematical concepts best when they have concrete manipulative experiences, their teaching behaviors did not always reflect these beliefs. In order to implement the NCTM standards, teachers must be well grounded in constructivist learning theory and practice their beliefs in their daily teaching. They must provide ample time, materials, and experiences for students to explore and discover concepts so they may be well integrated into their knowledge bases.

Likewise, although these teachers believed mathematics is best learned through specific sequenced steps, they seemed not to have had sufficient knowledge of the subject area or knowledge of how best to teach it in order to ensure a logical sequencing of topics that fosters the building of new mathematical knowledge upon previous knowledge. They must provide the tasks, discourse, and environment to foster the integration of new knowledge, and do so in logical and sequential ways that address the developmental needs of students, not the expectations of textbook publishers or the students' future teachers.

Providing mathematical tasks and learning in a logical and sequential order, requires teachers to have a good knowledge of the subject matter itself, including the various branches of mathematics and how they are related to one another. Teachers must understand the language of mathematics and its power to communicate ideas. They must know and understand how to teach more than computation; estimation, relationships, patterns, measurement, reasoning skills, problem solving, geometry, and probability are all areas of mathematics important to elementary school curricula. There are many ways to relate measurement to number lines, addition, place value, and tasks relevant to students' lives. These kinds of activities are valuable to students' developing coherent ways of expressing the physical world in mathematical terms.

This study showed that some teachers spend considerable time on practice and repetition, as opposed to providing students with sufficient time to experience and explore various concepts and to apply mathematical concepts to common situations. Teachers also viewed students' self-concepts and successful experiences as critical to their mathematical development.

Professional development in the theory and practice of constructivism and of mathematics could enhance the effectiveness of mathematics education in the Kirby School. Teachers need to know how children learn. Equally important, they need to know and understand mathematics - how it can be used to communicate parts of our world to others, how one branch relates to another, and how it can be used to express and solve problems. Readings, discussions, workshops, conferences, and courses are useful professional development activities to expand these teachers' knowledge bases on child development and learning theory and on mathematics as a content area.

While most teacher training programs include child development and some Piaget in their course of study, for teachers in these classrooms, their studies were many years ago. Recent research on constructivism may shed new light on existing notions about children's learning and alter teachers' views and beliefs.

Few elementary teachers have had extensive mathematics training or know much about mathematics as a science. For teachers, mathematics education at the elementary level centered on the development of computation skills. Expanding teachers' information bases about mathematics, its logic, its effectiveness at communicating ideas, and its expressiveness and flexibility, could be a critical step towards more effective mathematical education. Understanding how mathematics works helps to understand the sequential logic within the subject. With a comprehensive knowledge of mathematics, these teachers would be better able to convey the importance of mathematics to students and show them how it is a sense-making branch of knowledge.

Areas of knowledge, skills, and attitudes form the basis of much professional development. Numerous opportunities created and taken advantage of by elementary school teachers could help them refine and better define their beliefs and practices so that they may change their teaching and learning of mathematics.

Professional development activities can provide opportunities for new learning through retraining, role playing, practice, coaching, and receiving feedback on what teachers do in the classroom. Feedback from students through student assessment, from peers as a result of observation and discussion, from oneself through the use of audiotape or videotape, and from administrators as a result of observation are invaluable sources for identifying and improving teaching behaviors and their effectiveness. Whether professional development is individually tailored and guided, self-initiated, based on observation and assessment, or based on inquiry, teachers can change and improve their teaching of mathematics.

Next steps may include active leadership and change within the corps of teachers, school administrations, state departments of education, professional schools of education, and professional organizations in order to spearhead new thinking for the teaching of mathematics. Revising curricula, an all too common approach to effecting change, is insignificant without fundamental change in the beliefs and behaviors of teachers, school structures, and bureaucratic structures.

Teachers have an important role to play to ensure that their classrooms are endowed with the language and thought processes of mathematics. Individually and collectively, teachers have power over their classrooms, curricula, and the teaching/learning processes. Daily choices of pedagogical activities and materials, content, pace, sequence, and delivery affect student learning. Teachers select what is purchased, what texts are chosen, and serve on committees which determine curriculum. They have an obligation to educate themselves about how children learn mathematics and how to effectively teach mathematics, so they can ensure major changes in our schools. Through courses, discussions, workshops, lectures, readings, and conferences, teachers can begin to form new ideas not just about how children learn mathematics, but how they can act on their beliefs when they are in the classroom.

Textbook authors and publishers do not know the specific needs of a school or school district and thus are ill-equipped to write curricula. Teachers must take responsibility for identifying and articulating the needs of their students, setting the standards, and designing their students' curricula with clear expectations and outcomes for their students.

By getting involved with their local teacher organization and school district planning, teachers can promote enlightened decision making at the school and district levels. Based on professional development experiences and recent research, teachers can volunteer for, influence, and serve on committees to promote staff development activities, teacher incentives for innovation and change, curriculum evaluation and revision, materials selection, school restructuring, personnel evaluation and hiring, and innovative proposal development.

Within teacher unions and union contracts recognition for quality teaching can complement recognition for years of service. Unions can serve their profession by valuing and promoting new learning and research. They must encourage improved teaching and learning for both teachers and students. They can cooperate and collaborate with administrations to move from the status quo to new ground in mathematics education from kindergarten on up. Incentives and recognition, financial or otherwise, can be provided to those who break the old molds and demonstrate leadership and excellence.

School administrators willing to support changes in teaching, in curricula and materials, in what is expected of students, and in what is assessed for mathematics learning will think of mathematics as a communication tool rather than an isolated set of algorithms. They can support the teaching of mathematics as a process, as an integral part of all curricula, and as a noisy activity. Principals involved in teachers' professional development can increase implementation of a program's objectives (Gall & DeBevoise, 1984).

There are many ways administrators who are effective change agents provide opportunities for staff to learn and grow including encouraging the trying out of new ideas, the planning and participating in reforms, visitations to other classrooms, attendance at conferences and workshops, and experimentation with new expectations for students. Not all efforts need to cost money; encouragement, time, and the reallocation of existing resources can provide for major shifts in priorities. If changing the outcomes of mathematics education is important to them, school administrators must provide leadership to that effort.

School districts must also reflect on the present and future needs of their communities. Today's students are tomorrow's work force. The economic and business futures must be forecasted and translated into appropriate curricula. Without doubt, mathematics will play an evermore increasing and invaluable role in the lives of today's students as they progress into the world of work. Algebra will no longer be a gate keeping course or a luxury for the college bound student, but a necessity for all as users of computers. And the elementary student must be prepared to think logically, solve problems, and have the mathematical skills to communicate, record, and analyze data. They must be provided with the foundations for higher level mathematics instruction.

State departments of educations, well-known for their bureaucratic rules and regulations, frequently burden teachers and school districts as they attempt change. Mandates on requisite number of minutes to be spent teaching each subject area neglect the notion that curricula can be integrated so that mathematics lives as a part of science exploration or as a means of expressing sociological phenomena. State tests generally assess students' knowledge of answers to questions rather than methods of inquiry. Required textbooks or specific curricula dampen reform activities. These outmoded mindsets must change.

State certification requirements fail to screen teachers with appropriate knowledge, beliefs, and behaviors from those without them. By certifying teachers based on their passing a course of study rather than on their performance, states have perpetuated their focus on teachers' knowledge and beliefs rather than on teachers' behaviors. Certification officers have assumed a passing grade indicates the teacher knows and can do what is needed for his or her teaching tasks.

Since *Tomorrow's Teachers*, the 1986 report of the Holmes Group, many state departments of education and post secondary schools of education have promoted liberal arts education, research on learning and teaching, and good practice for teachers. While these new directions may have the potential to enhance some teaching and learning processes in particular content areas, especially at the secondary level, there is no evidence that the teaching profession has yet improved its teaching of mathematics. Since many potential elementary teachers are now majoring in psychology or child development instead of elementary education, there is hope that these future teachers will be well grounded in a constructivist theory of learning. Research in this area could serve the profession and our elementary students well.

Tomorrow's Schools, the Holmes Group's 1990 report for the design of professional development schools, sets a direction of collaboration for post secondary schools of education and public schools. It explores ways to provide meaningful professional development opportunities for teachers in the field which promote new ways to teach, learn, and assess. When ideas from this Holmes Report are implemented, old ideas can be broken apart, inspected, and jettisoned for newer practices that work.

Professional organizations such as NCTM have begun to conduct their own research, publish their recommendations, and provide professional development guidelines and activities. It is important that members of the profession explore the rich literature that is available to them and act on it. Elementary teachers are not well known for being involved in content area professional activities, except perhaps for reading. If our students are to move into the twenty-first century with the skills necessary to compete in the global economy, it is imperative that teachers, teachers unions, administrators, departments of education, and schools of education actively and collaboratively engage in various professional organizations and professional development activities.

In the area of mathematics, collectively we must move forward on NCTM's agenda to implement the curriculum and professional standards for school mathematics. Elementary teachers must broaden their knowledge bases about how children learn mathematics and about mathematics as a field of study and an area of inquiry, and then bring their behaviors in concert with knowledge and beliefs. The opportunity for reform in mathematics education is upon us. With widespread coordinated effort, our society can be mathematically literate as we approach the twenty-first century.

APPENDIX A

PARTICIPANT AGREEMENTS

11 Chestnut Hill
Brattleboro, VT 05301
March 10, 1991

Ms.xxxxx
xxxxxx
xxxxxxxx VT 05xxx

Dear Ms.xxxxx:

As you know, I am currently a doctoral student at the School of Education at University of Massachusetts in Amherst. My interests in the effective teaching and learning of elementary school mathematics contribute to the focus I have chosen for my dissertation topic: What is the relationship between what teachers believe about how children learn mathematics and how those teachers teach mathematics?

To investigate the issues, I will engage in a case study of four elementary school teachers, interviewing them about their beliefs about how children learn mathematics and observing their practices in the classroom.

You are one of the elementary school teachers I would like to observe and interview. I hope that you will agree to take part in this study. If you do, you will be asked to be interviewed an hour or an hour and one half 3 or 4 times on audiotape and be observed teaching math approximately 10 times between March and June. The audiotapes will be transcribed and analyzed for common themes and notions about the teaching of math. You will have the opportunity to review the transcripts.

In all written and oral products of the research, pseudonyms will be used for all participants including teachers, students, administrators, associates, friends, schools, and communities. If you consent to participate in this study, you may withdraw up until June 15, 1991.

You must also agree to make no financial claims for the use of the material in your interviews and agree that no medical treatment will be required by you from the University of Massachusetts should any physical injury result from participating in this study.

I hope you will join me in exploring issues around the teaching of math by signing the form below and returning it to me. There is an additional copy for you to keep for your records. Of course, if you have any questions, do not hesitate to contact me.

Sincerely yours,

Sarah F. Carter
(phone: 254-6630 or
885-5183

I, ______________________________, have read the above statement and agree to participate in the study under the conditions stated above.

____
signature of participant

______________________________ ______________________________
signature of interviewer date

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11 Chestnut Hill
Brattleboro, VT 05301
March 10, 1991

Ms.xxxxx
xxxxxx
xxxxxxxx VT 05xxx

Dear Ms.xxxxx:

As you know, I am currently a doctoral student at the School of Education at University of Massachusetts in Amherst. My interests in the effective teaching and learning of elementary school mathematics contribute to the focus I have chosen for my dissertation topic: What is the relationship between what teachers believe about how children learn mathematics and how those teachers teach mathematics?

To investigate the issues, I will interview elementary school teachers about their beliefs about how children learn mathematics, and observe their practices in the classroom.

As a part of my study, I wish to talk to your child for approximately 30 minutes to understand students' perspectives on mathematics and the teaching of mathematics. Your child will not be required to answer any questions he or she does not wish to answer.

In all written and oral products of the research, pseudonyms will be used for all participants including teachers, students, administrators, associates, friends, schools, and communities. If you consent to have your child participate in this study, you may withdraw up until June 15, 1991.

You must also agree to make no financial claims for the use of the material in your child's interviews and agree that no medical treatment will be required by you or your child from the University of Massachusetts should any physical injury result from participating in this study.

I hope you will join me in exploring issues around the teaching of math by consenting to my talking with your child, signing the form below, and returning it to me. There is an additional copy for you to keep for your records. Of course, if you have any questions, do not hesitate to contact me.

Sincerely yours,

Sarah F. Carter
(phone: 254-6630 or
885-5183

I, ______________________________, have read the above statement and agree that my child may participate in the study under the conditions stated above.

______________________ ______________________________
child's name signature of parent or guardian

______________________________ ______________________________
signature of interviewer date

APPENDIX B

GUIDING QUESTIONS

Guiding Questions for Teachers

Interviews with teachers will be in-depth and open ended. The following questions will serve to direct the interviews and to prompt and promote discussion. The interviews will be informal and comfortable, conducive to much elaboration of ideas and extensions of thought. Teachers will be encouraged to speak as much as they like around their beliefs about how children learn mathematics and how they teach mathematics. Additional questions will evolve during the interviews.

Ultimate question:

What is the relationship between what teachers believe about how children learn mathematics and how those teachers teach mathematics?

Investigative questions:

A. What activities and teaching strategies do you use when you are teaching math?

1. What do you think you're best at in teaching math?
   a. Why?
   b. What makes you good at that?
   c. Are there other things?
2. What curriculum or guidelines do you follow as you teach math?
   a. How would you describe them?
   b. Are they useful?
   c. What changes to them would you recommend?
3. What are your favorite math activities?
   a. Why?
4. What kinds of resources do you like to use?
   a. Why?
   b. Can you explain for me the ways you use them?
5. What strategies do you think are most effective for you?
   a. Why?
   b. Can you give me one or two examples?

6. How much do you use the textbook?
   a. for teaching ideas?
   b. for assignments?
   c. what else do you use it for?
7. If you could have anything you wanted to have to use to teach math, what things would be included?
   a. Why? What difference would they make in your teaching?
   b. How would you make use of them?

B. What factors do you consider when you plan your math lessons?

1. How do you select/determine your math lessons?
   a. Are there other considerations?
   b. What's a typical math lesson like?
2. What things do you consider as you plan?
   a. Are there other considerations?
3. How do you determine the sequence of activities for students?
4. How do you fit math into your day?
   a. Why do you teach it when you do?
   b. Are you pretty regular about how much time you spend on math?
   c. Or when you teach it each day? (Why/why not?)
5. How do you decide with which children you will work, and in which groups?
   a. What kinds of changes do you make?
   b. Why?
6. What do you think are the most important things for children to gain from math?
   a. Why?
7. What was your feeling about math as a child? Has it changed?
   a. What things made it change?
   b. Are there other things? or categories of things?
8. What things influence your attitudes about math?
   a. as a teacher?
   b. in your personal life?

C. How do children learn math?

1. What's your philosophy about how children learn?
   a. Can you give me some examples of how you do this in your classroom?
2. On what do you base your thinking?
3. What do you think is the best way for third (fourth) grade children to learn math?
   a. Why do you think that's best?
   b. Can you illustrate how you might do this in the classroom?

4. What are your views about the way math is taught in schools today?
  a. Are there changes you would recommend?
5. What do you think makes for effective teaching of math?
  a. Can you cite some examples for me?

Guiding Questions for Students

1. What are your favorite subjects?
  a. Why?
  b. What do you like about them?
2. How do you feel about math?
  a. Have your feelings about math changed since first grade?
  b. In what ways?
3. Are there things in math you're good at?
  a. How do you know you're good at them?
  b. Do you like them? Why?
  c. What activities do you do when you're doing that kind of math?
4. When do you use math outside of math class?
  a. Can you give me examples or show me?
5. What kinds of activities do you like to do best in school?
  a. Why?
  b. Can you explain exactly what you do?
6. How about in Math - what kinds of cativities do you like to best in math?
  a. Why?
  b. Can you explain these activities to me?
7. Think about when you learn new things. What things do you do when you learn things well?
  a. Can you give me a couple of examples of when you've learned things this way?
  b. How do you think you learn best?
  c. What things do you do?
  d. Can you give me some examples?

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