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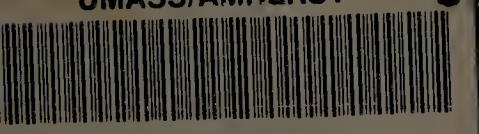# Evaluation of the weighted least squares method for the analysis of categorical data.

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EVALUATION OF THE WEIGHTED LEAST SQUARES METHOD
FOR THE ANALYSIS OF CATEGORICAL DATA

A Dissertation Presented

by

ROSEMARY A. RESHETAR

Submitted to the Graduate School of the
University of Massachusetts in partial fulfillment
of the requirements for the degree of

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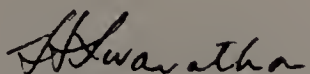
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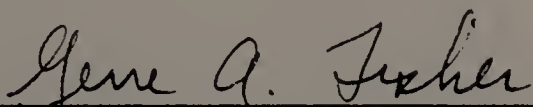
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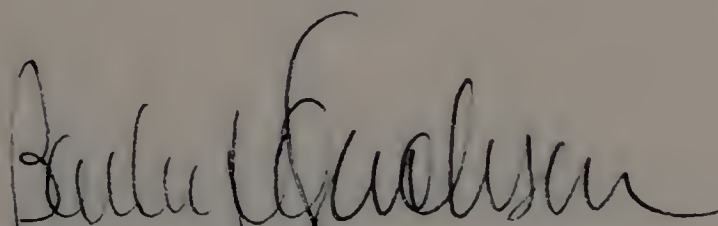
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ABSTRACT

EVALUATION OF THE WEIGHTED LEAST SQUARES METHOD

FOR THE ANALYSIS OF CATEGORICAL DATA

SEPTEMBER 1991

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Hypotheses about the relationship among variables in a multiway contingency table may be tested by analysis of the probability distribution of observed frequencies or transformation of these frequencies. Two model-based approaches for the testing of structural hypotheses are the log-linear model, using iterative maximum-likelihood (ML) estimation procedures and the weighted least squares (WLS) linear model method of Grizzle, Starmer and Koch (GSK), a general noniterative procedure. Both methods asymptotically provide the same estimates and test statistics.

This study compared the GSK and log-linear approaches for testing hypotheses in r x c contingency tables. Tables were simulated under various conditions of table, sample, row-, and column-effect sizes. Test statistics for row and column effects, and interaction were calculated using: (i) GSK linear model, untransformed proportion (p); (ii) GSK linear model, logarithm of the proportion (log p); (iii) GSK linear model, log-odds (log p/(1-p)); and (iv) log-linear model. Type I error rates were examined, and the relative power of the procedures was studied.

The log-linear model yielded Type I error rates close to the expected values; all GSK models yielded error rates higher than expected, with smallest error rates associated with logarithmic transformations. Sample size and table size had no effect on Type I error rates.

All GSK procedures were uniformly more powerful than the log-linear procedure. Differences were most noticeable with medium effect sizes and diminished as sample and effect sizes increased. There were no systematic differences due to table size.

Findings from this study are pertinent to applied researchers who wish to test hypotheses other than those of independence with categorical data. Hypothesis testing and interpretation of results are straightforward with a model-based approach and are thus encouraged. The results indicate that GSK methods provide the most powerful tests. Since the GSK method is easily implemented and can be understood by researchers familiar with linear regression analysis, it is recommended that the GSK method be used to analyze categorical data.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

MODELS FOR THE ANALYSIS OF CATEGORICAL DATA

Introduction

Multiway contingency tables may arise in experimental work when data on two or more variables can be assigned among nominal (or ordinal) categories. This type of data occurs, for example, when the members of a sample are subdivided into categories representing different characteristics.

Once a table is constructed based on assignment to categories, a researcher may test hypotheses about the relationship between two or more variables by analysis of the probability distribution of the observed frequencies or of some transformation of these frequencies. Several methods have been applied to categorical data analysis. These include the Pearson chi-square, the log-linear model approach using maximum likelihood (ML) estimation procedures, and the analysis of linear models approach presented by Grizzle, Starmer and Koch (1969).

In this study, categorical data will refer to information which is measured such that it is classified into discrete categories. These categories may be ordered (ordinal) or unordered (nominal). The focus of this study will be on the situation where all dependent and independent variables are nominal categorical. Methods for analyzing contingency tables will be discussed next. First, the Pearson chi-square test of independence will be reviewed. Then log-linear models and ML estimation procedures will be discussed. Finally, discussion of linear model methods will be presented.

Pearson Chi-Square

By far, the most common analysis for data in the form of a contingency table is the Pearson chi-square test for independence. Pearson chi-square is a test for independence or no association between variables. The null hypothesis for chi-square tests in a table with i rows and j columns is H_o: $P_{ij} = P_i P_j$, which states that the

responses in each column follow the same probability distribution over the rows (Light, 1973).

The Pearson chi-square statistic is defined as

$$\sum_{i=1}^{k} \frac{(x_i - n\pi_{0i})^2}{n\pi_{0i}} \qquad (1.1)$$

where k = the number of cells

x_i = the number of responses in cell c_i

n = the total sample size

and π_{0i} = the expected probability for cell c_i.

Thus $n\pi_{0i}$ = the expected frequency in cell c_i. The degrees of freedom (d.f.) is $(R - 1) \times (C - 1)$, where C is the number of columns and R is the number of rows. If there is substantial discrepancy between the observed frequencies and the expected frequencies, the chi-square test would be significant and the null hypothesis of independence would be rejected.

The Pearson chi-square test is a general test of independence (analogous to the test for interaction with the ANOVA model), and is thus appropriate only when the research question is that of association between the categories of interest. In terms of a hypothesized model, it may be helpful to consider the possible linear model for a 2x2 table.

$$p_{ij} = M + R_i + C_j + RC_{ij} \qquad (1.2)$$

where p_{ij} is the probability of belonging to cell ij, M is an overall mean, R_i characterized the row effect, C_j characterizes the column effect, and RC_{ij} characterizes the interaction effect. With the 2x2 table there are four p_{ij}; however the model given above has nine parameters: M, R_1, R_2, C_1, C_2, RC_{11}, RC_{12}, RC_{21}, and RC_{22}. This model is then

2

overparameterized, and an arbitrary set of constraints can be imposed to allow solving
(1.2). Using summation notation, these constraints may be written as

$$\sum_i^I R_i = 0 \qquad \sum_j^J C_j = 0 \qquad \sum_i^I RC_{ij} = 0 \qquad \sum_j^J RC_{ij} = 0 \qquad (1.3)$$

With these constraints, four model parameters will need to be estimated. Since there
are only four cells in the table, the only solution will be an exact solution which would
not simplify the interpretation of the data. The Pearson χ^2 test of the hypothesis of no
association tests the hypothesis that the interaction parameter is zero and thus the
hypothesized model for the 2x2 table could be written as

$$p_{ij} = M + R_i + C_j \qquad (1.4)$$

For tables with more than two rows or columns, the above models given in (1.1) and
(1.2) would be extended to include the necessary R_i, C_j and RC_{ij} parameters with the
Pearson χ^2 always testing the hypothesis that the RC_{ij} parameters are equal to zero and
that the hypothesized model includes both row and column main effects.

While testing this hypothesis with categorical data is analogous to the ANOVA
test for interaction, the researcher may also be interested in testing hypotheses
pertaining to row and/or column effects analogous to main effects hypotheses with the
ANOVA model. In other words, rather than testing the hypothesis of no association, a
model-based approach useful for the testing of structural hypotheses may be desired.
The log-linear model can be used in this case.

<div align="center">Log-Linear Models</div>

As with the Pearson chi-square test, one is again interested in comparing
expected cell frequencies with observed frequencies. Maximum-Likelihood (ML)
estimation methods may be used when all variables are measured categorically (log-
linear or logit model) or when a dichotomous response variable is identified and all

<div align="center">3</div>

independent variables are measured on a continuous scale (logistic regression). This discussion will focus on the log-linear case and on the logit model.

The general log-linear model does not distinguish between independent and dependent variables. The criteria to be analyzed are the expected cell frequencies, F_{ij}'s, as a function of all the variables in a model.

With the general log-linear model, each variable measured is considered as a factor. Effects of each of these factors and interactions between each of these factors (interactive effects) on the expected cell frequencies may be examined. In general, the expected cell frequencies (F_{ij}) can be modeled as a function of the various effects (τ's), where effects are multiplicative, as

$$F_{ij} = \mu \, \tau_1 \tau_2 \cdots \tau_k \qquad (1.5)$$

where μ is an overall baseline effect, and τ_i refers to a specific factor effect or an interactive effect. For example, a 3-factor model with factors i, j and k and interactive effects would be written as

$$F_{ij} = \mu \, \tau_i \tau_j \tau_k \tau_{ij} \tau_{ik} \tau_{jk} \tau_{ijk} \qquad (1.6)$$

As with the model given in (1.2), all possible effects are included; and this model is thus referred to as a saturated model. Since there are as many parameters in the model as there are cells in the table, some constraints are imposed when estimating the effect parameters. (For complete discussion see e.g., Knoke and Burke, 1980.)

While a saturated model includes all possible effects, a non-saturated model is one which does not include all effects. With a saturated model, the resulting chi-square test would have 0 degrees of freedom; and thus testing of hypotheses requires a non-saturated model. An example of a non-saturated model was given for the Pearson χ^2 test of no association by (1.3). With log-linear analysis, hierarchical models are most commonly utilized; i.e., in a given model, if a lower-order effect is assumed to be zero,

then any higher-order interactive effect involving the same factor(s) plus others must also be assumed to be zero. It is possible to test hypotheses pertaining to nonhierarchical models; however this leads to computational complexities.

While the multiplicative model given in (1.5) is suitable, it is computationally more convenient and more conducive to statistical testing to use a somewhat different model. By taking the natural logarithm of each side of equation (1.5) we have:

$$\ln(F_{ij}) = \ln(\mu) + \ln(\tau_1) + \ln(\tau_2) + \dots + \ln(\tau_k) \qquad (1.7)$$

which is commonly written with different symbols as

$$G_{ij} = \theta + \delta_1 + \delta_2 + \dots + \delta_k \qquad (1.8)$$

where θ is the natural log of μ and δ_i is the natural log of τ_i. We now have a linear model. Since the logarithms are used to form this linear model, we hence have the name log-linear models.

Parameter Estimation

Given a hypothesized non-saturated model where various effects are included or excluded, in order to test hypotheses it is then necessary to estimate the effect parameters and to test for the goodness of fit of the model. With a ML estimation procedure the task is to find values for the parameters that lead to estimates which are closest to the observed frequencies in the sense that given the observed frequencies, these parameters are more likely than any others to have produced them.

For a simple model, such as one in a 2x2 table, simple formulas exist which permit direct estimates for nonsaturated models to be written. But for larger tables and more complex models closed-form expressions for the estimated expected frequencies cannot be written. Some type of algorithm is then required to generate the expected frequencies. The two commonly used iterative procedures are the iterative proportional fitting algorithm and the Newton-Raphson algorithm. With these procedures,

preliminary estimates of expected cell frequencies are successively adjusted to fit each of the marginal subtables specified in the model. Each iteration results in some improvement, until an arbitrarily small difference between the current and previous estimate is reached, at which point the process concludes. Once the expected cell frequencies are produced (F_{ij}'s) for a given model specification, the numbers are entered into the appropriate formulas to produce the effect parameter estimates for the variables and their interactions.

Goodness of Fit Tests

Once parameters are estimated with ML methods, these parameter estimates are used in statistical tests of the model's adequacy. As mentioned earlier, with a two-dimensional table the Pearson chi-square test can be used. When ML estimation is used the likelihood ratio chi-square test statistic, L^2, is often used. L^2 is defined as

$$L^2 = 2\Sigma\ x_i\ \ln(x_i/n\pi_{0i}) \tag{1.9}$$

where x_i = the number of responses in cell c_i, and $n\pi_{0i}$ = the expected frequency in cell c_i. L^2 is asymptotically distributed as chi-square with degrees of freedom equal to the number of tau parameters set equal to 1.00 (no effect on expected cell frequencies).

L^2 can be used with contingency tables of any dimensions and is more generally applicable than the Pearson chi-square test. L^2 is also preferable to Pearson chi-square because (1) the expected frequencies are estimated by ML methods and (2) L^2 can be partitioned uniquely for more powerful tests of conditional independence in multiway tables (Knoke & Burke, 1980).

L^2 tests can be used in two ways. One is to test hypotheses about the coefficients given a saturated model, by testing the null hypotheses that the coefficients are zero, or asking if the coefficient is statistically significant. If L^2 is greater than the tabled chi-square value with α and d.f., the null hypothesis that a given coefficient is zero is rejected.

6

The second use of the L^2 test is for testing the goodness of fit of a hypothesized nonsaturated null model. This is analogous to comparing a restricted model to a full model, where the saturated (full) model serves as a reference point of best fit with which other restricted models may be compared. In this situation, we are seeking to prove the null hypothesis in order to accept that the reduced model adequately describes the data. In general, the larger the L^2 relative to the available d.f., the more the expected frequencies depart from the actual cell entries. Hence we want to find a low L^2 value relative to d.f. Since we are seeking to accept the null hypothesis, Type II error is of interest. To control for Type II error it is recommended that Type I error be set between .10 and .50, with the null hypothesis being accepted if L^2 is less than the tabled chi-square value at the specified α level.

The Logit Model

The log-linear model is a specialization of the logit model. In the logit case, one factor can be considered a dichotomous dependent variable and the remaining factors can be considered explanatory variables. Thus with the logit model one is interested in predicting the probability of a subject being in one of two groups of the dependent variable, based on knowledge of the subject's responses to other attributes. That is, the logit or log odds that the dependent variable has a specified value is a linear function of the independent variable(s). The logit is defined as

$$\text{logit}(\pi) = \ln \frac{\pi}{1 - \pi} \tag{1.10}$$

Since there are only two categories within each subpopulation, we can define two possible parameters, π_1 and π_2, for each subpopulation. Given that π_1 and π_2 sum to one, the logit can be rewritten as

$$\text{logit}(\pi) = \ln (\pi_1/\pi_2) \tag{1.11}$$
$$= \ln(\pi_1) - \ln(\pi_2).$$

7

As with the general log-linear model, once a restricted model of interest is specified the common procedure is to estimate the parameters and to test for the goodness of fit of the model. In the logit case this may be done with ML or weighted least squares (WLS) procedures. ML equations are similar to those used for log-linear models and are give by Haberman (1978). A simplified version of the Newton-Raphson algorithm may be used in the logit case. L^2 may be used to test a model for goodness of fit, where L^2 is asymptotically distributed as a chi-square random variable. WLS procedures will be discussed next in the context of the linear model method.

Linear Model Method

Grizzle, et al. (1969) proposed a methodology for the analysis of categorical data based on applications of the general linear model (referred to as the GSK method). While the general linear model had been the basis for both regression analysis and analysis of variance for continuous data, no similar procedure had been developed for analysis of categorical data. Using estimation and testing procedures based on weighted least squares (WLS) and the Neyman (1949, cited in Freeman, 1987) chi-square or Wald (1943, cited in Freeman, 1987) test statistics, the GSK method can be applied to categorical data.

The WLS approach is most useful in situations where both the dependent and independent variables are categorical; yet it may also be used in cases where the dependent variables are all categorical and the independent variables are all continuous or both continuous and categorical. This discussion will focus on the case where all dependent and independent variables are categorical, as no ideal general WLS method exists for analyzing problems which include both continuous and categorical variables.

The Model

In contingency table analysis, the researcher is often interested in finding a parsimonious model which accounts for variability in the data set. In developing a linear model, relationships between the function of the probabilities as the dependent variable,

and the independent or factor variables may be examined. This can be written as $F = X\beta$ where X is a $u \times s$ known design matrix of rank $v \le u$ and β is an $s \times 1$ vector of unknown parameters.

It should be noted that although this form resembles the analysis of variance model, there is one main difference. In ANOVA, homoscedasticity is assumed. In this situation, the assumption of homoscedasticity is unreasonable for the elements of F. Therefore, instead of using least squares analysis to estimate β, that is, $b = (X'X)^{-1}X'F$, the weighted least squares method is used. The WLS estimate of β is $b = (X'S^{-1}X)^{-1}X'S^{-1}F$, where S is the variance-covariance matrix of F.

Estimation. Consider a contingency table with s rows and r columns (see Table 1.1). The rows, called subpopulations, represent combinations of independent or factor variables from which independent random samples of apriori fixed sizes $n_1, n_2, ..., n_s$ (n_i is the total of the ith subpopulation) have been selected. The columns represent the levels of the dependent or response variables. Thus the n_{ij} are the cell frequencies or the number of subjects in the ith subpopulation with response level j.

Table 1.1 An s x r Contingency Table

Subpopulations	Response Categories				Total
	1	2	...	r	
1	n_{11}	n_{12}	...	n_{1r}	$n_{1.}$
2	n_{21}	n_{22}	...	n_{2r}	$n_{2.}$
:	:	:	: :		:
s	n_{s1}	n_{s2}	...	n_{sr}	$n_{s.}$

Given the above frequency table, we can construct a corresponding table of probabilities (Table 1.2) where π_{ij} is the probability that a subject in the ith subpopulation has the jth response.

Table 1.2 Table of Probabilities

| | Response Categories | | | | |
Subpopulations	1	2	...	r	Total
1	π_{11}	π_{12}	...	π_{1r}	$\pi_{1.}$
2	π_{21}	π_{22}	...	π_{2r}	$\pi_{2.}$
.	.	.		. .	.
.	.	.		. .	.
s	π_{s1}	π_{s2}	...	π_{sr}	$\pi_{s.}$

Since each level of the factor combinations (each row) is viewed as a distinct subpopulation, we can let the sum of the probabilities for each row equal 1. We can then define

$$\pi_i' = [\pi_{i1}, \pi_{i2}, ..., \pi_{ir}]$$

and

$$\pi' = [\pi_1', \pi_2', ..., \pi_s'].$$

Given

$$p_{ij} = n_{ij}/n_{i.}$$

$$p_i' = [p_{i1}, p_{i2}, ..., p_{ir}]$$

and

$$p' = [p_1', p_2', ..., p_r'],$$

π' is estimated by p'.

Once the probabilities are estimated, relationships between functions of the probabilities $f(\pi)$ or F, and the independent or factor variables may be examined. The

10

functions formed may be used as the dependent variables in a linear model analysis and also for testing hypotheses directly. The functions that can be formed involve linear, logarithmic, exponential or combined transformations of the cell probabilities (p_{ij}). Most applications are covered by linear and logarithmic functions; and these will be discussed next.

A linear transformation of the probabilities can be represented by $F(\pi) = A\pi$ where A (dimension u x rs) is of rank $u \leq s(r-1)$. The elements in the ith row of A are the coefficients for the π_{ij} of the ith subpopulation.

The A matrix is selected to examine relationships of interest. For example, comparison of cell proportions are possible, cell probabilities may be selected, or a weighted sum of probabilities or mean score can be used.

While logarithmic transformations were discussed in the context of ML estimation, they are also applicable to GSK methods. Logarithmic transformations of the data may be defined as $F(\pi) = K\ln(A\pi)$. In the case where all of the π_{ij} are to be used, A is then equal to I and this simplifies to $F = K\ln(\pi)$.

In the case where the response variable is dichotomous, the logit may be used. Given a table of probabilities of dimension s x 2, the logit is defined as by equations 1.10 and 1.11. Thus the linear combination of the logarithm of the probabilities for each subpopulation is given as:

$$f_i = \ln(\pi_{i1}) - \ln(\pi_{i2}). \tag{1.12}$$

and for the entire set of subpopulations is expressed in matrix notation as

$$F(\pi) = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_s \end{bmatrix} = K\ln(A\pi) \tag{1.13}$$

where if all π_{ij} are of interest,

$$K = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}$$

and $A = I$.

<u>Variance of a Function</u>. We use the vector $\mathbf{p}$ as an estimate of the vector π and then use the variance-covariance matrix of $\mathbf{p}$ as the sample estimate of the variance-covariance matrix of π $[V(\pi)]$. The variance-covariance matrix of $\mathbf{F}$ can then be calculated using this information.

Recall that an element of $\mathbf{p}$, p_{ij}, is defined as

$$p_{ij} = n_{ij}/n_{i.}$$

Since the data follow the multinomial distribution, the variance of p_{ij} is

$$var(p_{ij}) = \pi_{ij}(1 - \pi_{ij}) / n_{i.} \tag{1.14}$$

and the covariance of p_{ij} and p_{ik} is defined as

$$cov(p_{ij}, p_{ik}) = -\pi_{ij}\pi_{ik} / n_{i.} \qquad \text{where } j = k. \tag{1.15}$$

The estimates of the variances and covariances are obtained by substituting the p's for π's, and thus the variance-covariance matrix for the ith subpopulation may be written as:

$$V_i = \begin{bmatrix} p_{i1}(1-p_{i1}) & -p_{i1}p_{i2} & \cdots & -p_{i1}p_{ir} \\ -p_{i1}p_{i2} & -p_{i2}(1-p_{i2}) & \cdots & -p_{i2}p_{ir} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ -p_{i1}p_{ir} & -p_{i2}p_{ir} & \cdots & -p_{ir}(1-p_{ir}) \end{bmatrix}$$

The estimated variance-covariance matrix for all s subpopulations is

12

$$V_p = \begin{bmatrix} V_1 & 0 & \cdots & 0 \\ 0 & V_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & V_s \end{bmatrix}$$

V_p is a block-diagonal matrix with V_{pi} on the main diagonal and zeros in the off-diagonal positions. This structure results from considering the subpopulations uncorrelated with one another; hence there are zero covariances between the subpopulations.

For a linear function, $F = A\pi$, and a linear combination of the logarithm of the probabilities, $F = K\ln(\pi)$, the A matrix is a matrix of constants. The estimated sample variance-covariance matrix of the estimated function $F = Ap$ is

$$\text{var}(F) = S = AV_pA' \tag{1.16}$$

The estimated variance-covariance matrix for $F = A\ln(p)$ is

$$\text{var}(F) = AD^{-1}V_pD^{-1}A' \tag{1.17}$$

where D is an rs x rs diagonal matrix with the elements of p on the main diagonal.

After estimating the variance-covariance matrix of F, it is possible to obtain the WLS estimator of ß as

$$b = (X'S^{-1}X)^{-1}X'S^{-1}F. \tag{1.18}$$

This procedure gives more weight to the elements in F that have smaller variances. The variance-covariance matrix of b is

$$\text{var}(b) = V_b = (X'S^{-1}X)^{-1}. \tag{1.19}$$

Hypothesis Testing. The fit of the model may be tested using a chi-square test statistic (χ^2). χ^2 represents a statistic that asymptotically follows a central chi-square

13

distribution with u - v degrees of freedom (d.f.) if the hypothesis being tested is true (i.e., if the model fits). The goodness of fit test statistic is:

$$\chi^2_{GOF} = (F - Xb)'S^{-1}(F - Xb). \tag{1.20}$$

If the model fits, we can proceed to test linear hypotheses about the β. This is expressed as H_o: $C\beta = 0$, and is produced by conventional methods of weighted multiple regression, where C is a (d x v) matrix of arbitrary constants of full rank d≤v. The test statistic of the hypothesis H_o: $C\beta = 0$ is given by

$$\chi^2 = SS[C\beta = 0]$$
$$= (Cb)'[C(X'S^{-1}X)^{-1}C']^{-1}Cb \tag{1.21}$$

which has asymptotically a chi-square distribution with d d.f. in large samples if the hypothesis is true.

In the case of one population with the objective of studying relationships among several ways of classification of the sample units, many tests can be formulated as $F(\pi)$ = 0. This fits into the general framework by setting X = 0, the null matrix. Thus the test statistic is $F'S^{-1}F$, which has asymptotically a chi-square distribution with u d.f. if Ho is true.

Summary

The methods discussed for the analysis of categorical data are the Pearson χ^2, the log-linear method, the logit and the linear model method. The Pearson χ^2 is the most commonly used procedure and may be used to test the hypothesis of no association between variables which is analogous to the test of no interaction with the ANOVA model. With the log-linear, the logit and the linear models a model-based approach for testing of hypotheses is available for categorical data. Hypothesized structural models are formed which explain the observed cell frequencies in terms of their relation to the variables or categories of interest.

The log-linear model referenced is estimated with ML procedures and with the exception of the 2x2 table, an iterative estimation method is required. With the general log-linear model dependent and independent variables are not distinguished and the criteria to be analyzed are the expected cell frequencies, F_{ij}. Hierarchical models of the F_{ij} are most commonly hypothesized and tested, where if a lower-order effect is set to zero, any higher-order interactive effect involving the same factor(s) is also set to zero.

With the logit model one factor can be considered a dichotomous dependent variable and the remaining factors may be considered explanatory variables. Logit models permit estimation of the parameters by using ML or WLS. ML procedures are similar to those used for the log-linear model, and WLS procedures were discussed in the context of the linear model.

The linear model method presented by Grizzle, et al (1969) utilizes the WLS procedure to estimate parameters and the χ^2 test statistic. A linear model is hypothesized in which relationships between the function of the cell probabilities as the dependent variable, and the independent factor variables may be examined. The WLS approach provides flexibility in choosing a function of the dependent variable for analysis. The function may be formed by a linear, logarithmic, or exponential transformation or an combination of these three operations. The WLS procedure also provides a closed-form solution in contrast to the ML procedure which is iterative.

CHAPTER 2

BACKGROUND OF THE RESEARCH PROBLEM

The GSK general approach to the analysis of categorical data provides the analyst with latitude in choosing models and testing hypotheses tailored to specific data. Grizzle, et al. (1969) presented a noniterative procedure for fitting functions of categorical data to a linear model, for testing the goodness of fit of the model, and for testing hypotheses about the parameters in the linear model. The GSK procedure utilizes estimation and testing procedures based on WLS and the Neyman (1949, cited in Freeman, 1987) chi-square or Wald (1943, cited in Freeman, 1987) test statistic. When the sample size is large, this procedure gives estimates and test statistics which converge on true parameter values, as do the maximum-likelihood (ML) and Pearson's χ^2 for a variety of problems.

Comparisons Between WLS and ML Methods

The WLS method of GSK was presented as an alternative to ML estimation procedures for categorical data analysis. Comparisons of these two methods is presented next.

Computational Differences

ML estimation requires maximization of a given equation with respect to each expected cell frequency under the constraints imposed by the hypothesized model and sample design. Often it is not possible to write a closed-form expression for the estimated expected frequencies. In this situation it is common to use the iterative proportional fitting algorithm or the Newton-Raphson algorithm. These methods may pose computational difficulties when the number of parameters is large. For complex models, assurance of convergence to global rather than local maximum may be difficult to achieve. Also, totally general algorithms for solutions have not been disseminated (Koch & Imrey, 1985).

16

Hypothesis Testing

In the situation of a symmetric model (one in which no response variable is specified) ML procedures prove advantageous. The GSK approach is straightforward when a response variable is selected. The GSK approach can however handle symmetric problems. To do so the analyst must systematically rotate through a problems variables choosing different variables, individually, as the response measure. Therefore, ML procedures are preferable in this situation.

Given an asymmetric model, the GSK approach provides somewhat greater flexibility to test hypotheses. With the GSK method, the researcher may establish nearly any linear combination on nearly any transformation of the response measure. The log-linear approach forces definition of the response function in terms of logged proportions. While the analyst is permitted to establish any desired transformation and linear combination on the expected frequencies, this is not typically done and is more mechanically difficult.

Response Functions

Given the ability to select response functions, there are two important implications. One is for the procedure used to estimate model parameters, and the other is for the interpretation of model results. As discussed above, more complex functions may be analyzed with the WLS method. Application of various functions include problems relating to paired comparisons, observer agreement, repeated measures, complex sample surveys, partial association and rank correlation methods (Forthofer & Lehnen, 1981).

The choice of function also affects the interpretation of the results. Differences between response functions have been discussed in detail with respect to hypotheses of no interaction. Bhapkar and Koch (1968) provide an overview of general hypotheses of no interaction that may be tested when working with categorical data. The distinction is drawn between fixed and random marginal totals and the appropriateness of an additive

or multiplicative model for a variety of cases. A linear response function corresponds to an additive model and a logarithmic response function corresponds to a multiplicative model. Results of analyses on a linear scale are expected to differ from results based on a logarithmic scale, and these differences must be considered when interpreting results.

Comparisons of Estimates and Test Statistics

As mentioned earlier, WLS and ML estimates and test statistics are asymptotically equivalent in large samples. Given a finite sample, WLS and ML estimates may differ and possess different properties. Freeman (1987) notes that when the results differ, the assumption of a large sample is probably inappropriate. Some examples of comparisons between ML and WLS estimates will be discussed next.

Forthofer and Lehnen (1981) and Freeman (1987) give examples of analyses using both WLS and ML methods. Similar results for parameter estimation and significance tests were found in their examples. In a univariate analysis of infant mortality rates, Freeman (1987) used likelihood ratio, Neyman χ^2 and Pearson χ^2 goodness-of-fit tests to test hypotheses regarding the grouping of US census data. The same conclusions were drawn from all three goodness-of-fit statistics, but the Neyman χ^2 was systematically the largest. In this example, the sample size was quite large and there were no observed zeros.

Forthofer and Lehnen (1981) compare WLS and ML results using the logit model. Again the similarity of results for parameter estimation and test statistics was demonstrated, as would be expected with a reasonable sample size. In this example however, the large-sample situation did not apply to three of the subpopulations (with $n < 25$), yet reasonable agreement still exists for ML and WLS approaches.

Smith, Savin and Robertson (1984) performed a Monte Carlo comparison of ML and minimum chi square (MCS) sampling distributions using a dichotomous logit regression model. They examined symmetric and asymmetric designs which are commonly used in insecticide research. They found in most cases MCS was superior to

ML in point estimation when mean square error was used as the basis for comparison. The exception to this was in the asymmetric case. With regard to inference, Smith, et al. (1984) found that ML is superior to MCS. In general, the MCS test statistics for the regression coefficient showed larger biases and the MCS variances were often further from expected values than those of ML. Also when less than satisfactory confidence intervals for effective doses were found, MCS showed greater deviation in coverage from the nominal.

Sample Size

Parameter estimation results and test statistics for the GSK method are well behaved when sample size is large. When a test relies on asymptotic results for computing the critical value, an important question is how well the test performs for a finite sample.

Some general recommendations for sample size have been suggested. Forthofer and Lehnen (1981) give the following guidelines when a single function is to be constructed for each subpopulation.

1. Ideally, no more than one-quarter of the functions would be based on subpopulation samples of less than 25 cases.

2. No subpopulation sample size should fall below 10 cases (p. 13).

In the case of extreme events, $\pi < 0.2$ or $\pi > 0.8$, the following is suggested (Forthofer & Lehnen, 1981).

Given the subpopulation parameter π_i (the probability of success for subpopulation i) and sample sizes n_i, the following rule should apply:

$n_i \pi_i \geq 5$ and $n_i(1 - \pi_i) \geq 5$ (p. 13)

Other discussions of small sample situations exist. Koch and Imrey (1985) state that some small sample situations may allow application of the asymptotic WLS results to models for suitable chosen functions. As mentioned above, Forthofer and Lehnen (1981) cite findings where similar results for ML and WLS estimators were obtained with small samples (n < 25).

Read and Cressie (1988) suggest that ML methods are advantageous in the small sample case. Specifically they mention studies which indicate the exact distribution of the Neyman modified χ^2 statistics is less well approximated by the chi-squared distributions than are the likelihood ratio and Pearson χ^2 statistics. Bush (1987) notes that as sample sizes decrease, GSK and ML estimates may differ, with ML estimates tending to have smaller variance. He also mentions that the question of how large a sample is necessary for either method is not precisely known.

Smith, et al. (1984) studied the effect of sample size on MCS and ML estimators for logistic regression. In addition to results mentioned earlier, they found that with asymptotic theory, the approximation begins to deteriorate with N < 120. For example, with a symmetric 8 cell design with N = 64, confidence-interval coverage was eroded. They suggest that in small to moderate samples, a better distribution of the test statistic is needed.

Drew (1985) conducted a simulation study of the validity of the distributional approximation for small samples of the WLS method, in terms of accuracy and power associated with the chi-square statistic. A balanced factorial design with a single dichotomous response variable was employed. Three transformations to the proportion of successes in each subpopulation were applied: the logit function, the complementary log-log (CLL) and the log complement (LC). Subpopulation sizes of n=4 and n=12, and probabilities of success π = 0.5, 0.7 and 0.9 were studied. Findings indicated that the logit and CLL response functions provided conservative tests in the sense that rejection of the null hypothesis of no factor effect at a nominal α level is actually a rejection at a lower level. For LC functions, most tests were found to be conservative; however, sometimes a liberal test was found. The logit function proved less accurate than the LC or CLL functions. Variations in n made large differences in accuracy. As would be expected, with n=12, the accuracy of all simulated situations is much greater than when n=4. Also, the accuracy of the chi-square tests increases as n increases.

Empty Cells and Extreme Values

A limitation of the WLS approach occurs in the presence of observed zeros, i.e., when the observed value of the proportions is 0 or 1. In this case, the estimated variance of the observed function is zero, which yields a test statistic (χ^2) that is undefined. Also, an estimated variance of 0 implies certainty about the value of a function, which never exists when the observed value is based on a sample of observations (Forthofer & Lehnen, 1981). In this instance it is common practice to replace the observed zeros with 1/r or 1/rn, where r is the number of responses and n is the sample size for the subpopulation. This procedure will introduce slight bias into the estimate which may make it more difficult to detect significant effects.

A similar problem exists for ML methods. That is, when the probabilities equal 0 or 1, finite values of ß which satisfy the normal equations do not exist. It has been recommended that 0.5 be added to all cells for improving the convergence of ML iterative algorithms (Goodman, 1970, cited in Forthofer & Lehnen; Dixon, 1981, cited in Freeman, 1987). This is a more extreme procedure than is used in the case of zero cells for WLS estimators.

As with empty cells, in the case of extreme values of π, i.e., $\pi < 0.2$ or $\pi > 0.8$, both ML and WLS estimators may be biased. This is because both procedures rely on large sample size to effect robustness in the statistical properties of estimators. Bush (1987) suggests that neither procedure has the edge in this situation. Given extreme π values, he does advise using a log transformation on proportions for WLS estimates and for follow-up contrasts on ML estimates.

The above section highlights some of the differences found when ML and WLS procedures were compared. Limits of both estimation methods were noted in the case of small sample sizes, empty cells and extreme probability values. Differences between methods were discussed regarding computational differences and hypothesis testing.

Significance of the Study

When data are collected in contingency table format, a linear model approach to data analysis may be desirable due to its ease of implementation in comparison to maximum likelihood procedures. Since the introduction of the linear model approach by Grizzle, et al. (1969), there have been many applications of this method.

One area where the linear model approach has been used in that of public health and public program analysis. In this research application, categorical data are often collected in multidimensional contingency tables with many subpopulations. An example is a Health Maintenance Organization study done by Greenlick, et al. (1968, cited in Forthofer & Lehnen, 1981) where the response factor has seven categories and there are three other factors with four, two, and five categories, respectively. In situations like these, where the linear model approach is employed, it is important to know properties of estimators where expected cell frequencies are low.

In social sciences and education, similar concerns arise. For example, it would not be uncommon in an educational assessment study to have more than five subgroups of interest and three or more response categories. Even with a large total sample size, some of the cell frequencies may prove smaller than is generally recommended. Again, one would want to know how the linear model estimators and test statistics are expected to perform.

A possible application of WLS analysis may be in testing applications where the researcher is interested in characterizing differential item performance between subgroups. Differential item performance may be uniform or nonuniform. These have been defined by Mellenbergh (1982). Uniform bias exists when there is no interaction between ability level and group membership. Non-uniform bias exists when there is interaction between ability level and group membership: that is, the difference in the probabilities of a correct answer for the groups is not the same at all ability levels. The Mantel-Haenszel procedure has been shown to be effective for the detection of uniform

bias, and the logistic regression method has been shown effective for detecting both uniform and non-uniform bias (Swaminathan & Rogers, 1990). Since the logistic regression method is iterative, there is the possibility of nonconvergence. The WLS method utilizes a closed-form solution, and thus may provide an alternative approach.

CHAPTER 3

DESIGN OF THE STUDY

The purpose of this study is to compare the GSK linear model approach using weighted least squares (WLS) procedures with the log-linear model approach for testing hypotheses of interest in r x c contingency tables. Drew (1985) examined the performance of χ^2 tests associated with the WLS approach with 2 x c tables and small sample sizes. He found that tests using logits or complementary log-logs were conservative. As expected, it was also noted that accuracy of the χ^2 distribution increased as sample size increased. Drew's investigation was limited to 2 x c tables (the binomial case). Since the situation may be different in the case of general r x c tables, it is important to study the relative merits of the two procedures in the general case.

A further consideration with respect to the linear model is its appropriateness and its ability for detecting multiplicative interaction. The test statistics obtained with both linear and logarithmic transformations will also be examined with respect to tests of the hypothesis of multiplicative interaction. For all the analyses, the following four estimation methods and response function pairs will be studied: (i) GSK linear model, untransformed proportion (p); (ii) GSK linear model, log (p); (iii) GSK linear model, log-odds: log (p/(1-p)); and (iv) Maximum-likelihood log-linear model.

Design

In this study, performance of WLS and ML test statistics with r x c contingency tables are examined. Total sample size and subpopulation sample size are expected to affect the accuracy of results. The variation of expected cell probability parameters (π_{ij}), or the effect size, may affect the power and accuracy of the statistical tests, particularly when $\pi_{ij} < 0.2$ or $\pi_{ij} > 0.8$. When null hypotheses are true, the estimates and test statistics are distributed as expected; however the power and accuracy of these distributions fluctuates given a false null hypothesis. As the size of the contingency table

24

and the corresponding total number of cells increase, more complexity is introduced as structural models are developed since more parameters are involved in the initial saturated model. Greater variation between expected cell probabilities may also be expected with increased table size, and the occurrence of smaller π_{ij} would be more common. General recommendations of minimum subpopulation sample sizes have been given (see e.g. Forthofer & Lehnen 1981), and recommendations for expected counts for each cell (based on n_i and π_{ij}) have also been given (see e.g. Freeman 1987). In general, when expected cell counts are lower, more uncertainty exists with regard to stability of parameter estimation.

Given specified cell probability parameters and sample sizes, r x c contingency tables will be generated using the Gauss statistical program. Tables with column effects only, and tables with both row and column effects will be simulated.

Tables with Only Column Effects

The first set of tables simulated with have only column effects present and no row effects. The first factor manipulated will be table size. For tables with only column effects, the following nine table sizes were selected.

Table 3.1 Table Sizes of Simulated Tables with Only Column Effects

2 Column	3 Column	4 Column
2x2	2x3	2x4
3x2	3x3	3x4
4x2	4x3	4x4

The total sample size (N) will also be controlled. Three sample sizes of 250, 500 and 1000 will be used for each table. The smallest sample size of 250 was selected because anything less than 250 would not result in convergence of ML procedures with the larger table sizes.

With these tables, no row effect will be present, and this will be designated as row effect size (1). The row marginal probability parameters ($\pi_{i.}$) used for the simulations

25

are as follows. For the two row tables, $\pi_i = .50$; for the three row tables, $\pi_i = .33$; and for the four row tables $\pi_i = .25$. Note that for each table the π_i parameters are equal.

Within each table size, nine levels of column effect size will also be produced with the column marginal probabilities as shown in Table 3.2.

<div align="center">Table 3.2 Column Marginal Probability Parameters</div>

Column Effect Sizes: $\pi_{.j}$ parameters, where $\pi_{.j}$ = column marginal probability parameter for column j

	2 Column		3 Column			4 Column			
	$\pi_{.1}$	$\pi_{.2}$	$\pi_{.1}$	$\pi_{.2}$	$\pi_{.3}$	$\pi_{.1}$	$\pi_{.2}$	$\pi_{.3}$	$\pi_{.4}$
1)*	.5000	.5000	.3333	.3333	.3333	.2500	.2500	.2500	.2500
2)	.4925	.5075	.3258	.3333	.3408	.2425	.2475	.2525	.2575
3)	.4850	.5150	.3183	.3333	.3483	.2350	.2450	.2550	.2650
4)	.4775	.5225	.3108	.3333	.3558	.2275	.2425	.2575	.2725
5)	.4700	.5300	.3033	.3333	.3633	.2200	.2400	.2600	.2800
6)	.4625	.5375	.2958	.3333	.3705	.2125	.2375	.2625	.2875
7)	.4550	.5450	.2883	.3333	.3783	.2050	.2350	.2650	.2950
8)	.4475	.5525	.2808	.3333	.3858	.1975	.2325	.2675	.3025
9)	.4400	.5600	.2733	.3333	.3933	.1900	.2300	.2700	.3100

*Note: Size 1 = No column effect

For tables with only column effects these factors will be completely crossed yielding a total of 243 tables (i.e., nine tables sizes by three sample sizes by nine column effect sizes). Three hundred replications of each table will be done.

For example, with the 2x2 table and a total sample size of 250, nine sets of tables with the parameters shown in Table 3.3 (see next page) will be simulated.

Table 3.3 Cell Probability Parameters (π_{ij}) Used For Simulation of 2x2 Tables with Only Column Effects, Row Effect Size (1).
($\pi_{i.}$ and $\pi_{.j}$ parameters shown in bold type.)

Column effect size 1:

.2500	.2500	**.5000**
.2500	.2500	**.5000**
.5000	**.5000**	

Column effect size 2:

.2462	.2538	**.5000**
.2462	.2538	**.5000**
.4925	**.5075**	

Column effect size 3:

.2425	.2575	**.5000**
.2425	.2575	**.5000**
.4850	**.5150**	

Column effect size 4:

.2388	.2612	**.5000**
.2388	.2612	**.5000**
.4775	**.5225**	

Column effect size 5:

.2350	.2650	**.5000**
.2350	.2650	**.5000**
.4700	**.5300**	

Column effect size 6:

.2312	.2686	**.5000**
.2312	.2686	**.5000**
.4625	**.5375**	

Column effect size 7:

.2275	.2725	**.5000**
.2275	.2725	**.5000**
.4550	**.5450**	

Column effect size 8:

.2238	.2763	**.5000**
.2238	.2763	**.5000**
.4475	**.5525**	

Column effect size 9:

.2200	.2800	**.5000**
.2200	.2800	**.5000**
.4400	**.5600**	

The expected cell frequency (n_{ij}) can be computed by multiplying the cell probability (π_{ij}) by the total sample size. Thus, when column effect size is 5, the expected frequencies with N=250 are:

58.75	66.25	125
58.75	66.25	125
117.5	132.5	(250)

The same nine sets of 2x2 tables will be simulated using sample sizes of 500 and 1000. The remaining two-column tables were created as above substituting the appropriate row marginal probabilities (π_i) corresponding to three and four row tables. Simulations for three and four column tables took place substituting the three column or four column marginal probability values ($\pi_{.j}$) for the two column $\pi_{.j}$ values.

Tables with Row and Column Effects

The research design for tables with row and column effects is similar to that for tables with only column effects. Six table sizes will be used: 2x2 and 4x2 for two column tables; 2x3 and 4x3 for three column tables; and 2x4 and 4x4 for four column tables.

The three sample sizes of 250, 500 and 1000 will again be utilized. The column effect sizes used for the tables with only column effects will also be used for the tables with row and column effects (see Table 3.2, page 26).

In addition, the two levels of row effect sizes designated as (2) and (3) will be created as shown in Table 3.4 (see next page). (Recall that row effect size 1 refers to no row effect.)

As with the tables with only column effects, all factors were completely crossed (six table sizes, by three sample sizes, by nine column effect sizes, by two row effect sizes). For example, the nine 2x2 tables for row effect sizes (2) and (3) will be created as shown in Tables 3.6 and 3.7. (See pages 30 and 31.)

Table 3.4 Row Effect Sizes and Row Marginal Parameters

$\pi_{i.}$ = row marginal probability parameter for row i

| | Two Rows | | | Four Rows | | |
	$\pi_{1.}$	$\pi_{2.}$	$\pi_{1.}$	$\pi_{2.}$	$\pi_{3.}$	$\pi_{4.}$
1)	.50	.50	.25	.25	.25	.25
2)	.47	.53	.22	.24	.26	.28
3)	.44	.56	.19	.23	.27	.31

Simulations for each of the 18 tables shown in Tables 3.6 and 3.7 will be done using the three sample sizes. As with the tables with only column effects, expected cell frequencies can be computed by multiplying the total sample size by π_{ij}. Eighteen tables will also be created for each of the remaining table sizes, again using sample sizes of 250, 500 and 1000. Thus a total of 324 tables with row and column effects will be generated with 300 replications each.

Comparison of Linear and Logarithmic Methods for Detecting Multiplicative Interaction

In order to examine the difference between the linear and logarithmic models when detecting interaction effects for tables with row and column effects generated without multiplicative interaction, the following two levels of column effect size (CES = 10 and CES = 11) were added to the research design. (See Table 3.5.)

Table 3.5 Additional Column Effect Sizes and Column Marginal Parameters

$\pi_{.j}$ parameters, where $\pi_{.j}$ = column marginal probability parameter for column j

| | 2 Column | | 3 Column | | | 4 Column | | | |
	$\pi_{.1}$	$\pi_{.2}$	$\pi_{.1}$	$\pi_{.2}$	$\pi_{.3}$	$\pi_{.1}$	$\pi_{.2}$	$\pi_{.3}$	$\pi_{.4}$
10)	.41	.59	.243	.333	.423	.16	.22	.28	.34
11)	.38	.62	.213	.333	.453	.13	.21	.29	.37

Table 3.6 Cell Probability Parameters (π_{ij}) Used For Simulation of 2x2 Tables with Row and Column Effects, Row Effect Size (2). ($\pi_{i.}$ and $\pi_{.j}$ parameters shown in bold type.)

Column effect size 1:

.2350	.2350	**.4700**
.2650	.2650	**.5300**
.5000	**.5000**	

Column effect size 2:

.2315	.2385	**.4700**
.2610	.2690	**.5300**
.4925	**.5075**	

Column effect size 3:

.2280	.2420	**.4700**
.2571	.2729	**.5300**
.4850	**.5150**	

Column effect size 4:

.2244	.2456	**.4700**
.2531	.2769	**.5300**
.4775	**.5225**	

Column effect size 5:

.2209	.2491	**.4700**
.2491	.2809	**.5300**
.4700	**.5300**	

Column effect size 6:

.2174	.2526	**.4700**
.2451	.2849	**.5300**
.4625	**.5375**	

Column effect size 7:

.2139	.2561	**.4700**
.2412	.2888	**.5300**
.4550	**.5450**	

Column effect size 8:

.2103	.2597	**.4700**
.2372	.2928	**.5300**
.4475	**.5525**	

Column effect size 9:

.2068	.2632	**.4700**
.2332	.2968	**.5300**
.4400	**.5600**	

Table 3.7 Cell Probability Parameters (π_{ij}) Used For Simulation of 2x2 Tables with Row and Column Effects, Row Effect Size (3). (π_i and π_j parameters shown in bold type.)

Column effect size 1:

.2200	.2200	**.4400**
.2800	.2800	**.5600**
.5000	**.5000**	

Column effect size 2:

.2167	.2233	**.4400**
.2758	.2842	**.5600**
.4925	**.5075**	

Column effect size 3:

.2134	.2266	**.4400**
.2716	.2884	**.5600**
.4850	**.5150**	

Column effect size 4:

.2101	.2299	**.4400**
.2674	.2926	**.5600**
.4775	**.5225**	

Column effect size 5:

.2068	.2332	**.4400**
.2632	.2968	**.5600**
.4700	**.5300**	

Column effect size 6:

.2035	.2365	**.4400**
.2590	.3010	**.5600**
.4625	**.5375**	

Column effect size 7:

.2002	.2398	**.4400**
.2548	.3052	**.5600**
.4550	**.5450**	

Column effect size 8:

.1969	.2431	**.4400**
.2506	.3094	**.5600**
.4475	**.5525**	

Column effect size 9:

.1936	.2464	**.4400**
.2464	.3136	**.5600**
.4400	**.5600**	

31

The added simulations will be done for three table sizes (4x2, 4x3, and 4x4), three sample sizes (250, 500 and 1000), two new column effect sizes (10 and 11) and two row effect sizes (2 and 3). These factors will be completely crossed yielding a total of 36 new tables with 300 replications each.

Simulation

Data will be simulated using programs written with the Gauss statistical package. An example of the simulation program can be found in Appendix A.

The simulated tables will be analyzed using the following four estimation methods and response function pairs.

1) GSK linear model, untransformed proportion (p)

2) GSK linear model, log (p)

3) GSK linear model, log-odds: log (p/(1-p))

4) Maximum-likelihood log-linear model

For each table simulated, three χ^2 test statistics will be computed using each of the four estimation methods. These will be referred to as χ^2 column, test of no column effect; χ^2 row, test of no row effect; and χ^2 interaction, test of no interaction effect. The programs used to estimate the b-parameters and test statistics were written with the Gauss programming package and an example of the estimation program is given in Appendix B.

Data Analysis

The STATA (Computing Resource Ctr., 1990) program will be used to analyze the simulated data. Statistical analyses will be conducted to examine the distribution of the χ^2 test statistics. Descriptive analyses will be presented to examine in greater detail the effects of sample size, effect size and table size. The percentage of significant χ^2 values at $\alpha = .05$ are tabulated. Comparisons between methods are highlighted.

A distinction should be noted between instances were the null hypothesis is true and where the null hypothesis is false. In cases where the null hypothesis is true, we would expect to reject the null hypothesis 5% of the time, and discussion will focus on

type I error rates. The corresponding discussion when the null hypothesis is false pertains to statistical power.

For the tables with only column effects, we would not expect to reject the null hypothesis for tests of no column effects when CES=1, and all tests of no row effects and no interaction effects. We would expect to reject the null hypothesis when testing for column effects if CES>1.

For the tables with row and column effects, we would not expect to reject the null hypothesis for tests of no column effects when CES=1, and all tests of no interaction effects. We would expect to reject the null hypothesis when testing for column effects if CES>1, or when testing for row effects.

Presentation of the results follows. Discussion of the descriptive analyses will be given, first for the tables with only column effects, then for the tables with row and column effects. Finally, the results for the research design comparing linear and logarithmic models for χ^2 interaction tests will be presented.

CHAPTER 4

ANALYSIS AND RESULTS

Tables with Only Column Effects: Percentage of Significant χ^2 Values

For each table simulated, χ^2 values for χ^2 tests of no column effect, no row

effect, and no interaction effect were computed using each of the four estimation

methods. The percentage of χ^2 values significant at α = .05 were then tabulated. When

the null hypothesis is true, i.e., all tests of no column effect when no column effect exists,

and all tests of no row effect and no interaction, this percentage would be equal to the

Type I error rate and we would expect the percentage of significant χ^2 values to equal

.05. Results for these tables will be discussed next, followed by discussion of χ^2 column

values found to be significant when the null hypothesis of no column effect is false.

Tests of No Column Effect When the Null Hypothesis Is True

For each of the tables generated with only column effects, when the column

effect size (CES) is 1, the null hypothesis of no column effect is true. In these situations

the percentage of significant χ^2 values corresponds to the type I error rate, and the

expected percentage is five. Table 4.1 gives summaries of the type I error rates for the

four methods.

Table 4.1 Summary of Type I Error Rates
χ^2 Column, Tables with Only Column Effects, CES=1

Method	Range of Type I Error			Mean Type I Error
GSK Linear	4.7%	to	13.7%	7.73%
GSK Log(p)	2.0%	to	12.0%	6.72%
GSK Log-Odds	2.3%	to	12.0%	6.77%
ML Log-Linear	2.3%	to	8.0%	4.80%

With the GSK linear model, the rates exceeded the expected 5% level in most

cases, and the mean rate of 7.73% was much higher than expected. In contrast the log-

34

linear model provided lower Type I error rates with a mean value of 4.8% very close to the expected value. No systematic variations due to sample size or table size were noted for either of these methods.

Since findings were similar for the two-, three- and four-column tables, the four-column tables are shown as a representative example in Table 4.3 on pages 36 and 37. (Results for the two- and three-column tables are given in Appendix C.) The first three columns of Table 4.3 give results for the 2x4 table. Note that for sample sizes 250, 500 and 1000 the type I error rates are 7.7, 4.7 and 8.3 respectively for the GSK linear model and 5.3, 3.0 and 5.0 respectively for the ML log-linear model. In these tables and for all the tables, with both methods similar patterns of type I error rates were found; i.e., increases with one method corresponded to increases with the other method and decreases with one method corresponded to decreases with the other method. However, none of these changes were systematically related to changes in sample size or table size.

Summaries of the ratios of the GSK model Type I error rates to the log-linear model Type I error rates are shown in Table 4.2. The ratios of the GSK linear model type I error rates to the corresponding ML log-linear model Type I error rates are also illustrative. For example, with the 4x4 table these ratios are 2.63, 1.57, and 1.33 for sample sizes of 250, 500 and 1000 respectively. In examining Table 4.2 it is notable that for the GSK linear model all ratios exceed 1, indicating the GSK linear model consistently provides type I error rates greater than does the ML log-linear model.

Table 4.2 Summary of Ratios of Type I Error Rates:
Rate for GSK Method/Rate for Log-Linear Method
χ^2 Column, Tables with Only Column Effects, CES = 1

Method	Range of Ratios			Mean Ratios
GSK Linear	1.16	to	2.64	1.66
GSK Log(p)	.67	to	2.43	1.50
GSK Log-Odds	.77	to	2.43	1.41

Table 4.3 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (1)], Four Column.

Column Effect Size	Method	Size: 2x4			Size: 3x4			Size: 4x4		
	Sample Size:	250	500	1000	250	500	1000	250	500	1000
1	GSK Linear	7.7	4.7	8.3	6.0	9.0	5.0	8.7	6.3	5.7
	GSK Log(p)	7.3	5.0	6.7	3.0	8.0	3.0	4.0	4.7	6.0
	GSK Log-odds	7.7	5.0	6.7	3.0	8.0	2.7	4.3	4.7	5.7
	ML Log-linear	5.3	3.0	5.0	3.0	6.0	2.3	3.3	4.0	4.3
2	GSK Linear	10.7	10.3	10.3	9.3	10.0	12.7	8.3	8.3	10.7
	GSK Log(p)	9.3	8.3	9.3	7.7	10.0	12.3	3.7	7.3	10.7
	GSK Log-odds	10.0	8.7	10.0	7.7	9.7	11.7	3.7	7.3	11.0
	ML Log-linear	5.3	5.3	5.0	6.7	9.0	8.7	4.0	6.3	9.7
3	GSK Linear	11.7	13.3	32.3	13.7	18.7	22.3	13.0	15.7	21.3
	GSK Log(p)	11.0	13.0	30.3	8.7	17.3	22.0	11.3	13.3	21.3
	GSK Log-odds	11.0	13.0	30.7	9.7	17.3	22.3	11.3	13.3	20.7
	ML Log-linear	8.0	9.3	24.3	8.7	15.3	19.0	9.7	12.3	19.0
4	GSK Linear	17.3	26.0	50.0	16.0	25.3	47.0	17.7	30.7	47.3
	GSK Log(p)	16.0	24.3	48.7	12.3	24.0	42.7	13.7	28.3	44.0
	GSK Log-odds	14.7	24.3	49.7	13.3	23.7	42.7	14.0	28.3	45.0
	ML Log-Linear	11.0	18.3	40.3	11.3	20.0	37.3	12.7	27.3	44.3

Table 4.3 Continued

Column Effect Size	Method	Size: 2x4			Size: 3x4			Size: 4x4		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	27.3	43.7	75.0	23.3	47.0	76.0	21.0	43.0	68.7
	GSK Log(p)	24.7	40.3	73.7	21.0	43.3	73.0	17.0	40.0	67.0
	GSK Log-odds	24.7	40.3	74.0	21.7	43.7	73.3	17.0	39.3	67.0
	ML Log-linear	20.0	34.0	65.7	18.7	40.7	70.0	16.3	37.7	65.7
6	GSK Linear	34.3	58.7	87.0	34.7	53.3	86.7	36.3	52.0	87.7
	GSK Log(p)	32.0	56.3	86.7	28.3	52.3	86.3	27.3	50.0	88.7
	GSK Log-odds	32.7	56.7	87.0	28.7	53.0	86.0	28.0	50.0	88.3
	ML Log-Linear	27.7	50.0	84.3	24.7	49.3	84.0	28.7	47.3	85.7
7	GSK Linear	42.7	78.0	97.3	43.7	78.0	96.0	43.7	70.7	96.0
	GSK Log(p)	44.0	76.3	97.0	38.7	76.0	95.7	39.7	67.7	96.0
	GSK Log-odds	42.7	77.3	97.0	38.3	75.7	95.7	39.7	67.7	96.0
	ML Log-linear	35.7	69.0	94.7	33.7	73.7	94.7	37.3	66.0	95.7
8	GSK Linear	60.3	89.3	99.3	61.0	88.0	99.0	61.0	85.7	99.7
	GSK Log(p)	56.3	89.7	99.3	54.7	84.7	98.7	53.3	84.0	99.7
	GSK Log-odds	55.7	89.7	99.3	54.0	86.0	98.7	53.3	84.0	99.7
	ML Log-linear	51.0	87.0	99.0	51.3	83.3	98.7	54.3	82.7	99.7
9	GSK Linear	72.7	96.3	100.0	69.0	97.0	100.0	70.0	94.3	100.0
	GSK Log(p)	71.0	96.0	100.0	65.0	95.7	100.0	63.7	93.7	100.0
	GSK Log-odds	71.3	96.0	100.0	66.0	96.0	100.0	64.3	93.7	100.0
	ML Log-Linear	65.0	94.7	100.0	63.3	94.3	100.0	63.0	93.3	100.0

The GSK log and the GSK log-odds methods provided nearly identical results with respect to the Type I error rates for each table (see Table 4.1, page 34). With both of these methods, the rates were as high as 12%, and the mean rates of 6.72% and 6.77% were higher than the expected 5%. As with the other two methods, no systematic variations due to sample size or table size were noted. Comparison of these methods to the log-linear method revealed that for all but two tables the type I error rate was higher for these methods. The ratios shown in Table 4.2 (page 35) are lower for these methods than for the GSK linear method, but the mean values are still well above 1 indicating these methods generally yield type I error rates greater than the log-linear model.

In general for χ^2 values when the null hypothesis is true, all three GSK methods provided type I error rates greater than the expect 5%, and greater than the ML log-linear method, with the GSK linear model giving the highest type I error rates. For all the tables, the magnitude of the type I error rates covaried between all four methods.

Tests of No Row Effect

For all χ^2 row values, the expected percentage of significant values is equal to the Type I error rate since the null hypothesis is true. Detailed results are presented in Appendix D. Within all column effect sizes, results were similar to those obtained above for χ^2 column values. Ranges of the Type I error rates are given in Table 4.4.

Table 4.4 Range of Type I Error Rates
χ^2 Row, Tables with Only Column Effects, All CES

Method	Range of Type I Error		
GSK Linear	3.7%	to	13.3%
GSK Log(p)	3.0%	to	12.3%
GSK Log-Odds	3.0%	to	12.3%
ML Log-Linear	1.7%	to	8.7%

The GSK linear model yielded the highest values, with most greater than 5%. In contrast, with the log-linear model most values were close to 5%. Again all GSK linear model Type I error rates were greater than the ML log-linear model error rates. Similar

patterns between methods was noted, with no systematic variations due to sample size or column effect size detected.

As with χ^2 column, both GSK logarithmic methods gave similar type I error rates for each of the tables. Again no systematic variations attributable to sample size, table size, or column effect size were detected. Both these methods gave type I error rates generally higher than the expected 5% and greater than did the ML log-linear method. However both of these logarithmic transformations fared better in terms of type I error than did the linear response function. Again, for all tables similar variations between methods was seen.

Tests of No Interaction Effect

For all tables with only column effects, the null hypothesis of no interaction is true and the corresponding type I error rates are given in Appendix E. Results similar to those for χ^2 column and χ^2 row were obtained for χ^2 interaction. Ranges of the Type I error rates are given in Table 4.5. With the GSK linear model, the rates were generally higher than expected. In contrast, the log-linear model yielded rates much closer to the expected value of 5%.

Table 4.5 Range of Type I Error Rates
χ^2 Interaction, Tables with Only Column Effects, All CES

Method	Range of Type I Error		
GSK Linear	3.7%	to	13.3%
GSK Log(p)	3.3%	to	14.0%
GSK Log-Odds	3.7%	to	14.0%
ML Log-Linear	1.7%	to	8.7%

The GSK logarithmic methods again showed comparable type I error rates for each of the tables. For both of these methods, rates greater than 5% frequently occurred. Again, no systematic variations due to sample size, table size or column effect size were apparent.

Summary of Type I Error Rates. With the tables generated with only column effects, three situations existed where null hypotheses were true: no column effects when the column effect size was one, and no row effects and no interaction effects for all tables. In each of these situations, a trend was detected such that the patterns of change in the type I error rates were similar between all methods, where increases and decreases in one method corresponded to increases and decreases in the other methods. Also each of the GSK methods were typically biased in that the observed type I error rates were greater than the expected 5%. This bias was greater when the linear response function was used than when either logarithmic responses function was used. In most cases, the GSK methods gave type I error rates greater than the ML log-linear method which overall proved to be much closer to the expected 5%.

Tests of No Column Effect When the Null Hypothesis is False

When the column effect size is not equal to 1, the null hypothesis of no column effect is false. As described in the research design section, the larger the column effect size (CES), the more false the null hypothesis. The percentage of χ^2 column values significant at the .05 level, within each column effect size, is given in Table 4.3 (pages 36-37) for the four-column tables and in Appendix C for the two- and three-column tables. Figures F.1 through F.9 in Appendix F graphically depict these results. A perusal of these results reveals that the three GSK methods generally yielded rejection rates greater than the ML log-linear method did.

For each table simulated when all methods do not reject the null hypothesis at a rate > 98%, the GSK linear method gave higher percentages of rejection than did the ML log-linear method. This is most evident for tables where the rates of rejection ranged from 20% to 80% and can be seen clearly with medium effect sizes (e.g., CES $\geq$ 4 and CES $\leq$ 7).

The GSK log and the GSK log-odds methods performed similar in these situations where the null hypothesis is false as they did when the null hypothesis was

true. Both methods were closely matched in the percentage of times the null hypothesis was rejected for each table. This percentage was usually greater than that given by the ML log-linear method and slightly less than that given by the GSK linear method. Again the differences between methods is most noticeable for medium effect sizes and where rejection rates are between 20% and 80%.

In these situations where the null hypothesis is false, two factors are influential in the percentage rejection rates. As expected, when the sample size increases and the CES increases the percentage of significant values increases. Along with these increases in the percentage of significant values, comes a change in the ratio of each of the GSK methods to the ML log-linear method. These ratios all approach 1 as the power to detect a false null hypothesis increases; i.e., the difference between methods decreases as sample size increases and the null hypothesis becomes more false.

The 4x4 table (see Table 4.3 and Figure F.9) can be referred to as a representative example of this trend. For CES=3, the ratios of the GSK linear model rate of rejection to the ML log-linear model rate of rejection are 1.32, 1.28 and 1.12 for the three sample sizes, while these corresponding ratios for CES=9 are 1.11, 1.01 and 1.00. Thus differences in power to detect a false null hypothesis diminish as the sample size and the effect size increase.

In examining Tables 4.3, C.1 to E.3 and Figures F.1 to F.9 in conjunction with the research design, it should be noted that the number of columns and the corresponding $\pi_{.c}$ parameters differ. Tables 4.3, C.1 and C.2 and Figures F.1 to F.3 give results for two-column tables; Tables D.1 to D.3 and Figures F.4 to F.6 give the corresponding results for three-column tables; and Tables E.1 to E.3 and Figures F.1 to F.9 give results for four-column tables. Within each set of tables having the same number of columns, and within each CES, similarities can be seen in the percentage of significant values.

In summary, trends similar to those noted when the null hypothesis is true were found when the null hypothesis is false; i.e., similar patterns of change in the percentage

of hypotheses rejected were observed between methods and the three GSK methods gave greater percentage rejection rates than did the ML log-linear model. In this situation rather than increasing the error rate, the GSK methods show more power to detect a false null hypothesis. Also, as expected the sample size and column effect size influenced the percentage of significant χ^2 values detected, with increases in sample size and column effect size corresponding to increases in the percentage rejection rate.

Tables with Row and Column Effects: Percentage of Significant χ^2 Values

For tables with row and column effects, the null hypothesis is true for tests of no column effect when CES is one, and for all tests of no interaction. The remaining tests of no column effect and all tests of no row effect are tests of false null hypotheses.

Tests of No Column Effect When the Null Hypothesis is True

Data for the tests of no column effect when the null hypothesis is true is given under CES = 1 in Appendix G for row effect (2); these data are graphically presented in Appendix H. Appendices I and J give the corresponding tabled and graphic results when the row effect size is (3). The expected percentage of significant values is five. Summaries of the Type I error rates for the four methods are given in Table 4.6.

Table 4.6 Summary of Type I Error Rates
χ^2 Column, Tables with Row and Column Effects, CES = 1

Method	Range of Type I Error			Mean Type I Error
Row Effect (2)				
GSK Linear	4.7%	to	11.0%	8.08%
GSK Log(p)	3.3%	to	11.0%	7.19%
GSK Log-Odds	3.3%	to	11.3%	7.12%
ML Log-Linear	3.3%	to	7.3%	5.16%
Row Effect (3)				
GSK Linear	5.7%	to	12.3%	8.43%
GSK Log(p)	3.3%	to	10.7%	7.57%
GSK Log-Odds	3.0%	to	11.0%	7.70%
ML Log-Linear	2.7%	to	7.0%	5.58%

Effects similar to those for the tables with only column effects are apparent. Most notably all the GSK methods provided type I error rates greater than the ML log-linear method, and greater than the expected 5%.

As with the tables with only column effects, the patterns of change of the type I error rate were similar between methods and the ML log-linear model proved most accurate in the percentage of true null hypotheses rejected, while the GSK linear model proved least accurate.

Tests of No Interaction

Ranges of Type I error rates for tests of no interaction are given in Table 4.7. Detailed results for tests of no interaction are given in Appendix K for row effect size (2) and Appendix L for row effect size (3). Analysis of type I error rates for these tables showed the ML log-linear model differing from the GSK model as before: with the three GSK methods producing error rates greater than the expected 5%. Tables K.1 to K.3 show that all three of the GSK methods generally performed alike with the two logarithmic functions being most similar.

Table 4.7 Range of Type I Error Rates
χ^2 Interaction, Tables with Row and Column Effects, All CES

Method	Range of Type I Error
Row Effect (2)	
GSK Linear	5.0% to 12.7%
GSK Log(p)	4.3% to 11.7%
GSK Log-Odds	4.3% to 12.0%
ML Log-Linear	3.0% to 9.3%
Row Effect (3)	
GSK Linear	5.0% to 15.0%
GSK Log(p)	3.7% to 12.7%
GSK Log-Odds	4.0% to 12.4%
ML Log-Linear	2.7% to 9.0%

43

The GSK linear model's type I error is often noticeably higher than the other three methods, especially with the larger tables, larger sample sizes and larger column effect sizes. In Tables L.1 to L.3 where the row effect size is larger, this trend is more apparent. Descriptions of additive and multiplicative association as well as comparison of linear and logarithmic response functions given by Forthofer and Lehnen (1981) help to clarify these results. The authors note that results of analyses on the logarithmic scale may differ from results based on the linear scale, with the logarithmic model sometimes yielding fewer interaction terms, and in other cases with the linear model yielding the simpler model. The distinction is made between additive and multiplicative association. (See Forthofer & Lehnen, 1981, pp. 30-35.) While the GSK linear model tests the hypothesis of no additive association, the GSK log, GSK log-odds and the ML log-linear models test the hypotheses of no multiplicative association. The tables simulated in this study were generated without multiplicative association, but when both the CES and row effect size do not equal one, additive association is present. Thus the GSK linear model, while seemingly producing high type I error rates, is seen to perform as expected in detecting a false null hypothesis of "no additive interaction". In order to investigate the differences in χ^2 interaction rates of rejection more fully, a further comparison of linear and logarithmic models is discussed later.

Turning our attention toward the three logarithmic models reveals patterns similar to those found for tables with only column effects. The GSK log and GSK log-odds models are very similar in their type I error rates for each table, and these rates are generally higher than 5%. In comparison the type I error rates for the ML log-linear model are generally closer to the expected 5%. Again, CES and sample size did not seem to affect the type I error rate.

In summary, for the GSK logarithmic models and the ML log-linear model results were comparable to results found for χ^2 interaction when tables were generated with only column effects. Similar patterns of type I error rates were found between

44

methods, and GSK methods yielded error rates higher than 5%. Sample size and CES were not found to effect the error rates for these three methods. For the tables with row and column effects, the GSK linear model detected occurrences of additive interaction and thus rejected the null hypothesis more often in instances where CES and the row effect size were not one.

Tests of No Column Effect When the Null Hypothesis is False

When the column effect size does not equal 1, the null hypothesis of no column effect is false. Tables G.1 to G.3 give the percentage of χ_2 column values significant at the .05 level within each column effect size for row effect size (2); and Figures H.1 to H.6 graphically represent these results. Tables I.1 to I.3 and Figures J.1 to J.6 give the corresponding results for row effect size (3).

Results for tests of no column effect are similar for both row effect sizes. As expected, for all methods the power to detect a false null hypothesis increases as the sample size and the column effect size increase. In most cases the three GSK methods are more powerful in detecting a false null hypothesis than is the ML log-linear method. In general, results for the three GSK methods are comparable.

Data for the 2x4 table, row effect size (2) (see Table G.3 and Figure H.5) are selected as a representative example. For all sample sizes, when CES$\geq$3, the rates of rejection for the ML log-linear model are apparently lower than those rates for the GSK models, until the rates for all models converge at 100% with the larger sample size and effect sizes. With a sample size of 1000, the three GSK methods provide similar results. This is also shown for large CES (i.e., $\geq$5) with a sample size of 500.

With a sample size of 250, and with the smaller CES and N=500, the GSK linear method provides higher rates of rejection than either of the GSK logarithmic methods and the ML log-linear method, thus indicating the power of the linear method is greater in these situations.

This difference between the GSK linear method and the other methods is also noted with table sizes of 4x3 and 4x4 and N=250, with row effect sizes (2) and (3). For example, see Table G.3 and Figure H.6. For these tables, the three logarithmic models yield similar percentage of significant values, while only the GSK linear model yields noticeably higher percentage of significant values.

In summary, trends similar to those for tables with only column effects are seen for tables with row and column effects. For all methods the percentage of significant χ^2 values increased as the sample size and column effect size increased. In many cases similarities are shown for the three GSK models, with the ML log-linear model being less powerful. With the 4x3 and 4x4 tables, and N=250, the GSK linear model is most powerful in detecting a false null hypothesis, while the three logarithmic methods give similar results.

Tests of No Row Effect

For the tables with row and column effects, the null hypothesis of no row effect is false and row marginal probability parameters (π_j.) used in these simulations are given in the research design section (see Table 3.4). Since overlap between column effects and row effects is inherent in the design for these tables, results for χ^2 tests of no row effect are expected to be similar to those for χ^2 tests of no column effect with similar effect sizes. Results for these tests are given in Appendix K for row effect size (2) and Appendix L for row effect size (3).

The results of the tests of no row effect are similar to those for tests of no column effects and serve to confirm the findings of specific trends. In general, the percentage of significant values increased as the effect size and sample size increased. The power of the GSK linear model was greatest and the power of the ML log-linear model was lowest, until all methods converged at a rejection rate at or near 100% with a large sample size and large row effect size.

46

Comparison of Linear and Transformed Models for Detecting Multiplicative Interaction

In order to examine the difference between the linear and logarithmic models in detecting interaction effects for tables with row and column effects generated without multiplicative interaction, two levels of column effect size (CES=10 and CES=11) were added to the research design. For row effect size (2), results of the percentage of significant χ^2 interaction values for these tables are given in Table 4.8 (see next page) and are graphically presented in Appendix O. In order to illustrate comparisons with smaller column effect sizes, Figures O.1 to O.3 also show results for CES=7, CES=8 and CES=9. When the row effect size is 2, the GSK linear model shows evidence of detecting more significant χ^2 values as the column effect size increases. This is more so when the sample size is 1000 and CES=11. For example, with the 4x2 table, (N=1000) the GSK linear model type I error rate is 12.0%, and the highest type I error rate for the logarithmic models is 6.7%. With the 4x4 table (N=1000) these rates are 13.7% for the GSK linear model, contrasted to the highest rate of 8% with any of the logarithmic models.

The results for row effect size (3) are given in Table 4.9 (see page 49) and are graphically presented in Appendix P. When the row effect size is (3) the difference between the GSK linear model and the logarithmic models is more apparent. With a sample size of 1000 and CES=11 the GSK linear model produced type I error rates three to four times as high as the logarithmic models. Thus, even though the tables were created with no multiplicative interaction, as the column and row effect sizes increase the inherent additive interaction is increased. Therefore, as expected with the GSK linear model the percentage of significant χ^2 interaction values increases as the sample size and CES increase.

Table 4.8 Comparison of Linear and Transformed Models, Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (2)].

Column Effect Size	Method	Size: 4x2			Size: 4x3			Size: 4x4		
	Sample Size:	250	500	1000	250	500	1000	250	500	1000
10	GSK Linear	8.7	8.7	7.7	9.0	9.0	12.0	10.3	7.0	7.3
	GSK Log(p)	8.0	6.3	6.0	5.3	7.0	8.3	5.0	3.0	5.0
	GSK Log-odds	8.3	7.0	5.7	5.3	8.0	8.7	5.3	3.0	5.0
	ML Log-linear	5.7	4.0	3.7	5.3	6.3	6.0	4.0	3.0	4.7
11	GSK Linear	8.0	7.0	12.0	10.3	9.0	8.3	9.3	9.7	13.7
	GSK Log(p)	4.7	6.3	5.3	5.3	6.7	5.3	4.0	4.7	8.0
	GSK Log-odds	5.0	6.3	6.7	5.7	6.7	5.3	4.3	4.7	8.0
	ML Log-linear	3.0	4.7	4.0	5.0	4.7	4.3	4.7	4.3	6.0

Table 4.9 Comparison of Linear and Transformed Models, Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (3)].

Column Effect Size	Method	Size: 4x2			Size: 4x3			Size: 4x4		
	Sample Size:	250	500	1000	250	500	1000	250	500	1000
10	GSK Linear	12.0	8.3	18.0	9.0	11.3	18.3	12.0	15.3	17.0
	GSK Log(p)	8.0	6.3	6.7	7.0	6.3	6.3	5.3	8.3	5.3
	GSK Log-odds	8.0	5.7	7.0	7.0	6.7	6.3	5.3	8.3	5.3
	ML Log-linear	5.0	4.3	4.3	5.3	5.0	4.7	5.0	7.0	4.7
11	GSK Linear	10.7	19.0	23.7	14.7	18.0	23.0	12.7	17.0	26.0
	GSK Log(p)	5.7	9.3	7.0	6.3	4.0	6.0	4.7	6.3	6.3
	GSK Log-odds	6.3	10.0	7.7	6.3	5.3	6.0	4.7	6.3	6.3
	ML Log-linear	4.0	7.7	5.0	4.7	4.0	5.0	5.0	4.7	5.0

CHAPTER 5

DISCUSSION AND CONCLUSIONS

The simulation studies conducted examined the use of the GSK linear, GSK transformed and ML log-linear models for the analysis of categorical data under varying conditions of sample size, table size, and effect size. Some general trends were noted in the study and recommendations based on these findings follow.

The influence of the first factor of sample size was as expected. In cases where the null hypothesis was true, sample size did not appear to affect type I error rates. When the null hypothesis was false, the percentage of times the null hypothesis was rejected increased as sample size increased for all situations. Thus for all methods, an increase in sample size corresponded to an increase in power.

When generalizing findings, it is important to keep in mind that the smallest sample size used in this study was 250, which is larger than often available for educational research studies. The findings may not hold true with smaller sample sizes and previously recommended observed cell frequencies should be followed when analyzing data and interpreting results (see page 19).

The second factor of effect size also appeared to systematically influence results as expected. As the null hypothesis of no column or row effect became more false, the power to reject the null hypothesis increased for all of the models tested, until all models converged near 100% rates of rejection with a large sample size (e.g., N=1000) and large effect sizes.

The third factor studied is that of table size. Systematic differences related to table size are more difficult to detect and interpret. In general, systematic differences between χ^2 statistics attributable to table dimensions were not detected. One exception is with the larger tables (e.g., 4x3 and 4x4) and N=250. In these instances, the GSK linear method provided larger rates of rejection of the null hypotheses.

Given the influences of the above three factors controlled in the study, differences between the four methods were apparent. The two GSK transformed models performed almost identically in all situations, therefore differences observed were between the GSK linear, the GSK transformed and the ML log-linear methods.

For tables where the null hypothesis was true, the GSK linear model usually yielded type I error rates greater than the expected 5%. In contrast, the ML log-linear model usually yielded type I error rates near the expected 5%, and lower than the GSK methods. The GSK logarithmic methods yielded type I error rates generally lower than the GSK linear method but usually greater than the ML log-linear method and greater than the expected value of 5%. In summary, all GSK methods tended to incorrectly reject a true null hypothesis greater than 5% of the time, thus providing an inflated type I error rate, while the type I error rate for the ML log-linear method was generally close to the expected value of 5%.

In situations where the null hypothesis was known to be false for tests of column effects, differences between the methods similar to those described above were found. In general the GSK linear model most often rejected the false hypothesis and the ML log-linear model least often rejected this hypothesis, with the GSK logarithmic methods falling between. Differences between methods were most noticeable with medium effect sizes; these differences diminished as sample size and effect size increased.

The important finding of greater power with the GSK methods is made under the assumption of the χ^2 distribution. With limited sample sizes, this distributional assumption may not hold and corrections to asymptotical distributional results for tabled χ^2 values would be necessary. If the observed distributions for GSK and log-linear methods are examined and appropriate changes to tabled χ^2 values are made, the differences in power between methods may diminish. Further research regarding corrections to the asymptotical distributions is recommended.

Detecting Multiplicative Interaction

An added factor contributing to differences between methods was introduced with χ^2 tests of no interaction when both column and row effects were present. Differences between hypotheses of no multiplicative interaction and no additive interaction were discussed; and the corresponding appropriateness of the linear or logarithmic model for these hypothesis tests was also covered. In this study, tables with row and column effects were simulated with no multiplicative interaction and only the logarithmic models provided reasonable type I error rates when the column and row effects became increasingly large. The linear model detected the presence of additive interaction which was inherent in the tables, and provided increasingly large rates of rejection of the null hypothesis as the sample sizes and effect sizes increase. In light of the differences in linear and logarithmic response functions, these findings were as expected.

Recommendations and Discussion

In this study some differences were noted between the GSK and ML log-linear estimation methods and between linear and logistic models. Based on the findings, in most cases either a GSK logarithmic model or the log-linear model could be used and similar results would be expected. If a GSK logarithmic model is selected, it should be kept in mind that when the null hypothesis is true the χ^2 tests tend to be slightly liberal; i.e., the null hypothesis is incorrectly rejected more often than expected thus producing an inflated type I error rate. When the effect sizes are greater, and the sample size is increased, differences between the methods diminishes.

For tests of main effects hypotheses, the linear model provided results similar to the three logarithmic models. The distinction between the linear and logarithmic models became apparent when χ^2 tests of no interaction were studied. When selecting a model and interpreting results for tests of no interaction it is important to consider the difference between additive and multiplicative interaction. Most often researchers are

interested in testing hypotheses of multiplicative interaction. Therefore it is recommended that logarithmic transformations be used with the GSK approach. An elaborate discussion of hypotheses of no interaction is given by Bhapkar and Koch (1968). In their study, fixed and random marginal effects are discussed and appropriate underlying probability models are presented for a variety of cases.

Along the same lines, this study points toward the possibilities that exist in terms of selecting response functions and testing specific hypotheses using a model-based approach. The GSK approach may be used with any linear, logarithmic or exponential transformation of the response function and examples of these applications have been discussed. This may also be advantageous when questions about ordered response data arise. The GSK model-based approach easily lends itself to representation and testing of research questions.

Log-linear models may also be applied to tests of specific hypotheses using a model-based approach. Discussion of procedures for nonstandard log-linear models is given by Rindskopf (1990) where log-linear models analogous to ANOVA models are presented as special cases of the generalized linear model. Within this context the flexibility of model building discussed for WLS procedures is available. Flexibility does not exist for changing the response function; the logarithm of the cell frequencies is the dependent variable.

When any model-based approach is utilized, the selection of a model and response function that tests the stated hypothesis of interest is important. For example, models have been presented that model ordered categorical responses (Agresti, 1989; Forthofer and Lehnen, 1981) and given ordinal data these may be more desirable than the general categorical data models. When building a model for analysis, cross-validation of the findings is advised especially when a large number of models have been tested (Rindskopf, 1990).

Summary

This study revealed some differences between GSK linear, GSK logarithmic, and log-linear models when χ^2 tests of main and interaction effects were examined. The log-linear model yielded Type I error rates close to the expected values, and the three GSK models yielded error rates higher than expected. Sample size and table size had no effect on Type I error rates.

In cases where the null hypothesis was false, the three GSK procedures were uniformly more powerful than the log-linear procedure. Differences between methods were most noticeable with medium effect sizes; these differences diminished as sample size and effect size increased. There were no systematic differences due to table size.

The GSK linear model, an additive model, was not appropriate for tests of no multiplicative interaction. Therefore, in the general case where a researcher is interested in testing for multiplicative interaction, a logarithmic transformation of the observed proportions is necessary.

Findings of this study are pertinent to applied researchers who wish to expand their analysis of categorical data to test hypotheses other than those of independence. Hypothesis testing and interpretation of results are relatively straightforward with a model-based approach and are thus encouraged. The results indicate that GSK methods provide the most powerful tests. Since the GSK method can be understood by researchers familiar with linear regression alaysis and is easily implemented, it is recommended that the GSK method be used to analyze categorical data. Since it was found that untransformed proportions yielded a higher Type I error rate that logarithmic transformations, it is also recommended that logarithmic transformations be used with GSK methods for the analysis fo categorical data.

APPENDIX A

GAUSS CODE FOR SIMULATION PROGRAM

```
/*To simulate data for tables with only column effects, 4x2 tables
        (Column effect sizes 1 through 9, row effect size 1)*/

/* Set up files, etc. */

rndseed 10000000 * rndu(1,1);
loadp mktabl2 = c:\mktabl2;
nout = 0;
closeall;
let varnames = tsize n coltype t;
create f1 = sim4x2 with ^varnames,11,2;

let size = 3;              /* size of table, 3 = 4 x 2 */
let r = 4;                 /* no. of rows in table*/
let c = 2;                 /* no. of columns in table */

let ss = 250 500 1000;

let colp[9,2] = .50 .50
                .4925 .5075
                .4850 .5150
                .4775 .5225
                .47   .53
                .4625 .5375
                .455  .545
                .4475 .5525
                .44   .56;

  let rp[1,4] = .25 .25 .25 .25;

        /* Do loop i for indexing sample size */

i = 1;

    do until i > 3;
    n = ss[i,1];

        /* Do loop j for indexing selection of col. % */

j = 1;
    do until j > 5;
    cp = colp[j,.];


    /* Construct table probabilities to send to procedure */
```

```
    /* Do loop k to get 300 simulations per table */

k = 1;
   do until k >300;

   locate 1,1; print "n " i ", col%" j  ", table # " k ".";

   cellp = vec(rp' * cp);
   cellp = cumsumc(cellp);        /* get cumulative probabilities */

   t = mktabl2(cellp,n);        /* call procedure to generate table */


   /* Write out table to data file */


nout = nout + writer(f1,size~i~j~t);


k = k+1;
endo;

   j = j + 1;
   endo;

      i = i + 1;
      endo;

closeall f1;
end;


MKTABL2 PROCEDURE

proc 1 = mktabl2(cp,nc);

local rx,rndval,t,cpp;

rx = rows(cp);
cpp = 0|cp; cpp = cpp[1:rx,1];
rndval = rndu(nc,1);
t = sumc(rndval .> cpp' .and rndval .<= cp')';

retp(t);

endp;
```

APPENDIX B

GAUSS CODE FOR ESTIMATION PROGRAM

```
/*To estimate data from the simulation program
4x2 data used, row effect (1) */

/* Set-up */

closeall;
clearg b,sb,chi2,chirow,chicol,meth;
let rz=1;
          /* WLS Procedure */

proc (0)=wls(y,x,w,ccol,crow);
local yw,xw,xwinv,xwy,ew;

yw= y.*w;
xw=x.*w;
xwinv = invpd(xw'*xw);
xwy= xw'*yw;
b=xwinv*xwy;
sb= sqrt(diag(xwinv));
ew = (y - x*b).*w;
chi2=ew'*ew;              /* chi2, Goodness of fit*/
                    /* Calc. chi2 for row and col effect */

chirow = (crow * b)' * invpd(crow*xwinv*crow') * crow * b;

chicol = (ccol * b)' * invpd(ccol*xwinv*ccol') * ccol * b;

endp;

     /* Max. Lik. Procedure */

proc (0)=mloft(y,x,ccol,crow);
clearg b,sb,chi2,chirow,chicol,meth;
local w,iter,bs,cov,ew,tol,bdiff,yhat;


bs=ln(y + .005)/x;
iter=1;
tol=.00001;
bdiff=1;

do until bdiff <tol;
   yhat=exp(x*bs);
   w=sqrt(yhat);
   yhat=x*bs + (y-yhat)./yhat;
   b=(yhat.*w)/(w.*x);
   bdiff= maxc(abs(b-bs));
   bs=b;
   iter=iter +1;
endo;
cov=invpd((w.*x)'(w.*x));
sb=sqrt(diag(cov));
ew=(ln(y) - x*b).*w;
```

```
chi2 = 2*sumc((ln(y) - x*b).*y);
chirow = (crow*b)'*invpd(crow*cov*crow')*crow*b;
chicol = (ccol*b)'*invpd(ccol*cov*ccol')*ccol*b;
meth=4;
endp;

          /* Set up output file */

let size=3;

closeall f1,f2;

nout1=0;

let id = size n colp method;
let chi = gof row col;


    let bname = b1 b2 b3 b4 b5;
    let sename = se1 se2 se3 se4 se5;
    varnames=id|bname|sename|chi;
    open f1=sim4x2;
    load x=x4x2;
    load cc=cc4x2;
    load cr=cr4x2;
    create f2=est4x2 with ^varnames,17,4;


          /* Read data, compute ests. */

do until eof(f1);
   dat=readr(f1,1);

          /* ID Variable recoding */

   ss = dat[.,2];
   colp=dat[.,3];

   ni=dat[.,4:11]';
   nii=ni.*(ni.>0) + .5*(ni.==0);
   n=sumc(nii);
   rf1=nii/n;
   rf2=ln(rf1);
   rf3=rf1./(1-rf1); rf3=ln(rf3);

          /* Set up Y and weights. */

   j=1;
   do until j>3;
       if j==1;
           y=rf1;
           wi=n / (rf1 .*(1-rf1));
           wi=sqrt(wi);
           meth=1;
```

60

```
        elseif j = =2;
            y=rf2;
            wi=n*(rf1 ./(1-rf1));
            wi = sqrt(wi);
            meth=2;

        elseif j = =3;
            y=rf3;
            wi=n*(rf1 .*(1-rf1));
            wi = sqrt(wi);
            meth=3;
    endif;

            /* Call WLS procedure to compute ests. */

call wls(y,x,wi,cc,cr);

        /* Write out results from WLS proc. */
locate 1,1; print "n" ss "col%" colp  "table #" rz;

nout1=nout1+ writer(f2,size~ss~colp~meth~b'~sb'~chi2~chirow~chicol);

j=j+1;
endo;
        /* Call MLofT proc. and write out results */

call mloft(nii,x,cc,cr);
nout1=nout1+ writer(f2,size~ss~colp~meth~b'~sb'~chi2~chirow~chicol);

rz = rz + 1;
endo;
closeall f1, f2;
end;
```

APPENDIX C

PERCENTAGE OF SIGNIFICANT χ^2 COLUMN VALUES FOR TABLES
WITH ONLY COLUMN EFFECTS

Table C.1 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (1)], Two Column.

Column Effect Size	Method	Size: 2x2			Size: 3x2			Size: 4x2		
	Sample Size:	250	500	1000	250	500	1000	250	500	1000
1	GSK Linear	7.7	9.3	12.0	8.7	4.7	9.3	7.3	5.3	7.3
	GSK Log(p)	7.3	8.7	11.7	9.0	4.3	9.7	7.3	5.7	7.0
	GSK Log-odds	7.3	9.3	12.3	8.7	4.3	9.7	7.3	6.0	7.0
	ML Log-linear	3.0	4.3	6.7	7.0	3.7	5.7	6.3	4.3	5.0
2	GSK Linear	9.3	12.3	11.7	8.3	9.3	8.3	3.7	8.7	11.0
	GSK Log(p)	8.7	11.0	12.7	7.0	8.7	8.3	3.7	7.7	10.0
	GSK Log-odds	9.0	11.7	12.7	7.0	8.3	8.3	3.7	8.0	10.3
	ML Log-linear	4.3	8.7	6.7	5.3	7.0	7.3	3.0	7.3	8.7
3	GSK Linear	10.3	14.0	26.0	13.0	13.7	19.0	9.3	15.7	16.0
	GSK Log(p)	9.7	12.7	25.3	11.3	13.3	18.7	9.0	17.3	16.3
	GSK Log-odds	9.7	13.7	26.0	11.3	13.3	18.7	9.3	17.0	15.7
	ML Log-linear	5.7	9.7	15.3	8.0	11.7	14.0	6.7	14.7	13.0
4	GSK Linear	15.7	23.0	38.0	17.0	21.7	32.7	15.0	21.0	36.7
	GSK Log(p)	15.3	23.3	38.0	14.7	20.7	32.7	15.0	20.0	34.7
	GSK Log-odds	15.3	23.7	38.3	15.0	20.3	32.3	15.0	20.0	34.3
	ML Log-linear	9.7	18.0	25.3	12.0	18.0	26.0	13.3	17.3	28.3

63

Table C.1 Continued

Column Effect Size	Method	Size: 2x2			Size: 3x2			Size: 4x2		
	Sample Size:	250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	25.7	34.0	57.0	19.3	37.7	55.3	19.0	34.7	57.7
	GSK Log(p)	25.3	33.3	57.0	19.0	36.0	55.0	18.3	34.0	56.0
	GSK Log-odds	25.3	33.3	58.0	19.0	35.7	55.0	19.0	34.3	56.0
	ML Log-linear	17.7	27.3	45.3	14.7	31.0	47.0	15.3	30.0	52.3
6	GSK Linear	29.7	49.7	69.7	25.0	45.3	72.0	30.3	39.3	70.7
	GSK Log(p)	29.0	48.0	69.7	23.3	44.0	70.7	28.0	38.0	69.7
	GSK Log-odds	29.0	48.3	70.0	23.3	44.0	70.7	28.7	38.0	69.7
	ML Log-Linear	22.7	42.3	57.3	21.3	41.0	64.0	23.0	36.0	67.0
7	GSK Linear	46.3	64.0	88.0	33.3	60.7	85.0	32.7	50.7	82.3
	GSK Log(p)	45.3	62.3	88.0	32.7	58.7	85.0	30.7	50.7	82.0
	GSK Log-odds	45.7	62.0	88.3	33.0	58.3	85.0	31.7	51.3	81.3
	ML Log-Linear	33.7	58.3	82.0	27.0	53.3	81.7	27.7	48.0	78.0
8	GSK Linear	55.7	76.7	96.3	48.7	67.3	95.7	38.7	71.7	92.7
	GSK Log(p)	54.0	75.7	96.3	47.0	66.3	95.3	38.0	71.0	92.0
	GSK Log-odds	54.7	76.7	96.3	47.3	66.0	95.3	38.3	71.3	92.0
	ML Log-Linear	44.0	65.7	91.7	43.0	64.0	93.0	34.7	68.0	89.7
9	GSK Linear	56.7	83.0	98.0	61.7	77.7	97.3	49.3	83.0	96.7
	GSK Log(p)	55.7	84.3	98.3	59.3	77.3	97.3	48.0	82.0	96.7
	GSK Log-odds	56.3	84.0	97.0	59.3	77.0	97.3	49.0	82.0	96.7
	ML Log-Linear	45.0	77.0	97.0	55.7	75.3	96.0	44.0	81.3	95.0

Table C.2 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (1)], Three Column.

Column Effect Size	Method	Sample Size:	Size: 2x3			Size: 3x3			Size: 4x3	
		250	500	1000	250	500	1000	250	500	1000
1	GSK Linear	10.0	13.7	9.0	7.3	9.0	5.3	4.7	9.3	7.3
	GSK Log(p)	8.0	12.0	9.0	7.3	7.0	5.3	2.0	6.7	5.7
	GSK Log-odds	8.0	12.0	9.0	6.7	7.0	5.7	2.3	6.7	5.7
	ML Log-linear	4.7	8.0	6.7	5.0	6.0	3.3	3.0	5.7	5.0
2	GSK Linear	10.3	8.3	11.0	12.3	9.0	6.7	8.7	8.0	9.0
	GSK Log(p)	7.7	8.0	10.7	11.3	8.7	6.0	5.3	7.0	9.3
	GSK Log-odds	8.3	8.0	11.0	12.0	9.0	6.0	5.3	7.0	9.3
	ML Log-linear	5.3	6.3	7.3	8.3	5.7	5.3	4.7	4.3	6.7
3	GSK Linear	10.7	13.3	22.7	11.3	13.0	21.3	10.0	16.0	21.3
	GSK Log(p)	9.3	12.0	20.7	10.3	12.0	21.0	6.3	14.0	19.7
	GSK Log-odds	9.3	12.3	21.3	10.3	12.3	21.0	6.0	14.3	19.7
	ML Log-linear	6.7	8.0	15.7	8.7	10.3	17.0	5.7	13.3	15.3
4	GSK Linear	17.0	23.0	38.3	14.0	22.0	39.7	13.3	18.7	36.0
	GSK Log(p)	16.0	22.7	39.0	12.3	20.7	39.0	12.3	18.0	33.3
	GSK Log-odds	16.0	22.3	39.7	12.7	20.3	39.0	12.3	18.0	34.0
	ML Log-Linear	9.3	15.7	31.0	11.7	17.7	35.0	11.3	15.3	30.7

65

Table C.2 Continued

Column Effect Size	Method	Size: 2x3			Size: 3x3			Size: 4x3		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	26.7	42.3	66.3	17.7	33.0	60.3	19.0	32.0	56.7
	GSK Log(p)	23.7	40.3	66.3	15.3	31.0	58.3	16.0	32.3	54.7
	GSK Log-odds	24.3	40.3	66.3	15.3	31.3	58.3	16.7	32.7	54.7
	ML Log-linear	16.3	33.0	56.7	13.7	27.7	53.3	14.3	28.7	52.0
6	GSK Linear	31.7	49.0	78.7	23.3	44.7	78.7	26.7	48.7	75.0
	GSK Log(p)	30.3	49.7	79.3	22.0	43.3	79.0	22.7	43.3	74.3
	GSK Log-odds	30.7	50.0	79.0	22.3	44.0	79.3	23.0	43.7	74.0
	ML Log-Linear	24.3	42.3	71.7	19.7	39.7	74.7	21.0	41.7	70.7
7	GSK Linear	41.3	67.7	93.0	35.3	67.0	93.7	32.7	67.3	93.0
	GSK Log(p)	38.3	67.7	93.0	31.3	65.7	93.7	31.0	64.0	91.0
	GSK Log-odds	40.0	68.0	93.0	31.3	66.0	94.0	31.0	64.0	91.0
	ML Log-linear	30.3	58.7	89.3	27.3	61.3	91.7	29.7	60.3	90.3
8	GSK Linear	51.0	79.7	98.0	48.0	76.0	98.3	54.3	78.7	99.7
	GSK Log(p)	48.3	76.3	98.3	48.7	73.7	98.7	49.3	76.7	99.7
	GSK Log-odds	49.0	77.0	98.3	48.7	74.3	98.7	50.0	77.0	99.7
	ML Log-linear	42.0	70.3	96.3	43.3	70.3	98.3	47.7	74.3	99.0
9	GSK Linear	63.0	84.7	99.3	64.3	86.7	99.0	59.3	87.3	98.7
	GSK Log(p)	60.3	84.7	99.0	60.7	87.0	99.0	56.0	85.3	98.3
	GSK Log-odds	60.0	84.7	99.0	61.0	87.0	99.0	56.3	85.7	98.3
	ML Log-Linear	53.0	81.0	99.0	56.7	84.0	99.0	55.0	83.7	98.0

APPENDIX D

PERCENTAGE OF SIGNIFICANT χ^2 ROW VALUES FOR TABLES
WITH ONLY COLUMN EFFECTS

Table D.1 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (1)], Two Column.

Column Effect Size	Method	Size: 2x2			Size: 3x2			Size: 4x2		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
1	GSK Linear	8.7	7.3	9.7	9.3	7.7	9.0	8.3	9.0	7.3
	GSK Log(p)	8.3	8.3	9.3	8.3	7.3	8.7	6.0	6.3	7.3
	GSK Log-odds	8.7	8.0	9.7	8.3	7.3	9.0	6.3	6.3	7.0
	ML Log-linear	5.3	5.0	3.7	6.3	4.0	6.0	4.3	4.7	5.0
2	GSK Linear	9.3	11.7	11.7	11.3	7.0	6.7	7.3	6.3	8.3
	GSK Log(p)	9.3	10.7	11.7	10.7	7.3	7.0	6.0	5.0	8.3
	GSK Log-odds	9.3	12.0	12.0	11.0	7.0	7.0	6.0	5.3	8.3
	ML Log-linear	6.3	6.3	7.0	7.0	4.0	3.3	3.7	2.7	4.7
3	GSK Linear	9.0	10.3	8.7	8.0	8.3	7.3	5.3	8.0	7.0
	GSK Log(p)	9.0	9.7	8.3	7.7	8.7	7.0	3.7	6.0	5.7
	GSK Log-odds	9.3	10.0	9.0	7.0	8.7	7.3	3.7	6.0	6.0
	ML Log-linear	4.7	7.0	4.0	5.7	4.7	4.3	1.7	4.3	3.7
4	GSK Linear	7.3	13.3	9.3	9.3	5.3	5.7	8.3	5.0	6.3
	GSK Log(p)	7.0	11.3	9.7	9.0	4.7	7.0	5.7	4.7	5.3
	GSK Log-odds	7.0	12.3	10.3	9.0	4.7	6.7	6.3	4.7	5.7
	ML Log-linear	4.3	7.3	4.0	6.7	2.0	3.7	5.3	2.3	5.0

Continued, next page

68

Table D.1 Continued

Column Effect Size	Method	Size: 2x2			Size: 3x2			Size: 4x2		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	10.0	10.0	7.7	6.7	9.0	6.0	7.7	9.0	9.3
	GSK Log(p)	9.0	10.0	8.7	5.3	10.3	5.0	5.0	8.7	9.7
	GSK Log-odds	9.0	9.0	9.0	5.3	10.3	5.0	5.0	8.7	9.3
	ML Log-linear	5.3	6.7	5.3	3.3	6.3	2.0	3.3	5.7	4.7
6	GSK Linear	6.7	11.0	9.7	6.3	7.7	6.0	9.3	9.3	8.3
	GSK Log(p)	6.3	8.7	9.7	6.3	8.3	6.0	7.3	7.3	7.0
	GSK Log-odds	6.7	9.3	9.7	6.3	8.0	6.0	7.3	7.7	7.0
	ML Log-Linear	5.7	6.3	4.7	3.0	5.3	4.7	5.0	5.7	3.7
7	GSK Linear	8.7	7.7	6.0	8.0	6.7	8.7	12.0	8.0	6.7
	GSK Log(p)	8.3	6.7	7.3	6.0	6.0	9.3	9.7	7.3	6.7
	GSK Log-odds	8.3	7.0	7.3	6.0	6.7	9.3	9.7	7.7	6.7
	ML Log-Linear	4.0	3.7	3.3	4.0	3.3	5.3	7.0	4.7	4.3
8	GSK Linear	9.0	8.7	7.0	8.3	12.0	7.7	9.7	6.0	10.7
	GSK Log(p)	9.0	8.0	7.0	9.7	10.0	7.0	8.0	6.3	8.7
	GSK Log-odds	9.0	8.3	7.3	9.3	10.3	7.3	8.7	6.0	8.3
	ML Log-Linear	5.3	5.0	3.3	4.7	6.7	5.7	5.3	3.7	4.7
9	GSK Linear	8.7	10.3	9.7	7.7	7.7	8.7	8.7	9.3	6.0
	GSK Log(p)	8.3	11.0	10.0	7.3	7.0	9.3	7.0	8.7	5.3
	GSK Log-odds	8.3	10.3	10.3	8.0	7.3	9.0	7.7	8.3	5.7
	ML Log-Linear	4.0	6.3	5.0	3.7	3.7	6.3	5.0	4.0	4.3

69

Table D.2 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (1)], Three Column.

Column Effect Size	Method	Size: 2x3			Size: 3x3			Size: 4x3		
Sample Size:		250	500	1000	250	500.	1000	250	500	1000
1	GSK Linear	6.3	12.0	8.7	9.7	8.3	5.3	11.3	6.0	8.7
	GSK Log(p)	5.7	10.7	8.7	7.7	7.7	5.7	7.0	5.0	8.0
	GSK Log-odds	6.0	10.3	8.7	8.0	8.0	5.7	7.3	5.0	7.7
	ML Log-linear	5.0	8.7	5.3	7.7	5.7	3.3	7.3	4.3	5.0
2	GSK Linear	7.7	7.3	7.7	8.7	7.7	6.0	7.3	6.7	5.0
	GSK Log(p)	6.3	7.0	7.7	7.3	6.7	5.3	5.3	7.0	5.3
	GSK Log-odds	6.3	7.0	7.7	8.0	6.3	5.3	5.3	7.0	5.3
	ML Log-linear	4.3	5.3	4.7	6.3	4.7	3.3	4.7	5.3	3.7
3	GSK Linear	7.7	10.7	7.3	9.0	6.3	7.0	10.3	5.3	10.0
	GSK Log(p)	6.0	10.7	7.0	7.7	5.3	5.3	7.3	4.7	9.0
	GSK Log-odds	6.3	10.7	7.0	7.3	5.7	5.3	7.7	4.3	9.0
	ML Log-linear	5.0	8.0	4.7	4.7	4.3	4.0	6.7	3.7	7.0
4	GSK Linear	7.0	9.3	5.7	7.3	10.7	5.3	5.0	8.3	10.0
	GSK Log(p)	6.7	8.0	6.3	6.3	11.3	4.7	3.3	6.7	9.3
	GSK Log-odds	6.7	8.0	6.3	6.3	11.3	4.7	3.7	6.7	9.3
	ML Log-linear	6.0	7.0	4.3	6.0	8.0	3.7	3.0	6.3	8.3

70

Table D.2 Continued

Column Effect Size	Method	Size: 2x3			Size: 3x3			Size: 4x3		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	7.0	7.0	7.7	8.0	5.3	9.0	8.7	7.0	9.3
	GSK Log(p)	6.0	6.7	7.7	7.3	5.7	9.0	5.3	6.0	9.0
	GSK Log-odds	6.0	7.0	7.3	7.3	5.3	9.0	5.0	6.0	9.0
	ML Log-linear	4.7	6.0	5.0	6.3	4.0	8.0	4.3	4.3	7.7
6	GSK Linear	5.7	6.3	5.7	6.3	7.3	4.0	8.7	6.0	5.7
	GSK Log(p)	5.7	6.0	6.0	5.0	6.7	4.7	5.3	5.0	5.0
	GSK Log-odds	6.0	6.0	6.0	5.0	7.0	4.7	5.3	5.0	5.0
	ML Log-Linear	4.3	5.0	4.7	5.0	5.7	3.0	4.7	4.0	3.3
7	GSK Linear	6.7	6.3	5.3	8.0	5.0	6.7	6.7	5.3	7.7
	GSK Log(p)	6.3	6.7	5.7	6.3	5.7	6.7	5.0	5.3	6.7
	GSK Log-odds	6.3	6.3	5.7	6.0	5.3	6.7	5.0	5.3	6.3
	ML Log-Linear	3.0	4.3	3.7	5.7	3.7	4.7	4.3	4.7	4.7
8	GSK Linear	8.7	6.0	8.3	8.3	5.7	6.3	8.3	5.0	7.7
	GSK Log(p)	7.3	6.7	8.3	7.7	5.7	6.0	5.7	4.7	6.7
	GSK Log-odds	8.0	6.3	8.3	8.0	5.3	6.0	5.7	5.0	7.0
	ML Log-Linear	4.7	5.0	4.7	6.3	4.0	3.3	5.3	3.3	5.7
9	GSK Linear	7.0	7.7	7.7	5.3	7.0	8.0	6.7	7.0	8.0
	GSK Log(p)	6.3	8.0	7.7	6.0	6.3	8.3	5.0	5.7	7.3
	GSK Log-odds	6.3	7.7	8.0	5.3	6.3	8.3	5.0	5.3	7.3
	ML Log-Linear	5.0	6.7	5.7	4.7	4.7	5.3	5.0	4.3	5.0

71

Table D.3 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (1)], Four Column.

Column Effect Size	Method	Size: 2x4			Size: 3x4			Size: 4x4		
	Sample Size:	250	500	1000	250	500	1000	250	500	1000
1	GSK Linear	8.7	9.3	7.3	4.7	6.7	8.0	9.7	4.3	7.0
	GSK Log(p)	8.7	9.0	6.3	4.0	5.3	7.7	4.0	3.0	6.3
	GSK Log-odds	8.7	9.3	6.0	4.0	5.3	7.7	4.0	3.0	6.3
	ML Log-linear	7.0	8.0	4.7	3.0	4.7	6.0	4.3	2.7	5.3
2	GSK Linear	7.7	6.7	5.0	7.0	6.0	7.0	9.7	5.0	7.0
	GSK Log(p)	7.7	6.7	4.0	6.3	4.7	6.0	5.0	3.0	6.3
	GSK Log-odds	7.7	6.7	4.0	6.0	4.7	6.0	5.0	3.0	6.3
	ML Log-linear	6.7	6.0	1.7	5.0	4.3	5.0	3.7	2.0	5.3
3	GSK Linear	8.3	6.3	8.7	6.3	8.3	9.0	8.0	8.0	7.3
	GSK Log(p)	7.3	6.7	8.0	5.0	6.7	8.3	5.3	6.0	6.7
	GSK Log-odds	7.7	6.7	8.0	5.0	6.7	8.7	5.7	6.3	6.3
	ML Log-linear	6.0	6.3	5.7	4.0	5.7	6.7	5.7	6.3	5.7
4	GSK Linear	7.0	6.7	6.3	9.0	6.3	5.7	8.7	10.7	7.0
	GSK Log(p)	6.3	6.3	6.7	7.0	6.0	5.3	5.3	7.0	6.0
	GSK Log-odds	6.3	6.3	6.7	7.7	6.3	5.7	5.7	7.3	6.3
	ML Log-linear	5.7	4.7	4.0	7.0	4.3	4.0	5.3	7.0	6.0

72

Table D.3 Continued

Column Effect Size	Method	Size: 2x4			Size: 3x4			Size: 4x4		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	7.7	5.3	9.3	10.3	8.0	5.7	10.0	7.0	9.3
	GSK Log(p)	6.7	5.7	8.3	8.3	5.3	5.3	7.0	5.3	10.0
	GSK Log-odds	6.7	5.7	8.3	8.3	5.3	5.3	7.0	5.3	10.0
	ML Log-linear	5.3	4.3	5.3	7.0	4.3	3.3	5.3	4.3	8.0
6	GSK Linear	10.0	5.7	5.7	6.3	6.3	8.0	8.7	9.0	11.0
	GSK Log(p)	9.3	5.3	5.0	5.0	5.7	6.0	6.0	7.7	7.7
	GSK Log-odds	9.7	5.7	5.0	5.3	5.7	6.7	5.7	8.0	7.7
	ML Log-Linear	8.0	4.3	3.3	3.0	5.0	3.3	5.7	6.3	6.7
7	GSK Linear	3.7	5.7	5.7	7.7	6.0	3.7	6.3	8.0	6.0
	GSK Log(p)	4.3	6.3	6.0	6.0	5.0	4.3	3.7	5.7	7.0
	GSK Log-odds	4.3	6.7	5.7	6.0	5.3	4.0	3.7	5.3	6.7
	ML Log-Linear	3.7	4.7	3.7	6.0	4.7	3.7	2.7	5.0	5.0
8	GSK Linear	6.3	5.7	8.3	7.7	6.0	9.0	11.3	5.0	4.7
	GSK Log(p)	5.3	5.3	8.3	4.0	5.7	8.3	7.7	4.7	5.3
	GSK Log-odds	5.3	5.7	8.0	4.0	6.0	8.3	7.7	4.7	5.3
	ML Log-Linear	4.7	4.3	5.7	4.7	4.0	7.7	6.3	4.0	4.0
9	GSK Linear	6.3	6.0	7.3	5.7	7.0	7.0	8.3	9.0	4.7
	GSK Log(p)	6.0	5.7	6.3	4.0	5.3	6.3	6.3	7.3	4.3
	GSK Log-odds	6.3	5.7	6.3	4.0	5.3	6.3	6.3	7.3	4.3
	ML Log-Linear	4.7	5.0	4.3	3.7	5.0	5.0	6.7	6.7	3.3

73

APPENDIX E

PERCENTAGE OF SIGNIFICANT χ^2 INTERACTION VALUES FOR TABLES
WITH ONLY COLUMN EFFECTS

Table E.1 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (1)], Two Column.

Column Effect Size	Method	Size: 2x2			Size: 3x2			Size: 4x2		
	Sample Size:	250	500	1000	250	500	1000	250	500	1000
1	GSK Linear	9.0	10.3	4.7	10.0	7.3	10.7	8.7	9.7	4.3
	GSK Log(p)	9.3	9.7	4.7	9.0	7.3	11.7	7.7	9.0	4.3
	GSK Log-odds	9.3	10.0	4.7	9.3	7.3	11.7	7.7	9.3	4.3
	ML Log-linear	4.7	5.7	3.0	5.3	4.7	5.0	6.0	7.3	3.7
2	GSK Linear	7.3	8.7	7.3	10.3	5.0	7.0	8.7	7.0	9.3
	GSK Log(p)	8.0	8.0	7.7	10.7	5.0	7.0	7.7	6.7	9.7
	GSK Log-odds	7.3	8.0	8.0	10.7	5.0	7.0	8.0	6.7	9.7
	ML Log-linear	4.0	6.0	4.3	5.3	3.0	5.0	5.0	5.7	4.7
3	GSK Linear	5.3	8.0	10.0	9.3	9.0	6.3	9.3	7.7	7.3
	GSK Log(p)	6.0	7.3	9.0	9.7	8.3	6.7	8.0	6.3	6.7
	GSK Log-odds	6.0	7.7	9.0	9.7	8.3	6.7	8.0	6.7	6.7
	ML Log-linear	2.7	5.7	4.7	6.0	6.7	2.7	4.7	4.3	5.7
4	GSK Linear	9.3	9.7	11.0	9.3	10.0	8.0	8.3	7.0	8.7
	GSK Log(p)	9.7	9.3	10.0	8.3	9.7	7.3	6.7	6.0	8.3
	GSK Log-odds	9.3	9.3	10.3	9.0	9.7	7.3	7.0	6.0	8.3
	ML Log-linear	5.0	5.3	6.0	5.7	6.3	5.3	3.7	4.7	6.3

Table E.1 Continued

Column Effect Size	Method	Size: 2x2			Size: 3x2			Size: 4x2		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	10.7	9.3	9.0	8.7	6.7	7.7	9.0	9.0	6.3
	GSK Log(p)	10.0	8.3	9.0	8.0	6.3	7.3	7.3	9.0	6.7
	GSK Log-odds	10.0	8.3	9.0	8.0	6.7	7.3	7.7	9.0	6.7
	ML Log-linear	6.3	3.3	6.0	5.0	4.7	4.3	5.6	6.7	3.0
6	GSK Linear	11.7	10.7	8.3	9.7	6.3	9.7	8.3	6.0	8.0
	GSK Log(p)	11.3	9.7	8.0	8.3	5.7	10.0	6.7	6.3	7.3
	GSK Log-odds	11.0	9.7	8.0	8.3	6.0	10.0	7.0	6.3	7.7
	ML Log-Linear	8.7	5.7	5.3	6.7	3.3	5.3	4.3	4.7	5.0
7	GSK Linear	6.7	10.3	9.0	5.0	9.0	9.7	9.0	8.0	7.0
	GSK Log(p)	7.7	9.7	8.3	4.0	7.7	9.0	8.0	7.0	6.7
	GSK Log-odds	7.3	9.3	8.0	4.3	7.7	9.0	8.7	7.7	6.7
	ML Log-Linear	3.7	5.3	4.3	2.7	5.7	7.0	6.7	5.3	6.0
8	GSK Linear	9.3	15.0	9.3	7.0	9.7	7.0	7.7	11.3	8.0
	GSK Log(p)	9.0	14.0	9.7	6.3	9.0	7.7	8.0	10.0	7.7
	GSK Log-odds	9.0	14.0	9.3	6.7	8.7	8.0	8.3	9.7	7.7
	ML Log-Linear	5.7	8.7	7.0	5.0	4.7	4.3	4.3	7.0	6.0
9	GSK Linear	5.0	9.0	8.7	9.0	9.3	11.0	9.3	8.3	11.3
	GSK Log(p)	6.7	6.7	8.7	8.0	8.7	11.3	8.3	8.7	10.3
	GSK Log-odds	6.3	7.7	8.7	8.3	9.0	11.0	9.3	8.7	10.7
	ML Log-Linear	3.3	5.0	4.7	6.3	6.0	8.3	7.0	6.0	6.3

Table E.2 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (1)], Three Column.

Column Effect Size	Method	Size: 2x3 250	500	1000	Size: 3x3 250	500	1000	Size: 4x3 250	500	1000
1	GSK Linear	6.3	7.3	8.3	8.3	5.7	7.7	9.3	7.7	9.0
	GSK Log(p)	6.3	7.3	9.0	8.3	5.7	7.7	7.7	7.3	9.3
	GSK Log-odds	6.3	7.3	9.0	8.3	5.7	7.7	7.7	7.7	9.3
	ML Log-linear	3.7	5.0	5.3	6.7	3.3	5.0	7.7	5.7	5.7
2	GSK Linear	9.7	9.7	8.7	8.0	5.3	9.7	7.3	5.3	6.0
	GSK Log(p)	9.3	9.0	8.7	5.3	5.3	9.0	7.3	3.3	6.3
	GSK Log-odds	9.3	9.3	8.7	5.7	5.3	8.7	7.7	3.7	6.3
	ML Log-linear	6.7	5.0	5.0	3.7	4.3	5.3	6.7	3.3	4.3
3	GSK Linear	8.3	8.7	9.7	11.3	7.3	9.3	8.0	6.0	5.3
	GSK Log(p)	7.3	7.3	9.7	8.0	6.7	9.0	5.0	5.3	4.3
	GSK Log-odds	7.3	7.3	9.3	8.3	6.7	9.7	5.7	5.3	4.3
	ML Log-linear	5.3	3.7	5.3	7.3	5.3	6.7	3.7	3.7	3.0
4	GSK Linear	10.0	7.3	9.7	9.3	7.3	5.3	8.3	4.7	8.0
	GSK Log(p)	9.0	7.7	9.7	7.3	6.7	5.0	8.0	4.7	8.0
	GSK Log-odds	9.0	7.3	10.0	7.7	6.7	5.0	8.3	4.7	8.3
	ML Log-linear	5.3	5.7	5.3	5.0	5.3	3.7	6.3	3.7	6.3

Continued, next page

77

Table E.2 Continued

Column Effect Size	Method	Size: 2x3			Size: 3x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	6.3	5.3	10.3	8.3	5.3	7.0	5.7	8.0	8.3
	GSK Log(p)	6.3	5.3	10.3	6.3	5.7	6.3	4.7	7.7	8.3
	GSK Log-odds	6.3	5.3	10.3	7.0	5.7	6.3	5.3	7.7	8.3
	ML Log-linear	3.7	3.7	7.0	5.0	3.3	4.3	3.7	6.3	4.7
6	GSK Linear	10.0	9.3	9.3	8.7	8.7	9.0	9.0	6.7	7.0
	GSK Log(p)	9.7	10.0	8.7	5.3	6.7	9.0	5.7	7.3	6.0
	GSK Log-odds	10.0	10.0	8.7	5.7	7.3	9.0	6.3	7.3	6.3
	ML Log-Linear	5.7	5.7	5.3	3.7	4.3	4.7	5.7	4.3	5.3
7	GSK Linear	7.0	9.0	6.7	9.3	13.7	8.3	7.7	5.3	7.0
	GSK Log(p)	7.3	8.0	6.0	7.7	12.0	8.0	6.3	5.0	5.7
	GSK Log-odds	7.3	9.0	6.0	8.0	12.3	8.0	6.3	5.0	5.7
	ML Log-Linear	4.3	4.7	2.3	5.3	9.7	5.7	5.7	4.0	4.0
8	GSK Linear	6.7	9.0	7.3	6.7	8.0	8.3	7.3	9.0	7.0
	GSK Log(p)	6.0	9.3	7.7	5.3	8.0	8.7	6.0	8.3	6.3
	GSK Log-odds	6.3	9.3	7.7	6.3	8.0	8.7	6.0	8.3	6.7
	ML Log-Linear	4.3	5.7	5.7	4.0	5.3	6.0	4.7	5.3	5.0
9	GSK Linear	10.7	10.0	6.0	9.3	6.0	7.0	7.0	8.3	5.0
	GSK Log(p)	10.3	10.0	5.7	9.3	5.3	6.0	4.3	7.7	5.3
	GSK Log-odds	10.3	10.0	6.0	9.3	5.7	6.0	4.7	8.3	5.0
	ML Log-Linear	7.0	6.0	3.7	7.0	2.3	5.3	4.0	6.0	3.0

78

Table E.3 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (1)], Four Column.

Column Effect Size	Method	Size: 2x4			Size: 3x4			Size: 4x4		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
1	GSK Linear	7.3	9.0	8.7	10.0	9.0	7.0	7.0	6.3	6.3
	GSK Log(p)	7.3	9.0	9.3	8.0	7.7	6.3	4.7	5.7	6.3
	GSK Log-odds	7.3	9.3	9.3	8.0	7.7	6.7	4.7	5.3	6.3
	ML Log-linear	5.7	4.7	6.3	6.3	5.0	4.7	5.0	4.0	5.7
2	GSK Linear	8.7	7.3	6.7	6.0	8.0	8.3	6.3	7.7	7.0
	GSK Log(p)	7.7	7.0	5.3	4.0	7.3	7.3	4.3	5.0	5.7
	GSK Log-odds	8.0	7.3	5.3	4.7	7.7	7.3	4.3	5.0	5.7
	ML Log-linear	6.0	5.7	4.3	4.3	4.7	5.7	4.0	4.7	4.0
3	GSK Linear	7.7	5.0	7.7	9.0	8.0	6.0	7.7	8.0	8.7
	GSK Log(p)	7.0	4.3	7.3	7.3	8.7	5.7	5.7	7.0	7.7
	GSK Log-odds	7.0	4.3	7.3	7.7	8.7	5.7	5.7	7.0	7.7
	ML Log-Linear	5.3	3.0	4.7	4.0	6.7	4.3	5.7	6.3	7.3
4	GSK Linear	6.7	7.0	8.3	9.3	8.0	7.7	10.0	8.0	7.7
	GSK Log(p)	6.3	6.3	7.7	7.0	7.3	7.0	7.0	6.0	7.0
	GSK Log-odds	6.3	6.3	7.7	7.3	7.3	7.0	7.7	7.0	7.0
	ML Log-Linear	5.3	4.3	4.3	6.0	5.3	4.7	8.3	5.7	6.0

Continued, next page

79

Table E.3 Continued

Column Effect Size	Method	Size: 2x4			Size: 3x4			Size: 4x4		
Sample Size:		250	500	1000	250	500	1000	250	500	1000
5	GSK Linear	5.7	5.7	4.7	10.3	8.7	6.0	5.7	8.7	4.7
	GSK Log(p)	5.3	5.3	5.0	8.0	8.3	6.0	5.0	4.7	3.7
	GSK Log-odds	5.7	5.7	5.0	8.0	8.3	6.0	5.0	5.0	3.7
	ML Log-linear	4.3	4.0	2.3	6.3	6.0	4.0	3.7	4.0	2.7
6	GSK Linear	8.0	7.3	7.3	8.7	7.3	8.3	7.7	8.0	6.3
	GSK Log (p)	7.7	6.3	7.0	5.3	6.7	7.7	6.0	6.7	6.0
	GSK Log-odds	7.7	6.7	7.0	5.3	6.7	7.7	6.0	6.3	6.0
	ML Log-linear	6.0	4.3	5.0	5.0	4.3	6.0	5.7	6.0	4.3
7	GSK Linear	9.7	9.7	5.7	9.0	6.7	8.3	10.3	8.3	8.0
	GSK Log (p)	9.0	8.7	5.7	6.3	5.7	8.3	5.3	5.3	9.0
	GSK Log-odds	9.3	9.0	5.7	6.3	5.7	8.7	6.0	5.3	9.0
	ML Log-linear	7.0	5.0	3.7	6.0	4.7	6.7	6.7	4.7	6.7
8	GSK Linear	8.7	7.7	8.3	7.7	7.7	7.3	8.3	8.7	10.7
	GSK Log (p)	7.3	8.7	8.0	7.7	6.3	7.0	3.7	6.0	8.0
	GSK Log-odds	7.3	8.0	8.0	8.0	6.7	7.0	4.3	6.7	8.0
	ML Log-linear	5.7	6.3	6.3	6.7	5.0	5.0	4.3	5.7	5.7
9	GSK Linear	11.0	7.3	9.0	7.7	7.0	8.0	7.3	10.0	8.7
	GSK Log (p)	10.0	7.0	9.0	6.7	5.3	7.7	4.0	6.7	7.0
	GSK Log-odds	11.0	7.3	8.7	7.0	5.3	7.7	4.0	6.7	7.0
	ML Log-linear	7.0	4.7	5.3	6.0	4.3	4.7	4.3	7.0	5.7

APPENDIX F

FIGURES SHOWING PERCENTAGE OF SIGNIFICANT χ^2 COLUMN
VALUES FOR TABLES WITH ONLY COLUMN EFFECTS
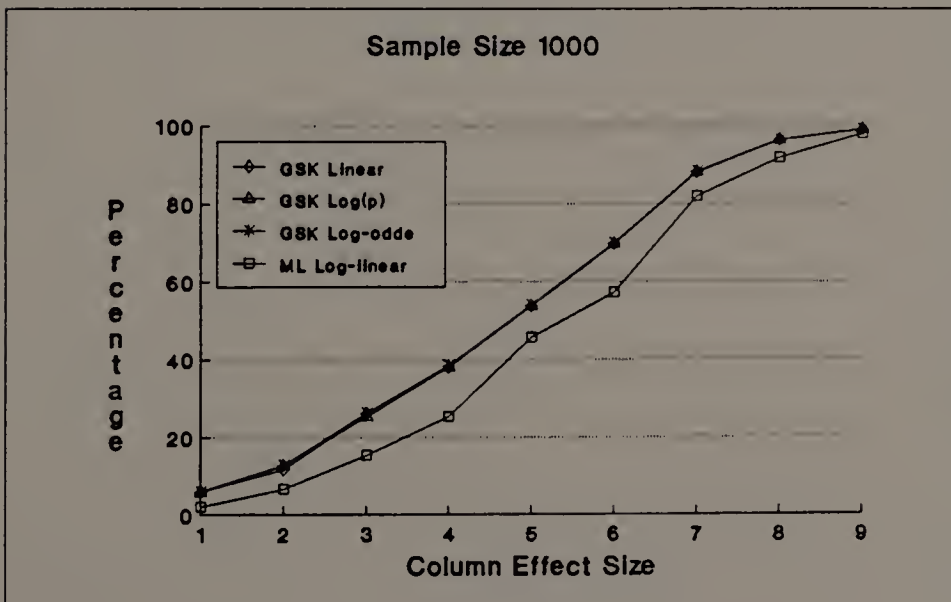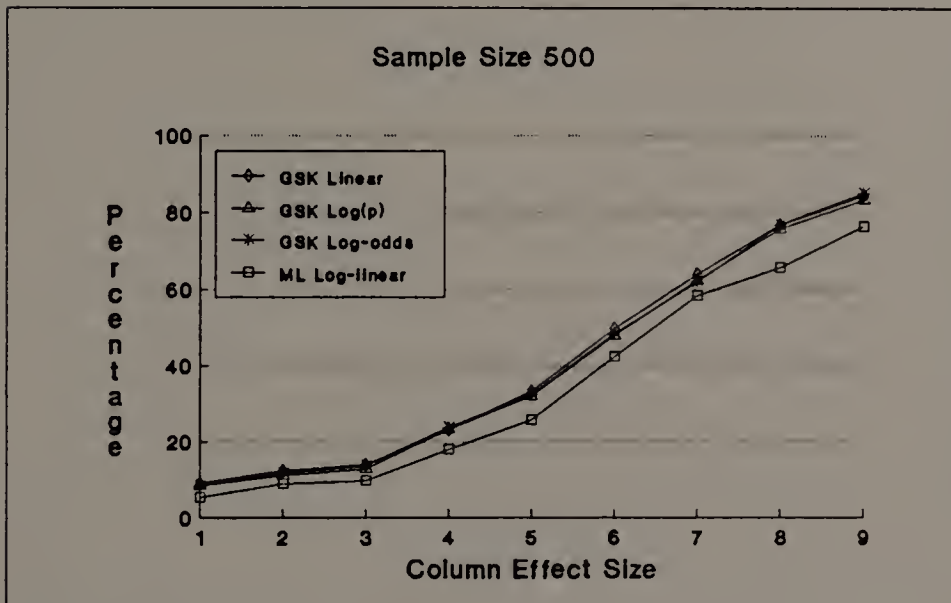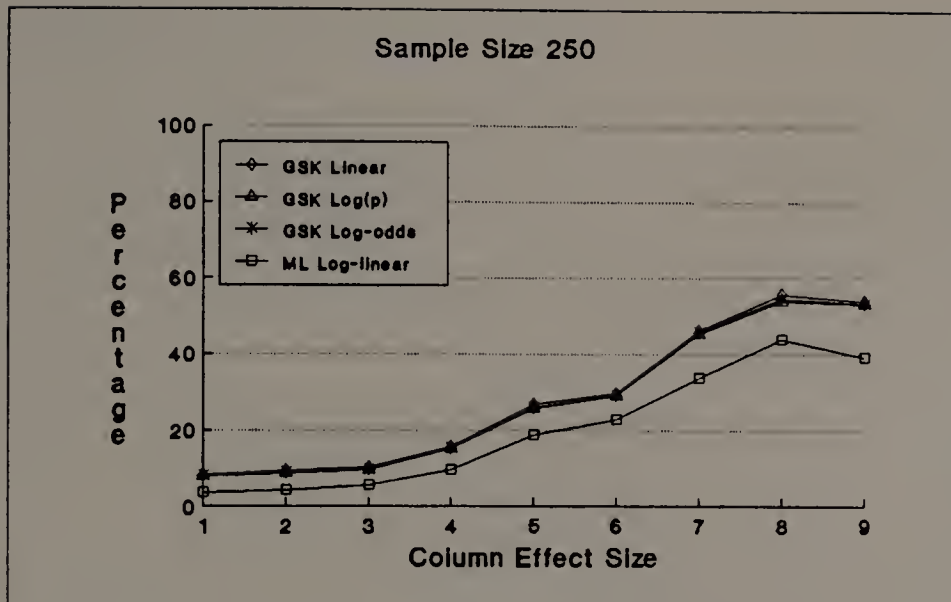
Figure F.1 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 2x2.
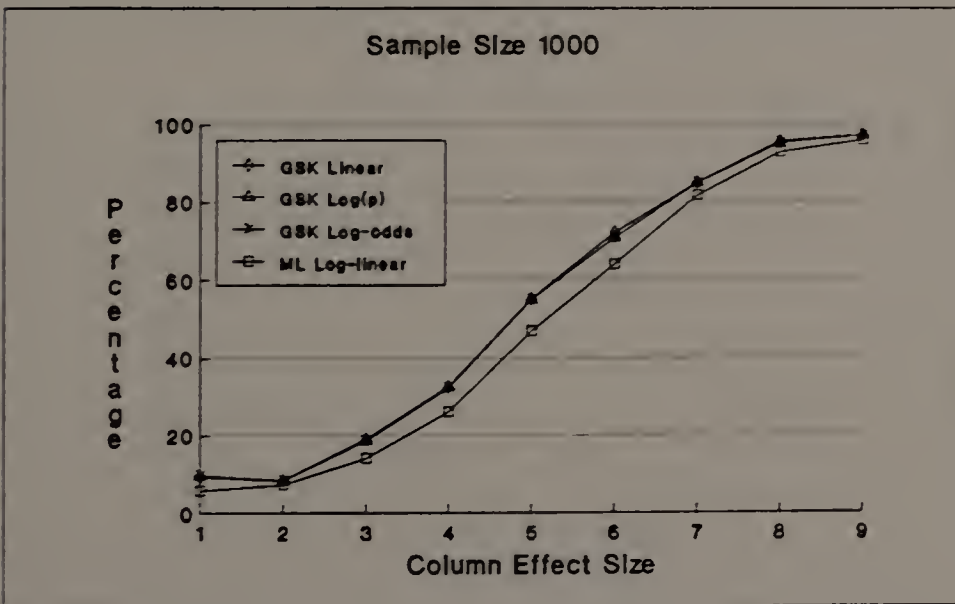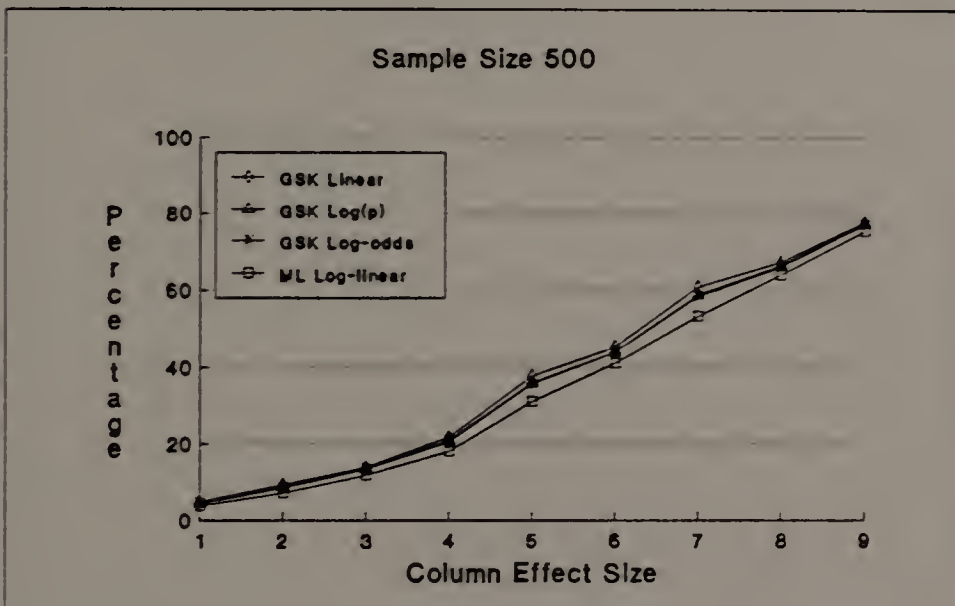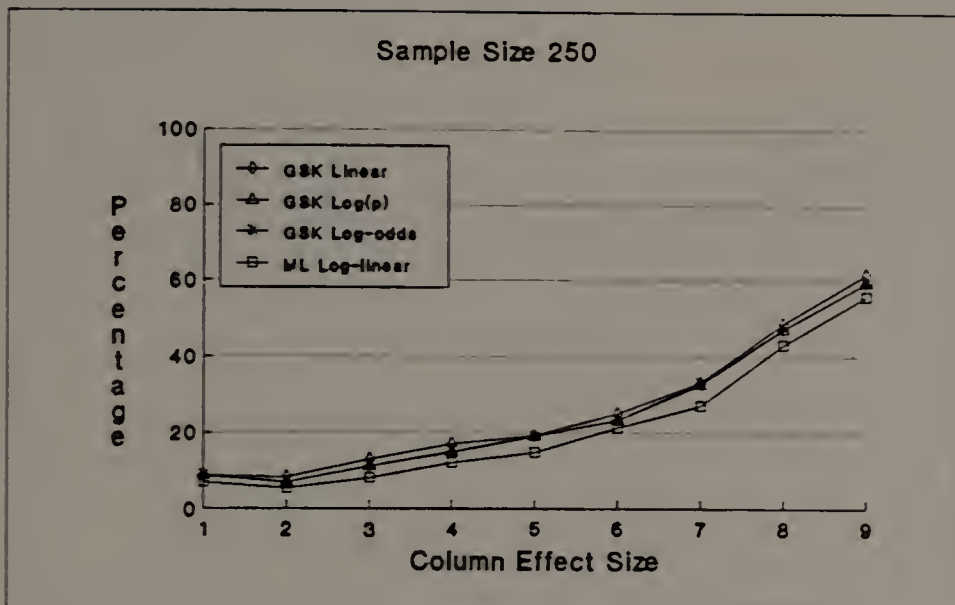
Figure F.2 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 3x2.
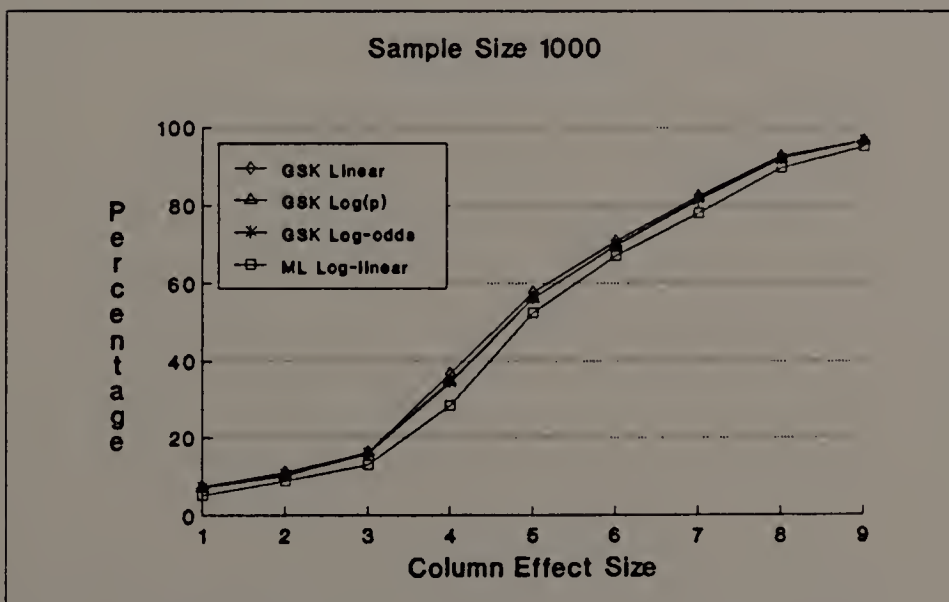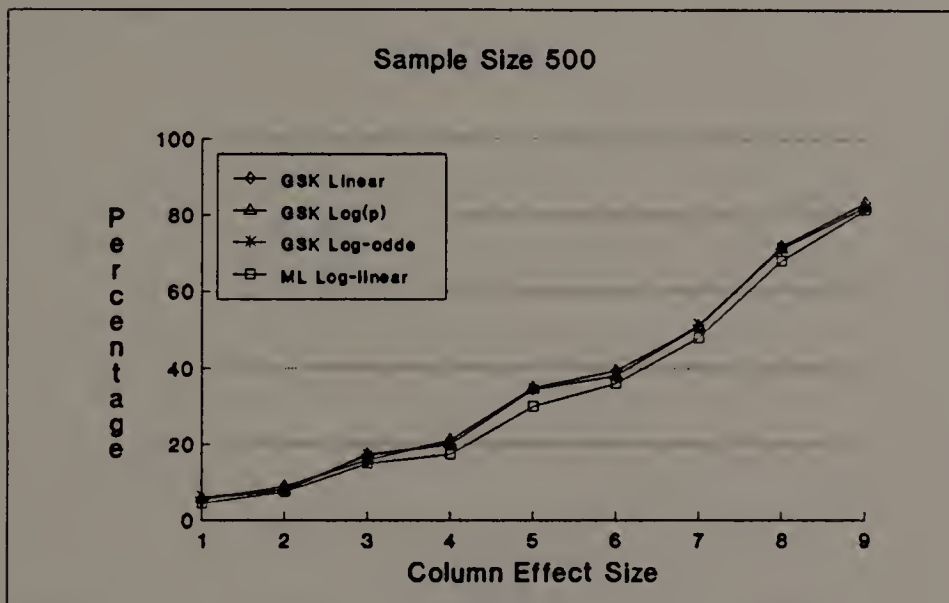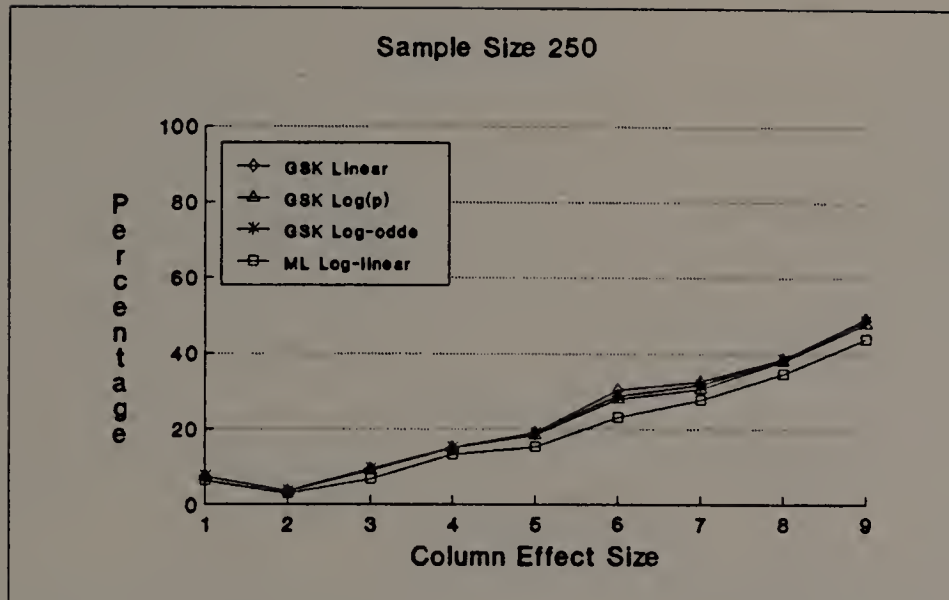
Figure F.3 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 4x2.
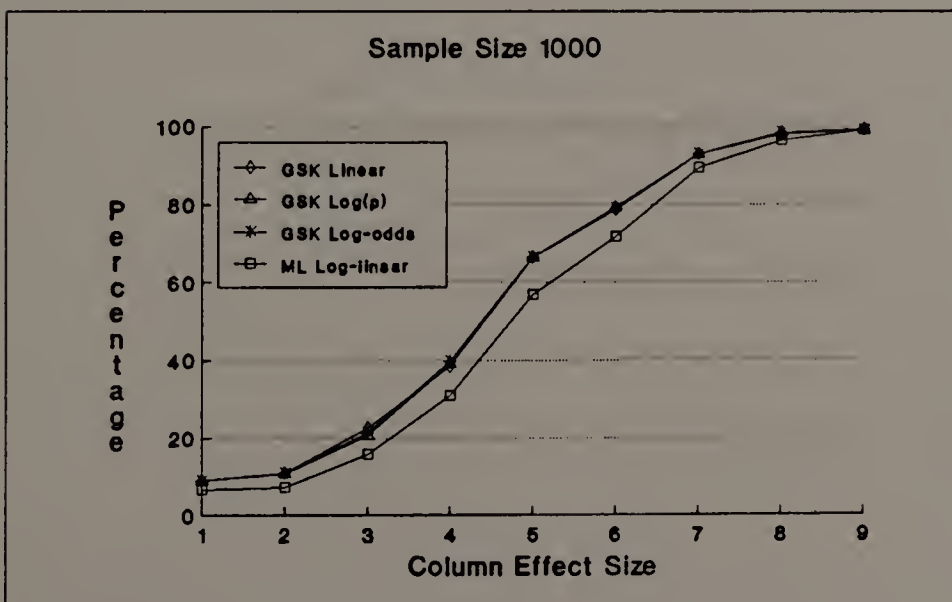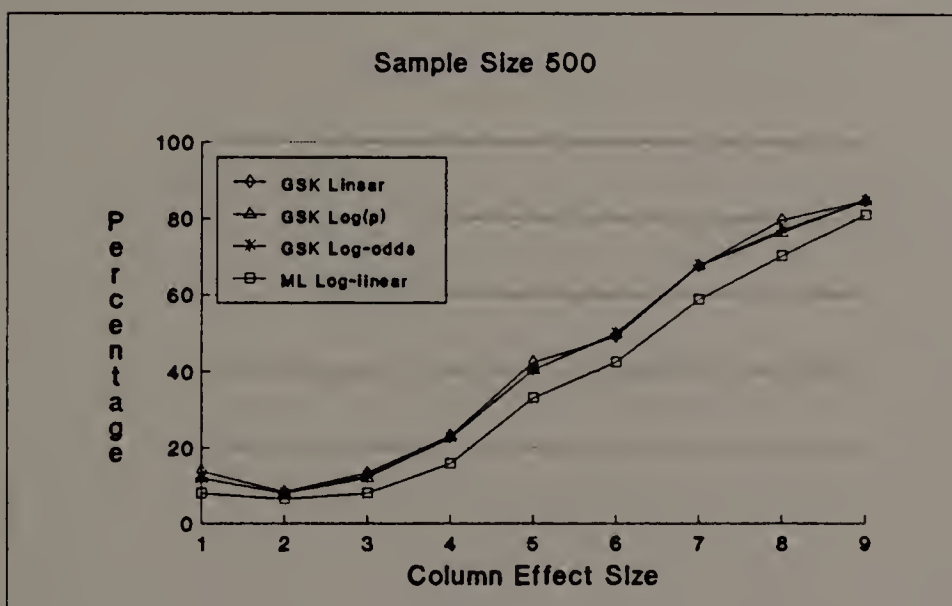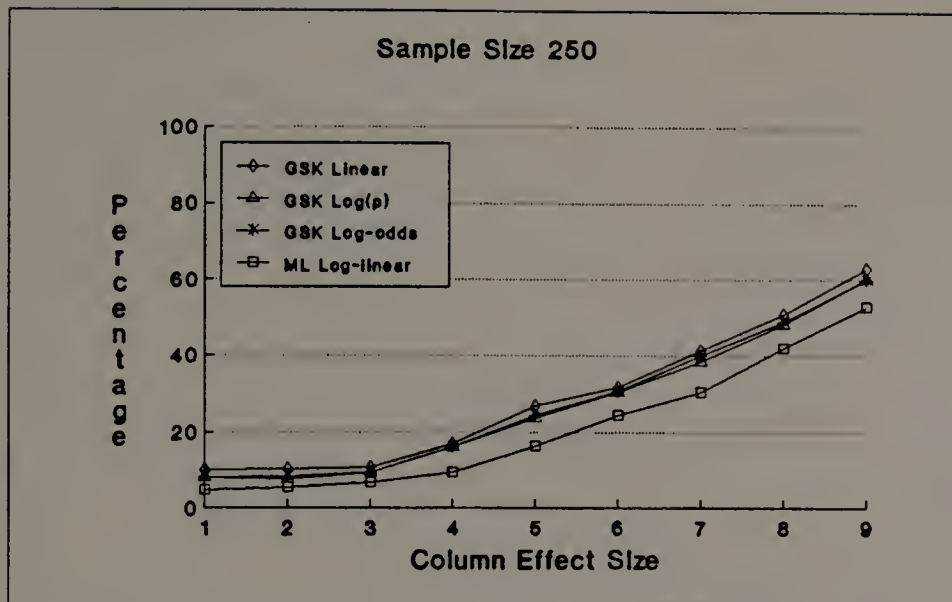
Figure F.4 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 2x3.
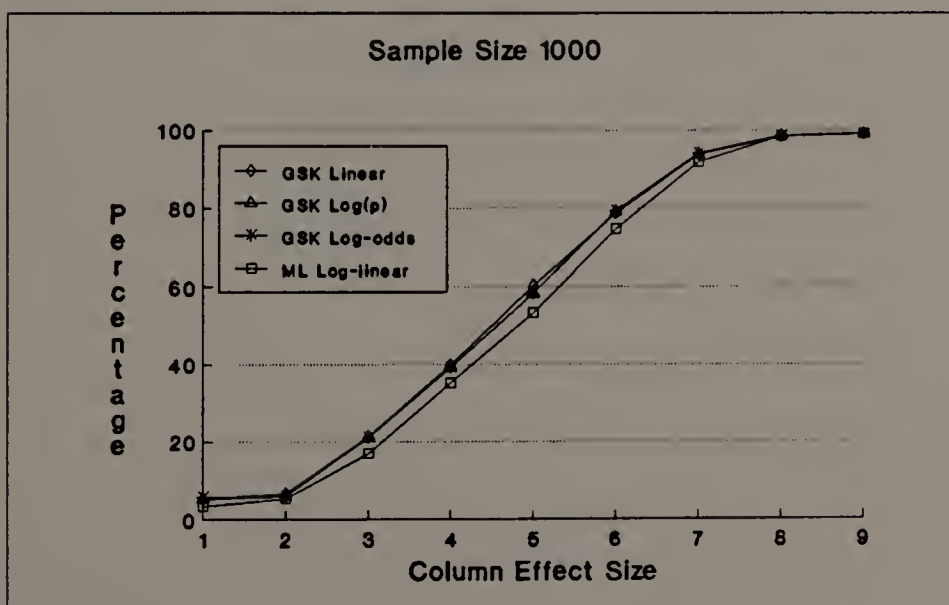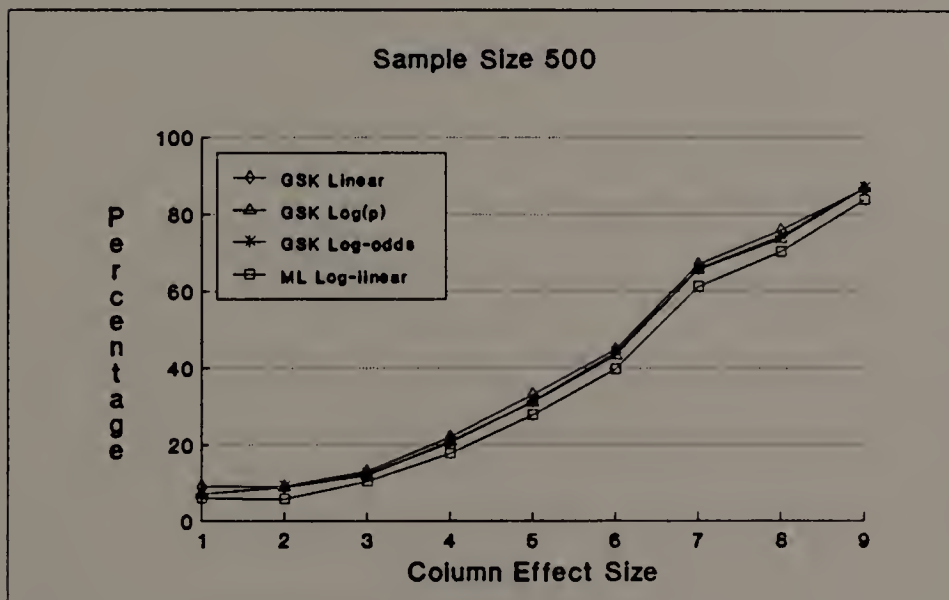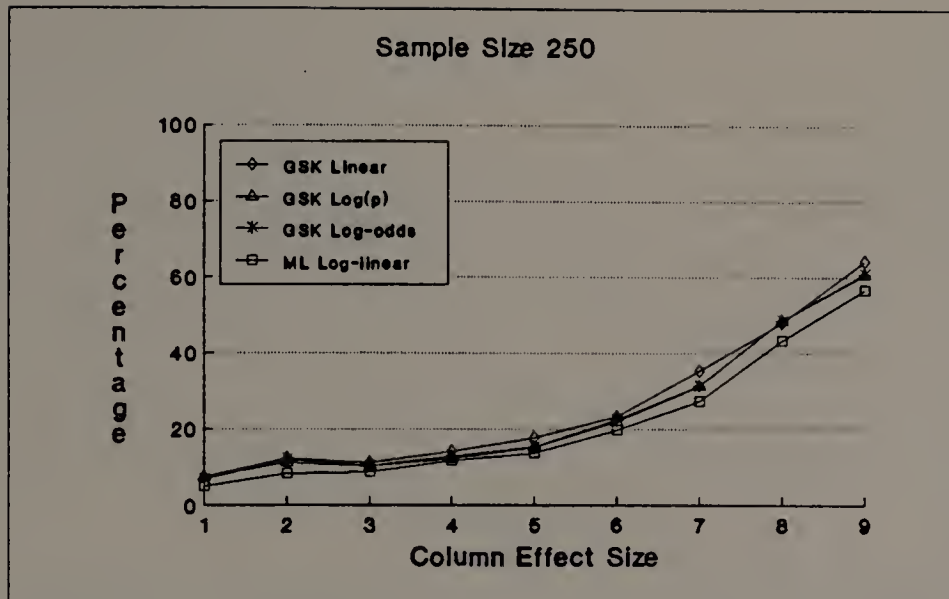
Figure F.5 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 3x3.
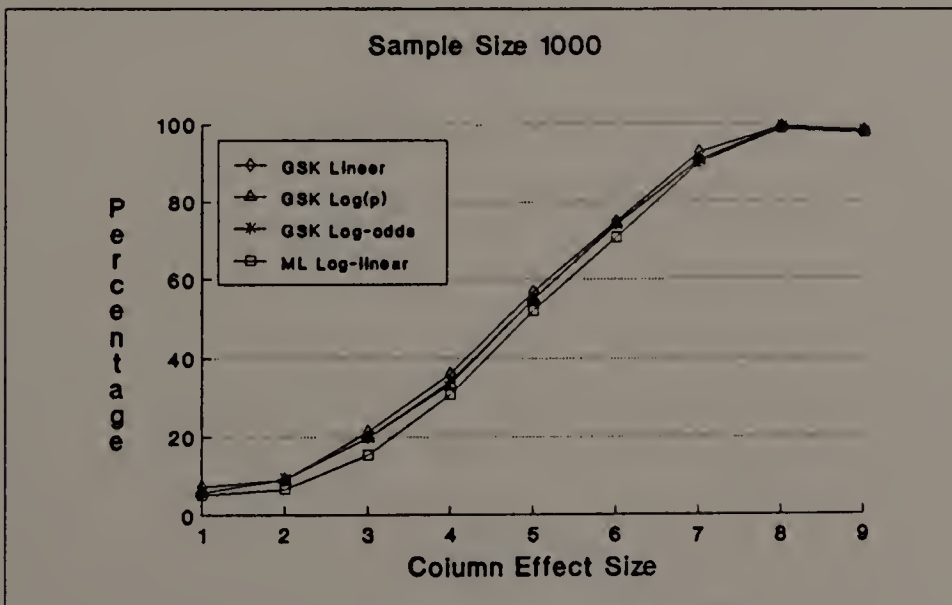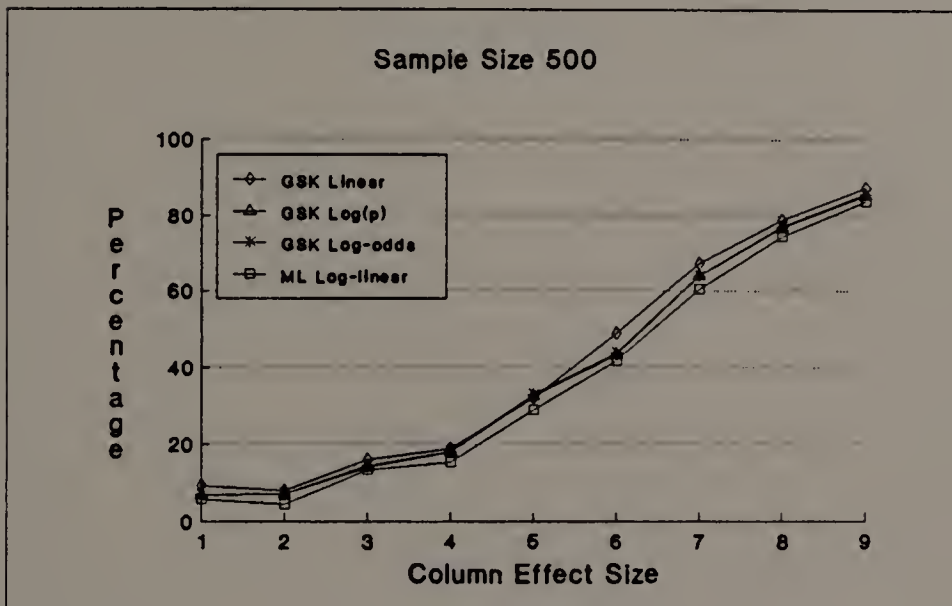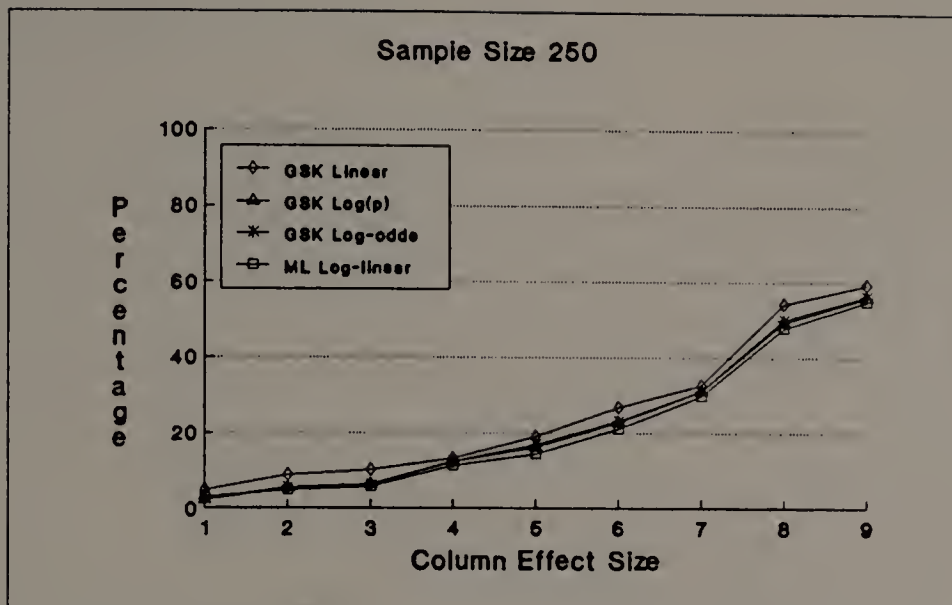
Figure F.6 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 4x3.
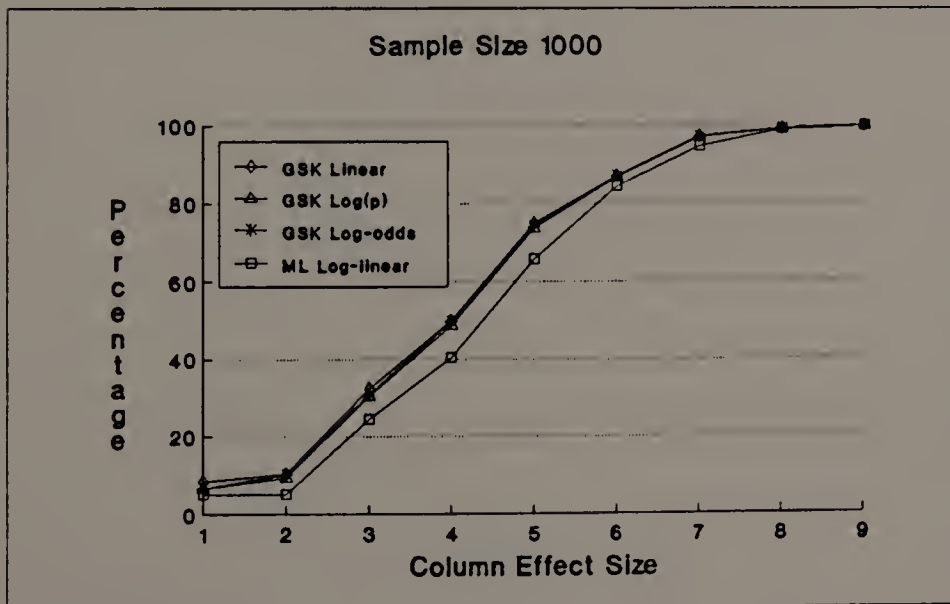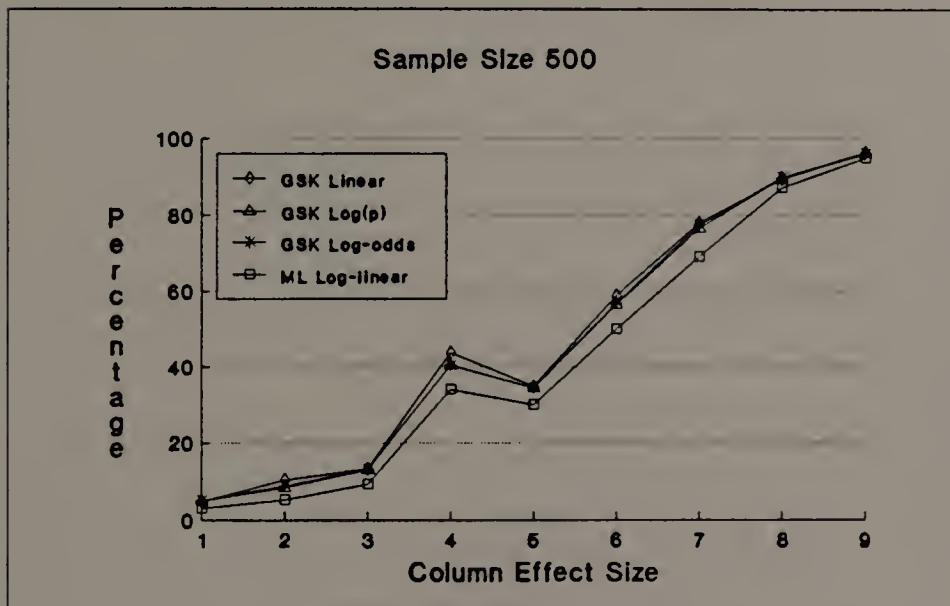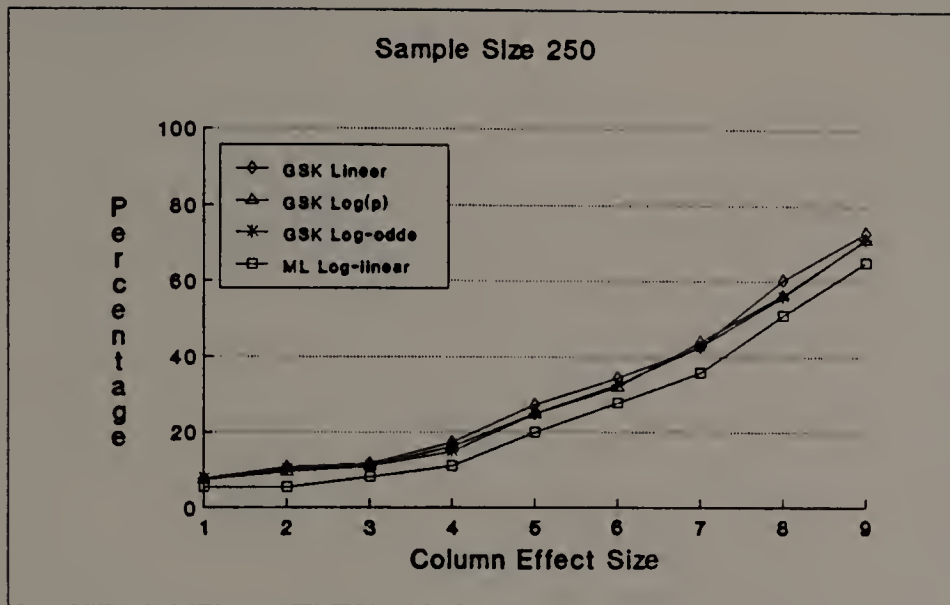
Figure F.7 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 2x4.
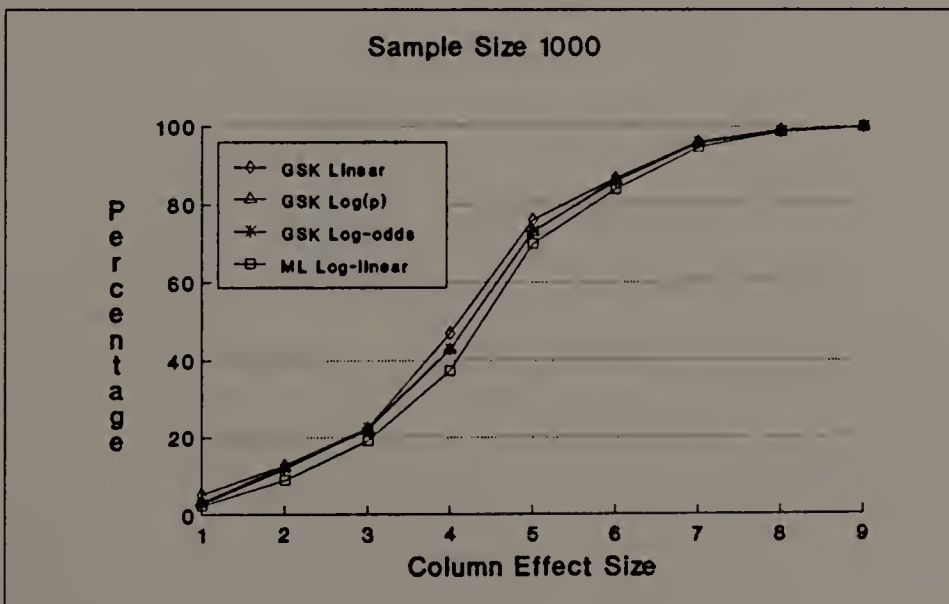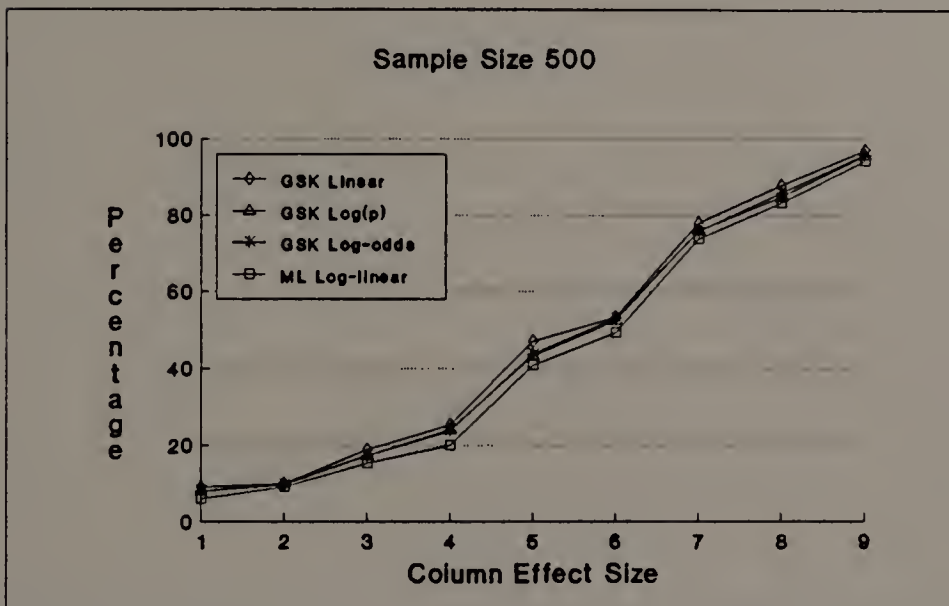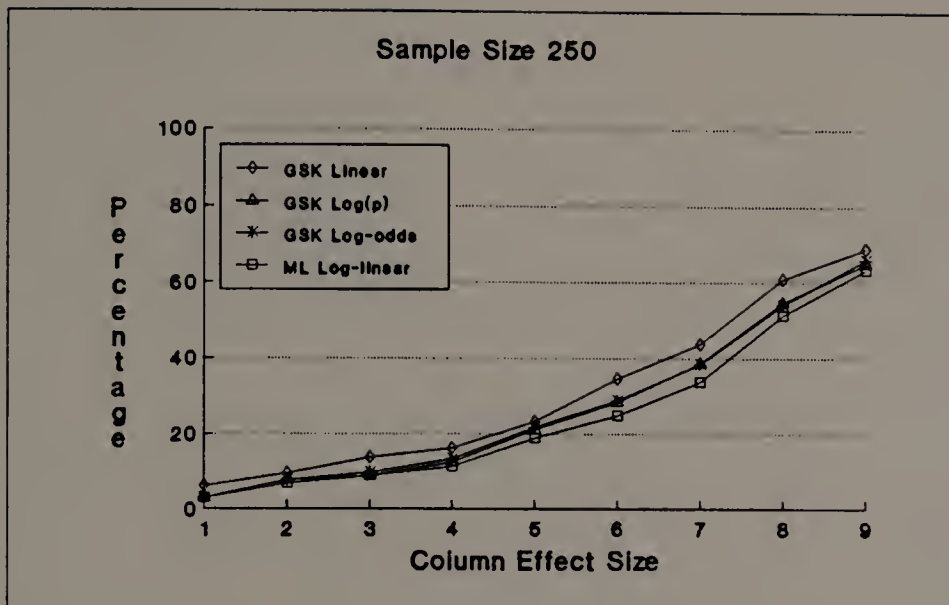
Figure F.8 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 3x4.

89
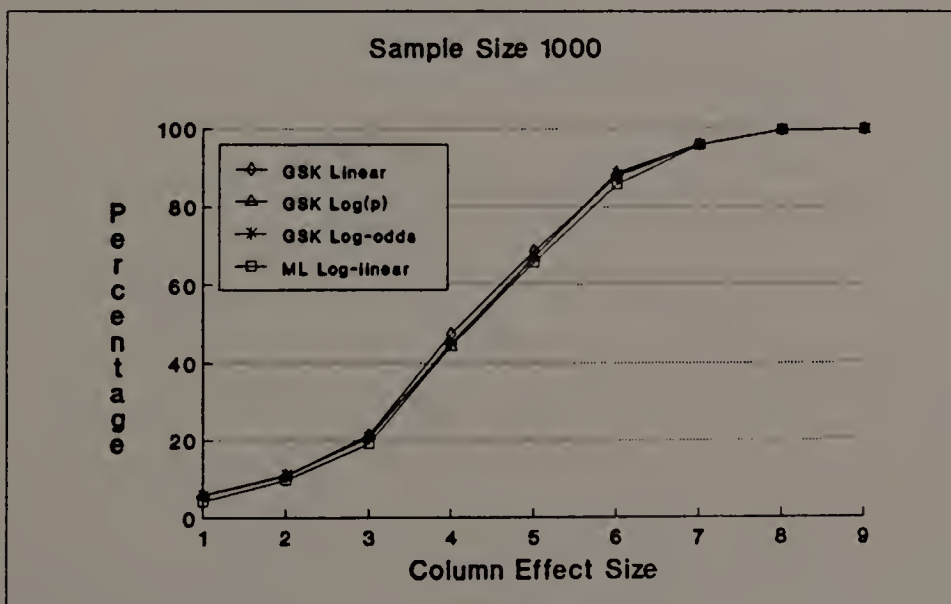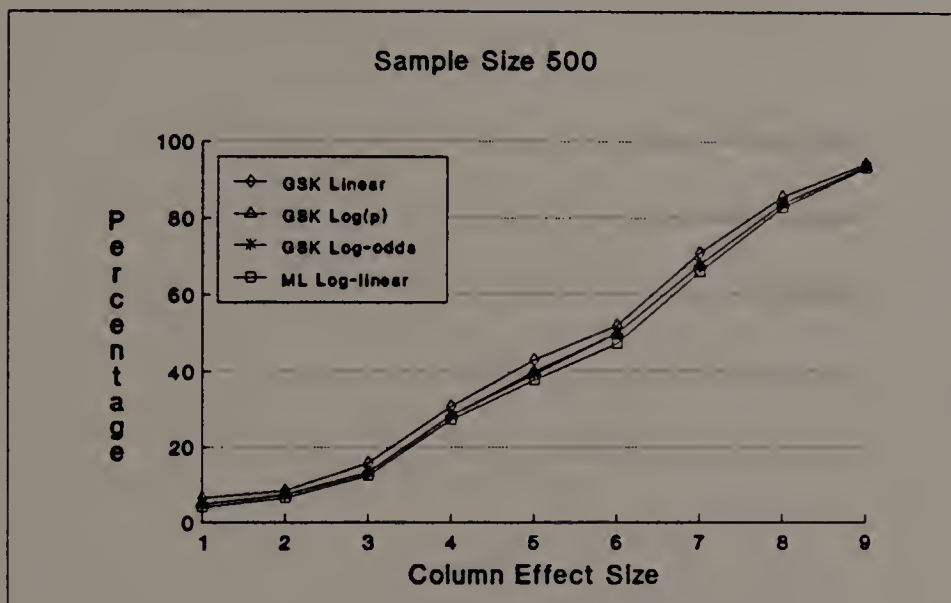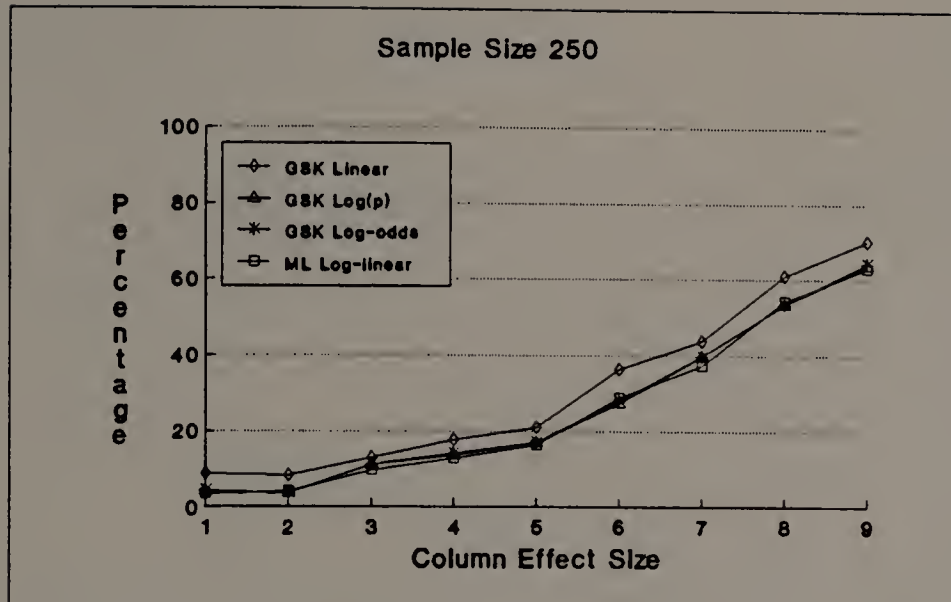
Figure F.9 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (1)], 4x4.

APPENDIX G

PERCENTAGE OF SIGNIFICANT χ^2 COLUMN VALUES FOR TABLES WITH ROW AND COLUMN EFFECTS (ROW EFFECT SIZE 2)

Table G.1 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (2)], Two Column.

Column Effect Size	Method	Sample Size:	Size: 2x2			Size: 4x2		
		250	500	1000	250	500	1000
1	GSK Linear	11.0	9.0	8.3	8.7	8.0	7.7
	GSK Log(p)	11.0	9.3	8.7	8.0	5.7	8.6
	GSK Log-odds	11.3	9.0	8.7	8.0	5.7	7.7
	ML Log-linear	7.3	5.3	5.3	7.0	4.7	6.0
2	GSK Linear	10.7	14.3	12.0	5.3	8.3	5.0
	GSK Log(p)	10.0	15.3	11.7	4.3	7.0	5.3
	GSK Log-odds	10.0	14.7	11.7	3.3	7.3	5.3
	ML Log-linear	5.3	8.7	7.0	3.3	6.0	3.7
3	GSK Linear	12.7	15.3	23.0	11.0	14.7	19.7
	GSK Log(p)	12.3	16.3	22.7	11.3	14.0	20.3
	GSK Log-odds	12.3	16.3	23.0	12.0	15.0	20.3
	ML Log-linear	8.0	11.0	13.0	7.3	13.0	16.3
4	GSK Linear	15.7	26.0	35.0	12.0	20.3	34.3
	GSK Log(p)	15.3	25.0	34.7	10.7	19.3	33.7
	GSK Log-odds	15.7	25.0	35.3	11.0	20.0	33.0
	ML Log-Linear	9.3	19.7	22.7	9.3	17.7	27.7

Continued, next page

Table G.1 Continued

Column Effect Size	Method	Size: 2x2			Size: 4x2		
	Sample Size:	250	500	1000	250	500	1000
5	GSK Linear	25.7	34.3	60.3	22.3	35.0	52.3
	GSK Log(p)	24.7	34.0	59.3	19.3	34.7	50.3
	GSK Log-odds	25.0	35.0	61.3	20.0	34.7	50.0
	ML Log-linear	15.7	26.3	43.7	18.3	32.0	43.3
6	GSK Linear	31.0	46.0	76.7	29.3	43.3	72.7
	GSK Log(p)	30.7	46.3	76.3	29.0	44.3	74.0
	GSK Log-odds	31.0	46.3	76.7	29.3	44.3	73.0
	ML Log-Linear	21.7	36.3	66.0	26.7	40.7	69.7
7	GSK Linear	37.7	60.0	89.3	28.3	56.7	85.0
	GSK Log(p)	36.3	59.7	89.3	27.7	56.3	85.0
	GSK Log-odds	37.0	59.7	89.3	28.0	56.7	84.7
	ML Log-linear	27.3	49.7	82.0	24.3	53.7	82.3
8	GSK Linear	46.0	76.0	95.7	44.7	69.3	94.3
	GSK Log (p)	45.0	74.7	95.7	44.3	69.3	93.3
	GSK Log-odds	45.3	75.0	95.7	45.0	69.7	93.3
	ML Log-linear	35.7	71.3	90.3	38.7	66.7	90.7
9	GSK Linear	54.3	85.7	95.0	55.3	80.3	97.7
	GSK Log(p)	54.7	86.7	95.3	53.0	83.0	98.3
	GSK Log-odds	55.0	86.3	95.3	54.3	83.0	98.0
	ML Log-Linear	44.7	80.0	93.7	51.0	80.0	97.0

Table G.2 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (2)], Three Column.

Column Effect Size	Method	Size: 2x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000
1	GSK Linear	11.0	8.0	7.0	8.3	7.3	7.7
	GSK Log(p)	10.3	8.0	7.3	4.3	4.7	7.7
	GSK Log-odds	10.7	8.0	7.0	4.7	4.7	7.7
	ML Log-linear	5.7	4.3	4.0	4.0	4.3	6.0
2	GSK Linear	8.3	10.3	14.0	10.3	6.3	11.0
	GSK Log(p)	8.0	8.7	13.7	7.7	7.0	9.7
	GSK Log-odds	8.0	9.0	14.0	8.0	6.7	10.0
	ML Log-linear	5.3	6.0	8.7	7.3	5.3	7.3
3	GSK Linear	12.7	15.7	20.7	8.3	15.0	14.3
	GSK Log(p)	11.0	15.3	19.3	7.7	14.7	14.0
	GSK Log-odds	11.0	15.3	20.3	7.7	15.0	13.7
	ML Log-linear	9.0	10.0	14.7	7.3	14.0	10.7
4	GSK Linear	21.3	26.7	38.0	14.7	23.0	33.3
	GSK Log(p)	18.7	26.7	37.7	12.3	20.7	32.3
	GSK Log-odds	19.3	26.3	37.7	12.0	21.3	32.0
	ML Log-Linear	16.0	18.7	30.7	11.0	19.0	28.0

Table G.2 Continued

Column Effect Size	Method	Size: 2x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000
5	GSK Linear	23.7	41.0	64.0	21.7	32.7	52.7
	GSK Log(p)	22.7	39.0	62.0	17.3	32.3	54.3
	GSK Log-odds	23.0	39.3	63.0	17.7	32.7	54.3
	ML Log-linear	14.7	31.3	56.0	17.0	27.3	49.3
6	GSK Linear	31.7	43.3	82.0	27.7	50.0	81.0
	GSK Log(p)	30.3	43.0	81.3	22.0	48.3	81.0
	GSK Log-odds	30.7	43.0	81.3	23.3	48.7	81.0
	ML Log-Linear	21.3	34.7	77.7	22.3	43.7	79.7
7	GSK Linear	44.3	66.0	93.0	36.3	65.0	89.3
	GSK Log(p)	44.0	65.0	92.7	33.3	62.3	88.7
	GSK Log-odds	44.0	66.0	96.0	32.7	62.7	88.7
	ML Log-linear	35.0	56.7	89.3	30.3	60.7	86.0
8	GSK Linear	52.3	79.7	99.0	49.3	77.7	98.3
	GSK Log (p)	51.3	78.7	99.0	42.3	77.7	97.7
	GSK Log-odds	52.7	78.7	98.7	42.7	77.3	97.7
	ML Log-linear	42.7	73.3	98.0	41.7	74.7	97.3
9	GSK Linear	63.0	90.0	99.7	61.7	87.0	99.7
	GSK Log(p)	62.0	89.3	99.7	59.3	86.3	99.7
	GSK Log-odds	61.0	89.7	99.7	59.3	86.3	99.7
	ML Log-Linear	53.7	85.7	99.3	57.7	85.0	99.3

Table G.3 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (2)], Four Column.

Column Effect Size	Method	Sample Size:	Size: 2x4			Size: 4x4	
		250	500	1000	250	500	1000
1	GSK Linear	5.7	7.7	10.0	7.7	7.7	4.7
	GSK Log(p)	3.7	7.7	9.3	3.3	7.0	4.7
	GSK Log-odds	4.3	7.7	8.7	3.3	7.0	4.7
	ML Log-linear	3.3	5.0	7.0	3.3	6.7	3.7
2	GSK Linear	11.0	10.0	11.0	10.0	11.0	7.3
	GSK Log(p)	12.0	7.7	10.0	5.7	10.0	6.0
	GSK Log-odds	12.0	8.3	10.3	5.7	10.0	6.3
	ML Log-linear	7.7	5.3	6.7	5.7	8.3	4.7
3	GSK Linear	14.3	19.0	25.3	12.7	16.3	20.0
	GSK Log(p)	10.3	17.7	25.0	6.0	13.3	19.3
	GSK Log-odds	11.0	17.7	25.0	5.7	13.0	19.3
	ML Log-linear	7.3	12.7	18.3	6.0	12.3	16.0
4	GSK Linear	16.7	30.0	49.7	18.7	22.7	48.7
	GSK Log(p)	15.0	28.7	49.3	15.3	20.3	45.7
	GSK Log-odds	15.0	28.7	49.7	14.7	21.0	46.0
	ML Log-Linear	12.0	21.3	42.0	15.3	20.0	43.3

Table G.3 Continued

		Size: 2x4			Size: 4x4		
Column Effect Size	Sample Size: Method	250	500	1000	250	500	1000
5	GSK Linear	28.0	42.3	70.7	23.7	39.7	73.3
	GSK Log(p)	25.7	43.7	72.3	20.0	36.7	73.0
	GSK Log-odds	25.3	43.0	71.7	20.0	37.0	73.0
	ML Log-linear	19.0	35.0	64.3	19.3	34.3	70.3
6	GSK Linear	35.0	61.0	90.3	33.7	53.3	90.3
	GSK Log(p)	31.3	60.0	89.3	28.7	50.3	90.7
	GSK Log-odds	31.0	59.7	89.7	28.7	50.3	90.3
	ML Log-Linear	26.3	52.0	85.7	30.0	49.0	88.3
7	GSK Linear	49.3	76.0	99.0	45.3	75.7	96.3
	GSK Log(p)	47.3	76.0	98.7	38.3	73.0	96.3
	GSK Log-odds	47.3	75.7	98.7	38.7	73.3	96.7
	ML Log-linear	39.7	69.7	96.7	37.7	70.7	96.0
8	GSK Linear	55.7	88.7	99.0	61.3	88.3	99.3
	GSK Log (p)	54.0	87.7	99.3	56.7	87.3	99.3
	GSK Log-odds	53.3	88.0	99.3	57.3	86.7	99.3
	ML Log-linear	45.7	84.3	98.0	56.0	85.7	99.3
9	GSK Linear	73.0	96.7	100.0	73.3	95.3	99.7
	GSK Log(p)	70.3	95.3	100.0	67.7	94.7	100.0
	GSK Log-odds	70.3	95.7	100.0	67.0	95.0	100.0
	ML Log-Linear	65.3	94.0	100.0	65.7	94.7	99.7

APPENDIX H

FIGURES SHOWING PERCENTAGE OF SIGNIFICANT χ^2 COLUMN
VALUES FOR TABLES WITH ROW AND COLUMN EFFECTS
(ROW EFFECT SIZE 2)
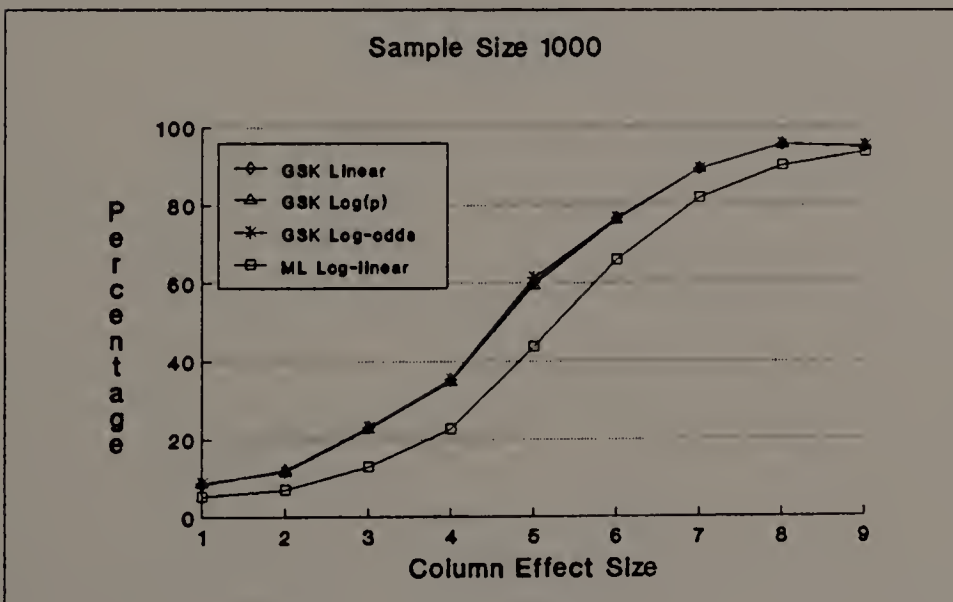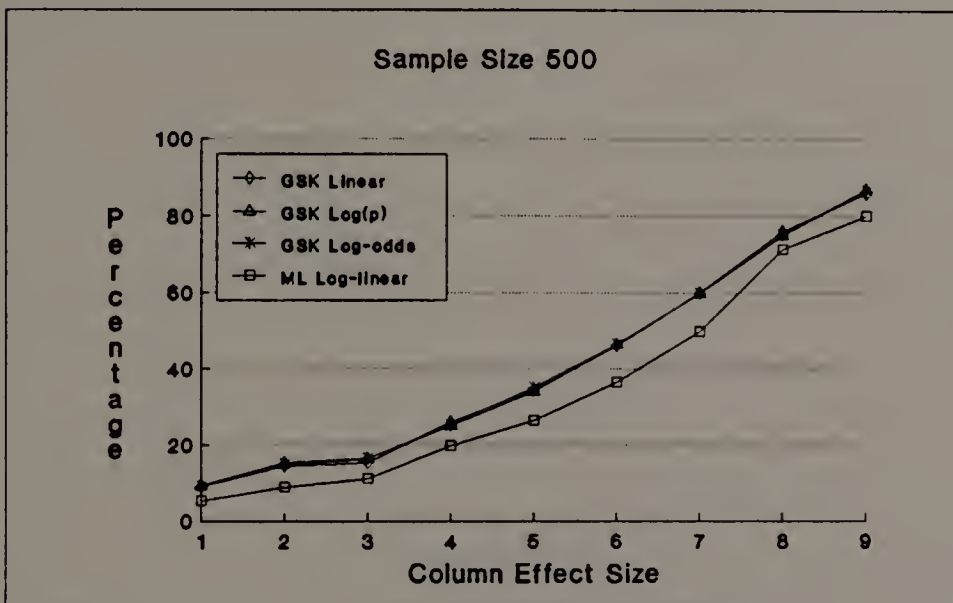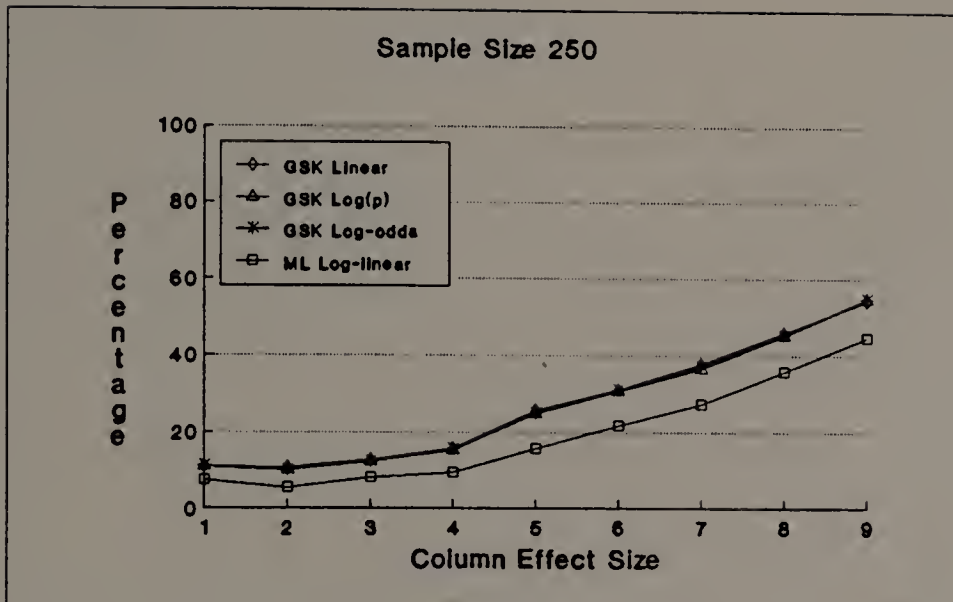
Figure H.1 Percentage of Significant Chi-Square Column Values,
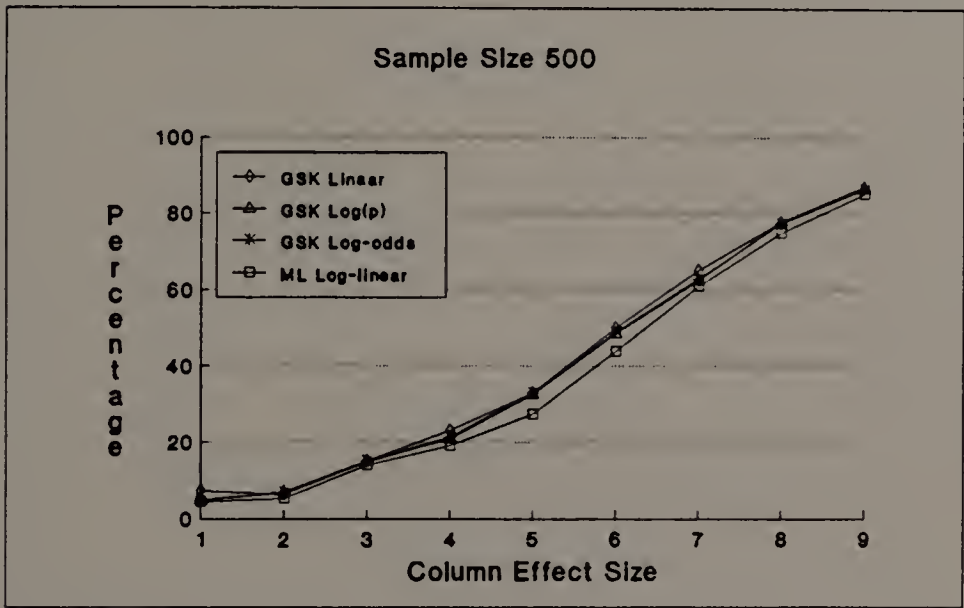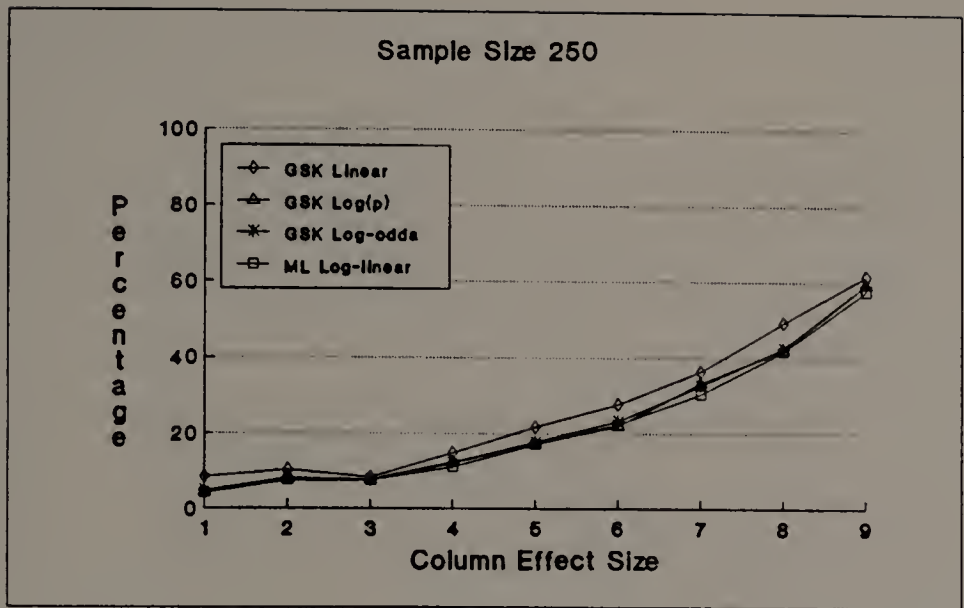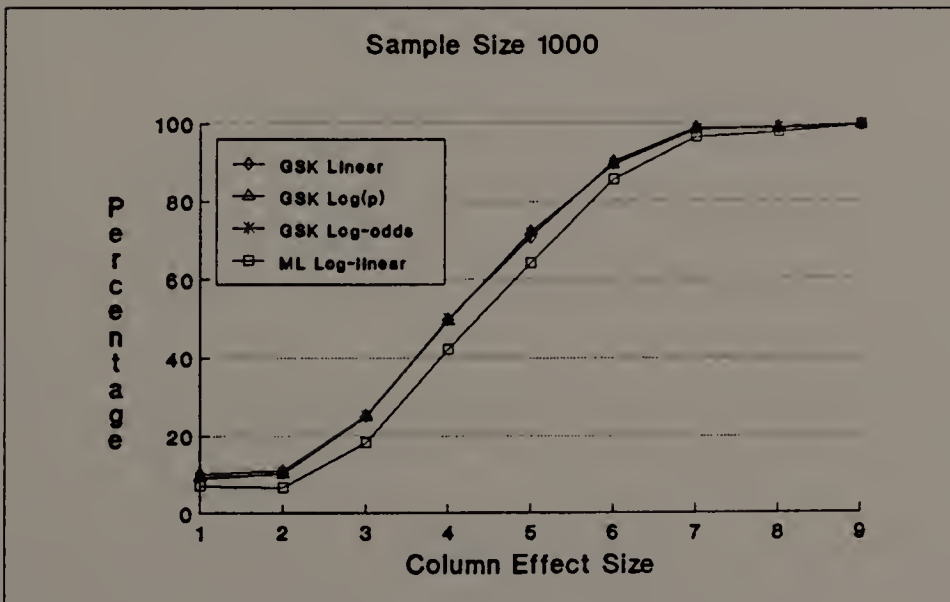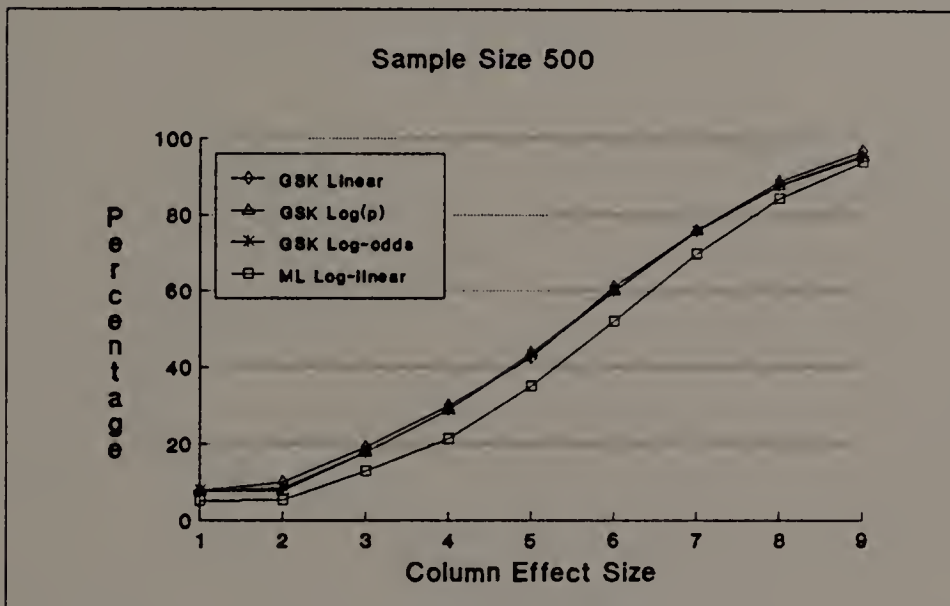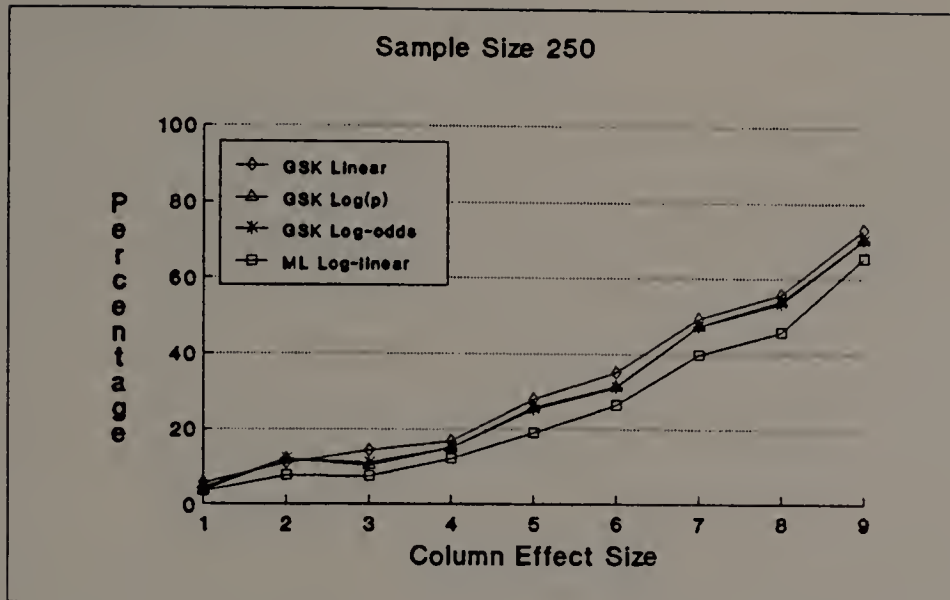Table Type: [Row Effect (2)], 2x2.
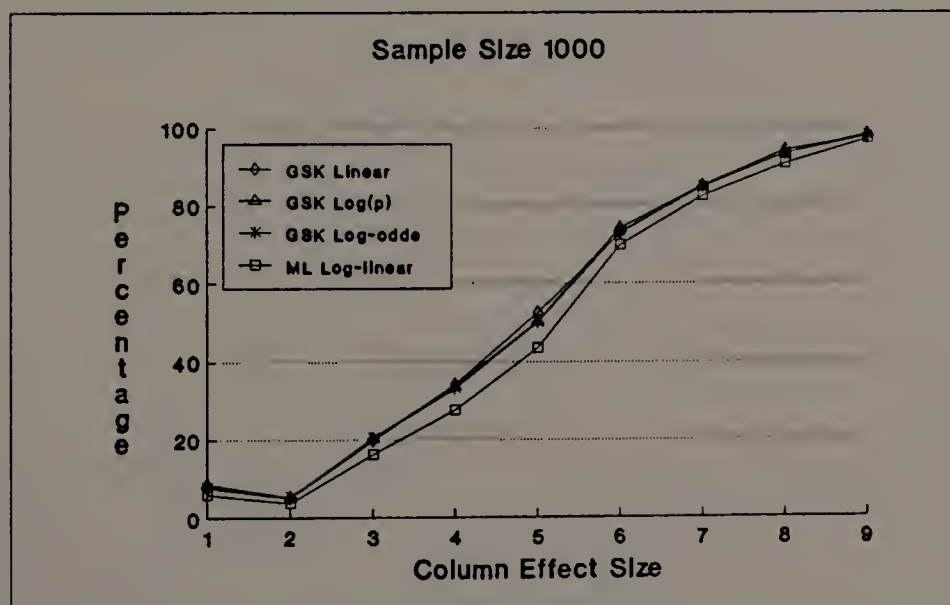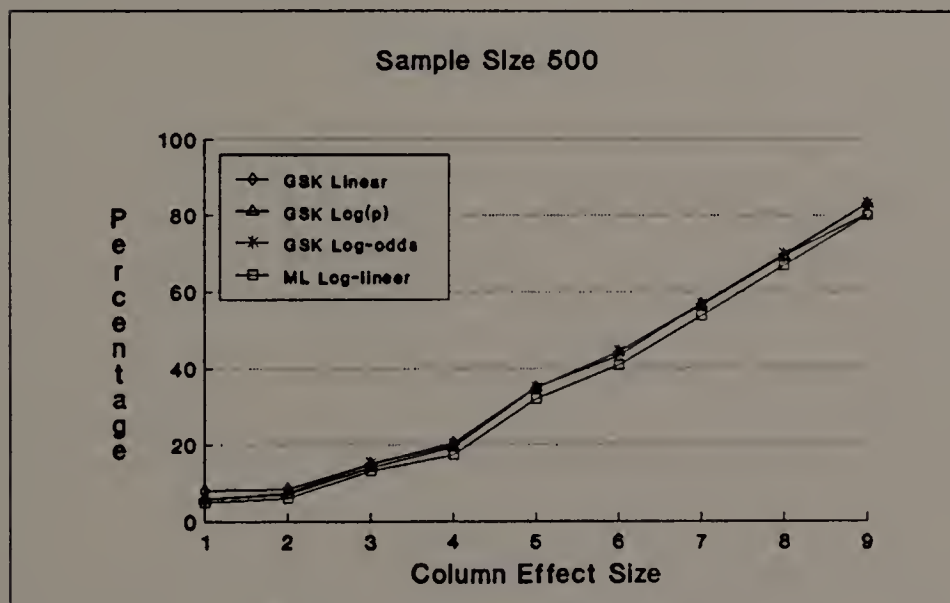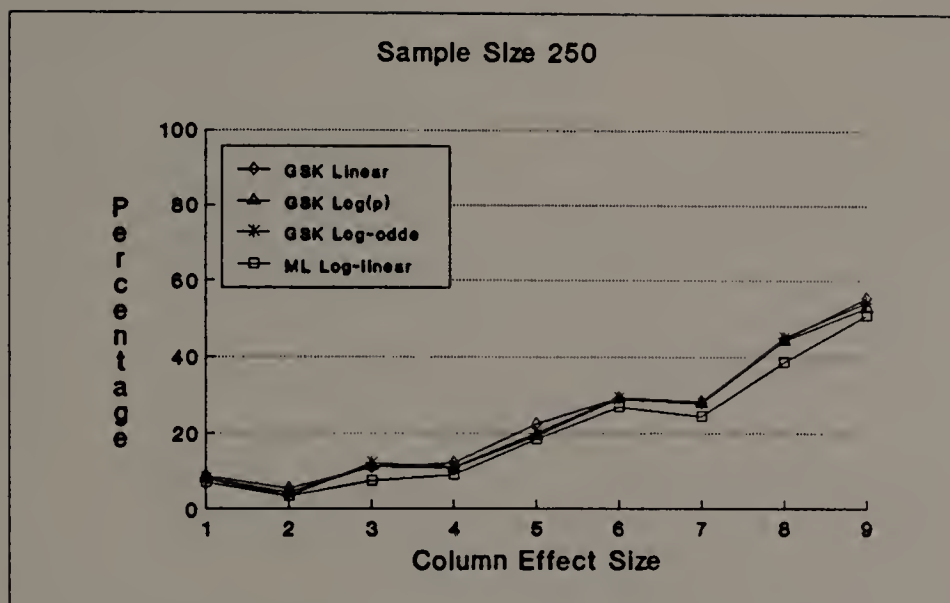
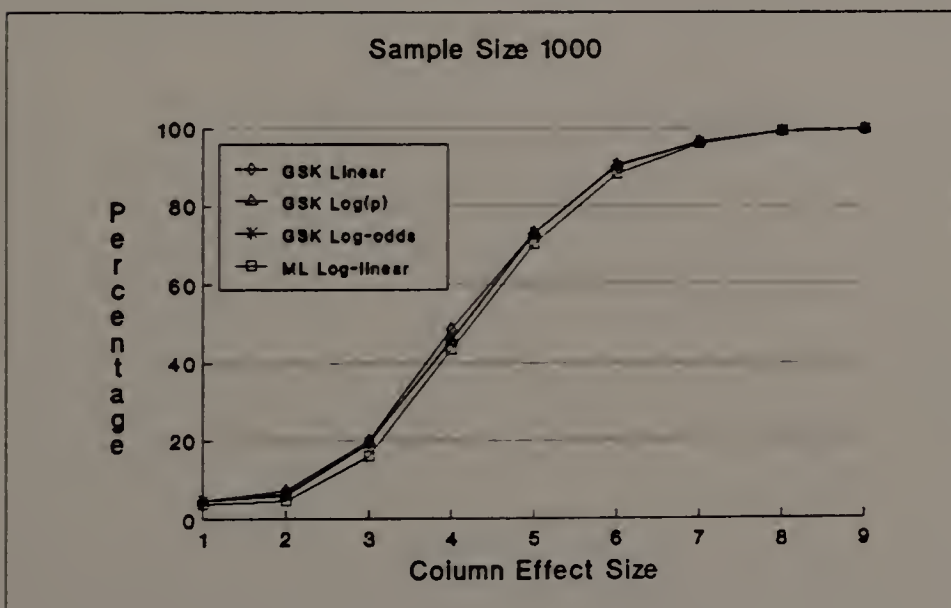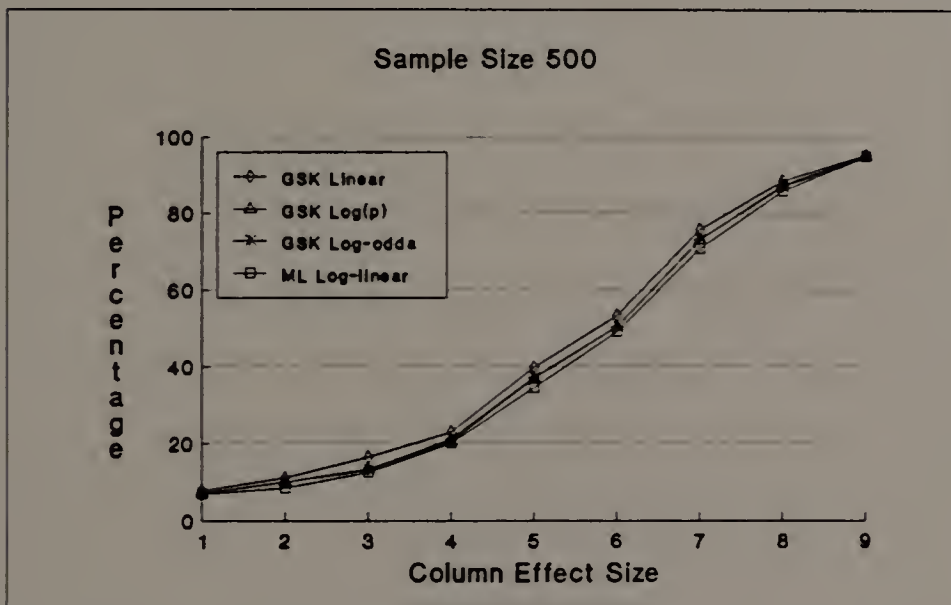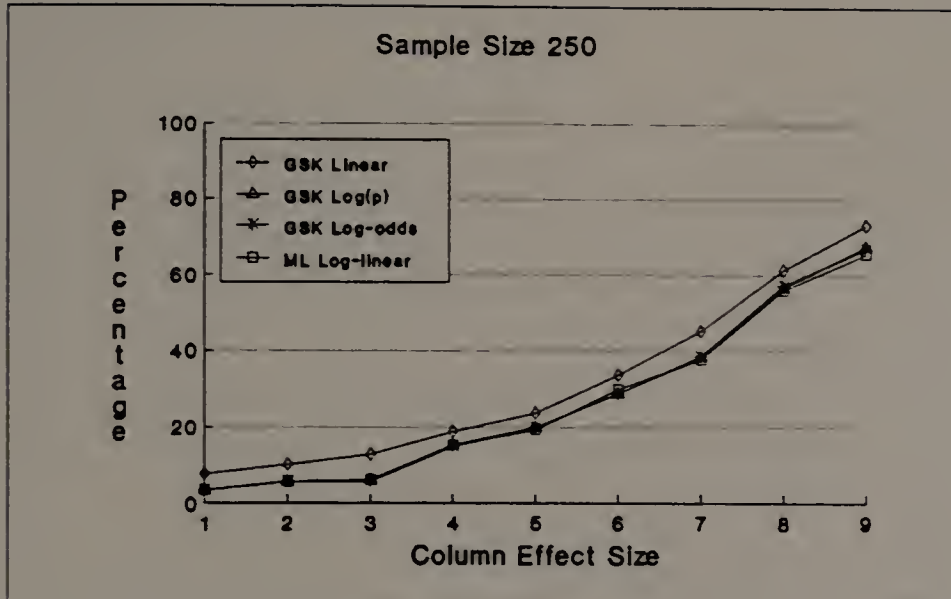Figure H.2 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (2)], 4x2.
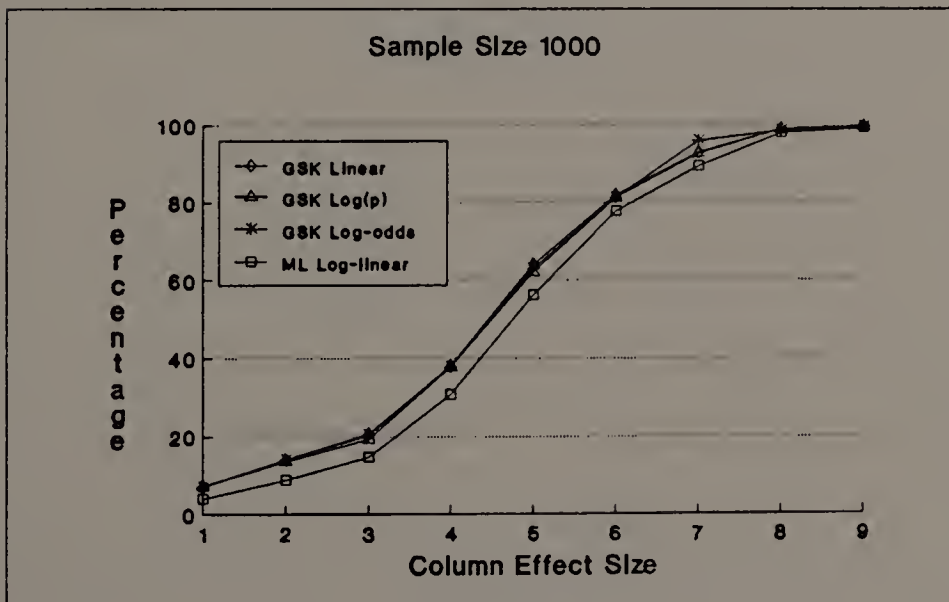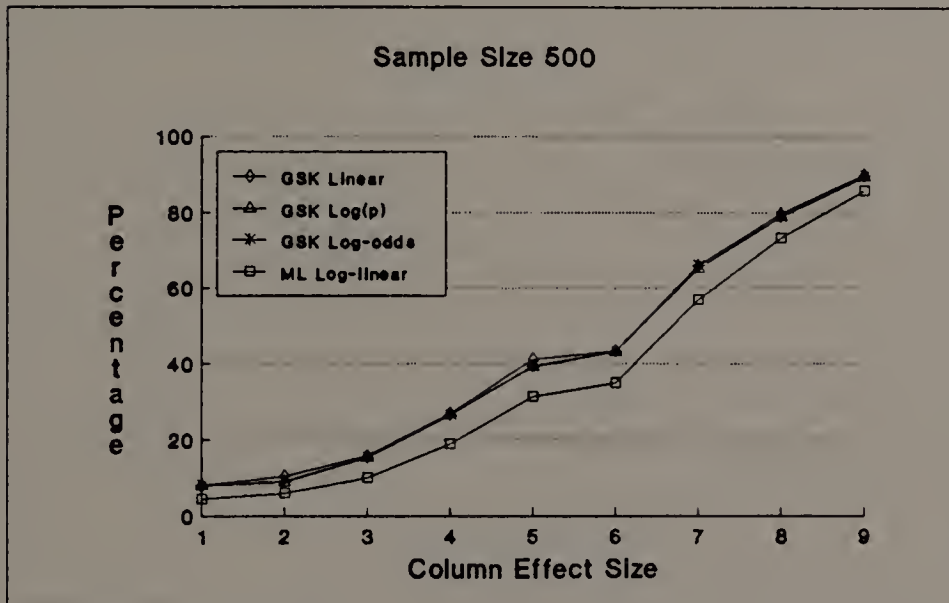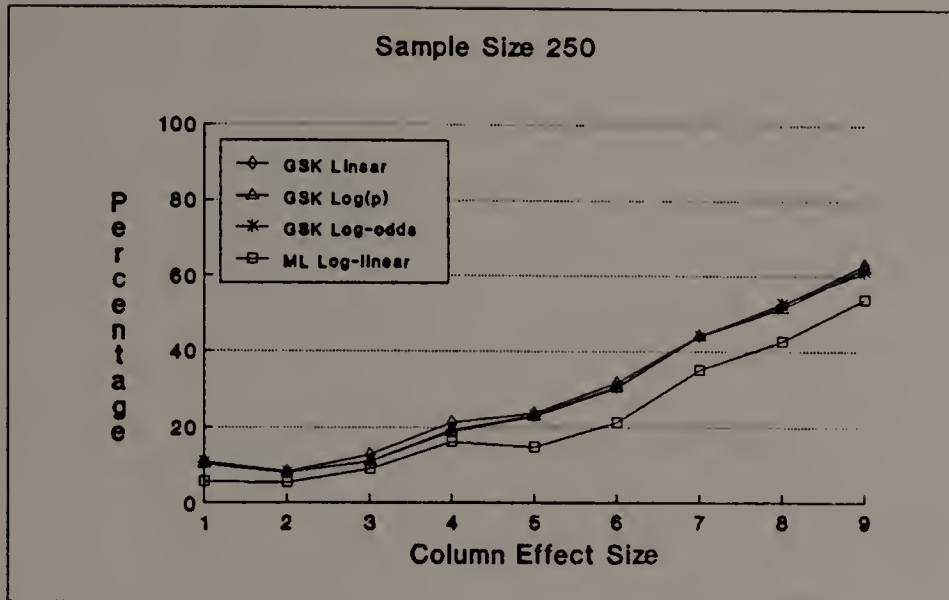
Figure H.3 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (2)], 2x3.
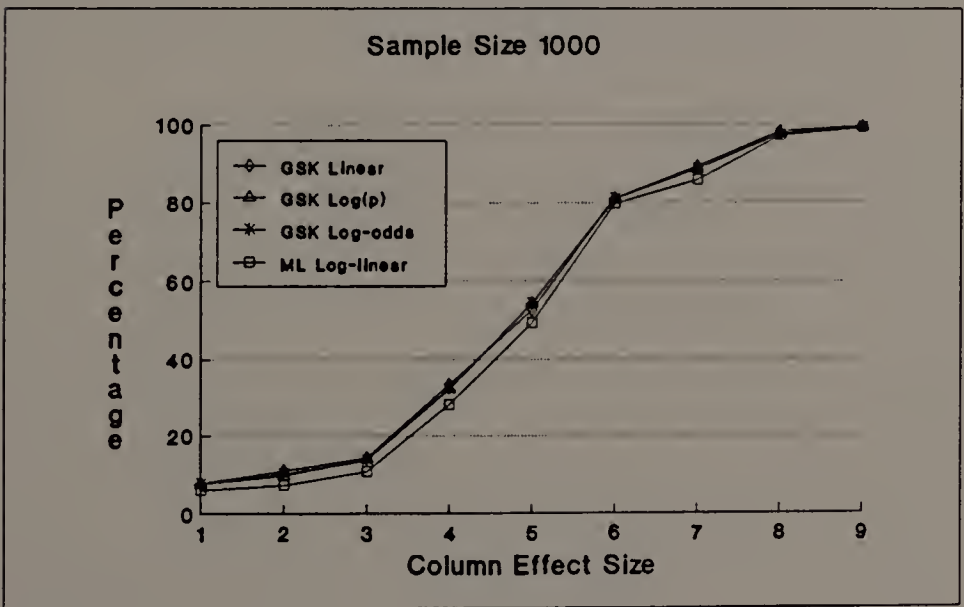
Figure H.4 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (2)], 4x3.

Figure H.5 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (2)], 2x4.

103

Figure H.6 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (2)], 4x4.

APPENDIX I

PERCENTAGE OF SIGNIFICANT χ^2 COLUMN
VALUES FOR TABLES WITH ROW AND COLUMN EFFECTS
(ROW EFFECT SIZE 3)

Table I.1 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (3)], Two Column.

Column Effect Size	Method	Sample Size:	Size: 2x2			Size: 4x2	
		250	500	1000	250	500	1000
1	GSK Linear	8.0	10.0	11.3	7.3	6.3	6.7
	GSK Log(p)	8.0	10.3	10.7	6.7	6.0	6.0
	GSK Log-odds	8.0	11.0	11.0	6.7	6.0	6.0
	ML Log-linear	4.3	5.3	5.7	5.7	5.3	4.3
2	GSK Linear	9.0	14.3	15.0	6.0	8.0	9.7
	GSK Log(p)	9.0	14.3	15.7	5.3	7.3	8.0
	GSK Log-odds	9.0	15.0	15.3	5.3	7.7	8.0
	ML Log-linear	6.7	8.3	9.7	4.3	6.7	6.7
3	GSK Linear	10.3	14.7	22.3	7.0	13.3	16.0
	GSK Log(p)	10.7	13.7	22.3	5.7	13.7	17.0
	GSK Log-odds	10.7	14.3	22.7	6.0	14.3	17.0
	ML Log-linear	4.7	10.3	14.7	4.7	13.0	13.3
4	GSK Linear	19.7	24.3	38.3	13.0	20.7	29.7
	GSK Log(p)	19.0	24.0	39.3	12.3	21.0	29.0
	GSK Log-odds	19.0	24.7	39.0	12.7	20.7	29.7
	ML Log-Linear	10.7	17.7	30.3	11.0	18.7	25.3

Table I.1 Continued

Column Effect Size	Method	Size: 2x2			Size: 4x2		
Sample Size:		250	500	1000	250	500	1000
5	GSK Linear	23.7	36.0	60.7	17.3	29.3	55.0
	GSK Log(p)	22.0	37.0	61.7	17.3	31.0	55.0
	GSK Log-odds	23.0	37.3	62.0	17.7	31.3	55.0
	ML Log-linear	17.3	30.3	47.7	14.0	27.0	50.7
6	GSK Linear	31.3	46.3	72.0	26.3	40.7	69.0
	GSK Log(p)	32.3	46.3	73.3	27.3	42.0	67.7
	GSK Log-odds	31.7	48.0	73.7	28.3	42.3	68.0
	ML Log-Linear	22.3	37.7	62.3	24.7	39.0	64.0
7	GSK Linear	35.7	60.3	89.0	32.7	60.0	81.0
	GSK Log(p)	35.3	61.0	89.0	29.0	59.0	84.0
	GSK Log-odds	35.3	62.0	89.3	29.7	59.3	84.3
	ML Log-linear	26.3	50.3	84.0	26.7	55.0	78.3
8	GSK Linear	53.7	75.7	94.3	43.0	70.3	92.3
	GSK Log (p)	53.0	75.7	94.7	41.0	71.0	93.7
	GSK Log-odds	53.3	75.7	95.0	41.7	72.0	93.7
	ML Log-linear	44.3	65.7	89.0	39.0	68.0	90.7
9	GSK Linear	51.7	82.3	99.3	50.7	78.3	96.3
	GSK Log(p)	50.3	81.7	99.3	52.0	77.7	96.0
	GSK Log-odds	50.7	83.0	99.3	53.3	77.7	96.0
	ML Log-Linear	42.3	74.7	98.0	44.0	76.0	96.0

Table I.2 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (3)], Three Column.

Column Effect Size	Method	Sample Size:		Size: 2x3				Size: 4x3	
		250	500	1000	250	500	1000		
1	GSK Linear	8.7	10.0	11.3	8.3	5.7	8.7		
	GSK Log(p)	7.7	8.7	10.3	6.7	5.3	8.0		
	GSK Log-odds	8.0	8.7	10.7	7.0	5.3	8.0		
	ML Log-linear	5.3	6.0	5.7	6.0	5.3	6.3		
2	GSK Linear	9.3	12.7	11.7	8.0	10.3	9.0		
	GSK Log(p)	7.7	11.3	12.0	5.3	9.7	9.3		
	GSK Log-odds	8.0	11.7	12.0	5.0	9.7	9.3		
	ML Log-linear	6.7	7.0	7.3	5.0	7.7	8.3		
3	GSK Linear	13.0	14.3	25.0	9.3	17.0	18.0		
	GSK Log(p)	10.3	15.3	25.7	6.3	15.0	18.0		
	GSK Log-odds	11.0	14.7	25.7	6.7	16.0	17.7		
	ML Log-linear	7.3	8.7	19.0	6.3	12.7	15.0		
4	GSK Linear	20.0	25.7	39.0	15.0	23.3	38.7		
	GSK Log(p)	19.7	24.3	39.0	13.0	20.7	36.0		
	GSK Log-odds	19.7	24.0	39.0	13.0	20.7	36.7		
	ML Log-Linear	14.3	17.7	30.0	12.3	19.3	34.0		

Table I.2 Continued

Column Effect Size	Method	Sample Size:	Size: 2x3			Size: 4x3	
		250	500	1000	250	500	1000
5	GSK Linear	22.7	33.3	60.3	21.0	31.7	55.0
	GSK Log(p)	22.0	36.0	58.0	18.0	29.0	57.0
	GSK Log-odds	22.7	35.3	60.0	18.7	29.3	57.0
	ML Log-linear	17.3	24.7	48.3	17.7	27.3	54.0
6	GSK Linear	31.7	50.7	81.0	25.7	47.0	81.7
	GSK Log(p)	30.7	52.3	83.0	26.7	46.7	81.3
	GSK Log-odds	30.7	52.0	82.7	27.3	47.0	80.7
	ML Log-Linear	23.3	41.0	78.7	24.7	41.3	77.7
7	GSK Linear	41.0	70.7	92.0	37.0	65.0	91.7
	GSK Log(p)	37.0	70.7	91.7	34.7	64.3	91.3
	GSK Log-odds	38.0	70.7	91.7	34.7	64.0	91.7
	ML Log-linear	30.3	63.0	91.3	33.0	61.0	90.3
8	GSK Linear	55.7	81.7	99.0	48.0	74.7	97.0
	GSK Log (p)	53.0	79.3	99.0	44.0	74.0	98.3
	GSK Log-odds	53.7	79.7	99.0	44.7	73.7	98.3
	ML Log-linear	46.7	73.0	98.0	40.7	70.3	98.0
9	GSK Linear	61.7	90.0	99.0	54.7	88.3	99.3
	GSK Log(p)	61.0	90.0	99.7	56.3	90.0	99.3
	GSK Log-odds	60.7	90.7	99.7	56.3	90.3	99.3
	ML Log-Linear	52.0	85.7	98.7	53.7	88.7	99.3

Table I.3 Percentage of χ^2 Column Values Significant at .05 Level, Table Type: [Row Effect (3)], Four Column.

Column Effect Size	Method	Sample Size:	Size: 2x4			Size: 4x4		
		250	500	1000	250	500	1000
1	GSK Linear	12.3	7.7	8.0	6.0	8.7	6.7
	GSK Log(p)	9.0	8.3	8.0	3.3	7.3	6.0
	GSK Log-odds	9.0	8.3	8.3	3.0	7.7	6.0
	ML Log-linear	6.0	5.0	5.3	2.7	7.0	5.3
2	GSK Linear	9.0	8.3	12.0	9.0	12.0	10.0
	GSK Log(p)	8.3	8.3	13.0	4.3	7.0	8.7
	GSK Log-odds	8.3	8.3	13.0	4.3	6.7	8.7
	ML Log-linear	4.0	4.3	7.3	4.7	6.7	7.0
3	GSK Linear	11.0	14.3	27.0	8.3	14.3	22.7
	GSK Log(p)	10.0	15.7	28.0	7.3	10.7	20.7
	GSK Log-odds	10.3	15.0	28.3	7.3	11.3	21.0
	ML Log-linear	6.0	11.0	19.7	5.7	9.0	18.0
4	GSK Linear	19.3	32.0	50.0	20.7	21.7	45.3
	GSK Log(p)	16.0	30.7	50.7	13.0	20.0	42.7
	GSK Log-odds	16.7	30.7	50.3	14.0	19.7	42.7
	ML Log-Linear	13.0	22.3	43.3	12.7	18.0	39.3

Continued, next page

110

Table I.3 Continued

Column Effect Size	Method	Size: 2x4			Size: 4x4		
	Sample Size:	250	500	1000	250	500	1000
5	GSK Linear	29.0	48.0	68.0	23.7	40.3	65.0
	GSK Log(p)	28.7	46.0	70.0	20.3	40.7	67.3
	GSK Log-odds	29.0	46.3	69.7	20.7	40.3	66.7
	ML Log-linear	21.0	37.3	61.0	20.0	38.3	64.3
6	GSK Linear	35.3	65.0	89.0	36.0	61.0	85.7
	GSK Log(p)	32.0	65.0	89.0	29.0	58.0	86.0
	GSK Log-odds	33.7	65.0	89.3	29.0	58.0	86.0
	ML Log-Linear	26.0	60.7	84.0	28.0	56.0	84.0
7	GSK Linear	47.0	75.3	98.0	49.7	71.3	96.0
	GSK Log(p)	44.7	74.7	98.0	41.0	69.7	96.0
	GSK Log-odds	44.7	75.0	98.0	41.0	69.3	96.0
	ML Log-linear	37.3	70.7	97.0	42.0	68.0	96.0
8	GSK Linear	61.0	90.3	98.7	59.3	84.0	100.0
	GSK Log(p)	57.3	89.7	98.3	55.7	83.0	99.7
	GSK Log-odds	57.7	90.0	98.3	56.3	83.0	99.7
	ML Log-linear	51.3	86.3	97.7	55.0	82.7	99.7
9	GSK Linear	74.7	97.3	100.0	69.3	94.0	99.7
	GSK Log(p)	71.3	97.7	100.0	63.0	94.0	99.7
	GSK Log-odds	71.3	97.7	100.0	63.3	94.0	99.7
	ML Log-Linear	68.7	95.7	100.0	64.0	93.7	99.7

APPENDIX J

FIGURES SHOWING PERCENTAGE OF SIGNIFICANT χ^2 COLUMN
VALUES FOR TABLES WITH ROW AND COLUMN EFFECTS
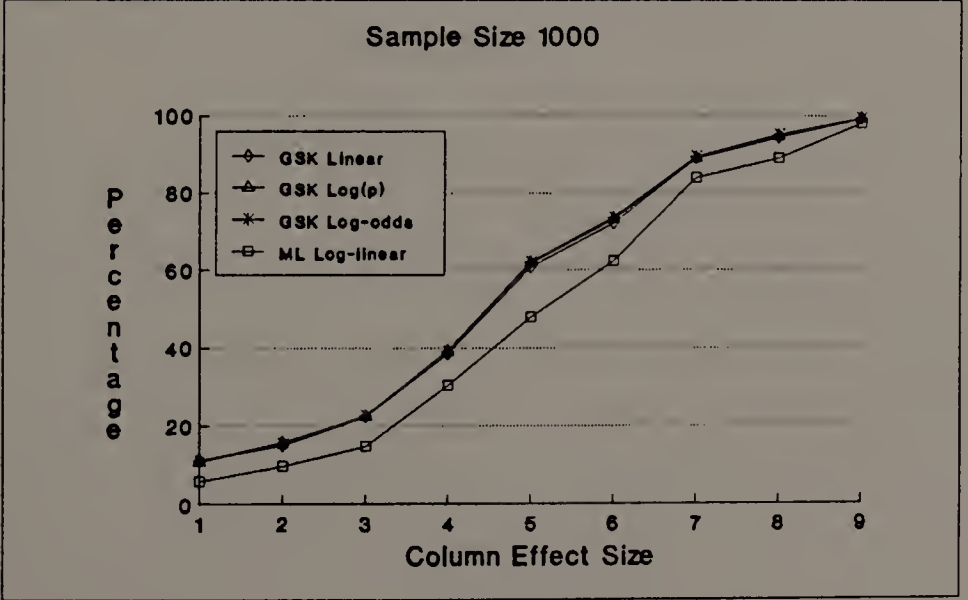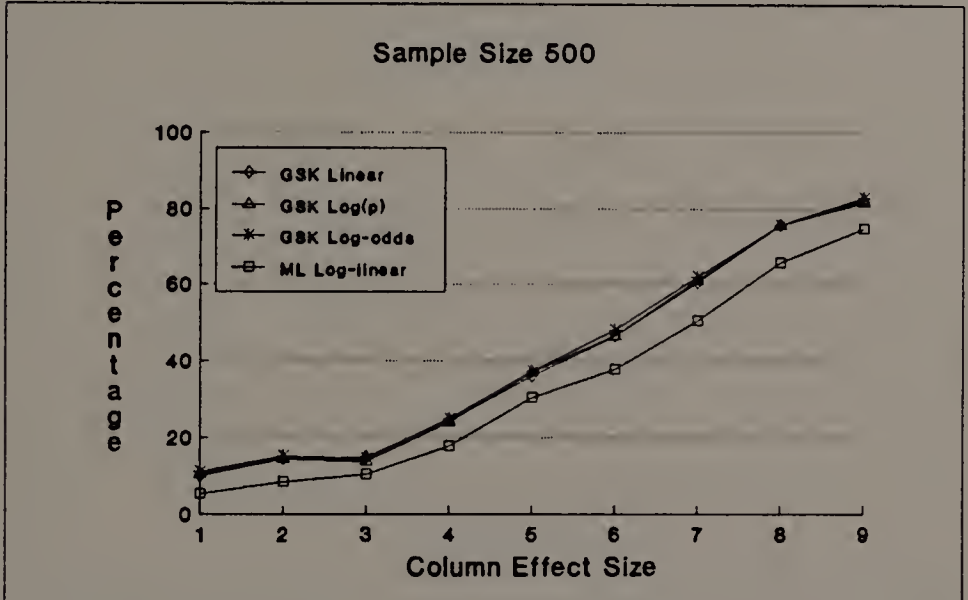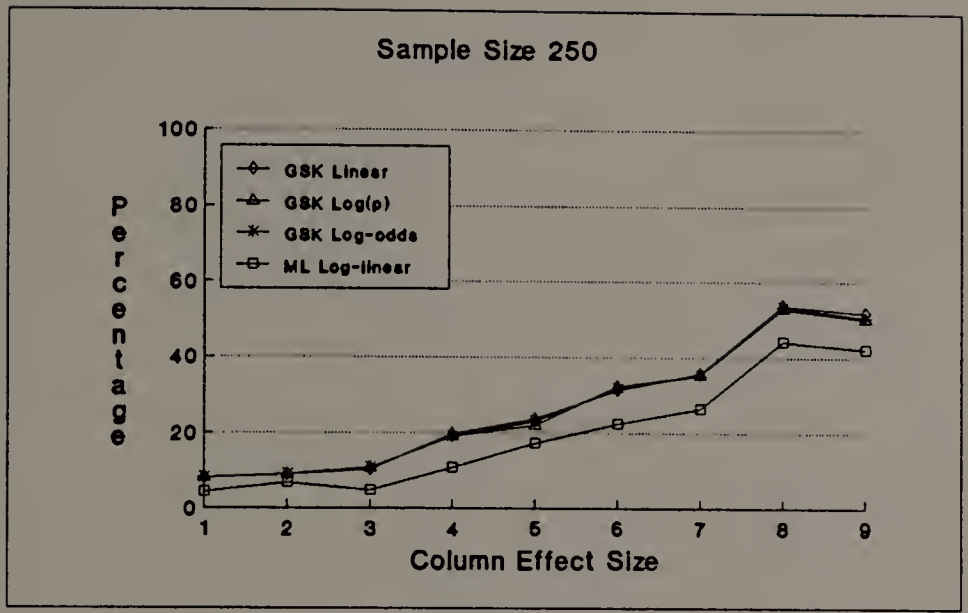(ROW EFFECT SIZE 3)

Figure J.1 Percentage of Significant Chi-Square Column Values,
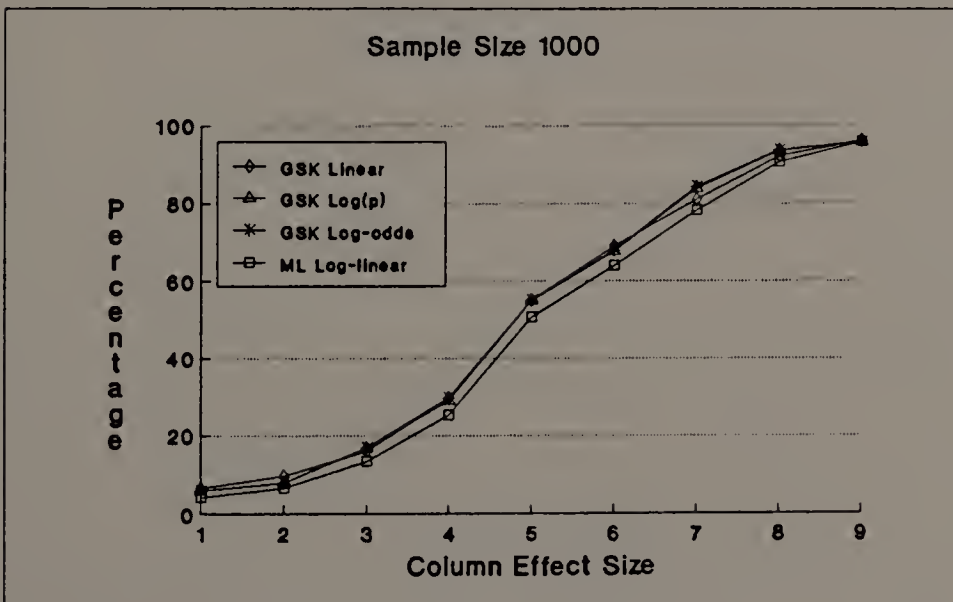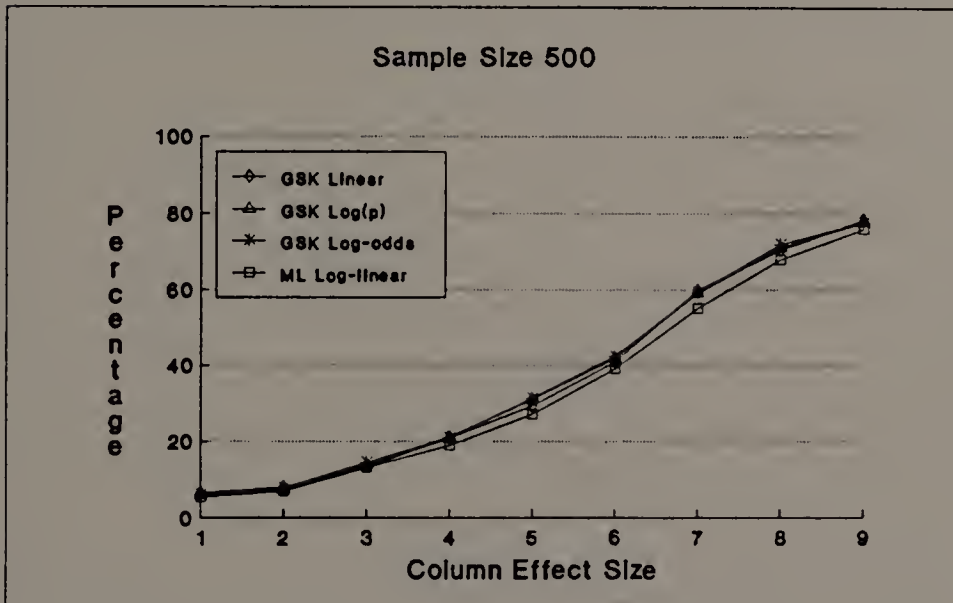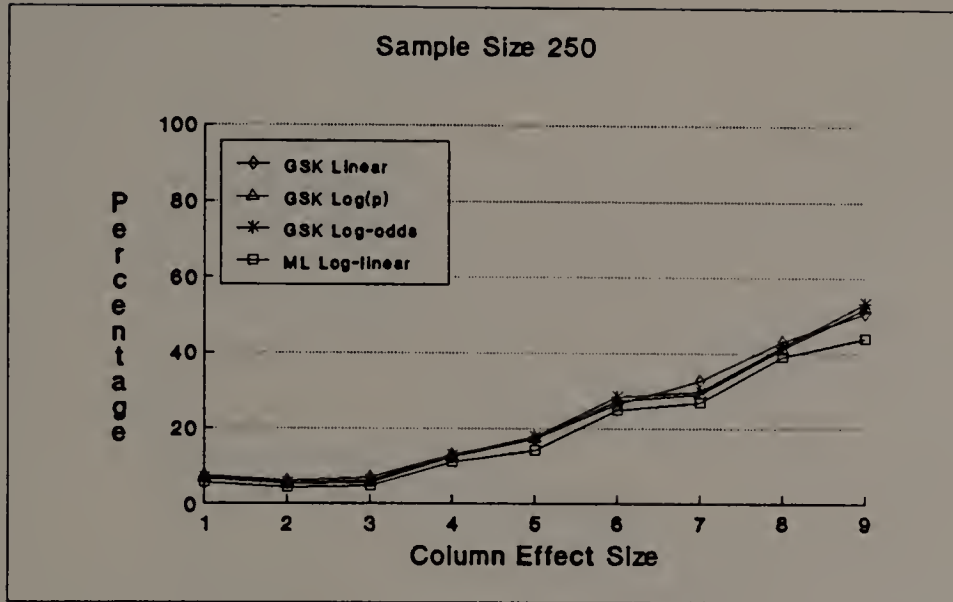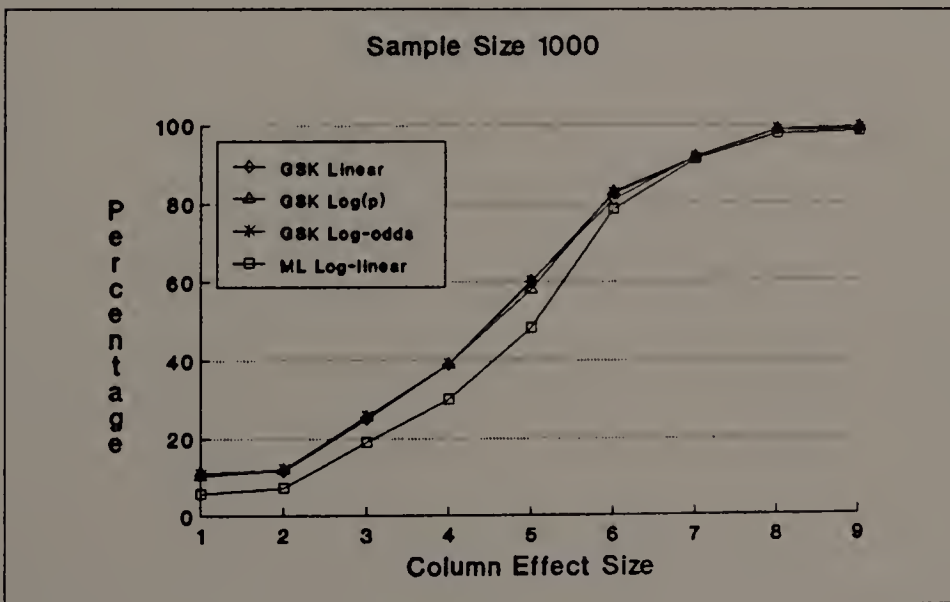Table Type: [Row Effect (3)], 2x2.

Figure J.2 Percentage of Significant Chi-Square Column Values,
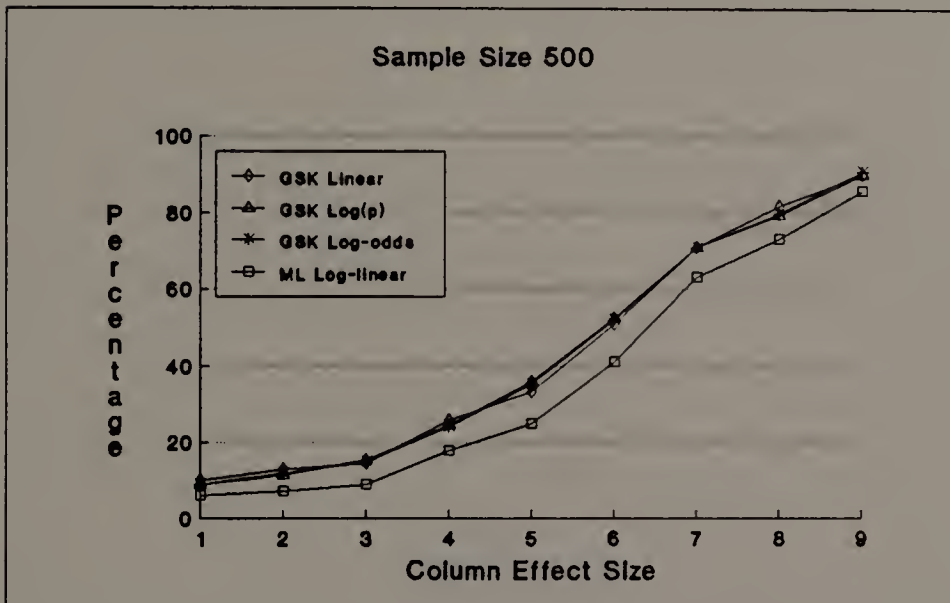Table Type: [Row Effect (3)], 4x2.

114

Figure J.3 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (3)], 2x3.

115

Figure J.4 Percentage of Significant Chi-Square Column Values,
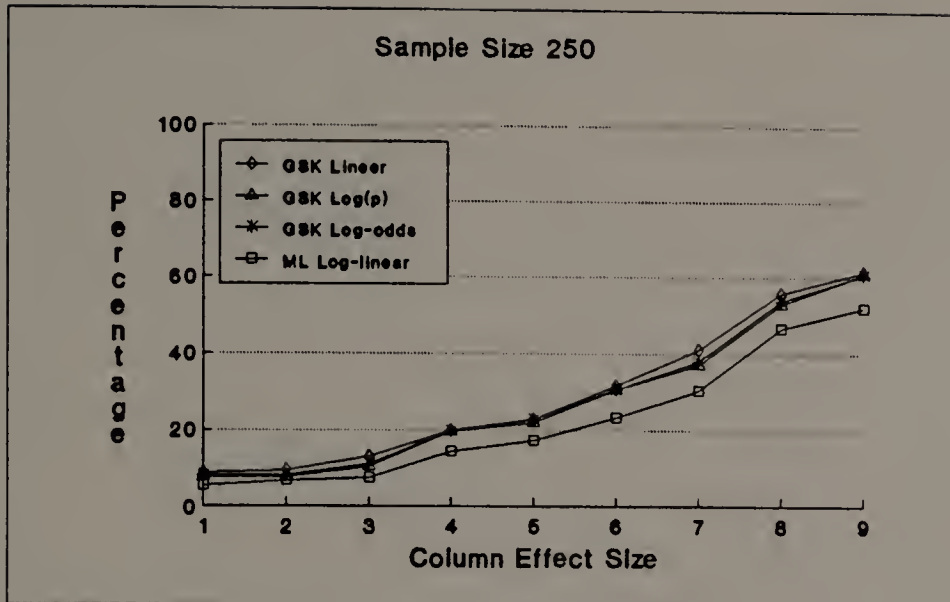Table Type: [Row Effect (3)], 4x3.

Figure J.5 Percentage of Significant Chi-Square Column Values,
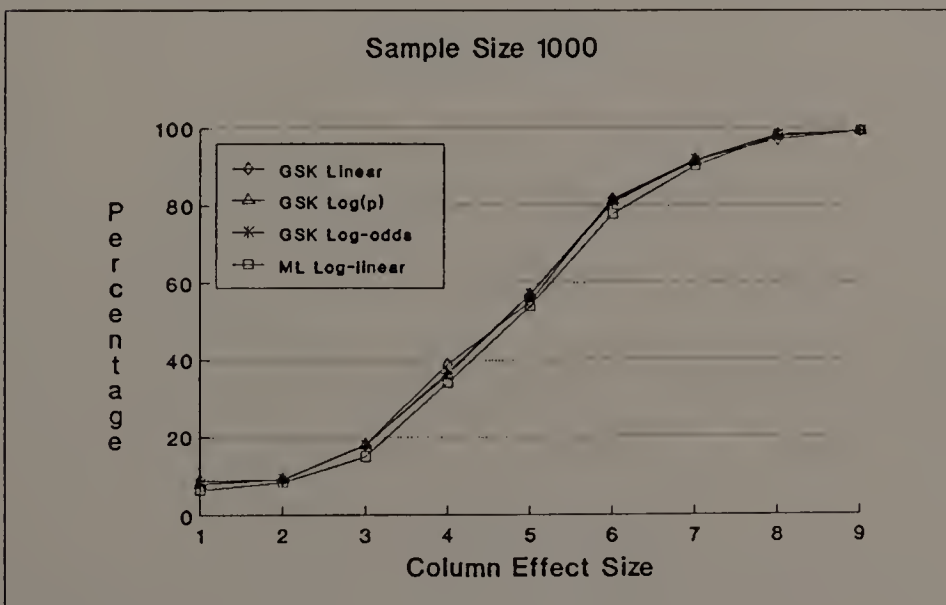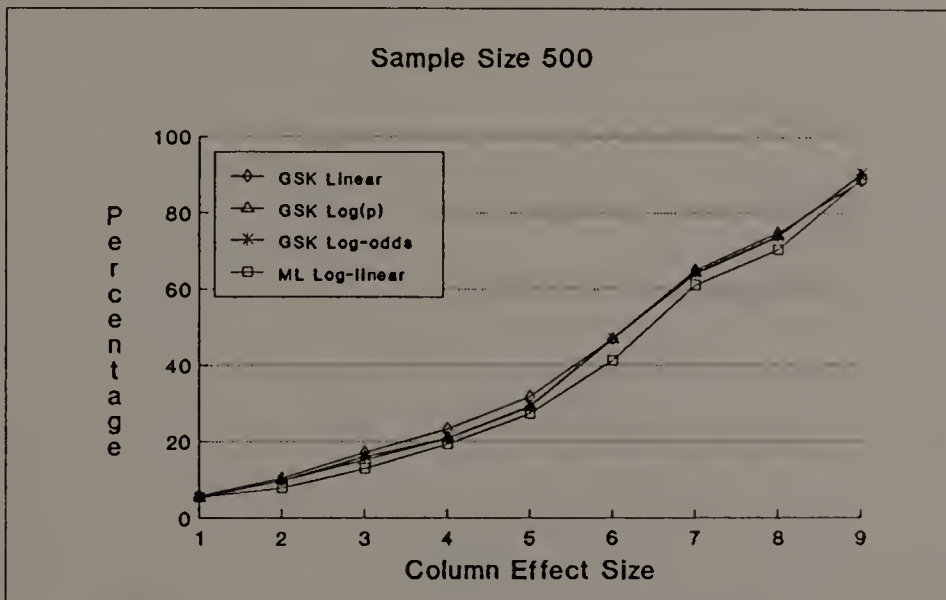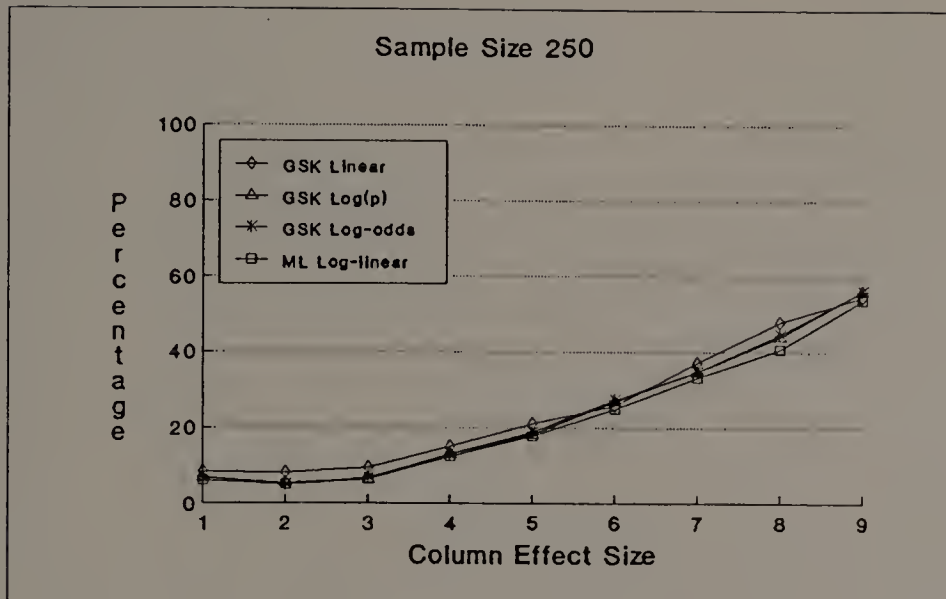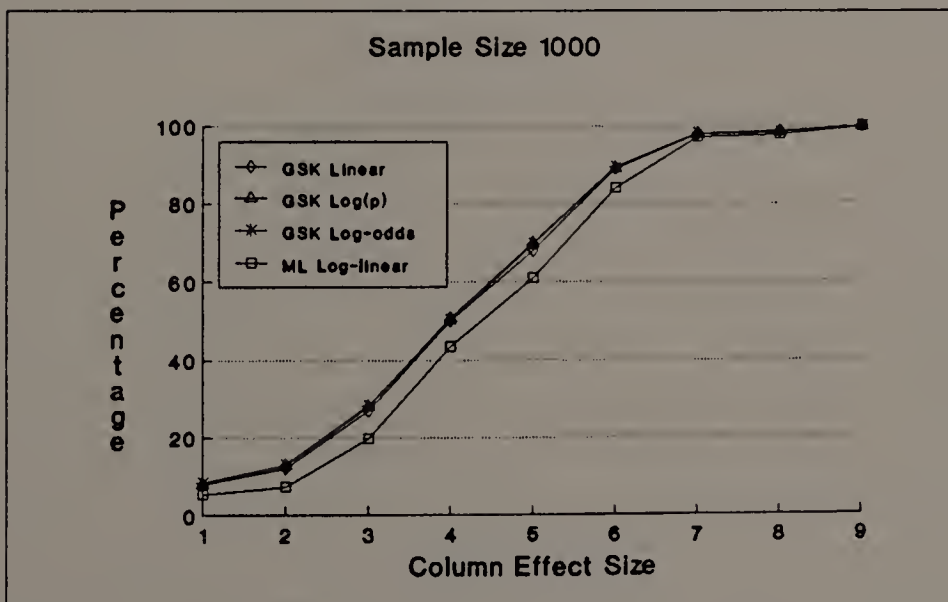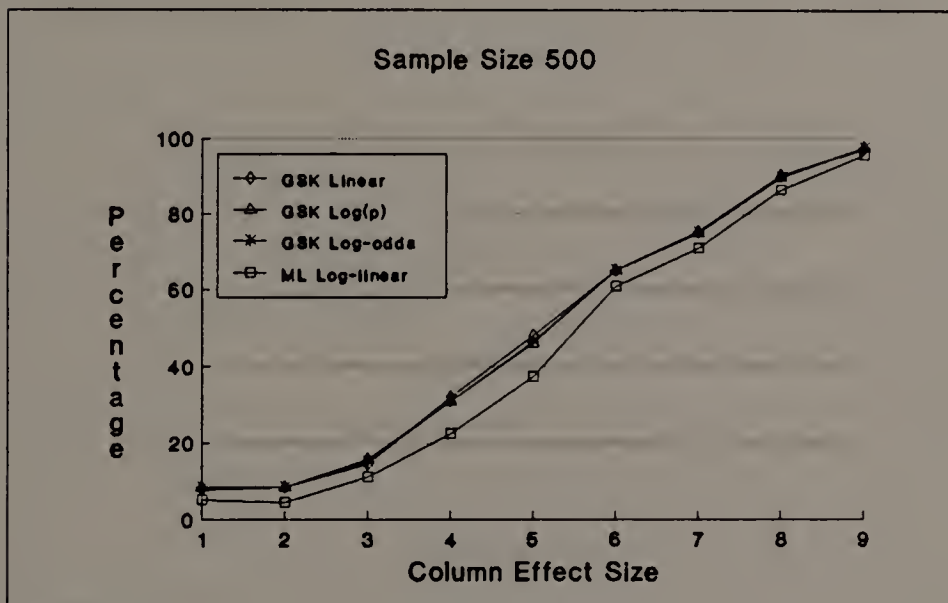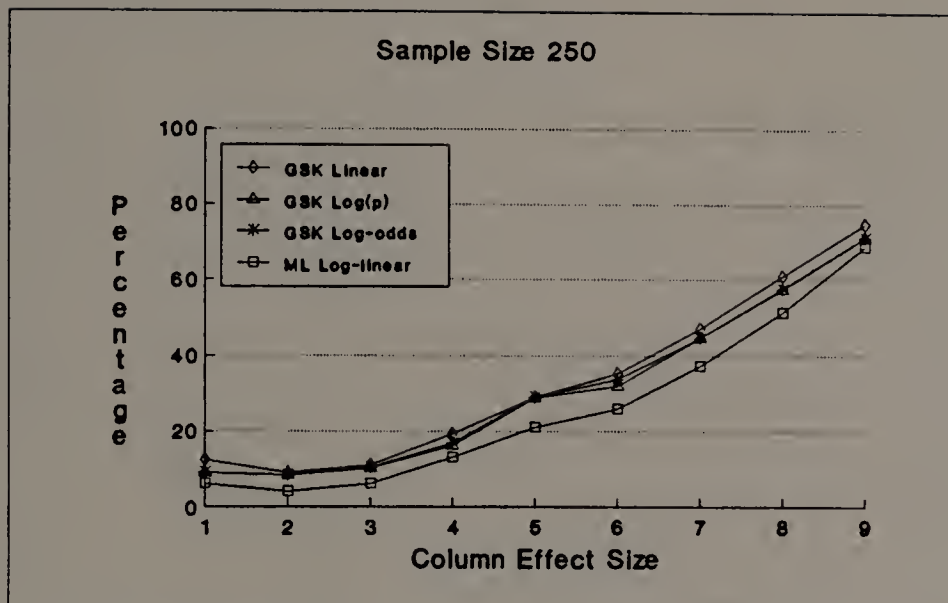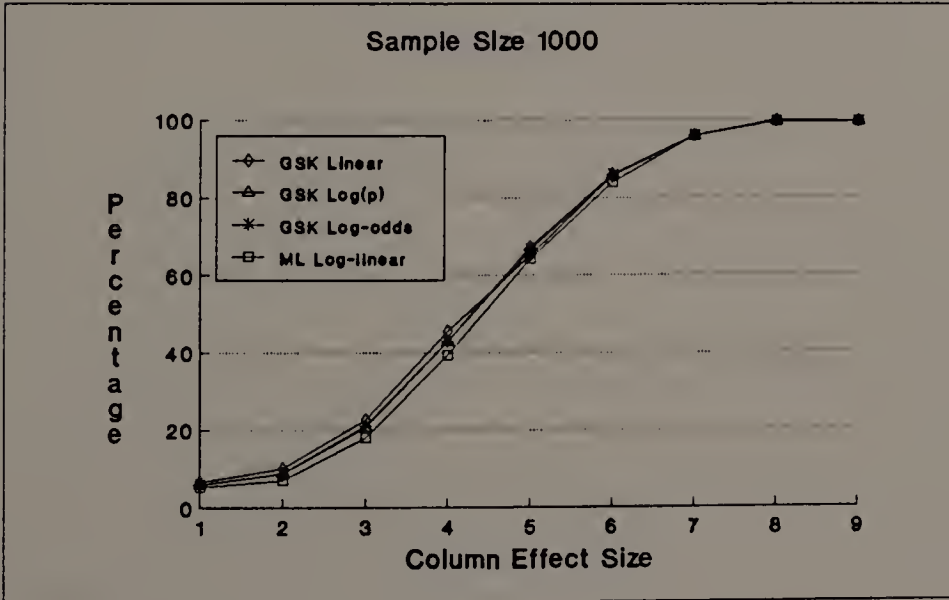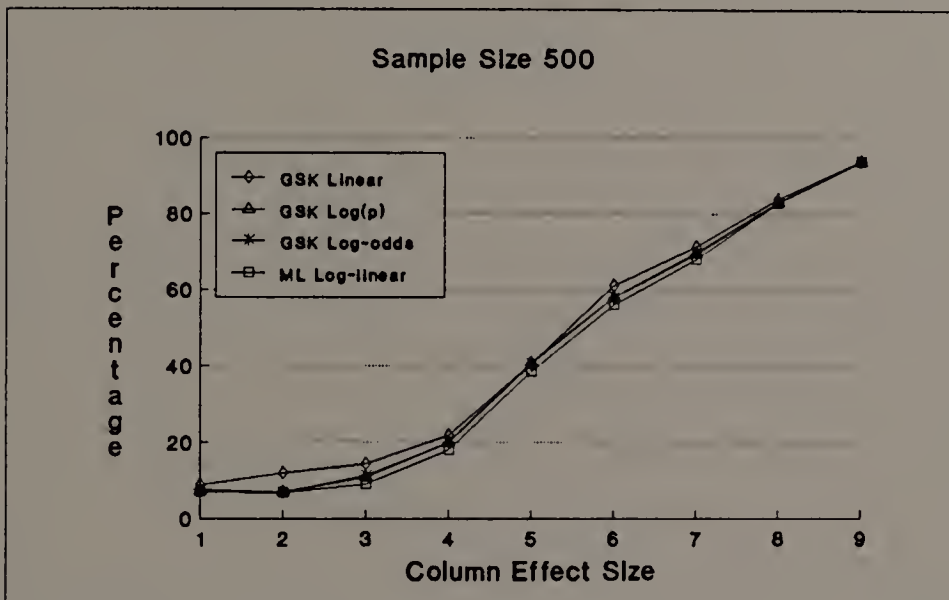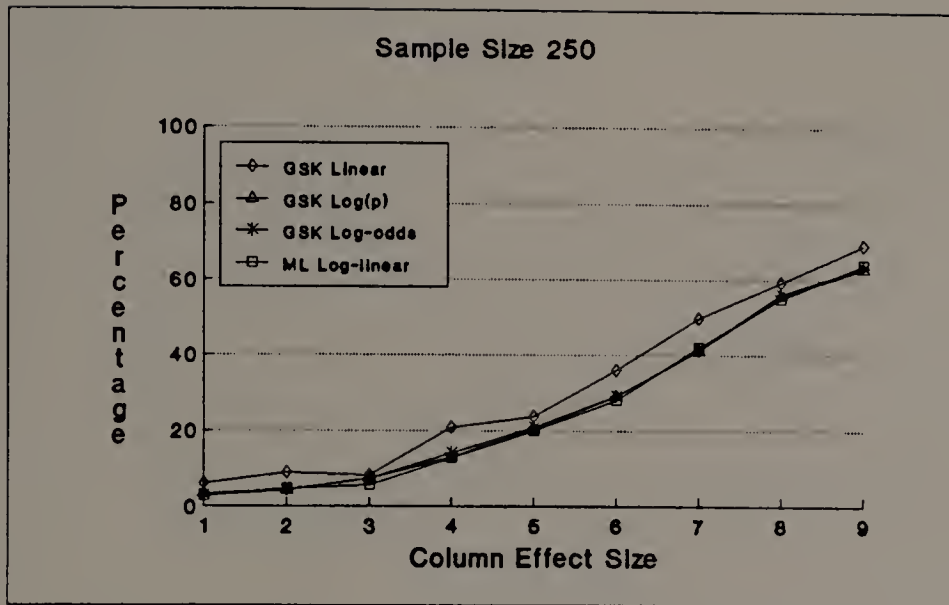Table Type: [Row Effect (3)], 2x4.

117

Figure J.6 Percentage of Significant Chi-Square Column Values,
Table Type: [Row Effect (3)], 4x4.

APPENDIX K

PERCENTAGE OF SIGNIFICANT χ^2 ROW VALUES FOR TABLES WITH ROW AND COLUMN EFFECTS (ROW EFFECT SIZE 2)

Table K.1 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (2)], Two Column.

Column Effect Size	Method	Sample Size:	Size: 2x2			Size: 4x2		
			250	500	1000	250	500	1000
1	GSK Linear		28.0	36.3	57.0	24.0	42.7	75.0
	GSK Log(p)		28.0	36.3	57.3	21.7	40.3	72.7
	GSK Log-odds		28.0	36.7	57.7	21.3	40.7	73.3
	ML Log-linear		21.3	30.3	46.7	15.3	33.3	66.3
2	GSK Linear		24.3	31.7	59.0	26.7	45.0	71.7
	GSK Log(p)		23.7	31.0	57.3	23.3	44.7	72.3
	GSK Log-odds		24.0	31.7	59.0	23.3	44.7	72.3
	ML Log-linear		16.7	22.0	47.0	18.7	37.3	62.3
3	GSK Linear		17.3	34.3	56.3	28.7	47.3	69.7
	GSK Log(p)		17.7	34.0	58.3	22.3	46.3	70.0
	GSK Log-odds		17.7	34.7	58.3	23.0	47.0	70.3
	ML Log-linear		11.7	25.7	48.0	17.3	39.3	64.0
4	GSK Linear		20.7	37.3	56.3	24.7	48.0	71.7
	GSK Log(p)		20.3	39.3	55.3	19.0	44.3	71.7
	GSK Log-odds		20.3	38.0	56.3	19.3	44.3	71.7
	ML Log-linear		14.3	29.0	45.0	15.7	38.7	66.3

120

Table K.1 Continued

Column Effect Size	Method	Sample Size:	Size: 2x2			Size: 4x2		
			250	500	1000	250	500	1000
5	GSK Linear		20.7	36.7	59.7	24.0	42.7	73.3
	GSK Log(p)		19.3	36.3	59.3	23.3	41.3	73.0
	GSK Log-odds		20.0	36.0	60.0	23.7	41.0	72.7
	ML Log-linear		15.7	28.3	47.0	16.7	35.3	64.3
6	GSK Linear		25.0	35.0	58.7	26.0	45.7	72.3
	GSK Log(p)		25.3	33.0	58.3	21.3	45.0	72.0
	GSK Log-odds		25.0	34.7	59.0	21.3	45.0	72.0
	ML Log-Linear		19.0	27.3	44.0	19.0	37.0	66.3
7	GSK Linear		19.7	34.7	60.7	25.0	49.3	71.0
	GSK Log(p)		18.7	33.3	61.0	23.0	49.0	71.7
	GSK Log-odds		19.0	34.7	61.7	22.3	49.3	71.7
	ML Log-Linear		13.7	26.7	48.0	18.3	40.7	64.0
8	GSK Linear		24.7	37.3	57.3	25.7	42.3	74.0
	GSK Log(p)		22.0	37.7	57.0	25.7	43.3	74.7
	GSK Log-odds		23.7	37.7	58.3	24.7	43.3	75.0
	ML Log-Linear		15.3	26.0	45.3	19.3	35.3	68.0
9	GSK Linear		22.3	33.3	56.7	24.7	46.3	70.3
	GSK Log(p)		21.3	34.0	57.0	22.3	45.7	70.0
	GSK Log-odds		22.0	34.7	57.3	23.0	45.3	70.7
	ML Log-Linear		17.3	26.0	45.0	17.0	40.0	61.7

Table K.2 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (2)], Three Column.

Column Effect Size	Method	Size: 2x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000
1	GSK Linear	21.0	32.3	58.0	23.0	44.3	75.0
	GSK Log(p)	19.0	59.7	57.7	18.7	43.7	72.7
	GSK Log-odds	19.0	29.7	57.7	18.0	44.7	72.0
	ML Log-linear	16.0	25.0	47.0	17.7	39.7	69.3
2	GSK Linear	21.0	31.3	50.3	25.0	37.7	72.0
	GSK Log(p)	20.0	30.0	49.7	21.0	34.7	72.7
	GSK Log-odds	20.0	30.0	49.7	20.3	33.7	71.7
	ML Log-linear	17.0	25.7	43.0	20.7	30.7	68.0
3	GSK Linear	17.0	33.0	53.3	27.7	35.7	72.0
	GSK Log(p)	16.0	31.3	52.3	20.3	32.0	71.0
	GSK Log-odds	16.0	31.0	53.0	20.7	32.3	70.0
	ML Log-linear	13.7	27.3	45.0	18.0	30.3	66.3
4	GSK Linear	24.7	37.0	55.3	27.0	41.3	73.3
	GSK Log(p)	23.3	35.0	54.3	21.3	36.3	73.0
	GSK Log-odds	23.3	35.3	54.3	21.3	36.3	73.0
	ML Log-linear	19.7	32.7	48.0	18.7	34.0	67.7

122

Table K.2 Continued

Column Effect Size	Method	Size: 2x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000
5	GSK Linear	18.7	33.7	57.3	23.3	48.7	71.0
	GSK Log(p)	16.0	32.7	56.3	20.3	47.0	68.3
	GSK Log-odds	16.3	30.3	56.7	19.7	47.7	68.3
	ML Log-linear	13.7	27.7	46.3	15.7	41.7	62.7
6	GSK Linear	18.3	31.3	48.0	21.3	41.3	67.3
	GSK Log(p)	17.7	30.0	48.0	19.7	39.3	67.0
	GSK Log-odds	17.7	29.7	48.3	19.3	39.0	67.0
	ML Log-Linear	15.0	25.7	41.7	15.3	34.3	65.0
7	GSK Linear	19.7	37.3	52.3	24.0	39.7	67.0
	GSK Log(p)	18.3	36.0	53.3	22.0	40.7	66.0
	GSK Log-odds	18.3	35.3	53.7	23.0	40.3	65.0
	ML Log-Linear	16.7	32.3	43.0	19.3	35.0	62.3
8	GSK Linear	19.0	34.7	59.7	26.7	37.3	67.7
	GSK Log(p)	17.7	35.7	60.0	18.3	36.0	69.0
	GSK Log-odds	18.0	34.7	60.0	18.0	36.0	68.3
	ML Log-Linear	15.0	29.7	53.3	16.3	31.0	64.0
9	GSK Linear	24.3	27.7	54.3	22.7	35.0	68.0
	GSK Log(p)	22.0	29.3	53.7	17.0	35.0	70.3
	GSK Log-odds	22.3	29.0	53.3	17.3	35.3	71.0
	ML Log-Linear	20.3	24.3	46.0	16.0	30.7	67.7

Table K.3 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (2)], Four Column.

Column Effect Size	Method	Size: 2x4			Size: 4x4		
	Sample Size:	250	500	1000	250	500	1000
1	GSK Linear	16.3	30.7	51.7	20.7	41.0	66.0
	GSK Log(p)	16.0	29.3	49.3	14.7	37.7	63.0
	GSK Log-odds	16.0	29.3	49.3	14.7	38.3	63.0
	ML Log-linear	14.7	28.3	43.0	16.3	35.3	59.7
2	GSK Linear	19.0	33.7	57.0	25.3	47.0	73.3
	GSK Log(p)	18.3	34.7	56.3	19.3	41.7	71.3
	GSK Log-odds	18.3	35.0	56.3	20.0	42.0	71.3
	ML Log-linear	16.7	31.3	50.7	18.7	41.3	69.7
3	GSK Linear	20.0	28.0	53.7	26.0	43.7	68.3
	GSK Log(p)	19.0	27.7	53.0	18.7	39.0	66.3
	GSK Log-odds	19.3	27.3	53.0	19.0	39.0	66.7
	ML Log-linear	16.7	26.7	47.7	18.7	37.7	64.3
4	GSK Linear	24.3	34.3	53.3	24.7	48.0	70.0
	GSK Log(p)	23.7	34.0	54.0	18.7	44.7	70.0
	GSK Log-odds	23.7	34.3	53.7	18.7	44.7	70.0
	ML Log-linear	20.7	31.0	46.0	18.0	43.0	67.7

Table K.3 Continued

Column Effect Size	Method	Size: 2x4			Size: 4x4		
	Sample Size:	250	500	1000	250	500	1000
5	GSK Linear	18.7	31.3	49.3	24.7	43.7	72.0
	GSK Log(p)	18.3	30.3	48.3	20.0	43.3	69.7
	GSK Log-odds	18.7	31.3	48.0	19.7	43.0	70.0
	ML Log-linear	17.3	27.7	42.3	17.0	39.3	64.7
6	GSK Linear	20.3	32.0	57.3	25.3	42.3	64.3
	GSK Log(p)	19.7	30.3	55.7	18.3	38.7	63.7
	GSK Log-odds	20.3	31.3	55.3	18.3	38.3	64.0
	ML Log-Linear	17.0	27.7	50.3	17.0	35.3	61.3
7	GSK Linear	17.7	35.3	48.0	22.0	41.3	68.0
	GSK Log(p)	17.0	33.3	48.0	14.0	37.3	67.0
	GSK Log-odds	18.0	34.0	47.7	14.7	37.0	67.7
	ML Log-Linear	14.7	31.3	42.0	13.7	36.0	65.3
8	GSK Linear	16.7	28.7	52.3	24.0	39.7	65.7
	GSK Log(p)	15.3	27.3	53.3	18.7	36.7	67.0
	GSK Log-odds	15.7	28.0	53.0	19.0	36.3	67.0
	ML Log-Linear	12.7	25.7	47.0	17.7	36.0	65.3
9	GSK Linear	18.3	30.7	50.3	25.7	34.7	69.0
	GSK Log(p)	16.7	31.0	52.3	23.0	36.3	69.0
	GSK Log-odds	17.3	31.3	52.0	23.0	36.3	69.0
	ML Log-Linear	14.7	29.3	46.3	20.3	34.3	67.0

APPENDIX L

PERCENTAGE OF SIGNIFICANT χ^2 ROW VALUES FOR TABLES WITH ROW AND COLUMN EFFECTS (ROW EFFECT SIZE 3)

Table L.1 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (3)], Two Column.

Column Effect Size	Method	Sample Size:	Size: 2x2			Size: 4x2		
		250	500	1000	250	500	1000
1	GSK Linear	57.7	77.0	98.0	66.3	97.3	100.0
	GSK Log(p)	57.7	75.3	98.0	66.7	97.7	100.0
	GSK Log-odds	58.0	76.3	98.0	67.7	97.7	100.0
	ML Log-linear	48.7	71.3	94.7	60.0	95.3	100.0
2	GSK Linear	58.7	84.0	99.0	77.3	95.0	100.0
	GSK Log(p)	58.7	82.3	99.0	75.7	92.7	100.0
	GSK Log-odds	59.0	82.3	99.0	76.0	93.0	100.0
	ML Log-linear	47.7	77.0	96.0	66.0	92.0	100.0
3	GSK Linear	53.7	83.7	97.0	73.0	95.0	100.0
	GSK Log(p)	53.0	84.0	97.0	70.3	94.7	100.0
	GSK Log-odds	53.3	84.0	97.0	70.3	94.7	100.0
	ML Log-linear	44.7	77.7	95.3	64.3	93.0	100.0
4	GSK Linear	54.0	82.3	98.0	71.7	97.3	100.0
	GSK Log(p)	53.7	83.3	98.0	69.0	97.3	100.0
	GSK Log-odds	54.0	83.7	98.0	69.7	97.3	100.0
	ML Log-linear	44.7	75.7	97.0	63.0	96.0	100.0

127

Table L.1 Continued

Column Effect Size	Method	Sample Size:	Size: 2x2			Size: 4x2	
		250	500	1000	250	500	1000
5	GSK Linear	60.0	82.0	99.0	70.7	96.0	100.0
	GSK Log(p)	58.7	81.3	99.0	69.0	95.7	100.0
	GSK Log-odds	60.0	82.0	99.0	70.0	95.7	100.0
	ML Log-linear	49.7	77.7	97.7	63.7	94.3	100.3
6	GSK Linear	60.7	85.7	97.7	70.3	96.7	100.0
	GSK Log(p)	59.7	85.0	98.3	67.7	95.7	100.0
	GSK Log-odds	59.7	85.3	98.3	68.0	95.7	100.0
	ML Log-Linear	49.3	78.0	95.7	63.0	93.7	100.0
7	GSK Linear	58.7	83.7	97.7	73.3	95.3	100.0
	GSK Log(p)	58.3	82.7	98.3	71.3	94.7	100.0
	GSK Log-odds	58.3	83.7	98.3	71.7	94.3	100.0
	ML Log-Linear	48.0	78.7	96.7	65.0	92.0	100.0
8	GSK Linear	60.7	83.0	99.3	77.7	96.7	100.0
	GSK Log(p)	59.7	82.0	99.3	74.3	95.0	100.0
	GSK Log-odds	61.0	82.7	99.3	74.3	95.3	100.0
	ML Log-Linear	49.3	77.7	97.7	68.3	93.3	100.0
9	GSK Linear	61.3	80.7	98.7	75.0	97.0	100.0
	GSK Log(p)	58.7	80.3	98.3	73.3	95.3	100.0
	GSK Log-odds	58.7	81.3	98.7	73.3	95.3	100.0
	ML Log-Linear	51.7	76.3	96.7	68.7	92.7	100.0

128

Table L.2 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (3)], Three Column.

Column Effect Size	Method	Size: 2x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000
1	GSK Linear	52.0	83.0	99.0	66.3	94.3	100.0
	GSK Log(p)	51.3	81.7	99.0	66.3	93.3	100.0
	GSK Log-odds	51.3	81.0	99.0	66.7	93.3	100.0
	ML Log-linear	46.3	78.0	97.3	62.0	92.7	100.0
2	GSK Linear	51.0	80.7	96.7	69.7	96.3	100.0
	GSK Log(p)	49.0	80.0	96.7	67.3	95.3	100.0
	GSK Log-odds	49.0	80.0	96.7	68.0	95.3	100.0
	ML Log-linear	44.3	77.0	95.3	65.3	94.7	100.0
3	GSK Linear	54.7	83.3	97.3	72.0	96.7	100.0
	GSK Log(p)	51.3	82.0	97.3	65.0	96.3	100.0
	GSK Log-odds	52.3	82.0	97.3	65.3	96.7	100.0
	ML Log-linear	47.7	79.7	96.3	64.0	94.7	100.0
4	GSK Linear	57.3	79.7	98.7	73.3	97.0	100.0
	GSK Log(p)	54.0	78.7	98.7	70.3	97.7	100.0
	GSK Log-odds	54.0	78.7	98.7	70.3	97.7	100.0
	ML Log-linear	50.0	77.0	98.3	65.3	97.0	100.0

Table L.2 Continued

Column Effect Size	Method	Size: 2x3			Size: 4x3		
Sample Size:		250	500	1000	250	500	1000
5	GSK Linear	59.0	79.0	97.3	67.3	95.7	100.0
	GSK Log(p)	56.0	79.0	97.7	65.0	95.3	100.0
	GSK Log-odds	57.0	78.3	97.7	65.0	95.3	100.0
	ML Log-linear	51.3	75.7	96.3	62.3	93.7	100.0
6	GSK Linear	58.3	83.7	98.7	71.0	96.0	100.0
	GSK Log(p)	56.0	82.7	98.7	66.7	95.7	99.7
	GSK Log-odds	56.3	82.7	98.7	67.0	95.7	99.7
	ML Log-Linear	50.7	79.3	96.7	65.3	94.3	99.7
7	GSK Linear	50.0	81.3	99.3	74.0	94.7	10.0
	GSK Log(p)	48.0	81.0	99.3	70.3	95.3	100.0
	GSK Log-odds	48.3	80.3	99.3	70.3	95.3	100.0
	ML Log-Linear	42.0	77.3	98.0	68.0	93.7	100.0
8	GSK Linear	58.0	85.0	98.7	67.7	95.7	99.7
	GSK Log(p)	58.7	83.3	99.0	64.7	96.0	99.7
	GSK Log-odds	58.0	84.0	99.0	64.7	96.3	99.7
	ML Log-Linear	50.3	80.7	97.7	62.0	94.7	99.7
9	GSK Linear	54.0	83.3	97.0	74.3	95.7	100.0
	GSK Log(p)	52.3	84.0	97.7	68.3	94.3	100.0
	GSK Log-odds	52.0	84.0	97.0	68.3	94.3	100.0
	ML Log-Linear	46.3	80.7	96.7	66.7	93.3	99.7

Table L.3 Percentage of χ^2 Row Values Significant at .05 Level, Table Type: [Row Effect (3)], Four Column.

Column Effect Size	Method	Sample Size:	Size: 2x4			Size: 4x4		
		250	500	1000	250	500	1000
1	GSK Linear	50.7	75.7	97.3	65.7	91.3	100.0
	GSK Log(p)	49.0	75.3	97.3	61.7	91.3	100.0
	GSK Log-odds	50.0	75.3	97.3	62.0	91.3	100.0
	ML Log-linear	45.3	73.0	96.0	61.7	90.7	100.0
2	GSK Linear	54.3	77.3	97.3	69.0	94.0	100.0
	GSK Log(p)	53.0	76.0	97.0	61.3	93.3	100.0
	GSK Log-odds	53.3	77.0	97.0	61.7	93.3	100.0
	ML Log-linear	51.0	74.0	96.0	62.3	92.7	100.0
3	GSK Linear	52.0	80.7	97.7	71.3	97.0	100.0
	GSK Log(p)	51.3	80.3	97.7	63.7	96.7	99.7
	GSK Log-odds	52.3	80.7	97.7	64.7	96.7	99.7
	ML Log-linear	48.0	78.7	97.3	64.7	95.7	99.7
4	GSK Linear	51.3	86.3	96.7	70.7	94.0	100.0
	GSK Log(p)	49.3	85.7	96.3	66.0	92.7	99.7
	GSK Log-odds	50.3	85.7	96.3	65.3	92.7	99.7
	ML Log-linear	47.3	84.3	95.7	65.0	92.0	99.7

Continued, next page

131

Table L.3 Continued

Column Effect Size	Method	Sample Size:	Size: 2x4			Size: 4x4	
		250	500	1000	250	500	1000
5	GSK Linear	58.7	78.0	95.7	70.0	97.0	100.0
	GSK Log(p)	56.3	76.3	96.0	63.3	95.3	100.0
	GSK Log-odds	56.7	76.7	96.0	63.3	95.3	100.0
	ML Log-linear	50.3	74.0	95.0	63.7	95.0	100.0
6	GSK Linear	53.7	78.7	98.3	69.0	93.7	100.0
	GSK Log(p)	54.0	77.3	98.7	61.0	93.7	100.0
	GSK Log-odds	54.3	78.0	98.7	61.3	93.7	100.0
	ML Log-Linear	51.3	76.0	98.7	61.7	93.3	100.0
7	GSK Linear	48.7	79.3	97.3	70.3	95.0	100.0
	GSK Log(p)	48.7	82.3	97.3	66.3	96.3	99.7
	GSK Log-odds	49.0	82.3	97.3	66.7	96.7	99.7
	ML Log-Linear	45.3	80.0	96.3	67.3	95.3	99.7
8	GSK Linear	49.7	78.7	97.3	66.0	94.7	100.0
	GSK Log(p)	49.7	77.3	98.0	59.0	94.7	99.7
	GSK Log-odds	50.3	77.7	97.7	59.3	94.7	99.7
	ML Log-Linear	45.0	77.0	96.3	60.0	94.3	99.7
9	GSK Linear	54.7	82.2	95.7	70.0	94.7	99.7
	GSK Log(p)	57.0	82.0	96.0	65.3	93.0	100.0
	GSK Log-odds	57.3	82.3	96.0	65.3	93.3	100.0
	ML Log-Linear	52.0	80.0	95.7	65.7	93.7	99.7

APPENDIX M

PERCENTAGE OF SIGNIFICANT χ^2 INTERACTION VALUES FOR TABLES
WITH ROW AND COLUMN EFFECTS (ROW EFFECT SIZE 2)

Table M.1 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (2)], Two Column.

Column Effect Size	Method	Size: 2x2			Size: 4x2		
	Sample Size	50	500	1000	250	500	1000
1	GSK Linear	8.7	9.0	8.0	9.0	6.3	7.0
	GSK Log(p)	9.3	8.3	7.7	7.7	6.7	7.0
	GSK Log-odds	9.0	8.7	7.7	7.7	6.7	7.0
	ML Log-linear	4.7	4.7	5.3	4.3	4.7	4.3
2	GSK Linear	10.7	10.0	9.7	8.0	6.7	9.0
	GSK Log(p)	10.0	9.7	10.0	7.7	6.0	9.0
	GSK Log-odds	10.3	9.0	9.7	7.7	6.0	9.0
	ML Log-linear	6.3	3.7	5.0	5.7	4.3	6.0
3	GSK Linear	10.0	10.7	12.3	10.0	8.0	7.0
	GSK Log(p)	10.0	10.0	11.3	8.0	8.7	7.0
	GSK Log-odds	10.0	10.3	11.3	8.7	8.7	7.3
	ML Log-linear	6.3	4.7	5.7	5.7	7.0	4.3
4	GSK Linear	8.0	7.7	10.0	10.3	10.3	9.0
	GSK Log(p)	8.7	8.0	9.7	9.3	10.0	8.7
	GSK Log-odds	8.7	7.3	9.7	10.0	10.0	8.3
	ML Log-linear	6.0	3.0	6.0	6.3	7.0	6.7

Table M.1 Continued

Column Effect Size	Method	Sample Size:	Size: 2x2			Size: 4x2		
			250	500	1000	250	500	1000
5	GSK Linear		11.3	7.7	12.0	9.7	7.3	9.3
	GSK Log(p)		11.7	6.3	11.0	8.7	7.0	7.7
	GSK Log-odds		12.0	7.0	11.3	8.7	7.0	8.0
	ML Log-linear		9.3	4.3	7.7	6.7	3.7	5.3
6	GSK Linear		9.3	9.0	10.3	6.3	8.3	6.7
	GSK Log(p)		8.7	8.7	10.7	5.7	7.7	5.7
	GSK Log-odds		9.0	8.7	10.7	5.7	8.0	5.7
	ML Log-Linear		5.3	4.7	5.3	3.3	4.7	5.0
7	GSK Linear		8.3	12.7	10.0	6.7	8.7	8.0
	GSK Log(p)		8.0	10.0	8.0	4.7	8.0	6.3
	GSK Log-odds		8.7	10.7	8.3	5.0	8.0	6.3
	ML Log-Linear		5.0	6.3	3.3	3.7	5.7	3.0
8	GSK Linear		7.7	9.3	8.0	6.0	6.7	7.0
	GSK Log(p)		7.3	9.7	6.7	5.7	6.0	5.7
	GSK Log-odds		8.3	9.3	7.0	6.3	6.0	6.0
	ML Log-Linear		4.7	3.7	2.7	3.3	5.0	4.7
9	GSK Linear		10.0	11.7	10.0	5.7	8.7	7.3
	GSK Log(p)		10.0	10.0	8.3	5.7	6.7	6.0
	GSK Log-odds		10.0	10.0	8.3	5.7	7.0	6.0
	ML Log-Linear		6.0	6.3	5.0	3.7	5.0	3.3

135

Table M.2 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (2)], Three Column.

Column Effect Size	Method	Size: 2x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000
1	GSK Linear	5.3	8.7	10.3	5.3	10.3	8.7
	GSK Log(p)	5.7	8.7	10.7	4.7	7.3	7.3
	GSK Log-odds	5.7	9.0	10.7	4.7	7.7	7.7
	ML Log-linear	3.0	5.3	7.0	4.0	5.3	6.0
2	GSK Linear	5.3	8.7	10.0	8.0	8.7	10.0
	GSK Log(p)	5.3	9.0	10.0	5.3	8.3	9.3
	GSK Log-odds	5.3	8.7	10.3	5.7	8.3	9.7
	ML Log-linear	4.0	6.3	7.0	4.3	7.0	7.3
3	GSK Linear	5.0	8.3	8.3	12.3	12.3	6.0
	GSK Log(p)	5.0	8.3	7.7	9.0	9.7	5.3
	GSK Log-odds	5.7	8.3	8.0	9.0	9.7	5.3
	ML Log-linear	3.0	5.0	5.3	8.7	7.7	4.0
4	GSK Linear	8.0	8.7	7.3	8.0	7.0	7.0
	GSK Log(p)	7.7	8.3	8.7	7.0	6.7	5.3
	GSK Log-odds	8.3	8.7	8.7	7.3	6.0	5.7
	ML Log-linear	6.3	6.0	5.3	5.7	5.3	4.0

Table M.2 Continued

Column Effect Size	Method	Size: 2x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000
5	GSK Linear	10.3	8.3	8.3	10.7	5.7	6.3
	GSK Log(p)	9.7	8.0	7.7	9.0	5.7	6.7
	GSK Log-odds	9.7	8.3	8.0	9.3	5.7	6.7
	ML Log-linear	6.7	5.3	4.7	8.0	3.0	5.0
6	GSK Linear	6.0	7.7	7.0	5.0	5.7	10.7
	GSK Log(p)	6.0	7.7	6.0	4.7	6.0	10.0
	GSK Log-odds	6.3	8.0	6.0	4.7	6.0	10.3
	ML Log-Linear	3.3	4.7	3.7	3.3	3.3	7.7
7	GSK Linear	5.3	6.7	10.3	10.3	9.0	7.3
	GSK Log(p)	4.7	5.7	8.7	9.3	6.7	6.3
	GSK Log-odds	5.0	5.7	9.0	9.3	7.0	6.7
	ML Log-Linear	3.7	3.7	5.0	7.3	5.3	4.3
8	GSK Linear	10.0	8.3	10.3	9.3	6.0	7.0
	GSK Log(p)	8.7	7.7	7.3	8.3	4.3	7.7
	GSK Log-odds	9.3	8.0	9.0	8.3	4.7	7.7
	ML Log-Linear	6.3	6.0	3.3	7.0	3.0	4.7
9	GSK Linear	9.0	7.0	8.0	8.3	7.7	7.7
	GSK Log(p)	7.7	6.3	7.0	6.3	7.0	6.3
	GSK Log-odds	8.3	6.3	7.7	6.3	7.3	6.7
	ML Log-Linear	5.3	4.3	3.0	6.7	4.0	4.3

Table M.3 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (2)], Four Column.

Column Effect Size	Method	Sample Size:	Size: 2x4			Size: 4x4		
			250	500	1000	250	500	1000
1	GSK Linear		6.7	8.3	6.0	7.7	7.0	7.7
	GSK Log(p)		6.7	7.3	6.3	5.0	5.0	7.3
	GSK Log-odds		6.7	7.3	6.3	6.3	5.3	5.0
	ML Log-linear		5.0	5.7	4.0	5.7	4.3	6.7
2	GSK Linear		9.3	8.0	9.7	8.0	8.3	6.7
	GSK Log(p)		8.7	7.0	8.7	5.0	6.3	6.0
	GSK Log-odds		8.7	7.3	8.7	5.0	6.7	5.7
	ML Log-linear		5.7	5.0	5.3	4.7	6.3	5.0
3	GSK Linear		10.0	5.0	5.7	7.7	9.3	7.3
	GSK Log(p)		8.3	5.3	6.0	6.0	8.7	6.0
	GSK Log-odds		8.3	5.3	6.3	6.0	8.3	6.7
	ML Log-linear		5.7	4.0	3.7	7.0	6.7	5.0
4	GSK Linear		9.0	10.3	9.0	11.0	6.7	6.3
	GSK Log(p)		8.3	10.3	7.7	7.3	4.7	6.7
	GSK Log-odds		8.7	10.3	7.7	7.7	5.0	7.0
	ML Log-linear		6.0	7.3	4.3	8.3	4.0	5.3

138

Table M.3 Continued

Column Effect Size	Method	Sample Size:	Size: 2x4			Size: 4x4		
			250	500	1000	250	500	1000
5	GSK Linear		7.3	8.7	9.0	8.7	6.7	8.7
	GSK Log(p)		7.0	7.7	9.7	5.3	5.0	8.0
	GSK Log-odds		7.3	8.0	9.7	5.7	5.0	8.0
	ML Log-linear		4.3	5.3	7.3	5.7	3.7	6.3
6	GSK Linear		6.7	9.7	10.7	8.7	7.7	7.7
	GSK Log(p)		6.0	8.3	9.7	5.3	5.3	7.3
	GSK Log-odds		6.0	8.7	9.7	5.3	5.7	7.7
	ML Log-Linear		4.3	6.7	6.7	6.7	4.7	6.3
7	GSK Linear		5.0	8.0	8.3	8.7	8.7	7.0
	GSK Log(p)		4.3	7.3	7.7	5.0	6.0	7.0
	GSK Log-odds		4.3	7.7	7.3	5.0	6.0	6.7
	ML Log-Linear		3.3	6.0	3.7	5.3	5.7	5.0
8	GSK Linear		8.0	5.7	10.7	10.0	6.3	8.0
	GSK Log(p)		7.3	5.3	9.7	4.3	5.0	6.3
	GSK Log-odds		7.7	5.3	10.0	4.7	5.0	6.7
	ML Log-Linear		5.3	4.0	6.7	6.3	4.3	4.3
9	GSK Linear		8.7	9.7	9.7	6.3	7.0	8.7
	GSK Log(p)		8.0	7.3	7.3	4.7	5.7	6.3
	GSK Log-odds		8.0	7.7	7.0	4.7	5.7	6.0
	ML Log-Linear		5.3	5.7	5.0	5.0	4.3	5.0

APPENDIX N

PERCENTAGE OF SIGNIFICANT χ^2 INTERACTION VALUES FOR TABLES
WITH ROW AND COLUMN EFFECTS (ROW EFFECT SIZE 3)

Table N.1 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (3)], Two Column.

Column Effect Size	Method	Size: 2x2			Size: 4x2		
	Sample Size:	250	500	1000	250	500	1000
1	GSK Linear	9.0	11.7	10.7	5.3	6.7	8.0
	GSK Log(p)	8.3	10.7	10.7	5.0	6.7	7.3
	GSK Log-odds	9.0	11.7	10.3	4.7	6.7	7.3
	ML Log-linear	5.3	5.7	5.3	3.0	4.3	5.7
2	GSK Linear	8.0	6.7	8.0	11.3	7.7	10.3
	GSK Log(p)	9.3	6.3	7.7	11.3	7.3	10.3
	GSK Log-odds	9.3	7.0	7.3	11.3	7.3	10.3
	ML Log-linear	5.0	3.3	4.0	7.3	5.3	9.0
3	GSK Linear	10.7	8.3	10.0	8.3	8.0	7.3
	GSK Log(p)	10.7	7.7	8.3	6.7	8.0	6.7
	GSK Log-odds	10.3	8.0	8.7	6.7	8.0	7.0
	ML Log-linear	6.3	5.0	6.0	5.0	5.3	3.7
4	GSK Linear	7.0	6.7	11.0	7.3	7.3	8.0
	GSK Log(p)	6.0	6.7	9.7	7.7	6.7	8.0
	GSK Log-odds	6.7	6.0	10.0	7.3	7.0	7.3
	ML Log-linear	4.0	2.7	7.0	7.3	5.0	5.3

Table N.1 Continued

Column Effect Size	Method	Sample Size: Size: 2x2			Size: 4x2		
		250	500	1000	250	500	1000
5	GSK Linear	11.3	10.7	14.3	7.0	9.7	7.3
	GSK Log(p)	10.7	8.3	12.7	5.0	6.3	6.3
	GSK Log-odds	10.7	8.3	13.3	5.3	7.0	6.7
	ML Log-linear	6.0	4.3	9.0	3.0	4.7	6.0
6	GSK Linear	8.0	11.0	12.7	9.0	11.0	9.7
	GSK Log(p)	7.0	8.0	10.3	7.0	10.3	8.7
	GSK Log-odds	7.3	8.0	11.0	7.3	10.3	9.0
	ML Log-Linear	3.7	4.0	6.3	5.7	8.0	6.7
7	GSK Linear	12.3	9.7	9.7	8.0	9.3	9.7
	GSK Log(p)	10.0	9.7	8.7	6.3	7.3	7.7
	GSK Log-odds	11.0	8.7	9.0	6.0	7.7	7.3
	ML Log-Linear	5.3	3.0	4.3	6.0	5.7	5.0
8	GSK Linear	8.0	12.0	10.7	12.0	10.3	9.0
	GSK Log(p)	8.0	9.7	7.7	10.3	9.0	7.7
	GSK Log-odds	8.3	9.0	7.3	10.7	9.3	8.3
	ML Log-Linear	5.3	5.0	4.7	6.3	6.7	5.0
9	GSK Linear	12.7	15.0	13.3	11.7	11.7	12.7
	GSK Log(p)	9.3	9.3	11.3	8.0	6.7	7.3
	GSK Log-odds	11.0	11.0	11.3	8.3	7.0	7.7
	ML Log-Linear	6.7	6.3	6.7	5.3	6.3	5.3

Table N.2 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (3)], Three Column.

Column Effect Size	Method	Sample Size:	Size: 2x3			Size: 4x3	
		250	500	1000	250	500	1000
1	GSK Linear	8.3	8.0	9.0	10.3	5.7	8.7
	GSK Log(p)	5.7	7.3	9.0	6.0	4.7	7.7
	GSK Log-odds	6.3	7.7	9.0	6.7	4.7	7.7
	ML Log-linear	3.3	3.7	6.0	6.7	4.0	6.0
2	GSK Linear	9.0	11.0	9.0	10.0	8.3	7.0
	GSK Log(p)	8.0	11.0	8.7	8.0	7.3	6.3
	GSK Log-odds	8.3	10.7	9.0	8.0	7.3	6.3
	ML Log-linear	5.0	7.3	6.0	6.7	6.3	4.0
3	GSK Linear	7.7	9.3	9.0	8.7	7.7	6.7
	GSK Log(p)	7.7	10.0	8.3	7.0	5.7	6.0
	GSK Log-odds	7.7	9.7	8.7	7.7	6.0	6.3
	ML Log-linear	5.0	7.3	2.7	5.3	4.7	4.0
4	GSK Linear	8.0	5.0	7.7	11.7	6.7	6.0
	GSK Log(p)	7.3	6.0	6.3	8.0	6.3	4.7
	GSK Log-odds	7.7	6.3	6.3	8.3	6.3	4.7
	ML Log-linear	3.3	3.3	3.7	7.3	3.7	4.7

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143

Table N.2 Continued

Column Effect Size	Method	Size: 2x3			Size: 4x3		
	Sample Size:	250	500	1000	250	500	1000
5	GSK Linear	6.3	9.3	11.3	9.3	5.3	7.3
	GSK Log(p)	7.0	9.7	10.0	7.3	3.7	6.0
	GSK Log-odds	6.7	9.0	10.3	7.7	4.0	6.0
	ML Log-linear	4.0	4.3	6.7	7.3	2.7	4.7
6	GSK Linear	10.3	12.7	11.0	8.7	9.3	10.7
	GSK Log(p)	8.0	10.3	9.3	5.3	8.3	9.3
	GSK Log-odds	8.7	10.0	9.3	5.3	8.0	9.7
	ML Log-Linear	5.0	6.3	3.7	4.3	7.3	6.7
7	GSK Linear	5.0	6.7	9.7	6.7	7.0	9.3
	GSK Log(p)	5.3	6.3	7.7	4.0	4.7	7.7
	GSK Log-odds	5.7	6.7	7.7	4.0	4.7	7.7
	ML Log-Linear	3.3	3.3	4.0	3.3	4.0	5.3
8	GSK Linear	11.7	9.0	8.3	8.3	7.7	12.3
	GSK Log(p)	10.7	7.3	9.3	6.3	6.7	7.3
	GSK Log-odds	11.0	7.7	9.0	6.3	6.7	7.3
	ML Log-Linear	5.7	6.0	3.7	5.7	4.3	5.3
9	GSK Linear	7.0	8.0	12.3	9.0	9.7	10.7
	GSK Log(p)	6.7	6.7	8.0	7.0	7.0	7.7
	GSK Log-odds	6.7	6.3	7.7	7.0	7.7	7.3
	ML Log-Linear	4.3	3.3	5.0	7.0	6.0	5.7

Table N.3 Percentage of χ^2 Interaction Values Significant at .05 Level, Table Type: [Row Effect (3)], Four Column.

Column Effect Size	Method	Sample Size:	Size: 2x4			Size: 4x4		
		250	500	1000	250	500	1000
1	GSK Linear	5.3	6.7	8.0	8.7	8.3	6.0
	GSK Log(p)	5.7	6.7	7.7	5.3	6.3	6.0
	GSK Log-odds	5.7	6.3	7.3	5.7	6.3	6.0
	ML Log-linear	3.7	4.3	4.3	6.7	5.7	5.0
2	GSK Linear	6.7	7.7	10.7	7.0	8.3	9.0
	GSK Log(p)	6.0	8.0	9.0	4.3	5.3	8.0
	GSK Log-odds	6.3	8.0	9.0	5.0	5.3	8.0
	ML Log-linear	4.7	5.7	6.7	4.0	4.7	5.7
3	GSK Linear	8.7	5.3	8.7	7.7	6.7	8.0
	GSK Log(p)	8.0	5.0	9.0	4.0	5.7	6.3
	GSK Log-odds	8.3	5.3	8.7	4.7	5.7	6.0
	ML Log-linear	5.3	4.7	6.0	5.3	3.7	5.3
4	GSK Linear	9.7	6.7	8.0	7.3	10.7	8.0
	GSK Log(p)	7.3	5.7	6.3	4.3	8.7	6.3
	GSK Log-odds	8.0	6.0	6.0	4.7	8.3	6.7
	ML Log-linear	6.0	4.3	4.3	5.0	6.3	3.7

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145

Table N.3 Continued

Column Effect Size	Method	Size: 2x4			Size: 4x4		
	Sample Size:	250	500	1000	250	500	1000
5	GSK Linear	11.3	10.0	11.0	8.0	9.0	8.3
	GSK Log(p)	9.3	7.7	8.0	6.0	7.0	8.0
	GSK Log-odds	10.0	7.7	9.3	6.0	7.3	8.0
	ML Log-linear	7.3	5.3	3.7	6.0	6.7	6.3
6	GSK Linear	8.0	8.7	8.7	8.7	10.0	11.7
	GSK Log(p)	4.3	7.3	8.0	5.3	7.3	8.7
	GSK Log-odds	4.7	7.3	7.3	5.7	7.7	8.7
	ML Log-Linear	3.3	4.3	5.3	5.7	7.0	8.0
7	GSK Linear	7.0	9.0	9.7	11.7	7.7	8.7
	GSK Log(p)	5.0	5.7	6.7	6.0	6.7	6.0
	GSK Log-odds	6.0	6.3	7.3	7.0	6.7	6.3
	ML Log-Linear	2.7	3.7	5.0	7.3	4.3	3.7
8	GSK Linear	7.7	7.3	11.0	6.3	10.7	9.3
	GSK Log(p)	7.0	5.7	7.7	4.3	6.0	6.3
	GSK Log-odds	8.0	5.0	8.3	4.7	6.0	6.3
	ML Log-Linear	6.0	2.7	4.3	4.7	4.7	5.3
9	GSK Linear	9.7	10.3	12.7	8.0	11.7	14.0
	GSK Log(p)	6.0	7.3	5.7	5.3	6.7	5.7
	GSK Log-odds	6.7	8.0	5.7	5.3	6.3	6.0
	ML Log-Linear	5.0	5.3	4.0	5.0	5.3	4.7

APPENDIX O

FIGURES SHOWING PERCENTAGE OF SIGNIFICANT χ^2 INTERACTION
VALUES FOR TABLES WITH ROW AND COLUMN EFFECTS
(ROW EFFECT SIZE 2)
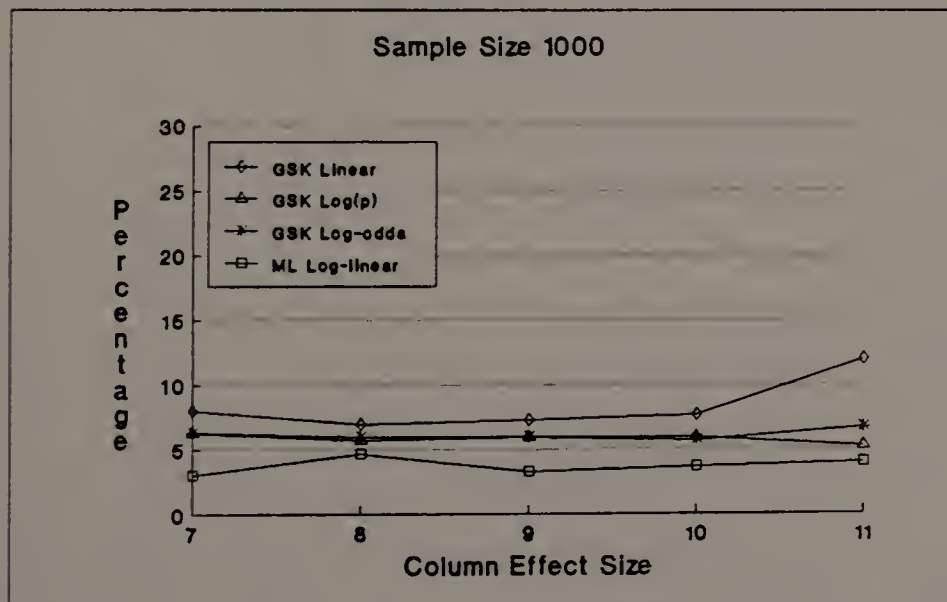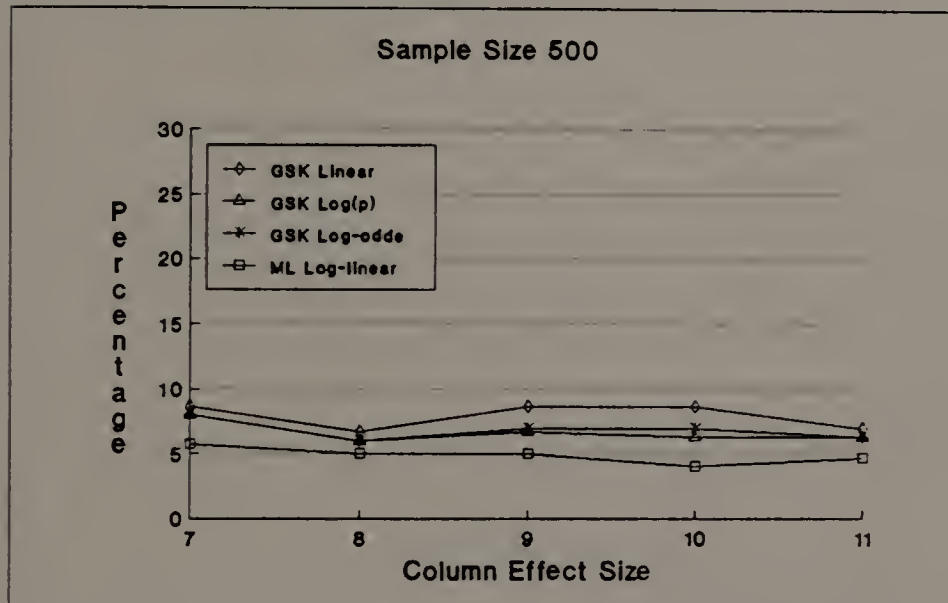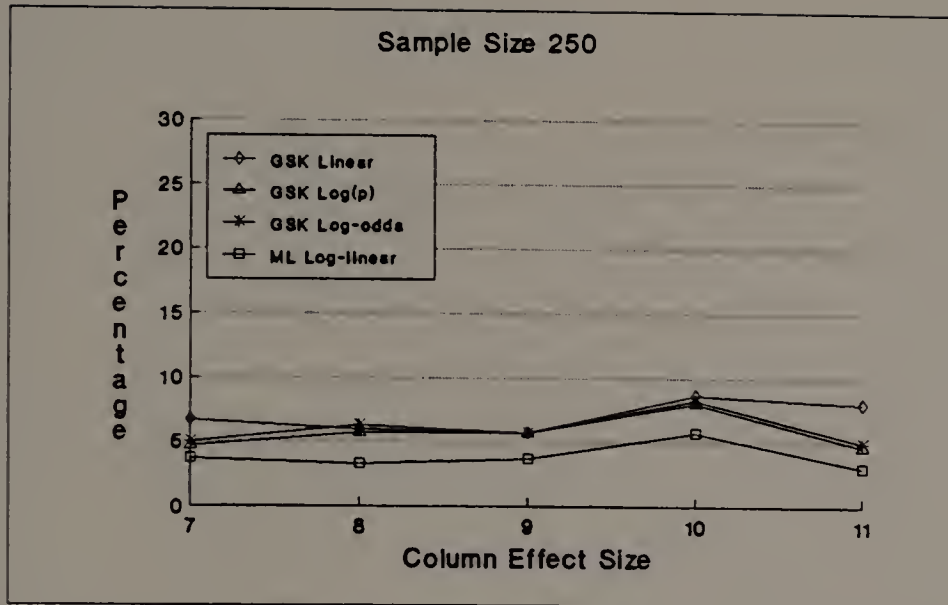
Figure O.1 Percentage of Significant Chi-Square Interaction Values,
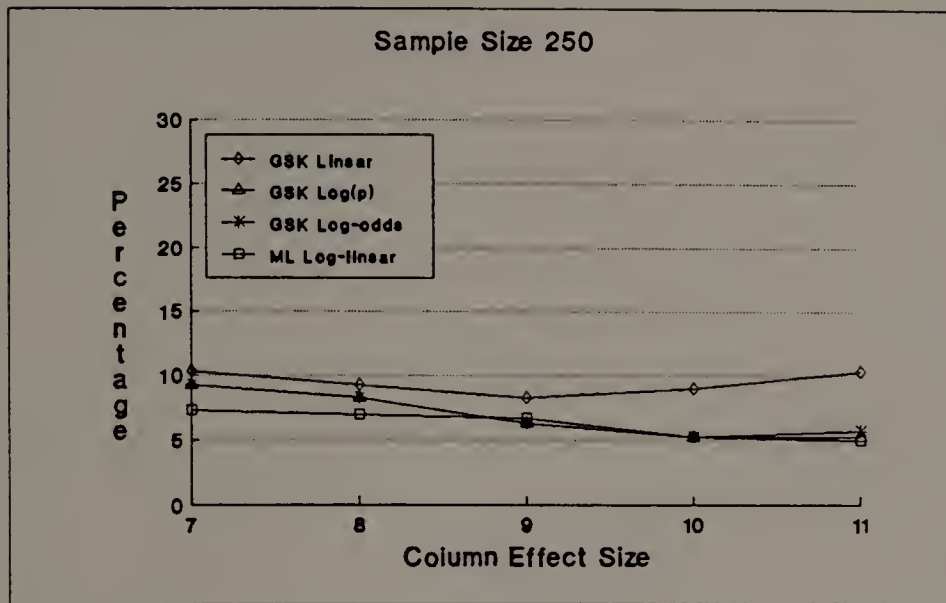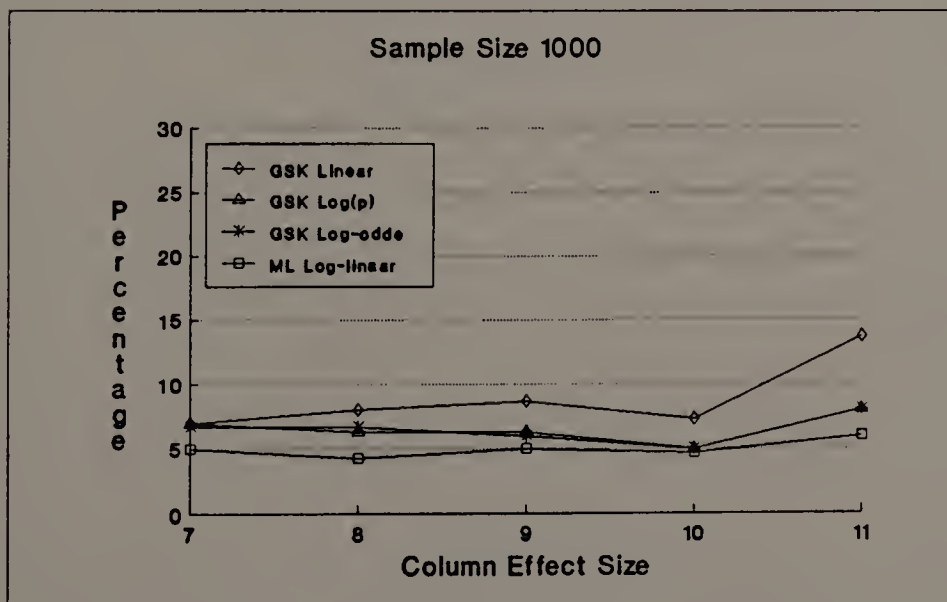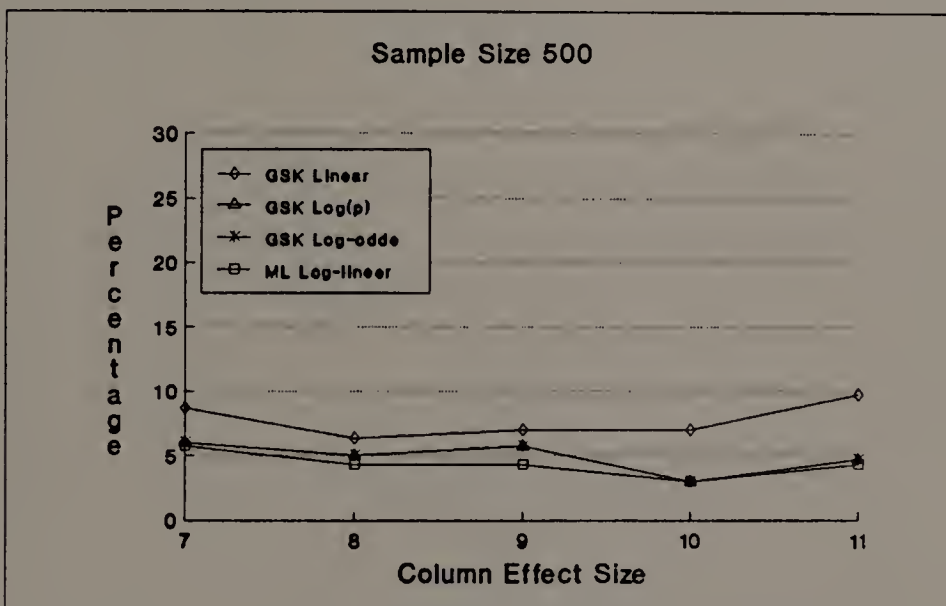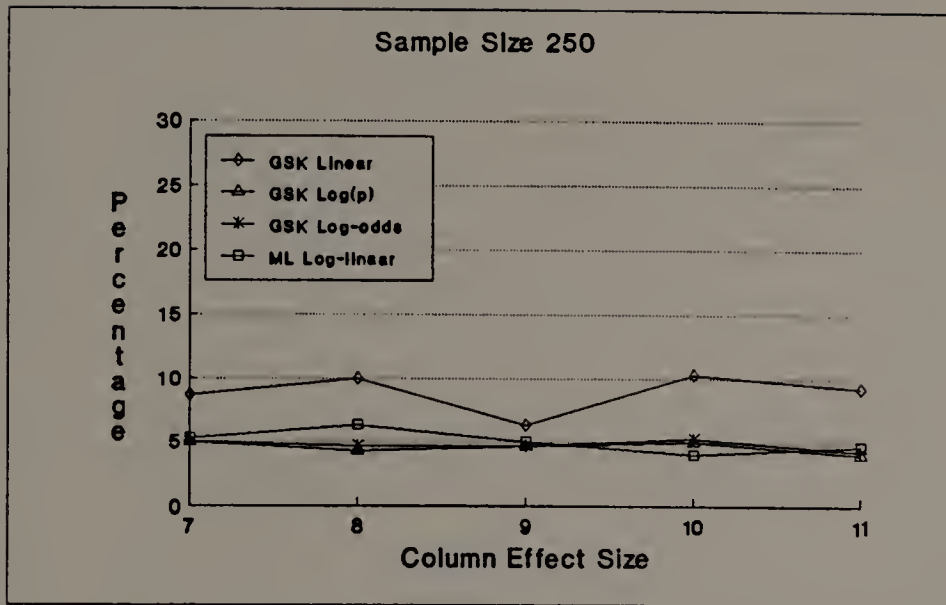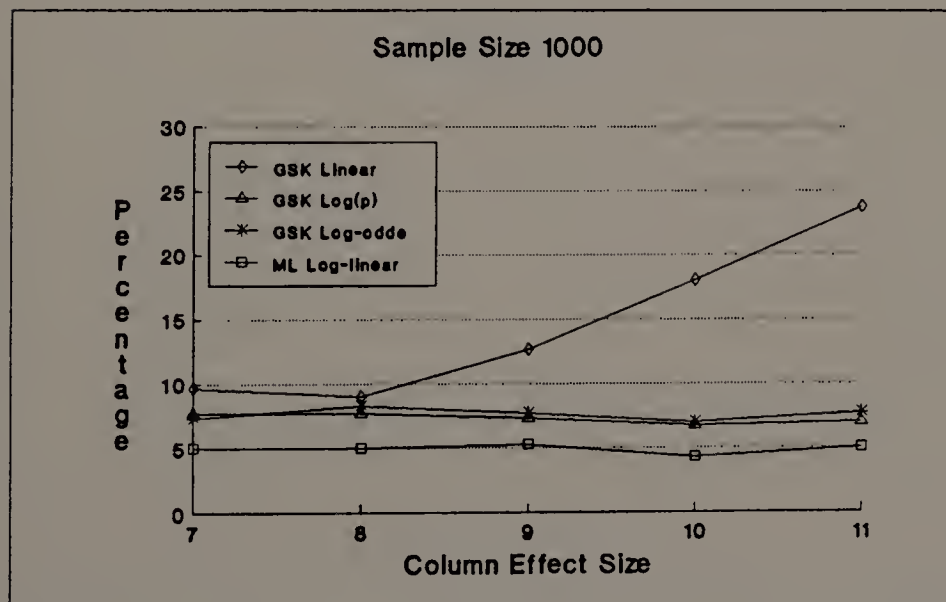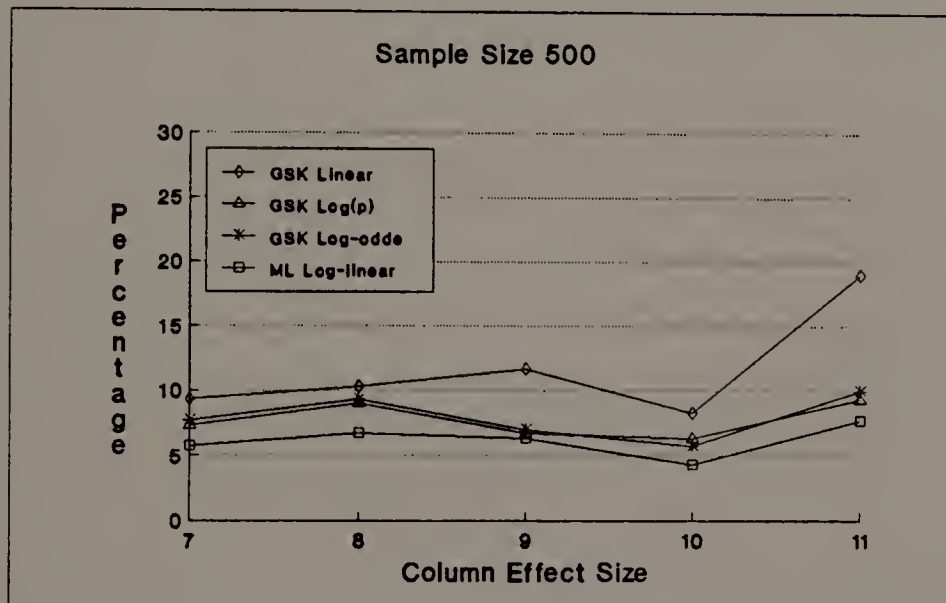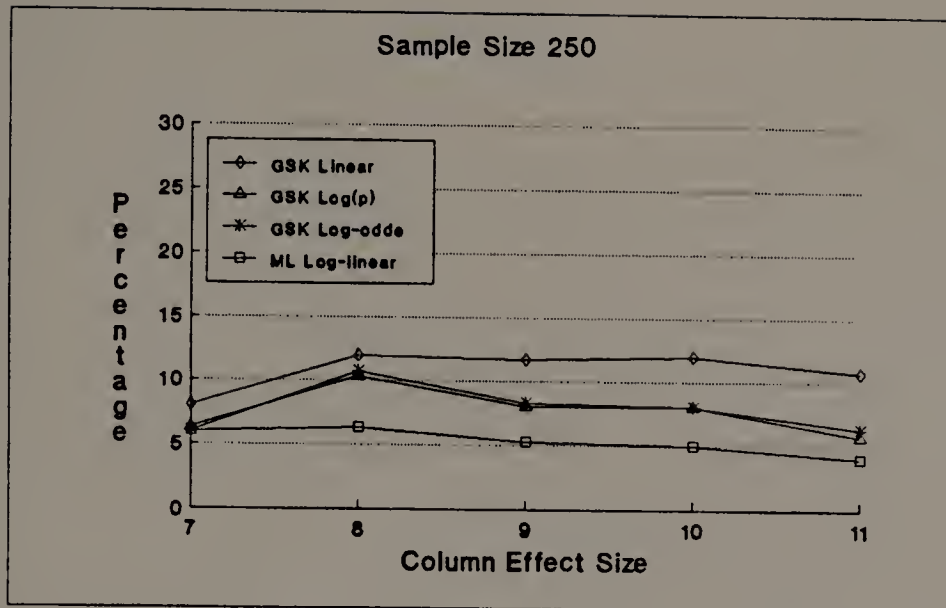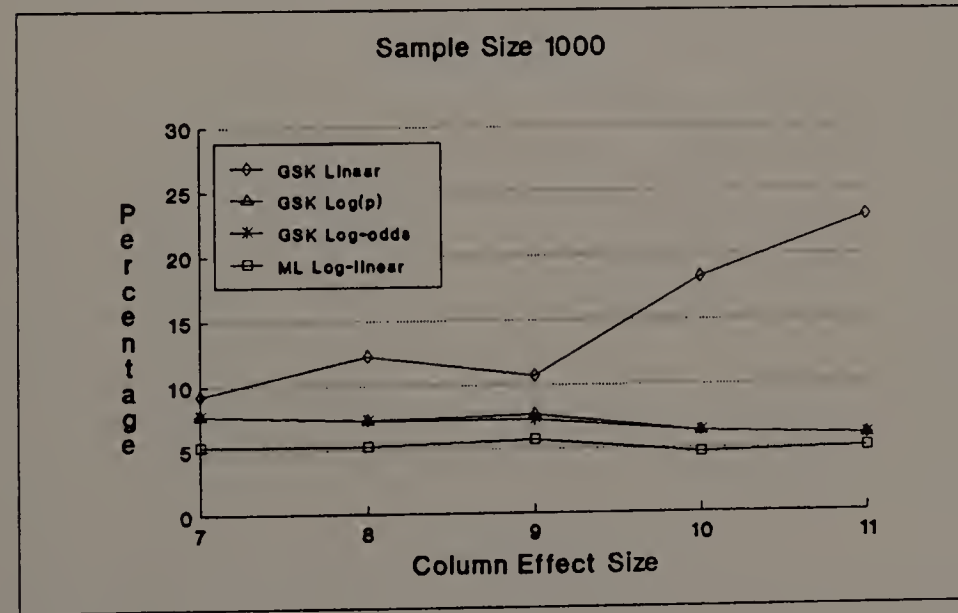Table Type: [Row Effect (2)], 4x2.
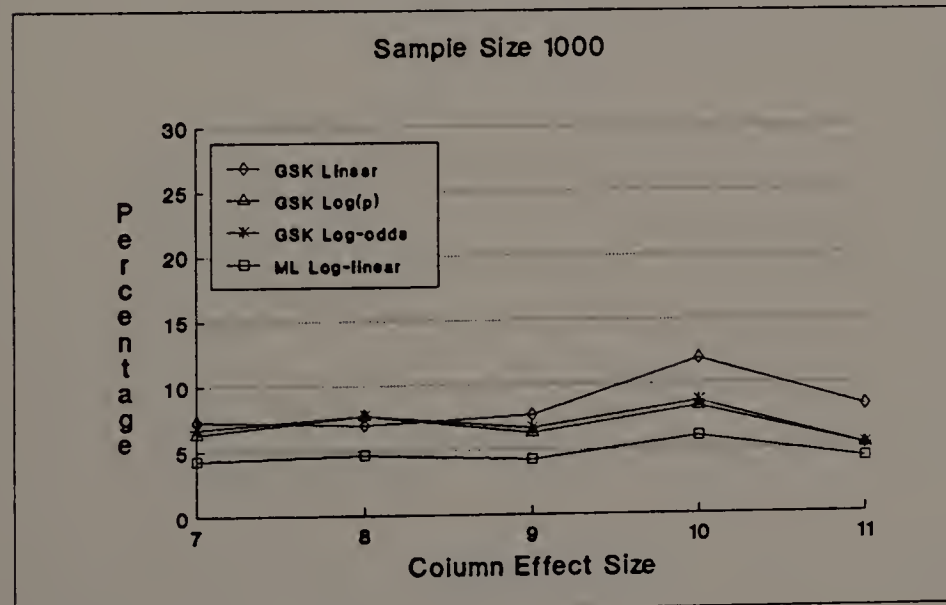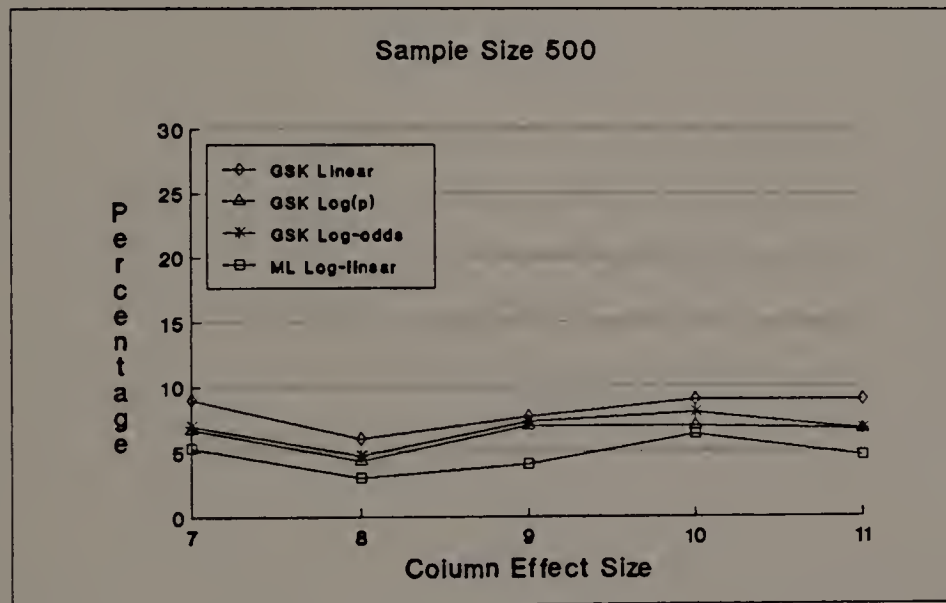
Figure O.2 Percentage of Significant Chi-Square Interaction Values,
Table Type: [Row Effect (2)], 4x3.

Figure O.3 Percentage of Significant Chi-Square Interaction Values,
Table Type: [Row Effect (3)], 4x4.

APPENDIX P

FIGURES SHOWING PERCENTAGE OF SIGNIFICANT χ^2 INTERACTION
VALUES FOR TABLES WITH ROW AND COLUMN EFFECTS
(ROW EFFECT SIZE 3)

Figure P.1 Percentage of Significant Chi-Square Interaction Values,
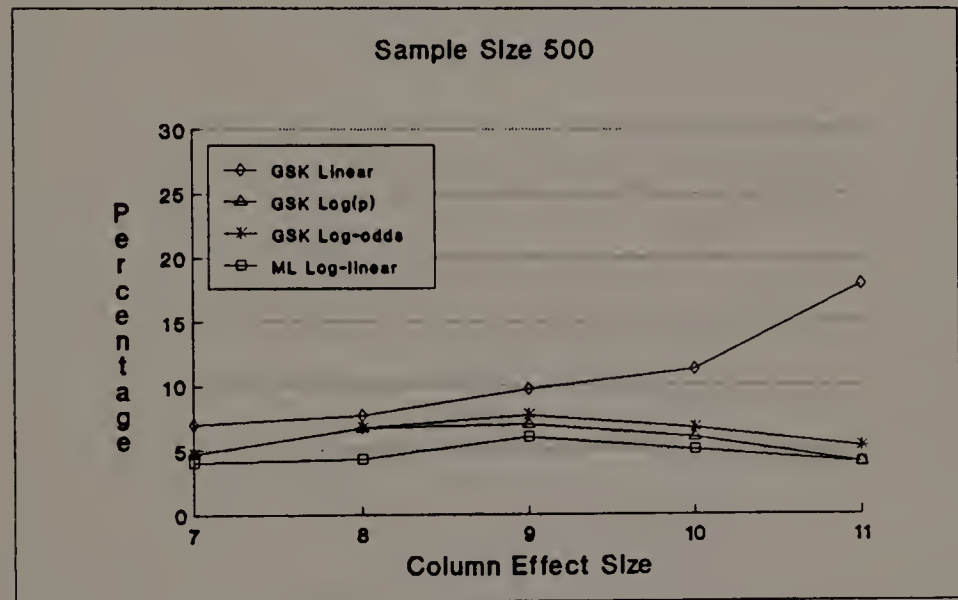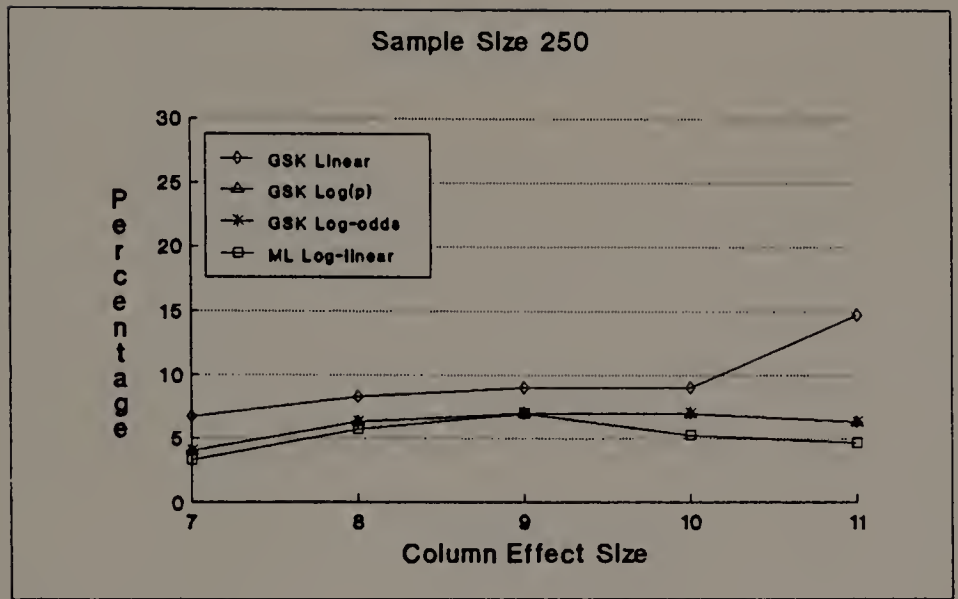Table Type: [Row Effect (3)], 4x2.

Figure P.2 Percentage of Significant Chi-Square Interaction Values,
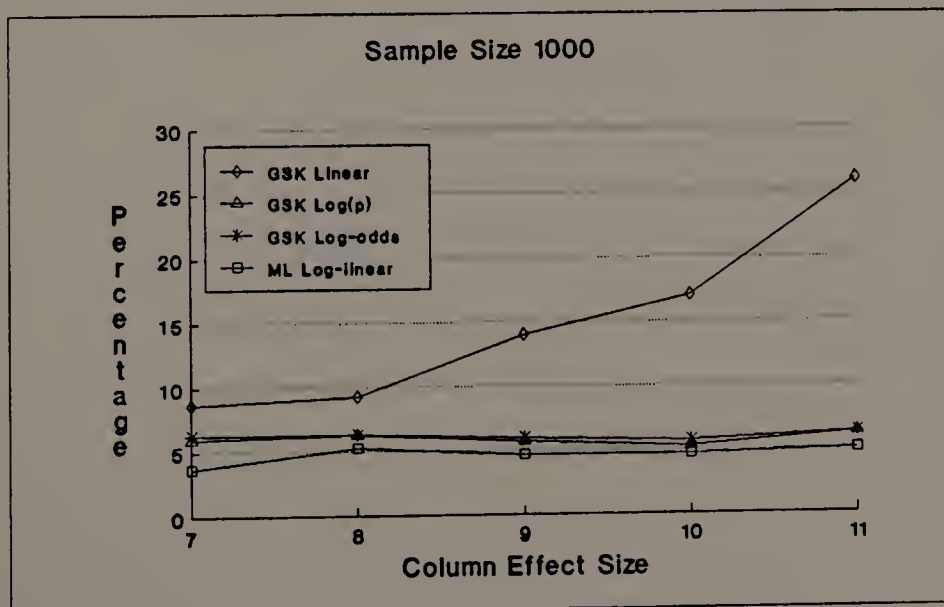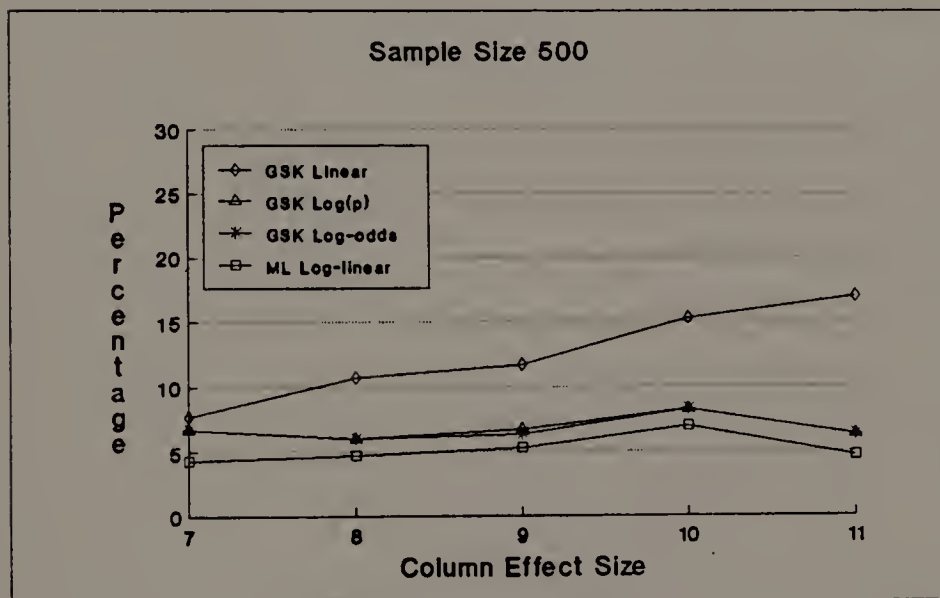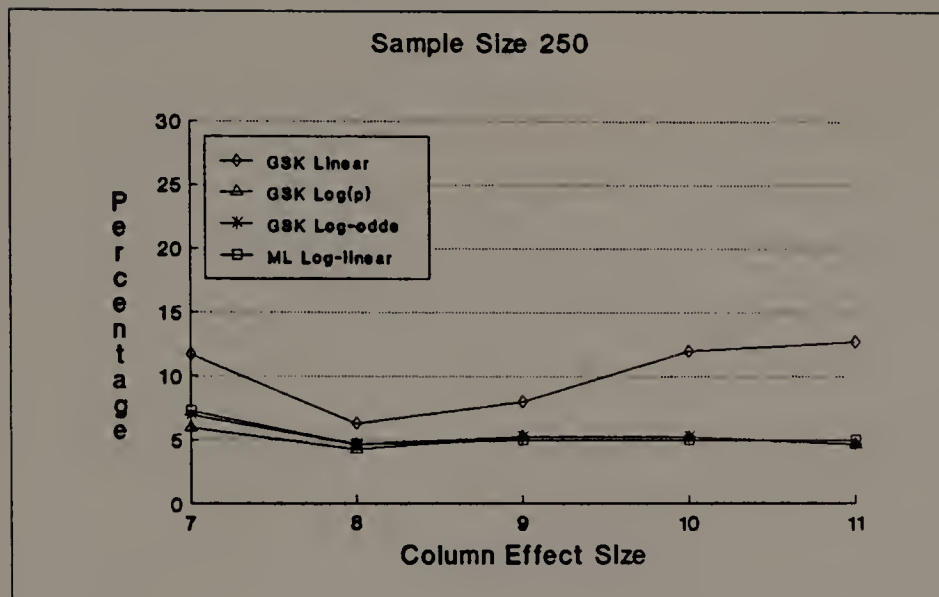Table Type: [Row Effect (3)], 4x3.

153

Figure P.3 Percentage of Significant Chi-Square Interaction Values,
        Table Type: [Row Effect (3)], 4x4.

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