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The development and field testing of geometry units using a learning cycle approach and ten question guide as a framework for lesson design and classroom methodology.

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THE DEVELOPMENT AND FIELD TESTING OF GEOMETRY UNITS USING A LEARNING CYCLE APPROACH AND TEN QUESTION GUIDE AS A FRAMEWORK FOR LESSON DESIGN AND CLASSROOM METHODOLOGY

A Dissertation Presented

by

VIRGINIA M. BASTABLE

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ABSTRACT

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Geometry units for a secondary school program were created and field tested. The lessons were developed according to a ten question format for lesson design that insured the materials would be at all levels of geometric reasoning, were concrete as well as abstract, and included the use of the computer as a tool for learning mathematics.

These units were taught using a methodology which incorporated the investigator's learning cycle approach. This teaching method starts with the learner's intuitive understandings and proceeds through levels of exploration and deduction until the learner has constructed a new belief.

The purpose of this study was to create geometry units and to implement a teaching methodology which would integrate a problem solving approach with the principles of Piaget and the van Hieles. This study described the development of the lessons, the implementation of this methodology, the day to day impact of this teaching style on the students, and provided a comparison of male and female student opinions regarding these materials.

This approach was field tested at a public secondary school. Data were gathered to determine the students' views toward learning in four formats which were embedded in the materials and methodology: working in groups, using computers, using writing, and using manipulatives. In addition, student reaction to differing teacher roles, facilitator and explainer, was studied. Male and female students were compared to determine if the results of this teaching style were constant or if they varied with gender.

The indications from this work are that the question guide and learning cycle were powerful constructs for devising, planning and implementing lessons in geometry. The field testing, student evaluation forms, and summative evaluation forms indicated that some components of this teaching style were considered positively: use of groups, use of computers, and differing teacher roles. The use of manipulatives was received with mixed feelings. The use of writing was considered a negative feature in this study. Three of these strategies, use of group work, use of writing, and teacher as facilitator showed no gender related differences. Use of computers and use of manipulatives indicated a male preference. Overall student views to the teaching methods were positive.

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CHAPTER I

INTRODUCTION

Introduction to the Problem

"Lessons should be designed around problem situations posed in an environment that encourages students to explore, to formulate and test conjectures, to prove generalizations, and to communicate and apply the results of their investigations." This statement from the National Council of Teachers of Mathematics' recently published document, Curriculum and Evaluation Standards for School Mathematics (Romberg et al., 1987, p. 90), sets the tone for this study.

Geometry units for a secondary school program were created and field tested. The lessons were developed according to a ten question format for lesson design that insured the materials would be at all levels of geometric reasoning as determined by the van Hieles, were concrete as well as abstract, and included the use of the computer and calculator as tools for learning mathematics.

These units were taught using a methodology which incorporated the investigator's learning cycle approach. This teaching method starts with the learner's intuitive understandings and proceeds through levels of exploration and deduction until the learner has constructed a new belief.

The intent of the study was to produce curriculum units of geometry in keeping with the principles of the van Hieles and Piaget. The teaching methodology incorporated a problem solving approach. The study includes the development of the lessons, the implementation of this methodology, the day to day impact of this teaching style on the students, and a comparison of male and female student opinions regarding these materials.

This approach was field tested at a public secondary school. The majority of the students using these materials were in homogeneously grouped classes which represented the three levels of achievement recognized by the school district. In addition, one class included students who had been designated as needing remedial help in mathematics. This class contained students at all grouping levels.

Data were gathered to determine the students' views toward learning in four formats which were embedded in the materials and methodology: working in groups, using computers, using writing, and using manipulatives. In addition, student reaction to differing teacher roles, facilitator and explainer, was studied. Male and female students were compared to determine if the results of this teaching style were constant or if they varied with gender.

Background of the Problem

"Almost all writings on school geometry are derived from two major problems: the poor performance of students and an outdated curriculum" (Usiskin, 1987, p. 17). The lack of success that students display is a continuing problem. The results of the "Fourth National Assessment of Educational Progress" indicate poor student performance on geometry items. For example, 45% of eleventh grade students taking the test could not find the area of a square when given the length of one side. These results are not significantly different from previous assessments (Brown, et al., 1988).

The issues in geometry curriculum are made even more complicated by the fact that there is no consistent opinion on what geometry is or what approach should be taken toward it. In 1969 Allendorfer identified three styles of geometry: synthetic, analytic, and vector. In the 1973 Yearbook of the National Council of Teachers of Mathematics, published articles represented six different approaches to geometry: conventional, affine, transformational, coordinate, vector, and eclectic.

The curricular confusion continues unresolved in spite of the efforts of committees that were charged with formulating recommendations for mathematics curriculum. In NCTM's An Agenda for Action (1980) geometry is discussed in a single paragraph. In School Mathematics: Options for the 1990's, the only recommendation for the geometry course was that "the topics should be unified and integrated so that the interrelationships of algebra, geometry, and applications are made" (Romberg, 1984, p. 12).

Two reports do address the issue of geometry curriculum in a new light. The 1983 statement from the Conference Board of the Mathematical Sciences contains a paragraph which argues for geometry students spending less time on writing formal two column proof and more time studying algebraic methods in geometry, three dimensional relationships, and using computer graphics packages to get a visual sense of geometric concepts (CBMS, 1983).

In NCTM's Curriculum and Evaluation Standards for School Mathematics (Romberg et al., 1987) recommendations to restructure the geometry curriculum are made. These suggestions include: incorporating algebraic methods in geometry such as transformations and coordinates; requiring that students represent problem situations with geometric models; deducing through short sequences of logical relationships between figures; using computer programs which will allow students to create and manipulate two and three dimensional objects; and designing classes so that students form connections between the geometry studied and their real world experiences.

While it is true that the CBMS and NCTM reports do treat the issue of geometry curriculum more extensively than did the reports cited earlier, what is more important to notice is the tone of these reports. Both of these statements do more than discuss what content is being covered. They include discussions of what the students are doing in class and descibe how the students are learning geometry. These reports do not contain merely a listing of curriculum topics but carry implicit and explicit messages about the teaching practices that are needed.

This new awareness of incorporating the learner into the process demands some background from educational theorists. Piaget and the van Hieles have each made significant contributions to educational theory that should be reflected in secondary school mathematics education practice. The implications of the work of these educational theorists for classroom instruction is now addressed.

In his work Piaget described developmental levels of reasoning through which children progress. These levels are summarized as reported in the work of Hedden (1984):

Level 1 <u>Sensory-Motor Knowledge</u> (ages 0-2) The child at this level displays a knowledge of objects as they exist in space and time. The mental constructs are preverbal and presymbolic.

Level 2 <u>Preoperational</u> (ages 2-6) During this level language is acquired. The child can understand signs and symbols as representations of the real world. The child can distinguish between the reality and the symbols, however he/she cannot operate on those mental symbols.

Level 3 <u>Concrete Operational</u> (ages 7-12?) The child at this level can reason about the world of objects. The child can appreciate relations between real objects.

Level 4 <u>Formal Operational</u> (ages 13?- on) The child at this level can reason in the manner of a scientist, can use propositional logic, can reason on abstract as well as concrete objects.

The debate over the specific ages for each level and the search for the mechanism that causes change continues, but there is little debate over the fact that Piaget's conception has formed the foundation on which much of mathematics education has been built. In particular, the explicit construct that children are consciously trying to make sense of the world they experience is central and critical to mathematics learning.

In order to succeed in a secondary school geometry class, students must be able to hypothesize, reason deductively, understand the role of mathematical models, and understand the difference between defining and deducing

(Farrell and Farmer, 1979). All of these abilities are characteristic of Piaget's formal operational stage (Farrell, 1987). But it has been shown that many students in geometry classrooms are not at this Piagetian level. For example, Renner (1977) reports that students he studied in grades ten through twelve exhibited concrete thought 57% of the time.

Another study indicates that at least 30% of students in geometry class reason at the concrete operational level with another 30% to 40% being labeled as transitional reasoners, sometimes reasoning concretely, other times displaying formal reasoning abilities. This leaves only 30% at the formal stage (McDonald, 1982; Farrell and Farmer, 1985).

Such a high percentage of non-formal reasoners indicates that teaching styles must take into account the needs of the concrete learner. "The presence of concrete reasoning means that actual experience with those concepts that are to be learned is the only way understanding develops" (Renner and Marek, 1988, p. 22).

These students who are concrete operational need opportunities to learn geometric concepts through manipulation of concrete objects, not mental abstractions. The value of manipulatives in elementary and middle schools has been established. In her review of the research on the use of manipulatives, Suydam (1984) states that lessons which incorporate the use of materials to manipulate have a higher probability of producing greater learning than lessons which do not. She also notes that the achievement-enhancing effect of manipulatives occurs at every grade level, with a variety of mathematical topics, and with every ability level of student.

Traditional formal geometry instruction has not provided this opportunity. In spite of research evidence indicating that gains are made by using manipulative materials, the tendency in elementary schools is for that use to taper off as the students proceed to higher level elementary grades (Scott, 1983). In secondary school geometry courses teachers rely heavily on the text book (Brown, 1974) and spend most the class time, 80%, talking at the board (Beaulieu, 1979). This would indicate that students in geometry classes at this level spend little time manipulating objects in order to learn geometry.

Dina and Pierre van Hiele have studied geometry learning extensively and have proposed a model of the developmental levels involved in learning this subject matter. In the description below each of the five levels is illustrated by a response typical of a student reasoning at that level.

Visualization (level 0) in which the learner reasons from a whole visual image. For example, a rectangle is something that looks like a door.

Analysis (level 1) in which the learner lists properties of a figure. A rectangle has two pairs of equal sides, four right angles, etc.

Abstraction (level 2) in which the learner uses classification schemes. A rectangle is a special type of parallelogram.

Formal Deduction (level 3) in which the learner relies on deductive proof. Prove all squares are rectangles.

Rigor (level 4) in which the learner can consider alternate axioms. Define a rectangle on the surface of a sphere.

The van Hiele model is still developing with some points open to debate. However, it provides a framework for consideration to classroom teachers (Schoenfeld, 1986). This model states that a qualitatively different kind of reasoning is displayed at each level. Therefore full conceptual understanding develops over time through a variety of reasoning styles. The van Hiele model states that students who are not yet capable of formal reasoning which occurs at level three are in fact reasoning but in a different mode.

It is important to note that the "informal deduction" characteristic of a learner operating at level 2 of the van Hiele model is reasoning. This kind of thinking should not be regarded as the naive work of an unsophisticated student, but rather as a different mode of reasoning. Erich Wittman (1981) uses the phrase "intuitive activities" to describe this kind of logic. Within informal mathematics intuitive activities are a natural mode of mathematical thinking. They must not be understood as a concession to the students not yet mature for proper mathematics.

How well does this model of geometry learning fit with the reality of the classroom? One study listed the following results (Burger and Shaugnessy, 1986):

1. The van Hiele levels were good descriptors of students' reasoning processes.
2. No secondary school student reasoned consistently at level 3.
3. There is often a mismatch between the level of the teacher's work and the level of student thinking.
4. Students may actually regress to a lower van Hiele level after taking a full year course in geometry.

Taken together these conclusions indicate the problem. Geometry students are operating on van Hiele levels 0, 1 and possibly 2, but the materials and teaching approach in traditional geometry are at level 3 or 4.

These results have not gone unnoticed. The Oregon Mathematics Educational Council has prepared concept papers describing a geometry course which has been created in response to some of these concerns. The suggested course is designed to integrate informal geometry and proof at the high school level. "The emphasis of this course is in a visual approach to problem solving in geometry" (Morgan, 1986, p. 104). The Oregon project is an attempt to provide materials which will connect the students' intuition with the formal geometry study.

In the traditional geometry course, significant time is spent teaching students how to write proofs. Yet one study showed that only 31% of students in 85 classes were judged as being competent in this area of the curriculum (Senk, 1985). Schoenfeld noted that even students who were competent at doing proofs could not use that knowledge in a problem solving situation (Schoenfeld, in press).

What kind of teaching does impart the value and meaning of proof? According to Alan Bell,

> ... pupils will not use formal proof with appreciation of its purpose until they are aware of the public status of knowledge and the value of public verification. The most potent accelerator towards achievement of this is likely to be cooperative research-type activity by the class. In this, investigation of a situation would lead to different conjectures by different pupils, and the resolution of conflicts by arguments and evidence. (1976, p. 25)

This summary of geometry curriculum issues indicates the scope of the problem. Student performance in the content of geometry is poor. Student understanding of the process, value, and meaning of proof is poor. As a partial explanation for this poor performance and as a partial formulation of a solution to this problem, note that the work of Piaget and the van Hieles illustrates a gap between the content and practices of the traditional course and the cognitive structures of the students in those courses.

Statement of the Problem Situation

The work of Piaget and the van Hieles provides the framework upon which to consider classroom practices. Both models of understanding indicate the need for students to be actively involved in the classroom, manipulating objects and formulating conjectures as is appropriate to each student's level of reasoning.

Yet the findings of the "Fourth National Assessment of Educational Progress" illustrate that "typical mathematics instruction consists of listening to teacher explanations, watching the teacher work problems at the blackboard, using a mathematics textbook, and working alone to solve problems on worksheets" (Silver et al., 1988, p. 725).

A gap exists between the current realities of classroom instruction and the implementation of the theories of Piaget and the van Hieles which conceptually designed curriculum materials can help bridge. But curriculum design is only part of the answer. The need for an interactive teaching methodology is also necessary.

Geometry instruction based on the perspectives offered by Piaget, the van Hieles and Bell requires materials which are designed with these principles and must be implemented with a teaching style which encourages student exploration and dialogue.

Curriculum design cannot be separated from instructional methods. The content and the process are interwoven. "Curriculum has become less about merely the classroom realization of syllabuses and teaching materials, having more to do with pupil's learning as a complex interaction between teacher and pupil, materials and experiences, schools and society, time, place, and intent" (Tripp, 1986, p. 3).

So it becomes clear that curriculum materials and an instructional process for their implementation which are based on the principles of learning stated above are required. During the course of this study, curriculum content was developed, linked to methodology and field tested.

Purpose of the Study

The purpose of this study was to create geometry units and to implement a teaching methodology which would integrate a problem solving approach with the principles of Piaget and the van Hieles. Materials were based on the following premises:

One: Students need exploratory concrete activities to provide them with information before formal work on the concepts can begin.

Two: Geometry study should contain a variety of types of exercises so that students at every van Hiele level have an opportunity to reason about the concepts.

Three: Students must understand proof as a way of communicating not only what they know is true but also how they came to know it.

Developing lessons in geometry which are conceptually based was therefore the main objective of this work. The investigator developed a question guide as a format for designing such lessons. This question guide is embodied within the learning cycle approach recommended by the investigator as a teaching methodology.

Definition of Terms

The Geometric Supposer(s): The term The Geometric Supposer(s) is used to refer to a set of computer software discs titled: Pre-Supposer, Triangles, Quadrilaterals, and Circles. This software has been created and developed by Dr. Judah L. Schwartz and Dr. Michal Yerushalmy and is marketed by Sunburst Inc. The software allows the user to construct geometric objects and measure various attributes such as length, area, angle size, and perimeter. It can be used in the classroom as a demonstrator or as part of an interactive process where students can gather inductive evidence to form and test conjectures.

Grouping (heterogeneous/homogeneous): All geometry students at the school where this study was conducted are identified as being in the basic, the standard or the advanced level. This categorization is an achievement based system with original recommendations made by the students' previous mathematics teacher. Parents have the right to disagree and may override a teacher's recommendation.

Most classes in the mathematics department are singly grouped, that is each class contains students who have received the same designation. The term homogeneous will be used to refer to these classes.

One class in the study contained students from all three levels. This class which was best described as a mixed class will be referred to as being heterogeneous for the purposes of this study.

Learning Cycle Approach: The teaching methodology used during this study was based on a conception of student learning which asserted that students gain understanding by encountering concepts in a cyclical fashion. This learning cycle contains four stages: intuition, exploration, formalization, and again intuition.

At the first level of the cycle, intuition, the teacher's objective is to create exercises which allow the learner to verbalize their current understanding or belief about the concept. During the second stage of the cycle, exploration, the learner "plays with" the idea in various formats. The third stage is deduction during which the learner formalizes their understanding of the geometric concept. The final stage is again labeled intuition because it now describes the new belief held by the learner.

This completes one cycle of learning. The learner has moved from pre-existing belief through exploration and deduction to a new belief. Now once again the learner is at stage 1 with their current level of belief about the concept. New periods of exploration and deduction would lead to more sophisticated levels of understanding.

Although the learning cycle can be considered a four stage process, it does not fit a linear pattern. Each stage 4 returns the learner to a new level of stage 1. The appropriate visual image would be a spiral, every level 4 spiralling into a new level 1.

<u>Question Guide</u>: At each stage of the cycle specific questions are the appropriate tools for focusing learning.

The ten questions which provide the guidelines for lesson design and their relationship within the learning cycle are listed below.

Each question is identified with a particular mode or set of materials which are suggested as the vehicle for student reasoning within this level of the cycle.

Stage 1
Questions which engage the intuition:

1.	What is suggested?	intuition/quick sketch
2.	What is apparent?	scale diagram/graph paper

Stage 2
Questions which provide a forum for exploration:

3.	What can be constructed?	physical model
4.	What can be calculated?	arithmetic/calculators
5.	What can be expressed?	algebra
6.	What can be explored?	computer analysis
7.	What can be changed?	sequenced drawing/models

Stage 3
Questions which compel formal thinking:

8.	What can be deduced?	formal reasoning

Stage 4
Questions which engage the intuition:

9.	How can this be employed?	problem solving
10.	What does this mean?	belief

Materials developed using this question guide present each concept through a variety of exercises using concrete models, calculators, computers, and problem solving techniques. Therefore students at all Piaget levels, students reasoning at all van Hiele levels and students at every level of understanding about problem solving and proof find geometry accessible to them.

Rationale

Current cognitive science research has added significantly to our understanding of how mathematics skills and concepts are acquired. However this work has been carried out in very structured environments and sheds little light on the learning that occurs in the classroom. Classroom educational research has focused on topics such as time on task, wait time, and teacher differentiated behavior with little regard to the content being taught.

"Research is needed that blends the strengths of current cognitive research with a concern for the realities of the classroom and focuses on students' learning from instruction over extended periods of time" (Romberg and Carpenter, 1986, p. 868).

The present study moved toward that end. Geometry units were created and instruction given according to the learning cycle. The impact of these materials and this teaching approach was documented and described. Revision of the materials was undertaken according to these results. The revised materials with teaching suggestions were the final outcome of this study.

The intent of the study was to document both teacher views and student views throughout this process. Static models for research were not sufficient to describe these changes. "Dynamic models are needed that capture the ways meanings are constructed in classroom settings on specific mathematical topics" (Romberg and Carpenter, 1986, p. 868). The teacher as researcher was an integral part of this work since the teacher had the final responsibility in creating, implementing, and revising methodology and curricula. Toward this end the teacher/researcher kept a journal and the students filled out periodic evaluation forms as a way to assess day to day activities.

Questions to be Answered

Implementation of the suggested teaching methodology involved many aspects. The integration of the van Hiele theory, the work of Piaget and the problem solving approach recommended by Bell within the traditional geometry course was the key feature of this dissertation. This by its very nature was a very expansive and consuming project. The intent was to create units of geometry study which incorporated all aspects of mathematics and reached a variety of learning styles, making geometry study sensible and meaningful to students of all levels.

Within the methodology recommended by the learning cycle approach there lay several specific aspects of teaching style upon which to base research questions. The suggested classroom style involved five aspects which are generally new to students in secondary school mathematics: group work, computer use, use of writing, the use of manipulative materials and the role of the teacher as a facilitator not as a fact giver.

In this study the materials and the teaching style were field tested by students in all three grouping levels recognized by the school system. The opinions of both male and female students toward specific components of those materials was determined and analyzed.

Field testing the materials written for this study and implementing the suggested teaching methodology provided the investigator with information concerning the effects of this conceptually based geometry program. What is the effect of this style of teaching on male and female students? Do student views towards components of this teaching style change over the course of the project? Students in the various grouping levels and of both genders were studied in order to determine if the impact of this style of teaching is the same for these groups or if it varies.

Information was gathered to help answer the following questions:

1. Do students identify group work as a positive, neutral, or negative influence on their learning of geometry?

2. Do students identify the use of the computer software, the Supposers and LOGO, as a positive, neutral, or negative influence on their learning of geometry?

3. Do students identify the use of writing as a positive, neutral, or negative influence on their learning of geometry?

4. Do students identify the use of manipulating actual objects as a positive, neutral, or negative influence on their learning of geometry?

5. Do students note the role of the teacher as a facilitator not as a giver of fact as a positive, negative, or neutral influence on their learning of geometry?

Data were gathered on these questions before and after the field testing experience and also analyzed according to the gender of the students.

Scope and Delimitation of the Study

Since the study was small in scale, it did not resolve these issues for the general education audience. This study was limited in scope. It involved just one teacher, who was also the investigator, and a set of four classes during the fall semester in one school. Therefore the results were not generalizable beyond that population.

Importance of the Study

This study was a beginning step toward developing a mathematics curriculum which is conceptually based. A set of geometry concepts were considered. The materials to teach these concepts were developed and a methodology for presenting the concepts were discussed and implemented. The effect of this conceptually based learning on different students was examined to determine trends in the answers to these questions:

1. Is this style more appropriate to certain levels of students than others?
2. Are components of this style more accepted by female or male students?

The sharing of this information with colleagues will provide a basis for discussion concerning the geometry program in secondary school mathematics.

As valuable as initiating such discussion, the study also highlights the importance of the role played by the teacher/investigator. Blending the roles of researcher and teacher into one allows the teacher/researcher to bring into the classroom situation the formal work of the researcher and also to integrate that work with the practical realities of daily classroom life. Cameron-Jones reports that Whitehead (1982) spoke of the

> ... educational potential of individual teachers' motivations for engaging in the improvement of their professional practice. Whitehead saw these motivations to action as stemming, for all thoughtful and self-critical educators, from their responses to continued discrepancies between their espoused principles and their habitual practices. (1985, p. 4)

By serving as both teacher and researcher the teacher has an opportunity to analyze his/her own daily work in terms of his/her own ideals.

A final aspect of the importance of this study to the school community is that the teacher/researcher provides an example to professional educators of how a fresh attitude toward educational research can be of value to the practitioner and also promotes a better understanding of the work of educational researchers in general.

Outline of the Remainder of the Dissertation

The second chapter of this study will include three sections. One will be a review of the research on mathematics learning in general and geometry learning in particular. The second will contain a review of work done on the role of teacher as researcher. The third section of Chapter Two will report on gender issues in the learning of mathematics as related to the following components of learning: use of writing, use of computers, use of manipulative materials, use of small group work, and the role of the teacher.

Chapter Three will describe the procedures involved in the study including the field testing. Chapter Four will contain the lessons which were created, a summary of the teaching process used, and the results of the surveys and tests given. Chapter Five will interpret the data and state recommendations for future work. Appendices will contain the units of study, teaching guides, evaluation forms and a list of teacher resources.

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

This study has three main components, the development of the lessons and the methodology for geometry instruction, the field testing of the lessons, and the investigation of male and female student views towards five pedagogical tools used in the approach. The organization of this chapter reflects this sectioning.

The first part of this chapter will be devoted to summarizing aspects of the literature on the learning of mathematics, in particular the learning of geometry. The findings of the theorists in this field will be compared with the findings of the researchers who describe the reality of the classroom situation. This is the background necessary for the formation of the lessons and the pedagogical style implemented during the work of this study.

The second section will be a discussion of the work regarding the role of the teacher as researcher. This is the methodology used in the field testing of the materials.

The third part of the chapter will summarize the work of educational researchers concerning gender differences found within the study of mathematics. Implications for pedagogy will also be explored. Five specific teaching tools will be examined to see if student views towards these learning methods display gender differences. The following questions state these areas of concern explicitly:

1. Do students identify group work as a positive, neutral, or negative influence on their learning?

2. Do students identify the use of the computer, ie Logo and the Geometric Supposers, as a positive, neutral, or negative influence on their learning?

3. Do students identify the use of writing as a positive, neutral, or negative influence on their learning?

4. Do students identify the use of manipulating actual objects as a positive, neutral, or negative influence on their learning?

5. Do students note the role of the teacher as a facilitator not as a giver of fact as a positive, neutral or negative influence on their learning?

Learning Mathematics

Theories of learning and theories of instruction should be interrelated, each informing the other to improve the models of understanding from which teachers make their daily and long term decisions. However, these theories must be based on a philosophy which states one's beliefs about how mathematical knowledge is acquired.

It is necessary to state a position on this question before continuing. This excursion will be brief since this is not the explicit purpose of this chapter, however the belief to be stated provides the context for work of this study and to omit it would limit the understanding of the reader.

It is the belief of the investigator that people create their own understanding of mathematics by forming conjectures based on their experiences and the mental representations from which they form them. With this constructivist view of mathematics learning, the investigator believes that each of us seeks to understand mathematics, to abstract it in our own way, according to the mental structure we currently have.

Theoretical Framework

The work of Piaget forms the theoretical basis of this philosophy. His work began when he resolved that educators were missing important information by focussing on the quantity of right vs wrong answers on IQ tests. His approach was to determine the reasons why people chose the answers they did. From this viewpoint a rich field of analysis was born.

As he studied children in detail, his theory grew more complex, incorporating the constructs of accommodation and assimilation. He postulated and described the levels of development (as outlined in Chapter One). In this way

Piaget further refined his basic premise: that individuals are always trying to understand the world and that this understanding is tested, modified and expressed through the individuals' own experiences.

This theory is employed as a basis for many researchers. As reported in Steffe, et al. (1983), one of the basic assumptions underlying much current research is that children actively construct knowledge for themselves through interaction with the environment and reorganization of their own mental constructs.

The implication of this theory for teachers can be subtle. Although instruction clearly affects what children learn, it does not determine it. Children are not passive recipients of knowledge; they interpret it, put structure into it, and assimilate it in the light of their own mental framework (Wittrock, 1974).

The testing of Piaget's theories continues. Points to be investigated which will have impact on school curriculum and practices in the future include questions concerning the levels that Piaget has hypothesized: what is the relationship between age and Piagetian level and can instruction move a child from one level to another?

In Piaget's early work he identified ages 12 to 13 as marking the beginning of the change from concrete operational to formal operational thinking. However in recent years there has been evidence that this transition is a very slow process. In one study students in grades ten through twelve were found to demonstrate concrete thought 57% of the time (Renner, 1977). Work done by Farrell (1967), Farrell and Farmer (1979) and McDonald (1982) support the statement that students in geometry classes display a spectrum of Piagetian levels; approximately 30% of these students are at the concrete operational level, approximately 30% are at the formal operational level and the remaining 40% are labeled transitional reasoners displaying both concrete and formal thought at various times.

Thus we see that the age at which a formal operational level is attained can not be determined accurately. Can instruction affect this process? Are the levels purely developmental or are they amenable to instruction? Klausmeier has tested his own Cognitive Learning and Development (CLD) theory and disagrees with those who are content to wait for children to develop without stimulating that development. According to his CLD theory, the transitional period between concrete thought and formal thought may be longer than many previously imagined, and instruction can hasten the transition for many individuals (Klausmeier, 1979).

Within the specific field of geometry, the van Hiele model of levels of reasoning (described in Chapter One) includes a description of the pedagogy necessary to support student understanding in geometry. In this model the van Hieles describe the different phases of instruction which help move the students through the levels of reasoning. These phases are described by Crowley (1987).

Information: working with materials presented by the teacher, students become familiar with the structure of the concept.

Guided Orientation: investigating the material guided by the teacher's questions.

Explicitation: learning to express concepts in correct mathematical language.

Free orientation: allowing students to explore the concept through the use of open-ended questions.

Integration: connecting this new knowledge with existing cognitive representations.

The van Hiele model states that movement through the levels is accomplished by the teacher guiding the students using these phases of instruction. This part of the theory has not yet been tested by researchers but does provide the stage for the next level of inquiry.

Understanding Mathematics

If each person constructs their own meaning and that meaning is grounded in and expressed by their own experiences and mental structures, then how do teachers define mathematical understanding in a learner? Robert Davis provides this statement,

> ... when we say that a student is good at mathematics, we mean that he or she deals with a wide range of mathematical situations powerfully and flexibly. This includes coping with things that may be novel and unexpected. Understanding what you are doing is an important part of this capability. (1983, p. 103)

This definition of mathematical understanding precludes rote learning as the major vehicle for teaching mathematics. It is useful to consider mathematical understanding as containing two components, conceptual and procedural.

Conceptual and Procedural Knowledge

Understanding is a word that can be defined in many ways. One useful partition of the meanings of this word is to separate the knowledge components into two categories, that of conceptual knowledge and that of procedural knowledge. The descriptions and comments that follow appear in the work of James Hiebert (1986).

> Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (pp. 3-4)
>
> Procedural knowledge, as we define it here, is made up of two distinct parts. One part is composed of the formal language, or symbol representation system, of mathematics. The

> other part consists of the algorithms, or rules, for completing mathematical tasks...they are step-by-step instructions that prescribe how to complete tasks. (p. 6)

The type of learning demanded by these kinds of knowledge varies. "Conceptual knowledge, by our definition, must be learned meaningfully. Procedures, on the other hand, may or may not be learned with meaning." (p. 8) Although this separation of types of knowledge and the learning approach needed to foster each of them is clear and distinct, it would be a mistake to believe that these forms of knowledge are complete unto themselves. There is clear evidence that the related interplay of these two forms of knowledge support understanding.

> Linking conceptual knowledge and procedural knowledge has many advantages. Usually the advantages are claimed for procedural knowledge. Procedural knowledge that is informed by conceptual knowledge results in symbols that have meaning and procedures that can be remembered better and used more effectively. A closer look reveals advantages for conceptual knowledge. Procedural knowledge provides a formal language and action sequences that raise the level and applicability of conceptual knowledge (Hiebert, p. 10).

There have been many studies that indicate conceptual knowledge "helps" in the selection of procedures. There are indications that having a strong conceptual base from which to build procedural and algorithmic skills gives positive results (Lesh et al, 1983; Greeno, 1980; Gelman and Meck, 1986).

There have also been indications that procedural knowledge can be used as a base from which to expand conceptual knowledge (Byers and Erlwanger, 1984; Kotovsky et al, 1985; Baroody and Ginsburg, 1986).

These studies show a need for two kinds of knowledge for complete mathematical understanding. The link between these two is essential. van Hiele points out that it is possible to teach "... a skillful pupil abilities above his actual level, like one can train young children in the arithmetic of fractions without telling them what fractions mean..." (Freudenthal, 1973, p. 25). Geometrical examples include students who can calculate the area of a rectangle but who have no understanding of what area is and students who simply memorize "a square is a rectangle" but who do not understand the nested quality of these definitions.

The phases of instruction model designed by the van Hieles attempts to provide a structure for teaching that will result in the integration of procedural and conceptual knowledge. This aspect of the model has not been researched. The research that has been done has focused on validating the hierarchical nature of the levels (Burger and Shaughnessy, 1986), the appropriateness of the model for characterizing geometrical thinking (Fuys, Geddes, and Tischler, 1985; 1988; Usiskin, 1982), and connections between the model and geometry textbooks (Fuys, Geddes, and Tischler, 1985, 1988). Fuys and Lehrer call for further research to determine the relation of this model to broader psychological contexts such as Piagetian theory and cognitive psychology (Fuys and Lehrer, 1988).

Individual Differences

Other work shows the wide range of individual differences that occur in every classroom. Students display a variety of attributes according to the developmental levels of Piaget, the learning style they prefer and the mental structure they have formed. Mark Driscoll (1986) summarizes the results of the studies he reviewed. He reports that every secondary level mathematics student:

> (1) is somewhere on the continuum between concrete thinking and full formal thinking (Phillips, 1978),
> (2) has a position on each of several cognitive style continuums (Fennema and Behr, 1980), and
> (3) differs from many other students in the kind of bridge he or she has built-- with language, intuition, and the formation of personal rules-- between mathematics and the real world (Peck and Jencks, 1979; Erlwanger, 1975).

To summarize the points made regarding the learning of mathematics, mathematical knowledge is constructed from conceptual and procedural knowledge by building on intuitive understandings. Students in mathematics classrooms construct meaning in their own ways and display a variety of reasoning styles.

The writings of Dina and Pierre van Hiele contain descriptions of the five levels of geometric understanding they have postulated. Students in geometry classes vary along this dimension in addition to displaying the spectrum of individual differences already noted.

The van Hieles note the importance of meeting each learner at their own level of understanding. According to this theory it is important to make contact with the learner's meanings before instruction can begin.

> The geometric figures have already obtained certain meanings. These meanings can lead to inappropriate actions during the initial stages of geometry instruction because the mathematician considers appropriate only those actions that are based on certain logical rules of the game. By starting geometry instruction using the logical structure of thought one really puts the child into an ambiguous learning situation; the meanings which the material possesses for the children do not fit the operations that have to be carried out with the material. This undesirable situation can be avoided by taking care that already existing meanings are utilized as much as possible in the initial learning situations (van Hiele in Fuys, 1984, p. 34).

These statements are made with regard to geometry. However, little has been done to study the role of intuition in the learning of geometry. There are many studies which indicate that it is easy for students to be "successful" in a formal school setting and yet retain many misconceptions. The research indicating students may have "formal" knowledge without real understanding comes from several academic areas. Studies in arithmetic by Erlwanger (1973), in algebra by Rosnick and Clement (1980) and in physics by di Sessa (1982) all indicate that students can learn to imitate without acquiring internal meanings.

Robert Davis describes the situation in these words:

> In our view a strong instructional program operates both at the formal level (i.e. with algorithms, definitions, notations, etc.) and also at the experiential level, taking pains to make contact with the student's existing representational structures, and helping the student to build, revise and extend these structures by the process of 'assembly' (including the process of 'educating your intuition') (Davis, 1985, p. 371).

Madeleine Coutant (1987) reports that students develop intuition by encountering phenomena in everyday life. The way to build that intuition is to have students control those experiences and thus lead the way to insight.

Classroom Realities

Several studies have been conducted to determine if the van Hiele model of geometry understanding is appropriate to American secondary school students. In summing up these studies Burger and Shaugnessy (1986) noted the following:

1. The van Hiele levels were good descriptors of students' reasoning processes.
2. No secondary school student reasoned consistently at level 3.

3. There was often a mismatch between the level of the teacher's work and the level of student thinking.

4. Students may actually regress to a lower van Hiele level after taking a full year course in geometry.

5. Over 70 per cent of students beginning a geometry course were at levels 0 or 1 (Usiskin, 1982).

6. Geometry text books contained exercises at levels 0 and levels 3 only; that is there were no questions of an analytical or informal nature, only strictly visual or totally proof oriented (Geddes et al, 1982).

These studies indicate that the van Hiele model is an appropriate construct for analyzing geometry materials, instruction, and student understanding. They also indicate that a gap exists between the suggested approaches of the model and the actual work of teachers and students in the classroom.

Further difficulties with geometry instruction can be found by looking at the results of researchers studying student understanding of proof. Several studies provide information. The Cognitive Development and Achievement in Secondary School Geometry Project (CDASSG) addressed many issues. One of them was proof. These results are summarized as follows:

> ...at the end of a full year course in geometry in which proof writing is studied, about 25% of the students have virtually no competence in writing proofs; another 25% can do only trivial proofs; about 20% can do some proofs of greater complexity; and only 30% master proofs similar to the theorems and exercises in standard textbooks (Senk, 1985, pp. 453-454).

These results indicate poor achievement in proof writing. Then what do we know about student understanding of the concept of proof? Edgar Williams surveyed eleventh graders and found that "fewer than 30% exhibited any understanding of the meaning of proof in mathematics, that approximately 60% were unwilling to argue, for the sake of argument, from any hypothesis they considered false." (1980, p. 166)

Is this how students see the role of proof? Alan Schoenfeld finds that students form "beliefs" about mathematics as a result of their schooling. He states students believe "that 'proof' was used either to confirm (a) what they already believed was obvious for intuitive reasons, or (b) what the teacher attested to be true, which they were to <u>verify</u>. In either case, mathematical argumentation was never used to discover anything." (1983, p. 21) This does not agree with the reasons the professionals give for doing proofs.

In a further study Schoenfeld noted that "... students can be competent at deduction, and competent at constructions, but that they will often compartmentalize their knowledge in inappropriate ways. The result is that much of their knowledge goes unused ..." (1985, p. 259). This indicates the poor results stemming from current practice.

Many students are not becoming competent at the task of writing proofs and among those who are competent this knowledge is a dead end because the value and meaning of proof are not understood.

Common Practice

What kind of geometry teaching is common? Several studies provide clues. Brown (1974) found that geometry teachers depend very heavily on the textbook. Beaulieu (1979) discovered that teachers in a geometry classroom talk about 80% of the time. Another study noted that "The majority of student time in mathematics class was spent listening to teacher presentations, doing seatwork or taking tests. Little time was spent in small group work" (Crosswhite et al., 1985, p. 56). These results show that common practice in the geometry class does not provide opportunities for open-ended exploratory activities that would give students a good understanding of the meaning and value of proof.

Cognitive Process Research

What do researchers in cognitive development have to offer geometry teachers? Much current cognitive work has been concerned with identifying students' misconceptions. This kind of research has many attributes that distinguish it from other educational research paradigms. It is process oriented; that is, it is concerned more with how students are thinking, not just with the correctness of the final result. It is domain specific, "...it lays much greater emphasis on the particulars of the subject matter being studied" (Schoenfeld, in press). One common format is that of protocol analysis in which students talk to an interviewer as they solve problems explaining what they are doing and why they are doing it. These studies provide information of two types: how people think about what they are doing when they are solving problems and the impact of schooling on those reasoning processes.

Three of these studies are of interest. One has been referred to earlier, that of Alan Schoenfeld (1985), in which it was noted that students compartmentalize their understanding and do not use proof knowledge in solving non-proof problems. A second finding of interest is in the field of arithmetic. Erlwanger (1973) found that sixth grade students were studying a formalistic system of one kind or another, were thought to be successful and turned out to have gross misconceptions at a fundamental level. The persistence of these misconceptions is so strong that researchers can predict the errors that students will make based on their experimental evidence.

A last example from this kind of research will indicate the scope of the problem. Within the realms of arithmetic and algebra, teaching students to solve word problems has always been difficult. One solution widely accepted is to teach students to identify "key words". That is, to encourage students to pick out of a problem a word such as "left" and connect the operation of subtraction with this key word. But what has this really accomplished? "It has allowed students to obtain the right answers without understanding-- and gave them the option of not seeking understanding at all" (Schoenfeld, in press).

This misconception research indicates that students are being taught by rote and as a consequence have developed much incorrect though very persistent and consistent methods of solving math problems. Robert Davis argues that some educators have created a system of learning which allows students "...to create formalistic knowledge of verbal statements (that can be memorized and repeated without being understood) and rote algorithms. Many curricula today do precisely this. When this happens, students' knowledge is fragile and superficial, allowing for severe misconceptions" (Davis, 1983, p. 106).

Summary

A classroom situation that allows students to relate what they learn to their own belief systems, to engage their own intuitions, and to discuss and argue with each other about mathematics will prevent this memorized learning.

A curriculum that is multifaceted, incorporating the levels of Piaget and the van Hieles, integrating conceptual and procedural knowledge, and providing opportunities for small group discoveries and justifications is indicated by the work of the theorists.

Yet, what is the classroom reality? Teachers talk 80% of the class time. Teachers spend most class time presenting material or discussing homework exercises. Teachers depend very heavily on the text book. Text books are at van Hiele level 3.

The predominant model of current instruction is based on what Romberg and Carpenter call the "absorption" theory of learning. "The traditional classroom focuses on competition, management, and group attitudes; the mathematics taught is assumed to be a fixed body of knowledge, and it is taught under the assumption that learners absorb what has been covered." (1985, p. 868)

The need for curriculum restructuring as well as pedagogical tools for classroom implementation has been established.

Teacher as Researcher

> All teachers think about what happens in the classroom, but these thoughts are largely undocumented and unreported, and if they are

reported they are usually anecdotal and only for lunchroom discussion. In brief, teacher research, because it is unplanned and undocumented, has no institutional standing, and, as a result, few districts provide paid time for teachers to do it; thus education is one of the few professions where expertise in 'how' to do its tasks is assigned to people who do not in fact do them (Myers, 1985, p. 2).

This statement describes a situation in which teachers are always researching but are rarely researchers. There is a new concern in supporting teachers who wish to take on this dual role (Myers, 1985). This resurgence of interest has led to a rediscovery of a process originally labeled "action research" which was prominent in the 1940's.

Historical Perspective

Historically, action research was classroom based and involved either the teacher as a researcher or, more often, a team approach with teachers and researchers collaborating. The degree of teacher involvement varied considerably. Some teachers were active participants in the entire process, designing the experiment, analyzing the data, and interpreting the findings with the researcher. Other teachers participated only by allowing their classroom, their students and themselves to be observed and studied without active involvement between the teacher and the reseacher. Lewin (1948) saw action research as the application of tools and methods of the social sciences to understand and improve practice in schools.

The goals of such research were to learn how to improve school practice. Action research focused on immediate application of techniques or approaches to be tested, not the development of a theory or the construction of a model for nationwide applicability.

In the 1950's Hodgkinson (1957) attacked the principle behind teachers doing action research. His powerful statements decrying amateurs doing research resulted in a loss of academic respectability for this kind of investigation.

> Because of the critiques of action research as unscientific and unproductive and the emphasis on social sciences and federal funding agencies on the separation of research and practice, action research in the 1960's and the early 1970's became inquiry done by practitioner with the help of a consultant. During these years, action research was used to provide inservice training and to improve practice rather than to produce generalizable results (Smulyan, 1984, p. 7).

Recently there has been more interest in this kind of research model.

> Many observers have deprecated action research as nothing more than the application of common sense or good management. But whether or not it is worthy of the term 'research', it does apply scientific thinking and methods to real life problems and represents great improvement over teachers' subjective judgements and decisions based on folklore and limited personal experience (Best and Kahn, 1986, p. 22).

Bolster is in the almost unique position of being both a practicing teacher (He spends his mornings as a social studies teacher at a secondary school in Massachusetts.) and a professor of education. (He spends his afternoons at Harvard in that role.) In his consideration of this kind of research he states, "The more I became aware of and experienced with this methodology the more I became convinced that of all the models of research, I knew, this method had the greatest potential for generating knowledge that is both useful and interesting to teachers" (Bolster, 1983, p. 305). What is it that makes this model of research so appealing for teachers? Contrasting this model of research with others will help to elucidate the reasons.

Characteristics of Teacher Research Work

> Academic research is defined as the process of discovering the relationships between two or more variables. It requires careful disciplined procedures. However, the classroom teacher has usually neither the time nor the money to engage in rigidly designed, carefully controlled research. Rather we can think in terms of Webster's definition of research, 'a studious inquiry, examination, or investigation,' in our case investigation into what is really going on in our classrooms (Klinghammer, 1986, p. 1).

This statement sets out in broad terms some of the contrasts between the research work done by teachers and that done by educational researchers who are not scrutinizing their own classrooms. In the discussion that follows, these two styles of research will be described. According to Simon (1981), the main differences between teacher research and pure educational research occur in three areas: the kind of data and its analysis, the purposes of the results, and the amount of control over the experiment itself.

Data

In teacher research, the data are likely to be extensive and inclusive, based on the multitude of experiences in a classroom every day. Data are often reported in terms of general trends rather than analyzed by statistics. The findings are generally believed if they are corroborated by the experiences of other teachers rather than verified by the significance tests of statistics.

On the other hand, in pure research the data would be tightly defined and limited to a small number of variables for a specific prestated purpose. Once obtained this data would be subjected to careful analysis using statistical tests to determine the validity and the generalizability of the findings.

Purposes

In teacher research, the purpose of the investigation is classroom related, for example; to determine a solution to a class problem, to study a type of lesson to see if it promotes learning in the students, or to determine the effect of certain teacher behaviors on the students. The object of the study is to gain information on what is happening in the teacher's own classroom so that the teacher can make better educational decisions.

Researchers are interested in more general information. They are not just concerned with the specific classroom which produced the data. The purpose of the experiment is to produce knowledge which can be shared with others and generalized to other situations. The results may not be related to school structures at all, but may be associated with life experiences in other settings.

Control of the Experiment

Teachers have few opportunities to design an experimental structure; they must use the existing classroom situation as the setting for their study. They are limited by circumstance to a particular time period, place and subjects. They are also constrained by their role as teacher. In that role they must make decisions based on what they think is best for each student, not what is best for the experiment. The design of the experiment is not rigidly controlled, but must be in keeping with the general atmosphere of the classroom. "In such a study, the designer, 'satisfices', selects a solution, that is which 'suffices', to get the job done and at the same time 'satisfies' the need for a solution which, if not the best, is at least better than the other alternative" (Simon, 1981, p. 138).

Researchers can and do create specific experimental designs, often choosing their subjects according to some preset criteria, using control groups, acting on these groups in different ways, and rarely having contact with the subjects once the experiment is over.

In summary, the teacher researcher and academic researcher differ in three main areas; the form and analysis of the data, the purpose of the experiment, and the control of the experimental situation. While it is useful to note these differences, it is important to remember that most research will not completely fit either description, but rather will display characteristics of both. A more realistic picture would be a spectrum of research types with these descriptions providing the endpoints and with most studies lying somewhere in between.

Definition of Teacher Research

What then is the definition of teacher research? Myers notes that "Dixie Goswami, for one, solves the problem by defining teacher research as naturalistic inquiry procedures which do not result in statistical data toward which journals of education are so heavily biased." (1985, p. 4) But many studies display such characteristics and some teacher studies do include statistical analysis. So this definition is clearly lacking. One could consider the style of Oswald Veblen who, when asked to explain what geometry was, said, "geometry is what geometers do" (Allendoerfer, 1969, p. 165). So perhaps teacher research is simply research that teachers do.

The Value of Teacher Research

Elliott speaks of the value to the teaching profession of this kind of research. "Educational action research is not only practical but emancipatory." (1987, p. 165) In reporting about the experiences of teachers who were taking graduate course work in educational research and were encouraged to do a research project, Williams and Loertscher (1986) noted that naturalistic methods lent themselves to the study of education. They suggest teachers can learn to use naturalistic approaches in dealing with their students and evaluating their own effectiveness.

Margaret McIntosh, a teacher who has been involved in several self-designed research studies summarizes her experiences in this way:

> I do not recommend that all educational research be conducted in intact classrooms

by teachers, but I do propose more such 'action' research be encouraged - especially as opposed to the research that involves what Eisner (1984, p. 451) calls 'educational commando raids to get the data and get out.' Being a teacher researcher has had a positive impact on colleagues, (who appreciated the boost), students (who are the raison d'etre for teachers), and for the researchers themselves. (1984, p.8)

Teachers involved in this aspect of classroom life are natural participant observers in their own classrooms and are committed to using their results to improve educational practice in the future. In fact, Erickson calls for more work of this kind.

> If classroom teaching in the elementary and secondary schools is to come of age as a profession - if the role of the teacher is not to continue to be institutionally infantilized - then teachers need to take the adult responsiblity of investigating their own practices systematically and critically by methods that are appropriate to their practice. (1986, p. 157)

Teachers who define their own problem, design a reseach project to test a solution, and then adapt their practices according to these results demonstrate the potential power for educational change embedded in the construct of teacher as researcher.

Gender Issues and Teaching Methodology in Mathematics

The growing crisis in the effectiveness of mathematics education, most severe among females and minorities, has been well documented (Carpenter et al., 1983). Furthermore, there is a substantial body of evidence which indicates that although young women may excel during the elementary grades, by the high school years males perform significantly better on most measures of mathematical achievement (Lee and Ware, 1986; Peterson and Fennema, 1985). These differences tend to show up by ages 13 to 17 and are independent of formal educational experiences (Benbow and Stanley, 1980; Carpenter et al., 1984; Lewis and Hoover, 1983).

In the report on the Fourth National Assessment of Educational Progress it was noted that gender related differences between thirteen-year-old male and female students is small, but when comparing seventeen-year-old male and female students this gap, though less serious than in the past, is still significant. The same report showed that course-taking behavior varied for the seventeen-year-olds. "Significantly more males than females, however, reported taking precalculus or calculus courses..." (Silver et al., 1988, p. 724).

In order to determine specific information on how these differences are played out in classrooms, many aspects of students' mathematical lives have been studied.

The results below provide a flavor of these studies.

Women enroll in fewer advanced mathematics classes (DeBoer, 1984).

Among students who are doing well in advanced math and science courses, women are less confident and have a lower self concept than the men students (Fennema and Sherman, 1978).

Women students are more likely to characterize mathematics as less interesting and less useful than other subjects than are men students (Fox, 1981; Lips, 1984; Fennema and Sherman, 1978).

Women display more negative attitudes towards mathematics than do their male counterparts (Meece, 1981).

Women suffer from math anxiety to a greater and deeper extent than do men (Tobias and Weissbrod, 1980).

This group of studies and many more like them have made clear the differences that exist between the achievement and attitude of male and female students.

Many points are less clear. Are these differences due to purely physical states? Are these differences a sociological phenomenon, having more to do with our culture than physical selves? Are these differences amenable to changes created by our educational system? Some research has been done on these issues.

One means of attack is to determine if there is a strictly phsical explanation to these differences. In studies reported by Peterson (1983), Luchins (1981), and Fennema and Ayer (1984), it has been documented that biology alone is not the cause of these differences.

It would seem then that affective variables and the effect of socialization are more likely to explain these differences. This avenue of study has led to many projects.

The impact of differential parental expectations has been explored by Petersen (1983) and Stage et al. (1985).

The role of attribution and its differences between men and women has been explored in studies by Enemark and Wise (1981) and Wolleat et al. (1980).

Wigfield (1984) has noted the importance of beliefs and attitude toward success in mathematics classes.

Pedro (1981) noted that students' perceptions of the usefulness of mathematics to them is a factor on which males and females vary.

Another construct under investigation is sex role conflict. This occurs when male or female students are successful in domains that are traditionally associated with the opposite sex (Smead and Chase, 1981; Petersen, 1983).

These studies all point to various influences which impact male and female students differently and may be implicated in causing the gaps noted. However it is not at all clear if these influences are the root of the problem or simply another sympton.

Taken as a whole these studies provide a picture of an educational system which is a reflection of the culture. The explanations that are most likely to be helpful to educators in the future are those which take the socialization aspects into account.

Pedagogical Considerations

None of the studies cited provides a hint as to what pedagogical tools could help to lessen these disparities. There are some indications that the educational system, the curriculum, and prevalent teaching styles favor the males in our schools. "Traditional teaching may tap boys' strengths more effectively than girls'" (Featherstone, 1986, p. 2).

It is the investigator's belief that the teaching methodology which supports geometric understanding discussed in the first part of this chapter will also provide an atmosphere to help counteract the negative influence of these gender imbalances.

The learning cycle proposed as a curriculum format involves several teaching tools. Five of these will be discussed as they apply to geometry instruction. The possible impact of each tool on female students will be explored.

Use of Group Work

To implement the discovery-type geometry curriculum being recommended, small group exploration, discussion and argument is essential. The cooperative group method of instruction is the most appropriate means to teach this kind of conceptual mathematics.

A summary of the research on discovery learning notes, "The general conclusion is that discovery is often less effective than exposition for immediate learning, but is better for retention and for transfer to new situations" (Bell et al., 1983, p. 171).

Small group work is most appropriate during the exploration phase of the learning cycle. It can also be used effectively during the intuition phases by providing a forum for students to discuss with each other what they believe to be true about the concept. In this way they challenge each others' beliefs.

The cooperative learning models of Slavin, and Johnson and Johnson have been tested extensively in the elementary schools to determine their effects. In his review of the research literature on cooperative learning, Slavin (1987) reports that when considering cooperative learning studies which involve two particular aspects, group goals and individual accountability, effects on achievement have been consistently positive; 34 out of 41 such studies (83%) found significantly positive achievement effects.

Johnson and Johnson (1984) summarize the work of well over a thousand studies which indicate that cooperative learning promotes greater mastery and retention of facts and concepts, greater interpersonal and small group skills, greater development of higher level reasoning, greater motivation for success, greater affinity for classmates, more positive attitude toward subject matter, higher self esteem, and greater social maturity.

In spite of this overwhelming research evidence indicating the positive effects of cooperative learning, little work has been done on the secondary level. A few studies in the realm of mathematics learning are noted here. One study does indicate that this teaching tool has positive effects for older students as well as those in elementary grades. Sherman and Thomas conducted a study in general mathematics classes. In this study two classes were taught the same content unit; one class was taught in the traditional individualistic fashion, the second was taught using Slavin's cooperative group approach. They report "...the cooperative group demonstrated significantly higher achievement on the post-test than the individualistic group" (Sherman and Thomas, 1986, p. 6).

Small group work has been shown to be an effective means of involving female students. "In elementary schools competitive math activities tend to favor the boys, cooperative math activities tend to favor the girls" (Berliner, 1987, p. 11).

Use of Computers

Another recommendation for geometry exploration is to incorporate the use of the computer into the geometry curriculum. The LOGO language and the use of the Geometric Supposer(s) allow students to make and test their own conjectures.

Previous work on this topic has been done by Susan Scally using the LOGO language to support geometric understanding. Scally's research study attempts to bridge the gap between middle school study of shapes and properties and the tenth grade treatment of formal deductive geometry by providing ninth grade students with a better understanding of geometric relationships using the LOGO language (Scally, 1986). Another study (Olive and Lankenau, 1987) indicates the power of the LOGO computer language to enhance students non-verbal cognitive abilities.

Studying the effects of teaching with the <u>Geometric Supposer(s)</u>, Yerushalmy (1987) notes two aspects of student behavior which change after they study with the computer software: their attitude toward geometric diagrams and their method of attacking a problem.

There is some indication in the literature that computer use may favor males (Fuchs, 1986), but those results were not based on computer activities in geometry but rather were related to mathematical games on the computer or Computer Assisted Instruction. It is not clear if the same factors will be present within a geometry curriculum.

<u>Use of Writing</u>

Using writing as a pedagogical tool for learning is appropriate at all levels of the learning cycle. During the intuitive stage it allows the students to explicitly state their current belief. At the exploration phase it is a way of recording and reporting their results. During the formalization phase it provides a way of stating what they have proved and sharing that proof with others. At the final intuitive phase, writing is a means of stating their understanding.

Johnson reports that writing can help the mathematics student in several ways. "Students who are required to write must do considerable thinking and organizing of their thoughts before they write, thus crystallizing in their minds the concepts studied." (1983, p. 117)

In work by Nahrgang and Petersen (1986), journal writing in mathematics class was studied. They did not find any strong relationship among the variables studied (test scores, attitudes and writing), but they did note that the students used the journals extensively to think about the mathematics being explored in class and that the students viewed the journals in a positive light.

Another study investigating the usefulness of journal writing and mathematics was conducted at Michigan State University. In this work it was shown that students who did journal writing about the mathematics presented in class scored equally well on examinations as those who were assigned traditional drill and practice problems (Young, 1985).

In secondary school mathematics, one study of ninth graders indicated that "... writing about mathematics had made an impact upon the math averages of even students who were marginal or below average academically" (Gladstone, 1987, p. 4).

Sandra Keith describes the importance of using writing in a mathematics class in this way. "Short explorative writing assignments can transform the mathematics classroom into a dynamic and exciting learning laboratory. In explorative writing, students explore their knowledge about a topic by writing what they know about it in their own language." (1988, p. 714)

In a summer program designed to improve both the math competence and confidence of young women, instructors who incorporated writing into their course work report that

> ...informal analysis of this pilot project suggests that a portion of the success achieved by thesestudents can be attributed to the writing exercises and subsequent discussions. Students' attitudes towards themselves and mathematics improved, and they now felt more comfortable and competent in their math classes (Morrow and Schifter, 1988, p. 384).

<u>Use of Manipulatives</u>

Use of manipulatives is necessary within the learning cycle approach during the exploration level. Exercises using manipulatives provide the link between the intuitive ideas of the learner and the concepts he/she is to formalize. The use of these materials will provide all students the opportunity to act on objects concretely before performing mental abstractions.

Many studies document the power of manipulatives to provide the link between concrete and abstract reasoning. From an analysis of sixty-four research studies at the elementary level, Parham (1983) reported that students who had used manipulative materials outperformed those who did not as evidenced by achievement scores. This finding is in agreement with earlier work of Suydam and Higgins (1977) who found that lessons involving manipulatives are more likely to produce greater achievement in mathematics than lessons which do not involve manipulatives. Canny (1984) found that the fourth graders she studied displayed significant improvement in problem solving scores when manipulatives were used to introduce content.

> Manipulatives should be used not only at the elementary level but also at the secondary level. Many adolescents and even adults find science and math difficult because they lack the concrete experience from which to make sense of the concepts" (Skolnick et al., 1982, p. 52).

In spite of this research evidence, studies show that manipulatives are not used very frequently. Hunting (1984) concluded that a lack of use of manipulatives resulted in poor student performance on equivalent fractions tasks. This study included students at fourth, sixth, and eighth-grade levels.

Scott (1983) has also deplored this lack of the use of manipulatives. In his survey of teachers from kindergarten to grade five, he found that few teachers reported using manipulatives more than five times a year. The percentage of first grade teachers using materials was only about 60%. This percentage dropped off each year after first grade.

In a research study performed in Ontario grade 8 teachers were surveyed to determine their use of instructional aids for teaching geometry. It was found that,

> The traditional ruler and compass, protractor, and graph paper were the basic tools for instructional purposes. Laboratory-oriented teaching materials such as mirrors and models were used infrequently, and more specialized manipulative or interactive resources were used not at all (Raphael and Wahlstrom, 1989, p. 175).

There are some middle school teachers who do use manipulatives. Herbert reports that "manipulatives allow teachers to create situations that draw mathematical responses from the children. Such situations result in improvements in motivation, involvement, understanding, and achievement -- overwhelming reasons to believe that manipulatives are good mathematics." (1985, p. 4)

The usefulness of manipulative materials in secondary school mathematics has not been documented. Largely, this investigator suspects, because they are so rarely used. Given the earlier documentation on the number of students at secondary level who still display concrete reasoning at least half the time, a ripe field for study would be the connection between the lack of instructional approaches involving manipulatives and the poor performance of secondary students on mathematics achievement exams. But this study has yet to be done.

The poor performance of female students in particular could be related to the lack of manipulative approaches at the secondary level. Fennema and Sherman (1978) have noted that female difficulties with spatial visualization may be the result of less knowledge and experience with manipulative materials. Incorporating the use of manipulative materials within the classroom could help dispel any differences caused by previous experience.

The factors most critical to the development of spatial visualization skills are experience with manipulative materials such as constructing and examining three dimensional structures, graphing, and modeling (Skolnick et al., 1982). Creating classroom environments where these activities are included would insure that women students will have the experiences needed to form mental abstractions.

Teacher as Facilitator

> The role of the teacher is not to transfer knowledge, nor to command an action, but to show and explain reality. The teacher should be a companion in the perception of reality, and a supplier of techniques to retrieve desired information. In this approach the teacher's power as manager of the educational experience is replaced by his participation in a joint intellectual venture, his relation with the student is not based on authority, but rather on partnership in the pursuit of understanding and change (D'Ambrioso, 1981, p. 41).

The teaching style necessary for successful implementation of the learning cycle requires the teacher to take on a different role than one traditionally sees. The teacher is not a giver of facts, but one who facilitates discovery, exploration, and discussion.

In this style of teaching, "... the teacher no longer acts as the embodiment of knowledge or the container of secret criteria and so becomes less 'important,' less the authority, more a 'coach' or an 'ally' in Elbow's (1979) terms, more a 'partner' in the language of Paulo Friere (1971)" (Belenky et al., 1986, p. 208).

In their discussion of the classroom process using the Geometric Supposer(s), Yerushalmy and Houde note that new roles for both student and teacher are necessary for this kind of inductive teaching. They summarized student activities in this way:

Students worked in pairs at the computer laboratory and wrote reports of their work together. Students reported their results to the class. Students spent the majority of class time discussing and doing geometry rather than listening to the teacher talk about it" (Yerushalmy and Houde, 1986, p. 422).

From these statements, one can infer the role of the teacher. The teacher is there to pose the questions, to serve as a facilitator of discussion and to encourage students to pursue the next steps. This model of teaching is also recommended for the implementation of the learning cycle.

But this style of teaching is being suggested for more than computer laboratories and geometry classes. The Educational Testing Service recently issued a report based of twenty years of testing done by the US Department of Education termed the National Assessment of Educational Progress. The results of these assessments have been discussed elsewhere in this paper, but of interest here is the interpretation of those results. The report titled "Crossroads in American Education" (1989) calls for a revolution in the classroom. "Students must become doers and thinkers rather than passive learners, and teachers must serve as guides rather than continuing in their traditional, authoritarian roles" (The Boston Globe, February 15, 1989, p. 4).

In the NCTM document Curriculum and Evaluation Standards for School Mathematics it is noted that "For instruction to result in the student outcomes specified in the NCTM Standards, it is necessary that students frequently be given opportunities to explore and investigate mathematical ideas, either as part of a whole-class discussion, in small groups, or independently, and use mathematics to communicate their ideas" (Romberg et al., 1989, p. 169).

Even though the call for this style of teaching is coming from many fronts and the need for this change in teacher role is implicit in the work of many learning theorists, the investigator could find no direct classroom research in secondary schools to study the effect of this style of teaching on either male or female students. The NCTM report cited above calls for "sustained classroom observations and ... teacher reports" in order to obtain the necessary information about how the mathematical content is actually treated during instruction and to determine the effect of this approach on student achievement and attitudes (Romberg et al., 1989, p. 169).

Summary

The first part of this chapter reviewed the materials concerning the learning of mathematics in general and that of geometry in particular. It was noted that the geometry curriculum should reflect the levels of Piaget and those of the van Hieles. The interplay of conceptual and procedural knowledge was examined. The classroom realities of individual differences, overemphasis on the text book, and a preponderance of teacher lecture were described.

In the second section the background and current climate regarding the role of the teacher as a researcher was discussed. The value of this type of research was established.

Next a summary of the literature on gender issues in mathematics learning was presented. The overall picture shows that these differences still exist and that our understanding of what socialization factors contribute to this problem has grown but is not complete.

Finally, the results of studies regarding five specific pedagogical tools were summarized. What stands out is that few of these studies were performed in secondary schools. The studies quoted on using cooperative groups and using manipulatives were almost entirely based on elementary school subjects. The studies quoted on using writing were predominantly done at the college level. Even the research results for using the computer were generally middle school students. The consideration of the role of the teacher as a facilitator in the classroom as opposed to a fact giver has not yet been studied.

Based on what research has been done, it is likely that these five tools could have a positive impact on student understanding at the secondary schools; however, it is clear that much research in these areas is yet to be done.

CHAPTER III

METHODOLOGY

Overview

This study has three main components, the development of the geometry units and the methodology for instruction, the field testing of the lessons, and the investigation of male and female student views towards five pedagogical tools used in the approach. The organization of this chapter reflects this partitioning.

The first section will discuss the process and procedures used during the first stage of this work, the development of the lessons and the teaching guides. The second part of the chapter will explain the procedures used during the field testing phase of the work. The third section of the chapter will describe the methods used to determine student views on five teaching tools: the use of small group instruction, the use of computers, the use of writing, the use of manipulatives, and the role of the teacher as a facilitator.

The Development of the Units

The objective of the first part of this work was to produce curriculum units of geometry which were designed using the question guide as a format. This question guide had been developed by the investigator to serve as a structure for analyzing and creating instructional units. Determining the usefulness of the question guide as a model for lesson design was one goal of the study. Each unit consisted of six to eight individual lessons. Four of the units were field tested during the second phase of this work. These four units are: Determined, Triangle Congruence, Angles In and Out of Polygons, and Quadrilaterals. These are to be found in Appendix A.

Teaching guides were written to connect these lessons with the investigator's learning cycle model for classroom methodology. Appendix B contains the teaching guides for the four units under discussion.

Procedure

These lessons were designed to be incorporated into a public school course in geometry, not an experimental version of such a course. It is important to the investigator, who is also a public school teacher, that the materials be useful within a traditional school setting; also that the methodology being suggested be one which could reasonably be implemented within a conventional school environment. Therefore the lessons were designed to take into account the accepted textbook and 45 minute class period usually available for mathematics instruction.

It was the belief of the investigator that units designed with this question guide and implemented with the learning cycle approach would fill the gap between current practice in secondary school mathematics classrooms and the work of the theorists in mathematics education such as Piaget and the van Hieles. The design of both the materials and the methodology was informed by the work of cognitive psychologists, educational researchers studying student misconceptions in mathematics, and the current work being done on implementing problem solving strategies in the classroom. These were the goals of the investigator in the creation of the units and teaching guides.

Data Collection

In order to gather anecdotal information on the process of developing the units, the investigator kept a journal to record her thoughts and reflect upon the interplay of curriculum materials, the question guide, and the learning cycle as they affected the decisions concerning lesson design. The journal entries provided a basis for the written natural history of the process. This record serves as the data for the first component of this study, the description of the process of the development of the materials.

Data Analysis

Thus the first stage, the development of the lessons, was documented by the journal of the teacher/investigator. These subjective data were analyzed by the investigator by rereading the journal. Then the investigator wrote an account of the development of one of the units. This account will be the first section of Chapter Four. It will describe the process starting with the content and pedagogical goals of the unit, illustrating the use of the ten question guide, and examining the relationship of the learning cycle in the production of the teaching guide. The result of this part of the study was to produce a natural history of the process of the creation of one of these units.

Limitations

The teacher/investigator both originated data and interpreted it. The results of this work, the lessons that were created and the description of the process of their design, was produced by the teacher/investigator using her own journal. This subjective process provides no guarantee of bias-free reporting; it does however provide a record of curriculum design from the point of view of the teacher.

Field Testing

The second aspect of this study was the field testing of the materials and methodolgy. The objective of the field testing was two-fold, to actually use the materials and methodology with secondary school geometry students and to determine what revisions were necessary to improve these materials and/or the teaching approach being suggested. The field testing was accomplished by the teacher/investigator within the structure of the school day.

The implementation of the teaching guide and the field testing of these materials was part of the naturally occuring work of the teacher. The decision to evaluate the lessons according to the teacher's reading of their success was a conscious one, motivated by the desire of the teacher/investigator to parallel the usual process of teacher curriculum design and revision.

Subjects

This approach to teaching geometry was field tested at a public secondary school in a suburban setting. The school has a strong academic bent with 80% of its 300 yearly graduates attending further schooling. The investigator is a teacher at this school.

The students in the investigator's geometry classes served as subjects in this study. Students were assigned to one of these classes by routine computer scheduling. For the most part this scheduling was a random process. Approximately 60 students were involved. About two-thirds of these students were in classes which were grouped homogeneously according to their prior achievement in mathematics.

One exception to this random process was made for a selected group of students who had been identified as remedial. These 7 students were placed in one section along with other students who had been randomly selected. This section was co-taught by the remedial mathematics teacher along with the teacher/investigator. This class contained students who had been identified by the school system at all levels and is best described as a heterogeneous class.

Procedure

The students participated in two types of data gathering in the initial phase of the study. During the first two weeks of school, class time was devoted to the following two instruments for the purpose of collecting entry level data: The STEP Level IJ test of Mathematics Computation by Addison Wesley Testing Service and the STEP Level I test of Basic Concepts of Mathematics by Addison

Wesley Testing Service. The objective of this testing was to provide a description of the student population in the study in terms of their understanding of mathematical concepts and their abilities with mathematical skills.

During the fall semester these students learned geometry using the materials and methodology of the learning cycle and the ten question guide. The students were asked to evaluate the materials and the teaching approach being used. Each class was informed that their teacher had been working on constructing geometry lessons and that they would be working on those lessons as part of their regular class assignments. In addition they were told that their teacher would solicit their reactions to the materials and teaching style.

Data Collection

At the end of each unit being field tested the students were asked to complete an evaluation form concerning the materials and teaching style for that particular unit. An example of this form is in Appendix C. During the completion of this form the teacher/investigator reminded students that she would not be able to identify them by name, encouraged them to be honest, and stated that she was interested in using their input to modify the materials and methodology for future use. She also answered clarifying questions concerning the form itself and the lessons it identified.

The teacher/investigator was the primary source for collecting information. Throughout data collection and recording she worked at maintaining the balance between guaranteeing student anonymity and valuing their opinions. The students were assured that the comments on the forms would remain confidential and could not have any effect on their grades.

At the same time it was important for the students to know that the teacher/investigator had a genuine interest in what they had written as a group. In one instance a lesson that had been planned but not yet presented was modified by the investigator because of the comments on evaluation forms of similar lessons in a previous unit. This incident illustrates the fluid nature of this work. The results from one stage impacted the work of the next.

To gather additional information as to the success of the lessons, the investigator took field notes. She recorded her interpretation of the consequences of the lessons, how the class felt about the materials, and what suggestions for change were indicated. These notes were started at school and then reflected upon and completed in the journal. In order to differentiate between this use of the journal and the one previously described I will refer to these writings as field notes.

Data Analysis

During this second stage of the work, both the teacher and the students generated data. The field notes of the teacher and the evaluation forms of the students each described classroom activities from their own point of view. The teacher analyzed this data by summarizing the comments on the forms and comparing them with her own notes. Similarities and differences in the views of the teacher and those of the students were noted. Suggestions for revising the lessons were made as a result of this analysis.

Working from the field notes and the student evaluation forms, the teacher/investigator summarized the class activity and evaluation form results for one of these units. The second section of Chapter Four will contain a day by day reconstruction of a class working through one of the units and their comments on the evaluation forms. This description is one of the results of the field testing portion of this study. The suggested changes in the lessons and teaching guides were compiled as a result of the field testing.

Limitations

In order for this study to be considered in the proper perspective, the following limitations must be kept in mind.

The study was small in scale involving as it did approximately 75 subjects in one school over a time span of four months. It was limited to one teacher/investigator who presented the classroom material, gathered the data and performed the data analysis.

This fact that the investigator was also the classroom teacher makes the data obtained idiosyncratic and limited to the study itself. The input of the students may have been affected by the fact that the investigator was also their teacher.

The subjective data from the student evaluation forms and the classroom field notes were analyzed by the investigator, not an impartial evaluator. The results and findings from the study are therefore not free from the subjective bias of the investigator.

These limitations indicate that the results of this study are not generalizable to the population as a whole.

Five Pedagogical Tools

The third component of this study involved student opinions concerning the use of five specific teaching tools. Field testing the materials written for this study and implementing the suggested teaching methodology provided the investigator with information concerning this teaching approach. Two questions were considered. What is the effect of this style of teaching on male and female students? Do student views towards components of this teaching style change over the course of time? In order to answer these broad questions, students were queried to determine if the impact of this style of teaching was the same for all students or if it varied with gender.

The suggested teaching approach involved five teaching techniques not commonly used in secondary school mathematics classes: use of small groups to solve problems cooperatively, use of computer software such as LOGO and The Geometric Supposer(s), use of writing to learn mathematics, use of manipulative materials, and a change in the role of the teacher from one who explains and gives information to one who asks questions, listens, and facilitates discussion.

Information was gathered to help answer the following more specific questions. Each question was designed to elicit information concerning one of the five pedagogical tools used in the implementation of this approach.

1. Do students identify group work as a positive, neutral, or negative influence on their learning of geometry?

2. Do students identify the use of the computer software (The Geometric Supposer(s) and LOGO) as a positive, neutral, or negative influence on their learning of geometry?

3. Do students identify the use of writing as a positive, neutral, or negative influence on their learning of geometry?

4. Do students identify the use of manipulating actual objects as a positive, neutral, or negative influence on their learning of geometry?

5. Do students note the role of the teacher as a facilitator not as a giver of fact as a positive, negative, or neutral influence on their learning of geometry?

Data Collection

To gather data on these questions the investigator asked students to complete a questionnaire expressing their views toward these five pedagogical tools. This form was titled "Summative Evaluation". (See Appendix D.)

The first questionnaire was distributed early in the term before any of the field tested materials were used. This initial form provided a base line from which to measure differences over time. The same questionnaire was used at the end of this work, after the fourth unit was completed.

The students were asked not to write their names on the form but to indicate their gender. The investigator explained that she was interested in knowing if the results showed any differences between the opinions of male and female students in her classes taken as a whole.

During the completion of this questionnaire the teacher answered clarifying questions and encouraged the students to express their true feelings, reminding them that she would not be able to identify anyone by name. During the discussion the teacher/investigator explained that the blank line was provided in order that students would be able to write in a class activity not previously listed. She also indicated her interest and willingness to use the information provided to modify or adapt class practices.

Data Analysis

During this third phase of the study, data were derived from the student questionnaires and was analyzed by looking for general trends and sorted by gender. The investigator was particularly interested in noting any differences between the perceptions of male and female students as to the usefulness of these teaching methods.

In order to determine any change of opinion, a comparison of the data received at the beginning of the study and the data recorded after the fourth unit was field tested was made. This information was used to describe the general feelings of the students towards these methods. The results of this analysis will form the third section of Chapter Four.

The data were not subjected to any statistical analysis beyond noting general trends. This was in keeping with the objective of this work, to produce a natural history of the process of developing and field testing materials for a geometry unit in a public school setting.

Limitations

Given the small number of students involved and the complicating factor that the investigator was also the classroom teacher for these students, there was no attempt to subject the data that these questionnaires provided to statistical tests. The data were used to note the general feelings of the students. This questionnaire, while not statistically significant, was however a part of the process, a means of recording whatever opinions the students were willing to express at the time. The questionnaire also provided the students with a specific vocabulary with which to discuss details of classroom methodology. The results from this phase of the study are not generalizable beyond the study itself.

Summary

In summary, each of the three stages of this study created data of a different kind. Each kind of data required its own method of analysis. Each stage also required a format for reporting the findings which was appropriate to the kinds of data gathered and analyzed.

I. The Development Stage: Data were in the form of a journal and was analyzed by the investigator who summarized the process and described the materials created. The first section of Chapter Four will report the findings of this stage of the work. This section is a detailed account of the development of one of the four units that was field tested. The description provides a natural history of the design of the lessons, the lessons themselves, and the creation of the teaching guide which illustrates the process of using the learning cycle approach.

II. The Field Testing Stage: Data were from student evaluation forms and the field notes of the investigator. This was analyzed by the investigator comparing the comments of the students on the student evaluation forms to the field notes made after those classes. The second section of Chapter Four includes a characterization of the classroom implementation of the lessons of one unit, the results of the student evaluation forms, a summary of the teacher/investigator's field notes, and the suggestions for the revised lessons.

III. The Five Pedagogical Tools: Data from student questionnaires were analyzed by looking for trends. Data were also sorted by gender. The third section of Chapter Four contains a summary of the data concerning the points of methodology that were investigated. These results will be examined as a whole and also sorted and reported according to gender. The data will also be examined to see if any change occurred over the course of the study.

CHAPTER IV

RESULTS

This study had three components, the development of the units, the field testing of the units, and a survey of students' views on teaching methodology. The results of these components are discussed in this chapter. The first section contains a detailed description of the process of the development of one of the units. The second section is a characterization of a class working through this unit as part of the field testing experience. The last section contains the data from the student questionnaires concerning the use of five specific pedagogical tools.

The Development of the Units

The first component of this work was the development of the units to be field tested. In previous work, the investigator had formulated a learning cycle approach to instruction. This construct of learning asserted that the teacher must first elicit the learner's intuitive beliefs about a concept, then provide experiences for exploration of the concept in various formats and modes so as to move the learner to formalize a new belief about the concept. This learning cycle is discussed in Chapter One.

The investigator organized a curriculum around this principle so as to implement and test this construct of instruction.

Geometry

The curriculum area of geometry was chosen for several reasons. The investigator had broad experience teaching geometry to students of differing ability levels over a period of time. She had used a variety of textbooks and problem solving materials, and was familar with the available computer software. This background meant that the investigator was building on a knowledge base of previous curriculum work.

Another reason for working with geometry was the flexibility it offered. The content of a geometry course is less defined than that of many other secondary school mathematics courses. The course content varies from text to text and school to school. The demands of the curriculum are therefore reduced. This makes it easier to plan a variety of lessons. The teacher can feel a sense of freedom in this curriculum area that is not often felt in other courses.

As a subject matter, geometry is especially interesting. It displays the extremes of being both very physical and also very abstract. It can be approached from a practical, down-to-earth perspective by emphasizing the ways shapes are used in daily life. Equally it can be highly formal and abstract with teacher emphasis on the deductive nature of mathematics. Elucidating the connections between these approaches was appealing to the investigator.

An additional factor in choosing this subject was that most students at the secondary level enroll in this course. So the materials that were to be developed would be used by students of all abilities. This would also indicate that the materials and approach, if successful, are appropriate to students of all ability levels.

Finally, geometry was chosen because there was an extensive amount of theoretical work that had yet to be applied to the geometry classroom. The learning theories of Piaget and the van Hieles provides a solid foundation on which to plan this curriculum. The work of the researchers in cognitive psychology indicated the direction for a teaching methodology. The investigator was interested in determining how to apply these ideas to her classroom.

The Question Guide

In thinking about the learning cycle and reflecting on the work of the theorists in mathematics and geometry education, the investigator was convinced that there was a need for a variety of exercises at the exploration level. The question guide was developed in response to the need of the lesson designer to evaluate the materials being made. This question guide is explained in Chapter One. The guide would help to determine if some aspect had been neglected in the construction of the lessons. It is meant to provide a structure for the lesson writer to be sure that every concept being considered is discussed in several modes. The investigator wanted to test her belief that the question guide would function in this way during the development of the materials.

Classroom Considerations

The lessons that were developed imply a teaching style. The investigator made several assumptions about the style of teaching and kind of classroom that served as a background for this work. This will be discussed here in general and made more specific by the lessons and teaching guides that follow.

A basic premise was that students construct their own understandings based on their previous knowledge and current experiences using the mental representations they prefer. This belief ruled out careful explaining as the primary teaching method.

An implication derived from this premise is that students need to be active in the classroom, actually carrying out experiments on objects, before they can be expected to formalize knowledge and understand it abstractly.

The teacher's role is determined by these beliefs. The teacher's main job is to listen. That is not to say that the teacher does not set the agenda, control the classroom, and organize the materials; but that the teacher must give the students the power to define how to do the task in their own (or their group's) way.

Not only must students be active in the classroom, but they must learn to judge for themselves the correctness or reasonableness of an answer or an explanation. It is not the teacher's job to say "right" or "wrong" but rather to see that the students work on a problem until they are convinced they understand it.

The principles implied in these statements served as the framework for the investigator in the planning of the lessons. What follows is an account of the use of the learning cycle to create the lessons to be field tested. This description is based on a review of the investigator's journal writings. One of those units, Angles In and Out of Polygons, serves as an example of how the lessons were devised using the question guide to implement the learning cycle approach. In keeping with the informal nature of journal writings, the natural history of the development of this unit on polygons will be written in the first person.

Course Constraints

The first activity in the development of the lessons was to take a whole year view and to identify the main units that would be taught. In planning these units, I accepted several constraints. These units should contain material from the geometry course traditionally taught so that my students could take the departmental exams in January and June along with their schoolmates.

The units had to be written so they could be taught to students of all levels of ability. I wanted to use the same materials regardless of the achievement level designation of each class. This was important to me because I believed that the lowest level students had many abilities that were not brought out by traditional teaching. Often curriculum goals are "reduced" for these students. I wanted to make no assumptions about the "limitations" of basic level students but rather I wanted to present the same questions to all my students. My response beyond that would then be based on their method of working with the materials.

Additionally, the lessons had to be completed within the structure of the usual schedule, 45 minute mathematics classes meeting seven out of every eight days throughout the year.

Textbooks if assigned would be either of two commonly used at school. There was to be no sizable purchase of new resources.

All students in every class would do all the lessons in each unit. Even though I did not believe a linear approach through the materials was necessary, I had planned on implementing the units in that fashion.

Given these constraints I started the planning process by perusing the two commonly used text books in my school system, <u>Geometry For Enjoyment and Challenge</u> by Rhoad, Milauskas, & Whipple published by McDougal, Littell and <u>Geometry</u> by Jacobs published by Freeman. A complete list of all the resource materials I used in the development of these units is included in Appendix E.

In the rotating schedule of the school system each class meets for seven days and then skips a day. I determined that the second quarter would contain five of these eight-day time blocks. Assuming that one of these periods would be taken up with semester reviews and testing, I planned on being able to field test four units in the second quarter.

The Content for the Units

I considered the usual content for that time period in geometry; triangle congruence, types of triangles, formal proofs of triangle congruence, parallelism of lines, and quadrilaterals. In keeping with the findings of the researchers I had determined that formal two column proof would be done late in the year (if at all). I also knew that triangle congruence, which is the building block for the rest of the course, was not usually understood by students. So I planned the following content areas for second quarter:

1. Determined: During which the students would build up a physical understanding of the principles behind congruence.

2. Triangle Congruence: During which we would explore the conditions that determine a triangle and the various kinds of triangles.

3. Angles In and Out of Polygons: During which students would investigate the relations among angle measure and shape.

4. Quadrilaterals: During which we would investigate the conditions for making various quadrilaterals and the relations between them.

These units were not developed in sequence. I started with Angles In and Out of Polygons. I made that decision because I felt that I had many ideas on how to approach this topic and I wanted to see if the question guide would help me in organizing those thoughts. I did need to consider what students would be likely to know as they entered this unit, so I jotted down what I thought first quarter work would cover: classification, the use of Venn diagrams, similar figures, geometric vocabularly, and problem solving.

Assuming that Angles In and Out of Polygons would be the third unit in the second quarter meant I would plan on students already completing the units on Determined and Triangle Congruence. It bothered me a little to be working on the units out of sequence, but I was sure I knew where to begin with Angles In and Out of Polygons and I felt a little vague about the Determined unit. I wanted to see how the format would help me and I assumed that what I learned in designing this unit would provide a basis for working on the others.

I did decide to work on one unit at a time. I knew I would be distracted by other ideas I might want to use later, so I created files on my computer for all the topics I planned to field test. This system provided a way of recording useful exercises whenever I came across them and at the same time it kept me focused on my main goal.

In the discussion that follows I will refer to the lessons and teaching guide for Angles In and Out of Polygons. They are in Appendices A and B respectively. I will abbreviate this unit as A in P.

Tiling Lesson #1

I knew how I wanted to open the A in P unit. The introductory lesson would have students physically determine if given shapes tile. This lesson which I called "Tiling" was intended to provide students with an informal sense of the meaning of this concept. By referring to squares and circles, shapes that are very recognizable to the student and ones which fairly obviously do and do not tile, I believed that students could intuit the meaning of tiling. During the unit I would work on developing their abilities to determine a rule for tiling in more complicated figures.

The objective of this lesson was to get students thinking about how figures fit together, to notice the patterns in the world in which they live, and to make them wonder if there is a rule to determine what figures tile and which do not.

Possible Text Book

As I was looking through text books to get ideas, I came upon a text that I had used some ten years before, Geometry A Guided Inquiry by Chakerian, Crabill, & Stein published by Houghton Mifflin. It was intended to be used as a guide for small groups of students working through the geometry content. It does not have pages of text followed by practice problems, but rather the text itself is full of questions for students to answer as they read the book. The students are often directed to discuss what they think with their classmates or to compare their answers to those of others in the class.

This book has another feature which I found compelling. It starts every chapter with a problem to be solved physically, by a scale diagram or actual measurement. This problem is then eventually returned to later in the chapter when more formal knowledge can clarify it. This motivating problem shared some of the traits that I felt were important at the intuition stage. It could make the learner aware of their current understanding of the topic and it called into question what they knew, providing the impetus for further work and a possible change in this belief.

I wondered if this book was still available in class-sized quantities at my school. I was quite committed to working without a textbook, because I knew how dominating a book can be. Two years before when I was implementing a problem solving component in my geometry course, I felt pressure to cover the regular content as well as provide time for my students to work and discuss the non-routine problems. By designing my own lessons away from any text, I felt I had more freedom to decide what the course content was. It would also ensure a greater flexibility in responding to the work of the class. Yet I was intrigued with the idea of using this book and connecting it to my approach. At this point I decided to use the book as a resource of my own but not issue them to students.

Angle Measures Lesson #5

The next lesson I wrote was Angle Measures. I wanted students to form a link between the shape of a figure and its angle measures. I also wanted measurement to convince students that the sum of the exterior angles was always 360 degrees. As I envisioned the lesson, I considered a whole class activity. The data would be recorded on the blackboard and we would analyze it as a group. However, I knew I wanted to use at least one computer exercise during this unit. At this time I had only three Apple computers which I could use and classes of 18 to 24 students. Since I wanted groups of two of three at the computers, I needed to split my class in thirds. I decided to write the Angle Measures lesson in enough detail so that student groups could work through it with a minimal amount of teacher intervention.

Computer Lessons Lessons #2 & #3

I wanted to integrate the use of The Geometric Supposer(s) into the curriculum I was designing, but I felt restricted by having only three computers available. I chose the Supposer lessons from the suggested materials that the school system had purchased. Both of these exercises accomplished my goals. They reinforced the definition of exterior angle, provided a structure for data gathering and analysis, and they anticipated theorems the class would develop in the future. Those were the content goals. I also wanted to have lessons that were easy for the students to accomplish without my help. I also assumed that by this time in the course, students would be familar with the computer system and software and would be able to concentrate on the geometry of the problem.

The LOGO lesson was written as an aside. I was not even sure if the school system owned LOGO. I was somewhat familar with LOGO due to summer coursework I had done. I wanted to design a lesson myself that used LOGO. In reading textbook problems, I noted that the sum of the degrees around a point equalling 360 was implied in many problems but never addressed outright. The motivation for the LOGO lesson grew out of that notion and out of my own experience as a learner in the LOGO environment.

Regular Polygons Lesson #4

The shape of this lesson was motivated as much by the physical constraints of the teaching situation I was in as by pedagogical concerns. In order to make the computer lesson available to groups of three, I had decided to use three class days and to divide the class among three activities. One third of the class would work at the computer, one third would work on Angle Measures Lesson #5, and the remaining groups would construct regular and nonregular shapes. Then over the next two class days the groups would change places, until all students had accomplished all three tasks.

From the readings in cognitive psychology, I knew that most people associate stereotypical shapes with words such as hexagon and quadrilateral. I wanted this lesson to broaden the students' views of these terms. I wanted them to see a spectrum of shapes all labeled hexagons. I also knew that in order for that to happen the students must create the shapes for themselves. The opening questions on the assignment using the Venn diagram leave these questions unresolved. Exercises 6 and 7 are designed to allow students to explore the variety of shapes which satisfy a given condition.

In addition I wanted my students to question the connection between equal sides and equal angles. Intuition might tell them that one demands the other, but that intuition is based on triangles and only triangles. The questions #6 and #7 provide students with an experience to make them question this assumption.

I designed the homework questions to reinforce learnings from the lesson, to introduce questions in case the students did not pose them during the classwork, and to provide an opportunity for informal deduction. The issue of proof bothered me. I had no trouble discarding two column proof from the curriculum. It had been clear to me for a long time that students understand little of the power and meaning of this form of proof and the research evidence supported my experience.

At this point the term 'informal deduction' was appealing and yet meaningless. My work studying the theorists was good background, but it was up to me to interpret their findings. Informal deduction had the sound of something solid, real, and sensible, I could agree that it was a vital step for students when I thought abstractly about pedagogical issues. But the term also had no connections for me at the concrete level, actually working with students. What would informal deduction 'look like'?

I did not resolve this issue as I wrote the lesson. I decided to think about it more later. So I responded to the content oriented goals and wrote true/false questions.

At this stage I was feeling confused. I had created lessons based on my initial ideas but I was out of ideas and clearly the work was not complete. I did like the fact that I had not depended on a text book but I was concerned that I did not know how to implement the informal deduction concept which made so much sense to me intellectually. I decided to analyze the completed lessons according to the question guide. As I did so I recorded those thoughts in the form of a teaching guide for each lesson which would elucidate that lesson's function in the learning cycle. The complete set of teaching guides is in Appendix B.

It was easy to categorize Tiling and the computer lessons. Angle Measure was based on measurement and involved only arithmetic in creating the number patterns. Once I noted that, I realized that it would be necessary to have a lesson which would turn the arithmetic into algebraic statements. The question "What can be expressed?" had not yet been included.

Polygons Formulae Lesson #7

This question motivated lesson #7, Polygons Formulae. I noted that all the lessons written so far were for small groups. I wanted to introduce some variety into the class structure and I also wanted to provide a forum to allow students to show what they had learned to each other. So I planned this lesson to be conducted in whole class format.

The content of the lesson would be determined by the classwork done in the Angle Measure work. This class would be the bridge between the arithmetic and the algebra of the formulae. I wanted students to connect their arithmetic statements ("Subtract 2 from the number of sides.") to the diagrams they made ("There are two less triangles than sides."). There was no way to write this out to guarantee that what I wanted to happen in the class would happen. I wrote out my intention for the lesson, gave myself some hints, but the rest would depend on what the students did in response.

The difference between the lessons that I had written earlier and this one struck me. The other lessons, which I still liked, were very teacher controlled. Students were following my lead and answering my questions. Even the computer work was quite guided. This class lesson did have a specific place to end; that is, when each student had a scheme to determine the number of degrees in a polygon of some fixed number of sides, but I did not write step by step directions for getting there. It would depend what happened in class. I liked that.

I also noticed that "What can be constructed?" had not been addressed. The lessons that I had written did not provide any opportunity, except briefly in the intuitive stage, to work physically with the concept. I was wondering what might be appropriate as I was writing the whole class exercise #7, Polygons Formulae. In a moment of inspiration I noted that the basis for deriving the formulae was the partition of the polygons into triangles.

I wanted to have a physical exercise related to that, but not specific to it. The answer was to develop a lesson using the seven Tangram pieces.

<u>Tangrams Lesson #6</u>

I have used the Tangram pieces before as an example of recreational mathematics and problem solving. As I thought about them now, I planned a way to use them to embody the two principles of decomposing a figure into separate pieces and its reverse, constructing a figure out of smaller figures. While the particular compositions and decompositions in the lesson do not directly relate to the other A in P lessons, I felt the exercise would give my students experience with this principle.

I did not intend to address this principle explicitly, but wanted the students to perform the actions with the figures and to intuit a sense of the principle involved. I was not interested in their ability to articulate this concept. I wanted the experience to speak for itself directly to the physical domain. Therefore the content of the questions was not related specifically to the A in P unit but was designed to have students note properties which remain unchanged and those which do change when figures are cut apart and rearranged.

As I checked the learning cycle and question guide I noted several points. I did have a variety of exploration exercises. I did have a good opening intuitive question. I was lacking deduction, except for the derivation of the formulae, and a final intuitive lesson which would pull things together for students who were ready and which would serve as another level of stage one for those who were not. But I felt that this analysis was not complete.

I decided to see if I could put the lessons in a sequence and see what was missing. After a few tries, I settled on the sequence in Appendix A. I was not sure what computer lesson I would use but I knew it would have to be linked with #4 and #5. I noted that even though I did not plan on writing the units in any kind of order that I had in fact covered stage 1 and stage 2 of the learning cycle but that stage 3 and stage 4 were lacking.

Solve It Lesson #9

As I pondered the sequence I chose to work on a problem solving exercise which would demand integration of the concept of shape and angle measure and "fit". At this point I wrote lesson #9, Solve It, questions #1 through #4. As I imagined my students answering these questions, I realized two points. One, they needed some practice problems on which to base their work here, and two, that these questions are more a test of belief than problem solving. So I moved the category for this lesson to stage 4 and I planned on a new lesson to follow up the Formulae class which would serve as arithmetic practice of the relationships.

Question #5 of lesson #9 was created by a colleague, the Chapter One teacher with whom I was planning to team teach. She found the pattern in a coloring book but realized that it would make an interesting way to test students' understanding of the relations between tiling, regular polygons, and angle measure. I agreed and added her question based on the design to this lesson.

Chart It Lesson #8

Lesson #8, Chart It, was written from old materials I had used in the past. It started out as a drill and practice lesson, but I was intrigued with the idea of having students notice that all mathematics questions do not have an unique answer. So I included that possibility in #4. As I pondered the categorization of the exercise I alternated between arithmetic and problem solving. I did not find this a problem of the question guide but it pointed out that the categorization had more to do with how a student solved the problem rather than the problem itself. For some, this would be an arithmetic exercise, for others, this would be problem solving.

I was still left with the deduction concern. I turned back to lesson #4, Regular Polygons. I thought some more about what proof was. I decided that it was a form of communication, a way to tell someone else how it is you have come to understand something. I added the phrase, "Say how you know." to the true/false questions. This simple instruction struck me as quite profound. Not only would it mean that students could not just guess at an answer, but the form of their answers would indicate the level of their reasoning. They might back up their answer with physical examples from the work they had done or they might refer to the relationships embedded in the Venn diagram. Thus their answers would serve a diagnostic function for me.

I made a final check of the lessons and the learning cycle/question guide. I noted that I did not use the question "What can be changed?". I spent some time thinking what this might mean. How could sequenced drawings be used? I visualized a sequence of polygons with an increasing number of sides, yet roughly the same area. I quickly realized that this lesson, while it was intriguing here would be a great opener for the circle measure unit. I decided to save it. The Angles In and Out of Polygons Unit was completed. Now I wanted to see what my students would do with it.

Field Testing

The four units that were developed using the question guide were field tested in a public secondary school in four geometry classes. The investigator was the teacher in these classes. The students involved in the field testing had been assigned grouping levels based on their previous work in math class according to school guidelines. In order to provide a picture of the mathematical competence of this student population, the investigator administered the standardized exams described in Chapter Three.

The results of the basic mathematics computation test indicated that 48 of the students were at or above grade level and that 17 were below grade level with regard to arithmetic skills. The test results for mathematics concepts showed a slightly more negative picture. 42 students tested at or above grade level and 26 were below grade level concerning their understanding of mathematical concepts. (The difference in these totals was due to student absences and changes in students' schedules.)

This description of the population, that about one fourth were below grade level in mathematical skills and that one-third were below grade level in understanding mathematical concepts was actually more positive than the NAEP results of secondary school students in general (Carpenter at al., 1987). This difference is likely due to the fact that geometry is a course taken by college-intending students. This assessment was accomplished before the field testing took place.

The first quarter work was also completed before the field testing. The topics covered during this time period included the following: classification, the use of Venn diagrams, similar figures, geometric vocabulary, use of protractors, and problem solving. The second quarter units, Determined, Angles In and Out of Polygons, Triangle Congruence, and Quadrilaterals were presented by the teacher/investigator using the methodology made clear by the teaching guides.

During these weeks of field testing, the teacher/investigator kept field notes. After each class, she would write a brief statement concerning the activity and the student reaction to it. At the end of the day these comments were collected and expanded on in the journal writing. At the end of the unit (all lessons) the class was asked to complete the student evaluation form concerning the lessons. (Appendix C contains a sample of this form.) This information was compiled and compared with the teacher's field notes. On the basis of this information suggestions for revisions were noted.

The account that follows is a characterization of a class working through the Angles In and Out of Polygons

Unit. In order for this description to be as rich as possible, it does not follow a single class through this material but rather blends events that happened in each of the classes. The teacher/student dialogue is reconstructed from the field notes of the teacher. It is not intended to be a word for word account of the conversation, but rather to provide the reader with the flavor of the interaction.

This description was written by the teacher/investigator after reading the field notes, the journal entries, and the student evaluation forms for the unit. This description will be in the first person from the point of view of the teacher .

Classroom Implementation

There were fifteen minutes left to a class when I distributed the Tiling exercise. I suggested to the students that they work with a partner to start the assignment and then complete it at home. The immediate student response was: "I don't know how to do this. You haven't taught us anything yet." I asked if they had read the directions. Most said, "No." I asked them to read the directions and then ask questions as they start the work.

As the period ended students were discussing with humor the implications of a floor covered with circles. I felt that I had accomplished the main objective of this lesson. I wanted them to note the real world relations of geometric shape and begin to think about the issues involved in tiling.

On the second day I started the three day round robin of lessons #3, #4, and #5. This series of exploration lessons was designed to provide experiences to prepare students to formulate an algebraic method to determine the relationship between the number of sides of a polygon and the number of degrees in its interior and exterior angles. The computer exercise provided a special case of triangles with which most students were familar and introduced the vocabulary of exterior angle in this familiar setting. The Regular Polygons exercise was designed to help students note the relationships and non-relationships involving the side lengths, angle measures, and shape of polygons. It also communicated some important vocabulary. The Angle Measures lesson would provide the inductive evidence from which the formulae could be generated. At this time I did not have LOGO available so lesson #2 was not assigned.

As I considered this plan I was confident that I could supervise all three activities. My students had done some previous work on the Supposer, so I knew they could use the software appropriately. But I wondered if it would seem confusing to the class to have so much going on at once. In order to relieve that concern, I came in early to prepare the room so that the opening of class would be smooth. I wanted the physical arrangement to support the activities. I placed three chairs around each computer in the back of the room. I set up the middle section to accommodate three groups of three and the Regular Polygons materials. The front of the room would be for everyone else and Angle Measures.

As the students entered class, I collected the Tiling assignment and informed the class that we would be working in groups of three for the next three days. I allowed them to choose their own groups. I also let them choose which activity they were to do today. While some students were eager to use the computers and some were not, I noted no discernible pattern by gender.

Interior and Exterior Angles Lesson #3

Ten minutes into the class, everyone was busy on their own activity. I noted that each of the computer groups did work well together, but they did no talking across groups. I did intervene once in this lesson to help students with the Extend Option on the Supposer menu. The Supposer exercise took the whole class period for my students. Some groups did not finish all parts. I was disappointed that there was no time left for discussion, but pleased with the involvement of each group in this activity.

Regular Polygons Lesson #4

The students working on this lesson found it very hard to work on problems #5 and #6. I spent considerable time interacting with these groups. They were to construct accurate models of the shapes required, either by drawing them with ruler and protractor or by building them with wood strips. The students wanted to draw a diagram free hand and just to mark it to show that it met the constraints.

As I watched their work I was struck by the fact that they were really doing what is common in text books and in my own teaching. I often drew diagrams that were not to scale and simply marked the attributes I knew. I knew I could ignore the unimportant information it might contain, but I was not convinced that my students' abstract ability was as refined. I felt they would not fully comprehend the unconnectedness of the ideas of equal sides and equal angles unless they had explored the range of possibilities physically.

I drew a triangle and marked the angles 40 degrees, 70 degrees, and 60 degrees. I indicated that the side connecting the two smaller angles was 6 cm long. I requested that they make this triangle. Some students tried and failed. They looked at their attempts and explained why the shape I had drawn was not possible. I used this opportunity to talk about the misleading information on the diagram and to reemphasize that when they physically tried to build the object, not only did they find out that it could not be made, but also they were able to explain why not. I asked a student in each group to explain what process the group would use to solve #5 and #6, asked if all group members concurred and left them, now that the process of how to complete the assignment was clear to all.

Angle Measures Lesson #5

As I checked with the groups working on Angle Measures, I noted that many of then had not followed the directions step by step but that they had completed columns four and five as they went along. They had no trouble second guessing what those columns were for. I decided to ignore this at the time being but made a note to revise the exercise.

I sat in with each group as they began to work with generalizing the arithmetic to further cases. Every student was able to continue the pattern by adding some fixed amount of 180 degrees, but extending this pattern to 102 sides was difficult for more than half the students. I listened in to a conversation similar to this exchange:

S1: Look, it's just 102 times 180.
S2: Wait, how can that be? A triangle

isn't 3 times 180

S1: We already know that a triangle is 180. We don't have to worry about that.

S2: I guess that's right then. What else could it be. Write that down, 102 times 180.

These students paid little attention to any patterns built in the chart. They focused on the specific question and were looking for a specific mathematical operation that would produce it. When one of them did try to produce a counter example by looking at the extreme case with which he was familiar, it was clear that both students were satisfied by considering that as a special case. Since they already knew this fact, it was not related to the new work.

As I listened to this I was intrigued by the interplay of logical and nonlogical thinking that this exchange provided. I tried to imagine what sense these students would have made out of my reasoning had I taught this unit by presenting the formulae at the board. This one small conversation reinforced my belief in a style of teaching that demanded that the students do mathematics, not listen to it being done by the teacher.

But it was difficult to accept where they really were. Just as I would have expected that they would have followed my presentation, I did expect that they would see all the useful relationships. But their rules include eliminating special cases from consideration instead of building from them. As a teacher I realized that I would not have considered this kind of logical misconception in a teacher explanation mode of presentation.

Another group was having more success, I turned to them in time to see 18,000 degrees being recorded as the answer. I asked them how they arrived at that answer and one student showed me. He turned over his paper and I saw that they had added 180 repeatedly until they had arrived at 102 sides. One group member asked me why I would ask them to do such a dumb, time wasting thing. I asked if they had found any shortcuts to their work and the first student admitted that they had added up groups of the 180 degrees five at a time. He offered this to me as if I would think they were cheating. Every few additions they had added 900 degrees and five sides. I pointed out that that was a time saver and asked if there was a way to simplify the work even more. In spite of my prompting they were unable to proceed further.

I indicated that I would like them to share their approach to the class when it came time for a whole class discussion. I was hoping that the class would be able to expand on this procedure and recognize the multiplicative nature of the problem. While my comment to consider further shortcuts did not produce much, my request that they share this method with the class did. This group was now quite confident and went over to the first group to explain how to do the problem. The first group decided to change their answer to 18,000. As far as I could determine, the reason for this change was simply because I had indicated interest in this second group's work.

No one had yet noticed that 18,000 was 100 times 180, nor had anyone paid any attention to any geometric objects such as the polygons on the exercise sheet or a triangle. I decided to wait until they had the results from the exterior angles to press them for a more geometric solution. I was hopeful that the degrees of 360 would trigger a geometric relationship. The summary questions also asked them to state their conclusions in words. I hoped that this effort would help students note the relationship.

The next two days, different groups worked through the same three activities. I made a special effort to inform the Angle Measure groups to follow directions exactly. I suggested that each group appoint a reader who would read the directions aloud one at a time and not proceed to the next step until the previous one was complete. In these classes the work went smoother. However the content was similar to what happened on the first day. The computer activity continued to be too long and the Regular Polygon lesson was challenging but useful. They were all serving as exploration exercises.

I was frustrated at this time. I did enjoy running the round robin type of class. I was satisfied with the small group process. I enjoyed the interactions I had with the groups and was pleased to note that the students accepted me in this less "talky" role. But I was frustrated with the lack of discussion among the groups. There had been no opportunity for the groups to analyze what they had done and share it with the whole class. I decided not to include the follow up questions after the computer work since some groups did not finish. I felt those questions prompted further thought and exploration and did not want to give them up, but I also wondered if my students were ready for them at this point.

I was also confused about what I wanted from these exercises. Was I falling into the trap of expecting students to generate abstract understanding too quickly? If I meant them as exploration, didn't they really accomplish that task? The thoughful analysis which would lead to deduction could and perhaps should come later. I had been quite convinced that students need sufficient time to explore in several modes before formalizing. What occured to me now was that I had this belief intellectually but that I needed to pay attention to what was happening in my class to act on this knowledge concretely.

As I considered the lessons I wrote and the response of my students, I began to feel that my intuition about the classroom had led me to the correct response as a teacher. The students should not be rushed through the exercises and asked to analyze when the level of thinking they were illustrating in the daily work showed that these exercises were challenging for them. The discussion and formalization could wait. I decided to give my students the time to gain valuable physical experiences without the urge to sum up in a formula within 30 minutes. So I decided that the round robin was successful.

One of the difficulties with the round robin was that it took close to a week for all students to complete the activities and this delayed any whole group discussion of this work. At first I was quite concerned about this, but after noting the level of thinking in the classroom I came to the conclusion that the delay would actually be beneficial. They would be able to look at the results of their work from a distance of time. I collected the results of their work daily, read through the papers and held on to them, planning to return them at the opening of the whole group discussion which I hoped would lead to the formulae.

Tangrams Lesson #6

Before that whole class discussion, I had planned for the class to work with the tangrams. I remembered that I added this lesson as an after thought to this unit. Now that the class had actually worked through the first 5 lessons, I came to see how absolutely necessary it was. Reflecting on the discussions the groups had concerning the 102 sided figure and their mostly arithmetic approach, I knew that the process of partitioning a polygon into triangles would be alien to them without some experience of that type.

I had also decided that I would include discussion time in this lesson, even if it meant that all students would not complete all questions. I would stop the activity after 30 minutes to provide students with time for a whole class discussion. My reason for wanting this had more to do with pedagogy than content. I was not even concerned with the topic for discussion, that would depend on the class, but after three days of all small group work, I wanted to provide a time when the class would operate as a whole, listen to each other, and participate in a discussion.

To begin this activity I drew sketches of the seven pieces on the front board and labeled them. A and B were the two large triangles, C and E were the two small triangles, G was the medium size triangle, D was the square and F was the parallelogram. I asked each pair to form a triangle out of pieces C, E and F. I have found in the past that this is a good preliminary exercise. Even though it involves only three pieces, it brings out a number of good points. One is that there is more than one way to accomplish the task. Another is that this particular example illustrates how the pieces fit together compactly, and it also provides a model to illustrate how to record the answers.

After a brief discussion of their answers, they worked in pairs on the worksheet. Most students enjoyed this activity. This exercise can be very frustrating. My main job was to circulate and determine which pairs were getting stuck and which needed clues to continue. I discovered much variation in the room concerning the giving of clues. I made a quick numerical check of student reaction to clue-giving. About two-thirds of the class were adament, "Don't tell me. I don't want to see how it is done. Let me try some more." I liked that. I wished they accepted that attitude as part of learning mathematics. I was trying to figure why they had this attitude here. Part of the answer was that they were sure they could solve the problem. All they had to do is move the pieces and eventually the solution would be there. Most students do not believe that this is true in math in general. That is, they do not believe that they could figure it out without the teacher giving them the answer.

The remaining third of the class was equally split among groups who would ask for a clue after working for a while and those who simply said, "I never could do these kinds of things." I changed some of the pairs to split up those who felt completely unable to do anything. That seemed to help, but a lot depended on the willingness of the partner to talk about their train of thought.

Many students did work on all the questions in the 30 minutes. Of those who did number 8, about half said that the perimeters were the same. I decided to focus our whole class discussion on that question. It was difficult for the groups who had not finished to stop working and join in the group discussion, but eventually everyone did.

I asked a student to read problem #8 aloud to focus everyone's attention on it. Various groups came to the board and drew their diagrams for the four shapes. I asked for a show of hands, "How many say they all have the same area?" It was unanimous. "How do you know?"

S1 They are all made up out of the same pieces.
S2 They have to be same, I just took one part of the rectangle and moved it to make the triangle.
S3 They are all the same as 16 little triangles

Next I posed the same question about the perimeters, "How many say they all have the same perimeters?" About two-thirds of the class said yes.

S1 If they have the same areas then they have the same perimeters.
S2 If they are made up out of the same pieces then they would have

to have the same perimeter.
S3 They are all made up out of the same pieces.
S4 What is perimeter, anyway?

I was intrigued by the fourth response. I believed that it indicated a crack in this student's rather unsophisticated beliefs about measure. I was sure that this student was aware of what perimeter was, but was beginning to realize that he had answered my question without considering that. He had simply decided that all measures would be the same in this case. Now he was wondering if that was true.

I asked the class, "What is perimeter?" They responded with the distance around the figure. I asked how to measure that and they decided to use the length of the hypotenuse of the small triangle. Each section of the room made one of the four figures and then measured the perimeter in terms of the given length. They decided the perimeters were not the same.

Suddenly the class was over and everyone left. I had wanted to pose a homework question that would ask them to note which figures had the larger perimeters and which the smaller. I wanted them to explain why. That would have to wait until another day.

Angles with LOGO Lesson #2

I was checking in the tangrams work, and thinking about the whole class discussion, when a colleague came into the math teachers' office with the news that she had found a way to make the LOGO software work on some of the computers in our computer room. It would only work on nine of the machines, but since pairs could work together I decided to test it out the next day.

The next class I asked students to work in pairs, or triples, if necessary. I told them to be sure that someone in their group knew what LOGO was. It turned out that this was all that was needed. LOGO is so easy to learn, at least the graphics commands we would use, that I never gave any instruction in the language itself.

The students found it easy to work through these problems. They enjoyed the sense of control they had over the machine. They could make it do what they wanted. I overheard students noting, "Look, it just goes around 360 and then it's back where it started." Most students finished the tasks on the lesson and then spent time trying to make their initials and other designs. A student who wanted to make an "N" said he thought it was pretty tricky of me to assign work with LOGO. I asked him what he meant. He said, "It seems like just a game, just fooling around, but you have to pay attention to the angles if you want it to come out right." I was sure that the student evaluation forms for this exercise would be highly positive, but I was wondering how all these exercises would come together into a whole concept. The next class was planned to accomplish that.

Polygon Formulae Lesson #7

Several days had passed since students had worked on the Angle Measures assignment, I decided to focus this class on a new question, rather than consider it as a discussion of that work. I did not return their Angle Measure assignment because I did not want them to refer to the numerical results from Angle Measures. I had them sit in groups of four but started with a question to the whole class.

I drew a convex but irregular seven sided figure on the board. I marked every one of the seven angles, then asked, "If I measured these angles and added them up, how many degrees would I get? How would you convince someone that you were right, if you couldn't just measure?" In a few minutes, they were convinced that the answer was 900 degrees. I challenged each group to provide an explanation at the board and gave them fifteen minutes to work together.

During this time I checked with each group. About half of them were partitioning the figure into triangles. In one of these, there was an argument about whether you could do this starting with an interior point or whether you had to draw diagonals. When I came over they asked me which was right. I told them they had to decide as a group.

One of the other groups was working from a triangle and adding segments until they had a seven sided figure which looked something like what I had drawn. I drew a different looking seven sided figure and asked them if their method would still work. They said they would check it out and started to draw again.

The other two groups were stumped. I sat in with them and asked them to list all they knew about angle measures. One of the things they listed was that a triangle had 180 degrees. I asked them if there was any way they could use this fact to help them and one person said, "Maybe we could see how many triangle we have?" As I left they were drawing in random diagonals to make triangles.

I called time to end the small group work and asked each group to present their solution. A student from one group volunteered. She drew all the diagonals from one vertex and counted the triangles. (Case 1) The students in the group which had been arguing about how to partition the figure said that they had done it a different way. A student from that group drew triangles from a point in the 'center' of the figure making seven triangles and sat down.

S1 But that's not 900 degrees.
S2 You don't count all that extra stuff. You subtract 360 degrees. (Case 2)

A third student who seemed very confused said that they had done it very differently. He showed their method of drawing line segments on a triangle and adding 180 each time. (Case 3)

I summarized by asking them to help me write out arithmetic sentences for each case:

Case 1 (7 - 2) times 180 degrees
Case 2 (7 x 180 degrees) minus 360 degrees
Case 3 180 degrees + 4 times 180 degrees

Next I posed the question about a figure of some unknown number of sides. What if we called the number of sides n? I told them to give an explanation which referred to their method of calculation. We ran out of time so this became a homework question.

The next day we continued this discussion. Now each method had its advocate.

Case 1 For n sides, I can make n - 2 triangles. Each triangle has 180 degrees, so the figure has (n - 2) times 180 degrees.

Case 2 For n sides, I can make n triangles. But I have some extra, 360 degrees. So the figure has 180 degrees times n minus the extra 360 degrees.

Case 3 I start with a triangle. It has three sides and 180 degrees. For each side more I add 180 degrees, so the figure has 180 degrees plus (n - 3) times 180 degrees.

I was impressed. Not everyone in the room understood all three ways, but the explanations were presented based on the triangles and polygons that the students had drawn. Each generaliztion was backed up by a specific diagram. I asked about the exterior angles.

S1 Just like LOGO, its just 360.
S2 I noticed when we measured, the more angles there were the smaller each one was. They added up the same, 360 degrees.
S3 It's because you are getting to a circle.

The step to formalization was made by many students at this class. I was not sure if the students who had been quiet were making the same conclusions. I planned on listening to them during this next assignment to check that out.

Chart It Lesson #8

As students were working on this assignment I noted that it implied the use of the (n-2) version of the formula. Even though I had listed openendedness as one of my objectives, I had not anticipated the variety of responses my classes created. I had them eliminate column two. I wondered how they would recieve the last two columns. I had not addressed the connection between regular and angle size in the discussion at all. However, it was not a problem. As one student said, "It is just a division problem."

Students had little trouble picking out #4 as the problem which could not be done. I had imagined some confusion regarding the difference between no solution and no unique solution, but a student referred to the earlier work on the Determined Unit to explain to her partner, "It has an infinite number of solutions, you remember, like the triangle with only two sides given." I was glad to have heard this conversation because it provided me with a connection I had not made before. Not only did I have a better way to help students who were having trouble here, but I learned something new myself. The class ended before students had finished so this assignment became homework.

As they came in the next day I saw that students had varying success with problems #5 and #6. Someone who had them figured out offered to explain. "You work with the outside angles. They are easier since they always equal 360 degrees. That is the trick."

Solve It Lesson #9

The questions in this lesson were good follow up questions to the previous days work. Students who were still unclear about the relationships had an opportunity to manipulate them. I did wish that I had asked the question in a different order though, #3 and #4 should have been first. Some students tried to answer #1 and #2 without referring to the formulae at all. They wanted to try to physically make them and see what would happen. The difficulty with this solution was expressed by one student, "I can't tell if I can't make it or if it can't be made."

This lesson brought out the differences among students who had integrated the concepts of this unit and those who had not. The question about the design was especially telling. Student responses varied from:

S1 How can I tell? Do you want me to measure?
S2 Yes, they all look it.
S3 They can't be. It doesn't make 360 degrees.
S4 Not regular but they are equilateral.

Several points struck me as I listened to the students explain their beliefs to each other. I noted how much of what was important in the class would really be considered problem solving. Students with skills at approaching problems were able to make more sense out of the exercises than those who had poor problem solving skills. This led me to wonder what was the difference between learning mathematics and learning problem solving?

I noticed the variety of understandings that they displayed. It was clear that even though they all had done the same work, the results were different. I also noted that students who did not intuitively see the answer to the design question were unimpressed by the correct solution. They simply were not ready to understand that yet.

Some students had proceeded through the stages of the learning cycle: intuition, exploration, deduction, and intuition and had reached a new level of belief concerning this concept. Other students were still at the exploring stage and would need some additional experiences before they reached the next level.

Lesson Revisions

Now that the unit was over, I had a list of changes that I wanted to make before I used the lessons again. I decided to list the changes before investigating the student evaluation forms.

Tiling Lesson #1: This needs to be changed so that students will not be bound by the rectangular edges. Some students rejected the Greek Cross because of this. Students need an image of an infinite floor. As an addition to this or as a followup to it, I'll have students find tiling patterns at home or at school, copy them and display them in the classroom.

Tangrams Lesson #6: I would like to add a writing assignment as a homework, to create a question that would give students a chance to talk about feeling frustrated in math class and what they do about it. "How did you feel today when you were making figures out of the tangrams and were having difficulty? Do you feel the same way in class? What do you do when you feel like that?"

Interior and Exterior Angles Lesson #3: This lesson should be modified so that the class will have some discussion time and time to share results.

Angle Measures Lesson #5: I want to change the table so that directions and format are easier for students to follow. I will add to question #5 that students should check with another group here to see if they agree. I will add a teacher checkpoint after question #8 so that I will interact with every group here.

Chart It Lesson #8: I will change or remove the second column so that all possible formulae can be accommodated. The chart should reflect the variety of possible approaches to the question.

Those were my suggestions for changes but I also wanted to see what the students wrote on the evaluation forms. I wondered if we would agree on any of the same points and I was curious to see on which points we would disagree.

Student Comments

The overall impression I have when reading the forms is that students are not used to noticing what happens in class except in very broad terms. Many of the comments are of the order, "I liked this, it was easy." or the partner comment, "I didn't like this, it was hard." No students made specific suggestions for change, other than "Make it easier." or "Make it shorter." This was a little disappointing to me as I had hoped for more substantive comments on the specific lessons.

However, student comments concerning the teaching style were more interesting. The comments that follow indicate the variety of responses from all four classes.

Several students commented on working in groups.

> "She lets us get involved together by letting us work as a team together in groups."

> "It helps because if I can't do the homework I can hear how other people solved it instead of just finding out the answer."

> "I like it when we work in groups. We can share ideas and explain problem answers to each other."

No students spoke negatively of the group work but several expressed frustration at some of the tasks and the teaching style.

> "She (the teacher) half explained something and then ran off."
>
> "I feel that she thinks that we should already know what to do."
>
> "I find it frustrating when we start a new section. You give us problems to work on before you explain them."

These students were reacting negatively to the stage 1 intuition level of this process. As a teacher it is important to be aware of student frustration levels and to provide emotional support for those who need it, without reducing the educational value of the task itself. Determining the appropriate amount of teacher intervention is not easy and the comments on the evaluation forms indicate this difficulty.

Another set of comments were related to the computer lessons. S will be used to indicate comments directed to <u>The Geometric Supposer</u> lesson and L will indicate comments made specifically concerning the LOGO activity.

> S "I was able to experiment with different angles with ease."
>
> S "I hate computers. I work on them anyway."
>
> S "There weren't enough computers to go around."
>
> L "Watching things on the computer where you can do it yourself is better."

L "I could use my logic to rationalize the computer actions."

The computer views expressed were in agreement with the field notes from the computer lessons. Most students found a sense of freedom and power in the activity. A few students would prefer not to use them at all.

Several students spoke of the value of building models and drawing diagrams. However, others did not like this physical approach. The comments below indicate the spectrum of opinion.

"I like things I can get my hands on."

"I love to draw my own shapes."

"Hand out objects to work with instead of just paper."

"Activities with actual objects take too long."

"I hate dealing with objects in real life."

"I can't draw well so it doesn't help me."

Finally some of the comments spoke generally about what students felt was happening in the class.

"She lets us do our own work. We teach ourselves."

"She's creative, but it is hard to ask questions about everything I don't know."

"It is nice, being able to know why we were doing it."

"I didn't not like anything, except maybe too much writing."

This set of comments, while not providing any clear direction for changes, made me feel encouraged in another way. As I read the comments and studied my field notes and journal summaries, it was clear that the students and I were all in the same room. I felt that my interpretation of the class was on base with their comments. Some of the changes I created would deal with the difficulties we had determined. Some of the changes highlighted the positive aspects we agreed on.

At the same time some of the difficulties would remain. There were areas on which we would not agree. This style of teaching was still unusual; some students still had the conception that I should just tell them how to do the work and let them practice. Some of their comments spoke to this difference of opinion concerning the definition of teaching.

It should be noted though that many of the comments quoted indicate student awareness of teaching methods. Data on student views toward five specific teaching tools are reported on in the final part of this chapter.

Five Pedagogical Tools

The third component of this study involved student opinions concerning the use of five specific teaching tools. These teaching techniques (which are are not commonly used in secondary school mathematics classes) are: use of small groups to solve problems cooperatively, use of computer software such as LOGO and <u>The Geometric Supposer(s)</u>, use of writing to learn mathematics, use of manipulative materials, and a change in the role of the teacher from one who explains and gives information to one who asks questions, listens, and facilitates discussion.

The implementation of the learning cycle approach to teaching geometry included these five pedagogical tools. Units created according to the format of the question guide would necessarily include these teaching styles. The field testing experiences would expose the students to these approaches in the context of the lessons. The investigator was interested in determining if student views towards these teaching methods would be the same before and after the field testing experience.

Additionally, the investigator wanted to determine if student opinion on these techniques varied with gender. Results reported in chapter two provided some indication that these tools might be more accepted by female than male students.

To gather data on these questions the investigator asked students to complete a questionnaire expressing their views toward various class activities. This questionnaire, "Summative Evaluation", can be found in Appendix D. The title was chosen so that students would understand they were to evaluate each type of class activity on the basis of its relation to their learning.

The questionnaire was designed so that each of the five teaching methods would be included. Question #1 and Question #6 were meant to point out the difference in teacher behavior. These two questions were intended to reflect student opinion concerning the role of the teacher.

Question #2 dealt with computer use. No distinction was made between the use of LOGO and <u>The Geometric Supposer(s)</u>. Question #3 recorded student opinion toward working in cooperative groups. Question #4 elicited responses concerning the use of writing to learn mathematics. Question #5 provided students the opportunity to express their views on using actual objects, constructions, or manipulatives.

The students were asked to indicate which of these class activities had a positive influence on their learning, which had a negative influence, and which they considered as neutral.

The first questionnaire was distributed before any of the field tested materials were used. The table that follows contains the results of that initial survey.

Table 1 Initial Evaluation Form Results

	Positive	Negative	Neutral	Total
Explainer	26	5	24	55
Computer	21	14	21	56
Groups	39	4	13	56
Writing	36	11	9	56
Manipulatives	30	5	22	57
Facilitator	36	4	15	55

Several points can be made by considering these results. First, the student responses indicate a generally positive feeling toward all of these activities. Considering both positive and neutral responses shows that most of these students felt that the class activities supported their learning. Given that overall feeling, it is not clear if they effectively differentiated between these activities.

Second, both teacher roles are accepted, with some preference shown for the teacher as facilitator. This question was placed in the context of small group instruction and some student response may have been made on that basis without much analysis of how the teacher was responding. These positive responses may also be interpreted as students indicating overall satisfaction with the teacher. The data does not indicate if the students noted differing teacher behavior.

Third, the most negative responses occurred in reference to the computer. This activity created the most divergent views. As many felt neutral concerning the computer as felt positive. The computer activity also drew the largest negative response.

Fourth, the responses to the writing question showed more negatives than any other except the computer question. Also less students felt neutral concerning this question than any other.

Table 1 indicates the views of the students on each of the pedagogical tools. In Table 2 and Table 3 these results are separated by gender.

Table 2 Initial Evaluation Form Results Males

	Positive	Negative	Neutral	Total
Explainer	16	3	15	34
Computer	15	8	11	34
Groups	20	3	11	34
Writing	21	9	5	35
Manipulatives	20	3	13	36
Facilitator	22	0	11	33

Table 3 Initial Evaluation Form Results Females

	Positive	Negative	Neutral	Total
Explainer	10	2	9	21
Computer	6	6	10	22
Groups	19	1	2	22
Writing	15	2	4	21
Manipulatives	10	2	9	21
Facilitator	14	4	4	22

Several points are clear from this breakdown of the data. First there was a 3 to 2 ratio of males to females in the students questioned. Since this questionnaire was completed in regularly scheduled geometry classes, it indicates that more males than females were enrolled in these classes.

Second, one difference noted is in the reports concerning the use of small group work. The female students were strongly positive about this teaching device. Only a few did not indicate positive, yet the males were more split. Only a few of the males indicated a negative response, but about one-third responded in the neutral category. It would seem then that this activity was more important to the females than to the males.

Third, the response to the computer question showed the largest difference between the two groups. As many females indicated this activity as positive as indicated that it was negative. About half of the females marked the neutral response to the computer use question. The males also displayed a variation of opinion on this question, with one-fourth of them stating a negative reaction, and about half indicating a positive one.

Fourth, the use of writing brought out negative responses from approximately one-fourth of the male but only one-tenth of the female students. The positive and neutral responses were about the same for the two groups.

In considering the teacher as facilitator issue, the male students were all positive and neutral, yet the female students indicated a negative response in one-fifth of the responses.

The use of manipulatives showed differences between the two groups. In the male responses about two-thirds indicated positive and the remaining indicated neutral, yet the female response was much more equally split. About one-half were positive and one-half were neutral.

Table 4 Final Evaluation Form Results

	Positive	Negative	Neutral	Total
Explainer	26	6	23	55
Computer	30	10	16	56
Groups	40	3	14	57
Writing	20	17	19	56
Manipulatives	30	7	19	56
Facilitator	45	3	7	55

The data in Tables 4, 5, and 6 indicate the responses of students after the field testing component of this study. The numbers show some variation from tables 1, 2, and 3 due to changes in students' schedules. The time interval between the two surveys was four months.

In comparing the results of Table 1 and Table 4 three differences are apparent. The biggest change was in student feeling towards writing. More than one-third of the students changed their view in this category. That change was decidedly negative. The responses in the neutral and negative columns were increased from what they had been at the expense of the positive responses. This indicates that students see little connection between the writing they do and what they learn in mathematics. Their view may have grown more negative after the field testing since they were asked to do more writing during those experiences.

A second trend occurs in the computer oriented question. A more positive response to computers can be seen after the field testing work. Still, it is important to note that one-sixth of the students indicated a negative reaction to the computer component.

The third result of interest is found in the category of teacher as facilitator. One-seventh of the students moved from a neutral view of the teacher in this role to a positive one. This shift resulted in the teacher as facilitator receiving the highest proportion of positive responses. Almost 90% of the students had a positive response to this item on the final form.

Tables 5 and 6 indicate the data from Table 4 broken into male and female groups.

Table 5 Final Evaluation Form Results Males

	Positive	Negative	Neutral	Total
Explainer	16	3	14	33
Computer	17	5	11	33
Groups	23	2	9	34
Writing	11	10	12	33
Manipulatives	18	4	11	33
Facilitator	27	2	4	33

Table 6 Final Evaluation Form Results Females

	Positive	Negative	Neutral	Total
Explainer	10	3	9	22
Computer	13	5	5	23
Groups	17	1	5	23
Writing	9	7	7	23
Manipulatives	12	3	8	23
Facilitator	18	1	3	22

These results show some changes from the earlier data. In the teacher as facilitator question both the males and females shifted views to a more positive one. This change showed no difference due to gender.

The computer question did indicate a shift that was different for the two groups. The male students started out positive in this category and they made a small positive shift noted in Table 5. The female students however changed more noticeably. Table 6 indicates that the number of female students marking this positive after the field testing had doubled. It is interesting to note however, that these changes came from the neutral responses. The one-quarter of the females who had marked negative originally did not change.

Writing was the category that displayed the most negative change. There was little difference between the male and female students, this shift to the negative was over the whole population.

Remaining relatively unchanged over time were student views towards manipulatives and small groups. The differences between the male and female students that were noted after the initial survey remained the same.

Summary

Question 1. Do students identify group work as a positive, neutral, or negative influence on their learning of geometry? The data indicated a generally positive response to this question with female students more strongly positive than the males.

Question 2. Do students identify the use of the computer software (The Geometric Supposer(s) and LOGO) as a positive, neutral, or negative influence on their learning of geometry? The overall data indicated a mixed response prior to the field testing and a more positive view after the field testing experience. Male students were positive intially and remained so. There was a trend in the female group to react more positively to computer use after the field testing.

Question 3. Do students identify the use of writing as a positive, neutral, or negative influence on their learning of geometry? The overall data indicated a negative trend in the entire population, both male and female students giving more negative views after the field testing.

Question 4. Do students identify the use of manipulating actual objects as a positive, neutral, or negative influence on their learning of geometry? The data indicated some differences between male and female students on this question. Male students were more strongly positive while female student responses were more neutral. This did not change after the field testing experience.

Question 5. Do students note the role of the teacher as a facilitator not as a giver of fact as a positive, negative, or neutral influence on their learning of geometry? The data is confusing on this question. There is an overall postitive view towards both teacher roles discussed. There was a shift of the population to an even more positive response to the teacher as facilitator after the field testing. This shift was the same for both male and female students.

These results provide a sense of student views toward the teaching styles they encountered. Analysis of these results and suggestions for further work are contained in Chapter Five.

CHAPTER V

CONCLUSIONS

The Development of the Units

The first part of this study involved the development of units of curriculum based on the question guide which had been devised by the investigator. One objective of the study was to determine the usefulness of the question guide as a format for lesson design.

The application of this question guide to geometry lesson construction was illustrated in the work reported in Chapter Four. The purpose of the guide was two-fold: one, to provide an overall structure for an entire set of lessons based on one concept, and two, to describe the mathematical format and the physical context of each lesson within that concept.

The guide was found to be useful in several ways. It enabled the teacher/investigator to review the lessons that had been designed to teach a particular concept by providing an analysis of each lesson according to the type of mathematics it used and the type of educational context necessary for its application. This analysis resulted in a description of the lessons that had been constructed.

Classifying the completed lessons in this way and comparing them to the overall structure of the question guide indicated to the investigator the mathematical formats and physical contexts which had not yet been addressed in the group of lessons. Thus the question guide served the function of alerting the lesson designer to the type and content of lessons which should be created in order that the group of lessons approach the concept in a variety of mathematical formats and physical contexts.

The analysis that the guide provided served another function as well. The lesson designer was able to generate lessons by considering the relationship of the question from the guide to the concept to be addressed. This process was illustrated by the application of the "What can be changed?" question to the concept of Angles in Polygons. The analysis indicated that this question had not been included in the lessons developed at that point. As the investigator considered the meaning of the question relative to the concept, the idea for a lesson involving changing polygons into circles was formed. The role of the question guide here was to focus the concept into a particular format and a new lesson was formed as a result. Thus the question guide was used to help generate lesson ideas, not just analyze them.

The question guide also provided a structure for coherence without forcing a rigid daily pattern. This guide provided the curriculum planner with a format for designing lessons concerning a concept without imposing a linear structure. A new model of curriculum grew as a result of this work. A flexible curriculum plan could be devised in this format. The concept to be addressed would be identified. Lessons for each question would be designed using the guide as a format. Once the curriculum had been created, it would be the teacher's choice of lessons that would determine the daily work for the class. The question guide provided a sense of freedom for planning each lesson while insuring that each lesson was connected to the overall concept. The question guide gave a structure with flexibility.

A model of this flexibly designed geometry curriculum could serve as the basis of a school system inservice plan. All teachers would add lessons to the curriculum and all teachers would have access to all the ideas in the unit. In this way teachers would encounter not only the content of their colleagues' lessons but also the classroom format and structure which applies to the lesson.

Sharing the results of applying the lessons, suggesting revisions in them, and creating additional lessons would provide a natural, informal, and practical method for teacher development which would emphasize the cooperation among teachers and which would allow each teacher to build on their existing strengths and teaching personality.

Field Testing

The second component of this study was field testing the lessons and implementing the learning cycle as a structure for teaching methodology. The classroom described in Chapter Four illustrated a class working through the learning cycle on one unit of lessons. Field testing provided the teacher/investigator with actual experience in implementing these constructs in the classroom.

The most significant conclusion was that the learning cycle categorization of the lessons, while helpful, blurred in the reality of the classroom. A lesson which was at the intuition stage for one student could have been exploration or even formalization for another. The student response to the lesson determined the category, not the lesson itself. This indicated the need for the teacher to have access to a variety of lessons designed for all stages. It highlighted the usefulness of the flexibly designed curriculum described above.

This variation in student response also illustrated that the learning cycle analysis could be used as a diagnostic tool. Teachers could apply this in two ways. One, by considering the class as a whole, this analysis would provide the teacher with information on which to plan the next lesson. The decision making of lesson organization would be enhanced. Second, teachers could use this analysis on an individual basis, to determine the level of understanding of each student in order to decide the most appropriate format and context for the next lesson. Thus the learning cycle provided not only a structure for classroom methodology but also more sophisticated information for the teacher on which to make decisions.

The investigator found the learning cycle a useful construct of learning. It provided a format for the implementation of lessons, guided daily classroom decisions, and indicated student progress. The connectedness of the learning cycle and the question guide was reinforced by this study. The model of curriculum proposed above would be based on the premises of this learning cycle. This study indicated the power of these constructs of learning to the teacher. It demonstrated that this adaptable curriculum implemented through the learning cycle approach provided a teacher with a structure for conceptually based mathematics classes.

The Five Pedagogical Tools

The third component of this study involved gathering student opinion on five specific teaching methods. These teaching strategies were critical to the implementation of the learning cycle approach to the classroom. The intent was to determine if the views of the students changed after the field testing stage. In addition the investigator was interested in knowing if the views of the female and male students varied.

In Chapter Four the investigator described the data that was gathered on the five questions. The conclusions for each question are discussed below.

1. Do students identify group work as a positive, neutral, or negative influence on their learning of geometry?

This study indicated an overall positive response from the total population to this teaching technique. This response was stable, it did not change after the field testing. The response of the female students was more strongly positive than the males.

The implementation of a teaching methodology using group work would be favorable received according to these results. The data indicated that cooperative small group work may support the learning styles of female students.

This teaching technique may be helpful in bridging the gap between the achievement levels of male and female students.

2. Do students identify the use of the computer software (The Geometric Supposer(s) and LOGO) as a positive, neutral, or negative influence on their learning of geometry?

The initial results showed a clear difference between male and female students on this question. The data after the field testing experience showed no change for the males but a trend to the positive for the females.

The study indicates a postive feeling on the part of most students towards integrating computer use into the geometry class. The difference between the male and female views changed after the field testing indicating that after experience with the The Geometric Supposer and LOGO, female students responded positively to the effect of computers on their learning.

The data also showed, however, that this positive shift was from the female students who had indicated an initial neutral view. The female students who indicated a negative response originally remained negative even after the computer experiences. The conclusions from this aspect of the study remain mixed. Further work is needed to determine the role of the computer in the mathematics learning of the female students.

3. Do students identify the use of writing as a positive, neutral, or negative influence on their learning of geometry?

The data indicated a neutral to positive view toward this aspect at the beginning and a trend to the negative after the field testing experiences. This shift was common throughout the population of the study and showed no clear differences between male and female students.

A possible conclusion from this study is that these students did not find that the use of writing impacted positively on their learning of mathematics. The shift toward the negative would be accounted for by the observation that they were required to do more of this task during the field testing stage. It is important to note however that writing is hard work. Many students commented on difficulties they have writing in a mathematics class. It is not clear from these responses if students were reacting to the difficulty of the work or to its impact on their learning.

Another difficulty in interpreting this data is that this study involved a short time period. It may well be true that students did not have sufficient time to develop their writing skills to the point where the writing was helpful to them. The results of this study are inconclusive on this question of using writing to teach mathematics.

4. Do students identify the use of manipulating actual objects as a positive, neutral, or negative influence on their learning of? geometry?

The data was stable on this question, no change was noted after the field testing experience. There was a difference in the views of the male and female students. The female students were neutral, the males positive.

Evidence from the background reading had led the investigator to the conclusion that the female students would benefit from manipulation with actual objects. One interpretation of this data is that the female students did not perceive the benefit from these activities. Another possible interpretation is that the differences between males and females were reinforced rather than reduced by these activities. Research to determine the gender-related use of manipulatives to teach mathematics at the secondary school is needed.

5. Do students note the role of the teacher as a facilitator not as a giver of fact as a positive, negative, or neutral influence on their learning of geometry?

Students reacted positively to both teacher roles noted in the initial survey. After the field testing there was a shift to an even more positive view of this teacher role. These views were similar for both female and male students.

The data showed that students do not appear to contrast these two teacher roles. It may be that students do not differentiate betweeen the roles or that they react positively to both of them. It was clear that there was no gender-related difference noted in student response to teacher roles. Since students accepted the teacher in the facilitator role even more positively after the field testing experience, it is likely that this style of teaching would be received in a positive manner after students have had experience with it.

Suggestions for Further Research

The usefulness of this question guide should be explored further. It would be interesting to see if other teachers find it as helpful as the investigator did. Research to determine the value of this guide and learning cycle to mathematics teachers in general would be an important next step.

A further area of study would be to determine how to use this question guide and learning cycle approach in non-geometrical aspects of mathematics. For instance, what modifications in the guestion guide would be necessary to design a conceptually based algebra or precalculus course.

The issue of evaluating student understanding has not yet been addressed. The implementation of a conceptually based mathematics program has ramifications for the process of assessment. Is it possible to create mathematics tests which are wholistically scored, similar to the tests being used for assesssment in the writing process? Research investigating this possibility will be needed as classroom teachers implement a conceptaully based program.

This study looked at five specific teaching strategies and indicated the reaction of the male and female students to these approaches. Further research should be done to determine if any of these teaching tools support female learning and can be used to reduce the gap in the achievement and attitudes of male and female students.

Summary

This study resulted in a set of geometry lessons and teaching guides which were designed using the question guide and were implemented by the learning cycle approach to methodology. The indications from this work are that the question guide and learning cycle are powerful constructs for devising, planning and implementing lessons in geometry.

The field testing, student evaluation forms, and summative evaluation forms provided student reaction to this teaching style and materials. It is clear that components of this style are considered favorably: use of groups, use of computers, and differing teacher roles. Use of manipulatives was received with mixed feelings by the students. The use of writing was not considered as a positive aspect in this study. Yet, when considering the teaching style as a whole, student response was positive.

In conclusion, it is important to realize that the key to the cohesiveness of the three phases of this study is that they are interwoven and are designed to be so. The development of the lessons was the curriculum content, the field testing was the methodology component, and student opinions of the teaching tools represented the learners themselves. Life in the classroom is an amalgam of these elements: the content, the pedagogy, and the learner. The learning cycle and question guide approach to conceptually based teaching integrates all three aspects into the structure of curriculum planning. In order for conceptually based teaching to be effective these three must be dynamically connected. This study points the direction on a bold new path.

APPENDIX A

UNITS OF STUDY

Types of Triangles

Use rulers, protractors, compasses, graph paper, etc. to construct two different examples for each triangle described below. Your drawings should include the measure of each angle and the length of each side of every triangle.

Warning: Some of these descriptions are actually impossible to create. In those cases show what happens when you try to make them and explain why no such triangle can exist.

Reminder: Draw and measure two triangles for each case.

1) A right-scalene triangle

2) A right-isosceles triangle

3) A right-equilateral triangle

4) An obtuse-scalene triangle

5) An obtuse-isosceles triangle

6) An obtuse-equilateral triangle

7) An acute-scalene triangle

8) An acute-isosceles triangle

9) An acute-equilateral triangle

THE DETERMINATOR

What does it mean to say something is "determined"?

In today's experiment, you will be using pieces of wood (The Determinator!) to build some simple structures. In each case you want to figure out how many solutions exist; that is, how many different structures are possible which fit the requirements of the given situation?

- Are there <u>no</u> solutions?
- Is there only <u>one</u> solution?
- Are there <u>two or more</u> solutions (some definite number)?
- Is there an <u>infinite</u> number of solutions (within a range)?
- Is there an <u>infinite</u> number of solutions (no limitations?)

For each set of given conditions, state how many solutions are possible. Include sketches or descriptions of the solutions. Label the measurements on your sketches.

1) Given conditions: four-sided figure with side lengths of 18", 14", 12", and 10" (in that order). Leave the pieces together to use for #2.

2) Given conditions: five-sided figure with side lengths of 18", 14", 12", 10", and 15" (in that order).

3) Given conditions: triangle with side lengths of 18", 14", and 12".

4) Given conditions: triangle with a side length of 18", an angle of 50, and a side length of 6". Be sure the 50 angle is <u>between</u> the two known sides.

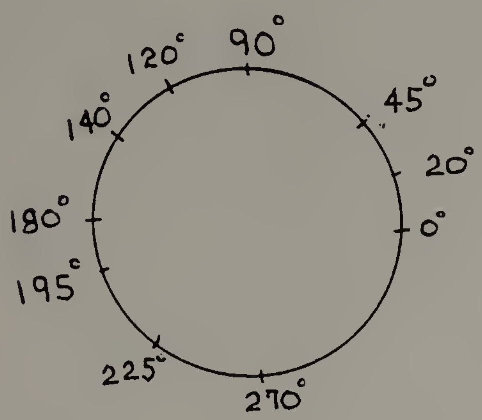
5) Given conditions: triangle with a 50 angle, 14" side, and 12" side. Be sure the 50 angle is <u>not</u> between the two known sides.

Suggestions for using The Determinator

- The pegboard holes are exactly one inch apart (center to center); use the holes to help you measure lengths.

- When one of your sides is an unknown length, use a fairly long piece of wood for that side.

- On the angle-fixing blocks, the settings are as follows:



You can use various combinations to get different sizes of angles.

Examples: the angle between 20° and 45° is 25°; from 140° to 195° is 55°.

- To mark off a length, you can insert a thumbscrew just far enough to stay in the piece of pegboard.

- Once you've marked off a particular length or fixed an angle, use a sticker (A or S) to show that it's fixed.

Algebra-Determined

WARM-UPS

Find numbers for X that make each statement true. List up to five for each statement.

1) X + 3 = 5

2) X + 3 = 3 + X

3) X + 3 = X + 5

4) X = 9

5) X < 3

Remember the POSSIBILITIES

1) no solution
2) only one solution (determined)
3) two or more solutions (some definite number)
4) an infinite number (within a range)
5) an infinite number (no limitations)

Look at the warm-ups and decide which category describes their solutions.
Do this now...write it down...have it checked!

INVESTIGATE each statement given to decide which category of numbers of solutions it belongs to.

1) 4X > 3X

2) 3(X + 9) = 3X + 27

3) 3(X + 9) = X - 18

4) 3(X + 9) = (-54 + 3X)

5) 6X = 5X

6) X(X - 1) = 0

HOMEWORK

In today's classwork we used five categories to describe the number of solutions to algebraic statements. In the past we have used Always, Sometimes, Never, as a system of categories.

1) Study your Warmups and Class Exercises and identify them as Always, Sometimes or Never True.

2) Show how to regroup our five categories by using Always, Sometimes, and Never; ie, which belong with A, which with S, which with N.

3) Make a geometric statement which illustrates each type.

UNIT 1 DETERMINED

Triangles-Determined

Remember the possibilities for how many different structures you can build?

1) no solution
2) only one solution (determined)
3) two or more (some definite number)
4) an infinite number (within a range)
5) an infinite number (no limitations)

Use each set of information below to try to construct triangles. Be sure to check if more than one kind of triangle can be made. Then choose one of the five cases above to describe the situation. Hand in drawings and answers.

1) RED $\angle E = 90^\circ$, $\angle R = 30^\circ$, RE = 4 cm

2) GRN $\angle G = 35^\circ$, GR = 6 cm, RN = 4 cm

3) BLU $\angle B = 70^\circ$, $\angle L = 60^\circ$, $\angle U = 50^\circ$

4) YEL $\angle Y = 70^\circ$, $\angle E = 60^\circ$, $\angle L = 60^\circ$

5) BRN NR = 7 cm, BR = 6 cm, BN = 5 cm

6) WIN $\angle W = 80^\circ$, $\angle I = 120^\circ$, WI = 2 cm

7) YES $\angle Y = 60^\circ$, YE = 7 cm, YS = 4 cm

Homework Questions

1) Explain what happens in each no solution case.

2) Explain what happens in each infinite case.

3) Explain what happens in each more than one solution case.

4) List here the sets of information which <u>determine</u> triangles on the basis of this work.

UNIT 1 DETERMINED

The Geometry of Laundry Racks

If nothing else, your geometry course ought to give you a deeper understanding of laundry. Today you will be examining one of several laundry-related structures. When folded up and put away these structures are not determined. Only when you set them up to be used do you get something that won't move around.

Your job is to figure out what it is that changes the structure from its undetermined state to its determined state. Why does it go from being movable to being rigid?

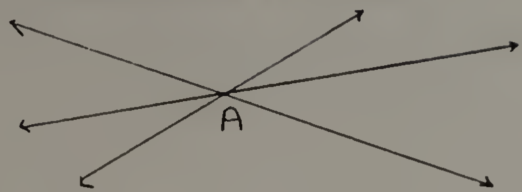
To do this analysis use The Determinator to make a model of the structure and then make sketches from that. Don't forget to use the S and A stickers to show when a side or an angle is fixed.

Then, base your discussions on one or several of the handy little observations listed below. Remember, one observation might have lots of implications for your structure. Follow the argument all the way through.

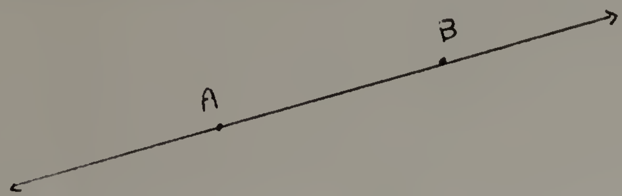
You will turn in one report for your entire group. (Put everyone's name on it.) Keep this sheet and take it home to use with your homework.

Handy Little Observations

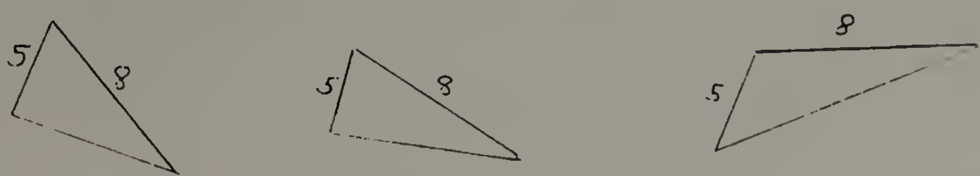
. One point does not determine a line.



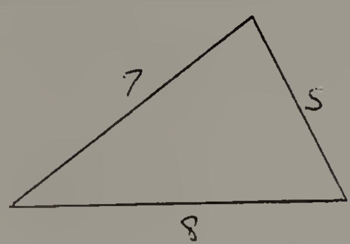
. Two points determine a line.



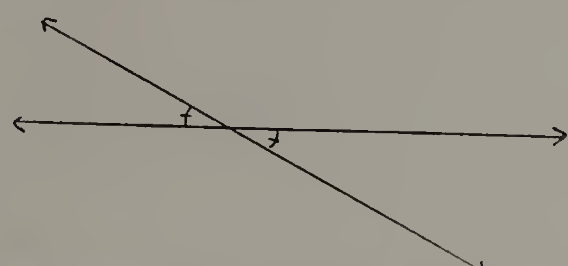
. Two fixed side lengths do not determine a triangle.



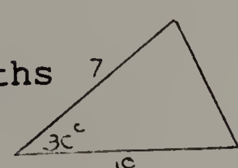
. Three fixed side lengths determine a triangle (SSS).



. Fixing an angle automatically fixes the one opposite it (Vertical Angles).



. A fixed angle between two fixed side lengths determines a triangle (SAS).



UNIT 1 DETERMINED

Two Sides of a Triangle

I. Create some (at least four) triangles with sides of 5 and 8. Make scale diagrams of them. (Everyone needs to have them.)

1) Measure the lengths of the third side in each case.

2) What is the longest measurement you found?

3) Do you think it is possible to make a 5-8 triangle with a third side longer than the one you found in problem 2?

4) Is it possible to make a 5-8 triangle with a side of 100?

5) Is there a limit on the largeness of the third side of a 5-8 triangle. If so, what is it?

6) What is the shortest measurement you found?

7) Is it possible to create a 5-8 triangle with a third side shorter than the one you found in # 6?

8) Is it possible to create a 5-8 triangle with a third side of 1 1/2?

9) Is there a limit on the smallness of the third side of a 5-8 triangle. If so, what it is?

II. Repeat these steps for a triangle with sides of 8, 8. Answer questions 1-9 for that case. How did your answers change in the isosceles case?

Homework

1) Look at the lengths of the third side in each triangle you made. Write a rule that states what you know about the third side of a triangle if you know two sides are 6 and 12.

2) Write a rule about the third side of a triangle when know the two sides are 7 and 7.

3) Measure the angles in your triangles. You should get 180. Right? Now notice the size of the angles and size of the sides. Write <u>two</u> statements describing the relations you see about the size of the angles and the size of the sides.

UNIT 2 TRIANGLE CONGRUENCE

Another Look at Determined

1) Use the wood strips to build a figure with these measurements: 6 inches, 9 inches, 8 inches, and 10 inches. How many different structures can you build that meet these conditions? Do the lengths of four sides determine the angles of a figure?

2) Use the wood strips to build a five sided figure by adding a side of 7 inches to the figure you made in number 1 above? How many different structures can you make now? Are the size of the angles determined by the lengths of the sides in this five sided figure?

3) In problem 1, you looked at a figure with four given side lengths. Problem 2 had five given side lengths. Based on what you saw with those figures, do you believe that a figure with <u>six</u> given side lengths would be determined? Say why or why not.

4) Look at these two sketches:

Are these two different triangles? Explain carefully.

5) Given conditions: Point B is 3 cm away from Point A.

• A

How many solutions can you find to satisfy the given conditions for Point B? Is point B determined?

6) Given conditions: a triangle has one side length of 5 cm and another side length of 3 cm. How many solutions can you find for the triangle? Is the triangle determined by the length of two of its sides?

Solving Triangle Problems

Solve each problem by making a careful scale drawing (let 1 inch = 1 mile). In each problem, compare your drawing with those of other students.

Problem A. Dave and Ann are 4 miles apart. Dave sees Ann and he also sees a certain oak tree. The angle formed by drawing the line from Dave to Ann and drawing the line from Dave to the tree is 43 degrees. The tree is 3 miles from Ann. How far is the tree from Dave?

Problem B. Bill and Al are 3 miles apart, but they can see each other. Each can see a statue. The angle between the line from Bill to Al and the line from Bill to the statue is 73 degrees. The angle between the line from Al to Bill and the line from Al to the statue is 51 degrees. How far is the statue from each person?

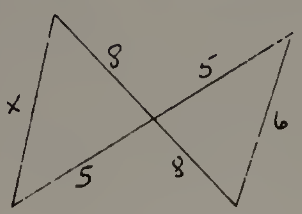
Problem C. Mike can see an elephant 4 miles away and a donkey 3 miles away. The angle between the line from Mike to the elephant and the line from Mike to the donkey is 126 degrees. How far apart are the two beasts?

Problem D. Jane and Mary are 2 miles apart; each can see the other. Both see a ship at sea. The ship is 1.75 miles from Jane and 3 miles from Mary. What is the angle between the lines drawn from the ship to Jane and Mary respectively?

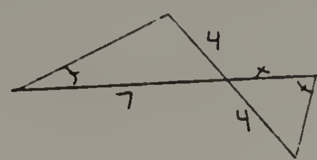
Finding Missing Parts

I. Use what you have learned so far to find x without measuring. Explain how you arrived at your answers.

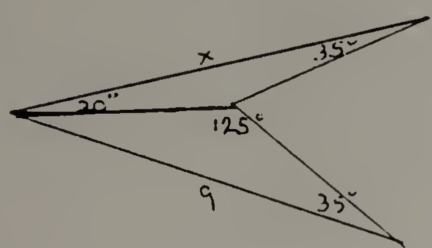
1)



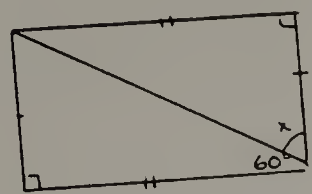
2)



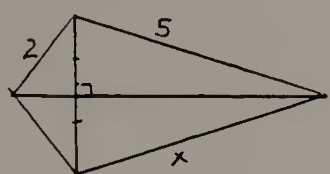
3)



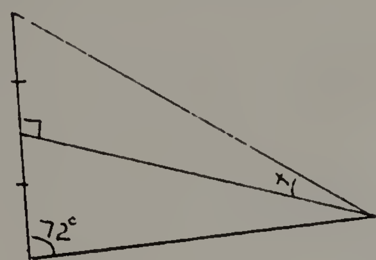
4)



5)



6)



II. Triangle FAT is congruent to Triangle PIN

For each case determine the value for the variable and calculate the missing part. Explain the geometric reason behind each equation that you write.

7. FA = 2x + 3   TA = x + 1   FT = 3x + 3   PI = x + 4
   Find x, IN, and FT

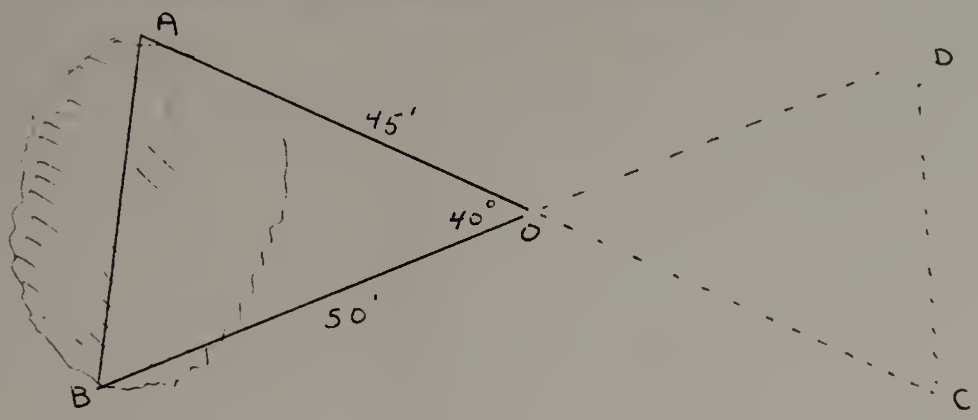
8. <A = y - 6   <P = 3y   <I = 2y - 16   <N = y + 7
   Find y, <F, and <T

9. FA = 4x + 18   PN = 2x + 4y   PI = 12x + 2
   FT = 3y + 4x   <A = 6x + 10y
   Find x , y, and <I

10. TF = 1 - x   NP = 19 + 5x   <T = 90 + 10x
    <F = 45 - 5x   <A = 5x + 75
    Find x. What do you know about triangle PIN?

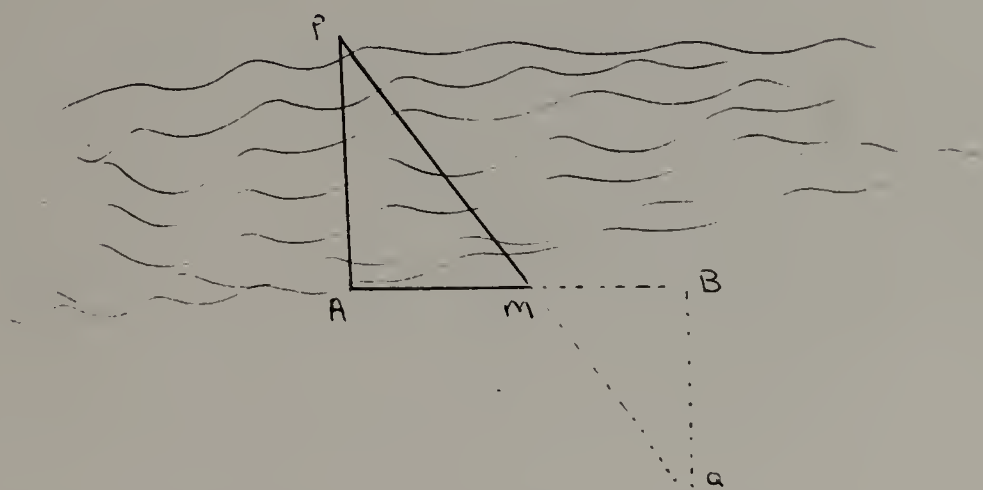
Using Congruent Triangles

1) Explain how to use congruent triangles to construct triangle DOC so it could be used to find the distance AB across the lake.

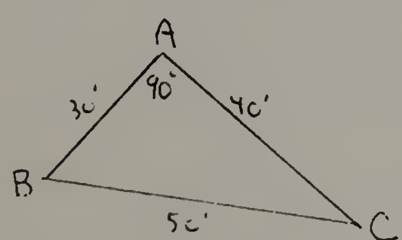


2) Explain how to construct triangle MBQ so that it could be used to find the distance AP across the river.

<A = 90°
AM = 80m
<PMA = 40°



3) Explain how to construct triangle BCD so that angle BDC is guaranteed to be 90.



UNIT 2 TRIANGLE CONGRUENCE

Line Segments in a Triangle

1) Cut out each triangle on the attached sheet.

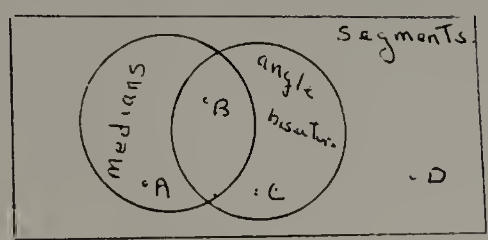
2) Use a protractor and a ruler to figure out which triangles are scalene, isosceles, or equilateral; which are acute, right, or obtuse. Then, on the <u>back</u> of each triangle write "scalene isosceles" or whatever is appropriate for that particular triangle.

3) Use folding to help you draw the following medians:

in triangle FBI, median from vertex B to segment FI
in triangle COW, median from vertex O to segment CW
in triangle USA, median from vertex S to segment UA
in triangle ZPG, median from vertex Z to segment PG
in triangle EQL, median from vertex E to segment QL

Write the word "median" in fairly small letters somewhere along each median.

4) Now go back to each triangle listed above and use a different kind of folding to help you draw an angle bisector from the indicated vertex. Label it also. Try to be very careful about labeling so that it's quite clear which segment is a median, and which is an angle bisector.



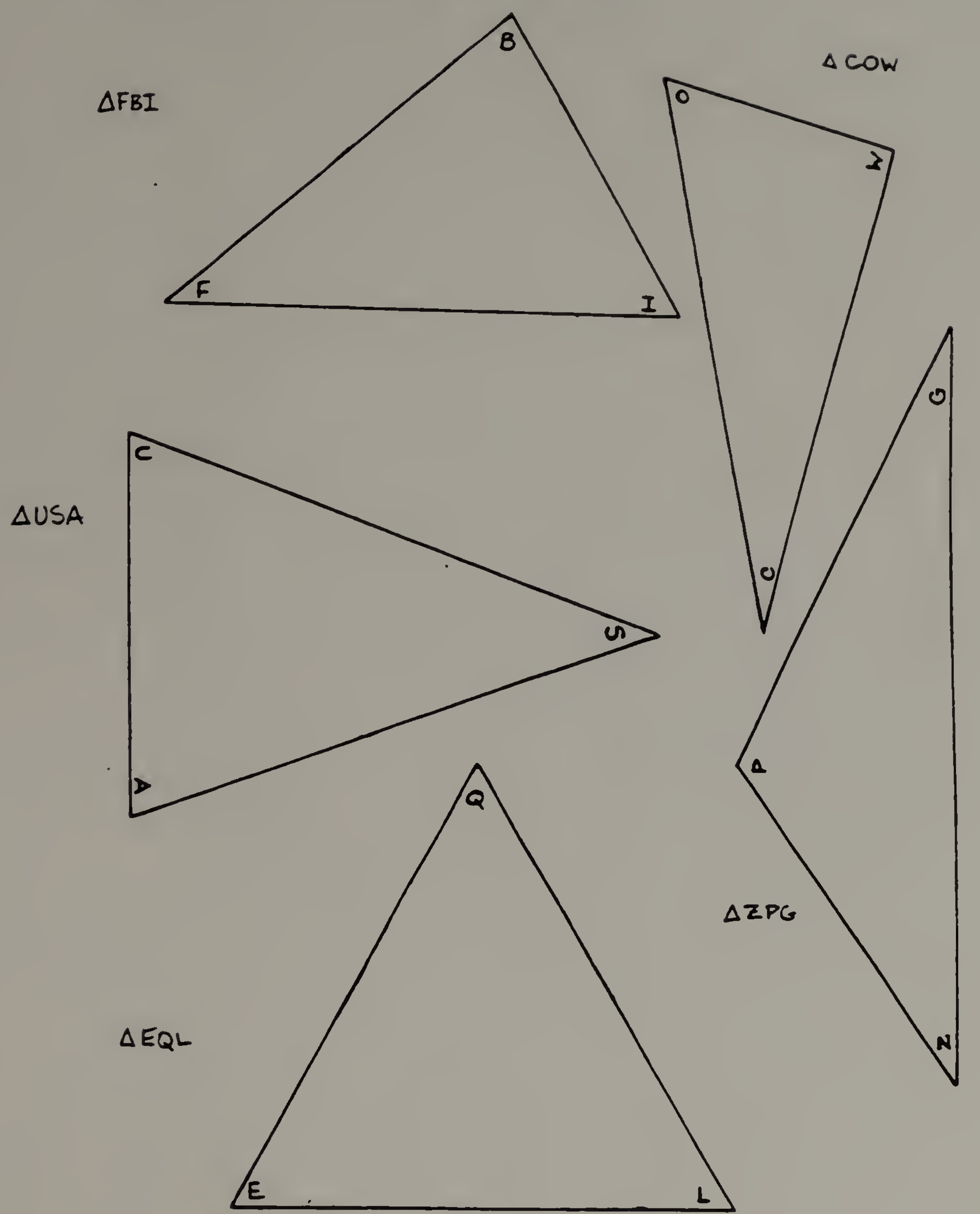
5) What is this diagram saying about medians and angle bisectors?

6) Is it true, according to what you found with your triangles? Give examples from each category.

7) Now go back to triangles EQL and USA. Draw another set of medians and angle bisectors but this time start from vertex Q in triangle EQL and from vertex A in triangle USA. Does it make any difference which vertex you start from?

8) When is a median also an angle bisector? Answer ALWAYS, SOMETIMES, or NEVER and explain on a separate sheet.

CUT OUT EACH TRIANGLE



UNIT 2 TRIANGLE CONGRUENCE

Segments in Triangles-Proof

1) Draw and label a triangle in which an angle bisector is also a median. How do you know you are right?

2) Congruent triangles can also be used to verify this. For example, we start with an equilateral triangle and an angle bisector. We identify the given conditions:

1.

2.

Draw an equilateral triangle.

Draw an angle bisector.

Mark the given conditions on the diagram.

Are there any congruent triangles here?

How do you know?

What does that tell you about the other parts of the small triangles?

What does that tell you about the angle bisector?

3) Follow the format of problem 2 for this situation:

In an isosceles triangle the median to the base is also an angle bisector.

Start this way:

Draw an isosceles triangle.

Draw the median to the base.

Continue in the format of problem 2.

Tiling

This unit of study is about geometric figures which do and which do not fit together nicely. Notice the floor of the classroom. It is covered with square tiles. The squares fit together nicely at each corner. They do not overlap. They do not leave any space uncovered. We say then that a square "tiles".

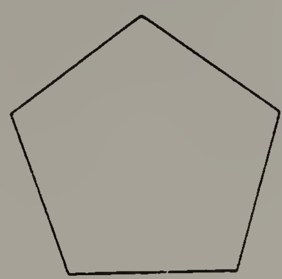
Imagine that the floor was covered instead with circular shaped objects. This would not be very efficient, would it? There were be lots of area to be filled in. We say therefore the circles do not "tile".

Your job is to study the figures drawn below and to determine if they tile or not. Remember for these examples you must use only shapes congruent to the one given. You may find that tracing paper will help you decide.

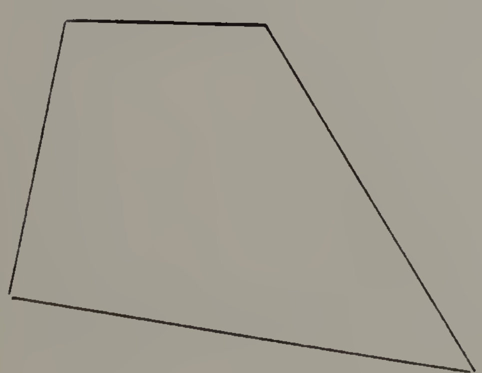
For each figure given, write YES if it does tile and NO if it does not.

GREEK CROSS

PENTAGON



QUADRILATERAL



TRIANGLE

UNIT 3 ANGLES IN AND OUT OF POLYGONS

Interior and Exterior Angles of a Triangles

I. One Exterior Angle in Each Triangle

1. Construct an acute triangle ABC.

2. Draw an extension of side BC such that BA = AD.

3. Measure all angles and record the measures in a diagram.

4. Use Repeat to do this for several triangles.

5. For each triangle, measure the interior angles of the triangle and the exterior angle CAD.

6. State your conjectures about the relationship between the exterior angle and the interior angles of the triangle.

II. All Three Exterior Angles in a Triangle

7. Construct an acute triangle.

8. Draw all three exterior angles.

9. Measure all three exterior angles.

10. Record your drawings and measurements.

11. Repeat the steps for other types of triangles. (Use the repeat feature.)

12. State your conjectures.

13. What is the sum of the measures of the three exterior angles for an acute triangle?

14. Is this sum the same for all types of triangles?

Tangrams

1) Identify each of the seven pieces as precisely as you can. i.e., A is an isosceles right triangle.

2) How many small triangles cover square D _______?
The area of square D is _________ small triangles.
The area of parallelogram F is ______ piece C. The area of triangle G is _______ small triangles.
What do you know about the areas of pieces D, F and G?

3) Form a trapezoid using pieces C, D, E.
Sketch a diagram to show how the pieces fit.
The area of this trapezoid is ________ small triangles.

4) Form a rectangle using pieces C, D, E.
Sketch a diagram to show how the pieces fit.
What do you know about the area of this rectangle?

5) Form a square using the pieces C, D, E, F, G.
Show a sketch.
How many triangles G would it take to cover the square?
How many of the small triangles?
Make a square using the pieces A and B.
How many triangles C would it take to cover the square?
How many of the small triangles?
How would you describe this square and the square you made from pieces C, D, E, F and G?

6) Consider pieces A and B.
How are they the same?
How are they different?

7) Consider pieces A and C.
How are they the same?
How are they different?
What other tangram piece is like A and C? Why?
Each side of A = _________ each side of C.
The perimeter of A is ________ the perimeter of C.
The area of A is _________ the area of C.

8) Use all seven pieces to form a square, rectangle, a trapezoid, and a triangle. Show sketches to illustrate how the pieces fit together.
What do you know about the areas of these shapes?
What do you know about their perimeters?

UNIT 3 ANGLES IN AND OUT OF POLYGONS

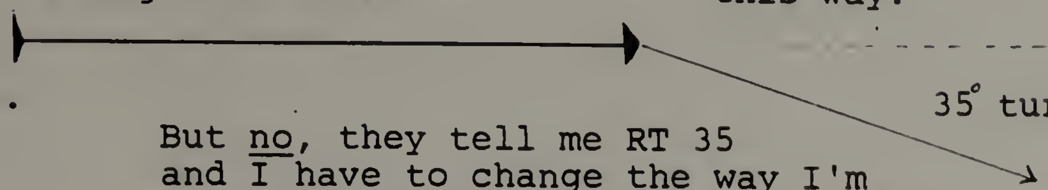
Angles with LOGO

Here's how the LOGO turtle thinks about angles:

They said FD 100, so I'm moving straight along this line.

I want to keep going straight this way.

I start here.



35° turn

But no, they tell me RT 35 and I have to change the way I'm pointing. Then when they tell me FD 50, I go off in a new direction.

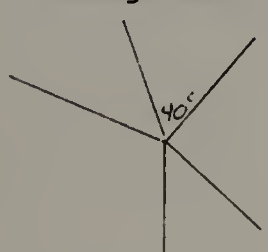
1) If you command FD 100 RT 35 FD 50 do you get a picture of a 35° angle? If it's not a 35° angle, what size is it?

2) List the commands you would use to get this picture:

3) Find a way to draw that 30 angle using FD, BK, and RT for your commands (no LT allowed). Then keep going with the same method and put another 30 right next to it.

4) If you kept going with that method, how many 30° angles could you fit (before you start to retrace)?

5) In this figure the angles are not necessarily congruent. Just be sure you have five spikes coming out, and that one angle is 40°. (Again, no LT and no RT > 180)



In your version of the figure, what is the measure of each angle? (Write the degrees in the picture.)

6) Now do the same type of five-spike figure, but this time make all five angles congruent. What is the measure of each angle?

UNIT 3 ANGLES IN AND OUT OF POLYGONS

Angle Measures

1) Measure all the angles in the figures A - J. This is a good time for shared group work!

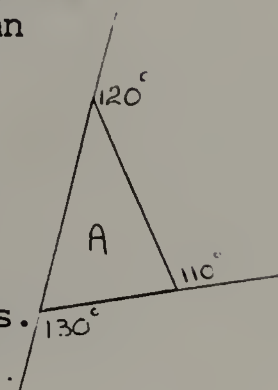
2) Record your measurements in the chart provided. This should complete the first three columns of the chart. Problem A has been done for you.

3) Study the information in the chart. Can you extend the chart to an entry for a figure K which would have 8 sides? (Careful I did not say 7 sides!)

4) What would column three be if the figure L had 9 sides?

5) Imagine a figure M with 102 sides. What would the sum of its interior angles be? Complete column three for M.

6) Remember the work we did with exterior angles? Here is triangle A with an exterior angle drawn at each vertex. I listed these measures in the fourth column. Label the fourth column: Measures of the exterior angles. Complete the fourth column for figures B - J.



7) The fifth column should be labeled: Total degrees in exterior angles (one at each vertex). Complete the fifth column.

8) Study your results: summarize what you found to be true.

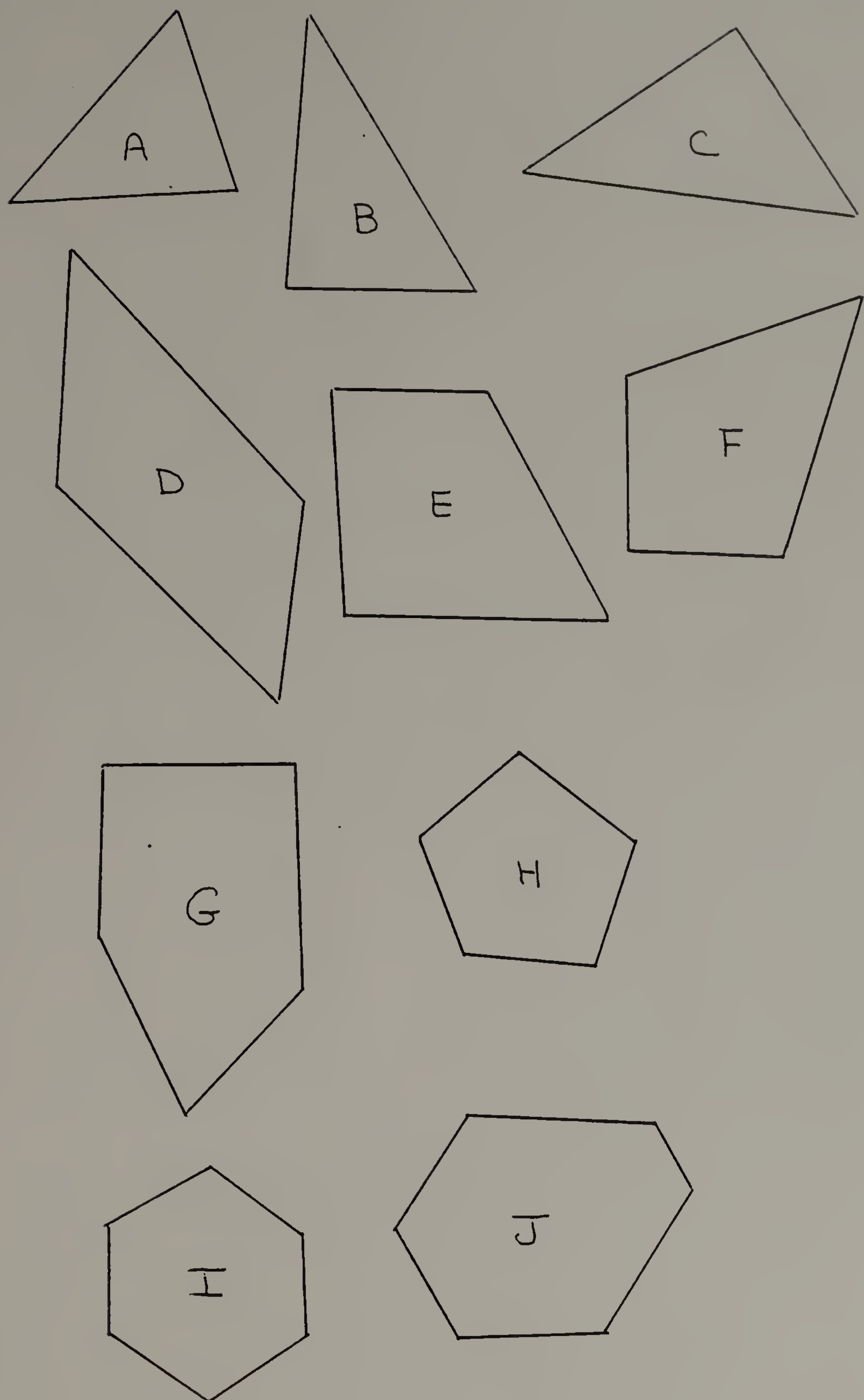
9) Are these results surprising to you?

UNIT 3 ANGLES IN AND OUT OF POLYGONS

Angle Measure Chart

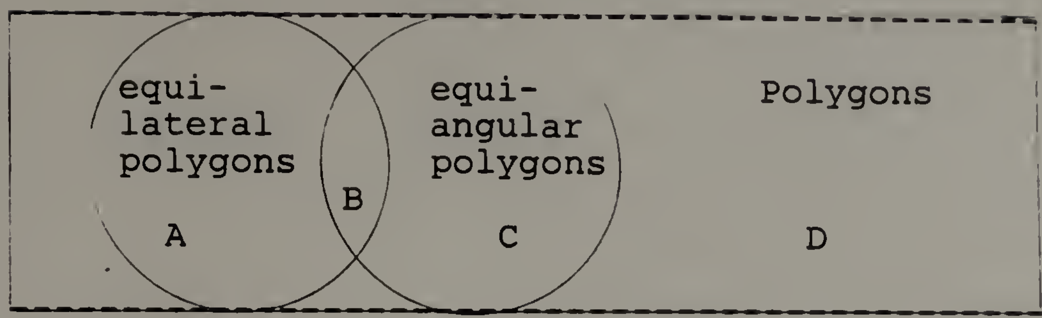
	1	2	3	4	5
	Number of sides	Angle Measures	Sum of angle measures		
A	3	50, 60, 70	180	130,120,110	360
B					
C					
D					
E					
F					
G					
H					
I					
J					
K					
L					
M					

Hand in this table and the related question sheet as your classroom assignment for today.


A
B
C
D
E
F
G
H
I
J

UNIT 3 ANGLES IN AND OUT OF POLYGONS

Regular Polygons

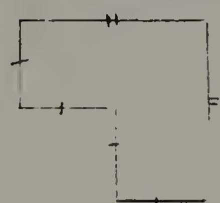


1) To which category does this pentagon belong? Say why.

2) What about this triangle? Say why.

3) How come this shape does not belong to category C?

4) Is this a hexagon?



5) Briefly summarize the characteristics of each category of polygons.

Now get some blank paper, rulers, protractors, compasses, and pieces of wood so that you can design some polygons according to these categories. Divide the work within your group.

6) Create a quadrilateral from each category.

7) Create a hexagon from each category.

In geometry, there is a special name for polygons from category B: Regular Polygons.

Homework Questions

1) Look at the Venn diagram on your classwork sheet. Working with your group, you created quadrilaterals and hexagons in all four different catagories. Why is it <u>impossible</u> to draw a triangle for each category?

2) Does a regular quadrilateral have to be square? Explain.

3) Are all regular hexagons congruent? Explain.

4) Based on the work you've done so far with regular polygons, answer True or False. Say how you know.

   a) All regular pentagons are equilateral.

   b) The only way to make a polygon equilateral is to make it equiangular also.

   c) An equiangular quadrilateral is really just another name for a rectangle.

   d) All squares are equiangular quadrilaterals.

   e) All squares are rectangles.

UNIT 3 ANGLES IN AND OUT OF POLYGONS

Chart It

Note: one of these problems can't be done. When you find it, explain why.

REGULAR

Number of sides	Number of triangles	Total degrees interior	Total degrees exterior	Each interior angle	Each exterior angle
1) 6					
2)	3				
3)		1800			
4)			360		
5)					40
6)				144	

Which one could not be done? Why not?

UNIT 3 ANGLES IN AND OUT OF POLYGONS

Solve It

Explain your answer to each question.

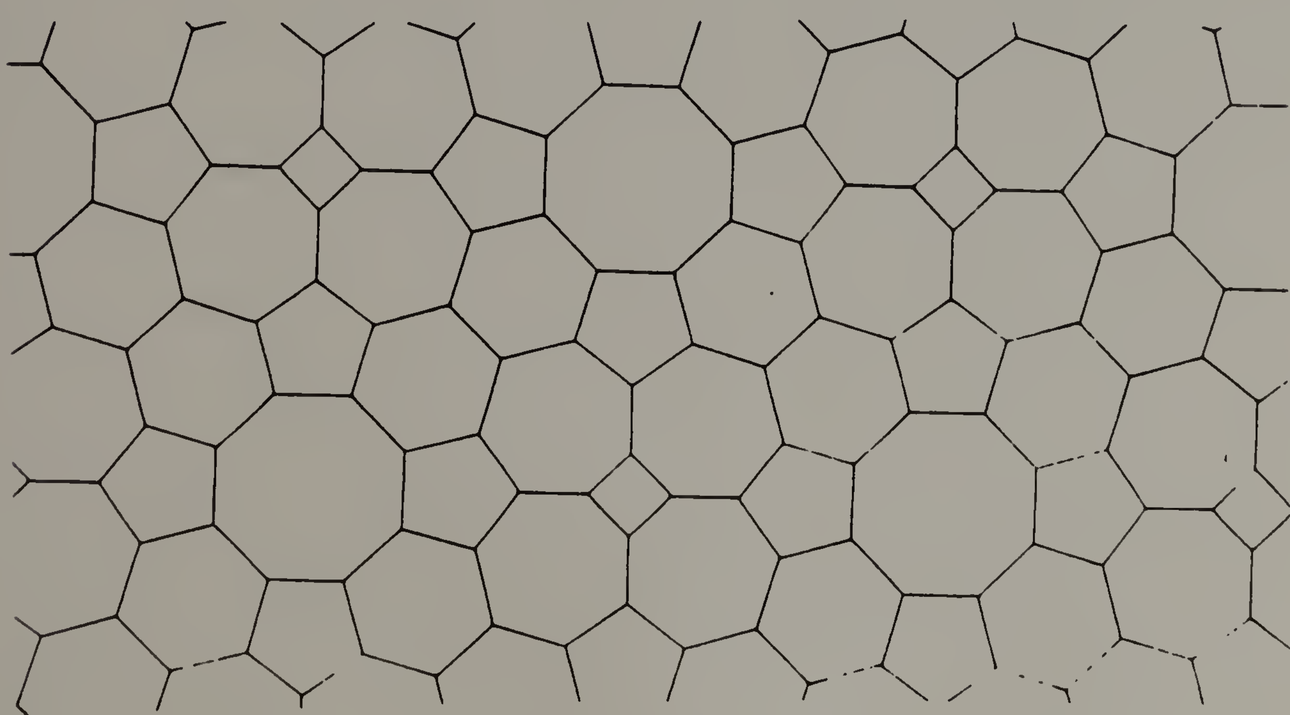
1) Can a regular polygon have an exterior angle of 20 degrees?

2) Can a regular polygon have an exterior angle of 22 degrees?

3) How many sides does a regular polygon have if each of its exterior angles is 6 degrees?

4) How many sides does a regular polygon have if each of its interior angles is 144 degrees?

5) Can all the polygons in this design be regular?



UNIT 4 QUADRILATERALS

Quadrilaterals-Finding the Range of Solutions

To do this exercise you'll need to review a few things.

1) What is a quadrilateral?

2) Draw a convex quadrilateral.

3) Draw a non-convex quadrilateral.

4) What does it mean to say that something is determined?

5) How come three fixed angles (AAA) are not good enough to determine a triangle?

For each set of given conditions, make a model with The Determinator. Then, experiment with the model to get a sense of the range of solutions for that set of conditions. Once you've seen how the model behaves explain carefully:

- .in what way(s) is the quadrilateral restricted?

- .in what way(s) is the quadrilateral free to move or change?

For example, perhaps the quadrilateral is restricted in that it must be convex, or maybe it's not allowed to have more than one pair of parallel sides. Maybe it's free to move in that it can lean at any angle, or in that it can be any size. Diagrams will be crucial to help you explain what you mean.

1) Given Conditions: the quadrilateral is scalene.

2) Given Conditions: the quadrilateral has two pairs of congruent opposite sides.

3) Given Conditions: the quadrilateral has four congruent angles.

4) Given Conditions: the quadrilateral has four congruent sides.

5) Given Conditions: the quadrilateral is regular.

Homework Questions

Use your classwork to help you explain why each of these statements is FALSE.

1) If a quadrilateral has four congruent sides, then it also has four congruent angles.

2) If two quadrilaterals have exactly the same corresponding side lengths, the two quadrilaterals are congruent.

3) If a quadrilateral is equiangular, then it is also equilateral.

4) If a quadrilateral is scalene, then none of its sides will be parallel to each other.

5) If a quadrilateral has three congruent angles, then the fourth angle must be the same measure as the other three.*

* Hint: on #5, I said congruent angles---not 90 degree angles.

Quadrilaterals-How Many Kinds are There?

Look up the definition of parallelogram and write it here:

Make accurate drawings or models of quadrilaterals which satisfy the following conditions. Show sketches. If not possible, explain.

I A quadrilateral with diagonals that are perpendicular <u>and</u>
1. diagonals bisect each other.
2. one diagonal bisects the other <u>and</u> the second one does not.
3. neither diagonal bisects the other.
4. diagonals are congruent to each other.

II A quadrilateral with congruent diagonals <u>and</u>
5. diagonals bisect each other.
6. one diagonal bisects, the other does not.
7. neither diagonal bisects the other.
8. diagonals are perpendicular to each other.
9. diagonals are <u>not</u> perpendicular to each other.

III A quadrilateral with $\cong$ diagonals and no $\cong$ sides.

IV A parallelogram with $\cong$ diagonals and no $\cong$ sides.

Homework Questions

I Look up the definitions of these words and write them here:

Trapezoid:

Rectangle:

Rhombus:

Square:

II Look at your work from class today. Identify each shape you made as a) parallelogram b) trapezoid c) rectangle d) rhombus d) square.

UNIT 4 QUADRILATERALS

Quadrilaterals-Special Types

1) ABCD is a parallelogram. Angle A = 32 degrees
AB = 8 cm. BC = 3 cm. Draw a sketch.
Find as many angle measures as you can.
Find as many side measures as you can.

2) EFGH is a rectangle EF = 8 cm. FG = 3 cm.
Draw a sketch.
Find as many angle measures as you can.
Find as many side measures as you can.

3) IJKL is a rhombus. Angle I = 32 degrees.
IJ = 8 cm. Draw a sketch.
Find as many side measures as you can.
Find as many angle measures as you can.

4) MNOP is a square. MN = 8 cm.
Draw a sketch.
Find as many angle measures as you can.
Find as many side measures as you can.

5) Draw a parallelogram. Label it QRST.
Write down as many true statements as you can about the sides and angles of the figure you drew. Use terms like congruent, supplementary, parallel, and perpendicular.

UNIT 4 QUADRILATERALS

Properties of Quadrilaterals

Use the figures from class today to complete this chart.
Write <u>yes</u> or <u>no</u>.

	Parallelograms	Rectangles	Rhombus	Square
opposite sides congruent				
opposite angles congruent				
all sides congruent				
all angles congruent				
all angles right				
adjacent angles congruent				
adjacent angles supple-mentary				
diagonals congruent				
diagonals perpen-dicular				
diagonals parallel				
diagonals bisect				

Reversibility

You can always take a definition and write it in "if, then" form. Furthermore, the "if, then" statement you write for your definition can always be reversed. This is helpful when you want to check to see if something fits a definition.

DEFINITIONS ARE ALWAYS REVERSIBLE!

1) Definition: a parallelogram is a quadrilateral with two pairs of parallel opposite sides.

If-then:

Reversed:

According to the definition, is a rectangle a parallelogram?

____

2) Definition: a rectangle is a quadrilateral with four right angles.

If-then:

Reversed:

According to the definition, is a square a rectangle?

____

3) Definition: a trapezoid is a quadrilateral with exactly one pair of parallel sides.

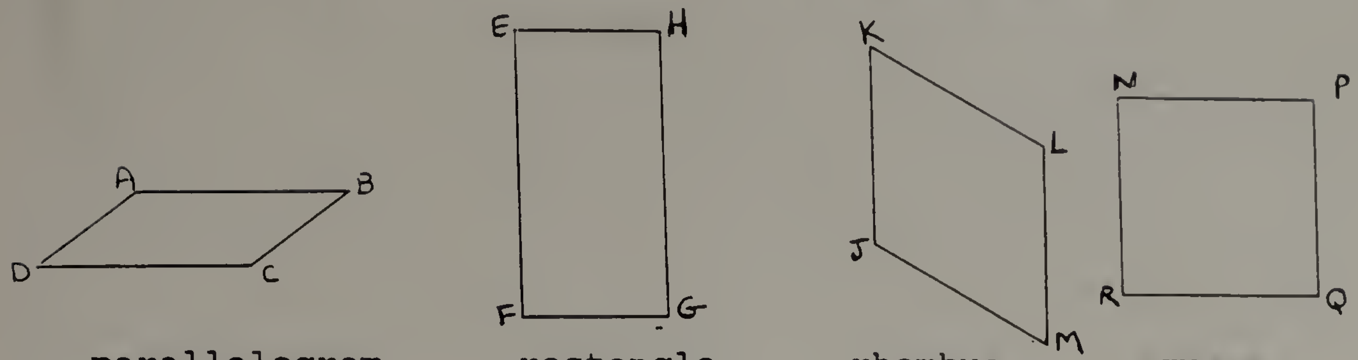
If-then:

Reversed:

According to the definition, is a parallelogram a trapezoid?

____

Quadrilaterals-Finding Missing Parts



parallelogram rectangle rhombus square

---

1) angle BAD = 120, angle J = 130, BC = 4 cm, CD = 7 cm

HG = 10 cm, FG = 3 cm, JK = 4 cm, RQ = 2 cm

Find these angle and side measures using the diagrams above. Explain how you know for each case.

angle B___ angle C___ angle D___ AD___ AB___

angle E___ angle F___ angle H___ angle G___

EF___ EH___ angle K___ angle L___ angle M___

KL___ LM___ JM___ angle N___ angle P___ NP___

Use the diagrams above to write algebraic sentences for each problem. Explain why each equation you wrote is correct. Solve for x. Find the required part.

2) angle ABC = 3x - 20
angle ADC = x + 5
find x, angle DAB

3) EH = 2x
EF = 3x + 18
HG = 4x - 12
find x, FG

4) EF = 2x - 3
angle G = 10x

5) angle MLK = 4x + 20
angle LKJ = x + 20
find x, angle KJM

6) angle NPQ = 4x + 10
angle NP = 1/2x - 5
find x, NR

7) NR = 2 1/2x - 3
RQ = 2x
find x, NP

8) JK = 1/2x + 4
JM = 2x - 26
find x, ML

UNIT 4 QUADRILATERALS

Quadrilaterals and LOGO

I 1) What will the computer draw if you type:

FD 50   RT 150   FD 70?

2) Type it in. Were you right?

3) What will the computer draw if you type:

FD 50   RT 90   FD 70?

4) Type it in. Were you right?

5) What will be drawn if you type:

REPEAT 4[FD 50 RT 90 FD 60]

6) Check it out. Were you right?

II For each exercise that follows (7-12), write commands that will have the computer draw each object. Record your commands.

7) A square.

8) A rectangle which is not a square.

9) A parallelogram which is not a rectangle and not a rhombus.

10) A rhombus which is not a square.

11) This figure: a parallelogram with both diagonals.

12) This figure: an equilateral triangle with its midlines.

APPENDIX B

TEACHING GUIDES

Unit 1: Determined

Note: Unlike the other units in this series, the Determined unit does not complete the entire sequence of the Learning Cycle. This unit is constructed to serve as the opening stages of the cycle: Intuition and Exploration. The Formalization and the second level of Intuition for this concept in the special case of triangles are included in the Triangle Congruence Unit.

1. TYPES OF TRIANGLES

Stage: Intuition
Question: What Can Be Constructed?
Format: Physical Drawings
Class Structure: Groups of Three
Materials Needed: Rulers
Protractors
Compasses
Graph Paper

Notes to the Teacher:

This lesson provides both a summary of vocabulary terms and an introduction to the determined unit. The content objective is to review the terms commonly used to describe triangles. The lesson also encourages students to investigate the relationship between the sides and angle of triangles.

The lesson includes descriptions of triangles which cannot exist. It is important for students to note that not all sequences of mathematical sounding words are meaningful. When a figure cannot be made, students are asked to explain what happens when they try to build it. Then they explain why no such triangle exists. These questions provide a start for reasoning. Students who have trouble explaining why such a triangle cannot exist should be asked to look again at their explanation of what happens when they try to make such a triangle. The physical connections seen there can often be translatd into reasons.

This lesson also introduces the notion of drawing careful diagrams and analyzing them to make conclusions. The role of induction is often undervalued in geometry. If each group makes two triangles of different sorts, the class will have many examples to consider before making any generalizations. The discussion of special cases will be incorporated in a future lesson.

Unit 1: Determined

2. THE DETERMINATOR

Stage: Exploration
Question: What Can Be Constructed?
Format: Physical Models
Class Structure: Groups of Four
Materials Needed: Wood strips with holes an inch apart
Blocks with angle markings drilled
Thumbscrews and paper fasteners

Notes to the Teacher:

This lesson starts out with a whole class discussion of the word "Determined". Ask everyone in the class to write a sentence using this word. Collect these and write several of them on the board. Ask the class to explain some of them. Once the word has been explored, present the mathematical meaning of the term.

To present a geometrical example, attach two wood strips so they form a pair of supplementary angles. Ask the students what is determined if one angle in this pair is "fixed"? This question defines the word "fixed" as well as provides an opportunity to discuss "determined" in a geometric situation. After this discussion, distribute the Determinator materials and the question sheet.

Students may need some help in their groups as to how to use the materials. Sticky tape with S and A can be used to denote the object that is fixed and that which can still vary. The use of these symbols will be built on later in the triangle congruence unit.

Unit 1: Determined

3. ALGEBRA-DETERMINED

Stage: Exploration
Question: What Can Be Expressed?
Format: Use of Algebra
Class Structure: Whole Class/Pairs
Materials Needed: Calculators

Notes to the Teacher:

This exercise is designed to provide students with an algebraic context for the meaning of determined. It has the additional quality of helping students make sense of the variety of ways variables are used.

Start the class with the warm ups. Record their answers on the board. Remind the class of the scheme used previously to categorize the number of solutions. Have students individually decide on the categories for each. As students provide their answers discuss any differences of opinions. This part of class should end with the five statements, solutions for each and the category left on the board so that students may refer to this as they do their work.

There are at least two ways to approach this task. One is to look at these statements as sentences in arithmetic. In that case, students find it interesting to use a calculator to try to find numbers that work. If pairs do use calculators it is worthwhile to create a list of "kinds" of numbers with them. Ask them to tell you what category of numbers they plan to try. Together they can generate a list something like this: positive, negative, large, small, zero, one, fractions less than one, and so on. Determining when how many numbers is enough to try to use the label Always is a fascinating question. For some students this will motivate the use of algebra.

Another way to start this problem is to use algebra manipulation. Geometry students need to maintain their algebra skills. This exercise can be considered a review of skills already learned or an opportunity to teach some concepts that have not yet been mastered. For some students, for instance, problem #2 is a mystery until they investigate some numbers.

The suggested homework questions are appropriate if the class has worked extensively with the Always, Sometimes, and Never format of questions. It is important for students to note that these statements can be categorized in more than one way. Learning that classification schemes can vary and that the user must decide which scheme works for the content of the problem is important.

Students will also see the category of Sometimes is pretty vague. It includes several different situations. This is very different from the Always and Never case which are clearly unique.

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4. TRIANGLES-DETERMINED

Stage: Exploration
Question: What Can Be Constructed?
Format: Physical Drawings
Class Structure: Groups of Three
Materials Needed: Rulers
Protractors
Compasses

Notes to the Teacher:

This is another exploration exercise. It will form the basis for the triangle congruence unit. The previous two lessons had students investigate geometric and algrabraic statements and introduced categorization for the number of solutions. This lesson focuses on how these ideas apply to triangles.

Even though groups of three are suggested, be sure each individual actually constructs the triangles. Students find that constructing these triangles can be a powerful experience. Many students miss one of the triangles in #2, yet I have found no clear pattern in which one they leave out. Since that is the case, encourage them to share their work with other students to check if they are all using the "same" triangle. Some students will make the acute and some the obtuse triangle, so they will encounter both possibilities.

Some students will consider all the triangles made in exercise #3 as the same even though they notice they have different length sides. Clarifying the difference between same, similar, and congruence will help them.

This exercise can be extended to include SSS cases for triangles that cannot exist due to contradiction of the triangle inequality.

5. THE GEOMETRY OF LAUNDRY RACKS

Stage: Exploration
Question: What Can Be Changed?
Format: Physical Models
Class Structure: Groups of Four
Materials Needed: The Determinators: Wood
Angle Fixers
Folding structures:
wooden clothes dryers
folding clotheslines

Notes to the Teacher:

This is a fun, though unusual, activity. Provide the class a variety of objects that change structure, such as wooden clothes racks, sweater dryers, or folding clotheslines. (If your objects are not all to do with laundry, a different title may be appropriate.) Students are to analyze the structure in both the determined and undetermined state.

Building a model of the structures with the wooden pieces and angle fixers is a necessary part of this exercise. As students work on building their model, they must notice which aspects of the structure are important features in the change process and which are irrelevant. The model serves as a stripped down version of the structure. As students label the fixed and unfixed sides and angles, they are able to connect the motion of these objects with the earlier information on determining a triangle.

Unit 1: Determined

6. TWO SIDES OF A TRIANGLE

Stage: Exploration/Deduction
Question: What Can Be Changed?
What Can Be Deduced?
Format: Physical Models
Class Structure: Pairs
Materials Needed: Rulers
Protractors
Compasses

Notes to the Teacher:

This exercise is designed to help students note several inequalities concerning triangles. As the students work through these questions they will be drawing, checking their measurements against their intuition, and determining the limitations of one measure when other measures are fixed. In this case the fixed measures do not determine a unique value for the third side but they do provide a limit for the range of solutions.

Students are then asked to determine if they must change their analysis in the case that two sides of the triangle are the same. The answer to this question points out the role of zero as a number. Determining that a segment must be greater than zero is the same as saying that it must exist. This rather powerful meaning of zero can be noted here.

The homework questions are designed to help students organize the results of the two experiments. Questions 1 and 2 are phrased in arithmetic terms. Generalizing these two solutions as the sum and the difference of the fixed sides should be part of the class discussion following this assignment. Some students who understand this concept physically and arithmetically, may still have trouble with the general rule. These students may need extra practice with arithmetic cases to help them see the connection.

The third homework problem allows students to note the isosceles triangle angle equality and its more general case, the theorem stating that the larger side in a triangle is opposite the larger angle.

Building these triangles out of the wood strips and changing the angle between the sides is an effective alternate or additional activity for this conept. The Geometric Supposer can also be used to analyze various cases of SAS. Students who need additional experience with these concepts would find those exercises beneficial.

Unit 2: Triangle Congruence

1. ANOTHER LOOK AT DETERMINED

Stage: Intuition
Question: What Can Be Constructed?
Format: Physical Diagram
Class Structure: Groups of Three
Materials Needed: Wood strips
Paper Fasteners
Compasses
Rulers

Notes to the Teacher:

This lesson is designed to provide the link between the concepts of determined and congruence. Question #1, #2, and #3 review the work done earlier in the Determined Unit. Be sure students build these models. The connection between many figures, many different possible angles and the non-determined state of the structures is important. Give students an opportunity to write about the relationships and non-relationships they see.

Question #4 is designed to elicit student intuitive responses. The teacher's job here is to note what meaning the students give to the word "different". Language is used casually in everyday life. The context of the sentence indicates what meaning to ascribe to a given word. This principle of English, which makes our language interesting, is often a hindrance in mathematical understanding. Students may interpret the word "different" to mean different shape, or different size, or different object. The definition of congruence depends on a commonly held meaning to the word different. Discussing the answers to question #4 provide the opportunities to bring out all these meanings and to agree on the meaning of "different" as in different object.

Question #5 is another example of determining the number of solutions. It is also designed to help students move away from a strictly horizontal and vertical orientation. Once students see there are more than four answers to this question, they often settle on 360 as the number. Other students will argure for an infinite amount. Letting these students argue with each other is effective.

Question #6 reviews the results of the two sides of a triangle exercise which closed the Determined Unit. For students who formalized that work, this is a review question and provides a link between old knowledge and the new work. For students who are still forming this concept, it is additional exploration.

<u>Unit 2: Triangle Congruence</u>

2. SOLVING TRIANGLE PROBLEMS

Stage: Exploration
Question: What Can Be Constructed?
Format: Physical Diagram
Class Structure: Groups of Three
Materials Needed: Protractors
Compasses
Rulers

Notes to the Teacher:

This lesson narrows the scope of the study of determined to the case of triangles. The exercises include the three triangle congruence cases (SSS, SAS, ASA) and the ambiguous case (SSA) in which two answers are possible.

Drawing scale diagrams is an important task in and of itself. The skill to physically construct these triangles is as important as the formalization of the rules that will take place later. These questions also provide practice with interpreting verbal information and using the protractors and compasses.

Even though students are working in groups it is important that every student draw their own diagrams. Alert students to the possibilty of more than one answer by referring to the Determined unit work. The difference between different diagrams providing the same answer, as may happen in questions B, C, and D, and different possible diagrams providing different answers as in A, is important to be discussed.

Many students do not notice the second case in problem A. Some will draw the acute triangle version. Others will draw the obtuse case. As students present their solutions to the class, both cases should be noted. Provide the determinator wood strips so that students may build a model of this case as part of this discussion. Seeing the two answers on paper is one format. Building the wood strip model shows the two solution case in another format. Both of these experiences are worthwhile.

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3. TRIANGLE CONGRUENCE RULES

Stage: Formalization
Question: What Can Be Deduced?
Format: Reasoning
Class Structure: Individual/Whole Class
Materials Needed: None

Notes to the Teacher:

After the class discussion of the problems A - D from lesson #2, ask the students to identify what information was given in each problem. Provide the wood strips so they can build models of each case, marking what is given or fixed. The language of the Determined unit should help them express the situations. As a summary, the class should be asked to list what information is necessary to determine a triangle. Abbreviating their conclusions should end up with the traditional sequences of SSS, ASA, and SAS.

The following are good follow-up questions which help to link the determined concept to that of triangle congrunce:

1. I have two triangles. I know that all three sides of one are the same length as all three sides of the other. Are the two triangles the same size and shape? Do you know anything about their angles? What else would you need to know to be guaranteed that the triangles are identical copies of each other?

2. I have two triangles. I know that two of the sides of one are the same length as two of the sides of the other. Are the two triangles the same size and shape? Do you know anything about their angles? What else would you need to know to be guaranteed that the triangles are identical copies of each other?

3. I have two triangles. I know that one side of one is the same length as one side of the other. Are the two triangles the same size and shape? Do you know anything about their angles? What else would you need to know to be guaranteed that the triangles are identical copies of each other?

4. FINDING MISSING PARTS

Stage: Exploration
Question: What Can Be Calculated?
What Can Be Expressed?
Format: Use of Arithmetic
Use of Algebra
Class Structure: Individual
Materials Needed: None

Notes to the Teacher:

This exercise provides arithmetic and algebraic practice in applying the triangle congruence rules. The directions require that students supply reasons for each conclusion they make with the numerical examples. These questions help students notice the connection between this work which appears intuitively easy, and proof which is often difficult. Students who develop a sense of justifying their work here will find the justifications in non-numerical cases similar.

The algebra examples serve two goals. For many students algebra means solving a given equation by some set of steps. Few understand the meaning of the equation itself (which means they do not understand the reason behind the steps of the solution as well). In these examples the students must form their own equation. This means that they must notice that an equation is a statement that two representations have equal value. This setting provides an opportunity for students to note the meaning of equation. They are asked to provide the geometric reason for each algebraic statement. This helps them build up a notion of proof as indicative of why they know something is true.

The solutions to the algebraic exercises do not end with finding the value of x. The questions require that the students calculate some geometric object as well. This is done so that students will see that algebra is not an end unto itself, but rather a tool for solving other problems.

Several variations are included in the set of exercises. There are problems in which the value of x is a negative number, but the solution of the geometric problem is sensible. There are problems in which the value of x is a positive number but the situation in geometry represents an impossible geometric case. Students will need to analyze the geometry of the solution in order to make sense out of their solutions.

Students need to understand that one possible answer is that the situation described cannot happen. Explaining why not is the appropriate answer to such a question. Students will add this to their list of possible answers only if they encounter such questions frequently enough. Most text books include only questions for which answers as possible.

Unit 2: Triangle Congruence

5. USING CONGRUENT TRIANGLES

Stage: Intuition
Question: How Can This Be Used?
Format: Problem Solving
Class Structure: Pairs/Whole Class
Materials Needed: None

Notes to the Teacher:

This exercise is designed to provide further practice with the triangle congruence rules and their meaning. It also provides a non-proof setting to discuss the statement frequently used in proofs: Corresponding Sides of Congruent Triangle are Congruent. Students use it here as a natural part of the definition of triangle congruence.

These problems are best solved in pairs. Many students who feel they understand this concept have difficulty explaining their ideas. The process of writing out these solutions helps clarify the concepts involved. A useful classroom technique to help them with this task is to regroup the pairs into groups of fours once each set of pairs has made an attempt to write a clear explanation. In the groups of four have them compare results by reading each other's explantions and then write one explanation to represent their group's work.

Have each group of four present their solution for one of the three problems to the whole class. With the whole class sharing, students will see both approaches to the geometrical problem and other formats for the explanations.

---

6. LINE SEGMENTS IN TRIANGLES

Stage: Exploration
Question: What Can Be Constructed?
Format: Drawings/Models
Class Structure: Pairs
Materials Needed: Rulers
Protractors
Scissors

Notes to the Teacher:

This lesson provides physical meaning to two line segments commonly studied in geometry and is the basis for the formal proofs to follow in the next lesson. The intial step of measurement and triangle description is important. The reason for the classification of triangles in these categories is that we can state geometrical information about a whole class of triangles not just a particular triangle. That is what the proofs in the next lesson will accomplish. In order for students to comprehend this value of proof they need to see that describing triangles by the relations of the sides and angles is not just a trivial labeling exercise. This will only happen if students are asked to make conclusions on the basis of the categories.

Note that the directions do not include how to do the paper folding. Students are to be given the definitions of these terms and to work out for themselves how to fold the paper to perform the action. The paper folding exercise provides students with physical evidence which indicates the difference between the median and the angle bisector. Even when they coincide the physical acts of creating them are different.

Question #5 provides an abstract format for the conclusions of this exercise. Questions #5 and #6 ask them to interpret this Venn diagram and to connect it with the physical exercise they performed. This question is designed to help students articulate what they noticed in the paper folding exercises. The very physical nature of the task can create a situation where students do the exercise but do not reflect on what happened and therefore do not learn from the activity. Performing the action is not enough; if the lesson is to have impact, students must think about what they did, and then verbalize in formal geometric terms what they saw .

Question #7 is designed to help students increase their precision and to note all the conditions necessary to the geometrical situation. Many students will conclude the median and the angle bisector coincide in an isosceles triangle. This statement is true but not precise enough. The function of question #7 is to alert students to the modifications necessary to improve the statement. This process illustrates the scientific method: collecting data, forming a conclusion, checking with further study of the data, reforming the conclusion.

Question #8 provides the forum for students to write their final conclusions. Not all students will note the difficulty in the isosceles case. This should be brought out as students share their answers with the whole class.

Unit 2: Triangle Congruence

7. SEGMENTS IN TRIANGLES-PROOF

Stage: Formalization
Question: What Can Be Deduced?
Format: Reasoning
Class Structure: Individual/Whole Class
Materials Needed: None

Notes to the Teacher:

This lesson should be organized as an interactive class with students working individually at their seats, but not at their own pace. The class should operate as a whole. The content of this lesson follows from the previous lesson and illustrates the connection between the physical reality of geometrical relations and the way that congruent triangles are used to verify them in general. Question #1 is used to focus the class attention on the situation to be discussed. It is a review of the work of the previous day's lesson. Be sure students write down an answer to "How do you know you are right?" for later use.

Work through Question #2 step by step with student suggestions and have students record the board work on their own papers. This will give them a format to follow for other exercises of this type. Some students have trouble noting all three triangles present. In order to comprehend the use of congruent triangles in proof it is necessary for students to have the ability to refocus in the middle of a problem. Sometimes we see one triangle with an angle bisector, sometimes we see two triangles. Drawing separate diagrams with the appropriate labels can help make this explicit.

Once question #2 has been completed through this interactive class, ask students to compare their answers to question #1 to the work in question #2. It is important for them to note the similarities and differences between the two answers. Both accomplish the task. One is informal, based on physical knowledge, the other is formal, based on deductive results. One is the way we convince ourselves, the other is the way mathematicians convince each other.

Unit 3: Angles In and Out of Polygons

1. TILING

Stage 1: Intuition
Question: What Is Suggested?
Format: Scale diagrams
Class Structure: Pairs or Individual as Homework
Materials Needed: Tracing Paper
Scissors (optional)

Notes to the Teacher:

"Tiling" is intended to provide students with an informal sense of the meaning of regular tesselation. In this introductory lesson students start with squares and circles. These shapes are not only very familar but also serve as base examples of the meaning of tiling. It is "obvious" to students that a square tiles and a circle does not. These shapes connect with students' intuitive beliefs. Using this as a basis the students are able to move from what is very familar and obvious to shapes that are more complex.

Another significant point to note in the written instructions is that students are directed to look at the world around them, in this case the floor, and to analyze its form. The transition from paper and pencil exercise to reality and back to paper and pencil illustrates the interrelationship between mathematics and the real world.

Teachers should avoid restating the instructions. Have the students read them. Our goals as teachers are not only content related, but also to make our students independent learners. Yet often we subvert our own goals by doing what seems to be more "efficient" such as simply telling the students what directions mean. In fact this robs them of the chance to be in charge of their own learning.

Do not tell students how to accomplish this task. If they come to wrong conclusions, challenge them to show you their diagrams and to compare their work with others.

2. ANGLES WITH LOGO

Stage 1: Exploration
Question: What Is Apparent?
Format: Computers
Class Structure: Pairs
Materials Needed: Computers
LOGO

Notes to the Teacher:

This LOGO lesson is designed to help students understand that there are 360 degrees around a point. It also makes clear the significance of the exterior angle. Students need little experience with the LOGO language to complete this task. However students do need to act out the role of the turtle themselves in order to comprehend the turtle moves. Having students walk and turn according to the LOGO commands in the introduction of the lesson, provides them with a physical meaning to the terms.

The problem solving process implicit in working with LOGO can be described as follows:

Try something.
If it doesn't work, see why not.
Change what you did.
Try it again.
Repeat this until it comes out right.

The LOGO exercises on this sheet have two important attributes that teachers can capitalize on. One is that there are many ways to do most of the tasks. Letting students share solutions with each other broadens the approaches that students have and also provides satisfaction to the students. Also the tasks are self correcting. The students know when they are right without having the teacher take on the role of authority. It is right when you accomplish the task, when the physical reality is satisfied.

3. INTERIOR & EXTERIOR ANGLES
EXTERIOR ANGLES

Stage 2: Exploration
Question: What Can Be Explored?
Format: Computers
Class Structure: Groups of three
Materials Needed: Computers
The Geometric Supposer Triangles

Notes to the Teacher:

This lesson is started in groups at the computers. As groups finish the assignment have them get together to compare conjectures. Let them discuss similarities and differences between their own work and the work of the other groups. Good follow-up questions: Can you convince me that your conjectures are always true? Do similar conjectures hold for figures or more than three sides?

Here students are introduced to the concept of the exterior angle of a triangle. The exploration may bring out the inequality concerning exterior angles or the equality involved. One benefit of having different groups compare notes is to disseminate all this information.

This lesson starts in small groups, then combined groups, and by the time the class is done may well end up as a whole class discussion.

One of the suggested follow-up questions asks students to imagine what an exterior angle would be in a figure of more than three sides. This kind of question has two purposes. First, it provides a basis for a future lesson. Second, it is an illustration of generalizing. If we want our students to generalize, we need to model that behavior in our questions.

A useful construct for organizing geometric statements is to determine which are always true (a theorem), which are sometimes true (a theorem with conditions), and which are never true (a statement that contradicts known fact). This device can be used here to form homework questions:

Determine which of these statements are always true, sometimes true, or never true. Refer to your computer work to help you decide. Write out your explanations.

1. The sum of the exterior angles of a triangle (one at each vertex) is 360 degrees.

2. An exterior angle in a triangle is greater than any one of the interior angles of the triangle.

3. In a right triangle, an exterior angle is equal to one of the interior angles of the triangle.

Unit 3: Angles In and Out of Polygons

4. REGULAR POLYGONS

Stage: Exploration
Question: What Can Be Deduced?
Format: Diagrams/Physical models
Class Structure: Groups of four
Materials Needed: Rulers
Compasses
Wood Strips with holes
Paper fasteners

Notes to the Teacher:

The object of this lesson is to help students understand the independence of the terms equilateral and equiangular. This single lesson (classwork and homework) actually illustrates the learning cycle itself. Even though students are given the Venn diagram to interpret, many of them will answer questions 1 through 4 based on their own "gut level" definitions. For example, students who say that a hexagon is a figure with six sides will also argue that the figure in number 3 is not a hexagon because of the "corner". Problems #1 - #4 then bring out the students' beliefs and are at the stage one level of intuition.

Problem #5 is designed to have students state these beliefs explicitly and connect them with the Venn diagram. This problem can serve to indicate conflicts between the informal definition of the students and the formal meanings implied in the diagram.

Problems #6 and #7 are physical exploration exercises. Students either build or draw objects which satisfy the given conditions. This provides another opportunity for them to refine their belief and connect it with physical reality. Thus this exercise serves as the exploration stage.

The homework questions are designed to accomplish two tasks. The first is content related. These questions ask them to reflect on the class activity and to interpret it in terms of the principles embedded in the Venn Diagram. As importantly, these questions also allow the student to justify their beliefs. The directions, "Say how you know.", create a format for informal deduction. Students learn to reason from known facts in small steps in this way.

Correcting this assignment is an important and difficult task. Most students have not been asked to explain their answers in mathematics class. They need to learn what this task involves. Their first attempts are usually inadequate, but they will not improve unless they get clear feedback. The teacher plays a vital role here as your expectations will define what is acceptable.

One way to make the importance of communication clear is to have the students read and critique each other's explanations. This accomplishes two objectives. First, each student receives feedback from their peers. This can be less threatening than feedback from a teacher who is the evaluator. Second, as critiquers, the students note unstated assumptions, missing links in the arguments, and different approaches to the task. This helps to make the task less arbitrary; that is, it's done to communicate an idea, not just to please the teacher.

These homework questions, then, are the formal deduction stage of the learning cycle. The student has now constructed a new belief about the relationship between equilateral and equiangular. This is the end of one cycle of learning within this single lesson which is part of a broader cycle illustrated by the whole unit.

Unit 3: Angles In and Out of Polygons

5. ANGLE MEASURES

Stage 1: Exploration
Question: What Can Be Calculated?
Format: Diagrams
Class Structure: Groups of four
Materials Needed: Protractors

Notes to the Teacher:

This lesson is a guided discovery lesson. The students compile data from their actual measurements, analyze this data, and extend the patterns they found to cases beyond the original data sources. This lesson is designed to motivate the angle meassure formulae.

This lesson also illustrates two different uses for groups. At first the group functions as a work sharer; that is, each member of the group generates some part of the required data. The value of this is simple. It reduces the amount of time necessary to complete the task. However, the function of the group changes after the inital data collection. The group should now function as a problem solving (two heads are better than one) entity. The questions #3 - #7 require pattern finding and generalizing tasks quite appropriate for group problem solving.

Remind the students to follow the directions carefully. One device is to have each group appoint a "reader" whose job is to read the directions out loud one at a time. The reader does not move to the next direction until the work of the first step is completed. This technique helps the group stay on task and avoid confusion.

Unit 3: Angles In and Out of Polygons

6. TANGRAMS

Stage 2: Exploration
Question: What Can Be Constructed?
Format: Physical model
Class Structure: Pairs
Materials Needed: Tangram Pieces

Notes to the Teacher:

For the purposes of this lesson, the tangram pieces are labeled this way: A and B are the large triangles, C and E are the small triangles, G is the medium sized triangle, D is the square and F is the parallelogram.

This exercise allows students to have physical knowledge of how shapes connect with each other and form other shapes. Many students think of this as a puzzle day and are not aware of the mathematics behind the work they are doing. The ability to build complicated shapes out of simple ones and the reverse process of partitioning a solid shape into smaller sections is quite important in understanding not only tiling, but also the theorems which support most of the familar area formulas. Do not assume these exercises are merely play simply because students can not articulate this kind of knowledge. Physical experiences such as these provide the students with understandings that can be built on later.

This work with tangrams can also include a discussion of the problem solving process. Many students note that in solving problem #10 once any one of the figures has been made the others can be created by slight modifications of that one structure. This provides an excellent starting point for talking about solving mathematics questions by building on to previous work rather than starting from the beginning each time a problem is posed.

Note also in problem #10 students encounter a question to which the correct response is to say there is no relationship. This question asks what they know about the areas and the perimeters of the figures they have made. While they can be assured the areas of the figures are all the same, there is no such obvious relationship for the perimeters. It is just as important for students to note this non-relationship of perimeters as it is for them to note that areas relate consistently, yet we often ignore this kind of question.

Unit 3: Angles In and Out of Polygons

7. POLYGONS FORMULAE

Stage: Formalization
Question: What Can Be Expressed?
Format: Reasoning
Class Structure: Whole Class/Groups of Four
Materials Needed: None

Notes to the Teacher:

This exercise starts out as a whole class discussion of lesson #5, Angle Measures. Each group presents the number patterns they found in the tables. Possible student results: "You add 180 degrees each time you add a side." "The exterior angles are always the same, 360 degrees." "For 102 sides you take away two and times by 180 degrees." Once all these statements have been collected on the front board, ask each group to create a picture of the rule they created. Have them work in groups of four on this task for a period of time and then share their results.

If the groups need more direction to accomplish the work, the teacher may draw a 9 sided figure on the board, ask the class how many degrees are contained in the interior angles, and then ask them to find a way to verify that without measuring.

The teacher's role during the small group work is to challenge the students to give physical meaning to the written expression of the number patterns. Some groups may need concrete reminders of the partitioning work done earlier in this unit before they can make sense out of the number patterns.

After the small group work, have the students report their findings. It usually happens that some students will partition the nine sided figure into triangles by drawing diagonals from one vertex and will arrive at the (9 - 2) times 180 degrees version of the formula. Others will draw from an interior point and make 9 triangles times 180 degrees minus the extra 360 degrees. Having students verify the equivalence of these expressions is worthwhile.

Unit 3: Angles In and Out of Polygons

8. CHART IT

Stage: Exploration
Question: What Can Be Calculated?
Format: Use of Arithmetic
Class Structure: Individual/Whole Class
Materials Needed: None

Notes to the Teacher:

This exercise is a drill and practice type with a touch of problem solving. As students work through these problems, have them verbalize what they are doing in terms of the polygon and its triangles. This approach helps them to connect each arithmetic step with the physical picture. In this way students will be able to integrate this formal statement with the physical actions from which it was derived. This prevents rote memorization of the formulae.

One of the exercises does not have a unique solution. Warning students that this may happen is important, since this is not routinely the case in school work assignments. It is not here as just a trick question, but rather it illustrates an important mathematical principle. This problem can also be used to illustrate the difference between problems which cannot be solved and those which do not have a unique solution.

9. LVE IT

Stage: Intuition
Question: What Does This Mean?
Format: Problem Solving
Class Structure: Groups of Three/Whole Class
Materials Needed: None

Notes to the Teacher:

This lesson is designed to help students integrate their knowledge of angle size and shape of a figure. It also provides the teacher with information as to how well the students have been able to make these connections. If they seem surprised there is no regular polygon with an exterior angle of 50 degrees and can only verify it numerically, the teacher knows that more work must be done. In that case this exercise can be considered as the level 1 intuition. On the other hand, students who can see that the angle sizes are restricted have an intuitive knowledge of shape that has been formalized in the previous work.

The last question, the pattern built out of shapes, is designed to refer back to the original issue of tiling. Here are some shapes which do tile. Are they all regular? They certainly seem regular but measurement will not help decide the issue. The ability to apply the abstract concept of angle size and polygon shape to correctly interpret this quesstion indicates to the teacher that students have moved from a previous belief to a newer, more complex understanding. A student would then be at level 4 intuition and ready to explore new levels of this concept such as tesselations that are not regular and three dimensional versions of "tiling".

1. QUADRILATERALS - FINDING THE RANGE OF SOLUTIONS

Stage: Intuition
Question: What Can Be Constructed?
Format: Physical Diagram
Class Structure: Whole Class/ Groups of Three
Materials Needed: Wood strips
Paper Fasteners
Compasses
Rulers

Notes to the Teacher:

This lesson is designed to set the stage for the study of quadrilaterals in general and special types of quadrilaterals in particular. The questions are planned to accomplish two goals at once. One is to engage the students' intution and the second is to start the process of exploration.

The lesson uses model building as a format to be sure students picture the wide variety of shapes that the word quadrilateral includes. The language of given conditions is suggestive of the language used in the proof exercises in order to help students form the link between the physical models and the abstract proofs. The lesson also reviews the geometrical vocabularly used in this unit.

Be sure to have groups share their results with the whole class. Many different approaches can be used and it is fruitful for the whole group to see the variety. Sharing conclusions and determining similarities and differences in the results of the approaches provide valuable experience for the class in how to solve problems.

The homework questions are further examples of informal deduction. Students use the results of their own work to back up their conclusions. Some of these questions are difficult so do not expect complete solutions from everyone. Have students show what they have done as a starting point. The whole class can then continue to refine the answer until they are satisfied with the result.

Unit 4: Quadrilaterals

2. QUADRILATERALS-HOW MANY KINDS ARE THERE

Stage: Exploration
Question: What Can Be Constructed?
Format: Physical Diagram
Class Structure: Groups of Three/Four
Materials Needed: Protractors
Compasses
Rulers

Notes to the Teacher:

This exercise continues the exploration that had begun in the earlier lesson, determining shape as a function of conditions. This work is the building block for the formal stage where students will be deducing the properties from given conditions.

The questions in this lesson also sharpen the problem solving skills of students. Many groups will start this task by drawing quadrilaterals and measuring to see if they meet the conditions. A class discussion concerning the problem solving strategy, "Start with the given conditions, start with what you know.", may be necessary before students will draw the diagonals and work from them.

As students present their results, a discussion of special cases is likely to occur. Some students will understand the task to be: make a quadrilateral with diagonals which are perpendicular and congruent. Others will interpret it as: make a quadrilateral with diagonals which are perpendicular and congruent and nothing else in particular. Students in the first category should be encouraged to draw a variety of figures which meet the conditions. As the groups share their results the variety of cases will occur.

Unit 4: Quadrilaterals

3. QUADRILATERALS-SPECIAL TYPES

Stage: Exploration
Question: What Is Apparent?
Format: Diagrams
Class Structure: Pairs
Materials Needed: Rulers
Protractors

Notes to the Teacher:

This lesson provides a link between the construction of the figures and the properties that will be derived. Constructing the quadrilaterals is an excellent group task that reinforces earlier work on parallel. Group problem solving on how to build the figures is a worthwhile task. Sharing the procedures used to construct the figures will illustrate a variety of geometric principles. Be sure each group analyzes their method as they present it so the connection between the physical task and the concepts of parallel are made explicit.

The last question is purposedly open ended. Have students share their list of conclusions with each other. Sorting this list into facts that are true by definition and facts that can be noted as a result of the definition is a good closing class activity which will be used later in the proof work.

Unit 4: Quadrilaterals

4. PROPERTIES OF QUADRILATERALS

Stage: Formalization
Question: What Can Be Deduced?
Format: Reasoning
Class Structure: Individual
Materials Needed: None

Notes to the Teacher:

This lesson provides a chart for students to record the results of their measurements from the class activity. Initially have the students complete the chart by using the figures they made. Once the chart is complete, looking for patterns and the relationships within the chart can be profitable. The classification of the figures into nested and intersecting categories will connect with this task.

Once the chart is complete and some analyis has been done by the whole group, ask students to work in twos or threes to create a Venn diagram representation of the quadrilaterals: Trapezoids, Rectangles, Squares, Parallelograms, and Rhombuses. This is a difficult task. As groups present their solutions, let students find cases which support the diagram and create cases which require refinements. The whole class should arrive at an acceptable solution. Note: the diagram will vary depending on the definition of trapezoid used.

Unit 4: Quadrilaterals

5. REVERSIBILITY

Stage: Formalization
Question: What Can Be Deduced?
Format: Reasoning
Class Structure: Individual
Materials Needed: None

Notes to the Teacher:

This lesson extends the informal deduction of the earlier work in lesson #1 by linking that content with the if ... then ...sentence format used in earlier lessons. In essence this lesson provides the definition of the word "definition".

The reversibility issue will be used to help make clear the reason why the statement, "All squares are rectangles." is true, while the statement, "All rectangles are squares." is not.

Unit 4: Quadrilaterals

6. QUADRILATERALS-FINDING MISSING PARTS

Stage: Exploration
Question: What Can Be Calculated?
What Can Be Expressed?
Format: Use of Arithmetic
Use of Algebra
Class Structure: Groups of Three
Materials Needed: None

Notes to the Teacher:

This lesson uses the properties of quadrilaterals to provide drill and practice exercises in both arithmetic and algebra. The work that students have done with the physical objects should help them make the abstract connections necessary for these problems. As with the triangle congruence exercise of the same type, expect students to provide a reason for each conclusion they make. Justifications made here in formal language but familiar contexts will help students connect their intuitive knowledge of the figures with the formal theorems concerning them.

The algebra problems have the same quality as they did in the Congruent Triangle Unit; students must be in charge of the algebra. Students must determine from geometry what relationship is useful. They must use that relationship to form an equation. Next, they must solve the equation and use the value for x to find the missing segment or angle. Solving equations using algebra can then be seen as a tool to solve other problems, not as just an end in itself.

The difficulty level of the algebraic statements has been increased by one level. The co-efficients of some of the terms involve some fractions. Many students will assert that the answer to $1/2\ x = 50$ is 25 because they perform operations rather than solve equations. They see 1/2 and 50 and multiply. Having students draw diagrams to illustrate their work, checking solutions, and verbalizing what they are doing as they work are techniques that will help them reach correct conclusions.

Also, some of the solutions may not be possible; that is, the algebraic statements do not make sense in the physical geometric examples. Alert students to consider this possibity.

Unit 4: Quadrilaterals

7. QUADRILATERALS AND LOGO

Stage: Exploration
Question: What Can Be Explored?
Format: Use of Computers
Class Structure: Pairs
Materials Needed: Computers
LOGO

Notes to the Teacher:

This LOGO exercise is designed to provide another format for exploring the conditions that make various types of quadrilaterals. The lesson introduces the use of the repeat command. Be sure students note that problem #5 does not make a rectangle. Students may need to act this out by walking around the room according to the directions.

Sharing methods of solution is valuable because LOGO offers so many methods of attack. Some students solve each problem separately, others modify a result to get the next required figure. Not only are the geometric principles made clear but also problem solving strategies can be discussed.

APPENDIX C

STUDENT EVALUATION FORM

STUDENT EVALUATION FORM (SAMPLE)

Unit ___ Geometry Class Period____ Male or Female_____

Now that this unit of study is complete, please discuss each lesson that we did. Your comments will be kept confidential and will be considered carefully.

Lesson 1 TILING

I liked this because________________________________

I did not like this because__________________________

I would change this by ______________________________

Comments____

Teaching Approach

I liked what the teacher did because__________________

I did not like what the teacher did because___________

I would have liked the teacher to_____________________

Comments____

Lesson 2 ANGLES AND LOGO

I liked this because________________________________

I did not like this because__________________________

I would change this by ______________________________

Comments____

Teaching Approach

I liked what the teacher did because__________________

I did not like what the teacher did because___________

I would have liked the teacher to_____________________

Comments____

(Note: This form continues in the same manner and includes all lessons in the unit.)

APPENDIX D

SUMMATIVE EVALUATION FORM

SUMMATIVE EVALUATION FORM

Unit ___ Geometry Class Period____ Male or Female_____

The unit we have just completed was presented to you in various formats. I would like you to indicate which of the following activities you felt influenced your learning in a positive way (were very helpful), which were negative influences (were confusing and not helpful) and which had no effect at all (were neutral).

1. Whole class with teacher explaining at the blackboard.

   Positive Negative Neutral

2. Exploring ideas at the computer.

   Positive Negative Neutral

3. Working in groups with classmates.

   Positive Negative Neutral

4. Writing out explanations, not just answers.

   Positive Negative Neutral

5. Activities involving actual objects.

   Positive Negative Neutral

6. Teacher help during small group work.

   Positive Negative Neutral

7. ____
   (Your Choice)

   Positive Negative Neutral

APPENDIX E

TEACHER RESOURCES

Resources Used in Developing the Units

TEXT BOOKS

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