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# THE COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP) <br> A MODEL PROJECT TO INCREASE ACHIEVEMENT IN A FIRST COURSE IN HIGH SCHOOL ALGEBRA (1979-1983) 

A Dissertation Presented
by
GILBERT J. LOPEZ

Submitted to the Graduate School of the University of Massachusetts in partial fulfillment of the requirements for the degree of

DOCTOR OF EDUCATION
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School of Education
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THE COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)
A MODEL PROJECT TO INCREASE ACHIEVEMENT IN A FIRST COURSE IN HIGH SCHOOL ALGEBRA (1979-1983)

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by
GILBERT J. LOPEZ

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## To

my wife Mary
and daugther Catherine
for their love,
patience and understanding.

## ACKNOWLEDGEMENTS

Sincere appreciation and special thanks to members of my dissertation committee for their guidance, encouragement and support over the extended period of time consumed by project and dissertation development: William J. Masalski chairman of the committee and committe members Harvey Scribner, Doris Stockton and Ernest Washington.

The CMSP model research and development work described in this project case study originated at the School of Engineering and Applied Science at Columbia University and operated as a collaborative project blending the resources from a diversity of institutions, including schools, colleges and industry. Many individuals from these institutions made special contributions during the project period that enabled the CMSP to establish a firm base and to mature as a viable research and development effort. In particular, many thanks are extended to Peter Likins, Dean of the School of Engineering and Applied Science at Columbia University, Nathan Quiñones, Executive Director of the High School Division of the New York City Board of Education, Chor Weng Tan, Dean of the The Cooper Union School of Engineering, Arsete Lucchesi, Professor of Mathematics at The Cooper Union, David Reyes-Guerra, Executive Director of the Accreditation Board for Engineering and Technology, Richard Neblett, James Nixon and George Aguirre of the Exxon Corporation, Louis Gonzalez of the International Paper Co., Sandra Kuntz, Vice-President of the International Paper Co. Foundation, Charles Bowen of the IBM Corporation and Howard Burpo of the Union Carbide Corporation .

The research and development of the Comprehensive Math \& Science Program (CMSP) model of curriulum and instruction during the 1978-1983 project period was made possible by grants from the private sector, including: the Alfred P. Sloan Foundation, the Exxon Education Foundation, IBM, the International Paper Company

Foundation, Union Carbide Corporation, Stauffer Chemical Company, Con Edison, the General Electric Foundation, Mobil Oil Foundation and the Pfizer Foundation. Grants from the private sector enabled the research and development of curriculum and instructional models that were central to the overall project effort. Private sector funds were matched by budget support from the New York City Board of Education which covered expense for school instruction, materials and coordination.

The CMSP model project was a structured and extensive effort that involved dozens of teachers and hundreds of students during each of the four years of project work. In contrast to the magnitude of the project, the central project staff was small, consisting of three full-time staff members, the Principal Investigator/Project Director Gilbert Lopez, the Administrative Coordinator Virginia Sawyer and the Program Coordinator Chester Singer. Virginia Sawyer, through a concentrated and tireless effort, handled with veracity the ever developing flow of curriculum, revisions and concurrent administrative and logistical details that characterized the active CMSP research project. On the programmatic side, Chester Singer, an experienced high school mathematics teacher, worked with a keen sense of awareness to lay the groundwork for initial project test implementation and coordinate the often difficult schedule of CMSP instructional program activity at the participant schools. A fourth member of the CMSP project team was Joan Diller, a highly creative and talented mathematics teacher who served the project first as curriculum consultant (1978-1981) and in later years, (1981-1983), served as a full-time program coordinator. Virginia, Chester and Joan represented a dedicated project core staff who provided the creative energies and technical support to assist the Project Director in managing and directing the CMSP model project as reported in this project case study. Without their hard work, commitment and attention to detail, the CMSP model would have remained a theoretical consideration.

ABSTRACT<br>THE COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP): A MODEL PROJECT TO INCREASE MATHEMATICS ACHIEVEMENT<br>IN A FIRST COURSE IN HIGH SCHOOL ALGEBRA (1979-1983)<br>MAY 1987<br>GILBERT J. LOPEZ, M.A., UNIVERSITY OF MASSACHUSETTS Ed.D., UNIVERSITY OF MASSACHUSETTS<br>Directed by: Professor William J. Masalski

The Comprehensive Math \& Science Program (CMSP) is an action research project aimed at developing model curriculum and organizational strategies to rebuild and establish students' foundation for high school algebra. The CMSP research effort was initiated as part of a national effort to significantly increase minority student representation in colleges of engineering which at the time of national project impetus in 1973 was well below parity. The underrepresentation appears to stem from an insufficient pool of minority students who graduate from high school with the requisite mathematics and science background. The problem is compounded by the apparent inadequate mathematics instruction that minority students receive in the middle and junior high school which leaves them largely underprepared to enroll and achieve in high school algebra coursework.

A founding assumption that guided CMSP work was that all students can learn mathematics very well given the foundation and academic support for the mathematics they are expected to learn in the classroom. This precept led to the development and design of a three semeter Prealgebra and Algebra model curriculum that was test implemented in three sequential cycles of model project activity during the period from 1979 to 1983. With each succeeding cycle of project activity the curriculum model was shaped and modified by timely and continual feedback from participating teachers and students. In all, eleven public schools in New York City, 70 teachers and over 2,000 randomly selected students participated in the development and test implementation of the mathematics
curriculum model. All of the schools participating in the CMSP had a predominant Black and Hispanic student enrollment and all but one were characterized by low enrollment and achievement in a first course in high school algebra. The CMSP model curriculum that was developed and test implemented allowed students entering high school with inadequate mathematics background to build a foundation for algebra in the space of a single semester. This provided students with the preparation and opportunity to enroll and achieve in the study of a first course in algebra as prescribed by the New York State Board of Regents.

Students who studied the first course in algebra utilizing the CMSP model curriculum outperformed similar student groups by better than two-to-one margins on New York State Regents Algebra Examinations. This better Regents examination performance was consistent across the diversity of participant schools and in the repeated cycles of model test implementation. Objective assessment of the model was hampered by the very high attrition rate of students which reduced the randomness of the participating student population. Nevertheless, the CMSP model project demonstrated that inadequate mathematics instruction at the middle and junior high school need not preclude students entering high school from enrolling and achieving in a first course in high school algebra.

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## CHAPTER 1

## OVERVIEW AND RESEARCH PROJECT SUMMARY

### 1.1 Introduction

Low student achievement in mathematics at the elementary and secondary school levels is a matter of record and is widely acknowledged to be a serious and pervasive problem in the nation's school systems. ${ }^{1}$ The problem is especially acute in the inner city schools of large metropolitan areas where complex socioeconomic factors aggravate the process of education as a whole.

For example, students in the inner city schools of New York City score markedly lower on standardized mathematics achievement tests compared to the school populations in outlying fringe and suburban areas. ${ }^{2}$ This low level of performance becomes magnified as students progress through junior and senior high school, leaving an extremely small pool of students who eventually complete and are successful in a traditional academic mathematics program before their high school graduation. In many of these New York City high schools (with enrollments of over 2,000 ) there is barely one class at the 12 th grade level that has successfully completed a traditional three year mathematics sequence of algebra, geometry, and trigonometry.

As in many other large cities, the inner city schools of New York are populated largely by minority students* and are located in low income neighborhoods isolated from any convenient interaction with modern industrial, business and higher education institutions. As such, minority students have little opportunity to meet personally with scientists and engineers with whom they may discover and learn about the challenging and rewarding

[^0]and rewarding nature of scientific and engineering vocations. Nor are there available the opportunities to see or experience how products and services of modern technology accrue from the study and applications of mathematics and science. The problem is further exacerbated by the very severe shortage of qualified mathematics teachers that appears to exist in the junior high and middle schools located in the inner city of large metropolitan areas. Students in these schools may not be getting the necessary mathematics foundation to enroll in and successfully pursue more rigorous mathematics study in high schools.

This lack of exposure to the necessary constituents, coupled with an inadequate background in mathematics and science, places students from inner city schools in the difficult position of trying to master school subjects which may appear to have no purpose or application to their lives. This double barrier to learning is reflected in the small number of minority high school graduates who have the inclination and proficiency to pursue and succeed in the study of engineering or the physical sciences at the college level. The consequence of this is the marked underrepresentation of minority persons currently employed in the engineering and science professions. ${ }^{3}$

### 1.2 A Direction for Study and Solution

Any significant advances that minorities make in engineering and science professions both now and in the forseeeable future are ultimately tied to the quality of secondary education, in particular, the quality of mathematics education that minorities receive. Only with a very strong mathematics foundation acquired in high school can students be expected to successfully complete the rigorous mathematics course sequences that make up engineering and science college programs of study. This fact underlies the research and development to be described and examined in this model project case study: a project effort that has sought directions for study of and solutions to the problem of minority underrepresentation in the nation's engineering colleges. The project is described in terms
of a long term process of action research and model development and test implementation that has taken place in several New York City high schools during the period from September 1979 to June 1983. During this period of research and development activity, a model of mathematics curriculum and instruction has evolved that differs significantly from conventional high school mathematics course offerings and student evaluation procedures currently utilized in New York City high schools.

Using a field based and systems approach, the model has been researched, developed and tested extensively in eleven New York City schools (three junior high schools and eight high schools) where over 2,000 student participants were selected at random from the schools' incoming seventh and ninth grade student populations. The participant schools test-implemented the model in lieu of regular school day mathematics programs, providing the necessary personnel and institutional resources to allow for objective and detailed comparisons of participant student mathematics achievement both within and across schools. Data and findings from the four years of project research and development show the model's promise to bring about a substantial increase in the pool of entering ninth year high school students who enroll and achieve in the first course in algebra as prescribed by the New York State Board of Regents.

The model of mathematics curriculum and instruction reported in this model project case study has, over the years, taken on the designation "CMSP", an abbreviation for Comprehensive Math \& Science Program which is the official name of the project. Hereafter, all references to the model of mathematics curriculum and instruction will appear as CMSP model, model project, or CMSP.

The CMSP research and developmental work directed at the first course in high school algebra represents the first phase of a larger, more comprehensive model building effort that will encompass the full four years of high school mathematics study in New York City. The model project work during the first phase was directed at demonstrating
the feasibility and utility of a curriculum model that could be used to restructure precollege mathematics programs in high schools where higher student mathematics achievement was desired.

The CMSP project efforts were promulgated on the belief that the essential and core subject in high school mathematics is algebra. Unless a student has a solid foundation and achieves at a high level in the very first course in algebra, enrollment in and completion of a traditional three to four year high school mathematics sequence (Algebra 1, Geometry, Algebra 2/Trigonometry and Precalculus/Calculus) is unlikely. This belief stems from the notion that fragmented and insufficient achievement in high school mathematics is one of the major obstacles preventing minority students from considering and successfully pursuing engineering or science based college study. Until this obstacle is overcome, the quest for parity by minorities in the engineering and science professions will be seriously hampered. The CMSP model project experience to be described in this chapter, as an overview, and in later chapters, in detail, provides a base for study of the factors which impede high school student mathematics achievement together with a model that can be further researched and explored as a pedagogical and curriculum strategy to enhance mathematics learning.

The problem being addressed by the CMSP is a highly complex one and is compounded because it is immersed within the larger context of the New York City high school population and the nation's high school system as a whole. The latter in its present state has been deemed by several national commissions and task forces as being less than adequate to meet the nation's future need for a strong technical workforce and educated citizenry. The "Nation At Risk" report outlines recommendations and plans of action which include increasing high school graduation requirements to three years of mathematics "to equip graduates to understand geometric and algebraic concepts" as well as a host of other mathematical principles and topics. ${ }^{4}$ This is in sharp contrast to the
minimum mathematics requirement for graduation from many of the nation's high schools which, for the most part, is well below those recommended by the national commissions. ${ }^{5}$ Resolving these contradictions in standards will need considerable discourse and time. And higher academic standards in the nation's high school systems will probably require legislation at the state and, possibly, federal levels. Commensurate with the enactment of higher academic standards, comprehensive support programs must be put in place to insure that students in need obtain the necessary academic assistance to meet the new standards. In the interim, large populations of high school students will continue to have inadequate schooling in the study of mathematics either to prepare them for subsequent high school and college study in mathematics and science or for entry level positions in the growing technical marketplace.

### 1.2.1 Perspectives on Past Development Efforts

The irony of the current dilemma is that similar national concerns about high school mathematics and science education were raised soon after the launching of Sputnik by the USSR in 1957. This spectacular event was followed by a deluge of federal and private foundation sponsored programs aimed specifically at increasing high school student achievement in mathematics and science. The primary goal of these programs was in developing new mathematics and science curricula and in teacher training, the aim of which was to keep schools abreast of new pedagogical techniques and to introduce aspects of modern sciences and technologies emerging during the post-Sputnik era.

It is a paradox that the proliferation of mathematics and science developmental programs, funded heavily over a period of more than twenty years, paralleled the decline in student achievement as measured by the National Assessment of Educational Progress (NAEP) studies and SAT-Math scores. ${ }^{6}$ Inferences can be made from this coincidence that the programs themselves were not broad enough in scope nor sufficient in duration to
offset the many complex factors contributing to the decline in student achievement. However, what is of greater importance than claims or conjectures of the programs' and their effect on student achievement is the fact that few, if any, of those developmental programs or their spinoffs are in existence in the high schools today. ${ }^{?}$

What is to be gained foremost from these past developmental program experiences is that making curriculum changes in a high school system that is steeped in tradition is a highly complex business. History tells us we must go beyond the accepted theories and methods of curriculum development and teacher training which, as strategies, have not been sufficient to effect large scale improvement in mathematics and science education. If there are to be comprehensive efforts to improve high school mathematics education significantly, we must broaden our view when investigating the problem. Essential to this is obtaining a better understanding of the nature of the problem in all its aspects, including the variabilities in institutional culture and the non-linear and dynamic processes of teaching and learning. There must also be a realization that current traditional models of educational research and theoretical inquiry may be inadequate to deal with the enormous complexity of the problem. The very small return on the huge federal investment in educational research and development over the last twenty years supports this argument. The National Science Foundation (NSF) alone spent over 800 million dollars from 1962 to 1980 specifically on precollege mathematics and science education, primarily in the area of mathematics and science curriculum development and teacher training. ${ }^{8}$

In the quest for a direction for study and solution, the CMSP pursued an experimental and field based approach largely because of the ineffectiveness of previous federally and state subsidized mathematics programs of remediation. These were created to stem the severe decline in student mathematics achievement in high schools with predominant minority student populations. Model projects that were developed to address the problem of minority students' underachievement, including diagnostic/prescriptives, Mastery

Learning, School Improvement Programs and Project SEED concentrated their efforts at the elementary and junior high school levels where the more centralized organizational qualities of these institutions lent themselves to the methodological approaches that were inherent in the model project strategies. However, the departmentalization along specific academic disciplines that characterizes urban high schools makes them almost impervious to methodological approaches to change. It can be argued with some conviction that high schools (and colleges also) will respond to significant curriculum changes only where it can be shown and demonstrated conclusively that such changes are practical and will bring about a marked and long term improvement in student achievement as a whole. It was with this contention that the CMSP first initiated its research and development efforts to create and test generalizable models of mathematics curriculum and instruction within the working environment of large high schools in New York City in 1978.

### 1.2.2 The National Minority Engineering Effort

The systems and field based approach taken by the CMSP in its research and development efforts has enabled examination of, at close range, the diversity and interdependence of school related factors which preclude or deter minority high school students from developing high levels of mathematics proficiency. In addition, the model project has sought to integrate and enrich the high school mathematics curriculum with personal and practical examples of science and technology in order to offset the scarcity of such learning experiences in the inner city high school and community environment.

Examples of work in these contexts can be found in the many precollege and college intervention programs around the country which have been developed in the decade of the 1970s to increase minority student enrollment in engineering colleges. Since 1973, a national effort has been in effect to identify, recruit and nurture minority students who have the interest and background for engineering college study. These efforts have taken place
both at the high school and college levels where students participating in special programs have obtained technical career counseling, academic enrichment, support services and financial aid incentives.9, 10 Since their inception, the special programs in operation across the country have made a dramatic impact on minority student enrollment in engineering colleges.

In the $1981 / 82$ academic year, over $11,000(10 \%)$ of the more than 110,000 engineering college freshmen were identified as underrepresented minorities--Black, Hispanic and Native American students. This is in contrast to 1973/74 (the first year such data were compiled) when just over 3,000 minority students (or $6 \%$ ) were part of the total freshman engineering population of $50,000 .{ }^{11}$ Most of these freshman enrollment gains are directly attributable to the special minority engineering programs and student service organizations in place at engineering colleges. The gains were also bolstered by regionally established precollege consortia (consisting of high schools, colleges and industry) which help prepare and assist students in making the connections between high school and engineering college. ${ }^{12}$

The increase in minority engineering enrollment over the last decade, while substantial, have been overshadowed by the markedly lower rate of minority engineering graduates as compared to the graduation rate of the general engineering student population. In 1981, the rate at which of Blacks, Hispanics and Native Americans graduating from engineering college was 4.7 percent. This figure is far below the $25 \%$ percent that minorities comprise in the nation's college age population on the basis of the 1980 census. ${ }^{13}$ Cognizant of these data, the national minority engineering effort revised its earlier goal of achieving parity in minority engineering student enrollment by 1984 to one of graduating 8,000 minority engineering students by $1988 .{ }^{14}$

Underlying this new goal and all other attempts to gain parity for minorities in engineering and science based occupations and professions is the extremely small pool of
minority students who achieve in the study of mathematics and science at the precollege level. Unless this pool is increased significantly, minorities will continue to lose ground not only in their quest for parity in higher education but also in professional and technical career opportunities beyond high school.

While the current precollege and extracurricular efforts are needed to continue to identify, nurture and produce greater numbers of minority students with an adequate mathematics foundation for engineering, the ultimate solution will be realized through the implementation of major program strategies to substantially increase minority student achievement in high school mathematics. The CMSP model project effort is one such strategy that is addressing this issue.

### 1.3 The CMSP: A Systems And Field Based Approach To The Problem

The dilemma in education that minorities and the general student population face has no precedent. Any attempt to correct the situation must invariably use an approach that leads to the creation and building of curriculum models that take into account the multiplicity of variables that interplay in the high school mathematics classroom. The CMSP has adhered to this research doctrine by experimentation and development within the high school environment itself. This field work and experimentation has been aimed at investigating curriculum and instructional practices under real world conditions. Through an empirical process the CMSP has researched and developed curriculum based models that appear to foster student achievement in fundamental coursework in prealgebra and algebra, both of which are prerequisite for the traditional high school courses in geometry and trigonometry which follow. The project work has been conducted utilizing scientific and engineering management principles, and the model project case study will show that significant progress has been made in finding ways to reduce the complexity of our current problems in secondary school mathematics education.

The systems and field based approach which has been utilized in the CMSP has great potential for inquiry and creating models to better understand the ways teachers and students interact in the process of teaching and learning. The approach rests on the belief that current techniques and developmental strategies for making change in education are prescriptive and authoritative in nature and thus their impact can be only transitory. This has been shown to be true by the great failure of the "new Math and Science" education reform movements of the 1960 's. ${ }^{15}$ In contrast, the field based and systems approach utilized in the CMSP model project is evolutionary and dynamic where in the real world environment of the schools, students and teachers play a decisive role in the process of curriculum change and development. It is through their immediate and continual feedback and assessment of program elements that a model project evolves. And ultimately it is the consensus of opinion of participant students and teachers (however arrived at--either though higher classroom achievement or long term use of curriculum materials) that ultimately determines the effectiveness and utility of model project efforts over the long term. Herein lies the nature and reality of systems and field based project efforts--success can neither be prescribed nor instituted; it must be proven without reservation prior to acceptance and wholescale use by the school community.

### 1.3.1 Goals and Premises

The primary goal of the CMSP, since its inception in 1979, has been to research, develop and test models of mathematics curriculum and instruction aimed at significantly increasing the pool of students in the inner city high schools of New York City who enroll and achieve in the study of the traditional 3-year high school mathematics program. In the attainment of this goal, the CMSP focused its initial efforts on the first course in algebra generally taken in the ninth grade. Because of the prerequisite and sequential nature of the traditional high school precollege mathematics curriculum, the first course in algebra must
be mastered if students are to have any opportunity to continue achieving at the next level and subsequent mathematics courses--geometry and trigonometry--prior to high school graduation.

Setting forth the project goal: to create a model that would insure the mastery of a first course in algebra, two interelated premises were established as cornerstones on which project efforts would be directed and assessed:

1) the major deterrent to the successful learning and completion of a first course in algebra is the lack of student preparation in the basic arithmetic upon which algebraic concepts and algorithms are founded.
2) for most entering ninth year high school students, preparedness for a first course in algebra can be attained in one semester, independently of students' prior mathematics proficiency and background.

The first premise is grounded on the sequential nature of the high school mathematics curriculum where advancement to higher level courses is highly dependent on student mastery of preceding coursework. The second is based on the concept that the fundamentals of mathematics can be learned well by most students in a relatively short period of time--provided the mathematics curriculum and instruction has the structure and continuity to foster and reinforce student concentration and effort.

Both of these premises have been tested by the CMSP through the research, development and test-implementation of a model system of mathematics curriculum and instruction which focuses on building strong academic foundations as a precursor to the study of algebra. Research and development work undertaken by the CMSP is based on experimental project efforts at eleven New York City inner-city junior high and secondary schools over a four-year period where definition, feasibility and building of the model prototype took place during regular school day hours within the high school environment.

This experimental work has yielded strong indications that significant achievement gains in the study of algebra can accrue when students have a well grounded foundation in arithmetic skills and problem solving routines. The project work was conducted on a sizable scale with a pool of randomly selected high school students at the seventh and ninth grade levels who would have otherwise been programmed and tracked in "remedial mathematics" programs of study. Preliminary data and findings show that the students who participated in the CMSP project outperformed similar student groups at the same participant schools by two-to-one margins on standardized mathematics competency and algebra examinations administered by the New York State Board of Regents.

### 1.3.2 Systems and Field Based Research and Development

The organization and design of the CMSP model was predicated on the proposition that the curriculum (in the broadest sense of the term) could be more realistically developed while the effort was undertaken in the very environment in which it was to be used--namely, in the working day classroom. The process undertaken by CMSP is evolutionary rather than prescriptive as in traditional educational research where projects and curricula tend to be fully developed prior to large scale implementation. The evolutionary research process utilized by the CMSP is akin to the methods and techniques used in engineering in the development of new products and systems. Using the engineering approach, the growth of a new product is tempered by a dynamic sequence of events that calls into play the tenets of Research, Design, Development, Test \& Evaluation (R,D,D,T\&E) at each stage of its maturation. The inherent value of this approach is that a model prototype can be shaped and modified in stages and on the basis of steady and timely feedback. This includes full-scale testing of model prototypes directly in the environments in which they will be used. In this mode of systems development, all of the elements of the model system are developed and continually tested in a parallel and
hierarchical project arrangement that insures controlled and evolutionary growth.
The systems and field based approach utilized by CMSP has been largely avoided in much of the mathematics and science educational research and development efforts of the past. This may be because the process of gaining access to a typical urban high school to conduct experimentation and develop and research models with the intention of making substantial change in the curriculum and instructional programs is both time consuming and complex. The process requires organization, sustained planning and collaborative negotiations with public school and higher education officials. This organizational process can readily tax the often limited resources of traditional education research projects and stretch project time lines well beyond reasonable limits. However, the collaboration of high schools and colleges is an essential quality of systems and field based projects and unless joint institutional commitments are firmly established prior to model project effort, coordinated and useful modes of research and development of inquiry are unlikely.

Creating and developing a curriculum model prototype under field based conditions carries with it the responsibility of recording and describing the evolving chain of events and organizational strategies that are used to develop and test-implement the model. This is where evolutionary field based and traditional top down approaches to curriculum development differ sharply. In the former, project implementation strategies become an inherent ongoing part of the development of a model curriculum, while the latter assumes that diffusion of the model curriculum that is developed will take place automatically or within the realm of school administrative practice. Traditional curriculum development and research practices, because of their prescriptive nature, do not provide any information and the wealth of data that is generally available in the diffusion and model test-implementation process. This is a research limitation because it is as important to know why a model curriculum or instructional strategy does not work as it is to know why
it succeeds.
In many respects, the development of curricula is only part of the process of creating a model for educational change. Creating the structure for implementation is equally as important as the curriculum product itself; and the lack thereof may mean the difference between acceptance or rejection by a school or school system. The mathematics and science curriculum development of the 1960's and 1970's has shown that even the most highly regarded curriculum can go underutilized for lack of an entry point into the school system.

The systems and field based experiences of the CMSP have demonstrated that curriculum development and implementation strategies are mutually supportive and interdependent. Both these strategies need to be researched, developed and fine tuned in consonance. As integrated parts of a complete system they can be tried and field tested and evaluated almost simultaneously across a wide range of environments and conditions. Through this integrated process of systems research and field based development, the model's effectiveness is heightened and replication difficulties minimized substantially.

In conducting research and development under field based conditions, model project activity is immersed into the real world environment of the school with no assurances or guarantees of project outcomes or continuance in a given school. Uncontrollable factors such as cuts in budgets, excessive student absences or dropouts, teacher layoffs, changes in attitude on the part of administrators toward the project, experience and background of the teaching staff, professional relationships between teachers and department chairpersons and general administrative stability of the school in particular can greatly influence the normal operation and longevity of a project. Any of these factors peculiar to a school are completely beyond the control of the research investigator and all are very difficult to ascertain or predict prior to project implementation. During the course of project implementation, the occurrence of any one of the factors can seriously disrupt or even lead
to the eventual termination of the project in a school.
It is often with this air of uncertainty that field based projects such as the CMSP must operate on a day-to-day basis. Over a prolonged period of time of working in the school, however, if the model project is found to establish a curriculum structure that contributes to student mathematics achievement, it will be perceived by the school community as being useful and an integral part of the school's developmental and instructional resources. With this recognition and acceptance, the model project is less apt to be disrupted or cut. Because of the intimate collaboration and acceptance by the school, curriculum models can be researched and developed as comprehensive educational systems and with far more clarity and depth than projects which operate outside of the school environment. This is the basic strength of the systems and field based project approach.

### 1.3.3 Curriculum Model Design

The curriculum model design that has evolved over the four-year project period of CMSP research and development has been predicated on creating classroom and curriculum strategies that give all incoming ninth grade students the opportunity to work to their highest level of academic mathematics potential. The model was designed to provide a framework for a highly structured prealgebra and algebra curriculum that has four key elements:

1. A Zero-Base Start: The incorporation of a highly structured and intensive one-semester program of study that sequences basic arithmetic operations with a heavy emphasis on word problem solving and geometric applications.
2. A Complementary Mathematics Curriculum: The development of a parallel, interlocking set of mathematics courses taken over a three-semester period (students take two mathematics courses each semester for three semesters) that
substantially increases the rate of mathematics instruction and gives students considerably more time to apply and reinforce their mathematics learning.
3. Random Student Selection and Heterogeneous Class Groupings: Students who are enrolled to study mathematics utilizing the CMSP model are selected at random from the entire incoming 9 th grade student body and are grouped heterogeneously for each of the two CMSP complementary courses.
4. Uniform Pace and External Testing: All participating classes move at the same instructional pace and are evaluated on the basis of uniform tests constructed outside the classroom but administered by the classroom teacher.

Each of these elements serves to intensify and broaden both the teaching and learning of mathematics. The elements have also provided the foundation for the development of curriculum materials that have a problem solving orientation and a structure that promotes class mastery of given mathematics topics as the class progresses throughout the term. This is accomplished by organizing the curricula of both courses in parallel so that a particular mathematics topic is seen and studied by the class twice, doubling the length of time generally assigned. This parallel arrangement of the two courses interlocks the continuum of topics over a full semester, providing students with continuous instruction and reinforcement of learning. Redundancy and saturation of learning in a given mathematics topic is minimized because of the complementary way in which both courses present problems for study and review. In one course, problems for a given topic are numerical in form, while in the other, the same topic is presented geometrically. In both courses there is a heavy emphasis on word problems which are constructed based on real world situations and within the context of important mathematical themes and concepts.

The complementary curriculum is supported by a zero based start which makes no assumptions on what a student's mathematics background is prior to program enrollment.

This gives every student an opportunity to review thoroughly and to strengthen arithmetic operations and problem solving routines during a single semester prealgebra course. The same complementary parallel course approach is continued for two additional semesters in the first year algebra course as prescribed by the New York State Board of Regents. Figure 1 is a block diagram representation of the complementary courses organized over a three-semester period. Each of the complementary semester course blocks indicates the mathematics topics which are taught in parallel.

The achievement level of all CMSP classes in a given school is measured by the administration of uniform tests on a bi-weekly basis. The uniform tests help regulate the pace of both courses and assure that progression to the next course topic is consistent with mastery of the previous unit. The external construction of unit tests minimizes teachers' teaching to the test and makes determination of class mastery a more objective process.

### 1.3.4 Parallel Approach to Project Test-Implementation

The CMSP organized its research and development efforts into four distinct and overlapping three-semester cycles during the September 1979 to June 1983 model project period. This organizational design has allowed the model to be researched, developed and field testedwith a continuously renewed student and teacher population. In all, thirteen schools, 70 teachers and over 2,000 randomly selected students participated in the development and test-implementation of the model. The flow of the cycles of project activity as well as the number of schools and students involved in each cycle is illustrated in Figure 2. The first two cycles of project activity were essentially periods of problem definition and feasibility testing, where a process of almost continuous testing and modification allowed a rudimentary form of the model to emerge. In the third and fourth cycles a process of design and development, coupled with a close scrutiny of data and teacher feedback helped shape the model's complementary course structure. And, finally,
COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP THREE-SEMESTER MATH ACHIEVEMENT MODEL .. FALL 1982
ETRST SEMESTER
SECONDSEMESTER

NOTE: The topics listed in the Algebra sequence represent the curriculum format used by the CMSP in 1982 to satisfy the curriculum include salient Algebra topics and also topics in Logic, pronent efforts.
Figure 1
COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)

Figure 2
the experience gained from the first four cycles provided an organizational base to build a framework for the development of a model prototype which would be test implemented on a large scale within a network of seven high schools beginning in the Fall of 1983.

The CMSP began its first cycle of research and development and test-implementation in New York City in the Fall of 1979 with eight high schools and three junior high schools which agreed to participate in the CMSP effort over a three semester period. Each of the participant schools selected approximately 60 students (two classes) at random from their incoming student populations to study prealgebra and algebra using the CMSP model in lieu of the school's regular mathematics program. One of the schools, Brooklyn Technical High School, started with four classes. A total of about 700 students and 24 mathematics teachers participated in the first cycle of project activity.

The selection of eleven schools to become involved in the very first cycle of model experimentation and development was a key decision that laid the foundation for parallel field based operations. The parallel approach to model project research and development established a research mechanism that insured substantive and timely feedback from a variety of sources. In addition, conducting project activity in several different schools allowed the CMSP to interact with a critical mass of teachers, department chairpersons and principals. These relationships with the staff of the eleven participant schools quickly established project acceptance and provided an initial sense of whether the model in its rudimentary form was progressing.

In the final analysis, the ultimate value of the parallel approach was in maintaining model project stability over a prolonged period of time in the face of the uncertainties that occur regularly in large city school systems. Operation within a single school or just a few schools would have naade the project vulnerable to factors beyond its control. Although taking the parallel approach added complexity to the project management overall, the advantages gained in project endurance and time saved in obtaining knowledge and results
from many sources of experimentation far outweighed the complexity of organization.
The parallel approach proved its value in CMSP implementation on a number of occasions. After the first year of model project activity, three of the original eleven schools were phased out of the project for a variety of reasons which included high student dropout rates, lack of faculty consensus, and school budget difficulties.

In the second cycle (September 1980-January 1981) three of the eight remaining schools (two high schools and one junior high school) and one new high school enrolled a new group of randomly selected incoming students ( 60 students at each school), for a total of approximately 240 students. Difficulties arose once more that were again beyond the project's control, except in this instance it was a pedagogical issue that surfaced gradually at the three junior high schools as course content became more involved.

Almost a year after the inception of the first cycle, it became apparent that most of the junior high school teachers participating in the CMSP lacked the experience and background in mathematics to cross the pedagogical threshold from prealgebra to algebra. This had a serious negative impact on student achievement and at each of the three junior high schools there was almost no progress in mathematics coursework beyond prealgebra. At that point in time, a situation had arisen that none of the three junior high schools nor the CMSP could alter with the instructional resources available. And, as the project continued, the problem became more accentuated and created an impasse that led to the termination of project activity at all three junior high schools.

This experience, which occurred well into the project's second cycle, again demonstrated the uncertainties which are prevalent in field based projects. After two cycles of project activity, seven of the original eleven schools had ceased participation in the project for reasons that could not have been predicted at project inception nor could they be controlled during project test-implementation.

The value of taking a parallel approach in implementing field based educational
projects cannot be overstated. And, in the CMSP experience described, the experimental efforts over the two-year period would have been totally forfeited had only a few schools been selected to participate at project inception in 1979. Because eleven schools were selected, the project was able to withstand the severe attrition of participating schools and continue its research and development with the four remaining high schools in the third cycle.

Following the three cycles of model project activity, a more thorough understanding of the variables that influence project stability in the schools was gained. The experience enabled the CMSP to plan and organize a larger scale and long term project effort that began in the Fall of 1983. In this later phase of CMSP work (to be chronicled in a future project case study) a network of seven high schools participated in the development and implementation of a model prototype and tested the premises and effectiveness of the CMSP model on a larger scale.

### 1.4 New York City: A Microcosm

Ideally, a model intended to produce a specific outcome should be created and tested within environments that allow the model builder to generalize the model to the real world. The test environments should be as diverse and as rigorous as possible so that a process of worst case analysis can be implemented in the progressive stages of model development. Adhering to these principles of worst case analysis minimizes design faults and provides a strong and viable base for model replication and assessment of results.

In creating and testing curriculum materials within a school environment, the school itself plays a decisive role in regulating and controlling the pace of the project effort. Very little can be accomplished if the school environment is unstable or if the school is not supportive of the work to be undertaken. Given these two basic requirements, stability and a supportive environment, the investigator must insure that model building efforts take
place in as many different schools as can be managed effectively. The more schools, the better the chances for eventual model generalizability and project maturation. The schools that participated in the CMSP represented a broad cross section of the New York City schools, and as a block, provided the diversity of school characteristics that made test implementation of the CMSP model a comprehensive project effort.

The New York City high school system provides a large and diverse arena from which to study and analyze the decline in high school student mathematics achievement. Much of the difficulty that might be encountered in problem definition and analysis is removed because of the traditional and highly uniform precollege mathematics curriculum structure that is in place at the state and local district level. In terms of traditional precollege mathematics courses offered and student enrollment in these courses at comprehensive/ academic high schools with a predominance of minority students, there also does not appear to be a shortage of qualified mathematics teachers.

The New York City public school system is the largest in the country. In 1979/80 (when the project started) close to one million students were enrolled in grades $\mathrm{K}-12$ in 984 separate school buildings. ${ }^{16}$ The system is decentralized at grade levels K-9 with thirty-two community school boards governing elementary and junior high school education in given local school districts. The high school system is one enormous enterprise that is administered by the city's central board of education. In 1979/80 it was comprised of nearly 300,000 students in grades 9-12 enrolled in 112 high schools. ${ }^{17}$ (See Appendix A.)

Because of its magnitude and diversity of population, the New York City public school system can be viewed as a microcosm of the nation's high school system. Within its boundaries a full range of academic, social, economic and political considerations exist to characterize and influence the education of adolescent youth. These same considerations are notable and in operation in many of the large urban high school systems throughout the
country. While each school system is unique with its own set of regional characteristics, there are also common demographic and academic traits. Prominent among these is the large minority student population that dominates high school enrollment. Insufficient preparedness in basic mathematics skills among minority high school students is another. The high dropout rate is still another. As in New York City, minority students in large U.S. cities are advancing to secondary schools without the necessary preparation to pursue traditional academic high school mathematics coursework successfully.

There are contrasts in student mathematics performance among the high schools in the New York City school system which bear a striking resemblance to the wide variation in SAT median mathematics scores in state school systems nationwide. At one extreme, there are high schools in New York City where an essential course in 10th or 11th Year Mathematics cannot be offered because there is an insufficient number of students with the prerequisite knowledge base to fill a single class. In contrast, at the other end of the spectrum, in two of the nation's most successful public high schools, Bronx High School of Science and Stuyvesant High School, all juniors take 11th Year mathematics (as many as 20 full-size classes) and both schools have consistently dominated the proportion of National Merit Scholarship and Westinghouse Science awards. ${ }^{18}$ The irony of these mathematics achievement comparisons is that at both extremes the traditional three-year mathematics sequence of courses is in place with a sufficiency of qualified mathematics teachers to provide the necessary instruction. What appears to be the basic deterrent to improved mathematics performance for schools at the low end of the achievement scale is the increasing shortage of students who are academically prepared to engage successfully in the traditional mathematics courses of study as they enter high school.

The problem is the most serious in New York City high schools with predominant minority populations because of the apparent severe mathematics teacher shortage that prevails at the local middle and junior high schools from which the high schools draw their
entering student populations.* Invariably, students from these feeder schools enter their neighborhood high schools with a very poor foundation in mathematics and have almost no chance of being enrolled or succeeding in traditional 3-year high school mathematics programs. Instead, students' poor mathematics preparation inevitably leads them to a trail of high school "remediation" which rarely provides the basis for continued and advanced mathematics coursework beyond the two years (mathematics competency) required for high school graduation in New York City.

The CMSP, focusing its research and development efforts on the high school level, has recognized the enormous difficulty of trying to rectify the problems of mathematics teacher shortages at inner city junior high and middle schools, a problem of enormous proportion. Solving this problem will require huge investments of funds for teacher training over long periods of time with no assurance that such efforts (which have been tried before) will provide a workable solution. In contrast, the high schools provide a working academic setting and a sufficiency of qualified mathematics teachers to research, develop, and test curriculum based models.

* Student underprepareness for precollege mathematics upon high school entry strongly suggests inadequate or discontinuities in mathematics instruction at middle and junior high school levels


## CHAPTER 2

## A LOOK AT RELEVANT NATIONAL AND NEW YORK CITY MATHEMATICS ACHIEVEMENT DATA

### 2.1 Minority Engineering Data and Their Implication

The underrepresentation of minorities in the professional fields is a reflection of students choice of college major, the foundations they attained in high school, and their progress and achievement in their chosen four to eight year program of college study. Students who elect to pursue engineering college study invariably take and excel in three years of science (biology, chemistry and physics) and four years of mathematics including algebra, geometry, trigonometry, precalculus and the rudiments of calculus. Not only is four years of high school mathematics study required for enrollment in engineering and science based college study, it is also required for the fundamental core of mathemathics coursework in most college programs of study during the first two years of college.

In comparison to other undergraduate programs of study, engineering is a rigorous four year program course of study whose structure and content is built upon a mathematics core. Success in the first two years, for engineering college majors, is highly dependent on students' mathematics proficiency gained in high school and their capacity to endure the rigor of the four consecutive college semesters of the Calculus, Differential Equations and Engineering Mathematics. Coincident with mastery of this mathematics core, engineering majors must also enroll and achieve in Chemistry and Physics courses whose principles and concepts are bound in the abstractions of the Calculus and higher order mathematics.

The high demand for technical and scientific personnel during the last decade has been paralleled by a sharp increase in the number of students who major in engineering
and science. ${ }^{1}$ Both minority high school students and the greater high school populations have recognized the value of a rewarding and high paying scientific and technical career. The average starting salary for new engineering graduates with no work experience was $\$ 25,000$ in 1985-and students flocked to the nation's engineering colleges in unprecedented numbers in the last decade. ${ }^{2}$ Minority students' gravitation to engineering colleges has been carefully documented by the Engineering Manpower Commission and analyzed by the National Action Council for Minorities in Engineering (NACME) since 1973. These longitudinal data plus National Assessment of Educational Progress (NAEP) data and the Profiles of College Bound Seniors, compiled annually since 1971 by the College Board, in addition to data on high school mathematics enrollment and achievement in New York City public high schools provide a broad data base on which to make comparisons and draw conclusions on the disparity in mathematics course enrollment and achievement by minority high school students.

Because the engineering college program is so closely aligned to mathematics and because of the uniformity of the curriculum among engineering colleges, engineering college enrollment and graduation data can provide a stable statistical context in which to examine minority student enrollment and graduation as compared to the general student population. The Engineering Manpower Commission (EMC), under the aegis of the American Society of Engineering Societies (AAES), documents engineering college enrollment and graduation at colleges accredited by the Accreditation Board for Engineering and Technology (ABET) on a yearly basis. ${ }^{3}$

The EMC has been compiling engineering enrollment and graduation data for Black, Hispanic and American Indian students since 1973. In January of 1982, a comprehensive report by the American Association of Engineering Societies (AAES) examined EMC data in the context of Black, Hispanic American Indian and Asian/Pacific Islander enrollment and graduation over a nine year period--1973 to $1981 .{ }^{4}$ The longitudinal data enabled
the analysis of enrollment versus graduation for each of the four ethnic groups. In the report enrollment data show that the retention of Blacks at the nation's engineering colleges "appears to be a serious problem". The report continues:

Although it is not possible to tell from the gross numbers exactly how many individual students who enter as freshmen in engineering receive their baccalaureate degrees four or five years later, one can look at the total number of entering freshmen and compare that with the number of graduates, say, five years later, bearing in mind that the graduating class may not be made up of only those students who entered as freshman engineering students four years earlier.

For Black students, the numbers show that 1978 graduates were only 42 percent of the entering freshman class in 1973 and 1981 graduates were only 33 percent of the entering freshman class four years earlier in $1976 .{ }^{5}$

While the report did not draw any similar conclusions about Hispanic or American Indian engineering students, the enrollment and graduation data presented in the report indicate that for these two ethnic groups a serious retention problem also exists. The report states:

Of the nearly 63,000 graduates who received baccalaureate degrees in engineering in 1981, all minorities accounted for 8.3 percent, while Blacks, Hispanics and American Indians taken as a group accounted for only 4.7 percent of the group. ${ }^{6}$

The disparity between Black, Hispanic, American Indian and Asian/Pacific Islander engineering students is clearly shown in the above data analysis. On the basis of 1980 census data, Black, Hispanic and American Indian students accounted for $25 \%$ of the college age population, yet their representation as engineering graduates in 1981 was just 4.7\%. By contrast, students classified as Asian/Pacific Islanders in the report were overrepresented in engineering degrees earned in 1981 with $3.6 \%$ of the total compared
with their $2 \%$ representation in the college age population--again on the basis of 1980 census data.

The AAES report does not make any general comparisons on retention between Black, Hispanic and American Indian students to the larger population, however, sufficient data are available from the NACME 1985 Annual Report to assemble a graphical account. ${ }^{7}$ Figure 3 is a graphical representation of engineering enrollment and graduation from the academic years 1970/71 to 1984/85.

The lower half of the graph details freshman enrollment of Black, Hispanic and American Indian students as individual groups and also their figures are combined to give totals for minority freshman enrollment and graduation. By utilizing straight line comparisons between freshman enrollment and graduation four years later--e.g., 1974 and 1978, 1975 and 1979, 1976 and 1980, etc.--an average graduation rate for minority students of approximately $35 \%$ is obtained. This average graduation figure has to be qualified as not being wholly accurate because, as noted previously, the students counted in their freshman year and those graduating may not be entirely the same students. If transfers into and out of the four-year engineering program affect a substantial part of the total enrollment, this information could have a marked affect on the legitimacy of the data as it is presently compiled. Taking this factor into account and making the assumption that the transfer rate is not appreciably different between minority students and the larger engineering population, (this may be a large assumption) a general comparison can be made on minority student retention in engineering colleges.

The top half of Figure 3 shows total freshman engineering enrollments and graduates (including the minority students represented in the bottom part of the graph-but not including students from the University of Puerto Rico). Utilizing the sanie technique for calculating minority students graduation rate it is found that the average graduation rate for the larger engineering population was $70 \%$ or twice that of minority students whose


- Based on 4 years of study. UPR $=$ Univ. of Puerto Rico.

Graph by G. Lopez CMSP-7/86

Figure 3
graduation rate was $35 \%$. Actually, the difference is probably somewhat higher because the larger population data include minority students whose lower graduation rate reduced the overall graduation rate.

In looking at the graphical data, a number of statistical patterns and trends come to bear:

1) Freshman enrollments for Black, Hispanic and America Indian students increased substantially over the academic year periods from $1973 / 74$ to 1980/81. The almost fourfold increase by minority students-from 2,987 to 11,116--during this time was more than twice as great as the increase for the larger population which during the same period of time grew from 51,925 to 115,280 . The same dramatic increases occurred with minority graduates which rose from 1,256 in 1973 to 3,817 in 1985, a rise of greater than $200 \%$. This is in contrast to a $77 \%$ increase (from 43,086 graduates in 1974 to 76,576 in 1985) by the larger engineering student population.
2) Freshman enrollments for both minority students and the larger population peaked in the 1981/82 academic year to 115,280 for the larger population to 11,116 ( $9.6 \%$ of the total) for minority students.
3) There has been a steady decline in engineering freshman enrollments since the peak year of $1981 / 82$. In the larger population, freshman enrollments decreased 10,906 students from 115,280 to 104,374 in $1984 / 85$, a $9.5 \%$ drop. For minority students the percentage decrease was less. During the three years following the peak at 11,116 in $1981 / 82$, freshman enrollments declined by 522 students to 10,594 , the equivalent of a $4.7 \%$ decrease. Almost all of the decreases in the total minority student freshman enrollment are accounted for by declines in Black student enrollment--from 7,016 in the peak year of $1981 / 82$ to 6,245 in $1984 / 85$ for a reduction of 722 students, an $11 \%$ drop. This number is larger than the 522 drop registered for the total minority enrollment figure, however slight increases in Hispanic student freshman enrollments account for the differences in the Black and total minority student freshman declines.
4) During the period of declining enrollments from $1982 / 83$ to $1984 / 85$, the graduation rate for the larger student population increased from 66,652 to 76,576 a $15 \%$ increase. At the same time minority student graduates increased from 3,007 to 3,817 , a $26 \%$ increase.

Looking at the overall data there are signs of both encouragement and dismay. The encouraging signs are the dramatic increases in minority freshman enrollment and graduations that have occurred since 1973 in the nation's engineering colleges--over a threefold increase was realized with each minority group. Much of the progress in increasing minorities in engineering can be atributed to a national concern expressed by the private sector in consort with the nation's engineering colleges.

In 1973, the national minority engineering effort (as it has been called since its inception), supported almost exclusively by a group of large industrial corporations and the Alfred P. Sloan Foundation, initiated a comprehensive program effort to identify and counsel minority students towards engineering college study. ${ }^{7}$ Since then a myriad of precollege and college oriented programs have evolved, each with a common purpose-to increase minority engineering student enrollment and graduation. These programs have operated under the aegis of over 150 engineering colleges located in every major geographic region of the country. The national effort has been focused by the National Action Council for Minorities in Engineering (NACME), which in 1986 had on its Board of Directors the executives of 19 industrial corporations in the top 100 of The Fortune 500 listing, including E.I. du Pont de Nemours, Exxon, AT\&T, United States Steel, Hewlett Packard, General Electric, General Motors, RCA and General Dynamics, among others. ${ }^{8}$

While NACME has provided a central focus and a national outlook for the minority engineering effort (including the organization of an annual forum to highlight the national efforts, program accomplishments and goals, the establishment of a network of resource and program development and the administration and award of NACME scholarship funds
to minority students enrolled in engineering colleges), the actual work with students has been undertaken by college and precollege programs guided by the principles and contexts of the National Association of Minority Program Administrators (NAMEPA) and the National Association of Precollege Directors (NAPD). ${ }^{9,10}$ These two national organizations, whose programs are supported almost exclusively by funds and grants from the private sector, provide guidance and academic supportive services to a significant portion of the minority students enrolled in engineering colleges and to thousands of minority high school students with an interest in pursuing engineering college programs of study. These two national program efforts are quite substantial and are the direct result of the formation of the national minority engineering effort, itself initiated in 1973. It is generally recognized in the engineering education community that the significant rise in minority engineering college enrollment and graduation would not have occurred without the concerted efforts at the national level, through NACME, and at the regional and local levels through the work of NAMEPA and NAPD programs.

The less encouraging sign of the last decade in national minority engineering program efforts is the persistent two-to-one difference in graduation rate between engineering minority students and the larger majority engineering student population. The steady (and average) $35 \%$ graduation rate for minority engineering students coupled with the decreases in minority freshman enrollment in engineering colleges in the last three years is bound to adversely impact the significant progress that has been made to gain parity in minority engineering graduates since 1973. With fewer minority students coming into the pipeline and a continually lower rate of graduation, closing the wide gap in the forseeable future between minority and non-minority engineering students would appear to be difficult. The lower enrollments will eventually have an effect on the number of students graduating and unless some new approach is taken to better prepare minority students for engineering college study, the national minority engineering effort will be characterized by a leveling
off of minority engineering graduates. Minority engineering data show that this is already happening.

The successful completion of an engineering college program is fundamentally a function of a student's mathematical proficiency. Other factors such as social adaptation, financial considerations, perserverance and a general interest in science and technology may also contribute to success or failure at engineering college. However, the heavy mathematics conceptual framework and emphasis on mathematical problem solving embodied in engineering college coursework requires that prospective engineering students have both the mathematics proficiency and the capacity to compete academically in a rigorous and demanding four years college study.

Reviewing the minority engineering graduation data, it can be argued that, in large part, the problem of lower retention stems from a general underpreparedness of minority freshmen as they begin their course of engineering college study, in particular, their mathematics background and proficiency. Given the predominance of mathematics in engineering college coursework, the argument is consistent with the widely accepted fact that the quality of a student's academic preparation at the time of college entry is the major determinant for success in college. ${ }^{11}$ The argument, if valid, has serious consequences for future efforts directed at increasing minority engineering student enrollment and graduates.

The implications are clear--unless the pool of minority high school students who enroll and achieve in precollege mathematics is substantially increased, progress towards the goal of fair representation of minority students in engineering college, and ultimately in the engineering profession, will be difficult to realize. It also follows that increasing the pool of minority students proticient in precollege mathematics will be dependent on the public school systems' increasing student mathematics achievement overall. In the long term, a genuine solution to the problem will require that disparities in mathematics achievement
between minority and non-minority students be reduced to insignificance.
The current state of minority student enrollment and achievement in precollege mathematics is not encouraging either at the high school level or in the mathematics foundation building years at the elementary and junior high schools. National mathematics assessment data tend to be general and give only slight indications of achievement differences between minority and non-minority students in mathematics. Taken as a whole the national data do not present the extremes which are demonstrated in engineering college performance--e.g., the two-to-one differences in minority and non- minority graduation rates.

Disparities in performance in mathematics do indeed exist and correlate with the wide differerences in minority and non-minority graduation rates when examined in the context of mathematics achievement at the high end (scores of $90 \%$ and above) rather than general or average mathematics performance data. Engineering college majors are more likely to be high mathematics achievers. For example, comparing the number of minority students who score above 600 on the SAT-Math and College Board mathematics Achievement Tests with the performance of non-minority students on the same tests would yield data that would be relevant to minority engineering student retention and graduation. Likewise, inferences could also be made by looking at national and local mathematics assessment data and determining high mathematics performance differences between minority and non-minority students. And thirdly, examining mathematics exam data of New York City students, where the structure and organization of of the high school system are along racial lines, a clearer picture of the disparity in mathematics achievement for minority students can be obtained.

By analysis of upper level precollege mathematics achievement data at national and local levels it may be surmised that the current enrollment and retention patterns of minority students in engineering colleges is commensurate with the inadequate
levels of mathematics achievement that prevail for minority students with much less academic preparation at the precollege level. And because of this, a larger percentages of minority students is opting to pursue engineering college study than their non-minority counterparts. There is currently national concern about the growing number of students in need of mathematics remediation who are applying to engineering colleges--some estimates are as high as $30 \%$ of the entering freshman population ${ }^{12}$. What proportion of these underprepared students are minority is currently not available from existing minority engineering data.

### 2.2 National Mathematics Assessment Data, SAT-Math and Achievement Tests

A national perspective on the differences in mathematics achievement between minority and non-minority students can be obtained by examining the results of the mathematics assessment of the National Assessment of Educational Progress (NAEP), and the annual SAT and Achievement Data from the College Board.

## National Assessment of Educational Progress Mathematics Assessment:

In an analysis of the results of the third NAEP Mathematics Assessment at the secondary school, mathematics achievement differences are apparent between White, Black and Hispanic students. ${ }^{13}$ And the differences widen as student age increases. Table 1 summarizes the NAEP data, showing the mathematics performance of White, Black and Hispanic students taking NAEP test exercises in 1978 and again in 1982, including net change in performance in the two test years. While the 15 and 10 point differences that exist between 17-year old Black, Hispanic and White students respectively show that White students do better on the NAEP tests, the NAEP data in itself is not conclusive in demonstrating wide disparity in math performance. In fact the gains made by Black and Hispanic 13 year olds on the 1982 test were significant $(+6.5 \%)$ compared to

13 year old White students whose performance gains were limited to $3.2 \%$. This suggests that the gap in math performance narrowed for 13-year olds.


## TABLE 1

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However, the weakness of the NAEP statistics in trying to show disparity is that the data are averaged on the "basis of the mathematical performance of a representative national sample of over 70,000 9-, 13- and 17-year olds taking NAEP tests that include 250-450 mathematics exercises covering a wide range of basic mathematics objectives". ${ }^{14}$ Because of this, 10-15 point differences in White/Black mathematics performance can take on a variety of meaning and not necessarily show that a severe problem or wide educational disparity exists. If taken as a trend that will continue in later years, the gains made by Black and Hispanic 13 year olds could be construed to mean that the difference in math performance of White versus Black and Hispanic students that currently exists in the NAEP data will be narrowed considerably in the future. This scenario is a possibility, but is unlikely because of the nation's growing shortages of licensed and qualified math teachers across the spectrum of grade levels. ${ }^{15}$ The complexities of inner city
socioeconomics will tend to amplify difficulties in school systems that are predominantly Black and Hispanic, and because of this, future gains in student mathematical achievement in these schools generally will be less prevalent and more difficult to attain.

The NAEP data are useful for compiling a longitudinal study which, over the course of time, might indicate student age groups that are vulnerable to lower performance in mathematics. For example, in Table 1 the data for 13 year olds show a much larger net change in math performance than for 17 year olds. If the 1982 gains for 13 year olds do not hold constant when the next NAEP math assessment is done (when the new group of students is 17 years old) it might suggest that the transition from the middle school to high school is a break point in data compilation and reduction that needs to be further refined or changed considerably. For example, is reasonable comparative data obtained when similar mathematical exercises are given to students of different age groups even though the curriculum content and emphasis given to each age group are quite different? In fact, one of the reasons why the 13 year old group may have done better than either the 9 or 17 year olds is because the NAEP mathematics exercises more closely resemble the type of mathematical coursework that 13 year olds are taking at the time of the NAEP mathematics assessment. This is especially true for arithmetic topics where memorization and familiarity with the content of the material plays a major role in how students perform on tests. In this context, the computational aspects of middle school mathematics instruction and classroom practices which lead to "teaching to the test" can weigh heavily on objective type exercises such as those used in the NAEP mathematics assessment exercises.

## College Board Data

SAT data looked at from a longitudinal perspective show that during the period from 1976 to 1985 Black college bound seniors gained 14 points on the SAT-Math ( 332 to 346). In the same period White students had a net loss of 2 points on SAT-Math (493 to
491). There were also reasonable large gains made on the SAT-Math by Puerto Rican students ( 401 to 409 ) and Mexican American students ( 410 to 426 ). Although the gains made by minority students were substantial, the relatively small change in SAT-Math scores of all students ( 472 to 475 ) over the 1976-1985 period indicates that a threshold in SAT-Math performance has been reached and that incremental gains by Black/Hispanic students in the future will lessen. (Table 2 shows College Board longitudinal data by ethnic group.)

As in the NAEP data, the SAT-Math performance gains for Black/Hispanic students are notable, however, when direct comparisons of the SAT-Math test scores are made between White and Black/Hispanic students, the wide disparities in student math test performance become very apparent. In 1985 there was a 115 -point difference in SAT-Math scores between White and Black students--491 vs. 376. The differences in scores were smaller between White and Puerto Rican students--491 vs. 409 and between White and Mexican American students-491 vs. 426, but as a measure of comparison the differences show a significantly wide disparity.

When the SAT-Math scores were examined in the context of the college bound student profiles compiled by the College Board, the differences between and Black/Hispanic SAT-Math scores become more focused and considerably more serious when analyzed. In the College Board's Profiles. College Bound Seniors, 1985, the latest and final annual College Board report summarizes students' backgrounds and performance by racial/ethnic group and sex, based on the Admissions Testing Program (ATP). ${ }^{16}$

The comprehensiveness of the College Board "Profiles 1985" report allows the issue of disparity in Black/Hispanic math achievement to be examined from a number of different perspectives, especially as it relates to high scores on SAT-Math and Mathematics Achievement Tests Level 1 and 2. Comparison and analysis of these high math score intervals, plus examination of the scores within the group of students who stated that

For Release at Noon, Monday, September 23, 1985
SAT ${ }^{\circ}$ Averages by Ethnic Group, 1976-1985
SAT-Verbal

|  | $76^{\circ}$ | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| American Indian | 388 | 390 | 387 | 386 | 390 | 391 | 388 | 388 | 390 | 392 |
| AsianAmerican | 414 | 405 | 401 | 396 | 396 | 397 | 398 | 395 | 398 | 404 |
| Black | 332 | 330 | 332 | 330 | 330 | 332 | 341 | 339 | 342 | 346 |
| MexicanAmerican | 371 | 370 | 370 | 370 | 372 | 373 | 377 | 375 | 376 | 382 |
| Pueno Rican | 364 | 355 | 349 | 345 | 350 | 353 | 360 | 358 | 358 | 368 |
| White | 451 | 448 | 446 | 44 | 442 | 442 | 444 | 443 | 445 | 499 |
| Other | 410 | 402 | 399 | 393 | 394 | 388 | 392 | 386 | 388 | 391 |
| All Siudents | 431 | 429 | 429 | 427 | 424 | 424 | 426 | 425 | 426 | 431 |

SAT-Mathematical

|  | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| American Indian | 420 | 421 | 419 | 421 | 426 | 425 | 424 | 425 | 427 | 428 |
| Asian-American | 518 | 514 | 510 | 511 | 509 | 513 | 513 | 514 | 519 | 518 |
| Black | 354 | 357 | 354 | 358 | 360 | 362 | 366 | 369 | 373 | 376 |
| MexicanAmerican | 410 | 408 | 402 | 410 | 413 | 415 | 416 | 417 | 420 | 426 |
| Puent Rican | 401 | 397 | 388 | 388 | 394 | 398 | 403 | 403 | 405 | 409 |
| White | 493 | 489 | 485 | 483 | 482 | 483 | 483 | 484 | 487 | 491 |
| Oner | 458 | 457 | 450 | 447 | 449 | 447 | 449 | 446 | 450 | 448 |
| All Siudents | 472 | 470 | 468 | 467 | 466 | 466 | 467 | 468 | 471 | 475 |

1976 is the first year for which SAT scores by ethnic group are available.
Source: The College Board.
TABLE 2
engineering was their first choice of an intended area of college study, solidifies the data which show wide disparity in math achievement for minorities and partially explains their lower graduation rates in engineering colleges--i.e., $35 \%$.

Direct comparisons of SAT-Math achievement over a range of test intervals is shown in Table 3 categorized by White, Black, Puerto Rican and Mexican American college bound high school seniors. The one difference that is immediately apparent in the data is the larger number of White SAT test takers $(678,942)$ as compared to Blacks and Hispanics $(94,867)$. The Black/Hispanic total represents less than $10 \%$ of the total college bound senior population--977,361 (including White, Asian/Pacific American and remaining test takers who did not report their ethnicity and those who classified themselves as Other and were not assigned to the designated ethnic groups that were included in Profiles, College Bound Seniors 1985). The smaller number of Black/Hispanic SAT takers is of importance in the analysis of data, because on the basis of the 1980 census, Blacks and Hispanics made up almost $21 \%$ of the nation's 15 to 19 year old population. ${ }^{17}$

In particular, the much smaller percentage of Black students taking the SAT argues strongly that academic underpreparation was a key factor for not taking the SAT while national data indicate an increase in the percentage of Black high school graduates. In 1982 the percent of Black and White students age 18-19 years graduating from high school showed only a twelve point difference- $64 \%$ for Whites and $52 \%$ for Blacks--while in 1974 there was a 17 point spread. ${ }^{18}$ It can be assumed that students who complete high school and satisfy requirements for graduation and understand the benefits that accrue with a college education will probably consider college as an option beyond high school and would take the SAT. Using this assumption, then the major deterrent to taking the SAT is student underpreparedness for the subject matter on the test. If this assumption is true, then the disparity between Black and White SAT test takers will continue to

## 1285 SAT-MATH DATA COMPARISONS

| IESTSCORES | WHCLE | BLACK | MEX AMER. | $\begin{aligned} & \text { B. BICAN } \\ & \text { N g } \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | N \% | N q |  | $N \text { q }$ |
| $750-800$ | 5,804 (1\%) | 27 (0\%) | 25 (0\%) | 12 (0\%) | 64 (.07\%) |
| $700-749$ | 20,318 (3\%) | 157 (0\%) | 131 (1\%) | 55 (1\%) | 343 (.4\%) |
| 650-- 699 | 38,257 (6\%) | 511 (1\%) | 328 (2\%) | 121 (2\%) | 960 (1.0\%) |
| 600--649 | 64,716 (10\%) | 1,212 (2\%) | 746 (4\%) | 284 (4\%) | 2,242 (2.3\%) |
| 550--599 | 92,991 (14\%) | 2,654 (4\%) | 1,368 (8\%) | 495 (7\%) | 4,517 (4.7\%) |
| 500--549 | 105,104 (15\%) | 4,499 (6\%) | 1,913 (11\%) | 663 (9\%) | 7,075 (7.5\%) |
|  | $\overline{327,190}(\overline{49 \%})$ | $\overline{9,060}$ (13\%) | $\overline{4,511} \overline{(26 \%)}$ | 1,630 (23\%) | 15,201 $\overline{(16 \%)}$ |

## 1985SAT-MATH COMPOSUTE DATA

| TEST SCORES | WHITE | BLACK | MEX.AMER | RRICAN | TOTALBLAMSR |
| :--- | ---: | ---: | ---: | ---: | :---: |
| N | 678,942 | 70,156 | 17,246 | 7,465 | 94,889 |
| Mean | 490 | 376 | 426 | 405 |  |
| S.D. | 113 | 97 | 107 | 111 |  |

occur--e.g., the College Board Profiles of College Bound Seniors for 1984 and 1985 showed 1,100 less Black students took the SAT in 1985 than in 1984.19

The disparity in math achievement between White and Black/Hispanic students is compelling when the upper end SAT-Math achievement data shown in Table 3 are examined. In the analysis of this high end SAT-Math test data, the 500--549 range is used as an arbitrary reference score for two basic reasons: 1) it is above the 475 national average of all students and 2) it is slightly below the mean score (556) of all students who selected engineering as a first choice of intended area for college study.

The SAT-Math achievement differences for White and Black/Hispanic students are dramatic in comparing test scores of 500 and over and become extremely poignant when comparisons are made at each higher level score interval. The composite data shown in Table 3 show that the ratio between White student test takers $(678,942)$ and Black/Hispanic test takers $(94,889)$ is approximately seven to one. Using this ratio a comparison can be made showing the SAT-Math percentage margins between White and Black/Hispanic students. Accordingly, the percentage of White students scoring 500 and above was approximately three times the percentage of Black/Hispanic students scoring 500 and above. At test scores of 600 and above the percentage margin was five times and the disparity grew more extreme in scores of 700 and above where the percentage margin between White and Black students was nine-to-one. In absolute terms, there were only 407 Black/Hispanic students nationwide that scored 700 and above on the SAT-Math.

The same vast contrasts appeared in a tabulation of the College Board's 1985 Mathematics Achievement Tests, Levels 1 and 2. Table 4 shows comparisons of aggregate scores for White and Black/Hispanic students for test score intervals of 500 and above, 600 and above, and 700 and above. As with the SAT-Math, there are extremely wide disparities between White and Black/Hispanic students. It is interesting to note that although only 79 Black/Hispanic students scored 700 and above on the Mathematics Achievement Test Level 1, about three times that number, 243,

1985 MATH ACHIEVEMENT LEVEL $1 \& 2$ COMPARISONS BETWEEN WHITE \& BLACK/HISPANIC STUDENTS

$\frac{\text { TOT. BL/HISR }}{\text { N } q}$

Math Level 1

| $750-800$ | 909 | (1\%) | 6 | (0\%) | 7 | (0\%) | 4 | (1\%) | 17 | (.2\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 700--749 | 3,487 | (4\%) | 27 | (1\%) | 21 | (1\%) | 14 | (2\%) | 62 | (.7\%) |
| 650--699 | 9,881 | (10\%) | 107 | (2\%) | 81 | (3\%) | 22 | (3\%) | 210 | (3\%) |
| 600--649 | 15,988 | (16\%) | 328 | (7\%) | 191 | (7\%) | 82 | (13\%) | 601 | (7\%) |
| 550--599 | 19,529 | (19\%) | 571 | (12\%) | 321 | (12\%) | 108 | (17\%) | 1,000 | (12\%) |
| 500-- 549 | 21,362 | (21\%) | 946 | (20\%) | 534 | (20\%) | 147 | (23\%) | 1,627 | (20\%) |
|  | 71,156 | (71\%) | 1,985 | (42\%) | $\overline{1,155}$ | $\overline{(43 \%)}$ | 377 | $\overline{(59 \%)}$ | $\overline{3,517}$ | $\overline{(42.9 \%)}$ |



TABLE 4
did so on the Mathematics Achievement Test Level 2. For White students the ratio was 5 -to-1 for test scores of 750 and over between the Level 1 and Level 2 test. This was probably an indication of the confidence level of thestudents who elected to take the Level 2 test. Since high achievement on the more difficult Level 2 test is of greater value to students as they seek admission to competitive colleges, there is little need to take the less rigorous Level 1 test. This supposition is confirmed by the fact that $78 \%$ of the contingent of all students who took the Mathematics Achievement Test Level 2 scored 600 or higher, while on the Mathematics Achievement Test Level 1 only $29 \%$ scored in the 600 and above range.

Another problematic occurrence in the Mathematics Achievement Level I test is that 483 less Black students took the Mathematics Achievement Test Level 1 in 1985 as in 1984 $(4,746$ vs. 5,229$)--10 \%$ less--which is a substantial drop in test takers. (See Tables 5 and 6.) It remains to be seen in future years whether this drop off is the start of a downward trend or simply an isolated anomaly in 1985. It is a statistic that needs to be closely monitored because it could be indicative of further weakness in secondary school mathematics programs for Black students.

In contrast to the Mathematics portion of the SAT, the College Board Mathematics Achievement Tests Levels 1 and 2 are more reflective of the content of the secondary school curriculum, in particular algebra, geometry and trigonometry. Therefore, student performance on these tests is more dependent on student enrollment and achievement in the traditional mathematics courses given in high school as part of the academic and precollege programs. Because of the closer tie to the secondary math curriculum, it is possible that students who take the achievement tests are more likely to score higher than they do on the SAT-Math.

The lower performance of Black/Hispanic students on the Mathematics Achievement Tests is a better indicator than the SAT that underpreparation for the test is a key factor

## 1984 COMPOSITE DATA EOR MATH ACHUEYEMENT TESTS LEYELS 1\&2

## MATH LEVEL 1

|  | WHITE | BLACK | MEX. AMER | P. RICAN | TOT. BL/HISP |
| :--- | ---: | ---: | :---: | :---: | :---: | :---: |
| N | 102,855 | 5,229 | 2,438 | 630 | 8,297 |
| Mean | 546 | 481 | 486 | 510 |  |
| S.D. | 90 | 87 | 89 | 93 |  |

## MATHLEYEL 2

|  | WHITE | BLACK | MEX, AMER |  | P. RICAN |  |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: |
| N |  |  |  |  |  |  |
| TOT. BL/HISP. |  |  |  |  |  |  |

## 1985 COMPOSITE DATA EOR MATHACHUEYEMENTTESTS LEYELS $1 \& 2$

## MATH LEVEL.

|  | WHITE | BLACK | MEX. AMER | P. RICAN | TOT. BL/HISP |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: |
| N | 100,458 | 4,746 | 2,964 | 640 | 8,080 |
| Mean | 544 | 478 | 483 | 511 |  |
| S.D. | 89 | 85 | 87 | 91 |  |

## MATHLEVEL 2

|  | WHITE | BLACK | MEX. AMER | P. RICAN | TOT. BL/HISP. |
| :--- | ---: | ---: | :---: | :---: | :---: | :---: |
| N | 30,768 | 1,023 | 539 | 128 | 1,890 |
| Mean | 660 | 581 | 598 | 620 |  |
| S.D. | 87 | 106 | 99 | 101 |  |

TABLE 6
explaining the lower test scores. However, although the SAT is described as a test of developed ability and not necessarily a measure of what has been learned in the mathematics classroom, students also need to have a basic knowledge and proficiency in arithmetic, algebra and geometry in order to test well on the mathematics portion of the SAT. Thus it can be surmised that the great disparity in performance noted above, that exists between Black/Hispanic students and their White counterparts on the SAT-Math and the Mathematics Achievement Tests is primarily a function of differences in the mathematics instructional programs available to both constituents of students--which, in the major urban areas of the country, are racially, economically and ethnically segregated. ${ }^{20}$

Another important set of data included in the College Board's Profiles, College Bound Seniors 1985 are student responses to their first choice of college major. Table 7 shows first choice of an intended college major among groups of White, Black, Mexican-American and Puerto Rican students and their respective SAT-Math scores in the $25 \mathrm{th}, 50$ th and 75 th percentiles. The data are revealing in that a greater proportion of male students than female students selected engineering as their first choice of a college major. This was the case for both White and Black/Hispanic students where $82 \%$ of the males chose engineering in contrast to less than $14 \%$ of the females. Engineering still remains a field of college study which is predominantly male.

A major clue as to why Black/Hispanic students graduate from engineering colleges at less than half the rate than their White counterparts can be obtained by comparing the SAT-Math percentiles. However, before developing an argument along these lines, the data must be qualified. Because of the thrust of the national minority engineering effort and the vigorous affirmative action recruitment by engineering colleges in the last decade it must be assumed that most minority students who selected engineering as their first choice college major eventually enrolled as engineering college majors. This is a fairly safe assumption considering that the 1984/85 first year enrollment figure of 10,594 for minority

## 1985 SAT MATH SCORES FOR SENIORS WHO CHOSE ENGINEERING FIRST AS THEIR INTENDED AREA OF COLLEGE STUDY

|  | PERCENTAGES |  |  | SAT-MATH PERCENTILES |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MALE | FEMALE | TOTAL | 25TH | 50TH | 75TH |
| WHITE | $\begin{aligned} & 66,855 \\ & (20.2 \%) \end{aligned}$ | $\begin{aligned} & 11,263 \\ & (3.1 \%) \end{aligned}$ | $\begin{aligned} & 78,118 \\ & (11.2 \%) \end{aligned}$ | 488 | 570 | 596 |
| BLACK | $\begin{array}{r} 6,081 \\ (20.1) \end{array}$ | $\begin{aligned} & 2,126 \\ & (4.6) \end{aligned}$ | $\begin{array}{r} 8,207 \\ (10.7) \end{array}$ | 345 | 430 | 514 |
| MEXICAN AMER. | $\begin{aligned} & 2,000 \\ & (22.7) \end{aligned}$ | $\begin{gathered} 423 \\ (4.2) \end{gathered}$ | $\begin{aligned} & 2,423 \\ & (12.9) \end{aligned}$ | 408 | 491 | 571 |
| PUERTO <br> RICAN | $\begin{gathered} 678 \\ (19.2) \end{gathered}$ | $\begin{aligned} & 112 \\ & (2.5) \end{aligned}$ | $\begin{array}{r} 790 \\ (9.8) \end{array}$ | 381 | 478 | 570 |

students in the nation's engineering colleges approximated the 1984 minority SAT takers $(11,168)$ who selected engineering as their first choice of college major. Note that in this approximation that two-thirds of the students who entered full time college study each year took the SAT ${ }^{21}$ which increased the pool of available minority engineering candidates to approximately 16,000 . Assuming further that acceptance in the chosen major was at a $70 \%$ rate, then the number of intended engineering college majors was close to the number that actually enrolled as first year engineering college students. These assumptions appear to hold for White students also.

It is a given that the stronger a student's mathematics background and proficiency the better the student's chances are for successfully completing an engineering college program of study. By examining the data it can be seen that $50 \%$ of Black students who considered engineering as their first choice did not score higher than 430 on the SAT-Math. This is 140 points below that of White students- $-50 \%$ of whom scored 570 or higher. This is a strong indicator that Black students who went on to engineering college did so with much less mathematics preparation and proficiency than their White counterparts and therefore were more vulnerable to academic difficulties in the heavily weighted mathematics based courses that occur in first and second year engineering college study. The same is true for Puerto-Rican and Mexican American students where the 50th percentile figure on the SAT-Math is also considerably lower than for White students--478 and 491 respectively.

What may be inferred from the vastly lower SAT-Math scores for prospective minority student engineering college majors is that more than half enter engineering college with much less mathematics preparation than White students. This weaker mathematics background can put minority students in serious academic jeopardy at the very start of engineering college study. Thereafter, they must struggle to keep up with requisite math and science based coursework and compete academically with a larger surrounding White population that has a stronger mathematics foundation. The mathematics focus is
especially intensive during the first two years of engineering college study where mastery of four semesters of calculus establishes the base for almost all of the science based and engineering coursework required for the Bachelor's degree in engineering. A large number of Black/Hispanic students who enter engineering college with a less than adequate mathematics foundation must, as a consequence, experience more difficulty in mastering engineering coursework. And over a four year period it is reflected in a retention rate for minority engineering students that is half ( $35 \%$ ) that of the larger majority population's (70\%).

### 2.3 Profile of the New York City Public School System and the Reporting of School Data Along Ethnic/Racial Lines

The New York City public school system provides a rich and large educational environment in which to examine student mathematics achievement. The data can be explored from a racial/ethnic perspective that highlights the disparities in mathematics achievement for minority students across all grade levels in which standardized testing is administered. As the nation's largest public school system, New York City had an enrollment in the 1982/83 academic year of 918,384 students dispersed throughout the city's 983 different schools located in thirty-three separate community school districts. Coupled with a diverse student population that extends across economic, ethnic and racial lines, many of the individual school communities within New York City share a resemblance to the diversity of school systems that can be found throughout the country. New York City can indeed be looked at as a microcosm of the nation's urban public school systems.

At one end of the spectrum, in the New York City public school system, there are public schools located in middle class neighborhoods where an overwhelming majority of students are reading and performing in mathematics at or above grade level, and where
daily attendance is extremely high and truancy and drop out rates are insignificant. In contrast, in schools located in lower income neighborhoods just the opposite is true. Across grade levels, and starting as early as the second grade, students who attend these schools which have predominant Black and Hispanic student populations are behind grade level in reading and in mathematics. And the achievement gaps widen as students continue through the middle grades and high school with successively weaker foundations at each step in the formal process of schooling. The outcome of this steady decline in student achievement is a prevalence of low attendance, high truancy and extremely low rates of completion of high school. And for many of the students who do complete high school, a lack of academic competency severely limits their options for the more rigorous majors in college or for meaningful employment in the job market place. New York City high schools with predominant Black and Hispanic populations graduate few students with the highly regarded New York State "Regents" endorsed diploma that indicates mastery of traditional precollege courses has been attained.

Demographically, the schools with long histories of lower student achievement are those which have predominant Black/Hispanic student populations and are located in the city's low income neighborhoods which are overwhelmingly populated by Black and Hispanic people. This demographic pattern exists in New York City as it does in many of the largest metropolitan areas of the country. In the last decade, in these major urban areas there has been an increase in the proportion of Black and Hispanic students that makes up the total school system enrollment.

The phenomenon of a predominant minority public school enrollment can be attributed to a more youthful Black and Hispanic population and is due also to a White exodus from the urban public school system. For example, in the New York City public school system, the proportion of White students in academic comprehensive high schools decreased from $44 \%(115,180)$ in 1973 to $29 \%(68,344)$ in 1982 --reduction of almost $40 \%$. During this
period, overall academic comprehensive high school enrollment decreased by less than $10 \%$, from 263,214 to $238,299 .{ }^{22}$ Similar reductions in the White student population occurred in the Boston public school system during the period between 1970 and 1982 where the White student enrollment diminished from $70 \%$ to $36 \%{ }^{23}$

This pattern of increasing proportion of Black and Hispanic students in the public school systems of the nation's major urban areas is compelling and is likely to continue in the future as the nation's school age population becomes increasingly Black and Hispanic. The trend is suggestive of a distressing turn of events that is establishing racially divided school communities and an ironic return to the segregated system of schooling that the United States Supreme Court found to be unequal and discriminatory in the Brown vs. Board of Education ruling in 1954. ${ }^{24}$

In 1975, the New York City Board of Education began compiling data in a comprehensive annual report entitled, School Profiles. The initial School Profiles report, published in April 1975, provided concise and detailed information on the schools and student population that comprised K-9 for the years 1973/74. Subsequent reports have enlarged on the population and have included data and information on the high school level--reported first in the School Profiles 1974/75 report.

The data compiled in the New York City School Profiles annual series include information on school enrollment detailed by five ethnic/racial categories: Black, American Indian, Asian, Hispanic and White. The data and information on K-12 enrollment are presented in a hierarchy that lists enrollment for: 1) each of the five boroughs of the city, and 2) each of the school districts within a borough, and 3) each of the schools within the school district. At each of these areas of compilation, pupil data reported at grade levels 2-9 include test scores in reading and mathematics in terms of percentage of students and their grade equivalents: at grade level, one year below grade level, and two years below grade level. Also included are data on attendance, admissions, and departures,
promotions, normal aid to families with dependent children, eligibility for free lunch, staffing patterns, and salary scales. A page of the data reported in School Profiles 1982/83 for a sample school is shown in Appendix B.

In grades K-9, students enrolled in the New York City public school system attend school in thirty-three community school districts that are located in the five boroughs of the city: Manhattan, the Bronx, Brooklyn, Queens and Staten Island. (See map in Figure 4.) The high school system consisting of 112 high schools, is not part of the thirty-threecommunity school district arrangement, but is organized as a separate division that is governed directly by the New York City Central Board of Education.

In the 1982/83 academic year student enrollment in the K-9 grade levels totaled 627,448 , spread among the thirty-three community school districts. The average school district enrollment was 19,000 , bounded by a range of 10,920 at the low end (District 1 in Manhattan) to 32,608 at the high end (District 10 in the Bronx). To provide an appreciation for how the system is patterned along ethnic/racial lines, Table 8 lists students by total district, enrollment and percent ethnic composition in each of the five boroughs of the city. A further breakdown of the ethnic/racial student composition at each of the thirty-three community school districts is shown in Appendices C1 and C2. As shown in Table 8, the boroughs of Manhattan, the Bronx and Brooklyn had a higher proportion of Black/Hispanic student enrollments at the elementary (K-6) and junior high/middle school level (5-9)--82\% and $76 \%$ respectively than did the boroughs of Queens and Staten Island whose enrollment of Black/Hispanic students at the same grade levels were $53 \%$ and $18 \%$ respectively. The proportion of Black/Hispanic students in a particular district or borough follows the housing patterns of the city along racial/ethnic lines. For example, District 5 in Manhattan, located in the heart of Harlem has a population which is almost entirely Black and Hispanic and its district enrollment of 11,218 students is $99.6 \%$ minority with a composition of 9,253 Black students and 1,910 Hispanic students. Located just above

## THE COMMUNITY SCHOOL DISTRICTS



Figure 4

## 1982/83 SCHOOL DISTRICT ENROLLMENT IN THE FIVE BOROUGHS OF NEW YORK CITY INCLUDING NUMBER AND PERCENTAGE OF BLACK/HISPANIC STUDENTS

|  | TOTAL ENROLL | BLACK \& HISPANIC | \% BLACK \& HISPANIC |
| :---: | :---: | :---: | :---: |
| 5 NYC BOROUGHS |  |  |  |
| Total Elementary | 435,056 | 310,324 | 71.3\% |
| Total JHS/IS | 192.392 | 134.214 | 69.7\% |
| Total | 627,448 | 444,538 | 70.8\% |
| MANHATTAN DIST, 1-6 |  |  |  |
| Total Elementary | 60,409 | 49,319 | 81.6\% |
| Total JHS/IS | 24,662 | 20.302 | 82.3\% |
| Total | 85,071 | 69,621 | 81.8\% |
| BRONX DIST. $7-12$ |  |  |  |
| Total Elementary | 90,755 | 80,555 | 88.7\% |
| Total JHS/IS | 40,960 | 36.511 | 89.1\% |
| Total | 131,715 | 117,066 | 88.8\% |
| BROOKLYN DIST, 13--23 \& 32 |  |  |  |
| Total Elementary | 162,161 | 123,266 | 76.0\% |
| Total JHS/S | 66.438 | 49.744 | 74.8\% |
| Total | 228,599 | 173,010 | 75.6\% |
| QUEENS DIST, 24--30 \& 33 |  |  |  |
| Total Elementary | 101,215 | 53,454 | 52.8\% |
| Total JHS/IS | 48.055 | 25,472 | 53.0\% |
| Total | 149,270 | 78,926 | 52.9\% |
| STATEN ISLAND DIST, 31 |  |  |  |
| Total Elementary | 20,516 | 3,730 | 18.2\% |
| Total JHS/IS | 12.277 | $\underline{2,185}$ | 17.8\% |
| Total | 32,893 | 5,915 | 18.0\% |

SOURCE: School Profiles 1982/83. New York City Board of Education

District 5 is District 6 in the Washington Heights area which also has a heavy Hispanic and Black population. District 6's enrollment of 19,391 students is $94.6 \%$ minority with 14,811 Hispanic students and 3,538 Black students. The same pattern of heavy Black/Hispanic student enrollments occurs in the community school districts of the boroughs of the Bronx and Brooklyn where there are predominantly Black and Hispanic populated neighborhoods.

A similar pattern of enrollment by racial concentration of White students is found in the boroughs of Queens and Staten Island where a large majority of the city's White population resides. Community School District 26, located in the middle class and largely White populated neighborhood of Flushing, Queens which had a White student enrollment of 16,523 or $72 \%$ of the total enrollment. The pattern of heavy White student enrollment was the same in the borough of Staten Island where Community School District 31 (the only community school district in the borough) the White student population represents $82 \%$ of the total.

### 2.4 Standardized Mathematics Testing and Achievement in New York City Community

## School Districts 5 and 26

The racial/ethnic data provided by the School Profiles report establishes a base with which to make comparisons of mathematics achievement data. The data to be examined are organized by the selection of a school district and a particular junior high school within the district whose student enrollment is predominantly Black and Hispanic and a comparably sized district and school enrollment which is predominantly White.

Two districts that fit the demographic characteristics of the data comparisons are District 5 located in Harlem and District 26 located in the Bayside section of Queens. In 1982/83, both districts had approximately the same size student enrollments--11,2128 in District 5 and 12,101 in District 26. However, all seventeen elementary and junior
high/intermediate schools located in District 5 were designated as Chapter 1 schools while none of the twenty-five schools in District 26 had a Chapter 1 designation.

Chapter 1 designated schools in the New York City public school system qualify to "...receive additional educational services under Chapter 1 of the Educational Consolidation Improvement Act (ECIA), if its percentage of low income pupils is equal to or greater than the citywide percentage of low income pupils." $25^{*}$ In District 5 the percentage of students eligible to receive reduced cost or free lunches is $85 \%$ as compared to District 26 where it is $28 \%$. Students in District 5 who come from familities receiving AFDC payments number 6,641 out of the 11,218 total district population--while at District 26 , students in this category number only 267 out of the 12,101 total district population--2\%. See Appendices D1 and D2 for more detailed information on characteristics of Districts 5 and 26 as reproduced from the pages of the School Profiles 1982/83 report.

From an instructional staffing standpoint, there appears to be little difference between the percent of certified teachers who were teaching mathematics in either of the two districts. In District 5 it was $94 \%$ and in District 26 it was $95 \% .{ }^{26}$ However these data are deceptive, since certification simply indicates that the districts' teachers who teach mathematics are certified to teach in the districts' classrooms. Information is not specific as to whether the licenses of the teachers are for teaching the subject of mathematics, or whether they are academically qualified to teach the mathematics courses they are scheduled for. This is an important point and must be kept in mind in the examination of the comparative mathematics achievement data.

Table 9 shows comparative mathematics achievement for all of the students in Community School District 5 and 26 at grade levels 7,8 and 9. The data are aggregate

[^1]1982/83 STANDARDIZED MATH TEST SCORE COMPARISONS BETWEEN COMMUNITY SCHOOL DISTRICTS 5 AND 26


TABLE 9

1982/83 STANDARDIZED MATH TEST SCORE COMPARISONS BETWEEN MANHATTAN AND QUEENS

MANHATTAN
QUEENS

| GRADE | N | $\begin{gathered} \text { AT GRADE } \\ \text { LEYEL } \end{gathered}$ | $\frac{1 Y R}{\text { BELOW }}$ | $\frac{2 \text { YRS. }}{\text { BELOW }}$ | N | $\begin{gathered} \text { ATGRADE } \\ \text { LEVEL } \end{gathered}$ | $\frac{1 Y R}{\text { BELOW }}$ | 2 YRS <br> BELOW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 10,568 | 37.6\% | 51.6\% | 38.2\% | 18,479 | 53.6\% | 35.7\% | 23.8\% |
| 8 | 6,860 | 44.5\% | 40.7\% | 26.8\% | 15,239 | 60.2\% | 26.8\% | 15.8\% |
| 9 | 4,442 | 49.4\% | 47.5\% | 32.1\% | 8,010 | 68.0\% | 29.1\% | 18.1\% |

TABLE 10

SOURCE: School Profiles 1983-83. New York City Board of Education

1982/83 school year and are organized to show: 1) the number of students taking the test, 2) the percent of students at or above grade level, 3) the percent of students one year below grade level, and 4) the percent of students two years below grade level.

The mathematics test used by New York City for grade level testing during the three designated testing years was the Stanford Diagnostic Mathematics Test, a three-part test that essentially tests a student's proficiency in whole number arithmetic with some problems that involve basic geometric shapes and properties, simple tables and graphs and rudimentary measurement problems. ${ }^{27}$ The disparity in mathematics test performance between the students in District 5 and 26 is evident in Table 9. At the 9th grade level in District 5 , over $42 \%$ of the students tested two years or more below grade level while at District 26 only $12.9 \%$ of the 9 th graders showed the same deficiency in test results. The comparative figures for test scores one year below grade level are just as disparate, with $51.4 \%$ of the 8 th graders in District 5 scoring in this range as compared to only $16.7 \%$ of the 8 th graders in District 26. The differences in test scores for 7 th graders show the same wide contrasts. And a comparison of the average percentage of students at or above grade level in District 26 shows that it is twice that of District 5 ( $70 \%$ vs. $32 \%$ ).

The low levels of student mathematics test performance shown in the District 5 mathematics test data are not reflected with the same acuteness when viewed at the borough level. Table 10 shows the same mathematics test score statistics but with the larger junior high school population of the borough of Manhattan that includes Community School District 2. District 2 in Manhattan is noteworthy because of its relatively high White and Asian student population-- $28.7 \%$ and $29.1 \%$ respectively--for a total of $61 \%$. This is in contrast to the other five largely Black and Hispanic populated districts in Manhattan where the White and Asian student population combined does not exceed $13 \%$ of the total student enrollment. The higher mathematics test scores of District 2 alone are sufficient to skew the data of the total junior high school enrollment in the borough of Manhattan. The
relatively higher level of mathematics performance of District 2 is indicated in Table 11 along with the singling out of Junior High School 167 located in the middle class neighborhood of the upper eastside of Manhattan where student mathematics performance is comparable to the largely White populated junior high schools located in District 26 in the borough of Queens. The racial/ethnic enrollment proportions of Junior High School 167 are $42 \%$ White, $26 \%$ Black, $10 \%$ Asian, and $22 \%$ Hispanic.

The comparison of two selected junior high schools located in Districts 5 and 26 is shown in Table 12. The two schools, P.S. 43 in District 5 and P.S. 216 in District 26 have almost the same size enrollments: 1,044 students at P.S. 43 and 1,045 students at P.S. 216. The mathematics test data show the very wide disparity that exists between the two schools. At the 9th grade level more than six times as many students in P.S. 43 tested two years below grade level than at P.S. 216 ( $47.2 \%$ vs. $6.9 \%$ ). The same extreme differences exist at the 7 th and 8 th grades. In the comparison of number of students who tested at or above grade level, only one-third of the population at P.S. 43 attained this mean equivalent score whereas greater than $80 \%$ of the students at P.S. 216 had test scores at this level. The disparity in mathematics test scores that exists between these two schools is consistent when comparisons are made of other junior high schools in school districts which have predominant Black and Hispanic student populations compared to predominantly White student populations.

As a further illustration of the low level of Black/Hispanic student mathematics test performance, Table 13 shows a listing of New York City junior high schools in districts where Black and Hispanic student enrollment exceeds $94 \%$ of the total student enrollment. The schools have been selected on the basis of size and each has a total student enrollment in the vicinity of 1,000 students. As can be seen, all of the schools listed have mathematics test scores in the same low range as exhibited by P.S. 43 in District 5 in Manhattan. Typically, the at or above grade level scores range from $28 \%$ to

1982/83 STANDARDIZED MATH TEST SCORE COMPARISONS BETWEEN COMMUNITY SCHOOL DISTRICT 2 AND PUBLIC SCHOOL 167

| DISTRICT 2 |  |  |  |  |  | RUBLIC SCHOOL 167 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| GRADE | N | $\frac{\text { AT GRADE }}{\text { LEVEL }}$ | $\frac{1 \mathrm{YR}}{\text { BELOW }}$ | $\frac{2 \text { YRS }}{\text { BELOW }}$ | N | $\begin{gathered} \text { ATGRADE } \\ \text { LEYEL } \end{gathered}$ | $\frac{1 \mathrm{YR}}{\text { BELOW }}$ | $\begin{aligned} & 2 \text { YRS. } \\ & \text { BELOW } \end{aligned}$ |
| 7 | 2,042 | 59.3\% | 32.3\% | 22.8\% | 457 | 67.0\% | 26.3\% | 16.2\% |
| 8 | 1,442 | 68.2\% | 22.6\% | 14.1\% | 336 | 75.6\% | 17.6\% | 11.0\% |
| 9 | 1,227 | 69.4\% | 28.3\% | 18.9\% | 283 | 62.9\% | 33.6\% | 23.0\% |

TABLE 11

## 1982/83 STANDARDIZED MATH TEST SCORE COMPARISONS BETWEEN PUBLIC SCHOOLS 43 AND 216

|  | PUBLIC SCHOOL 43 |  |  |  | PUBLIC SCHOOL 216 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| GRADE | N | $\begin{gathered} \text { AT GRADE } \\ \text { LEVEL } \end{gathered}$ | $\frac{1 \mathrm{YR}}{\text { BELOW }}$ | $\frac{2 Y R S}{\text { BELOW }}$ | N | $\begin{gathered} \text { AT GRADE } \\ \text { LEVEL } \end{gathered}$ | $\frac{1 \mathrm{YR}}{\text { BELOW }}$ | 2 YRS. BELOW |
| 7 | 460 | 32.0\% | 51.3\% | 34.3\% | 446 | 80.5\% | 13.9\% | 10.3\% |
| 8 | 275 | 34.2\% | 51.6\% | 33.5\% | 386 | 85.1\% | 10.7\% | 7.1\% |
| 9 | 300 | 36.3\% | 60.7\% | 42.7\% | 233 | 84.5\% | 13.7\% | 6.9\% |

TABLE 12

SOURCE: School Profiles 1982-83. New York City Boand of Education

CHAPTER 1 JUNIOR HIGH/INTERMEDIATE SCHOOLS IN DISTRICTS WITH GREATER THAN 94\% BLACK/HISPANIC ENROLLMENT

| $\begin{aligned} & \text { DISTRICT } \\ & \quad \# \end{aligned}$ | $\begin{aligned} & \text { SCHOOL } \\ & \# \end{aligned}$ | TOTAL ENROLLED | N | GRADE <br> AT GRADE LEVEL | LEVEL <br> 1 YR. BELOW | 2 YRS. BELOW |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | 383 | 1,061 | 285 | 84.6\% | 7.7\% | 4.2\% |
| 32 | 111 | 947 | 214 | 33.6\% | 45.3\% | 27.6\% |
| 23 | 263 | 949 | 327 | 38.8\% | 41.6\% | 25.1\% |
| 16 | 324 | 933 | 174 | 28.7\% | 56.3\% | 36.2\% |
| 13 | 265 | 1,048 | 401 | 42.9\% | 43.1\% | 29.7\% |
| 12 | 167 | 951 | 226 | 38.1\% | 42.1\% | 29.6\% |
| 9 | 145 | 1,054 | 254 | 37.8\% | 41.3\% | 22.8\% |
| 7 | 162 | 902 | 193 | 33.7\% | 50.3\% | 34.2\% |
| 6 | 164 | 998 | 442 | 28.4\% | 56.2\% | 42.4\% |
| 5 | 43 | 1,044 | 275 | 34.2\% | 51.6\% | 33.5\% |
| 4 | 117 | 942 | 260 | 42.3\% | 43.5\% | 28.5\% |

TABLE 13
year or below grade level scores fall within the $40 \%$ to $50 \%$ range, and the two years or below grade level scores are in the $22 \%$ to $42 \%$ range. The single exception in the schools listed is P.S. 383 located in District 32 in Brooklyn which is a school designated for the "gifted and talented" and where admission to the school is based on competitive academic examinations. ${ }^{28}$

### 2.5 Arithmetic Test Data for a Sample of Entering 9th Year Students at Eight CMSP

## Participating High Schools

The problem of Black and Hispanic student underpreparation in mathematics upon high school entry is exemplified by mathematics test data collected by the CMSP in its research and development efforts to find curriculum model alternatives to the standardized mathematics testing currently utilized as an administrative mechanism for mathematics course placement. The mathematics test data shown in Table 14 were compiled in three separate and successive cycles of students who participated in CMSP model development activity in the Fall of 1983, 1984 and 1985. The students tested were selected randomly from the incoming 9th grade population at seven CMSP participant schools in September 1983 and from eight CMSP participant schools in September of 1984 and 1985. All but one of the eight schools are designated as Chapter 1 schools.

The mathematics test scores summarized are the results of three preliminary arithmetic tests that were administered to CMSP participant classes at each of the eight high schools. The tests were given primarily to verify class heterogeneity and random selection of students and also as a broad measure for comparative school analyses. The test was not used as a diagnostic instrument nor as a predictor for subsequent mathematics performance since all students began their mathematics coursework at "ground zero" utilizing the CMSP Model curriculum in prealgebra.

Prior to taking the preliminary arithmetic test, the CMSP-designated classes were
given an extensive review of arithmetic topics over a three-day period-the equivalent of five forty-minute periods. This was done just to refresh and jog students' memories but not to teach the students the topics of the test. After the long summer vacation it was felt

## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)

Preliminary Arithmetic Test Data for Incoming 9th Grade Students Participating in the CMSP Model Implementation Cycles in the Fall of 1983, 1984 and 1985

| Year | Total Taking Test | $0-19$ | $Z_{20-39} \text { Test }$ | $\begin{aligned} & \text { Score I } \\ & 40-59 \end{aligned}$ | $\begin{gathered} \text { ervals - } \\ 60-84 \end{gathered}$ | $85-100$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1983 | 1132 | $\begin{gathered} 79 \\ (7 \%) \end{gathered}$ | $\begin{aligned} & 234 \\ & (21 \%) \end{aligned}$ | $\begin{aligned} & 303 \\ & (27 \%) \end{aligned}$ | $\begin{aligned} & 396 \\ & (35 \%) \end{aligned}$ | $\begin{gathered} 120 \\ (11 \%) \end{gathered}$ |
| 1984 | 1253 | $\begin{gathered} 123 \\ (10 \%) \end{gathered}$ | $\begin{aligned} & 218 \\ & (17 \%) \end{aligned}$ | $\begin{gathered} 322 \\ (26 \%) \end{gathered}$ | $\begin{gathered} 403 \\ (32 \%) \end{gathered}$ | $\begin{gathered} 187 \\ (15 \%) \end{gathered}$ |
| 1985 | 1547 | $\begin{gathered} 167 \\ (11 \%) \end{gathered}$ | $\begin{gathered} 345 \\ (23 \%) \end{gathered}$ | $\begin{gathered} 378 \\ (24 \%) \end{gathered}$ | $\begin{gathered} 483 \\ (31 \%) \end{gathered}$ | $\begin{gathered} 174 \\ (11 \%) \end{gathered}$ |
| Total | 3932 | $\begin{aligned} & 369 \\ & (9 \%) \end{aligned}$ | $\begin{aligned} & 797 \\ & (20 \%) \end{aligned}$ | $\begin{aligned} & 1003 \\ & (26 \%) \end{aligned}$ | $\begin{gathered} 1282 \\ (33 \%) \end{gathered}$ | $\begin{gathered} 481 \\ (12 \%) \end{gathered}$ |

TABLE 14
that an extensive review of test topics would minimize students' doing problems incorrectly because of memory blocks and thus more clearly reveal the basic deficiencies in arithmetic which the test was seeking to ascertain on a class by class basis.

The preliminary CMSP arithmetic test consisted of twenty problems covering
arithmetic topics in whole numbers, fractions and decimals. Twelve of the test problems were straight computation and eight were straightforward word problems, seven of which involved a single arithmetic operation and one which required two operational steps. There were no algebra or geometry problems nor were there multiple choice selections or true/false questions. Students were required to work out the solution to each of the problems in a space provided on the test paper. No partial credit was given for any of the 20 problems and each problem had an equal weight of 5 points--a perfect score therefore was 100 . A parallel version of the preliminary arithmetic test was utilized for each of the three years that the test was administered. A sample of the test is shown in Appendix E.

By examining the data, it can be seen that student test performance at the high end is limited--an average of $12 \%$ of the incoming class scored in the $85-100$ range for the three years tested. In contrast, $55 \%$ of the students scored less than 60 ( 12 or fewer problems correct) and $29 \%$ scored less than 40 ( 8 or fewer problems correct). Again, the mathematics test results tended to agree with the standardized test results obtained when students were in junior high school. It is clear from the results on the CMSP preliminary arithmetic test administered that only a very small percent of the students had demonstrated a proficiency in arithmetic. Experience in the CMSP has shown that a preliminary test score of 90 or higher is usually indicative of a student's having had adequate preparation in arithmetic, but the preparation was still insufficient to achieve at a high level in a traditional two term first course in high school algebra at the onset. On the average, not more than $5 \%$ of the incoming population at the eight participant schools fell into this high level of academic preparedness for first year algebra coursework.

While the eight schools participating in the CMSP Model are not fully representative of all of the high schools in New York City which are designated Chapter 1, they do share the following characteristics: 1) the schools are large, with enrollments of 2,500 students or more, 2) student enrollment is predominantly Black and Hispanic--greater than $95 \%$ for
seven of the schools and $80 \%$ for the remaining one and 3 ) seven of the designated Chapter 1 schools are neighborhood high schools located in the boroughs of Manhattan, the Bronx and Brooklyn and thus draw their entering 9th grade student population from the heavily Black and Hispanic community school districts previously described in Section 2.4. The eighth school is a non-Chapter 1 school located in the borough of Queens and is organized as a magnet or educational option school and which accepts entering 9th grade students from all boroughs of the city.

Thus the eight CMSP participant high schools face the dilemma that many of the predominantly Black and Hispanic school in New York City and elsewhere in the nation face--a small and limited pool of students with the academic preparation to excel in the traditional precollege mathematics courses that the high schools have to offer. Instead, the schools must increasingly revert to general and remedial mathematics programs as the primary option for entering 9 th grade student populations. Students at predominant Black and Hispanic schools who are selected for the traditional Regents mathematics coursework may find that the regular two-semester coursework is stretched out over three or four semesters, leaving little chance for students to complete the study of three years of Regents mathematics coursework before high school graduation.

### 2.6 Regents 11 th Year Mathematics Examination Comparative Data in a Sample of 13

## Chapter 1 and 13 Non-Chapter 1 High Schools

Ultimately, it is the number of students who graduate from high school having completed the study of three years of precollege mathematics at a high level that reveals the true nature of the disparity between White and Black/Hispanic students. On a national scale, mathematics data of this sort are not available, except as presented in the form of the NAEP and SAT studies. Unfortunately, the SAT and NAEP studies data do not indicate the specific number of students nationally who are studying the higher levels of high
school mathematics at the time the NAEP and SAT examinations are given. However, even if data on traditional precollege mathematics course enrollment were available they would have little value because of the large variations in mathematics course content that exists between the state and district levels across the country.

A two-term algebra course, as offered in New York City, may be substantially different from what is offered in Atlanta, Georgia for instance. And the variations from state to state and between schools in a given school district can also be immense. Considering the range of elements: the plethora of textbooks available for a given high school mathematics subject, the content and length of a given precollege mathematics course, the structure by which the course is organized and taught, all are so varied, and there is no standard for meaningful analysis of national achievement.

The generally accepted model for a traditional high school precollege mathematics program is the three year sequence that covers a three-to four-year period. The sequence includes a two-term course in algebra given in the 9th year, a two-term course in geometry in the tenth year and a two-term eleventh-year course which covers higher level algebra in the first term and trigonometry in the second term. This high school mathematics curriculum sequence is the basic one for which high school mathematics textbooks are written and is the one that has formed the basis for the Regents Mathematics program utilized by New York City high schools. Figure 5 is a diagram that shows the major curriculum topics of the Regents mathematics sequence for grades 9 through 11 in the New York City high school system. The New York State Regents High School Mathematics Curriculum shown, was modified in 1974 to include topics in Logic, Statistics, Probability and Transformation Systems, but the City of New York did not adopt this new curriculum sequence until June 1984 and high school mathematics achievement data to be examined below do not include test results from this newer Regents Mathematics curriculum.

Students coming into New York City high schools at the 9th grade are directed at

## NEW YORK STATE EDUCATION DEPARTMENT 3-YEAR REGENTS MATHEMATICS SEQUENCE* (NEW YORK CITY 1979-1983)

## REGENTS ALGEBRA: 9TH YEAR

## Term 1

Fundamental Operations
First Degree Equations in One Variable Systems of First Degree Equations
Monomials \& Polynomials

## Term 2

Quadratic Equations
Algebraic Fractions Inequalities
Radicals \& Pythagorean Theorem Introduction to Trigonomerry

## REGENTS GEOMETRY: 10TH YEAR

Term 1
Lines \& Angles
Congruent Triangles
Parallel Lines \& Quadrilaterals
Circles

Term 2
Similar Triangles
Area
Inequalities
Locus \& Coordinate Geometry

## REGENTS ALGEBRA II/TRIGONOMETRY: 11TH YEAR

## Term 1

Operations on Rational Expressions
First Degree Equations \& Inequalities
Linear Relations \& Functions
Operations on Radicals \& Complex Numbers
Quadratic Equations \& Inequalities

Term 2
Conic Sections
Trigonomerry
Exponents \& Logarithms
Variation

## FIGURE 5

[^2]either of two high school mathematics programs: 1) the three-year Regents mathematics program which is closely allied to science coursework in biology, chemistry and physics or 2) a Fundamentals of Mathematics (FM) program that extends over a two-year period and satisfies the two-year mathematics course requirement for a local high school diploma. The differences in the mathematics programs will be examined in detail in Chapter 3, but a short comparison here is noteworthy.

The differences in content between the two mathematics programs are enormous. Students taking the Fundamentals of Mathematics path rarely receive the preparation to advance to a Regents Mathematics course beyond the first year of Algebra, and the opportunity to enroll in science courses beyond the general science in the 9 th and 10th year is limited. The end result of the two-year Fundamentals of Mathematics program is a curtailment of mathematics coursework after the two years of mathematics study required for graduation are completed and the student has passed the mathematics portion of the Regents Competency Test (RCT) which does not involve mathematics above the 8th grade level.

In contrast, a student enrolled in the three year Regents mathematics sequence will study Algebra, Geometry and Trigonometry, subjects which are at the core of mathematics learning and necessary preparation for the SAT, Mathematics Achievement Test and future college mathematics study. Taking the three-year Regents mathematics path also gives students the opportunity to enroll in advanced mathematics courses like Precalculus and the Calculus during their senior year, giving them a decided edge and foundation if they pursue college study in science and engineering.

The New York State three-year Regents mathematics sequence is based upon a highly structured curriculum that is uniform throughout the state. Each course in the three-year Regents mathematics sequence carries with it a three-hour examination which is administered by the New York State Board of Regents three times each year: in January,

June and August. The Regents Examinations, as they are called, are a long standing tradition in New York and have been administered by the State Board of Regents since 1897, not only in mathematics, but in all of the high school academic disciplines for which the State Board of Regents establishes curriculum standards. ${ }^{29}$ As a result of the uniformity of the Regents examinations, there is a wealth of mathematics data available to analyze trends and make comparisons at the state, city and district levels.

The Regents examinations provide important data when trying to gain a perspective of student mathematics achievement because the examinations are a reliable and objective measure of a student's classroom performance in a given Regents mathematics course. The examinations reflect the State Regents mathematics curriculum and the mathematics exam data obtained is largely independent of classroom grading practices since the tests are uniform and explicit in the point value to be given for each test problem.

The Regents mathematics test data to be examined in Table 15 are the exam results of the 11 th Year Mathematics Regents Examination given during the $1983 / 84$ and 1984/85 academic years. The examination covered topics included in the Algebra II and Trigonometry mathematics coursework as outlined by the New York State Board of Regents. An outline of the topics covered in this third course in the Regents three year mathematics course sequence is as shown in Figure 5.

The 11th Year Mathematics Regents Examination data are being analyzed because they represent the critical mass of students in the high school system who have the mathematics background and preparation to pursue mathematics study beyond the two years necessary for a local high school diploma. The prerequisite and sequential nature of the traditional mathematics curriculum makes student performance in 11th Year Mathematics highly dependent on the achievement and the level of mathematics confidence that students bring with them from the study of mathematics at the 9 th and 10th grades. Because of this, the pool of students in a given high school who enroll in 11th Year Mathematics and
who achieve at high levels on the 11th Year Mathematics Regents Examination is a fairly accurate measure of the strength and effectiveness of a school's mathematics program. Thus, looking at 11 th Year Regents Mathematics Exam results provides a strong data base by which to make school comparisons of 11 th Year Mathematics achievement. And because of the sequential organization of the three year Regents mathematics courses, reliable inferences can be drawn on the mathematics achievement at the 9 th and 10 th years where students gain the foundation and complete the prerequisites for the mathematics coursework to be taken in the 11th year.

## New York City Public High School System

13 Chapter 1 Vs. 13 non-Chapter 1 Schools Comparison of 11th Year Regents Mathematics Scores -- 1983/84 and 1984/85


| 1984/85 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Enroll | \% Black \& Hispanic | Total <br> Taking | $\begin{aligned} & \text { \# Pass } \\ & >65(\%) \end{aligned}$ | $\begin{gathered} \# 75 \\ (\%) \\ \hline \end{gathered}$ | $\begin{array}{r} \#>85 \\ (\%) \\ \hline \end{array}$ | $\begin{gathered} \hline \text { \#>95 } \\ (\%) \end{gathered}$ |
| $40,588$ | 33\% | 2,926 | $\begin{array}{r} 2,177 \\ (74 \%) \end{array}$ | $\begin{aligned} & 1,402 \\ & (48 \%) \end{aligned}$ | $\begin{aligned} & 748 \\ & (26 \%) \end{aligned}$ | $\begin{aligned} & 220 \\ & (8 \%) \end{aligned}$ |
| 39,783 | 98\% | 527 | $\begin{gathered} 300 \\ (57 \%) \end{gathered}$ | $\begin{gathered} 163 \\ (31 \%) \end{gathered}$ | $\begin{gathered} 80 \\ (15 \%) \end{gathered}$ | $\begin{gathered} 14 \\ (3 \%) \end{gathered}$ |

TABLE $\qquad$

Nationally, the percent of students who took advanced high school mathematics courses such as trigonometry decline markedly as compared to the percent of students who studied a first-year course in algebra ( $25 \%$ vs. $79 \%$ ). ${ }^{30}$ The enrollment decline in upper level mathematics courses suggests the possibility that students in high school are not likely to pursue the study of mathematics on a year to year basis, unless they achieve and master prerequisite mathematics coursework at a high level.

The mathematics data shown in Table 15 are organized to show comparative achievement for the 11th Year Regents Mathematics Examination taken by students at thirteen Chapter 1 high schools and thirteen non-Chapter 1 high schools. The twenty-six schools selected were designated as academic comprehensive high schools and offered the full range of Regents coursework that leads to Regents endorsed diplomas. Selection was on the basis of student population size and racial/ethnic composition. As indicated, the Chapter 1 schools selected have a total population for both academic years of close to $40,000-$ an average of 3,000 students at each of the thirteen schools. The lowest school enrollment figure was 2,018 and the highest was 4,672 . For the non-Chapter 1 schools, total enrollment was slightly more than 40,000 with the lowest school enrollment being 2,373 and the highest being 4,288 .

The twenty-six schools selected are part of the larger New York City high school system, which in the 1983/84 school years had seventy-eight schools designated as comprehensive academic high schools with a total student enrollment of 223,882 students. Thirty-six of the schools had Chapter 1 status and a total student enrollment of 105,979, $88.4 \%$ or 93,705 of whom were Black and Hispanic students. The remaining 42 non-Chapter 1 schools had a total enrollment of 117,884 , with a Black and Hispanic student enrollment of 51,955 students or $44.7 \%$ of the total. On balance, the 80,000 students that comprise the population of the twenty-six schools selected for mathematics data comparison is greater than one-third of the total academic comprehensive student
population and can be looked upon as being fairly representative of what occurs city wide.

The racial/ethnic composition of the twenty-six Chapter 1 and non-Chapter 1 high schools selected for the 11th Year Mathematics Regents Exam comparisons are much more pronounced than the city wide total of seventy-eight high schools. For the thirteen Chapter 1 high schools selected the percentage of Black and Hispanic student enrollment is $98 \%$, while for the non-Chapter 1 schools it is $33 \%$. The selection of Chapter 1 high schools with an almost exclusive Black and Hispanic student population was important in order to insure that all of the students counted as having taken the 11th Year Regents Mathematics Examination were, to a high degree of probability, Black and Hispanic.

The Regents exam results provided by the New York City Board of Education are aggregate data and do not carry with it racial/ethnic identifiers. The selection of Chapter 1 high schools with $98 \%$ Black and Hispanic student populations largely avoids the problem of counting students in Chapter 1 students who are not Black or Hispanic. Experience in the New York City high school system has shown that a high school can often have a substantial majority of White and Asian students enrolled in 11th Year Regents Mathematics courses at schools which have a predominance of Black and Hispanic students. In some cases, high schools with Black and Hispanic enrollments as high as $70 \%$ of the total school population have only a minute fraction of its percentage of Black and Hispanic students enrolled in 11th Year Regents Mathematics classes. Again, this is a consequence of the lower level of mathematics preparation of Black and Hispanic students and a system of mathematics course placement which may overlook students who test low on standardized mathematics tests but who otherwise could achieve in Regents mathematics coursework given the opportunity and academic support.

Table 15 indicates the sharp differences in precollege mathematics enrollment and
achievement that exists between Chapter 1 and non-Chapter 1 high schools in the New York City public school system. The predominance of Black and Hispanic student population ( $98 \%$ ) at the selected Chapter 1 schools allows racial/ethnic comparisons that parallel the SAT achievement data comparisons made in Section 2.2.

The most notable differences between the Chapter 1 schools and the non-Chapter 1 schools can be found in the achievement levels in exam scores of 85 and higher. In the 1983/84 exam year, 807 students from the non-Chapter 1 schools scored 85 or higher, while at the Chapter 1 schools only 66 students scored at this level. This a ratio of more than 12-to-1. The differences become much more acute with exam scores at or above 95 where only seventeen students in the Chapter 1 schools scored at this level as compared to 294 at the non-Chapter 1 schools--a 17-to-1 ratio! The comparative data for the 1984/85 data show the same marked differences in exam test performance. In exam scores 85 or higher, there is a 9-to-1 ratio, and for exam scores above 95, a 16-to-1 ratio prevails. The significant differences in Regents exam scores in 11th Year Mathematics leaves little doubt of the extremely difficult problems that Chapter 1 schools face in developing a critical mass of students who enroll and achieve in the traditional three year Regents mathematics sequence. The sixty-six students at the thirteen Chapter 1 schools who scored 85 or higher in the 1983/84 year represent an average of only three students per school, or only a handful of high achieving students that precludes the school from building a critical mass. And even if the exam level score of identifying the critical mass of students were lowered to 75 , there would still be an average of only seven students at each of the thirteen Chapter 1 schools, hardly enough to program a meaningful fraction of a class beyond 11 th Year Regents Mathematics.

The data also show that large differences exist between the number of students taking the Regents exam and the number of students passing the exam. There were four and a half times as many students taking the exam in 1983/84 in non-Chapter 1 schools as there
were in Chapter 1 schools ( 2,764 vs. 591). And in 1984/85 the ratio of test takers at non-Chapter 1 and Chapter 1 schools was 5.5 ( 2,926 vs. 527). On the average this meant that less than one full sized class was in place (an average of 22 students) to take the Regents exam in Chapter 1 schools while there were three full sized classes (an average of 107 students) at the non-Chapter 1 schools.

The 11th Year Regents Mathematics data are significant because they establish with one exam score a fairly accurate picture of the level of student mathematics attainment at a given school. It follows that if a only a handful of students are achieving high scores on the 11 th Year Regents Mathematics Examination, then the achievement at the 9 th and 10th year levels is low also since these courses provide the foundation for the 11th Year Regents Mathematics course. And because the Regents mathematics courses are so closely tied to Regents science course offerings, the pool of students achieving at a high level in 11th Year Regents mathematics will generally establish the number of students who are available to enroll in Regents Chemistry and Physics courses. The low number of high achievers in 11th Year Regents Mathematics at Chapter 1 schools also means that it is unlikely that advanced mathematics courses beyond 11th Year Regents Mathematics will be offered. Thus, important precollege mathematics learning opportunities may be denied even to the handful of students with the mathematics background and interest in pursuing science and engineering college study.

Mathematics, more so than other subjects that students learn in high school, is highly dependent on a student's performance on a year to year basis. In order for a student to successfully complete the three-year Regents mathematics sequence and have that learning form a base for advanced mathematics coursework either at high school and at college, a student's performance should be at a level of at least 80 or better rather than the 65 that connotes a passing course grade. For the most part, in the design of mathematics tests, and on a Regents examination, a score of 65 can usually be obtained by memorization and
with repeated practice test taking. In these instances students' test performance may not provide the core of mathematics learning necessary for high achievement in subsequent mathematics courses.

If a high school is to function effectively as an academic institution, it must maintain a critical mass of students who are high achievers in 11th Year Regents Mathematics. This is important both from the standpoint of giving students a basic foundation for future college study and also for solidifying the quality of instruction by the school's mathematics and science teaching staff. As the number of high achieving students in upper level mathematics courses declines, so does the opportunity for teachers to practice and sharpen their teaching skills. From the data presented, the Chapter 1 schools in New York City face the dilemma of the continued arrival of students in the 9 th and 10 th grades who are underprepared to enroll and achieve in the Regents mathematics courses that the high schools have to offer. And the situation becomes more acute at the upper grade levels as the pool of high achieving students becomes less than is required to program students for the more advanced mathematics courses, thus denying teachers and students, the rewarding teaching and learning experiences that both need for academic maintenance and growth.

## CHAPTER 3

## ESTABLISHING A RATIONALE AND FRAMEWORK FOR MODEL RESEARCH AND DEVELOPMENT

### 3.1 The Inconsistencies of Standardized Diagnostic Tests in Mathematics

The low standardized test scores in mathematics for the Chapter 1 middle and junior high schools listed in Section 2.4, have profound and adverse impact on math enrollment and achievement levels at the high school level. Most of these schools serve as "feeders" for the academic comprehensive and vocational high schools that draw students from the surrounding neighborhood. However, the low math test scores in and of themselves do not convey the full extent of the adverse impact of low math achievement. The Stanford Diagnostic Mathematics Test is, for all intents and purposes, a basic arithmetic test, and, upon the basis of its content, the test is far removed from the level of academic coursework that students traditionally take in a New York State high school Regents mathematics program. Taken in this context, the test itself is a very inexact measure of a student's preparation or foundation for mathematics course enrollment upon high school entry.

It is entirely conceivable that a student who tests at or above grade level on the Stanford Diagnostic Mathematics Test can enter a first course in Regents mathematics in high school and be ill-prepared for the much higher level coursework and algebraic content that is to be learned and mastered. Obviously the 8th or 9 th grade students in junior high school who test one year or two years below grade level--and who bring this "label of math deficiency" with them as they enter high school--are destined to be placed in high school mathematics programs which are remedial in nature. And, as a consequence, these students have little chance of completing the three-year Regents high school mathematics program of study that provides the mathematics foundation for future college study in science and engineering. By virtue of inadequate mathematics preparation at the
college study in science and engineering. By virtue of inadequate mathematics preparation at the junior high school level and/or by poor test results on standardized tests designed to diagnose student mathematics proficiency at one point in time, students' options for advancement along a more competitive precollege mathematics program are essentially closed.

But how accurate are the results of the Stanford Diagnostic Mathematics Test or any other standardized test designed to measure mathematics preparedness? Do the test scores measure with a high degree of reliability what a student is capable of achieving in mathematics in later years or even in the immediate future? Conventional wisdom and a long histoy of educational testing within psychometric and statistical domains have, unfortunately, established a frame of mind that gives the standardized testing mechanisms far more credibility than are deserved. To have the test scores provide a measure (a rudimentary one at best) of students' appreciation and skill in basic mathematical exercises at a particular point in time is one thing, but to use the test score to label students' "mathematical ability" or to use the test score as a criterion for enrollment in the more academically rigorous high school mathematics programs goes far beyond what standardized mathematics tests or any test (including classroom tests) are designed to do.

Mathematics tests, for the most part, are little more than incidental measures of a student's understanding and skill in handling mathematical procedures or algorithms that have been recently taught. How accurate the results of the test are is dependent on a host of interrelated factors--not the least of which is whether the test reflects the material that has been previously taught and learned. Other factors include the quality of mathematics instruction at the school, students' familiarity with the format of the test, students' repeated practice or experience with similar tests, the length of time given for the test, the classroom test and proctoring environment under which the test is given. Any of these factors, if not in keeping with reasonable conditions for testing and/or preparation for the
test, can impede the testing process, contribute to producing a low score and still reveal little of the students' true mathematics proficiency at the time of test taking.

Unfortunately, with standardized tests there is no follow through (until the next test year) nor is there any opportunity for personal examination of the test to see what type of errors the student made. Lacking this additional information, it is virtually impossible to separate the mathematics proficiency of two students who may, for example, have tested two years below grade level on a standardized test like the Stanford Diagnostic Mathematics Test. One student may have simply made errors in computation while the other may have had little or no knowledge of how to solve the problems presented on the test.

To assume that on the basis of a single test score received in the 8th or 9 th year that students scoring two years below grade level are forever incapable of performing well in 9th grade mathematics coursework (whether it is Regents mathematics or coursework prerequisite to Regents mathematics) or that these students can never be considered seriously for higher level mathematics is a very poor value judgement at best. But in large part this is generally how standardized mathematics test scores are used--to judge or diagnose students' "math ability" and to place students in mathematics courses in high school. The practice is widespread and seriously undermines and curtails the opportunity for many students to enroll in the more rigorous Regents mathematics coursework. This is especially true in junior high schools and high schools with predominant Black and Hispanic student populations.

It can be argued that the standardized mathematics test is the only instrument available at the present time and that it is better than nothing. However, any psychometric test instrument which is unable to distinguish why a student obtained an incorrect answer on the test is really of little value in determining whether the student is really unable to do the mathematics presented on the test. At the extreme ends of testing-the high end, i.e., the
upper 90th percentile and the low end i.e., the lower 20 th percentile--there may be some value in initially identifying students for special programs, but for the remaining $70 \%$ of the students the results of the test can have a variety of implications, such as whether a student is judged capable of doing higher level mathematics in high school. The field based research and development work of the CMSP model has demonstrated that utilization of standardized mathematics tests for course placement is not only unreliable but also puts students who test low in serious academic jeopardy by placing them in general mathematics programs in high school that are little more than the mathematics coursework experienced in junior high schools. The problem is exacerbated by the lack of structure in middle school and junior high school level mathematics programs which makes it difficult for high school counselors to determine the relevancy of the mathematics courses taken by incoming high school students. This compounds the problem of determining whether entering high school students have the preparation for the mathematics courses that the high school has to offer. Under these clouded circumstances of trying to assess student proficiency there will be a natural tendency by high school counselors and mathematics departments to rely more heavily on standardized mathematics test scores for mathematics course placement as students enter high school. Is it possible to tell from aggregate test scores that improvement in mathematics learning is being made? How reliable are standardized tests for diagnosing student mathematics preparation for enrollment in higher level mathematics or for determining mathematics achievement trends for a school or school district on a year to year basis? And is an increase in standardized test scores sufficient to state that a student, school or district has made improvements in mathematics
learning? These are extremely important questions for which definitive answers are ultimately necessary in order to insure that decisions about students' and schools' mathematics programs are sound and promote learning and academic progression.

The perception of the the elementary and secondary education communities regarding the reliability and accuracy of standardized mathematics tests to predict and to diagnose students in mathematics is very strong. These strongly held beliefs in standardized mathematics tests place an unusually heavy burden on students who test one or two years below grade level at the junior high school level--and thereafter are deemed incapable of mathematics learning beyond that of RCT mathematics.

But suppose, because of "unusual circumstances", that the standardized tests being administered are not producing test results that can be considered reliable. And suppose it can be shown that for a substantial number of students the test scores are clearly not providing information that is indicative of the students' capacity to learn mathematics both in the present and in the future. What then is the responsibility of the educational community? And to what extent can standardized diagnostic mathematics tests be rightly used if, because of unusual circumstances, the tests do not provide useful information as originally intended or designed? Obviously, an instrument designed for scientific or educational measurement which gives unreliable data or, because of the nature of the test environment, provides erroneous data is really worse than no instrument. In medicine and engineering incorrect measurements of biological and scientific conditions can lead to undesirable or even harmful consequences unless there is a process in place that allows the measurement to be repeatedly checked from a variety of sources. Second and third opinions are common in medicine and law and the very nature of engineering places an extremely high value on the accuracy and repeatability of measurements across a wide range of environmental conditions.

A measurement device used in engineering that does not give a measurement over its full range in accordance within its accuracy specifications is either discarded or not used. It is not enough for the instrument to be accurate only at the high or low end of the scale; to be useful to the scientist or the engineer it must maintain a "linearity of measurement" that
is accurate across the entire range in which the phenomena to be measured varies. Should the educational community or the general public accept any less from psychometric instrumentation and their measurement accuracy in determining who should study Regents mathematics or be placed in minimum competency mathematics or remedial mathematics programs? For a significant portion of the Black and Hispanic population in Chapter 1 junior high schools and high schools, standardized diagnostic mathematics tests--which may have extremely limited value and accuracy in assessing students preparedness to learn mathematics--play a decisive role in determining the students' future mathematics education both in high school and beyond.

Like the faulty engineering instrument, standardized diagnostic mathematics tests' accuracy may be limited to students who test at the very high and low end of the scale, leaving the great majority of students with very general indications of mathematics performance at best and clearly inaccurate mathematics profiles at worst. It may be argued that the measurement of biological or scientific phenomena is much less complex than the measurement of students' mathematics aptitude or their capacity to perform intellectually on a test of "mathematics ability". However, the analogies between scientific and educational measurement are legitimate in that both are concerned with accuracy, linearity and range over which measurements are to be made and the consistency of repeated measurements over the long term. In keeping with these accepted principles of measurement, it may be that the simpler of the measurements, as currently practiced, is the educational one, rather than those which are scientific.

No matter how accurate an instrument of measurement is specified to be, the accuracy and reliability of the measurement taken is primarily a function of the stability of the phenomena measured. If there are gross fluctuations in the phenomena then the measurements to be recorded will be characterized by "peaks and valleys" which, in order to have meaning, must be studied carefully in chart form after the measurements are taken.

Lacking either the time or resource for this post-analysis of test results, the measurements are generally "averaged" to give some signs of usefulness and trend. However, when this is done, the essence of the fluctuating measurement is masked and subtle and important information may be lost.

This "instability of phenomena" is an element of the "unusual circumstances" that fosters the gross inaccuracies in standardized diagnostic testing in mathematics. This instability occurs because of the apparent lack of uniform and adequate mathematics instructional programs at Chapter 1 elementary and junior high schools. As a result, a great proportion of students attending these schools do not receive the fundamental core of mathematics learning that students need to respond adequately to the problems on the standardized diagnostic mathematics tests.

Scores on standardized tests are generally reported as mean grade equivalents and, presumably, the assumption is made that for a given score the standardized test instrument is able to determine whether a student is doing mathematics at or above or below a particular grade level. For this assumption to have meaning a further assumption must be made that the students tested were adequately prepared to take the test. If this is the case, then the standardized mathematics test may be a reflection of what students have learned at the point in time that the test was administered. Given the very low scores on standardized tests that prevail at Chapter 1 junior high schools, the assumption of adequate preparedness of students prior to test administration is highly questionable.

In its most useful application, the standardized test can provide a measurement of what students remember or what they may know on the day the test is given. If a student scores high, there is a degree of certainty that the student's knowledge of the test problems exists. However, if the student performs poorly, there is no way of knowing whether the poor performance is due to memory blocks, inaccuracies in calculation or lack of knowing how to solve the test problems. A student who does not remember an arithmetic algorithm
at the time of the test or miscalculates is in a far different position academically than a student who has limited working knowledge of the mathematical concepts and procedures asked for on the test. Although both students may have obtained the same mean equivalent score, the mathematics program that should be prescribed for each of the students is completely different.

And this is the crux of the problem with standardized tests--and especially so in schools where there exists an inadequate programs of mathematics instruction. It would appear that a standardized diagnostic test which cannot distinguish between students who know and don't know how to solve a particular problem or a set of mathematics problems on a test has little value as a criteria for determining whether a student should be placed in one mathematics program or another.

The very notion of mean grade equivalents contributes to the unrealistic thinking that somehow mathematics learning is a linear process--that if we give students the appropriate remediation, students will gradually respond and improve in their mean grade equivalent score on standardized tests. However, teachers are primarily interested in whether students understand and are able to do the mathematics problems that make up the coursework. It is ambiguous to state that students can almost do an arithmetic problem. The students either know how to do the problem or they don't! And yet the standardized diagnostic system of testing labels 8th grade students in a quasi-proportional way that suggests that their 6th grade mean equivalent score indicates that they have only mastered part of the mathematics that 8th grade students should know. Is the part not known by the students the multiplication algorithm, place value, addition and subtraction or a combination of these? The standardized diagnostic test does not give this information-instead, what is established is a label for students that they are or are not performing in accordance with some norm reference. And this information has little bearing on whether students can do mathematics problems of a particular kind and level.

What teachers and counselors in high school really want to know with a high degree of confidence is the level of mathematics preparation that students have for the courses that the high school has to offer--especially Regents (precollege) mathematics coursework. Because of its inability to distinguish how and why students committed errors, the standardized diagnostic mathematics test is a very poor assessment device for determining mathematics preparation for the great majority of students who enter high school in the 9th and 10 th grade.

The concept of mean equivalent scores, besides unfairly labeling students who test low, is also at odds with the realities of inadequate schooling. If students do not receive appropriate mathematics instruction during the K-8 school years and test low on a standardized test at the 8th grade, does this mean that students themselves are unable to learn mathematics or that the schools were unable to provide the foundation for students to learn mathematics well? At present, the former is accepted as a given and students who test low on standardized tests must bear the burden for a consequence that is, for the most part, beyond their own and their parents' capacity to control.

The standardized diagnostic mathematics test and its manifestations is clouded by a host of irregularities that makes its continued use as an assessment device for individual students, schools and districts questionable. It is a product whose time for serious re-evaluation as to its usefulness has come. And its merit as a device for quantifying the levels of student progress in mathematics must be closely examined.

From the perspective of high schools with predominant Black and Hispanic students populations, the use of the standardized diagnostic test scores as criteria for mathematics course placement should be discontinued. Their use as an assessment instrument has contributed to a litany of high school mathematics courses that are little more than a review of pre-high school mathematics topics. But more importantly, the use of the test scores unfairly places "low mathematics ability" labels on incoming 9th grade high school
students who, through little fault of their own, did not receive the mathematics instruction needed to prepare them for high school mathematics coursework. This mathematics "ability" labeling is unjust even when high, as it creates a false sense of academic accomplishment. Its continued use as a mathematics diagnostic instrument may be denying many students their right to obtain the best mathematics education that the high schools have to offer, namely, Regents mathematics coursework.

Given the fact that federal and state governments rely on standardized test scores to determine educational need--and therefore the allocations of funds-it is not likely that standardized test usage will be curtailed at the elementary and junior high school levels in the foreseeable future. However, the diagnostic format of the test and the fact that large proportions of students may not be getting the appropriate mathematics instruction creates an unstable testing environment which makes meaningful measurement difficult at best and erroneous at worst. Because of this, large swings in test performance on a year-to-year basis are possible, which, if not taken into account, can confound the interpretation of test results.

The standardized diagnostic mathematics test should be used primarily as a very general group measure only in school and district environments where there is a structured and continuous program of mathematics that insures that all attending students are receiving the mathematics foundation that prepares them for subsequent higher level mathematics courses. And when the standardized tests are given, the movement and variability of group test scores within a district or school should be looked at carefully to insure that test score improvements are not simply a function of repeated practice in the school's mathematics classrooms where the principal aim may be "teaching to the test" at the expense of true mathematics learning.

### 3.2 Mathematics Course Enrollment as a Function of Standardized

## Test Scores in New York City High Schools

High schools as well as colleges are constantly faced with the task of determining the level of student preparation for mathematics course placement. And the dilemma deepens as greater numbers of students enter high school and college with deficiencies in their mathematics background. It may well be that the heavy reliance on standardized tests is creating a problem by grade level labeling students at the high school level.

As more students have tested across a wide range of grade levels, high schools must create remedial mathematics coursework that is "consistent" with the grade levels tested. Given the low achievement levels reached at Chapter 1 junior high schools, greater numbers of Black and Hispanic students enter high school with test scores below grade level. And, as a consequence, the high schools they enter are inclined to offer mathematics courses to match the low grade levels tested as a "remediation strategy." And the cycle of decline in mathematics performance continues as more and more students test low, and still more remedial classes or general mathematics courses to fulfill students' high school graduation requirements are offered until such time that the majority of the mathematics programs in high school are largely remedial in nature.

Over the years, the cyclical process of standardized diagnostic mathematics testing and placement of students in mathematics remedial courses has become the norm in New York City high schools with predominant Black and Hispanic student populations. The process of mathematics remediation, currently in widespread practice at the Chapter 1 schools, not only seriously limits students' opportunities for learning higher level mathematics coursework, but also precludes experienced mathematics teachers at these schools from teaching the mathematics courses they were trained in college to teach. And it also precludes new and younger mathematics teachers from gaining the classroom mathematics teaching experience that they need to become proficient in teaching precollege mathematics.

Given the low achievement test scores on standardized tests that prevail for Black and Hispanic students in Chapter 1 middle and junior high schools in New York City, the practice of placing students in remedial courses or non-Regents mathematics courses on the basis of these low scores as they enter high school continues. The practice is reinforced because of the wide variations in the mathematics curriculum and the quality of instruction that characterize the Chapter 1 junior high schools. Because of this, mathematics courses taken by junior high school students and the grades attained can show large differences from school to school. This lack of uniformity in the students' academic records leaves high school counselors and mathematics department chairpersons with no stable references for appropriate mathematics course placement. Thus, test scores on standardized mathematics tests become the instrument of measure.

The placement of students in high school mathematics courses as a function of their test scores on the standardized diagnostic mathematics tests is a rather serious and undertaking that can be detrimental to students. For all intents and purposes, once students are programmed for a general mathematics course or "math remediation", their future mathematics learning is essentially curtailed after a period of two years in high school and the students are often unaware of the future consequences. There are two factors that contribute to this predicament of students in Chapter 1 schools. The first is the fact that to graduate, New York City requires the successful completion of two years of mathematics-either General Mathematics or Regents Mathematics is accepted to fulfill this mathematics requirement. The second factor, and the more important one, is the requirement that a student studying General Mathematics must pass the Regents Competency Test (RCT) in mathematics in order to graduate from high school.

The RCT graduation requirement, in particular, places an unusual amount of attention on a specific three hour test that is given twice a year. The number of students passing the RCT in mathematics is one of the criteria that New York State Education Department uses
to judge a school's effectiveness. Because of this, schools, and in particular Chapter 1 schools, are under great pressure to have their students pass the RCT in mathematics and, as a consequence, will organize general mathematics courses and remedial mathematics programs to closely resemble the mathematics topics covered on RCT mathematics tests. Thus a tendency arises where instruction in these general mathematics classes is directed to teaching to the RCT, not so much as an instructional practice but as a result of the RCT based course structure. In effect, the RCT becomes, in many schools, the curriculum of necessity.

The RCT based curriculum, which for many Chapter 1 students may cover a period of two years, could offer students an opportunity to build a foundation for future mathematics if its content were in keeping with prerequisites for a first course in Regents mathematics. However the RCT is a minimum competency test and the mathematics topics and problems are at a level much lower than needed to prepare students for a first course in algebra.

Another factor that contributes to lower level mathematics learning for students enrolled in RCT based high school mathematics courses is the fact that the passing grade for the RCT is set at $65 \%$. Thus what is established is a level of mathematics performance which is not much higher than that which might be experienced by students in the 7 th grade. An analysis of the test items on a typical test shows that students can obtain a test score of $65 \%$ by correctly solving problems that do not require knowledge above the level of mathematics which is described in official New York City curriculum guides for the 8 th grade. ${ }^{1}$

The minimum competency aspects of the RCT also raises questions as to whether enrollment in an RCT based mathematics course may actually hinder a student's opportunity to learn mathematics. Mathematics, like the study of music and foreign language, requires constant practice to build a knowledge base which students can use to learn more mathematics and at a higher level. Without constant practice to regulate
achievement in progressive stages in basic mathematics, students may be subjected to a learning process which is circular rather than sequential.

The possible inhibition of mathematics learning by minimum competency testing was found during the study and analysis of the mathematics assessment of the second National Assessment of Educational Progress (NAEP) conducted in the 1977/78 academic year. ${ }^{2}$ The authors of the study noted the continual improvement of students' performance on arithmetic exercises with increasing age. They also point to the fact that:
... many fundamental errors also disappear as students progress in school. Although over $30 \%$ of 9 year olds subtracted the smaller digit from the larger in a subtraction exercise that required regrouping, only $5 \%$ of the 13 year olds and 1 percent of the 17 year olds committed the error.

## And they go on to state:

These results have profound implications for minimum competency programs. Rigid minimum competency programs which hold children back until they have demonstrated mastery of a given set of skills may in fact, be depriving them of the very experiences that would lead to mastery of the particular skills.

To what extent this premise may be operating to detract from school mathematics learning needs to be further investigated, however, as stated by the researchers, the results of the NAEP mathematics assessment in the area of basic arithmetic have profound implications and may be exacerbating the current population of students in minimum competency programs in mathematics. In large measure, these are students in the Chapter 1 junior high and high schools who have been placed in minimum competency programs which are terminal and circular in nature and offer little chance of gaining further experience to do higher level mathematics coursework.

Although high schools consider other factors such as attendance and mathematics
grades, the placement of entry level students in high school mathematics courses is largely influenced by students' performance on the mathematics portion of standardized diagnostic mathematics tests. This practice is not likely to change unless the quality of instruction improves dramatically at the junior high schools and a uniform and structured program is put in place that provides sound preparation for the Regents mathematics coursework to be taken in high school. For this to occur there must be a corresponding increase in the number of teachers who are qualified to teach the Regents preparatory mathematics course at the Chapter 1 middle and junior high schools. The junior high school mathematics test score data presented in Chapter 2 of this study would appear to indicate that such improvements in teaching are not likely to occur in the near future as there are no signs or trends that indicate the situation is improving in Chapter 1 junior high schools.

Given the questionable value of using standardized diagnostic mathematics test scores as a major criteria for mathematics course placement in high school, what other options do high school counselors have, if as suggested, the use of the standardized diagnostic mathematics test is discontinued? The apparent lack of structure and uniformity of the junior high school mathematics curriculum among junior high schools that feed a given high school makes the examination of students records highly unreliable. And students' academic history is clouded by the low quality of mathematics instruction that may have prevailed in the students' Chapter 1 junior high school setting. The lack of a suitable mathematics assessment mechanism for entering students does present a dilemma for high schools, and a variety of strategies have been utilized to circumvent the inadequacies of the information that high schools have to work with. Two of the major strategies are:

1) the design of pre-evaluation tests by the high school which are tailored for the mathematics programs that the high school offers-e.g., various cut off scores are established on the pre-evaluation test and incoming students who score above or below these cutoffs are placed in the school's mathematics courses accordingly, and
2) all incoming students (except those who test very low on the standardized tests) are placed in the first year Regents mathematics courses for a fixed period of time during the early part of students' first semester in high school. At the end of this "probationary" period students classroom performance is evaluated and students who are passing stay in the Regents mathematics classes and those who aren't are reprogrammed for general or remedial mathematics courses.

Both of these assessment strategies have shortcomings. The first suffers from the same major weakness of the standardized diagnostic mathematics test in that it is a single shot event. And a host a variables exist to affect student performance, including memory blocks, unfamiliarity with the content and format of the test and the usual anxiety that often occurs in test taking. Anxiety is amplified considerably when students take the test in new and unfamiliar situational environments ${ }^{3}$ as high school can be upon first entry. Student mathematics assessment using this pre-evaluation test strategy could be improved by making the test taking conditions similar to those the CMSP uses in administering preliminary mathematics tests to students who have participated in the model program. CMSP allows time for review of mathematics topics on the test before the test is administered, does not have multiple choice or true/false questions on the test, allows students more than enough time to complete the test and insures that the test is graded by experienced mathematics teachers who can qualify student errors on the test.

The second strategy of assessing students, although it allows more time for students to demonstrate their preparedness for a first course in Regents mathematics, and is probably worthwhile for the students who pass, plays emotional and educational havoc on students who fail. In Chapter 1 schools the failing students would be in the overwhelming majority. There is probably nothing that can destroy students' academic confidence more than placement in a course for which they are largely underprepared. The "sink or swim"
strategy, while intended to give all incoming students an opportunity to enroll in Regents mathematics, unfortunately often operates as a screen which selects a few at the expense of the many. This strategy is sometimes modified to select students for various versions of the first year Regents mathematics course--e.g., giving the higher performing students the traditional two term course sequence, while students performing lower are placed respectively in three and four term course sequences that cover the same mathematics topics but at a slower instructional pace.

The high school system in New York City and other urban school systems where there are large Black and Hispanic student enrollments suffer from the effects of mismatched mathematics course placement for entry level students. Given the lack of an instrument or strategy that can provide more meaningful information on incoming students' mathematics preparation for traditional high school mathematics course offerings, the high schools, to a large extent, have ameliorated the situation by reducing their mathematics programs for incoming students to the lowest common denominator. This is reflected in New York City Chapter 1 high schools by the extremely small number of students who take and achieve at a high level on the 11th Year Mathematics Regents Examination, and nationally, by low SAT-Math and Achievement Test scores by Black and Hispanic high school seniors and their declining enrollment in the nation's engineering colleges.

### 3.3 The Fundamentals of Mathematics Track Versus Regents Mathematics

As students enter 9th or 10th grade in New York City public high schools there are essentially two mathematics program paths they can follow in their high school education,

1) Fundamentals of Mathematics or 2) Regents Mathematics. The two programs are substantially different in terms of course content, structure, length, and academic regard among the high school mathematics teaching staff.

The Fundamental of Mathematics (FM) program is essentially a two year program
which, when completed successfully, satisfies the two-year mathematics requirement for high school graduation in New York City. In addition however, students who complete the two year FM program must also pass the Regents Competency Test in Mathematics. The FM course is terminal in nature, and upon completion in the 10th grade, there is little incentive for students to continue the study of mathematics since high school mathematics graduation requirements for a local diploma have been met. Thus, students enrolled in the FM program are not likely to graduate with more than the two years of mathematics. Completion of mathematics by the 10 th grade leaves students with the prospect of taking no mathematics courses during the 11th and 12th year--a full two years before high school graduation.

In contrast, the Regents mathematics program is a traditional three-year course sequence which provides students with the mathematics foundation they will need to pursue mathematics and science coursework beyond high school in order to be competitive either in college study or in entry level service oriented job positions. ${ }^{4}$ Students who successfully complete the three-year Regents mathematics sequence will have satisfied one of the rigorous requirements needed to obtain a Regents endorsed high school diploma. Knowing the value of these courses, as they apply to college admission, students in Regents mathematics programs are also more likely to continue the study of advanced mathematics and college level courses in their senior year. In addition to the higher level mathematics coursework that students experience in Regents mathematics courses, those who achieve in upper level Regents mathematics coursework (10th and 11th year) gain the opportunity of being taught by the school's more experienced and qualified mathematics teachers.

Figure 6 is a diagram that shows the curriculum paths of the two mathematics programs that students can enroll in as they enter the 9 th grade in New York City. The two program outlines shown are as prescribed by the New York City Board of Education
NEVY YORK CITY MATH AND SCIENCE CURBICULUM SEQUENCE
9TH-12TH GRade and 1ST yEar of college

Figure 6
and approved by the New York State Education Department to meet diploma and graduation requirements. Both programs are offered by all of the high schools in the New York City high school system (except the three specialized high schools--Brooklyn Technical, Stuyvesant and Bronx Science) where only Regents mathematics programs are offered. However, the number of Regents mathematics program course offerings are disproportionately low at Chapter 1 high schools. This is shown to be the case in the data presented in Chapter 2, Table 15, where almost six times as many students at non-Chapter 1 high schools had taken the 11th Year Mathematics Regents Examination compared to the number at Chapter 1 high schools during the same year. This is an indication that student enrollment in prerequisite Regents courses in the 9th and 10th grades at Chapter 1 high schools is correspondingly low. For the most part, the major mathematics program of students in Chapter 1 schools is Fundamentals of Mathematics (FM).

The Fundamentals of Mathematics program was adopted by the New York City Board of Education in response to New York State regulations that require all high school students in New York State to pass the Regents Competency Test (RCT) as one of the conditions for graduation and receiving a high school diploma. The latter also requires that students accumulate two years of Fundamentals of Mathematics course credit in order to receive a diploma from the local school district. This diploma requirement is different from receiving a Regents endorsed diploma that requires students to take a three-year Regents mathematics sequence and pass Regents examinations administered traditionally in the 9th, 10th and 11th grades.

The Regents Competency Test in Mathematics was first administered officially in New York City in June 1980 and is of the same genre as standardized tests, consisting of two parts--Reading/Writing and Mathematics. The three-hour RCT Mathematics examination is given twice each year, in January and in June. The content of the mathematics portion of the test is essentially arithmetic, including problems involving geometry, graphs, statistics
and probability-- all presented at a very rudimentary level. Problem difficulty and level of rigor is about the same as that found in traditional 7th grade mathematics textbooks.

Students are exempt from taking the RCT in mathematics if they enroll in a first course in Regents mathematics and if they pass a corresponding Regents examination after the completion of coursework which, as traditionally offered, covers two semesters. This ruling by The New York State Education Department is a very important one because it has a profound influence on the availability/choice of mathematics programs as students enter high school at the 9 th grade. At Chapter 1 high schools the choice for most students is at once Fundamentals of Mathematics and the concomitant goal of passing the RCT Mathematics test as a requirement for graduation.

In effect, there is little choice for Chapter 1 students: on the one hand they are faced with the inadequate mathematics preparation received while in junior high school, and secondly, when they arrive at high school, they find that the major high school mathematics course offering is not much different from what they had been studying in junior high school. In addition, the FM program to be taken in high school carries the added requirement of being closely tied to an examination that must be passed in order to graduate. Thus, if there is student choice in the matter, they are faced with the major decision to pursue either the FM track (a relatively easy two year mathematics program which satisfies the two-year mathematics requirement in preparation for the RCT and a local diploma) or pursuing the Regents track (a much more rigorous mathematics program which is sequenced over a three-year period which satisfies the Regents endorsed diploma requirement, wherein students must pass a Regents examination at the completion of each of the three mathematics courses).

Unless students in Chapter 1 high schools are able to clearly see the long term value of enrolling in a Regents mathematics program, the incentives of a much shorter and easier Fundamentals of Mathematics program of study plus the impact of the RCT will have a
decided influence on the choice of mathematics program. The value and incentives for enrolling in a Regents mathematics program are not as clear to a young student in high school who may not understand why the more rigorous and demanding Regents program is the program of merit. Students at Chapter 1 high schools are not generally aware that the completion of at least two Regents mathematics courses (Algebra and Geometry) is necessary even if there is the slightest consideration that the student will attend college or work in the increasingly technical and service oriented economy. At the present time neither home nor school counseling seems to be raising students' awareness and perception of the long term and somewhat irreversable effects of opting for the less rigorous Fundamentals of Mathematics program. One consequence of this lack of advisement is the very few students at Chapter 1 high schools who take the College Board Mathematics Achievement Tests. Table 16 shows the handful of students ( 44 total) who took the 1984/85 Mathematics Achievement Test Level 1 at seven Chapter 1 schools listed. In comparison the three specialized high schools had 1,206 students who took the tests.

To a large extent, course enrollment in the Fundamentals of Mathematics programs is reinforced at the school level and district level. At the school level--and especially at Chapter 1 high schools where success in Regents mathematics programs is minimal-- there is a strong tendency to make the school mathematics program tie directly to the RCT because the number of students passing the RCT has become a very important academic indicator of the school's effectiveness. In effect, schools are held "accountable" by the strength or weakness of the "RCT" mathematics programs they offer. It follows that schools, in order to reduce the chances of being designated ineffective, will be inclined to offer mathematics programs that maximize students' passing the RCT--and probably at the expense of students establishing a base for learning higher level mathematics.

At a higher administrative level, the number of high schools in a given school district
1984/85 MATH ACHIEVEMENT TEST LEVEL 1 COMPARISONSAT SELECTED CHAPTER 1 AND NON-CHAPTER 1 HIGH SCHOOLS
\# TEST ..... MEAN
TAKERS ..... SCORE
CHAPTER 1 HIGH SCHOOLS
Evander Childs ..... 7 ..... 557
Erasmus Hall ..... 6 ..... 465
Washington Irving ..... 5 ..... 488
Martin Luther King ..... 6 ..... 523
George Washington ..... 6 ..... 522
George Wingate ..... 5 ..... 404
Julia Richman ..... 9 ..... 516
NON-CHAPTER 1 HIGH SCHOOLS
Midwóod 117 ..... 591
Cardoza ..... 67 ..... 597
Bronx Science ..... 402 ..... 603
Stuyvesant ..... 532 ..... 640
Brooklyn Tech ..... 272 ..... 553
that fail to meet the RCT standard may be looked upon as reflecting the school district's effectiveness. And, as a consequence, the tendency towards the RCT standard shifts to the point where it establishes the foundation and core for a high school mathematics curriculum standard. In essence, the New York State Education Department RCT dictates and district compliances thereof provide a compelling rationale for creating a mathematics curriculum whose outline and content is structured around the topics and problem sets that appear in the semi-annual administration of the RCT. This has essentially been the evolution of the Fundamentals of Mathematics program--a curriculum strategy whose basic goal is to maximize students' passing the RCT.

The tendency toward the RCT is amplified by the focus and attention that schools and districts receive both in the dictates from the New York State Education Department and through the media and press which give substantial coverage of "lists of ineffective schools" not meeting RCT standards and the results of the statewide testing program. ${ }^{5}$ With this media coverage, it is hard for schools, parents of students and students in the schools not to be conscious of the ranking of "school effectiveness" and be sensitized to the importance of the RCT as a condition for fulfilling both high school coursework and graduation requirements. No such statewide or local school district attention is given to Regents mathematics coursework or the results of Regents examination, however.

Since there is no Regents mathematics track requirement for high school graduation with a local high school diploma, the only incentive for pursuing Regents mathematics courses is students' realization that a three-year sequence in Regents mathematics will provide a strong foundation for the SAT's and for future higher level mathematics learning both in high school and in college. The increasing number of high school graduates who arrive at college with inadequate mathematics preparation indicates that students are opting for the less rigorous one or two year General Mathematics courses rather than the stronger traditional three-year mathematics programs of study -- i.e., the "Regents" in New York

City.
What is ironic in New York City is, that traditionally, the major mathematics program offering in academic comprehensive high schools was the Regents mathematics program. This is still the case in the specialized high schools of New York City: Stuyvesant, Bronx High School of Science and Brooklyn Technical High School. However, at the remaining New York City high schools there has been an enormous decline in the number of students who take the Regents mathematics examination. Less than half the number of the students took the 11th Year Regents Mathematics Examination in 1982 as did in 1970.6 During this period high school enrollment decreased by only $10 \%$. Whether this is attributable to the complexity of factors associated with the general nationwide decline in mathematics achievement is not clear. However, since the New York City public school system has traditionally had a strong mathematics program in place and a sufficient number of high school mathematics teachers to teach the courses at a high standard, the sharp declines in Regents mathematics present somewhat of a paradox.

Besides the declines in Regents mathematics participation, there has been an extensive softening of the general mathematics curriculum during the same period of time. The Fundamentals of Mathematics program that is currently in practice is essentially a general mathematics program to the extent that the label infers the learning of mathematics found in traditional 8th grade textbooks. In support of the FM program, the New York City Board of Education issued two curriculum guides entitled Fundamental of Mathematics, Part 1. Preparing Students for the RCT and Fundamental of Mathematics Part 2. Preparing Students for the RCT. ${ }^{7,8}$ The two FM guides were published as experimental editions in 1981 and have since been utilized by New York City high schools to develop general mathematics programs that conform to the content and structure of problems that are seen on the RCT.

For students who do complete the FM program in one year, there is the option of
enrolling in a first course in Regents mathematics or satisfying the second year mathematics requirement by enrolling in a one-year computer oriented course or consumer mathematics course that builds on the mathematics coursework learned in Fundamentals of Mathematics. This second year mathematics course presents an interesting set of options for students: either to satisfy the two-year mathematics requirement by taking a traditional course in Regents mathematics which introduces students to the abstractions of algebra or taking a course where students have the opportunity to work with computers, while at the same time, continuing their learning of mathematics through BASIC programming.

As described in the bulletin Computer Mathematics: An Introduction, published by the New York City Board of Education, the computer oriented course "is designed to engage students in using the computer to solve mathematical problems." The bulletin further goes on to state that the course "has been prepared to be used with students who have completed a year of general mathematics or for those who are not meeting success in the more traditional mathematics programs." 9 As indicated, the course becomes an attractive one year option for students who want to continue on a non Regents mathematics track or for those students who experience difficulty with Regents mathematics and want to complete their second year mathematics requirement in a less demanding course of study and "get a chance to use computers".

The Computer Mathematics course essentially extends the Fundamentals of Mathematics program for a second year as an optional means of satisfying the two year mathematics requirement of a general mathematics program. The course emphasis is on simple BASIC programming, the writing of algorithms and problem applications of FM topics learned in the first year. Given the choice, a student who has satisfactorily completed a year Fundamental of Mathematics program of study is more likely to enroll in a subsequent mathematics course for which a prerequisite base of knowledge has been established and where students can get to use computers in class.

Fundamental or General Mathematics course offerings in high school has been a nationwide trend that has narrowed students options for enrollment in precollege mathematics courses considerably-especially for students in Chapter 1 high schools. The student who arrives at high school and is "found" to be unprepared for Regents mathematics is placed in the Fundamentals of Mathematics program sequence. Once in this program sequence, the chances are slim that the student will elevate to a Regents mathematics program. However, if enrollment in a Regents mathematics class does occur, the students will probably experience great difficulty in mastering course material because of the lack of topic coverage and inadequate preparation received in the earlier Fundamental of Mathematics course.

In effect, the Fundamentals of Mathematics course followed by Computer Mathematics (or Consumer Math) course, is a two-year course of study that is terminal in nature. Besides routing students toward the RCT and providing the schedule for students to accumulate the mathematics course credit needed for graduation, FM has little value or substance for providing the foundation or core of learning required for students to continue their mathematics learning after the two-year FM coursework has been completed. In an age of science and technology where there is an increasing awareness that students need to be more mathematically adept rather than less, the limited two-year RCT mathematics option falls far short. In order to meet the occupational and technical demands of the future, the traditional three-year Regents Mathematics sequence must be the curriculum utilized in the mathematics programs offered at the high schools. The FM program and all other mathematics programs which are tied to minimum competency tests, or dwell in "generalities" of prealgebra mathematics, are not consistent with the times nor have they the intrinsic value for providing students with the foundation needed to learn higher level mathematics both at and beyond high school.

### 3.4 Mathematics: Its Distinction and Potential for Student Learning

What is characteristic about mathematics that makes it distinct from other subjects that students study in high school? And how is learning mathematics different from learning the other academic core subjects--science or english or social studies? Both of these questions raise pedagogical as well as organizational education issues that have a direct bearing on academic learning not only from the standpoint of accumulating school course credit but also as a basis for future learning opportunities beyond high school.

Mathematics, as an individual course of study, carries the same weight of course credit as other academic subjects, so its level of importance in the school day curriculum for a given semester is on an equal footing with english and social studies. However, from the perspective of New York City Board of Education and New York State Education Department course credit requirements, mathematics has less importance than English or Social Studies. Students in New York City need only complete two years of mathematics (either Fundamentals of Mathematics, Regents Mathematics or a combination of the two) to satisfy the requirements for a local diploma, or three years of Regents Mathematics to qualify for a Regents endorsed diploma. In comparison, four years of english and four years of social studies are needed to satisfy both the requirements for either a local or a Regents endorsed diploma.

This lower number course credit required in mathematics than in english and social studies for graduation is somewhat of a contradiction considering the significance of mathematics in standardized diagnostic tests where attempts are made to determine students' verbal and mathematical competencies even as early as the first grade. Standardized diagnostic tests are continuously administered to students at all levels of their schooling; at elementary and middle schools included are grade level tests and IQ Tests, while high schools administer minimum competency tests and the Scholastic Aptitude Test (SAT). The assessment of students' verbal and mathematics proficiency at any given point
in time is paramount. It is not students' recollection of historical facts nor their current knowledge of political or socio-economic events that are tested, but instead, students' ability to solve mathematical problems and comprehend and decipher written passages and word meanings.

Given the importance that is attributed to standardized tests that are aimed at assessing students' verbal and mathematics proficiency and the fact that mathematics constitutes one-half of most standardized tests' value, it is odd that mathematics does not occupy a larger segment of the school curriculum and course structure in high school. From the standpoint of academic course time allocation during the regular high school day, mathematics' single course offering occupies only $25 \%$ of academic instructional time while English, Social Studies and Science make up 75\%. The latter subjects, as taught in high school, can all be categorized as non-mathematical (including Science) because of their emphasis on reading and the recollection of facts and events. And taken together, they are inclined to contribute more to students' achievement on the verbal portion of standardized tests than on the mathematics portion. In actuality, high school students receive one-quarter the preparation time in mathematics that they do for the verbal as it applies to standardized tests. And yet, $50 \%$ the content of most standardized tests (including the two-part diagnostic, predictive and minimum competency tests) is based on students' ability to perform mathematics operations and solve mathematical problems.

The fact that there is a disproportionate amount of instructional time for mathematics is due serious consideration. Standardized diagnostic or aptitude tests attempt to measure students' accumulated or developed verbal and mathematical skills and "reasoning abilities." 10 By this definition, the standardized tests can be said to be tests of general knowledge and are therefore not directly related to the subject matter being tested or studied by students at the time of the test. This is probably true for the verbal portion of the test, given the broad topics found in written passages and the grammatical nature of verbal
exercises. However, in the mathematics portion of the test, what is being tested is whether students can solve specific mathematical problems. And these problems are directly related to only one subject that students are studying in school--Mathematics. And whether students do well on the mathematics portion of the standardized test is again directly related to the mathematics course being taken and the quality of instruction received. If the mathematics course includes topics and problem exercises which are consistent with the problems on the standardized test, then there is a high likelihood that students will test well simply because of the direct relationship between what is being tested and what has been taught.

For example, a typical standardized diagnostic mathematics test given to high school students will include mathematical problems that are found in 7th and 8th grade mathematics textbooks. Thus, in taking the test, the student is confronted first with a format that includes the exercise of problem recognition and recall and then actual solution of the problem. The more closely the mathematics course taken by students is tied to the test, the better students' chances of recall and subsequent solution of the problems will be. In mathematics this is a problem unto itself because teaching to the test becomes a distinct reality, especially given the rudimentary mathematics levels and skills which are tested in standardized diagnostic and minimum competency tests at the secondary levels. And because of the highly structured and objective nature of the mathematics portion of the standardized tests, memorization, as preparation for the test, becomes a useful and pervasive classroom practice, especially if the mathematics course being taught is similar in content to the examination to be taken.

The problem of teaching to the test is reinforced New York City Chapter 1 high schools where a majority of students are enrolled in Fundamentals of Mathematics courses the content of which is inherent in the RCT Mathematics exam. The problem is further exacerbated by the tendency for textbook publishers to gear new textbook development to
educational markets that are perceived to have growth potential. And in recent years the increased usage and emphasis on standardized diagnostic and minimum competency testing has been fertile ground for textbook and workbook development.

From the perspective of standardized diagnostic testing, mathematics is different from other academic subjects, both as it compares in instructional time for student test preparation and the specificity of the accumulated knowledge being tested. In mathematics, it is the subject of mathematics per se that is being tested, whatever the level may be, whereas performance on the verbal is a more general consequence of schooling and home environment and the host of academic subjects including English, Social Studies and Science. The 3R's cliché, that reading, writing and arithmetic are the three basic skills that society upholds and desires students to attain in school--only one of the skills, arithmetic, is tied directly to a school's single subject and course offering.

One of the distinctions that mathematics has from other academic subjects in high school is in the very close relationship that mathematics courses have in and among themselves in content and in the prerequisite and sequential nature of their course structure and organization. The tradition of building a foundation for mastery at progressively higher academic levels in subsequent mathematics courses is the central pedagogical design of high school mathematics. This is true in high school as it is in college where students are required to master the prerequisite mathematics topics before proceeding to the next higher level mathematics course. If this tightly structured sequence of mathematics coursework is viewed over the elementary, secondary and post secondary continuum, then what is obtained is at least a 14 -year long concentration in mathematics that begins with arithmetic and ends with some level of the calculus. Students majoring in engineering or science will necessarily complete 16 to 18 years of mathematics study. In between there is plethora of mathematical content that can either impede or propel a student's academic progress in the school mathematical course sequence.

The serial organization of mathematics in high school is clearly unique when compared to other subjects like English, Social studies and Science. Although high school courses in Biology, Chemistry and Physics are organized as a three-year science sequence, the courses have little bearing on one another in terms of relative content and conceptual framework. The same can be said of the four-year sequences of English and Social Studies where term to term course content is more dependent on students' ability to read and recall information than it is to study any underlying principle or concepts taught in the courses.

Learning mathematics is somewhat like learning music or a foreign language in that it is akin to a process of deciphering code. All three disciplines have distinctive syntax and forms of expression that are different from English. And all three are sequentially structured courses of study in which mastery levels or established stages of achievement are necessary before students can progress to higher performance levels. And constant practice and testing is the rule by which all three dictate acceptable topic and/or course performance levels. Where mathematics parts company from music, foreign language and any other high school course offering, taught as a subject or skill to be learned, is in its abstractness and lack of cultural ties. The major distinction between mathematics and all other high school subjects is the abstract nature of its language and symbolic structure. Mathematics bears no resemblance to any language or cultural norm either present or past, yet it has a universal acceptance that is enjoyed by no other academic discipline.

Mathematics is the universal language of science and commerce, used in the same unaltered form by all countries and modern societies of the world. Primary usage in the economic world includes the statistical and probability functions that are utilized by businesses and governments to calculate budgets and predict project expenditures and returns on investments. In the engineering profession the calculus and linear programming are widely used as tools in design and development and in the efficient manufacture of
mechanical and electronic systems. It is the abstraction of mathematics and its generalizability that allows its use across cultures and in many societal applications.

Throughout modern history the utilization of mathematics in scientific inquiries has established a research doctrine of acceptable "scientific truths" where investigations of natural phenomena and theoretical concepts in all branches of science can be tested for completeness and consistency. Rarely are scientific theories or discoveries accepted as sound in principle unless they have been presented in mathematical terms and subjected to the scrutiny and rigorous mathematical/analysis by members of the scientific community. This mathematical analytic process of creating scientific theory and subsequently testing its applications in the industrial and consumer market has propelled modern society forward by explosive and exponential growth in industrial and technological developments in the last century.

The principles of aerodynamic flight discovered by the German scientist Theodore von Karman several years after the Wright brothers' first flight at Kitty Hawk in 1904 is a splendid example of how mathematical analysis and design turned a little understood and long sought invention into a major transportation industry that has since literally transformed the once separated world into a community of nations. ${ }^{11}$ The aviation and space technology systems that have evolved are now orders of magnitude larger and more complex than when they were first originated at the turn of the century. Not only was a new industry created but an aerodynamic science as well. This has led to important advances and discoveries in the fields of geography, meteorology and astronomy. These scientific advances and the enormous progress of aerospace and aviation technology made in the last 80 years would not have occurred without the precision and analytical power of mathematics that scientists and engineers used as a tool better research and develop the science and technology of flight.

Mathematics as a subject to be learned in New York City public high schools has
remained virtually unchanged for the last 100 years, and the content of specific courses like Geometry still closely follow the classical work of Euclid that was done almost 2,400 years ago. ${ }^{14}$ While there have been infusions of topical treatments in logic, probability and statistics, the traditional high school mathematics course sequence is still centered around algebra, geometry and trigonometry, which as a core of study, is intended to prepare students academically for the higher level mathematics to be encountered in college. As a utility for practical life applications, traditional high school mathematics study has little purpose other than being a prerequisite for higher level college mathematics study. While its almost exclusive academic focus may make mathematics a less compelling course of study than English, Social Studies or Science, its academic nature is its inherent strength in the school day curriculum. This abstractness, while making it more difficult for teachers to find applications for its teaching (which might heighten students' interest) in the end is what sets mathematics apart from other high school subjects. Its abstractness as a subject is noted by Whitehead in his Introduction to Mathematics:

> Thus we write down as the leading characteristic of mathematics that it deals with properties and ideas which are applicable to things just because they are things, and apart from any particular feelings, or emotions, or sensations, in any way connected with them. This is what is meant by calling mathematics an abstract science. ${ }^{13}$

Perhaps mathematics' most important asset as a subject to be learned is the precision and uniformity of its content across the wide spectrum of courses that are taken from elementary school through college. A beginning course in algebra taught in China will cover the same concepts and principles as one that is taught in the United States. And the manner in which a simple linear equation is solved by students will follow essentially the same procedures in an algorithmic format. Regardless of the spoken language or culture of
the student the symbolic expressions and equation solving that make up the study of a first course in algebra are essentially the same. Although the organization and structure of the algebra course may differ from school system to school system, the concepts and principles of the algebra to be learned are in a mathematical form that can be understood and interpreted largely independently of the accompanying written language. For example, the equation $y=m x+b$ will be recognized as an equation of a straight line in texts printed in all languages.

A case in point which highlights the universal nature of mathematics learning despite language differences is the performance of Asian/Pacific American senior high school students on the Scholastic Aptitude Test (SAT). During the 1984 test year, data on college bound seniors indicated that $28 \%$ of the 20,364 Asian Pacific Americans who took the test responded "no" to the question, "English as best language". 14 In comparing the verbal performance of these students to the mathematics part of the SAT, the power and universal quality of mathematics learning is clearly shown. Although the "limited English" Asian Pacific American students scored 155 points below the median score for all students on the verbal portion of the test-- 271 vs. 426--their median score on the mathematics portion of the test was 56 points higher than the median mathematics score for all students-- 527 vs. 471.

The median SAT-Math score for the "limited English" Asian Pacific American seniors was also higher than that of Asian Pacific American students who responded "yes" to the question, "English as best language"--527 vs. 522. In comparison to this group of Asian Pacific American students, the "limited English" students' verbal performance was 159 points lower in median score--271 vs. 430 . This striking imbalance between verbal and mathematics SAT performance for the "limited English" Asian Pacific American student could be attributed to a variety of factors including having received stronger mathematics schooling. However, the evidence is compelling that mathematics learning and
achievement can be attained even with limited English proficiency.
The data make a case both for the universality of mathematics and the academic strength that high mathematics achievement brings to students for educational opportunities beyond high school. Despite the "limited English" Asian Pacific American students' very low scores on the Verbal portion of the SAT, their place in higher education is assured by their sterling performance on the mathematics portion of the SAT. There is a compelling sense of academic discipline about high achievement in mathematics that overrides a limited proficiency in English. This is shown to be the case not only in the higher than average enrollments of Asian Pacific American students in the nation's engineering colleges, but also in their growing faculty and graduate level presence in programs of science and engineering. ${ }^{15}$

Achievement in mathematics is more than just having the skill to solve mathematics problems or conceptualizing an algorithmic process; it is also a way of thinking that epitomizes academic discipline and behavior. Mathematics cannot be learned well unless there is a conscientious effort on the part of the student to concentrate in the classroom and be consistent in the completion of homework assignments. It matters little whether the topic of study is arithmetic or the calculus. Disciplined academic behavior is an essential element for students' high mathematics achievement. But classroom concentration and disciplined study are academic qualities which are held in high stead by faculty in any subject area who are seeking to impart knowledge and understanding to students in a classroom setting. It is just more difficult to quantify these qualities with achievement in subject areas which are non-mathematical.

Mathematics as a subject to be learned involves a process of memorizing symbolic notations and procedures, recognizing numerical and geometric patterns and developing algebraic, geometric and graphical realtionships. All of these elements of learning are abstract and have little cultural tie to the learner. As such, it is much more of a purely
academic and intellectual pursuit than all other academic subjects. And almost all mathematics material that is new to students must be taught by a qualified mathematics teacher and learned in the classroom. Little if any learning will take place at home except by the determined efforts that students themselves make in doing assigned homework. This is due mainly to mathematics' lack of cultural ties and abstractness that make mathematical topics and algorithms not easy to apply to the outside world at the time of learning. Except in instances where parents or siblings are proficient in the mathematics being studied at the time, mathematics achievement for most students is almost exclusively a school-dependent learning experience. This is far different from the formal learning of English, Social Studies and Science where the ability to read in English is of primary importance and where socio-economic status and out of school experiences can have a profound influence on achievement in these subject areas.

Besides being largely inclusive to formal school day learning, mathematics has also been highly regarded by the general public as a subject to be learned in school. In the 11th Annual Gallup Poll of the Public's Attitude Towards Public Schools administered in 1979, mathematics was viewed as "essential by more people than any other school subject." ${ }^{16}$ In response to questions on eleven school subjects that were represented in the poll, $97 \%$ of those surveyed cited mathematics as being essential, followed by English Grammar and Composition--94\%, Civics/Government--88\%, U.S. History--86\% and Science--83\%. The general public is acutely aware of the importance of learning mathematics and perhaps this view has been reinforced in the public eye by the proliferation of standardized testing where mathematics stands up as a single school subject.

For the college viewpoint it could be argued that if students have the capacity to achieve in the study of mathematics, then other subjects will also be learned well, given the opportunity, time and academic support. This thinking is bolstered by 1984 College Board data which showed that $37 \%$ of the Asian Pacific American students surveyed
selected the Physical Sciences and related areas (Computer Science, Systems Analysis, Engineering and Mathematics) as the first choice of their intended area of college study, even though their median Verbal SAT score was 377. This verbal test score was 82 points below the 459 median score for White students, $23.2 \%$ who selected the Physical Sciences and Other Related Areas as their first choice of intended area of college study. However, the low verbal score for the Asian Pacific American students was balanced by a median SAT-Math score which was 13 points above the median for White students--557 vs. 546. The substantially larger enrollment of Asian Pacific students in engineeering colleges is a true indication of the extremely important role that the SAT-Math has in the college admission for this group of students whose proficiency in English may not be as high as the general student population's.

The enrollment and achievement in mathematics coursework presently appears to be distinctive, and, as an academic subject, is seemingly far less dependent on English proficiency than are other high school subjects. As has been shown by the Asian Pacific American students, mathematics achievement can greatly expand and influence students' academic opportunities beyond high school. However, the same forces that prevail for foreign born students (with limited English proficiency) who are proficient in mathematics should be applicable to Black and Hispanic students who, for socio-economic reasons, may find their societal experiences limited and language skills lacking, more so than their White student counterparts.

Because mathematics is abstract and lacks cultural ties, its learning is probably influenced much less by socioeconomic factors and language than are other school courses. And, as a result, enrollment and achievement in the subject of mathematics can be more directly related to both course placement and the quality of instruction that takes place in the classroom. If this is true and school models can be organized and developed to assure proper placement and quality mathematics instruction, it would open up and yield
substantial opportunities for Black and Hispanic students to achieve while still in high school and, at the same time, provide the base to further their education beyond high school.

A schoolwide mathematics impetus could also set in motion, an intervention strategy to create a critical mass of student academic leadership that is currently lacking in public high schools with predominant Black and Hispanic student populations or in high schools where there is a prevalence of low mathematics achieving students. By their increased mathematics achievement, students in Chapter 1 high schools can establish levels of academic performance that can be used as standards of measure in other academic subject areas and in so doing lead to general school improvement. This would be beneficial not only to students but to teachers as well, who, as a consequence of higher student mathematics achievement, can participate more often in the teaching of higher level mathematics and science courses.

### 3.5 The Establishment of a Strong Learning Foundation:

## A Major Key to Effective Growth in Mathematics Learning

The primary goal in the process of student mathematics assessment is to determine students' level of preparation for entry level mathematics courses in high school. This is also a growing issue for colleges where increasing numbers of students arrive in need of mathematics and English remediation upon to enrollment in college. It is essentially the same situation that colleges and high schools are facing. The major difference is that students who elect to attend college with severe deficiencies in their academic background have more coursework to make up and there is a high probability of their not completing their four year course of college study. In high school non-completion is less likely, as the high school, in response to the problem of student underpreparation for high school coursework reduces its academic standards and the problem has, in effect, been
compromised. This "solution" to the problem is widespread, and although not conducive to promoting student achievement or teacher competitiveness, does indicate that alternative strategies for assessing students and building stronger high school mathematics programs are possible and needed.

It follows that if major changes in a high school program can occur which drastically reduce the quality and level of mathematics education, then the opposite can also be true. The usual result of not maintaining or supporting a functioning program or institution that is in place is the consistent deterioration of performance over a period of time. Conversely, with continual maintenance and occasional redesign and restructuring to address changing environmental and socioeconomic conditions, the program or institution can not only become stronger but more versatile in adapting to the new situation. This alternative strategy can invigorate the institution or program, reinforce purpose, and insure longevity through continuous cycles of change and adaptation.

The major problem that needs to be addressed is the rebuilding of traditional high school mathematics programs at Chapter 1 high schools in order to achieve a significant increase in the 9 th grade student population prepared to enroll and achieve in Regents mathematics coursework. This immediately presents organizational and pedagogical challenges-organizational in the sense that the placement of students in mathematics courses must be consistent with their potential to learn if significant progress in student achievement is to be made. The current pool of high achieving students in Chapter 1 schools is so small that traditional student selection strategies are unlikely to make any impact that is measureable. Twenty years of remedial mathematics program model trials have elapsed with a persistent and continuing decline in traditional precollege mathematics enrollment. Low mathematics achievement levels in high schools with predominant Black and Hispanic enrollment provides strong evidence that new and different school organizational approaches to student course selection are desperately required. The current
practice of utilizing standardized diagnostic testing for determining a student's academic future must be reappraised as a school criterion for mathematics course placement. Using standardized test scores as a means of assessing students' mathematics "ability" appears to be frought with serious error and may well be exacerbating the problem rather than offering any genuine or long lasting solutions. The problem is pedagogical because, presently, the extremely small pool of high achieving mathematics students in a given Chapter 1 school has created an instructional vacuum for teachers whose talents go largely underutilized. And except for one or two senior teachers who occasionally have an opportunity to teach higher level Regents mathematics courses, the majority of the school's teachers are relegated to teaching courses that are far below the academic level of their mathematics background and teaching license. In addition, teaching remedial or general mathematics courses to students who enter high school with poor mathematics preparation may be beyond the pedagogical training of teachers who had traditionally taught the more rigorous Regents mathematics courses. As a result, the quality of instruction may be lacking even in the lower level remedial or general mathematics coursework because of the pedagogical mismatch of students and teachers.

Prior to high school entry at the 9th grade, students in New York City public schools should have followed a mathematics curriculum sequence to prepare them for the mathematics coursework they would encounter in high school. Currently, the mathematics preparation's being either along the lines that is prerequisite for the high school Regents mathematics program or for the less demanding Fundamental of Mathematics program is a function of the schools and the quality of the mathematics programs offered in grades K-8. The fact that large numbers of Black and Hispanic students enter Chapter 1 high schools with severe deficiencies in their mathematics background would suggest that the quality of mathematics instruction at the feeder schools is seriously lacking. The arguments about poor student preparation in Chapter 1 middle and junior high schools
range from the often heard socioeconomic factors to the poor quality of teaching. While there may be some value in investigating these arguments and a multitude of others, the fact remains that an overwhelming number of Black and Hispanic students arrive at Chapter 1 high schools each year with extremely weak mathematics foundations. As a result, the major mathematics course offering at the 9th and 10th grade in Chapter 1 high schools has become the Fundamentals of Mathematics program.

The position taken by the Comprehensive Math \& Science Program (CMSP) in the research and development of a model to significantly increase the pool of students who achieve in high school mathematics is one that addresses the poorly prepared entering students at the time of high school entry, rather than their past mathematics learning experiences. The fact that students are poorly prepared for high school mathematics and that this condition has prevailed for over a decade in New York City and elsewhere is sufficient to prove that a serious problem exists.

Traditional research studies that have taken place over the last decade have provided little if any guidance or direction toward solutions other than programs of mathematics remediation. And thus, Chapter 1 high schools remain with the dilemma of trying to adapt their instructional resources and traditional mathematics course structure to an entering student population that is vastly different in mathematics preparation from their student counterparts of a previous generation. The result has been the continuing deterioration of the Regents mathematics program structure and a corresponding increase in Fundamentals of Mathematics programs. And there appears to be no sustained effort at the federal or state level (other than programs of remediation) that would counter this downward trend in Chapter 1 high schools. The problem may grow still more acute when larger proportions of qualified teachers of high school Regents Mathematics courses become eligible to retire and leave the system. It is estimated by the United Federation of Teachers (UFT) that over one half of the nation's teaching force will have to be replaced
"by the early 1990's." 17 And without a commensurate number of new qualified teachers to replace the more experienced teachers that leave, an inadequate school staffing pattern may develop that can seriously impede future efforts to solve the problem.

The difficulties of staffing that loom in the not too distant future brings an added urgency to the problem because student mathematics achievement at any grade level is heavily dependent on the formal presentation given in class by qualified mathematics teachers. The CMSP, in developing curriculum models, has operated on the assumption that students with weak mathematics foundations entering Chapter 1 high school enter weak because of discontinuities in their mathematics learning at the 7th and 8th grades. And these discontinuities in mathematics learning are largely a consequence of a poorly structured curriculum and the shortage of qualified mathematics teachers at Chapter 1 middle and junior high schools that are designated feeders of the local (neighborhood) Chapter 1 high schools.

The persistence of student underpreparedness in mathematics upon high school entry over the last decade would indicate that solving the problem at the middle and junior high school level may indeed be a difficult if not an impossible task. This will be especially difficult if the student underpreparedness found is mainly attributable to severe shortages of qualified mathematics teachers. Increasing the number of qualified mathematics teachers in Chapter 1 middle and junior high schools would require a massive and costly effort that includes intensive mathematics training and teacher certification in mathematics. In addition, a successful solution by a massive teacher training program would aptly require newly trained teachers' making long term commitments to teach in middle and junior high schools located in the city's low income and predominantly Black and Hispanic neighborhoods.

The irony of the current problem is that there does not appear to be a shortage of qualified teachers in the New York City Chapter 1 high schools. In fact what appears to be
the case is a great underutilization of mathematics teachers who, instead of teaching Regents Geometry and 11th Year Mathematics, are, for the most part, teaching Fundamentals of Mathematics or beginning courses in Algebra (which in some cases are stretched out to three and four semesters). If there exists a severe shortage of any kind, it is the apparent one of students' preparedness to enroll in Regents mathematics courses as they enter Chapter 1 high schools. Student underpreparedness but not inability to do mathematics is the primary assumption upon which CMSP model development work was initiated in the Fall of 1978. In the continuing years of CMSP efforts, curriculum-based actions have been developed and test implemented that are counter to the prevailing methods and strategies of student mathematics assessment--in particular, those that are heavily dependent on standardized diagnostic tests for course placement.

In its model development work with students in high school in the last decade, the CMSP has, on numerous occasions, found that significant growth in mathematics learning is possible in a relatively short time once a strong mathematics foundation has been acquired. Positive project experiences like this with participant students have reinforced the premise that all students can learn mathematics very well provided they have the foundation and academic support for the mathematics they are expected to learn in the classroom. The proposition that mathematics can be learned by all is not new and was advanced by Morris Kline, the noted mathematician, in his book, Mathematics: A Cultural Approach. He states convincingly that students:
... can be assured that the subject is within their grasp and that no special gifts or qualities of mind are needed to learn mathematics. ${ }^{18}$

Establishing a fact that all students can learn mathematics very well is intimately tied to seeking solutions to the problem of mathematics underachievement among Black and Hispanic students. A basic goal of the CMSP model project was to research and develop
models that significantly increase the number of Black and Hispanic students at Chapter 1 high schools who enroll and achieve in a first course in Regents Algebra. In addition, the CMSP goal has to be attained in no more than three semesters by students who enter the high school at the 9 th grade. In this way, students participating in CMSP could continue Regents mathematics study beyond algebra for the remainder of their high school years and graduate with at least the minimum of three and half years of the Regents mathematics coursework that is prerequisite for college study in engineering and science. Electing to enroll in the three-year Regents mathematics course sequence, the students would invariably be drawn to enroll in parallel Regents courses in chemistry and physics that further bolster their precollege mathematics and science education.

A basic and logical question that arises upon the pronouncement of the CMSP project goals is, "Where are the 9th grade Regents mathematics students going to be drawn from, given the mathematics underpreparedness that is so prevalent among entering Chapter 1 high school students?" If the CMSP were to use standardized test scores as a basis for student selection, the assumption would be that there is little chance for a practical solution. That was true for remedial high school courses and placement strategies which were shown to have little ability to increase the proportions of students who go on to study Regents algebra. The outcome of these past and current school diagnostic testing efforts may have encouraged a general sentiment that if students had not learned mathematics well enough by the the time they reached the 9th grade, they are simply incapable of learning algebra, geometry and trigonometry in high school. Given the small pool of students who achieve in Regents mathematics coursework in Chapter 1 high schools, this may be a prevailing thought that is reinforced in the schools themselves, thereby adding to the burden of students who would otherwise seek to enroll in Regents mathematics coursework.

The major impediment of larger enrollments in the first course in Regents Algebra is
the student assessment process which, as currently practiced, gives few students in Chapter 1 high schools the opportunity to qualify for enrollment. However, as argued, the unreliability of students' middle and junior high school records and the inaccuracies of standardized diagnostic tests used in high school are limited for assessing mathematics capacity and student academic potential. And because of their inherent diagnostic weaknesses, these assessment strategies may, in fact, be depriving students from realizing the academic mathematics experiences that they require to learn mathematics well.

One solution is to have no selection process and designate that all incoming students be programmed for the Regents mathematics sequence. This approach is in keeping with the CMSP goal of significantly increasing the enrollment of students in Regents algebra, but it may not necessarily assure achievement. The uneven and varied mathematical experiences and backgrounds of entering high school students would be a major deterrent to general achievement in a Regents algebra course. And enrolling all entering 9th year students immediately in Regents algebra or in a stretched out three- or four-semester course has not proven to be an effective way to raise either enrollment or achievement in Chapter 1 high schools.

Suppose, however, that the Regents course enrollment is delayed by a single semester--during which time students are given the opportunity to review and refresh the mathematics foundation coursework that is prerequisite for achievement in algebra. Would this not be a possible solution to the Regents course placement problem? The strategy, if successful, would provide time for students to complete the three year Regents mathematics sequence and still leave one semester in the senior year to engage in college level mathematics coursework before high school graduation. This idea surfaced back in 1978 when the CMSP began experiencing difficulty finding enough eligible high school juniors and seniors at several New York City high schools to participate in CMSP academic enrichment activities--in collaboration with the six New York City based
engineering colleges. At the same time, several of the high school principals participating in the CMSP model enrichment programs expressed interest in the idea of an in-school mathematics achievement model. The idea was also presented to grant making corporations and foundations to fund experimentation on a small scale to test the in-school model concept as a prelude to larger scale model development.

At the time of idea's inception there was little discussion or exchange (philosophic or educational) on the feasibilty of developing the idea into a practical school strategy for mathematics course placement. Difficulties in Regents course enrollment existed in Chapter 1 schools in 1978 as they do currently in 1987, and there was an urgency and a fundamental need to follow any lead or idea that appeared to have promise. The CMSP model offered a glimmer of promise and its implementation was not questioned. However, in retrospect, a dialogue could have taken place at the time which, on the basis of tradition and convention, could have seriously questioned the soundness of the idea. Two basic questions: How is it possible for students to review and refresh eight years of mathematics in one semester, especially if prior schooling may have been inadequate? Secondly, how will students whose standardized diagnostic test scores are two years or below at the time of high school entry be affected; can we expect them also to make up the mathematics work in one semester and then enroll and be successful in a first course in

## Regents Algebra?

As is the case in most research and development efforts, at the time of their inception, work proceeds in spirited fashion on the capital of good hunches. The questions, as posed, were never looked at seriously as major considerations or project deterrents in the early stages of model development. This was partly due to the fact that in the previous four years (1974-1978) of working with high school students there was sufficient anecdotal evidence of the remarkable academic growth that can take place when students are given the strong foundation and personal attention for the mathematics subjects that
they are required to learn for engineering college admission. Another reason for non-inquiry to the posed questions was the general good feeling and enthusiasm of the project participants who, as a collaborative group from high schools, colleges and industry worked together with a single purpose of mind: to get more students to do well in precollege mathematics and to increase their awareness of college study in engineering and science. For the most part the efforts of the participating staff paid off and many high school students went on to study successfully at engineering colleges. These early successes established a solid base which encouraged further investigation of more complex high school matters such as the issue of Regents Mathematics course placement. However encouraging and promising the signs of CMSP project success may have been in 1978, the two fundamental questions concerning student underpreparedness, as cited above, remain.

The first question centers on the notion of eight years of mathematics schooling and its importance in providing students with the foundation to enroll and be successful in a first course in Regents algebra. The first thought in response to this question is: What basic knowledge is required to prepare students properly for a beginning course in algebra? If one examines the mathematical operations that are required for students to manipulate algebraic expressions, it is recognized that basic arithmetic is paramount. In fact, algebra, as a branch of mathematics study, is arithmetic in form except that letters are substituted for numbers in the process of designating unknown quantities and solving equations--which are core concepts of a first course in algebra.

It follows that students' knowledge of arithmetic operations is fundamentally important in learning basic algebraic concepts. And these operations include whole numbers, fractions and decimals, which occur repeatedly in all algebra problems and equation solving in first year algebra coursework. Mathematics teachers are sensitive to the inordinate amount of difficulty that students experience in learning algebra for the first time
if there are fundamental weaknesses in arithmetic.
How important are other mathematics topics such as geometry, graphing or the newly added course topis of statistics and probability for success in algebra? Fortunately, none of these topics stand in isolation in the school curriculum at the middle and junior high school levels and are themselves based in large part on a fundamental knowledge of arithmetic. In the end, a knowledge of place value and the four arithmetic operations plus fractions and decimals are what students need to know well as a base for their continued study of mathematics in high school. If the assumption of the dependency of algebra on arithmetic learning is correct, then the amount of mathematics work that must be reviewed and refreshed as students enter high school is reduced considerably. Secondly, most students, even those who test poorly, bring a considerable amount of mathematical knowledge and experience with them upon high school enrollment at the 9th grade. Almost all students have mastered addition and subtraction of whole numbers and have conceptualized place value. In addition, the symbols of arithmetic operations have been well learned as well as basic geometric forms and some elements of ratios and proportions, as they have been introduced in general science as fundamental measurements of area and volume.

Further, 9th year students regardless of their mathematics achievement in middle and junior high schools, have great facility with the manipulation of money and therefore, in this domain, have a good working knowledge of decimals and fractions and a good sense of place value--i.e., making change with pennies, nickels, dimes, quarters and half dollars, and the differences between a dollar bill and larger denominations. And lastly, all students understand very well the grading practices of tests and courses which are expressed in percentages or in values that reflect pass and fail and partial credit. Taken together, it is an enormous amount of mathematics knowledge that most students have acquired throughout their eight years of schooling prior to high school admission. Not
only is the knowledge abundant but is sufficiently learned by students to be used as a curriculum base upon which to build students foundation for a first course in algebra.

Approaching the problem from the perspective of arithmetic as a base of preparation for algebra and refreshing the previous eight years of mathematics schooling, creating strategies for a solution becomes manageable and not nearly as formidable as might be expected. Students arrive with more than a sufficient amount of knowledge and experience in arithmetic, however for the majority of students in Chapter 1 high schools, this mathematics background is fragmented. And this fragmentation appears to have been caused by the discontinuities in mathematics learning that are encountered in Chapter 1 middle and junior high schools where irregularities in the mathematics that students took may have occured on a term-to-term basis. If the middle and junior high schools do not have a mathematics staff that is fully qualified to teach the mathematics prerequisite to the study of a first course in algebra--and student enrollment and achievement in these courses in Chapter 1 high schools appears to strongly suggest this--then, without intervention, students will find their future learning of mathematics inconsistent and susceptible to failure. Junior high and middle school students may have taken a mathematics course in one term where instruction was solid and convincing and had that followed by one whose content was not matched to the previous term's and where instruction was unstructured and of poor quality. An experience of this sort, which may be typical in Chapter 1 middle and junior high schools, can seriously erode students' previous mathematics learning and impair their academic confidence in the classroom. A single term's experience of poor teaching, however short it might seem along the eight year mathematics continuum, can have a serious impact on the sequential buildup of students' mathematics foundations prior to their high school entry.

### 3.6 Conceptualizing a Model Program To Increase Mathematics Achievement

Formal schooling can be said to be a process of learning how to learn more about particular courses of study. This is especially so with pre-high school mathematics where a significant amount of rote learning takes place and memorization is required because of the abstractness and prerequisite/sequential nature of the subject matter. Learning how to learn more in mathematics means gaining the capacity to grasp and understand new material connected to that which was previously learned. This previously learned material, in order to be a useful base for future mathematics learning, must have been acquired and learned as a meaningful whole body of knowledge. ${ }^{19}$ And this is the basic difficulty that is encountered when students are programmed for courses for which they are insufficiently prepared, especially as it applies to a first course in algebra.

In learning the fundamental concepts for a first course in algebra it is not enough to be able to add and subtract. Students must know whole number arithmetic as a meaningful and complete body of knowledge. This includes using multiplication and division in a systematic form that allows an interplay of all four algorithms in the solution of single variable equations requiring arithmetic manipulations. The whole number knowledge base holds true for fractions and decimals where students must demonstrate a proficiency in all four arithmetic operations.

In the consideration of a model strategy designed to refresh and review K-8 arithmetic, where there is no selection criteria for enrollment in a single semester course, the following applies. Entering student profiles can range from those students on the low end who are able to perform with facility at least whole number arithmetic with two operations (addition and subtraction) to students at the high end who have a fairly complete knowledge of arithmetic and some fundamental learning experiences in algebra (signed numbers and formula evaluations).

However wide the range of heterogeneity of student mathematical backgrounds may
seem, it is well within the boundaries of a refresher course if the major elements of the course are confined to whole numbers, fractions and decimals. These concepts, if structured properly in a curriculum sequence that begins at ground zero, can give all the students an opportunity to refresh or reinforce topics they previously learned. And this intensive review allows the students to assemble their learning of the mathematics prerequisites into a whole body of knowledge that they can thereafter use as a foundation to learn algebra. Even students who score high on standardized diagnostic tests can benefit from this process of renewing their arithmetic foundations as it allows time to reflect on arithmetic concepts or topics which may have been learned only at a rudimentary or cursory level. Keeping the better prepared students' interest levels high in an arithmetic review course is a challenge in curriculum design which requires building in enriching experiences to maintain classroom participation for these students.

For the vast majority of students, including those who test poorly on standardized diagnostic tests, a one-semester foundations course in the arithmetic that is prerequisite to a first course in algebra will circumvent the inaccuracies of standardized testing and current school diagnostic and pre-evaluation strategies which severely limit student enrollment in Regents mathematics courses in Chapter 1 schools. Ironically, for students who have tested two years below grade level on standardized tests, the opportunity to learn thoroughly the mathematics in a prealgebra course will enable them to get the very experience they need to break the continuous cycle of mathematics remediation enrollment. Their participation in a formal prealgebra program of study in the ninth grade consequently removes the stigma of their being designated (once again) "below grade level" and thereby "unable to learn mathematics." And in schools where students are grouped heterogenecusly within and across mathematics classes in schools that offer a one-term arithmetic review course, "math ability" designations are meaningless.

The reduced number of topics in the arithmetic review course and the fact that the
course begins at ground zero helps students who may have tested poorly on standardized diagnostic tests in two ways: 1) the review course gives students an opportunity to begin with a fair amount of knowledge that has been previously acquired--i.e., a student who tests two years below grade level on a standardized diagnostic test given to 8th graders has done enough problems correctly on the test to demonstrate mastery of addition and subtraction, and 2) the course, over the length of the term, becomes an extremely useful tool for assessing students' mathematics strengths and weaknesses. With the course providing timely and continuous feedback, additional help can be given to students at the time it is needed and thus make an immediate impact on their learning and mastery of course material.

The content of the arithmetic review course must be organized to insure the establishment of a strong leaming foundation for all students in the class. This can be done by structuring the course to utilize the additional periods for mathematics remediation which are usually allocated and in place in substantial amounts at Chapter 1 high schools. This additional time will give the class significantly more opportunity to learn thoroughly and review all aspects of arithmetic that come into play as a prerequisite for algebra. Problem solving must permeate all written course material and should be the major classroom practice that determines whether the students are "able to do" the mathematics that is being taught. When the curriculum is structured to begin at ground zero and the initial instruction is tempered to identify students whom immediate tutorials over and above normal class instruction can assist, then the class can progress as a heterogeneous unit and pick up an instructional/learning pace as student mathematics foundations become strengthened through a process of topic review and reinforcement.

There is nothing inherent in the study of mathematics that requires it to be first learned formally in the early elementary grades. Because of its abstractness and lack of cultural ties it can be learned at any age since the mathematical concepts and symbolic notations of
the language have little relation to previously learned material. If anything, learning basic arithmetic concepts and developing computational skills may be more effectively learned when the child is older and reading skills have developed to a point where the student can tackle the more abstract reading of mathematics texts with more reasoning power and confidence. ${ }^{20}$

To a large extent, the growing population of adults who are attending college for the first time after a long absence from secondary school is an example where coursework in mathematics starts at ground zero and continues thereafter in the course sequences required for graduation. ${ }^{21}$ There is little evidence that the adults returning to college are not learning mathematics, so there is no reason why the same strategy of ground zero approach cannot be applied at the 9th grade level and also be effective.

Mathematics can play a very important role in revitalizing students' academic performance in Chapter 1 high schools. It can also have a marked affect on the school itself if the pool of students who takes and completes a three-year sequence of Regents mathematics is significantly increased. As a natural consequence of course programming, increasing the pool of Regents mathematics students will increase student enrollment in Regents science courses as well. And the basis for greatly expanding the student academic leadership of the school will have also been established. This will have a positive impact on the school's senior mathematics and science faculty who will again be teaching the higher level Regents courses consistent with their educational training and prior classroom experience.

Through the implementation of a model program which gives all students the opportunity to enroll in a Regents mathematics program, a multiplying effect may take hold that leads to the general improvement of the school's academic atmosphere. The high regard that mathematics occupies in the public eye and the weight that the subject itself carries on the SAT examination provides a strong rationale for embarking on a model
mathematics assessment program effort that allows all incoming 9th grade students to build a strong mathematics foundation for subsequent coursework in Regents algebra. For students at Chapter 1 schools, the model becomes a unique opportunity to excel in a subject which is abstract and forms the base for conceptualizations in logic and reasoning. Achievement in Regents mathematics thereafter can lead to academic growth in high school and contribute to the building of a strong academic foundation in preparation for the greater level of rigor and depth found in college programs of study.

## CHAPTER 4

## ORGANIZING AND SHAPING THE ELEMENTS FOR A MODEL MATHEMATICS ACHIEVEMENT PROGRAM

### 4.1 Research Applications and Models of Mathematics Education

In 1976 a small booklet of 26 pages entitled Minorities in Engineering: The Chatham Summer Study on Pre-engineering Education was published by the Alfred P. Sloan Foundation. The summer study was initiated by the Alfred P. Sloan Foundation, a philanthropic institution that has played a major role in laying the groundwork and formality for underwriting a host of organizational and programmatic initiatives to increase the representation of minorities in the engineering professions. Two years earlier, in 1974, the Sloan Foundation sponsored the publication of Minorities in Engineering: A Blueprint for Action, a comprehensive report on the things that needed to be done in order to develop and maintain a long term national minority engineering effort. A Blueprint for Action is still widely recognized as the reference work by the community of people involved in the broad based effort at engineering colleges and at precollege levels. ${ }^{1}$ The Chatham Summer Study which followed A Blueprint for Action was a more sharply focused report that examined and analyzed the problems of secondary schools in the inner city:
... upon the view that the minorities were heavily represented in such schools, and that the needs for intervention were greatest in such schools, and that the narrowing of the target would produce both a more coherent summer study and a more coherent plan of action. ${ }^{2}$

The work that led to The Chatham Summer Study was carried out by individuals that were brought together for a two week study session convened in Chatham, Massachusetts
in July of 1975. The twenty six participants were, for the most part, administrators and faculty from engineering colleges and teachers of mathematics and science from secondary schools. The wide background and range and experience represented by the participants allowed a short but intensive study of the secondary school curriculum and structure, factors which motivate students to pursue engineering college study, and how precollege efforts should proceed.

Although the charge of The Chatham Study participants represented consideration of a wide sweep of major elements that characterize the inner city secondary school, the position and recommendations, as set forth by the Steering Committee, became guideposts for future action and still serve as well in the current climate of the inner city secondary school systems. The Steering Committee composed of Robert Jahn, Dean, and William R, Schowalter, Associate Dean, School of Engineering and Applied Science, Princeton University, Marvin Feldman, President, Fashion Institute of Technology, James W. Mayo, Head, Instructional Improvement Implementation Section, National Science Foundation and John G. Truxal, Dean, College of Engineering and Applied Science, State University of New York at Stony Brook best summed up the work of the Chatham Summer Study participants in the overview of the report:
.... it is not possible, at this stage of knowledge and practice, to prescribe any single course of action that will lead in the context of the secondary school to the goals to which participants were committed. If minority students are to be brought in significant numbers into the professions of engineering, it may be necessary to put in place and test a number of new structural arrangements, in part because no one at this moment knows which of those structural changes will work out in the end to be most efficient, and in part because no one knows whether there exists one single optimum structure that will serve all capable students in all circumstances; a similar statement might be made about curriculum, and motivation, and modalities.

The steering committee went on to state that a broad range of recommended actions needs to be:
... undertaken if a serious effort is to be made to bring more disadvantaged minority students into the secondary school years in route to engineering education. If that broad range of actions is carried out, we believe there will emerge one or a few best practices which can thereafter be intensively pursued. In between there must be a process of trial and error: we see no alternative to that process. ${ }^{3}$

The broad range of actions recommended by The Chatham Summer Study cover a variety of school related activities that address relevant areas for improving the education of students enrolled in secondary schools in the inner city. The recommendations included:

- seeking methods other than standardized tests and formal guidance procedures to guide students towards precollege engineering study,
- assuring that students enroll in the algebra coursework no later than the 9th year,
- the creation of project oriented courses which give students opportunities to engage in building models and analyzing specific outcomes,
- the development of real world applications that would complement mathematics and science courses and corresponding teacher training efforts to introduce teachers to engineering techniques and formats of instruction,
- guidelines to insure that special programs do not focus on a select group of students at the expense of providing resources to the larger school population.

Although The Chatham Summer Study lacked the depth or comprehensiveness of $\underline{A}$

Blueprint for Action, its viewpoints and concise recommendations and plans of action served as the base for many of the precollege efforts that are currently in place attempting to increase the pool of minority students who enroll in college programs in engineering and science. But perhaps the steering committee's greatest contribution to program developers seeking to chart new directions for program research and development was the recognition that the problem of revitalizing inner city high schools is indeed complex and that "a process of trial and error", followed by close attention to a "few best practices" may be the only alternative to making significant gains in the future. These insights have had a profound influence and have guided CMSP model research and development efforts since its first inception and test implementation in 1978 at Chelsea High School in New York City.

Thirteen years have passed since The Chatham Summer Study was written and although there has been a proliferation of precollege program efforts which have significantly increased the resources available to participant schools, the "few best practices" that can be generalized and replicated on a large scale have yet to emerge. This is true not only for precollege programs associated with the national minority engineering effort but in the extensive project efforts that have also taken place at the federal, state, district and school levels. The billions of dollars of federal appropriations emanating from both the U. S. Department of Education and the National Science Foundation are convincing evidence that a considerable investment of time, money and effort has been made to increase educational opportunities for Black and Hispanic students and, indeed, all students at the secondary level during the past two decades. That this investment of time, money and effort by a diligent community of researchers and program developers has had a limited return is exemplified by the renewed concern to improve schools, as citted in the publication of A Nation At Risk by the President's Commission on Excellence in Education in 1983.

The publications A Nation at Risk and Educating Americans for the 21st Century, commissioned by the National Science Board, cast a shadow on past research and development efforts to improve mathematics and science education at the secondary school levels. Reflecting on the weakening of mathematics and science instructional programs their recommendations cover a broad range of solutions which are not unlike those offered by The Chatham Summer Study and which have been put into practice by many of the precollege programs operating under the aegis of the national minority engineering effort. The spate of national reports and studies that have emerged since A Nation at Risk have been directed at giving new impetus to school improvement, but at the same time, may have had a stronger but less obvious message--that past efforts of the educational research and development community have had limited value in bringing about noticeable positive change in the nation's schools. From the perspective of inner city high schools with predominant Black and Hispanic student populations, the situation in these schools may be worse than two decades ago--achievement is down, dropout rates are too high and the enrollment and retention of Black and Hispanic students in colleges is markedly lower than the general student population. ${ }^{4}$ Taken as a whole, the condition of education in the system of inner city high schools across the nation is highly unstable.

At the time of The Chatham Summer Study, in 1976, researchers seeking to investigate and test models to improve mathematics education for students at Chapter 1 schools had a variety of directions to pursue. The recommendations and broad plan of actions by The Chatham Summer Study was one, but there was a host of others including Personal Systems of Instruction (PSI), Mastery Learning, Discovery Learning, Time on Task Projects, Peer Tutoring and the models of mathematics remediation with or without the resources of media and computer technology. However, at the time of CMSP research inquiries in 1976, few if any of these models had had any experience at the high school level.

PSI and Mastery Learning can be said to be variations of the same pedagogical theme. ${ }^{5,6,7}$ Both are teaching methodologies that seek to regulate the pace of learning by timely and incremental feedback allowing students to correct their defficiencies in the mastery of specific topics in a given course. PSI has been extensively used in engineering colleges while Mastery Learning programs had their genesis in elementary and middle schools. Both are subject-independent, however their structural and frequent feedback arrangements lend them to the topic-specific and sequential format of high school mathematics courses. The major difference between the two methodologies is that PSI is an individualized program of instruction wherein students themselves control their learning pace, while mastery learning is group oriented and it is the teacher (consistent with class mastery of the specific course topic) that guides the instructional pace.

There were two serious limitations in considering both of these models for possible application in mathematics courses in Chapter 1 high schools. The first and most important is time. The salient quality of PSI and Mastery Learning techniques is being able to provide students sufficient time to attain levels of mastery in a given course. While this time-independent approach (modified somewhat in mastery learning) has been shown to work quite well with students who are moderately prepared for the course they are taking, it has limited value for students who have serious mathematics deficiencies as found among entering 9th grade students in Chapter 1 high schools.

The second limitation, somewhat related to the first, is one of methodology, which makes no allowances for differences in subject curriculum. Both approaches are designed to be subject independent and operate very much like programmed instruction in that course material must be divided into small learning units for which behavioral objectives are established. In arranging course material in this tightly structured format both approaches make the major assumption that different subjects offered in schools are organized, taught and tested in the same way. In essence, it assumes that learning
mathematics is the same as learning English. High school teachers as well as college professors would both have strong arguments against the proposition that teaching mathematics is the same as teaching a course in English literature. The distinctions between the humanities and science as presented by C. P. Snow's Two Cultures stand as the classical argument on why and how these two disciplines of inquiry are set apart. ${ }^{8}$ The mechanistic aspects of both PSI and Mastery learning although useful in reinforcing the factual and objective content of a course can make it difficult for students to grasp "conceptual wholes" and preclude meaningful learning so essential to foundation building.

PSI gets around the subject matter limitation somewhat by confining its objective orientation to the study of mathematics, science and engineering courses--a major reason for its popularity in engineering colleges. PSI also avoids the concepts limitation by its individualized format that gives the major responsibility for learning course content and concepts to the student while the teacher serves to evaluate final course mastery.

Mastery Learning as an approach to teaching a full size class is in a different milieu and must be able to overcome the limitations of time, objectivity and conceptualizations for acceptance in a high school system. It might be argued that its genesis and popularity in elementary schools has been due to the absence of course departmentalization and the fact that a single teacher licensed in common branches is responsible for instruction in all subject areas. This is an ideal situation for implementing Mastery Learning since the solitary teacher can, in fact, cut across subject material as the approach dictates--instructional strategies for all course subjects being the same because only one teacher is involved.

Wide scale implementation of the Mastery Learning techniques in large city public school systems has not taken hold. This may be found to be ultimately due to inadequate schooling and resultant student deficiencies that make the approach difficult to
operationalize. Its philosophy that most students can learn well may be conditional on a set of minimum competencies that students must bring with them when they enter a new course of instruction for the first time. The city of Chicago is an example where this set of minimum competency conditions may have tempered the benefits of Mastery Learning after it was implemented in the city's public school system as the primary form of instruction in 1979. Progress in student achievement was not as high as expected and the approach was severly criticized by an independent school research group that claimed that mastery learning "... is not consistent with research about effective reading instruction and effective teaching." 9 On the basis of this critical evaluation Mastery Learning was discontinued as an institutional strategy in the Chicago public school system in $1985 .{ }^{10}$

While these two approaches have limitations for modeling mathematics courses at Chapter 1 high schools, they do have one element that can serve a useful purpose and is in keeping with the content and sequential arrangement of mathematics courses--and that is the process of frequent and repetitive testing. Unlike courses in English and social studies where instruction and learning are subjective and may be centered on a period of history or in literary works of a given genre, mathematics has specific topics that students must master, in prerequisite order, to build the foundation for continuity of learning at higher levels. This course arrangement in high school mathematics is such that in most instances 5 to 10 major topics can be easily isolated as specific concepts that students must learn in the process of mastering the full term course material. Although not necessarily organized as objectives for study (as in Mastery Learning) the specific topics of a given term could be structured as segments of instruction for which unit examinations could be prepared to test both student and class mastery of course materials. The multiplicity of topics appears to be the case in all of the mathematics courses as they constitute the three-year New York State Regents mathematics sequence. It would also be the case for an arithmetic review course which would precede the beginning course in Regents Algebra and which
would contain the topics of Whole Numbers, Fractions and Decimals. For such a prealgebra course at least 10 separate and sequential topics can be singled out for testing. And if the course is organized to give a heterogeneous class of students an opportunity to attain mastery of all the topics in a given term, then the framework of a potential model for Chapter 1 school would begin to emerge.

Another model of mathematics education that has enjoyed wide acceptance in public school systems across the country where it has been tried is Project SEED. SEED is an acronym for Special Elementary Education for the Disadvantaged, a project that was initiated in 1963 by William F. Jontz in Berkeley, California. ${ }^{11}$ The basic tenet of Project SEED is Discovery Learning where a Socratic approach is employed to encourage student inquiry and the discovery of mathematics concepts and principles through an open dialogue with the class instructor. Project SEED's unique instructional approach is centered on the availability of mathematicians and scientists from universities and private industry who can visit and teach elementary school classes on a daily basis and as a supplement to regular coursework in arithmetic. As a matter of curriculum structure, the supplementary topics taught by SEED mathematicians and scientists do not reinforce students regular arithmetic school program but instead concentrate on topics found in high school and college algebra coursework.

According to its proponents, Project SEED has enjoyed great success and has been tested on a wide scale in many inner city school systems in the U.S. and in South America. Around 15,000 elementary and junior high school students have participated in Project SEED since its inception in 1963. In statewide evaluations in California and Michigan conducted by the California Institute of Technology and the American Institute for Research, students participating in Project SEED had significantly higher scores on tests of computational skills than comparable groups of students. And, in addition, the students were able to demonstrate facility with the abstractions of "conceptually oriented
mathematics. ${ }^{12}$
Given these noted successes in schools which have a predominance of "educationally needy" and "disadvantaged" students, Project SEED is a model that is worthy of replication, especially at locations where professional mathematicians and scientists are available and have the time and willingness to participate in a daily school instructional program. However, the need for a reliable and committed professional staff that is well trained in Discovery Learning appears to be Project SEED's biggest limitation for making greater gains in the number of students the model can serve in a given location. It would be an impractical task in the City of New York where replication of the model on a large scale would exhaust the reasonable supply of professional scientists and mathematicians willing to serve. New York City has not been a site for Project SEED's Discovery Learning instructional approach.

In the years prior to CMSP's 9th grade mathematics model development from 1974 to 1978, the CMSP did utilize a cadre of engineering college professors and engineers from industry to visit schools to conduct engineering career awareness seminars and work in consort with mathematics teachers teaching Precalculus courses in an after school program. Much like Project SEED, this project activity was highly dependent on the time and availability of scientists from the university and industry. At its peak in 1978, more than two dozen professional staff members from six universities and three technology based business firms participated by sharing their experiences both as role models and as experts in their fields. However useful and rewarding students found these meetings with professionals to be, the logistics of scheduling meetings and presentations were difficult to implement given the heavy workloads of the professional visitors and staff resources of the CMSP at the time. As CMSP student enrollments grew and more schools were added, the difficulties in scheduling made the professional visitations unwieldy and difficult to manage with available resources and the program in its original context was curtailed.

In retrospect, although visiting scientists and engineers represent an important strategy for increasing students' career awareness and assist in the reinforcement of school subjects, it is one that is difficult to organize and manage unless there are program resources available for the specific tasks involved. For example, any reasonably sized program that would serve around 200 students with a variety of structured learning activities on a biweekly basis would require at least one full-time staff person to coordinate visiting professor schedules and program implementation. The consideration of a full scale program involving thousands of students in an urban school setting on a daily basis as in the Project SEED approach would accordingly be a logistically complex undertaking. The utilization of such techniques has to be carefully evaluated prior to implementation to insure that large scale costs and logistics don't outweigh the benefits that an extracurricular program brings to students.

The replication of models like Mastery Learning, PSI and Project SEED are time-consuming and complicated tasks given the wide variability that is contained in methodological approaches. Since a curriculum is not available for these models, implementation is subject to wide interpretation by the users of the methodology. In one case the model can be highly successful, and in another, no differences in academic achievement may be attained. This is probably what occurred with Mastery Learning in the Chicago school system, where its implementation was discontinued; whereas in Red Bank, New Jersey the Mastery Learning program since its system-wide inception, has been flourishing with significant and consistent higher student achievement in mathematics at the elementary, middle and secondary school levels. ${ }^{13}$ This difference in participant performance may be due to personality driven effects or may be characteristic of the school environment that, if peculiar in such programs, would make them hard to replicate with any degree of uniformity that is meaningful.

The applications of researchers in mathematics education is even more obtuse and
difficult to apply simply because almost all the research that has been conducted is theoretically based and very narrowly focused. Further, research investigators that seek to examine the pedagogical environment and instructional practices in mathematics classrooms (or in schools) comprise a very small proportion of the research in mathematics education as a whole. Perry E. Lanier, in an article addressing the lack of mathematics classroom inquiry, notes that:

A review of the 580 entries in the tenth annual listing of research on mathematics education by Suydam and Weaver (1980) showed that 25 studies, slightly more than $4 \%$, were conducted to address questions of classroom practice.

In one sense, this last piece of information is encouraging: That there are nearly 600 people studying some aspect of mathematics education in a given year is commendable. Yet one wonders about the apparent imbalance when the need for practical/action research has been noted by scholars, teachers, and study groups for at least five or ten years. Only 25 of the 580 studies were directed toward investigating the quality and nature of life in mathematics classroom. The remainder can be categorized as being primarily concerned with the theoretic. ${ }^{14}$

The small proportion of practical/action research as described by Lanier provides a limited pool of research upon which to draw for developing a framework for mathematics curriculum model development in Chapter 1 high schools. Add to this the fact that much of the action research itself is confined to the practice of classroom observations in classifying and analyzing teacher and student behaviors as determined by an "objective" investigator ${ }^{15}$, and the pool of information that might be gained in building Chapter 1 school curriculum models is reduced further still. Although classroom investigations can provide important questions and insights on the variables that influence student achievement and teacher/student interactions, it does not address the central question of the
wide disparities in mathematics performance between Black/Hispanic students and their White student counterparts in high school. Large scale research investigations focusing on this complex issue are rare in the literature except to acknowledge that a deep and pervasive problem exists and that more research on the issue is required. ${ }^{16}$

The Ford Foundation, cognizant of the need to improve the state of mathematics education for Black and Hispanic students, initiated a grant program entited "Minorities and Mathematics" in 1981 designed specifically to address issues that included student achievement, preparation for college and the establishment of networks for information and model program exchanges. Grants totaling $\$ 1.7$ million were awarded to a "consortium of community colleges, several predominantly Black universities, public school districts in various parts of the country, an Ivy League college, a state university and the American Association for the Advancement of Science (AAAS)." 17 Although a whole range of activities were explored by the Ford grant program, none addressed the specific issue of mathematics achievement of Black and Hispanic students in Chapter 1 high schools in urban settings. In terms of relevant information and model applications that have since ensued from the Ford grant program, very few have application in the creation of broad-based curriculum models for students at Chapter 1 high schools.

Another program that was potentially useful in building a curriculum base for 9th Year Regents Mathematics was the $\$ 700,000$ grant awarded to Lehman College by the Fund for the Improvement of Post Secondary Education (FIPSE). The goal of the grant program was "to improve the quality of students' performance in high school" as it applied to reinforcing the study of algebra through the use of computer technology. ${ }^{18}$ The Lehman College program was of interest to the CMSP during its inauguration in 1981 because one of its participant schools was John F. Kennedy which, at the time, was involved in the test implementation of the first stages of CMSP curriculum model research and development. The Lehman College model involved a variety of classroom strategies to
improve achievement in algebra including computer-assisted instruction, algebra instruction on the computer and instructional games. The program was organized to give students conventional instruction in algebra several times a week and the students would visit a computer lab once or twice a week (with computers provided by Lehman College) where the class could reinforce and apply their algebra learning with courseware specifically designed by the Lehman College grant program.

The length and implementation of the Lehman College program at John F. Kennedy High School coincided with CMSP model curriculum efforts. The irony is that neither proved useful to the school and both programs were discontinued in 1983. For the Lehman College program no differences in student performance in Regents Algebra examinations were attained and the foreclosure of the program was a natural consequence. For the CMSP, however, there were significant student improvements on the Regents Algebra Examination as a result of students' participation in the CMSP model. However, the termination of the CMSP model development program activity was discontinued for more complex reasons that dealt with teacher concerns and a variety of program administrative difficulties that could not be resolved. These issues of school acceptance of the CMSP model are taken up in more detail in Chapter 5 in an examination of the first stages of CMSP model research and development.

Another project emanating from Lehman College that had special appeal and connection to the work of the CMSP was a special school project that operated under the aegis of the the College Discovery and Development Program (CDDP), a consortium of the City University of New York and the New York City Board of Education. The CDDP, which has been in place since 1963, has as its "long range goal the improvement of students' skills to enable them to succeed in college."19 The special project directed by CDDP/Lehman College staff took place at about the same time as the FIPSE sponsored project described above. The specific aims of both projects were also alike with their focus
on the improvement of student achievement in a first course in Regents Algebra. However, the CDDP project was more clearly focused on increasing the number of students who enrolled and achieved in the study of Regents Algebra as it applied to incoming 10th graders at Seward Park High School (a Chapter 1 high school) who were considered marginally prepared as measured by their previous coursework in prealgebra.

In the realm of CMSP model development efforts, the CDDP project also bore a close resemblance in terms of instructional organization and the utilization of resources allocations available at a Chapter 1 high school. Seward Park High School is located on the lower east side of Manhattan in New York City and its student population is predominantly Black and Hispanic with the remainder being largely of Asian origin.

The CDDP organized its mathematics instructional program under the doctrine of Mastery Learning. One teacher was selected and trained in Mastery Learning prior to program implementation in the Fall of 1981. The teacher was then assigned two experimental classes in which a double period was available for course instruction in a traditional two term algebra program. This additional period allowed the arithmetically weak students to review and reinforce basic skills and gave the class more operating time to spend on "correctives" through peer tutoring sessions. It also gave the teacher considerably more time to delve more deeply into conceptual aspects of algebraic expressions and their manipulations.

The results of the Regents examination the following year in June 1982 were substantially higher with CDDP students passing the test at a $76 \%$ rate as compared to $46 \%$ for the 280 students at Seward Park who took the examination at the same time. The program was repeated again a year later and the Regents results were equivalent and plans were being made to offer the program again in the 1983/84 year. No specific data are available on the 1982/83 results, however the program was curtailed in 1985 when the single teacher originally involved in the CDDP project left the school. ${ }^{20}$

The author of the article states at the onset in describing the CDDP project that the "success of the special project is due in greatest measure to Herbert Stender, the teacher selected for this experiment, and the implementation of Mastery Learning techniques in his classes. If it is the case that the teacher had a significant impact on student performance then the experimental project would have limited value and may account for why the program was not continued, i.e., other teachers were not available at the school who could devote as much time and energy as Mr. Stender apparently did in the first stages of experimentation.

The CDDP project is another example of the difficulty encountered in conducting research and development within the environment of a working school. From the program description and the promising results that were obtained by a group of students who exhibited underpreparedness for first course in algebra, one could surmise that the program would be expanded and would be made available to a larger student population of the school. This did not happen and persons who may be interested in further research or in replication the experimental program are left to start anew the process of program development and test-implemenation. Following through with a practical/action research project which is not proceeding according to a designated plan may at times cause frustrations and not be consistent with traditional research considerations. This is unfortunate because in both the CDDP and Lehman College projects there existed potentially rich pedagogical and administrative events and experiences which if examined in depth could have revealed elements and/or combinations of classroom practices that contributed to or deterred student achievement. In addition, further research inquiries could have analyzed the difficulties encountered in program organization and development and the factors that could be attributed to the curtailment of both programs. Obtaining this information in historical form is still possible, however it would lack the immediacy and dynamic qualities that are characteristic of practical research and development programs.

The subject of mathematics education and its consequences for Black and Hispanic students is taking on increasing importance as the proportion of these students become the greater part of the student population in the nation's largest metropolitan areas. It is clear that substantial progress in the search for a solution to the problem of Black and Hispanic students' underachievement in mathematics and other academic subjects must be made before the problem reaches levels of indeterminancy. The persistence of the problem and its multi-dimensional aspects makes it almost impervious to theoretical research considerations. Traditional research studies which examine the problem from controlled and narrowly focused conditions have been shown to be ineffective in shedding any light on how to proceed. If anything, conclusions are reached which are incomplete, but provide theoretical models for the continuance of such research in the hope that the bits and pieces examined will evolve into a rationale worthy of consideration. Two decades of such theoretical research have shown the problem is much more complicated than can be ascertained in the often isolated and self contained environments of the university and research agencies. Practical/action research and development project efforts which are organized directly in the schools experiencing difficulty must be the mainstay in the search for realistic and long lasting solutions. As stated by the steering committee of The Chatham Summer Study, project efforts must necessarily be those that are conducted in real world school environment and utilize a system of trial and error which can ultimately yield to a set of "best practices" which thereafter can be intensively pursued.

The shape of future research and development, as outlined over a decade ago by The Chatham Summer Study, still has relevance today as the nation begins anew the quest for models that will significantly improve mathematics education, not only for Black and Hispanic students but for all students. The need for students with a strong background in mathematics and science to face the technological challenges of a future society is imminent. The CMSP model development effort represents such practical research which
has, since 1978, organized a process of intervention that has demonstrated that major structural changes in mathematics curriculum can take place in school settings given the resources and general support and involvement of the participating school staff. Its history of development, organizational principles, records of accomplishment and project difficulties will be described in greater detail later in this chapter and in Chapters 5 where examples of the expediency and utility of action oriented research in comparison to theoretical approaches.

### 4.2 Precepts and Evolution of a Preliminary Curriculum Model

The major goal of the CMSP research and development effort is to create and test implement models of mathematics instruction and curriculum that significantly increase the pool of students at a given school who enroll and achieve in the first course in Regents Algebra. Implicit in this goal is the plan that students who complete their study of algebra will continue and complete three years of traditional Regents mathematics coursework that include the topics of geometry and trigonometry before graduation. This broad goal and range of project accomplishment as it applies to participant Chapter 1 schools immediately establishes a priori conditions and assumptions (both philosphical and programmatic) that the investigator must seriously consider adopting in formulating a set of principles upon which research and development will be guided and conducted.

The first and foremost of the founding assumptions is that all students can learn mathematics very well given the foundation and academic support for the mathematics that they are expected to learn in the classroom. This is a not a new idea and has been expressed previously in a number of different ways:

- through an ancient Simean proverb that heeds:
"What one fool can do, another can,"21
- Morris Kline's assurances:
" the subject is within students' grasp and that no special gifts or qualities of mind are needed to learn mathematics," 22
- Jerome Bruner's proclamation:
" any subject can be taught effectively in some intellectually honest form to any child at any stage of development," 23
- and Benjamin Bloom's proposition:
" what any person in the world can learn, almost all persons can if provided with appropriate prior and current conditions of learning." ${ }^{24}$

These expressions of faith that all students can learn is especially important concerning the subject of mathematics. Mathematics' abstractness and lack of cultural ties gives special meaning to the statements because it seems certain that learning can start anew at any given time and fairly independently of english proficiency and past school experiences. Bloom's qualifications about appropriate prior and current conditions are noteworthy in that learning, as a human trait, is based on formal schooling where the foundations for learning are, in effect, a continual process--i.e., learning how to learn more by virtue of what has been previously learned and with the support of what is currently being learned. Taken in the context of mathematics learning, Bloom's proposition has become a principle that has guided CMSP model efforts. That all persons can learn is a given, that all have not learned mathematics well is an argument that is evidenced by the recorded disparity of of achievement between White and Black/Hispanic students. The CMSP's assertion that all students can learn mathematics well given the foundation and academic support for the mathematics they are expected to learn in the classroom is further qualification of Bloom's
proposition and points to the curriculum and classroom instruction as the major influences on student achievement.

Bloom's perspective and his belief in all students' ability to learn have led to the methodological approaches of Mastery Learning where all subjects are organized to be taught in essentially the same manner. While this approach has been shown to have value at the elementary school level, its rigidity in classroom format makes its implementation somewhat awkward in institutions which are departmentalized like high schools and colleges. And although Mastery Learning could be viewed as having special value in the teaching of mathematics (because its objective content lends itself to course modularization) its methodological and "programmed" instruction approach may detract from students' gaining the conceptual base that is so important for learning how to learn more mathematics. This is because Mastery Learning programs are highly dependent on structured teaching practices that are regulated by "objectives and subsequent correctives" modules of instruction. While this teaching arrangement may be suitable in instances where the learning of facts and figures is the course objective, it is not a pedagogical strategy that is particularly useful where conceptualization of mathematical principles are paramount.

The quantification of mathematics course material in accordance with Mastery Learning methodologies, if not controlled and arranged sequentially to build on fundamental concepts, can easily overide and minimize students' understanding of the larger and global quality of mathematics course material. Mastery Learning, taken in this larger context, while offering the inspirational input that all students can learn, was deemed somewhat indeterminable as a strategy upon which to build the framework of initial CMSP model efforts during its early stage in 1978.

At the outset, in trying the build a realistic base for researching and developing the problem of student mathematics underpreparedness at Chapter 1 high schools, the CMSP
was driven by two compelling thoughts:

1) that traditional methods of instruction and "educational innovations" which had surfaced in the late 1960's and 1970's were having little impact on student mathematics achievement in Chapter 1 high schools, and
2) that the methods of trial and error of field based curriculum model development activity were not strategies that were in common use by the educational research community and school administrations.

Both of these considerations strongly influenced the initial direction of how best to proceed during the early efforts of shaping the CMSP curriculum model. Given the very low mathematics achievement that Black and Hispanic students were experiencing at the time there was nothing tangible to take hold of from past research and development except the constant reminder that the problem was highly complex and that past efforts offered few clues on which way to proceed. The only real alternative at the time was to employ, as recommended by The Chatham Summer Study, a system of "trial and error" that would hopefully lead to a "few best practices that could be more intensively pursued". From this perspective, the CMSP began investigating a solution through a series of educational experiments that had their beginning in 1978. The key approach taken by CMSP in building a rudimentary curriculum model was essentially that used by engineers in the research and development of new products and systems.

From 1974 to 1978, the CMSP had a well established program of extracurricular activities that was aimed at providing/science enrichment and pre-engineering college orientation experiences to high school juniors and seniors with the proficiency and inclination to consider future study at engineering colleges. At its peak in that period, the CMSP had programs operating in Springfield, Massachusetts, Hartford and Bridgeport, Connecticut, and New York City, involving over 400 students in a given year. Eight
engineering colleges, one technical community college and six industrial concerns working in conjunction with 15 participating high schools established the operating environment for program activity. Initiated under a grant from the National Science Foundation that was awarded jointly to the Schools of Engineering and Education at the University of Massachusetts and subsequently supported by grants from the private sector along with budget support from the New York City Board of Education, the CMSP was organized as a collaborative from its inception. Its special quality was in the blending of institutional resources of colleges and industry to support model program enrichment activity at participant high schools.

The full scale CMSP extracurricular enrichment program in place in 1978 was organized and designed around the concept of triumverate model program networks in which high school(s), an engineering college, and an industrial concern in a given locale worked cooperatively on behalf of the participating students. The engineering college represented the central core of program enrichment activity and the enrichment activity was focused on increasing students' mathematics preparation and awareness of the prerequisites and nature of engineering college study. One or two participant high schools in a given locale were paired with an engineering college for academic and project oriented learning experiences and a locally based business formed the third leg of the model network which functioned to heighten student awareness of careers in engineering. Table 17 shows the model program network arrangement and the listing of collaborative institutions as they were constituted in the Spring of 1978.

The CMSP coordinated the program efforts at each of the model networks from the environs of the School of Education and the School of Engineering at the University of Massachusetts. The program enrichment expariences and the staffing patterns were designed and organized to be uniformly implemented at each of the sites. This parallel arrangement provided structured program guidelines for the program networks that

## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP) <br> HIGH SCHOOL AND COLLEGE PARTICIPANTS IN THE 12TH YEAR AFTER SCHOOL PROGRAM

## HIGH SCHOOLS

Boys \& Girls
Louis Brandeis
Brooklyn Tech
Chelsea
East New York
A.I Prince
(Springfield, Ma.)

## Putnam <br> (Hartord, Ct.)

Samuel Gompers
Washington Irving
John F. Kennedy
New York Printing
George Washington
Bullard Havens
(Bridgeport, Ct .)

Manhattan College
Polytechnic Institute of N.Y.
Pratt Institute
University of Hartford
University of Massachusetts

Springfield Technical Community College
facilitated both program coordination and the building and testing of curriculum materials. The organization of a structured framework upon which curriculum and program enrichment activities were developed and tested played a crucial role in initial CMSP model program developments and has since become a major strategy in the research and development of a subsequent model of mathematics curriculum and instruction.

Uniformity in the implementation of the earlier CMSP program enrichment activity was especially important especially for test purposes, considering that participant high schools were located in three states and had considerably different academic and administrative qualities. The guiding force in the maintenance of program uniformity was the model program curriculum around which academic and experiential activity was centered. The curriculum in this instance was devoted to a design project and a course in precalculus (both conducted after school) that paralleled the mathematics courses that participating students were taking during the regular school day. The model enrichment program was organized and staffed as an after school program activity and was scheduled in four phases over a twelve-month period that began in the second term of the 11th year and ended in the middle of the last term of the senior year. Figure 7 shows, in schematic form, the structure and flow of the CMSP model enrichment program as it had evolved in the Spring of 1978.

The major goal of the CMSP model enrichment program was in keeping with the goal of the national minority engineering effort at the time: to increase the enrollment of minority students in engineering colleges. In this context the CMSP was one of the earliest precollege efforts to formulate academic strategies at the high school level with the focus on bolstering student mathematics proficiency as a major strategy for preparing students for enrollment in engineering colleges. The model enrichment program was designed to instill and build three qualities in students which were identified as the most important factors in the pursuit and successful completion of engineering college study:
COMPREHENSIVE MATH_\& SCIENCE PROGRAM (CMSPI
MODEL ENRICHMENT PROGRAM $-{ }^{-} 1978$

## 気昜

Program Network Team
 SPRING SEMESTER FALL/SPRING SEMESTER $\begin{aligned} & \text { SPRING SEMESTER } \\ & \text { 12TH GRADE }\end{aligned}$
Figure 7

1) mathematics proficiency, 2) general interest in technical matters and 3) perserverance in completing program enrichment activities. Each of these qualities was measurable by the various elements of the CMSP model enrichment program activity. And these qualities formed the basis by which students were assessed and counseled and recommended appropriately for admission to colleges that included the four-year engineering college major, the two-year technology college program as well as two-year pre-engineering programs at technical community colleges.

The major criteria in assessing student preparedness for each of the three levels of post-secondary education was their performance in the CMSP precalculus course. The offering of the precalculus course to students participating in the CMSP model enrichment program was borne out of curriculum work done in collaboration with Doris Stockton, Professor of Mathematics of the Department of Mathematics and Statistics at the University of Massachusetts, in 1978. Professor Stockton was largely responsible for the design of the CMSP precalculus course which was related to her work at the university, with entering students who needed to study calculus but were not prepared for it.

In her design of the CMSP Precalculus course, Professor Stockton was called upon to modularize the curriculum in accordance with PSI and Mastery Learning models, and she also constructed tests that reflected important concepts that students were expected to learn in a given module. There were seven unit tests in all for the precalculus course, and overall course achievement was assessed by cumulative midterm and final examinations. The textbook Precalculus by Salas and Salas* was used as the text for the course.

The structured curriculum design with accompanying module tests and the corresponding textbook formed the core of the precalculus instructional program. Not on!'y was the precalculus course used to bolster students' foundation for future study

[^3]in calculus at college but the course was also organized to assess students on a periodic basis in their attainment of the mathematical topics and concepts. The organization of the course was unusual in that instruction and testing were separated and delegated to high school mathematics teachers and college professors as distinct program responsibilities.

At each of the individual program networks, a professor and a cadre of engineering college students worked in consort with a high school mathematics teacher from a local participant high school. The mathematics teacher was responsible for teaching the course at the school site in accordance with the modularized format developed by Professor Stockton, and the college professor was responsible for administering a bi-weekly module test to CMSP participant high school students at the engineering college site. During the testing and evaluation session which lasted two hours at the engineering college, the professor had on hand four to six engineering college students who assisted in proctoring, grading and providing tutorials for the high school students. The objective of the testing session was to administer the test in the first hour, grade it immediately, and conduct an intensive review which included topic reinforcement by the professor and individual tutorial sessions by the college students. When implemented properly at the engineering college the test and evaluation sessions were a powerful technique for reinforcing the precalculus mathematics taught at the high school site.

The separation of mathematics instruction and testing in a mathematics course was a crucial element in assessing student achievement in the study of the precalculus as a prerequisite for the study of calculus at engineering college. The task of determining student performance was delegated to the college professor through the administration of module tests as well as subsequent class mentoring interaction. This process of assessment through external testing was all the more objective because the module test and its content was not seen by the high school mathematics teacher until after test administration. This external testing scheme has proven itself to be invaluable because it
eliminated the possibilties of teaching to the test and provided a platform whereby participating students could receive objective assessment by another person apart from instruction and also get immediate reinforcement on mathematical areas that were troubling to the student or were insufficiently learned the first time.

The external testing scheme has become one of the notable mainstays of the CMSP model of mathematics and instruction and is one of the three elements that differentiates the model from traditional programs of mathematics instruction (the other two are the zero-based start and the complementary curricula--both described in Chapter 1). The external testing scheme created an initial concern among mathematics teachers who first participated in the CMSP precalculus course offering. As is often the case, difficulties in adaptation accompany model programs when there are departures from the participating teacher's customary instructional experiences. After a period of time and initial adjustments, the mathematics teachers were for the most part comfortable and became used to the external test arrangement and structured their teaching in accordance with the modularized topic arrangement designed by Professor Stockton.

Not withstanding teachers' initial concern about the absence of classroom testing, the process of CMSP external testing is exactly the same as in the administration of New York State Regents Examinations that occur at the end of a full year's course of Regents mathematics instruction. Teachers who administer the Regents examination have no advance knowledge of the content of the examination until the time of the examination. The major difference between the annual Regents test and the CMSP external testing scheme is in the one-time/long term test scheduling of the Regents versus the CMSP frequent/short term module testing. In the CMSP Precalculus model curriculum format, external test administration occurs on a bi-weekly basis. The idea for the CMSP external testing scheme was derived from a description of a visiting examiner's program at Swarthmore College that appeared in a 1978 monograph, "The Testing and Grading of

Students," published by Change Magazine. The article challenged the inseparatibility of instruction and examination and implied that instruction and learning are intensified as "faculty members and students work together to meet and impress a sort of common foe, the visiting examiner. ${ }^{25}$ In practice, the scheme worked very well for the CMSP, not only in heightening student and teacher academic competitiveness, but in the involvement of a third party in the process of student mathematics achievement over the duration of the precalculus course. This team effort was an important by-product of the external testing scheme and provided a framework upon which to design and build a model of mathematics curriculum and instruction.

Most of the curriculum design and test implementation of the precalculus course was done in a period of time (1977-78) when there appeared to be troubling signs that jhigh school uniors and seniors participating in the CMSP did not demonstrate the calibre of mathematics proficiency that students did at their school who had participated in earlier project cycles. This may have been due to the larger number of students who were participating, (the CMSP had doubled program enrollment from 100 to 200 between 1977 and 1978). However there was a general feeling amongst the high school and college faculty that students were arriving less prepared to engage in the design project or in the precalculus course.

As a strategy to remedy the apparent arithmetic weaknesses of students, a mathematics review module was inserted in the project design course (for juniors) to better prepare them for the algebraic manipulations that were required in the design project. Although this remediation helped, it was troubling because the model was deviating from its intended purpose, plus, in offering remediation, the CMSP was becoming involved with instructional issues that were relevant to regular school day mathematics instruction.

It was clear that CMSP efforts to recruit juniors and seniors for its model enrichment program activities were beginning to suffer from the limited pool of students who were
mathematically proficient at the participant high schools. It also became obvious in the early Spring of 1978, that if the CMSP were to continue its efforts in the model enrichment program, student recruitment had to be extremely selective or the CMSP had to work more closely with participant schools at earlier grade levels to create a larger pool of mathematically prepared students. Being more selective in student recruitment was against the philosophy of CMSP model efforts and it was inconsistent with the limited staff resources at the time. Maintaining a program enrollment of 300 students (which was the capacity of the CMSP model enrichment program at the time) would have required the participation of a greater number of high schools and the added logistical burden would have strained the overall management of the project. Since expanding the program to other schools was not a workable solution, serious consideration was given to exploring the possibilities of working with the schools at earlier grade levels.

The CMSP would be entering this new project venture with some organizational and pedagogical experience which had been shown to promote mathematics learning at the precalculus level--in particular, the external testing scheme and the teaching team concept that blended the personnel resources from participant engineering colleges and high schools. It remained to be seen whether these elements could somehow be woven together in the development of a mathematics instructional model that would increase the mathematics achievement at earlier grade levels.

In the Spring of 1978, the concept of a new CMSP mathematics instructional model was discussed with the and Science Chairman of Chelsea High School who indicated he was interested in the concept and would bring it to the attention of the Principal of Chelsea High School. After discussion, both agreed the model was worthy of consideration and if grant funds were available, they would program two classes in the Fall semester of 1978 to test-implement the model.

With this agreement to go ahead, the CMSP Project Director worked with Arsete

Lucchesi, Associate Professor of Mathematics at The Cooper Union School of Engineering and the Chelsea and Science Chairman to develop organizational and curriculum schemes to be used in the classroom. Professor Lucchesi also made arrangements to have Cooper Union students available to visit Chelsea High School on a weekly basis to work with the two classes of students.

Because of the short time available for scheduling two classes for the program, a random sample could not be made of the total entering 9th year student body. Instead, two classes of students who were close to the low end in standardized diagnostic test scores were selected to participate in the fall program activity. These two classes were given pre-evaluation examinations consisting of twenty arithmetic problems and the average score for both classes was six correct.

At the start of this initial model development effort, a prescriptive/diagnostic approach was used along with short modules of instruction that stressed computational arithmetic. The course was taught to the two participating classes by one teacher who had taught precalculus in the CMSP model enrichment program. Mathematics instruction was backed by an after school tutorial program that was coordinated by Professor Lucchesi with five Cooper Union engineering college students who visited Chelsea High School twice a week.

The after school tutorial program was organized to complement school day instruction through the CMSP external testing scheme which identified specific student topic deficiencies, and the tutorial sessions were utilized to correct them before moving onto a succeeding topic of mathematics instruction. Although this model scheme did provide impressive achievement gains for some students, overall class attainment of the specified achievement goal was disappointing and fell far short of what was expected given the additional tutorial resources available from The Cooper Union. The basic problem appeared to be very low class attendance at the after-school tutorial sessions. In its
structural arrangement, the after school tutorial schedule proved to be an inopportune resource for the majority of students. The average attendance rate was about $50 \%$ and this lack of attendance made maintaining effective continuity and structure between school day mathematics instruction and after school tutorial reinforcement improbable.

Because of the lack of student attendance at the after school tutorial sessions and the very poor gains in arithmetic achievement in the school day program during the Fall semester, it was decided to abandon the diagnostic/prescriptive approach and concentrate on building student foundations as if they had little or no prior learning in arithmetic--and this appeared to be the case with the participating students. This foundation building program was offered to the same students in the Spring 1979 term in an effort to have them master Whole Number Arithmetic topics at the very least. In essence, this was a ground zero approach that stipulated that progress in mathematics learning is seriously hampered unless a strong foundation is in place for the mathematics that is to be learned in a given term. The ground zero start became the second major element (after the external testing scheme) in the evolution of the CMSP preliminary model of instruction.

As a result of inadequate mathematics instruction in their elementary and junior high schools, the students arrived at Chelsea High School with a fragmented knowledge base in arithmetic. And this weak arithmetic foundation seemed to deter students' progress in the Fall 1987 term and also minimized their benefiting from the structure of the diagnostic/prescriptive program and the additional tutorial resources available from The Cooper Union. Since all students participating in the program displayed this same weakness in their arithmetic foundation it seemed unwarranted to continue to utilize a diagnostic/prescriptive instructional approach.

In keeping with this model program assessment, which occurred in the late Fall of 1978, the instructional model was revised to include an additional period of mathematics during the regular school day. The two periods of instruction were structured so that in a
given day students received a period of instruction that focused on specific arithmetic topics and in the additional period that followed, the specific topics introduced or covered were reinforced and enlarged upon. Each of the periods was taught by a different teacher, allowing students to learn though different teaching styles and perspectives. This instructional model arrangement gave students substantially more time to learn arithmetic fundamentals and also gave them the opportunity to interact with two teachers in their learning of the same subject material. This involvement of two teachers, in the instruction and reinforcement of mathematics with the same group of students, was to be the third and final element in the evolution of a preliminary model of instruction. By late January, CMSP had completed the development of a framework for a model of curriculum and instruction which incorporated the following three elements:

1) a ground zero start in which students begin their learning of arithmetic with little reference to their past mathematics background and academic record,
2) a structured curriculum that allows two teachers to provide coordinated instruction and reinforcement on the same topics to students block scheduled for each of the two classes.
3) a scheme of frequent external testing that utilizes unit tests on specific mathematics topics that are constructed externally from the school but administered by the classroom teacher.

In addition to the two mathematics periods, instruction and reinforcement, another instructional period was set aside for a science project oriented activity that would give students an opportunity to work together in small project teams and build measurement devices and scale plastic model automobile engines. This period was included to complement mathematics instruction by giving students a structured science project that
would require measurement and simple arithmetic problem solving. The intention was to both heighten student interest through hands-on experiences and develop relationships between the arithmetic being learned and science principles that could be drawn upon from the model building.

This more comprehensive program of mathematics and science instructional activity was organized around a team of three teachers--two mathematics teachers and one science teacher. One of the mathematics teachers taught the instructional course and the second teacher taught the reinforcement course. The third member of the teaching team was responsible for science instruction and the technical aspects of model building in the science project course. In preparation for test implementing the newly constructed CMSP model of curriculum and instruction, a CMSP project team consisting of high school and college staff prepared curriculum materials and module tests to initiate and support Spring 1979 mathematics and science course program activity.

Development of a curriculum in modular structure and corresponding tests was ongoing and scheduled to keep ahead of formal instruction by about a month. In this way curriculum revisions and enhancements could be made on a timely basis by virtue of immediate classroom feedback. This field based approach to model curriculum development is in keeping with the research, development, test and evaluation (R,D,T\&E) techniques used in the engineering development of a new product or system. Since its inception in the Spring of 1978, this process of continual development and testing has served CMSP model curriculum development efforts well. It has proven to be an excellent educational research and development strategy by which to build and test implement curriculum models in school environments where continual revision and modification is the rule rather than the exception.

The CMSP model of mathematics curriculum and instruction in its revised and preliminary form was shown and discussed at length with the Principal of Chelsea High

School. He approved of the new model and plans for implementation and arranged to have the same students programmed for the three periods of mathematics and science instruction for the Spring 1979 term. Student course schedules were organized in a blocked sequence that allowed each of the two classes to be kept together as a group for each of the three instructional periods. Figure 8 illustrates the structure of three block programmed courses over a week time period. In addition to regular school day mathematics and science instruction, arrangements were made by Professor Lucchesi to have a team of five Cooper Union college students visit the mathematics classes on a twice-a-week basis. Their visits were timed to coincide with the mathematics reinforcement period where the class was divided into five groups for tutorial sessions.

The increased instructional and tutorial resources that combined starting at ground zero with a highly structured course format had an immediate and very positive impact on student learning. Chelsea students' performance on the initial tests on Addition and Subtraction of Whole Numbers was well above the achievement levels that were established as goals for each of the module tests-- $50 \%$ of the class scoring $80 \%$ or above-and much higher than the students had demonstrated in the same topics in the fall term. This much heightened student performance on succeeding modules continued throughout the Spring term with $80 \%$ of both classes typically achieving $80 \%$ and better on the module tests in all of the topics in Whole Number Arithmetic.

The students' mathematics performance was so impressive, considering their low level of arithmetic performance a term earlier that it provided the impetus to explore the possibility of testing the potential of the CMSP model program in other schools. Based on initial student achievement at Chelsea High school, the concept of a ground zero start appeared to be a sound and effective alternative to the standardized method of diagnostic testing for assessing student preparedness and mathematics course placement in high school. Since all students started at the same point in mathematics coursework when they
COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)
WEEKLY PROGRAM SCHEDULE FOR THREE PERIOD MATH \& SCIENCE COURSE
UTILZING COLLEGE STUDENTS TO ENHANCE STUDENT LEARNING

Figure 8
entered high school and continued thereafter at the same instructional pace (instruction was regulated by the administration of bi-weekly module tests), reference to students' prior mathematics learning experiences and history was not necessary. Exceptions to this rule were later found, however the ground zero approach appeared to benefit all students even those who might have entered better prepared in the fundamental concepts of arithmetic.

The fine performance and the significant mathematical progress of the participating students made the preliminary CMSP model a potentially useful strategy that could widely impact student mathematics achievement at Chapter 1 high schools. Even though the mathematics achievement was limited to basic arithmetic topics and the model test was confined to a relatively small group of students at one school, all of the staff and teachers participating in this initial project venture realized that there was something special happening that was different. This was mainly due to the mathematics achievement of the participating students at Chelsea High School, who, in everyone's opinion, were performing at a level much higher than would have been expected had they been programmed for the regular school program of remedial mathematics instruction.

Based on these early promising results, plans were explored to further develop and test implement the model in other schools beginning in the Fall of 1979. In mid-Spring of 1979, the model program at Chelsea High School was discussed with Nathan Quiñones, Executive Director of the Division of High Schools of the New York City Board of Education. Mr. Quiñones was very interested in the Chelsea model experience and agreed to visit and spend time at the school to observe the program in action. The visit to Chelsea High School impressed Mr. Quiñones and he agreed that the model was worthy of further testing with larger populations at several other high school sites in addition to Chelsea.

At the same time as planning and program negotiations were being carried out with Mr. Quiñones, consideration was being given to develop and test a parallel program at the

7th and 8th grades of Chapter 1 junior high schools. The thinking here was that if the CMSP model was useful in building student foundations in arithmetic as they entered high school at the 9th grade level, the model would have the same intrinsic value for students as they crossed the academic boundaries between elementary and junior high school. Discussions along these lines involved the late Ronald Edmonds who was then serving as assistant to Chancellor Macchiarola for curriculum and instruction. Mr. Edmonds also expressed an interest in the CMSP model and recommended that Chancellor's funds be made available to help test implement the model in two community school districts located in the East Harlem and Bedford-Stuyvesant sections of New York City. Subsequent meetings with Anthony Alvarado, Superintendent of Community District \#4 in East Harlem and Jerome Harris, Superintendent of Community District \#13 in Bedford-Stuyvesant led to agreements that established a plan and schedule for test implementing the CMSP model at junior high schools in these two community school districts.

With these agreements secured, a comprehensive master plan was developed in the Spring of 1979 that would cover a four-year project test implementation period from the Summer of 1979 to the Summer of 1983. This comprehensive plan was submitted for funding consideration to the consortium of private foundations and corporate grant making institutions which had been supporting the work of the CMSP model enrichment program during the period 1976 to 1978.

The larger scale program effort plan, to be implemented in the Fall of 1979, involved moving the two CMSP staff members (Gilbert Lopez, the Project Director and Virginia Sawyer, the Project Administrative Coordinator) from the University of Massachusetts to the School of Engineering and Applied Science at Columbia University. This was effected partially by a planning grant from the Exxon Corporation and a two-year grant from the Alfred P. Sloan Foundation that was awarded to the School of Engineering and Applied

Science at Columbia University in the Fall of 1978. Both of these grants, supplemented with grants from other private sector institutions, including IBM, International Paper Company Foundation, Union Carbide, Stauffer Chemical, Con Edison and General Electric supported preliminary CMSP model efforts at Chelsea High School and the development of the comprehensive master plan that would guide CMSP model development efforts on a larger scale in the years ahead.

### 4.3 Initial Project Guidelines and Curriculum Planning

The decision to undertake a large scale replication of the preliminary model of instruction that had been only slightly tested with a small number of students at Chelsea High School required that a well established long term plan be developed and approved by a number of institutions that would be collaborating in the project. The grants from the Exxon Corporation and the Alfred P. Sloan Foundation set the process in motion by enabling the organization of a consortium of institutions that would support the CMSP development activity at the high school sites to be selected. Institutions of higher education included all six engineering colleges in New York City: Columbia University, Pratt Institute, City College, Polytechnic University (then named Polytechnic Institute of New York), Manhattan College and The Cooper Union. In addition, arrangements were made with several industrial concerns to provide grant support and sites for student visitations. These included AT\&T, General Electric Foundation, International Paper Company Foundation, IBM, Union Carbide, Stauffer Chemical Company and Con Edison.

In late Fall of 1978 efforts were made to establish a project advisory panel which could provide leadership and counsel to the CMSP as it moved forward with model test implementation. The diverse and collaborative nature of the CMSP model made it imperative that constituencies involved in the broad array of project activity be represented on the advisory panel. Panel member considerations were given to the High School

Division and participant high schools, engineering colleges, the private sector, the military and the United Federation of Teachers (UFT). Involvement of the UFT was especially important because the framework of the model involved the restructuring of traditional programs of mathematics and science and the UFT's advisement would minimize the probability of pursuing courses of action which could prove to be impractical on a school or city wide scale basis.

In the establishment of an advisory panel, discussions were held with persons who were involved in some way with the CMSP model enrichment program including, officials and administrators from the high school, deans of engineering colleges, representatives of private industry and the military. These discussions led to a pool of likely candidates who were then contacted for membership on the advisory panel. The CMSP advisory panel was formed in October of 1978 and its first meeting hosted by the Exxon Corporation was held on December 15, 1978. The membership of the CMSP advisory panel as covened for 1979/80 is shown in Appendix F.

With the advisory panel in place and with the supportive structure from the private industry and engineering colleges in place, a series of meetings with Nathan Quiñones, Executive Director of the Division of High Schools of the New York City Board of Education established budgetary plans whereby costs of the project in the first year would be shared through the combination of budgetary allocations available from the High School Division and grant funds received from the private sector. The High School Division funds would be used to cover instructional expense and program coordination at the high schools and private sector funds would support research and development activity and the creation of curriculum models and materials.

The same budgetary arrangements were made with Ronald Edmonds, Senior Assistant to the Chancellor and with Superintendents Anthony Alvarado of District \#4 and Dr. Jerome Harris of Community School District \#13 where the CMSP instructional model
would be tested at selected junior high schools. The funding at the district levels was a bit more involved than the High School Division's as it required that Chancellor Macchiarola allocate funds directly to the districts to cover a portion of the instructional expense and the districts would appropriate the remaining funds needed.

The establishment of project guidelines was a process that grew out of meetings with principals and supervisors at schools that were being considered as possible sites for project test implementation. These informal discussions led to a series of planning meetings in the first part of 1979 with a group of teachers and CMSP staff where important questions and issues were raised. How many schools should participate? How many students should be enrolled and what would be the selection process? When and how were curriculum materials to be developed? And who would coordinate activity at the individual school sites? All of these were important questions which needed to be addressed and shaped for the project guidelines and became part of the comprehensive master plan submitted to the foundations and industrial concerns which supported CMSP model development efforts in the 1979/80 program year and beyond.

The first issue which had to be resolved was the question of student selection. Feelings among the teachers and CMSP staff at the planning meetings were leaning toward a plan that would use standardized diagnostic tests to differentiate students at the top tenth and bottom tenth percentiles. These students would have been taken out of the "random" pool and would thereafter programmed for mathematics classes consistent with their test scores. This would have left $80 \%$ of the student population upon which to draw and would have still permitted reasonable testing of the model. The arguments against this plan included the fact that the very Chelsea High School students who were then participating in the CMSP model and achieving success would have been excluded. In addition, several members of the planning team expressed serious concern about the legitimacy of random selection unless it included all students i.e., there would always be questions about the
selection process unless students were drawn randomly from the entire entering school population without regard to test scores on standardized tests. prior academic records and attendance. The argument was persuasive and the guideline for selection of students from then on would be a completely random selection from the entire student population entering the 7th grade at participating junior high schools and 9th grade at participating high schools in the Fall of 1979.

The number of schools and the number of students at each school that would participate was a function of the personnel and institutional resources that would be made available to the project in its first year of operation. In the Spring of 1979, the budgetary plans had not yet been approved and the question of participating schools and student populations was, at best, a value judgement based on previous experience in the CMSP model enrichment program. The question rested on the degree of project management that was required to insure that model test implementation would be conducted as uniformly as possible. This was necessary in order to maintain a research and a systems development quality in the model activity taking place at each of the schools.

In the Chelsea High School experience, collaborative project staffing was partially accomplished by the participation of Professor Lucchesi of The Cooper Union and his cadre of engineering college students who visited Chelsea on a twice-a-week basis. If the same arrangement could be made with other New York City based engineering colleges then the process of program coordination would be minimized as a factor in the selection of the number of schools that might participate. In further discussions along these lines with the Deans of the six schools of engineering, the High School Division and Districts \#4 and \#13, it appeared feasible to work with a total of nine schools and with a beginning population of 450 students--approximately 60 students at each school.

The question of program coordination was still a problem--was it better being done centrally as part of a CMSP staff function or locally by supervisory staff at the participant
schools? It was apparent that mathematics and science supervisors who might be counted on to supervise the implementation of the project within the participating schools might find it difficult to find the time to guide CMSP model activity in accordance with established project guidelines. This was also true of the college professors who, through their participation, would be serving in the capacity of special lecturers and coordinators of tutorial sessions at the school. Because the CMSP model of instruction was to be a field based activity that would take place directly in the participant schools and during the regular school day, it was important that program coordination--if it were to take place centrally as part of a CMSP staff function--be assumed by a person with classroom teaching experience in mathematics or science.

This idea of a teacher serving as program coordinator, assigned as part of the central CMSP staff was presented to the Nathan Quiñones for consideration and he agreed that the program would benefit with the assignment of an experienced high school teacher who would serve as central project coordinator. He approved the allocation of one full time position and it was left to the CMSP to find a person with the interest and appropriate background.

Discussions with administrators and recommendations by teachers led to a meeting with Chester Singer, an experienced mathematics teacher at John Jay High School, a Chapter 1 high school. Chester expressed serious interest in the project and was well aware of the severe weaknesses in mathematics of entering 9th year students at John Jay High School. After further discussion and agreement with the principal of John Jay High School, Chester Singer assumed the position of CMSP program coordinator starting in the Fall 1979 term.

During the Spring of 1979, other important work was taking place to organize and prepare for the larger scale model program to commence in the Fall of 1979. The first of these undertakings was the organization of a curriculum planning team that would
commence work on building a framework for the start of Fall model program activity. The CMSP, in its exploratory project efforts at Chelsea High School and at the planning meetings that were held in the early part of 1979 , had identified a group of junior high school and high school mathematics and science teachers who had expressed a desire to be part of the initial planning process.

In an effort to organize a planning team that would serve a dual purpose, teachers were recruited to both serve as members of the project planning team and also to participate in a curriculum writing effort that was scheduled for a three week period in July of 1979. The summer curriculum development effort took place at Media Center facilities provided by the School of Education of the University of Massachusetts at Amherst and expenses for teacher efforts and accommodations were covered by private sector grant funds from the Alfred P. Sloan Foundation and the Exxon Corporation.

### 4.4 The Collaboration of Colleges and Industry

The collaboration of engineering colleges was an important part of the initial CMSP organizational development. Their project participation provided an institutional resource that principals and mathematics and science chairpersons found extremely worthwhile, both in terms of academic support and the natural applications that engineering has to mathematics and science coursework. The engineering college's participation in the CMSP model enrichment program provided many opportunities for teachers and students to work with engineering college professors and students in project oriented activities that reinforced and gave applications for mathematics being studied. The college professors also played a very important role in the establishment of a project staff team at each of the participating schools. The team effort was promoted by the instructional and evaluative roles that both played in the precalculus course. The course collaboration was given cohesion by the organization of the model enrichment program for high school seniors.

In the program, students visited local colleges on a weekly basis for a variety of experiences which included course testing, laboratory projects and seminars in problem solving--as illustrated in Figure 7. The latter gave students the opportunity to explore mathematical problem strategies not often experienced in the high school classroom.

In the organizational development of the new model project the intent was to maintain close ties between participating schools and local engineering colleges by greater involvement of college personnel and students during the regular school day mathematics program. This required that professors recruit and supervise a larger number of college students who would serve as teaching assistants and tutors and also establish a schedule of regular visitations to the participant schools. The CMSP staff worked with the professors at each of the engineering colleges to develop a visitation program that would not impose on professorial time nor further burden the work of the usually active engineering college student carrying a full course load. In particular, college student recruitment for participation in the new project was directed at juniors and seniors who were in good standing with grade point averages that hovered around 3.0.

The number of college students that were assigned to each of the participant schools was based on a five-to-one ratio. This ratio was found to be effective in the CMSP model enrichment program as it enabled individualized instruction in the classroom without undue administration by teacher or professor. The ratio was also in keeping with project budget allocations and appeared to be a reasonable proportion of total project costs (10\%) that could be accommodated as the model project grew larger in the years ahead. As planned, teams of five college students visited the schools on a twice-a-week basis over an eight-week period during a given semester. The time period was scheduled to begin about two weeks after the start of college classes (this was usually about three weeks after the beginning of school in the Fall and Spring) and end before the week of final examinations. During this scheduled visitation period, the professors from each of the engineering
colleges would visit the schools on a once-a-week basis to supervise the college students and also to give short presentations on the value of mathematics and how its study relates to science, engineering and technology. In practice, the engineering colleges were paired with one or two participant high schools, and a team consisting of one college professor with a cadre of engineering college students provided academic support for the mathematics teachers and students.

Industry collaboration in the new project was patterned after their involvement in the CMSP model enrichment program. During the period 1974 to 1978 a wide range of corporate and research organizations had made their facilities and personnel available to support the CMSP in its efforts to heighten student awareness for careers in science and engineering. These institutional resources were invaluable in giving students an opportunity to see the technologies that accrue from the application of mathematics and science principles. Industry involvement also gave students a chance to leave their schools and inner city home communities and travel by chartered buses to the suburbs to visit research and manufacturing facilities. On these visits they met workers, engineers and scientists and experienced the systematic processes of research, development and manufacturing that produced new ideas, products and services.

Industry collaboration in the new model project were confined to industrial visitations, however the trips were organized to permit host engineers to visit the schools a day or two before for the purpose of orienting students on how the visitation day would be scheduled and what they would be expected to see. These pre-trip visits by engineers served a useful purpose in that students became acquainted with a person who gave an informative briefing on the sponsoring company later guided them through the industry visitation. For the most part, the company profile was a new strata of informaiion for students i.e., size of the company in terms of gross revenues, number of plants, employees, wage scales and mix of products and services. This company information was presented to students through
annual reports and media and advertising publications.
Contacts with administrative personnel of several industries and utilities were made to arrange for student visitations during the Fall and Spring terms of the 1979/80 academic year. The institutions included Con Edison, New York Telephone Co., Ford Motor Company, General Motors Corporation, IBM Corporation, International Paper Co., and the Nassau Recycling Corporation. The plan was to make arrangements with each of these companies so that each of the classes participating in the new project would have at least one industry visitation in a given term.

The trip arrangement was coordinated by a full-time staff person of the CMSP who contacted the industries and organized the trips for 1979/80--as many as twenty individual trips were made. The tasks involved in organizing and administrating industry visitations were substantial. They included arranging for company personnel to visit the schools, insuring that teacher supervision was in place on the day of the trip, establishing bus transportation schedules and working with company officials to structure the visitation agenda for student interest and optimal patterns of company staff support.

This level of project work went beyond the limits of the CMSP staff resources at the time of idea inception, however, Charles Bowen of the IBM Corporation suggested that the CMSP apply to IBM's Faculty Loan Program which might consider assigning a full time IBM employee to the CMSP on a year-to-year basis. The IBM Faculty Loan Program is a public service program that IBM developed to bolster the academic staff of Black colleges. Since its inception in 1969 and over the intervening ten-year period over two-hundred IBM scientists, engineers and professional staff have been assigned to the faculty of Black colleges to teach courses in business, mathematics, and computer science. ${ }^{26}$

Up until the time of the CMSP request, the IBM Faculty Loan Program had been limited to college level assignments where the IBM Faculty Loan person's basic
responsibility was teaching college courses. In contrast, the CMSP industry related component was directed at the precollege level and primary program tasks were in the coordination of career and college awareness activities for 9 th and 10 th grade students studying algebra. While the precollege efforts of the CMSP were not consistent with the original guidelines of the IBM Faculty Loan Program at the time, IBM was aware of the limiting effects of the small pool of minority high school students headed for the college pipeline. Because of this they made special considerations to assign a full-time person to the CMSP and Dr. Richard Sha, a computer specialist joined the CMSP staff in the Summer of 1979. This addition to the staff brought the number of the CMSP full-time staff to four including the Project Director, Gil Lopez, the Administrative Coordinator, Virginia Sawyer, the Academic Coordinator, Chester Singer and Dr. Richard Sha, Industry Coordinator (on Faculty Loan from IBM).

With a fully complemented staff, and the industries and colleges organized to collaborate in the new CMSP project, the CMSP was in a position to begin the important process of high school selection and the outlining of a master plan and model developmental outline which could be presented to the advisory panel. School selection at the high school level would be guided by previous experiences of the CMSP model enrichment program. However, work at the junior high school levels would be a completely new experience for the CMSP and the selection of schools would be undertaken by the district superintendents. Junior high school selection was influenced somewhat by location in reference to the collaborating engineering colleges. To the extent possible the district school selected would be located within a one-to-two mile radius of each other to insure that travel time and arrangements for college professors and college students would be logistically feasible.

### 4.5 The Selection of Participant Schools

The steps and sequences of events involved in the organizational and curriculum development of a field based research and development project are by no means linear or structured. The process can be described as non-linear, dynamic, and compounded by the diversity of people who administer schools and school districts. The task is made more challenging by the open environment that characterizes field based work where a multiplicity of project variables prevails and where assurances of collaboration by project constituents is tempered by a host of factors which are beyond the control of the central project staff. This degree of uncertainty is what makes field based systems research and development far different from theoretical educational research practices and also what makes it so compelling and fertile as a strategy to get at the heart of complex systems problems--as is the case with Chapter 1 schools and their low records of student mathematics achievement.

The events that have been heretofore described in the organizational development of the CMSP model mathematics project took place during the period from August 1978 to August 1979. There was a multitude of meetings involving principals, deans of engineering, mathematics and science department heads, New York City Board of Education officials, district superintendents and members of their central staff, and foundation and corporate sponsors. The meetings themselves were interrelated and involved obtaining agreements from several different sources before proceeding to the next step in the organizational development. The sequence of events was often convoluted, leading to open ended questions that needed to be resolved by further planning and field research and by repeated meetings with project constituents. For example, agreements had to be obtained from the engineering college deans before high school administrators could be assured of this source of academic support, the foundations and corporate sponsors had to know of schools' willingness to participate in the CMSP field based
activity, and matching budget contributions had to be in place from the New York City Board of Education before private sector grants could be awarded to support project activity. By the same token, the same degree of financial support had to be shown as being forthcoming from the private sector before the New York City Board of Education would authorize budget appropriations to cover instructional expense at participating schools. From the perspective of project cost sharing, private sector grant funds would be used to cover research, development and management activity (including faculty and college student participation) and the New York City Board of Education would cover those costs associated with school instruction and traditional classroom materials.

The degree of success in organizing and starting a field based project rests on the level of interest and cooperation that can be obtained from officials and school administrators where project activity will take place. On the fundamental issue of student mathematics achievement, it could be assumed that the educational community, both at the secondary school level and higher education, would express interest and be desirous of significant project gains that would result in the increase of the student pool taking higher level mathematics courses. However, addressing the questions of costs, the restructuring of the school's mathematics programs, student selection, teacher and class assignments and the adherence to a schedule of project activity that is, for the most part managed by an "outside agency" requires that participating institutions make a major commitment to support the project and become actively involved over the long term.

It was to be expected that school participation and long term involvement would continue until such time that project efforts cease to show any significant differences in student achievement or until competing priorities at the school outweighed the benefits of the project regardless of its success in promoting student mathematics achievement. In practical terms, public school and district officials' making long term project commitments rests on the premise that year-to-year budget allocations will be sufficiently stable to afford
reasonable support of school instructional activity connected with the CMSP model. It was with this level of understanding that the process of school selection began in the early Spring of 1979.

In March of 1979, a meeting with the principals of the high schools which were then particpating in the CMSP model enrichment program was held at The Cooper Union School of Engineering. The agenda was centered on the new CMSP model project, its goals and consideration by the principals to have their schools participate. It was made clear that participation in the project required adherence and commitment to guidelines and course formats that were, for the most part, a departure from traditional mathematics programs.

First and foremost of the required commitments was the random selection of two classes of students (sixty), who would be entering their school's 9 th grade. The second was the block programming of these students for two periods of mathematics with both class periods taught by a different teacher. Agreeing to this double period of mathematics instruction meant that the school would have to make an allocation of four-tenths of a teaching position in order to cover the additional course--normally not offered except to students who entered high school testing two or more years below grade level. Since the student population to be selected for CMSP model participation was to be random--covering the full range of standardized test scores--the cost of the second period could only be covered partially by Federal or State remedial education funds. The remaining costs of offering the courses had to be borne by the school through tax levy allocations.

While a budget commitment of four-tenths of a teaching position to cover the expense of twu additional mathematics classes had only small bearing on the total school costs, it could rise to a significant amount if the school decided to implement the model program on a full scale basis. For example, offering a double period of mathematics over the academic
year for the entire entering class would cost the school a full-time teaching position for each five classes offered. The schools considering the model had entering 9th year populations that ranged from 250 to 600 students and therefore full scale implementation of the model could cost the school up to four full-time teaching positions (assuming thirty students per class). The cost of the additional course could either be supported by lengthening the students' school day--in which case, the school would receive additional budget allocation from the central school district--or by offering the additional course in lieu of another subject (the subject not taken would be deferred). The latter was the strategy that the CMSP preferred because it decreased the number of different subjects taken by the student participants and allowed time for after-school tutorials.

The principals had difficulty with the peculiarities of the proposed CMSP model project, especially the requirement for the random selection of students and the budgeting constraints that would appear with the offering of the second mathematics course on a larger scale. Cost was a particularly worrisome concern for schools which had an array of different programs to offer entering high school students. The choice in the end came down to a matter of school priority in course offerings. Was student enrollment and achievement in Regents mathematics in the 9th grade more important than a technical course offering or foreign language or other school subjects taken at the 9th grade? This is the question that the high school principals were being asked to consider as they pondered on whether to participate in the CMSP model project.

Although there was general agreement amongst the principals that the model project was interesting and that the early results from Chelsea High School were a positive indication that the model could affect achievement, they viewed participation in the new school day project with reservation. Towards the end of the lively and often raucous two hour meeting, one of the principals commented that the CMSP model represented an effort that was "different" and worthy of serious consideration. Earlier in the meeting the

Principal of Chelsea High School, had interjected that the 9th grade students currently enjoying success in the rudimentary CMSP model at Chelsea would probably be failing in the school's regular program of mathematics remediation. Somehow the ground zero start and the academic support that Chelsea was receiving from the CMSP and The Cooper Union School of Engineering in the way of professorial time and college student assistance made the project worthy of consideration.

The two principals' comments had an influencing affect, and further discussions at the meeting led to agreements by all five of the principals present to participate in the project. The schools included Washingtom Irving, John F. Kennedy, Chelsea, Benjamin Franklin and East New York. Each of the schools had participated previously in the CMSP model enrichment program and all had collaborated with local engineering colleges in the implementation of CMSP model enrichment project activity. Chelsea and Washington Irving had worked closely with The Cooper Union School of Enginnering, East New York with Polytechnic Institute of New York (now Polytechnic University), John F. Kennedy with Manhattan College's School of Engineering, and Benjamin Franklin with Columbia University.

With the five high schools selected as sites for CMSP model project work in the Fall of 1979, attention turned to Community School Districts \#4 and \#13 for the selection of junior high schools that would participate. Meetings were held with District \#4 Superintendent Anthony Alvarado and he recommended that the program be implemented at the Rafael Cordero Bilingual School and in Intermediate School (IS) 117. Later meetings with principals of both schools at the school sites allowed the CMSP staff to discuss the details of the project and the commitment that had to be made in order to implement the model project as planned and designed. The points made were the same as those with high school principals except that the pace of mathematics instruction and the content covered at the junior high schools would be considerably less than in the high
schools.
The Director of the Rafael Cordero Bilingual School saw the CMSP project as a welcome addition to the school and readily agreed to participate, indicating further that the school would adapt its schedule and mathematics program consistent with CMSP model guidelines. The Principal of IS 117 was less receptive and did not view the model with the same degree of interest as at Rafael Cordero. The CMSP in its collaboration and project experiences with NYC schools has found that in beginning programs a very strong endorsement of the program and its aims is required by the principal if the program is to have any chance of overcoming the inertia in getting started. There are too many obstacles at the beginning and too many things that can go wrong, which unless acted upon by the highest authority of the school, can damage a program beyond recovery in its earliest and potentially most fertile stages of growth. The lack of enthuisasm for the project on the part of the principal was a clear sign that IS 117 was not a wise choice for participation for the CMSP model project. This was brought to the attention of Mr. Alvarado and it was decided that the Rafael Cordero Bilingual School would be the only school in District \#4 that would participate in the CMSP model project efforts.

A similar process of junior high school selection took place at Community School District \#13 except that the meetings with the two principals of the schools that were selected, IS 258 and IS 117, were held jointly in the office of District Superintendent, Dr. Jerome Harris. The CMSP model project was described to the Principals of IS 258 and IS 117 and they agreed that the project was worthy of consideration and that their schools would participate. Before the meeting was over Dr. Harris reinforced the notion that the schools implement the project in accordance with CMSP model guidelines and conditions. The point made by Dr. Harris was accepted by the two principals, and before the meeting ended, plans were made to visit both schools to meet and discuss the program with the school's mathematics and science staff.

With the meetings of the high school principals and the two district superintendents, a total of five high schools and three junior high schools had been selected to participate in the project in the Fall term. However there remained two schools that needed to be seriously considered for CMSP model project participation. The two schools, John Jay High School and Brooklyn Technical High School represented unusual situations that could provide answers to two important questions: 1) Could the CMSP model project survive in a school where there was no visible support from the school's mathematics chairperson? and, 2) Would the CMSP model with its structured curriculum design have a positive impact on students who had the background and proficiency to enroll in a first course in algebra?

The first question was directed at John Jay High School, where Chester Singer the new CMSP academic coordinator, had taught mathematics as a classroom teacher. Chester Singer's teaching experience at the school and a good relationship with the Principal led to discussions that made John Jay High School a likely participant in the CMSP model project. However a serious obstacle for John Jay's participation arose when the mathematics and science chairpersons felt that their busy schedules would not permit them to supervise the program properly. Hence they could not actively participate in the CMSP project test implementation in the 1979 Fall term.

The CMSP has always adhered to the doctrine that direct involvement by the mathematics chairperson was mandatory in implementing a mathematics based program within a school setting. The pronouncements made by the John Jay mathematics and science chairpersons disagreed with this doctrine and a decision was made not to enlist John Jay for CMSP model project participation. Although this decision seemed to be final, the principal of the high school felt that despite the lack of involvement by the mathematics and science chairpersons, the CMSP should still consider John Jay as a viable participant school for the project. To this end he offered to provide some degree of supervision of

CMSP activity at the school by recruiting a mathematics teacher who was then serving as assistant to the Mathematics Chairperson.

Although the situation at John Jay High School was contrary to the general operating principles of the CMSP, it did present a challenge for the project and would further test the premise that support and direct involvement by the mathematics chairperson is fundamental for project success and acceptance by the teaching staff. Given the strong desire by the principal for John Jay participation, plus Chester Singer's experience at the school and the interest displayed by the mathematics teacher assigned to supervise CMSP project activity, it was decided to add John Jay to the list of schools that would participate in the CMSP model project in the 1979 Fall term.

The second question concerned Brooklyn Technical High School where the CMSP model enrichment program had been in place since the Spring of 1978. Brooklyn Technical High School is one of three specialized high schools in New York City (the other two are Stuyvesant High Schools and Bronx High School of Science). Students who gain admission to the three specialized high schools are amongst the most academically prepared in the New York City public school system. Despite the academic preparedness of its students, Brooklyn Technical High School has long suffered from an unusually high rate of student departures- $27 \%$ of the average daily register as compared to departure rates of $8 \%$ at Stuyvesant and $12 \%$ at the Bronx High School of Science. ${ }^{27}$

In 1979/80 student enrollment at Brooklyn Technical High School stood at 5,173-much larger than the 2,646 student enrollment at Stuyvesant and the 3,181 student enrollment at Bronx Science. In addition, the economic status of the students at Brooklyn Technical High School was considerably lower than the Stuyvesant and Bronx Science students'. Almost $67 \%$ of Brooklyn Tech students were eligible for free lunch, while at Bronx Science and Stuyvesant the figures were $25 \%$ and $22 \%$ respectively.

The ethnic compostion of the three schools is also quite different and has changed
considerably in the period from 1971/72 to 1979/80. Table 18 indicates the marked shifts in the ethnic composition of the student body that has taken place in the eight year period.

## Student Ethnic Compostion at Stuyvesant. Bronx Science and Brooklyn Technical High Schools for the Academic Years 1971/72 and 1979/80

|  | Year | Black | Hispanic | Asian | White |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Stuyvesant | $1971 / 72$ | $10.8 \%$ | $3.7 \%$ | $7.9 \%$ | $77.6 \%$ |
|  | $1979 / 80$ | $8.7 \%$ | $2.9 \%$ | $24.4 \%$ | $64 \%$ |
| Bronx Science | $1971 / 72$ | $11.2 \%$ | $5.1 \%$ | $4.9 \%$ | $78.8 \%$ |
|  | $1979 / 80$ | $15.5 \%$ | $7.9 \%$ | $12.2 \%$ | $64.3 \%$ |
|  |  |  |  |  |  |
| Brooklyn Tech | $1971 / 72$ | $16.8 \%$ | $7.8 \%$ | $8.0 \%$ | $67.8 \%$ |
|  | $1979 / 80$ | $48.7 \%$ | $11 . \%$ | $16.6 \%$ | $23.7 \%$ |

TABLE 18

The shifts in ethnic population of the three schools reflect the changing housing patterns of New York City in the decade of the 1970's. As a consequence of the changing populations, the student body of schools, even when selective, will tend to reflect the surrounding neighborhood. This "neighborhood effect" appears to have influenced the student ethnic composition at Brooklyn Technical High School, which is located in the predominantly Black neighborhood of Fort Greene in Brooklyn, New York and that of Stuyvesant High School which is located on the lower east side of Manhattan, close to Chinatown.

The larger student enrollment and its changed ethnic composition has made it difficuit for Brooklyn Tech to enroll students with the same academic preparedness as the students who enter Stuyvesant and Bronx Science. The limited pool of Black and Hispanic
students in the Chapter 1 junior high school who can qualify for entrance to the specialized high schools serves as an admissions deterrent for Brooklyn Tech in particular. Because of this situation, the cutoff scores on the admissions test given at Brooklyn Technical High school have, over the years, become considerably lower than at Stuyvesant and the Bronx High School of Science. The lowering of cutoff scores by Brooklyn Technical High School occurred during the years after a court order required that the three specialized schools increase their enrollment of "disadvantaged" students. As shown in Table 18, the Black and Hispanic student population at Brooklyn Tech increased by greater than two-to-one--from $24.6 \%$ in $1971 / 72$ to $59.7 \%$ in 1979/80. At Stuyvesant during the same period there was actually a considerable decrease in the Black and Hispanic student population from $14.5 \%$ in $1971 / 72$ to $11.6 \%$ in 1979/80.

The situation at Brooklyn Technical High School is not unlike that being experienced by colleges around the country. The decline in preparedness of entering freshmen for traditional college study has induced colleges to reduce admission standards in order to maintain the stable enrollments that impact on institutional resources and faculty utilization. In response to the lower student preparedness, however, the colleges have implemented a variety of programs designed for entering freshmen including remedial programs in english and mathematics and reduced course loads during the freshman year. The basic remedial and reduced course load strategies designed to gird students' academic foundations for college study have no equivalency at Brooklyn Technical High School or at Stuyvesant and Bronx Science--all which under the court order accept students who test below the academic standard for regular admission. Instead, what is in place is a six week summer program prior to school entry that provides english and mathematics remediation for the coursework that will follow in high school. However, once enrolled at the school, these less prepared students must carry the same course load as students who have been admitted under the regular admissions standards.

It is clear why the student departure rate is so much higher at Brooklyn Tech than it is at Stuyvesant and Bronx Science. The limited pool of high achieving Black and Hispanic students from feeder junior high schools is at odds with the larger entering student population that Brooklyn Tech required to maintain yearly student enrollment stability. Add to this the lack of an ongoing academic year program of academic support and remediation that could help students with marginal academic backgrounds, and conditions exist that can lead to academic failure and drop out as has been the case with Brooklyn Technical High School.

For the CMSP, Brooklyn Technical High School represented an opportunity to implement an academic support program that could bolster students' mathematics foundations and provide the additional time to have them achieve in the study of a first course in Regents Algebra. The CMSP model was be tested more rigorously at Brooklyn Technical High School precisely because the entering students were better prepared academically than students entering Chapter 1 schools. In addition, average daily attendance at the school was high and all entering 9th year students were required to enroll in the first course in Regents Algebra (except for students who have passed the course in the 8 th grade). This more stable population of 9 th grade students who entered much better prepared than students at the other participant schools provided the setting to test the question whether the CMSP model could benefit high achieving students as well as those who arrived at Chapter 1 high schools with inadequate mathematics background.

The CMSP model project implemented at Brooklyn Tech was the same as in the other six participant schools, however the starting point in the CMSP three semester model curriculum was advanced by one term--students at Brooklyn Tech did not take the one-semester course in prealgebra. This was essential because all Brooklyn Tech freshmen are required to take Regents algebra upon school entry in the Fall term. There are no other mathematics courses offered to Brooklyn Tech students at the 9th year. Thus,
in the scheduling of the CMSP model for the seven particpating high schools, Brooklyn Tech would test implement the curriculum model over a two-semester period while the other six high schools would test implement the model over a three-semester period. Appendix A is a profile for each of the selected schools as they are portrayed in School Profiles, 79/80.

With the selection of schools to participate in CMSP project activity in the Fall of 1979, the latter part of the Spring and the Summer of 1979 were spent on organization, curriculum planning and staff development. In meetings with the individual principals of the selected schools agreements were worked out to have:

1) the selection of the school's mathematics or science chairperson to serve as school project supervisor,
2) the selection of two mathematics teachers and one science teacher who would teach CMSP designed mathematics and science courses and also be willing to participate and make a commitment to engage in after-school meetings with students and CMSP staff,
3) the random selection and heterogeneous class grouping of entering 7th and 9th grade students who would study mathematics and science utilizing the CMSP model, and
4) the participant students programmed for an additional period of mathematics in lieu of a non-technical subject, and scheduling the mathematics and science periods scheduled in a 1-3-5 sequence as shown in Figure 8 on Page 168.

The above four items represent critical components of the CMSP model and agreement was needed on all four items from the principals and mathematics and science chairpersons of each of the selected schools as a condition for participating in CMSP model test implemenation in the Fall of 1979. All of the principals and mathematics and science
chairpersons agreed, and meeting schedules were organized with the chairpersons to meet with the teachers at each school who would participate in the CMSP model project. The selection of teachers was made by the chairpersons.

During the latter part of the Spring semester of 1979 , the CMSP staff had several opportunities to meet with all of the mathematics and science teachers who were selected and agreed to serve as the CMSP instructional team at their respective schools. In all, thirty-three mathematics and science (two mathematics teachers and one science teacher from each of the three junior high schools and six high schools; the tenth school, Brooklyn Technical High School had four mathematics teachers and two science teachers) would be participating in the model project in the Fall 1979 term. As part of the support team for each of the school, an engineering college professor was assigned to visit the participant schools on a weekly basis with a cadre of engineering college students. The project activity at each school would be coordinated by the mathematics or science chairperson at the school who would also serve as a member of a committee of school project supervisors which would meet on a weekly basis with the central CMSP staff to provide feedback and orchestrate the progress of the project. Table 19 lists the number of teachers, supervisors and college professors at each of the institutions that participated in the project in the Fall of 1979.

The meetings with the teachers were arranged to outline the CMSP model curriculum, the developmental aspects of the project and its goals and premises. Most of the teachers were enthusiastic about the project and all felt that the goals of the project were within reach given the additional mathematics instructional time in class and the academic support that would be forthcoming from the engineering colleges and the CMSP staff. Staff development strategies included reviewing the CMSP prealgebra curriculum (at Brooklyn Tech curriulum review focused on the CMSP algebra curriculum), outlining the project experiences at the Chelsea High School, describing the unusual elements of the CMSP

## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)

## Listing of the Number of Teachers. Supervisors \& College Faculty Particinating in the First Cycle of the Model Project Activity (1979/80

## HIGH SCHOOLS

Brooklyn Tech
Supervisors -- 2
Math Teachers -- 4
Science Teachers -- 2
Chelsea
Supervisor -- 1
Math Teachers -- 2
Science Teachers --1
East New York
Supervisor -- 1
Math Teachers -- 2
Science Teachers -- 1
Benjamin Franklin
Supervisor -- 1
Math Teachers -- 2
Science Teachers -- 1
Washington Irving
Supervisor -- 1
Math Teachers -- 2
Science Teachers -- 1
John Jay
Math Teachers -- 2
Science Teachers -- 1
John F. Kennedy
Supervisor -- 1
Math Teachers -- 2
Science Teachers --1

## JUNIOR HIGH SCHOOLS

Rafael Cordero
Supervisor -- 1
Math Teachers -- 2
Science Teachers -- 1
IS 117
Supervisor -- 1
Math Teachers -- 2
Science Teachers -- 1
IS 258
Supervisor --1
Math Teachers --2
Science Teachers --1

## COLLEGES

Columbia University
William T. Sanders
Professor, Mech. Engineering
Cooper Union
Arsette Lucchesi
Professor, Mathematics
Manhattan College
Br. Peter Drake
Professor, Elect. Engineering
Medgar Evers
Joshua Berenbaum
Professor, Mathematics
Polytechnic Institute of N.Y
Frank Lupo
Professor, Elect. Engineering
Pratt Institute
Esmet Kamil
Professor, Mech. Engineering
model including random selection, uniform instructional pace and external testing, demonstrating how the the two mathematics teachers would work together as a teaching team to instill and reinforce student learning, and showing how the science course would be used to provide project oriented applications to specific mathematics topic leamed in the mathematics courses.

The meetings with the teachers provided the CMSP staff not only with an opportunity to review and establish instructional and curriculum strategies, they also provided a forum that aided model development. Many of the teachers had years of experience in the classroom and suggestions were made that were very helpful in strenghtening the design and organization of the curriculum model. In particular, a number of teachers were recruited to develop specific modules in mathematics, and other teachers, along with school project supervisors, participated in the continuing project planning and the particulars of getting the project started at their respective schools including the procedures for the random selection of students, the scheduling classes in the $1-3-5$ period arrangement and the distribution of curriculum materials.

## CHAPTER 5

## THE SYSTEMS AND FIELD BASED DEVELOPMENT AND TEST IMPLEMENTATION OF A MODEL MATHEMATICS ACHIEVEMENT PROGRAM

### 5.1 Model Assessment Considerations

Program "success" is an often used term to describe the positive status or outcome of educational projects. In the best case, success of a project or the validity of an associative research argument is based on "objective" statistical comparisons of student achievement using psychometric instrumentation. Hopefully the compiled data fall into neat correlated patterns from which strong positive inferences can be drawn. At the other extreme, success can also be judged by the enthusiasm and good feelings displayed by the project participants (including project directors and researchers) in a process generally classified as being "subjective", however measured, i.e., by survey questionnaires, personal interviews, etc.

In either case, the "evaluative" strategies fall far short in giving a full or reasonably accurate account as to whether an educational project or process of research and development is sound or is making useful progress toward stated goals and intentions. The CMSP, in its field based efforts to research and develop a model curriculum program aimed at significantly increasing student achievement in the study of a first course in algebra, required a broad based and reasonably accurate means upon which its progress and effectiveness could be assessed. While higher mathematics test scores represented a "necessary but insufficient condition" for determining CMSP model effectiveness for increasing student achievement, there were other factors that became important, particularly, because the CMSP was a field based project effort.

The physical and academic environment of a school, in large part, characterizes its institutional culture. The school's academic course offerings, the experience and background of its faculty, the general academic profiles and socioeconomic status of the student body, the leadership qualities of the school's management team, the location of the school and the condition of its physical plant are all factors that interdependently create the school's institutional culture and tradition. In conducting field based research and development within a school setting, the model project is immersed in the school's culture and tradition, and over a period of time its operation will tend to take on the qualities of the school if progress in model test implementation is being maintained. When this occurs, quantitative measures of student achievement take on less importance than whether the model program itself is being assimilated into the everyday fabric of the school. In the end, assessing the effectiveness or success of the model program is reduced to:

- whether the model program has gained acceptance by a majority of the faculty, and
- whether the school administration deems the model program to be viable and consistent with the tradition and institutional resources of the school.

Neither of these two project outcomes can be easily obtained by an outside observer because both are influenced by the culture of the school and are, therefore, difficult to track and quantify, if at all possible. It is in this milieu of interrelated factors that the systems and field based approach stands apart from traditional theoretical educational research practices. The traditional education research approach centers on the belief that an "objective observer" can determine what is occurring or what has occurred as a result of an intervention program or "treatment." It assumes that the process of objective observation can be isolated from the surrounding culture of the school (or classroom) and that the
reported observations, for the most part, reflect the realities taking place in the school.
In field based projects, just the opposite is assumed, especially where the project itself is driven by an outside collaborative agency, as is the case of the CMSP model project effort. The field based process itself is one of change, and the model project's very presence in the school environment creates a synergetic condition that influences the nature of both the model project and that of the school's culture--however small at the beginning of project activity. The mere start of a program within the school environment is already a major step in the process of affecting change in the structure of the school's academic program. The model project becomes part of the school and vice-versa, and over the period of time the two become indistinguishable. And this is the way it should be if the goal of the model project is to effect positive and permanent academic change over the long term. In field based projects the role of the "objective observer" is not one of assessment, but instead one of overseeing program development, providing specific resources and serving as a central agency to compile and analyze the plethora of quantitative data that accumulate over the long gestation periods required for program assimilation.

Given this philosophy of model program assessment, what then should the CMSP utilize as measure of model program progress toward proving the value of the two interdependent premises as stated in Chapter 1 and repeated here?

1) the major deterrent to the successful learning and completion of a first course in algebra is the lack of preparation in the basic arithmetic upon which algebraic concepts and algorithms are founded, and
2) for almost all entering 9th year high school students, preparedness for a first course in algebra can be attained in one semester, independent of students' prior mathematics proficiency and background.

Both of these premises are couched in the previously stated belief that "all students can
learn mathematics very well, given the foundation and academic support for the mathematics they are expected to learn in the classroom" and in the major goal of the CMSP model efforts to affect significant increases in the pool of students at Chapter 1 high schools who enroll in and complete the three year Regents mathematics sequence prior to high school graduation. For the purposes of this project study, the measures to be adopted to test the premises were based on:

- the degree to which students who study prealgebra for a single term are prepared to enroll in a first course in algebra in the succeeding term,
- whether there is a significant increase in mathematics test scores by students participating in the CMSP model as compared to similar group of students who are studying or have studied the same mathematics in conventional school mathematics programs,
- the acceptance of the model project efforts by the faculty and their consensus to become further involved if the initial model test implementation shows promise in affecting student achievement, and
- the general support of the model program by the school's administrative staff and their willingness to reallocate school resources to allow for model test implementation and possible future program expansion.

These four assessment parameters go far beyond that necessary to measure model program effectiveness as qualified by the two interdependent premises previously stated. To prove the arguments raised, it would probably suffice to look at quantitative data as they reflect student continuance and achievement in Regents mathematics coursework over a three semester period. Taking this course of action, however, would defeat the purpose of field based research and development, which in this instance was to develop and test implement the model project within the full range of variables that characterize a working
school environment.
The power of the field based and systems approach is that the problem is investigated and models are developed within the environment and dynamics of a working school and, as a result, project outcomes have a systems based quality. Not only is something learned about specific elements of the model, but the model developed brings with it a global quality that embodies the complete process of schooling in the given mathematics subject area, including course placement, class organization, curriculum and instructional pace, uniform class testing and student course evaluation. In addition to these elements which, taken together, make up a system of curriculum and instruction, the field based and systems approach must also take into account the organizational and administrative aspects of the school in which model project efforts are taking place. Can the model be shaped consistent with school resources? And can it survive or compete with the changing priorities and peculiarities of the school? It is this evolutionary and dynamic quality that makes the system and field based approach an efficient strategy for investigating complex educational problems such as that being pursued by the CMSP.

Because of the global quality of the CMSP model project efforts, it is useful to enlarge on the four basic assessment parameters (noted above) to gain greater insight on why model efforts work or don't work at the participant schools. In the end what was needed and desired as a result of the CMSP model project effort was a more complete understanding of the problem, plus having the strategies and organizational constructs from which systems and field based research and development with a sharper focus could have continued beyond the initial phase--e.g., possible larger scale project efforts that would have commenced in the Fall term of 1983.

The work of the CMSP model project study when originally conceived in 1978 established the foundation for pursuing systems and field based development and research over the long term in New York City Chapter 1 high schools. However, future efforts
could only proceed on the merits of the progress made in creating a model achievement program that increased student enrollment and achievement in precollege mathematics during the initial phase of CMSP model project activity. Basically, whether this was possible was more a function of the participating school than any model curriculum or instructional strategy that the CMSP could have created. In the end the participant schools had to feel strongly that utilization of the CMSP model was beneficial to students and teachers and that the resources to implement the model were reasonable and consistent with what could be allocated to mathematics as a course of school study.

From the perspective of model systems development and research, the CMSP had a primary interest in schools' acceptance of the four primary elements of the CMSP model: random selection and heterogeneous class grouping, the zero based start, the double period of mathematics instruction, external testing and uniform pacing. The success in the launching and the continuance of model test implementation hinged upon the schools' acceptance and utilization of all four of these model elements. Prior to model project test implementation in the Fall of 1979, and in meetings with the principals and mathematics chairpersons, it was agreed that the four elements would be the cornerstones by which the model project would be guided and conducted at each of the participant schools. In effect, these four CMSP model elements were accepted as "non-negotiable" elements until such time they were shown to be unworkable or inconsistent with students' mathematics achievement.

The element of the CMSP model that is a significant departure from traditional school practice is the random selection of students and the heterogeneous grouping of classes (these two elements are considered as one since they are so closely intertwined-- although random selection ceases to be an issue when the model is fully adopted by the school, however, class heterogeneity remains in effect). It was felt from the start that this element of student selection and class grouping was the first major program hurdle that would
establish the essential working foundation with which model project activity would proceed. Adherence to the doctrine of random selection was considered essential in creating a base for the comparison of student achievement within and across participant schools. It was also consistent with the larger goal of increasing the pool of high school students in Chapter 1 schools who enroll and achieve in precollege mathematics.

The random selection of students for participation in the model project also carried with it the consideration of a "control group" with which "objective" comparisons of student achievement could be made. Under traditional educational research practices, presumably a comparable number of students (to those selected randomly) could be selected and used as a "control" to verify whether the CMSP model "treatment" had any "significant outcomes." There were two major problems with the selection of a control group besides being completely against the philosophy of the CMSP model effort. The first was political: How would you justify to a Black or Hispanic or any parent with children in Chapter 1 schools that their child has been selected to serve as a "control" for a model project in which substantial resources would be provided to advance the prospects of mathematics achievement for another group of students that their child would not be part of? Secondly, even in the unlikely event that consent was given by the parent, what parameters could be used to serve as a control, is it the students' socioeconomic status? or maybe standardized test scores? (which have already been shown to be misleading for students whose mathematical backgrounds are fragmented by virtue of inadequate mathematics schooling), or perhaps the students could be paired by originating junior high/middle schools and the mathematics courses they took there? These three parameters and more could be used as controls if there was any certainty they were stable. However, they are not stable to any degree of confidence, and this is the very crux of the problem. These non-linearities and instability of variables are what makes the problem complex and indeterminable and which seriously limits the value of linear and traditional educational
research approaches to find solutions.
The multivariate and dynamic quality of the working school environment makes it almost impervious to research methods that seek to compare student achievement though experimental and control group designs. It was from this rationale that thought was given to viewing the participant students' achievement as part of the school's tradition and history of mathematical programs. In particular, a salient characteristic along these lines was the school's performance on Regents mathematics examinations-- e.g., How many students took the 9th Year Regents Mathematics Examination? And how many scored over $85 \%$ ? And how did the similar school perform on the 10 th and 11 th Year Regents Mathematics Examinations? This information is available from the New York City Board of Education, and compiled over several years, could provide a rather comprehensive and accurate longitudinal profile of student achievement at a given school. In addition, because the Regents examinations are administered statewide on the same day each year by the New York State Education Department, comparisons of student test performance could be made with other Chapter 1 schools not participating in the CMSP model project and also across school districts outside of New York City.

In essence, the basis for comparing participating student mathematics achievement would be the school itself. And this comparison was made compelling by establishing a standard by which schools could determine whether progress was made by participating in the CMSP model project and increasing the pool of students who achieved in the study of a first course in Regents Algebra. The standard established had to be one that--under the guidelines which the CMSP model project was being conducted--had a fair chance of being attained. It also had to be a standard that all involved with the project (and this included students, parents, teachers, school administrative staff, collaborating organizations and the supporting private sector institutions) could understand and accept as being a legitimate and worthwhile project accomplishment. To this end, the traditional psychometric
sampling and probability functions were avoided, and instead straightforward percentage comparisons were utilized.

In the engineering profession there is an old adage which states that new product development is not worthwhile unless performance of the new product is a least twice that of existing comparable products. This engineering adage of twice the performance had merit for the CMSP model project, as it provided a standard that everyone could understand and also accept as a project accomplishment worthy of serious consideration. Given the college based resources, the additional instructional time and the tight structure of the model curriculum, the doubling of student mathematics performance, as measured by Regents mathematics examinations, appeared to be a reasonable goal and challenge for the participating teachers, students and staff. By agreement with the teaching and administrative staff of the participant schools a two-to-one difference in the pass rate of Regents mathematics examinations was established as the standard by which mathematics achievement by students participating in the CMSP model project would be compared. The achievement comparisons would be made within the school utilizing current and past student populations taking the same level of Regents examinations.

The zero-based start was another non-traditional aspect of the CMSP model that needed to be carefully watched in order to insure that beginning instruction evolved in a sequence that gave all of the students an opportunity to refresh and relearn mathematics material they had previously encountered in one form or another. The CMSP was already aware of a perception shared by some teachers and mathematics chairpersons that the ground zero start was unfair to the better prepared student because "it held them back". In retrospect this notion of "holding the good kids back" was the most persistent issue of the CMSP model throughout its developmental cycles of project activity. Even in the face of evidence that showed otherwise, the perception persisted. This may be an indication that firmly held educational beliefs are not likely to change regardless of their apparent conflict
with the student achievement data, however significant.
The ground zero start is at the heart of the CMSP model curriculum structure and from it emanates the legitimacy of external testing and uniform pacing of instruction, both of which are elements that allow objective comparisons of student achievement within and across schools. If students did not start at the same point in the curriculum model and move at a reasonably consistent pace of classroom instruction, then the program would have ceased to be one aimed at increasing the pool, and eventually would have gravitated toward one that was selective which would have destroyed the intent and goals of the CMSP model effort. From the perspective of CMSP model test implementation, the screening of students on the basis of their work in the prealgebra program was to be avoided, and if instances did arise where students were clearly prepared to do mathematics at levels above prealgebra, they would be handled on a case-by-case basis. The process of testing all students prior to the start of formal class instruction allowed a measure by which such "advanced level mathematics" students could be identified. On the whole, however, students in this category never exceeded more than $5 \%$ of the incoming population. And this small number appears to be typical at the Chapter 1 schools in New York City.

Another concern that would have compromised the integrity of the CMSP model project elements was the organization and scheduling of two periods of mathematics for each of the students participating in the program. Besides the two periods, the CMSP model required that the classes be "block programmed" and that each class be taught by a different teacher. In order to reduce the perception that the two classes were one and the same, it was further required that the class not be scheduled "back to back". Again, these program organizational requirements were a departure from traditional administrative practice, and it was important to know whether they could be carried out without serious disruptions and within the administrative constraints normally experienced at the start of the school year.

And finally, the major consideration in attempting to assess the quality and effectiveness of the CMSP model efforts to research and develop models to increase student mathematics achievement, were the attitudes and feelings of the participating teachers and mathematics chairpersons. It would hardly matter if student achievement was two or three times that of comparable students if the mathematics chairperson or the faculty at large (or both) questioned the value and potential of the program. Whether faculty resistance to the model occurred because of competing departmental priorities or because there was a general disbelief that "all students can learn mathematics very well" mattered little, for in the end a model project cannot survive with a lack of consensus on the part of the departmental faculty. This is true even if there is overwhelming support for the model project from the principal, mathematics chairperson or high administrative officials outside of the school.

In order for a model project to be judged as being of value and effective, it must be perceived as such by the school's departmental faculty. And this is correct because in the final analysis it is the school's constituents who employ the model-the mathematics teachers, chairpersons and students in their classroom experience over long periods of time--who must inevitably determine whether the model is better than existing programs of mathematics instruction. From the perspective of CMSP model assessment this acceptance by teachers and mathematics chairpersons is intimately tied to the energies they devote to school project activity and their desire to continue with succeeding cycles of model test implementation. The value of the systems and field based approach is such that personal and subtle faculty perceptions--enormously important assessment factors--can be ascertained over the course of model test implementation because of the model project staff's close working relationship with the school and its mathematics department faculty.

From a comprehensive model assessment perspective, what was sought in the CMSP model project efforts were the elements of curriculum and program structure that could be
used as the "raw material and fabric" for building a foundation and framework for future model research and development efforts. The model framework had to rest on a foundation that was reasonable and consistent with the traditions and resources of the school. And the framework also had to be able to withstand the competing school priorities over long periods of time. Meeting these changing and demanding school environmental qualities meant that the model framework had to be created and test implemented cyclically over long periods of time with different sets of teachers and student populations. Given this longitudinal and evolving process in the school setting, proving stated premises, or realizing the model project's intended goals, or judging its value to students and teachers could be ascertained with a reasonable degree of confidence.

### 5.2 Test Implementation Cycles and Milestones in Model Project Development

The systems and field based approach being utilized by the CMSP to develop and research a curriculum model entails repeated testing of the model with different groups of students and teachers. As organized and presented to Nathan Quiñones, then Executive Director of the New York City High School Division, and to the supporting foundations and companies, the CMSP model would evolve over a period of four years in which a cyclical process of model test implementation would take place at the participant schools. The test implementation cycles of model project activity would be structured to allow the three junior high schools, the six Chapter 1 high schools and Brooklyn Technical High School to function as three independent programs. Over the four-year period, the three junior high schools would implement two full cycles of two-year durations, the six high schools would participate in three full cycles of three-term durations and Brooklyn Tech would test implement four two-semester cycles. This cyclical schedule of model test implementation for the three categories of schools is shown in Figure 9, indicating how the cycles overlapped to take advantage of what was learned in the previous year of the
COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSPI
MODEL PROJECT TEST IMPLEMENATION CYCLES .- 1979/80 TO 1982/83

| 1979/80 | 1980/81 | 1981/82 | 1982/83 |
| :---: | :---: | :---: | :---: |
| 1st Cycle |  |  |  |
| PREALGEBRA | ALGEBRA -- TERM 1 |  |  |
| RAFAEL CORDERO | 2nd Cycle |  |  |
| $\begin{array}{ll} \text { I S } & 258 \\ \text { I S } & 117 \end{array}$ | PREALGEBRA | ALGEBRA -- TERM 1 |  |

1st Cycle
PREALGEBRA
JJAY
W. IRVING
B. FRANKLIN
EAST N.Y.
CHELSEA
J.F.KENNEDY
1st Cycle
BROOKLYN TECH
FIGURE 9
preceding cycle. Except for Brooklyn Tech, where the cycles occurred sequentially, this overlapping of project cycles had proven useful in previous CMSP model development efforts as it provided a paralleling of project activity and a greater number of participating teachers. This increased the level of project development activity and heightened curriculum feedback and teacher interaction considerably. The greater participation at the school also gave the schools an opportunity to shape the administrative procedures to match the peculiar elements of the CMSP model including the random selection of students and the block programming of mathematics and science classes.

What was expected with the repeated cycles of model test implementation over the four year period was a structured evolution of a model curriculum that would be shaped and refined with each cycle of project activity. It was assumed at the time of model program inception in the Fall of 1979 that the programs at the individual schools would remain intact and that the model development and research process would be continuous and with sufficient stabilty to test the premises as originally conceived. This sense of optimism was reinforced in the Fall term as the model program at each of the participating schools was started and proceeded throughout the Fall term with no major problems and with a shared commitment by the teachers and the mathematics and science chairpersons that the project activity had taken hold and that students appeared to be accepting to the instructional approach. The optimism was bolstered by the academic support that was provided by the college professors who provided teachers with college student tutorial assistance. Table 20 shows the number of college students that were involved at each of the ten high schools and their relationship with the engineering colleges.

The Fall term concluded with the teachers in each of the participant schools feeling that student achievement was progressing in accordance with the curriculum schedules that had been developed for their assigned school category i.e., the three junior high schools, the six Chapter 1 high schools and Brooklyn Tech. As shown in Figure 9, in $t$ he first

COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)
COLLEGE-PUBLIC SCHOOL PAIRINGS FOR CMSP TUTORIAL COMPONENT FALL SEMESTER 1979
\# OF COLLEGESTUDENT TUTORS ${ }^{(1)}$
COLUMBIA UNIVERSITY
Rafael Cordero ..... 11
Benjamin Franklin ..... 10
THE COOPER UNION
Brooklyn Tech* ..... 6
Chelsea ..... 23
John Jay* ..... 6
Washington Irving ..... 20
LONG ISLAND UNIVERSITY
John Jay* ..... 3
MANHATTAN COLLEGE
John F. Kennedy ..... 12
MEDGAR EVERS
IS 258 ..... 10
POLYTECHNIC INSTITUTE OF NEW YORK
Brooklyn Tech ..... 8
East New York ..... 8
PRATT INSTITUTE
IS 117 ..... 11
(1) If college student academic program does not conflict with CMSP
school day schedule, only 5 college students for each class are
needed.

* Spring semester 1980.

TABLE 20
cycle, the students at the three junior high schools were expected to complete and master prealgebra coursework over a three semester period, students at the six high schools were scheduled to complete and master the prealgebra course in the Fall term and complete the study of the first term of algebra in the following Spring term, and the Brooklyn Tech students would take and complete the algebra course in the normal two-term period.

The first year course schedule arrangement allowed the students at the six Chapter 1 high schools to take the Regents Competency Test (RCT) in Mathematics at the completion of the year's mathematics coursework which covered arithmetic and the fundamentals of algebra (similar to the topics covered on the RCT) and permitted the Brooklyn Tech students to take the 9th Year Regents Algebra Examination as scheduled with the rest of the student body at Brooklyn Tech. In both cases, the administation of the RCT and the Algebra Regents Examination allowed a comparison of student mathematics test performance within and across participant schools.

While the Fall semester project activity appeared to show that the project effort was off to a good start and that high school faculty and supporting college staff were working together to affect the model's goals, there were some aspects of the project at particular Chapter 1 high schools that were disturbing but interesting from the standpoint of developing a comprehensive model that could withstand the perturbations and dynamics of a working school setting. The first of these project disturbances was student attendance and attrition, and as the project continued in later cycles, chairpersons support and faculty acceptance of the project became a source of real concern as it affected the viabilty of the project in the participating school.

To a large extent the CMSP model development and research has been an empirical process of trial and error, with the errors and difficulties of model implementation providing the actuating forces and clues as to what direction to proceed in succeeding cycles of project activity. Each of the three categories of schools shared its own set of
problems and peculiar sets of circumstances. It also appeared that the problems were unique according to the three categories of schools. For the junior high schools it was the level and stability of mathematics instruction; student retention and the relationships between faculty and supervisors were the major concerns at the Chapter 1 high schools; and acceptance of the model by the faculty at large became an issue at Brooklyn Technical High School. Each of the problems that developed and their ultimate outcome at the participant schools was an important project experience that helped shape the CMSP model curriculum and also contributed to a better understanding of the complex process of implementing new programs of mathematics instruction in the context of an ongoing school program and operating environment. Because of the differences in project cycle length and diversity of the programs at the junior high schools and the high schools, the project experiences gained bear a recount from three points of view. These follow as short project perspectives which focus on the salient qualities and outcomes as they determined the course of project participation at each of the participant schools.

The Junior High Schools: The three participant junior high schools represented the strongest test of the CMSP model concept because the original model concepts were based on the weaknesses of mathematics instruction at the junior high school. The argument was that students arrived at high school from junior high school mathematics course experiences which were too unstuctured and weak to build students' arithmetic foundations. With their particpation in the model project, an opportunity would be gained to work with students and teachers with the intent of developing and test implementing a model program that would prepare students for the traditional Regents mathematics courses offered at high school.

It was very clear from the inception of CMSP project activity at the junior high schools that they were very interested in seeing that the model project work. Their participation
was much more personal than in the high schools and the relationships that the teachers developed with the visiting college professors and students at the start of program activity was a positive indication that students would gain considerably from this extra measure of academic support. In the scheduled meetings with the teachers, the school project supervisors were always present and the teachers were eager to cover the week's materials and ask for assistance where they felt unsure.

The progress in all three junior high schools in covering the materials in whole number arithmetic was much slower than expected. The project schedule was organized to give the junior high school students twice the time to cover the prealgebra course as was scheduled for the Chapter 1 high schools. The mastery of a given body of mathematics course materials in a set period of time was not a serious concern because of the start at the 7th grade. Because of this the instructional pace was tempered by student mastery of specific mathematics course topics--which in this instance was set at $80 \%$ of the class achieving a grade of $80 \%$ on the unit tests before proceeding to the next mathematics topic. All three schools found it difficult to achieve at this level of mastery and the level was lowered to reflect the confidence of the teachers which, as the program progressed through the topics in whole number arithmetic, varied between $40 \%$ and $60 \%$ of the classes achieving $80 \%$ or higher on the unit tests.

At the end of the Fall term all three junior high schools seemed to proceeding at a relatively equal pace and the mean unit test achievement levels on course topics in addition, subtraction and multiplication of whole numbers were high and within ten percentage points of one another as follows: Rafael Cordero--82, IS 117--84 and IS 258--76. However, even as early as the Fall term it was apparent there was a notable difference in the instructional quality at IS 258 as compared to the other two schools. As a result, the CMSP staff spent more time at IS 258 working with the teachers at rudimentary levels of instruction that were cause for concern. Further inquiry revealed that the teachers were not
licensed to teach mathematics and had minimal experience teaching mathematics, and almost none of the experience tied to the traditional coursework in prealgebra. Mathematics background inquiries of the teachers participating at the other two junior high schools showed similar degrees of mathematics teaching inexperience and a lack of a formal mathematics academic background (except for one mathematics teacher). None of the nine mathematics and science teachers had experience teaching algebra and it appeared that the CMSP model was their first venture in a structured mathematics program that prepared students for high school mathematics. The lack of mathematics teaching experience at the junior high schools began to surface in earnest as coursework moved on to the division of whole numbers and work in fractions. Student achievement in whole number division at IS 258 suffered as compared to the other two junior high schools and three schools ended their coursework in whole number arithmetic (addition, subtraction, multiplication and division) with the following percentages of students scoring $80 \%$ or higher on a cumulative examination on whole number arithmetic: Rafael Cordero--65\%, IS $117-36 \%$ and IS $258-17 \%$. Clearly, there was a serious problem developing at IS 258 and the small CMSP staff and available academic support from the colleges resources were insufficient to institute the type of fundamental teacher training that was required. It had always been assumed that the ground zero approach of the CMSP model would have special value at the junior high school level because it would give teachers the structure and students the time to build a strong student mathematics foundation. However this assumption was based on teachers' having had the appropriate background and teaching experience to take advantage of the structured curriculum and uniform pace of instruction. There was never a consideration that the teachers themselves would have difficulty teaching fundamental arithmetic topics.

The burgeoning problem at IS 258 became worse and began to surface at IS 117 as coursework advanced to the topics of fractions. Rafael Cordero seemed to be proceeding
with few apparent obstacles, except the course material covered at the schools was well below what was expected for a full of year mathematics instruction in which students had twice the time of the regular one-semester prealgebra program of instruction. At the end of the first year of model test implementation it was apparent that the quality of instruction at the junior high school would be a serious impediment to demonstrating students could master the content of the prealgebra course in a three-term period. With varying degrees of success, the participating students at the three junior high schools had completed coursework in whole number arithmetic and had begun introductory work on the multiplication of fractions and further coursework in prealgebra was relegated to the Fall of 1980 .

The second year of the program at IS 258 and at IS 117 became increasingly frustrating for the teachers as they moved on to teaching topics in fractions that they were unsure of, and this was reflected in low level students achievement on unit tests. The performance was low enough to preclude any type of mastery of course advancement. The amount of time the small CMSP staff could spend with the teachers was not enough to overcome the serious deficiencies in the teachers' mathematics backgrounds. In effect, the teachers were having difficulty with the content of the mathematics course topics they were expected to teach to students in their classes. This created an obstacle in model project activity at IS 258 and IS 117 that the CMSP could not remediate with its available resources.

Discussions were held with the principals and school project supervisors of IS 258 and IS 117 and they agreed that the problem was serious. However, they countered that the availability of experienced and licensed mathematics teachers was a luxury that Chapter 1 junior and middle high schools did not enjoy and it was not likely to get any better in the years ahead. They had hoped that the resources and the structure of the CMSP model would help alleviate the problem and student achievement would improve steadily as the
participating teachers gained confidence and experience. The CMSP continued working with IS 258 and IS 117 for another academic year, 1980/81, however the difficulties being experienced in the mathematics classroom were beyond the resources and means of the CMSP or participating colleges. In the spring, the project reached an impasse when it was realized that CMSP model project activity was being reduced to a program of remediation that was not much different from that of the school regular mathematics programs of instruction. Under these circumstances, little could be learned or contributed to the building of a model program that would have value in preparing junior high school students for traditional Regents mathematics coursework. By agreement with the principals of both schools and acknowledgement from the Superintendent of Community School District \#13, the CMSP concluded its model project activity at the end of the Spring semester of 1981.

The demise of the CMSP model efforts at IS 258 and IS 117 reinforced the assumption that poor mathematics teaching quality at Chapter 1 junior high schools was the major cause for the mathematics weaknesses displayed by students from these schools as they entered Chapter 1 high schools. But yet, there remained Rafael Cordero a junior high school that appeared to be enjoying a measure of success in mastering the topics of prealgeabra even if it was at a slower than expected pace. The enthusiasm of the participating teachers was still very high in the second year of project activity. And the twice a week visitations by the Columbia University students, supervised by an engineering professor, were in place and continued to be a source of true academic support and encouragement for both students and teachers. The Acting Director of the school (he replaced the Director who took a new administrative assignment at the central district office) was sufficiently impressed with the model project activity in the first year that he elected to start a second group in the Fall of 1980. This expansion of the program was accomodated by increasing the course load of the teachers who were currently teaching the
second year of the CMSP model.
The first group of students who began in the Fall of 1979 made progress in their CMSP mathematics studies in the second year and were able to complete all of the topics of fractions and some parts of the units on decimals before most of the students graduated to high schools at the end of their eight years of schooling. Sufficient material was covered and mastered to enable the Rafael Cordero students to take an equivalent of the RCT Mathematics examination that was given in June of 1981. The 8th grade Rafael Cordero students who had participated in the CMSP passed the test at a $50 \%$ rate which, although not as high as expected, approximated the pass rate at Chapter 1 high schools. The experience gained in the first year of the project at Rafael Cordero gave the teachers more insight and confidence with the second group of students. The mathematics course topics were covered with higher student achievement and with a more reasonable pace of instruction. By the end of the first year of this second group, the teachers were feeling that they would probably be able to reach the study of algebra in the third semester as originally planned in the program schedule for the junior high schools. However, this did not materialize because the problems of teacher inexperience arose once again. Although much later than experienced at IS 258 and at IS 117, the teachers appeared to be treading on new instructional ground for which they lacked preparation, background and experience. The difficulties in program continuance occured late in the Fall semester of 1981 when the participating class had completed most of the CMSP prealgebra coursework with a fair measure of success, but not at a sufficiently high level that would have enabled students to proceed in the study of algebra. However, the teachers were eager to begin the teaching of algebra as were the students to learn it--even if it meant that a sizable proportion of the students would have difficulty. This was the conclusion of the CMSP which felt that the work in prealgebra should be continued to assure that more students had the arithmetic foundation for the higher level coursework in algebra. The desires of the teachers
prevailed and the program at Rafael Cordero proceeded with the study of algebra in late Fall of 1981.

Because there had been teacher questions about algebra course content and specific inquiries about the level of instruction, the CMSP established a regular schedule of weekly meetings specifically to review topics before they were introduced in class instruction. While these teacher training sessions helped, the course of program study in algebra began to falter. And this was evident in the very first topics of fundamental operations and in first degree equation solving in one variable. Lacking the requisites of simple equation solving, students' progress in higher level algebra topics was impeded and the model program began to show the same disparaging signs that precluded CMSP continuance at IS 258 and IS 117. The problem at Rafael Cordero was a bit more involved because there had been progress in prealgebra, however not with the full pool of students that originally began. There was an unevenness in the performance of the students in the two participating classes, and it was later found out that in the programming of classes at the beginning of the second cycle that the students were not hetrogeneously grouped. As a result, one of the classes had students enrolled that were substantially better prepared academically than the other. The separation of students at Rafael Cordero by academic preparedness was an avoidance of the CMSP model guideline by the return to the school's tradition of "ability" grouping. The realization of Rafael Cordero's difficulty to advance an entire class of heterogeneously grouped students beyond the topics of fractions and decimals of prealgebra and the beginnings of algebra was especially troubling because it demanded CMSP staff attention and diverted competing priorities from the several high schools which were participating in CMSP model test implementation at the time. Given the limited staff resources and realizing that continued efforts at the Rafael Cordero school would create demands that could not be met without seriously overextending the entire project, it was decided to curtail CMSP project activity at the school at the conclusion of
the Spring semester of 1982. Thereafter, the CMSP would focus its program efforts completely at the high school level where there was a sufficiency of qualified high schools teachers who could take advantage of the CMSP model structure throughout its three semester course period.

In retrospect, the project experience at Rafael Cordero, viewed from a prealgebra perspective was quite encouraging as a model program. In two cycles of model test implementation, heterogeneous groups of students selected at the 7 th grade were able to make substantial progress in completing the topics that were central to the CMSP prealgebra curriculum. The fact that students took almost three semesters to reach the required levels of achievement did not diminish the the importance of building students' foundations for algebra at the 7th and 8th grade levels. Had resources and the time been available, it would probably would have been expedient to continue the model project at Rafael Cordero with a redesigned prealgebra curriculum that was consistent with the instructional pace that had been established in the first two cycles. This could have been done by restructuring the prealgebra topics over a three semester period, leaving the fourth semester for a comprehensive prealgebra review and an introduction to first year algebra. In this way, the junior high school program could have stood on its own as a structured curriculum precursor to high school mathematics. While this curriculum design would not have overcome the problems of teacher inexperience with mathematics content, with the appropriate resources it would have provided the basis for timely staff development that could have lessened the problem considerably.

The relationship between student underpreparedness for high school mathematics and inadequate instruction at the middle and junior high school levels is a strong one and points to the pervasive problem of teachers teaching sutject matter they have not been trained for and do not know well. While it may be possible for students to make up for inadequate instruction in other subject areas, it is especially difficult in mathematics. This is mainly
because of the abstract and sequential learning aspects of the subject where discontinuities in instruction or course failure can cause students to doubt their academic ability and diminish their concentration and efforts towards higher level mathematics.

Ted Sizer in his book, Horace's Compromise. The Dilemma Of The American High School speaks poignantly about his own experience as a high school English teacher who is given the responsibility to teach two sections of algebra with little prior knowledge of the subject.

The students in my clases leamed mathematical operations pretty well. They learned virtually nothing about mathematical inquiry or mathematical thinking, because I knew virtually nothing about these things. Certainly, my pupils were not inspired by the subject. In a word they became competent algebraic drones. However, if I had not had good texts, an ability to keep discipline with a tough administration behind me, and a supportive spouse, the year would have been a total disaster. Competent drones were the best I could hope for. Fortunately for high school youths, I have not taught mathematics since. My experience would be irrelevant except that it represents a sadly common situation. Many high school teachers do not know their subjects. They teach, as I did, from day to day, and the textbook is the source of everything. ${ }^{1}$

The CMSP junior high school experience added credence to the premise that the major deterrent to student success in high school mathematics was students' lack of proficiency in the basic arithmetic upon which algebraic concepts and algorithms were founded. And, at least, from the perspectives and experiences of the CMSP in its involvement with the three junior high schools in Community School Distrcict \#13 and \#4 that the student deficiencies in prealgebra were primarily the result of inadequate mathematics instruction. It can be inferred with a reasonable degree of confidence that the same situation prevails at other junior high schools in the two districts and at other school districts with predominant Black and Hispanic students populations. The disparate

Chapter 1 junior high school achievement data presented in Chapter 2 also provide convincing evidence to support the original CMSP premise of students' inadequate mathematics foundation for traditional high school mathematics coursework.

The Chapter 1 High Schools: Model test implementation in the participant Chapter 1 high schools was concurrent with that of the junior high schools, and because of this, direct comparisons of the quality of mathematics instruction could be made. In the six high schools which participated there were twelve mathematics teachers involved and all were licensed in mathematics and experienced (just the opposite of the junior high schools). In addition, all had taught upper level Regents high school mathematics courses and were prepared to work with the incoming 9th year students at the ground zero level required by the model guidelines. The distinction in mathematics teaching between the participating high schools and the junior high schools was clear--the continuity of the three semester program cycle at the high schools would not be affected because of the quality of instruction. However, quality of mathematics instruction and corresponding student achievement, while important and in keeping with the major goals of the CMSP, were not in and of themselves the overriding issues in the test implemenation of the model in the six high schools.

The major problem in the Chapter 1 high schools was high student absenteeism and attrition, neither of which were noticeable problems at the junior high schools. At two of the schools John Jay and Benjamin Franklin, the attendance was so poor that the programs deteriorated to a dysfunctional state by the end of the Spring term. The particpating student population at the Benjamin Franklin had been reduced to one third of the original students who took the initial pre-evaluation exam at the beginning of the program in September 1979. The same high level of attrition occurred at John Jay High School with the 64 starting students dwindling down to less than a class of 30 students. The problem at both
schools combined high absentee rates with class cutting and the eventual reduction of class size to levels that made it difficult for teacher to manage instruction effectively. These events were a completely new experience to the CMSP and were so acute that they dwarfed the teaching issues that were faced at the junior high schools. It was evident that no instructional program or form of academic support could hope to benefit students if the students themselves were not present to participate. At both schools there were periods of time when absences amongst students were so pervasive that continuity of class instruction was impossible--in the space of two week instructional period practically every one of the students had been absent or had cut class at least one time. Average daily class attendance was about $50 \%$ at both schools.

The high absentee rate at Benjamin Franklin had a disheartening effect on the participating teachers and also affected tutorial efforts of the college student team from Columbia University. Eventually, the college students felt that their efforts in the classroom were not realizing intended benefits to the high school students and their own participation at the high school became erratic. The attrition and, thus, lack of participation on the part of the high school students at Benjamin Franklin proved to be too great of a hurdle for the CMSP and the school administration to overcome, and model program efforts at the school eroded to the point of diminished return.

Subsequent discussions with the Principal were centered on the viability and value of the CMSP model test implementation effort at Benjamin Franklin, given the high student attrition. There was agreement that resources provided by the CMSP were insufficient to remedy the student drop out problem at Benjamin Franklin and that the CMSP model project, while worthwhile for the few remaining students, could not affect an increase in the pool of students who achieve in the study of precollege mathematics. The CMSP did not continue model program activity at Benjamin Franklin High School beyond the Spring term of 1980. The departure of the CMSP in June 1980 preceded by one year the official
closing of Benjamin Franklin by the New York City Board of Education as a result of the very high student attrition rate and poor student achievement.

The situation at John Jay High School was not much different from Benjamin Franklin's. Student absences and attrition made the program unstable and almost impossible to control. The uneven class attendance was exacerbated by the fact that the school was on double session, with one of the CMSP classes beginning at noon. The attendance in this first starting class was very low as compared to the second class which would provide reinforcement for work covered in the first class. Because of the disparate attendance in both classes, only a handful of students received the benefit of a double period of structured mathematics instruction. Of all the five schools which particpated in the Fall term prealgebra program, John Jay made the least progress, completing the topics only through multiplication of fractions. The severe attrition of students at John Jay High School had the same dysfunctional effect that eventually led to the program's demise there in June of 1980.

There were some aspects of the experience at John Jay High School that were useful indicators of the viability of a school environment for the implementation of intervention programs such as the CMSP. What factors in a school can thwart the implementation of a new program concept or model of instruction? Certainly, attendance and student attrition are important key factors. Both have to be carefully examined to insure that resources brought to the schools by the intervention program are not squandered or made ineffective by the lack of student participation. And then there is the question of school and departmental leadership. In the case of John Jay, the mathematics chairperson had indicated that other school priorities prevented him from giving the program the time and effort it needed. And, therefore, in order for the program to operate, another member of the department with supervisory experience, a teacher was called upon to supervise the model test implementation. The school's choice of a program coordinator was excellent
as was the selection of the two mathematics teachers who would instruct the two CMSP participating classes. All three gave the program the best of their efforts, and even in the face of the severe student attrition, maintained a spirit of enthusiam in the classroom. In effect, the program became teacher driven and the teachers' energies and resolve lessened the impact of not having a mathematics chairperson directly involved with the project. In the end, however, student attrition became too great an obstacle to overcome and CMSP project activity ended at the conclusion of the 1980 Spring term. The question of the necessity of the mathematics chairperson's direct project involvement remained unresolved, however there were strong indications from the other participant schools that it was a critical factor to effective model test implementation and program continuance.

Attendance and student attrition remained a problem at the other four schools participating in the first model project cycle--Chelsea, John F. Kennedy, Washington Irving and George Washington. By the end of the Spring term of 1980 the student population at each of these four schools had been reduced to half. This attrition occurred despite the fact that there was no attempt to hold back students from the Fall to Spring terms for academic reasons. In general, students who attended class regularly did fairly well in their mathematics coursework and their participation in class provided the impetus for movement forward in the program. Students who left the program were esentially students who were excessively absent or were school dropouts. As much as possible the students who remained with the program were given the resources both during school and after school to keep up with the pace of the course and to insure the slow build up of the foundation for algebra coursework.

In two of the schools, Chelsea and John F. Kennedy, the problem of faculty interaction with the mathematics chairperson led to circumstances that were extremely difficult to control as an outside intervention program. In fact, the presence of the CMSP model project at the schools appeared to exacerbate the problem as both faculty participants
and the chairperson utilized the project and its peculiarities as a platform for departmental reactions. The biggest frustration was at Chelsea High School where the CMSP pilot model had originated and whose program exerience in the Spring of 1979 became a model for other schools to emulate. Students' fine performance in the initial whole number arithmetic topics of the prealgebra course carried over to the Fall term.

In the preevaluation examination in whole number arithmetic that was given to the five high schools starting in September of 1979, Chelsea scored the highest, with a mean test score of 75. This score was more than 20 points higher than the student performance at all of the other schools, with the exception of John F. Kennedy, whose students' mean test score was 74. What was significant about the Chelsea students' performance was the fact that they had not been randomly selected and represented the bottom third of the school's 9th year students' ranking in terms of standardized mathematics test scores. The other important element of their high test performance was the fact that the students had retained much of what they had learned the previous Spring when the material on the test was covered. This was an indication that their mathematics knowledge of whole number arithmetic was, for the most part, intact and the process of building a foundation for algebra coursework could move forward smoothly.

Because of their head start in whole number arithmetic Chelsea students completed work in fractions and decimals during the Fall 1979 term and were in a position to begin coursework in algebra in the following Spring term. Student achievement on the twelve unit tests that were given in the Fall prealgebra course averaged around $80 \%$. This performance by the students was very encouraging and lent some credence to the premise that students could master prealgebra coursework in the space of one semester independently of their prior mathemetics proficiency and background.

Surrounding this fine student performance was a rising tide of resistance to the program that was being demonstrated by the participating mathematics teachers. The
resistance was manifested by "communication problems" with the school's mathematics and science chairperson and with the CMSP staff regarding programmatic details of the model project. Teacher resistance to the CMSP led to their gradual non-participation in after school teacher meetings which the CMSP held regularly with all of the participant teachers to share experiences, discuss problems and to plan for the coming weeks' work. The situation with the teachers became progressively worse in the Spring 1980 semester. In late Spring teacher dissatisfaction reached the point where their continuance in the program was in jeopardy. The unstable situation that had developed at Chelsea High School was creating a strain on student performance and their achievement levels began to falter. The Spring term ended with the two teachers declaring that they no longer wished to continue in the CMSP model development effort. This essentially signaled the end of CMSP model project activity at the school. Chelsea is a small school in comparison to the other five schools and the mathematics department consisted of only five teachers. The fact that two of the five teachers, who were senior members of the department, had expressed dissatisfaction with the program colored the perception of other teachers who might have been willing to participate. The CMSP had always operated on the principle that new intervention programs needed the general support of the faculty, and any attempt to implement new programs without that support is futile, especially where it concerned teachers who were or would be directly involved.

This was the situation at Chelsea High School as understood by the CMSP staff, the mathematics and science chairperson, the principal and the dissenting teachers. The principal assessed the situation by explaining that perhaps the teachers had been with the same group of students for too long and they had grown tired of the overwhelming structure of the CMSP model. This seemed to be a valid point because there were indications from other schools that the block programming of students for two periods of mathematics was causing student behavior problems.

At the end of the Spring 1980 semester, Chelsea High School became the third high school to drop from the CMSP model project. The experiences at each of the three schools demonstrated the unpredictable nature of field based model project development and the degree to which program continuance is function of events and and institutional qualities that are completely beyond the control of the program staff--or the available resources that program might bring to the school. In discussions of these turns of events with Nathan Quiñones, he suggested that a certain "air of stability" must be in place at the school in order for a new program or intervention to take hold. The stability he noted further has to do with the presence of the principal and a collaborative team of school department heads and administrators who, together, establish a supportive school environment in which teachers and students can pursue the process of teaching and learning. Interest in students' academic achievement is paramount as should be the support of faculty and chairpersons' initiatives in working towards these academic aims. If these elements of the schools are in place and functioning to the good of the students, then the ensuing "air of stability" would nurture the growth of intervention programs. The experiences at Benjamin Franklin, John Jay, and Chelsea High School were in reality, more complex examples of the consequences of high student attrition and/or teacher resistance, which, over the short life of the intervention, may have created a situation that impeded the "air of stability" to which Nathan Quiñones referred.

With the conclusion of the project activity at the three high schools in June of 1980, four high schools were left with which the CMSP model could continue to work in the development of a model mathematics achievement program, East New York, Washington Irving, John F. Kennedy and George Washington. George Washington was a newcomer to the CMSP 9th grade model project effort, first participating on the Fall of 1980. Each of the schools represented was unique in its school character and all four had former program association with the CMSP in its 11th and 12th year model enrichment program.

Of the four schools, John F. Kennedy had the most stable program environment in terms of student program retention and supporting supervisory staff. The situation at Washington Irving High School was precarious as the mathematics chairperson there became increasingly dissatisfied with the program and its operating principles. In this instance the impediment was the chairperson clashing with the CMSP's philosophical view that all students could learn mathematics very well. At East New York High School it was the principal who indicated that the model project activity was too costly and that continued school participation would require budget assistance from the High School Division of the New York City Board of Education. Since this was not possible it put future CMSP project efforts at East New York High School in doubt. And finally at George Washington High School the project was proceeding as scheduled but student attrition loomed as a potential problem.

Starting a new year with a new set of students and some new teachers at the four high schools provided the CMSP with an opportunity to update the curriculum model and incorporate changes that reflected feedback provided by the students and teachers who had participated in the previous year. These changes included a more balanced arrangement of mathematics course topics in the two prealgebra courses and also refinement of the unit examinations. This curriculum work was done in the summer of 1980 in preparation for the new group of students that would study prealgebra in the Fall of 1980.

The Fall 1980 term proceeded without major incident in the four participating schools except for Washington Irving High School where the philosophical differences voiced by the mathematics chairperson became a source of rising concern. The situation at the school grew worse towards the end of the Fall term and became somewhat chaotic with a rapid turnover of mathematics teachers teaching the CMSP class that was then studying algebra. Within the space of two months, students in this first cycle of model project activity had the continuity of algebra instruction disturbed by a changeover of four different
mathematics teachers. The effect on the students was devastating and they never fully recovered from the experience. By the end of the Fall term it was clear that the situation had become intolerable and that model project collaboration between the CMSP and Washington Irving could not continue. This was assured by the mathematics chairperson's decision to terminate all CMSP model project activity at the end of the Fall semester. Thus, another negative school outcome was added to the field based experiences of the CMSP. In this case, however, not much was learned outside the fact that collaborative programs may be seriously impeded by philosophical differences as they pertain to programmatic goals.

The Spring semester with three remaining high schools went smoothly and according to schedule. One class at John F. Kennedy completed the first three-semester cycle and took the Regents Algebra Examination. The 28 students who took the examination passed at a $68 \%$ rate, which was significantly higher than the $21 \%$ recorded by the 200 students who studied the same subject in the school's regular mathematics program. This student performance on the first Algebra Regents Examination was very encouraging and revitalized the school's participant teachers and provided a hopeful sign to the teachers at the other two participant schools. It also provided the CMSP with another indication that the higher rate of mathematics instruction (afforded by the double period) was affecting student achievement in a positive way. Six months earlier, in June of 1980, students participating in the CMSP at three schools--Washington Irving, George Washington and John F. Kennedy--did very well on the Regents Competency Test (RCT) in Mathematics. Their pass rate on this basic arithmetic test averaged $80 \%$, which was significantly higher test performance than scored by other students at the same schools who took the test. For example, at Washington Irving, $86 \%$ of the CMSP students passed the test as compared to non CMSP students who passed the test at a $29 \%$ rate.

The fine performance of the John F. Kennedy students on the Regents Algebra

Examination also provided the CMSP with its first indication that the three term model (one term of prealgebra course followed by a two-term algebra sequence) offered students sufficient time to complete course material and contributed to student mathematics achievement. The slow pace of instruction at the other participating schools left the impression that the model curriculum was scheduled in too short a period of time and that possibly another semester was required to cover the course topics in the prealgebra and algebra course sequence. For example, East New York High School was behind in the completion of algebra coursework and CMSP students at the school would not take the Regents Algebra Examination until June of 1981--four semesters after they had started. However, the pace of instruction at John F. Kennedy High School was taken as a reference indicator and curriculum revisions centered on restructuring course topics to solidify the three semester curriculum model. In June of 1981, the model was given additional feedback of a positive nature when 15 of the 19 CMSP students at East New York High School remaining from the first cycle ( 60 students had been enrolled in the CMSP model project two years earlier) passed the Regents Algebra Examination-two students had perfect scores of 100 ! The fine student performance on the Regents Algebra Examination was primarily a function of the the exemplary teaching efforts of one of the participating mathematics teachers. The mathematics teacher, Joan Diller, was part of the CMSP staff that worked on CMSP model development and organizational structure during the Summer of 1980 and planning for the first and second cycles of model test implementation, and later joined the CMSP in 1982 as a full-time staff member, coordinating model program efforts at new participating schools.

The 1980/81 program year ended with the second cycle of John F. Kennedy students taking the RCT Mathematics and passing it at an $89 \%$ rate. This was higher than the $81 \%$ that CMSP students first cycle registered on the RCT in June of 1980 and considerably higher than the $40 \%$ pass rate registered by non-CMSP students studying comparable
mathematics courses at John F. Kennedy in both test years. The high CMSP student achievement on the RCT was another indication that the coursework in prealgebra was strengthening students' arithmetic foundation. It should be noted that little time during the Spring semester was spent reviewing for the RCT Mathematics test. The students passed the test with high scores on the basis of prealgebra and algebra knowledge gained by their participation in the CMSP model project.

The Summer of 1981 was spent revising the model curriculum once again in preparation for the third cycle of students who participated in the model project at John F. Kennedy and at East New York High Schools and a second cycle at George Washington High School. A major change in the curriculum model was made by stopping work on the science curriculum. This was done because of the overwhelming priorities of the mathematics program. It was felt that an effective mathematics curriculum model must first be created before a complementary science program could be developed. Based on the experiences of the first cycle of project activity the mathematics curriculum model would undergo many changes before it was finalized. The development of a matching science curriculum would be put on hold until a structured mathematics curriculum unfolded.

The original textbook that was used in the first cycle for the prealgebra course Quick Arithmetic by Carman and Carman was discarded and a traditional textbook Refresher Mathematics by Stein was substituted. This was done because of negative teacher reaction to the Quick Arithmetic text. Their basic complaints were that there were not enough problems in the book and the reading levels created difficulty for a fair number of students. There was some apprehension about using Refresher Mathematics because it was widely used in the New York City public schools and around the country. ${ }^{2}$ Because of this there was the possibility that classroom instruction would be guided by teacher's previous teaching experiences with the book rather than by the course structure of the CMSP model. However, Refresher Mathematics had a very large number of problems and the
book's outline was consistent with the CMSP prealgebra course outline.
Changes were also made in the content of the prealgebra course and the blocked reinforcement course that gave students an additional period of mathematics instruction. The prealgebra course was restructured to have fewer arithmetic topics and was keyed to the Refresher Mathematics text. The reinforcement was correspondingly restructured to match the topic sequence of the prealgebra course and an array of word problems and geometric configurations were added to give arithmetic applications. These changes were made to further balance the distribution of topics between the two courses and reduce the perception of students that the second course was unofficial or remedial in nature. With these structural changes a whole new set of unit tests was developed and schedules of instruction organized. This development work was followed by the staff development meetings with participant teachers where the changes were discussed and reviewed in preparation for the 1981/82 program year.

Program activity during 1981/82 focused on the test implementation of the model that was beginning to take shape as a complement of six courses that were scheduled over a three-semester period. In each semester the courses were structured to provide students with instruction, reinforcement and applications of a set of mathematics topics, thus giving students and teachers a significant increase in teaching and learning time in a given time period. The course materials developed by the CMSP included problem sets and unit tests that matched and reinforced the content of the two textbooks used in the classroom, Refresher Mathematics by Stein and Elementary Algebra by Jacobs.

From the perspective of curriculum model testing the CMSP project had stabilized to the point where a fair test of the curriculum model could take place. John F. Kennedy, while having some internal disagreements about the program between participant staff and mathematics chairperson, would be completing the second three semester model cycle in January of 1982, and a second group of students would be taking a Regents Algebra

Examination. At East New York High School, the principal indicated that continued participation in CMSP model project activity in the following year 1982/83 required budgetary assistance from outside the school. As per original agreements with the principals at all of the participating schools, the CMSP did not provide any budget support for the second mathematics course offering. The cost of the additional period of mathematics for two participating classes amounted to four tenths of a teaching position. As the program would increase in size naturally, covering the second mathematics class became an increasing burden for the schools, however it was assumed that the commensurate rise in student mathematics achievement would make the CMSP second class allocation a worthwhile school investment. However, from the East New York principal's point of view, there were competing priorities at the school and the allocation for CMSP was a drain on the school's budget; and unless supporting funds for the program were forthcoming from outside sources, East New York High School could not afford to continue participation in the CMSP model project effort. Since this was not possible within the budget structure of the CMSP and its resource allocations, the third cycle of students who began their study of mathematics using the CMSP model in September of 1981 was the last. The CMSP students at East New York would continue in the program for three semesters and take the RCT in June of 1982 and the Regents Algebra Examination in January of 1983. This was the same Regents testing schedule that would be used by third cycle students at John F. Kennedy High School and second cycle students at George Washington High School. All of the schools would start prealgebra coursework with two heterogeneously grouped classes of students selected randomly from the entire incoming 9 th year student population.

Faculty resistance to the CMSP model project activity was becoming evident at John F. Kennedy High School even though seven teachers in the department of about 30 teachers had participated. This turn of events at the school was curious because student
performance on the RCT mathematics and the Regents Algebra Examination had already demonstrated (on two occasions for the Regents Algebra Examination and three for the RCT mathematics examination) that students participating in the CMSP model did significantly better than comparable students on the tests--exceeding the two-to-one differences that were established as a CMSP reference standard for comparison. It was expected that this fine performance would foster greater teacher participation and an interest on the part of the mathematics department faculty to expand the program to include more students. There did not appear to be budgetary problems as the principal--who was very supportive of the program and impressed with student test achievement--made the commitment to support an expansion to four classes for the third cycle of project activity. However, in accordance with the CMSP organizational model four teachers were required to teach the four model classes. CMSP and the department chairperson's efforts to recruit two additional teachers who would teach in the expanded model undertaking (four classes) failed and the third cycle project activity proceeded with two classes.

The reasons for the lack of greater teacher participation at John F. Kennedy appeared to be similar to the situation at Chelsea High School although not as acute because of the greater size of the John F. Kennedy High School mathematics department staff. Beginning intervention programs require a great deal of time and effort on the part of participating teachers. There is a certain amount of inertia that has to be overcome whenever something new is started, especially if the new task differs considerably from one's previous experience. The new task becomes a burden to the mind and requires steady concentration, and, over a period, of time can tax the patience and enthusiasm of teachers who already carry a great responsibility to teach adolescent students the abstractions of high school mathematics--especially given the highly structured format of the CMSP model.

Another reason for faculty resistance at John F. Kennedy High School may have been
the faculty's perceived need for the CMSP model in particular. Intervention programs are created to solve problems or to fill a need. However, "need" is a term that can have many facets, and from the faculty's viewpoint, the CMSP model wasn't needed because the school, although classified as Chapter 1, shared few of the problems of the other participant schools. Attendance and retention were high, and the proportion of students enrolling in the three-year sequence of Regents mathematics was substantially higher than the other schools--e.g., there were eight classes of Regents Geometry at John F. Kennedy compared to two at George Washington and one at East New York. Perceived need, therefore, can play a rather important role in faculty acceptance of an intervention program. While this point of view is speculative, it may be plausible for John F. Kennedy High School, given the the resistance of the faculty towards CMSP model program expansion, in the light of significant mathematics student achievement on the RCT mathematics and Regents Algebra Examinations.

The situation at John F. Kennedy became more curious as the second cycle students took the Regents Algebra Examination in January 1982. As their first cycle CMSP counterparts did a year earlier the students in the second cycle passed the Regents Algebra Examination with a greater than two-to-one ratio in comparison to non-CMSP students at John F. Kennedy- $-68 \%$ vs. $29 \%$. The results at George Washington High School showed the same substantial differences with CMSP students outperforming non-CMSP students by margins of almost three-to-one- $64 \%$ vs. $22 \%$. The significantly better Regents Algebra results for CMSP students at John F. Kennedy (for the second time) failed to influence faculty acceptance of the model, and continuance of the program with a fourth cycle of students starting in the Fall of 1982 was questionable.

In the Spring of 1982 a new mathematics chairperson joined the department at John F. Kennedy. The new mathematics chairperson's appointment provided fresh department leadership and it was expected that CMSP model activity would continue on a more solid
footing. Discussions were held with principal and mathematics chairperson to explore the possibility of expanding the program to include eight classes in the Fall of 1982. In accordance with the model's staffing pattern, eight teachers were required, limiting each teacher to teaching two CMSP classes. Because this was to be the first large scale test of the CMSP in a Chapter 1 school, the CMSP held firm to this teaching arrangement for two basic reasons: 1) it was important that the larger model test be supported by the larger faculty and eight teachers volunteering to take part would be an affirmation of faculty acceptance of the CMSP model, and 2) testing the model with less than eight teachers would cause an imbalance in the staffing pattern (each teacher teaching two classes of the CMSP same course) of the eight paired classes and would thus introduce variables in CMSP teaching load that the CMSP wanted to minimize.

The CMSP program requirements were presented to the principal and mathematics chairperson and an effort was begun to recruit eight teachers who would participate in the Fall of 1982. The ensuing weeks were not fruiful as faculty resistance to the program continued unabated. The principal recognized that the recruitment of eight teachers, given the mathematics department's resistance to the CMSP model, would not be possible. In light of the situation, the chairperson suggested that the eight paired class program be implemented with four teachers rather than eight. However, this plan would increase the CMSP teaching load to four classes which would be too much of a burden on the teachers given the structure of the CMSP model. It was felt that such a change in the composition of model staffing pattern would confound the issues of model program development.

However, the major concern for the CMSP in this instance was whether a model program intervention serves any useful or valid purpose if it could not gain a consensus from the mathematics department faculty at large. On this point, the John F. Kennedy mathematics department faculty were not accepting the model and continuance of the CMSP model project activity would be in vain and not contribute any more to model
program development than it had already in its three cycles of model test implementation. And the contributions had been substantial in terms of demonstrating the feasibility of the model elements including the ground zero start, the random selection and heterogencous grouping classes, the benefits of uniform pacing and external testing and the usefulness of a coordinated double period of mathematics instruction. The three groups of students and teachers which participated in the three cycles of model test implementation at John F. Kennedy High School had proven the usefulness and viability of the three-semester model. And the students' repeated achievement on the RCT mathematics and Regents Algebra Examinations was evidence that their test achievements were not merely chance occurrences. In the end it was the time and efforts of the seven participating mathematics teachers that provided the impetus and energy to overcome the inertia of the model project and to follow the individual program through to conclusion. It was unfortunate the CMSP model project activity could not have reached a greater part of the faculty. This was a reality, however, and the problem of faculty consensus of the CMSP model that surfaced at John F. Kennedy and Chelsea High Schools (and later at other high participant high schools) had to be confronted as a possible major impediment for wide scale replication of the model when such dissemination efforts are organized.

By mid-Spring of 1982 it was clear that George Washington High School would be the only Chapter 1 high school participating in the model test implementation with a new group of students. Both John F. Kennedy and East New York High Schools would not be continuing in the project past the current group of students who would complete the CMSP three-semester model sequence in January of 1983 with the taking of the Regents Algebra Examination. This presented a problem because the virtue of systems and field based model project development is a paralleling of the development and testing in different schools sites. The value of this parallel approach rests on obtaining similar outcomes at schools sites which have widely different school characteristics. When a common
outcome appears there is a high likelihood that the project is on course in its model development and systems organization. Given this necessity, efforts were made in the Spring of 1982 to recruit two more schools for the Fall 1982 term.

Chester Singer, who was part of the CMSP central staff and served the project as academic program coordinator, knew of two former colleagues at John Jay High School who were currently chairpersons at Park West High School and Eastern District High School. In the analysis of Regents data it was found that student Regents mathematics achievement at these two Chapter 1 schools was sufficiently low to warrant attention to the CMSP model by the mathematics chairpersons of Park West and Eastern District High Schools. Meetings were scheduled with the two mathematics chairpersons and the particulars of the CMSP model were described, citing the conditions for participation, including random student selection and the double period requirement. In addition to the standard model, the CMSP asked that the beginning student population be set at four classes with each of four teachers scheduled to teach two classes. Further, because of the increase in program size the school would have to designate a school coordinator from the pool of four teachers who would teaching in the CMSP model program. This school program coordinator would have his or her teaching load reduced by one period and the additional time during the school day would be used to coordinate program activity at the school and also serve as liaison to the central CMSP staff. With this new organizational plan--which was a prelude to the networking of schools--the participant schools would have to allocate the equivalent of one full teaching position to the program, eight-tenths of which would be used to cover the cost of staffing the four additional mathematics classes and two-tenths for the school program coordinator.

The new plan was agreed to by the mathematics chairperson and presented to the principal of Eastern District and Park West High Schools. They consented to participate and preparation for the Fall model test implementation began in earnest with the selection
of teachers and starting the administrative processes for the random selection of students and the scheduling of the four classes. The selection of teachers created a little bit of concern at Eastern District where there were misunderstandings about the developmental nature of the model project effort. The chairperson had interpreted the program as being one of service rather than development and, as such, had assigned a number of inexperienced teachers to the initial effort when just the opposite was expected. Because of the inertia required in beginning intervention programs it is extremely important that senior experienced teachers participate in the first cycle of model test implementation. This is essential for two reasons: 1) there are many aspects of the model that take "getting use to" and often the model program must call on the participant teachers' long classroom experience to adapt to the program peculiarities or to overcome program hurdles that appear frequently during the first cycle of project activity, and 2) model program expansion is dependent on teachers' perception of the value of the model; and this is more effectively disseminated to other teachers in the department by a senior faculty member than by a less experienced teacher.

The influence of the senior faculty member which would take place at the very inception of project activity was seen as part of the solution to the problem of faculty acceptance of the model project and its test implementation strategies. This would help alleviate faculty skepticism to the detriment of the project before it got started. And thereafter, the project could be judged on its merits to promote student mathematics achievement and eventually increase teacher opportunity to teach higher level mathematics courses. The latter is a long term consequence of an effective intervention mathematics program and must be considered seriously by the entire department if the model project efforts are to take root and operate in a stable departmental environment.

In Eastern District's case the CMSP model program was already jeopardized by the misinterpretation by the mathematics chairperson that the model project needed no special
attention, when in fact it did. Teacher selection was a very important part of the initial processes of program organization. Even before project efforts began, the skepticism within the department at Eastern District High School was keeping senior faculty away from volunteering to participate in the program. As events developed only two classes were selected to begin the first cycle of project activity. A senior mathematics teacher agreed to serve as school program coordinator along with a less experienced teacher. The senior teacher would teach the prealgeabra course and the less experienced teacher would teach the mathematics reinforcement course. The CMSP entered in this agreement with some reservation, knowing of the obstacles that would confront the teachers (especially the less experienced teacher) as they became involved with teaching in accordance with the model structure.

The situation at Park West High School was similar to that at Eastern District High School except that the mathematics chairperson seemed to have a genuine understanding of the CMSP model effort, the reason for its highly structured format and its intended goals. He was very eager to see the program work, but, upon reflection, did not think that starting with four classes was appropriate. Accordingly, he assigned two teachers to teach two classes in the model project. One of the teachers was mathematics licensed and had over ten years of teaching experience while the other teacher had less experience and was not licensed. The selection of the less experienced teacher to participate in the first model project cycle was based on the chairperson's belief that the inexperienced teacher could do justice to the program because of his sensitivity to young students. And also teaching the second mathematics "reinforcement and applications" course would be good experience because of the CMSP teaching "partners" and block programming arrangements which would promote interaction between teachers. This is a form of experiential teacher training that would later prove to have great value as the program expanded within the school and at other school locations.

In late Spring of 1982, plans were made to do a major revision of the CMSP model program format during the Summer of 1982 based on the experiences of the second and third cycles of model test implementation. One of the major criticisms of the model was the second reinforcement period which teachers felt was perceived by students as not being as important as the first course. As originally conceived it was thought that the block programming of classes and the teaching team arrangement would foster interaction and discussion between the two teachers. But this did not happen as often as expected and was dependent on the personal teaching styles and sociability of the two teachers-characteristics which were difficult to predict or arrange. One solution to this problem would be to tie the two courses together so they could complement one another by curriculum themes. If this was done with sufficient structure it would minimize the need for frequent interaction between teachers and also separate the courses as two distinct mathematics classroom learning experiences. In this complementary course format, treatment of a single arithmetic topic was numerical as developed in the first course and geometric as reinforced in the second course. This complementary course arrangement would also fulfill another criticism of the second course, the fact that there was no regular course testing. In the new curriculum design the second course with its geometric theme would stand on its own and have its own set of unit tests that would be tied to the testing program of the first course. If this complementary course design was sound in practice, students and teachers would look at the second course as official and just as important as the first course. And, hopefully, student effort and concentration in both courses would strike an even balance.

The new curriculum design work was carried out over the summer with a sense of anticipation that a framework of the CMSP model would be developed and test implemented in the 1982/83 year that would give further evidence that "all students can learn mathematics very well given the foundation and academic support for the
mathematics they are expected to learn in the classroom". The complementary course structure in prealgebra would significantly increase student learning time in the classroom and this would strengthen students' foundations for subsequent coursework. The two courses would also give the students the necessary academic support because they would be taught the same mathematical topics from different perspectives from two different teachers. If students have difficulty learning the mathematics topics from one teacher they will generally learn from the paired teacher. But more importantly the duality of the courses provided two uniform course records that could be assembled for students enabling diagnosis on a much broader scale than can be done by a single course. Through frequent compilation of achievement data, a longitudinal student profile could be organized and utilized to identify trends in student achievement as they progress through each of the CMSP courses.

The 1982/83 program year was a watershed of project activity. This included testing the new model concept and organizing a large scale development of a model curriculum prototype that would be test implemented in seven NYC high schools beginning in the Fall of 1983 and continuing thereafter in three overlapping cycles of two year duration. The new larger scale effort was based on the model project experiences at the three schools that were originally involved in CMSP project activity and still were participating in the Fall of 1982--John F. Kennedy, George Washington and East New York high schools. These three schools, in particular, had provided the consistency of effort and ensuing student achievment that helped shape the format and structure of the complementary course model mathematics program. With this new design the CMSP could move forward and assemble the elements for a comprehensive curriculum design that would be subjected to a wide scale test in later years with increasing student populations.

Brooklyn Technical High School: Model project activity at Brooklyn Technical High

School represented a distinct and separate project effort that was primarily testing the question of whether the model and its constructs would be beneficial to students who were academically prepared to enroll in a first course in Regents Algebra. Would these students achieve at higher level than their non-CMSP counterparts who would study algebra without using the CMSP model? Model test implementation would be the same as in the other Chapter 1 schools. There would be a random selection of students-- however, because of increased resources made available by the school, there would be four classes rather than two. Students would be heterogeneously grouped in classes that would be block programmed for a double period of mathematics and one period of science. The major difference in the model program at Brooklyn Tech was that students would not enroll in the prealgebra course but instead be enrolled in a traditional two term Regents Algebra program that would be structured for two courses in mathematics --with each of the courses taught by a different mathematics teacher. Another difference was the science program which was structured around the school's Material Science course but was modified with application modules and projects that was tied to mathematics topics.

The primary reason for CMSP model project activity at Brooklyn Technical High School was the school's tremendous potential for enlarging the national pool of Black and Hispanic students with the mathematics background to pursue engineering college study. The school's Black and Hispanic population is large enough so that, under ideal circumstances, it is possible to significantly impact the total Black and Hispanic first year enrollments in the nation's engineering colleges which in $1979 / 80$ hovered around 10,000 students. To obtain a perspective of this possibility, the following logic applies. In 1979 when the CMSP started working with Brooklyn Technical High School, its student enrollment stood at 5,173 with 3,088 of these students, (or close to $60 \%$ ) being Black and Hispanic. The graduating student population in that year was 1,051 with an estimated 500 students being Black and Hispanic. If it can be assumed that each of these

Black and Hispanic students had successfully completed the school's rigorous program of Regents mathematics and science, then a sizable pool of students would be eligible to consider college study in engineering. Given the school's tradition for preparing its graduates for engineering college, it is not speculative to state that $50 \%$ of the graduating class would select engineering as their first choice of intended major, the school's technical programs are structured towards this aim. ${ }^{3}$ Using this logic, then Brooklyn Technical High School could effectively contribute 250 students or $2.5 \%$ of the total national pool of Black and Hispanic students who enroll as freshmen in the nation's engineering colleges. This is a significant pool of potential Black and Hispanic engineering students emanating from one school--Brooklyn Technical High School.

However, the reality that prevails at Brooklyn Tech does not support the logic, because Black and Hispanic students who complete the three-year Regents mathematics and science sequence and who excel in their mathematics studies represent only a fraction of the ideal as presented above. In working with the school, in years previous, in the after school CMSP model enrichment program, what was found to be the major issue was the low number of Black and Hispanic students who achieved at a high level in the study of 11th Year Regents Mathematics. These were the students who were to be prime candidates for engineering colleges, but yet were not found in the numbers that the total Black and Hispanic 11th year student population at Brooklyn Technical High School would be expected to yield.

When the idea of CMSP model project and the mathematics and science chairpersons' participation first arose in discussions with the Principal in the Spring of 1979, there was agreement that working with incoming 9th year students in their first Regent mathematics course experience might help increase the overall pool of Brooklyn Tech students who achieve at a high level in 11th Year Mathematics. One reason for this viewpoint was the fact that pass rates in Regents Algebra examinations at Brooklyn Technical High School
were well below those of the two other specialized high schools. For example, in June of 1979, Stuyvesant and Bronx Science had pass rates of $98 \%$ and $94 \%$ respectively, as compared to Brooklyn Tech which had a pass rate of $81 \%$. For a specialized high school, failure rates of $19 \%$ in 9th Year Regents algebra can have a marked effect on the school's mathematics programs. At Brooklyn Tech this was especially so because of its large enrollment and the fact that 993 students took the June Regents Algebra Examination as compared to 177 at Stuyvesant and 377 at Bronx. These numbers of algebra exam takers reflect both school size and also the fact that a higher proportion of Stuyvesant and Bronx Science students had completed the Regents algebra course of study prior to high school entry. A failure rate of $19 \%$ on the Regents Algebra Examination at Brooklyn Technical High School translated to 240 students who would have to repeat the course in the following year. 240 students is equivalent to 8 full-size classes which would have to be staffed by the school's experienced mathematics teachers who, under more favorable circumstances, would be teaching higher level courses or courses with students on track in their mathematics studies. This high failure rate in a first course in Regents Algebra not only is a drain on the school's instructional resources, but is demoralizing for teachers as well. Few teachers look forward to teaching a class which has a history of failure, even a basic course in algebra. In addition, high failure rates in the first course in the traditional three year Regents mathematics sequence establishes an off-rrack precedent for a large pool of students at the school, which, over a period of time can preclude their graduating with sufficient course credit to earn the specialized high school diploma.

It is important to note that at the three specialized high schools there are no RCT or general mathematics course offerings, and students at these schools must enroll and pass the Regents examinations in each of the 9th, 10th and 11th Year Regents mathematics courses in order to qualify for the specialized school high school diploma upon graduation. The fact that all students at the three schools are required to take the same Regents
sequence of courses made the CMSP model test implementation at Brooklyn Technical High School a special project undertaking. This was because, unlike the Chapter 1 high schools, there was a group of students within the school with which to make objective and direct comparisons of Regents Algebra exam performance.

The four classes that were selected to participate in CMSP model project activity at Brooklyn Technical High School were not chosen randomly in the same manner as in the Chapter 1 high schools. At the Chapter 1 high schools, the students were chosen by random number assignment, while at Brooklyn Technical High School, four of the classes of the thirty classes of incoming 9th year students that were scheduled for 9th year Regents algebra courses were assigned to the four mathematics teachers who were selected to participate in model project activity. The four mathematics teachers were all licensed and had considerable experience teaching Regents mathematics. One of the teachers would serve as school program coordinator in the later cycles of the model project. Regular meetings were held with the four teachers during the Spring semester of 1979 and they were fully prepared to engage in the model test implementation when it began in the Fall of 1979.

The CMSP model program at Brooklyn Technical High School differed significantly from the CMSP program at the Chapter 1 high schools. There were none of the serious problems that afflicted the other schools. Attendance was very high as was retention and teacher participation was energetic and directed at implementing the program as designed and scheduled. Even before the end of the first semester, it was clear that the two mathematics class periods, driven by a paired teaching team, was having an affect on student mathematics performance. One of the teachers observed that students in his two CMSP classes appeared to be learning algebra with greater depth than other students (not in the CMSP) in algebra that he was teaching. However, his comments were tempered with the first sign of teacher fatigue that seemed to be experienced by teachers who were
teaching the second course, mathematics reinforcement. This second course effect appeared to be a common criticism that extended itself to the Chapter 1 high school but not the junior high schools.

The two mathematics teachers of the second course were also involved with utilizing personal computers to provide algebraic applications by teaching students BASIC programming. These efforts with computers were not fruitful primarily because (it was thought) that six computers were available in the class which created logistical problems and made classroom instruction difficult. However, one of the mathematics teachers noted that learning how to program in BASIC seemed to be at odds with students' learning algebra. In his terms, "it was putting the cart before the horse" because students first needed to be fully conversant with algebraic operations and expressions before they could be prepared to fully appreciate the algebraic syntax inherent in BASIC programming. However interesting, these initial problems with computers did not detract from the academic progress of the participating students.

The first year of model test implementation at Brooklyn Technical High School went very smoothly and the students did surprisingly well on the June 1980 Regents Algebra Examination, with 106 of the 107 students passing the examination--a $99 \%$ pass rate! This was in comparison to $88 \%$ for the 528 students who studied algebra in the school's regular mathematics program. While the pass rates (a passing score is 65 or higher) between the two groups of students did not differ widely, there was a significant difference in the number of CMSP students who scored 90 or higher on the examination. For the CMSP students it was $58 \%$ and for the non-CMSP students it was $33 \%$--not exactly a two-to-one difference, but approaching it.

Because there was a significant number of CMSP students who scored 90 and higher on the examination, the school decided to create a special class of CMSP students who would take honors geometry in the following year. In retrospect, this was probably not a
good idea because, as matters developed the students selected did not fare as well in the honors geometry course as they did a year earlier when they studied algebra. Their performance on the Regents Geometry Examination showed no appreciable difference between the school's larger population that took the test. The larger pool of CMSP students who were mainstreamed into the school's regular geometry program also did not achieve as well on the Regents Geometry Examination as they did on the Regents Algebra Examination The subjects of high school algebra and geometry are sufficiently different in course content to preclude direct learning transferences, however it was felt that because of the students' high test achievement in algebra that their self confidence in mathematics would extend to continued high achievement in geometry. This did not occur and, in fact, may have given the students a false sense of security because of the additional course time experienced in algebra. Nevertheless, the study of geometry loomed as the next step in the effort to develop curriculum models to increase the pool of students at Brooklyn Technical High School who achieve at high levels in the three year Regents mathematics sequence. However the primary issue in June of 1980 was to prepare for a second cycle of CMSP model test implementation that would determine whether the high student Regents Algebra test performance in the first cycle was legitimate and the basis for future model program expansion. Plans were made to implement the model program with slight modifications with the same number of students--four classes.

In the 1980/81 year there was a sizable increase in 9 th year student enrollment with over 1,200 students from which to choose the four classes of students who would participate in CMSP model project activity. In the second year there was a second attempt to incorporate BASIC computer programming in the second algebra reinforcement class, except this time a classroom equipped with 15 personal computers was available to give students greater opportunity to work with the computers in class. Again the chief deterrent for their effective usage appeared to be students' lack of understanding of algebraic
operations which they were learning concurrently. Although the students enjoyed working with the computers, there was no compelling evidence to show that it was helping the majority of students learn algebra-- and learning algebra was the basic objective of the CMSP model project activity.

As in the previous year the model project test implementation ran smoothly and in June of 1981 the CMSP students took the Regents Algebra Examination and again CMSP student performance was impressive and significantly higher than non-CMSP students' at the school. The much larger entering 9th year population lowered the pass rate at the school significantly with only $72 \%$ of the 1,162 students who took the test obtaining a passing grade. This compared with a $91 \%$ pass rate for CMSP students. At the higher end, test score differences were much wider, with $48 \%$ of the CMSP students scoring $\mathbf{9 0 \%}$ or higher as compared to $\mathbf{2 8 \%}$ for non-CMSP students. The New York State Regents Mathematics Examinations can differ in their level of difficulty on year-to-year basis, therefore, making comparisons from year to year can only be done in relative terms. However the real value of the student performance was the near majority of students who achieved 90 or better on the Algebra Regents Examination. This was important because it showed that the model appeared to be promoting high level mathematics achievement and thus could be a useful strategy for all students. One of the initial concerns at Brooklyn Technical High School was that the second period of mathematics might be construed by students as being remedial and, thus, actually be counterproductive. This concern did not materialize. In fact most of the students liked the idea of having the two mathematics courses and two teachers that were part of the parallel course model.

In the third and fourth cycles that took place in the 1981/82 and 1982/83 academic years, the model program size at Bruoklyn Technical High School was increased in steps, with over 200 students participating in the third cycle and over 300 in the fourth cycle. These larger numbers of students provided opportunities for more teachers in the
mathematics department to participate and with it came murmurs of faculty concerns that were similar to those voiced by the faculty at John F. Kennedy High School--Why is the CMSP needed at the school? While there were no major impediments in implementing the model program on a larger scale during the third and fourth cycles, the concerns of the faculty raised the key issue of the value of an intervention program that leads to higher student mathematics achievement but fails to gain a consensus of the faculty. From the CMSP's view point this was a problem that had to be resolved if model project activity was to continue at Brooklyn Technical High School.

The CMSP model project was given a boost in both the third and fourth cycles of test implementation by the continued exemplary performance of the CMSP students on the Regents Algebra Examinations. On the June 1982 Regents Algebra Examination, the 218 CMSP students in the third cycle passed the test at an $88 \%$ rate as compared to $60 \%$ of the 894 non-CMSP students. Regents performance on this particular test was down city wide, as there was a general recognition amongst New York City mathematics chairpersons that the test was more difficult than in previous years. With scores of 90 and above, the $29 \%$ for CMSP students was more than twice the $13 \%$ posted by non-CMSP students. In June of 1983, 318 CMSP students passed the Regents Algebra Examination with a $96 \%$ pass rate as compared with 436 non-CMSP students who passed the exam with a $78 \%$ rate. As on previous exams CMSP student performance with scores of 90 and above was significantly higher than non-CMSP students-- $50 \%$ for CMSP students versus $22 \%$ for non CMSP students, a better than two-to-one margin.

Over the four cycles of CMSP model test implementation there was a consistency of high student achievement on the Regents Algebra Examination that was clearly superior to students who did not participate in the model project. Curriculum concerns had been ameliorated to the general satisfaction of a majority of the participating faculty. And the student selection process was revised to allow a special honors class to be formed on the
basis of student mathematics scores on the school's admission test and on the CMSP preevaluation test given to students upon their arrival at the school in September. These changes in program format did help to increase faculty acceptance of the model, however, at the end of the fourth cycle of CMSP model project activity, there still remained an air of skeptism about the program and its value for increasing student test achievement above Regents Algebra. In the tracking of student achievement as they progressed in mathematics at the 10 th and 11 th grades there seemed to be little indication that students who participated in the CMSP model project at the 9th year did any better than students who did not participate. Solving the problem at these upper level mathematics courses--which was the crux of the problem at Brooklyn Technical High School--would require intervention over a longer term scale than was being explored by CMSP model project efforts in 1983.

### 5.3 The Compilation and Analysis of Mathematics Achievement Data

Almost all student academic achievement in schools is measured by some form of classroom testing. There is cause for argument that a single test may not always be indicative of what a student has learned in the classroom. However it is the common school instrument for assessing student achievement and its intrinsic value is governed by how closely the test reflects what is taught in the classroom and by the logistics that surround test administration. Testing, both in the classroom and year end Regents examinations have been the measures that have guided CMSP model development and research. The CMSP testing program was given impetus and new meaning because of the parallel arrangement of the participant schools and their involvement in adhering to the practice of administering classroom tests that were constructed by the CMSP.

The major advantage of the parallel school arrangement was that a process was arranged whereby a common school outcome--student achievement on a given
examination--could be sought as a measure of whether the program was having an effect on increasing student achievement. If a common outcome in student test achievement was obtained from several participant schools which had different school characteristics it could be surmised that the model program was an important factor in producing the outcome. It has been this assessment rationale that has driven the CMSP in its model development and research efforts.

The abundance of class room testing and reporting of results at the participant schools sites contributed to the development of curriculum as it enabled short term revisions with little delay as the model was being test implemented during the academic year. During the first cycle, for example, any single unit test administration covering a specific topic in prealgebra was taken by students at participating schools within the space of a single week. These data were reported quickly by their schools to the central CMSP offices and compiled and analyzed to determine trends in student achievement. This constant flow of test data provided almost immediate feedback and allowed the CMSP staff to make adjustments in the model curriculum as deemed necessary and warranted. Besides being of value to model development and research, it also gave the schools a sense of how the model program at their schools was proceeding, and because of the uniform pacing and external testing strategy, they could make comparisons of student test achievement with other schools participating in the CMSP. Since many unit tests were given throughout a term a rather extensive longitudinal profile was developed for each school. At the end of the first term, data summaries were constructed to indicate how participating schools performed on the unit tests and cumulative tests. Table 21 shows the summary that was constructed at the end of the Fall 1979 term of the first cycle of model test implementation.

V!hile the CMSP repetitive unit tests serve to guide short term model development. it is the New York State Regents Competency Test (RCT) in Mathematics and the Regents Algebra Examination that established the growth and progress of CMSP model

COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP) SUMMARY OF HIGH SCHOOL PERFORMANCE FALL 1979

|  | $\frac{\text { CHELSEA }}{(\mathrm{n}=45)}$ | $\frac{\frac{\text { EAST }}{N . X_{0}}}{(\mathrm{n}=57)}$ | $\frac{\text { BEN }}{\text { FR'NKLIN }} \begin{aligned} & (\mathrm{n}=46) \end{aligned}$ | WASH. <br> IRVING <br> ( $\mathrm{n}=53$ ) | $\frac{\mathrm{IOHN}}{\frac{\mathrm{JAY}}{(\mathrm{n}=64)}}$ | $\frac{\text { IOHN E }}{\frac{\text { KENNEDY }}{(\mathrm{n}=51)}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { PRE-EVALUAT } \\ & \text { MEAN }(X) \\ & S D \end{aligned}$ | $\begin{aligned} & 75.23 \\ & 20.43 \end{aligned}$ | $\begin{aligned} & 54.65 \\ & 27.12 \end{aligned}$ | $\begin{aligned} & 42.30 \\ & 23.90 \end{aligned}$ | $\begin{aligned} & 55.50 \\ & 19.40 \end{aligned}$ | $\begin{aligned} & 49.45 \\ & 25.76 \end{aligned}$ | $\begin{aligned} & 74.02 \\ & 21.77 \end{aligned}$ |
| $\begin{aligned} & \text { CUM WHOLE } \\ & \text { MEAN (X) } \\ & \text { SD } \end{aligned}$ |  | $\begin{aligned} & 75.47 \\ & 20.47 \end{aligned}$ | $\begin{aligned} & 52.80 \\ & 27.40 \end{aligned}$ | $\begin{aligned} & 71.16 \\ & 18.97 \end{aligned}$ | $\begin{aligned} & 64.60 \\ & 25.07 \end{aligned}$ | $\begin{aligned} & 82.86^{*} \\ & 21.36^{*} \end{aligned}$ |
| LATEST L.U. EXAM WITH MASTERY | D1 <br> (Meaning of Percent) |  | B3 <br> (Div.of <br> Fractions) | B1-B5 <br> (Cum. <br> (Fractions) |  |  |

L.U.EXAM AVE.

|  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| EXAMS TAKEN | 12 | 10 | 8 | 11 | 9 | 10 |
|  |  |  |  |  |  |  |
|  |  | 77.38 | 74.63 | 66.70 | 83.09 | 72.54 |
| MEAN $(X)$ | 15.50 | 19.71 | 20.10 | 10.70 | 19.82 | 17.81 |
| SD | $47 \%$ | $60 \%$ | $30 \%$ | $68 \%$ | $53 \%$ | $65 \%$ |
| \% ABOVE 80\% | $82 \%$ | $77 \%$ | $61 \%$ | $94 \%$ | $73 \%$ | $84 \%$ |
| \% ABOVE 65\% | $18 \%$ | $23 \%$ | $39 \%$ | $6 \%$ | $27 \%$ | $16 \%$ |
| \% BELOW 65\% | $18 \%$ |  |  |  |  |  |

[^4]TABLE 21
development and research over the long term. Both of these examinations are administered statewide and are generally accepted by schools and school systems as standard reference measures by which to gauge and evaluate students as they progress in their study of mathematics. The Regents mathematics examinations are a tradition in New York State junior and senior high schools and are administered three times each year for each course in the three-year mathematics course sequence. In particular the examinations are constructed by a revolving committee of mathematics school teachers to reflect the scope and content of the New York State Regents Mathematics course syllabus, and therefore student achievement on the Regents examinations has been traditionally used to determine both student and school Regents course performance. Because the examination is administered statewide on the same day, three times a year, frequent and timely test comparisons can be made within schools, across schools, within a district and between districts. The achievement data are reported centrally to the New York City Board of Education and the New York State Education Department where current and longitudinal data are available for research documentation and school administrative purposes.

The Regents Competency Test (RCT) in Mathematics is a more recent test and has been been administered by the New York State Education Department only since 1980. Unlike the traditional Regents mathematics examinations, which are course specific, the RCT does not reflect any particular mathematics course of study but is constructed to measure students' knowledge of general mathematics as a requirement for high school graduation. The basic content of the RCT mathematics is arithmetic at a 7 th or 8th year level. The RCT mathematics examination's appearance as a statewide mathematics testing instrument proved to be of value to the CMSP primarily because of its arithmetic format. At the time of the first cycle of CMSP model test implementation in the Fall of 1979, the reference measure to be used to gauge student achievement over the three-semester CMSP model curriculum period was to be the Regents Algebra Examination. While this was a
useful measure of student achievement in algebra there was no equivalent measure of students' prealgebra achievement, except the inferences that could be drawn by their commensurate achievement in algebra. It would have been possible to use the CMSP's array of unit tests in the prealgebra course to assess the longitudinal course performance, but these were internally constructed tests that could not be used to compare arithmetic student performance within and between participating schools. The RCT mathematics test served this purpose.

The three-semester configuration of the CMSP model curriculum provides two distinct milestones upon which to assess student mathematics achievement, 1) at the end of the first year in which students complete a semester of prealgebra and a semester of algebra and 2) at the end of the third semester when students have competed the coursework in algebra as prescribed by the New York State Board of Regents. New York State regulations require that students complete a year of mathematics before the RCT mathematics can be administered. This requirement fits nicely with the CMSP's plans for the model testing schedule after the completion of two terms of coursework because it provides a mechanism by which the participant schools can administer the test to other students at the school studying RCT or general mathematics.

With the RCT mathematics test as a measure for CMSP prealgebra coursework and the traditional Regents Algebra Examination as a measure for algebra coursework a legitimate and widely recognized testing system was in place by which to compile and structure achievement data for the cycles of CMSP model test implementation that would ensue from September 1979 to June of 1983. Before the four years elapsed, the CMSP had accumulated and compiled RCT mathematics achievement data for the June RCT mathematics test administrations of 1980, 1981, 1982 and 1983. For the Regents Algebra Examinations test achievement data were available for the participant Chapter 1 high schools for the January test administrations of 1981, 1982 and 1983, (plus a June 1981
test administration at East New York High School) and for Brooklyn Technical High School for the June test administrations for 1980, 1981, 1982 and 1983. The data were compiled for the CMSP participating classes and other classes within the schools that were studying mathematics courses that culminated in the RCT mathematics tests or the Regents Algebra Examination.

Only at Brooklyn Technical High School were comparisons with non-CMSP students made with students who entered the school at the same grade and time as the CMSP students-i.e., all of the students who took the Regents Algebra Examination were true 9th graders. At the Chapter 1 schools there was no clearly defined grade level by which to separate non-CMSP students. The achievement data comparisons made with non-CMSP students include students from 9th through 12th grades. In the analysis of RCT test scores between grade levels at one school, it was found that RCT mathematics pass rates increased with higher grades--this tends to be a trend with the RCT mathematics because of repeated testing to qualify for high school graduation. Thus the RCT comparisons between CMSP and non-CMSP, if singled out for just 9th year student comparisons, would be more widely separated in test performance levels. Because of the very small pool of students who enroll in Regents Algebra coursework at Chapter 1 high schools, non-CMSP student enrollment in Regents Algebra courses are invariably mixed with students from all grade levels. Add to this the common practice of offering the traditional two-term Regents Algebra sequence over three and four terms and a mixture of students result which are difficult to disentangle for data comparisons.

RCT Mathematics Achievement Data. Tables 22 and 23 show the RCT mathematics achievement test data for the years 1980 to 1983 (for RCTs administered in June). The achievement data compare CMSP students who had completed two terms of mathematics study utilizing the CMSP model curriculum with non-CMSP students who had studied

## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)

NEW YORK STATE REGENTS COMPETENCY TEST (RCT) PERFORMANCE COMPARISONS OF CMSP \& NON-CMSP STUDENTS AT PARTICIPATING HIGH SCHOOLS--JUNE 1980 \& JUNE 1981

JUNE 1980

|  | $\underline{N}$ | NO.PASS | NO. $>77$ | NO. >89 |
| :--- | ---: | ---: | :--- | ---: |
| John F. Kennedy |  |  |  |  |
| CMSP | 36 | $29(81 \%)$ | $19(31 \%)$ | $8(22 \%)$ |
| Non-CMSP | 396 | $168(42 \%)$ | $47(12 \%)$ | $19(5 \%)$ |
| Washington Irving |  |  |  |  |
| CMSP | 29 | $25(86 \%)$ | $12(38 \%)$ | $1(3 \%)$ |
| Non-CMSP | 566 | $164(29 \%)$ | $52(9 \%)$ | $12(2 \%)$ |
| East New York |  |  |  |  |
| CMSP | 32 | $25(78 \%)$ | $18(44 \%)$ | $4(13 \%)$ |
| Non-CMSP | 269 | $81(30 \%)$ | $10(4 \%)$ | $5(2 \%)$ |

JUNE 1981
$\underline{N} \quad \underline{N O} . P A S S \quad$ NO. $>89$

| John F. Kennedy |  |  |  |  |
| :--- | ---: | :--- | :--- | :--- |
| CMSP | 28 | $25(89 \%)$ | $23(82 \%)$ | $7(25 \%)$ |
| Non-CMSP | 223 | $82(37 \%)$ | $24(11 \%)$ | $3(1 \%)$ |

## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)

NEW YORK STATE REGENTS COMPETENCY TEST (RCT) PERFORMANCE COMPARISONS OF CMSP \& NON-CMSP STUDENTS AT PARTICIPATING HIGH SCHOOLS--JUNE 1982 \& JUNE 1983

JUNE 1982

|  | N | NO. PASS | NO. $>77$ | NO, > 89 |
| :---: | :---: | :---: | :---: | :---: |
| John F, Kennedy |  |  |  |  |
| CMSP | 56 | 32 (57\%) | 17 (30\%) | 1 (2\%) |
| Non-CMSP | 875 | 329 (38\%) | 112 (13\%) | 30 (3\%) |
| George Washington |  |  |  |  |
| CMSP | 27 | 12 (44\%) | 6 (22\%) | 1 (4\%) |
| Non-CMSP | 480 | 97 (20\%) | 20 (4\%) | 5 (1\%) |
| East New York |  |  |  |  |
| CMSP | 30 | 20 (68\%) | 10 (33\%) | 1 (3\%) |
| Non-CMSP | 372 | 134 (36\%) | 39 (10\%) | 5 (1\%) |

JUNE 1983

## Park West <br> CMSP

NO. PASS

| NO. $>77$ | NO. $>89$ |
| :--- | ---: |
| $40(54 \%)$ | $6(8 \%)$ |
| $114(15 \%)$ | $23(3 \%)$ |

George Washington
CMSP 66

| $52(79 \%)$ | $30(45 \%)$ | $7(11 \%)$ |
| ---: | :--- | ---: |
| $273(45 \%)$ | $74(12 \%)$ | $12(2 \%)$ |


| Eastern District | 27 |
| :--- | ---: |
| CMSP | 541 |


| $21(78 \%)$ | $11(41 \%)$ | $2(7 \%)$ |
| ---: | ---: | ---: |
| $301(56 \%)$ | $132(24 \%)$ | $24(4 \%)$ |

TABLE 23

Fundamentals of Mathematics (the RCT-directed course) or general mathematics or some other form of non-Regents track mathematics. Data have been compiled and structured to show test achievement at three test score references: number and percent of students with test scores equal to or greater than: 65 (passing), 77 , and 89 . The test reference values of 77 and 89 are in keeping with the standard intervals reported by the New York State Education Department.

In the analysis of test data in the first two years, 1980 and 1981, the CMSP student pass rate was at least twice that of non-CMSP students. In test scores at or above 77 and 89 the differences grew so large as to question the validity of the data. For example, at East New York High School in June 1980, the percentage of CMSP students who scored at or above 77 was $44 \%$ which was ten times higher than the $4 \%$ scored by non-CMSP students. The achievement data at this reference level are made even more pointed by the fact that the absolute number of CMSP students (18) who scored at or above 77, was almost twice that of non-CMSP students (10) even though the number of non-CMSP test takers was eight times higher than the CMSP test takers-- 269 vs. 32 . The same wide variability between CMSP and non-CMSP student achievement occurred at John F. Kennedy High School in June 1981 where the percentage of CMSP students scoring 77 or higher was eight times as high as non-CMSP students.

In the 1981 and 1982 test years the differences in RCT test performance between CMSP and non-CMSP students were still quite substantial with three-to-one differences appearing in test scores at or above 77 at five of the six schools listed. However, the pass rates appear to be somewhat less than two to one but yet are substantially higher for CMSP students.

The wide differences could be attributed to a number of factors: 1) the Cl MSP model project was experiencing a Hawthorne Effect, or first time trial effect, where teacher energy and staff enthusiasm greatly influenced student achievement, 2) the newness of the

RCT (it was administered for the first time in January of 1980) made non-CMSP students less prepared for the test because an RCT curriculum was not yet in place in the New York City school system, and 3) the CMSP prealgebra course and ground zero start had indeed strengthened students' arithmetic foundation and their high test performance on the RCT was reflective of this renewed and stronger knowledge base. Conceivably, CMSP students' high test performance was a combination of these factors, however the consistency of high test performance over the four-year period belies the Hawthorne Effect and argues strongly for the achievement effects induced by students' participation in the CMSP model experience.

What was of utmost interest to the CMSP was the consistency by which the CMSP students at the participant schools outperformed a comparable group of students by two-to-one margins. These schools have widely different characteristics and yet there was a common output. This was a very important indicator that the CMSP model curriculum, as implemented, was having an effect on student arithmetic achievement and that it may be setting the stage for continued achievement in the study of algebra.

However significant the CMSP RCT test achievement data might seem, the one variable that could have been influencing student achievement independently of the CMSP model, was the effect of student attrition. In almost all of the schools the number of students who were tested on the RCT were about one-half the number of students who started a year earlier. Because of this high rate of student departure from the school it could be inferred that CMSP students were self-selecting and that at the end of the year the random distribution of students that was in place at the start of the program was skewed toward the high end of student mathematics preparedness. However, if this were the case, then student attrition for CMSP students would have been higher than for non-CMSP students, which was found not be the case. At all of the schools, student participation in the CMSP had little effect on retention.

Regents Algebra Achievement Data-Chapter 1 High Schools Tables 24 and 25 show Regents achievement data comparisons for CMSP and non-CMSP students. The data are structured in three test score intervals with the number and percentage of students indicated scoring equal to or greater than: 65 (passing), 75 and 85 . The achievement data reflect the participation of the three high schools--John F. Kennedy, George Washington and East New York-that persisted in model project activity over a two-year test period interval--January 1981 to January 1983--in which the Regents Algebra Examinations were administered as part of the model assessment process. John F. Kennedy took the examination in three consecutive years--in January 1981, 1982 and 1983. The examination was taken twice by George Washington High School--in January 1982 and 1983. And East New York High School took the examination in June 1981 and in January 1983. The Regents Algebra Examination was administered to CMSP students after they had completed the program sequence of one semester of prealgebra and two semesters of algebra in accordance with the two-course mathematics model. The only exception to this was the first cycle of students at East New York High School who took the Regents exam after four semesters of coursework.

Comparisons of Regents examination performance were made with students at the same schools who had completed course work in Regents Algebra using the schools' traditional mathematics program. There was no attempt to differentiate between the variations in the schools' Regents Algebra programs, which could range from the conventional two term program consisting of 9th and 10th graders to the three- and four-term programs which included students from grade levels 9 through 12. At East New York High School, there were no comparisons possible because the school traditionally did not administer the Regents Algebra Examination.

The number of CMSP students who took the test at each of the three participating schools ranged from a high of 44 at John F. Kennedy in January 1983 to a low of 19 at

## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)

NEW YORK STATE REGENTS ALGEBRA EXAMINATION PERFORMANCE COMPARISONS OF CMSP \& NON-CMSP STUDENTS AT PARTICIPATING HIGH SCHOOLS

JANUARY 1981

| John F. Kennedy | N | NO. PASS |  | NO. $>75$ | NO. $>85$ |
| :--- | ---: | :--- | :--- | :--- | :--- |
| CMSP | 28 | $18(64 \%)$ |  | $10(36 \%)$ | $6(21 \%)$ |
| Non-CMSP ${ }^{(1)}$ | 200 | $42(21 \%)$ | $14(7 \%)$ | $3(2 \%)$ |  |

JUNE 1981

|  | N | NO. PASS | NO. $>75$ | NO. $>85$ |
| :---: | :---: | :---: | :---: | :---: |
| East New York |  |  |  |  |
| CMSP | 19 | 15 (79\%) | 8 (42\%) | $4(21 \%)^{(2)}$ |
| Non-CMSP ${ }^{(1)}$ | none ${ }^{(3)}$ | ---- | -.-- |  |

JANUARY 1982

| JANUARY 1282 | N | NO. PASS | NO. $>75$ | NO, $>85$ |
| :---: | :---: | :---: | :---: | :---: |
| George Washington |  |  |  |  |
| CMSP | 22 | 14 (64\%) | 5 (23\%) | 3 (14\%) |
| Non-CMSP ${ }^{(1)}$ | 45 | 10 (22\%) | 2 (4\%) | 0 (0\%) |
| John F. Kennedy |  |  |  |  |
| CMSP | 25 | 17 (68\%) | 7 (28\%) | 6 (24\%) |
| Non-CMSP ${ }^{(1)}$ | 188 | 55 (29\%) | 25 (13\%) | 10 (5\%) |


| COMPOSITE DATA | N | NO. PASS | NO. $>75$ | NO. $>85$ |
| :---: | :---: | :---: | :---: | :---: |
| CMSP | 94 | 64 (68\%) | 30 (32\%) | 19 (20\%) |
| Non-CMSP | 433 | 107 (25\%) | 41 (9\%) | 13 (3\%) |

(1) Tenth grade students only.
(2) Two students obtained perfect scores of $100 \%$.
(3) Only CMSP students take Regents Algebra Exam at East New York.

TABLE 24

## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP) <br> NEW YORK STATE REGENTS ALGEBRA EXAMINATION PERFORMANCE COMPARISONS OF CMSP \& NON-CMSP STUDENTS AT PARTICIPATING HIGH SCHOOLS

JANUARY 1983

|  | N | NO.PASS | $\mathrm{NO},>75$ | NO. $>85$ |
| :---: | :---: | :---: | :---: | :---: |
| CMSP | 21 | 12 (57\%) | 4 (19\%) | 2 (10\%) |
| Non-CMSP | 133 | 40 (30\%) | 16 (12\%) | 5 (4\%) |
| John F. Kennedy |  |  |  |  |
| CMSP | 44 | 18 (41\%) | 10 (23\%) | 4 (9\%) |
| Non-CMSP | 345 | 58 (17\%) | 23 (7\%) | 3 (1\%) |
| East New York |  |  |  |  |
| CMSP | 24 | 7 (29\%) | 2 (8\%) | 1 (4\%) |
| Non-CMSP | 0 |  |  |  |


| COMPOSITEDATA | $\underline{N}$ |  | NO. PASS |  | NO. $>75$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | 18 | $101(55 \%)$ |  | NO. $>85(25 \%)$ | $26(14 \%)$ |
| CMSP | 18 |  | $205(23 \%)$ | $80(9 \%)$ | $21(2 \%)$ |

TABLE 25

East New York in June of 1981. This number of CMSP Regents Algebra test takers reflects students' persistence in the three-semester CMSP model for each given cycle. Student scores on the Regents Algebra Examination are not weighed heavily (if at all) in the students' Algebra course grade. Therefore their taking the test could be seen as an expression of their confidence in their previous Algebra course learning. In comparing the number of RCT test takers (Tables 22 and 23) with those who took the Regents Algebra Examination six months later, it can be seen that there had been attrition over the sixth month period--although not as severe as in the first two terms of the three model cycles. Except for John F. Kennedy High School, on the January 1983 exam, which tested 44 students, all of the schools had less than one full size class taking the Regents Algebra Examination in each of the three test administrations. Three semesters earlier for each of the test implementation cycles, all of the schools had started with at least two classes with a live student register of about 25 students in each class. There was no attempt in the model project to screen students and they were always given the benefit of the doubt when it came to course promotion. The number of students listed in Tables 24 and 25 therefore represent the maximum number of students who could have participated in the Regents Algebra Examination.

In the analysis of the Regents Algebra Examination data it can be seen that CMSP students maintained their 2-to-1 margins over non-CMSP students in test performance, except for George Washington High School in January 1983 where the differences in student test performance was slightly less than 2-to-1--CMSP 57\%, non-CMSP 30\%. In the 1981 and 1982 Regents Algebra Exam administrations, there appeared to be a consistency of CMSP student performance that was not evident in the third and fourth test administration in January 1983 and 1984. In the first two cycles, the pass rate of CMSP students was all above $63 \%$, whereas in the last cycle only George Washington High School approached this pass rate with $57 \%$, while less than a majority of students passed
at John F. Kennedy-41\%, and the results from East New York showed almost no sign of having participated in a special mathematics program.

Although the margins of student performance between CMSP and non-CMSP students at John F. Kennedy remained wide- $-41 \%$ vs. $17 \%$, the $41 \%$ pass rate was sufficiently different from the past two cycles to indicate that the phase out of CMSP model project activity at the school may have effected test scores. The lower pass margin for CMSP students may have also been influenced by the larger number of students taking the test-- 44 students in January 1983 as compared to 28 and 26 in 1981 and 1982 respectively. This appeared to be the case with the RCT mathematics Test where the 36 and 28 students respectively who took the RCT in 1980 and 1981 did considerably better (respectively $81 \%$ and $89 \%$ pass) than the third group of 56 students who took the test in 1982 and only passed the test at a $57 \%$ rate.

As on the RCT mathematics Test, performance by CMSP students on the Regents Algebra Examination at test score intervals at or above 75 was much higher than non-CMSP students'. At John F. Kennedy High School in January 1981, six of twenty-eight CMSP students ( $21 \%$ ) scored 85 or higher while only three of the two-hundred non-CMSP students ( $2 \%$ ) scored at the 85 or higher level. These absolute achievement differences occurred even though the number of non-CMSP test takers was seven times the number of CMSP test takers ( 200 vs . 28). The same wide margins of higher achievement (in test scores at or above 75) were evident for CMSP students at all of the schools with Regents Algebra Exam administration. At East New York High School two of the students had perfect scores of 100 !

The composite exam scores listed in Tables 24 and 25 give a more complete and generalized picture of CMSP student performance on the Regents Algebra Examination as compared to the non-CMSP students. In the first two years (Table 24), the cumulative totals show that CMSP students passed the examination at a $68 \%$ versus the $25 \%$
non-CMSP student rate--a margin of 2.7 -to-1. In scores at or above 75 the margins became larger, $32 \%$ vs. $9 \%$ ( 3.6 -to-1). The same held for scores at or above 85 (margins of 6.7 to 1 ) where the absolute numbers of CMSP students scoring at this level is greater than non-CMSP students ( 19 CMSP vs. 13 non-CMSP) even though the latter number of test takers exceeded CMSP students by more than four-to-one (433 vs. 94).

The composite margins of exam performance for CMSP students on the January 1983 exam are less dramatic than in the two previous years, but in relative terms are still considerably higher than for non-CMSP students. The pass rates still show greater than two-to-one margins in favor of CMSP students ( $55 \%$ vs. $23 \%$ ) as do the margins at or above $75(25 \%$ vs. $9 \%)$. In scores at or above 85 there was a greater number of CMSP students than non-CMSP scoring at this level even though the non-CMSP test takers exceeded CMSP test takers by five-to-one (911 vs. 183).

When the exam performance is looked at from a comparative viewpoint, the achievement of CMSP students on the Regents Algebra Examination is significant and indicative that elements of the model project or the intervention project itself may have contributed to the much better Regents performance by CMSP students. However, when examined in absolute terms, CMSP student achievement is still much below what is needed to initiate a sustained student movement to upper level mathematics courses. The fact that only an average of $25 \%$ of the CMSP students scored higher than 75 on the exam is suggestive that no more than this number of students is likely to enroll and achieve in higher level Regents mathematics courses. While CMSP student algebra exam performance is much higher than non-CMSP students', scores are not at a level or in sufficient number to begin the process of building a critical mass of students who can pursue the three year Regents mathematics sequence. The wide margins in Algebra Regents Exam performance, however, demonstrated that significant gains can be made by students given the foundation and structure provided by the CMSP model project.

In the third cycle, the Regents Algebra pass margins for CMSP students at John F. Kennedy and at East New York High School were much lower and not as consistent between schools as they were in previous project cycles. This may in part have been due to the phase out of CMSP model project activity at both schools. To a certain extent, intervention programs (especially when new and driven by an outside agency) tend to be more structured and goal oriented than traditional school programs. When the impetus of the intervention program is reduced, teachers' collaborative input may wane. The removal of project goals, thus, may have contributed to the fall off in student exam performance at the two schools. This is an area of project experience that requires further study and investigation as it may suggest that the merit of intervention programs may lie in the associations that the intervention programs themselves bring to the schools. It may follow, that in order for structured mathematics program change to take place in participant schools, the association between an intervention program and a school may have to become a permanent part of the model prototype that is finally developed and adopted by the schools.

Regents Algebra Achievement Data--Brooklyn Technical High School Table 26 shows data comparing the performance of CMSP students with non-CMSP students on the Regents Algebra Examination at Brooklyn Technical High School over four cycles of model test implementation--June 1980 to June 1983. The data presented are more distinct than the Chapter 1 high school data because of 1) the stable and growing population of CMSP students over the four cycle period and because 2) both CMSP and non-CMSP students studied Algebra over a two term period (rather than 3 or 4 terms) and almost all of the test takers were ninth graders.

The Brooklyn Tech data presented in Table 26 are structured to show the number of and percentage of students who scored at or above 65,80 and 90 . The test score intervals

## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)

9TH YEAR ALGEBRA REGENTS CYCLE I-IV PERFORMANCE COMPARISONS OF CMSP \& NON-CMSP STUDENTS AT BROOKLYN TECHNICAL HIGH SCHOOL

|  | \# TAKING $\qquad$ | $\begin{gathered} \text { \# PASSING } \\ \text { TEST } \\ \hline \end{gathered}$ | $\begin{gathered} \# \\ \pm 80 \\ >8 \end{gathered}$ | $\begin{gathered} \# \\ \geq 90 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| CYCLE I (ending 6/80) |  |  |  |  |
| CMSP | 107 | $\begin{aligned} & 106 \\ & (99 \%) \end{aligned}$ | $\begin{aligned} & 91 \\ & (85 \%) \end{aligned}$ | $\begin{gathered} 62 \\ (58 \%) \end{gathered}$ |
| Non-CMSP | 528 | $\begin{aligned} & 462 \\ & (88 \%) \end{aligned}$ | $\begin{aligned} & 310 \\ & (59 \%) \end{aligned}$ | $\begin{gathered} 176 \\ (33 \%) \end{gathered}$ |
| CYCLE II (ending 6/81) |  |  |  |  |
| CMSP | 106 | $\begin{gathered} 96 \\ (91 \%) \end{gathered}$ | $\begin{gathered} 77 \\ (73 \%) \end{gathered}$ | $\begin{gathered} 51 \\ (48 \%) \end{gathered}$ |
| Non-CMSP | 1162 | $\begin{aligned} & 832 \\ & (72 \%) \end{aligned}$ | $\begin{aligned} & 543 \\ & (47 \%) \end{aligned}$ | $\begin{gathered} 324 \\ (28 \%) \end{gathered}$ |
| CYCLE III (ending 6/82) |  |  |  |  |
| CMSP | - 218 | $\begin{aligned} & 187 \\ & (86 \%) \end{aligned}$ | $\begin{gathered} 128 \\ (59 \%) \end{gathered}$ | $\begin{gathered} 64 \\ (29 \%) \end{gathered}$ |
| Non-CMSP | 894 | $\begin{aligned} & 533 \\ & (60 \%) \end{aligned}$ | $\begin{gathered} 252 \\ (28 \%) \end{gathered}$ | $\begin{gathered} 120 \\ (13 \%) \end{gathered}$ |
| CYCLE IV (ending 6/83) 293 |  |  |  |  |
| CMSP | 303 | $\begin{aligned} & 292 \\ & (96 \%) \end{aligned}$ | $\begin{gathered} 236 \\ (78 \%) \end{gathered}$ | $\begin{array}{r} 151 \\ (50 \%) \end{array}$ |
| Non-CMSP | 436 | $\begin{aligned} & 338 \\ & (78 \%) \end{aligned}$ | $\begin{gathered} 203 \\ (47 \%) \end{gathered}$ | $\begin{gathered} 97 \\ (22 \%) \end{gathered}$ |

COMPOSITE (CYCLES I-IV)

| CMSP | 734 | 681 <br> $(93 \%)$ | 532 <br> $(72 \%)$ | 328 <br> $(45 \%)$ |
| :--- | :---: | :--- | :---: | :---: |
| Non-CMSP |  | 3020 | 2165 1308 | 717 |
|  |  | $(72 \%)$ | $(43 \%)$ | $(24 \%)$ |

TABLE 26
of 80 and 90 are higher than the 75 and 85 test score intervals used for the Chapter 1 high schools. This was done because Brooklyn Technical High School traditionally has pass rates in the Regents Algebra Examination in the 70-80 range, and if two-to-one differences occurred in the test data, it would be reflected at the upper end of the achievement scale.

Over the four-year period the test taking population at Brooklyn Technical High School varied widely from a low of 635 in 1980 to a high of 1,268 in 1981. In the first two cycles of CMSP model test implementation (1979/80 and 1980/81) four randomly assigned classes (approximately 120 students each year) studied Regents algebra coursework using the CMSP model. In the first cycle the performance of students on the Regents Algebra Examination was outstanding, with all but one of the 105 students passing. In addition, 90 students ( $86 \%$ of the 105 students) scored 80 or higher and 61 students ( $58 \%$ of the students) scored 90 or higher. The performance of non-CMSP students was reasonably high also with $86 \%$ of the students passing the examination, $59 \%$ scoring 80 or higher and $33 \%$ scoring 90 or higher. While the two-to-one margins are not apparent (or possible) at the 65 and 80 test score intervals, there is close to a two-to-one margin between CMSP and non-CMSP students in test scores of 90 and above.

When the program was initiated at Brooklyn Technical High School in the Fall of 1979, there was a strong feeling among the CMSP staff that the structure and the additional instructional time provided by the CMSP model would have a significant impact on student performance on the Regents Algebra Examination. Because of this expectation, a high standard was established as a reference goal which the model project would work towards. The participating teachers and staff generally agreed that a goal of $80 \%$ of CMSP students scoring 80 or higher on the Regents Algebra Examination was a reasonably high standard, and if incoming students could be attain this level of performance over the course of several cycles of model test implementation, then the model project would prove itself worthy of serious consideration. The students in the first cycle did very well, indeed, on
the Regents Algebra Examination. They exceeded the $80 / 80$ standard on the first model trial- $-85 \%$ of the students scored 80 or higher and $58 \%$ scored 90 or higher. While better than average performance was expected on the part of the participating students, it was not anticipated that the $80 / 80$ goal set would be reached in the first cycle of model test implementation.

Not only had the CMSP students achieved at a significantly higher level than nonCMSP students, but the participating teachers felt that students had learned the subject matter in greater depth. This they felt was mainly due to the additional time provided by the two blocked periods of mathematics and the fact that the two courses were taught by different teachers performing as instructional partners. While the excellent student performance on the Algebra Regents Examination was a clear signal that model test implementation was proceeding in the right direction, it also raised the concern that the outcome was influenced by first time trial effects and the exemplary efforts of the teachers.

In order to further test the model and its influence on student achievement, it was decided to conduct the second cycle as closely as possible to the first cycle. Participating classes were held to four and there was a change in one of the four teachers. All four of the teachers who participated in the second cycle were certified mathematics teachers and had extensive teaching experience. There was, however, a noticeable difference in the size of the incoming student population from which the four classes for the second cycle were to be randomly drawn-it was almost twice the size as the first cycle--around 1,300 vs 700 . Because of this larger student enrollment the levels of student preparation in mathematics were lower--based both on entrance examination cutoff scores and the CMSP pre-evaluation scores. The cut-off score for entrance to Brooklyn Technical High School for the Fall of 1980 was 17 points lower than for the Fall of $1979--94$ vs. 111 ( 180 was the maximum score on the entrance examination). In the distribution of CMSP pre-evaluation test scores, $58 \%$ of the second cycle students scored below 60 , as compared
to $37 \%$ below 60 in the previous year ( 100 was the maximum score on the CMSP pre-evaluation test).

The Regents Algebra Examination was administered at the end of the second cycle of CMSP model test implementation activity to the considerably larger student population of 9th graders. Although not scoring as high as in the first cycle, the 106 CMSP students passed the Regents Algebra Examination at a level that was much higher than the 1,162 non-CMSP students-- $91 \%$ vs. $72 \%$. In scores at and above 80 and 90 , the CMSP students again out-performed the non-CMSP students- $73 \%$ vs. $47 \%$ with exam scores at or above 80 and $48 \%$ vs. $28 \%$ with exam scores at or above 90 . In neither of the test score comparisons for the first two cycles was a two-to-one margin achieved between CMSP and non-CMSP students; however, the margins obtained were sufficiently large to warrant further development and testing of the model on a larger scale. In the Fall of 1981, a third cycle of model test implementation was started with the random assignment of eight classes of students drawn from an entering student population of approximately 1,200 students.

This third cycle of CMSP students continued achieving at a relatively high performance level on the Regents Algebra Examination as compared to non-CMSP students. Overall, the exam results were lower for CMSP and non-CMSP students than in the previous two cycles. And this was probably due to the increased difficulty of the exam (which teachers acknowledged) as compared to the exams given in 1980 and 1981. While the pass rates and absolute scores of third cycle CMSP students were lower than on the two previous cycles for CMSP students--86\% (cycle 3), $91 \%$ (cycle 2) and $99 \%$ (cycle 1), the margins of passing and high achievement exam scores were much wider between CMSP and non-CMSP students. There was a 26 point percentage spread between CMSP and non-CMSP students in pass rates-- $86 \%$ vs. $60 \%$. And, on both exams, scores at or above 80 and 90 , two-to-one margins were achieved: for exam scores at or above 80 ,

CMSP $59 \%$ vs. $28 \%$ non-CMSP; and for exam scores at or above 90 , CMSP $29 \%$ vs. $13 \%$ non-CMSP. The wide exam achievement margins attained by CMSP students over non-CMSP continued to demonstrate the usefulness and value of the model. This was more apparent in the third cycle because of the greater number of teachers that participated--eight rather than four in the previous two cycles. Their wider range of teaching styles and teacher experience allowed a more comprehensive test of the model.

In the fourth cycle of the CMSP model project, implemented in the Fall of 1982, it was decided to increase the participating student population to twelve classes or approximately $40 \%$ of the entering 9 th grade student population. This substantially larger student population was approaching the level at which the model could be tested under conditions that would closely reflect the administrative and academic realities of the school. This included the difficulties encountered in course programming, the orientation of teachers, the distribution of materials, the administration, grading and analysis of unit tests on a bi-weekly basis, among other considerations.

The fourth cycle of model project activity, although large, ran smoothly, primarily because of the experience gained in the three previous cycles and the strong support provided from the mathematics chairperson, Dr. Melvin Klein, and the exemplary efforts of the school project coordinator, Sheldon Pasner. The apparently more stable and larger model project in the fourth cycle reflected continued excellent performance by CMSP students on the June 1983 Regents Algebra Examination. CMSP students passed the exam at a $96 \%$ rate (second only to the $99 \%$ rate achieved in the first cycle) as compared to $78 \%$ for the non-CMSP students. And in exam scores at or above 80 and 90 , the margins of exam performance were, $78 \%$ vs. $47 \%$ and $50 \%$ vs. $22 \%$, respectively. In exam scores at or above 90 the two-to-one margins of achievement were obtained for the second time. It is important to note that the absolute number of CMSP students scoring at or above 80 and 90 was higher than non-CMSP students, even though non-CMSP student enrollment
was larger than that of CMSP students. The difference in absolute numbers is especially noticeable in exam scores at or above 90, where more than $50 \%$ more CMSP students scored in this range as compared to non CMSP students--151 vs. 98.

The composite scores over the four cycles of model project activity provide substantial exam data to indicate the superior performance of CMSP students on the Regents Algebra Examination. Overall, the pass rate for the 734 CMSP students was $93 \%$ as compared to $72 \%$ for the 3,024 non-CMSP students. In exam scores at or above 80 and 90 , the overall margins in favor of CMSP students were $72 \%$ vs. $43 \%$ and $45 \%$ vs. $24 \%$, respectively. The most important aspect of the higher Regents Algebra Exam performance by CMSP students is its consistency when compared to non-CMSP students. On each of the examinations there were close to two-to-one differences in exam performance at exam scores at or above 90.

Taken in the context of model development and research within a participating school, the data comparisons of Regents exam performance at Brooklyn Tech were similar to the Chapter 1 schools'. However, because of the much larger number of CMSP participant students at Brooklyn Tech and their higher retention in the model project, the Regents exam data had more depth and value by which to assess the impact of the model project on student mathematics achievement. To this end there was general agreement amongst CMSP staff, participating teachers and school administrators that the model project was providing a structure that contributed to higher student mathematics achievement. The larger questions, however, were: To what extent would higher student achievement in Algebra lead to similarly high achievement in upper level mathematics courses? And was the model consistent with the resources and tradition of the school? The answers to both of these questions were elusive at the end of the fourth cycle and remained largely unanswered as the CMSP model project activity continued large scale model test implementation during the period 1983 to 1986.

### 5.4 Perspectives on the Factors Influencing the Acceptance of the CMSP

Model Project and Its Test Implementation
How successful was the CMSP model project in the attainment of established goals? Did the ground zero start and double the mathematics instructional time provide students with the arithmetic foundation to succeed in a first course in Regents Algebra? And was the model project perceived by the school's mathematics faculty and administration as being viable and consistent with the school's resources and priorities? These are the difficult questions that surface in the implementation of a systems and field based project whose goal is the creation of curriculum models directed at the wholescale improvement of a school's mathematics program. There are no clear and direct answers for any of the questions posed because the questions themselves represent the dynamics and complexities inherent in large educational institutions--which include urban high schools located in the midst of metropolitan life. Just as the CMSP model project effort was systems oriented, so must be the assessment of its effectiveness.

Taken singly, the questions posed have little meaning or value beyond establishing a base upon which further research or development can continue. While this is important in keeping with research tradition it does not address the major project issue, whether the model project was proven useful to the participating school. This is the overriding issue not only for the CMSP, but for all intervention strategies which are undertaken to improve school effectiveness--either academically or administratively. It is not enough to show that student achievement has been significantly raised by the program intervention, although this academic outcome must be an inherent part of the overall process of program assessment. There must also be a consensus by the faculty who feel that the implementation of the model project is in their own best interest and offers a program of study that is more effective than the existing instructional program.

Certainly, student mathematics achievement in the CMSP model project has been
impressive and there appeared to be consistency of student performance at all of the participating schools despite the wide differences in school characteristics. The diversity of participant schools ranged from relatively small vocational high schools (East New York and Chelsea) to the large urban Chapter 1 high schools which included a school with an all-female school population (Washington Irving) to a specialized high school where admission was based on competitive examination (Brooklyn Technical). For those schools that persisted in the model test implementation, RCT Mathematics and Regents Algebra test scores were substantially higher for CMSP participating students than for comparable students studying mathematics in the school's regular mathematics programs.

However, student achievement at all of the schools was tempered by a combination of factors that reduced its significance and impact on the school's mathematics department faculty. The first and most compelling of these was the size of the model project test implementation at each of the participating schools. In retrospect, the size of the model program turned out to be of extreme importance not only in demonstrating the viability of the model curriculum but also in testing how the model would impact the school's budget and administrative processes, including student course placement, course scheduling and faculty utilization.

At Brooklyn Technical High School where the model project started with four classes and four experienced faculty members, the project's viability was being continually demonstrated with each repeated cycle of model test implementation that reached its peak enrollment of 24 classes in June 1983. The increase in size at Brooklyn Technical High School brought with it increased conjecture and criticism of the program by the department faculty as more teachers participated. CMSP student achievement, as high and consistent as it was at Brooklyn Technical High School, became less important than what the model project represented in terms of school priorities and the allocation of school resources to the mathematics department. A double period of mathematics for a student population of

350 students or 24 classes represents a budget investment by the school of almost three full teacher positions (given a five class teaching load per teacher) that are assigned to 9th year mathematics programs. In implementing a model project of this magnitude the school must decide to supplement its budget by increasing the number of courses offered during the school day or reallocating the school's teaching positions to cover the costs of offering the additional period of mathematics. If the latter is the school's decision then another department will have less flexibility in course offerings because of its reduced teacher allotments. For a full scale program at Brooklyn Technical High School the additional allocations to the mathematics department may be as many as seven full time teaching positions. Thus, size of the model program and its impact on the school's resources is a primary concern that the school must weigh against the potential benefits of student achievement.

At the other Chapter 1 high schools, size was a programmatic issue that was influenced by the decision to conduct parallel model project operations at several different schools at the same time. Available project resources and a realization that a beginning program is better started small--and increased gradually as the model program demonstrates that it is beneficial to students--kept the classes to just two at each of the participating classes. While this number of students was appropriate for initial project implementation it did create a "a scale modeling effect" that served to limit and qualify participating students' mathematics test achievement as the model project progressed over several cycles of test implementation.

Because of the small number of classes at each of the participant schools, the model project was subject to the influences of one or two teachers (unlike Brooklyn Technical High Schocl). In addition, the teachers volunteered to participate in the project and therefore interest and enthusiasm were added as factors that had to be taken into consideration in assessing the impact of the model program on student achievement. While
there is a myriad of factors that influences student achievement in the classroom, there is none so great as the teacher. Teachers can devote time and energy to a classroom that far outweigh any intervention program or special curriculum and their students will consistently achieve at high levels. Such teachers appear to have three important qualities, they have:

1) a high scholarly interest in the subject they teach,
2) a great respect for their students and
3) a keen sense of classroom organization and management.

Probably only the last of the three qualities is modifiable. High school teachers in possession of the three qualities can make almost any student rise to the occasion of learning and they are generally recognized for this and held in high regard by their teacher colleagues and administrative staff at the schools. Their presence in a model project can greatly influence project outcomes and must be taken into account in assessing the model project's viability and effectiveness in promoting student achievement.

The CMSP model project was designed as a parallel effort to minimize the effects of small starting populations and single teacher influences. Presumably conducting the project in many different school settings over repeated cycles of test implementation with a growing student and teacher population would allow the model to be tested across a wide range of school variables. A common outcome--high student achievement--under these long term and parallel school project circumstances would be a fair indication that the model project was working as intended. However, the project at the Chapter 1 schools did not grow as expected and in the third cycle of model project test implementation, (the Fall of 1982); the CMSP was still operating with two beginning classes at each of the three remaining participant schools. Budgetary problems, teacher resistance and lack of support from some school administrators combined to keep the school's participating student
populations constant at two beginning classes. Thus at the end of the four year period it could still be surmised that the one or two teachers that were involved in the model project in each of the participating cycles were influencing student achievement as much as the structural elements of the CMSP model.

This uncertainty of the effects of the CMSP model because of program size was complicated still more by the severe problem of student attrition that was experienced at each of the seven Chapter 1 schools that participated during the three cycles of model test implementation. From the drastic reductions in student participants at Benjamin Franklin and John Jay High Schools to the gradual decreases that took place at John F. Kennedy High School, the CMSP model project was tested under conditions that made it seem as if half of the participating student population was being selected out of the program. It mattered little that the attrition of CMSP students was no different from other comparable students at the school. The fact remained that the students who stayed with the program could be looked at as a special group of students who, at the end of a particular cycle of model test implementation, were vastly different in student characteristics from the original group of randomly selected students. However difficult this makes the process of judging the value and effectiveness of the CMSP model project, it is a fact of life in Chapter 1 schools in the New York City public high school system (as in other large urban school systems) and must be reckoned with in designing and implementing programs of intervention aimed at wholescale student achievement in a particular subject area.

Another factor that lessened the impact of high student mathematics achievement in the CMSP model project was the fact that there appeared to be little transference from algebra mathematics achievement to the succeeding Regents course in geometry when the students were mainstreamed with all other students at the school. At Brooklyn Technical High School where objective comparisons could be made over several cycles, students who participated in the CMSP algebra program a year earlier did no better on the Geometry

Regents Exam than the larger student population. The situation was much worse at the Chapter 1 schools where the invariably low number of students enrolled in Regents Geometry precluded the formation of classes where a critical masses of high achieving students. The critical mass of high achieving students is necessary in Regents mathematics classes because it provides the teacher with a reference pool upon which the teacher can plan and manage the pace and depth of instruction for the entire class. Regents Geometry, in particular, in its early course excursions on formal proofs can be frustrating to teachers if only a few students in a class are able to make the conceptual leaps in logic and reasoning.

The combination of factors that were experienced in the CMSP model project including program size, student attrition and student achievement transference from algebra to geometry, all served to dilute the academic accomplishments of the participating CMSP students on the RCT and Regents Algebra Examinations. From this perspective the CMSP model project could be viewed by the school's mathematics faculty as a useful program intervention for 9 th year students, but it hardly touched on the more complex problem of student enrollment and achievement in upper level Regents mathematics courses. In this context, the advances that might have been made on a particular educational problem (in this case Regents Algebra achievement) created an issue for the faculty to seriously consider: whether the intervention program, if successfully implemented, will lead to greater positive change in school achievement.

Change prescribed by intervention programs may often be resisted by the senior faculty of educational institutions (especially those which are departmentalized like high schools and colleges) if they are not involved in the planning and development process or participate in the early stages of program experimentation. Through their subject matter expertise and their long experience at the school, the senior faculty and department heads largely influence the tradition and inherent qualities of the institution. They, in effect,
represent the culture and collective consciousness of the institution and, as such, need to be consulted and relied on concerning matters that pertain to potential major changes in the school's departmental programs. It is important to minimize the perception among the senior faculty that changes to be implemented will undermine their long years of school experience and academic standing at the institution.

Senior faculty must be able to question changes in a curriculum that they have worked long and hard to master and/or develop and which they feel works to the best affect possible. How can it really be shown the new intervention program will work any better than the school's existing program over the long term? At best, this is a value judgement that must be assumed by the faculty at large. Two decades of educational research have shown that educational interventions and reforms can be initiated by outside or administrative forces. However, it is the school's faculty that must eventually support and nurture the new intervention processes to fruition. It is highly improbable that such long term support can be sustained without the full consensus of the faculty and with their belief that the new program is in the best interest of the school's teachers and students and is in keeping with the school's tradition and available resources.

In the end, therefore, it was not so much what had been accomplished in the CMSP model project as whether the accomplishment was acknowledged by a consensus of the mathematics department faculty and the school administrators. The attainment of department faculty consensus is in itself complex because it revolves around the basic support that is given the program by the mathematics chairperson and the relationship that exists between the chairperson and senior members of the department faculty. Strong support for the program can be forthcoming from both the mathematics chairperson and principal. However, this is still insufficient if a consensus of program endorsement is not also reached by the senior faculty. This was the situation at Chelsea and John F. Kennedy High Schools where the CMSP model project activity eventually dissolved due to lack of a
consensus from the senior faculty. Despite the hierarchical structure of the traditional urban high school, departmental senior faculty wield considerable power and influence when it comes to academic matters that relate to their background and expertise. The ultimate decision to continue with model implementation beyond that established for experimentation must reside with the department's senior faculty because it is they who will be called upon to carry the burden of its future implementation and in the training of less experienced teachers who will later participate.

The CMSP model project and its development and test implementation in the participant schools represented one of the many precollege efforts which have been undertaken throughout the country to increase the pool of minority students who enroll and achieve in high school mathematics and science study. The CMSP research and developement work, conducted during the period from the Fall of 1978 to the Spring of 1983, was an attempt to create models that would overcome the obstacles that prevent students in Chapter 1 schools from enrolling and achieving in a first course in Regents algebra. In the pursuit of this goal, schools participating in the model project made provisions to implement the model in accordance with guidelines that were a departure from traditional mathematics programs. The random sampling of students, the heterogeneous class groupings, the offering of two periods of mathematics (with two teachers) and the uniform pacing of instruction tied to bi-weekly unit tests administered by teachers but constructed by the CMSP are all elements of a systems and field based effort to increase the pool of students who enroll and achieve in the study of Regents algebra.

The testing of the CMSP model project was administrative as well as academic, for, in adhering to the project guidelines, the participant schools demonstrated their capacity to alter traditional administrative practices of student course placement and course scheduling. The fact that CMSP participant students were selected randomly with no reference to their junior high school records and standardized test scores was a significant step in the model
development and research process. And besides providing heterogeneous classes of students that could participate in the project without the selective aspects of "ability grouping", it also demonstrated to the schools that there are alternatives to the reliance on standardized diagnostic mathematics test scores for mathematics course placement. The fact that the administrative changes were facilitated by the participant schools over several cycles of model test implementation was evidence that the process of organizing the CMSP model on a larger scale was possible. The project effort at Brooklyn Technical High School, while focusing on a different student population from the Chapter 1 schools, was an example of the large scale under which the model project can be implemented.

The testing of the premises and the assessment of model project effectiveness for realizing the stated goal at Chapter 1 high schools was hampered primarily by the severe attrition of CMSP students at each of the participant schools. However, the repeated and consistent CMSP student achievement on the RCT and Regents Algebra Examinations provided the timely and continuous feedback that allowed that the model curriculum to be shaped and refined as a prototype. There was also a strong feeling amongst the CMSP staff and several of the participant school administrators and teachers that feasibility of the model project had been proven.

At the completion of model project activity in the Spring of 1983, the CMSP had established both organizational and programmatic constructs that solidified the model project for continued resaerch, development and testing. This provided the foundation for a second phase of model project activity that would be implemented in seven Chapter 1 high schools over a four-year period (1983 to 1987) in three overlapping cycles of two-year duration.

Access to higher education has always been determined by the strength of student applicant's academic background, particularly by their performance record in high school. Historically the uneven elementary and junior high school experiences of Black and

Hispanic students in science and mathematics has limited their enrollment in precollege mathematics in high school that is prerequisite for engineering and science programs of college study. This complex problem has been the focus of CMSP model project efforts over the initial four year period (1979-1983) in which repeated cycles of model test implementation enabled significant improvements in student mathematics achievement in Algebra coursework and the formation of a model curriculum prototype. What appears to be evident in the CMSP model project work is that despite the inadequate mathematics schooling prior to their high school entry participating students were able to secure a solid mathematics foundation upon which to enroll and achieve in a first course in Regents Algebra. If students' mathematics achievement continues to hold in the larger scale testing of the model curriculum prototype, then a significant step will have been made to better understand the pervasive problem that has for too long limited Black and Hispanic students enrollment in traditional high school mathematics courses of study and their access to programs of college study in engineering and science.

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SOARD OF EDUCATION OF THE CITY OF NEW YORK
$1979-1980$ SCHOOL PROFILES SUMMARY EY SCHOOLS CITYWIDE


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| 4 | 81,217 | 4.9 | 5.1 |  | 5.0 | 50.4 | 25.5 20.2 | 3.6 |
| 8 | 58.364 | 6.0 | 0.3 |  | 0.1 | 52.9 | 20.7 | 6.7 |
| 6 | 57,713 | 6.7 | 7.3 |  | 7.0 | 51.0 | 20.7 31.9 | 14.8 |
| 7 | 36.745 | 7.3 | 7.8 |  | 7.6 | 45.6 | 39.3 | 22.9 |
| ? | 58.629 | 8.4 | 9.1 |  | 8.8 | 49.4 | 34.6 | 21.4 |
| - | 20,469 | 9.4 | 10.1 |  | 9.8 | 54.3 | 35.9 | 20.8 |
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| 4 | 7.702 | 5.0 | 4.4 |  | 4.7 | 45.0 | 23.1 | 2.0 |
| 5 | 58.656 | 6.5 | 5.9 |  | 6.2 | 58.7 | 17.7 | 4.1 |
| 1 | 58,832 | 9.1 | 8.6 |  | 8.8 | 45.8 | 31.5 | 16.9 |



Appendix A

COARO Of EDUCATION Of TME CTY OF NEW YORK LEVIL - JUMOR MOH/MNTEMORIDATE SCHOOL
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| PHYSICAL FACILITY DATA |  |  |  |  |  |  |
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Appendix B

1982/83 COMMUNITY SCHOOL DISTRICT ENROLLMENT IN THE FIVE BOROUGHS OF NEW YORK CITY INCLUDING NUMBER AND PERCENTAGE OF BLACK \& HISPANIC STUDENTS

|  | TOTAL ENROLL. | BLACK \& HISPANIC | \% BLACK \& HISPANIC |
| :---: | :---: | :---: | :---: |
| MANHATTAN |  |  |  |
| District \# |  |  |  |
| 1 | 10,920 | 9,654 | 88.4\% |
| 2 | 17,658 | 6,864 | 38.9\% |
| 3 | 12,127 | 10,636 | 87.7\% |
| 4 | 13,757 | 12,955 | 94.2\% |
| 5 | 11,218 | 11,163 | 99.6\% |
| 6 | 19,391 | 18.349 | 94.6\% |
| Total Elementary | 60,409 | 49,319 | 81.6\% |
| Total JHS/IS | 24,662 | 20,302 | 82.3\% |
| Total | 85,071 | 69,621 | 81.8\% |
| BRONX |  |  |  |
| District \# |  |  |  |
| -7 | 14,238 | 14,152 | 99.4\% |
| 8 | 21,117 | 18,166 | 86.0\% |
| 9 | 26,849 | 26,292 | 97.9\% |
| 10 | 32,608 | 26,879 | 82.4\% |
| 11 | 21,580 | 16,490 | 76.4\% |
| 12 | 15,323 | 15.087 | 98,4\% |
| Total Elementary |  |  | 88.7\% |
| Total JHS/IS | 40,960 | -36,511 | $\frac{89.1 \%}{88.8 \%}$ |
| Total | 131,715 | 117,066 | 88.8\% |
| BROOKLYN |  |  |  |
| $\frac{\text { District\#\# }}{13} 16.638$ |  |  |  |
| 13 | 16,638 | 16,214 | $97.4 \%$ |
| 14 | 18,470 | 16,654 | 90.2\% |
| 15 | 20,360 | 15,490 | 76.1\% |
| 16 | 10,513 | 10,494 | 99.8\% |
| 17 | 25,879 | 25,228 | 97.5\% |
| 18 | 17,276 | 12,525 | 72.5\% |
| 19 | 24,136 | 22,193 | 91.9\% |
| 20 | 23,199 | 8,034 | 34.6\% |
| 21 | 20,155 | 7,732 | 32.9\% |
| 22 | 23,503 | 10,591 | 45.1\% |
| 23 | 12,450 | 12,417 | $99.7 \%$ $96.4 \%$ |
| 32 | 16.020 | 15,438 | 96.4\% |
| Total Elementary | 162,161 |  |  |
| Total JHS/IS Total | $\begin{array}{r}66,438 \\ \hline 228,599\end{array}$ | $\frac{49,744}{173,010}$ | $\frac{74,8 \%}{75.6 \%}$ |

SOURCE: School Profiles 1982-83. New York City Board of Education

# 1982/83 COMMUNITY SCHOOL DISTRICT ENROLLMENT IN THE FIVE BOROUGHS OF NEW YORK CITY INCLUDING NUMBER AND PERCENTAGE OF BLACK \& HISPANIC STUDENTS 

## (cont.)

|  | TOTAL ENROLL. | $\begin{aligned} & \text { BLACK \& } \\ & \text { HISPANIC } \end{aligned}$ | \% BLACK \& HISPANIC |
| :---: | :---: | :---: | :---: |
| OUEENS |  |  |  |
| District.\# |  |  |  |
| 24 | 24,662 | 11,567 | 46.9\% |
| 25 | 20,052 | 5,540 | 27.6\% |
| 26 | 12,101 | 2,992 | 24.7\% |
| 27 | 27,604 | 15,943 | 78.5\% |
| 28 | 20,299 | 13,410 | 66.1\% |
| 29 | 21,397 | 18,052 | 84.4\% |
| 30 | 21,818 | 10,803 | 49.5\% |
| 33 | 1,337 | 619 | 46.3\% |
| Total Elementary | 101,215 | 53,454 | 52.8\% |
| Total JHS/IS | 48,055 | 25,472 | $\frac{53.0 \%}{52.9 \%}$ |
| Total | 149,270 | 78,926 | 52.9\% |
| STATEN ISLAND |  |  |  |
| $\frac{\text { District \# }}{31}$ | 32,893 | 5.915 | 18.0\% |
| Total Elementary | 20,516 |  |  |
| Total JHS/IS | $\frac{12,277}{32,893}$ | $2.185$ | $\frac{17.8 \%}{18.0 \%}$ |
| Total | 32,893 | 5,915 | 18.0\% |

SOURCE: School Profiles 1982-83. New York City Board of Education

## APPENDIX C2

## SOARD OF EDUCAIION OF THE CTV OF MEW YORK 1902-193 satoot mornis

DISTRICT 5
summanr ar oistact


| PUPIL DATA |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | anmawn alim |  |  | mename |  | avecen on |  | 12 | \% 8 | \%000 | rem 1 | 11.133 |
|  | maca |  |  |  |  | rotal |  |  |  |  | necer | ave a |
|  | $\cdots$ | romen |  |  |  | mer | (17\% | Hmer | Pamen | +10 | Nomem |  | Hamolt | 0 |
| xmoeer | 112 | 00.2 |  | ; | 3 |  |  | 120 | 19.3 | 1 | 1 | $\pm$ | 10.0 |  |  | 348 |
| arem: | $\infty$ | 33.1 |  | ! | 1 | 213 | 17.7 | 1 | 1 | 1.116 | 100 |  | 02.9 | 32.0 |
| cheo | 970 | es. ${ }^{\text {ch }}$ |  | 3 | , | 161 | 14.2 | 1 | 1 | 1,13 | 100 |  | 1. | \% |
| Ghat | 91 | ess. |  | 4 | 4 | 131 | 13.9 |  |  | $1 \times$ | 10.0 |  | 2ss | 310 |
| and | 80 | 25.3 |  | 2 | 3 | 10 | 14.3 |  |  | m | 10. |  | $\infty 0.1$ | 241 |
| anos | 201 | 81.0 | 1.1 | 5 | , | 170 | 10.2 | 1 | 1 | \% | 1080 |  | 0.3 | 20.5 |
| anct | 207 | Ss. 4 |  | 3 | 1 | 140 | 14.0 | 1 | . | 183 | 10.0 |  | 0.7 | 315 |
| aract | 1.203 | 0.1 | 1.1 | - | 4 | 308 | 19.1 | 4 | 3 |  | 100 |  | 11. | 37.6 |
| graid | 4 | 11.2 |  | ! | 1 | 10 | 16.4 | 1 | 1 | 120 | 1008 |  | 0.1 | 20 |
| grater | 103 | 70.4 |  | 3 | 1.1 | 14 | 27.2 |  |  | 23 | 1008 |  | -4.0 | 2.3 |
| seram |  | 81.9 |  | ${ }^{3}$ | 2 | 1,91 | 173 | - | - | 11210 | 190 |  |  | :3 |
| rotas | -138 | ces | 2 | 28 | 3 | 1,910 | 17. | 16 | . | 11.216 | 10.0 |  |  | m |




Appendix D1

## CONRD OP EDNCATION OP TME CIT OF NTW YORE 10t2-194s sotoot momiss

summary or orstact


| TEst sconts |  | Catabing |  |  |  | MATMIMATCO |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | pains | ming benet | © aviacor |  | *2 rians |  |  | $\pm 15$ | ${ }^{2} 2$ rum |
| ceam | Vmp | rotal | Lne | UnE | uve | T-10 | und | une | M |
| : | 1.07 |  |  | 9 | $i^{2}$ | 1.0101 | $1{ }^{18}$ | 28 | 20 |
| , | 1,107 | 4.7 | 7 | 0.7 | 1.2 | 1.101 | 298 | 2.0 | , |
| : | i,042 | \% | 710 | \% | 23 | 1.200 | 73 | 10.7 | 4 |
| - | 1,143 | 18 | 32.0 | 18 | 29 | 1,237 | 4.1 | is | 20 |
| 7 | 1.801 | P. | 120 | 120 | 1.2 | 1.201 | 70 | ${ }^{21}$ | 12.1 |
| : | 1.200 | 118 | 32.2 | 72 | 29 | 1.237 | 89.2 | 10 | -1 |
| - | 0 | 123 | 320 | 11. | 23 | 130 | 73 | 80 | 12. |



Appendix D2

| TEST SCORE | COMPREHENSIVE MATH \& SCIENCE PROGRAY (OMSP) PREALGEBRA: PREEVALUATION EXMM |  | DATE |
| :---: | :---: | :---: | :---: |
| STUDENT MAME: |  | CuSs: ${ }^{\text {S }}$ | Schoot: |
|  |  |  |  |
| ) Add:$86,347+473,845+69,682+329$ |  | $\begin{array}{r} 620,046 \\ -387,294 \\ \hline \end{array}$ |  |
| 3) | $\begin{array}{r} 938 \\ \times 607 \\ \hline \end{array}$ | $\text { 4) } 2 9 \longdiv { 2 0 , 4 }$ |  |
|  | $4 \frac{4}{5} \times \frac{5}{6}$ | 6) $\frac{5}{12} \div \frac{3}{4}$ |  |
|  | $\frac{2}{5}+\frac{1}{2}$ | 8) $4 \frac{1}{8} \cdot 2 \frac{3}{4}$ |  |
| 9) | $82.4+9.36+21+8.702$ | 10) 61.3 - 8.79 |  |

Appendix E1


## COMPREHENSIVE MATH \& SCIENCE PROGRAM (CMSP)

## LISTING OF CMSP ADVISORY PANEL MEMBERS 1979 / 80

| George Altomare, Vice President <br> United Federation of Teachers | Karen Nicholls, Student <br> Washington Irving High School |
| :--- | :--- |
| Richard Brucato, Principal <br> Brooklyn Technical High School | Nydia Novoa, Director <br> Rafael Cordero Bilingual School |
| Linda Carnes, Contributions Advisor <br> Exxon Corporation | George Quarles, Chief Administrator <br> Office of Career Education |
| David B. Easson, Major General <br> United States Air Force | Nathan Quinones, Executive Director <br> Division of High Schools |
| Ronald Edmonds, Senior Assistant to | New York City Board of Education |
| the Chancellor for Instruction Reyes-Guerra, Executive Director |  |
| New York City Board of Education | Engineer's Council for Professional <br> Development |
| Sandra Kuntz, Vice-President <br> International Paper Co. Foundation | Chor Weng Tan, Dean <br> School of Engineering <br> The Cooper Union |
| Peter Likins, Dean | Melvin Taylor, Principal |
| School of Engineering \& Applied Science |  |
| Columbia University |  |$\quad$| Benjamin Franklin High School |
| :--- |

## Appendix F

COARD OF EDUCATION OF THE CTY OF NEW YORK
OISTRICT
UVEL - MNOOR MOM/LNTERMEDIATE SCHOOL
TME 1
SOHOOL 45
GRADE SPAN $07-08$

| PHYSICAL FACIITY DATA |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| ADORESS | 2351 First Ave 10035 | TEL 60-5838 | wo of awnexes | 2 |
| reat of construction | 1958 |  | a |  |
| Tre Of COMSTRUCTION | Firspreof thruout |  |  |  |
| cepacty | 1,413 |  |  |  |
| nimomt uturanion | 80.4 |  |  |  |




Appendix G1

SOARD OF EOUCATHON OF THE CTTY OF NEW YORK 197e-190 sChOOL PROFILES
DISTRMCT 13
LEVEL - NHNOR MOW/HNTEMEDATE SCHOOL

| PHYSICAL FACILITY DATA |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A007Es3 | 300 Willoudthy Avo | 11205 | TE 134-6904 | Ho | Anumes |
| Year of construction | 1955 |  |  |  |  |
| TYPE OF CONSTEUCTION | Firupreot Tirueut |  |  |  |  |
| capacity | 1.295 |  |  |  |  |
| mincent umuzanow | 70.6 |  |  |  |  |




Appendix G2

# COARD OF EDLCATION OF THE CIY OF MEW YORK 1974-190 sCHOOL PROFILS <br> IMEL - MMNOR MOM/WTERMEDATE sCHOOL 

Destact 15
Imu: 1
CORO EXOOKIYK
scrool $25{ }^{\circ}$
ceade span of-as

| PHYSICAL FACILTY DATA |  |  |  |
| :---: | :---: | :---: | :---: |
| Mecitss | 141 macen 3111216 | TEL 134-6916 | Mo Of Aminxes |
| The Of construction | 1953 |  |  |
| Trit of consiruction | Propeot tinuout |  |  |
| cepactr | 1,261 |  |  |
| Fmant utmeation | 100.0 |  |  |




Appendix G3

# COARD OF EDUCATION OF TAE CTY OF NEW YORK 1979-1960 satool moples UVES - ACAOEMC COMPREMENSVE HON savOOL <br> SORO MANEATTAN <br> SOMOOL 435 RENJAMMN FRANKUN <br> axade sPan en-12 

Destukt 73
min 1




Mrosition (SV CLASs)

$-$ miti unow wavis (porem of 10/21/70 liedtion) 4.51
1.043


| STAFF DATA |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mumet op nownows/mouns |  |  | ANMUAS salatiss (\%) |  |  |  | Mrustar Ranlo | $\begin{aligned} & \text { STAF cont } \\ & \text { PR NHE } \end{aligned}$ |
|  | TE Lev | Retmbursedide | Total | Teis Levy | Pedmexrectio | Totel | Avereso |  |  |
| nosmon imits |  |  |  |  |  |  |  |  |  |
| Pumiond | 1.0 |  | 1.0 | 39,389 |  | 37.309 | 39,369 | 1.827 .0 | 2 |
| Amen Mr(atm) | 1.0 |  | 1.0 | 29.290 |  | 29.290 | 29,290 | 1.827 .0 | 16 |
| Asen Mar (in) | 5.0 |  | 5.0 | 143,962 |  | 143,962 | 28,792 | 345.4 | $\pi$ |
| Teetrep | 70.0 | 13.0 | 15.0 | 1.518,643 | 309,503 | 1,820,146 | 21,500 | 21.5 | 1,001 |
| -as Commedor | 2.0 | 3.0 | 3.0 | 50,905 | 75,100 | 126.125 | 25,223 | 308.4 | 4 |
| teretery | 0.0 |  | $\bigcirc .0$ | 136,369 |  | 13.369 | 15,152 | 200.0 | 73 |
| Oner Prumed | 1.0 |  | 1.0 | 17.539 |  | 17,559 | 17.559 | 1,827.0 | 10 |
| 8unum | 1.0 |  | 1.0 | 10.391 |  | 10,3P1 | 10,391 | $1,027.0$ | 1250 |
| Eunotal | $\pm$ | 18. | 100 | 1,24, 510 | 204, ${ }^{\text {as }}$ | 2881,21 | 21.80 | 16. | 1510 |
| HOUATY TILES | 20,50 | 14.113 | 37.111 | 116,073 | 100,14 | 216,021 | 5.4.4 |  | 11 |
| 4umer Cuad | 7.350 |  | 7.534 | 39.077 |  | 39,077 | 3.18 | 2 | 21 |
| Onom Morry | 25,418 | 6.094 | 41,502 | 100.790 | 35.127 | 215,926 | 3.20 |  | 11 |
|  | 4,484 | 22,17 | 0,131 | 843.599 | 185.275 | 41.894 | 5.40 |  | 1080 |
| rotas |  |  |  | 2393,117 | 319.03 | 2000.085 |  |  | 180 |
| JNA 00 T <br> AVLuN GAS | R1 $\mathbf{1 . 2 5 0}$ | Q 200 | T1. 1.590 | num Tacme mano (Oved) <br>  |  |  |  |  | 20.3 |
|  |  |  |  |  |  |  |  |  | 246 |
|  |  |  |  |  |  |  |  |  | 85. |
|  |  |  | Number | Hereent |  |  |  |  |  |
| men max | Y sarsua | $C 1$ |  | 9.8 |  |  |  |  |  |
| truMa ruor | T sonbut | C-2 |  | 28.0 |  |  |  |  |  |
| -unar ruok | T sonum | c-4 | 5 | 62.2 |  |  |  |  |  |
| TOTA | \% |  |  | 1000 |  |  |  |  |  |

Appendix G4
minuct ts
Inve 1

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mLs 1

COARD OF EDUCATION OF THE CTY OF HTW YORK 197R-186 satoot PROFILE
LVEL - ACABEMIC COMPAEMENEMV MOH sCroot
sarool 465 cronos Wasmmaton




STAFF DATA


COARD OF EDUCAINON OF THE CIT OF NEW YORK
1979-1940 SCNOOL PROFILES
Destrict 78
TIIL 1
LEVEL - ACAOEMIC COMPREMENSIVE MOH SCHOOL
SOAOOL 475 JOWM F KENNEDY HS

PHYSICAL FACIITY DATA





Appendix G7

COARD OF EDUCATION OF THE CIT OF NEW YORK 1979-1940 schoot mofiles
DISTRICT 78
TITLE 1
LEVE - ACADEMIC COMPREHENSIVE HIOH SCHOOL
EORO EROOKITM
GRADE SPAN OP-12

| PHYSICAL FACILITY DATA |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ADORES3 | 29 For Groene M | 11217 |  |  | TEL B50-5150 |  | 100 | Amexes |
| YEAR OF CONSTRUCTION TVPE OF COMSTRUCTION | 1933 |  |  |  | MODERMZATION | 1960 |  |  |
| CAPACTY | 5.782 |  |  |  |  |  |  |  |
| PEACEMT UTULZATION | 89.5 |  | averact | mistructional | Meriods 6.9 |  |  |  |





## COARD OF EDUCATHON OF TNE CIT OF NEW YORK

0estaict 78 LVEL - ACADEMIC COMPREMENSUVE MOH SCHOOL

EORO EROOKIYM ITHE 1

SOHOOL 460 JOWN JAY HS
ORADE SPAM OR-12

|  | PHYSICAL FACILITY DATA |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nicaliss | 2377 Ave 11215 |  |  | TEL 78-1514 | NO 0 | ANMEXS |
| riak of constauction | 1903 | adomion | 1939 | TIL Jozisia | NO | dunex |
| eapadir | Finoproof thruour |  |  |  |  |  |
| Hmomi umbrantion | 122. | avizaer | NSth | Hatoos 6.2 |  |  |





Appendix G9

# COARD OF EDUCATION OF THE CIY OF NEW YORK <br> 1578-1800 school mofilss LVEL - VOCATIOHAL TICNWCAL MOH sCHOOL 

DETRICT 78
sono manmattam
NON-ITIE 1
sayOOS 15 CHELSA VHS
CAAD SPAN ER-12





Appendix G10

## COARD OF EDUCATION OF TME CTY OF NEW YORK 197P-1980 sCHOOL PROFILES

Castact 78 LEVEL - VOCATOMAL TECHNTCAL MEH sOHOOL

DORO MOOKIYA
NON-TITLE 1
SCHOOL 615 EAST NEW YORK VTHS





Appendix G11

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[^0]:    * For the purposes of this project study, the term "minority" refers to Black and Hispanic persons.

[^1]:    *The percentage of low income pupils is determined by dividing the school's October 291982 register into the weighted sum of free and reduced cost lunch-eligible pupils enrolled in the school and children aged 5 to 17 receiving Aid to Families with Dependent Children (AFCD) payments.

[^2]:    * In 1984, New York City adopted a new Integrated Mathematics Sequence, prescribed by the Board of Regents, that added topics in probability, logic, statistics and transformations.

[^3]:    *John Wiley and Sons, 1975.

[^4]:    * Based on results of only one class.
    $\mathrm{LU}=$ Leaming Unit.

