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1 Overview

We propose a new way to model derivations, based on a category (Mac Lane, 1971) $\mathbf{Der}$, building on the work of Boston et al. (2010) and Ehrig et al. (1973; 1997). $\mathbf{Der}$ is a generalization of $\mathbf{FPos}$, the category of finite partial orders (fposets) and order-preserving maps. The objects of $\mathbf{Der}$ are 'variable' fposets which keep track of derived syntactic objects (DSOs), parameterized over a sequence of steps. As with Boston, et al. we consider derivations of dependency trees, here modeled as fposets. However, we represent structural changes explicitly in derivations as order-preserving maps. In this sense, we represent structural changes like the "derivations" of Ehrig et al. (1997), in that they are morphisms between dependency structures (explicit mappings between nodes which preserve dependency). This gives information relating the structure of each DSO to each DSO depending on it. These data are sufficient to characterize projection, constituency, grammatical relations, and many other properties. A morphism $\phi : \Delta \to \Gamma$ in $\mathbf{Der}$ is a generalization of an order-preserving function which induces order-preserving functions between DSOs associated to points of Δ and Γ. In this sense, a morphism of derivations piecewise maps (portions of) DSOs in Δ to (portions of) DSOs in Γ.

Since we in principle allow general posets to be derived objects, and general order-preserving maps to represent "structural changes", it is an immediate consequence that we can model feature-geometry, feature sharing, etc. in derivations and derived objects, not expressible in prior models (such as Boston, et al.).[1] Since we encode the structural change information in the derivation itself, morphisms between derivations will keep track of this information. Similarly, we can describe grammatical relations just using derivational structure, since the explicit structural changes are part of the model.

2 Formal properties of $\mathbf{Der}$

A derivation Δ consists of: (1) a set X of points; (2) a partial ordering $\leq$ on X; and (3) for each point $x \in X$, a partial order $\top_x$ and an order-preserving surjection $!_x : U_x \to \top_x$, where $U_x = \{y \in X \mid x \leq y\}$. The assignments in (3) must meet a 'compatibility condition': if $y \leq x$, then there exists an order-preserving function $f_{x,y} : \top_x \to \top_y$ such that $!_y \circ i_{x,y} = f_{x,y} \circ !_x$, where $i_{x,y} : U_x \hookrightarrow U_y$ is just the subspace inclusion. If such a function exists, it is unique. Thus, to every point $x \in X$, we have an associated partial DSO $\top_x$, and $f_{x,y} : \top_x \to \top_y$ represents the net structural change from the partial DSO at x to the partial DSO at y whenever $y \leq x$. We write $|\Delta|$ for the set underlying Δ, and $\leq_\Delta$ for the underlying partial ordering on $|\Delta|$. Note that $\top_x$ has root (least element) $!_x(x)$.

We define a morphism of derivations $\phi : \Delta \to \Gamma$ to be a function $|\phi| : |\Delta| \to |\Gamma|$ such that $a \leq_\Delta b$ implies that $\phi(a) \leq_\Gamma \phi(b)$. This function must induce order-preserving maps between partial DSOs, in that for every $x \in \Delta$, there must exist a (necessarily unique) order-preserving function $\phi_x : \top_x \to$

[1] Feature sharing and its implementations in syntax have been described by (Frampton and Gutmann, 2000) and (Pesetsky and Torrego, 2007); feature geometry in (Harley and Ritter, 2002), (Preminger, 2014), and (Svenonius and Bye, 2011).

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$\top_{\phi(x)}$ such that $!_{\phi(x)} \circ \phi = \phi_x \circ !_x : U_x \to \top_{\phi(x)}$.

This category is *concrete* in the sense of Adámek, et al. (2004) using the obvious (representable) functor $|\cdot| : \mathbf{Der} \to \mathbf{Set}$ mapping Δ to $|\Delta|$ and morphisms to their underlying functions.[2]

All categories have an induced notion of isomorphism. A morphism $\phi : \Delta \to \Gamma$ is an iso if there is a morphism $\psi : \Gamma \to \Delta$ such that $\psi \circ \phi = 1_\Delta$ and $\phi \circ \psi = 1_\Gamma$, where 1_Δ and 1_Γ are the identity functions on Δ and Γ (Mac Lane, 1971). Two derivations Δ and Γ are then isomorphic iff their points are in a bijective correspondence, such that for each point $x \in \Delta$, $\psi_{\phi(x)} \circ \phi_x = 1_{\top_x}$ and $\phi_x \circ \psi_{\phi(x)} = 1_{\top_{\phi(x)}}$ are isos of partial orders, such that the following diagram of fposets commutes for any $x \leq_\Delta y \in \Delta$:

$$\begin{tikzcd} \top_x \arrow[r, leftrightarrow, "\sim"] & \top_{\phi(x)} \\ \top_y \arrow[u, "!_{y,x}"] \arrow[r, leftrightarrow, "\sim"] & \top_{\phi(y)} \arrow[u, "!_{\phi y, \phi x}"'] \end{tikzcd}$$

That is, the bijection between Δ and Γ extends to an isomorphism between corresponding DSOs, such that these isomorphisms induce isomorphisms of structural changes (order-preserving functions). For example, the derivations for 'I saw the dog' and 'I saw the cat' will be isomorphic, so long as the feature structure of each word and the dependencies introduced are in exact correspondence. We can say that $x \leq y$ is a projection relation if $\top_y \to \top_x$ maps $!_y(y)$ to $!_x(x)$; in this case we write $x \sqsubset y$. It is immediate that projection is preserved not only under isomorphism, but arbitrary morphisms.

Concrete categories have an induced notion of embedding. A morphism $\iota : \mathcal{S} \to \Delta$ is an *embedding* if the underlying function is injective, and for any $\phi : \Gamma \to \Delta$ such that for all $g \in \Gamma$, $\phi(g) \in \mathcal{S}$, the function $\phi|^{\mathcal{S}} : \Gamma \to \mathcal{S}$ is a morphism (Adámek et al., 2004). We can prove that for any subset $S \subset |\Delta|$ of any Δ, there is a unique embedding $\mathcal{S} \to \Delta$ whose underlying set function is the subset inclusion $S \subset |\Delta|$. We can show that for $a, b \in S$, $a \leq_{\mathcal{S}} b$ iff $a \leq_\Delta b$. The induced map $\top^{\mathcal{S}}_a \to \top^\Delta_a$ from the DSO at a in $\mathcal{S}$ to the DSO at a in Δ is an embedding of fposets for each $a \in \mathcal{S}$. An important class of subderivations are those associated to a subset of a derived object: for any subset of a derived object $K \subset \top_x$, $x \in \Delta$, there is an induced subderivation structure on $\{y \in \Delta \mid x \leq y \text{ such that } !_x(y) \in K\}$. Both 'derivational' constituents and 'derived' constituents are special cases of these. Such substructures are useful for 'structured' implementations of 'copying the derivation tree' along the lines of Kobele (2006). More general subderivations show what dependencies remain if certain objects are removed from the derivation.

Similarly, every pair of derivations (Δ, Γ) has a product with underlying set $|\Delta| \times |\Gamma|$ and coproduct with underlying set $|\Delta| + |\Gamma|$, where $\times$ and $+$ are Cartesian product and disjoint union, respectively. In fact, $\mathbf{Der}$ has all finite limits and colimits.[3] Coproducts can be used for representing workspaces, but they are especially useful for describing structural changes taking in tuples of objects.

3 Rules and recursive construction of derivations

We have formalized structural changes as order-preserving maps $f : P \to Q$ between derived objects, as in (Ehrig et al., 1997). Since $\mathbf{FPos}$, like the category of directed graphs, has coproducts, a tuple of maps $f_i : P_i \to Z$ can be represented by a single map $+_i f_i : P_1 + \ldots + P_n \to Z$; so, adapting Ehrig et al.'s method to tuples of syntactic objects, like in (Boston et al., 2010), doesn't require any new constructions. However, in describing a *grammar*, it is useful to think of many structural changes as arising from a common operation, applied in different contexts. Ehrig et al.'s 'single pushout method' does exactly this. In any category, given objects A, B, and C and maps $f : A \to B$ and $g : A \to C$, a *pushout* of f and g is an object E together with maps $k_1 : B \to E$ and $k_2 : C \to E$ such that $k_1 \circ f = k_2 \circ g$, such that for any F with maps $s_1 : B \to F$ and $s_2 : C \to F$ such that $s_1 \circ f = s_2 \circ g$, there is a unique morphism $u : E \to F$ such that $u \circ k_1 = s_1$ and $u \circ k_2 = s_2$ (Borceux, 1994). As with all universal constructions, if (E, k_1, k_2) and (E', k'_1, k'_2) are any two pushouts of f and g, then there is a uniquely determined isomorphism $E \cong E'$ between them. We call k_1 the pushout of g along f, and conversely k_2 the pushout of f along g.

[2]The functor is represented by a terminal derivation $\mathbf{1}$, any singleton with the only possible derivation structure.

[3]For standard definitions of limits and colimits, see (Mac Lane, 1971).

As an example, we show that for any pair of rooted partial orders R and S, the map $f : R + S \to Z$ which attaches the root of S to the root of R, f arises as the pushout of a canonical root-attachment structural change along the unique map picking out the two roots, as shown in Fig. 1. The 'basic operation' is pushed out along the 'input-in-context' (structural description) to obtain the structural change on R and S.

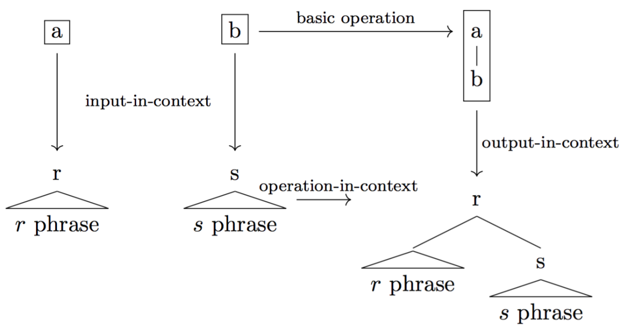

Figure 1: Example rule attaching one root to another.

To restrict application of a rule, restrict which vertical maps are admissible. For example, we can restrict to maps arising as a sum of two maps which must send a to the root of R and b to the root of S (if these roots exist).

We can generalize the construction to derivations. We first note that $\mathbf{FPos} \hookrightarrow \mathbf{Der}$ is a subcategory, taking P to the derivation with the same points and underlying order, where for each $x \in P$, we have $!_x : U_x \to \top_x$ an isomorphism. Consider where A, B, and C are derivations (say, Δ, Γ, and Σ) with morphisms $\phi : \Delta \to \Gamma$ and $\psi : \Delta \to \Sigma$. We ask for the universal *fposet* completing the diagram, that is, an fposet P (considered as a derivation) together with derivation morphisms (P, k_1, k_2), such that for any other poset and pair of morphisms (Q, s_1, s_2), we have a unique order-preserving map $u : P \to Q$ such that $u \circ k_1 = s_1$ and $u \circ k_2 = s_2$. If ϕ is the basic 'structural change' and ψ the structural description, we can obtain this 'pushout' of ϕ along ψ, an operation $h : \Sigma \to P$. Since $\mathbf{Der}$ has coproducts, this also handles operations acting on tuples.

Given an operation $h : \Sigma \to P$, we want a construction which returns a new derivation containing Σ as a subderivation, with P 'on top', incorporating the 'new' structural change h. This can also be given with a universal construction. Consider any derivation E and morphism $k : \Sigma + P \to E$. We say that k ***takes h-images to projection*** if for every $x \in \Sigma$, $k(h(x)) \sqsubset k(x)$. For any operation $h : \Sigma \to P$, there is a universal derivation $\mathrm{ext}(h)$ together with a map $k : \Sigma + P \to \mathrm{ext}(h)$ which takes h-images to projection, in that if $k' : \Sigma + P \to E$ takes h-images to projection, then there is a unique morphism $u : \mathrm{ext}(h) \to E$ such that $uk = k'$.

We can recursively construct derivations by starting with some base fposets, then taking pushouts to obtain operations, then extending along those operations using the construction above. The model given so far does not keep track of 'typing' of elements in the DSOs (N/V/wh/etc.), nor whether a feature is 'active', nor any ordering between the features (indicating the order they must be checked in). We may add these data to fully emulate Boston et al. (2010) and similar formalisms using objects of $\mathbf{Der}$ and operations as sketched above. Operations using feature geometry and feature sharing can be modeled identically to the root-attachment case using the same methods, just with different generating operations and admissible contexts.

4 Comparison to other models

The defining property of $\mathbf{Der}$ is that it emphasizes descriptions of derivations as 'structured sets'. We compare our notions of isos and substructures to those in other theories.

We start by looking at the relevant notions for DSOs. Stabler & Keenan (2003; 2010) view a grammar as a set of expressions, together with partial operations. Their method is general, accounting for many variants of formal MGs. Two objects are isomorphic in their sense if there is a permutation π of the expressions preserving the operations, such that π takes one object to the other. This notion is not with respect to the structure of the SOs: embedding a grammar in another can decrease the isomorphisms between objects, simply by virtue of there being more items to combine with. There is no corresponding notion of substructure of a DSO, though they do give a 'constituency' relation between DSOs. A is a constituent of B in their sense if there is some sequence of operations taking in an A and eventually producing a B. Since this quantifies over the whole grammar, items not occurring in particular derivations of B may still be constituents of it; e.g., if $\lambda ::= yx$ and $\lambda :: y$ are items of the grammar, they will be constituents of $\lambda : x$, despite not occurring in the derivation in Fig. 2L. In both cases, these definitions are about the combinatorial relations between objects under operations, not about the structure of the objects involved.

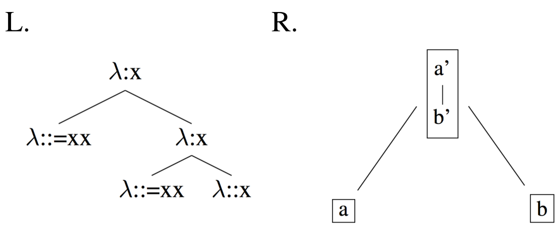

Figure 2: Example MG and $\mathbf{Der}$ derivations

We turn to isos of derivations. Kobele (2006) constructs labeled ordered trees which keep track of the sequence of steps of the derivation and the derived SO at each step. Notice if we take order- and label-preserving maps between them as morphisms, the isos are then very 'rigid' in that only identically labeled trees can be isomorphic, unable to compare the derivations of sentences like those for "I saw the dog" and "I saw the cat". We could weaken the morphisms, such that they do not have to preserve the labels 'on the nose' but rather only 'up to iso of DSOs/lexical items.' Such a notion is at least clear when DSOs are objects like graphs or fposets. For example, consider derivations of dependency trees like those in Boston et al., such as in Fig. 2R. However, consider the derivation just like that in Fig. 2R, except a' and b' have been reversed. We can isomorphically compare the steps of the two derivations, mapping a to a, and b to b, and bicontinuously compare the $a' < b'$ tree to the $b' < a'$ tree. However, this should not be an iso, since the attachment is reversed. $\mathbf{Der}$ does not suffer from this issue precisely because it explicitly keeps track of the relations between steps, even without making reference to the operation applied. Our notion of isos of derivations extends to a more robust notion of equivalence of languages, beyond the case of inclusions of languages such as the 'lexical extensions' of Keenan & Stabler (2003; 2010). None of the other models can talk about the general subderivations from §3.

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References

Jiří Adámek, Horst Herrlich, and George E Strecker. 2004. *Abstract and concrete categories. The Joy of Cats*. Citeseer.

Francis Borceux. 1994. *Handbook of Categorical Algebra 1: Basic category theory*. Cambridge Univ. Press.

Marisa Ferrara Boston, John T Hale, and Marco Kuhlmann. 2010. Dependency structures derived from minimalist grammars. In *The Mathematics of Language*, pages 1–12. Springer.

Hartmut Ehrig, Michael Pfender, and Hans Jürgen Schneider. 1973. Graph-grammars: An algebraic approach. In *Switching and Automata Theory, 1973. SWAT'08. IEEE Conference Record of 14th Annual Symposium on*, pages 167–180. IEEE.

Hartmut Ehrig, Reiko Heckel, Martin Korff, Michael Löwe, Leila Ribeiro, Annika Wagner, and Andrea Corradini. 1997. Algebraic approaches to graph transformation: part ii: single pushout approach and comparison with double pushout approach. In *Handbook of Graph Grammars*, pages 247–312.

John Frampton and Sam Gutmann. 2000. Agreement is feature sharing. *Ms., Northeastern University*.

Heidi Harley and Elizabeth Ritter. 2002. Person and number in pronouns: A feature-geometric analysis. *Language*, 78(3):482–526.

Edward L Keenan and Edward P Stabler. 2010. Language variation and linguistic invariants. *Lingua*, 120(12):2680–2685.

Gregory Michael Kobele. 2006. *Generating Copies: An investigation into structural identity in language and grammar*. Ph.D. thesis, Citeseer.

Saunders Mac Lane. 1971. *Categories for the working mathematician*. Springer-Verlag.

David Pesetsky and Esther Torrego. 2007. The syntax of valuation and the interpretability of features. *Phrasal and clausal architecture: Syntactic derivation and interpretation*, pages 262–294.

Omer Preminger. 2014. *Agreement and its failures*, volume 68. MIT Press.

Edward P Stabler and Edward L Keenan. 2003. Structural similarity within and among languages. *Theoretical Computer Science*, 293(2):345–363.

Peter Svenonius and Patrik Bye. 2011. Non-concatenative morphology as epiphenomenon.