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Technology Integration and Use in Elementary Mathematics Methods Courses for Pre-service Teachers

Elzbieta Manos

TECHNOLOGY INTEGRATION AND USE IN ELEMENTARY MATHEMATICS METHODS COURSES FOR PRE-SERVICE TEACHERS

A Dissertation Presented

By

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DEDICATION

To my son, Dean, my husband, Paul, my mother, Zofia, and my father, Piotr

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ABSTRACT

TECHNOLOGY INTEGRATION AND USE IN ELEMENTARY MATHEMATICS METHODS COURSES FOR PRE-SERVICE TEACHERS

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According to the National Educational Technology Plan 2010, technology should be incorporated into teaching methods courses and field experiences and not just in stand-alone technology courses. The teacher preparation programs would provide technology-based learning experiences to prepare pre-service teachers to effectively use technology to improve learning, assessment, and instructional practices. However, the problem is that graduate pre-service teacher preparation programs do not adequately prepare pre-service teachers to incorporate technology into their teaching. Furthermore, the teacher preparation programs lacked opportunities for the pre-service teachers to experience technology as learners beyond the stand-alone course in technology.

Research shows the need for pre-service teachers to experience technology as learners so that they can use their knowledge to create learning environments of greater understanding in their future classrooms, specifically in the area of mathematics. Technological pedagogical content knowledge is knowledge of how to incorporate technology into the teaching of content to promote student learning (Koehler & Mishra, 2009).

This study investigated the ways in which math methods courses that provide technology-based learning experiences for pre-service teachers enable them to gain the technological pedagogical content knowledge necessary for effective teaching. This study investigated two elementary methods courses where technology integration was in place. Informants included the instructors and pre-service teachers in each course. A qualitative multiple case study methodology utilizing observations of methods courses, interviews with faculty and pre-service teachers, and collection of teaching and learning artifacts was used. Additionally, this study focused on both the faculty and the students' use of instructional technology for enhancing the teaching and learning.

Furthermore, Massachusetts has a technology self-assessment tool that can be utilized by teachers to assess their own technology proficiency (Massachusetts Department of Elementary and Secondary Education, 2010). The criteria in Standard 3, Teaching and Learning with Technology, was used to assess the instructors. The analysis also described how faculty used and modeled instructional technology in the methods courses to enhance teaching and learning.

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CHAPTER 1

PROBLEM STATEMENT

As one of its goals for teaching, the National Education Technology Plan 2010 states, "Professional educators will be supported individually and in teams by technology that connects them to data, content, resources, expertise, and learning experiences that enable and inspire more effective teaching for all learners" (U.S. Department of Education, 2010, p. 49). One way to accomplish this goal is through teacher preparation programs. According to this plan, technology should be incorporated into teaching methods courses and field experiences and not just in stand-alone technology courses. The teacher preparation programs would provide technology-based learning experiences to prepare both pre-service and in-service teachers to effectively use technology to "improve learning, assessment, and instructional practices" (p. 50).

However, according to the National Center for Education Statistics, in 2009, only 33% of all public school elementary teachers reported that their graduate teacher education program prepared them to a moderate or major extent to use educational technology for instruction (Gray, Thomas, & Lewis, 2010). Another 32% reported that their graduate program was not applicable—in other words, not a contributing factor. In addition, Sutton (2011) found that novice teachers felt that the required stand-alone technology course, Introduction to Instructional Computing, in their graduate pre-service teacher preparation program did not adequately prepare them to incorporate technology into their classrooms, nor did it enable them to retain or transfer the skills learned in that course to other coursework and components of their program, such as the methods courses. Furthermore, the novice teachers indicated that their education program lacked opportunities to experience technology as learners beyond the one course.

Computer technology is an important classroom component for student learning and can be used to increase student achievement, comprehension, and problem-solving skills (Otero et al., 2005). However, the National Center for Education Statistics (2000) reported that almost two-thirds of teachers indicated that they were ill prepared to use computers and the Internet in their own classrooms. According to the International Reading Association (2009), "Students must become proficient in the new literacies of 21st-century technologies. As a result, literacy educators have a responsibility to effectively integrate these new technologies into the curriculum, preparing students for the literacy future they deserve" (p. 2). Lastly, technology skills are needed for an educated workforce—a goal of education in general (Marx, 2005; Okojie & Olinzock, 2006; Otero et al., 2005) and particularly for pre-service teachers (Otero et al., 2005)—but students are not learning these skills (Brown & Warschauer, 2006; Watson, 2006).

In addition, there are successful ways for pre-service teachers to acquire knowledge and skills to design effective instruction using technology aligned with constructivism, but again this is not found in teacher education programs (Brown & Warschauer, 2006; Murray & Zembal-Saul, 2008). Moreover, there seems to be a gap in the technology knowledge and skills that pre-service teachers do have and their confidence to effectively use that knowledge (Davis & Falba, 2002; Pope, Hare, & Howard, 2005). Thus, the fact that pre-service teachers know how to use such technologies as the Internet or word processing does not translate into effective classroom integration (Ertmer et al., 2003). In a 10-year study conducted in 1995, researchers found that, relative to other majors, education majors not only have greater computer anxiety but also less familiarity with computers (Reed, Ervin, & Oughton, 1995). This is not a very encouraging statistic for our future teachers. Furthermore, a 2001 study of pre-service teachers had similar findings: These college students, relative to their peers, had less familiarity and expertise with computers (Reehm, Long, & Dickey, 2001). The study's authors found that knowledge of computers was lacking in teacher education programs.

CHAPTER 2

LITERATURE REVIEW

The literature review addresses Vygotsky's sociocultural learning theory, the use of technology in learning new ideas including mathematics, how pre-service teachers learn to teach, as well as how they learn to teach with technology. The Technological Pedagogical Content Knowledge (TPACK) framework is also described in the teaching with technology section.

2.1 Sociocultural Learning Theory: Vygotsky

Vygotsky was both an educator in the classroom as well as a researcher (Jaramillo, 1996). He regarded psychology as the area between sociology and biology (Shalin, 2017). In other words, he believed that "every function in the cultural development of the child appears on the stage twice, in two planes, first, the social, then the psychological" and thus "first between people", then "within the child" (as cited in Shalin, 2017, p. 182).

Vygotsky believed that learning is social and proposed the theory of the "zone of proximal development." The focus of this theory is on "the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (as cited in Wertsch, 1979). For example, in an honors thesis course, the adult guidance consisted of the faculty reviewers, thesis committee members, librarians, theorists and researchers, and the general public [at the thesis defense] (Briggs, 2010). The students' problem-solving included "unpacking, translating, linking, [and] speculating" the research (Briggs, 2010, p.64). Thus ZPD applies not only to young children but also to college students.

This ZPD can be broken down into three contexts: "developmental, educational, and assessment oriented" (Kozulin, 2011, p.195). The developmental context refers to the surfacing of new psychological performance. Vygotsky considers that the optimal period for the development of a particular ability to be the emergence of that psychological ability as opposed to the full development. The educational context refers to it as the "motor" and driving force of a child's progress. Vygotsky believed that the attainment of "academic concepts" needs deliberate instructional activity (Vygotsky, 2012). Lastly, the assessment context denotes a process that is "assisted, presenting a complete problem-solving model, asking leading questions, starting solution, and then asking the child to continue" (Kozulin, 2011, p.196). An example of the ZPD might be how more tech-savvy students provide technical support and guidance to the less tech-savvy students in their coursework (Abbott & Faris, 2000). A tutoring writing center provides another example of ZPD. First, the tutor/instructor understands the current skills of the student and then tailors the tutoring session on practicing the developing skills of the student (Nordlof, 2014).

According to Vygotsky, a teacher determines/assesses a child's ZPD in the following ways:

1. Demonstrate the solution to see if the child can imitate the steps
2. Begin to solve the problems to see if the child can finish it
3. Have the child cooperate with another, more fully developed child (a child who has a higher IQ)
4. Explain the principles of solving the problem, ask leading questions, analyze the problem for the child, and so on (Gredler, 2012, p.118).

Then the teacher can design instruction for the child at the ZPD level.

Additionally, Vygotsky utilized the concept of "scaffolding" in his ZPD theory. He describes scaffolding as an "assisted learning process that supports ZPD, or getting to the next level of understanding, of each student from the assistance of teachers, peers, or other adults" (as cited in Powell & Kalina, 2009, p.244). In other words, the scaffolding provides a support system for the student/learner. In this ZPD, the teacher starts with concepts that are slightly above the students' current knowledgebase. The students then use manipulatives in a realistic activity to learn the concepts and "construct" meaning from their experience (Jaramillo, 1996). The teacher facilitates the activity in which the students interact with the manipulatives in small groups. Thus, in the classroom, the teacher understands the student's cognitive processes/activities according to Vygotsky: "conscious awareness of his or her own thinking" and "understanding of the psychological nature of the task" (Gredler, 2012, p.125).

Vygotsky stressed "the role of tools which mediate and control the relationship between subjects and object (goals)" (as cited in Keengwe & Kang, 2012, p. 85). Some psychological tools described by Vygotsky include "language, different forms of numeration and counting, mnemotechnic [vocabulary learning] techniques, algebraic symbolism, works of art, writing, schemes, diagrams, maps, blueprints, [and] all sorts of conventional signs" (as cited in Shalin, 2017, p. 183). These tools are internal or external (Pange & Kontozisis, 2001). These symbolic tools are social in nature, and mediate experiences and learning.

Vygotsky also emphasized the importance of "the social activity of speech or speaking rather than the structure of the language system" and that "speech can be understood only if it is viewed as being part of ongoing human activity" (as cited in Wertsch, 1979, p. 4). Language is the main "vehicle" of communication between humans and is therefore required in social interactions (as cited in Jaramillo, 1996). Additionally, Vygotsky believed that "language" enriches learning and that it comes before knowledge or thinking; it is a "correlative of consciousness" (as cited in Powell & Kalina, 2009). Specifically, Vygotsky (as cited in Nordlof, 2014, p.55-56) expressed the following:

> The acquisition of language can provide a paradigm for the entire problem of the relation between learning and development. Language arises initially as a means of communication between the child and the people in his environment. Only subsequently, upon conversion to internal speech, does it come to organize the child's thought, that is, become an internal mental function.

Technology can provide tools that facilitate speech and social discourse and thus can be used to co-construct knowledge and learning according to Vygotsky's philosophy.

Additionally, Vygotsky's sociocultural theory denotes that individual learning occurs in a social situation and cannot be separated (as cited in Jaramillo, 1996). Furthermore, the group [more knowledgeable peers and adults] plays a vital role in that individual learning process and thus in the construction of knowledge. In Vygotsky's view, this group teaches the "rules and norms of society", i.e. the "social", to individuals [learners] (as cited in Jaramillo, 1996, p.134). The individual needs to experience the learning and then "socially" negotiate their meaning in a real learning environment.

According to Vygotsky, in the classroom, the teacher facilitates student learning. The teacher needs to understand the prior experiences of his/her students and design instruction based upon those experiences, "a continuity of experience" (as cited in Jaramillo, 1996, p.134). Furthermore, Vygotsky stressed the need for experiential learning, i.e. "learning by doing", as did Dewey (1902/1971). This learning consists of both internal and external experiences as well as cognitive, emotional, and external interactions (as cited in Jaramillo, 1996). Additionally, this learning in the classroom should be consistent with the "real world" and vice versa (Pange & Knotozisis, 2001).

Moreover, in the classroom, Vygotsky emphasized peer collaboration. In small groups, students would work together on various learning activities including problem-solving. Again the teacher facilitates the learning activities and has each group and/or student explain his/her thinking, building on each other's' thinking on how they solved the problem. In the groups, the students exchange ideas, discuss their strategies as well as misconceptions, and communicate their thinking (as cited in Jaramillo, 1996; Henson, 2003). The teacher promotes the "dialogue" and encourages students to think critically (Powell & Kalina, 2009). In other words, everyone is learning from everyone. The students worked more efficiently problem-solving in groups as compared to individually. Vygotsky named this type of social learning as "negotiating meaning" and now referred to as cooperative learning (as cited in Henson, 2003, p.13; Shalin, 2017). It is student/learner-centered learning. Although each student internalizes this learning/knowledge, it happens differently for each student depending on his/her own experience. According to Vygotsky, this "internalization" is more effective in a social situation as described above (as cited in Powell & Kalina, 2009).

2.2 The Use of Technology – Learning of New Ideas and the Learner

Technology can be used to promote this cognitive growth in children as well as adults (Judson, 2006; Otero et al., 2005). Technology can be used to facilitate problem-solving (Hartsell, 2006) and foster collaboration and social interaction with the teacher and/or other students (Judson, 2006; Sherman & Kurshan, 2005). Technology provides an opportunity to form online communities of students and adults who work together to solve particular problems (Sherman & Kurshan, 2005). Technology in the form of cognitive tools can help pre-service teachers comprehend concepts and solve problems. For example, Inspiration software can facilitate the critical analysis of complex texts, such as Jonathan Kozol's *Savage Inequalities* (Otero et al., 2005).

In 2011, Strawn reviewed research on technology and learning and referenced an article by Tamim, Bernard, Borokhovski, Abrami, and Schmid (2011), a meta-analysis of 40 years of research. Tamim et al. (p. 16) presented three key findings:

1. A significant positive small to moderate effect size favoring the utilization of technology in the experimental condition over more traditional instruction (i.e., technology free) in the control group.
2. Computer technology that supports instruction has a marginally but significantly higher average effect size compared to technology applications that provide direct instruction.
3. The average effect size for K–12 applications of computer technology was higher than computer applications introduced in postsecondary classrooms.

Thus, as described above in the second finding, when students experience technology as a tool for learning, these experiences lead to greater understanding compared to when students use a website or some other technology to learn the content themselves. Furthermore, instruction with technology in K–12 shows students scoring “12 percentile points higher” than their counterparts without the technology (Strawn, 2011, p. 38). This is a significant difference and one that educators should be aware of.

Other educators also recognize the impact of technology at such a young age on greater achievement in school. Espinosa, Laffey, Whittaker, and Sheng (2006) documented a longitudinal study of children from kindergarten through fifth grade in private and public schools where their cognitive development was assessed as measured by their reading and math ability. Technology use at home and socioeconomic status (i.e., household income and parental educational attainment) were two variables that were studied. The researchers found technology use at home is so prevalent that almost half of the households in the lowest quintile of SES indicated as such. The assessments were conducted during the children's spring semesters in kindergarten, first grade, and third grade. Espinosa et al. (pp. 428–433) presented the following key findings:

1. Greater incomes and higher parental educational attainment (in this case as indicating higher SES) showed a greater access to computers, Internet access, and books at home.
2. Technology and Internet use at home were positively correlated with grade-level reading achievement in kindergarten and third grade, and SES was positively/significantly correlated to both reading achievement and reading growth rates.
3. Similar findings for math achievement correlated to technology use and Internet use at home.

Interestingly, technology use at home had a positive correlation in kindergarten but turned out to be negative in the third grade (p. 437). One possible explanation provided by the researcher is the educational software used in third grade is not related to the assessments used in the grade level.

It is essential that students develop skills to function and problem solve in groups. In the real world, problems are not solved in isolation but through collaboration with others. Technology can facilitate this student-teacher and student-student contact and communication, which in turn can enhance student learning.

For example, in an educational technology course focused on problem-based learning, pre-service teachers used various technologies to work on such activities as ”authentic” problem solving, discussion, and reflection (Park & Ertmer, 2008). In an elementary social studies methods course for pre-service teachers, a ”virtual field trip” provided students with opportunities for collaboration, communication, reflection, and observation (Gibson, 2002). These pre-service teachers indicated this constructivist learning experience was a significant teaching and learning tool.

In addition, Pope et al. (2005) described a constructivist model of technology incorporation where the focus is on the student, a ”learner-centered” classroom. One such example of this type of technology is multimedia or hypermedia. The instructor can use multimedia to teach content material to students whereby students can conduct research, investigate ideas, and then present their findings rather than the instructor lecturing to the students (Hartsell, 2006).

Technology can be used in the classroom to promote meaningful student learning in keeping with constructivist approaches, particularly where the absence of the technology would make learning difficult, impractical, or virtually impossible. Simulation software, such as the *Oregon Trail*, enables students to take the historical trail across the country to Oregon whereby they must actively participate and make various decisions regarding food, weather conditions, sickness, and other variables (Hartsell, 2006). This software provides a meaningful learning experience that, given its historical nature, otherwise would not be possible for students. CD-ROM software, such as *The Astronomy Village* developed by NASA, provides a virtual community whereby students conduct an investigation and search for such items as a supernova, earth-crossing objects, or stars. Then the students must present their results to the rest of the class (Harstell, 2006). This CD-ROM software provides a meaningful learning experience of astronomy exploration that otherwise would not be available to or practical for students.

Digital camera technology can also be used to facilitate student learning. In a science methods course as part of the plant growth project (discussed later), the pre-service teachers used digital cameras (and also traditional cameras) to illustrate the changes in the plants each week, and these photographs were then incorporated into PowerPoint presentations (Davis & Falba, 2002). It would have been nearly impossible for the students to effectively illustrate these changes in any other manner (including drawing them). Furthermore, these same students utilized spreadsheet software to record data and generate various graphs. They could then compare and contrast their data and findings (Davis & Falba, 2002). In other words, the spreadsheet software, particularly the graphing capability, facilitated the students' learning and understanding of plant growth.

Becker and Ravitz (1999) studied more than 400 elementary and secondary teachers in over 150 schools and looked at their teaching practices relative to computer technology and the Internet. They discovered that high school teachers and their teaching practices were the most influenced by utilizing computer technology toward constructivist pedagogy. Specifically, those teaching social studies, science, and nonacademic subjects utilized the computer technology to reflect real-world activities and practices (p. 381). These pedagogies included more project-based activities, more parallel activities, and more student input into the types of activities selected.

Blended online learning environments demonstrate the application of Vygotsky's zone of proximal development on student learning. Blended (sometimes referred to as hybrid) online environments are those in which students and instructors interact in both online and face-to-face (FtF) learning environments. Chen (2012) compared third graders in two blended online courses (one involved FtF peer interaction and the other FtF student-teacher interaction) and a totally online course (no interaction with either teacher or other students; this was the control group). The same instructional materials were provided to all students and a test was administered to measure their knowledge of the materials. The researcher found that students in both blended courses attained significantly higher scores in the fact portion of the test as compared to the students in the totally online course. Furthermore, there were no significant differences in findings between the two blended online environments. Thus, the guidance of the adult (teacher) or more capable peers (the other students in the course) contributed significantly to the students' learning and understanding.

Learning clubs and learning communities are also exemplars of Vygotsky's zone of proximal development as related to student learning (Hung & Nichani, 2002). Fifth Dimension is a learning club initiative involving public schools and other organizations, such as Boys and Girls Clubs, YWCA, and YMCA (pp. 175–176), and combines a 3-D maze and games (about three-quarters of which are computer related) in 20 rooms. As the children progress through the rooms and levels, the activities require more advanced problem-solving ability and guidance from an adult. In addition, the children are required to reflect on and document or communicate their strategies to others in the club. The activities and social interactions used in this learning club support the children's cognitive development and facilitate the children reaching Vygotsky's zone of proximal development. On the other hand, classroom learning communities provide an opportunity for children (students) to work with others in the classroom community (teams) in the construction of knowledge. The teacher's role is one of facilitator guiding the students. Discourse and collaboration are key elements as the students move toward understanding. Instructional technology can assist this discourse as well as collaboration. Bodomo (2010) described a specific type of learning community—a conversational learning community. In this model, there are three types of instructional interactivity that may or may not involve information and communication technology (ICT): instructor-learner interaction, learner-learner interaction, and learner-resource interaction (p. 20). The communication or conversation takes place in these interactions. In this study, students in linguistics courses were noted to be more open to conversation as the course progressed. In other words, as the students moved through and interacted with the course, they realized some of their potential development and so the conversation improved. The author concluded that the interactivity of the web, specifically ICT, such as course management systems, can positively influence teaching and learning, even in traditional classroom environments. Access to an adult expert (teacher or outside resource) and social interaction can lead to knowledge construction and are supportive of Vygotsky's zone of proximal development.

Collaborative technology tools, such as Google Docs, an online database environment, can help students bridge the difference in Vygotsky's zone of proximal development—that is, what they know and what they can potentially know. Rimor, Rosen, and Naser (2010) described a study of graduate students using Google Docs in an online course. The students were expected to collaborate and complete an online database of the Internet. As part of the assignment, the students worked in teams of three to four through the online discussion forum and had to classify the entries as one of the following categories of knowledge: "declarative, procedural, structural, meta-cognitive" (p. 358). Once each team reached consensus, the entry would be made in Google Docs. The authors concluded that this collaborative process, which required interaction and mediation, enabled knowledge construction, moving from potential to actualization.

Wikis are another collaborative technology that supports students in Vygotsky's zone of proximal development. Hazari and Penland (2010) presented a study in which wikis (www.wikispaces.com) were used in several business management courses. The students worked in teams of no more than five (the maximum recommendation by the authors) on real-life scenario cases where they had to analyze the issues in the case, engage in collaborative discourse, and provide a solution or resolution to the case. A template for the case analysis and rubric for assessment were provided to the students by the instructors. A peer evaluation form was also used in the assessment process. The authors found that wikis can be used effectively as a collaborative learning tool in a business curriculum where collaboration and teamwork are critical to success in the business world. The instructor's role in guiding and monitoring the interaction in and content of the wikis is consistent with Vygotsky's philosophy and the construction of knowledge. In a similar study of wikis, Hazari, North, and Moreland (2009) focused on the pedagogical value of wikis. Wiki assignments included group journal article critiques and group management consultant case reports with an online presentation. Gender and full-time work experience were moderately correlated in terms of the pedagogical value of wikis. Males scored higher than females. Students with fewer than five years of full-time work experience scored higher on the pedagogical value of wikis than those students with more than five years of experience—a surprising finding; I would have expected the opposite. In addition, age was only weakly correlated and not a significant factor. This is contrary to the general idea that younger students (sometimes referred to as digital natives) would score or perform better than older students. The authors concluded that wikis can be used as an effective pedagogical tool to create collaborative student teams and develop content and knowledge. However, the instructors must be comfortable with this technology for it to be used effectively as a teaching and learning tool.

Interactive whiteboards can be used by elementary teachers for pedagogical interactivity with their students consistent with Vygotsky's philosophy. Blau (2011) conducted a study investigating the implementation of lesson plans immediately following professional development in interactive whiteboards. The results showed a high level of pedagogical interactivity between the teacher and the students but a marginally lower than average pedagogical interactivity among the students (p. 285). This is evidenced by the preference for whole-class learning (50% of the time) and individual differentiated learning rather than small-group learning with the use of the interactive whiteboard. The students did not have the opportunity to collaborate and interact with more capable peers; therefore, the concept of Vygotsky's zone of proximal development was not actualized. However, the teachers did facilitate the individualized student learning and as such provided learning support.

This section will describe how technology tools can mediate relationships consistent with Vygotsky's philosophy. ICT tools can be used in elementary science classrooms to mediate between students and learning of science. Otrel-Cass, Cowie, and Khoo (2011) investigated how ICT tools are used to improve the teaching and learning of science to seven- and eight-year-old students. One such example is the use of time-lapse video from YouTube. In a lesson on condensation, the teacher showed the video to the class but only after the students conducted their own physical experiments with condensation. This enabled the students to make connections (mediate) between their own real-time experiences and what they were watching in the video and deepen their understanding of the science concept of condensation. In another example, students used Google Earth and an interactive whiteboard to explore how rivers affect the landscape. The interactivity afforded by both tools empowered the students to work collaboratively on their journey of discovery. The ICT tools proved to motivate and engage the students. The authors concluded that ICT tools can be used to support the learning of science, helping students in elementary classrooms relate and mediate science concepts to their own experiences and thereby deepen their understanding of science.

Web-Based Learning Tools (WBLTs) are another example of technology tools that can be used to mediate between students and learning in math and science. Kay (2011) explored how context and WBLTs influenced the learning of more than 800 students in middle and secondary schools. Kay defines WBLTs as "interactive web-based tools that support the learning of specific concepts by enhancing, amplifying, and/or guiding the cognitive processes of learners" (p. 125). The teachers of these middle and secondary school students received training and then implemented lesson plans in math or science from a database of predesigned WBLT-based lesson plans. Learning performance was measured by the difference in pre-tests and post-tests in terms of remembering, understanding, application, and analysis. The findings revealed that the science WBLT-based lessons were significantly higher than the math lessons in terms of engagement, layout, learning importance, and performance. In addition, the secondary school students were significantly higher than the middle school students in regard to the same contextual factors. Lastly, teacher-led lessons versus student-led, individual versus paired student learning, and lack of software glitches were significantly higher in terms of students' learning performance. The WBLTs and the context in which they are used can positively and meaningfully affect students' learning.

Web-Based Learning Resources (WBLRs) are third example of technology tools that can be used to mediate between students and learning. Hadjerrouit (2010) conducted a study of three middle school classes and how WBLRs can affect teaching and learning. WBLR is defined by Hadjerrouit as technology that "is delivered through the Web, . . . teaches content that meets specific learning objectives aligned with the curriculum, . . . is designed on the basis of a learning theory and pedagogical strategy, . . . [and] contains reusable elements" (p. 56). In a case study of three classes of middle school students, the author measured both student and teacher perceptions of the technical and pedagogical usability of WBLRs. The author found that most of the measures of pedagogical usability were positively correlated to technical usability apart from collaboration and variation. Both students and teachers found the technical usability to be well designed and user-friendly. In terms of pedagogical usability, the students indicated they were motivated and that the WBLRs enabled their understanding of the lesson material. The teachers had a similar view that the students learned from both the WBLRs and the textbooks. However, the teachers were concerned about the lack of student collaboration and the fact that the students worked independently (not interacting with them either). Thus, the students were missing opportunities to learn from the adult experts (teachers) and more capable peers (other students) and did not realize their potential in Vygotsky's zone of proximal development.

Mindtools are a group of cognitive technology tools that can be used to enable student learning—that is, mediate between students and knowledge construction. Jonassen (2000) defines mindtools that can be used for constructivist learning as "computer-based tools and learning environments that have been adapted or developed to function as intellectual partners with the learner in order to engage and facilitate critical thinking and higher order thinking" (p. 9). These mindtools can range from spreadsheets to modeling tools to multimedia publishing tools to asynchronous/synchronous communication tools. For example, spreadsheets are a problem-solving tool that allows students to analyze, ascertain the quantitative variables and connections between the variables, generate formulas to manipulate the quantitative data, and ultimately solve the problem. As the mindtool, the spreadsheet requires students to think critically and logically in new ways and therefore mediate between themselves and their learning as they progress through Vygotsky's zone of proximal development.

This section will describe how speech and social discourse can be used to co-construct knowledge and learning according to Vygotsky's philosophy. For example, in online discussions, the students reflect on the speech [written discussion on the topic] and then interact in the online discussion by posting and responding to others (Whiteside, 2015). This process repeats itself and thus the co-construction of knowledge and learning.

Online community of inquiry is consistent with Vygotsky's co-construction of knowledge and learning, as well as Dewey's concept of community and inquiry as established by Garrison, Anderson, and Archer (2000). The authors define the first area, cognitive presence, as the "construction of meaning" through continuous communication (p. 89). Dewey (1938/1982) believed that "the presence of reflection which is the mediating aspect of inquiry" (p. 530) is a key feature of knowledge and therefore learning. Online learning can be both reflective and interactive, taking advantage of the technology. This requires the student—the online learner—to be both independent and interdependent, as compared to the FtF learner. Garrison, Cleveland-Innes, and Fung (2004) conducted a study of graduate students enrolled in six different online courses that compared previous FtF learning experiences with the online learning experiences. The community of inquiry framework, as developed by Garrison et al. (2000) and described above, was used to structure the study. The researchers found that the students felt that online learning required more of a cognitive presence and a greater responsibility on their part. However, FtF learning involved the other elements of the community of inquiry model: social and teaching presence. In a comparative study (Akyol & Garrison, 2011) of blended learning and online learning, graduate students in a Masters of Education program were compared—one group was enrolled in a completely online course and the other was enrolled in a blend online course. Again, the community of inquiry model, particularly the cognitive presence, provided the framework for the study. The researchers found few differences in the students in both courses; even the average grade in each was almost the same. One difference, slight though it may be, was in regard to the cognitive presence: the students in the blended course believed there was a greater cognitive presence and corresponding perceived learning and satisfaction compared to the students in the online-only course. Another difference is the increased frequency at the integration phase of cognition—that is, the ability to integrate information from several different types of sources, formulate hypotheses, and generate answers to the problems—in the blended course compared to the totally online course. Thus, the blended environment may have afforded slightly better conditions, but both constructivist learning environments contributed to meaningful learning experiences for the students.

The second area of the community of inquiry model is social presence (Garrison et al., 2000, 2010). This can be defined as the participants' ability (students and teacher) to "project their personal characteristics into the community" as real people (2000, p. 89). This social presence also included open communication (in keeping with Dewey's view on communication mentioned previously) and group cohesion. In a study of graduate students in a totally online course with both asynchronous and synchronous formats, Akyol and Garrison (2008) focused on the element of social presence. Weekly online discussions were required, and a key element to their success was the instructor's model of facilitation during the first online discussion. Thereafter, the students led and facilitated the discussions. The researchers found a social presence emerging over time indicated mostly by open communication messages and an increase in group cohesion (p. 7). However, the social presence was not positively correlated to perceived learning but to overall satisfaction with the course. Students noted that the sense of community contributed greatly to their participation in the online discussions (p. 15).

Teaching presence is the third and final area of the community of inquiry framework (Garrison et al., 2000, 2010). Teaching presence in an online environment is similar to teaching presence in FtF environments in several ways: design and implementation of course content, assignments and assessments, and the overall management of the course to support and further student learning. In an online environment and in keeping with this framework, the teaching presence must also pay closer attention to and monitor the other areas of cognitive and social presence. As seen in the previously mentioned study (Akyol & Garrison, 2008), the teacher presence as noted in the facilitation and modeling of the online discussion was a contributing factor to the social presence and overall success of the course. Abdallah (2009) also found the role of the instructor to be critical. The instructor must be like a cheerleader— boosting and encouraging the students to participate and interact to succeed in the online environment. She goes on to say that how and when we guide the students in the online environment translates into enabling or inhibiting their online participation and learning (p. 18). Likewise, Whiteside (2015) identified the instructor's facilitation in blended courses as a key factor. In blended learning the instructor also had to facilitate the students' transition from face-to-face to online as well as online back to face-to-face learning. Shea (2006) also found that an actively involved instructor had a significant effect. In a study of online students across 32 college campuses in the SUNY system, Shea established that the instructor's guidance in discourse, building an "accepting climate," ensuring the students were participating and keeping up with the coursework, and correcting student misconceptions is positively correlated to the students' perceptions of community and learning (p. 41). Accordingly, in keeping with constructivist pedagogy, the instructor designs and teaches the course to include such activities as reflection, expression of both current and alternative views, and integration of new ideas and concepts to build on existing knowledge and further cognition and understanding (p. 37).

Collaborative online reading, along with a joint argumentative essay assignment, is another example of discourse leading to the co-construction of meaning and learning. Kiili, Laurinen, Marttunen, and Leu (2012) studied high school students working in self-selected pairs in a Finnish language and literature course. The student pairs were first required to discuss among themselves the topic and then search for any additional information online. Lastly, the student pairs discussed and composed the final essay. The results indicate that all the student essays received one of the three highest marks possible under the Finnish national content-focused evaluation system (p. 455). Furthermore, the teachers noted that these essays were significantly better than past essay assignments that had been completed individually. The authors concluded that the collaborative online reading, along with an argumentative assignment and self-selection of partners, led to a co-construction and deeper understanding of learning.

Asynchronous computer-mediated communication is another example of discourse leading to the co-construction of meaning and learning. Barab, Thomas, and Merrill (2001) explored how this type of communication among adult learners can support learning. In a graduate-level course in adult education, students were both studying and experiencing adult learning. In other words, the course content and course context overlapped (p. 114). The students accessed an online course website and "conversed" twice via video conferencing and the remainder asynchronously via a discussion board and chat room. As a result of examining the course discourse transcripts, the authors found three elements that supported the learning process. First, flexibility in the design of the assignments and activities took into account the different backgrounds, experiences (technical and nontechnical), and interests of the adult learners. Second, the assignments and the nature of the online learning environment provided opportunities for the adult learners to connect the course readings, their own personal experiences, and the other students' personal experiences to build collaborative meaning and learning. Third, this environment fostered in the adult learners an open disposition—to both share their personal experiences and to listen to the other students' personal experiences. The authors concluded that asynchronous computer-mediated communication environments, along with careful design of assignments and nonintrusive facilitation by the instructor, can lead to deeper understanding and learning.

Another study of asynchronous computer-mediated communications is presented by Schrire (2006). In this case, the author investigated asynchronous communication in a doctoral-level online program in computer technology in education. Three samples of computer conferences were selected: two were instructor moderated and one was not (p. 57). Analysis of the discourse in the three sample conferences indicates that the students both introduced discussions and connected to one another's discussions more so than to the instructor-led discussions. The author concluded that this type of interaction and discourse mediated by asynchronous computer communication can direct students to co-construction more meaningful learning and therefore is valuable.

Lastly, podcasting is another example of discourse leading to the co-construction of meaning and learning. Ng'ambi and Lombe (2012) explored how podcasts can be used in a post-graduate program in educational technology. Podcasting has a distinct advantage as students are already competent in the use of this technology due to today's pervasive use of MP3 players and iPods. In two sections of a blended course, students accessed podcasts and other course materials posted in the course learning management system. The podcasts were recorded during the FtF sessions. According to the authors, the students felt that the podcasts contributed to their learning. From an educational point of view, the podcasts served as tool that students could use to build on prior knowledge and incorporate new knowledge from the FtF sessions, take time to reflect on the material, and work together with the instructor and/or other students to support one another's learning. The key is to have the podcasts directly interwoven as part of the course pedagogy.

In summary, technology promotes and supports the learning process. The technology allow learners to build on prior knowledge, to continue to add to their experiences, to actively engage in the learning process, and to extend their learning in the zone of proximal development, as previously discussed by Dewey and Vygotsky. In particular, mathematics and science are two subject areas in which technology supports learning. The technology fosters greater understanding and knowledge. Furthermore, the communication aspects of the technology support the learning process through speech and social discourse, which is also consistent with Dewey and Vygotsky.

2.2.1 Learning Math with Technology

This section describes the use of technology in learning mathematics from a socio-cultural Vygotskian perspective. Rosen and Salomon (2007) compiled a meta-analysis of 32 studies and compared “constructivist technology-intensive oriented” and “traditional” didactic learning environments in the field of mathematics. The authors define constructivist learning and understanding of mathematics as occurring when “learners socially appropriate and actively construct knowledge” (p. 3). Computer tools are used by the students to facilitate this knowledge construction. These constructivist technologically intensive learning environments (CTILEs) in math include such learning objectives as “self-guided and team-based problem solving, participatory meaning appropriation, and active knowledge construction” (p. 3). The results of this meta-analysis indicated that math students in CTILEs demonstrate greater learning achievements when tested using constructivist-appropriate criteria relative to math students in traditional learning environments. Conversely, the math students in traditional learning environments did not demonstrate greater learning achievements when tested using traditional criteria. Thus, as students encounter these kinds of technology in their learning environments, the cognitive learning of mathematical concepts is greatly enhanced.

One can explore how experiences influence cognitive learning of mathematics in children. For instance, Martinez (2010) described a public school in New York, Quest to Learn School, in which the students work with video games to learn math. In an imaginary city, the students learn and apply math concepts through such activities as portraying a travel agent, changing foreign currencies, writing travel blogs, and determining and maintaining a travel budget. Problem-solving skills and system-thinking skills are learned by determining the reasons for the failure and collapse of the economy as well as brainstorming new revenue streams for economic recovery. In these classrooms, the teachers facilitate the learning by playing the role of mentors and individual learning coaches, and the students play the role of active and engaged learners constructing their own knowledge. These experiences are significant factors that contribute to the learning of mathematics.

Polly (2016) observed the following technologies in three elementary classrooms: projector, document camera, iPad, teacher computer, interactive whiteboard, and hand-held quiz device, i.e. clickers (p.114). The iPads enabled students to work on memorization and basic skill tasks. The hand-held quiz devices enabled students to work algorithms and procedures. Additionally, Polly (2016) wrote about the use of interactive white boards and the types of high-level tasks they allowed fourth grade elementary teachers to use in their classrooms. Specifically, these interactive boards enabled students in the class to demonstrate and discuss their strategies in challenging problem-solving.

Likewise, Muir, Callingham, and Beswick (2016) described technology instructional strategies in a first grade lesson on addition. The teacher utilized an interactive whiteboard and virtual ten frames (manipulatives) in her lesson. The students were actively involved in using both technologies. The researchers noted that this lesson enhanced the first-grade students' understanding of basic addition facts. For example, the teacher demonstrated the "bridging ten strategy," a thinking strategy in which 10 is used as a "bridge" in adding and subtracting numbers. Working with the numbers 7 and 8 (corresponding number of dots in each ten frame), the teacher guided the student who moved two dots from the 7 ten frame to the 8 ten frame. Now you have a ten frame with 10 dots and one with 5 dots, which is much easier to add.

Grandgenett, Harris, and Hofer (2009) provide concrete, specific examples of how technology is used to understand mathematical concepts and describe this as “grounded technology integration” into mathematics education. This integration focuses on “content, pedagogy, and how teachers plan instruction” rather than on the elements of the educational technology. Examples include “drill-and-practice software, virtual manipulatives, real-life data sets, interactive geometry programs, graphing calculators, robots, and computer-based laboratories” (p. 24). The authors have developed mathematics activities and taxonomies (a complete list can be found at http://activitytypes.wmwikis.net) along with recommendations for “grounded technology integration” based on National Council of Teachers of Mathematics’ process standards. These activities fall into seven categories: consider, practice, interpret, produce, apply, evaluate, and create (pp. 25–26). Consider activities are defined as activities providing direct foundational knowledge. Practice activities are characterized by the repetition of computations. Interpret activities are defined as activities concerned with deductions of mathematical connections. Produce activities are concerned about generating mathematical structures. Apply activities provide the link between mathematic theory and the real world. Evaluate activities require the assessment of students’ own work as well as that of other students. Create activities are characterized by imaginative and innovative thinking. The main focus is on the student rather than on the teacher—that is, the emphasis is on the individual. Student activities may be combined for higher-level learning, such as mathematical modeling. The authors described one such combination: a graphing calculator, a mechanical robot, and a digital video camera. Math students work on a mathematical expression/equation (i.e., distance = rate x time) using a graphing calculator that is connected to a robot. The digital video camera is used to record the students as they work and is then played back for the rest of the class (Grandgenett et al., 2009, p. 24). Ultimately, the students are learning to interpret, create, and apply mathematical concepts and relationships through working with these objects, or tools according to Vygotsky (Keengwe & Kang, 2012).

Shirvani (2009) also presented concrete, specific examples of how technology is used to understand and co-construct knowledge of mathematical concepts in elementary math methods courses. The first half of the methods courses were taught by direct instruction and the only technology used was PowerPoint by the instructor [researcher]. The second half of the methods courses were taught differently: group work, group projects, hands-on activities, test, and student presentations. Technology, i.e. manipulatives, such as Cuisenaire rods, base-ten blocks, Unifix cubes, tangrams, and Geoboards, were used by the pre-service teachers in the group hands-on activities. The mathematical concepts covered number sense, operations, area, and volume (p.248). The students worked on real-world problems in groups and had to explain and share their learning. Additionally the students had to teach and present a mathematical concept/activity to the rest of the class. The researcher found that this social interaction led to a deeper understanding of the mathematical concepts compared to the direct instruction (p.250).

Likewise, Abramovich and Brouwer (2004) provided concrete, specific examples of how technology, such as Geometer's Sketchpad, is used to understand mathematical concepts in a mathematics course for pre-service teachers. One example described a problem: "find all rectangles with integral sides whose areas are numerically equal to their perimeters" (p.306). The students used Geometer's Sketchpad, a dynamic geometry program, to solve the problem. Another example involved geometric constructions, "edge-to-edge" tiling with three regular polygons and the Geometer Sketchpad (p.312). The researchers noted that the geometry program facilitated the development of "conscious control over the conceptual system of geometry of regular polygons" and rather than the physical rotation/manipulation polygons (p.312). The students also wrote mathematics proofs of their solutions. Furthermore, the researchers mentioned that this control happened in the ZPD where students, with the guidance of the instructor, constructed a geometric solution.

Moreover, Maloy, Verock, Edwards, and Woolf (2017) described various web resources and apps that facilitate learning mathematics. The researchers described computer math learning games that facilitate practice of mathematical operations: Math Blaster, The House Series, Zoombinis, and Raft Race Challenge (p. 165-166). They also recommended iPad apps, such as Rocket Math, Splash Math, and Math Ninja (p.166). Another valuable resource, Common Sense Media (https://www.commonsense.org/education/reviews/all), reviews by subject, grade level, skills, and purpose as well as provides teaching tips.

Computer-mediated inquiry can be used in mathematics to enable co-construction of probability knowledge and learning. Enyedy (2003) explored how seventh-grade students engaged in social discourse to gain an understanding of basic probability concepts. Working in pairs, the students participated in computer simulations, hands-on games (such as rolling dice and flipping coins), and full-class discussions of findings (including the reasoning) and mathematical concepts. Pre-tests, post-tests, and final projects were used to measure how well students achieved an understanding of basic probability concepts. The author found that most students reached a level of understanding in which they could articulate their reasoning and apply it to new situations. Interestingly, peer interaction created productive disagreement/argumentation but the resolution and significance of these arguments were both "intrapersonal and interpersonal" (p. 402). Furthermore, the students were actively engaged in their own knowledge construction as well as that of their class community.

Additionally, Sharma (2016) investigated research on teaching probability from a socio-cultural Vygotskian perspective. In one study of middle school students, researchers Gürbüz, Erdem, and Firat (2014) designed learning activities in which students made “predictions, collected, collated, and compared data in groups” (as cited in Sharma, 2016, p.132). These groups then afforded students opportunities for social discourse and support in the learning activities. The researchers concluded that these activities had a positive impact on the students’ learning of probability. Sharma (2016) referenced another study by Joyce (2006) which described a strategy, “Predict, Observe and Explain”, to teach probability utilizing the ZPD. The teaching strategy consisted of the following elements:

- Unless students are asked to predict first what will happen during an experiment, they may not observer carefully.
- Writing down predictions motivates students to find the answer.
- Asking students to explain the reasons for their predictions allows the teacher to identify the students’ beliefs and theories about a given concept. This can be useful for uncovering misconceptions or building on the understandings that students already have.
- Explaining and evaluating their own predictions and listening to others’ predictions helps students to begin evaluating learning and constructing new meanings (as cited in Sharma, 2016, p.133).

The researcher maintains that this strategy can be used to teach probability to elementary through high school students. Sharma added that students should use technology tools, such as computers, graphing calculators, and other simulation tools, in their problem-solving and experimentation. Furthermore, Sharma indicated such experimentation can be used to address student misconceptions about probability, e.g. the concept of fairness [equal probability or chance] (p.134).

2.3 How Pre-Service Teachers Learn to Teach

This section describes how pre-service teachers learn to teach. Drawing from my own experience, I concur with such researchers as Borko and Putnam (1996) that teaching and, more specifically, learning to teach, is a very complex undertaking. Pre-service teachers should have experiences that “mirror the experiences we would like them to create in their own classrooms” (p. 701) whereby the learning of subject matter knowledge and pedagogical content knowledge can be enhanced. Borko and Putnam consider the following areas of knowledge as pertinent to learning to teach: “a) general pedagogical knowledge and beliefs, b) subject matter knowledge and beliefs, and c) pedagogical content knowledge and beliefs” (p. 675).

Looking at the first area, general pedagogical knowledge and beliefs includes an understanding of the teacher’s role as “a mediator of meaningful student learning, instructional strategies that promote active cognitive processing of academic content, classroom environments that foster learning for understanding and self-regulation, and methods of assessment that reveal students’ thinking” (Borko & Putnam, 1996, pp. 675–676).

This type of role that promotes meaningful student learning is consistent with the learning theorists. The teacher acts as the mediator between the students and their learning. According to Vygotsky, tools, including technology tools, can assist the teacher in this learning process (Keengwe & Kang, 2012). The teacher develops and implements instructional strategies, such as debates and experiments, which foster active engagement and learning (Darling-Hammond & Baratz-Snowden, 2007; Dewey, 1902/1971).

Case studies are another instructional strategy. Case studies are defined as narratives describing teaching and learning scenarios (Darling-Hammond & Baratz-Snowden, 2007). Pre-service teachers can analyze case studies and strategize toward solutions or even write their own cases. In this process, they are gaining an understanding of how particular teaching approaches can affect the learning of their students. In other words, the pre-service teachers are honing their skills and ability to link theory to practice. These instructional strategies may or may not involve technology. This all takes place in a classroom environment that is respectful and conducive to learning. The teacher employs routines, such as hand-raising to get attention, in an effort to manage the classroom environment to maximize students' learning (Darling-Hammond & Baratz-Snowden, 2007).

Pre-service teachers can analyze student learning through the use of various technologies (Murray & Zembal-Saul, 2008). In the methods course, the pre-service teachers recorded three science lessons they taught in their field placement. Using iMovie and Mac notebooks, the pre-service teachers selected small snippets of teaching that epitomized "teaching science as inquiry" (p. 55). In addition, the pre-service teachers were required to record the rationale for their selections. The assignment allowed the pre-service teachers to review and analyze their teaching and its effects on their students' learning.

Pre-service teachers and/or interns can also analyze the physical setting of the classroom and its influence on learning using technology (Murray & Zembal-Saul, 2008). Specifically, Apple notebooks were used in a pre-service teacher methods course, Classroom Learning Environments. In this course, digital photographs were taken of the elementary school classrooms in which the pre-service teachers interned. Using these photographs as well as readings, students created a slide show, using iMovie, which was then shared with classmates.

The second area, subject matter knowledge and beliefs, greatly affects how pre-service teachers present their fields, including what they teach, how they teach, and which textbooks they select for their students (Borko & Putnam, 1996). In the subject area of mathematics, the pre-service teacher's knowledge of mathematics via college math courses has been shown to positively influence and contribute to the mathematics learning and understanding of their students (Cochran-Smith & Zeichner, 2005). The Massachusetts Department of Education (2007) published Guidelines for the Mathematical Preparation of Elementary Teachers, which describe the mathematical coursework required for elementary teachers. Ultimately, elementary pre-service teachers “must demonstrate not only that they know how to do elementary mathematics, but that they understand and can explain to students, in multiple ways, why it makes sense” (Massachusetts Department of Education, 2007, p. 9). (This is also true in other subject areas. A pre-service teacher’s knowledge of science via college science courses and research experiences has been shown to contribute to a greater understanding of scientific inquiry by the students [Borko & Putnam, 1996].) Teacher preparation programs must provide opportunities for pre-service teachers to learn the subject matter in this manner.

Lastly, in terms of pedagogical content knowledge, which is “knowledge of a subject that is specifically related to teaching that subject” (Borko & Putnam, 1996, p. 676) and beliefs, pre-service teachers can enhance their knowledge in a variety of ways in teacher education programs. For example, in one science methods course, pre-service teachers are given the opportunity to consider model lessons and online video-based cases that provide insight into the teaching and learning of science subject matter before their field placements (Murray & Zembal-Saul, 2008). In another science methods course, pre-service teachers worked on a project focused on plant growth. As these students conducted ”meaningful scientific inquiry,” they were also investigating science content pedagogical practices (Davis & Falba, 2002, p. 312).

This section will further describe how pre-service teachers learn to teach. Feiman-Nemser (2008) describes a framework consisting of four themes related to learning to teach: “learning to think like a teacher, learning to know like a teacher, learning to feel like a teacher, and learning to act like a teacher” (p. 698). First, pre-service teachers must acquire the ability to think on their feet, think about their teaching, and then modify their teaching. Second, pre-service teachers need to learn subject matter as well as how to teach that subject matter, identify how children develop and learn, and understand how culture affects learning. Pre-service teachers should endeavor to become lifelong learners. Third, pre-service teachers must become emotionally invested in their teaching and feel that all students have the potential to learn. Lastly, pre-service teachers need to learn a "bag of tricks"—strategies and techniques that they can demonstrate inside and outside their classrooms. Grossman, Smagorinsky, and Valencia (1999) categorize this "bag of tricks" as pedagogical tools that can be employed by pre-service teachers. Conceptual pedagogical tools are the theories, conceptual frameworks, and thoughts about teaching and learning. Practical pedagogical tools are the lesson plans, classroom procedures, and curriculum materials.

Darling-Hammond and Baratz-Snowden (2007) describe a framework for learning to teach. This framework consists of a learning community with the following components:

> Vision—images of good practice that guide teaching; knowledge—understanding of content, pedagogy, students, and social contexts; dispositions—habits of thinking and action regarding teaching and children; practices—a beginning repertoire of instructional strategies; and tools—conceptual and practical resources for use in the classroom. (p. 121)

Research shows that pre-service teachers from teacher preparation programs who incorporate this framework are well prepared and more effective as they begin their teaching careers. These teacher preparation programs are structured in similar ways:

> (1) a common core curriculum grounded in knowledge of development, learning, subject-matter pedagogy, and assessment, taught in the context of practice; (2) well-defined standards of practice and performance used to guide the design and assessment of course work and clinical work; (3) extended clinical experiences (at least thirty weeks)

that are interwoven with course work and carefully mentored; (4) strong relationships between universities and schools that share standards of good teaching consistent across courses and clinical work; (5) use of case-study methods, teacher research, performance assessments, and portfolio examinations that relate teachers' learning to classroom practice. (Darling-Hammond & Baratz-Snowden, 2007, p. 120)

Learning to teach can be seen as a continuity of experience as previously described by Dewey (1902/1971). Experiences as learners, whether in methods courses, math courses, technology courses, or in field placements, all will affect the pre-service teachers and their future teaching experiences. Similarly, as described by Vygotsky, adult guidance provided by the teacher preparation faculty and the supervising teachers will contribute to the development of pre-service teachers learning to teach.

Accordingly, one key element of the programs is the clinical student-teaching experience. This is a partnership between the university and the classroom teacher. Sometimes the university employs graduate students or retired teachers rather than university faculty to supervise student teachers in their field placements (Zeichner, 2010). Thus, there is discord and inconsistency between what the pre-service teachers learned in their coursework and their student-teaching experiences. Darling-Hammond and Baratz-Snowden (2007) add that supervision should be provided by experienced teachers who are accessible and able to mentor these student teachers. The pivotal role of field placements in pre-service teacher education programs was stressed in an assessment of technology and its teacher preparation program. Mentor teachers, who modeled effective technology integration and teaching, had a significantly positive effect and influence on pre-service teachers' future use of technology (Brown & Warschauer, 2006). Thus, mentor teachers need to improve their technical skills and abilities to integrate technology into instruction. One such model, Project ImPACT

(Implementing Partnerships Across the Curriculum with Technology), proved to be effective. Three factors contributed to the success of this program: (1) access to technology in the field site, (2) professional development, and (3) support, both technical and instructional (O'Bannon & Judge, 2004).

College and university faculty also have to provide some level of technical support to pre-service teachers (Falba, Strudler, & Bean, 1999). Studies show that this modeling and support will enable these students to increase the use of technology in their coursework and improve their own learning. Students rely on their instructors to provide support and answer questions, and this is just another layer of that support.

Case studies are another element. As previously described, case studies are defined as narratives describing teaching and learning scenarios (Darling-Hammond & Baratz-Snowden, 2007). Pre-service teachers can analyze the cases and strategize toward solutions or even write their own cases. In this process, they are gaining an understanding of how particular teaching approaches can affect the learning of their students. In other words, the pre-service teachers are honing their skills and ability to link theory to practice.

In addition to analyzing cases, pre-service teachers can analyze teaching artifacts. Video records of teaching are one such artifact. The Carnegie Foundation's Knowledge Media Lab provides Web-based examples of expert K–12 teaching (Darling-Hammond & Baratz-Snowden, 2007). Furthermore, Zeichner (2010) describes how university faculty can incorporate these websites into pre-service teacher preparation programs. One such example is Stanford University's Pamela Grossman, who integrated a Los Angeles high school English teacher's website with her English methods course. The pre-service teachers were able to view classroom discussions, student work, lesson materials, and interviews with the English teacher. These videos show the complexities and intricacies of classroom teaching and allow the viewer to slow down and repeat the snippet that is being analyzed (Borko, Whitcomb, & Liston, 2009; Rosaen & Florio-Ruane, 2008). Advances in technology, particularly relating to the Internet, will provide access to a huge number of educational video archives and other types of educational resources that will further our understanding of teaching and learning (Dede, 2009).

Portfolios are also found in these teacher preparation programs. Teaching portfolios are compilations of the pre-service teachers' artifacts and typically are in a digital format. Artifacts may include video recordings of teaching, lesson plans, examples of student work, assessment plans, and assignments from university coursework (Darling-Hammond & Baratz-Snowden, 2007). Pre-service teachers and university faculty can analyze these artifacts, reflect on what worked and what did not, and strategize to make adjustments or improvements.

Wilson (2009) adds that workplace contexts are also important as pre-service teachers learn to teach. These contexts refer to the following areas: "recruitment, early preparation, retention, as well as professional development" (p. 1). Recruitment starts with attracting good teachers from highly qualified college graduates who have the knowledge and skills to succeed from the beginning. Preparation programs need to be structured to provide this knowledge and skills. Retention strategies range from financial inducements to professional development and mentoring. Professional development needs to focus on both content and pedagogy. Teacher learning communities are groups of teachers, new and experienced, that are centered on learning with and from each other (Westheimer, 2008). These teacher learning communities are shown to increase pre-service teachers' learning to teach as well as their retention as teachers. Furthermore, these learning communities provide pre-service teachers with connections to experienced teachers, and this mutual connection enables both to succeed in their teaching.

Rosaen and Florio-Ruane (2008) describe three metaphors and their effects on how pre-service teachers learn to teach. The first metaphor, field experience, refers to the hands-on classroom teaching experience of student teachers. The pre-service teachers feel that this is where they learn to teach and do not consider their other experiences and coursework in the teacher preparation program as contributing factors. In reality, they should not be separated, and the pre-service teachers need to understand their interconnectedness. The second metaphor, struggling reader, refers to students who are experiencing difficulties with learning to read. This metaphor illustrates the power of labels and language and how they can negatively affect children's learning. The teacher may limit the scope of the readings because it is too much for these students as they learn to read, thereby denying them wonderful reading opportunities. The pre-service teachers need to learn to be aware of the language they use and its influence on their students' learning. The last metaphor, at-risk learners, refers to students with problems or difficulties. The new teacher may perceive these students as likely to fail, but, in fact, they may be very capable but are dealing with problems outside of their control. Nonetheless, the pre-service teachers must be aware of this and not give up on these students and their opportunities to learn.

Cochran-Smith and Demers (2008) pointed out that pre-service teachers learn to teach over a period of time and through a variety of experiences, building on their previous knowledge and acquiring new knowledge. As a teacher, I believe that the process of learning to teach does not end. One should always be learning and striving to become an even better teacher.

2.4 How Pre-Service Teachers Learn to Teach Using Technology

This section describes how pre-service teachers learn to teach using technology. First, individual teacher preparation programs as well as a national survey of 1,000+ four-year initial licensure teacher preparation programs show what the most commonly taught technologies are. Second, pre-service teachers learn to teach using technology in stand-alone technology courses and in technology-intensive methods courses. Additionally, TPACK, Technological Pedagogical and Content Knowledge framework (Mishra & Koehler, 2006, 2009), is described for pre-service teachers learning to teach, as are instructional strategies that incorporate technology to promote the learning of specific content. Finally, teacher preparation programs can utilize a technology competency assessment of its students, as pre-service teachers need to be competent in the technology before they can incorporate it into their future teaching. These assessments can be self-designed by the teacher preparation program or can incorporate the National Educational Technology Standards for Teachers (International Society for Technology in Education, 2012).

Oliver, Osa, and Walker (2012) looked at how a teacher preparation program prepared its students to integrate technology into their future teaching. In this study, the faculty were surveyed. The findings showed that Blackboard and PowerPoint were the most frequently used technologies in their own classrooms and the most frequently taught to the education students (p. 289). These findings are not surprising given the expectations of higher education institutions to have their faculty use such learning management systems as Blackboard in their courses, even those not taught online. It also seems that every textbook publisher provides PowerPoint slides for faculty who adopt their texts. In addition, PowerPoint, Blackboard, and Videos were the most commonly taught instructional technologies for use in PreK–12 classrooms (p. 290). It is surprising to see Blackboard listed as a technology for use in PreK–12 classrooms. Teacher preparation program faculty have a "vital role to play in exposing preservice teachers to new technologies and in modeling the use and integration of these technologies into instructional activities" (Oliver et al., 2012, p. 294). The technology has to be experienced by these prospective teachers who themselves are learners and who are constructing their own knowledge, which they can share in their future classrooms. A comprehensive look at pre-service teacher preparation programs provides meaningful insight into the technology preparation of future teachers.

Gronseth et al. (2010, pp. 33–34) conducted an online survey of more than 1,000 four-year institutions with initial licensure teacher preparation programs to determine the technology experiences presented to their students. The study showed that personal productivity and information presentation technologies were the most common technologies taught. Furthermore, about one-third of the respondents cited the ability to use technology to support instruction as the most important technology focus. Professional growth and technological literacy were the most important foci for more than 20% of the respondents. Only 5% noted the ability to use technology to support diverse learners as the most important technology focus. Lastly, more than 50% of the respondents indicated a need for more technology integration throughout their programs, especially in the areas of field experiences and methods courses. Although pre-service teachers realized the importance of technology, too many felt a need to have more technology training and experiences.

Foulger, Buss, Wetzel, and Lindsey (2012) described a growing trend in pre-service teacher preparation programs: moving away from stand-alone technology courses to technology-intensive methods courses. The authors surveyed the last group of students required to enroll in the stand-alone technology course. Their findings showed that the students felt fairly prepared to teach with technology given time constraints to “play” with the technology (similar to previously mentioned studies) and a failure to connect the technology to the content area. Despite these findings, the students had an understanding of and could explain the distinction between teaching that involved ”technology” and teaching that was “reformed by the technology” (p. 54). Foulger et al. (2012) suggested a technology-intensive methods course in place of a stand-alone technology course in teacher preparation programs.

The technology-intensive methods courses would need to provide opportunities for the students to improve their technology skills. Given the ever-changing nature of technology, the methods courses should also prepare pre-service teachers to use new technologies in their future teaching of content courses to support and advance their students’ learning. Two issues surfaced that would affect the success of this technology experience for pre-service teachers: the college faculty teaching these methods courses are not necessarily experts in the technology, and field experiences may not provide opportunities for mentoring and technology integration into instruction. Table A.1 shows the kinds of technology found in a technology-empowered learning environment (U.S. Department of Education, 2010). This must be recognized and implemented by faculty in teacher preparation programs to allow these future teachers to create technology-rich experiences for greater understanding in their own classrooms. Many colleges and universities have participated in the U.S. Department of Education’s Preparing Tomorrow’s Teachers to Use Technology program, which provides funding to incorporate technology into teacher preparation programs (Otero et al., 2005).

Another study focused only on pre-service teachers in a graduate program. Sutton (2011) described this study of novice teachers and their technology experiences during their master’s program. Students are required to take a stand-alone technology course—Introduction to Instructional Computing—as part of the program. The study found a major disconnect between the technology training and the rest of the program. In other words, the pre-service teachers realized that they were supposed to create “student-centered, technology-rich lessons,” but most felt they were unprepared due to a lack of “authentic experiences using technology” (p. 43) in their master’s program. The one required course in technology did not “connect” to the other courses, such as the methods courses. The study also found that the software applications learned in the technology course had no relevance or applicability to the students' content area. Thus, the students did not learn how to use technology to enrich their content-area instruction. Lastly, the study established that the students required more time to "practice, reflect, and plan student-centered, technology rich lessons" (p. 44) and not just during the technology course. Retention and transferability of the technology skills were areas of concern for these pre-service teachers. These findings reinforce the need for pre-service teachers to experience technology as learners so that they can use their knowledge to create learning environments that foster greater understanding in their future students.

Hofer and Grandgenett (2012) also focused their study on a graduate teacher preparation program. The students in this master's program are required to take an educational technology course in their second semester. This course gave the students an opportunity to experience different kinds of general and content-specific technologies. The course culminated in a final project in which the students created a technology-rich, content-specific lesson plan. This lesson plan is also often submitted in their methods course. The authors found that this link between the two courses is significant and leads to greater technological pedagogical content knowledge or TPACK (p. 85). This framework (see Figure 2.1) was created by Mishra and Koehler in 2006 (see also Mishra & Koehler, 2009) and Hofer and Grandgenett used it as a basis for their survey of these graduate students. This technological pedagogical content knowledge is vital for pre-service teachers to acquire and then use in their future classrooms to enhance the understanding and learning of their students.

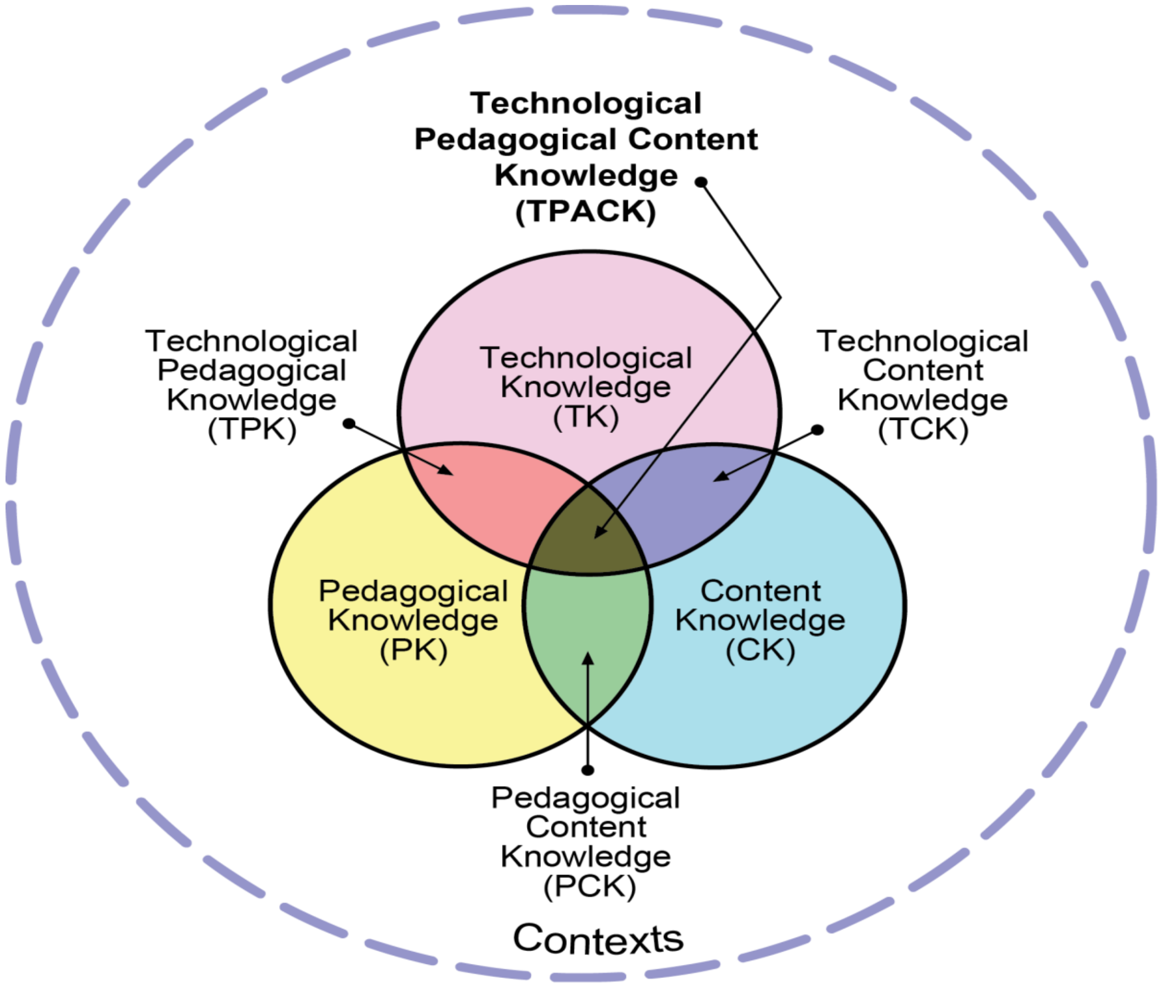


Figure 2.1: Technological Pedagogical and Content Knowledge

Note. Adapted from Technical Pedagogical and Content Knowledge (http://www.tpack.org/).

Furthermore, this experience in technology should not be limited to technology courses and faculty in teacher preparation programs; instead, it should be an integral part of the program and offered to the entire faculty in teacher preparation programs (Bai & Ertmer, 2008).

There appears to be a "disconnect" between teacher preparation programs and theory and practice that is consistent with Dewey's ideas. Dewey (1902/1971) stated that lectures and books (theory) seem to be more prevalent than "real" teaching (practice). He suggested that higher education teacher preparation programs become a repository for "theories and ideas demonstrated, tested, criticized, enforced, and the evolution of new truths" (p. 93). Dewey took this one step further by noting the need for a partnership between the universities and elementary schools where theory and practice work together. Putnam and Borko (2000) added that college coursework and activities should be synchronized with K–12 classroom field experiences, a kind of apprentice model. The college would be the setting for learning about new ideas, knowledge, and pedagogy. Technology, such as interactive multimedia cases, may be used to illustrate different classroom scenarios and pedagogy alternatives. The field setting would allow pre-service teachers to practice what they have learned. Both settings would provide occasions for reflection and feedback. In other words, the pre-service teachers must "learn to think, talk, and act as teachers" (p. 10). However, in actuality, the type of field experience available may limit this partnership.

Oigara and Keengwe (2011) examined how interactive whiteboards, specifically Smart Boards, are used by pre-service teachers. In an action research case study of pre-service teachers in elementary social studies methods courses, the authors investigated how the students integrated these interactive whiteboards into their teaching practice. Smart Boards were available in the college classroom as well as in the field placement K–8 school. Several themes and findings emerged from this investigation. First, the pre-service teachers improved their technology skills and expressed more interest in using Smart Boards in their own teaching. This was attributed to faculty modeling of the interactive technology in the methods courses. Second, in order for teacher preparation programs to adequately prepare their students to teach effectively with technology, the coursework must emphasize technology and provide numerous opportunities for pre-service teachers to gain experience in using technology more effectively for student learning. Third, in particular, the methods courses can provide pre-service teachers with contextual (content-based) technology experiences that they can incorporate into their future teaching practice (also recommended by Davis & Falba, 2002). Lastly, the pre-service teachers need field placements in which the cooperating teachers mentor them in effective technology use and integration. Thus, in this study, the mentoring by both faculty and cooperating teachers provided the pre-service teachers with opportunities to develop these skills and to use them to positively influence their future students' learning.

Technology tools can also be used in teacher preparation programs to mediate between the pre-service teachers and their learning. Keengwe and Kang (2012) conducted a literature review of blended learning in teacher preparation programs. One theme that emerged is the significance of both pedagogical and technological tools in blending learning, particularly to integrate FtF and online learning. Technology tools, such as multimedia CD-ROMs, Web-based models, and online lectures and discussion chats, can be used by pre-service teachers to integrate their ideas and technology skills into their teaching practices and classrooms. In addition, these technology tools can be used with a variety and combination of pedagogical methods and approaches to promote learning. The authors recommended that teacher preparation programs afford more blended learning experiences for their students, but that these experiences must be adapted and personalized for pre-service teachers who will then be applying their experiences to their own teaching.

Teacher preparation programs can assess the technology competency of their students—that is, prospective teachers. In the first example, Banister and Vanatta (2006) described college freshmen in an introductory education course and their technology competency assessment. This assessment accounted for 10% of their final course grade. An e-portfolio was used to document this competency. The following technologies were assessed: "word-processing, spreadsheet, presentation, and graphics software applications, and integrate Internet and file management expertise" (p. 213). Using the computer lab, the students were given two hours to complete the assessment. The results showed that only about 29% of the students passed all sections on the first try and that about 8% actually failed all the sections (p. 219). The lowest passing rates were in the area of the presentation software (this is surprising; I would have thought it to be the spreadsheet). The students performed markedly better in the retake, with about 74% passing all sections. This, however, still meant that 26% did not pass one or more sections—not a very promising statistic (p. 220). The education department faculty are now ensuring that they exhibit technology competency in their own teaching and not just insisting that their students have it. Students are required to use these technologies in subsequent coursework. The technology has to be experienced by these prospective teachers who themselves are learners. In turn, these prospective teachers can use their own experiences to create learning environments that enhance the cognitive development of their future students. As a result, this teacher preparation program trains its students to use technology to strongly guide the teaching and learning in their future classrooms.

In 2012, Banister and Reinhart updated the technology competency assessment for their students in the teacher preparation program. This time the authors used the Wayfind Teacher Assessment (Learning.com, 2013) instrument, an online assessment tool that evaluates the technology competency aligned with the NETS-T or National Educational Technology Standards for Teachers (International Society for Technology in Education, 2012).

The NETS-T has five standards of proficiency (Bannister & Reinhart, 2012, p. 61): (1) facilitate and inspire student learning and creativity, (2) design and develop digital-age learning experiences and assessments, (3) model digital-age work and learning, (4) promote and model digital citizenship and responsibility, and (5) engage in professional growth and leadership. Normally this instrument is used to assess current teachers; in this case, the authors administered this instrument to their undergraduate junior and senior students in the teacher preparation program. The reports from Learning.com indicate technology proficiency in all areas, although the fourth standard, digital citizenship, had the lowest mean scores. However, more careful analysis indicates that 5% of the students had basic or below basic scores in one or more areas (p. 62). These data also enabled faculty to provide differentiated instruction and additional support. Again, the focus on the individual is vital to understanding and learning. As these struggling students complete their teacher preparation program, it is hoped they become teachers who integrate technology into their instruction and curriculum.

These same standards were used by Koch, Heo, and Kush (2012) in the testing of their students in the teacher preparation program throughout each of their four years. The study's findings revealed that there are no noteworthy differences in the students' academic year relative to the standards (p. 5). However, students who experienced greater technology integration during their high school years had significantly better technology integration in their teacher preparation program experiences. These technology experiences, as well as the students' maturation, enhance the students' cognitive development as they progress through the teacher preparation program. Furthermore, the student teaching experience provided opportunities for these prospective teachers to create technology-rich learning experiences for their own students, which fostered active involvement in their learning. Interestingly, the students in the early childhood teacher preparation program experienced the lowest technology integration relative to the other areas.

This study investigated the ways in which math methods courses provide technology-based learning experiences for pre-service teachers to gain the technological pedagogical and content knowledge to effectively use technology in their future teaching.

2.5 Research Questions

The research questions are as follows:

1. In which ways did the use of technology in a math and/or science methods course facilitate the learning of elementary pre-service teachers and the integration of technology into classroom instruction?
2. How do faculty in the math and/or science methods course use instructional technology for enhancing teaching and learning?

CHAPTER 3

METHODOLOGY

The methodology for this qualitative dissertation research study was comparative case study. Specifically, this research study consisted of data collection and analysis from two research sites, i.e. case studies. This method helped me in my role as researcher to gain insight into and understanding of the field settings and thus to develop descriptive findings that can contribute to the preparation of pre-service teachers and to further research. The following elements are typically found in qualitative research studies: (1) fieldwork; (2) focus on the construction of meaning, insight, and understanding; (3) inductive analysis; and (4) descriptive and thematic findings (Merriam, 1998, p. 11). Observations, interviews, documents, and artifacts, as well as the researcher as the main instrument for data collection and analysis, are the primary methodologies for data collection in a qualitative research study (Merriam, 1998). In this study, I relied upon my role as researcher, as well as interviews, observations, documents, and artifacts, as the methodologies for data collection and analysis.

Semi-structured interviews were the first method employed for data collection. Schensul, Schensul, and LeCompte (1999) define semi-structured interviews as "predetermined questions related to domains of interest, administered to a representative sample of respondents to confirm study domains, and identify factors, variables, and items or attributes of variables for analysis or use in a survey" (p. 149). I formulated questions related to the domain of interest—technology integration in a math methods course.

The interviews were recorded and the data transcribed. Faculty in pre-service teacher preparation programs who teach elementary mathematics methods courses were interviewed. These faculty interviews totaled an hour each. In addition, a representative sample of students in the elementary mathematics methods courses was interviewed. Specifically, at Xever, all thirteen of the students participated in the interviews. At Yexer, eight out of nine students participated in the interviews. One interview of each participant was conducted usually lasting 10 to 15 minutes. As a result of class cancellations due to snow, the class meeting times were extended and thus limiting opportunities for longer student interviews.

Field observations were the second method of data collection. These field observations were exploratory or open ended, thus enabling the researcher to explore "who, what happened, where, when, why, for whom" in the research setting (Schensul et al., 1999, p. 101). The courses met once a week for an extensive time period (two and one half to three and one half hours), which provided a significant amount of time for the observations. In fact, the classes often met longer to make-up for cancellations due to snow. At Xever, I observed four class sessions – two classes were not held. At Yexer, I observed six class sessions. The observations (field notes) described in great detail the physical setting of the classroom, the participants, the activities and interactions, and the observer's role. The field notes allowed analysis of the ways that technology is used in a math methods course. As with the interviews, the identities of the participants were protected and pseudonyms were used.

Artifacts and documents were the third method of data collection. The following artifacts were collected: course descriptions, syllabi, handouts, assignments and rubrics, and pre-service teachers' sample lesson plans. These artifacts and documents assisted in the research process to "uncover meaning, develop understanding, and discover insights relevant to the research problem" (Merriam, 1998, p. 133) and to the study site. The syllabi provided a guide for when, how, and what will be happening during the classes and, more specifically, during the observations. In addition, the syllabi provided a context for the other artifacts and documents, including the assignments and lesson plans. The grading rubrics provided insight into the more significant elements of the assignments as well as the instructor's thinking regarding the course learning outcomes.

3.1 Research Setting

The first research site was a private nonprofit college, Xever College (a pseudonym), in Eastern Massachusetts. This Master's of Education program was a 14-month, cohort model, licensure, M.Ed., 36-credit program. It required a portfolio project, a practicum seminar, and a yearlong clinical residency with a PreK–12 school (instructor interview and website). All the courses were four-credit courses. Additionally (like all other elementary programs), students had to pass three Massachusetts Tests for Educator Licensure (MTEL) exams: communication and literacy; general curriculum, which includes the math subtest; and foundations of reading (see http://www.doe.mass.edu/mtel/testrequire.html). A math content course encompassing numbers and operations, functions, geometry, and algebra was required before the math methods course.

The class that I observed had 13 students, all of whom were female. The students have been assigned pseudonyms, which are reflected in this section. Some of the second-grade pre-practicum sites had document cameras and manipulatives, and another second grade had tablets. A third-grade and second-grade pre-practicum sites used websites.

The course instructor was a retired high school math teacher who worked part-time as a math coach in a nearby public elementary school system. The instructor had an MS in mathematics. The classroom had a ceiling-mounted projector, a Windows-based PC, a document reader, Apple TV (AirPlay via the Wi-Fi network– projection from iPad), and an iPad (instructor's personal one).

The second research site was a large public university, Yexer University (a pseudonym), in Eastern Massachusetts. This Master's of Education program was an initial licensure M.Ed. 34-credit program. It required a competency portfolio project, 80 hours of pre-practica, a 12 credit practicum, and a one-credit graduate program planning course (instructor interview and website). All the courses were three-credit courses. Additionally (like all other Massachusetts elementary programs), students had to pass three MTEL exams: communication and literacy; general curriculum, which includes the math subtest; and foundations of reading (http://www.doe.mass.edu/mtel/testrequire.html). Furthermore, each student was required to have a university=approved tablet device—an iPad or iPad Mini.

The class that I observed had nine students—seven females and two males. The students have been assigned pseudonyms, which are reflected in this section. One of the fourth-grade pre-practicum sites used iPad apps. Some of the pre-practicum sites had document cameras.

The classroom had a Smart Cart Extron Teaching Station with a ceiling-mounted projector, a desktop and a laptop Windows-based PC, a Wolfvision document reader, a DVD/VCR player, Apple TV (AirPlay via the Wi-Fi network–projection from iPad), and an iPad (instructor's personal one). The projector displayed to a pull-down white screen and not to a Smart Board. The classroom was designated for education courses and had two large storage cabinets with supplies and manipulatives, such as base ten blocks. The instructor also brought in her own manipulatives (N2.2).

The instructor was a K–8 math-science curriculum director in a nearby public elementary school district and a math consultant who provided professional development training/workshops for elementary teachers in school districts throughout the state. Generally, the instructor taught, as an adjunct faculty member, graduate courses for practicing teachers.

She had an M.Ed. degree and was planning on enrolling in a doctoral program in educational, instructional, and curriculum supervision in the fall (N2.1, N2.8).

3.2 Analysis

First, each research site was individually analyzed, and then analysis among the research sites was conducted as recommended by Merriam (1998). Individual analysis of each research site enabled me to gain insight into and understanding of that field setting. Cross analysis enabled me to compare the research sites, leading to a greater understanding of the domain of interest as a whole. Thus, I was able to develop descriptive findings that can contribute to the preparation of pre-service elementary teachers as well as future research. Ultimately, the goal of analysis in my role as researcher was to answer the research questions.

As mentioned previously, interviews, observations, documents, and artifacts were the primary data collection methods in this study. The data were organized and analyzed very much like a jigsaw puzzle coming slowly together (Schensul et al., 1999). Merriam (1998) recommends starting the first level of coding with assigning notations to the data collection methods: interviews and observations were designated as N [for Notes] and documents and artifacts were designated as A [for Artifacts] for each site. For example, A1.1 refers to the syllabus for Xever (site 1) and A2.1 refers to Yexer' (site 2) syllabus. Initially the data was organized chronologically.

The next level of coding involved classifying data into themes/categories which reflect the focus of this study (Merriam, 1998). Tables were constructed organizing the data by the themes/categories of instructional technology. The instructional technology categories were quantified into frequency tables. Furthermore, the data was linked to the elements of the TPACK conceptual framework (see Table A.2). Additionally, the high-level of TPACK in the instructors' teaching was assessed using the criteria in Standard 3 of the TSAT – Teaching and Learning with Technology (Massachusetts Department of Elementary and Secondary Education, 2010).

The constant comparative method of data analysis was utilized throughout (Merriam, 1998). First, comparisons within each site were made and described. Then, comparisons between the sites resulted in the cross-case analysis. Finally, analysis of data gathered from these collection methods described how the use of technology in math and/or science methods courses facilitated the learning of elementary pre-service teachers and the integration of technology into classroom instruction.

CHAPTER 4

CASE STUDY 1

4.1 Description of Setting

The first research site was a private nonprofit college, Xever College (a pseudonym), in Eastern Massachusetts. This Master's of Education program was a 14-month, cohort model, licensure, M.Ed., 36-credit program. It required a portfolio project, a practicum seminar, and a yearlong clinical residency with a PreK–12 school (instructor interview and website). All the courses were four-credit courses. Additionally (like all other Massachusetts elementary programs), students had to pass three MTEL exams: communication and literacy; general curriculum, which includes the math subtest; and foundations of reading (http://www.doe.mass.edu/mtel/testrequire.html). A math content course encompassing numbers and operations, functions, geometry, and algebra was required before the math methods course. The class that I observed had 13 students, all of whom were female. The students have been assigned pseudonyms, which are reflected in this section. The course instructor was a retired high school math teacher who worked part-time as a math coach in a nearby public elementary school system. The instructor had an MS in mathematics. The classroom had a ceiling-mounted projector, a Windows-based PC, a document reader, Apple TV (AirPlay via the Wi-Fi network–projection from iPad), and an iPad (instructor's personal one).

4.2 Research Question Two

The second research question will be addressed first and is as follows:

2. How do faculty in the math and/or science methods course use instructional technology for enhancing teaching and learning?

Descriptive findings as a result of data collection for the first site can be categorized into the following codes/domains of interest for how the instructor used and modeled these instructional technologies: videos (instructional), the iPad, websites linked to the Common Core State Standards (CCSS) and math curriculum frameworks, document camera, and other (see Table 4.1). These technologies are aligned with the Massachusetts Standards for Mathematical Practice in Table 4.2 and will be discussed in section 4.4.5 (TPACK). The technologies observed and the frequencies of the technologies used are shown in Tables 4.3 and 4.4, respectively. Each code/domain of interest (i.e., instructional technology) will now be described.

4.3 Instructor Use of Technologies

4.3.1 Videos

The instructor played a DVD on her computer and used the Smart Board to project it on the large screen (N1.10). This DVD featured exemplar teaching by an educator, Mahesh Sharma, part of the Center for Teaching Learning Mathematics in Framingham, Massachusetts (Sharma, 2000a). In one scene, the video showed how to teach the area model for multiplication using Cuisenaire rods, which is similar to finding the area of rectangles, as shown in Figure 4.1.

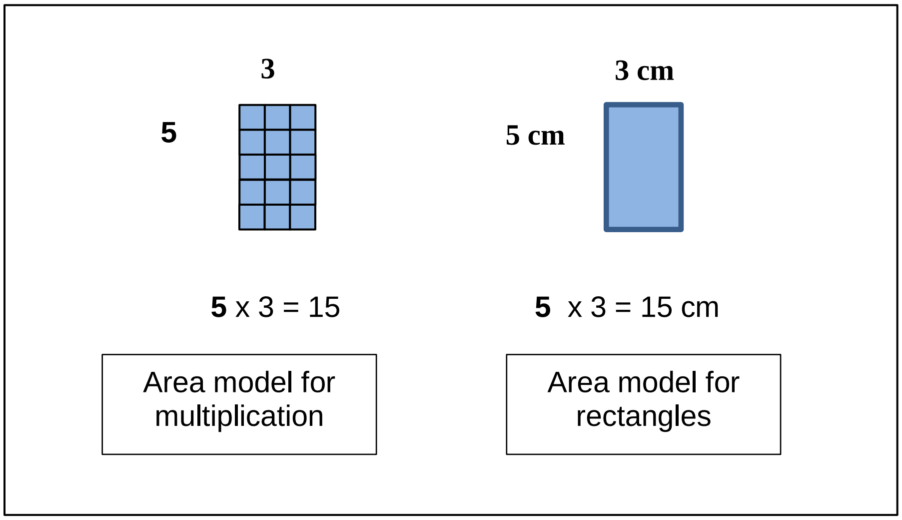


Figure 4.1: Area Models for Multiplication Using Cuisenaire Rods and for Rectangles

Misconceptions on perimeter and area were also presented from the DVD. For example, a student may memorize the formulas for area and perimeter of squares and rectangles. But he or she may not understand that all squares are rectangles and that the formulas for rectangles can be applied to squares. The math methods instructor stopped the DVD at key points to reinforce some of the concepts and strategies illustrated on the DVD. For example, she used the document reader to demonstrate how to use the Cuisenaire rods for place value and multiplication with two digits, in a manner similar to that on the DVD.

Another DVD scene depicted a third-grade classroom learning four parts to understanding fractions (Sharma, 2000b). The students used paper strips like a ruler as they worked through the fraction activity.

The instructor indicated that she included videos in her instruction to show the pre-service teachers exemplar teaching of math concepts (N1.3). An interview with the instructor revealed that the use of instructional videos was designed into the course by also incorporating them into an assignment (N1.6). The first part of the assignment required the students to work in groups to create a math skill book; the second part required the group to create a mini-lesson on math skills. Finally, the students had to present both. The assignment required the students to incorporate instructional videos of the math skills. Three groups presented this assignment (see Tables 4.5 and 4.6), which will be described later. The course syllabus described this assignment as the "Technology in the Math Classroom" section. The syllabus showed that this assignment served as the assessment for the course objective linked to Standard 7.08.2: "Students will be able to identify and share techniques and resources (including websites, apps, and manipulatives), that can be incorporated into their lessons" (A1.1). The instructor created such opportunities by requiring the pre-service teachers "to find a website [five websites/apps actually were required] that they would use either in the classroom or to plan lessons, and then they need to present at least one to the class" (N1.1). The presentations took place the last night of the class. The instructor further elaborated: "They use PowerPoint presentations, web-sites to do research and develop classroom activities and other manipulatives as appropriate" (N1.6). The websites and iPad apps will be discussed later in further detail.

4.3.2 Google Docs

The instructor created a spreadsheet in Google Docs for the students to access and to enter their research of apps and websites (see Tables 4.7 and 4.8), which all the pre-service teachers could use for future reference. In addition to the technology review assignment, the pre-service teachers also used Google Docs to collaborate for the group lesson plan presentations. Hannah shared the following: "Using PowerPoint, Google Docs, and Word doc[uments] helps our group collaborate for the lesson. The technology helps us to organize our info in a structured manner to display our information" (N1.11). Similarly, another group utilized the accessibility feature of Google Docs, as described by Kelly: “We used Google Docs to collaborate because we can’t meet in person” (N1.11).

4.3.3 iPad

The iPad was used extensively by the instructor (see Tables 4.3 and 4.4). In an interview, the instructor stated the following: “I use my iPad to present the class agenda, PowerPoint presentations, show YouTube videos, [and] call up applicable website[s]. I use the classroom computer to show the DVD[s]” (N1.1). The iPad was connected to a Smart Board to show an app or an instructional video to the whole class. At one point during a class, the instructor used the terms “math and technology” and showed the multitude of apps available. As will be discussed later, the pre-service teachers found the use of the iPad very helpful.

4.3.4 Websites

Websites linked to the CCSS and math curriculum frameworks were one instructional technology used by the instructor. In an interview, the instructor discussed websites, such as IXL (https://www.ixl.com/standards/massachusetts/math), which provides math concepts and practice skills by grade level and alignment with standards and curriculum frameworks (N1.3). Additionally, the IXL Analytics Reports track student progress. Another example, the Illuminations website (http://illuminations.nctm.org/), developed by the National Council of Teachers of Mathematics (NCTM), provides lessons and interactives by grade level, which are aligned to standards and curriculum frameworks. This website was used by the instructor for teaching counting and cardinality—that is, the number of values in a set (A1.2).

The students also mentioned these websites in the interviews/discussions. For example, Barbara disclosed: “We were shown a website, which was directly applicable to the standards so we can help enhance student learning” (N1.11). Also, Crystal indicated: “She showed us where to find activities and Common Core resources online." She added the following: "The professor has showed us numerous websites that outline the standards for mathematical practice and show sample activities/lessons that incorporate them" (N1.11). These websites served as a potential resource for the pre-service teachers.

4.3.5 Document Camera

The instructor used the document camera extensively (see Tables 4.3 and 4.4). She demonstrated on the document camera how to use physical manipulatives, such as Cuisenaire rods, to solve a mathematics problem, such as multiplication with two digits (see Figure 4.1) (N1.10). While the instructor modeled on the document camera, the students in the class worked and solved the problem at their desks with manipulatives. Additionally, the instructor used the document camera, rather than a whiteboard, to write out step-by-step a solution to a mathematics problem. For example, a "Sheep and Ducks" activity worksheet asked the following question: "Next to the barn is a pen with 2 sheep and 3 ducks. How many legs altogether? Show how you know your answer is correct." (A1.10). The instructor wrote the following solution:

4 + 4 = 8 [sheep]

2 + 2 + 2 + 2 = 6 [ducks]

8 + 6 = 14 [total number of legs]

As the instructor demonstrated the solution and provided a performance assessment rubric, the pre-service teachers worked on the problem (A1.10, N1.10).

4.4 TPACK

4.4.1 Content Knowledge

The TPACK framework and its components are evident in the instructor's teaching of the elementary math methods course (see Tables 4.9 and 4.10). Content knowledge is subject matter understanding of mathematics (see Table A.2). Standard 7.06.7 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations (Massachusetts Department of Elementary and Secondary Education, 2017) describes the subject matter knowledge requirements for elementary teachers specifically in mathematics: numbers and operations, functions and algebra, geometry and measurement, and statistics and probability. The syllabus outlines the following mathematical content in the course: number sense, addition, subtraction, place value, multiplication, division, fractions, decimals, percents, and geometry (A1.1). This content is aligned with the content domains found in the Massachusetts Curriculum Frameworks for Mathematics (2011) for elementary mathematics. This content knowledge was evident as the instructor explained and walked the students through various topics with examples and exercises (see Table 4.9). For example, the instructor used story problems to illustrate the quotative and partitive properties of division. Through repeated subtraction, the story objects were broken down into groups (i.e., quotative property), and the number in each group (equal amounts) was counted (i.e., partitive property). For example, Dean had nine Legos and two friends want to share the Legos with him; how many does each one get? (N1.9). The instructor asked the pre-service teachers to create and share their own story problems for the problem 27 ÷ 4, but they could not use food or people (the two most commonly used) as objects. The "exit ticket" was similar but involved fractions: "You have 7½ ft. of board and you will need to cut the board into ¾ ft. pieces. How many pieces will you have?" (N1.9).

Also, the instructor displayed mathematical knowledge for teaching pre-service teachers how to teach mathematics to children. This content knowledge includes “teachers’ knowledge about the subject matter to be learned or taught” (see Table A.2). For example, teachers need to correct student answers. But, in mathematics, they also need to recognize how and why the students make errors/give incorrect answers as well as how to explain these errors to the students. In another example, the instructor stressed the importance/understanding of place value for the ability to multiply with two digits. She used two Cuisenaire rods together for place value and then used the rods to multiply two digits (N1.10). Likewise in her parting thoughts, she stated that “giving them the algorithm without the conceptual understanding does not help students learn” (A1.11).

4.4.2 Pedagogical Knowledge

Pedagogical knowledge is knowledge of how to teach that fosters student learning (see Table A.2). High-quality and coherent instruction was consistently observed (see Tables 4.9 and 4.10). The instructor had college teaching experience, which requires curriculum planning and design. Thus, the syllabus described the course objectives with the corresponding assessments as well as a detailed outline of the weekly topics and assignments. The math methods course was a graduate-level course, and the instructor established high expectations of the pre-service teachers appropriate to the graduate level. The instructor used appropriate assessments, such as lesson plans, presentations, and papers. She assessed the first group’s presentation/materials and required them to make their presentation again (N1.4). She met with them privately and provided feedback/guidance.

Additionally, the instructor has experience as a math coach in the elementary schools. She was able to provide real-life examples and advice/tips from her own teaching that would be beneficial for the pre-service teachers. The instructor discussed and provided such resources such handouts on Bloom's Taxonomy (A1.13) and Poyla's Steps for Problem Solving (A1.15). The interviews with the pre-service teachers (noted earlier in the discussion of the instructor's use of technology) provide evidence of pedagogical knowledge. For example, the pre-service teachers, who indicated they were visual learners, valued the instructor's use of visual aids (i.e., videos), which supported their learning of mathematics and learning to teach mathematics with technology.

4.4.3 Technological Knowledge

Technological knowledge is knowledge of using various technologies—for example, a document camera, researching and using such video sites as YouTube as well as DVD videos, and websites/iPad apps (see Table A.2). Massachusetts developed a tool, the Technology Self-Assessment Tool (TSAT), for educators to assess their technology skills (Massachusetts Department of Elementary and Secondary Education, 2010). Specifically, criteria in Standard 3 of the TSAT, Teaching and Learning with Technology, will now be described and applied to analyze the presence and level of mastery of technology used by the instructor in the math methods course, as shown in Tables 4.11 through 4.14.

The instructor displayed confidence in using the various technologies in her teaching (see Table 4.9). For example, she demonstrated on the document camera how to use physical manipulatives, such as Unifix cubes, to solve a mathematics problem, such as addition of two-digit numbers (see TSAT, C3.4, C3.5; N1.3). iPad apps were used by the instructor to illustrate how to solve a mathematical problem using digital manipulatives, such as the Number Line app shown in Table 4.3 (see TSAT, C3.7). Furthermore, the instructor assisted the students to use the technology, such as Apple TV and AirPlay, so that the app on the student iPad would display on the whiteboard (see TSAT, C3.9; N1.8).

At the TSAT Early Technology Mastery Level, the instructor received a 100% skill percentage, as shown in Table 4.11. Curriculum-specific information, in the form of websites, resource lists, and apps, were presented and shared with students (see TSAT, A3.2; A1.12). In one class, videos of exemplar teaching highlighted best practices on teaching and learning with technology (see TSAT, A3.1; N1.3, N1.7, N1.9, N1.10). Communication tools, such as email, and digital tools, such as Google Docs, were utilized to communicate with students and to disseminate class materials (see TSAT, A3.4; A1.2, A1.12, N1.2).

Likewise, at the TSAT Developing Technology Mastery Level, the instructor received a 100% mastery skill percentage, as shown in Table 4.12. Technology resources were identified by the instructor (see TSAT, B3.3; A1.1, A1.12, N1.10). Many of these resources were tied to the CCSS and Massachusetts Math Curriculum Frameworks as well as to the Massachusetts Mathematics Standards of Practice. The instructor created an Excel template in Google Docs for the pre-service teachers to organize their review of apps and websites (see TSAT, B3.5; A1.12). She seamlessly moved back and forth between the desktop computer, iPad, document camera, and projector (see TSAT, B3.4; N1.4, N1.5, N1.8) and utilized these technologies to create/present multimedia presentations of the curriculum content (see TSAT, B3.6; N1.9, N1.10). As mentioned previously, the instructor assisted the students in setting up Apple TV for projection from the iPad (see TSAT, B3.4; N1.8).

In contrast, at the TSAT Proficient Technology Master Level, the instructor received an 85% mastery skill percentage, as shown in Table 4.13. At this percentage (minimum 80%) level, the instructor can move to the next mastery level. As mentioned before, the instructor facilitated student use of online tools, such as Google Docs, to collaborate on their technology assignment: review of five apps/websites (see TSAT, C3.14; A1.12). The instructor presented/discussed technology resources suitable for an elementary mathematics curriculum (see TSAT, C3.2; A1.12, N1.9, N1.10). Such apps as the number line, as well as such instructional videos as *Numeracy 4: Teaching Fractions* (Sharma 2000b), are suitable resources that provide virtual manipulatives and lesson plans, respectively, for elementary mathematics curriculum (see Table 4.3). These resources were demonstrated using such technology tools as the desktop computer, iPad, document camera, Apple TV and AirPlay, and projector (see TSAT, C3.4; N1.4, N1.5, N1.9, N1.10). Furthermore, these technology tools and resources facilitated the learning of the pre-service teachers who identified as visual learners (see TSAT, C3.5; N1.9, N1.10).

At the Advanced Technology Mastery Level, the instructor received a 56% mastery skill percentage, as shown in Table 4.14. This percentage is below the 80% needed for mastery at this level. As mentioned before, the instructor used the appropriate communication technologies to convey her ideas (see TSAT, D3.6; A1.2, A1.12, N1.2). Multimedia presentations, such as PowerPoints and the Sharma DVD, were utilized to present the class agenda, course content, and link to videos/websites and resources (see Table 4.3). In addition, the instructor singled-out effective design/presentation (see TSAT, D3.7; N1.4). In one case she required a student group to “re-present” their presentation, as it did not meet her expectations. Additionally, the instructor required the pre-service teachers to review and evaluate apps and websites in their assignments. Lastly, staff (i.e., the pre-service teachers) development regarding the technology integration was embraced by the instructor (see TSAT, D3.9; N1.4, N1.5, N1.9, N1.10). The instructor’s modeling and instructional strategies provided professional development of technology integration into an elementary mathematics curriculum. In essence, the entire math methods course was designed and delivered as professional development for the pre-service teachers.

4.4.4 Pedagogical Content Knowledge

Pedagogical content knowledge is knowledge of instructional strategies (i.e., pedagogy) as they relate to specific content, such as mathematics (see Table A.2). Standard 7.06.7 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations states that teachers "must demonstrate not only that they know how to do elementary mathematics, but that they understand and can explain to students, in multiple ways, why it makes sense" (Massachusetts Department of Elementary and Secondary Education, 2017). The instructor exemplified pedagogical content knowledge in action via her use of manipulatives and tools, such as rekenrek, Cuisenaire rods, and place value charts, to teach mathematical concepts (CK) (N1.7–N1.10). The rekenrek was used to show turn-around facts in addition. A rekenrek resembles an abacus with two rows of beads The Cuisenaire rods and place value charts were used to illustrate multiplication with two digits.

The instructor employed multiple strategies to teach math concepts, such as fractions and number sense (see Table 4.9). For example, she passed out a second-grade worksheet, rubric, and task analysis sheet on sheep and ducks (A1.9, N1.10). Several problems and sample student solutions were shown and related to how many sheep and ducks were on the farm given various numbers of legs seen by the farmer. The task analysis sheet asked the pre-service teachers to describe successful strategies used by the students and to identify how these strategies may be shared to help the whole class. Additionally, the pre-service teachers were asked to identify areas of difficulty for second graders. The instructor asked the pre-service teachers to look at the student work and to think about the following question: "What does this mean for instruction and not just for a grade"? (N1.10).

Mathematical models, such as arrays, were illustrated and used to explain decimals and multiplication strategies (N1.3). Area models were used to illustrate multiplication of two digits and the area of a rectangle (see Figure 4.1). Misconceptions about perimeter and area were presented (see Figure 4.2) so that the pre-service teachers understood and could address the mistaken beliefs and thinking of their students.

4.4.5 Technological Pedagogical Content Knowledge (TPACK)

TPACK is knowledge of instructional strategies (PK) that integrate technologies (TK) to constructively teach content (PCK), in this case mathematics content (CK), for greater understanding of that content (see Table A.2). Thus, TPACK, in the instructor's teaching, is the knowledge of instructional strategies that integrate technologies, such as an instructional video, to demonstrate how to use such models as arrays to perform mathematical operations; a website/iPad app to explain how to solve a mathematical problem with either a digital or nondigital tool, such as a double number line; and a document camera to demonstrate manipulatives, such as Cuisenaire rods (see Tables A.2 and 4.9).

Instructional videos of exemplar teaching of mathematical concepts were utilized by the instructor (see Table 4.9). As seen in Figure 4.2, the Xever instructor showed the Sharma DVD, which illustrated the area model for multiplication using Cuisenaire rods and then reinforced the model by using the document camera and manipulatives like Cuisenaire rods (see Figure 4.1) (N1.10). Figure 4.2 also shows another lesson from the Sharma DVD that illustrated the area model for rectangles using Cuisenaire rods. Similarly, the instructor explained the model by using the document camera and Cuisenaire rods (see Figure 4.1) (N1.10). These instructional strategies align with the Standards for Mathematical Practice (SMP) (see Table 4.2). For example, the instructor used appropriate tools, such as instructional videos, document camera, and Cuisenaire rods, strategically (SMP5) to support the pre-service teachers as they make sense of the problems (SMP1) and reason quantitatively (SMP2).

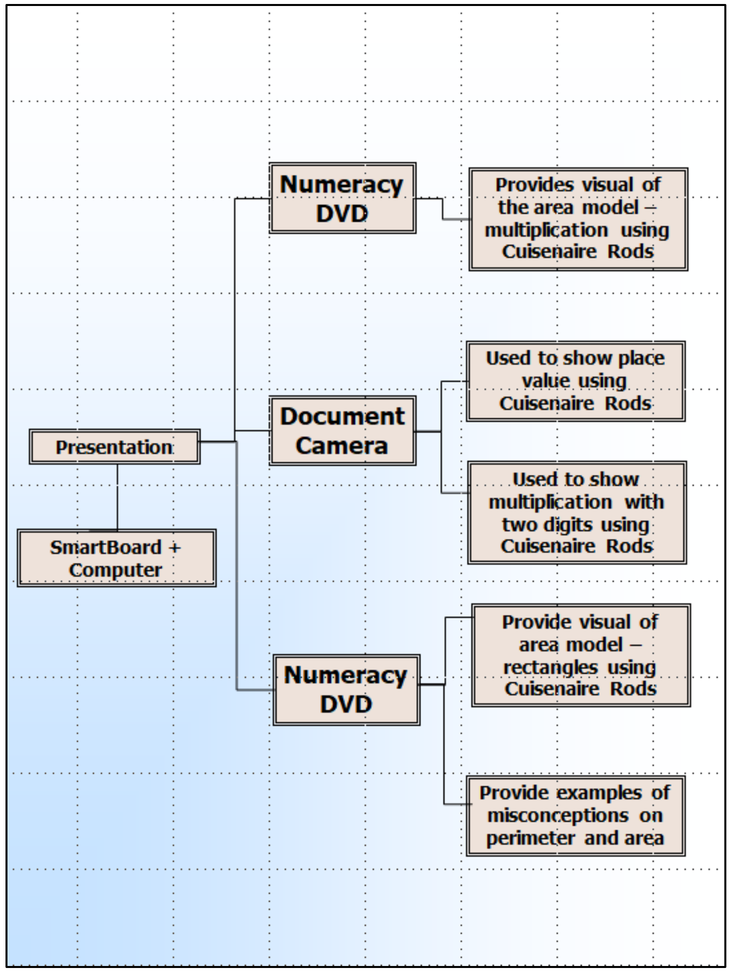


Figure 4.2: Flowchart of Site 1 Instructor and Numeracy Presentation

The iPad and its applications in mathematics was modeled by the instructor in her teaching. For example, the instructor used the Number Line app, via Apple TV, to explain how to solve multiplication problems (see Table 4.3) (N1.1). The instructor provided visual examples of multiplication on her iPad for the pre-service teachers to observe. These instructional strategies align with the SMP (see Table 4.2). For example, the instructor used appropriate tools, such as the Number Line app, strategically (SMP5) to support the pre-service teachers as they make sense of the problems (SMP1) and make use of structure and precision (i.e., the Number Line) (SMP6, SMP7). Likewise, the instructor incorporated websites into her teaching. Very often these websites were linked to the CCSS, the Massachusetts Math Curriculum Frameworks, and the Standards for Mathematical Practice (see Table 4.9). The daily class agendas referred to apps, YouTube videos, and websites used by the instructor in class (A1.12).

The document camera was used by the instructor to demonstrate manipulatives like Cuisenaire rods and Unifix cubes. As see in Figure 4.3, the document camera was used to show manipulatives, Cuisenaire rods, which provided visual examples for multiplication problem-solving and mathematical concepts like place value. Similarly, the instructor used Unifix Cubes on the document camera as visual examples of story problems and strategies in division (see Table 4.3) (N1.9). These instructional strategies align with the SMP (see Table 4.2). For example, the instructor used appropriate tools, such as a document camera, Cuisenaire rods, and Unifix cubes, strategically (SMP5) to support the pre-service teachers as they make sense of the problems (SMP1) and reason quantitatively (SMP2). Additionally, the instructor facilitated the development and critiquing of viable arguments (SMP3) and repeated reasoning (SMP8). The instructor consistently embodied technological pedagogical and content knowledge in this math methods course.

The interviews/discussions with the students provided insight into how the students perceived the role and value of the instructor using instructional technology in enhancing their learning of math content and teaching practices. Irene stated that "The instructional technology helps access different learning styles. As a visual learner, I appreciate videos to supplement the lessons being taught" (N1.11). Similarly, Hannah responds that "Videos and iPad engage us in the lesson. I am a visual learner so the videos and computer help" (N1.11). These two pre-service teachers understand their own learning styles and so the instructor's modeling is enhancing their learning.

Additionally, Barbara indicated "Watching our instructor work with technology on a weekly basis has allowed me to see the many ways I would be able to incorporate new methods into my classroom: iPad, PowerPoints, [and] websites" (N1.11). Joanna adds the following: "The professor uses her iPad to share videos pertaining to methods of teaching certain math subjects and also to generally share information with us" (N1.11). Similarly, Kelly states that "It shows us different websites/instructional videos to apply it to the classroom. We can see different ways to teach to different styles of learning" (N1.11). Likewise, Debra shared the following: "Access to real life students working through similar problems. [The instructor] shows examples of what I can do in the classroom to make my mathematic practice clearer" (N1.11). Crystal also mentioned that "we have watched educational videos and model classrooms" (N1.11). These pre-service teachers indicate that they have been able to connect the modeling by the instructor to their own teaching of mathematics.

Furthermore, the high level of TPACK, in the instructor's teaching, can be assessed using the criteria in Standard 3 of the TSAT – Teaching and Learning with Technology (see Tables 4.11 through 4.14). At the TSAT Early Technology Mastery Level, the instructor showed and discussed videos of exemplar teaching highlighting best practices on teaching and learning with technology (see TSAT, A3.1; N1.3, N1.7, N1.9, N1.10). For example, the instructor showed the Sharma Teaching Fractions DVD and then built on the lesson shown by explaining again the four parts to understanding fractions: whole is being divided, divided into certain parts, equal parts, and parts make up the whole (N1.10).

Likewise, at the TSAT Developing Technology Mastery Level, the instructor designed lessons and activities that explain how to solve a mathematical problem with either a digital or non-digital tool like a number line (see TSAT, B3.1, B3.2; N1.3). Multiplication problems were solved by using the number line app. Additionally, she used a document camera to demonstrate manipulatives (i.e., Cuisenaire rods) to illustrate the area model for multiplication, as shown in Figure 5.1 (see TSAT, B3.1, B3.2). Then the instructor reinforced the model by using the document camera and manipulatives as the pre-service teachers worked on the same activity. As mentioned previously, these instructional strategies used technology that was appropriate for visual learners.

Similarly, at the TSAT Proficient Technology Master Level, as mentioned before in the interview with the pre-service teachers, the instructor presented/discussed websites and apps that were linked to the CCSS, the Massachusetts Math Curriculum Frameworks and the Standards for Mathematical Practice (see TSAT, C3.2, C3.5; A1.1, A1.12).

Lastly, at the Advanced Technology Mastery Level, the instructor identified and used websites and apps that relate to teaching and learning of elementary mathematics (see TSAT, D3.1; A1.1, A1.12, N1.3). In conclusion, staff (i.e., the pre-service teachers) development regarding the technology integration was embraced by the instructor (see TSAT, D3.9; N1.1, N1.8). The instructor's modeling and instructional strategies provided professional development of technology integration into an elementary mathematics curriculum. In essence, the entire math methods course was designed and delivered as professional development for the pre-service teachers.

4.5 Research Question One

These pre-service teachers demonstrated confidence in their mathematics knowledge, pedagogies tied to the CCSS, the Massachusetts Curriculum Frameworks, and the Standards of Mathematical Practice, and instructional strategies that incorporated technology and other tools (manipulatives) to address the various learning styles of their students. The specifics of how the pre-service teachers demonstrated this will be discussed in the research question one section analysis.

The first research question is as follows:

1. In which ways did the use of technology in a math and/or science methods course facilitate the learning of elementary pre-service teachers and the integration of technology into classroom instruction?

Descriptive findings as a result of data collection for the first site can be categorized into the following codes/domains of interest (i.e., instructional technologies): videos (instructional), Google Docs, iPad apps/websites, document camera, and other (see Tables 4.1 and 4.5). Each code/domain of interest will now be described.

4.6 Pre-Service Teacher Use of Technologies

4.6.1 Videos

Videos, generally instructional digital videos, were utilized by the pre-service teachers. For example, as noted earlier, the videos were incorporated into a group assignment: create a math skill book, create a mini-lesson on the math skills, and make a presentation of both. The video technology generally was a YouTube video, as shown in Table 4.5. The frequency of each video technology is shown in Table 4.6.

Student Group 3 made a presentation and booklet on fractions, decimals, and percents A1.6, N1.8). The students showed YouTube videos: one on misconceptions naming decimals (https://www.youtube.com/watch?v=1z_xkfzCDoM) and one on the number line (https://www.youtube.com/watch?v=SE7AcLdU8NE&noredirect=1) (see Table 4.5). The misconceptions video explained that decimals have two names and that you “say the name as if it has no dot then say the name of the last place value spot.” Thus, .065 is not said as 65 tenths but should be 65 thousandths. The zero after the decimal often confuses students.

The number line video shows the percents on the top side of the number line and the numbers under the number line. Then the solution to the problem is modeled. Similarly, this can be seen in the model provided by the students (A1.6) illustrated in Figure 4.3. The problem is 15 is what percent of 75?

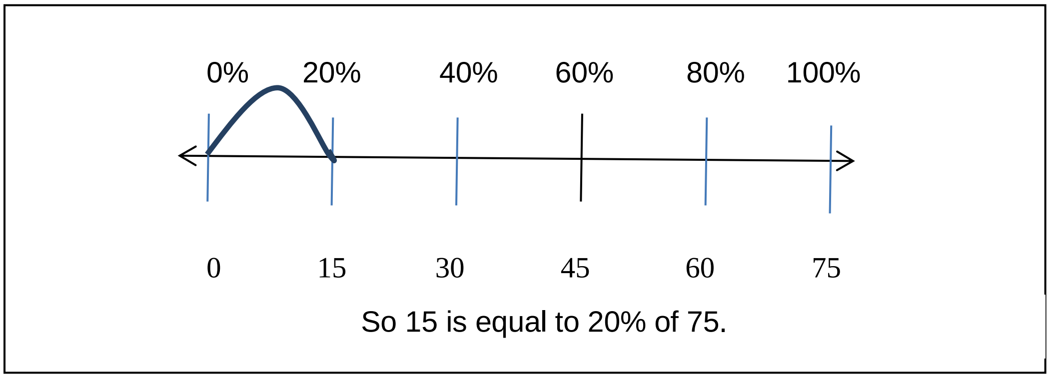


Figure 4.3: Double Number Line Showing Percents

The students (group 3) followed up with another video showing how to model decimals using arrays (https://www.showme.com/sh/?h=fxpU5Q). A demonstration of multiplying decimals using arrays was shown. The sample multiplication problem is: 3.4 x 2.2. The strategy utilized to solve the problem is to break apart the numbers/decimals. First, 3 x 2 is presented, then 3 x .2, followed .4 x 2, and finally .4 x .2. This array model is illustrated in Figure 4.4 using base ten blocks (N1.8).

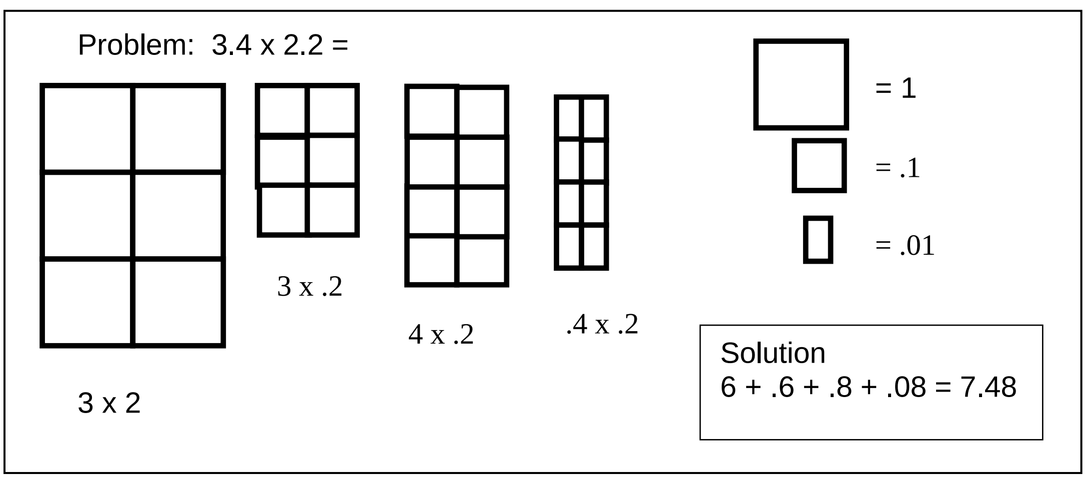


Figure 4.4: Multiplying Decimals Using an Array

Likewise, student group 2's presentation on multiplication and division (N1.5) also incorporated videos. The students showed YouTube videos: one for second grade called "Doubles Rap Baby" (https://www.youtube.com/watch?v=jG6n5xLt-0Q) and another called "Pre-Algebra7 Associative and Distributive Properties of Multiplication" (https://www.youtube.com/watch?v=CkVJ8xa63ow) (see Table 4.5).

The interviews/discussions with the students provided insight into how the students perceived the role and value of the video technology regarding their own learning as well as their teaching practice. Eva shared the following: "The videos help reinforce how to teach different skills" (N1.11). Similarly, Frances disclosed: "The technology helps us see real-life situations in videos" (N1.11). Likewise, Amanda indicated: "Implementation of videos, common core standard websites, iPad applications, etc., provides examples and how to make them effective for my classroom—where and when to use" (N1.11). Irene responded with the following: "My group used Word documents and YouTube videos. This kind of technology can help supplement math lessons by accessing students of all learning types and styles" (N1.11). Amanda and Irene clearly saw a direct link between their learning now and their future teaching with technology. Lastly, Linda shared the following: "As a visual/auditory learner, visual aids used in class [videos] help reinforce how important it is to utilize technology with [my] own students" (N1.11).

Thus, these interviews indicate that the pre-service teachers (students) understood how the video technology facilitated their own learning, particularly the visual learners, of how to teach mathematics and particular concepts, as well as how to teach mathematics with technology (i.e., TPACK) (see Table A.2).

4.6.2 Document Camera

The document camera was another instructional technology used in the student group presentation assignment (see Table 4.5). Student group 3 demonstrated how to match fractions and percents using popsicle sticks on the document camera. The popsicle sticks are of equal size and are labeled with various fractions and percents as shown. The students demonstrated how two ¼ sticks equal one 50% stick (N1.8), as shown in Figure 4.5.

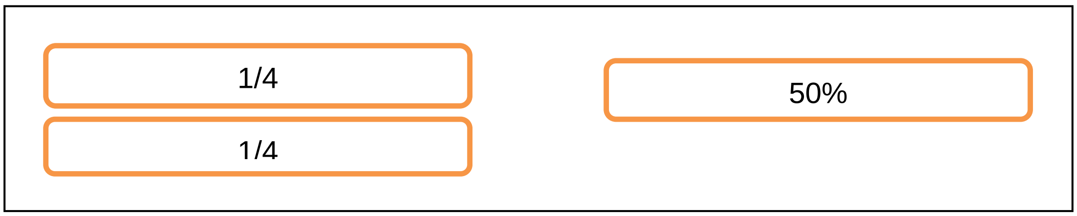


Figure 4.5: Popsicle Sticks Matching Fractions and Decimals

Note. The popsicle sticks are all the same size and only the labels indicate their numeric representation.

Likewise, student group 1 also used the document camera. One of the students brought in manipulatives, flipping cubes (different-colored cubes), which she used in her own second-grade class. She demonstrated turnaround facts with the flipping cubes, illustrating that 5 + 3 = 8 is the same as 3 + 5 = 8, as shown in Figure 4.6 (N1.4).

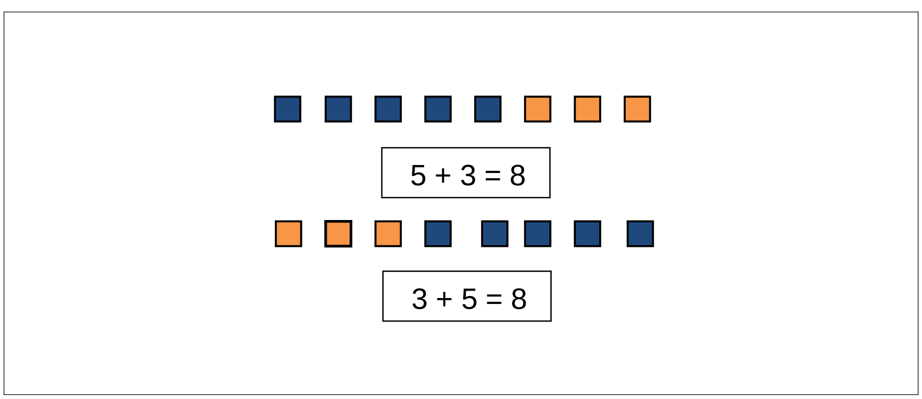


Figure 4.6: Turnaround Facts with Flipping Cubes

Another student (group 1) demonstrated ten frames and colored chips with the document camera and linked this demonstration to her own second-grade class. She demonstrated regrouping, which helps the second-graders to understand addition so that 10 + 4 is much easier to add than 8 + 6, as illustrated in Figure 4.7 (N1.4).

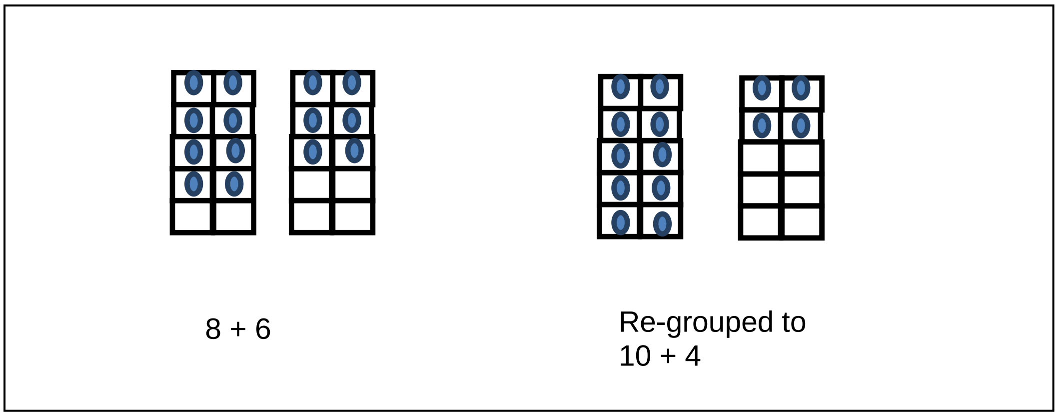


Figure 4.7: Regrouping in Addition Using Ten Frames

Similarly, one student (from student group 2) used Unifix cubes and snap cubes provided by the instructor. The student demonstrated how to use these manipulatives for repeated addition and doubling for multiplying by 2 (A1.3, A1.4). One specific example of a repeated addition problem shown by student group 2 was as follows: "Andrew had 3 friends over to hangout. Each friend, including Andrew, ate 3 slices of pizza each. How many slices of pizza did all 4 friends eat? The multiplication sentence for this problem would be (4 x 3). Students can use repeated addition to find the product for this problem by adding 3 + 3 + 3 + 3. By adding 3 to itself four times, the students would be able to find the product, or that altogether the 4 boys ate 12 slices of pizza" (A1.3, A1.4). The student used the Unifix cubes to illustrate this process, as shown blow in Figure 4.8.

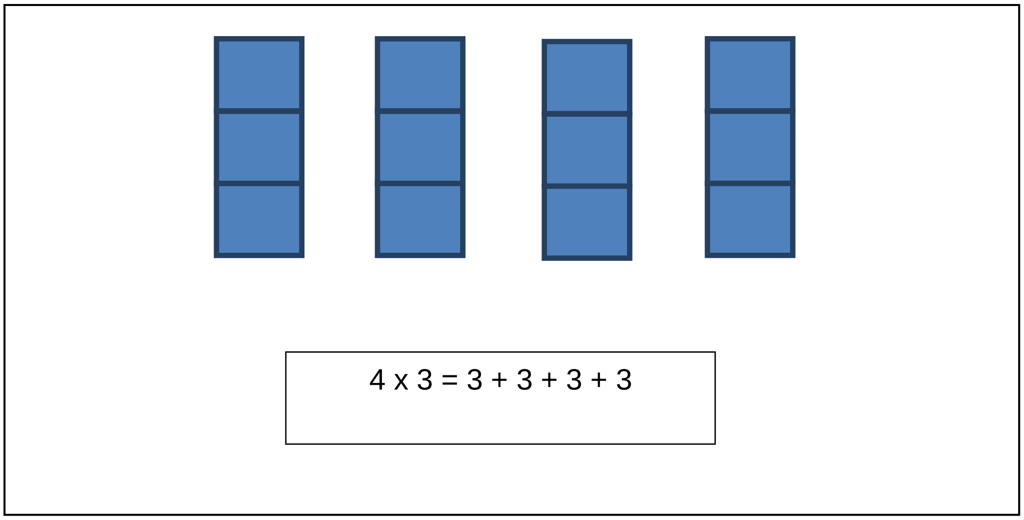


Figure 4.8: Repeated Addition Strategy for Multiplication

4.6.3 Websites and iPad Apps

The websites and iPad apps were another technology used by the pre-service teachers. These websites and apps provided different models/tools for the pre-service teachers to use to teach the mathematical concepts to their current as well as to their future students. As previously mentioned, in one of the assignments the students had to research five websites and/or iPad apps that they might use in their future teaching. This also served as a potentially valuable resource for all the pre-service teachers. This list was compiled and shared in a Google Docs Excel spreadsheet created by the instructor. Tables 4.7, 4.8, and 4.15 are adapted from that spreadsheet. These websites and apps demonstrate pedagogical strategies for teaching that particular mathematics content (i.e., TPACK). In many cases, this content was aligned to the CCSS, as can be seen in Figure 4.9 from the Hoodamath website (http://www.hoodamath.com/ccss1.pdf).

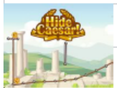
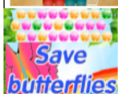
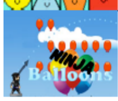
Game	Description	Math topics	Grades K & 1st – 3rd	Grades 4th–5th	Grades 6th – HS
	Oblong: Pick up all the tiles	Analyze, visualize & create path for rectangular prism. Develops & applies concept of Surface area & net.	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1	CCSS.Math.Content.6.G.A.4
			CCSS.Math.Practice.MP8	CCSS.Math.Practice.MP8	CCSS.Math.Content.7.G.A.3
	Blue Blox 2: Connect two blue blocks together to move to next round.	Analyze givens, constraints, relationships & goals. Players visualize & synthesize using transformations to choose their course of action.	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1	CCSS.Math.Content.8.G.A.1
			.CCSS.Math.Practice.MP7	.CCSS.Math.Practice.MP7	.CCSS.Math.Practice.MP7
	Hide Caesar: Protect Caesar from falling stones. Click and drag objects to cover Caesar	Use knowledge of geometric attributes, physics and reasoning to create the blockade.	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1	CCSS.Math.Content.8.G.A.1
			CCSS.Math.Content.1.G	CCSS.Math.Practice.MP7	CCSS.Math.Practice.MP7
	Fancy Constructor: Use translations to move puzzle pieces into their proper places	Transformations , spacial visualization and problem solving	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1	CCSS.Math.Content.8.G.A.1
			CCSS.Math.Content.1.G	CCSS.Math.Practice.MP7	CCSS.Math.Practice.MP7
	Save butterflies	Problem Solve	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1
			.CCSS.Math.Practice.MP7	.CCSS.Math.Practice.MP7	.CCSS.Math.Practice.MP7
	Shell Heroes: Bring the king and his soldiers to safety	Analyze givens, constraints, relationships & goals. Problem solve.	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1
			.CCSS.Math.Practice.MP7	.CCSS.Math.Practice.MP7	.CCSS.Math.Practice.MP7
	Germies: remove germs from the game board by creating squares and rectangles	Distinguishing shape attributes, strategy and problem solving	CCSS.Math.Content.1.G.A.1	CCSS.Math.Practice.MP1	CCSS.Math.Practice.MP1
			CCSS.Math.Content.K.G.B.6	.CCSS.Math.Practice.MP7	.CCSS.Math.Practice.MP7
					

Figure 4.9: Hoodamath Games Alignment to Common Core State Standards

Additionally, some of the websites/apps provided opportunities for differentiated learning based on grade level and ability. The Hoodamath website provided search capability by grades K through high school as well as math subjects, including addition, subtraction, multiplication, division, fractions, integers, and algebra. In some cases the pre-service teacher actually used the website/app in her own teaching, and these will be described in the following paragraphs on the websites/apps. Several of these websites/apps offered feedback reports so that students, teachers, and parents gained insight into the students' understanding (or lack thereof) of the various math content.

Often school districts may not have a lot of funds to spend on various math manipulatives and supplies, but they may provide access to the Internet in the classroom and/or library. In her technology presentation, one pre-service teacher, Debra, described an interesting online alternative—The National Library of Virtual Manipulatives website. The tools (i.e., manipulatives) are arranged by grade level. The students work with the manipulatives virtually to learn/reinforce the math concepts (N1.7).

In her technology presentation, the pre-service teacher, Amanda, described Hoodamath (http://www.hoodamath.com/games/), a website with over 500 math games searchable by grade or math subject. The site included tutorials, games, quizzes, and puzzles. She described how her second-grade students earn the use of a tablet as a reward—the students don't realize they are practicing math (N1.7).

The Starfall website (http://www.starfall.com) was described and used by Eva, a pre-service teacher, in her class as "great" for special education students to build and practice their math skills. The website is designed for adaptive learning to meet students' individual needs. Two others in the class also had used the website with special education students. Examples of math topics included numbers, shapes, addition, subtraction, and telling time. The website was motivational and fun for the students to use. Both teachers and parents can track student progress (N1.7).

A very different website, Mathcats (http://mathcats.com), was demonstrated by another pre-service teacher, Frances. This website explores open-ended concepts in math and presents problems for the students to solve. For example, the category "math explores the world" included whole-class activities for exploring how old are you whereby you create an age cake to solve the problem. The problems often took the form of brainteasers. These open-ended exploration problems build on students' prior knowledge, which the pre-service and/or classroom teacher would need to be aware of before incorporating this type of website into their teaching (PCK). The website incorporated a list of resources for teachers and parents that was updated often. Teachers could also contact them to submit their own problems for addition to the website (N1.7).

Another pre-service teacher, Crystal, working with second graders described the Math Playground website (https://www.mathplayground.com) for problem solving by grade level and topic. Her students would watch a math video and then play a game on this website to solve word problems related to the math video. Manipulatives, such as thinking blocks, could be used to solve the word problems. The math categories ranged from addition to geometry to logic and by grade level. She also indicated that it was good resource for parents to work at home with their child (N1.7).

The Ten Marks website (http://www.tenmarks.com) was used by Linda, a pre-service teacher, in her third-grade class. It is not game-based. A login and password is created for each student, allowing the teacher to assign different work and due dates. Written hints and video tutorials assist the students as the try to complete the 10 assignments in each module. Students can go back and fix their answers only once (N1.7). The ability to differentiate instruction through this website to enhance the learning of these third-grade students is characteristic of pedagogical content knowledge.

Accordingly, the iPad apps and websites presented by the pre-service teachers during class and listed in the Tables 4.7, 4.8, and 4.15 enriched their own learning. The pre-service teachers were building and showing their awareness of content knowledge, pedagogical content knowledge, and technological pedagogical content knowledge in order to positively influence their students' learning. These pre-service teachers determined which websites/apps "fit" with the particular mathematics content and instructional strategies (i.e., TPACK).

4.6.4 Google Docs

The pre-service teachers used Google Docs to collaborate for the group presentations (see Table 4.5). Google Docs is an online tool that enables teamwork whereby the students do not have to meet in person. All the students in the group were able to write and edit the same document. Additionally, the pre-service teachers used a spreadsheet in Google Docs created by the instructor to access and enter their website and iPad app research (see Tables 4.7 and 4.8), which all the pre-service teachers could then use for future reference.

4.7 TPACK

4.7.1 Content Knowledge

The TPACK framework and its components can be applied to this student group presentation. Content knowledge is subject matter understanding of mathematics (see Table A.2). In the student group 3 presentation, content knowledge is subject matter understanding of fifth-grade mathematics, such as place values, matching fractions, and decimals (see Table 4.16). The group tied all the above to content strands for the fifth-grade Massachusetts Curriculum Framework for Mathematics (Massachusetts Department of Elementary and Secondary Education, 2011), as shown in Table 4.17. For instance, the double number line activity aligns with the content strands of analyzing relationships and interpreting numerical expressions. The multiplication of decimals activity corresponds to understanding place values and performing operations with decimals. The popsicle stick and pizza activities align with using equivalent fractions and analyzing relationships.

Additionally, student group 3 asked questions of the class related to key vocabulary found in the booklet. For example, place value is defined as “the value of a digit in a number, based on the location of the digit” (A1.6). Another vocabulary term is *decimal*, which is the “fractional base-ten equivalents making use of place value” (A1.6). Other key vocabulary terms include *numerator*, *denominator*, *mixed numbers*, *improper fraction*, *common fraction*, and *percent* (A1.6). An understanding of these vocabulary terms is essential for the pre-service teachers so that they, in turn, can explain them to their students in various ways. The vocabulary is critical to interpreting and solving the mathematical problems. Furthermore, this student presentation aligns with the Massachusetts Teacher Standard 7.06.7 subject matter (content) knowledge requirement of numbers and operations (see Table 4.18).

Similarly, in the student group 1 presentation, content knowledge is subject matter understanding of mathematics, such as number sense, zero property, place values, partial sums, and plus 10, as shown in Table 4.19 (N1.4). This group presentation can also be tied to the content strands for the second-grade Massachusetts Curriculum Framework for Mathematics (Massachusetts Department of Elementary and Secondary Education, 2011) of addition and subtraction within 20, representing and solving problems, and working with equal groups of objects (see Table 4.20). The group described and demonstrated how to compare numbers rolling dice, how to use snap cubes to compare odd versus even numbers, and how to use base 10 blocks to illustrate place value, as shown in Figure 4.10 (N1.4).

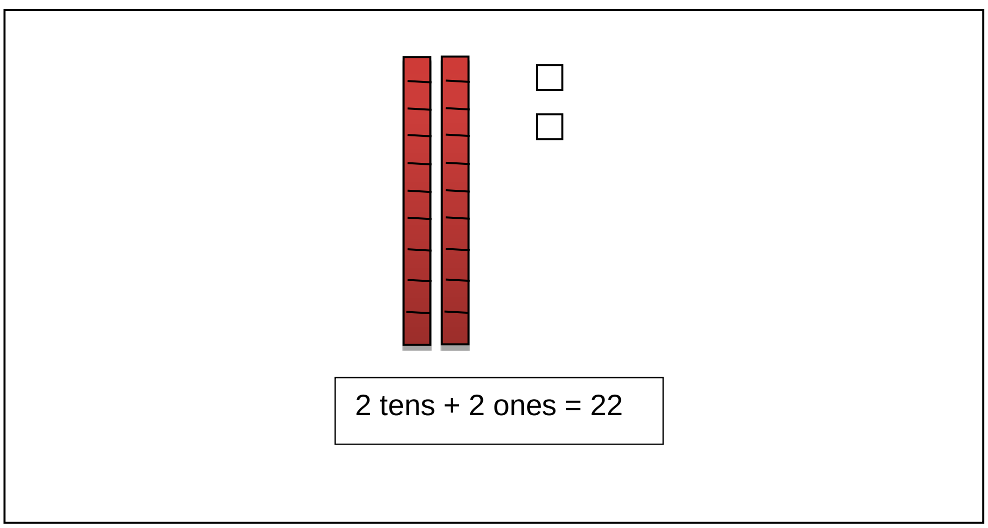


Figure 4.10: Using Base 10 Blocks to Illustrate Place Value

Correspondingly, in the student group 2 presentation, content knowledge is subject matter understanding of mathematics, such as multiplication, distributive property, associative property, and commutative property (see Table 4.21). This group presentation can also be tied to the content strand for the second- and sixth-grade Massachusetts Curriculum Framework for Mathematics (Massachusetts Department of Elementary and Secondary Education, 2011), as shown in Table 4.22. Second-grade content strands include working with equal groups of objects to gain foundations for multiplication, and sixth-grade content strands include computing fluently with multi-digit numbers—that is, numbers greater than nine, and finding common factors (e.g., whole numbers that are multiplied by one another, such as 4 and 3 for the product 12) (A1.3, A1.4, N1.4, and Table 4.22).

Additionally, student group 2 described the distributive property as one in which "each addend inside a set of parenthesis can be multiplied by a factor outside the parenthesis and the products are then added, i.e. A(B +C) = AB + AC" (A1.4). The associative property was described by the group as one if there is a parenthesis, then the grouping of factors that are multiplied is not relevant—that is, "A(BC) = (AB)C" (A1.4). Similarly, the commutative property does not take into account the order of the factors that are multiplied, such as "A x B = B x A" (A1.4). The group also discussed inverse operations—for example, multiplication is the inverse (opposite) operation of division and vice versa (A1.4). Next, division was presented by the group. Key vocabulary, such as *divisor*, *dividend*, and *quotient*, were defined. The divisor is "the number of groups you wish to divide an amount into," dividend is "the amount you are separating into equal groups," and quotient is "the equal amount within each group," also known as the answer (A1.3, A1.4). An understanding of these vocabulary terms is essential for the pre-service teachers so that they, in turn, can explain them to their students in various ways (see Table 4.18). The vocabulary is critical to interpreting and solving the mathematical problems.

4.7.2 Pedagogical Knowledge

Pedagogical knowledge is knowledge of how to teach that fosters student learning (see Table A.2). Student group 3 demonstrated pedagogical knowledge by starting with key vocabulary and progressing to fraction strategies, such as doubling and multiples. Multiple strategies were utilized in a progressive, coherent manner. For example, multiplication of fraction strategies started with explaining the part-whole relationship, followed by repeated addition strategy, and finally showing the array strategy (A1.8). Additionally, the group incorporated various manipulatives that the "students" in the class then worked with to address the tactile needs of some students. The group provided several models and strategies thereby addressing the different learning styles of their peers and their potential students' learning about fractions, decimals, and percents. As a result, the group was able to "share techniques and resources (including web-sites, apps, and manipulatives)" (A1.1) that can be utilized by the other students in the class in their future lesson planning and instruction (see Table 4.23).

Likewise, student group 1 demonstrated pedagogical knowledge by starting their presentation with a basic understanding of number sense and building on that understanding as they progressed to addition strategies, followed by subtraction strategies. Number sense was illustrated with base 10 blocks, shown in Figure 4.10, which demonstrated place value. One of the most popular addition strategies explained by the group is doubles plus or minus—for example, 6 + 7 is rewritten as 6 + 6 + 1 or 7 + 7 – 1—which is easier to solve (N1.4). One of the subtraction strategies explained by the group utilizes the hundreds chart, a chart showing numbers from 1 to 100 in rows of 10. The problem is 94 – 32, so the group made jumps of 10 backward—94 – 10, 94 – 20 (another 10), 94 – 30 (another 10)—and then a jump of 2 backward to arrive at the solution of 62 (N1.4). One student also mentioned that she had used the hundreds chart in a similar manner in her own class.

Correspondingly, student group 2 demonstrated pedagogical knowledge by starting their presentation with essential vocabulary and key points, followed by multiplication vocabulary and division vocabulary. Then multiplication strategies were presented and then progressed to division strategies (A1.3, A1.4). The pre-service teachers understood that students needed to learn and acquire skills in multiplication before moving on to division.

4.7.3 Pedagogical Content Knowledge

Pedagogical content knowledge is knowledge of instructional strategies (i.e., pedagogy) as they relate to specific content, such as mathematics (see Table A.2). Student group 3 presented multiple strategies to explain fractions, decimals, and percents. One fraction strategy, doubling, is a strategy that will preserve the ratio equivalent. For example, each student will have ¾ of a pizza, so how many persons can be in a group in relation to the number of pizzas? (A1.6). The solution looks like this:

3 pizzas , 4 students
6 pizzas, 8 students
(6 + 6) 12 pizzas, 16 students (8 + 8)
(12 + 12) 24 pizzas, 32 students (16 + 16)

Multiples is another fraction strategy that will keep the ratio constant and thereby provide more equivalent fractions. For example, what are the equivalent fractions for ¾? (A1.6). The solution involves finding multiples of 3 for the numerator and multiples of 4 for the denominator: $^3/_4 = {}^6/_8 = {}^{12}/_{16} = {}^{24}/_{36}$. Repeated addition is another fraction strategy presented by the group: $4 \text{ x } {}^1/_8 = {}^1/_8 + {}^1/_8 + {}^1/_8 + {}^1/_8 = {}^4/_8 \text{ or } {}^1/_2$ (A1.6). This was followed by decimal strategies and percent strategies.

Student group 3 presented decimal strategies that utilize the different ways to write a decimal (standard form, expanded form, and word form) as well as place value charts. One such decimal activity is the clothes line activity in which the group "hung up" a variety of decimal numbers with clothes pins on a string (line), and then the students in the class had to "rehang" the decimal numbers in order (A1.6), as shown in Figure 4.11 (N1.8).

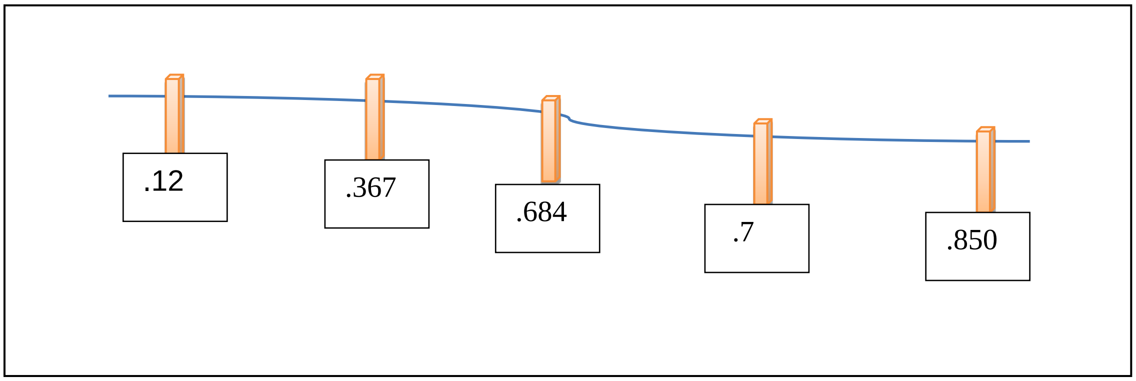


Figure 4.11: Decimals Clothes Line

Then student group 3 presented percent strategies. For example, a percent strategy is to use comfortable percentages to determine more obscure percents. For example, to find 60% of 80, you would start with a percentage that you know—50% of 80, which is 40. Then you can use another familiar percentage—10% of 80, which is 8. Finally, add the two familiar percentages together for a final answer of 48 (A1.8). Once these were understood, the group illustrated some mathematical models and finally posed a challenge problem. One such model is the double number line, which shows the relationship between whole numbers and fractions. For instance, a 24 kilometer race is taking place, and the volunteers need to arrange water stations at the quarter mark, halfway mark, and three-quarter mark (A1.8). The double number line would be used to identify the kilometer marks, as shown in Figure 4.12.

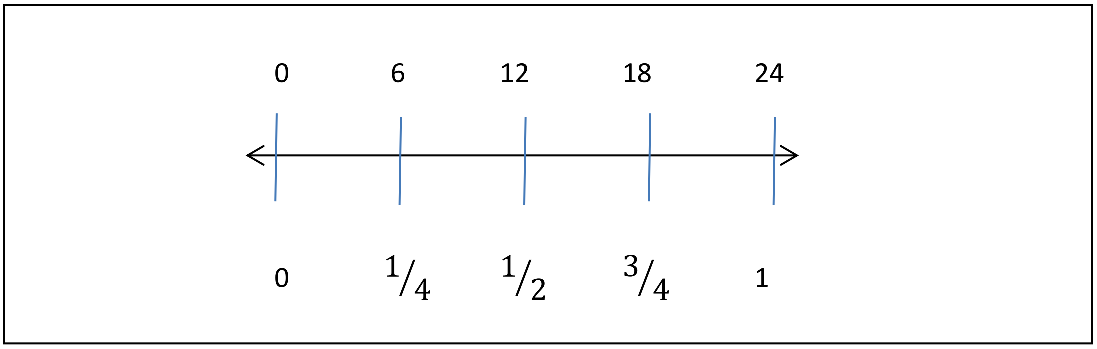


Figure 4.12: Double Number Line Showing Fractions and Whole Numbers

Student group 3 further demonstrated their pedagogical content knowledge via their use of manipulatives and tools, such as place value charts and arrays, to teach mathematical concepts and to address misconceptions about naming decimals (A1.6). For instance, Cuisenaire rods were used to provide concrete representations of fractions. Arrays, another mathematical model, were used to help visualize the multiplication problem. In one such example to multiply $^1/_2$ x 2, the group drew boxes in which each box in the array would represent $^1/_2$ of the total boxes, the two boxes are filled in, and so $^1/_2$ x 2 = 1 (whole) (A1.6).

Likewise, student group 1 demonstrated their pedagogical content knowledge. Addition strategies were presented before subtraction strategies (N1.4). Thus, the group understood that addition operations need to be understood in order for students to progress to subtraction operations. One such addition strategy, the part-part-whole strategy, breaks down the numbers to be added–for example, 53 + 34 can be broken down into 50 + 30 + 3 + 4 parts (N1.4).

Student group 1 further demonstrated their pedagogical content knowledge via their use of such manipulatives and tools as snap cubes, flipping cubes, base 10 blocks, ten frames, and rekenrek to teach mathematical concepts and instructional strategies, such as part-part-whole (as described above), regrouping, and place values. The group demonstrated regrouping via ten frames, shown previously in Figure 4.7, and place value via base 10 blocks, shown previously in Figure 4.10 (N1.4). The group also described how ten frames would be used as a subtraction strategy once students have mastered addition operations.

Similarly, student group 2 demonstrated pedagogical content knowledge. They presented the first multiplication strategy: repeated addition (see Figure 4.8), a strategy appropriate for the introduction of multiplication. This was followed by skip counting and doubling strategies (A1.3, A1.4, N1.5). Skip counting was described by the group as “counting forwards or backwards by a number other than one”—that is, skip counting by 10 shows the pattern of adding a zero: “2 x 10 = 20, 3 x 10 = 30, 40, 50, 60, etc.” (A1.3, A1.4). The presentation of doubling strategy for multiplication by student group 2 was similar to the presentation of doubling in addition by student group 1: “4 + 4 = 8” is the same as “4 x 2 = 8” (A1.3, A1.4). The first division strategy shown by the group was repeated subtraction, a strategy appropriate for the introduction of division. They explained the following example: “12 students at recess are dividing themselves into 3 teams in order to play a game. How many students are on each team? The solution can be found by repeatedly subtracting 3 from 12 until the difference is 0, and then counting how many times the number 3 was subtracted. 12 – 3 = 9, 9 – 3 = 6, 6 – 3 = 3, 3 – 3 = 0. The number 3 was subtracted from 12 4 times . . . 12 ÷ 3 = 4!” (A1.3, A1.4).

Student group 2 further demonstrated their pedagogical content knowledge via their use of such manipulatives and tools as Unifix cubes, shown in Figure 4.6, and arrays to teach mathematical concepts and instructional strategies, such as skip counting, doubling, repeated addition, repeated subtraction, inverse operations, and fact families (N1.5). The group described arrays as a visual representation and illustrated how to use it in the following example: “Mrs. Apple was setting up desks for the first day of school. She lined up her students’ desks in 6 rows. She put 4 desks in each of these rows. How many desks did Mrs. Apple set up?” (A1.3). They presented the following array (see Figure 4.13) as the solution to the problem.

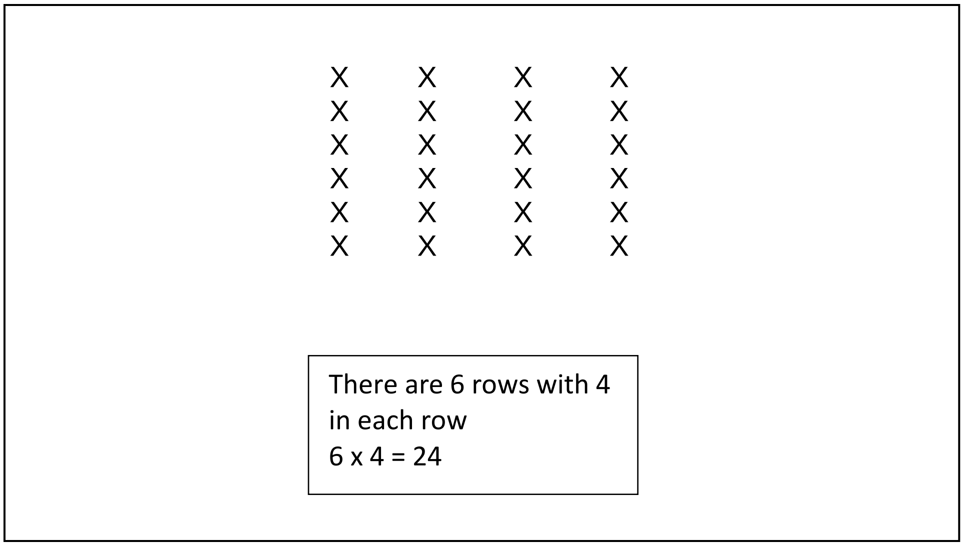


Figure 4.13: Array Used for Multiplication

Student group 2 also described inverse operations and fact families, which show the relationships between multiplication and division. They illustrated the following example with same three numbers: 3, 4, and 12 (A1.3):

$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

$$12 \div 3 = 4$$

$$12 \div 4 = 3$$

The group made a very interesting key point regarding multiplication and division: “important . . . necessary to make connections and understand the relationship between multiplication and division in order to understand both skills” (A1.3).

4.7.4 Technological Knowledge

Technological knowledge is knowledge of using various technologies—for example, a document camera, researching and using such video sites as YouTube as well as DVD videos, and websites/iPad apps (see Table 4.5). In this example, technological knowledge is knowledge of using a document camera and researching and using such video sites such as YouTube and websites/iPad apps. Student group 3 knew how to set up the document camera (connected to a ceiling-mounted projector) to demonstrate their activities, such as the popsicle sticks, and to project them on the whiteboard for the whole class. Student group 3 researched video sites using search engines and/or search functions to find mathematical instructional videos at the elementary school level, websites, and iPad apps. In this case, the instructor assisted the student group to use Apple TV and AirPlay to have the app on the student iPad display on the whiteboard (N1.8). The instructor connected Apple TV to the projector at the beginning of class to display her iPad on the whiteboard. Then she configured the student's iPad to enable AirPlay, which then connected that iPad to the wireless network.

Likewise, student group 1 knew how to set up the document camera (connected to a ceiling-mounted projector) to demonstrate their activities, such as the ten frames and base 10 blocks, and to project them on the whiteboard for the whole class. Student group 1 researched video sites using search engines and/or search functions to find mathematical instructional videos at the elementary school level, websites, and iPad apps. Similarly, the instructor assisted the student group to use Apple TV and AirPlay to have the app on the student iPad display on the whiteboard and connect to the wireless network (N1.4).

In the same manner, student group 2 knew how to set up the document camera (connected to a ceiling-mounted projector) to demonstrate their activities, such as the pizza problem with Unifix cubes, and to project them on the whiteboard for the whole class. Student group 2 researched video sites using search engines and/or search functions to find mathematical instructional videos at the elementary school level, websites, and iPad apps. In this case, the student group knew how to use Apple TV and AirPlay to display the app on the student iPad on the whiteboard (N1.5).

4.7.5 Technological Pedagogical Content Knowledge (TPACK)

TPACK is knowledge of instructional strategies (PK) that integrate technologies (TK) to constructively teach content (PCK), in this case mathematics content (CK), for greater understanding of that content (see Table A.2). Thus, TPACK incorporates instructional strategies that use technologies constructively and appropriately to teach mathematics (see Table A.2). Student group 3 used technologies, including videos, manipulatives, and a document camera, appropriately to enhance and support the teaching and learning of mathematics. Specifically, TPACK, in the student group 3 presentation is the knowledge of instructional strategies that use such technologies as a YouTube video to explain how to find percents using a double number line, thus building on previous knowledge of a single number line (see Figure 4.14). Another YouTube video was shown addressing misconceptions naming decimals. A website/iPad app, such as ShowMe, was utilized to describe how to use arrays (i.e., mathematical models) to multiply decimals and to visualize the multiplication problem. A document camera was used to demonstrate place value (see Table 4.16). Additionally, the document camera was utilized to explain more detailed fraction strategies and fraction models (see Figure 4.15).

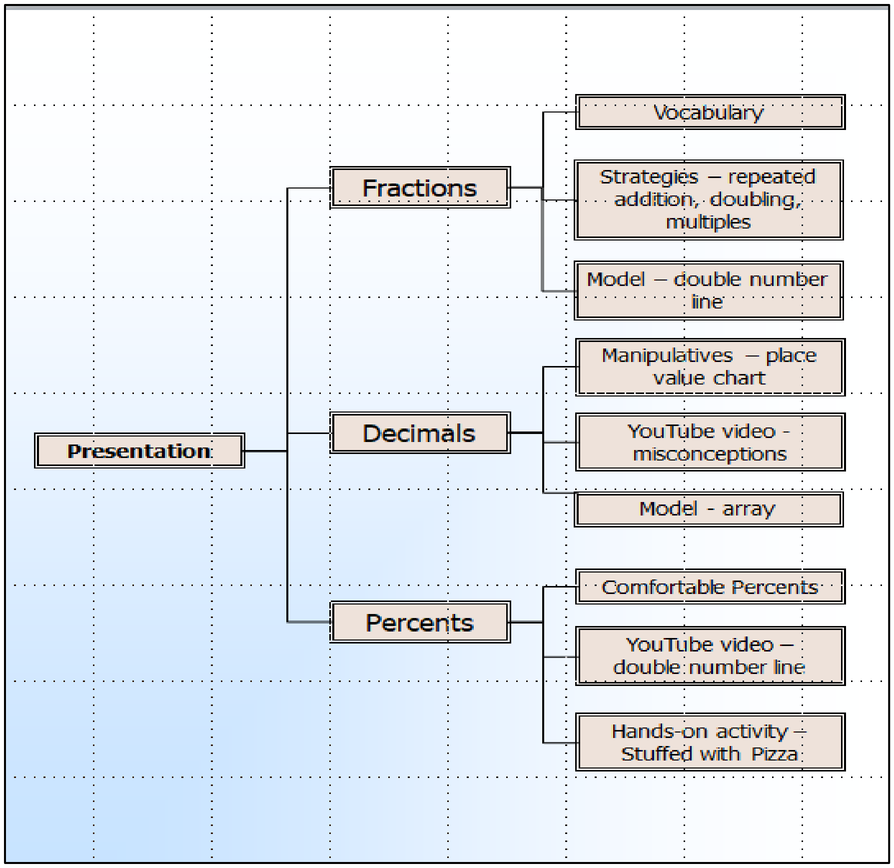


Figure 4.14: Flowchart of Student Group 3 Presentation

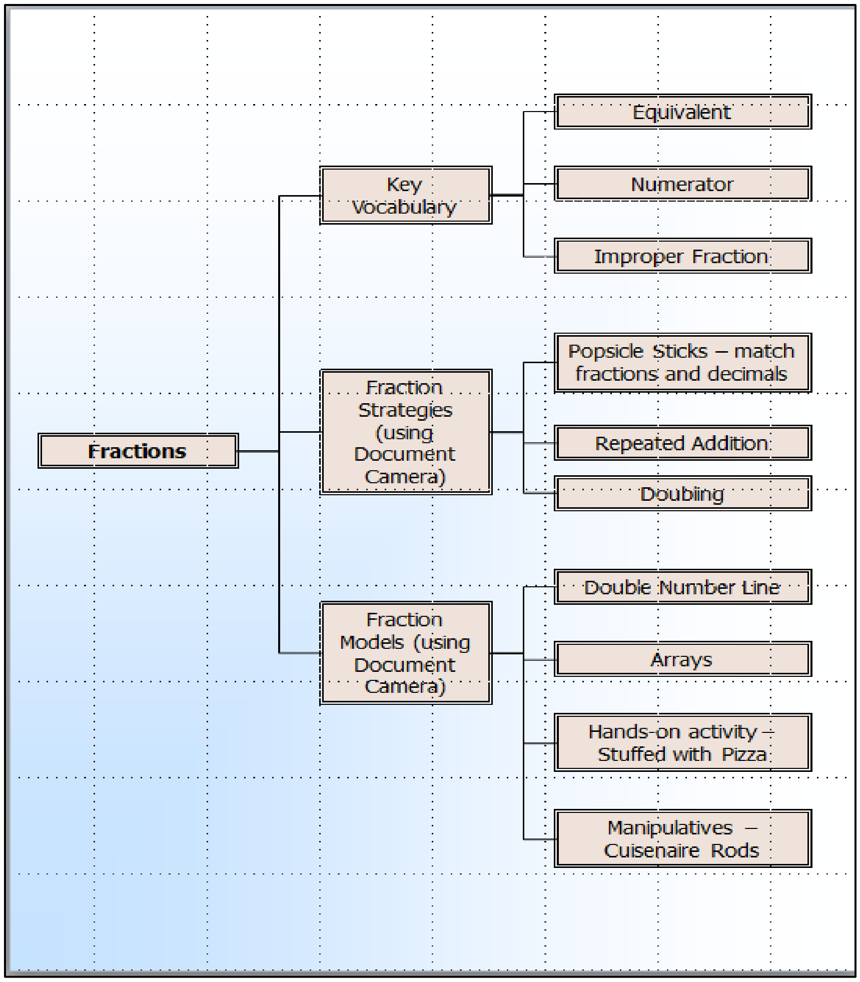


Figure 4.15: Flowchart of Student Group 3 Fractions Detailed Presentation

Accordingly, student group 3 used technologies, such as manipulatives, including physical ones shown on the document camera or in the instructional video and virtual ones used in an iPad app, videos, and document camera, appropriately to enhance and support the teaching and learning of mathematics.

Correspondingly, student group 1 used such technologies as videos, manipulatives, and a document camera appropriately to enhance and support the teaching and learning of mathematics. Specifically, TPACK, in the student group 1 presentation (see Figure 4.16), is knowledge of instructional strategies that use such technologies as YouTube videos and a rekenrek to elucidate fluency to 20—that is, building the conceptual understanding of numbers.

Another YouTube video was used to explain doubling as an addition strategy. A document camera was utilized to demonstrate regrouping via ten frames—that is, modeling of regrouping with nondigital manipulatives (see Tables A.2 and 4.19). Furthermore, student group 1 used technologies, such as manipulatives, including physical ones shown on the document camera or in the instructional video and virtual ones used in iPad apps, computers, and the Internet, appropriately to enhance and support the teaching and learning of mathematics.

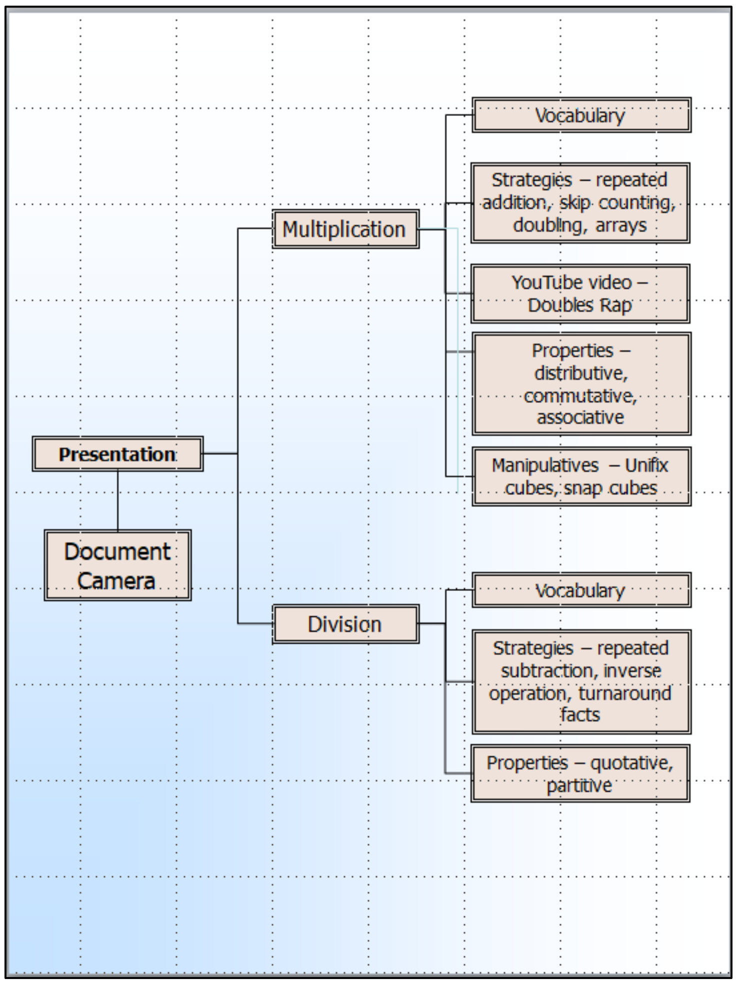


Figure 4.16: Flowchart of Student Group 1 Presentation

Likewise, student group 2 used technologies, such as videos, manipulatives, and a document camera, appropriately to enhance and support the teaching and learning of mathematics. Specifically, TPACK, in the student group 2 presentation, is knowledge of instructional strategies that use such technologies as YouTube videos to explain associative and distributive properties of multiplication, as shown in Figure 4.17. A document camera was used to demonstrate repeated addition—that is, multiplication strategy building on prior knowledge of addition. Additionally, the document camera was used to explain doubling (i.e., multiplication strategy, using physical such manipulatives as Unifix cubes and snap cubes) (see Tables A.2 and 4.21). Furthermore, multiplication strategies were presented before division strategies (i.e., understanding of multiplication facilitates an understanding of division). Student group 2 used technologies, such as manipulatives, including physical ones shown on the document camera or in the instructional video and virtual ones used in iPad apps, videos, and a document camera, appropriately to enhance and support the teaching and learning of mathematics.

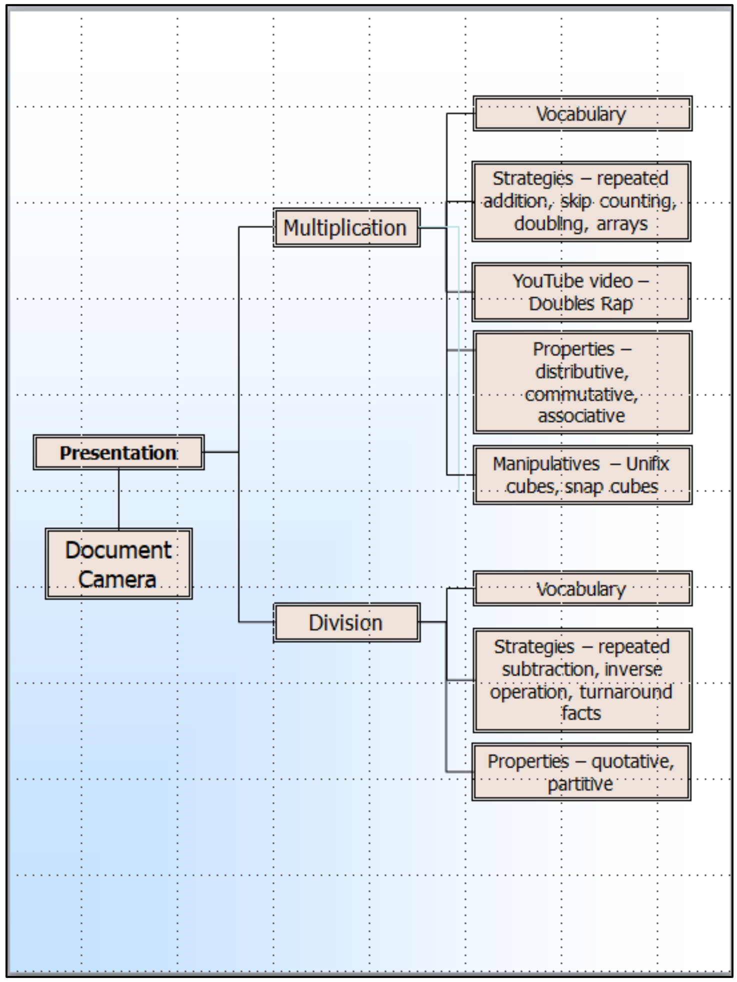


Figure 4.17: Flowchart of Student Group 2 Presentation

4.8 Summary

In the math methods course, the instructor used instructional technology for enhancing teaching and learning of the pre-service teachers. She used and modeled these instructional technologies: videos (instructional), iPad, websites linked to the CCSS and math curriculum frameworks, PowerPoints, and the document camera. The instructor demonstrated effective teaching with technology, pedagogical strategies that use technologies in constructive ways to teach math content at the elementary level, how technology can be used to redress some of the difficulties students have learning mathematics, and how technology can be used to build on prior knowledge (see Table A.2). In other words, the instructor demonstrated and exemplified the elements of TPACK in action. Assignments in the math methods course were designed to provide opportunities for the pre-service teachers to practice, experience, and use instructional technologies to effectively teach mathematics. The pre-service teachers understood how the instructor's modeling was valuable to their learning and would have a positive effect on their future teaching of mathematics and their students' learning.

The pre-service teachers used instructional technologies, including videos (instructional), Google Docs, iPad apps/websites, and the document camera, in various assignments and throughout the math methods course. Exemplar teaching video technology generally was a YouTube video. Google Docs enabled collaboration in the math skills group assignment as well as in the technology assignment. These websites and apps provided different models/tools for the pre-service teachers to use to teach the mathematical concepts to their current as well as to their future students. The document camera, along with manipulatives, was another instructional technology used by the pre-service teachers. Often the pre-service teachers saw a direct link between their learning now and their future teaching with technology to have a positive effect on their students' learning of mathematics.

Table 4.1: Summary of Findings/Domains of Interest for Site 1

	Instructor	Students	Observations	Artifacts
Videos	x	x	x	x
Websites	x	x	x	x
Google Docs	x	x	x	x
iPad Apps	x	x	x	x
Document Camera	x	x	x	
MS Publisher		x		x
Other: Projector with Apple TV/iPad	x	x	x	

Note. X represents the presence of the instructional technology.

Table 4.2: Technology and Meeting the Massachusetts Standards for Mathematical Practice for Site 1

	1. Make sense of problems and persevere in solving them.	2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics.	5. Use appropriate tools strategically.	6. Attend to precision.	7. Look for and make use of structure.	8. Look for and express regularity in repeated reasoning.
Videos	x	x	x	x	x	x	x	x
Websites	x	x	x	x	x	x	x	x
Google Docs					x			
iPad Apps	x	x	x	x	x	x	x	x
Document Camera	x	x	x	x	x		x	x
MS Publisher					x			
Other: Projector with Apple TV/iPad	x	x	x	x	x		x	x

Source Massachusetts Department of Elementary and Secondary Education (2011)..

Table 4.3: Instructor Using Technology from Observations for Site 1

	O1: November 18	O2: November 25	O3: December 2	O4: December 9
Videos			Sharma DVD	
Websites				List of resources
Google Docs				Excel: List of websites/apps
iPad Apps	Number Line app			List of resources
Document Camera	Unifix cubes	Story Problems – Partitive and Quotative Properties	Tally Marks, Cuisenaire Rods, Paper Strips (like the DVD)	Handshake problem, 5-Vertices problem
Other: Projector with Apple TV/iPad	Agenda PowerPoints	Agenda PowerPoints	Agenda PowerPoints	Agenda PowerPoints

Table 4.4: Frequency of Instructor Using Technology from Observations for Site 1

	O1: November 18	O2: November 25	O3: December 2	O4: December 9
Videos			2X	
Websites				List of resources
Google Docs				Excel: List of websites/apps
iPad Apps	X			List of resources
Document Camera	X	5X	4X	2X
Other: Projector with Apple TV/iPad	X	X	X	X

Note. X represents the frequency of the technology used.

Table 4.5: Technology Use in Student Presentations

	Student Group 1: Number Sense, Addition, and Subtraction	**Student Group 2: Multiplication and Division**	**Student Group 3: Fractions, Decimals, and Percents**
Videos	“Rekenrek Kindergarten” (fluency up to 20): https://www.youtube.com/watch?v=R4m6soJDVq8 “Double Baby Rap 2nd Grade”: https://www.youtube.com/watch?v=jG6n5xLt-0Q “Flocabulary—Addition and Subtraction—Know About 10s”: https://www.youtube.com/watch?v=Zl-Yvs-0dU8&list=RDZl-Yvs-0dU8	“Doubles Rap Baby 2nd Grade”: https://www.youtube.com/watch?v=jG6n5xLt-0Q Pre-Algebra7—Associative and Distributive Properties of Multiplication”: https://www.youtube.com/watch?v=CkVJ8xa63ow	“Finding a Percent of a Number Using a Double Number Line”: https://www.youtube.com/watch?v=SE7AcLdU8NE&noredirect=1 “Multiplying Decimals Using an Array”: https://www.showme.com/sh/?h=fxpU5Q (Premium Access) “Misconceptions Naming Decimals”: https://www.youtube.com/watch?v=1z_zkfzCDoM
iPad Apps	YouTube also available as an app	YouTube also available as an app	YouTube and ShowMe interactive whiteboard also available as apps
Document Camera	Demonstrated addition and subtraction using snap cubes, base 10 blocks, rekenrek, ten frame	Demonstrated using Unifix cubes	Demonstrated using place value chart Demonstrated matching fractions and decimals with popsicle sticks
Google Docs	Collaboration	Collaboration	Collaboration
MS Publisher	Create booklet	Create booklet	Create booklet
Projector with Apple TV/iPad	YouTube videos	YouTube videos	YouTube and ShowMe videos

Table 4.6: Frequency of Technology Use in Student Presentations

	Student Group 1: Number Sense, Addition, and Subtraction	**Student Group 2: Multiplication and Division**	**Student Group 3: Fractions, Decimals, and Percents**
Videos	3X	2X	3X
iPad Apps	X	X	X
Document Camera	4X	X	2X
Google Docs*	X	X	X
MS Publisher	X	X	X
YouTube videos	4X	3X	4X

Note. X represents the frequency of the technology used.

* Frequency most likely more and varied but exact number uncertain.

Table 4.7: Websites in Technology Assignment

Website Address	Description	Features
http://www.hoodamath.com/	Math games, quizzes, and puzzles	Grade level or math subject
http://www.multiplication.com/games/all-games	Multiplication games; can be adjusted to easy, medium, or hard	Multiplication facts, times tables, videos
http://www.softschools.com/math/games/	Math games and online practice	Operations, estimation, number sense, place value, decimals, fractions
http://www.math-play.com/	Math games online	Grade level, content, game type
http://www.studysmart.com/math-facts-apps.html	Learn math facts. Practice session video for fast facts. Audio and regular flashcards.	Math facts, four operations
https://www.splashmath.com/ (Also app)	An interactive math practice website at every grade level. Aligned with the CCSS.	Addition, subtraction, mixed operations, money, geometry
http://www.mathplayground.com	Math games of all levels and concepts. Movies and manipulatives. Aligned with the CCSS.	Grade level or math topic, logic, manipulatives, video
http://interactivesites.weebly.com/math.html	Covers all subjects, including holiday games and activities. Categorized by topic. Provides teacher tools, very user-friendly.	Math tools, such as number lines. Math topics
http://www.kenkenpuzzle.com/play_now	Create math puzzles for addition, subtraction, multiplication, and division. Choose dimensions and difficulty level. Helps a lot with math facts. *I gave this to my fifth graders and they loved it.*	Addition, subtraction, multiplication, and division.
http://www.mathsisfun.com/puzzles/	All grades up to 12th grade. Explains concepts and gives examples. FREE printable worksheets!	Math and logic puzzles, math dictionary
http://www.aaaknow.com/lessonFull.php?slug=addObjects1&menu=Addition	Students can choose their topic and quiz themselves and monitor their progress while practicing their math skills.	Math lessons by grade or topic
http://nlvm.usu.edu/en/nav/vlibrary.html	National Library of Virtual Manipulatives. Choose content and work with virtual manipulatives within that domain.	Virtual manipulatives by content topic/grade level

Continued on next page

Table 4.7 continued

http://www.cut-the-knot.org/Curriculum/index.shtml	Variety of different "quizzes" for students to check their understanding. Organized by grade level and standards.	Arithmetic, geometric fallacies, puzzles and games; over 1,500 math activities
http://www.starfall.com/	Pre–K to second grade, special education, and English language learners build number sense, practice basic math operations, geometry, and measurement.	Number sense, math operation, shapes, time
http://www.funbrain.com/brain/MathBrain/MathBrain.html	Twenty-five math games grades K–8. Students enter gender and skill level/grade.	Arithmetic operations, fractions
http://www.arcademics.com/ (Also an app)	Grades 1–6. Counting, shapes, operations, money, time, decimals, fractions, ratios and proportions. Multiplayer math games (ideal for a classroom). Track progress. Control content.	Arithmetic operations, money, time, fractions, ratios, proportions
http://www.sumdog.com/en/teachers/	Teachers/parents have full control over the games. Track progress/proficiency charts (individually or whole class). Aligned with the CCSS.	Adaptive learning, skills by grade level K–8
http://www.mathnook.com/stations/stations.html	Math games for grades PreK–8. Math worksheets (20,000+). Ideas/tools/tutorials for math centers/stations for teachers.	Grade level for math centers/stations
http://www.coolmath4kids.com	Fun math games and challenges, lessons, quizzes, manipulatives.	Operations, fractions, manipulatives. grade level
http://www.mathcats.com	Open-ended explorations of math; interesting ideas to get students thinking. Problem solving. Not game oriented.	Problem solving. Story problems. Open-ended concepts in math.
http://www.figurethis.org	Many challenges for children, printable. Parents' corner gives advice on how to talk to teachers and how to help at home. Teachers' resource lessons and ideas for classroom.	Algebra, geometry, measurement, numbers, statistics, probability
http://www.mathforum.org (part of NCTM)	A community of people learning about math. Problems of the day, ask Dr. Math, and other math ideas/problems. Resources for students, parents, and teachers.	Grade level, math tools, problems and puzzles, math tips and tricks

Continued on next page

Table 4.7 continued

http://illuminations.nctm.org/coinbox/#AC	Interactive online tool Grades 2–5. Experiment with/manipulate coins to practice counting. Collecting, exchanging, and making change modes to extend student thinking/learning. Lessons for money, grouping, part-whole or whole-part thinking strategies.	Counting, money, grouping, part-whole, whole-part
http://www.mathplayground.com/visual_fractions.html	Online math manipulative tool to visualize and explore equivalent fractions. Particularly useful for students who have a hard time thinking abstractly with fractions; create and change representation of fractions live on screen. Number line as a tool for part-whole thinking and LCD.	Equivalent fractions, number, part-whole, LCD (least common denominator)
http://www.sheppardsoftware.com/mathgames/mahjong/mahjongMath_addition_easy.htm	Interactive online game for reviewing addition facts. Like the traditional mahjong game. Use a fun review game, individually, whole class if projected.	Addition; site also has operations, fractions, money, place value, audio tutorials; PreK–8
http://Fastmath.com	Math fact fluency/automaticity in addition, subtraction, multiplication, and division. Aligned with the CCSS. Track progress to master math facts.	Adaptive instruction for math fluency and automaticity; Grades 2–9
http://STmath.com	Awesome interactive site for math comprehension and proficiency through visual learning. Aligned with the CCSS. Game based. Trial/error method allows for deeper mathematical thinking, conceptual understanding, and problem-solving skills.	Visually represent math concepts for problem solving; Grades K–12
http://Cliffsnotes.com/math	Homework help. Great advice and review for fourth to sixth graders. Numerous examples and explanations for completing at-home assignments.	Basic math to pre-algebra study guides, word problems
http://www.multiplication.com/games/play/fish-shop-warehouse-common-core	Multiplication is repeated addition. Aligned with the CCSS.	Multiplication

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Table 4.7 continued

http://www.kidsmathgamesonline.com/money/moneycounting.html	Pocket change. Different coin combinations that add to different amounts of money.	Addition, money
http://media.abcya.com/games/math_bingo/flash/math_bingo.swf	Math bingo. Addition, subtraction, multiplication, and division. Like traditional bingo.	Math operations
http://www.missmaggie.org/scholastic/roundtheworld_eng_launcher.html	Around the World in 80 Seconds: addition, subtraction, multiplication, and division. Correct answer moves to a different place on Earth. Goal is to make it around the word in 80 seconds or less, beat your time.	Math operations
http://www.ixl.com/math	Any grade level/any mathematical skill. Practice word problems or sets of individual problems. Great for feedback to students/teacher.	Grade level PreK–12 or topic; real-world scenarios
http://www.onlinemathlearning.com	Choose skills. Variety of videos/songs to learn math skills.	Grade level or topic; video lessons
http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html	Any tool to solve a problem. Use on a Smart Board to display different methods to solve problem or individually on iPads.	Grade level PreK–8 and manipulatives
http://www.hwtears.com/kwt	"Wet, Dry, Try." Math games/activities for individual play or for entire class.	Numbers and manipulatives
http://Xtramath.org (Also app)	In-school math quizzes and races.	Operations
http://www.mobymax.com	Online math quizzes and games, incremental building of math skills	Fact fluency, number sense
http://www.brainpop.com	Math movies explaining different mathematic topics.	Numbers, operations, practical math, geometry, measurement, movies
http://www.cobbk12.org/sites/literacy/math/math2.htm	Categories from number sense to data analysis. Activities to practice given skill.	Numbers, operations, fractions, money, measurements
http://mrnussbaum.com/mathcode/ (Also app)	Great resource. Many games, suggestions for apps, and videos.	Skill and drill, math tools, numbers, operations
http://pbskids.org/cyberchase/math-games/	Different skills. Practice in games, including doubling, tangrams, fractions, probability, and percents.	Number sense, fractions, percents. math tools

Note. Adapted from the instructor's Google Docs spreadsheet.

Table 4.8: iPad Apps in Technology Assignment

iPad App	Description	Features
Khan Academy (and website: https://www.khanacademy.org)	Grade levels higher than third grade. Step-by-step videos. Visuals to reinforce learning.	Grade level or topic, videos and virtual manipulatives
Times Tables app	Times tables application that is engaging and works on fluency and accuracy.	Times tables
Squeebles Addition and Subtraction app	Practicing addition and subtraction skills.	Addition and subtraction
Squeebles Math Bingo app	Math bingo to practice addition, subtraction, multiplication, and division. Like traditional bingo game but motivates to earn ice-cream ingredients to create a unique ice cream.	Addition, subtraction, multiplication, division
Math Bingo app	Interactive bingo games for addition, subtraction, multiplication, and division. Answer math problems to get bingo. Different difficulty levels, which is great for differentiation.	Addition, subtraction, multiplication, division
Slice-it app	Students use their fine-motor skills to “slice” or divide various geometric shapes into specific parts. Practice coordination, division, symmetry, area, and measurement with increasing levels of difficulty. Highly interactive, very visually friendly.	Shapes, symmetry, Measurement, division
Sylvan Play Apps	Math facts: adding and subtracting (bunny math), fractions (pizza party), and equator (partner game).	Addition, subtraction, fractions
Digi-make app	Fun way to visualize making 10 using a number’s complementary. Clear, colorful visual representation of each number and its sum. Encourages early mental math strategies if used consistently (maybe once a week).	Number sense

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Table 4.8 continued

Amazing Coin app	iPad application that goes over the names and look of coins.	Money
First Grade Learning Games app	Specifically patterns and ordering (more options for purchase).	Patterns
Animal Math app	Greater, less, equal, number sentences, subtraction (more for purchase).	Number sense, subtraction
My Math app	Different levels. Addition, subtraction, multiplication, and division flash cards. Solve in 30 seconds. Many different settings to help fit to any child's level.	Addition, subtraction, multiplication, division
Number Line app	Work with the number line to create addition and subtraction problems. Bar models to show the number of jumps made on number line (good way to make fact families).	Number line, addition, subtraction, bar models, fact families
Common Core Math for Teachers Front Row app	Aligned with the CCSS. Work at improving math skills. Students begin by taking a test to see what level they are at. The teacher receives the information and is able to better plan instruction for the class as a whole.	Math facts

Note. Adapted from the instructor's Google Docs spreadsheet.

Table 4.9: Instructor and TPACK

	O1: November 18	**O2: November 25**	**O3: December 2**	**O4: December 9**
Content Knowledge (CK)	Multiplication, division, addition, subtraction, numeracy	Fractions, decimals, percents	Fractions perimeter, area, geometry, place value	Graph theory Variables
Pedagogical Knowledge (PK)	Assessment of student group presentation	Story problems Exit tickets	Number line Sheep and ducks activities/strategies with rubric assessment	Bloom's taxonomy Parting thoughts Problem solving
Technological Knowledge (TK)	iPad apps Apple TV/projector Document camera	Document camera Apple TV/projector	Videos Document camera Apple TV/projector	Document camera Apple TV/projector Google Docs Websites
Pedagogical Content Knowledge (PCK)	Assessment leading to corrections/re-presentation	Strategies and models to teach fractions Manipulatives	Sheep and ducks: areas of difficulty and successful student strategies Number line: less or closer to 1 (magnitude) Clothes line problem	Handshake problem Strategies Konigsberg Bridge Problem strategies Real-world problems
Technological Pedagogical and Content Knowledge (TPACK)	Modeling using iPad, document camera	Document camera: using mathematical models and manipulatives Different learning styles	Exemplar teaching on DVD: mathematical models and misconceptions Document camera: Cuisenaire rods reinforce DVD Different learning styles	Google Docs: sharing resources Websites: using mathematical models and manipulatives Different learning styles

Table 4.10: Instructor and TPACK Summary

Content Knowledge (CK)	**Pedagogical Knowledge (PK)**	**Technological Knowledge (TK)**	**Pedagogical Content Knowledge (PCK)**	**Technological Pedagogical and Content Knowledge (TPACK)**
Fractions, decimals, and percents Numeracy Operations: addition, subtraction, multiplication, division Geometry: area, perimeter Graph theory	Development of syllabus, assignments, and assessment strategies High-quality and coherent instruction Instructional strategies that support learning and growth of all students, including visual learners Bloom's taxonomy Problem solving	DVD videos YouTube videos iPad apps Document camera Google Docs Websites: resources, activities, sample lessons Apple TV	Multiple strategies to teach a math concept like fractions Place value charts Arrays to teach decimals, multiplication Mathematical models to teach operations, fractions, and geometry Mathematical manipulatives to teach numeracy, fractions, operations, and geometry Understanding misconceptions	DVD of exemplar teaching and misconceptions Websites and instructional videos: different learning styles Modeling using iPad, document camera, websites Videos showing methods of teaching certain math concepts Videos , websites, apps: using mathematical models and manipulatives Document camera: mathematical models and manipulatives

Table 4.11: Site 1 Instructor and TSAT Standard 3 – Early Technology Mastery Level

	Technology Skill	Site 1 Instructor	Site 1 Data Sources
A3.1	Discuss current best practices on teaching and learning with technology in order to plan rich learning environments and experiences.	√	N1.3, N1.7, N1.9, N1.10
A3.2	Use technology to gather curriculum-specific information from online and/or local digital sources.	√	A1.12, N1.7, N1.9, N1.10
A3.3	Integrate technology into the curriculum of one's subject and/or grade level with assistance from a coach, mentor, or other staff member.	√	A1.12, N1.4, N1.5, N1.7, N1.8
A3.4	Use digital and online tools to communicate with teachers, parents, and other stakeholders and to create/distribute classroom materials.	√	A1.1, A1.12, N1.2
A3.5	Identify your personal technology professional development needs.	N/A	N/A
		100%	

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

N/A = not discussed in interviews.

√ represents the presence of the technology skill.

Table 4.12: Site 1 Instructor and TSAT Standard 3 – Developing Mastery Level

	Technology Skill	Site 1 Instructor	Site 1 Data Sources
B3.1	Design and develop lessons and activities that integrate technology into a variety of instructional settings for all students.	√	A1.1, A1.12, N1.4, N1.5, N1.7, N1.8, N1.10
B3.2	Use appropriate technology to differentiate instruction (e.g., multimedia presentations, concept maps) for all learners.	√	N1.4, N1.5, N1.9, N1.10
B3.3	Identify and locate technology resources, including online curriculum resources (Massachusetts Curriculum Frameworks and/or district curriculum guides), for planning.	√	A1.1, A1.12, N1.10
B3.4	Manage student technology activities to optimize learning with available resources (e.g., in a one-computer classroom, a computer lab, or with portable/wireless technology).	√	N1.4, N1.5, N1.7, N1.8, N1.9, N1.10
B3.5	Use applications (spreadsheets, databases, etc.) to organize curriculum-specific information into charts, tables, and diagrams.	√	A1.1, A1.12, N1.7
B3.6	Create multimedia presentations to communicate curriculum content.	√	N1.4, N1.5, N1.9, N1.10
B3.7	Integrate electronic research results into classroom instruction with proper citations as appropriate to the grade level.	√	A1.1, N1.4, N1.5, N1.9, N1.10
B3.8	Locate and participate in appropriate technology professional development activities offered by the district, local college/university, or online provider.	N/A	N/A
		100%	

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

N/A = not discussed in interviews.

√ represents the presence of the technology skill.

Table 4.13: Site 1 Instructor and TSAT Standard 3 – Proficient Mastery Level

	Technology Skill	Site 1 Instructor	Site 1 Data Sources
C3.1	Plan for the management of technology resources within the context of learning activities (e.g., schedule use of computer lab, wireless laptops, whiteboard).	√	N1.4, N1.5, N1.9, N1.10
C3.2	Evaluate technology resources, including online resources for accuracy and suitability, for your curriculum area and the students you teach.	√	A1.12, N1.4, N1.5, N1.9, N1.10
C3.3	Identify and discuss the technology proficiencies needed in the workplace, as well as strategies for acquiring these proficiencies.	√	N1.4, N1.5, N1.9, N1.10
C3.4	Use appropriate technology tools to enhance your curriculum (e.g., digital projectors, wireless laptops, handhelds, environmental probes).	√	N1.4, N1.5, N1.9, N1.10
C3.5	Facilitate technology-enhanced lessons that address content standards and student technology literacy standards while addressing a variety of learning styles.	√	N1.4, N1.5, N1.9, N1.10
C3.6	Use technology resources to collect and analyze data, interpret results, and communicate findings to improve instructional practice and maximize student learning.	√	A1.12, N1.4, N1.5, N1.9, N1.10,
C3.7	Identify and evaluate developing technologies as they relate to your subject area, grade level, and student population.	√	A1.12, N1.7, N1.10
C3.8	Assess student learning using a variety of district, school, or individual technology tools and strategies (e.g., the state Data Warehouse, progress spreadsheets, or commercial gradebook applications).	√	N1.4, N1.4, N1.7, N1.8
C3.9	Provide assistance to colleagues in using multimedia presentations, WebQuests, and other technology-rich lessons in the classroom.	√	N1.4, N1.5, N1.9, N1.10
C3.10	Manipulate data using charting tools and graphic organizers (e.g., concept mapping, outlining software) to connect ideas and organize information.		BLANK
C3.11	Use electronic communication tools (e.g., message boards, email, virtual classrooms) to enhance teaching and learning.	√	A1.1, A1.12, N1.2
C3.12	Use the Internet to network with other teachers and learn about effective use of technology in teaching your subject(s).		BLANK

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Table 4.13: continued

C3.13	Explain and correctly use terms related to online learning (e.g., upload, download, forum, journal, post, thread, intranet, drop box, account).	√	N1.4, N1.5, N1.9, N1.10
C3.14	Facilitate student use of online tools (e.g., blogs, wikis, message boards) to gather and share information collaboratively.	√	A1.12, N1.4, N1.5, N1.7, N1.9, N1.10
		85%	

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

√ represents the presence of the technology skill.

Table 4.14: Site 1 Instructor and TSAT Standard 3 – Advanced Mastery Level

	Technology Skill	Site 1 Instructor	Site 1 Data Sources
D3.1	Routinely and rigorously identify, evaluate, and apply emerging technologies as they relate to teaching and learning.	√	N1.4, N1.5, N1.9, N1.10
D3.2	Use specialized technology tools for problem solving, decision making, and creativity (e.g., simulation software, geographic information systems, dynamic geometric software, art and music composition software).		
D3.3	Develop tools and online content (e.g., webpages, blogs, wikis, mailing lists) for instruction and communication among students and faculty.	√	A1.12, N1.7
D3.4	Use technology (e.g., applets that require the use of logic to solve problems) to challenge students to develop higher-order thinking skills and creativity.		
D3.5	Plan and implement collaborative projects with other classrooms or schools using interactive tools (e.g., email, discussion forums, groupware, interactive websites, VoIP, videoconferencing).		
D3.6	Present ideas using the most appropriate communications technologies (e.g., multimedia presentations, webpages, desktop-published documents).	√	A1.12, N1.4, N1.5, N1.9, N1.10
D3.7	Distinguish between effective and ineffective design and presentation in electronic format (e.g., websites, multimedia, charts).	√	A1.3, A1.4, A1.6, N1.4, N1.5, N1.9, N1.10
D3.8	Explain and demonstrate the use of metadata (e.g., tagging, EXIF) to help students and teachers organize information on their computers and/or the Internet.		
D3.9	Design and deliver effective staff development in technology and its integration into the curriculum.	√	N1.4, N1.5, N1.9, N1.10
		56%	

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

√ represents the presence of the technology skill.

Table 4.15 CK and TPACK in Websites/Apps in Individual Student Presentations

Technology	Websites	iPad Apps	Content Knowledge	TPACK
Amanda	http://www.hoodamath.com/		Grade level or math subject	Aligned with the CCSS; Used in her second-grade class
Barbara		https://itunes.apple.com/us/app/squeebles-maths-bingo/id580882257?mt=8	Addition, subtraction, multiplication, division	Examples for fourth and fifth grades; Individually or with partners
Crystal	http://www.mathplayground.com		Problem solving, grade level/topic	Aligned with the CCSS; Used in her second-grade class
Debra	http://nlvm.usu.edu/en/nav/vlibrary.html		Grade level	Virtual manipulatives
Eva	http://www.starfall.com/		Numbers, shapes, time, math operations	Used in her class with special education students; aligned with the CCSS
Frances	http://www.mathcats.com		Problem solving	Exploring open-ended concepts in math
Gloria		https://www.graphite.org/app/slice-it	Shapes, symmetry, measurement Fine motor skills	Aligned with the CCSS; wants to try in her own class
Hannah	http://www.mobymax.com		Incremental building of math skills;	Upload own curriculum; Used in her school
Irene		My Math app	Addition, building automaticity	Used in her first-grade math center
Joanna	http://www.cobbk12.org/sites/literacy/math/math2.htm		Number sense, place value, measurement, fractions,	Used in her school; higher-order thinking

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Table 4.15 continued

Kelly	http://www.hwteears.com/kwt		Writing numbers, manipulatives	Aligned with the CCSS; used in her kindergarten class
Linda	http://www.tenmarks.com		Problem solving	Used in her third-grade class
Margaret	http://www.fun4thebrain.com/addition/coneFlurryAdd.swf		Addition, subtraction, fact families, fluency	Can customize

Table 4.16: Student Group 3 Presentation

Content Knowledge (CK)	Pedagogical Knowledge (PK)	Technological Knowledge (TK)	Pedagogical Content Knowledge (PCK)	Technological Pedagogical and Content Knowledge (TPACK)
Fractions, decimals, and percents Massachusetts Content Strands: patterns and relationship, numerical expressions, place value, operations, equivalent fractions (see Table 4.17)	Lesson planning High-quality and coherent instruction Instructional strategies (i.e., hands-on activities) that support learning and growth of all students Curriculum development that addresses different learning styles	Videos YouTube videos iPad apps Document camera MS Publisher Google Docs	Finding percent using double number line Place value charts Arrays Mathematical models Mathematical manipulatives Understanding misconceptions	YouTube video on finding percent using double number line Video on multiplying decimals using an array Document camera: place value chart, matching fractions and decimals

Table 4.17: TPACK in Student Group 3 Presentation Alignment with Massachusetts Mathematical Content Strands, Fifth Grade

TPACK	Massachusetts Content Standards Fifth Grade
YouTube video: finding percent using double number line	• Analyze patterns and relationships • Write and interpret numerical expressions
Video: multiplying decimals using an array	• Understand the place value system • Perform operations with multi-digit whole numbers and with decimals to hundredths • Write and interpret numerical expressions
Document camera: place value chart, matching fractions and decimals using popsicle sticks	• Understand the place value system • Use equivalent fractions as a strategy to add and subtract fractions • Analyze patterns and relationships

Source. Massachusetts Department of Elementary and Secondary Education (2011).

Table 4.18: Alignment of Massachusetts Teacher Standard 7.06.7 with Course Objectives and Assessment

Massachusetts Teacher Standard	Course Objective	Assessment
Standard 7.06.7: Subject matter knowledge requirements for teachers: Mathematics. a. Basic principles and concepts important for teaching elementary school mathematics in the following areas: 1. Number and operations (the foundation of topics in 603 CMR 7.06 (7) (b) 2. A. ii. – iv.). 2. Functions and algebra. 3. Geometry and measurement. 4. Statistics and probability. b. Candidates shall demonstrate that they possess both fundamental computation skills and comprehensive, in-depth understanding of K–8 mathematics. They must demonstrate not only that they know how to do elementary mathematics, but that they understand and can explain to students, in multiple ways, why it makes sense.	Students will be able to demonstrate a strong working knowledge of the content in the Massachusetts Curriculum Frameworks for Mathematics and the CCSS for Mathematics CCSM, Grades 1–6. **Standard 7.06.7**	Class participation Lesson plan Paper on Standards for Mathematical Practice

Source. Massachusetts Department of Elementary and Secondary Education (2017) and elementary math methods course syllabus.

Table 4.19: Student Group 1 Presentation

Content Knowledge (CK)	Pedagogical Knowledge (PK)	Technological Knowledge (TK)	Pedagogical Content Knowledge (PCK)	Technological Pedagogical and Content Knowledge (TPACK)
Number sense, addition, and subtraction Massachusetts Content Strands (see Table 4.20)	Lesson planning High-quality and coherent instruction Instructional strategies (i.e., hands-on activities) that support learning and growth of all students Curriculum development that addresses different learning styles	YouTube videos iPad apps Document camera MS Publisher Google Docs	Addition strategies, such as regrouping Subtraction strategies, such as constant difference Mathematical models Mathematical manipulatives	YouTube video: Rekenrek (fluency up to 20) YouTube video: Near doubles rap YouTube video: Flocabulary know about 10s Document camera: Base 10 blocks, snap cubes, rekenrek, ten frame

Table 4.20: TPACK in Student Group 1 Presentation Alignment with Massachusetts Mathematical Content Strands, Second Grade

TPACK	**Massachusetts Content Standards**
YouTube video: Rekenrek (Fluency up to 20)	Add and subtract within 20.
YouTube video: Near doubles rap	Work with equal groups of objects to gain foundations for multiplication.
YouTube video: Flocabulary know about 10s	Understand place value. Represent and solve problems involving addition and subtraction. Work with equal groups of objects to gain foundations for multiplication.
Document camera: Base 10 blocks, snap cubes, rekenrek, ten frame	Understand place value. Represent and solve problems involving addition and subtraction. Work with equal groups of objects to gain foundations for multiplication. Add and subtract within 20.

Source, Massachusetts Department of Elementary and Secondary Education (2011).

Table 4.21: Student Group 2 Presentation

Content Knowledge (CK)	Pedagogical Knowledge (PK)	Technological Knowledge (TK)	Pedagogical Content Knowledge (PCK)	Technological Pedagogical and Content Knowledge (TPACK)
Multiplication and division Massachusetts Content Strands (see Table 4.22)	Lesson planning High-quality and coherent instruction Instructional strategies (i.e., hands-on activities) that support learning and growth of all students Curriculum development that addresses different learning styles	YouTube videos iPad apps Document camera MS Publisher Google Docs	Unifix cubes Snap cubes Multiplication strategies Division strategies	YouTube video: doubles rap YouTube video: associative and distributive properties of multiplication Document camera: Unifix cubes, snap cubes

Table 4.22: TPACK in Student Group 2 Presentation Alignment with Massachusetts Mathematical Content Strands, Second and Sixth Grades

TPACK	**Massachusetts Content Standards**
YouTube video: doubles rap	Second Grade Work with equal groups of objects to gain foundations for multiplication.
YouTube video: associative and distributive properties of multiplication	Sixth Grade Compute fluently with multi-digit numbers and find common factors and multiples. Apply and extend previous understandings of arithmetic to algebraic expressions.
Document camera – Unifix cubes, snap cubes	Compute fluently with multi-digit numbers and find common factors and multiples. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Source. Massachusetts Department of Elementary and Secondary Education (2011).

Table 4.23: Alignment of Massachusetts Teacher Standard 7.08.2 with Course Objectives and Assessment

Massachusetts Teacher Standard	Course Objective	Assessment
Standard 7.08.2 – Professional Standards for Teachers a. **Curriculum, Planning, and Assessment:** Promotes the learning and growth of all students by providing high-quality and coherent instruction, designing and administering authentic and meaningful student assessments, analyzing student performance and growth data, using this data to improve instruction, providing students with constructive feedback on an on-going basis, and continuously refining learning objectives. b. **Teaching All Students:** Promotes the learning and growth of all students through instructional practices that establish high expectations, create a safe and effective classroom environment, and demonstrate cultural proficiency. c. **Family and Community Engagement:** Promotes the learning and growth of all students through effective partnerships with families, caregivers, community members, and organizations. d. **Professional Culture:** Promotes the learning and growth of all students through ethical, culturally proficient, skilled, and collaborative practice.	Students will learn how to develop lesson activities that address the **Content Standards** in the Massachusetts Frameworks for Mathematics. **Standard 7.08.2**	Lesson plan Math skills book Journal
	Students will be able to develop lessons that incorporate the use of math manipulatives and address various learning styles. **Standard 7.08.2**	Lesson plan Math game
	Students will learn how to develop lessons and assessments that incorporate the **Standards for Mathematical Practice** in the Common Core Standards for Mathematics and the Massachusetts Curriculum Frameworks for Mathematics. **Standard 7.08.2**	Paper on Standards for Mathematical Practice
	Students will be able to identify and share techniques and resources (including websites, apps, and manipulatives) that can be incorporated into their lessons. **Standard 7.08.2**	Journal Class participation Lesson plan Technology presentation
	Students will understand that assessment is a major component of the learning process. **Standard 7.08.2**	Lesson plan Mini-lesson Class participation

Source. Department of Elementary and Secondary Education (2017) and elementary math methods course syllabus.

CHAPTER 5

CASE STUDY 2

5.1 Description of Setting

The second research site was a large public university, Yexer University (a pseudonym), in Eastern Massachusetts. This Master's of Education program was an initial licensure M.Ed. 34-credit program. It required a competency portfolio project, 80 hours of pre-practica, a 12-credit practicum, and a one-credit graduate program planning course (instructor interview and website). All the courses are three-credit courses. Additionally (like all other Massachusetts elementary programs), students had to pass three MTEL exams: communication and literacy; general curriculum, which includes the math subtest; and foundations of reading (http://www.doe.mass.edu/mtel/testrequire.html). Furthermore, each student was required to have a university-approved tablet device—an iPad or iPad Mini.

The class that I observed had nine students—seven females and two males. The students have been assigned pseudonyms, which are reflected in this section.

The classroom had a Smart Cart Extron Teaching Station with a ceiling-mounted projector, a desktop and a laptop Windows-based PC, a Wolfvision document reader, a DVD/VCR player, Apple TV (AirPlay via the Wi-Fi network–projection from iPad), and an iPad (instructor's personal one). The projector displayed to a pull-down white screen and not to a Smart Board. The classroom was designated for education courses and had two large storage cabinets with supplies and manipulatives, such as base 10 blocks. The instructor also brought in her own manipulatives (N2.2).

The instructor was a K–8 math-science curriculum director in a nearby public elementary school district and a math consultant who provided professional development training/workshops for elementary teachers in school districts throughout the state. Generally, the instructor taught, as an adjunct faculty member, graduate courses for practicing teachers. She had an M.Ed. degree and was planning on enrolling in a doctoral program in educational, instructional, and curriculum supervision in the fall (N2.1, N2.8).

5.2 Research Question Two

The second research question is as follows:

2. How do faculty in the math methods course use instructional technology for enhancing teaching and learning?

Descriptive findings, as a result of data collection for the second site, can be categorized into the following codes/domains of interest for how the instructor used and modeled these instructional technologies: videos (instructional), the iPad, websites linked to the CCSS and math curriculum frameworks, document camera, and PowerPoints (see Table 5.1). These technologies, modeled by the instructor, are aligned with the Massachusetts Standards for Mathematical Practice, as shown in Table 5.2, and will be described with each instructional technology. Each code/domain of interest (i.e., instructional technology) will now be described.

5.3 Instructor Use of Technologies

5.3.1 Videos

Instructional videos from YouTube, PBS Learning Media, LearnZillion, and a DVD from Australia were shown in class (see Table 5.3). Three of the YouTube videos are from the popular television program *Who Wants to be a Millionaire* and depict instances where the contestant forgets basic math facts or fails at math. These illustrate very embarrassing instances but also the sad reality that many people, even famous ones, do not have an understanding of basic math. After showing the videos, the instructor asked the class the question: “Why is facility with mental strategies better than rote memorization?” (A2.11). The answers given were “better problem solving,” “more flexible,, and “you can work around it [the problem]” (N2.2). The instructor noted that “math thinking should be three Es: efficient, easy, and effective” (N2.2). Additionally, the instructor showed two YouTube videos related to division. The first video illustrated how to use base 10 blocks to model long division, and the second one showed how to use a number line for division (see Table 5.3), as shown in Figure 5.1 (N2.2).

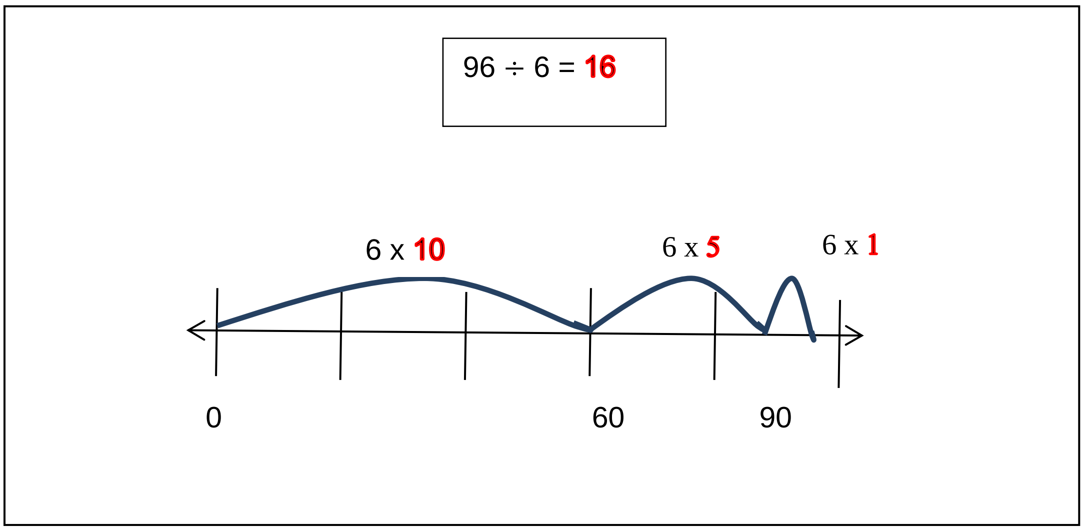

Figure 5.1: Division Using a Number Line

The last YouTube video dealt with geometry, specifically perimeter (see Table 5.3). The video explained how to find the perimeter of polygons like triangles and rectangles by adding the length of each side of the polygon (N2.5).

The instructor played a DVD on the laptop, the “Cathy Episode” from the DVD (see Table 5.3), which shows what happens when students use algorithms without understanding. The student, Cathy, had a deficit in her understanding of place value; she memorized the multiplication algorithm or procedure but did not have a way to solve the problem. Cathy also tried to use tallies to solve the problem unsuccessfully. The point of the video and the problem, the instructor emphasized, is that students are not learning the "why"—a mental strategy is better than rote. This leads to better problem solving and flexibility with working out solutions (N2.2). Math instructional videos from LearnZillion (also available on YouTube) (see Table 5.3) on multiplication and place value using number lines and on fractions were shown to the class. The instructor used a PowerPoint presentation and provided the example shown in Figure 5.2 (A2.11) related to the multiplication in the video.

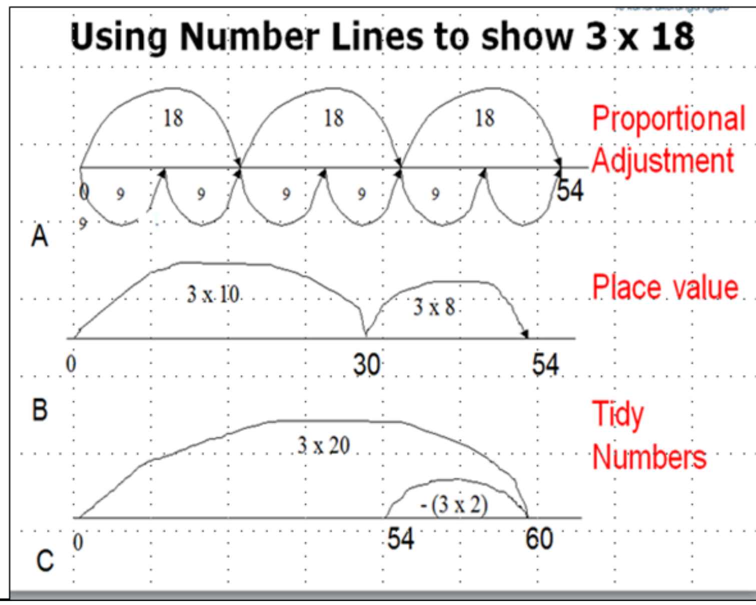

Figure 5.2: Using Number Lines to Show Multiplication

Part A shows proportional adjustment on a number line. Students are more familiar with using the number line with the digit 9, so the original number of 18 can be adjusted to 9. Part B shows the "decomposing" of the number 18 into 10 and 8, which builds on an understanding of place value. Part C shows the use of tidy numbers—that is, numbers that are easier to work with (N2.4).

The instructor used the laptop to display the video. The fractions video (see Table 5.3) showed multiple examples of dividing fractions by fractions. The instructor presented an example on the board explaining the invert-multiply strategy, as shown in Figure 5.3.

$$\frac{6}{8} \div \frac{1}{4} = \frac{\frac{6}{8} \times \frac{4}{1}}{\frac{1}{4} \times \frac{4}{1}} = \frac{\frac{24}{8}}{\frac{4}{4}} = 3$$

Figure 5.3: Division of Fractions

The PBS Learning Media fraction video corresponded with a custom activity created by the instructor titled “Bianca’s Chocolate Dilemma” (A2.2, N2.3). In the problem, Bianca wants to share a candy bar, but she keeps half for herself. She gives the other half to her sister, half of that piece to her brother, and then her parents split the other half. Bianca seems to feel that this was fair because everyone got a half. Is Bianca correct? The students then had to write two fractions: one to show each person’s share and one with a common denominator to show each person’s share. The video site has real-world problem videos by subject, grade level, and alignment to the CCSS but also includes the ability for teachers to customize and create their own questions with the Lesson Builder, Storyboard, and Quiz Maker tools, as the instructor did (N2.3).

In an interview, the instructor disclosed that “the instructional technology is a tool that makes instruction more dynamic” (N2.8). Furthermore, the instructor stated that “each one [video] showed a different model and multiple examples” (N2.4). Cassandra shared the following: “We get to watch videos of effective teaching strategies and also of students who are struggling. By being able to use the technology to see something like that, we remember it more than if we just read about it” (N2.7). Similarly, Faith stated that “the technology helps by illustrating the topics the instructor is covering and helps to clarify the subject matter” (N2.7). Danielle added that videos “allow me to see the multiple ways in which a topic can be addressed” (N2.7).

5.3.2 iPad

The iPad was used extensively by the instructor. One major usage was to demonstrate highly-rated (by Apple and the instructor) and beneficial apps from her experience (see Table 5.4). One such app, Number Pieces*,* helps students understand place value. First, the instructor presented a multiplication problem on the document camera and used the array model and place value to solve the problem. The multiplication problem was 16 x 13 (N2.2), and the solution is shown in Figure 5.4.

16 x 13 = 160 + 48 =

Rows		10	1	1	1	Columns
		10 x 10 =				
100	10	100	10	10	10	30
	1	10				
	1	10				
	1	10				
	1	10				
	1	10				
60	1	10	6	6	6	18

Figure 5.4: Array Model for Multiplication

Then she demonstrated the Number Pieces app to "build on what was modeled" (i.e., the array model) with the document camera. The instructor indicated that the multiplication problem/model can be extended with the app by changing colors for partial products (N2.2). The instructor indicated that this model was "tedious and time consuming" for teachers to use but was a "nice way to model [multiplication] with young students" (N2.2).

The National Council of Teachers of Mathematics (NCTM) Illuminations apps were also demonstrated and discussed. The instructor stated that these were "great for math teachers" (N2.3) and were arranged by grade level. She rated the NCTM apps as well as their website with “five stars” (N2.3). Geoboard was another app that was shown. This is a virtual version of geoboard (manipulative) that provides a blank-canvas board to demonstrate and solve geometry problems in perimeter, area, and so on. (N2.3).

In an interview, the instructor shared that “students are encouraged to use the iPad, document camera, and interactive websites, especially virtual manipulatives” (N2.8). She added that “the students are utilizing [instructional technology] to enhance instruction in the areas of engagement,” including the NCTM Illuminations website, apps like the Number Line, and reading aloud math children’s literature with the document camera (N2.8). Furthermore, the instructor stated that “some are tools for interactive instruction that also pose purposeful questions—important for novice practitioners,” such as the video about the student memorizing the algorithm but not learning why (N2.8). A list of apps for the iPad by topic and grade level was provided by the instructor as a resource for the pre-service teachers (see Tables 5.5 and 5.6).

The instructor arranged a “workshop” of apps for the pre-service teachers to participate in. For example, four stations were set up in which groups of two would work on an activity/exercise on the iPad for five minutes and then rotate to the next station (see Table 5.4, N2.3). At each station, the students had to identify the Standard of Math Practice (see Table A.4) the activity employed. The first station used the Versamate app involving fractions and played against the computer. The second station used the Thinking Blocks app and physical thinking blocks to solve word problems. The third station involved fraction games through the Math Playground app. The final station focused on problem solving using square colored tiles on blank sheets of paper (no technology). The next round/set of rotations involved the following apps: Oh No Fractions, Splash Math Grade 3, NCTM Illuminations, and Your Teacher. The final round/set of rotations consisted of two websites: Mathwire and Amy’s Electronic Classroom (part of LA

County Teaching Channel) and two apps: Mathmateer and Mathlands (see Tables 5.4 and 5.7 as well as N2.3). Moreover, some of the websites/apps provided opportunities for differentiated learning based on grade level and ability, such as Splash Math. The pre-service teachers enriched their content knowledge through actually doing these activities/problems on the iPad apps, which will be discussed later in the research question one section.

The pre-service teachers found the use of the iPad very helpful. Brittany stated the following: “[The instructor] shows us options of apps to use with our students to connect to technology” (N2.7). Harley also revealed that “[the] Professor uses technology constantly. I have learned about many techniques to use such as apps, websites, etc.” (N2.7). Edwina shared that “with many classrooms having SmartBoards and school systems having iPads and computers for students to use at the elementary level, it is important that we, as future educators, are familiar with it and learn ways to include it into our teaching practice” (N2.7). Not only did the instructor use the iPad and apps in her teaching but also she provided opportunities for her students to use the iPad, which will be discussed later in detail.

5.3.3 Websites

Websites linked to the CCSS and math curriculum frameworks were one instructional technology used by the instructor. In the classroom, I observed the instructor connecting her iPad and/or the classroom computer to the projector to show some of the websites (see Table 5.7, N2.2). The Enriching Mathematics website includes activities and resources to use concrete and virtual manipulatives, such as the number line and ten frames, to solve problems, including decomposing math values into smaller groups of numbers (see Figure 5.1, N2.2).

The National Library of Virtual Manipulatives (NLVM) website (also an app) provides mathematical activities/resources and interactive virtual manipulatives by grade level and by topic (Table 5.7). The instructor demonstrated a fraction activity through this website using the projector. She provided each student with a fraction card transparency and a small whiteboard. She modeled how to solve the problem: $^1/_3$ x $^2/_4$ (N2.4). The instructor wrote the equation on the whiteboard. Then she worked with the virtual manipulatives, projecting them on the whiteboard; the students could visualize the activity and its steps. The students then used the fraction transparencies to follow that process and solve the equation on the small whiteboards.

The Would You Rather website is another website demonstrated by the instructor (see Table 5.7). It is a very interesting site in which students choose between two options but then have to justify their answer with mathematics. In one example, the first scenario is "Would You Rather . . . RECEIVE 500 POUNDS OF PENNIES OR 40 POUNDS OF QUARTERS?" (http://www.wouldyourathermath.com/category/money/). The second scenario is "Would You Rather . . . share a small bag [1.5 oz. or 42.5 grams] of chips with 1 friend OR share a large bag [7 oz. or 198.4 grams] of chips with 7 friends?" (http://www.wouldyourathermath.com/category/comparison/). The site offers real-world scenarios and encourages students to reason mathematically as they justify their answers mathematically (N2.9). The instructor indicated that routines, such as "Would You Rather," are ones that teachers can use as they "line up" and transition to the next activity (N2.9). In other words, it is a good use of time, rather than a gap of silence, as the teacher prepares the next activity.

5.3.4 Document Camera

The document camera was another technology that was used often by the course instructor (see Table 5.8). The instructor used the document camera to read aloud and project from math books such as *Amanda Bean's Amazing Dream*, a story about counting that serves as a good transition from addition to multiplication, and *Sir Cumference*, a series about geometry topics (N2.2, N2.5).

Additionally, the instructor used the document camera to illustrate mathematical models, such as the array model (see Figure 5.4), while the students worked on the same problem on their iPads with the Number Pieces app (N2.2). Real-world problems, such as "I get paid," were also explained via the document camera (N2.3). The instructor wrote the problem and solution on a graphic organizer sheet as she used the document camera to display it for the whole class. This fifth- or sixth-grade problem asks which one you would prefer: You will work five days a week for four weeks and get paid $125 per week (4 x 125) for a total of $500. The second offer is to get $0.01 first day, $0.02 second day, $0.04 third day, $0.08 fourth day etc.—the pay is doubling per day for the 20 days. The second offer generates $5.12 on the 10th day, $163.84 on the 15th day, and a final total of $10,485.75 (N2.3).

Lastly, the instructor used the document camera with manipulatives, such as Anglegs, shapes that snap together, to demonstrate the difference between a square and a rhombus (N2.5). The instructor indicated that "most schools have a document camera for them [pre-service teachers] to use" (N2.8). Likewise, Edwina stated that "technology is the way of the future. I am already seeing with the practice in a school setting just how much students know about technology and how comfortable they are with it. It is best to embrace it and use it to help students. I plan to use it a lot as an elementary teacher" (N2.7).

5.4 TPACK

5.4.1 Content Knowledge

The TPACK framework and its components are reflected in the instructor's teaching of the elementary math methods course (see Tables 5.9 and 5.10). Content knowledge is subject matter understanding of mathematics (see Table A.2). Standard 7.06.7 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations (Massachusetts Department of Elementary and Secondary Education, 2017) describes the subject matter knowledge requirements for elementary teachers specifically in mathematics: numbers and operations, functions and algebra, geometry and measurement, and statistics and probability.

The syllabus outlines the following mathematical content in the course: number sense, operations, place value, measurement, fractions, decimals, percents, and geometry (A2.1). Fractions, decimals, and percents are part of numbers and operations but are listed separately because the instructor emphasized them in her course. Furthermore, the syllabus indicates that the subject matter, statistics and probability, would be presented in class, although the researcher did not observe this during her site visits. In fact, the syllabus states that the pre-service teachers would be quizzed on this content. Algebraic thinking concepts were also presented in the second site (A2.1, A2.11, N2.2). This mathematical content is aligned with the content domains found in the Massachusetts Curriculum Frameworks for Mathematics (2011) for elementary mathematics.

The Yexer instructor's content knowledge was evident as she explained and walked the students through various examples and exercises (see Table 5.9). For example, as noted earlier, the instructor wrote the following example—$^6/_8 \div {}^1/_4$—on the whiteboard to explain division of fractions. Then she illustrated the solution using the invert-multiply strategy, as shown in Figure 5.3 (N2.4). Additionally, the Yexer instructor demonstrated subject matter understanding by illustrating the difference between a square and a rhombus as well as place value, multiplication, division, and fractions, as noted earlier (see Table 5.9, N2.5).

Furthermore, the instructor also displayed mathematical knowledge for teaching pre-service teachers how to teach mathematics to children. This content knowledge includes "teachers' knowledge about the subject matter to be learned or taught" (see Table A.2). For example, teachers need to correct student answers. But, in mathematics, they also need to recognize how and why the students make errors/give incorrect answers as well as explain these errors to the students. For example, the instructor used a DVD video, the Cathy Episode, in which Cathy was incorrectly solving a multiplication problem (see Table 5.3). She explained the nature of the student error—namely, a deficit in understanding place value—even though Cathy, the student, had memorized the multiplication algorithm. She emphasized that students are not learning the "why" (N2.2). Similarly, the instructor reviewed some of the geometry quiz questions and clarified the pre-service teachers' misunderstandings/errors (N2.6).

5.4.2 Pedagogical Knowledge

Pedagogical knowledge is knowledge of how to teach that fosters student learning (see Table A.2). Standard 7.08.2 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations defines the "pedagogical and other professional knowledge and skills required of all teachers" (Massachusetts Department of Elementary and Secondary Education, 2017). Table 5.12 shows the syllabus course objectives and assessments alignment with Standard 7.08.2. The instructor has elementary teaching experience. She has an M.Ed. degree and is a K–8 math-science curriculum director in a nearby public elementary school district. Furthermore, she is a math consultant who provides training and workshops for elementary teachers in school districts throughout the state. The instructor provided real-life examples and advice/tips from her own teaching. One such "teaching nugget," as the instructor referred to it, was "teach with the end in mind—look at assessment" (N2.6). The lesson plan should have measurable outcomes (A2.1). Another example suggested that the pre-service teachers "label their classrooms so that the students become comfortable with the geometry vocabulary"

(N2.5). She added that pre-service teachers should “think about ELLs [English language learners] when looking at that vocabulary” (N2.5) and addressing the needs of all learners.

High-quality and coherent instruction was consistently observed. The instructor has college teaching experience, which requires curriculum planning and design. A variety of assignments/assessment tools (e.g., lesson plans, journals, and problem presentations) were utilized by the instructor (A2.1). Writing assignments, whether it was in a journal or part of a lesson plan, required the pre-service teachers to reflect “about theories of teaching mathematics, instructional methodologies, developmentally appropriate practices and teaching for understanding” (A2.1). As noted before, in-class activities included a “workshop” of apps that were completed in small groups (see Table 5.4, N2.3). In addition, she had the pre-service teachers write math autobiographies that provided her with insight into their math experiences. The math methods courses are graduate level, and the instructor established high expectations of the pre-service teachers appropriate to the graduate level. Furthermore, the instructor reviewed questions from the previous week’s quiz after realizing the students still had difficulty with the geometry subject matter (N2.5).

The instructor also added that “telling is not teaching . . . need modeling” (N2.6). Another thought-provoking question the pre-service teachers should ask themselves was “Am I doing the thinking or are they [the students] doing the thinking?” when the teacher is in front of the class (N2.5). In other words, teachers need to actually do the “math” and to get their students actively involved in their own learning. The instructor modeled this as she actively engaged the students in the classroom (see Figure 5.5). She required class participation from the pre-service teachers. The instructor assessed the class participation with the following evaluation criteria:

- Does the student participate actively in class discussion on a daily basis?

- Does the student become engaged in his or her own learning on a daily basis?
- Does the student take the intellectual risk (e.g., put ideas on the table and ask questions)?
- Is there evidence that the student works collaboratively with other students and contributes to their learning? (A2.1)

She discussed mathematical routines that could be used by the pre-service teachers to enhance their teaching of mathematics. These routines are useful as transitions between activities. She recommended the book *High-Yield Routines for Grades K-8* by Ann McCoy, Joann Barnett, and Emily Combs. The instructor demonstrated a few routines: "Click clack: What's my number if I am looking for a multiple of 2? Then click"; "Alike and different: What is alike and different for the numbers 11 and 17?"; "I was walking down the street and I heard Sally say 9. So what is the question?"; "Equation bubbles: Draw bubbles in which equations like 17 – 8 or 5 x 9 x2 are written. What is the answer?"; and "Eliminate one: Which number in the group of four numbers does not belong and why?" (N2.9). She was encouraging the students to get into the habit of mind and to use these routines in their instruction.

The instructional strategies employed by the instructor were designed to foster in the students a positive disposition toward the learning and teaching of mathematics (N2.9). Danielle, a pre-service teacher, commented on the class, with all its instructional technologies, that "it allows me to see the multiple ways in which a topic can be addressed. It provides me with multiple avenues for my students to learn and therefore comprehend" (N2.7). During one of the breaks, a pre-service teacher remarked that "[the instructor] really knows her stuff. . . . She knows what she's talking about" (N2. 5).

The instructor ended the course with the "Five [Actually Seven] Attributes of an Effective Teacher":

- Enjoy being with the students
- Compassion: understanding your students' abilities and challenges
- Effective communication skills with students, colleagues, parents
    - Knowing how to ask for help and constructive feedback
- Confidence in front of students
    - Confidence in the subject matter
    - Understand current research in teaching
    - Be open-minded to new ideas and feedback
    - Resilience
- Professionalism: ability to work effectively with students and adults
- Flexibility: ability to change your instruction to meet student needs
- Organizational skills and classroom management skills that promote community and student learning (A2.13)

5.4.3 Technological Knowledge

Technological knowledge is knowledge of using various technologies—for example, a document camera, researching and using such video sites as YouTube as well as DVD videos, and websites/iPad apps (see Table A.2). The instructor demonstrated confidence in using the various technologies in her teaching (see Tables 5.3, 5.4, 5.7, and 5.8). The frequency of these technologies used by the instructor is shown in Table 5.11. The students commented on the instructor's use of technology. Edwina mentioned, "My instructor is very comfortable with technology and uses it to our advantage by showing PowerPoints, videos, games, etc. that reinforce her teaching"; Gemma noted the following: "She uses a great variety of technology, including apps, videos, overhead projections, etc."; and Harley observed that "[the] Professor uses technology constantly" (N2.7).

The document camera is a significant and innovative technology used by the instructor. She demonstrated how to use physical manipulatives, such as base 10 blocks, to solve a mathematics problem, such as in Figure 5.4, the multiplication of two-digit numbers (N2.2), while the students in the class solved the problem at their desks. The instructor projected elementary-level math literature books through the document camera and read aloud to the class. The entire class, rather than a small group, followed along with the text and illustrations. As seen in Table 5.8, the instructor read aloud *Amanda Bean's Amazing Dream* and *Sir Cumference*.

Instructional videos, from a DVD, YouTube, or website, were utilized by the instructor. These engaging videos provided visual concrete models of exemplar teaching of elementary mathematics in real classrooms. For example, videos on teaching fractions were shown; the PBS Learning Media videos (N2.3) were also available on YouTube.

Websites and iPad apps were often used by the instructors to illustrate how to solve a mathematical problem using digital manipulatives. For example, the Yexer instructor used the Number Line app to show how to use a digital number line for solving multiplication problems (N1.3, N2.2). Moreover, the instructor mentioned in class that physical manipulatives "don't travel well"—that is, they can break—and so the digital versions are more practical (N2.2).

The instructor's teaching can be discussed using the criteria in Standard 3 of the TSAT, Teaching and Learning with Technology (see Tables 5.13–5.16). At the TSAT Early Technology Mastery Level, the instructor showed and discussed videos (YouTube, PBS Learning Media, LearnZillion, and DVD from Australia) (see Table 5.4) of exemplar teaching, highlighting best practices on teaching with instructional technology and learning with technology (see TSAT, A3.1). Furthermore, the instructor provided curriculum-specific information from online sources and created a resource list of iPad apps by grade level and topic (see Tables 5.5 and 5.6) for the pre-service teachers to potentially use (see TSAT, A3.2; A2.15).

Likewise, at the TSAT Developing Technology Mastery Level, the instructor provided such online curriculum resources as the Massachusetts Curriculum Framework for Mathematics, the CCSS, the NCTM lessons, the Georgia State Standards Math Resources, the University of Auckland, Faculty of Education and Social Work: Team Solutions, and the Los Angeles County Office of Education (see Table 5.7) (see TSAT, B3.3; A2.14, N2.2–N2.6). Additionally, the instructor used applications, such as Word and PowerPoint, to organize curriculum-specific information. Then she created multimedia presentations in PowerPoint to share that content with the pre-service teachers weekly (see Table 5.8) (see TSAT, B3.5, B3.6; A2.11).

Furthermore, the instructor also participated in professional development (see TSAT, B3.8; N2.4). She attended and participated in the NCTM annual meeting and workshops during that spring semester. The NCTM is the largest organization for mathematics education and provides resources for curriculum, instruction, assessment, and professional development. This conference included workshops on instructional strategies and technologies in mathematics, such as "Understanding Math with PBS Learning Media," a resource also used by the instructor (see Table 5.3); "Technology Used in the Flipped and Traditional Classroom" on using Google products; and "Teaching Number Sense with Math Buddies, the Singapore Online Resource," which uses the "concrete-pictorial-abstract approach" to number sense, to name a few.

Similarly, at the TSAT Proficient Technology Master Level, the instructor presented/discussed technology resources suitable for an elementary mathematics curriculum (see TSAT, C3.2, C3.5). iPad apps (see Table 5.4), websites (see Table 5.7), and instructional videos (see Table 5.3) are suitable resources that provide virtual manipulatives and lesson plans for elementary mathematics curriculum (N2.2–N2.6, N2.9).

Additionally, the instructor created the "Enhancing Math Instruction Using Apps and Websites" assignment in which the pre-service teachers had to identify and align to the CCSS, content focus, the possible uses in lessons, engagement level, tracking of results/reports, strength, weakness, and overall rating of apps (see Tables 5.17–5.21) and websites (see Tables 5.22–5.24) selected by her (see TSAT, C3.5; A2.10). The instructor indicated that the pre-service teachers should "think about your pre-practicum and how you might incorporate the technology into your lessons" as they worked on the assignment (N2.3).

Lastly, at the Advanced Technology Mastery Level, the instructor identified and used videos (see Table 5.3), websites (see Table 5.7), and apps (see Table 5.4) that relate to teaching and learning of elementary mathematics (see TSAT, D3.1; N2.2–N2.6). Multimedia presentations, such as PowerPoints, were utilized to present the class agenda, course content, and link to videos/websites and resources (see Table 5.8) (TSAT, D3.6; A2.11).

5.4.4 Pedagogical Content Knowledge

Pedagogical content knowledge is knowledge of instructional strategies (i.e., pedagogy) as they relate to specific content, such as mathematics (see Table A.2). Standard 7.06.7 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations states that teachers "must demonstrate not only that they know how to do elementary mathematics, but that they understand and can explain to students, in multiple ways, why it makes sense" (Massachusetts Department of Elementary and Secondary Education, 2017). Table 5.25 shows the alignment of the syllabus course objectives and assessments with Standard 7.06.7.

For example, as noted earlier, the instructor exemplified pedagogical content knowledge in action via her use of such manipulatives and tools as base 10 blocks to model long division, Angleg shapes to create geometric shapes, number lines to show multiplication and division, ten frames to decompose math values into smaller groups of numbers, Cuisenaire rods to clarify operations with fractions, lady bug rulers and yardsticks to measure various objects, small dry-erase whiteboards to solve problems, and real pizzas (one small and one large) to clarify a misconception regarding equality and the same number of pieces (CK) (N2.2–N2.6). The instructor utilized manipulatives in multiple ways to represent the mathematics content (see Table A.2).

She employed multiple strategies to teach such math concepts as operations, fractions, and geometry. As Figure 5.5 illustrates, the instructor used multiple instructional strategies for understanding operations (N2.2, A2.11). First, using a PowerPoint presentation, the instructor started with a review of tools explored for addition and subtraction. Next she showed a video on what happens when you “forget” basic math facts. Then the instructor used the document camera to real aloud a math literature book.

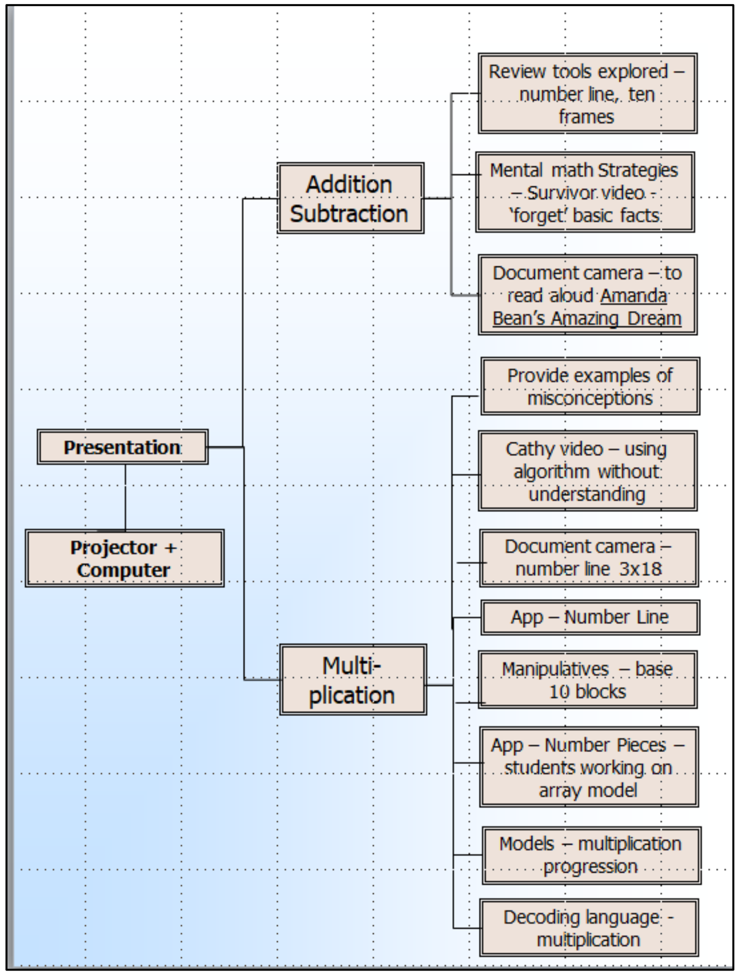

Figure 5.5: Flowchart of Site 2 Instructor and Understanding Operations Presentation

The instructor moved on to multiplication operations, as also shown in Figure 5.5. First she discussed misconceptions in multiplication. Next she showed a video that illustrates a student using an algorithm without understanding. Then she demonstrated a multiplication problem using a number line and several apps. Next she explained a progression of multiplication models. Finally, the instructor explained how to “decode the language” and to think about multiplication, as shown in Figure 5.6.

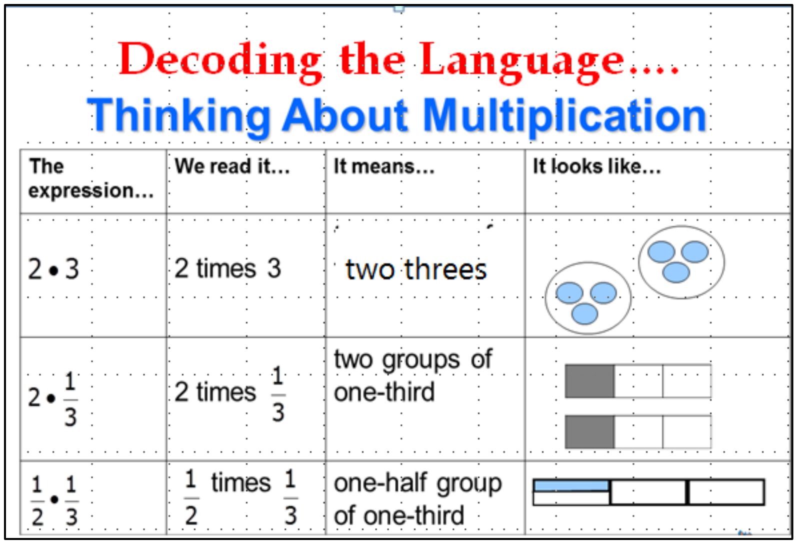

Figure 5.6: Thinking about Multiplication

Fraction instructional strategies started with a "body fractions game" in which the instructor and the pre-service teachers tried to illustrate various fractions using their body parts—for example, ½ is one arm and ¼ is part of the arm from the elbow to the shoulder (N2.3). Next the instructor accessed the University of Auckland website as a resource. Misconceptions about equality and number of pieces were illustrated with a real small and large pizza; both pizzas have the same number of pieces but are not equal in size. Such manipulatives as Cuisenaire rods, a clock, and fraction towers were illustrated. A fraction number line was used to illustrate multiplication. Finally, the instructor stressed the importance of "words first before symbols" when trying to explain fraction language.

Geometry instructional strategies started with a great opportunity to "explore the world around us" by looking around the classroom and coming up with three questions. An example of a question was: "Where are the intersections of a plane?" (N2.3). The instructor suggested that the pre-service teachers label their classrooms and create visuals to make their students comfortable with geometry vocabulary. They need to think about ELL students when looking at vocabulary. Then she advised the pre-service teachers to "make math meaningful by increasing the 3Rs—rigor, relevance, and reasoning" (N2.5). Next the instructor used websites, such as Illuminations from NCTM (see Table 5.7), to show hands-on activities and lessons. Specifically, the instructor demonstrated Venn diagrams through the website and then used physical manipulatives—plastic expandable circles—to demonstrate again the Venn diagrams. With the document camera, the instructor used other manipulatives—Anglegs and 3-D solid shapes—to make different shapes and to demonstrate the difference between a square and a rhombus. Lastly, the instructor showed a video on perimeter and read aloud, using the document camera, a book on circumference (N2.5).

Misconceptions about mathematics were presented so that the pre-service teachers understood and could address the mistaken beliefs and thinking of their students. The instructor referred to the misconceptions as "lies my teacher told me" (N2.2). One such "lie" was "to multiply by 10, just add a zero to the end of the number." The instructor addressed this misconception with this example: "5.2 x 10 ≠ 5.20" (N2.2). Another one was that "multiplication makes a value larger." The example was: "2 x ½ does not equal a larger number" (N2.2). A misconception regarding equality was also demonstrated with real pizzas, as noted earlier. The instructor brought in one small and one large pizza. Both pizzas have the same number of pieces but are not equal in size (N2.3).

Furthermore, after each activity and/or presentation, the instructor asked the pre-service teachers to identify the standard(s) of mathematical practice (see Table A.4) that were reflected in that activity or presentation (N2.2–N2.6).

5.4.5 Technological Pedagogical Content Knowledge (TPACK)

TPACK is knowledge of instructional strategies (PK) that integrate technologies (TK) to constructively teach content (PCK), in this case mathematics content (CK), for greater understanding of that content (see Table A.2). Thus, TPACK, in the instructor's teaching, is the knowledge of instructional strategies that integrates technologies to explain how to understand and solve a mathematical problem with either a digital nor nondigital tool, such as a number line; a YouTube video; a website/iPad app that demonstrates how to use models, such as arrays, to perform mathematical operations; or a document camera to demonstrate such manipulatives as base 10 blocks (see Table A.2 and Table 5.10).

As Figure 5.5 illustrates, TPACK is reflected in the multiple instructional technologies and strategies for understanding operations utilized by the instructor (N2.2, A2.11). First, using a PowerPoint presentation, the instructor built on prior knowledge with a review of tools/manipulatives, such as the number line and ten frames, explored for addition and subtraction. Next, through PowerPoint, she linked to a video on what happens when you "forget" basic math facts. Then the instructor used the document camera to read aloud a math literature book as she transitioned from addition to multiplication. This part of the lesson started with some misconceptions regarding multiplication, also shown in Figure 5.5. Next she showed a DVD video from the laptop that illustrates a student using an algorithm without understanding (see Table 5.3). This particular student in the video, Cathy, had memorized the algorithm but had a deficit in understanding place value, and as such, she was unable to solve the multiplication problem. The instructor segued to the document camera to demonstrate a multiplication problem using a number line; she then reinforced with several apps and manipulatives, such as base 10 blocks. Meanwhile, the pre-service teachers worked on similar multiplication problems on small whiteboards at their desks. Next the instructor explained a progression of multiplication models. Finally, she explained how to "decode the language" and to think about multiplication, as shown in Figure 5.6. These instructional strategies align with the SMP (see Table 5.2). For example, the instructor used appropriate tools, such as instructional videos, document camera, apps, number line, and base 10 blocks, strategically (SMP5) to support the pre-service teachers as they make sense of the problems (SMP1) and reason quantitatively (SMP2).

Likewise, TPACK is manifested in the multiple instructional technologies and strategies for teaching fractions utilized by the instructor. Fraction instructional strategies started with a "body fractions game" in which the instructor and the pre-service teachers tried to illustrate various fractions using their body parts—for example, ½ is one arm and ¼ is part of the arm from the elbow to the shoulder (N2.3). Next the instructor accessed, through a PowerPoint, the University of Auckland Faculty of Education website as a resource (see Table 5.7).

Misconceptions about equality and number of pieces were illustrated with a real small and large pizza; both pizzas have the same number of pieces but are not equal in size. Such manipulatives as Cuisenaire rods, a clock, fraction card transparencies, and fraction towers were illustrated. The instructor mentioned that she has gone to lectures on fractions by Sharma (a resource utilized at Xever). The instructor utilized Cuisenaire rods in a manner similar to Sharma's DVD (see Chapter 4). A fraction number line was used with a document camera to illustrate multiplication (e.g., ¾ of 80). Finally, the instructor stressed the importance of "words first before symbols" when trying to explain fraction language (N2.3). For example, the fraction ¼ should be written as one-fourth with the emphasis on the "th" before the digits of the fraction. These instructional strategies align with the SMP (see Table 5.2). For example, the instructor used appropriate tools, such as websites, fraction card transparencies and towers, and fraction number line, strategically (SMP5) to support the pre-service teachers as they make sense of the problems (SMP1) and make use of structure and precision (e.g., the fraction number line) (SMP6, SMP7).

Similarly, TPACK is exhibited in the multiple instructional technologies and strategies for teaching geometry utilized by the instructor. Again, she used PowerPoint to organize and start the geometry presentation. Geometry instructional strategies started with a great opportunity to “explore the world around us” by looking around the classroom and coming up with three questions. An example of a question was: “Where are the intersections of a plane?” (N2.5). The instructor suggested that the pre-service teachers label their classrooms and create visuals to make their students comfortable with geometry vocabulary. They need to think about ELL students when looking at vocabulary and the words that they use. For example, the windows and doors in the classroom would be labeled as “rectangles,” and the clock on the wall would be a “circle” (N2.5). Then she advised the pre-service teachers to “make math meaningful by increasing the 3Rs—rigor, relevance, and reasoning” (N2.5). Next the instructor linked to websites, such as Illuminations from NCTM (see Table 5.7), to show hands-on activities and lessons. Specifically, the instructor demonstrated Venn diagrams through the website and then used physical manipulatives—plastic expandable circles—to reinforce Venn diagrams. With the document camera, the instructor used other manipulatives—Anglegs and 3-D solid shapes—to make different shapes and to demonstrate the differences between shapes. Returning to the PowerPoint, she linked to another website, abcteach, to discuss a “family tree” of shapes (see Table 5.7). Lastly, the instructor showed a video on perimeter and read aloud, using the document camera, a book on circumference (N2.5). These instructional strategies align with the SMP (see Table 5.2). For example, the instructor used appropriate tools, such as the document camera, expandable circles, and Anglegs, strategically (SMP5) to support the pre-service teachers as they make sense of the problems (SMP1) and reason quantitatively (SMP2). Additionally, the instructor facilitated the development and critiquing of viable arguments (SMP3) and repeated reasoning (SMP8).

The instructor consistently embodied technological pedagogical and content knowledge in this math methods course. The interviews/discussions with the students provided insight into how the students perceived the role and value of the instructor using instructional technology in enhancing their learning of math content and teaching practices. Edwina indicated that "technology deeply enhances the experience of learning in this class. My instructor is very comfortable with technology and uses it to our advantage by showing PowerPoints, videos, games, etc. that reinforce her teaching" (N2.7). Edwina adds that "instructional technology used by my instructor has shown me that technology can enhance mathematics in many ways. She has offered many websites and apps that will help us as teachers and our students. . . . We were also assigned to explore websites and iPad apps for an assignment, which resulted in finding many ways to include technology into our teaching. This course has helped me with that very much" (N2.7).

Gemma indicated that "I get to observe all the different ways to use technology. I take those ideas and implement them in my lessons and interactions with students I work with" (N2.7). Faith responded that "the technology used help in modeling how I'll be able to use technology in my own classroom" (N2.7). Cassandra stated that "we get to watch videos of effective teaching strategies and also of students who are struggling. By being able to use the technology to see something like that, we remember it more than if we just read about it" (N2.7). Cassandra added that "we have learned about many useful apps and websites with lesson plans. The Internet is full of useful and not so useful sources, and the instruction in this class helps us distinguish them" (N2.7). These pre-service teachers have been able to connect the modeling by the instructor to their own teaching of mathematics.

Additionally, Gemma stated that "she [the instructor] uses a great variety of technology, including apps, videos, overhead projections, etc. It helps a lot for me because I am a visual learner, so the more I can look at, the better" (N2.7). Similarly, she included in her review of the Math Playground Thinking Blocks Fractions app that "being someone who needs a visual representation whenever possible in math, these models really helped paint a clear picture for me" (A2.12). Gemma described this app as having a high level of engagement, with its use of clear and colorful models of part-whole relationships (see Table 5.19). Each problem starts off with a written word problem, and then the app will either reaffirm correct answers or show the mistake in incorrect answers. She adds that the app provides a "clear road map with labels and representative blocks . . . really takes the mystery out of the problem and makes it accessible" and that the "part-whole relationship is clear and vividly displayed as a meaningful solution" (A2.12). This pre-service teacher understands her own learning style and how the instructor's modeling enhances her own learning. This was reinforced in the side note Gemma made in her review of the Xtra Math app: "I know I sound like a different person. You have made me see the basic facts memorization light and it all makes sense to me now. The torture of this app really flipped a switch for me as well" (A2.12). In other words, Gemma realized that knowledge of basic math facts, such as times tables, facilitates problem solving. She described this app as strictly "drill and kill" and the "definition of busy work," providing practice with math facts/operations, with very low engagement, and rather plain, with an overall rating of two stars on a scale of 1 to 5 (A2.12, Table 5.19). The instructor noted on the assignment that she "was thinking that" Gemma indeed sounded like a different person. This type of feedback from a student and effect on a student is at the heart of teaching. Interestingly, this app was assigned as homework for Gemma's son almost every day, and he complained constantly about it (unlike the rest of his homework) (A2.12).

Furthermore, the high level of TPACK, in the instructor's teaching, can be assessed using the criteria in Standard 3 of the TSAT, Teaching and Learning with Technology (see Tables 5.13–

5.16). At the TSAT Early Technology Mastery Level, the instructor showed and discussed videos of exemplar teaching, highlighting best practices on teaching and learning with technology (see TSAT, A3.1) (see Table 5.3).

Likewise, at the TSAT Developing Technology Mastery Level, the instructor designed lessons and activities that explained how to solve a mathematical problem with either digital or nondigital tools, such as number lines, to demonstrate multiplication, as shown in Figure 5.2 (see TSAT, B3.1, B3.2). Additionally, she used a document camera to demonstrate manipulatives, such as Anglegs, which illustrate geometry concepts—for example, the difference between a square and rhombus (see TSAT, B3.1, B3.2; N2.5). Also, the instructor illustrated mathematical models, such as the array model, by using the document camera while the students worked on their iPads with the Number Pieces app (see Figure 5.2). As mentioned previously, these instructional strategies used technology that was appropriate for visual learners.

Similarly, at the TSAT Proficient Technology Master Level, the instructor presented/discussed technology resources suitable for an elementary mathematics curriculum (see Table 5.7) (see TSAT, C3.2, C3.5). Such websites as the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) are suitable resources that provide virtual manipulatives for elementary mathematics curriculum (N2.4). Technology tools, such as a document camera, a laptop, an iPad, and a projector, were constantly used to enhance the curriculum (see TSAT, C3.4) (see Table 5.11). Each week the instructor emailed her PowerPoints and other resources to the pre-service teachers so that they could access and explore the many links in the presentations. Additionally, this allowed the pre-service teachers to focus on the presentations/activities rather than on taking copious notes (see TSAT, C3.11; A2.1, A2.11).

Lastly, at the Advanced Technology Mastery Level, the instructor identified and used videos (see Table 5.3), websites (see Table 5.7), and apps (see Table 5.4) that relate to teaching and learning of elementary mathematics (see TSAT, D3.1). In addition, the instructor identified effective design/presentation as the pre-service teachers presented both their group and their individual lesson plans (see TSAT, D3.7; N2.2–N2.6). The instructor required the pre-service teachers to create lesson plans that they would teach in their pre-practica; most integrated appropriate instructional technology that was also available at these elementary schools (see TSAT, D3.5).

In conclusion, staff (i.e., the pre-service teachers) development regarding the technology integration was embraced by the instructor (see TSAT, D3.9; N2.2–N2.9). The instructor's modeling and instructional strategies provided professional development of technology integration into an elementary mathematics curriculum. In essence, the entire math methods course was designed and delivered as professional development for the pre-service teachers.

5.5 Research Question One

These pre-service teachers demonstrated confidence in their mathematics knowledge, pedagogies tied to the CCSS, the Massachusetts Curriculum Frameworks for Mathematics, and Standards of Mathematical Practice, and instructional strategies that incorporated technology and other tools (manipulatives) to address their students' various learning styles. The specifics of how the pre-service teachers demonstrated this will be discussed in the research question one section analysis.

The first research question is as follows:

1. In which ways did the use of technology in a math methods course facilitate the learning of elementary pre-service teachers and the integration of technology into classroom instruction?

Descriptive findings as a result of data collection for the second site can be categorized into the following codes/domains of interest (i.e., instructional technologies): the use of videos (instructional), iPad apps/websites, and the document camera (see Table 5.1). Each code/domain of interest will now be described.

5.6 Student Use of Technologies

5.6.1 Videos

Videos, generally instructional digital videos, were utilized both by the instructor and/or the students almost every week. The video technology generally was a YouTube video or a DVD, as shown in Tables 5.3 (instructor) and 5.26 (students). The frequency of each video technology is shown in Tables 5.11 (instructor) and 5.27 (students). The instructor's use of video technology was discussed earlier, and the students' use will be discussed subsequently.

Edwina, the only pre-service teacher who demonstrated a video technology in class, presented a third-grade lesson plan on tangrams (N2.3). This pre-service teacher started the lesson plan with a YouTube video (see Table 5.26) that illustrated how the "sage" (the main character) rearranged tangrams, such as triangles and parallelograms, to make geometric figures, like a square, and objects, like a camel, boat, and window. Next she used the document camera and physical tangrams to illustrate how to rearrange the tangrams to make a square as well as a rabbit, mountain, and sailboat. Then the students in the class used the tangrams to make a picture, where all the pieces had to touch one another but could not overlap. The pre-service teacher ended the lesson plan with an exit ticket activity: Draw two pictures with the tangrams that are different but have the same area (N2.3).

The interviews/discussions with the students provided insight into how the students perceived the role and value of the video technology regarding their own learning as well as their teaching practice. Brittany noted that "technology helps to visualize concepts and skills for students. Sometimes students need to see and interact with a visual representation for more meaningful reasoning" (N2.7). Gemma also shared that "it [technology] helps a lot for me because I am a visual learner, so the more I can look at, the better" (N2.7). Thus, these interviews indicate that the pre-service teachers (students) understood how the video technology facilitated their own learning, particularly the visual learners, of how to teach mathematics and particular concepts (pedagogical content knowledge) as well as how to teach mathematics with technology (i.e., TPACK) (see Table A.2).

5.6.2 Websites and iPad Apps

The websites and iPad apps are another way the pre-service teachers enriched their math content knowledge and their teaching. As previously mentioned, in one of the assignments, the students had to research and evaluate websites and/or iPad apps that they might use in their future teaching. The "Enhancing Math Instruction Using Apps and Websites" assignment required the pre-service teachers to identify and align to the CCSS, content focus, the possible uses in lessons, engagement level, tracking of results/reports, strength, weakness, and overall rating of apps (see Tables 5.17–5.21, A2.12) and websites (see Tables 5.22–5.24, A2.12). Furthermore, three of the students incorporated websites into their lesson plan presentations (see Table 5.26). The instructor also provided a list of iPad apps by grade level and math topic as a valuable resource for all the pre-service teachers (see Tables 5.5 and 5.6). One student also mentioned these websites in the interviews/discussions. Cassandra shared that "the Internet is full of useful and not so useful sources, and the instruction in this class helps us distinguish them"(N2.7).

In some cases, the pre-service teacher actually used the app in her own teaching. Gemma used some of the apps in her fourth-grade class (see Table 5.19) (TPACK). Several websites/apps offered feedback reports so that students, teachers, and parents gained insight into the students' understanding of, or lack thereof, various mathematic content (see Tables 5.17–5.24).

Websites were also incorporated into the individual student lesson plan presentations (see Table 5.26). Gemma presented a lesson plan on metric mass for a fourth-grade class (N2.3). Each student in the class received a worksheet with three columns: milligrams, grams, and kilograms (A2.3). The pre-service teacher brought in hold-and-touch examples/objects, including a feather, petal, flower, paper clip, penny, box of jelly beans, kettle drum, and baseball bat. The students had to place the examples in the corresponding weight unit column. The lesson ended with a demonstration of the Frogs and Cupcakes website that provides many lessons and visuals (e.g., lessons on measurements).

Brittany also presented a lesson plan on measurements (N2.4). She used the second-grade lesson plan, Footprints, from the Georgia Department of Education website (see Table 5.26). Brittany used the document camera to demonstrate such manipulatives as a ladybug ruler, yardstick, and meter stick. The students in the class were given a worksheet, "Measuring with Different Units," and had to measure the following: width of the desk (inches and feet), height of a textbook (centimeters and inches), height of the door up to the doorknob (feet and yards), length of a marker (centimeters and inches), width of the window (centimeters and meters), and width of the computer monitor (inches and feet) (A2.8). The students had to walk around the classroom to fill in the worksheet. Brittany also practiced the lesson plan at home with her own child (a kindergartener).

Faith likewise presented a lesson plan from the Georgia Department of Education website (see Table 5.26). She presented a third-grade lesson on comparing and contrasting polygons. Faith started with a read aloud of the book *Greedy Triangle* by Marilyn Burns via the document camera. Then the students in the class were given a pattern blocks worksheet, “Shape Search,” in which they had to find and color different shapes, such as two trapezoids that look different, a rhombus, two hexagons that look different, a pentagon, and a quadrilateral that is not a rectangle (A2. 7). Another worksheet, “Shape Search Questions,” was given to the students. Questions included: “How do your shapes compare to your classmates, same, different, and can you all be right even if you drew shapes that look different?” (A2.7). The questions required the students to reason/think and to justify their answers.

Accordingly, the iPad apps and websites used and presented by the pre-service teachers during class were meant to encourage the integration of technology into these pre-service teachers’ classrooms. These websites and apps exemplify pedagogical strategies for teaching that particular mathematics content. In many cases, this content was aligned to the CCSS (see Tables 5.17–5.24).

5.6.3 Document Camera

The document camera was another technology that was used often by the students, either individually or in groups, in the course (see Tables 5.26–5.29). The second group presentation demonstrated math in the real world: “eating cookies” (N2.2). The question was: “How many cookies did Dad bake in all? One cookie is left” (N2.2). One student put a paper plate and some chips (for the cookies) on the document camera. The solution process was demonstrated very nicely—working backward by moving and counting the cookies until only one cookie was left and then checking the answer working forward. The presentation provided a concrete demonstration of math in the real world.

Similarly, the third group presentation demonstrated math in the real world: “Pets, Pets, Pets” (N2.2). In this case, the pets are dogs and birds. There are 10 legs altogether: “How many pets do I have?” Such manipulatives as snap cubes and a ten frame were illustrated via the document camera to show the number of ways to make “pets” with a total of 10 legs. The group indicated that this was a good partner activity to use in a kindergarten or first-grade classroom. They also noted that it was a good idea to have a picture of a dog with four legs and a bird with two legs in case a student says that a dog has three legs or a bird has one leg and therefore the solution would be different.

Individual students also used the document camera to present their lesson plans (see Table 5.26). Agatha illustrated a first- grade lesson plan on place value (A2.3). She had 24 buttons that she arranged into groups of five on the document camera. Then she introduced groups of 10 by stringing 24 beads—10 beads per pipe cleaner. Both of these are shown in Figure 5.7. Lastly, she used a ten frame, similar to the Xever instructor in Figure 5.7, to repeat the place value of the 44 beads.

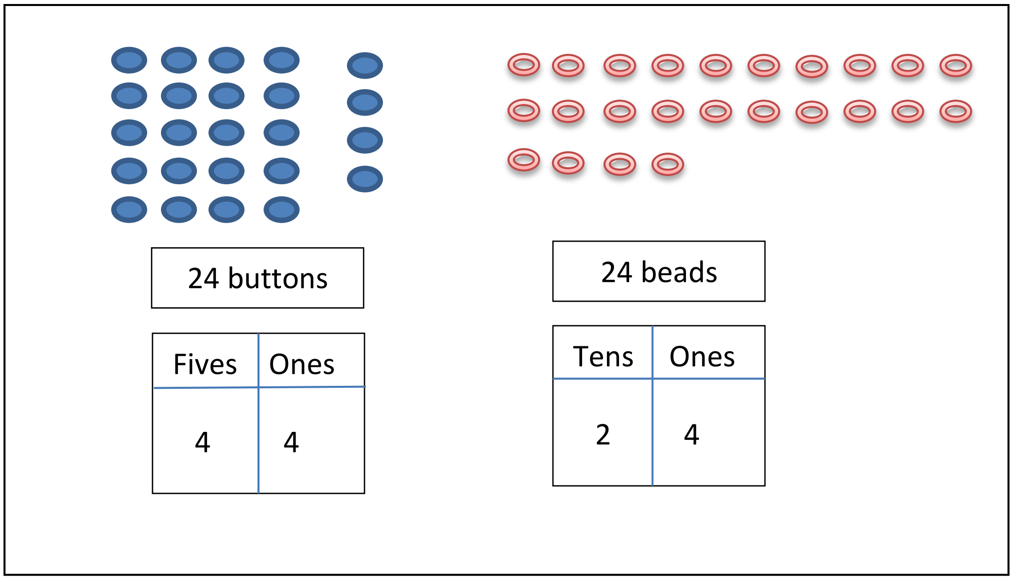

Figure 5.7: Place Value Using Buttons and Beads

Harley also used the document camera to present a fourth-grade lesson plan on fractions and decimals. The activity was called "Trash Can Basketball" (N2.3). The basketballs were paper balls made from scrap paper. Each pair of students made 10 paper balls. Then each student had to predict how many paper balls he or she would get in the basket (i.e., the trash can). Harley used a graphic organizer (A2.6), which was also distributed to each student in the class, on the document camera to record the baskets made by one group. Then he recorded the score as a fraction and then as a decimal for both players in the group.

5.7 TPACK

5.7.1 Content Knowledge

The TPACK framework and its components can be applied to the student lesson plan presentations (see Table 5.30). Content knowledge is subject matter understanding of mathematics (see Table A.2). Both Edwina and Faith illustrated an understanding of geometry and presented lessons for third graders. For example, Edwina showed an understanding of such shapes as triangles, squares, and parallelograms. She also understood how the various shapes related to one another to form/rearrange into other shapes (N2.3). Similarly, Faith showed an understanding of polygons, comparing and contrasting various polygons (N2.4).

Likewise, Agatha and Cassandra demonstrated an awareness of place value. Agatha presented a first-grade lesson plan in place value (see Figure 5.7) using ten frames to show groups of five and then groups of 10 (N2.3). Cassandra presented a third-grade lesson plan using money in envelopes to count by $1s, $10s, and $100s (N2.3).

Similarly, Brittany and Gemma illustrated an understanding of measurements and metric mass. Brittany presented a second-grade lesson that she practiced with her own child (a kindergartener). She used footprints as the unit of measurement to measure real-world objects in the room. She also used such manipulatives a ladybug ruler, yardstick, and meter stick (N2.4). Gemma presented a fourth-grade lesson on measurements, specifically metric mass. She used hold-and-touch examples, including a feather, paper clip, and baseball bat, to illustrate the differences between milligrams, grams, and kilograms (N2.3).

Harley and Ian demonstrated an awareness of decimals, fractions, and percents and presented lessons for fourth graders. Harley presented a lesson on fractions and decimals using trash can basketball. He used graphic organizers to record the number of baskets made, give fractions (out of 100), and give decimals (to the hundredth place) (N2.3). Ian presented a lesson on decimals, percents, and fractions using small whiteboards and such manipulatives as base 10 blocks at workstations throughout the classroom (N2.4).

An interview with the instructor indicated that “the instructional technology is a tool that makes instruction more dynamic” (N2.8). She added that the “students are utilizing [the technology] to enhance instruction in the areas of engagement. Some, however, are meant to deepen their own content knowledge” (N2.8). The instructor indicated that the “challenge was to give them [the pre-service teachers] math content; they don’t have a lot of math in the graduate program prior to this course" (N2.8). One of the pre-service teachers shared during a class break that she was going to take the math MTEL for the fourth time that weekend; another shared that she passed on the first time; and another shared that she passed on the second time (N2.6).

5.7.2 Pedagogical Knowledge

Pedagogical knowledge is knowledge of how to teach that fosters student learning (see Table A.2). In the lesson plan presentations, the pre-service teachers promoted active engagement and learning (see Table 5.30). For example, Faith engaged the students in the class with a shape search activity using pattern blocks and colored pencils (N2.4). Likewise, Brittany engaged the students in the class as they measured real-world objects in the room and recorded their measurements on worksheets (N2.4).

Multiple strategies were utilized and in a progressive, coherent manner. In his presentation, Harley used graphic organizers to facilitate trash can basketball (N2.3). Similarly, Agatha employed several strategies to illustrate place value. First she started with buttons and groups of five. Next she used beads on a pipe cleaner for groups of 10. Then she explained how to differentiate the lesson by using a variety of beads as well as fewer beads for those struggling with the concept. Finally, she ended the lesson with an exit ticket strategy: have the students correct with a marker as a teacher would (N2.3).

Edwina provided several models and strategies, thereby addressing the different learning styles of her peers and their potential students as they learn about geometry. She modeled for the students how to rearrange the tangrams to make the various figures and objects. Next she had the students draw a picture using tangrams—all the tangrams given to each student had to be used, and the tangrams needed to touch but not overlap one another.

Then the pre-service teacher asked questions, such as “Are the pictures the same?” and “What is common?” (N2.3). These types of questions encourage the students to think.

Likewise, Danielle designed several different models and center activities in her “fraction boot camp” lesson plan. Each center had small dry-erase boards. The first center was “fraction war,” with two decks of cards. The second center used fraction cards, which required the students to break down the fraction parts (e.g., $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$). The third center was “clothes pin fractions,” where the students to put them in order on the string and then arranged them with like denominators. The fourth and last center was the game of “Connect Four,” in which the tokens have fractions on them. The pre-service teacher also practiced this activity with her own children and was planning on teaching the lesson to her own students the following week (N2.3).

5.7.3 Technological Knowledge

Technological knowledge is knowledge of using various technologies—for example, a document camera, videos, and websites (see Table A.2). The pre-service teachers used the document camera in their student lesson plan presentations with manipulatives, graphic organizers, and read-aloud math literature books (see Table 5.26). For example, Edwina knew how to set up the document camera (connected to a ceiling-mounted projector) to demonstrate the tangram activity and project on the whiteboard for the whole class (N2.3). Likewise, Agatha used such manipulatives as beads, buttons, and ten frames on the document camera to show groupings (N2.3), and Brittany used such manipulatives as a ladybug ruler, yardstick and meter stick (N2.4). Lastly, Faith used the document camera to read aloud and project on the whiteboard the math literature book *Greedy Triangle* (N2.4).

Video technology was utilized only by Edwina (see Table 5.26). She researched and found an animated instructional video on YouTube (N2.3). This video illustrated the story of tangrams (i.e., manipulatives) appropriate for the third grade.

Websites were also researched and used in the lesson plan presentations (see Table 5.26). Both Brittany and Faith used the Georgia Department of Education website (N2.4). This website has resources, such as activities, worksheets, and lesson plans, that can be used in teaching mathematics. Brittany utilized a second-grade lesson plan, and Faith utilized a third-grade activity and worksheet from this website (N2.4).

5.7.4 Pedagogical Content Knowledge

Pedagogical content knowledge is knowledge of instructional strategies (i.e., pedagogy) as they relate to specific content, such as mathematics (see Table A.2). For example, Edwina showed an awareness of pedagogical content knowledge, such as knowledge of how to use manipulatives and tools like tangrams to teach mathematical concepts, as well as instructional strategies, such as hands-on activities (N2.3). The use of the animated video to introduce the topic of tangrams captured the students' interest (see Table 5.26). Then she designed the lesson plan so that the students would be actively involved in using the physical tangrams to create/rearrange geometric shapes and real objects. She demonstrated a clear understanding of the properties of parallelograms, squares, and triangles as she modeled for the students how to rearrange the tangrams to make the various figures and objects (N2.3).

Likewise, Faith displayed an awareness of pedagogical content knowledge in her lesson plan presentation. She started the lesson with a read-aloud of a book on triangles on the document camera. Then she reviewed the vocabulary that was on the back of the book. Next she engaged the students with a shape search activity using pattern blocks and colored pencils. Finally, the answers from the activity were recorded on a worksheet (N2.4).

Similarly, Brittany showed a familiarity with pedagogical content knowledge in her second-grade lesson plan. She practiced the lesson with her own child before presenting to the whole class. In the lesson, Brittany used a familiar unit of measurement, the footprint. She actively engaged the students as they measured real-world objects in the room and recorded their measurements on worksheets. Additionally, she used such manipulatives as the ladybug ruler, yardstick, and meter stick to illustrate different ways to measure real-world objects (N2.4).

5.7.5 Technological Pedagogical Content Knowledge

TPACK is knowledge of instructional strategies (PK) that integrate technologies (TK) to constructively teach content (PCK), in this case mathematics content (CK), for greater understanding of that content (see Table A.2). Thus, TPACK is the knowledge of instructional strategies that use technologies constructively and appropriately to teach mathematics (Table A.2). Specifically, TPACK, in Edwina's presentation, is the knowledge of instructional strategies that use such technologies as YouTube videos to explain how to form different shapes with tangrams and a document camera to demonstrate how to use the tangrams (i.e., manipulatives) to form geometric shapes and objects (see Figure 5.8, Table 5.26). Edwina used technologies, including videos, manipulatives, and a document camera, appropriately to enhance and support the teaching and learning of mathematics (N2.3).

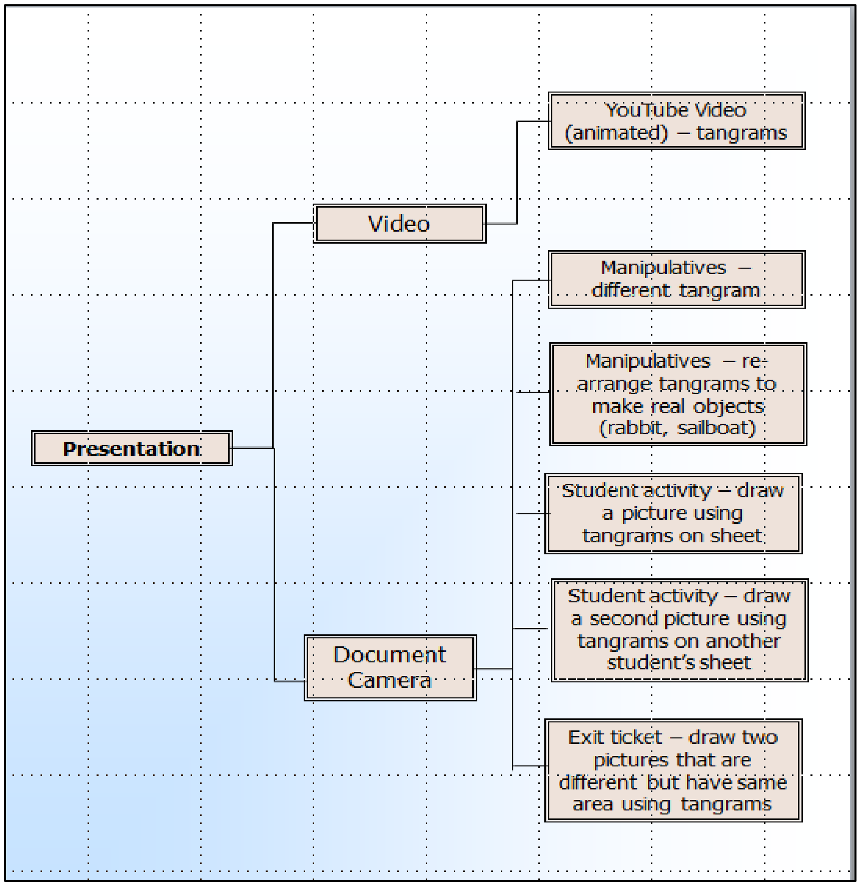

Figure 5.8: Flowchart of Student Presentation on Tangrams

Correspondingly, in Agatha's presentation, TPACK is knowledge of instructional strategies that use such technologies as the document camera to explain place value (see Table 5.26). She used such manipulatives as beads, buttons, and ten frames to model groups of five and then groups of 10. Agatha used technologies, including manipulatives and a document camera, appropriately to enhance and the support the teaching and learning of mathematics (N2.3).

Additionally, in the group presentations of Math in the Real World (see Tables 5.31–5.33), TPACK is knowledge of instructional strategies that also used the document camera. These presentations can be aligned to the appropriate grade level Massachusetts Mathematics

Content Strands, as shown in Tables 5.34–5.36. For example, student group 3 presented "Pets, Pets, Pets," a first- grade activity, in which the neighbors have pet dogs and birds (see Figure 6.9). If you have 10 legs altogether, how many pets do you have? The two pre-service teachers illustrated two solutions using snap cubes and ten frames on the document camera. They indicated this was a good partner activity but also to make sure you have a picture of a dog with four legs (not three) and a bird with two legs (not one). The pre-service teachers also mentioned that the activity could be differentiated by varying the number of legs.

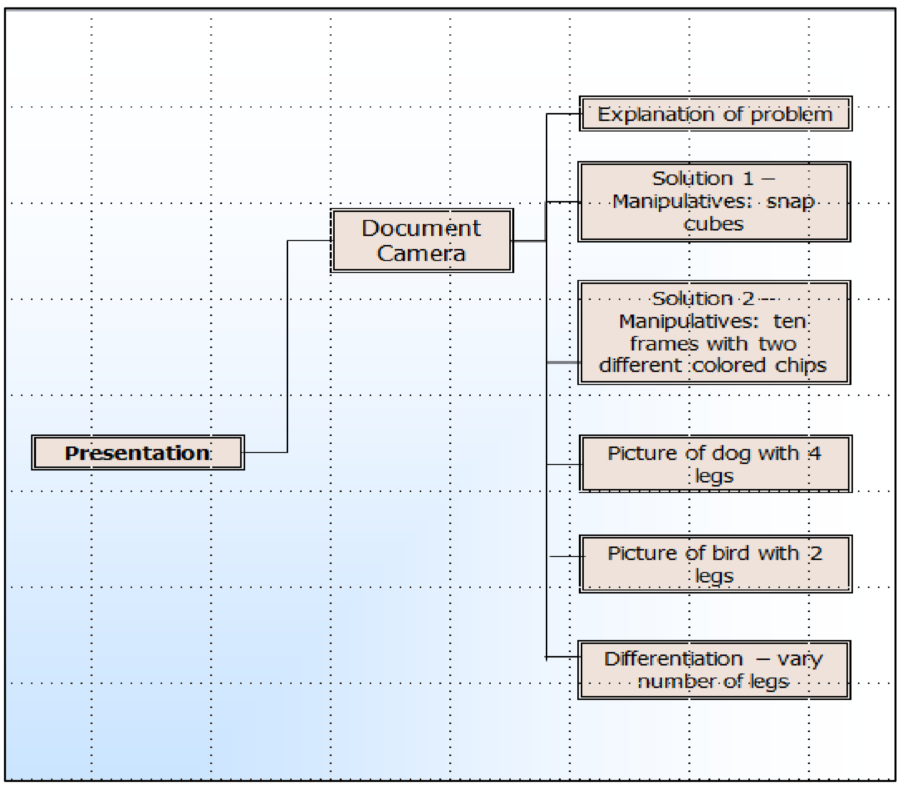

Figure 5.9: Flowchart of Student Group 3 Math in the Real-World Presentation

The interviews/discussions with the students provided insight into how the students perceived the role and value of the document camera technology regarding their own learning as well as their teaching practices. Faith shared that "the technology used help[s] in modeling how I'll be able to use technology in my own classroom" (N2.7). Brittany indicated that the "technology helps to visualize concepts and skills for students" and also added that "sometimes students need to see and interact with a visual representation for more meaningful reasoning" (N2.7). The document camera provides that "visualization" to enable/support and thus positively influences students' learning.

The pre-service teachers were building and showing their awareness of content knowledge, pedagogical content knowledge, and technological pedagogical content knowledge to have a positive effect on their students' learning (see Table A.2).

5.8 Summary

In the math methods course, the instructor used instructional technology to enhance the pre-service teachers' teaching and learning. She used and modeled these instructional technologies: videos (instructional), the iPad, websites linked to the CCSS and math curriculum frameworks, PowerPoints, and the document camera. The instructor demonstrated effective teaching with technology, pedagogical strategies that use technologies in constructive ways to teach math content at the elementary level, how technology can be used to redress some of the difficulties students have learning mathematics, and how technology can be used to build on prior knowledge (see Table A.2). In other words, the instructor demonstrated and exemplified the elements of TPACK in action. Assignments in the math methods course were designed to provide opportunities for the pre-service teachers to practice, experience, and use instructional technologies to effectively teach mathematics. As noted earlier in the interviews, the pre-service teachers understood how the instructors' modeling was valuable to their learning and would have a positive effect on their future teaching of mathematics and their students' learning.

The pre-service teachers used instructional technologies, including videos (instructional), iPad apps/websites, and the document camera, in various assignments and throughout the math methods course. Video technology generally was a YouTube video. These websites and apps provided different models/tools for the pre-service teachers to use to teach the mathematical concepts to their current as well as to their future students. The document camera, along with manipulatives, was another instructional technology used by the pre-service teachers. As noted in their interviews, often the pre-service teachers saw a direct link between their learning now and their future teaching with technology in regard to having a positive effect on their students' learning of mathematics.

These pre-service teachers showed confidence in their mathematics knowledge, pedagogies linked to the CCSS, the Massachusetts Curriculum Frameworks for Mathematics, and the Standards of Mathematical Practice, and instructional approaches that integrated technology and other tools (manipulatives) to address their students' various learning styles.

Table 5.1: Summary of Findings/Domains of Interest for Site 2

	Instructor	Students	Observations	Artifacts
Videos	x	x	x	x
Websites	x	x	x	x
PowerPoints	x		x	x
iPad Apps	x	x	x	x
Document Camera	x	x	x	
Other: Projector with Apple TV/iPad	x	x	x	

Note. X represents the presence of the instructional technology.

Table 5.2: Technology and Meeting the Mass. Standards for Mathematical Practice for Site 2

	1. Make sense of problems and persevere in solving them	2. Reason abstractly and quantitatively	3. Construct viable arguments and critique the reasoning of others	4. Model with mathematics	5. Use appropriate tools strategic-ally	6. Attend to precision	7. Look for and make use of structure	8. Look for and express regularity in repeated reasoning
Videos	x	x	x	x	x	x	x	x
Websites	x	x	x	x	x	x	x	x
PowerPoints	x	x		x	x		x	
iPad apps	x	x	x	x	x	x	x	x
Document Camera	x	x	x	x	x		x	x
Other	x	x		x	x		x	

Source. Massachusetts Department of Elementary and Secondary Education (2011).

Table 5.3: Instructor Using Video Technology from Observations for Site 2

O1: March 31	**O2: April 7**	**O3: April 14**	**O4: April 21**	**O5: April 28**	**O6: May 5**
"Richard Hatch on Aussie Who Wants to Be a Millionaire": http://www.youtube.com/watch?v=z0HpUgEVjGU "Patricia Eaton Fails at Math–Funny–Who Wants to Be a Millionaire": http://youtube.com/watch?v=Rn_OhPKBjB0 DVD accompaniment, *First Steps in Mathematics: Number*, book by Western Australia Department of Education "Math Instructional Videos" (on place values, number lines): https://learnzillion.com/resources/99913-math-instructional-videos (also on YouTube) "Using Base 10 Blocks to Model Long Division": http://www.youtube.com/watch?v=8IXAqXGDMXw "Division on a Number Line": http://www.youtube.com/watch?v=_K_EcLUnvGk "Millionaire—Math Is Hard": http://www.youtube.com/watch?v=7nG44X0LkR4	"Bianca's Chocolate Dilemma" (fractions) http://www.pbslearningmedia.org/about/products/teachers/	"Division of Fractions": https://learnzillion.com/resources/99913-math-instructional-videos (also on YouTube)		"Math Antics—Perimeter": http://mathantics.com/section/lesson-video/perimeter	

Table 5.4: Instructor Using iPad Apps from Observations for Site 2

O1: March 31	O2: April 7	O3: April 14	O4: April 21	O5: April 28	O6: May 5
Plickers app Number Pieces app Explain Everything app Number Line app	VersaMate app Oh No Fractions! app Splash Math Grade Three app NCTM Illuminations Math apps Math Your Teacher app Mathmateer app MathLand app Duprix Digital Brainwash app Montessori Geometry app Geoboard app (by the Math Learning Center)				

Table 5.5: Resource – List of iPad Apps by Grade Level

Kindergarten	First Grade	Second Grade	Third Grade	Fourth Grade
I Learn with Math	10 Frame Fill	Math Bugs	Amazing Time	Math Slide 1000
Concentration	Concentration	Coin Genius	Splash Math Grade 3	Splash Math Grade 4
Preschool Numbers	Preschool Numbers (lion level)	Money Bingo	Grade 3 Math (first step)	VersaMate
Native Numbers	Native Numbers	Math Ward Problems	Banana Math (hard)	Oh No Fractions!
Todo Math	Todo Math	Splash Math Grade 2	VersaMate (identifying fractions)	Chicken Coop Fractions
Grade 1 Math (first step)	Grade 1 Math	10 Frame Fill (difficult setting)	Math Bugs	Fractions on a Number Line
Animal Math	Line em Up	Tell Time Lite	zMath Grade 3	
10 Frame Fill	Number Magic 2	Kid's Clock	Math Slide 1000	
Kindergarten Splash Math	Math Slide 100	Number Lines		
Math Up K	Addimals	The Counting Game		
Line em Up	Animal Math			
Number Magic 2	Splash Math			
Math Slide 100	Dino Math			

Table 5.6: Resource – List of iPad Apps by Topic

Fact Fluency: Addition and Subtraction	Fact Fluency: Multiplication and Division	Teaching Tools: Multiplication and Division	Challenging Math
Splash Math Bingo	Splash Math Bingo	Number Line	Five-O
Math Dots (any theme)	Mathmateer	Number Rack	Sequential
Mathmateer	Sushi Monster	Number Pieces	2048
Sushi Monster	Penguin Math	Number Pieces Basic	5 Dice
Penguin Math	Multiples	Base 10	Pick-a-Path
Addimals	Fast Fact Math	Graph Cubes	MathLands
Fast Facts Early Add	Number Bubbles	ShowMe	Champions
Fast Facts Math	Math Circus	Notability	beHEXed (free)
Number Bubbles	Math Slide (x/÷)	Subtraction Table	Tangrams
Math Slide (+/-)			KENKEN
Mental Math- Addition and Subtraction			Zentominole
			Duprix

Table 5.7: Instructor Using Websites from Observations for Site 2

O1: March 31	**O2: April 7**	**O3: April 14**	**O4: April 21**	**O5: April 28**	**O6: May 5**
"Enriching Mathematics": http://nrich.maths.org Mathematics Georgia Standards of Excellence: https://www.georgiastandards.org/Georgia-Standards/Pages/Math.aspx	University of Auckland, Faculty of Education and Social Work: Team Solutions: http://www.education.auckland.ac.nz/en/about/professional-development/team-solutions-home.html Find Grampy: http://www.visualfractions.com/Games.htm Thinking Blocks: http://www.mathplayground.com/thinkingblocks.html Mathwire: http://mathwire.com/whohas/whohas.html Teams Distance Learning Math (LA County Office of Education): http://www.teachingchannel.org	ETA Real-World Manipulatives: http://www.hand2mind.com National Library of Virtual Manipulatives: http://nlvm.usu.edu/en/nav/vlibrary.html	Thinking Blocks (Ratios): http://www.mathplayground.com/thinkingblocks.html Give the Dog a Bone 100's Chart: http://www.primarygames.co.uk/pg2/dogbone/gamebone.html	National Council of Teachers of Mathematics Lessons: http://illuminations.nctm.org K–5 Math Teaching Resources: http://www.k-5mathteachingresources.com/ BBC Site: http://resources.woodlands-junior.kent.sch.uk/maths/ Links to Math Games and Lessons: http://www.adaptemind.com/ abcteach: http://www.abcteach.com (in multiple languages)	Shut the Box Dice Game: http://www.shut-the-box.net/ Would You Rather Math Routines? http://www.wyrmath.com

Table 5.8: Instructor Using Document Camera and Other Technology from Observations for Site 2

Date	Document Camera	Other: Projector with Apple TV/iPad
O1: March 31	Read aloud: *Amanda Bean's Amazing Dream* by Cindy Neuschwander Array Model for Multiplication (Figure 15)	Agenda PowerPoint
O2: April 7	Real-World Problem: I get paid	Agenda PowerPoint
O3: April 14		Agenda PowerPoint
O4: April 21	Review of last week's quiz questions on geometry	Agenda PowerPoint
O4: April 28	Anglegs: Shape Manipulatives Read aloud: *Sir Cumference* by Cindy Neuschwander	Agenda PowerPoint
O6: May 5		Agenda PowerPoint

Table 5.9: Instructor and TPACK

	O1: March 31	O2: April 7	O3: April 14
Content Knowledge (CK)	Place value, multiplication, division	Fractions	Fractions
Pedagogical Knowledge (PK)	Building on prior knowledge Understanding versus memorization	Language before symbols Small-group activities at different workstations Different learning styles	Visual fraction models Open-ended conversations
Technological Knowledge (TK)	iPad apps Videos Websites Document camera Apple TV/projector	iPad apps Videos Websites Document camera Apple TV/projector	Videos Websites Apple TV/projector
Pedagogical Content Knowledge (PCK)	Strategies and models to teach multiplication and division Manipulatives Misconceptions	Strategies and models to teach fractions Manipulatives Misconceptions	Strategies, models, and routines to teach fractions Manipulatives Multiple examples
Technological Pedagogical and Content Knowledge (TPACK)	Modeling using iPad, document camera Exemplar teaching on videos – mathematical models	Document camera – using mathematical models and manipulatives Different learning styles	Exemplar teaching on videos – mathematical models Websites – using mathematical models and manipulatives

Continued on next page

Table 5.9 continued

	O4: April 21	O5: April 28	O6: May 5
Content Knowledge (CK)	Decimals, percents, numeracy, fractions, geometry	Geometry	Math operations, numeracy, place value,
Pedagogical Knowledge (PK)	Review of quiz questions Revisit of fractions Different learning styles	Vocabulary for ELLs Visuals in classroom Assessment	Transition routines Visual models
Technological Knowledge (TK)	Websites Document camera Apple TV/projector	Videos Websites Apple TV/projector	Websites Apple TV/projector
Pedagogical Content Knowledge (PCK)	Strategies and models to teach decimals, percentages, fractions Manipulatives	Strategies and models to teach geometry Manipulatives Real-world problems Meaningful math – rigor, relevance, reasoning	Strategies and routines Real-world problems Manipulatives
Technological Pedagogical and Content Knowledge (TPACK)	Websites – using mathematical models and manipulatives Different learning styles	Document camera and websites – using mathematical models and manipulatives	

Table 5.10: Instructor and TPACK Summary

Content Knowledge (CK)	**Pedagogical Knowledge (PK)**	**Technological Knowledge (TK)**	**Pedagogical Content Knowledge (PCK)**	**Technological Pedagogical and Content Knowledge (TPACK)**
Fractions, decimals, and percents Numeracy Operations: addition, subtraction, multiplication, division Geometry: area, perimeter	Development of syllabus and assignments High-quality and coherent instruction Instructional strategies that support learning and growth of all students, including visual learners	DVD videos YouTube videos iPad apps Document camera Websites – resources, activities, sample lessons	Multiple strategies to teach a math concept like multiplication with two-digit numbers Mathematical models Mathematical manipulatives Understanding misconceptions	DVD teaching and misconceptions Websites and instructional videos – different learning styles Modeling using iPad, document camera, websites Videos showing methods of teaching certain math concepts Videos , websites, apps – using mathematical models and manipulatives Document camera – mathematical models and manipulatives

Table 5.11: Frequency of Instructor Using Technology from Observations for Site 2

	O1: March 31	**O2: April 7**	**O3: April 14**	**O4: April 21**	**O5: April 28**	**O6: May 5**
Videos	7X	X	X		X	
Websites	X	5X	4X	2X	5X	2X
iPad Apps	4X	7X				
Document Camera	2X	X			2X	
Other: Projector with Apple TV/iPad	2X	2X	2X	2X	2X	2X

Note. X represents the presence of the instructional technology.

Table 5.12: Alignment of Massachusetts Teacher Standard 7.08.2 with Course Objectives and Assessment

Massachusetts Teacher Standard	Course Objective	Assessment
Standard 7.08.2 – Professional Standards for Teachers **e. Curriculum, Planning, and Assessment:** Promotes the learning and growth of all students by providing high-quality and coherent instruction, designing and administering authentic and meaningful student assessments, analyzing student performance and growth data, using this data to improve instruction, providing students with constructive feedback on an on-going basis, and continuously refining learning objectives. **f. Teaching All Students:** Promotes the learning and growth of all students through instructional practices that establish high expectations, create a safe and effective classroom environment, and demonstrate cultural proficiency. **g. Family and Community Engagement:** Promotes the learning and growth of all students through effective partnerships with families, caregivers, community members, and organizations. **h. Professional Culture:** Promotes the learning and growth of all students through ethical, culturally proficient, skilled, and collaborative practice.	All students will write reflectively about theories of teaching mathematics, instructional methodologies, developmentally appropriate practices, and teaching for understanding.	Journal writing Mathematics Autobiography Thematic project/unit – PrePracticum Final exam
	All students will use developmentally appropriate methodologies to teach current research-based mathematics to small-group and whole-class elementary students.	Class participation Individual teaching lesson Thematic project/unit – PrePracticum Real-world math presentation Final exam
	All students will create lessons based on national and state standards (National Council of Teachers of Mathematics and the 2011 Massachusetts Curriculum Frameworks for Mathematics).	Class participation Individual teaching lesson Thematic project/unit – PrePracticum Final exam
	All students will write and deliver effective lesson plans with measurable outcomes.	Journal writing Class participation Individual teaching plan Real-world math presentation Thematic project/unit – PrePracticum Final exam
	All students will conduct various means of assessment (pre-assessment, formative and summative) and use the results of these assessments to make instructional decisions about lesson planning and teaching.	Journal writing Class participation Individual teaching plan Real-world math presentation Thematic project/unit – PrePracticum Final exam

Source. Massachusetts Department of Elementary and Secondary Education (2017) and elementary math methods course syllabus.

Table 5.13: Site 2 Instructor and TSAT Standard 3 – Early Technology Mastery Level

	Technology Skill	Site 2 Instructor	Site 2 Data Sources
A3.1	Discuss current best practices on teaching and learning with technology in order to plan rich learning environments and experiences.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
A3.2	Use technology to gather curriculum-specific information from online and/or local digital sources.	√	A2.14, A2.15, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
A3.3	Integrate technology into the curriculum of one's subject and/or grade level with assistance from a coach, mentor, or other staff member.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
A3.4	Use digital and online tools to communicate with teachers, parents, and other stakeholders and to create/distribute classroom materials.	√	A2.1, A2.11
A3.5	Identify your personal technology professional development needs.	√	N2.8
		100%	

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

√ represents the presence of the technology skill.

Table 5.14: Site 2 Instructor and TSAT Standard 3 – Developing Mastery Level

	Technology Skill	Site 2 Instructor	Site 2 Data Sources
B3.1	Design and develop lessons and activities that integrate technology into a variety of instructional settings for all students.	√	A2.1, A2.10, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
B3.2	Use appropriate technology to differentiate instruction (e.g., multimedia presentations, concept maps) for all learners.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
B3.3	Identify and locate technology resources, including online curriculum resources (Massachusetts Curriculum Frameworks and/or district curriculum guides), for planning.	√	A2.1, A2.11, A2.14, A2.15, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
B3.4	Manage student technology activities to optimize learning with available resources (e.g., in a one-computer classroom, a computer lab, or with portable/wireless technology).	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
B3.5	Use applications (spreadsheets, databases, etc.) to organize curriculum-specific information into charts, tables, and diagrams.	√	A2.1, A2.11, A2.14, A2.15
B3.6	Create multimedia presentations to communicate curriculum content.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
B3.7	Integrate electronic research results into classroom instruction with proper citations as appropriate to the grade level.	√	A2.1, A2.11, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
B3.8	Locate and participate in appropriate technology professional development activities offered by the district, local college/university, or online provider.	√	N2.4
		100%	

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

√ represents the presence of the technology skill.

Table 5.15: Site 2 Instructor and TSAT Standard 3 – Proficient Mastery Level

	Technology Skill	Site 2 Instructor	Site 2 Data Sources
C3.1	Plan for the management of technology resources within the context of learning activities (e.g., schedule use of computer lab, wireless laptops, whiteboard).	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.2	Evaluate technology resources, including online resources for accuracy and suitability, for your curriculum area and the students you teach.	√	A2.12, A2.14, A2.15, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.3	Identify and discuss the technology proficiencies needed in the workplace, as well as strategies for acquiring these proficiencies.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.4	Use appropriate technology tools to enhance your curriculum (e.g., digital projectors, wireless laptops, handhelds, environmental probes).	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.5	Facilitate technology-enhanced lessons that address content standards and student technology literacy standards while addressing a variety of learning styles.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.6	Use technology resources to collect and analyze data, interpret results, and communicate findings to improve instructional practice and maximize student learning.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.7	Identify and evaluate developing technologies as they relate to your subject area, grade level, and student population.	√	A2.14, A2.15, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.8	Assess student learning using a variety of district, school, or individual technology tools and strategies (e.g., the state Data Warehouse, progress spreadsheets or commercial gradebook applications).	√	A2.10, A2.12, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.9	Provide assistance to colleagues in using multimedia presentations, WebQuests, and other technology-rich lessons in the classroom.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.10	Manipulate data using charting tools and graphic organizers (e.g., concept mapping, outlining software) to connect ideas and organize information.		BLANK
C3.11	Use electronic communication tools (e.g., message boards, email, virtual classrooms) to enhance teaching and learning.	√	A2.1, A2.11

Continued on next page

Table 5.15 continued

C3.12	Use the Internet to network with other teachers and learn about effective use of technology in teaching your subject(s).	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.13	Explain and correctly use terms related to online learning (e.g., upload, download, forum, journal, post, thread, intranet, drop box, account).	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
C3.14	Facilitate student use of online tools (e.g., blogs, wikis, message boards) to gather and share information collaboratively.	√	A2.10, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
		93%	

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

√ represents the presence of the technology skill.

Table 5.16: Site 2 Instructor and TSAT Standard 3 – Advanced Mastery Level

	Technology Skill	Site 2 Instructor	Site 2 Data Sources
D3.1	Routinely and rigorously identify, evaluate, and apply emerging technologies as they relate to teaching and learning.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
D3.2	Use specialized technology tools for problem solving, decision making, and creativity (e.g., simulation software, geographic information systems, dynamic geometric software, art and music composition software).		BLANK
D3.3	Develop tools and online content (e.g., webpages, blogs, wikis, mailing lists) for instruction and communication among students and faculty.	√	A2.11
D3.4	Use technology (e.g., applets that require the use of logic to solve problems) to challenge students to develop higher-order thinking skills and creativity.		BLANK
D3.5	Plan and implement collaborative projects with other classrooms or schools using interactive tools (e.g., email, discussion forums, groupware, interactive websites, VoIP, videoconferencing).	√	N2.8
D3.6	Present ideas using the most appropriate communications technologies (e.g., multimedia presentations, webpages, desktop-published documents).	√	A2.11, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
D3.7	Distinguish between effective and ineffective design and presentation in electronic format (e.g., websites, multimedia, charts).	√	A2.10, A2.14, A2.15, N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
D3.8	Explain and demonstrate the use of metadata (e.g., tagging, EXIF) to help students and teachers organize information on their computers and/or the Internet.		BLANK
D3.9	Design and deliver effective staff development in technology and its integration into the curriculum.	√	N2.2, N2.3, N2.4, N2.5, N2.6, N2.9
		67%	

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

√ represents the presence of the technology skill.

Table 5.17: Enhancing Math Instruction Using Apps – Student Edwina

App	Splash Math Grade 3	Math Slide 1000	Banana Math	VersaMate	Amazing Time
CCSS Alignment	3.OA, 3.NBT, 3.NF, 3.MD, 3.G	Numbers and operation in Base 10	3.OA, 3.NBT, 3.NF, 3.MD, 3.G	3.NF	Time
Content Focus	Place value, number sense, addition, subtraction, multiplication, division, time, money, fractions, geometry	Place value, identifying numbers within a thousand, expanded form, words, addition and subtraction	Numbers, addition, subtraction, number sentences, algebra, geometry, money, time	Fractions, equivalent, compare and order, add and subtract, multiply and divide	Make a clock, set hour and minute hands, digital clocks
Engagement Level	High	Not high	High	High	Not high
Tracking of Results or Reports	Detailed tracking	None	Only immediate assessment	None	None
Possible Uses in Lessons (PCK)	Used to review and assess different units individually or whole class	Have students play together creating a competition	Real-world problems, chalkboard resource to write out algorithm	Great way to portray different ways fractions can be represented	Introduce unit on time then break students into groups for more problems
Strength	Detailed reports	Thorough coverage of numbers and operations	Teacher can adjust difficulty level for each topic	Immediate support for wrong answer	Demonstration of time and problems
Weakness	Subscription fee	No progress reports	No reports	No reports	No word problems
Overall Rating	Five stars	Three stars	Three stars	Four stars	Two stars

Table 5.18: Enhancing Math Instruction Using Apps – Student Agatha

App	iTooch by eduPad	Grade 3 Math	Splash Math Grade 3	Montessori Geometry	Geoboard
CCSS Alignment	Perimeter and quadrilaterals	Perimeter, area, quadrilaterals	All aspects of third-grade math	Shapes and quadrilaterals	Quadrilaterals
Content Focus	Geometry – polygons, quadrilaterals, perimeter	Geometry	Geometry – triangles, shapes	Identifying shapes as quadrilaterals	Quadrilaterals, area
Engagement Level	High	High	High	High	Not high
Tracking of Results or Reports	Only the most recent score	Four students at a time	Emails to parent and teacher	None	None
Possible Uses in Lessons (PCK)	Used in math stations	Differentiated instruction	Used in math stations, homework	Used to learn different shapes in very specific ways	Used as a geoboard to write area equations
Strength	Timed tests, lesson summary	Students create avatar	Questions can be read aloud	2-D and 3-D shapes, real-life shapes	Ability to write equations
Weakness	Fee after three lessons	Not good assessment tool	No explanation for wrong answers	Fee $3.99	Need instruction before use
Overall Rating	Four stars	Three stars	Four stars	Five stars	Two stars

Table 5.19: Enhancing Math Instruction Using Apps – Student Gemma

App	Oh No Fractions!	Splash Math	Xtra Math	Math Playground Thinking Blocks	Your Teacher
CCSS Alignment	4.NF: numbers and operations, fractions	4.NF: numbers and operations, fractions	4.OA: operations and algebraic thinking	4.OA, 4.NF	5.OA, 5.NBT, 5.NF, 5.MD, 5.G
Content Focus	Comparing fractions, multiplying, adding, subtracting, and dividing fractions	Comparing fractions, multiplying, adding, subtracting, dividing fractions, number sense, mixed numbers	Numbers, addition, subtraction, number sentences, algebra, geometry, money, time	Part-whole relationships	Formula and algorithms
Engagement Level	High	Very high	Very low	High	High
Tracking of Results or Reports	Email status report – all results	Separate portal for parents	Email to parents	Continuous tracking	Results shown but not tracked
Possible Uses in Lessons (PCK)	Used in center when students finish early, compete against others	Used at home or in class, used in centers for practice	Strictly drill and kill, used for homework	Modeling with thinking blocks, homework	Used independently for examples, practice, challenge, self-test, written worksheets
Strength	Stays on problem until correct answer	Different types of feedback	Practice of math facts	Starts with word problems, visual	Audio, visual, written, human help and feedback
Weakness	Rather plain visually	Not enough context in problems	Rather plain	No reports to parents	No bells and whistles
Overall Rating	Four stars	Four and a half stars	Two stars	Five stars	Four stars

Table 5.20: Enhancing Math Instruction Using Apps – Student Harley

App	Mathmateer	Amazing Time	Math Scale 1000	Pick-a-Path	VersaMate
CCSS Alignment	1.OA, 5.NBT	2.MD, 3.NF	3.NBT	Operations Grades 3–5	3.NF
Content Focus	Money, numbers and operations, time, basic geometry	Time on digital clocks, fractions	Addition, subtraction within 1,000	Operations using fractions, decimals, exponents	Fractions: equivalent, compare
Engagement Level	High	Only in the beginning	High	High	High
Tracking of Results or Reports	Yes	None	Tracks time	Tracks progress	None
Possible Uses in Lessons (PCK)	Used for reinforcement of math facts	Used for reinforcement, explains how to tell time	Small-group lesson	Used as a whole group for new concept	Used as students progress through fractions
Strength	Playability, covers a lot of material	Reinforcement, explanations	Various levels, multiplayer	Solid content	For all levels of fractional understanding
Weakness	Physics of rocket	Slightly repetitive	Players can cheat to get correct answer	Can be solved by process of elimination	No reports
Overall Rating	Four and a half stars	Three stars	Three and a half stars	Three stars	Four stars

Table 5.21: Enhancing Math Instruction Using Apps – Student Cassandra

App	Splash Math Grade 2	5 Dice	Mathlands	Digital Brainwash	Math Regrouping
CCSS Alignment	2.OA, 2.NBT	5.OA	3.OA, 6.G	4.NBT	2.OA, 2.NBT
Content Focus	Place value, number sense, addition, subtraction, multiplication, division, time, money, measurement, data, geometry	Addition, subtraction, multiplication, division, parentheses to write equation	Number sense, logic, geometry, word problems	Math skills and fluency in multiplication, place value, operations	Math regrouping, addition and subtraction within 20, 100, and 1,000
Engagement Level	High	High	High	High	High
Tracking of Results or Reports	Email to parents or teachers	None	None	Detailed but not sent to teacher	None
Possible Uses in Lessons (PCK)	Used to for test practice with scratch paper feature	Used to reinforce order of operations, whole class or small groups	Used to introduce math word problems	Used in small groups or individually to reinforce multiplication skills	Used to practice and have students show work
Strength	Alignment and very engaging	Reinforcement of order of operations	Different levels, visual	Good alternative to math drills on paper	Demonstration of time and problems
Weakness	None	Low-tech graphics	No reports	Report not sent to teacher	No tracking
Overall Rating	Five stars	Four stars	Five stars	N/A	N/A

Table 5.22: Enhancing Math Instruction Using Websites – Students Edwina and Agatha

	Student Edwina		Student Agatha	
Website	MathWire	NCTM Illuminations	Mathplayground Geoboard	K–5 math teaching resources
CCSS Alignment	Teacher selects standard	Searchable by standard	Third-grade solving area and perimeter problems	Third-grade geometry
Content Focus	Numbers and operations, algebra, geometry, measurement, data and probability	Based on grade level and math unit	Shapes, area and perimeter	Geometry, such as quadrilaterals, comparing shapes like square and rhombus
Engagement Level	High	High	High	High
Tracking of Results or Reports	N/A	N/A	None	Only through printed materials
Possible Uses in Lessons (PCK)	Lesson ideas, money activity in which students calculate cost of own name	Lesson plans with real-world problems	Works like a real geoboard to calculate area and perimeter, students could use in stations	Specific lessons that can be used for pre-assessment and assessment
Strength	Real-world problems, ideas for lessons and assessments	Ideas and resources, real-world problem modeling	Works like a real geoboard	Printable materials to correspond with lessons
Weakness	None	None	No reporting	Designed for teachers
Overall Rating	Five stars	Five stars	Two stars	Four stars

Table 5.23: Enhancing Math Instruction Using Websites – Students Gemma and Harley

	Student Gemma		Student Harley	
Website	Cool Math 4 Kids	Sheppard Software	Amy's Electronic Classroom	Math Playground
CCSS Alignment	4.NBT, 4.NF	4.NBT, 4.NF	Algebra, geometry, number sense Grades 3–8	Many areas
Content Focus	Numbers and operations in base 10, fractions	Based on grade level and math unit	Algebra, geometry, number sense	Addition, subtraction, multiplication, division, fractions, decimals, geometry, ratios, integers
Engagement Level	High	High	Not high	High
Tracking of Results or Reports	Parent tools but not tracking	Email of results of game only	N/A	Only through printed materials
Possible Uses in Lessons (PCK)	Independent or in groups, web quest, e-portfolios	Reinforcement of skill in class or homework	Alternative lessons to ones in textbooks	Used for homework, whole-class lesson
Strength	Lots of activities, skills explained	Volume of games, tutorial and review of content	Provides lesson plan ideas and activities	Use of virtual manipulatives, wide ranges of topics and games
Weakness	Too much going on	None	Links don't work	Not very flashy
Overall Rating	Three stars	Four stars	Two stars	Four stars

Table 5.24: Enhancing Math Instruction Using Websites – Student Cassandra

	Student Cassandra	
Website	MathWire	Amy's Electronic Classrooms
CCSS Alignment	2.0A, 2.NBT	3.OA, 4.NBT
Content Focus	Addition, subtraction, place value, count within 1,000	Geometry, algebra, number sense
Engagement Level	High	High
Tracking of Results or Reports	None	None
Possible Uses in Lessons (PCK)	Used in seasonal activities, templates	Used as guided lesson, student interactive activities
Strength	Learn through exploration, downloadable templates, links	Variety of games, different levels
Weakness	None	None
Overall Rating	Five stars	N/A

Table 5.25: Alignment of Massachusetts Teacher Standard 7.06.7 with Course Objectives and Assessment

Massachusetts Teacher Standard	Course Objective	Assessment
Standard 7.06.7 – Subject matter knowledge requirements for teachers: Mathematics. c. Basic principles and concepts important for teaching elementary school mathematics in the following areas: 1. Number and operations (the foundation of topics in 603 CMR 7.06 (7) (b) 2. A. ii. – iv.). 2. Functions and algebra. 3. Geometry and measurement. 4. Statistics and probability. d. Candidates shall demonstrate that they possess both fundamental computation skills and comprehensive, in-depth understanding of K-8 mathematics. They must demonstrate not only that they know how to do elementary mathematics, but that they understand and can explain to students, in multiple ways, why it makes sense.	• All students will gain confidence and flexibility to use mathematics meaningfully. • All students will engage in open-ended, real-world problem solving both as a learner and as a leader of mathematics. • All student will use multiple approaches to investigate and understand mathematical concepts and processes that are critical in developing elementary students' mathematical thinking. • All students will incorporate writing in mathematics using techniques designed to elaborate and explain mathematical thinking in a clear, written manner. • Examine and critique common misconceptions students have in their mathematical thinking.	Class participation Content quizzes Problem presentation Individual teaching lesson Real-world math presentation Final exam

Source. Massachusetts Department of Elementary and Secondary Education (2017) and elementary math methods course syllabus.

Table 5.26: Technology Use in Student Lesson Plan Presentations

Student/Math Topic	Videos	Websites	Apps	Document Camera
Gemma: Metric Mass		Tales of Frogs and Cupcakes: http://frogsandcupcakes.blogspot.com/		
Agatha: Place Value				Groupings of five and ten with beads, ten frames, buttons
Harley: Fractions/Decimals				Graphic organizer for "Trash Can Basketball"
Edwina: Tangrams	"A Sage's Journey: The Story of Tangrams": https://www.youtube.com/watch?v=X5mc-dkYLfI			Rearranged tangrams to make different shapes/pictures
Cassandra: Place Value	None	None	None	None
Danielle: Fractions	None	None	None	None
Brittany: Measurement		Georgia Department of Education: http://ccgpsmathematicsk-5.wikispaces.com/		Manipulatives: ladybug ruler, yardstick, meter stick
Faith: Polygons		Shape search activity, Georgia Department of Education: http://ccgpsmathematicsk-5.wikispaces.com/		Read aloud: *Greedy Triangle* by Marilyn Burns
Ian: Decimals, Percents, Fractions	None	None	None	None

Table 5.27: Frequency of Technology Use in Student Lesson Plan Presentations

Videos	Websites	Apps	Document Camera
1X	3X		5X

Note. X represents the frequency of the technology used.

Table 5.28: Technology Use in Student Presentations – Math in the Real World

	Student Group 1: Fourth Grade	**Student Group 2: Third to Fifth Grades**	**Student Group 3: Pre–K to First Grade**
Document Camera		Demonstrated eating cookies problem How many cookies did Dad bake in all? Used plate and chips as concrete demonstration	Demonstrated "Pets, Pets, Pets" problem – Dogs and Birds 10 Legs Altogether: How Many Pets? Used snap cubes, ten frames, two different-colored chips

Table 5.29: Frequency of Technology Use in Student Presentations – Math in the Real World

	Student Group 1: Fourth Grade	**Student Group 2: Third to Fifth Grades**	**Student Group 3: Pre–K to First Grade**
Document Camera	None	2X	X

Note. X represents the frequency of the technology used.

Table 5.30: Students' Lesson Plan Presentations and TPACK

Content Knowledge (CK)	Pedagogical Knowledge (PK)	Technological Knowledge (TK)	Pedagogical Content Knowledge (PCK)	Technological Pedagogical and Content Knowledge (TPACK)
Fractions, decimals, and percents Place value Geometry: shapes, polygons Measurements	Lesson planning High-quality and coherent instruction Instructional strategies that support learning and growth of all students, including visual learners	YouTube video Document camera Websites: resources, activities, sample lessons	Multiple strategies to teach a math concept like place value Mathematical models Mathematical manipulatives	Video showing geometric shapes with tangrams Websites: using mathematical models and manipulatives Document camera: read aloud math literature book, mathematical models and manipulatives

Table 5.31: Student Group 3 Presentation and TPACK

Content Knowledge (CK)	Pedagogical Knowledge (PK)	Technological Knowledge (TK)	Pedagogical Content Knowledge (PCK)	Technological Pedagogical and Content Knowledge (TPACK)
Number sense, addition Massachusetts Content Strands (see Table 5.29)	Lesson planning High-quality and coherent instruction Instructional strategies (i.e., hands-on activities) that support learning and growth of all students Curriculum development that addresses different learning styles	Document camera	Addition strategies like regrouping Mathematical models Mathematical manipulatives	Document camera: demonstrated "Pets, Pets, Pets" problem – Dogs and Birds 10 Legs Altogether: How Many Pets? Document camera: snap cubes, ten frames, two different-colored chips as concrete demonstration

Table 5.32: Student Group 2 Presentation and TPACK

Content Knowledge (CK)	Pedagogical Knowledge (PK)	Technological Knowledge (TK)	Pedagogical Content Knowledge (PCK)	Technological Pedagogical and Content Knowledge (TPACK)
Number sense, addition, subtraction Massachusetts Content Strands (see Table 5.28)	Lesson planning High-quality and coherent instruction Instructional strategies (i.e., hands-on activities) that support learning and growth of all students Curriculum development that addresses different learning styles	Document camera	Addition strategies like regrouping Subtraction strategies like constant difference Strategies based on the relationship between addition and subtraction Mathematical models Mathematical manipulatives	Document camera: demonstrated eating cookies problem Document camera: How many Cookies did Dad bake in all? Used plate and chips as concrete demonstration

Table 5.33: Student Group 1 Presentation and TPACK

Content Knowledge (CK)	**Pedagogical Knowledge (PK)**	**Technological Knowledge (TK)**	**Pedagogical Content Knowledge (PCK)**	**Technological Pedagogical and Content Knowledge (TPACK)**
Multiplication and division Massachusetts Content Strands (see Table 5.30)	Lesson planning High-quality and coherent instruction Instructional strategies (i.e., hands-on activities) that support learning and growth of all students Curriculum development that addresses different learning styles	N/A	Multiplication strategies Division strategies Mathematical manipulatives	N/A

Table 5.34: TPACK in Student Group 2 Presentation Alignment with Massachusetts Mathematical Content Strands, Third to Fifth Grades

TPACK	Massachusetts Content Standards
Document camera: demonstrated eating cookies problem	Third Grade • Solve problems involving the four operations, and identify and explain patterns in arithmetic. • Use place value understanding and properties of operations to perform multi-digit arithmetic. Fourth Grade • Generate and analyze patterns. • Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic. Fifth Grade • Analyze patterns and relationships. • Write and interpret numerical expressions. • Perform operations with multi-digit whole numbers and with decimals to hundredths. • Understand the place value system.
Document camera: How many cookies did Dad bake in all? Used plate and chips as concrete demonstration	Third Grade • Represent and interpret data. • Solve problems involving the four operations, and identify and explain patterns in arithmetic. • Use place value understanding and properties of operations to perform multi-digit arithmetic. Fourth Grade • Generate and analyze patterns. • Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic. Fifth Grade • Understand the place value system. • Perform operations with multi-digit whole numbers and with decimals to hundredths. • Write and interpret numerical expressions.

Source. Massachusetts Department of Elementary and Secondary Education (2011).

Table 5.35: TPACK in Student Group 3 Presentation Alignment with Massachusetts Mathematical Content Strands, Pre–K and First Grades

TPACK	**Massachusetts Content Standards**
Document camera: demonstrated “Pets, Pets, Pets” problem – Dogs and Birds 10 Legs Altogether: How Many Pets?	Pre-K Grade • Know number names and the count sequence. • Count to tell the number of objects. • Compare numbers. • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. • Classify objects and count the number of objects in each category. First Grade • Represent and solve problems involving addition and subtraction. • Understand and apply properties of operations and the relationship between addition and subtraction. • Add and subtract within 20. • Work with addition and subtraction equations. • Extend the counting sequence. • Use place value understanding and properties of operations to add and subtract.
Document camera: snap cubes, ten frames, two different-colored chips as concrete demonstration	Pre-K Grade • Know number names and the count sequence. • Count to tell the number of objects. • Compare numbers. • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. • Classify objects and count the number of objects in each category. First Grade • Represent and solve problems involving addition and subtraction. • Understand and apply properties of operations and the relationship between addition and subtraction. • Add and subtract within 20. • Extend the counting sequence. • Represent and interpret data.

Source. Massachusetts Department of Elementary and Secondary Education (2011).

Table 5.36: TPACK in Student Group 1 Presentation Alignment with Massachusetts Mathematical Content Strands, Fourth Grade

TPACK	Massachusetts Content Standards
No technology used	Fourth Grade • Use the four operations with whole numbers to solve problems. • Generate and analyze patterns. • Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic. • Gain familiarity with factors and multiples. • Represent and interpret data.

Source. Massachusetts Department of Elementary and Secondary Education (2011).

CHAPTER 6

CROSS-CASE ANALYSIS

As mentioned previously, each research site was individually analyzed. Now analysis among the research sites will be conducted as recommended by Merriam (1998). Cross analysis will enable comparison of the research sites, leading to a greater understanding of the domain of interest (i.e., instructional technology) as a whole. Furthermore, the cross analysis will compare how the use of technology in math methods courses facilitated the learning of elementary pre-service teachers and the integration of technology into classroom instruction. The TPACK (Koehler & Mishra, 2009) conceptual framework will be used to compare and contrast the two instructors as well as the pre-service teachers at the research sites. Additionally, the Massachusetts Technology Self-Assessment Tool (TSAT) will be utilized as an instrument for comparing the instructors of the math methods courses at the two research sites (Massachusetts Department of Elementary and Secondary Education, 2010).

6.1 Research Question Two

The second research question is as follows:

2. How do faculty in the math and/or science methods course use instructional technology for enhancing teaching and learning?

6.2 Instructors and TPACK

The TPACK framework and its components were previously discussed regarding each instructor. TPACK refers to technological pedagogical and content knowledge (see Table A.2). Specifically, TPACK is knowledge of instructional strategies (PK) that integrate technologies (TK) to constructively teach content (PCK), in this case mathematics content (CK), for greater understanding of that content (see TableA.2). Now this framework will be used to highlight similarities and differences between the instructors and research sites.

6.2.1 Content Knowledge

Content knowledge is subject matter understanding of mathematics (see Table A.2). Standard 7.06.7 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations (Massachusetts Department of Elementary and Secondary Education, 2017) describes the subject matter knowledge (CK) requirements for elementary teachers specifically in mathematics: numbers and operations, functions and algebra, geometry and measurement, and statistics and probability. Fractions, decimals, and percents are part of numbers and operations but are listed separately because the instructors emphasized them in their courses. This content knowledge is consistent with the Massachusetts Curriculum Frameworks for Mathematics (Massachusetts Department of Elementary and Secondary Education, 2011). Indeed, this fact was stressed by each of the instructors, as is evident in the individual syllabi (see Table A.5).

The respective syllabi show that statistics and probability were not topics or content covered in the course at Xever (A1.1). The other syllabus (A2.1) indicates that the subject matter would be presented in class, although the researcher did not observe this during her site visits. In fact, the syllabus states that the pre-service teachers would be quizzed on this content and so presumably questions related to this content would be discussed in class. Algebraic thinking concepts were presented in the second site (A2.1, A2.11, and N2.2).

Additionally, the instructor at Xever presumably held an in-depth understanding of mathematics as she has earned an MS in mathematics degree, and she taught high school mathematics course that required advanced coursework in mathematics (N1.3). Likewise, at

Yexer, the instructor presumably held an in-depth understanding of mathematics as a K–8 math and science curriculum director and math consultant.

In other words, content knowledge is knowledge of the mathematical concepts, principles, and curriculum frameworks, along with the recognized practices and routines of acquiring that knowledge (Koehler & Mishra, 2009; Shulman, 1986). Specifically, recall that the Xever instructor used story problems to illustrate the quotative and partitive properties of division (N1.9). Similarly, bring to mind, in Figure 5.1, how the Yexer instructor used a number line to show division of 96 ÷ 6 = 16 (N2.2).

Furthermore, both instructors also displayed mathematical knowledge for teaching pre-service teachers how to teach mathematics to children. This content knowledge includes “teachers’ knowledge about the subject matter to be learned or taught” (see Table A.2). For example, teachers need to correct student answers. But, in mathematics, they also need to recognize how and why the students make errors/give incorrect answers as well as explain these errors to the students. For example, the Yexer instructor used a DVD video, the Cathy Episode, in which Cathy was incorrectly solving a multiplication problem (see Table 6.3). She explained the nature of the student error—namely, a deficit in understanding place value—even though Cathy, the student, had memorized the multiplication algorithm. She emphasized that students are not learning the “why” (N2.2). Similarly, the Xever instructor stressed the importance of understanding place value in order to multiply two digits, and she demonstrated this with two Cuisenaire rods on the document camera (N1.10). Likewise, in her parting thoughts, she stated that “giving them the algorithm without the conceptual understanding does not help students learn” (A1.11). Correspondingly, the Yexer instructor reviewed some of the geometry quiz questions and clarified the pre-service teachers’ misunderstandings/errors (N2.6).

6.2.2 Pedagogical Knowledge

Pedagogical knowledge is knowledge of how to teach that fosters student learning (see Table A.2). Standard 7.08.2 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations defines the “pedagogical and other professional knowledge and skills required of all teachers” (Massachusetts Department of Elementary and Secondary Education, 2017). Tables 4.23 and 5.12 show the alignment of the syllabi course objectives and assessments with Standard 7.08.2. Both of the instructors are experienced teachers demonstrating pedagogical knowledge. They have college teaching experience that requires curriculum planning and design. A variety of assignments/assessment tools, including lesson plans, journals, and problem/game presentations, were utilized by both instructors, as shown in Table A.6. At Yexer, the writing assignments, whether it was in a journal or part of a lesson plan, required the pre-service teachers to reflect “about theories of teaching mathematics, instructional methodologies, developmentally appropriate practices and teaching for understanding” (A2.1). The Xever instructor provided the pre-service teachers with scoring rubrics for all the assessments (A1.1). Similarly, the Yexer instructor provided a scoring rubric with their graded “Enhancing Math Instruction using Apps/Websites” assessment (A2.12). The math methods courses are graduate level, and each instructor established high expectations of the pre-service teachers appropriate to the graduate level.

Additionally, both instructors have elementary teaching experience. Although one instructor taught mathematics at the high school level, she now is an elementary math coach. The Yexer instructor has an M.Ed. degree and is a K–8 math-science curriculum director in a nearby public elementary school district. Furthermore, she is a math consultant who provides training and workshops for elementary teachers in school districts throughout the state. Both instructors were able to provide real-life examples and advice/tips from their own teaching.

Recall one of Xever instructor's parting thoughts: "Believe that all children can learn. Some children learn at different rates and in different way from others" (A1.11). In the same way, recall one of the Yexer instructor's attributes of an effective teacher: "Flexibility: ability to change your instruction to meet student needs" (A2.13). She added that pre-service teachers should "think about ELL when looking at that vocabulary" (N2.5) and addressing the needs of all learners. These are valuable tips/advice for pre-service (novice) teachers from experienced teachers. One of the instructors provided a handout on Bloom's taxonomy and how it relates to critical thinking (A1.2, A1.13) as a resource for the pre-service teachers.

6.2.3 Technological Knowledge

Technological knowledge is knowledge of using various technologies—for example, a document camera, researching and using such video sites as YouTube as well as DVD videos, and websites/iPad apps (see Table A.7). The document camera proved to be a significant and innovative technology used by both of the instructors. First, the instructors demonstrated on the document camera how to use physical manipulatives: (1) ten blocks in Figure 4.13 to solve a mathematics problem, (2) multiplication of two-digit numbers (N1.3), and (3) in Figure 5.1, the division of two-digit numbers (N2.2), while the students in the class solved the problem at their desks. Second, one of the instructors projected elementary-level math literature books through the document camera and read aloud to the class. The entire class, rather than a small group, followed along with the text and illustrations. As seen in Table 5.8, the Yexer instructor read aloud *Amanda Bean's Amazing Dream* and *Sir Cumference*. On the other hand, analysis reveals that the Xever instructor did not use the document camera in this way.

Instructional videos, from a DVD, YouTube, or website, were utilized by all the instructors. For example, videos on teaching fractions were shown, whether it's the numeracy and fractions DVDs by Sharma (N1.10) or the PBS Learning Media videos (N2.3). The latter videos were also available on YouTube. Furthermore, these videos were particularly helpful for the visual learners in the courses (N1.11, N2.7).

Websites and iPad apps were often used by the instructors to illustrate how to solve a mathematical problem using digital manipulatives. Moreover, the Yexer instructor mentioned in class that physical manipulatives “don’t travel well”—that is, they can break easily—so the digital versions are more practical (N2.2).

Furthermore, the technological knowledge of the two instructors can be discussed and analyzed by using the criteria in Standard 3 of the TSAT, Teaching and Learning with Technology (see Tables 4.11–4.14, Tables 5.13–5.16, and Table A.6). The Massachusetts Technology Literacy Standards (http://www.doe.mass.edu/odl/standards/itstandards.docx) are standards developed for students. These standards are based on the technology standards of the International Society for Technology in Education (http://www.iste.org/standards/standards/standards-for-students) and the information and communication technology literacy skills of the Partnership for 21st Century Skills (http://www.nea.org/home/34888.htm). The three Massachusetts standards are as follows (Massachusetts Department of Elementary and Secondary Education, 2008, pp. 1–2):

- Standard 1. Demonstrate proficiency in the use of computers and applications, as well as an understanding of the concepts underlying hardware, software, and connectivity.
- Standard 2. Demonstrate the responsible use of technology and an understanding of ethics and safety issues in using electronic media at home, in school, and in society.

- Standard 3. Demonstrate the ability to use technology for research, critical thinking, problem solving, decision making, communication, collaboration, creativity, and innovation.

I focused on the third standard because it relates to teaching and learning with technology. The standard elaborates further (Massachusetts Department of Elementary and Secondary Education, 2008, p. 2):

- focuses on applying a wide range of technology tools to student learning and everyday life;
- aims to ensure that students will be able to use technology to process and analyze information;
- is to help students develop skills for effective technology-based communication;
- includes the use of technology to explore and create new ideas, identify trends, and forecast possibilities; and
- aims to provide students with an awareness of how technology is used in the real world.

Furthermore, the ISTE (2012) developed technology standards for teachers. The five standards are:

- Facilitate and inspire student learning and creativity
- Design and develop digital age learning experiences and assessments
- Model digital age work and learning
- Promote and model digital citizenship and responsibility
- Engage in professional growth and leadership.

Massachusetts developed a tool for educators to assess their technology skills related to these technology standards. The Technology Self-Assessment Tool (TSAT) is divided into three standards: technology operations and concepts, ethics and safety, and teaching and learning with technology (Massachusetts Department of Elementary and Secondary Education, 2010). In addition, the assessment tool includes four levels of mastery—early, developing, proficient, and advanced—and is aligned with the technology literacy standards (Massachusetts Department of Elementary and Secondary Education, 2008, 2010).The TSAT shows the skills needed at each of the four levels of mastery—early, developing, proficient, and advanced. The first level, early, requires 100% mastery before moving to the next level. The remaining three levels require 80% mastery to move to the next level.

Each of the instructors received a 100% skill percentage at the Early Technology Mastery Level (see Table A.6). Minimally, all were able to use technology for research and communication purposes. Curriculum-specific information, in the form of websites, resource lists, and apps, were presented and shared with students (TSAT, A3.2). Videos of exemplar teaching highlighted best practices (TSAT, A3.1). Communication tools, such as email, and digital tools, such as Google Docs and PowerPoints, were utilized to communicate with students and to disseminate class materials (TSAT, A3.4).

At the Developing Technology Mastery Level, the two instructors received 100% mastery skill percentage (see Table A.6). Technology resources were identified by each of the instructors (TSAT, B3.3). Many of these resources were tied to the CCSS, the Massachusetts Mathematics Curriculum Frameworks, and the Massachusetts Mathematics Standards of Practice (see Table A.4). Additionally, both instructors used applications, such as Word and PowerPoint, to organize and share curriculum-specific information (see Tables 5.3 and 5.8). The Xever instructor also used Google Docs to share information with the pre-service teachers (see Table 5.3) (TSAT, B3.5 and B3.6).

Likewise, at the Proficient Technology Mastery Level, the mastery percentages were different, 85% and 93%, respectively (see Table A.6). Both instructors presented/discussed technology resources suitable for an elementary mathematics curriculum (TSAT, C3.2). These resources were demonstrated using technology tools, including the desktop computer, iPad, document camera, Apple TV and AirPlay, and projector (TSAT, C3.4).

Similarly, at the Advanced Technology Mastery Level, the mastery percentages were different, 56% and 67%, respectively (see Table A.6). Thus, neither instructor has the 80% needed for this mastery level. Both instructors used the appropriate communication technologies to convey their ideas (TSAT, D3.6). Multimedia presentations, such as PowerPoints, were utilized to present the class agenda and course content and to link to videos/websites and resources (see Tables 5.3 and 5.8) (TSAT, D3.6).

6.2.4 Pedagogical Content Knowledge

Pedagogical content knowledge is knowledge of instructional strategies (i.e., pedagogy) as they relate to specific content, such as mathematics (see Table A.2). Standard 7.06.7 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations states that teachers "must demonstrate not only that they know how to do elementary mathematics, but that they understand and can explain to students, in multiple ways, why it makes sense" (Massachusetts Department of Elementary and Secondary Education, 2017). Both instructors exemplified pedagogical content knowledge as they modeled instructional strategies for teaching math to the pre-service teachers. Each of the instructors employed multiple strategies to teach math concepts. Additionally, the instructors adapted and customized the instructional activities (see Table A.2). At Xever College, the instructor passed out a second-grade worksheet, rubric, and task analysis sheet on sheep and ducks (A1.9, N1.10). Several problems and sample student solutions were shown and related to how many sheep and ducks were on the farm given various numbers of legs seen by the farmer. The task analysis sheet asked the pre-service teachers to describe successful strategies used by the students and to identify how these strategies may be shared to help the whole class. Additionally, the pre-service teachers were asked to identify areas of difficulty for second graders. The instructor asked the pre-service teachers to look at the student work and think about the following question: “What does this mean for instruction and not just for a grade?” (N1.10). Similarly, at Yexer University, the instructor used multiple strategies to illustrate division of fractions. First, she played a fractions video (see Table 5.3) showing multiple examples of dividing fractions by fractions. Next she presented an example of division, ${}^{6}/_{8} \div {}^{1}/_{4}$, on the board and explained the invert-multiply strategy (see Figure 5.3). Then the instructor followed up with a custom activity titled “Bianca’s Chocolate Dilemma” (A2.2, N2.3) on sharing (dividing) her chocolate bar. The pre-service teachers then had to write two fractions: one to show each person’s share and one with a common denominator to show each person’s share.

Mathematical models were illustrated and explained. These mathematical models are different representations of the mathematic content (see Table A.2). Recall that at Xever College, Figure 4.4 depicted area models for multiplication, using Cuisenaire rods, and for rectangles. Furthermore, the pre-service teachers, such as Joanna, remarked, “The professor uses her iPad to share videos pertaining to methods of teaching certain math subjects” (N1.11). Likewise, recall that at Yexer University, in Figure 5.1, the instructor demonstrated three different models using number lines to show multiplication: proportional adjustment, in which students are more familiar with using the number line with the digit 9, and so the original number of 18 can be adjusted to 9; place value, which shows the “decomposing” of the number

18 into 10 and 8; and tidy numbers, which are such numbers as 20 and 2 that are easier to work with (N2.4).

Physical (nondigital) and/or digital manipulatives (i.e., representations) were utilized to show different ways to solve the elementary mathematics problems and equations. Bring to mind that manipulatives, such as the rekenrek and place value charts, were used to show turnaround facts and to illustrate multiplication with two digits by the Xever instructor. The National Library of Virtual Manipulatives (NLVM) website (also an app) provides mathematical activities/resources and interactive virtual manipulatives by grade level and topic (see Table 5.7). The Yexer instructor demonstrated a fraction activity through this website and the projector. She provided each pre-service teacher with a fraction card transparency and a small whiteboard. She modeled how to solve the problem $^1/_3 \times {}^2/_4$ (N2.4). The instructor worked with the virtual manipulatives; the pre-service teachers could visualize the activity and its steps. The pre-service teachers then used the fraction transparencies to follow that process and to solve the equation on the small whiteboards.

Misconceptions about mathematics were presented so that the pre-service teachers understood and could address their own mistaken beliefs (if any) as well as the thinking of their students. Recall that the Yexer instructor referred to the misconceptions as “lies my teacher told me” (N2.2). One such “lie” was “multiplication makes values larger.” The instructor addressed this misconception with this example: “2 x ½ is not a larger number than 2” (N2.2). The Xever instructor showed the Sharma DVD on numeracy, which presented misconceptions about perimeter and the area of a rectangle (N1.10).

The Massachusetts Standards of Mathematical Practice (see Table A.4) align with the pedagogical content knowledge and the instructional practice of the instructors. Mathematical models (Standard 4) and manipulatives/tools (Standard 5) were used strategically as part of the instructional techniques. The instructors demonstrated and worked with the pre-service teachers to understand the mathematical problems/equations and to persevere in solving them (Standard 1). Furthermore, after each activity and/or presentation, the Yexer instructor asked the pre-service teachers to identify the standard(s) of mathematical practice (see Table A.4) that were reflected in that activity or presentation (N2.2–2.6).

The instructor at Xever provided this bit of wisdom that comes from years of teaching math: “Productive struggle: Allow students the time, resources, and support they need to engage in the mathematics, even if it is difficult. This will help them to persevere. Giving them the algorithm without the conceptual understanding does not help students learn” (A1.11). Likewise, the Yexer instructor offered “compassion: understanding your students’ abilities and challenges” (A2.13) as an attribute of an effective teacher.

6.2.5 Technological Pedagogical Content Knowledge

TPACK is knowledge of instructional strategies (PK) that integrate technologies (TK) to constructively teach content (PCK), in this case mathematics content (CK), for greater understanding of that content (see Table A.2). These instructional strategies incorporate such technologies as YouTube videos to explain how to solve a mathematical problem with either a digital or nondigital tool, such as a double number line; websites/iPad apps that demonstrate how to use models, such arrays, to perform mathematical operations; and a document camera to demonstrate manipulatives, such as Cuisenaire rods and base 10 blocks.

TPACK is reflected in the multiple instructional technologies and strategies for understanding operations utilized by each instructor. At Yexer, the instructor started with a PowerPoint presentation that built on prior knowledge with a review of tools/manipulatives explored for addition and subtraction (see Figure 5.5, N2.2, A2.11). Next, through PowerPoint, she linked to videos: one that shows what happens when you “forget” basic math facts and one that illustrates a student using an algorithm without understanding (see Table 5.3). Then the instructor used the document camera to read aloud a math literature book as she transitioned from addition to multiplication and to demonstrate a multiplication problem using a number line. Several iPad apps and manipulatives, such as base 10 blocks, were used by the instructor to reinforce the above-mentioned concepts. Misconceptions, a progression of multiplication models, and how to “decode the language” and to think about multiplication (see Figure 5.6) were explained by the instructor. Meanwhile, the pre-service teachers worked on similar multiplication problems on small whiteboards at their desks.

Similarly, as previously shown in Figure 4.1, the Xever instructor showed the Sharma DVD, which illustrated the area model for multiplication using Cuisenaire rods, and then reinforced the model by using the document camera and Cuisenaire rods, as seen in Figure 4.2 (N1.10). She stopped the DVD at key points to reinforce some of the concepts and strategies illustrated on the DVD. Misconceptions on perimeter and area were also presented in the video. This engaging video provided visual concrete models of exemplar teaching of elementary mathematics in real classrooms. The Yexer instructor mentioned that she also has gone to lectures on fractions by Sharma.

Likewise, TPACK is manifested in the multiple instructional technologies and strategies for teaching fractions utilized by each of the instructors. At Xever, the instructional strategies started with videos: *Teaching Fractions* by Sharma (see Table 4.3.). This video provided exemplar teaching of the four parts to understand fractions in a third-grade classroom: the whole is being divided, into certain parts, equal parts, and the parts make up the whole (N1.9). Next the instructor used a clothes line activity where she asked the pre-service teachers to “hang up” the fractions on the clothes line in order of magnitude. Then the instructor used the document camera and manipulatives to illustrate and reinforce the concepts mentioned previously: number lines, paper strips, and Unifix cubes. Finally, the instructor provided story problems for the pre-service teachers to solve and asked them to generate their own story problems.

At Yexer, fraction instructional strategies started with a concrete example, “body fractions game,” in which the instructor and the pre-service teachers tried to illustrate various fractions using their body parts (N2.3). Next, through PowerPoint, she linked to websites: the University of Auckland Faculty of Education website and the National Library of Virtual Manipulatives website (see Table 5.7, N2.4). Misconceptions, such as equality and number of pieces, were explained and illustrated with real small and large pizzas. Manipulatives, such Cuisenaire rods, a clock, fraction card transparencies, and fraction towers, were used by the instructor to reinforce the above-mentioned concepts. Then, the instructor used the document camera to demonstrate a multiplication problem using a fraction number line: ¾ of 80. Finally, the instructor stressed the importance of “words first before symbols” when trying to explain fraction language (N2.3).

Furthermore, TPACK, in the instructors’ teaching, can be discussed using the criteria in Standard 3 of the TSAT, Teaching and Learning with Technology (Tables 4.11–4.14, 5.13–5.16). At the TSAT Early Technology Mastery Level, each of the instructors showed and discussed videos of exemplar teaching, highlighting best practices on teaching and learning with technology (TSAT, A3.1).

Likewise, at the TSAT Developing Technology Mastery Level, both instructors designed lessons and activities that explained how to solve a mathematical problem, with either digital and/or nondigital tools, such as number lines (TSAT, B3.1 and B3.2). Additionally, each instructor used a document camera to demonstrate manipulatives, such as Anglegs, for geometry concepts, as well as Unifix cubes and base 10 blocks for operations (TSAT, B3.1 and B3.2). Also, both instructors illustrated mathematical models, including the array model and fraction model, by using the document camera while the pre-service teachers worked on their iPads with apps or on small whiteboards at their desks. Furthermore, both instructors designed activities for the pre-service teachers that integrated technology (TSAT, B3.1). These activities included the development of lesson plans and math content presentations. Technologies, including iPad apps, instructional videos, websites, the document camera, the Smart Board, and the projector, were integrated into these activities/presentations.

Likewise, at the Proficient Technology Mastery Level, each of the instructors presented/discussed technology resources suitable for an elementary mathematics curriculum (TSAT, C3.2). Such websites as the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) and NCTM (http://illuminations.ntcm.org), as well as instructional videos, such as LearnZillion and the Sharma *Teaching Fractions* DVD, are suitable resources that provide virtual manipulatives and lesson plans, respectively, for elementary mathematics curriculum (see Tables 4.3, 5.3, 5.4, and 5.7). Additionally, the instructor at Yexer University provided a folder for each pre-service teacher that contained lists of resources: websites and apps by grade level and topic (see Tables 5.5 and 5.6). Furthermore, both sites facilitated technology-enhanced curriculum tied to math content standards (TSAT, C3.5). The pre-service teachers, at each site, had to identify the math CCSS in their lesson plans and group presentations. The various technology resources presented in class by the instructors were usually aligned with the math CCSS.

Similarly, at the Advanced Technology Mastery Level, both instructors used multimedia presentations, such as PowerPoints, to present the class agenda and course content and to link to videos/websites and resources (TSAT, D3.6). In addition, both instructors singled out effective design/presentation (TSAT, D3.7). In one case (site 1), a group was required to "re-present" their presentation because it did not meet the instructor's expectations. In both cases, the pre-service teachers reviewed and evaluated apps, websites, and/or instructional videos in assignments. Lastly, staff (i.e., the pre-service teachers) development regarding the technology integration was embraced by both instructors (TSAT, D3.9). The modeling and instructional strategies of these two instructors provided professional development of technology integration into an elementary mathematics curriculum. In essence, the entire math methods courses were designed and delivered as professional development for the pre-service teachers.

In summary, the TPACK framework illustrates similarities and some differences in the various components of the framework as reflected in the instructors' teaching and modeling at the two research sites.

6.3 Research Question One

The first research question is as follows:

1. In which ways did the use of technology in a math methods course facilitate the learning of elementary pre-service teachers and the integration of technology into classroom instruction?

The TPACK framework and its components will be used to highlight similarities and differences between the pre-service teachers and the research sites.

6.4 TPACK

6.4.1 Content Knowledge

Content knowledge is subject matter understanding of mathematics (see Table A.2). As previously mentioned in the description of the research setting, Xever (site 1) is the only one that required math content course as part of the graduate curriculum. This course is taken during the summer before the math methods course. The pre-service teachers indicated that this was critical to their understanding of math and learning how to teach math.

In contrast, the instructor at Yexer brought up pre-service teachers' lack of math content knowledge. This was an issue for many of the students. In an interview, the instructor at Yexer stated that the "challenge was to give them [the pre-service teachers] math content" (N2.8). She added that "the students are utilizing [the technology] to enhance instruction in the areas of engagement. Some, however, are meant to deepen their own content knowledge" (N2.8). One of the pre-service teachers at Yexer shared during a class break that she was going to take the math MTEL for the fourth time that weekend (N2.6).

Despite these concerns, at both sites the students demonstrated specific content knowledge during presentations in class. At Xever, student group 1 presented on the content of number sense, addition, and subtraction, and related it to content strands for the second-grade Massachusetts Curriculum Framework for Mathematics (Massachusetts Department of Elementary and Secondary Education, 2011), as shown in Table 4.20. Student group 2 presented on multiplication and division and connected to content strands for second- and sixth-grade Massachusetts Curriculum Framework for Mathematics (Massachusetts Department of Elementary and Secondary Education, 2011), as shown in Table 4.22. Student group 3 presented on fractions, decimals, and percents and linked to content strands for fifth-grade Massachusetts Curriculum Framework for Mathematics (Massachusetts Department of Elementary and Secondary Education, 2011), as shown in Table 4.17.

Similarly, at Yexer, one of the pre-service teachers, Edwina, presented a lesson on third-grade geometry using tangrams (N2.3). She illustrated an understanding of such shapes as triangles, squares, and parallelograms. She also demonstrated how the various shapes related to one another to form/rearrange into other shapes. Agatha used the document camera to demonstrate a first-grade lesson on place value (A2.3), as seen in Figure 5.7.

6.4.2 Pedagogical Knowledge

As mentioned earlier, pedagogical knowledge includes skill and know-how in planning curriculum and thereby coherent instruction (Massachusetts Professional Standards for Teachers, Standard 7.08.2). At each research site, the pre-service teachers were required to plan and design lessons in elementary mathematics. At Xever, student group 3 presented on fractions, decimals, and percents. They started with key vocabulary and then progressed to fraction strategies, such as doubling and multiples. Multiple strategies were utilized and in a progressive coherent manner, as seen in Figure 4.14 (A1.6 and N1.8). The fraction presentation illustrating the strategies and technologies is explained in greater detail in Figure 4.15.

Student group 1 presented on the following content: number sense, addition, and subtraction (see Figure 4.16). They began with number sense and then presented addition strategies. This was followed up with subtraction strategies. Thus, these pre-service teachers understood that understanding addition supports an understanding of subtraction (A1.5 and N1.4).

Student group 2 presented on multiplication and division (see Figure 4.17). The group presented multiplication strategies first and then progressed to division strategies. Thus, these pre-service teachers understood that an understanding of multiplication supports an understanding of division (A1.3 and N1.5).

Similarly, at Yexer, the pre-service teachers presented lesson plans. These lessons included the promotion of active engagement and learning. Multiple strategies were utilized and in a progressive, coherent manner. Recall the example of Harvey, who presented “Trash Can

Basketball," a lesson on fractions and decimals (N2.3). The lesson started with pairs of students making 10 paper balls. He then distributed a graphic organizer to each student to record the baskets made (A2.6). Harvey then used the document camera to record and display each student's score by group in fraction and then decimal form. This lesson is appropriate for fourth graders. In another example, Brittany presented a lesson on measurement. She distributed the worksheet "Measuring with Different Units" (A2.8). She actively engaged the students in the lesson. The students had to measure in different units such things as the width of a desk, the height of a textbook, and the length of a marker. The students had to walk around the room and record their measurements. This lesson is appropriate for second graders (N2.3).

6.4.3 Technological Knowledge

Technological knowledge is knowledge of using various technologies (see Table A.2). As previously mentioned at Xever, the pre-service teachers used the following technologies in their presentations: videos, iPad apps, a document camera, Google Docs, MS Publisher, and a projector with Apple TV (see Table 4.5). The frequency of technology use is shown in Table 5.6. However, the instructor assisted two of the groups, student groups 1 and 3, with Apple TV and the projector (N1.4 and N1.8). Similarly, at Yexer, the pre-service teachers used some of the same technologies just mentioned in their presentations: videos, websites, iPad apps, and the document camera (see Tables 5.26 and 5.28). The frequency of technology use is shown in Tables 5.27 and 5.29.

6.4.4 Pedagogical Content Knowledge

In previous sections, pedagogical content knowledge was defined as knowledge of instructional strategies (i.e., pedagogy) as they relate to specific content, such as mathematics (see Table A.2). At Xever, the pre-service teachers used a variety of models and digital/nondigital manipulatives in their presentations. As illustrated in Figure 4.3, student group 3 used the double number line showing fractions and whole numbers in their presentation of fractions, decimals, and percents (A1.6 and N1.8). Figure 4.10 illustrates student group 1 using base 10 blocks to demonstrate place value (A1.5 and N1.4). Likewise, Figure 4.8 shows the repeated addition strategy for learning multiplication (A1.3 and N1.5). Furthermore, Table 4.5 shows the groups using such manipulatives as snap cubes, ten frames, Unifix cubes, place value charts, and popsicle sticks in their presentations. These manipulatives, visual representations, and models supported the teaching and learning of mathematic content.

In the same way, at Yexer, the pre-service teachers used a variety of models and digital/nondigital manipulatives in their presentations. Recall that Edwina used tangrams to present a third-grade lesson on geometric shapes, as shown in Figure 5.8. She started the lesson with an animated video from YouTube. Then she used manipulatives (i.e., tangrams) on the document camera to illustrate different pictures of a rabbit, a mountain, and a sailboat. Next Edwina had an activity for the students in the class: draw a picture using tangrams in which you have to use all of the tangrams. The tangrams needed to touch one another but could not overlap. Then the students switched sheets with another student and repeated the drawing activity. Finally, the students compared the two drawings (N2.3). Likewise, Brittany presented a second-grade lesson on measurements using such manipulatives as a ladybug ruler, a yardstick, and a meter stick (N2.4). Agatha also used manipulatives—buttons and beads (see Figure 5.7)—and then ten frames to demonstrate a first-grade lesson place value (A2.3). Similarly, these manipulatives, visual representations, and models supported the teaching and learning of mathematic content.

6.4.5 Technological Pedagogical Content Knowledge

TPACK is knowledge of instructional strategies (PK) that integrate technologies (TK) to constructively teach content (PCK), in this case mathematics content (CK), for greater understanding of that content (see Table A.2). As mentioned previously, instructional videos were utilized by some of the pre-service teachers at both of the research sites. Table 4.5 lists the instructional videos used by the pre-service teachers in their group presentations at Xever. These videos were aligned with Massachusetts Mathematical Content Strands as seen in Tables 4.17, 4.20, and 4.22. Similarly, at Yexer, Table 5.26 shows the instructional videos used by the pre-service teachers in their lesson plan presentations. These instructional videos often presented how to use digital and nondigital manipulatives, such as arrays and tangrams, to perform mathematical operations as well as to address misconceptions, such as in naming decimals. However, the pre-service teachers at Xever incorporated more instructional videos into their presentations compared to the Yexer pre-service teachers. The pre-service teachers incorporated the instructional videos into their chapter math content and/or lesson plan presentations.

In the same way, websites were utilized by the pre-service teachers at both research sites. Very often these websites were linked to the CCSS, the Massachusetts Mathematics Curriculum Frameworks, and the Standards for Mathematical Practice. At Xever, Table 4.15 shows the websites presented by the pre-service teachers and describes how they would use the websites in their own teaching. Furthermore, some of the pre-service teachers at Xever actually used these websites, including Starfall, Math Playground, and Ten Marks, at their pre-practicum sites. Correspondingly at Yexer, Tables 5.22–5.24 describe and evaluate websites, along with the content focus and potential uses in a lesson. Additionally, Table 5.26 shows the websites used by the pre-service teachers in their lesson plan presentations. However, the pre-service teachers at Xever incorporated more websites into their lesson plan presentations compared to the Yexer pre-service teachers.

Previous analysis revealed that the Xever and Yexer pre-service teachers used the iPad and apps. As with the websites just mentioned, the iPad apps were linked to the CCSS, the Massachusetts Mathematics Curriculum Frameworks, and the Standards for Mathematical Practice. At Xever, Table 4.15 shows the iPad apps presented by the pre-service teachers and describes how they would use these apps in their own teaching. Additionally, Tables 4.16, 4.19, and 4.21 show the iPad apps used in the group presentations. The iPad apps were also used in one of the second-grade pre-practicum sites. In a similar manner, at Yexer, Tables 5.17–5.21 describe and evaluate iPad apps, along with the content focus and potential uses in a lesson. However, the pre-service teachers at Yexer did not actually use apps in their presentations; rather, they used the apps as learners. The iPad apps were also used in one of the fourth-grade pre-practicum sites.

Lastly, the pre-service teachers at both of the research sites used the document camera in their presentations. As previously mentioned, the document camera was used extensively by the pre-service teachers. Whether it was to read aloud children's math literature books and/or to demonstrate how to use physical manipulatives to teach mathematics concepts and to solve mathematical problems, the document camera was a significant instructional technology in the math methods courses. At Xever, Figure 4.5 shows student group 3 illustrating how to use popsicle sticks to match fractions and decimals, and Figure 4.4 illustrates how they demonstrated multiplying decimals with an array. Similarly, student group 1 used the document camera to show how to use manipulatives, such as flipping cubes (see Figure 4.6), in addition strategies. Likewise, student group 2 showed how to use Unifix cubes and the document camera, also shown in Figure 4.6, in a multiplication strategy.

In a similar manner, at Yexer, the pre-service teachers used the document camera and manipulatives, such as ten frames and beads (see Figure 5.7), to illustrate such mathematical concepts as place value. Table 5.26 describes how the document camera was used with read-aloud math literature books. Additionally, Table 5.28 shows how the document camera was used in the group presentations of “Math in the Real World.” For example, student group 3 presented “Pets, Pets, Pets,” a first-grade activity, in which the neighbors have pet dogs and birds (Figure 5.9). If you have 10 legs altogether, how many pets do you have? The two pre-service teachers illustrated two solutions using snap cubes and ten frames on the document camera. They indicated this was a good partner activity but also to make sure you have a picture of a dog with four legs (not three) and a bird with two legs (not one). The pre-service teachers also mentioned that the activity could be differentiated by varying the number of legs.

In summary, the TPACK framework illustrates similarities and some differences in the various components of the framework as applied to the pre-service teachers in the two research sites. One of the significant differences is the content knowledge of the pre-service teachers. Another significant difference is the frequency of technology used by the pre-service teachers.

CHAPTER 7

FINDINGS

Faculty modeled instructional strategies that incorporated instructional technologies in the math methods courses. At both sites, the instructors were very knowledgeable about content-specific technology. This is not always the case with methods instructors (Foulger et al., 2015). For example, a math methods instructor was unable to explain to pre-service teachers the "value added features of integrating technology into instruction" (p. 139). However, through comprehensive professional development, Foulger et al. (2015) found that methods instructors, who integrated technology to support "content-rich engagement," were having a positive effect on the pre-service teachers (p. 142). In this study, one of the pre-service teachers, Hannah, described the effect of the instructional technology on her: "Videos and iPads engage us in the lesson. I am a visual learner, so the videos and computer help" (N1.11).

The instructional technologies included instructional videos, iPad apps, websites, a document camera, a Smart Board, Apple TV/Projector, PowerPoints, Google Docs, and MS Publisher (see Tables 4.3, 5.3, 5.4, 5.7, and 5.8). The instructors incorporated/utilized quality instructional technologies and resources versus quantity. Similarly, the instructors sorted through the plethora of resources available (see Tables 5.5 and 5.6). These instructional technologies allow learners, i.e. pre-service teachers, to build on prior knowledge, to expand their experiences, to actively engage in the learning process, and to extend their learning in the zone of proximal developments, as previously mentioned by Dewey and Vygotsky.

TPACK, Technological Pedagogical and Content Knowledge, is knowledge of instructional strategies (PK) that integrate technologies (TK) to constructively teach content (PCK), in this case mathematics content (CK), for greater understanding of that content (see Table A.2). These instructional strategies incorporate such technologies as YouTube videos to explain how to solve a mathematical problem with either a digital or nondigital tool, such as a double number line, a website/iPad app that demonstrate how to use such models as arrays to perform mathematical operations, and a document camera to demonstrate manipulatives, including Cuisenaire rods and base 10 blocks.

At Xever, TPACK can be seen in the way the instructor illustrated the area model (see Figure 4.2). First, she showed the *Numeracy* DVD by Sharma via the Smart Board and computer. This DVD illustrated the area model for multiplication using Cuisenaire rods. Then the instructor stopped the DVD and used the document camera to illustrate multiplication with two digits using Cuisenaire rods. She reinforced the concepts shown on the DVD. Next she showed the DVD again to illustrate the area model for rectangles using Cuisenaire rods as well as misconceptions on perimeter and area. This was then followed by a hands-on activity by the pre-service teachers, which also reinforced the concepts shown. These videos show the complexities and intricacies of classroom teaching and allow the instructor to slow down and repeat the snippet that is being analyzed (Borko et al., 2009; Rosaen & Florio-Ruane, 2008).

Likewise, at Yexer, TPACK can be seen in the way the instructor explained a PowerPoint lesson on understanding mathematical operations (see Figure 5.5). First, she started with addition and subtraction operations. The instructor reviewed tools/manipulatives previously explored: number line and ten frames. Then she talked about mental math strategies and showed a *Survivor* video from YouTube on what happens when you forget basic math facts. Next the instructor used the document camera to read aloud *Amanda Bean's Amazing Dream*, a fun and silly mathematical story, and pointed out that this served as a good transition from addition to multiplication.

Similarly, the instructor presented a multiplication lesson (see Figure 5.5). First, she addressed misconceptions related to multiplication. Then the instructor showed a video in which Cathy used the algorithm without understanding it. Next she used the document camera and number line to demonstrate 3 x 18. This was followed by the Number Line app to illustrate another way to solve the same problem. Next the instructor used physical manipulatives, base 10 blocks, to demonstrate how to solve a multiplication problem. Subsequently, she demonstrated the array model using the Number Pieces app while the pre-service teachers also worked on the same app. Several models for multiplication progression were then explained. Finally, the instructor clarified how to decode the language—thinking of multiplication (see Figure 6.6).

As Guerrero (2010) indicates, she uses technology in her mathematics classroom so that her students "see the math," make connections to the real world, and gain a deeper understanding of the math (p. 137). The Guiding Principles for Mathematics Programs in Massachusetts (2011) maintains that "technology is an essential tool that should be used strategically in mathematics education" (p. 10) (see Table A.3). Furthermore, such technologies as manipulatives, computers, and the Internet enrich the learning, development, and application of mathematics if properly used. Thus, TPACK incorporates instructional strategies that use technologies constructively and appropriately to teach mathematics (Koehler & Mishra, 2009). Hofer and Grandgenett (2012) succinctly describe TPACK as "enable[ing] a teacher to determine a 'fit' between the curriculum focus, pedagogical strategies, and digital or nondigital technologies" (p. 86).

Likewise, Muir, Callingham, and Beswick (2016) described TPACK instructional strategies in a first-grade lesson on addition. The teacher utilized an interactive whiteboard and virtual ten frames (manipulatives) in her lesson. The students were actively involved in using both technologies. The researchers noted that this lesson enhanced the first-grade students' understanding of basic addition facts. Specifically, they described a teacher who demonstrated confidence in using the technologies (TK), deep mathematics content knowledge (CK), instructional planning and design (PK), and sequential instructional design for addition (PCK). All these together comprise TPACK in a first-grade classroom where they engaged students effectively with the technology, encouraged student participation, and demonstrated mathematical strategies and skills for addition (p. 70).

Correspondingly, Angeli and Valanides (2013) also described TPACK instructional strategies for both pre-service and in-service teachers that were student centered. They recommend that teachers explore how tools can "transform" content for their particular students as well how technology can "transform" their instruction (p. 206). This is especially useful for content that is difficult to teach, such as mathematics and science.

At both sites, the instructor demonstrated/utilized instructional strategies integrating technologies, TPACK, to foster/support pre-service teachers in their learning of mathematics, learning to teach mathematics, and learning to integrate technology into their teaching of mathematics. Wetzel, Buss, Foulger, and Lindsey (2014) found that methods instructors in their study utilized similar instructional strategies to help "prepare them [pre-service teachers] to integrate technology in their future classrooms" (p. 96): demonstration of various technology tools, assignments that required the pre-service teachers to use the technology tools, and sharing/demonstrating to the other pre-service teachers in the class.

Researchers Davis and Falba (2002), Lu and Lei (2012), Oliver et al. (2012), and Virta (2002) indicate that faculty modeling has a strong effect on pre-service teachers. First, Lu and Lei suggest that faculty modeling in an "authentic setting" shows the pre-service teachers the "particulars of the teaching process" as well as furthers their understanding of the "appropriate context in which a strategy or a teaching behavior is executed" (p. 15). Guerrero (2010) further asserts that teachers need to have technological knowledge (TK) but also TPACK, a "thorough conceptualization of when and how to use them as instructional tools" (p. 135).

Additionally, pedagogical knowledge is knowledge of instructional strategies to promote active engagement and learning (Borko & Putnam, 1996; Darling-Hammond & Baratz-Snowden, 2007; Dewey, 1902/1971). Koehler and Mishra (2009) define pedagogical knowledge in the TPACK framework as the "teachers' deep knowledge about the processes and practices or methods of teaching and learning" (see Table A.2). The key word is *deep*, which implies a true understanding. These methods or instructional strategies need to "promote the learning and growth of all students by providing high quality and coherent instruction" (see Table 4.23, Massachusetts Standard 7.08.2).

Pre-service teachers should have experiences that "mirror the experiences we would like them to create in their own classrooms" (Borko & Putnam, 1996, p. 701). Both instructors are experienced teachers demonstrating pedagogical knowledge—that is, teachers' deep knowledge about the processes and practices or methods of teaching and learning (see Table A.2). According to the Massachusetts Professional Standards for Teachers, Standard 7.08.2 states that pedagogical knowledge is knowledge/skill in curriculum, planning, and assessment; teaching all students; family and community engagement; and professional culture (Massachusetts Department of Elementary and Secondary Education, 2015).

Each instructor has college teaching experience that requires curriculum planning and design. A variety of assignments/assessment tools, including lesson plans, journals, and problem/game presentations, were utilized by both instructors, as shown in Table A.6. The instructors understood the prior experiences of their students and designed instruction based upon those experiences, "a continuity of experience" as described by Vygotsky (as cited in Jaramillo, 1996, p.134). Additionally, these assignments/assessments provided experiential learning, i.e. “learning by doing”, opportunities for the pre-service teachers (Dewey, 1902/1971, Jaramillo, 1996). The math methods courses are graduate level, and each instructor established high expectations of the pre-service teachers appropriate to the graduate level.

Additionally, both instructors have elementary teaching experience. Although one instructor taught mathematics at the high school level, she now is an elementary math coach. They were able to provide real-life examples and advice/tips from their own teaching. The instructor at Xever provided this bit of wisdom that comes from years of teaching math: “Productive struggle: Allow students the time, resources and support they need to engage in the mathematics, even if it is difficult. This will help them to persevere. Giving them the algorithm without the conceptual understanding does not help students learn” (A1.11).

Elementary teachers teach mathematics to young children. This requires pedagogical content [mathematics] knowledge. Pedagogical content knowledge is knowledge of instructional strategies (i.e., pedagogy) as they relate to specific content, such as mathematics (see Table A.2). Standard 7.06.7 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations states that teachers “must demonstrate not only that they know how to do elementary mathematics, but that they understand and can explain to students, in multiple ways, why it makes sense” (Massachusetts Department of Elementary and Secondary Education, 2017). Both instructors employed multiple strategies to teach math concepts, such as fractions and number sense. This educational context provides the “motor” and driving force for the attainment of “academic concepts” (Vygotsky, 2012). In this study, the pre-service teachers presented and taught, in groups and individually, lessons on mathematical concepts. The pre-service teachers demonstrated multiple strategies to teach the mathematical concepts and often integrated technology to facilitate the learning of those mathematical concepts (see Tables 4.16, 4.19, 4.21, 5.31–5.33; Figures 4.14–4.19, 5.9).

Ball, Thames, and Phelps (2008, p. 400) describe mathematical tasks of teaching (i.e., pedagogical content knowledge) that are specific to the teaching of mathematics and include such tasks as:

- Responding to students' "why" questions
- Finding an example to make a specific mathematical point
- Modifying tasks to be either easier or harder
- Evaluating the plausibility of students' claims (often quickly)
- Giving or evaluating mathematical explanations
- Asking productive mathematical questions
- Selecting representations for particular purposes

Mathematical tasks require a different understanding or knowledge of mathematics that is not required of others, including mathematicians (Hill & Ball, 2009). For example, mathematics teachers have "decompressed" or "unpacked" mathematical knowledge that enables them to make "particular content visible to and learnable by students" (Ball et al., 2008, p. 400). In other words, they have to teach or explain the mathematics in ways that students will be able to understand. Knowing mathematics does not necessarily translate into an ability to effectively teach mathematics.

Both instructors clearly demonstrated this mathematical knowledge (i.e., content knowledge). Content knowledge is subject matter understanding of mathematics (see Table A.2). Standard 7.06.7 of the Massachusetts Educator Licensure and Preparation Program Approval Regulations (Massachusetts Department of Elementary and Secondary Education, 2017) describes the subject matter knowledge (CK) requirements for elementary teachers specifically in mathematics: numbers and operations, functions and algebra, geometry and measurement, and statistics and probability. Fractions, decimals, and percents are part of numbers and operations but are listed separately because the instructors emphasized them in their courses. This content knowledge is consistent with the Massachusetts Curriculum Frameworks for Mathematics (Massachusetts Department of Elementary and Secondary Education, 2011). Indeed, this fact was stressed by each of the instructors as is evident in the individual syllabi (see Table A.5, A1.1, and A2.1).

Thames and Ball (2010) describe mathematic content knowledge as knowledge of how to perform a procedure/calculation, the definition of a concept or term, and whether or not a student's answer/solution is correct. Guerrero (2010) states that depth in content knowledge "provides teachers with the ability and flexibility to explore, emphasize, or de-emphasize various mathematical topics that may arise in the course of instruction and investigation" (p. 136). Content knowledge is knowledge of the mathematical concepts, principles, and curriculum frameworks, along with the recognized practices and routines of acquiring that knowledge (Koehler & Mishra, 2009; Shulman, 1986). Both instructors continually presented an in-depth understanding of mathematics at the elementary level (i.e., content knowledge) in the math methods courses.

As previously mentioned in the description of the research setting, Xever (site 1) is the only one that required math content course as part of the graduate curriculum. This course is taken during the summer before the math methods course. The pre-service teachers indicated that this was critical to their understanding of math and learning how to teach mathematics.

LeSage (2012) also described the positive effect of an elective math content course in rational numbers, along with web-based video clips on the pre-service teachers' math content knowledge and confidence. This effect can be summed up by one of the pre-service teachers in the course: "I am really surprised at how well I [am] grasping decimals. I remember this as one of my worst math experiences; which usually ended in a lot of tears. But, watching the clips and using the manipulatives just made something click" (p. 26).

In contrast, the instructor at Yexer brought up pre-service teachers' lack of math content knowledge. This was an issue for many of the students. In an interview, the instructor at Yexer stated that the "challenge was to give them [the pre-service teachers] math content; they don't have a lot of math in the graduate program prior to this course" (N2.8). She added that "the students are utilizing [the technology] to enhance instruction in the areas of engagement. Some, however, are meant to deepen their own content knowledge" (N2.8). From a Vygotskian perspective, Rosen and Salomon noted that constructivist learning and understanding of mathematics occurs when "learners socially appropriate and actively construct knowledge" (2007, p.3). In this case, the technology facilitated the mathematics knowledge construction of the pre-service teachers. One pre-service teacher at Yexer shared that she was going to take the math MTEL that weekend for the fourth time. Despite these concerns, at both sites the students demonstrated content knowledge during presentations in class.

Similarly, Ball et al. (2008) noted that teachers' lack of content knowledge will be an impediment to their students' learning that content. In their investigation of videos of actual teaching, they observed instances where the teacher mispronounced vocabulary, made mistakes in calculations, or was unable to solve a problem on the board, all of which negatively affected student learning. Likewise, LeSage (2012) mentioned that pre-service teachers' lack of content knowledge coupled with substantial math anxiety can be overwhelming. In her research, she found that some pre-service teachers had deficits in content knowledge: decimal terminology, face value versus place value, and comparing decimal quantities. For example, one pre-service teacher in an elective math content course on rational numbers made the following statement: “I now know that the first position behind the decimal is called ‘tenths’, I would have called it the ‘ones’ position” (p. 25).

Likewise, in a seminal research study (cited in more than 1,030 articles), Hill, Rowan, and Ball (2005) researched teachers’ mathematical knowledge and student achievement. The study consisted of 115 elementary schools from 15 states over a 4-year period. Student achievement data and teacher data was collected in the first and third grades. The teachers averaged 12 years of experience, and 90% were fully certified (p. 380). Results of the study indicated that teachers’ mathematical knowledge for teaching (which includes math content knowledge and pedagogical content knowledge) was a “significant predictor of student gains” in mathematics achievement and at “both grade levels” (p. 396). Thus, the researchers found an effect even at the first-grade level.

In an interview, Shulman (as cited in Tell, 2001) succinctly summarized the issue regarding a teacher’s lack of content knowledge. He says that if “a teacher doesn’t have a deep understanding and affinity for mathematics or science . . . it’s hard to imagine how that teacher will help students understand and get excited by these subjects” (p. 6).

Furthermore, the final report of the National Mathematics Advisory Panel (2008, p. 65) found the following regarding effectiveness of mathematics teachers and student achievement:

- Differences in teachers account for 12% to 14% of total variability in students’ mathematics achievement gains during an elementary school year.
- When teachers are ranked according to their ability to produce student achievement gains, there is a 10 percentile point difference across the course of a school year between achievement gains of students of top-quartile teachers versus bottom-quartile teachers.

- The effects of teachers on student achievement compound dramatically if students receive a series of effective or ineffective teachers.

Thus, the knowledge of mathematics teachers appears to be significant for student learning of mathematics. Specifically, the panel looked at teachers' content knowledge. In terms of pre-service teachers, one of the recommendations from the panel is the following:

> The mathematics preparation of elementary and middle school teachers must be strengthened as one means for improving teachers' effectiveness in the classroom. This includes preservice teacher education, early career support, and professional development programs. A critical component of this recommendation is that teachers be given ample opportunities to learn mathematics for teaching. That is, teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach. (p. 66)

Cochran-Smith and Zeichner (2005) also described similar findings. The National Mathematics Advisory Panel's recommendation is consistent with the research mentioned previously as well as with the findings of this research study regarding the mathematics knowledge of pre-service teachers.

Furthermore, both instructors also displayed mathematical knowledge for teaching pre-service teachers. Castro Superfine and Li (2014) describe this mathematical knowledge as "knowledge of certain concepts related to preservice teachers' mathematics learning (i.e., student errors, multiplication algorithms, and place value) and knowledge of how these concepts connect to teaching practice" (p. 309). For example, the Yexer instructor used a DVD video, the Cathy Episode, in which Cathy was incorrectly solving a multiplication problem (see Table 5.3). She explained the nature of the student error—namely, a deficit in understanding place value—even though Cathy, the student, had memorized the multiplication algorithm. She emphasized that students are not learning the "why" (N2.2). Similarly, the Xever instructor stressed the importance of understanding place value in order to multiply two digits and demonstrated with two Cuisenaire rods on the document camera (N1.10). Likewise in her parting thoughts, she stated that "giving them the algorithm without the conceptual understanding does not help students learn" (A1.11).

Misconceptions about mathematics were presented so that the pre-service teachers could understand and address their own mistaken beliefs (if any) as well as the thinking of their students. Recall that the Yexer instructor referred to the misconceptions as "lies my teacher told me" (N2.2). One such "lie" was "multiplication makes values larger." The instructor addressed this misconception with this example: "2 x ½ is not a larger number than 2" (N2.2). Ball et al. (2008) also recognized the importance of knowledge of misconceptions about mathematics.

The Massachusetts Standards of Mathematical Practice (see Table A.4) align with the pedagogical content knowledge and the instructional practice of the instructors. Mathematical models (Standard 4) and manipulatives/tools (Standard 5) were used strategically as part of the instructional strategies/techniques. The instructors demonstrated and worked with the pre-service teachers to understand the mathematical problems/equations and to persevere in solving them (Standard 1).

Second, this live modeling—that is, the instructor actually teaching (modeling) the lesson—requires the pre-service teachers to be active participants. In other words, they participate and learn mathematics as elementary students in the lesson as well as learn about teaching mathematics. Both instructors consistently required the pre-service teachers to be active participants while they modeled instructional strategies, especially those that incorporated technology. Graham, Borup, and Smith (2012) "identified [a] need for exposing pre-service teachers to more content-specific technology integration examples (TPACK)" rather than technology examples linked to general pedagogical practices because the pre-service teachers had difficulty applying them to teaching with content-specific technology (pp. 179–180). The assignments/assessments in the math methods courses provided opportunities for the pre-service teachers to learn, use, practice, and demonstrate the mathematic instructional strategies and technologies (see Table A.6). Foulger et al. (2015) indicated that pre-service teachers gain confidence as they gain experience with the instructional technologies.

In this study, physical (nondigital) and/or digital manipulatives were utilized to show different ways to solve the elementary mathematics problems and equations. Bring to mind that manipulatives, such as the rekenrek and place value charts, were used to show turnaround facts and to illustrate multiplication with two digits by the Xever instructor. Vygotsky stressed the role of tools (as cited in Keengwe & Kang, 2012). Knowing which virtual manipulatives to use with a particular math concept (i.e., content knowledge) is characteristic of pedagogical content knowledge and can have a positive effect on students' learning (Muir et al., 2016). Similarly, Ball et al. (2008) refer to knowing how to use manipulatives, such as money, base 10 blocks, and Unifix cubes, to illustrate subtraction of multi-digit numbers. However, each manipulative signifies "different aspects of the content that make a difference at different points in students' learning" (p. 402). The pre-service teachers, at both sites, had to evaluate the appropriateness of the manipulatives and models for their lessons. In other words, they used manipulatives and models that they perceived would support the main point of the mathematics lesson (see Tables 4.5 and 5.12). Thames and Ball (2010) also suggest that mathematics teachers need to assess the appropriateness of manipulatives for a particular math lesson.

Similar to professional development programs for elementary teachers in mathematics, the instructors designed opportunities for the pre-service teachers to experience and to “simultaneously develop their knowledge of technology, pedagogy, and content . . . the intersection between each of these kinds of knowledge” (Polly & Orill, 2016, p. 265). This professional development experience will be beneficial to the pre-service teachers as they begin their careers in teaching and will be looked upon favorably by prospective employers (i.e., schools). Likewise the goals of professional development programs in mathematics can be applied to these elementary math methods courses as taught and modeled by the instructors: “preparing teachers to become more confident in their mathematics knowledge, enacting standards-based pedagogies, and designing instruction to best meet their students’ needs” (p. 263). In an online graduate course for in-service teachers, Niess (2016) examined in-service teachers’ professional development regarding TPACK and learning mathematics and science. Some of the technology tools used included PowerPoints, temperature probes, Jing videos (via PowerPoint), and Google Docs (p. 135). Niess found that a key element in the course that had a positive effect on the in-service teachers was their assignments/experiences in which they learned and used the technology as “students.” This allowed them to experience and understand “student thinking” because they were designing math and science instruction that incorporated technology within the TPACK framework (p. 140).

At each research site, the pre-service teachers were required to plan and design lessons in elementary mathematics. Furthermore, they, individually and/or in groups, had to present these lessons in a math methods class. Class discussions and reflections often followed the presentations. Vygotsky stressed the importance of “the social activity of speech” (as cited in Wertsch, 1970, p.4). This social discourse was used by the pre-service teachers to co-construct knowledge and learning according to Vygotsky’s philosophy.

For the pre-service teachers, TPACK is knowledge of what instructional technologies and teaching strategies would enhance the learning of math by their future students. Furthermore, they need to know when it is appropriate and how to use technologies in constructive ways as well as not to use technology just for technology's sake. When TPACK is integrated into the lesson plans, the pairing of TPACK with Vygotsky's ZPD makes the lesson planning robust.

At Xever, TPACK can be seen in the way the pre-service teachers in student group 3 made their presentation (see Figure 4.15). First, the group started off with explaining key vocabulary: equivalent, numerator, denominator, and improper fraction. Next the group used the document camera to demonstrate fraction strategies: popsicle sticks to match fractions and decimals, repeated addition, and doubling. Then the group used the document camera to illustrate fraction models: the double number line, arrays, a hands-on activity called "Stuffed with Pizza," and manipulatives (e.g., Cuisenaire rods and place value charts).

Likewise, at Yexer, TPACK can be seen in the way the pre-service teacher, Edwina, presented her geometry shapes lesson (see Figure 5.8). First, Edwina introduced tangrams by showing an animated YouTube video. Then she used the document camera and manipulatives (i.e., tangrams) to illustrate the different shapes: triangle, square, and rectangle. Next she rearranged the tangrams to make real objects: a rabbit, a mountain, and a sailboat. This was followed by a student activity in which they had to draw a picture on a sheet using all the tangrams they were given. Then the students swapped sheets and repeated the drawing activity. In the end, the students compared the two drawings and were given an exit ticket: draw two pictures that are different but have the same area using tangrams. The technology tools facilitated the learning of the geometry concepts and enabled the students to "create" meaning from their experiences, as described previously by Dewey and Vygotsky.

Feiman-Nemser (2008) describes a framework consisting of four themes related to learning to teach: "learning to *think* like a teacher, learning to *know* like a teacher, learning to *feel* like a teacher, and learning to *act* like a teacher" (p. 698). Cochran-Smith and Demers (2008) pointed out that pre-service teachers learn to teach over a period of time and through a variety of experiences, building on their previous knowledge and acquiring new knowledge. As a teacher, I believe that the process of learning to teach does not end. One should always be learning and striving to become an even better teacher.

Third, the pre-service teachers reflect on the teaching both during and after the observation (LeSage, 2012; Lu & Lei, 2012). The Xever instructor required six journal entries/writings on the following topics: mathematics or mathematizing, landscapes of learning, algorithms, assessment, math skills book presentation, and reflection on your attitude about teaching math as you complete this class (A1.1). Additionally, for the last journal entry, the instructor required the pre-service teachers to write and reflect on how they feel about teaching mathematics before and after taking the methods course (N1.10). Similarly, the Yexer instructor required weekly journal writings that were reflections on readings and the classes (A2.1). After a sheep and ducks activity, the instructor asked the pre-service teachers to look at the student work and to think about the following question: “What does this mean for instruction and not just for a grade?” (A1.9, N1.10). These reflections are about the pre-service teachers’ learning of mathematics as well as learning to teach mathematics.

The math methods courses are really important for pre-service teachers. These courses are tied to student teaching in pre-practica/practica simultaneously. Darling-Hammond and Baratz-Snowden (2007) indicate that teacher preparation programs should have “extended clinical experiences (at least thirty weeks) that are interwoven with course work and carefully mentored” (p. 120). By definition, pre-service teachers have limited experience in teaching and even more so in teaching with technology (Lei, 2009; Lu & Lei, 2012). Vygotsky considered learning as social and mentoring, i.e. adult guidance, to facilitate the pre-service teachers’ level of potential development, known as ZPD (as cited in Wertsch, 1979).

In this study, the pre-service teachers did, or did to some extent, incorporate what they saw and experienced in the math methods course in their own student teaching. For example, at Yexer, Danielle presented a fractions boot camp lesson plan that she was going to use with her fourth-grade class the following week (N2.3). Agatha presented a lesson plan on place value using ten frames that was designed for the first-grade class she was teaching (N2.3). Similarly, at Yexer, the instructor told the pre-service teachers to "think about your pre-practicum and how you might incorporate technology into your lessons" (N2.6) For example, Gemma indicated that her fourth-grade class was working on adding and subtracting fractions and that she chose apps/websites to review with that in mind (A2.12).

The pre-service teachers also brought back to the class their own experiences from student teaching. At Xever, the pre-service teachers demonstrated websites/apps in the technology assignment that they had used in their teaching at the elementary schools: Amanda—Hoodamath in her second-grade class; Eva—Starfall with her special education students; Crystal—Math Playground in her second-grade class; Irene—My Math with her first-grade class; Linda—Ten Marks in her third-grade class; and Kelly—Handwriting Without Tears in her kindergarten class (see Tables 5.7 and 5.8). Similarly, student group 1 pre-service teachers indicated in their presentation that they also used the following manipulatives in their teaching: flipping cubes in a second-grade class teaching turnaround facts, ten frames in a second-grade class as an addition regrouping strategy, and a hundreds chart in a second-grade class as part of a division strategy (N1.4). One of the student group 2 pre-service teachers used the Doubles Rap video in her second-grade teaching of multiplication (N1.5). Similarly, one of the student group 3 pre-service teachers used a place value chart in her fifth-grade class and then in the group presentation (N1.8). These pre-practicum sites, with the instructional technology mentioned earlier, are valuable settings in which the pre-service teachers can practice with actual students.

These experiences fostered the pre-service teachers' confidence because they were able to demonstrate TPACK in their teaching. Furthermore, the pairing of Vygotsky's ZPD with TPACK, i.e. TPACK is integrated into the lesson plans, makes the lesson planning robust.

Likewise, Debra shared the following: "Access to real-life students working through similar problems. [The instructor] shows examples of what I can do in the classroom to make my mathematic practice clearer" (N1.11). Likewise, Amanda indicated, "Implementation of videos, common core standard websites, iPad applications etc. provides examples and how to make effective for my classroom—where and when to use" (N1.11). Lastly, Linda shared the following: "As a visual/auditory learner, visual aids used in class [videos] help reinforce how important it is to utilize technology with [my] own students" (N1.11). These pre-service teachers indicate that they have been able to connect the modeling by the instructor to their own teaching of mathematics.

Lei (2009) (also Mouza, 2016) suggests that teacher preparation programs facilitate pre-service teachers in building connections between technology and teaching and in enabling the transition from "digital native students [i.e., grew up with technology] to digital native teachers," especially in using "subject-specific technologies" (p. 92). In this study, the technologies are, for the most part, the same technologies that elementary teachers utilize in their teaching (see Tables 4.1 and 5.1). Polly (2016) observed the following technologies in three elementary classrooms: projector, document camera, iPad, teacher computer, interactive whiteboard, and hand-held quiz device (i.e., clickers) (p. 114). None of the pre-service teachers at Xever and Yexer mentioned the presence of clickers in their field placements. As discussed previously, both teacher preparation programs required the pre-service teachers to purchase iPads or iPad Minis. Nguyen et al. (2016) recommend that pre-service teachers receive initial training from the university on using the features of the iPad as well as some apps. The majority of the pre-service teachers demonstrated an aptitude for the iPad, but the Xever instructor did assist one group with using Apple TV with the iPad.

7.1 Conclusions

The results of this study indicate the importance of faculty modeling in math methods courses. The two instructors seamlessly transitioned between the various technologies as they taught the content in the math methods courses. Both instructors consistently required the pre-service teachers to be active participants while they modeled instructional strategies, especially those that incorporated technology. Thus, the pre-service teachers' participation gave them opportunities to learn mathematics as elementary students and to learn about teaching mathematics as well as to reflect on both. I agree with the need to provide opportunities for the pre-service teachers to gain experience in TPACK, with the ultimate goal of designing and teaching mathematic lessons/activities in constructive ways using instructional technologies for greater understanding by their future students (see Table A.2). When TPACK is integrated in the lesson plans, the pairing of Vygotsky's ZPD with TPACK makes the lesson planning robust. These lessons/activities should be designed to reach all students, including the visual learners and ELLs. Additionally, the pre-service teachers need to have field experience simultaneously with the math methods courses. Thus, the pre-service teachers could incorporate what they saw and experienced in the math methods course into their own student teaching. The field experiences also contributed to the discussions in the math methods courses. These results are consistent with the research literature previously mentioned.

I concur with the significance of instructional technologies, such as the document camera, iPad apps, websites, digital manipulatives, and exemplar instructional videos; that can be used in the context of TPACK.

I have similar concerns about some of the pre-service teachers' lack of mathematic content knowledge. It might be difficult for them to teach math concepts that they are unsure of themselves. This could potentially have a negative effect on their students' understanding of mathematics during the elementary years—the formative years. These concerns/issues are also found in the research literature previously mentioned.

7.2 Implications for Further Research

Findings illuminate the need for further research in how to provide opportunities for pre-service teachers to gain math content knowledge in their graduate programs, which are so 'full' and heavily prescribed by the Massachusetts requirements for licensure at the elementary level. Further research could also be in the area of the pre-service pre-practica/practica—that is, to observe and research the pre-service teachers and their students' learning (when TPACK is integrated in the lesson plans) in the elementary classrooms. This might also include research into their reflections, which were not part of this study. Also, further research could follow up with the pre-service teachers during the first years of their teaching practice and their use of instructional technologies in mathematics, particularly at the elementary level. Most of the studies focus on the secondary level and not on the elementary level in terms of TPACK and mathematics.

APPENDIX

TABLES

Table A.1: Examples of Technology Found in a Technology-Empowered Learning Environment

Digital textbooks	Podcasts
Simulations	Interactive games
Tutoring systems	Interactive visualization
Audio/video capture/edit software	Electronic learning portfolios
Blogs	Learning applications for mobile devices
Wikis	Digital libraries
Learning management systems	Online videos

Note. Adapted from the U.S. Department of Education, Office of Educational Technology (2010).

Table A.2: TPACK and Knowledge Components Defined

Content Knowledge (CK)	• "teachers' knowledge about the subject matter to be learned or taught" (p.63) • "knowledge of concepts, theories, ideas, organizational frameworks, • knowledge of evidence and proof, as well as established practices and approaches toward developing such knowledge " (Shulman 1986 as found in Koehler and Mishra, 2009, p.63)
Pedagogical Knowledge (PK)	• "teachers' deep knowledge about the processes and practices or methods of teaching and learning" (p.64) • "understanding how students learn, • general classroom management skills, • lesson planning, • and student assessments" (p.64) • "understands how students construct knowledge and acquire skills • and how they develop habits of mind and positive dispositions toward learning" (p.64) • "understanding of cognitive, social, and developmental theories of learning and how they apply to students in the classroom" (p.64)
Pedagogical Content Knowledge (PCK)	• "similar to Shulman's (1986) idea of knowledge of pedagogy that is applicable to the teaching of specific content" (p.64) • "transformation of the subject matter for teaching … • the teacher interprets the subject matter, • finds multiple ways to represent it and • adapts and tailors the instructional materials to alternative conceptions and students' prior knowledge" (p.64) • "An awareness of common misconceptions and ways of looking at them • the importance of forging connections among different content-based ideas" (p.64)
Technology Knowledge (TK)	• Knowledge about "certain ways of thinking about and working with technology can apply to all technology tools and resources" (p,64) • "understand information technology … • to apply it productively at work and in their everyday lives, • to recognize when information technology can assist or impede the achievement of a goal, • and to continually adapt to changes in information technology" (NRC, 1999 in Koehler and Mishra, 2009, p.64)

Continued on next page

Table A.2: continued

Technological Pedagogical and Content Knowledge (TPACK)	• “the basis of effective teaching with technology, • requiring an understanding of the representation of concepts using technologies; • pedagogical techniques that use technologies in constructive ways to teach content; • knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; • knowledge of students’ prior knowledge and theories of epistemology; • and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones” (p.66)

Source. Koehler, M. J., & Mishra, P. (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher Education, 9*(1), 60–70.

Table A.3: Guiding Principles for Mathematics Programs in Massachusetts

1. Learning—Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.
2. Teaching—An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.
3. Technology—Technology is an essential tool that should be used strategically in mathematics education.
4. Equity—All students should have a high-quality mathematics program that prepares them for college and a career.
5. Literacy across the content areas—An effective mathematics program builds on and develops students' literacy skills and knowledge.
6. Assessment—Assessment of student learning in mathematics should take many forms to inform instruction and learning.

Source. Massachusetts Department of Elementary and Secondary Education (2011).

Table A.4: Massachusetts Standards of Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for an express regularity in repeated reasoning.

Source. Massachusetts Department of Elementary and Secondary Education (2011).

Table A.5: Instructors and Content Knowledge

Content Knowledge	Site 1 Instructor	Site 2 Instructor
Fractions, decimals, and percents	**X**	**X**
Operations: addition, subtraction, multiplication, division	**X**	**X**
Numeracy	**X**	**X**
Geometry and measurement	**X**	**X**
Functions and algebra		**X**
Statistics and probability		**X**

Note. X represents the presentation of content knowledge.

Table A.6: Instructors and Pedagogical Knowledge – Assessment Tools

Assessment Tools	Site 1 Instructor	Site 2 Instructor
Math autobiography		X
Lesson plan	X	X
Journal	X	X
Paper on Standards of Mathematical Practice	X	
Math topic/skill (presentation and/or book)	X	X
Content quizzes		X
Problem/game presentation	X	X

Note. X represents the presence of the assessment tool.

Table A.7: Instructors and Technological Knowledge

Technological Knowledge	Site 1 Instructor	Site 2 Instructor
Videos	X	X
Websites	X	X
Google Docs	X	
iPad apps	X	X
Document camera	X	X
Projector with Apple TV/iPad	X	X
PowerPoints	X	X

Note. X represents the presence of the instructional technology.

Table A.8: Summary of Technology Self-Assessment Tool (TSAT) Mastery Percentages

Mastery Level	Site 1 Instructor	Site 2 Instructor
Early Technology	100%	100%
Developing Technology	100%	100%
Proficient	85%	93%
Advanced	56%	67%

Note. Adapted from the Massachusetts Department of Elementary and Secondary Education (2010).

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