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## Reward Allocation For Maximizing Energy Savings In A Transportation System

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**REWARD ALLOCATION FOR MAXIMIZING ENERGY SAVINGS  
IN A TRANSPORTATION SYSTEM**

A Thesis Presented

by

ADEWALE O ODUWOLE

Submitted to the Graduate School of the  
University of Massachusetts Amherst in partial fulfillment  
of the requirements of the degree of

MASTER OF SCIENCE IN CIVIL AND ENVIRONMENTAL ENGINEERING

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Department of Civil Engineering and Environmental Engineering  
Transportation Engineering

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## **ABSTRACT**

### **REWARD ALLOCATION FOR MAXIMIING ENERGY SAVINGS IN A TRANSPORTATION SYSTEM**

MAY 2018

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Transportation has a major impact on our society and environment, contributing 70% of U.S petroleum use, 28% of U.S. greenhouse gas (GHG) emissions, over 34,000 fatalities and 2.2 million injuries in 2013. Punitive approaches to used to tackle environmental issues in the transportation sector, such as congestion pricing have been well documented, although the use of incentives or rewards lags behind in comparison. In addition to the use of more fuel-efficient, alternate energy vehicles and various other energy reduction strategies; energy consumption can be lowered through incentivizing alternative modes of transportation. This paper focused on modifying travelers' behavior by providing rewards to enable shifts to more energy-efficient modes, (e.g., from auto to either bus or bicycles). Optimization conditions are formulated for the problem to understand solution properties, and numerical tests are carried out to study the effects of system parameters (e.g., token budget and coefficient of tokens) on the optimal solutions (i.e., energy savings). The multinomial logit model is used to

formulate the full problem, comprised of an objective function and constraint of a token budget ranging from \$5,000-\$10,000. Comparably, the full problem is computationally reduced by various parameterization strategies, in that the number of tokens assigned to all travelers' is parameterized and proportional to the expected energy savings. An optimization solution algorithm is applied with a global and local solver to solve a lagrangian sub-problem and a duo of heuristic solution algorithms of the original problem. These were determined necessary to attain an optimal and feasible solution. Input data necessary for this analysis is obtained for the Town of Amherst, MA from the Pioneer Valley Planning Commission (PVPC). The results demonstrated strong evidence to conclude a positive correlation between the system's energy savings and the aforementioned system parameters. The local and global solvers solution algorithm reduced the average energy consumption by 11.48% - 19.91% and 12.79% - 21.09% consecutively for the identified token budget range from a base case scenario with no tokens assigned. The duo of lagrangian heuristic algorithms improved the full problems solution i.e., higher energy savings, when optimized over a local solver, while the parameterized problem formulations resulted in higher energy savings when compared to the full problems' formulation solution over local solver, but higher energy savings compared over the global solver. The Computational run-time for the global and local solvers solution algorithm for the full problem formulation required 43 hours and 24 minutes consecutively, while the local solver for the lagrangian heuristics and parameterized problem solution algorithm took 13 minutes and 7 minutes consecutively.

Future research on this paper will be comprised of a bi-level optimization problem formulation where a high level optimization aims at maximizing system-wide energy savings, while a low-level consumer surplus maximization problem is solved for each system user.

**Key Words:** Energy Savings, Fuel Consumption, Energy Optimization, Incentive-Based Pricing, and System Optimization.



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# CHAPTER I

## INTRODUCTION

### 1.1 Background

The motivation behind providing incentives for maximizing energy savings in a transportation system stems from the far-reaching impact of energy consumption on our society and environment. Transportation activities accounted for 33 percent of CO<sup>2</sup> emissions from fossil fuel combustion in 2011, while it was estimated that approximately 25% of household vehicle mile traveled (VMT) were commute trips and 79% of commuters drove alone to work (1). Virtually all of the energy consumed in this end-use sector came from petroleum products and nearly 65% of the emissions resulted from gasoline consumption for personal vehicle use. In the long term, energy consumption patterns respond to changes that affect the scale of consumption (e.g., population, number of cars), the efficiency with which energy is used in equipment (e.g., cars, power plants, steel mills) and behavioral choices (e.g., walking, bicycling, or telecommuting to work instead of driving). While technology has generally been used to increase fuel economy by improving efficiency (which has reduced CO<sub>2</sub> emissions) along the years(2). Historically, there has been a negative perception and public resistance conveyed by road users on the implementation of punitive approaches such as congestion pricing to reduce energy consumption. Inequality on the end-users side is one of many other major concerns highlighted in recent literature. Hence, reward-based approach is considered a more user-friendly policy in reducing road congestion and emissions.

It is widely recognized that governments could reduce congestion and its associated environmental effects by charging road users their marginal cost of excess demand during peak hours in transportation systems. This policy is commonly known as congestion pricing. Congestion pricing has been well researched, documented and implemented in a few major cities in the past decade, although it has faced major criticism and public resentment from an equitability point of view from connected neighborhoods where it is implemented. In the most recent field of sustainability, which is seen to be gaining more popularity in both the academic and professional career disciplines, equity is considered one of three major characteristics that define the quintessence of achieving a sustainable-based transportation policy. In sustainable transportation engineering and planning discipline, governments are starting to consider the efficient use of incentive policies as a primary substitute to congestion pricing, with the outlook that it has the potential to alleviate the most significant critic of congestion pricing, equity.

Recent literatures have highlighted governments intervention through incentive policies such as offering financial incentive in form of subsidies, gift cards, transit passes or monetary payment reimbursements to road users, to aide intelligent decisions on travel time and mode choice of transportation as potential substitute to congestion pricing policies externalities. As of March 2016, the United States government through the Department of the Navy (DON) and Department of Transportation (DOT) established a mass transit benefit program, the transit incentive program (TIP) for DON, part-time federal and intern employees to help reduce their daily contribution to traffic congestion and air pollution to and from

work, in conjunction with expanding their commuting alternatives(3). Applicable to civilian and military employees in the District of Columbia, Maryland, Virginia (DMV), all geographic counties within their associated boundaries and all DON facilities within the US territories, the TIP provides transit benefits of \$255/month (individual benefit) excluding parking fees for transit modes by commuter bus, commuter trains, subway/light rail, vanpool and ferries. Similarly, the Environmental Protection Agency (EPA) and the Department of Interior (DOI) adopted a similar program, the Transportation Subsidy Program (TSP) that provides financial incentives to encourage DOI employees nationwide to commute by means other than single-occupancy motor vehicles to and from work (4).

Several studies have reported the effects of incentive policies on commuter mode choices, significant to this research, Qing Su et al. 2012 developed a nested logit model to examine the impact of parking management and financial subsidies to alternative modes to drive alone, as well as travel demand management strategies on people's commute mode choices in Seattle based on the Washington state commute trip reduction dataset in 2005. A two-level nested logit model was selected to estimate the mode choices of (motor, non-motor and public) at the first level and (drive alone, share ride, transit and bike/walk) at the second level. A multinomial logit model was rejected after a hausman specification test of the independence of irrelevant alternatives (IIA) property violation at a level of 0.01(5). Focusing on financial subsidies as a public policy tool, findings from Qing Su and et al, 2012 suggests that commuters respond positively to direct financial subsidies to

alternative modes and represent an important contribution toward understanding how commuter's mode choice decisions respond to a variety of incentive policies.

In general, there is sufficient data, research and literature that suggests transit use may be affected by incentive policies that modify the relative costs of travel behavior. According to the US. Department of transportation, such incentives have been seen to increase public transportation use and use of active travel options such as walking and bicycling particularly among college students(6). Nevertheless, there is a gap in literatures on modeling framework for incentive policy optimizations to reduce energy consumption in a transportation system. This forms the basis of this study on the need for an efficacious modeling framework that effectively allocates these resources/incentives to end-users.

## **1.2 Literature Review**

It is of importance to understand factors that affect consumer's transportation mode choice, which calls for the adoption of a behavioral choice model while designing incentive optimization problems. A large body of literature exists on such models using discrete choice model approaches. For a review, (Jiang et al. 2009) conducted a study analyzing non-motorized transportation mode choice considering the influence of trip purpose, distance and car availability with a stated and revealed preference data survey in china(7).

In a researcher paper by Michael A. Silas et al. a mathematical formulation was established to gain insights into the most efficient way to allocate financial incentives to customers in an urban environment in order to maximize participation



and optimally distribute the incentives(8). The methodology involved the utilization of the Karush-Kuhn Tucker (KKT) condition to find the optimal amount of truck tours/trips shifted to non-peak hours. Two main important decisions were identified in this study for the optimal allocation of an incentive budget; the total budget to be distributed, and the allocation of the budget among different market segmentations. Focusing on the latter, mathematical formulations were developed, which involved the derivation of the optimal incentives distinguished by two types of constraints, namely, an exogenous and endogenous budget. The latter being a self-sustaining budget is formulated as a function of the total revenue generated by the system and the former an external budget distributed to recipients of the incentive, formulated to require the total amount of incentives distributed to the recipients be less than or equal to the budget constraint. While considering the exogenous budgets, numerical results showed that there was an increasing non-linear relationship between the number of tours transferred to the off-hours and the given incentive budget. This relationship was characterized by a rapid increase at the beginning followed by smaller increases in the number of tours transferred to off-hours. Numerical results of the endogenous budget showed that the number of off-hour tours increased as the regular-hour tour revenues increased. With a similar characteristic as the former, the off-hours tour increased rapidly at the beginning followed by an incremental reduction in the increments (8).

In a study of Intermodal Transportation Network Design with Emission Incentive and Mode Transfer cost by Peiyu Luo et al., the researchers considered the problem of designing an intermodal transportation network consisting of three

modes; highway, rail and inland waterway with a partial objective to incentivize shippers using a more energy efficient modes such as waterways and railways to promote intermodal transportation while satisfying a fixed annual demand from multiple origin-destination pairs(9). The problem was framed as a linear programming formulation to minimize the total transportation cost and time value, while including incentive rebates motivated by the objective of promoting the use of inland waterways and railways by the USDOT on a hypothesized super network. No incentives were awarded to roadways. Conclusions from numerical results outlined revealed incentives rebates for railways and waterways encouraged shippers to use railways and waterways which contribute to significant reductions emissions and total transportation cost while transportation time increased.

A study on the optimization of incentive policies for plug-in electric vehicles Nie Yu et al., proposed an optimization modeling framework that could potentially assist policy makers in deciding when and how much money should be invested on what incentive programs in order to achieve a desired goal, e.g reduce greenhouse gas or dependence on petroleum products, while accelerating the adoption of plug-in vehicles (10). The problem was formulated with two incentives over an analysis period, purchased rebates and publicly funded charging stations with the goal of promoting plug-in electric vehicles. Due to the highly non-linear, non-convex attribute of the problem, a non-linear optimization model was used to allocate resources to each of the incentives over the analysis period. Consumer choices were modeled using a simple logit-based vehicle choice model. To support policy making, a satisfactory improvement over existing decision was deemed a sufficient solution,

hence the use of the steepest descent direction algorithm was proposed for finding a local optimum while proving that the KKT conditions are necessary for a local optimum solution. Alternative policies were subsequently modeled against an optimal policy to conduct a sensitivity analysis on the effect of each proposed policies to the base case scenario. While the algorithm used in this study provided a satisfactory convergence and was reasonably efficient, in theory, a local optimum does not necessarily yield the best solution to the true solution.

A study by Wilfredo F. Yushimito et al. 2013 proposed a two-staged optimization model for staggered work hours of a firm with an aim to flatten peak hours of workers by assigning them to shoulders of peak hours under incentive policies, consequentially lowering peak trip demands. A quadratic objective function formulated as a nonlinear mathematical model as established by Ban et al. 2008 is used to solve the network optimization, constrained by arrival and departure times and benefits from government incentives. Hence, the first stage on the firm's workers assignment decision and second stage with workers decisions on arrival and departure times. The underlying model quantifies the effects of the firm's workers response to external incentives and evaluates departure schedules required to achieve a social optimum. The algorithm utilized in this study was heuristic and at best able to find a feasible local solution that achieved a social optimum in the system. The principle behind achieving a social optimum was to bound the government incentive budget by congestion savings so that the society will be better off. This guarantees that no extra fund will be required so as to ensure the amount of incentive will not offset the savings in travel time. The social objective

was ensured in the algorithms iterative process by setting the difference between the savings in the total systems travel time and budget to be greater than 0 (11).

Bie Jing et al. 2009 conducted a similar study using economic incentives to influence drivers' route choice for safety enhancement. The modeling framework was based of a route-based incentive structure introduced by a logistic company in co-operation with an insurance company where drivers get rewarded for taking the safest routes (12). In theory of this study, the incentive program adopts a win-win situation that bring benefits to all stakeholders, leading the formulation of a bi-level optimization designed to minimize/maximize cost and benefits whereby drivers choose a safer route and the logistics and insurance company are never at a loss. Hence safety in terms of equity on the incentive program is certainly guaranteed. The efficiency of the solution algorithms on the aforementioned study was exclusive.

All theses studies are fundamental to understanding various incentive-based optimization models but are limited in efficiency and equity of their solutions. This paper will be predominantly focused on the use of a global algorithm solution and its solution effect on a local algorithm solution along side an array of mathematical problem formulations.

In furtherance, we simplify our model assumptions using a multinomial discrete choice model with aggregated coefficients in predicting consumer's choice on the menu of mode choices provided. We take into account the baseline standard variables relevant to decision mode choice in to this case study; in vehicle travel time, out of vehicle travel time and cost in form of a token incentive parameter specific to this case study.

### 1.3 Statement of Contribution

In the studies discussed above, little attention has been given to near global and efficient solutions in terms of equity on incentive- based optimization, which is a reasonable objective. Contribution belongs to a limited body of literature on dealing with non-linear, non-convex incentive optimization problems on transportation systems, which is considered a dynamic and evolving field of investigation in transportation demand modeling.

We have a non-convex non -linear problem, which guarantees there could be multiple local minima (maxima) solutions to the problem with high computational time of obtaining a solution. Generally, finding a global optima solution is difficult and really only practical for relatively small problems. Hence, we explore both global and local solvers with a solution algorithm to obtain an optimal and feasible solution and establish the quality/efficiency of both solutions on the problem.

In the first problem variant we consider an incentive optimization problem that includes a lagrangian sub-problem where travelers are rewarded according to a multinomial logit model constrained by a reward budget to induce energy savings. For the first variant, we utilize a global and local solver algorithm to solve the non-linear, non-convex problem to attain optimal and feasible solutions in terms of energy savings. A unique feature of this paper is the second variant, which consists of a series of parameterized formulations of the first problem, solved with a local solver algorithm only. For this variant we establish a modeling framework and numerical results that necessitates equity in incentive-based optimizations such that the rewards assigned to travelers are proportional to energy savings while the

expected energy savings solution deviates from the optimal solutions of the original problem. Theoretically, if the numbers of tokens (rewards) assigned are directly proportional to the expected energy savings, the incentive policy is constrained to be more equitable. We demonstrate that a high-quality solution can be obtained quite fast with minor deviations on the solutions and a desired guarantee on the performance of this modeling framework on problems of similar characteristics.

#### **1.4 Research Objective**

The objective of this study is to maximize energy savings by allocating incentives to traveler's to enable shifts to more energy-efficient modes. Mathematical optimization conditions are formulated for the problem, first, a full-scale maximization problem followed by a series of parameterization formulations. An optimization solution algorithm is proposed utilizing global and local solution solvers to obtain optimal and feasible solutions. Transportation data is gathered for the Town of Amherst, MA and used as a hypothetical case study. Results are investigated through numerical tests, using sensitivity analyses to study the effects of the incentive budget and coefficient of incentives on the optimal solutions across the proposed problem formulations and solution algorithms. (Note: Incentive, reward and token are used interchangeably through out this paper).

## CHAPTER II

### METHODOLOGY

#### 2.1 Maximization Problem Formulation

In this section, the incentive policy design model is presented. The formulation for this problem is based from an inference of the MNL model, where  $n$  users exist in the transportation system, and each of them has a travel choice set of  $C_n$ . A multinomial logit-model is used to calculate  $P_{ni}$ , the probability of user  $n$  choosing alternative  $i \in C_n$  (Auto, Bus, and Bicycle) when making and OD trip  $j$  with the equation,

$$P_{ni} = \frac{\exp(V_{ni})}{\sum_{j \in C_n} \exp(V_{nj})}, \quad (1)$$

Where  $V_{ni}$  is the systematic utility for user  $n$  and alternative  $i$ . A linear-in-parameter specification is assumed for the systematic utility with two explanatory variables for this study, travel time  $t_{ni}$  (in-vehicle and out-vehicle travel of bus included) and the number of tokens  $\tau_{ni}$ , that is,  $V_{ni} = \alpha t_{ni} + \beta \tau_{ni}$ . The assumption of two explanatory variables is not restrictive and can be relaxed easily to account for more than two explanatory variables. Travel time  $t_{ni}$  is a constant for any given user and alternative across all origin-destination pair, and not a function of users' choices, as no congestion effect is considered given the homogeneous user behavior by an aggregate MNL model.

Let  $E_{ni}$  be the energy saving of alternative  $i$  for individual  $n$  when compared to a baseline energy consumption  $E$ , assumed a constant, i.e. not a function of users' choices and no tokens awarded.  $E_{ni}$  is the energy consumption attributed to

alternative  $i$  for individual  $n$ . The baseline energy consumption  $E$  is calculated as  $E_{ni} \times P_{ni}$  of the auto alternative. Auto energy consumption  $E_{ni}$  is calculated as the amount in gallons it takes to traverse each OD pair in day,  $\frac{1}{\text{Auto fuel economy } (\frac{g}{m})} \times \text{Distance } \frac{mi.}{day}$ . Bus and bicycle energy consumption  $E_{ni}$  are assumed to be 0 gallon/day based on a pre-assumption of a single occupancy auto vehicle. Hence, an additional occupancy on a bus transit vehicle greater than one will result in no additional energy consumption contribution to the system, therefore we can attribute a null constant value to bus energy consumption.

The expected energy savings for the system,  $Z(\tau)$ , is distinguished by the objective function, and calculated as,

$$Z(\tau) = \sum_{n=1}^N \sum_{i \in C_n} E_{ni} \frac{\exp(\alpha t_{ni} + \beta \tau_{ni})}{\sum_{j \in C_n} \exp(\alpha t_{nj} + \beta \tau_{nj})}, \quad (2)$$

Assumed a token budget of  $T$ , and the constraint is,

$$g(\tau) = \sum_{n=1}^N \sum_{i \in C_n} \tau_{ni} \frac{\exp(\alpha t_{ni} + \beta \tau_{ni})}{\sum_{j \in C_n} \exp(\alpha t_{nj} + \beta \tau_{nj})} \leq T. \quad (3)$$

The token allocation problem is then formulated as a constrained non-linear optimization problem,

$$\begin{aligned} \max_{\tau} \quad & Z(\tau) \\ \text{s. t.} \quad & g(\tau) \leq T \\ & \tau \geq 0 \end{aligned} \quad (4)$$

The first-order necessary condition of optimality is the KKT condition, that is,



$$\left\{ \begin{array}{l} \frac{\partial Z(\tau)}{\partial \tau_{ni}} = \lambda \frac{\partial g(\tau)}{\partial \tau_{ni}} - \mu_{ni}, \forall_n, \forall_i \in C_n \\ \lambda [g(\tau) - Y] = 0 \\ \mu_{ni} \tau_{ni} = 0 \\ \lambda \geq 0 \\ \mu_{ni} \geq 0 \\ g(\tau) \leq T \\ \tau \geq 0 \end{array} \right. , \forall_n, \forall_i \in C_n \quad (5)$$

$\lambda$ , can be interpreted as the shadow price of the budget constraint, that is, the increase of the system-wide expected energy saving if one more token is available.  $\mu_{ni}$  is zero, if a positive number of tokens is assigned to alternative  $i$  of person  $n$ .

The shadow price is zero, if the constraint is not binding, that is, there are more than enough tokens. In this case, the stationary condition is that any alternative of any user with a positive number of allocated tokens has zero marginal energy saving per allocated token. The shadow price is positive when the budget constraint is binding, that is, available tokens are used up. In this case, we have

$$\frac{\partial Z(\tau)/\partial \tau_{ni}}{\partial g(\tau)/\partial \tau_{ni}} = \lambda, \forall_n, \forall_i \in C_n | \tau_{ni} > 0. \quad (6)$$

The left-hand side of the equation can be interpreted as the marginal energy saving per used token, and the stationary condition given limited token budget is that any alternative of any user with a positive number of allocated tokens has the same marginal energy saving per used token.

### 2.1.1 Solution Algorithm

Due to the nature of a choice probability function, any solution with the same token differences across alternatives for a given person will give the same probabilities and thus same expected energy savings. The expected token

consumption is however different, and the constraint is less likely to be violated with smaller number of tokens assigned. It is therefore advantageous to set  $\tau_{nj} = 0$ , where  $E_{nj} = \min_{i \in C_n} E_{ni}, \forall n$ .

The LHS of the constraint of the objective function is non-convex and thus the feasible solution set is highly nonlinear but also very likely non-convex. Consequently, it is difficult to establish the uniqueness of a global optimal solution and a suitable algorithm is necessary to find a feasible and optimal solution. Preliminary tests on small problem instances using Matlab's optimization and global optimization functions show that the constraint is almost always violated when it is binding. It is therefore speculated that the constraint is "difficult" and a Lagrangian relaxation algorithm is proposed. Note: The objective function is non-concave either.

The Lagrangian is

$$\theta(\lambda) = \max_{\tau} \{Z(\tau) - \lambda(\tau) - T\}, \forall \lambda \geq 0. \quad (7)$$

$\tau^*$  is the optimal solution to the primal Problem (4). Due to weak duality,  $\theta \geq Z(\tau^*)$ .

$\theta(\lambda)$  for any value of  $\lambda$  provides an upper bound for the optimal energy saving and  $\theta^* = \min_{\lambda \geq 0} \theta(\lambda)$  is the tightest upper bound. Therefore, there exists a duality gap as the objective function is non-concave, that is,  $\theta^* > Z(\tau^*)$ . It is still valuable to solve  $\min_{\lambda} \theta(\lambda)$  as it measures the extent of the sub-optimality of any solution to the original problem, along with a good lower bound/feasible solution.

### 2.1.2 The sub-gradient algorithm

$\theta(\lambda)$  is convex but not smooth (with kinks) and thus not differentiable everywhere. The sub-gradient algorithm is adopted to solve  $\min_{\lambda \geq 0} \theta(\lambda)$ .

At iteration  $k$  of the algorithm, two critical pieces of information is needed: the sub-gradient  $s^k$  and step size  $\alpha_k$  and a new trial value  $\lambda^{k+1}$  is calculated as in

$$\lambda^{k+1} = \lambda^k - \alpha_k s^k. \quad (8)$$

A sub-gradient  $s^k$  is  $T - g(\tilde{\tau}^k)$  where  $\tilde{\tau}^k$  is the optimal solution to the relaxed Problem (7) with  $\lambda = \lambda^k$ .

The Polyak step size can be used, that is,

$$\alpha_k = \frac{\theta(\lambda^k) - \theta_{best}^k + \gamma_k}{(s^k)^2}, \quad (9)$$

where  $\theta_{best}^k - \gamma_k$  serves as an estimate of  $\theta^*$ , and  $\gamma_k$  is in the scale of the objective value with  $\sum_{k=1}^{\infty} \gamma_k = \infty, \sum_{k=1}^{\infty} \gamma_k^2 < \infty$ . A practical choice for  $\gamma_k$  could be  $b \frac{\alpha}{a+k}$ , where  $b$  is in the scale of the objective value and  $\alpha$  is a constant (usually 1, but other values can be tried) (13).

To effectively optimize the Lagrangian problem, an alternative approach for the step size is to used to calculate the step size when the polyak step sized failed to yield optimum results in the trial phase of the analysis.

$$\alpha_k = \frac{c_k(\theta(\lambda^k) - Z^*)}{(s^k)^2}, \quad (10)$$

where  $Z^*$  is the objective value of the best known feasible solution to the original problem, and  $c_k$  is a scalar chosen between 0 and 2. Frequently, the sequence  $c_k$  is determined by starting with  $c_k = 2$  and reducing  $c_k$  by a factor of two whenever  $\theta(\lambda^k)$  has failed to decrease in a specified number of iterations.  $c_k$  was

reduced by a factor of 2 after 10 consecutive iterations when  $\theta(\lambda^k)$  failed to decrease. The sub-gradient algorithm is implemented as follows:

**STEP 1**

*% Set feasible solution to objective function 'equation (4)'*  
*Z( $\tau^*$ ) lower bound/feasible solution*  
 *$\tau^*$  token original problem (4)*

**STEP 2**

*% Optimize Lagrangian function 'maximize equation (7)'*  
 *$\theta(\lambda^k)$  Upper bound*  
 *$\tau^k$  optimal token lagrangian problem (7)*

**STEP 3 – preliminary checks**

*% Check initial gap*  
*if  $\theta(\lambda^k) > Z(\tau^*)$*   
*else STEP 1*

**STEP 4**

*% Initial sub-gradient & step size estimate using at iteration  $k = 1$*   
 *$s^k$  sub-gradient*  
 *$\alpha_k = \text{step size}$*

**STEP 5**

*% Obtain solution for  $k+1$  iteration, Re-optimize Lagrangian function with new dual variable.*

*While iter < MaxIter*

*$\lambda^{k+1} = \max[0, \lambda^k - (\alpha_k * s^k)];$*   
*%optimize lagrangian*  
 *$\theta(\lambda^{k+1}) = \max(Z(\tau^k) + [\lambda^{k+1} * (g(\tau^k) - T)]);$*

*% update (sub gradient and step size)*  
*if  $g(\tau^{k+1}) \leq T$*   
*%calculate*  
*$Z(\tau^{k+1})$*   
*$s^{k+1} = T - g(\tau^{k+1});$*   
*$\alpha_k = c_k(\theta(\lambda^{k+1}) - Z(\tau^{k+1})) / (s^{k+1})^2;$*

*else  $s^{k+1} = T - g(\tau^k);$*   
*$\alpha_k = c_k(\theta(\lambda^{k+1}) - Z(\tau^k)) / (s^{k+1})^2;$*   
*end*

```

% update feasible solution
    if  $Z(\boldsymbol{\tau}^{k+1}) > Z(\boldsymbol{\tau}^k)$ 
         $Z(\boldsymbol{\tau}^{k+1}) = Z(\boldsymbol{\tau}^{k+1})$ 
    else  $Z(\boldsymbol{\tau}^{k+1}) = Z(\boldsymbol{\tau}^k)$ 
    end
    iter = iter + 1; loop STEP 4 (recalculate step size & sub gradient)
end

```

End While

Final solution  
 optimal  $\lambda^{k+1}$   
 optimal  $\boldsymbol{\tau}^{k+1}$   
 optimal  $\theta(\lambda^{k+1})$

### **STEP 6**

*%Perform checks*

*Upper bound should converge to lower bound (use upper bound iteration convergence as checking criteria)*

*% Check dual gap*

$$|\theta(\lambda^{k+1}) - Z(\boldsymbol{\tau}^{k+1})| < |\theta(\lambda^k) - Z(\boldsymbol{\tau}^k)|$$

### **2.1.3 Feasible solution**

Any feasible solution to Problem (4) serves as a lower bound to  $Z^*$ . A reasonably good (at least better than assigning zero tokens) feasible solution is to distribute  $\frac{T}{N}$  tokens to each of the alternatives, and zero tokens to the least energy-

efficient alternative, of each person. This ensures that the token budget constraint is satisfied (although not necessarily binding), as the expected token consumption is

$$\sum_{n=1}^N \sum_{i=1}^{C_n-1} \frac{T}{N} P_{ni} \leq \sum_{n=1}^N \sum_{i=1}^{C_n} \frac{T}{N} P_{ni} = \sum_{n=1}^N \frac{T}{N} \sum_{i=1}^{C_n} P_{ni} = \sum_{n=1}^N \frac{T}{N} = T, \text{ assuming}$$

without loss of generality that the least energy-efficient alternative is the last alternative in the choice set for any given person  $n$ .

The optimal solution to the parameterized problem described in the section below is a feasible solution to the original problem. At any iteration  $k$  of the

Lagrangian minimization problem, if  $g(\tilde{\tau}^k) \leq T$ ,  $Z(\tilde{\tau}^k)$  is used as a new lower bound if it improved the existing one.

## 2.2 Parameterized Token Allocation Strategies

Recent work has focused on optimizing incentives for end users but ignore equity related to allocating incentive. Its important to note the difficulty in determining an effective solution is due to the assumptions that the constraints are allocated in divisible portions of a constrained budget and involves utility functions that are complex mathematical forms involving probability measures.

Therefore, Instead of optimizing over the number of tokens for every user and alternative, parameterization of the token allocation reduces the number of decision variables and thus potentially reduces the computational time to obtain an optimal solution in that the numbers of tokens assigned are directly proportional to energy savings. This, however, comes with the price of solution sub-optimality.

In the simplest case, a single parameter,  $e$ , is defined as the token average energy saving (TEE), and the number of tokens assigned to alternative  $i$  of user  $n$  is:

$$\tau_{ni} = \max \left\{ \frac{E_{ni}}{e}, 0 \right\}. \quad (11)$$

Similar to the single parameterization strategy, a mode parameterization strategy is formulated, whereby a duo of mode parameters (e-bus and e-bicycle) are conceptualized as the TEE for bus and bicycle consecutively.

$$\tau_{ni} = \max \left\{ \frac{E_{ni}}{e_i}, 0 \right\}. \quad (12)$$

Furthermore, a more complex strategy is formulated for travel-time parameters  $e$ , based on  $w$  categories of the average travel-time between bus and

bicycle modes. The categories are broken down by a method of nearly equal frequency distribution of trips in the order e-(7-22)minutes, e-(23-33)minutes and e-(34-55)minutes.

$$\tau_{ni} = \max \left\{ \frac{E_{ni}}{e_{iw}}, 0 \right\}. \text{ If } \frac{\sum t_{ni}}{|C_n|} \text{ is in the } w\text{-th bracket,}$$

where  $w$  = average travel time category.

## CHAPTER IV

### COMPUTATIONAL TEST DESIGN

We systematically investigate how the changes in token budgets and token coefficients, i.e. token inflation rates, affect the system wide energy savings across the original and parameterized problem formulations with a static optimization model i.e. no congestion effects. Both the global and local 'fmincon' function in MATLAB's toolbox is used to solve the lagrangian function. (Figure 1) systematically illustrates the distribution of the problem formulations and solution algorithms evaluated. Duos of heuristic solution algorithms are further investigated to improve the local solvers algorithm solutions, while token, mode and travel-time parameterization strategies are investigated using the local solver solution algorithm.

Heuristic 1 & 2-solution algorithm are cloned versions of the local solvers solution algorithm. However, the iterative sub-gradient algorithm for Heuristic's 1 algorithm is modified to scale up the assigned tokens of all users (OD pairs) when a feasible solution is attained and the token constraint is not binding, while Heuristics 2 solution algorithm scales up when the token budget constraint is not binding and down when the token budget constraint is violated.

Through the optimization computations, we measure the computational run-time of each solver and formulation. All computational experiments were carried out on a PC running Windows with 8 GB memory and Core i7 3.4 GHz central processing unit CPU.



### 3.1 Token Budget/Coefficient

The token budget is designed based on an exogenous incentive budget assignment with an initial value selected based on the assumption that the decision maker's choice is to realize a 10% reduction in the auto mode share. Through a series of pretests, an assigned token budget of \$5,000 was observed to have reduced the choice probability of using auto mode by approximately 10% from the base case scenario i.e. no token incentives. This token budget was subsequently used as a starting point token budget. We further explore how changes in the token budget affect the system's energy savings by an increment of 10% corresponding to 10 different budgets.

Similarly, an initial is calculated using the logit model in parallel with the choice probability of each alternative  $C_n$  from the base case scenario with an assumed average value of time of 20\$/hr. The token coefficient is successively increased by (50, 70,90,110,130 and 150)%.

**Table 1: Solution Algorithms for the Full Problem and Parameterization Strategy**

Formulations	Full Problem			Parameterized		
	Lagrangian Heuristics			Lagrangian Solution		
	w/o	1	2	Single	Mode	TT
L.S						
Global	✓	×	×	×	×	×
Local	✓	✓	✓	✓	✓	✓

### 3.2 Input Data

Input data necessary for the analysis are obtained from the Pioneer Valley Planning Commission (PVPC) for the Town of Amherst, MA (See Figure 2). They

comprised a total of 21 traffic analysis zone's (TAZ), each representing an origin and destination, corresponding to 441 origin-destination pairs, of which encompassed a 24-Hr Origin Destination-demand matrix for each OD pair. The three modes of transportation's mode split; Auto 63.25 %, Bus 9.25 % and Bicycle 27.5 % are observed from the PVPC database (14). and the token coefficients were adjusted to match the observed mode split; Auto 62.48 %, Bus 9.78 % and Bicycle 27.73 %. A total of 9404 trips were observed between all OD-pairs and it is assumed that the demand through the analysis period is inelastic. Travel time  $t_{ni}$ , in minutes for all three modes of transportation are manually estimated using goggle maps for all OD-pairs, while the distance between each OD pair is computed as: (average-speed \* Travel time  $t_{ni}$ ). The networks computational average speed, auto fuel economy and value of time of bus VOT, is assumed at values 35 mph, 23.4 mpg<sup>1</sup> and 20 \$/hr. consecutively (15).

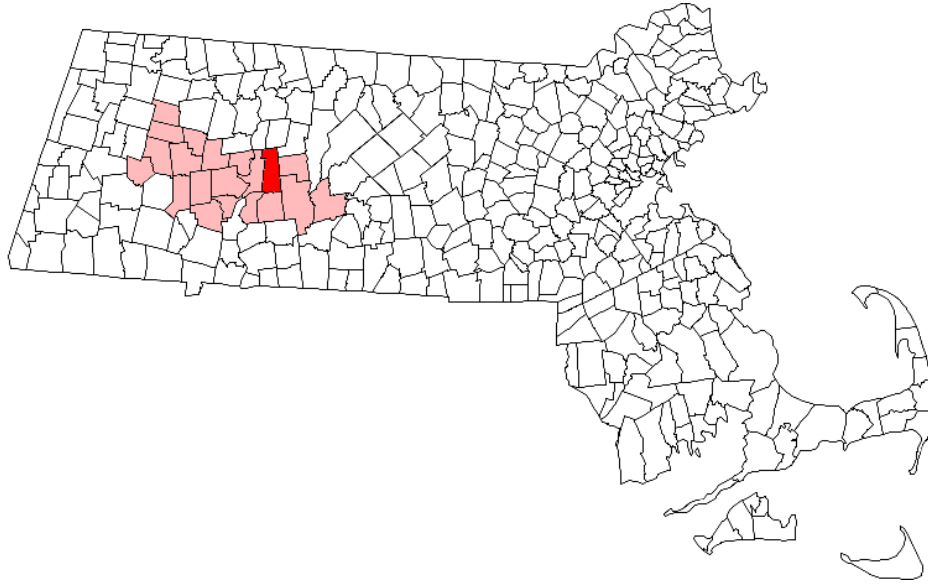
Using excel with previously obtained and assumed data, the travel time coefficients are adjusted as follows: Auto -0.195/min, Bus in-vehicle -0.061/min, Bus out-vehicle -0.122/min, Bicycle -0.15min, while the token coefficient of bus 0.183/\$, which equals that of auto and bicycle for computations is calculated as: bus in-vehicle travel time coefficient/ value of time of bus  $\frac{\beta_{nbus}}{VOT_{bus}}$ . This implies a monetary value of a token to a dollar. The Alternative specific constant for auto was interpolated to -0.05

Subsequently, the input data's are processed for application to the problem formulations as listed below:

---

<sup>1</sup> MAPC Data Services, Massachusetts Vehicle Census: <http://data.mapc.org/datasets>

**Figure 1: Amherst Network**



## CHAPTER V

### RESULTS

As described above, we conducted a series of analysis to study the performance of various problems formulations and solution algorithms with regards to the effects of token budgets and token coefficients on energy savings.

It is of importance to note that the scale of the optimality gaps in this analysis can be misleading and not necessarily accurate, as these gaps represent the percent difference between the solutions of lagrangian and objective function at a feasible region. As observed from the solutions of the local solver algorithms in Table 1, these values were lower than the results from the global solution algorithm, but results from the local solvers algorithms converged at an optimality gap less than 1%. An explanation for this behavior is related to evidence that the lagrangian function from the local solution algorithm might not be solved to a global optimum. Theoretically, a true solution to the problem lies between the lagrangian and objective function solutions. A non-global solution of the lagrangian will certainly result in a lower lagrangian solution, but still above the true solution. Hence an inaccurately underestimated optimality gap will be obtained at solutions that are categorized by non-global lagrangian function solutions.

The results of this analysis are detailed in the sections below.

## **4.1 Effectiveness and Efficiency of Solution Algorithms**

### **4.1.1 Local vs. Global Solver for Lagrangian Solutions**

When we consider the effectiveness of the global solver solution algorithm, the results show a significant reduction in the system wide energy savings from the base case scenario i.e. no token rewards in the system. The systems energy savings was observed to have initially increased rapidly with subsequent incremental increases as the token budget increased with a mildly non-linear relationship and positive correlation of +0.9753. The average energy consumption reduced by 12.79% - 21.09% for a token budget range of \$5,000-\$10,000. This corresponds to a percent increase of the bus and bicycle mode share by 16.70% - 30.46% and 17.56% - 30.67% consecutively (See Table 1). The solutions average computational run-time for this algorithm is 43hrs. The local solvers solution algorithm reduced the average energy consumption by 11.48% - 19.91% for a token budget range of \$5,000-\$10,000. This corresponds to a percent increase of the bus and bicycle mode share by 11.94% - 16.60% and 19.33% - 26.15% consecutively (see Table 1). The average computational run-time for this algorithm is 24 minutes. From these results above, it is evident that the global solution algorithm performs better than the local solution algorithm. For a range of \$5,000-\$10,000, the global solution algorithm improved by 10.22% - 5.58% from the local solution algorithm (see Table 1).

**Table 2: Results from Global vs. Local Solver Solution Algorithm**

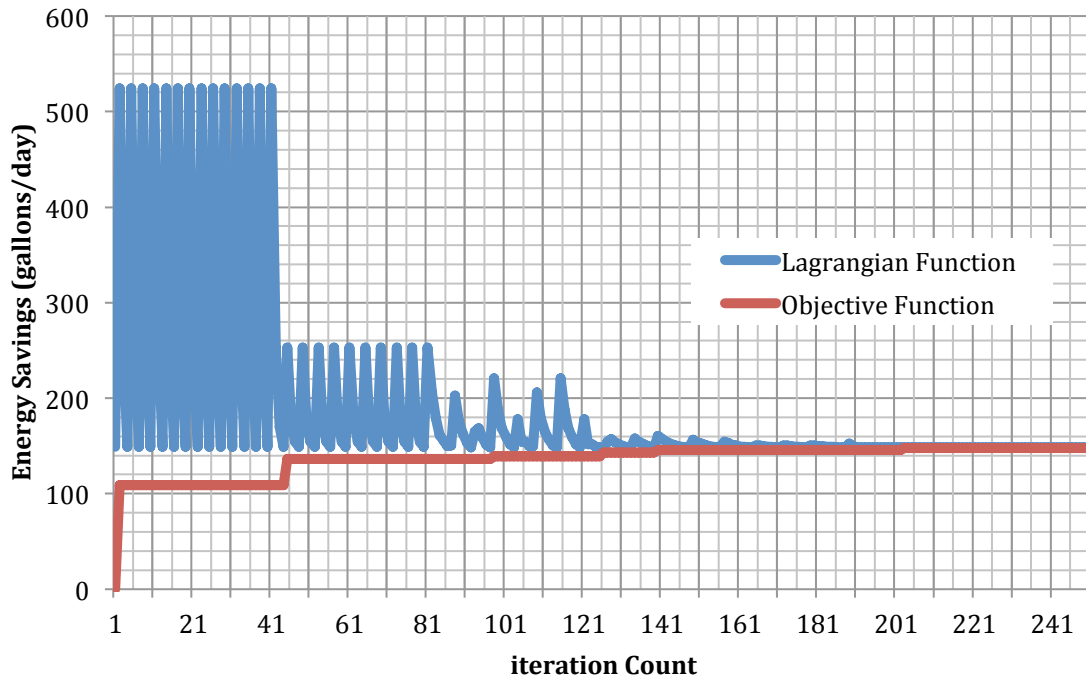
Token Budget (\$)	Global.S Energy Savings (gal./day)	Local.S Energy Savings (gal./day)	Global.S Bus Mode Share (%)	Global.S Bicycle Mode Share (%)	Local.S Bus Mode Share (%)	Local.S Bicycle Mode Share (%)	Global.S vs. Local (% Diff)
5000	148.1	133.0	11.79	32.57	11.31	33.06	-10.22
5500	157.9	132.9	11.96	32.95	11.31	33.06	-15.81
6000	168.5	132.8	12.03	33.32	11.30	33.05	-21.20
6500	179.9	167.2	12.25	33.73	11.57	33.87	-7.04
7000	189.9	177.9	12.38	34.10	11.61	34.07	-6.33
7500	199.5	187.0	12.49	34.47	11.64	34.23	-6.26
8000	208.8	196.3	12.65	34.85	11.68	34.39	-5.98
8500	218.0	204.7	12.81	35.24	11.70	34.53	-6.10
9000	227.0	213.8	13.02	35.65	11.73	34.68	-5.81
9500	235.4	222.2	13.08	35.93	11.75	34.81	-5.63
10000	244.2	230.6	13.18	36.20	11.78	34.94	-5.58

#### 4.1.2 Optimality Convergence of Global Solver Solution Algorithm

Observations indicates that all solution from the global solvers solution algorithm converges with an optimality gap less than 1%, which suggests the solutions have less than a 1% error margin to the true solution. (Figure 2) shows the convergence chart for the lagrangian function solution (upper bound) and the energy savings objective function (lower bound) for a token budget of \$5000. The convergence is verified through an iterative method using the sub-gradient algorithm. The 4 noticeable groups of oscillation on the chart are as a result of the changes in scale of the step size of the sub-gradient algorithm. During the course of the iterative process, as the lagrangian function fails to reduce after an initial preset iteration count 20, then subsequently 10, the step size scale is reduced by a factor of 2 until a lower lagrangian solution is obtained concurrently with a better (higher)

feasible solution of objective function. This sequence is repeated until both functions converge at an optimality gap less than 1%.

**Figure 2: Optimality Gap Convergence Chart of Energy Savings**



#### 4.1.3 Lagrangian Heuristics

Observations from the local solvers solution sets indicate that the token budgets \$5500 and \$6000 had optimal gaps of 8.66% and 15.02% consecutively, significantly higher than an acceptable threshold of 1%. A duo of lagrangian heuristic solution algorithms are proposed to improve the effectiveness and efficiency of the local solver's solution algorithm for token budgets were by the lagrangian and objective function failed to converge at an acceptable optimal gap, as described in the chapter above. This failure is attributed to the oscillation of the lagrangian function solutions at a local optimum.

For token budgets \$5500 and \$6000, the first heuristic solution algorithm (heuristic 1) is observed to improve the performance of the local solvers algorithm. Although still worse off than the global solution algorithms, the realized energy savings was 9.90% and 8.69% consecutively lower than the global solution algorithms, when compared to 15.81% and 21.20% from the local solution algorithm, while the computational run time was reduced by (25mins) a 99.03% change from the global solutions run time (43 hrs.). The Latter (Heuristic 2) results to a 9.22% and 6.64% reduction consecutively from the global solution algorithm, while the computational run time was reduced by 99.49% (13mins) from the global solution algorithm. Nevertheless, both heuristic solutions are noted to be worse off than the local solvers solutions for token budget solutions that already had a significantly low optimal gap (less than 1%), as observed for the solutions of all token budgets except 5500 and 6000. This failure in improvement is attributed to distortions in the iterative process as the objective functions are heuristically scaled.



**Table 3: Effects of Global Solver Algorithm vs. Local Optimization Solver Heuristic Solutions**

Token Budget (\$)	Global.S Energy Savings (gal. /day)	Local.S Optm. [No-Heuristic] (gal. /day)	Heuristic-1 (gal. /day)	Heuristic-2 (gal. /day)	Global.S vs. Heuristic-1 (% diff)	Global.S vs. Heuristic-2 (% diff)
5000	148.1	133.0	130.2	130.2	-12.09	-12.08
<b>5500</b>	<b>157.9</b>	<b>132.9</b>	<b>142.3</b>	<b>143.4</b>	<b>-9.90</b>	<b>-9.22</b>
<b>6000</b>	<b>168.5</b>	<b>132.8</b>	<b>153.9</b>	<b>157.3</b>	<b>-8.69</b>	<b>-6.64</b>
6500	179.9	167.2	166.6	166.8	-7.38	-7.29
7000	189.9	177.9	176.4	168.1	-7.09	-11.48
7500	199.5	187.0	178.5	187.3	-10.54	-6.14
8000	208.8	196.3	191.1	196.4	-8.47	-5.93
8500	218.0	204.7	203.5	204.6	-6.63	-6.13
9000	227.0	213.8	212.4	213.8	-6.44	-5.82
9500	235.4	222.2	222.7	221.5	-5.41	-5.90
10000	244.2	230.6	221.8	229.5	-9.17	-6.00

#### 4.2 Effectiveness and Efficiency of Parameterization Strategies

Results from the parameterized formulations were observed to have resulted in lower energy savings when compared to the original problem formulation with a global solver solution algorithm. The parameterization strategy analyses were conducted using a local solver algorithm for the lagrangian solutions. Although, from observation of the local vs. global solution algorithm above, it is evident that the local solvers solution algorithm did not solve the lagrangian functions to optimum solutions, which resulted in lower energy savings when compared to the global solvers solution algorithm. An exhaustive search method is used to validate the solutions of the token parameterization solutions of the lagrangian are solved to the optimal solution with the correct optimality gap to the objective function. This

selection was based on the analogy and can be theoretically proven that the solutions of the token parameterized formulations are convex at optimum.

Observations from this analysis showed a generic improvement in the energy savings across all three-parameterization strategies as token budget is increased. Overall, across all token budget scenarios, travel-time, mode and token parameterization solutions, were observed to be 3.2%, 13.4% and 13.7% consecutively less than the solutions obtained from the original formulation with a global solver. These results indicate road users are more sensitive to average travel-time over mode choice and token consumption. Hence, the travel time parameterization strategy can be considered as more efficient token allocation strategy from the menu of strategies observed.

Mode/token parameterization and travel time and strategies improved the problems computational efficiency by 99.81% (5mins) and 99.73% (7mins) consecutively. To save computational time, it is observed that the parameterized allocation strategy could be used in place of the original problem with a global solver, but not without an optimality penalty i.e. ranging between 4.5-18.3% for a token budget of 5000. (See Table 3).

**Table 4: Global Solver Algorithm vs. Parameterization Strategies Effect of Original Formulation**

Token Budget (\$)	Single P.Strategy (gal. /day)	Mode P.Strategy (gal. /day)	Travel-Time P.Strategy (gal. /day)	Global vs. Single P.Strategy (% Diff.)	Global vs. Mode P.Strategy (% Diff.)	Global vs. Travel-Time P.Strategy (% Diff.)
5000	121.0	121.0	141.5	-18.27	-18.27	-4.474
5500	131.7	131.7	152.3	-16.58	-16.58	-3.551
6000	142.2	143.3	162.6	-15.62	-14.98	-3.498
6500	152.5	153.5	171.4	-15.25	-14.67	-4.723
7000	162.5	163.6	183.2	-14.42	-13.86	-3.527
7500	172.5	173.0	193.4	-13.52	-13.30	-3.070
8000	182.3	183.1	203.0	-12.68	-12.29	-2.752
8500	191.6	191.8	212.4	-12.11	-12.00	-2.564
9000	200.8	202.0	221.2	-11.54	-11.04	-2.561
9500	210.5	211.3	230.6	-10.58	-10.27	-2.044
10000	219.3	220.3	239.1	-10.18	-9.78	-2.076

#### 4.2.1 Token Efficiency in Various Parameterization Strategies

As expected, the token energy efficiency parameter decreased as the token budget increased. This observation indicates that as more tokens are introduced into the system, the energy savings in the system increases while road users consume more tokens. It is of importance to note the inversely non-linear relationship between the token budgets and energy savings per token  $e$ . As the token budget increased, the average energy savings per token reduced. An explanation for this observation is in accordance with the allocation of more tokens to the same level of energy savings as the token budget increased, hence a smaller energy savings per token will be realized. (See Appendix A)

### 4.3 Sensitivity Analysis w.r.t Token Coefficient

Similarly, for a token budget of 5000, a 50-150% increase of the token coefficient resulted in a reduction ranging between 17.4% – 25.84% in energy savings (see Table 5). Changes in the token coefficient appear to be sensitive to the systems energy savings. Energy savings was observed to increase as token coefficient increased. The two variables are correlated by an index of +0.997, which suggest a strong correlation. The optimality gaps for all solution were less than 2%. The solution obtained when the coefficient of token are varied with a fixed base budget show that energy savings increases as the coefficient of tokens are increased. The energy savings per token increases as the coefficient of token increase with a non-linear relationship. The token coefficient has a proportional relationship with the value of time. Therefore, as the VOT is increased, the results indicate an increase in energy savings. This essentially means the model is somewhat sensitive to road users VOT.

**Table 5: Effect of Token Coefficient**

<b>Token Budget (\$)</b>	<b>Token coefficient (% Incr.)</b>	<b>Energy Savings (gal./day)</b>	<b>Energy Savings (% Change)</b>
5000	50	202	17.47
5000	70	227	19.58
5000	90	245	21.18
5000	110	267	23.01
5000	130	283	24.46
5000	150	299	25.84

## CHAPTER VI

### CONCLUSION AND RECOMMENDATIONS

#### 5.1 Summary of Findings

An incentive optimization method with global and local solvers is presented that maximizes logit model based formulations to attain optimal and feasible solutions w.r.t energy savings while satisfying budget constraints. Important parameters that affect energy saving's in a road transportation system have been studied, both the token budget and coefficients of token. We also present a parameterized formulation that necessitates equity in the allocation of a constrained budget and highlight the efficiency of this approach through numerical results. This formulation ensures that each unit of reward is equitable to a single unit of energy savings and provides alternative policy options to decision makers. Strong evidence exists to conclude that there is a positive correlation between the system's energy savings and the aforementioned parameters. An increase in the token budget resulted to an increase in expected energy savings as expected, while the travel-time parameterization provided a more cost effective computation as a substitute to the original problem formulation, saving significant computational time. Furthermore, the analysis demonstrated that a global solver algorithm is necessary to obtain a best attainable optimal solution when designing an incentive optimization problem. These observations substantiate the concept that incentives allocation in a transportation system is an alternative strategy to congestion pricing in reducing fuel consumption and ultimately increasing energy savings with less

resources expended while eliminating the often referenced social equity amongst road users.

While this research has been able to successfully demonstrate that assigning rewards to a transportation network can effectively result to the reduction of fuel/energy consumption, it is important to note that the same outcomes or attributes might not necessarily apply when formulated for a different city, as mode splits and travel time indexes will vary across different cities.

## **5.2 Limitations**

With these potential modeling framework contributions in mind, it is essential to recognize its associated limitation. The Town of Amherst is geographically constrained by a sprawled network with a moderate public transit service relative to its size. When considering transit options, on average the distance and travel time between a majority of the OD pairs is relatively large, which signals a higher likelihood of road users using auto mode to avoid high door to door walking distance. Similarly, bikers would be less willing to bike long distance in order to save travel time. Taking into consideration weather effects, road users will be less willing to use mass transit during inclement weather conditions. Another key limitation of this study is the omission of road users household and socioeconomic variables. The absence of these characteristic variables could potential increase the likelihood of model bias upwards on the solutions.

### 5.3 Future Work

Due to the complexity and non-linear characteristic of the problem studied. The model was simplified by using a naive and simplistic utility function to ensure proper understanding of the characteristics of the decision variables involved. Future studies on this research can be improved by expanding the vehicles classes (truck/cars etc.) used in the model to further reflect a more realistic representation and variability of energy consumptions by road users. The model can subsequently be expanded by increasing the number of decision variables that affect individual choice of modes of road transportation to improve the predictability and reduce bias of the choice models/probability in the utility function. The traffic conditions for the analysis are considered the best-case scenario based on the assumption of free flow travel speed, which accounts for a constant travel time. A dynamic optimization approach can be used to model the problem rather than a static optimization to reflect realistic congestion effects, which include dynamically varying rewards based on the current and predicted state of the system. Similarly, future research in this area of study will comprise of a bi-level optimization problem formulation where a high level optimization aims at maximizing system-wide energy savings, while a low-level consumer surplus maximization problem is solved for each system user.

## APPENDIX

### TABLE RESULTS OF TOKEN EFFICIENCY FOR PARAMETERIZED STRATEGIES

Parameterization -		Single				Mode				Average Travel-Time		
Token Budget (\$)	e	Energy Savings (gal./day)	Global vs. Single P. (% Diff.)	e-bus	e-bike	Energy Savings (gal./day)	Global vs. Mode P. (% Diff.)	e-[(7-22)min.]	e-[(23-33)min.]	e-[(34-55)min.]	Energy Savings (gal./day)	Global vs. Travel Time P. (% Diff.)
5000	0.095	121.0	-18.27	0.065	0.117	121.0	-18.27	*	0.257	0.054	141.5	-4.47
5500	0.087	131.7	-16.58	0.063	0.109	131.7	-16.58	*	0.196	0.052	152.3	-3.55
6000	0.081	142.2	-15.62	0.059	0.095	143.3	-14.98	*	0.160	0.050	162.6	-3.50
6500	0.076	152.5	-15.25	0.056	0.088	153.5	-14.67	*	0.138	0.048	171.4	-4.72
7000	0.072	162.5	-14.42	0.054	0.082	163.6	-13.86	*	0.117	0.046	183.2	-3.53
7500	0.068	172.5	-13.52	0.052	0.077	173.0	-13.30	*	0.103	0.044	193.4	-3.07
8000	0.064	182.3	-12.68	0.051	0.072	183.1	-12.29	*	0.093	0.043	203.0	-2.75
8500	0.062	191.6	-12.11	0.049	0.069	191.8	-12.00	*	0.084	0.041	212.4	-2.56
9000	0.059	200.8	-11.54	0.047	0.064	202.0	-11.04	*	0.078	0.040	221.2	-2.56
9500	0.056	210.5	-10.58	0.046	0.061	211.3	-10.27	*	0.072	0.039	230.6	-2.04
10000	0.054	219.3	-10.18	0.045	0.059	220.3	-9.777	*	0.067	0.038	239.1	-2.08

Note: \* indicates a very large number for e (no token assigned)



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