Aalto University

School of Science

Master's Programme in Life Science Technologies, Complex Systems

Alexander Gurevich

Comparing network-based taxation schemes to reduce systemic risk in banking systems

Master's Thesis

Espoo, 3 June 2019

Supervisor: Jari Saramäki, Professor

AALTO UNIVERSITY
School of Science
Master's Programme in Life Science Technologies

ABSTRACT OF THE MASTER´S THESIS

Author: Alexander Gurevich	
Title of the thesis: Comparing network-based taxation schemes to reduce systemic risk in banking systems	
Number of pages: 57	Date: 03 June 2019
Major or Minor: Complex Systems	
Supervisor: Jari Saramäki, Professor	
Thesis advisors: Jari Saramäki, Professor	

Difficulty in predicting financial crises may lead to attempts of getting ready to them instead. Authorities construct rules for financial institutions in order to secure the economies from catastrophic consequences of systemically important financial institutions defaults.

In order to create proper stimuli for the economic agents to avoid deals exposing the entire financial system, the tax on the systemic risk was proposed. While systemic importance of financial institutions might be measured in different ways, different metrics (namely: degree centrality, SinkRank, acyclic DebtRank, cyclic DebtRank and 2-step DebtRank) are compared as candidates for the systemic risk tax basis.

Economics, being in focus of complex systems disciplines, is very difficult to investigate due to the impossibility to set an experiment. In order to test a certain policy, one may use computer simulations. In this thesis, an agent-based model is used to investigate the simulated economy evolution under different credit taxation policies. For this special purpose, one more systemic risk metric was proposed (it is called 2-step DebtRank).

In addition, an equity layer for the interbank obligations network was introduced in order to investigate interlayer interaction.

As a result of the taxation schemes comparison, acyclic and cyclic DebtRanks demonstrated the best performance as systemic risk taxes; 2-step DebtRank appeared to perform closely to other two DebtRanks; SinkRank appeared to be inappropriate for this purpose.

The introduction of the equity layer does not create many essentially new results, however, the interlayer antiphase oscillation of the systemic risk was observed.

Keywords: Agent-based modelling, systemic risk, banking networks, DebtRank, SinkRank	Publishing language: English

Table of contents

I. Introduction

The Great Recession that has begun with 2007-2009 U.S. subprime mortgage crisis appeared to be unexpected by most of the world economic society like all the previous crises [1], spurring again interest for crisis management. And while one could think of how to predict the unpredictable, another approach [2] would focus on how to make failures unpredictable by their own nature to be less harmful. A failure of one component of the complex system, like an economic one, could lead to the failure of many other components in a domino effect fashion. In this work, we will discuss the theoretical foundations for designing a banking system less exposed to such events also considering economic growth. The desired effect would be a decrease in a failure cascade size, decrease of a probability of any bank to fail and the minimal decrease in output growth and consumption.

Arthur Cecil Pigou, an English economist, has developed a concept of externality [3] – cost or benefit affecting third parties. The most popular and illustrative example is environmental pollution, e.g. of air. Factories ejecting poisonous smoke produce more than they would do in case if they should have paid for all the harm caused to local people breathing polluted air. Other negative externalities (i.e. incurring not benefits but costs) examples are noise pollution, common property resources misuse, passive smoking, antibiotic resistance etc. Pigou proposed to levy taxes on the negative externalities producing actions in order to gauge social damage. This kind of taxation is called a Pigouvian tax.

An externality that is less obvious is the systemic risk incurred by the actions of banks. Every new transaction changes the interconnectedness of the assets (credits, CDSs, equity etc.) networks potentially exposing third parties to the defaults. The first problem is that the risk might be not embodied and the second is that there are different ways to measure the systemic risk. This is actually what this work would be dedicated to.

Another currently used alternative that was proposed by James Tobin (for a slightly different purpose though [4]) is taxation based on the volume of tax. It is much easier to calculate; however, its effectiveness is under dispute. We will discuss it too.

Summarizing, this work is dedicated to the Pigouvian tax applied to the banking system in order to decrease systemic risk. This idea was proposed with the DebtRank [5] in [6]. DebtRank is the systemic risk measure, denoting the fraction of the system's assets being exposed to the failure of the certain pool of institutions.

We model the economy in an agent-based fashion and try different systemic risk metrics. The reader can think of the agent-based modelling as a videogame development: different economic agents and the interaction between them are introduced, and then the economy is simulated. The economic agents behave as defined: give credits to each other, pay them back, sell and buy goods. The most important reason for the choice of agent-based modelling is that this fashion allows modeling very complex scenarios, what is done in this thesis. We also discuss other extensions of this model.

Network-based risk metrics and thus a tax on some of them involve the calculation all of the network nodes; in other words, this tax is based not only on the acting agent behaviour but also on the other agents' behaviour. Without taking into account technical difficulties in the real-world implementation, like computing these metrics for every transaction (one has to know all the up-to-date data on the obligations for all the financial institutions), we consider a legal issue: one usually cannot be taxed based on others behaviour. For this purpose, we introduce the truncated version of the original DebtRank and tested it.

As a result of this thesis, we obtained, that for the purpose of systemic risk tax different DebtRank versions work the best. We can generalize, that only metrics specially developed for measuring different systemic losses in response to the system's distress would work. Another metric, SinkRank, performed much worse and has demonstrated to be unsuitable for this purpose. 2-step DebtRank (that we suppose to be legal) introduced in this thesis, appeared to perform quite well; however, we think, that more investigation in this field is needed.

II. Background

This thesis is based on the paper [6], where authors introduce the systemic risk tax. Systemic risk tax is the tax levied on the systemic risk externality, and that is why authors call it Pigouvian. Every new transaction changes the systemic risk in the entire economy, and in order to manage it, authors propose to tax more risky transactions. They model it in an agent-based fashion and measure risk using the DebtRank (III.B.ii), [5]. This research is very interesting in the paradigm of the systemic risk management whipped up by the recent Great recession led to big alterations in the regulation of the financial world, like Basel III framework [7]. Instead of tightening remedies, like greater capital requirements to the systemically important institutions, authors propose a "smart tax", which is much smaller than the traditional Tobin tax, affects the economic activity the same way and stimulates economic agents to create much more robust and default-proof system. One of the aims of this research, however, not looking feasible to implement, is to begin the discussion of hypothetical methods to decrease systemic risks via economic policy.

We continue this research, describing the economy considered in the same fashion, introducing taxation rates based on some other systemic risk measures, and observe the simulated economy evolution. We will consider how long the system lives before the failure, the amount of assets lost due to the catastrophe and number of banks defaults caused by the first failure.

A. Systemic risk measures

Systemic risk is usually defined as a risk of the entire system to collapse; it is opposed to individual, idiosyncratic risk. The central idea of paper [6], on which this thesis is based, is to separate default risk from contagion risk. Default contagion is an event of a default being propagated from one institution to the other. So, the default is an idiosyncratic event, and contagion is a systemic one. In our thesis, we compare systemic risk metrics as the means for managing systemic risk while basing the tax rate on these metrics.

So, there are two main paradigms on how to measure this systemic risk exposure caused by any institution. First is "too big to fail" – idea, that some institutions are so large, that their failure would cause a significant domino effect and lead to many consequent failures. The second paradigm – "too interconnected to fail" – is similar, but in this case, the interconnectedness is considered to be a systemic risk factor. We will consider the second one. Let us consider the components of our model and their interaction in detail.

In the [8] paper, there are 5 different models (Eisenberg-Noe, Rogers-Veraart, Default Cascades, original DebtRank and cyclic Debtrank) for systemic risk calculation are presented, compared and discussed. Based on the balance sheets equations, these models differ in assumptions: e.g., DebtRanks are the only models there implemented with mark-to-market valuation (it is their advantage), and Rogers-Veraart differs from Eisenberg-Noe model only by bankruptcy costs. We do not consider the first three models, because it is proved that these models underestimate systemic risk compared to DebtRank models. All these models can be interpreted as a re-evaluation of the interbank claims [9]. First two models, unlike the latter ones, were initially introduced as clearing payment mechanisms for the banks networks with defaults, however, they also might be used for stress-testing [10]. All these models define the amount of assets exposed to the initial distress (which is defined here as a default of a certain institution). Based on this value, one can rank the institutions by the systemic risk they bring in the interbank network.

SinkRank systemic risk measure [11] also considered in this thesis is very different from these 5 models. Initially, it was developed for the interbank payments networks. Being more abstract, it denotes the inverse average number of steps in a random walk until the defaulted institution.

B. Agent-based approach

i. Motivation

The core motive of this thesis is the application of different systemic risk metrics for taxation in order to check the robustness of the system and consider the growth-sustainability trade-off. Traditional economic modelling approach (econometric models, considering "representative" agents or/and aggregated macroeconomic variables) would barely work here because of the following reasons:

- Systemic risk measures are too complicated for the analytical inference;
- The considered problem is focused on rare (tail) but destructive events;
- Lucas critique;

We confine the tail events discussion to the following quote:

> *Much of the real world is controlled as much by the "tails" of the distributions as by means or averages: by the exceptional, not the mean; by the catastrophe, not the steady drip; by the very rich, not the "middle class". We need to free ourselves from "average" thinking.* [12]

Concerning the last pitfall of the analytical approach, Lucas critique [13] of the traditional economic modelling blames it on that any economic policy based on the historical data relationships observation is naïve because the economic policy itself will affect economic agents' motivation thus probably leading to quite different results. This critique whipped up the discussion in an economic society and led to the idea of microfoundations, i.e., the individual behaviour description for the macro-perspective modelling. While our research is focused on changing this individual behaviour via taxation policy, this critique is relevant more than ever.

As an attempt to solve this issue, agent-based modelling was proposed. We also selected it for our purpose.

The model used is the one described in the book [14]. The model is called the Bottom-up Adaptive Macroeconomics (BAM) model, belonging to the agent-based computational (ABC) models class. ABC models are alternative to the mainstream dynamic stochastic general equilibrium models (DSGE – a class of models, based on the general economic equilibrium assumption, considering a temporal development of the system with stochastic random shocks) [15]. ABC economics is the use of computer simulations to grow and study evolving artificial economies composed of many autonomous interacting agents [14].

Their advantages include but are not limited to:

- Heterogeneity – it is computationally indifferent whether the model consists of similar objects or of heterogeneous ones.
- Explicit definitions – one can easily define explicitly the spaces where agents are placed, the law's interaction between agents, including, for example, the possibility of bounded rationality assumption (one of the core assumptions in New Institutional Economics, also quite a fruitful approach).
- Non-equilibrium dynamics – ABC models allow to investigate the whole market processes, instead of standard considerations on start-end equilibrium states.

For the sake of fairness, it is worth considering the main disadvantages:

- Hard to fit the model to the real data, i.e. not allowing to make predictions.
- The model tends to overfit in a sense that any desired dynamics could be modelled.

On the other hand, these models allow to qualitatively describe investigated phenomena and even to test governmental policies, e.g. [16] or [17], what will be demonstrated in this thesis.

ii. Brief model description

The model simulates the whole country (of course, a hypothetical one) economy as a closed one (i.e. no external trade or cash flows). There are 3 types of economic agents on the supply side: firms, bankers and hired employees. All of them are pursuing their interests. Firms hire employees, employees create goods within firms. Banks lend money to firms to help them execute their plans and to each other to help giving loans to firms and to manage their liquidity. We introduce taxes only for the interbank loans, and this is the only point of systemic risk management. The intuition is the following: if the increasing systemic risk transactions are taxed more, then banks are less motivated to make deals exposing the whole system to fail.

All these agents, on the other hand, behave as consumers on the market, i.e., demand side. They receive their wages (salary for workers, dividends for banks and firm holders). Then they construct their budgets and buy goods provided by firms.

Most of the variables used have a stochastic nature, so, firms and banks can fail. Instead of the bankrupt firm, a new one is created, and when the bank defaults, the simulation is stopped, and macroeconomic variables are saved. Due to the stochasticity, the simulation has always to be repeated many times, and the obtained sampled distributions of the macroeconomic variables are analyzed.

III. Methods

A. Description of the model

In this section, we give a rundown of the algorithm of modelling the economy in detail. First, we begin with a sequence of events description, and then we will consider all the economic agents and events. We will consider them in the chronological order of every time step, providing formulae describing their behaviour at every step. For instance, not all the formulae assigned to firms will be in section "Firms" (III.A.ii), some of them will be introduced in "Credit market" (III.A.v) section.

Note: the notation will be the same, as in the MATLAB code implementing the model; despite perhaps looking clumsy, we suppose the variables names to be useful, while it is plenty of them and ABM is actually developed for the computer implementation.

In this table, the miscellaneous variables not assigned to certain economic agents are described.

Variable	Description
T	Maximum number of simulation steps
t	The current step of the simulation
$Ftot$	Matrix of firm-bank borrowings at a current step.
τ	Rate of debt reimbursements, for both firms and banks.
div	The ratio of dividends to be paid from the profits of both banks and firms to owners.
$refi$	General refinancing rate, defined externally.
ζ	Taxation parameter defined externally.

Table 1. General variables description

i. Sequence of events

The simulation follows this sequence of events:

1. The system is initialized, firms, banks and consumers are set with their initial capitals.
2. Until any of banks defaults, or all of the firms fail, or the simulation time T is run out ($t > T$), repeat:
    a. Firms define expected demands, prices and their labour demand.
    b. Firms apply for credits to banks.
    c. Banks apply for credits to other banks.
    d. Banks receive credits.
    e. Banks pay taxes.
    f. Firms receive credits.
    g. Firms adjust their expectations, prices and labour demand according to credits received.
    h. Firms hire and fire employees.
    i. Firms produce goods.

j. Firms pay wages to their workers.

k. Consumers calculate their budgets and buy goods.

l. Firms maintain their accounting: compute their profits and losses, pay interests, debts and dividends.

m. Bankrupted firms vanish, new firms instead appear.

n. Banks maintain their accounting.

o. Banks default.

p. Non-bankrupt banks participate in the interbank credit market to manage their liquidity.

We illustrate this economic system with the following scheme:

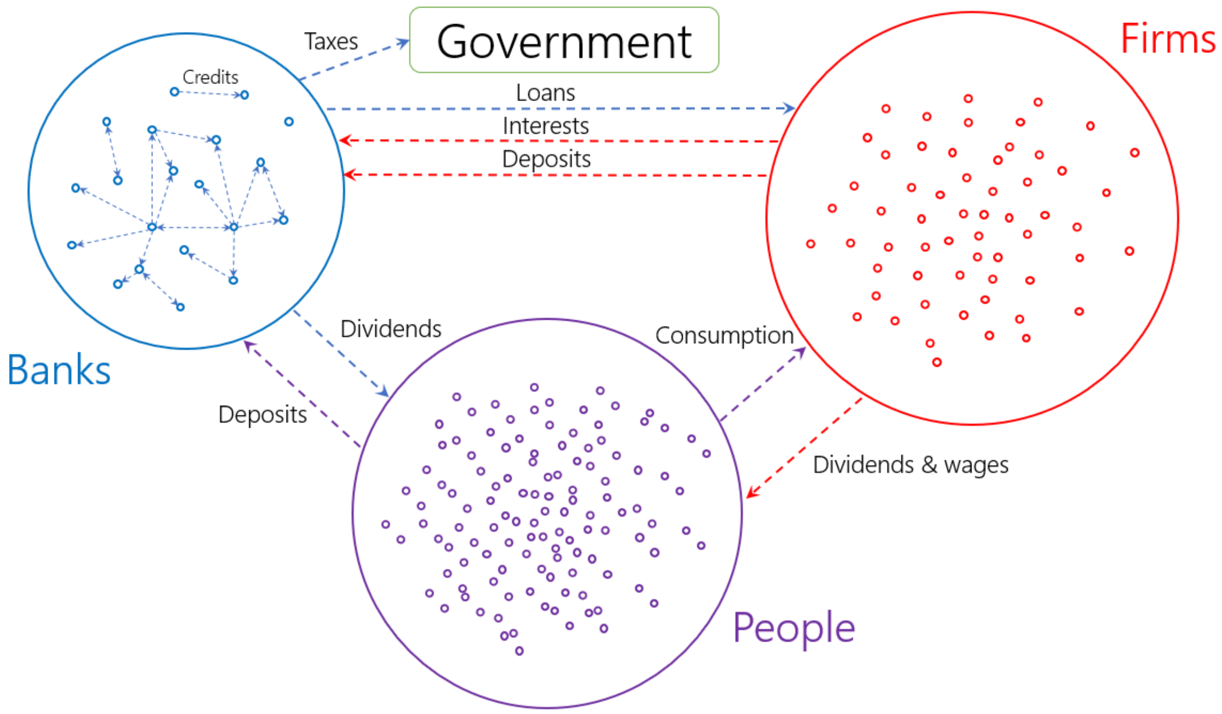


Figure 1. General scheme of the ABM

ii. Firms

Firms produce absolutely substitutable goods. They forecast the demand, the price they could charge, thus the amount of production and the number of workers needed to be hired for the production. In the case of a big number of workers to be hired, they need to raise capital (in this section, we consider only loans).

More precisely:

Variable	Description
I	The number of firms
$Leff_i(t)$	The current workforce of the i-th firm at the time step t.

$Y_i(t)$	Amount of production by the i-th firm at the time step t; in the beginning of every time step i,t is stock. So, in the beginning of time step t considered, $Y_i(t)$ is unsold goods, and $Y_i(t-1)$ is previously produced.
$De_i(t)$	Expected demand estimated by the i-th firm at the time step t.
$Q_i(t)$	Sales of the i-th firm at the time step t.
$price(t)$	Consumer price index (CPI) at the time step t.
$P_i(t)$	The price charged by the i-th firm at the time step t.
$pi_i(t)$	i-th firm profit at time step t.
$\delta_i(t)$	Random variable distributed uniformly between 0 and 0.1; modelled in order to reflect operating individual specifics of the i-th firm at the time step t (without considering the nature of this specifics).
wb	Wage rate.
$wages_i(t)$	Wages i-th firm pays to all workers at the time step t.
α	Labour productivity.
$Ld_i(t)$	Labour demand of i-th firm.
$liquidity_i(t)$	Liquidity of i-th firm at the time step t –the amount of cash resources they can spend, including profits and credits.
$A_i(t)$	i-th firm's equity (book value) at the moment t.

Table 2. Firms' variables.

The firms estimate their expected demand according to the following rule:

$Y_i(t) = 0$ & $P_i(t) \geq price(t)$	$Y_i(t) = 0$ & $P_i(t) < price(t)$	$Y_i(t) > 0$ & $P_i(t) \geq price(t)$	$Y_i(t) > 0$ & $P_i(t) < price(t)$
$De_i(t) = Y_i(t-1) \cdot (1+\delta_i(t))$ $P_i(t) = P_i(t-1)$	$De_i(t) = Y_i(t-1)$ $P_i(t) = P_i(t-1) \cdot (1+\delta_i(t))$	$De_i(t) = Y_i(t-1)$ $P_i(t) = P_i(t-1) \cdot (1-\delta_i(t))$	$De_i(t) = Y_i(t-1) \cdot (1-\delta_i(t))$ $P_i(t) = P_i(t-1)$

Equation 1. The rule for expected demand

In other words; if the stock $Y_i(t)$ is zero and the price $P_i(t)$ was greater than market average $price(t)$, then the demand $De_i(t)$ is expected to increase (stochastic variable $\delta_i(t)$ is for modelling of the decision making inside the firm) and the price is left untouched (Keynes' sticky prices); if everything was sold (zero stock), but the price was under average – same expected demand, increase price; if there were goods left and price was above average – same production, decrease price; otherwise decrease expected demand, leave price untouched. Firms do not want to idle, so, just to have at least 1 worker,

$$De_i(t) = \min(De_i(t), \alpha) \qquad (A.ii.2)$$

Firms estimate their labour demand:

$$Ld_i(t) = \left\lceil \frac{De_i(t)}{\alpha} \right\rceil, \qquad (A.ii.3)$$

where ⌈ ⌉ means roundup. (also, will be later redefined after credit market)

Firms have their equity, credits and interest rates, but we will consider other variables specific to firms in the context of firms' interaction with other economic agents.

Sum of wages to pay is also estimated (it will change later):

$$wages_i(t) = wb \cdot Ld_i(t) \quad (A.ii.4)$$

iii. Consumers

Consumers, or households, consist of banks owners, firms owners (we can also call them capitalists) and workers. After their work is done and they receive their income, they act as consumers on the goods market. In this section, we will just describe their variables.

Variable	Description
$I + J + Bk$	The number of consumers (J is the number of workers).
$PA_i(t)$	i-th household's personal assets at the time step t.
$cons\_budget_i(t)$	Consumption budget of the household i at the time step t
z	A number of applications in a consumption market.
$worker\_bank_i(t)$	A number of the bank where i-th consumer keeps his cash.
Oc_i	Denotes by which firm is i-th worker occupied.
c	A propensity to consume.

Table 3. Consumers variables

iv. Banks

Banks are described by the following variables:

Variable	Description
Bk	The number of banks.
$E_i(t)$	i-th bank's equity at moment t.
$loans_i(t)$	Loans given by bank i to all the firms at the moment t.
$Exp(t)$	Matrix of interbank exposures (row index is the creditor, and the column is the borrower bank).
$C_i(t)$	Bank's cash.
$deb(t)$	Matrix of firm-bank exposures (column index is the creditor, and row is the borrower firm)
$firm\_bank(t)$	Vector of length I, corresponding to the firm's bank (i.e., where the firm keeps its $liquidity$), that is, $firm\_bank_i(t)$ is the number of bank holding i-th firm's cash.
$piB_i(t)$	i-th bank profit at time step t.
$C_i(t)$	i-th bank cash resources (i.e. liquidity) at the time step t.

Table 4. Banks variables

v. Credit market

The following variables are needed to describe the process of credit market clearance:

Variable	Description

$Ftot_{ij}$	Total loans to the i-th firm from the j-th bank at the current step.
B_i	i-th firm's credit demand.
d_i	i-th firm's leverage.
n	The number of credit applications every firm can send to banks.
$maxrate$	The threshold for firms to agree for the bank's proposal – maximal interest rate they can afford.
$inflation\_rate$	Denotes inflation, i.e., change in CPI price.
$cash\_need_j$	The amount of cash bank j needs to satisfy a particular loan.
ν	Minimal equity/loans given ratio defined by financial authorities.
$Exp'(t)$	Temporary variable; denotes exposure matrix $Exp(t)$ after the considered transaction for the interest rate calculation.
lev_p	Leverage of the p-th bank; equals all the obligations to the bank divided by the bank's equity.
lev'_p	Leverage of the p-th bank after the considered credit transaction.
χ_j	Random variable, corresponding to j-th bank's operating specifics.
ψ_j	Random variable, corresponding to other operating specifics of the bank j.
ϵ_k	Another random variable, corresponding to operating specifics of the bank k; $\epsilon_k \sim Unif([0; 10^{-15}])$.
V	Amount of all interbank credits given (excluding the considered loan).
V'	Amount of all interbank credits given (including the considered loan).
tax	Temporary variable; denotes the rate of tax withheld from the interbank credit considered.
$tax_j(k) = tax$	Tax for the loan to the j-th bank from the k-th one.
tax_bank_j	Temporary variable to remember tax corresponding to the best credit proposal to the bank j.
$proposed\_rate_j(k)$	Interest rate proposed by the bank k to the bank j.
$proposed\_rate_{ij}$	Interest rate proposed by the bank j for the loan for the firm i.
ϕ	Coefficient describing the shrinkage of firms' credit appetite after exceeding the threshold.
$credit$	The size of credit given to the firm by the bank.
$ibloan$	The size of interbank credit granted to finance firm loan; in $(C.ii)$ it may also mean capital dilution, i.e. the money bank is raising.

Table 5. Credit market variables

The process sequence as follows: firms define, how much do they need for their production plan (i.e. their credit demand); then they send loan applications to a certain number of banks (this is how the asymmetry of information is modelled); then banks decide on how much do they need to satisfy the demand, and decide on their cash need (i.e. money they lack to give credits). Banks in need apply to each other for interbank loans, taxes for every transaction are calculated and based on this, they determine their rates for the firms applied. Firms choose the best proposals and correct their loan amounts again for the rates; based on this, banks take/give interbank loans, and then take/give loans to firms. A more detailed description is below.

First, we set total bank-firm loans at this step to zero:

$$Ftot = \mathbf{0} \qquad (A.v.1)$$

Firms define their credit demand:

$$B_i = wages_i - liquidity_i \qquad (A.v.2)$$

For the purposes we will discuss later, we calculate leverages for the firms, assuming the credit B_i is granted:

$$d_i = \frac{\sum_{j=1}^{Bk} deb_{ij}(t) + B_i}{liquidity_i + 0.01} \qquad (A.v.3)$$

0.01 term in the denominator needed to avoid division by zero. This leverage is just a ratio of debt and the firm's cash.

Firms with positive credit demand $B_i > 0$ send applications each to n banks. Firms also have a threshold of the interest rate at

$$maxrate = 0.03 + inflation\_rate \qquad (A.v.4)$$

So, every firm i with positive credit demand sends the application to randomly selected n banks. Every bank j received the application computes its cash need:

$$cash\_need_j = -\min\Bigg(0, \underbrace{E_j(t)}_{bank's\ equity} + \underbrace{\sum_{firm\_bank_i=j} liquidity_i(t)}_{firms'\ money\ held\ by\ bank} + \underbrace{\sum_{worker\_bank_i=j} PA_i(t)}_{people's\ money\ held\ by\ bank} - \underbrace{loans_j(t)}_{loans\ given\ by\ bank\ to\ firms} - \underbrace{\sum_i Exp_{ji}(t)}_{loans\ given\ and\ taken\ by\ bank\ to/from\ banks} - \underbrace{B_i}_{current\ loan\ application}\Bigg) \qquad (A.v.5)$$

In order to satisfy financial authorities' requirements, credit recipient bank's leverage ratio (equity divided by all the loans given including the claim from the firm) is computed:

$$lev\_ratio_j = \frac{E_j(t)}{loans_j(t) + \sum_i \max\big(0, Exp_{ji}(t)\big) + B_i} \qquad (A.v.6)$$

In case of leverage greater than a threshold ν (defined by financial authorities) and positive cash need, the considered bank sends applications to all the other banks. If the bank k considered to finance bank j is able to do this, i.e., if

$$E_k(t) + \sum_{firm\_bank_i=k} liquidity_i(t) + \sum_{worker\_bank_i=k} PA_i(t) - loans_k(t) - \sum_i Exp_{ki}(t) - cash\_need_j(t) > 0 \quad (A.v.7)$$

then we compute the following quantities (just like imagining the interbank loan was granted) to calculate the corresponding tax:

$$Exp'(t) = Exp(t) \quad (A.v.8)$$

$$Exp'_{kj}(t) = Exp_{kj}(t) + cash\_need_j \quad (A.v.9)$$

$$Exp'_{jk}(t) = Exp_{jk}(t) - cash\_need_j \quad (A.v.10)$$

$$lev_p = \frac{loans_p(t) + \sum_i \max\left(0, Exp_{pi}(t)\right)}{E_p(t)}, \forall p \in 1, .., Bk \quad (A.v.11)$$

$$lev'_p = \begin{cases} \dfrac{loans_p(t) + \sum_i \max\left(0, Exp_{pi}(t)\right)}{E_p(t)}, \forall p \neq j \\ \dfrac{loans_p(t) + B_i + \sum_i \max\left(0, Exp'_{pi}(t)\right)}{E_p(t)}, p = j \end{cases} \quad (A.v.12)$$

$$V = \sum_i \sum_j \max\left(Exp_{ij}, 0\right) \quad (A.v.13)$$

$$V' = \sum_i \sum_j \max\left(Exp'_{ij}, 0\right) \quad (A.v.14)$$

$$\chi_j \sim Unif(0,1), j \in 1, \dots, Bk \quad (A.v.15)$$

$$\psi_j \sim Unif(0,0.1), j \in 1, \dots, Bk \quad (A.v.16)$$

lev and lev' are needed to compute the tax, so are V and V' denoting the amount of interbank credits given (excluding and including the considered loan respectively). χ and ψ correspond to modelling of operating specifics of every bank. We will return to the moment of calculating the tax rate in the corresponding sections (III.B.vi – III.B.ix); we will reference this computed at this stage value as tax. Compute leverage ratio (but for the bank k now) once again:

$$lev\_ratio = \frac{E_k(t)}{loans_k(t) + \sum_i \max\left(0, Exp_{ki}(t)\right) + cash\_need_j} \quad (A.v.17)$$

And if this is greater than ν, the procedure continues. Based on the contractor characteristics (calculated above), the k-th bank calculates its proposed rate by the formula used in [6]:

$$proposed\_rate_j(k) = refi \cdot \left(1 + \chi_j \cdot tanh(d_i) + \frac{cash\_need_j}{B_i} \cdot \psi_k \cdot tanh(lev_j')\right) + \frac{cash\_need_j}{B_i} \cdot tax \qquad (A.v.18)$$

The tax computed here will be remembered as the tax for the loan to j from k: $tax_j(k) = tax$.

We return to the firm-bank transaction, and write down the proposed rate for the i-th firm (introducing $proposed\_rate$ matrix):

$$proposed\_rate_{ij} = \min_k \left(proposed\_rate_j(k) + \epsilon_k\right) \qquad (A.v.19)$$

In the following variable we will remember the corresponding tax – tax_bank_j.

In case when the bank j did not need to take interbank credit to give loan to the firm i,

$$proposed\_rate_{ij} = refi \cdot \left(1 + \chi_j \cdot \tanh(d_i)\right) \qquad (A.v.20)$$

After the firm gathered banks' proposals, it takes the best-proposed rate and remembers the bank that proposed it:

$$j = \arg\min_j proposed\_rate_{ij} \qquad (A.v.21)$$

If there were any of banks received applications that could give a loan, i.e., if the $proposed\_rate$ is defined, then there are 2 options:

$proposed\_rate_{ij} \leq maxrate$	$proposed\_rate_{ij} > maxrate$
$credit = B_i \qquad (A.v.22)$ $ibloan = cash\_need_j \qquad (A.v.23)$	$credit = \phi \cdot B_i \qquad (A.v.22')$ $ibloan = \max\left(0, cash\_need_j - (1-\phi)B_i\right) \qquad (A.v.23')$

where ϕ is the coefficient describing the shrinkage of firms' credit appetite after exceeding the threshold.

So, if $ibloan > 0$, i.e., if the bank has to take credit, then:

$$Exp_{kj}(t) = Exp_{kj}(t) + ibloan \qquad (A.v.24)$$

$$Exp_{jk}(t) = Exp_{jk}(t) - ibloan \qquad (A.v.25)$$

$$lev'_p = \frac{loans_p(t) + \sum_i \max\left(0, Exp_{pi}(t)\right)}{E_p(t)}, \forall p \in 1, \dots, Bk \qquad (A.v.26)$$

$$E_k(t) = E_k(t) + refi \cdot \left(1 + \psi_k \cdot \tanh(lev_j')\right) \cdot ibloan \qquad (A.v.27)$$

$$E_j(t) = E_j(t) - refi \cdot \left(1 + \psi_k \cdot \tanh(lev_j')\right) \cdot ibloan \qquad (A.v.28)$$

$$E_k(t) = E_k(t) - tax\_bank_j \cdot ibloan \qquad (A.v.29)$$

And then we update the following variables:

$$deb_{ij}(t) = deb_{ij}(t) + credit \qquad (A.v.30)$$

$$loans_j(t) = loans_j(t) + credit \qquad (A.v.31)$$

$$Ftot_{ij} = Ftot_{ij} + credit \qquad (A.v.32)$$

After all the credits are given and taken, we update the liquidity:

$$liquidity_j(t) = liquidity_j(t) + \sum_i Ftot_{ji}, \forall j \in 1,..,I \qquad (A.v.33)$$

vi. Job market clearance

Labour demand is redefined:

$$Ld_i(t) = \left\lfloor \min\left(Ld, \frac{liquidity_i(t)}{wb}\right) \right\rfloor, \qquad (A.vi.1)$$

where $\lfloor \quad \rfloor$ means integer part.

So, the firms also define the number of vacancies they are looking for, i.e., the difference between labour demand and existing workforce:

$$vacancies_i = Ld_i(t) - Leff_i(t) \qquad (A.vi.2)$$

So, firms with a surplus ($vacancies_i < 0$) fire randomly $-vacancies_i$ workers, setting corresponding Oc vector components to zero, and $Leff_i(t)$ to $Ld_i(t)$.

Firms with workforce deficit randomly hire either $vacancies_i$ people or as much as they can (in case of no unemployed left), changing corresponding variables.

vii. Production market clearance

Having certain market-defined conditions – first of all, workforce (firms might have corrected their initial plans because of the credit amount different from the desired one and the number of hired employees could also differ because of workforce deficit) – firms update their plans:

$$Y_i(t) = \min(De_i, \alpha \cdot Leff_i) \qquad (A.vii.1)$$

$$wages_i(t) = wb \cdot Leff_i(t) \qquad (A.vii.2)$$

Compute interests:

$$interests_i = \sum_{j=1}^{Bk} proposed\_rate_{ij} \cdot Ftot_{ij} \qquad (A.vii.3)$$

Redefine prices (they should not be less than break-even prices):

$$P_i(t) = \max\left(P_i(t), 1.05 \cdot \frac{wages_i(t) + interests_i(t)}{Y_i(t)}\right) \qquad (A.vii.4)$$

viii. Consumption market clearance

Workers receive a salary:

$$PA_i(t) = PA_i(t-1) + wb \cdot I(Oc_i(t) \neq 0), i \in 1, \dots, J \qquad (A.viii.1)$$

$I(\dots)$ is an indicator function, equals 1 in case of the correct statement in brackets or 0 otherwise.

All the households (including capitalists and bankers) define their consumption budgets:

$$cons\_budget_i(t) = PA_i(t) \cdot c, \qquad (A.viii.2)$$

where c is a propensity to consume.

And subtract it from their personal assets:

$$PA_i(t) = PA_i(t) - cons\_budget_i(t) \qquad (A.viii.3)$$

Consumers with positive $cons\_budget$ are picked randomly. Every consumer selects randomly z firms, sorts prices in ascending order and continues buying till the budget is positive and there are firms with any goods left.

ix. Firms accounting

We compute firms' profits and update their states:

$$liquidity_i(t) = liquidity_i(t) + P_i(t) \cdot Q_i(t) - wages_i(t) - interests_i(t) - \tau \cdot \sum_{j=1}^{Bk} deb_{ij}(t) \qquad (A.ix.1)$$

$$loans_j(t) = loans_j(t) - \sum_{i=1}^{I} deb_{ij}(t) \qquad (A.ix.2)$$

$$deb_{ij}(t) = (1-\tau) \cdot deb_{ij}(t) \qquad (A.ix.3)$$

Eventually, the profits of firms:

$$pi_i(t) = P_i(t) \cdot Q_i(t) - wages_i(t) - interests_i(t) \qquad (A.ix.4)$$

Firms with positive profits pay dividends (consider i-th firm):

$$PA_{J+i}(t) = PA_{J+i}(t) + \underbrace{div \cdot pi_i(t)}_{dividends\ amount\ from\ firm\ i} \qquad (A.ix.5)$$

$$liquidity_i(t) = liquidity_i(t) - div \cdot pi_i(t) \quad (A.ix.6)$$

$$pi_i(t) = pi_i(t) \cdot (1 - div) \quad (A.ix.7)$$

Update firms' equities:

$$A_i(t) = A_i(t) + pi_i(t) \quad (A.ix.8)$$

x. Firms defaults

Consider firms with negative liquidity. These firms (let us denote one of them with index i) are bailed-in by their owners:

$$A_i(t) = A_i(t) + PA_{J+i}(t) \quad (A.x.1)$$

$$liquidity_i(t) = liquidity_i(t) + PA_{J+i}(t) \quad (A.x.2)$$

$$PA_{J+i}(t) = 0 \quad (A.x.3)$$

If $liquidity_i(t)$ is still negative, the firm goes bankrupt. We update necessary macroparameters in order to account for this default in different time series for analyzing the system's evolution.

Banks lose their loans:

$$loans_j(t) = loans_j(t) - deb_{ij}(t) \quad (A.x.4)$$

Firm vanishes, and instead of it a new one appears with average market characteristics:

$$A_i(t) = PA_{J+i}(t) \quad (A.x.5)$$

$$PA_{J+i}(t) = 0 \quad (A.x.6)$$

$$liquidity_i(t) = A_i(t) \quad (A.x.7)$$

$$deb_{ij}(t) = 0, \forall j \in 1, \ldots, Bk \quad (A.x.8)$$

$$De_i(t) = Average\left(De_{A(i)>0}(t)\right) \quad (A.x.9)$$

$$P_i(t) = Average\left(P_{A(i)>0}(t)\right) \quad (A.x.10)$$

$$Y_i(t-1) = Average\left(Y_{A(i)>0}(t)\right) \quad (A.x.11)$$

$$Y_i(t) = 1 \quad (A.x.12)$$

xi. Banks accounting

Banks accounting is similar to firms accounting. Banks receive their profits:

$$piB_i(t) = piB_i(t) + \sum_{j=1}^{I} proposed\_rate_{ji} \cdot Ftot_{ji} \qquad (A.xi.1)$$

Banks (consider i-th bank) with positive profits pay dividends:

$$PA_{J+I+i}(t) = PA_{J+I+i}(t) + div \cdot piB_i(t) \qquad (A.xi.2)$$

$$piB_i(t) = (1 - div) \cdot piB_i(t) \qquad (A.xi.3)$$

$$E_i(t) = E_i(t) + piB_i(t) \qquad (A.xi.4)$$

Banks repay interbank loans:

$$Exp_{ij}(t) = (1 - \tau) \cdot Exp_{ij}(t) \qquad (A.xi.5)$$

xii. Banks defaults

If any of banks have negative equity ($E_j < 0$), then it is considered to be the default; its debts are loss in others' banks equities (this is the way defaults can propagate), and its loans are annulled:

$$E_i(t) = E_i(t) - \max\left(0, Exp_{ij}(t)\right) \qquad (A.xii.1)$$

$$E_j(t) = 0 \qquad (A.xii.2)$$

$$Exp_{ji}(t) = 0, i{:}\, Exp_{ji}(t) < 0 \qquad (A.xii.3)$$

$$Exp_{ij}(t) = 0, i{:}\, Exp_{ij}(t) > 0 \qquad (A.xii.4)$$

xiii. Interbank market

Banks cannot maintain their functions even with positive equity if they have a negative amount of cash:

$$C_j(t) = E_j(t) + \sum_{firm\_bank_i = j} liquidity_i(t) + \sum_{worker\_bank_i = j} PA_i(t) - loans_j(t) - \sum_{i=1}^{Bk} E_{ji}(t) \qquad (A.xiii.1)$$

For bank j with negative cash $C_j(t)$ pick up randomly a bank k with $C_k(t) > 0$. If both banks have positive equity, k borrows money to j trying to cover its cash need.

$$ibloan = \min\left(-C_j(t), C_k(t)\right) \qquad (A.xiii.2)$$

$$Exp_{kj}(t) = Exp_{kj}(t) + ibloan \qquad (A.xiii.3)$$

$$Exp_{jk}(t) = Exp_{jk}(t) - ibloan \qquad (A.xiii.4)$$

$$C_j(t) = C_j(t) + ibloan \qquad (A.xiii.5)$$

$$C_k(t) = C_k(t) - ibloan \qquad (A.xiii.6)$$

B. Network-based systemic risk metrics

In this section, we first discuss network-based systemic risk metrics used for taxation and then describe, how exactly the tax variable is calculated.

i. Degree

In network theory, there is a class of metrics called centralities. All of them are aimed to catch different "centrality" aspects of the considered node in the network. The most obvious and simple is degree centrality, which is:

- For the undirected unweighted graph, it is the number of incident edges of a vertex;
- For the undirected weighted graph, it is the sum of weights of incident edges of a vertex;
- For the directed weighted graph, it could be the sum of weights of incident incoming or outcoming (or their sum) edges. In this case, this is usually referred to as in-degree and out-degree.

In our case nodes are banks, \directed links are credits given, and weights are loans amounts.

ii. Original (acyclic) DebtRank

Different attempts have been tried in order to develop a model determining systemically important financial institutions, and one of them was DebtRank [5]. Compared to other models, like Eisenberg-Noe, Rogers-Veraart, Default Cascades, original DebtRank does not imply strictly greater vulnerabilities [8] like cyclic DebtRank but accounts more for network effects and in practice still usually demonstrates greater vulnerabilities.

The algorithm allows for obtaining more general results than it is used in this thesis. In general, one has to define the initial disturbances vector and then to obtain a vector of final disturbances with propagated stress.

As the initial stress vector, we use vector with only one component valued 1, and other components are zero. This means, that in an initial state only one bank defaults, and others are untouched. Then we obtain a vector of distress caused. Multiplying this vector by the vector of equities and dividing it by the sum of all equities, we obtain the fraction of assets in the system, that are distressed by the failure of a certain bank, corresponding to a single unit-component in the initial distress vector. Repeating this procedure for all the banks, we obtain a vector of exposed system's assets fractions.

Let us describe it in a mathematical fashion.

If the bank's equity is less than a certain positive threshold ($E_j \leq \gamma$), it defaults. If the node j defaults, node i loses $\max(0, Exp_{ij})$. In case of loss greater than capital the bank i has, the i-th bank also defaults: $\max(0, Exp_{ij}) \geq E_i$. So, the impact of the node i on the node j is define as the following matrix

$$W_{ij} = \min\left(1, \frac{\max(0, Exp_{ji})}{E_j}\right), \qquad (B.ii.1)$$

meaning, that in case of loss exceeding capital, the impact is 1. We introduce the following state parameters in a financial distress contagion process modelled: $h_i(t) \in [0,1]$ and $s_i(t) = \{U, D, I\}$ – undistressed, distressed and inactive (default). $h_i(t) = 0$ denotes undistressed node, while $h_i(t) = 1$ means default. Putting initial conditions on $h_i(1), s_i(1), i \in 1, \dots, Bk$, we iterate the contagion process:

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_{j:s_j(t-1)=D} W_{ji} \cdot h_j(t-1)\right\} \tag{B.ii.2}$$

$$s_i(t) = \begin{cases} D, & if\ h_i(t) > 0\ and\ s_i(t) \neq I \\ I, & if\ s_i(t-1) = D \\ s_i(t-1), & otherwise \end{cases} \tag{B.ii.3}$$

Please note, that "inactive" does not mean default, it only denotes nodes that are no more involved in the contagion process. State variables s_i are aimed to exclude walks with repeating edges in order to have impacts of a node not greater than one.

The economic value of any node is denoted by

$$v_i = \frac{\sum_j \max(0, Exp_{ji})}{\sum_j \sum_k \max(0, Exp_{kj})} \tag{B.ii.4}$$

After a certain number of iterations T all the nodes either belong to inactive or to undistressed. Originally the Debtrank of a set S_f was defined as follows:

$$R'_{S_f} = \sum_j h_j(T) \cdot v_j - \underbrace{\sum_j h_j(1) \cdot v_j}_{initial\ distress}, \tag{B.ii.5}$$

meaning only induced distress. For our purpose DebtRank also includes initial distress:

$$R'_{S_f} = \sum_j h_j(T) \cdot v_j \tag{B.ii.6}$$

In our case set S_f is a single node.

iii. Cyclic DebtRank

By introducing discrete state variables, the original DebtRank loses the amplification of distress in cyclic paths. Model fixing this problem was introduced in [18]. This extended model is called cyclic DebtRank, or differential DebtRank. The underlying process for vulnerabilities is the following:

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_{j=1}^{Bk} W_{ji}\left(h_j(t-1) - h_j(t-2)\right)\right\} \tag{B.iii.1}$$

The paper [8] contains an analytical proof, that vulnerabilities from cyclic DebtRank are greater than ones obtained by Eisenberg-Noe and Rogers-Veraart models regardless of the network topology.

iv. SinkRank

Another algorithm developed for determining systemically important banks was proposed in [11] specifically for payment systems.

Our network is considered as a graph. We define an absorbing random walk as a random walk terminated at a certain absorbing node – sink. The greater is the expected number of steps till absorbance, the less central (in terms of network's centrality measure) is the absorbing node.

Having an adjacency matrix of the network $M = [s_{ij}]_{n\times n}$, we define transition matrix as

$$P = \left[\frac{s_{ij}}{\sum_j s_{ij}}\right]_{n\times n}, \qquad (B.iv.1)$$

where matrix elements are transition probabilities for a random walk. In the original paper, this network was the network of banks, and link weights were payment value, while we will use credit amounts instead. Let m be the number of absorbing nodes; then define a matrix

$$S = [s_{ij}]_{(n-m)\times(n-m)}, \qquad (B.iv.2)$$

where columns and rows corresponding to the absorbing nodes in M are excluded. Also, let I to be $(n-m)\times(n-m)$ identity matrix; then

$$Q = (I - S)^{-1} \qquad (B.iv.3)$$

is the matrix, where q_{ij} defines the number of times starting at i a process is expected to visit j before the absorption. Sum of all entries of the matrix in the i-th row is called Sink Distance.

SinkRank is then defined as an inverted average Sink Distance for all the nodes:

$$SinkRank = \frac{n-m}{\sum_i \sum_j q_{ij}} \qquad (B.iv.4)$$

For our application, we always have $m = 1$, and we repeatedly calculate SinkRank for every node, considering it to be absorbing.

v. 2-step DebtRank

The idea of DebtRank used for taxation described in [6] is attractive: small but smart tax rapidly increases the banking system's robustness. Besides the real-world implementation difficulties – someone computing this tax has to know about all the interbank credits given every time the transaction is considered – this tax is unconstitutional. Authors of [6] were contacted by European authorities with interest to implement; however, lawyers claimed, that one cannot be taxed according to others' behaviour.

We decided if it is possible to "cut" DebtRank's propagation in order to take into account only neighbours of 2 counteragents.

Let us walk through the whole (original) DebtRank procedure. First, we slightly transform the vulnerability propagation process:

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_j W_{ji} \cdot h_j(t-1)\right\}, t = 2, \dots, 3 \qquad (B.v.1)$$

Say, initial condition on h is $h(1) = \left(0,0,\dots,\underbrace{1}_{i},\dots,0,0,0\right)$. Then, in the next step $h(2)$ has the vulnerability propagated to the neighbors of the bankrupt bank. Neighbors of neighbors are affected in the third step.

For the i-th bank, we calculate truncated losses, that might be induced by its failure:

$$R_i^{2'} = \sum_j h_j(3) \cdot v_j \qquad (B.v.2)$$

vi. Taxation: in- and out-degrees

Having a value quantifying systemic risk, the systemic risk tax is easy to calculate as a tax rate multiplied by the positive part of the difference in systemic risk induced by the transaction.

As it was mentioned in the previous section, the most obvious centrality measure is degree centrality. Just because in our model tax is withheld from the creditor, in-degree of the creditor and out-degree of the borrower are the same and stay unchanged; change in out-degree of the creditor (equal to in-degree of the borrower) is equal to Tobin tax (tax proportional to the transaction volume). We also considered tax based on the absolute value of the out-degree of the borrower (not its change) but declined this idea as destimulating the economy to evolve. However, this might have changed the whole topology of the interbank network.

As we can see, this tax is not a systemic risk tax. It equals the parameter defined externally:

$$tax = \zeta \qquad (B.vi.1)$$

vii. Taxation: DebtRanks

Defining an external variable ζ that means a fraction of systemic risk induced to be penalized by taxation, we follow the original paper [6] and derive the following rule:

$$SRT = \zeta \cdot \max\left[0, \sum_i p_i(t)\left(V' R_i^{(+k)} - V \cdot R_i\right)\right], \qquad (B.vii.1)$$

Where index $(+k)$ denotes "after transaction" variable, R_i is i-th DebtRank, $V = \sum_i \sum_j \max(0, Exp_{ij})$, $p_i(t)$ is the probability of i-th bank to default on the maturity date of the loan (for the sake of simplicity; [6] proposes valuation similar to [19], [20]). In our implementation, this probability is approximated with the hyperbolic tangent of the leverage lev_i or lev_i' (III.A.v), like it was done in [6], i.e.:

$$p_i(t) = 0.01 \cdot \tanh(lev_i) \qquad (B.vii.2)$$

While the transaction affects the probability of default, in the final expression there are two different probabilities. Taking into account all of these considerations, we obtain:

$$tax = \zeta \cdot \frac{\max\left[0, \sum_{i:} 0.01\left(\tanh(lev_i') \cdot V^{(+k)} R_i^{(+k)} - \tanh(lev_i) \cdot V \cdot R_i\right)\right]}{cash\_need_j} \qquad (B.vii.3)$$

viii. Taxation: 2-step DebtRank

Having a goal not to consider nodes 3 and more steps away from 2 contractors, we also have to make amends to the $(B.vii.3)$. For banks j and k we obtain:

$$tax = \zeta \cdot \frac{\max\left[0, \sum_{i:Exp_{ij}\neq 0 \text{ } or \text{ } Exp_{ik}\neq 0} 0.01\left(\tanh(lev_i') \cdot V' \cdot R^{2'}{}_i^{(+k)} - \tanh(lev_i) \cdot V \cdot R_i^{2'}\right)\right]}{cash\_need_j} (B.ix.1)$$

Here

$$V = \sum_i \max(0, Exp_{ij}) + \sum_i \max(0, Exp_{ik}), \qquad (B.ix.2)$$

And V' equals the same but using Exp'.

ix. Taxation: SinkRank

SinkRank has a different interpretation than DebtRank: this is "how close" is the bank to other banks via credit network. However, it has the interval of the same values $(0; 1]$, so, we are free to use it the same way as DebtRanks were used; instead of payment matrix, we used $\max(0, Exp)$.

C. Equity introduced in the model

As a side experiment, it was also tested, how the model behaviour would change, if a new layer in the network would be included. The first idea was to introduce equity relations: two major financing sources for businesses are credit and equity. Without certain expectations except for DebtRank behaviour (IV.B), we decided to model this, solving several particular issues, also penalizing systemic risk in a new layer.

i. Market pricing modelling

There were three approaches considered:

- Discounted cash flows (DCF) valuation [21];
- Geometric Brownian motion modelling;
- Valuation using multiples [22];

DCF has the advantage of imitation of the real valuation processes; this is how investment projects are evaluated by the business. Based on the history of dividends paid to the capitalist, one can use Gordon's formula to estimate share's price.

We decided to calculate the price of the company as follows:

$$PV_{company} = PV_{cash\ flows} + PV_{tax\ shield} + BankruptcyRate \cdot \Pr\{Bankruptcy\} \cdot PV_{company}, \quad (C.i.1)$$

where $BankruptcyRate$ denotes the average fraction of the company price needed to perform the bankruptcy procedure.

Then we expressed $PV_{company}$, and obtained:

$$PV_{company} = \frac{PV_{cash\ flows} + PV_{tax\ shield}}{1 + BankruptcyRate \cdot \Pr\{Bankruptcy\}}, \quad (C.i.2)$$

As the proxy for the bankruptcy probability we used the leverage of the company:

$$\Pr\{Bankruptcy\} \stackrel{\text{def}}{=} \frac{D}{E + D} = 1 - \frac{E}{E + D}, \quad (C.i.3)$$

where E is the firm's equity, and D is its debt.

$PV_{cash\ flows}$ was estimated as the infinite rent of profits equal to the last profit discounted by last return on equity:

$$PV_{cash\ flows} = \frac{profit}{return\_on\_equity} \quad (C.i.4)$$

Tax shield value was estimated as

$$PV_{tax\ shield} = D \cdot CapitalIncomeTaxRate \quad (C.i.5)$$

To avoid any problems with negative, infinite, NaN market values, there were boundaries introduced: $PV_{company} \in [0.1; 1000]$ – in practice, we were constantly facing this problem. This also refers to banks evaluation.

A problem with a warm-up period appeared: at a first step, firms were evaluated to be too expensive, thus selecting the source of financing to be equity, then their price went down, bankrupting banks. The system was too short-living to be investigated, and we could not create an adequate valuation model within DCF framework.

Valuation using multiples is also the way to imitate real business decision making, but it was not implemented due to geometric Brownian motion modelling was implemented, and we did not see any reason to implement another model: no essentially new results could be obtained.

Valuation with geometric Brownian motion is the easiest one and does not imply problems with volatile market prices we observed with DCF valuation. Moreover, it has explicit parameters to tune – drift rate and volatility. If we would like to model the effects of fat-tailed distributions of market prices, we could also easily do this within this framework.

Variables used for market pricing modelling:

Variable	Description
μ	Drift rate of the firms' market prices.
σ	The volatility of the firms' market prices.
μB	Drift rate of the banks' market prices.
σB	The volatility of the banks' market prices.
$market\_prices_i(t)$	i-th firm price at the time step t.
$market\_pricesB_j(t)$	j-th bank price at the time step t.
ξ_{it}	$\xi_{it} \sim N(0,1), i.i.d.$, i-th firm's market price random increment between $t-1$ and t time steps.
ϵ_{it}	$\epsilon_{it} \sim N(0,1), i.i.d.$, i-th bank's market price random increment between $t-1$ and t time steps.
$return\_on\_equity_i(t)$	Return on equity of i-th firm at time step t.
$return\_on\_equityB_i(t)$	Return on equity of i-th bank at time step t.
μ_{roe}	Mean value of return on equity for firms.
σ_{roe}	The standard deviation of return on equity for firms.
μB_{roe}	Mean value of return on equity for banks.
σB_{roe}	The standard deviation of return on equity for banks.
$eq_{ij}(t)$	Fraction of i^{th} firm held by j^{th} bank, $[I \times Bk]$ matrix.
$eqB_{ij}(t)$	Fraction of i^{th} bank held by j^{th} bank, $[Bk \times Bk]$ matrix.
E_f	$[I \times Bk]$ matrix of market values of firm fractions held by banks (i.e. $E_{f\,ij}(t) = eq_{ij}(t) \cdot market\_prices_i(t)$).
E_b	$[Bk \times Bk]$ matrix of market values of bank fractions held by banks, the analogy with Exp, but for equity.
eq_sold_i	All the equity sold by the i-th firm.
eq_soldB_i	All the equity sold by the i-th bank.
κ	Buyback parameter; the fraction of the equity firms aim to buy back from the banks having their shares.

Table 6. Market pricing variables

Equity was modelled in the following fashion:

$$market\_prices_i(t) = market\_prices_i(t-1) + \mu \cdot market\_prices_i(t-1) \cdot 1 + \sigma \cdot market\_prices_i(t-1) \cdot \xi_{it}, \quad (C.i.6)$$

$$market\_pricesB_i(t) = market\_pricesB_i(t-1) + \mu \cdot market\_pricesB_i(t-1) \cdot 1 + \sigma \cdot market\_pricesB_i(t-1) \cdot \epsilon_{it}, \quad (C.i.7)$$

Returns on equity modelled as independent identically distributed random values:

$$return\_on\_equity_i(t) \sim N(\mu_{roe}, \sigma_{roe}) \quad (C.i.8)$$

$$return\_on\_equityB_i(t) \sim N(\mu B_{roe}, \sigma B_{roe}) \tag{C.i.9}$$

Although being almost identical to the variables listed in Table 6, the following quantities are useful:

$$eq\_sold_i = \sum_j eq_{ij} \tag{C.i.10}$$

$$eq\_soldB_i = \sum_j eqB_{ij} \tag{C.i.11}$$

E_f is $[I \times Bk]$ matrix of market values of firm fractions held by banks, i.e.

$$E_{f_{ij}}(t) = eq_{ij}(t) \cdot market\_prices_i(t) \tag{C.i.12}$$

E_b is $[Bk \times Bk]$ matrix of market values of bank fractions held by banks.

Now the scheme slightly changed, and we illustrate it with the following picture:

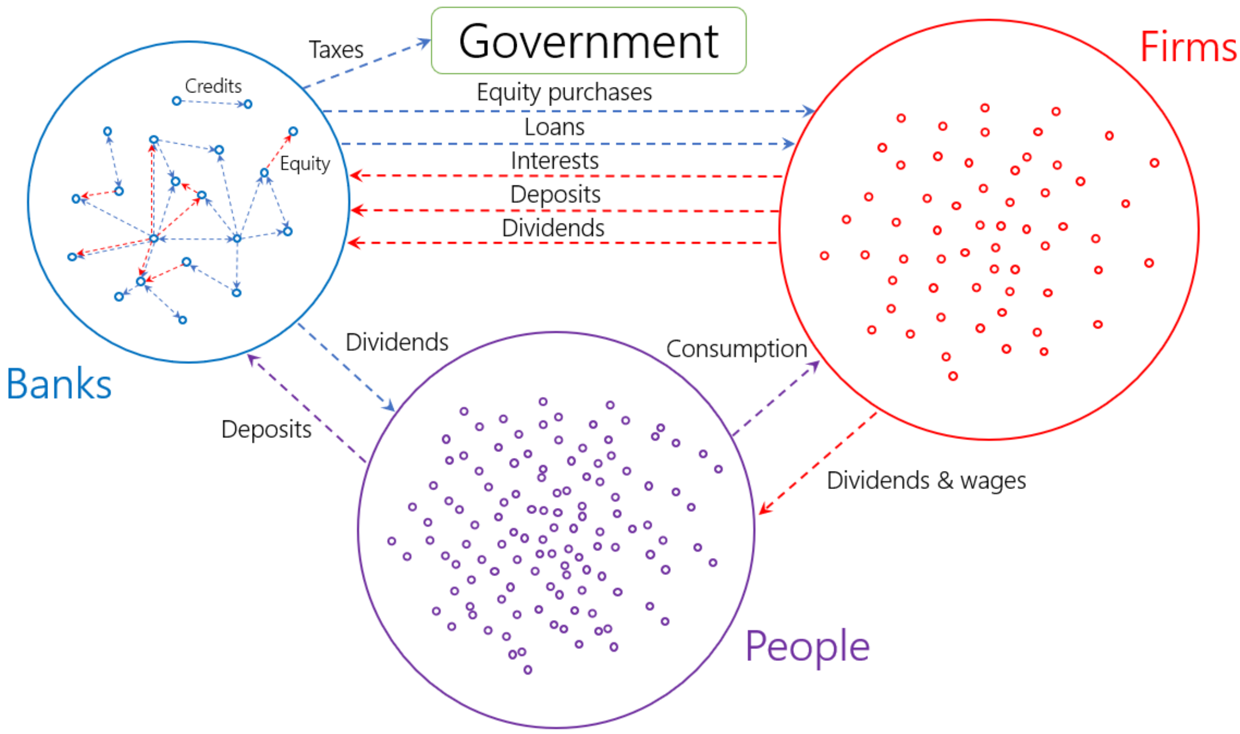


Figure 2. General scheme of ABM with equity layer

ii. Credit/equity firms' and banks' decision modelling

We introduce additional quantities and decision rules in the routine described in (III.A.v) part. First, let us think of why firms might raise capital via equity sell? If we do not take into account exceptional cases, like new ventures or firms with a low fraction of fixed assets (i.e. high price-to-book ratio) [23], then the only plausible reason we can implement (we are not willing to insert such things, as investors beliefs about managers beliefs [24]), is that it is more profitable. In other words, if the return on equity is less than the return on credit, then equity sell is preferred. It is fair for both banks and firms.

Having cash demand B_i, firms apply for capital raise to banks. Banks received applications may also have a need to draw on some money. After raising capital via debt/equity, banks prepare their proposals for the firms. Based on these rates, firms decide on their debt/equity capital raising.

New variables for decision modeling:

Variable	Description
CN	The maximal amount of money to be lent by the considered bank.
$maximal\_equity\_to\_sell$	Amount of equity can be sold by the considered bank.
$maximal\_fraction\_to\_sell$	Maximal share of the firm that could be sold to the considered bank.
$tax\_bank_{j_d}$	Temporary variable to remember tax corresponding to the best credit proposal to the bank j; refers to credit layer.
$tax\_bank_{j_e}$	Temporary variable to remember tax corresponding to the best credit proposal to the bank j; refers to the equity layer.
V_d	Sum of all credit liabilities in the system before the transaction.
$V_d^{(+k)}$	Sum of all credit liabilities in the system after the transaction.
V_e	Sum of all equity liabilities in the system before the transaction.
$V_e^{(+k)}$	Sum of all equity liabilities in the system after the transaction.
R_{i_d}	DebtRank of the i-th node in the network before the transaction; credit layer.
$R_{i\ \ d}^{(+k)}$	DebtRank of the i-th node in the network after the transaction; credit layer.
R_{i_e}	DebtRank of the i-th node in the network before the transaction; equity layer;
$R_{i\ \ e}^{(+k)}$	DebtRank of the i-th node in the network after the transaction; equity layer.

Table 7. Credit/equity decision modelling variables

After the same steps, as in $(A.ii.1) - (A.v.7)$ are executed, the banks in a cash need to solve the following problem:

$$CN \in [0, cash\_need_j]?: refi \cdot \left(1 + \frac{CN}{B_i} \cdot \psi_j \cdot tanh(lev_j')\right) = return\_on\_equityB(j) \qquad (C.ii.1)$$

Leverages whose hyperbolic tangents are proxies for default probabilities are adjusted in this case:

$$lev'_p = \begin{cases} \dfrac{loans_p(t) + \sum_i \max\left(0, Exp_{pi}(t)\right)}{E_p(t)}, \forall p \neq j \\ \dfrac{loans_p(t) + B_i + \sum_i \max\left(0, Exp'_{pi}(t) + CN\right)}{E_p(t)}, p = j \end{cases} \qquad (C.ii.2)$$

CN is the maximal amount of money to be lent (after this amount the interest rate provided to the bank would be greater than its return on equity, so, it would be less expensive to raise money via selling equity). In the case of

$$refi \cdot \left(1 + \frac{CN}{B_i} \cdot \psi_j \cdot tanh\left(lev_j'\right)\right) - return\_on\_equityB_j \Big|_{CN=cash\_need_j} < 0 \qquad (C.ii.3)$$

we set

$$maximal\_equity\_to\_sell = 0, \qquad (C.ii.4)$$

and if

$$refi \cdot \left(1 + \frac{CN}{B_i} \cdot \psi_j \cdot tanh\left(lev_j'\right)\right) - return\_on\_equityB_j \Big|_{CN=0} > 0, \qquad (C.ii.5)$$

we set

$$maximal\_equity\_to\_sell = cash\_need_j \qquad (C.ii.6)$$

This newly introduced variable determines, how much of the equity can be sold by the bank. In the first case credit interest rate would be less than return on equity even in case of all the money would be raised by credit – this would mean, that we do not need to solve the problem on determining CN – we see, that it would be profitable for the bank to apply for credit only; same thing is for the latter case. These boundary condition checks were made to avoid solving the equation above. In other case CN is maximal of zero and solution of the equation above.

After this problem is solved, and the upper bound for money raised as credit is determined, let us determine, how much of the equity should actually be sold:

$$equity\_to\_sell_j = max\left(0, min\left(\left[market\_pricesB_j \cdot \left(0.8 - eq\_soldB_j\right), maximal\_equity\_to\_sell, cash\_need_j\right]\right)\right) \qquad (C.ii.7)$$

Bank will not sell less than zero, the bank will not sell more than 80% of its shares, the bank will not sell more than it needs, the bank will not sell more than it is profitable.

Then the procedure is similar to one described before in $(A.v.8) - (A.v.16)$: we imagine transactions to be conducted in order to estimate tax levied, same transformations take place, taking into account, that we have two types of taxes for two layers of the interbank network:

$$Exp'(t) = Exp(t) \qquad (C.ii.8)$$

$$Exp_{kj}'(t) = Exp_{kj}(t) + cash\_need_j - equity\_to\_sell_j \qquad (C.ii.9)$$

$$Exp_{jk}'(t) = Exp_{jk}(t) - cash\_need_j + equity\_to\_sell_j \qquad (C.ii.10)$$

$$E' = E \quad (C.ii.11)$$

$$E_b' = E_b \quad (C.ii.12)$$

$$E_{b\,kj}' = E_{b\,kj}' + equity\_to\_sell_j \quad (C.ii.13)$$

$$E_k' = E_k' - equity\_to\_sell_j \quad (C.ii.14)$$

$$lev_p = \frac{loans_p(t) + \sum_i \max\left(0, Exp_{pi}(t)\right)}{E_p(t)}, \forall p \in 1,..,Bk \quad (C.ii.15)$$

$$lev'_p = \begin{cases} \dfrac{loans_p(t) + \sum_i \max\left(0, Exp_{pi}(t)\right)}{E_p'(t)}, \forall p \neq j \\ \dfrac{loans_p(t) + B_i + \sum_i \max\left(0, Exp_{pi}'(t)\right)}{E_p'(t)}, p = j \end{cases} \quad (C.ii.16)$$

$$V_d = \sum_i \sum_j \max\left(0, Exp_{ij}\right) \quad (C.ii.17)$$

$$V_d^{(+k)} = \sum_i \sum_j \max\left(0, Exp_{ij}'\right) \quad (C.ii.18)$$

$$V_e = \sum_i \sum_j \max\left(0, E_{b_{ij}}\right) \quad (C.ii.19)$$

$$V_d^{(+k)} = \sum_i \sum_j \max\left(0, E_{b_{ij}}'\right) \quad (C.ii.20)$$

$$\chi_j \sim Unif(0,1), j \in 1, \dots, Bk \quad (C.ii.21)$$

$$\psi_j \sim Unif(0,0.1), j \in 1, \dots, Bk \quad (C.ii.22)$$

To compute systemic risk taxes, we also need 4 DebtRanks (2 layers, before and after) – R_{i_d}, $R_i^{(+k)}{}_d$, R_{i_e} and $R_i^{(+k)}{}_e$.

Finally, we can compute both taxes:

$$tax_d = \zeta \cdot \frac{\max\left[0, \sum_i 0.01\left(\tanh(lev_i') \cdot V_d^{(+k)} R_i^{(+k)}{}_d - \tanh(lev_i) \cdot V_d \cdot R_{i_d}\right)\right]}{cash\_need_j - equity\_to\_sell_j} \quad (C.ii.23)$$

$$tax_e = \zeta \cdot \frac{\max\left[0, \sum_i 0.01\left(\tanh(lev_i') \cdot V_e^{(+k)} R_i^{(+k)}{}_e - \tanh(lev_i) \cdot V_e \cdot R_{i_e}\right)\right]}{equity\_to\_sell_j} \quad (C.ii.24)$$

After determining these taxes, the bank estimates its own proposal for the firm in need:

$$proposed\_rate_j(k) = refi \cdot \left(1 + \chi_j \cdot \tanh(d_i) + \frac{cash\_need_j}{B(i)} \cdot \psi_k \cdot \tanh\left(lev_j'\right)\right) + \frac{cash\_need_j - equity\_to\_sell_j}{B_i} \cdot tax_d + \frac{equity\_to\_sell_j}{B_i} \cdot tax_e \qquad (C.ii.25)$$

They are sorted and the bank j with the best proposal is chosen. Now the final amounts of equity to sell and credits to get are defined.

If there's a sense in selling equity, i.e.

$$proposed\_rate_{ij}(t) \; > \; return\_on\_equity_i(t), \qquad (C.ii.26)$$

firm together with the bank defines, how much it could sell:

$$maximal\_fraction\_to\_sell = \min\left(\left[max\left(0{,}1 - \frac{pi_i(t-1)}{P_i(t-1) \cdot De_i(t-1) - wages_i(t-1) - interests_i(t-1)}\right), max(0{,}0.8 - eq\_sold_i(t)), \frac{E_j(t)}{market\_prices_i(t)}\right]\right) \qquad (C.ii.27)$$

In case if this covers all the financial need of the firm, i.e.

$$(maximal\_fraction\_to\_sell - eq\_sold_i(t)) \cdot market\_prices_i(t) \geq \; B_i(t), \qquad (C.ii.28)$$

then the firm sells only equity:

$$eq_{ij}(t) = \; eq_{ij}(t) + \frac{B_i(t)}{market\_prices_i(t)} \qquad (C.ii.29)$$

$$E_j(t) = \; E_j(t) - \; B_i(t) \qquad (C.ii.30)$$

$$credit = 0 \qquad (C.ii.31)$$

$$ibloan = cash\_need_j(t) \qquad (C.ii.32)$$

In the opposite case, when

$$(maximal\_fraction\_to\_sell - eq\_sold_i(t)) \cdot market\_prices_i(t) \leq \; B_i(t), \qquad (C.ii.33)$$

the rest is raised by credit:

$$eq_{ij}(t) \; = \; eq_{ij}(t) \; + \; max(0, maximal\_fraction\_to\_sell - eq\_sold_i(t)) \qquad (C.ii.34)$$

$$E_j(t) = E_j(t) - max(0, maximal\_fraction\_to\_sell - eq\_sold_i(t)) \cdot market\_prices_i(t) \quad (C.ii.35)$$

$$B_i(t) = B_i(t) - max(0, maximal\_fraction\_to\_sell - eq\_sold_i(t)) \cdot market\_prices_i(t) \quad (C.ii.36)$$

In the case the firm does not sell any equity ($proposed\_rate_{ij}(t) \leq return\_on\_equity_i(t)$), steps $(C.ii.27) - (C.ii.36)$ are not executed. Finally, in both cases, we account for the too big credit rates, like we did in $(A.v.22) - (A.v.23)$:

$proposed\_rate_{ij} \leq maxrate$	$proposed\_rate_{ij} > maxrate$
$credit = B_i$ $(C.ii.37)$ $ibloan = cash\_need_j$ $(C.ii.38)$	$credit = \phi \cdot B_i$ $(C.ii.37')$ $ibloan = \max(0, cash\_need_j - (1-\phi)B_i)$ $(C.ii.38')$

If the bank raises capital ($ibloan > 0$ – now it can also denote the capital dilution, it is just an amount the bank needs for financing its investments), it selects the bank with the best proposal too, and the transaction is conducted as follows:

$$Exp_{kj}(t) = Exp_{kj}(t) + (ibloan - equity\_to\_sell_j) \quad (C.ii.39)$$

$$Exp_{jk}(t) = Exp_{jk}(t) - (ibloan - equity\_to\_sell_j) \quad (C.ii.40)$$

$$lev'_p = \frac{loans_p(t) + \sum_i \max\left(0, Exp_{pi}(t)\right)}{E_p(t)}, \forall p \in 1, \dots, Bk \quad (C.ii.41)$$

$$E_k(t) = E_k(t) + refi \cdot \left(1 + \psi_k \cdot \tanh\left(lev'_j\right)\right) \cdot \left(ibloan - equity\_to\_sell_j\right) \quad (C.ii.42)$$

$$E_j(t) = E_j(t) - refi \cdot \left(1 + \psi_k \cdot \tanh\left(lev'_j\right)\right) \cdot \left(ibloan - equity\_to\_sell_j\right) \quad (C.ii.43)$$

$$E_j(t) = E_j(t) + equity\_to\_sell_j \quad (C.ii.44)$$

$$E_k(t) = E_k(t) - tax\_bank_{j_d} \cdot \left(ibloan - equity\_to\_sell_j\right) - tax\_bank_{j_e} \cdot equity\_to\_sell_j \quad (C.ii.45)$$

And then we update the following variables, as we did previously:

$$deb_{ij}(t) = deb_{ij}(t) + credit \quad (C.ii.46)$$

$$loans_j(t) = loans_j(t) + credit \quad (C.ii.47)$$

$$Ftot_{ij} = Ftot_{ij} + credit \quad (C.ii.48)$$

After all the credits are given and taken, we update the liquidity:

$$liquidity_j(t) = liquidity_j(t) + \sum_i Ftot_{ji} + \sum_i \left(eq_{ji}(t) - eq_{ji}(t-1)\right) \cdot market\_prices_i, \forall j \in 1,..,I \qquad (C.ii.49)$$

E_f and E_b are updated in this step.

iii. Adjustments in firms and banks accounting

Nothing changes comparing to the sections (III.A.ix – III.A.xii), except for the dividend payments: firms and banks owners not necessarily hold 100% shares of their enterprises. So, we adjust for the firm's equation $(A.ix.5)$:

$$PA_{J+i}(t) = PA_{J+i}(t) + \underbrace{div \cdot pi_i(t)}_{dividends\ amount\ from\ firm\ i} \cdot \overbrace{(1 - eq\_sold_i(t))}^{share\ of\ the\ firm\ still\ belonging\ to\ the\ owner} \qquad (C.iii.1)$$

Firms also pay dividends to the banks holding their shares:

$$piB_j(t) = piB_j(t) + div \cdot pi_i(t) \cdot eq_{ij}(t), i \in 1, \dots, I, j \in 1, \dots, Bk \qquad (C.iii.2)$$

Firms defaults are the same, except for adding up equation describing banks losing their shares. If the firm i defaults, then:

$$eq_{ij}(t) = 0, \qquad j \in 1, \dots, Bk \qquad (C.iii.3)$$

Like with the loans, firm owners are willing to buy back their equity. If $pi_i(t) > 0$, and if

$$\kappa \cdot market\_prices_i(t) \leq PA_{J+i}(t), \qquad (C.iii.4)$$

(i.e., if the firm holder i can pay for a κ share of the company) then it either buys back all the rest of the company (if the share not belonging to the firm owner is less than κ) or κ share:

$$PA_{J+i}(t) = PA_{J+i}(t) - \min\left(eq_{ij}(t), \kappa\right) \cdot market\_prices_i(t), \qquad j \in 1, \dots, Bk \qquad (C.iii.5)$$

$$E_j(t) = E_j(t) + \min\left(eq_{ij}(t), \kappa\right) \cdot market\_prices_i(t), \qquad j \in 1, \dots, Bk \qquad (C.iii.6)$$

$$eq_{ij}(t) = \max(eq_{ij}(t) - \kappa, 0), j \in 1, \dots, Bk \qquad (C.iii.7)$$

Banks do the same. We amend the $(A.xi.2)$ for the dividends to other holders and include equation describing payments to them:

$$PA_{J+I+i}(t) = PA_{J+I+i}(t) + div \cdot piB_i(t) \cdot (1 - eq\_soldB_i(t)) \qquad (C.iii.8)$$

$$piB_j(t) = piB_j(t) + div \cdot piB_i(t) \cdot eqB_{ij}(t), i \in 1, \dots, I, j \in 1, \dots, Bk \qquad (C.iii.9)$$

Banks do buybacks in the same fashion, as firms. If $piB_i(t) > 0$, and if

$$\kappa \cdot market\_pricesB_i(t) \leq E_i(t), \qquad (C.iii.10)$$

(i.e., if the bank i has enough equity to pay for a κ share of itself) then it either buys back all the rest of the bank (if the share not belonging to the bank owner is less than κ) or κ share:

$$E_i(t) = E_i(t) - \min\left(eqB_{ij}(t), \kappa\right) \cdot market\_pricesB_i(t), \qquad j \in 1, \dots, Bk \qquad (C.iii.5)$$

$$E_j(t) = E_j(t) + \min\left(eqB_{ij}(t), \kappa\right) \cdot market\_pricesB_i(t), \qquad j \in 1, \dots, Bk \qquad (C.iii.6)$$

$$eqB_{ij}(t) = \max(eqB_{ij}(t) - \kappa, 0)\,, j \in 1, \dots, Bk \qquad (C.iii.7)$$

In the case of bank defaults, its equity in firms and other banks is distributed among other shareholders proportional to their shares. We do not disclose this mechanism here more precisely, because after banks defaults the simulation stops anyway.

IV. Results

A. Taxation schemes comparison

In this section, we present the results of the simulation for the taxation schemes comparison.

In this table initial values of non-temporary variables used are presented:

Variable	Initial value
T	500
τ	0.05
div	0.2
$refi$	0.02
ζ	Different values; a point for comparison.
I	100
$A_i(1)$	1
$Leff_i(1)$	0
$Y_i(0), Y_i(1)$	0
$De_i(1)$	1
$Q_i(1)$	0
$price(1)$	1
$P_i(1)$	1
$pi_i(1)$	0
wb	1
α	0.1
$liquidity_i(1)$	$A_i(1) + 10$
J	1300
$PA_i(1)$	0
z	2
$worker\_bank_i(1)$	Uniform integer between 1 and Bk
Oc_i	0
c	0.8
Bk	20
$E_i(1)$	17.5
$deb_{ij}(1)$	$\frac{10}{Bk} \cdot 1$
$loans_i(1)$	$\sum_j deb_{ji}(1)$
$Exp_{ij}(1)$	0
$C_i(1)$	0
$firm\_bank_i(1)$	Uniform integer between 1 and Bk
$piB_i(1)$	0
$C_i(1)$	0

$Ftot_{ij}$	0
n	5
$inflation\_rate$	0
ν	0
ϕ	0.8

Table 8. Variables initial values

i. Variables observed

For analyzing the evolution and death of the simulated economic system following variables are considered:

Variable	Initial value
Loss	Sum of all loans given by defaulted banks that are annulled because of default events.
Bad debts	Sum of all loans given to defaulted banks.
Defaults	A number of bank defaults in the final step (if simulation stop was caused by bank default).
Time	A number of steps system survives before stopping the simulation (i.e. before bank default or all the firms are bankrupt, or the number of steps exceeds the given threshold).
Credits	Time series of the sum of interbank loans given, characterizing the economic activity intensity.
DebtRanks	Time series of DebtRank (next chapter) sums over all the banks.
Taxes	Sum of taxes being levied on the interbank loans.

Table 9. Observed values

One should understand the limits of comparability of those values: cumulative values like tax strongly depend on the living time of the system. In addition, smaller loss/bad debts/number of defaults or greater living time does not imply better policy: one might simply stop any economic activity and enjoy the absence of any losses, defaults etc. So, here we have two major criteria, a trade-off between sustainability and economic activity. Economic activity is mostly measured via credits and taxes (one would not like to levy too much tax) and sustainability is measured via losses, bad debts, numbers of defaults, living times of the system and DebtRank sums series.

ii. The statistical description of the results

Repeating the simulation 200 times without any taxation, we recorded the state of exposure matrix Exp before the first bank failure, and then plotted a histogram of weighted degree distribution over these 200 simulations with 20 banks:

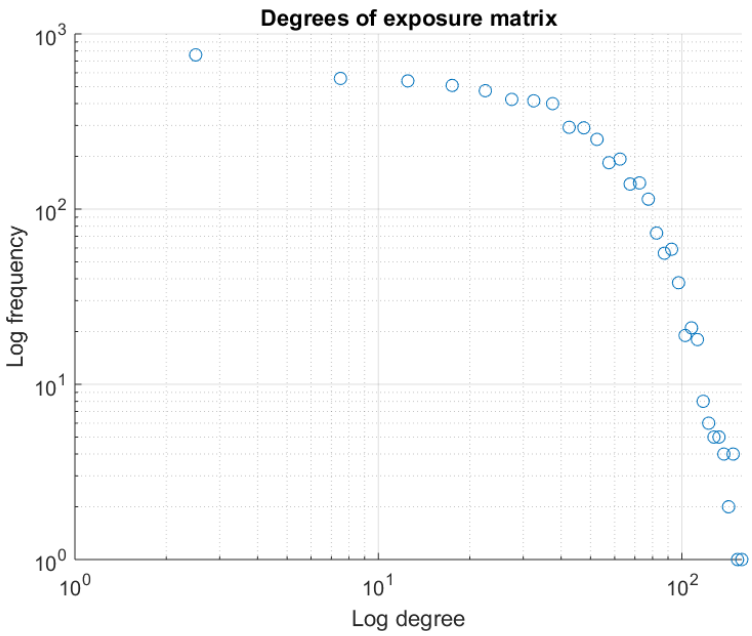


Figure 3. Degrees of the exposure matrix

We have plotted this distribution in order to check visually, whether the model reflects the power law common for many real-world networks. Having no preferential attachment feature in the algorithm for selecting counterparties, this model does not reflect a very important point in reconstructing real-world network: networks with degrees distributed by the power law are very sensitive to the central nodes' failures. This could be a point for further improvements to the model.

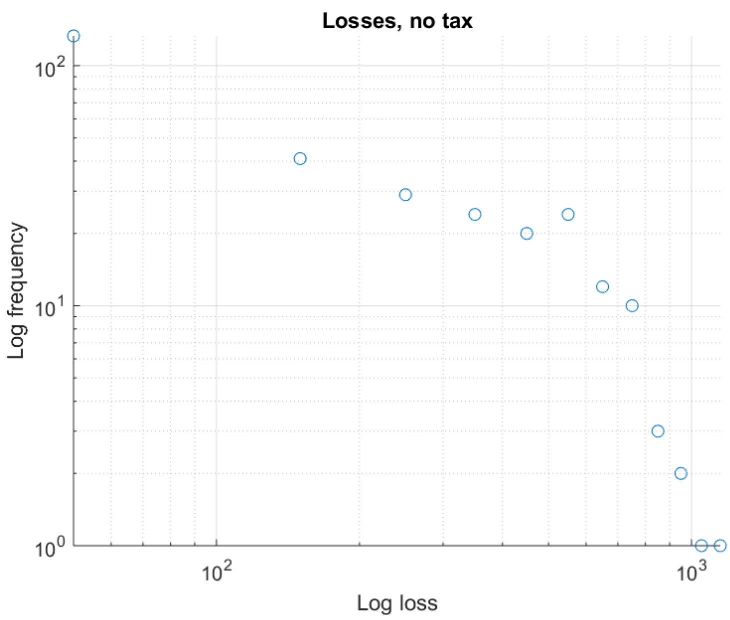


Figure 4. Losses in logarithmic scales

Log-log losses over the same simulations also do not reflect any power law. The average loss is ~225.

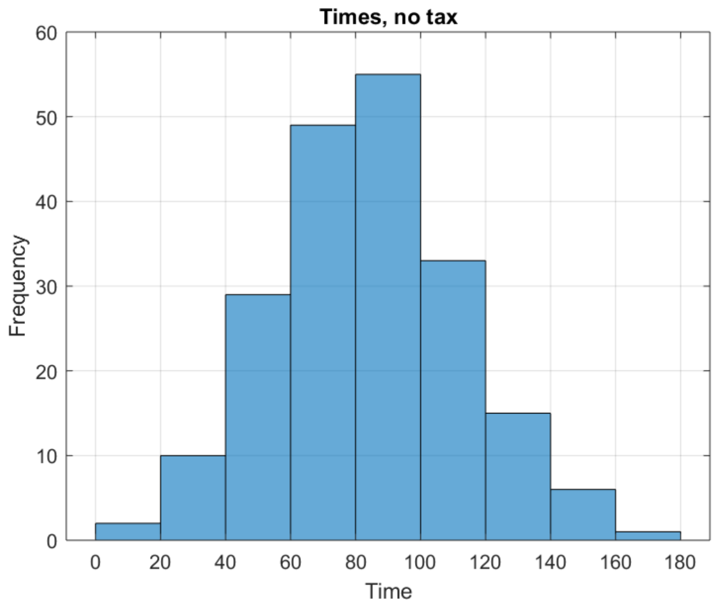


Figure 5. Times

Average time of the system life without taxation is ~83 steps.

Having this log-log plot of defaults numbers, we can neither decline nor assume a power law:

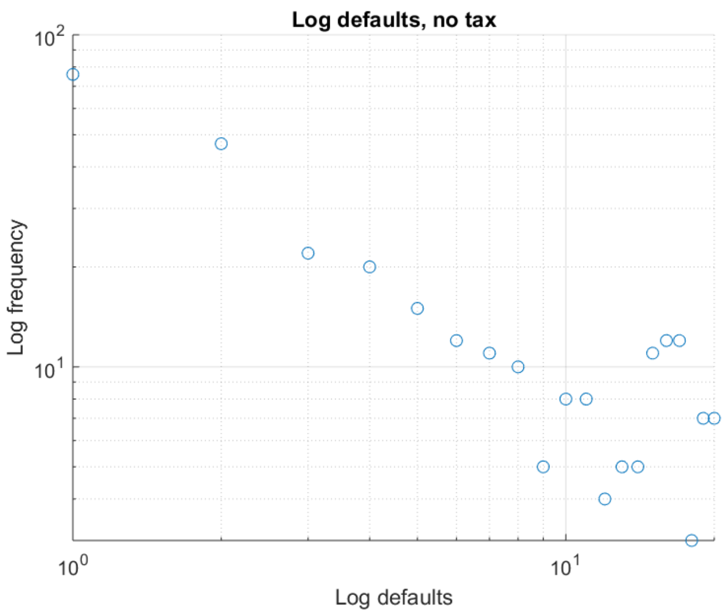


Figure 6. Defaults in logarithmic scales

Maybe, it will be clearer with more simulations. The histogram in linear axes:

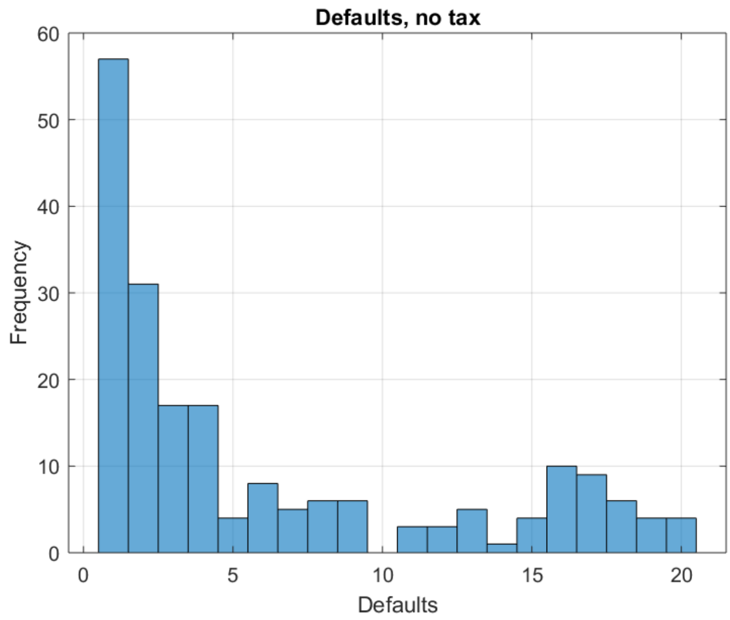


Figure 7. Defaults in normal axes

On average, there are ~5 defaults after the first failure.

Concerning bad debts, the distribution is broad but much more elaborate statistical analysis would be needed to confirm whether it follows a power law or not.

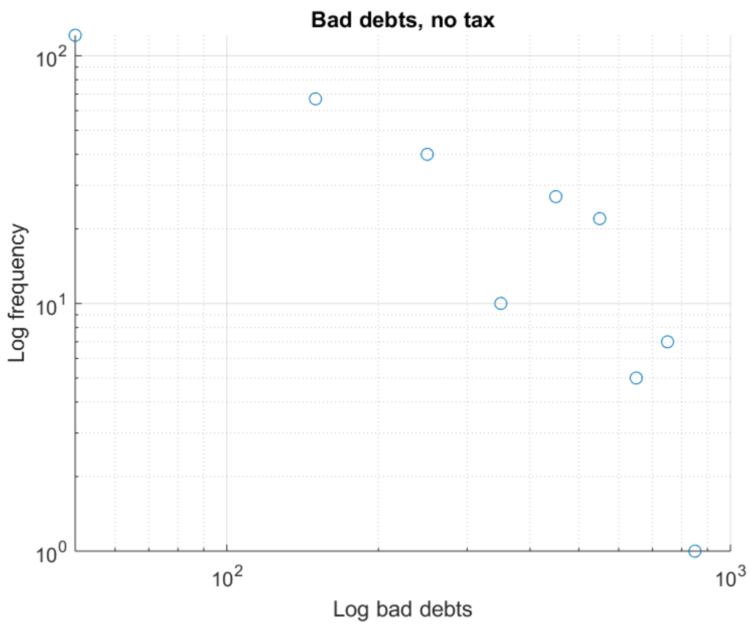


Figure 8. Bad debts in logarithmic scales

On average, bad debt amount constitutes up to ~209.

For every simulation, we also have a time series of the sum of credits given at every step. If we sum this up for every simulation, we will obtain some proxies for economic activity. On the other hand, given the variable time of the system's life, we cannot compare them directly. In order to be able to do this, we divide the sum of every credit by the time of the system's life, thus obtaining a "credit-per-step" value. Here we observe a thin-tailed distribution:

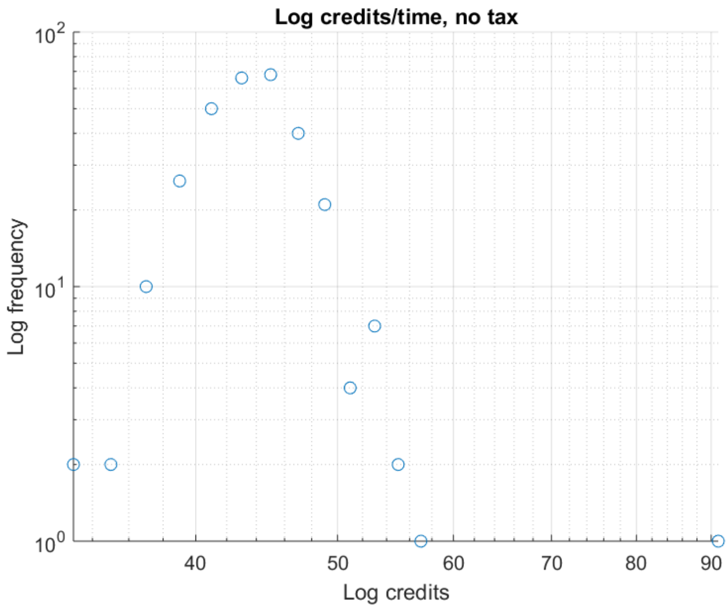


Figure 9. Credits per step in logarithmic scales

Average credits per step sum are 44.1728.

We measure risk in the system via the sum of DebtRanks of every bank; so, this sum has an upper bound of $Bk = 20$.

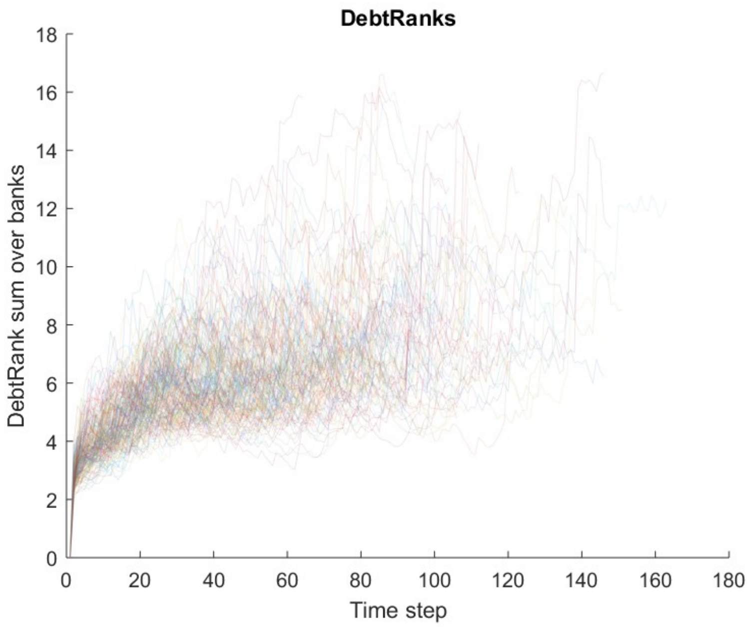


Figure 10. DebtRanks sums in one plot

We can observe that this value is mostly growing, and it is difficult to compare this for different taxation schemes, because of different living times of systems. We will grasp the last values before the failure for every simulation and compare their distributions:

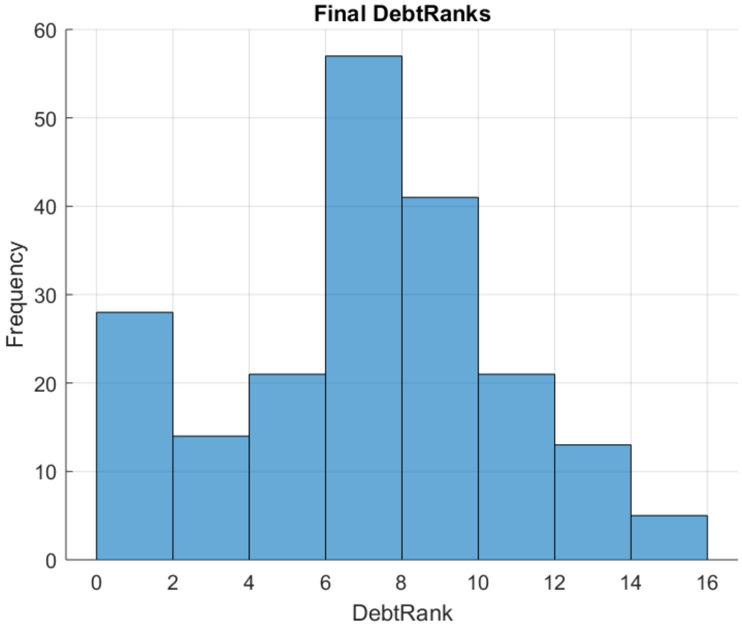


Figure 11. Final DebtRanks distribution

iii. Systemic risk tax check

In this section, we check the validity of the systemic risk tax proposal: is it Pareto-effective compared to others (improving one group of metrics without spoiling others). Losses with systemic risk tax are distributed better:

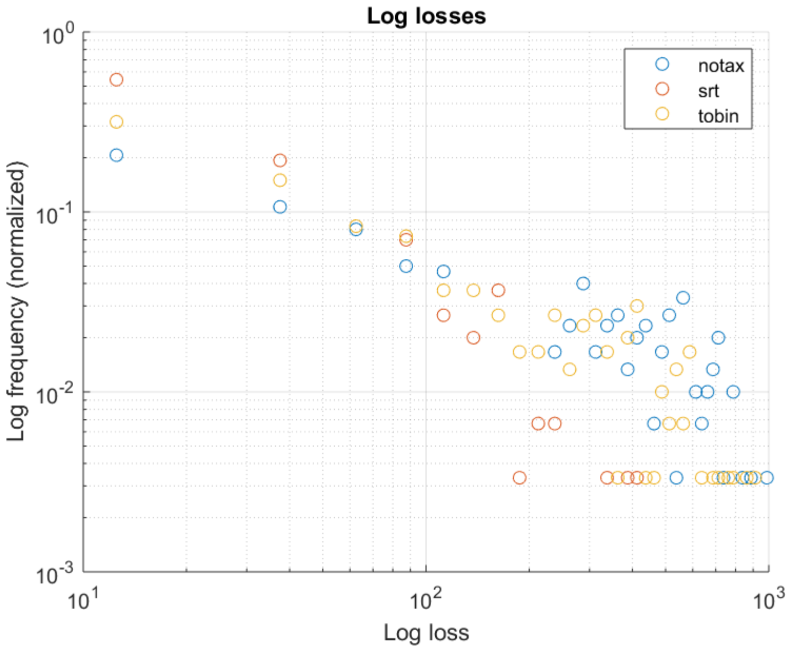


Figure 12. Losses for SRT check in logarithmic scales

Times of the system's life do not change much:

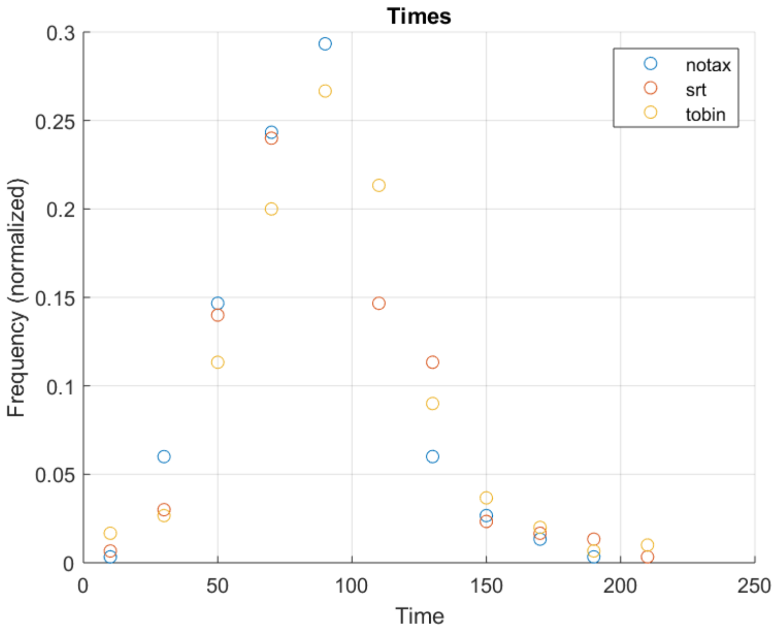


Figure 13. Times for SRT check

A number of defaults distribution shows us tails being much thinner with systemic risk tax:

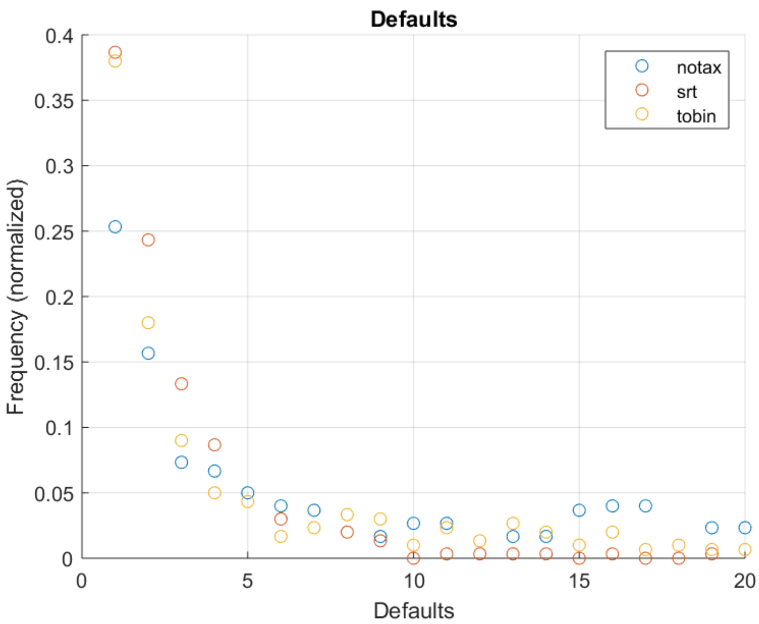


Figure 14. Defaults for SRT check

Bad debts (what is actually measured by DebtRank, so, SRT levies exactly these expected values) differ for all three schemes, being much less catastrophic with the systemic risk tax introduced:

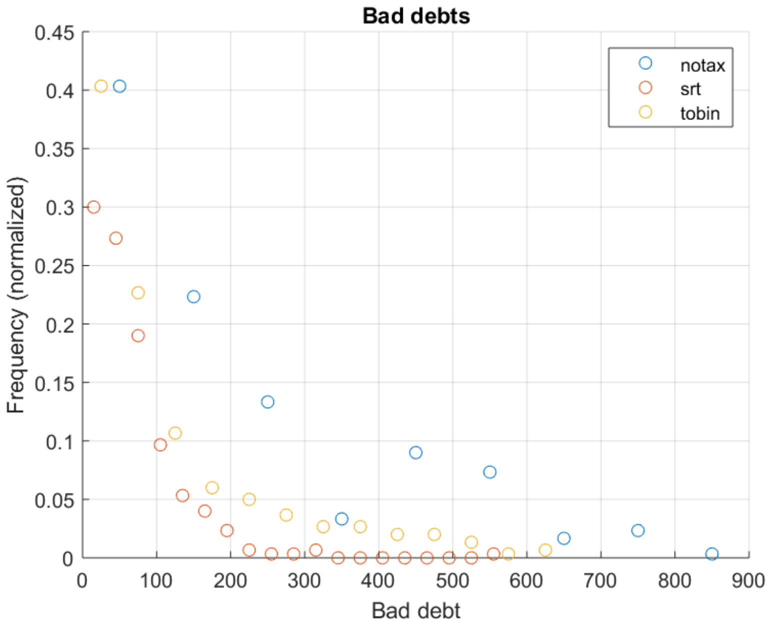


Figure 15. Bad debts for SRT check

Here we compare average credits volumes per step. This is where we visually can see a big difference between Pigouvian and Tobin taxes. This difference means, that systemic risk tax does not affect the economic activity much:

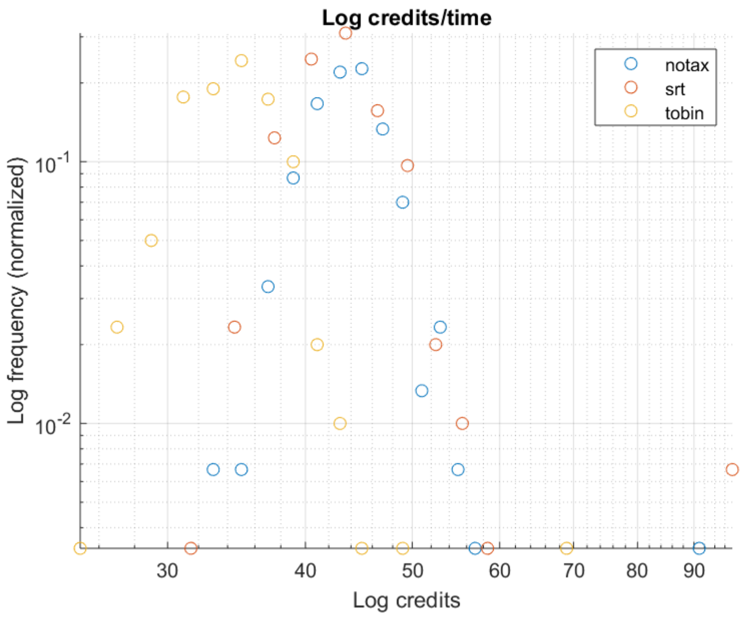


Figure 16. Credits per time step for SRT check in logarithmic scales

And now we compare taxation volumes per step – here we notice, that systemic risk tax is also much smaller than Tobin tax:

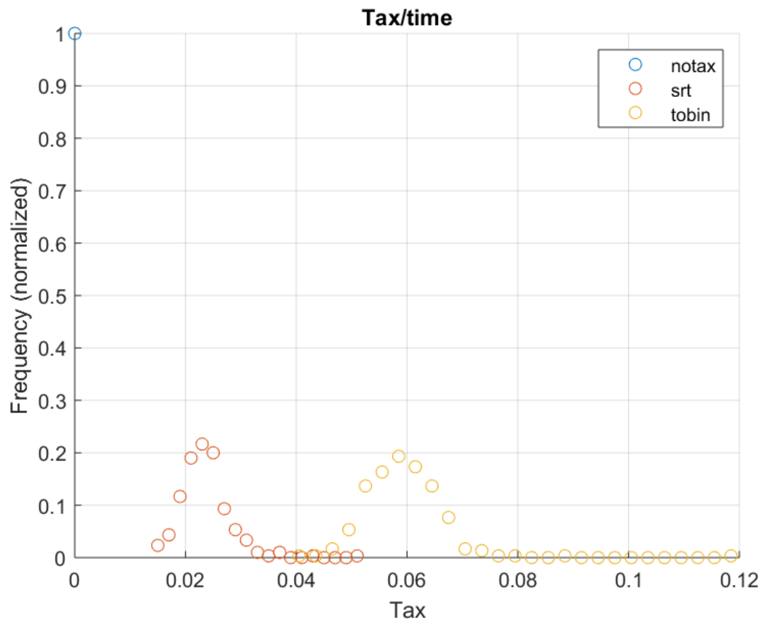


Figure 17. Tax per time step for SRT check

Final DebtRanks distributions demonstrate less default exposure in the systems developing with the systemic risk tax:

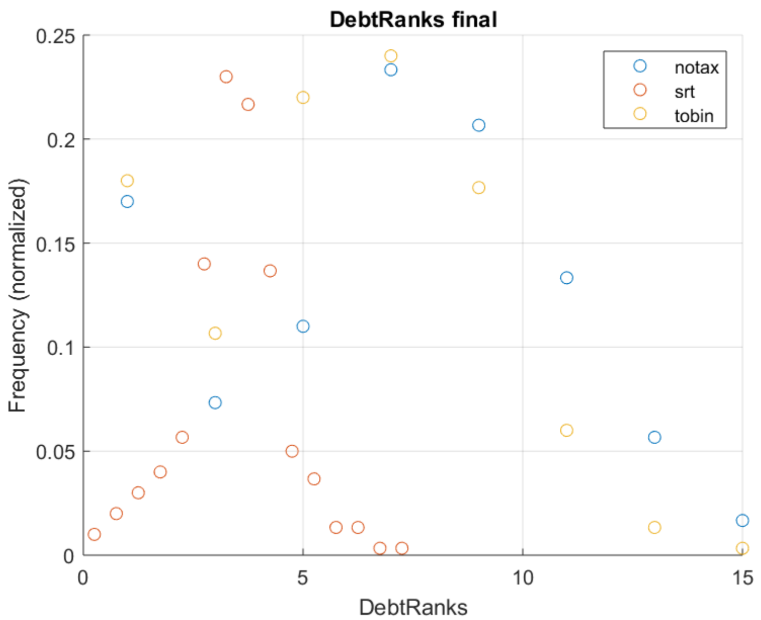


Figure 18. Final DebtRanks distributions for SRT check

Final degrees distributions (in log-log axes):

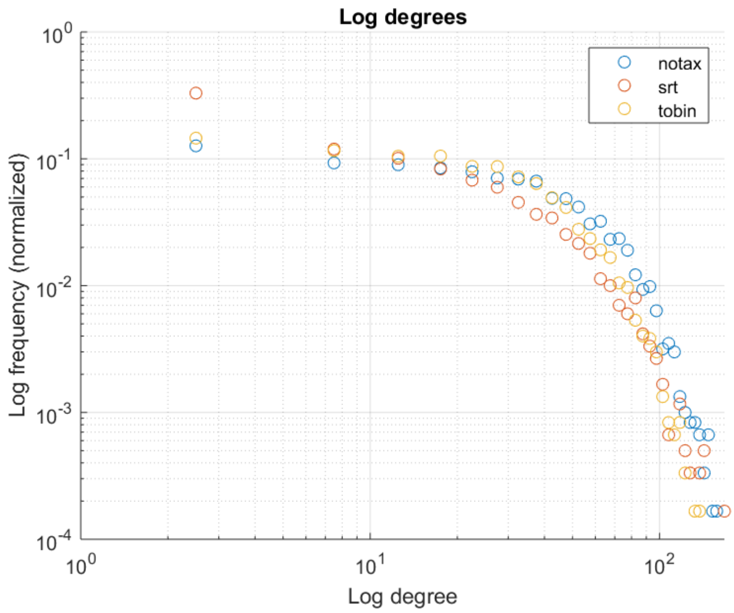


Figure 19. Degrees for SRT check in logarithmic scales

Degrees in normal axes:

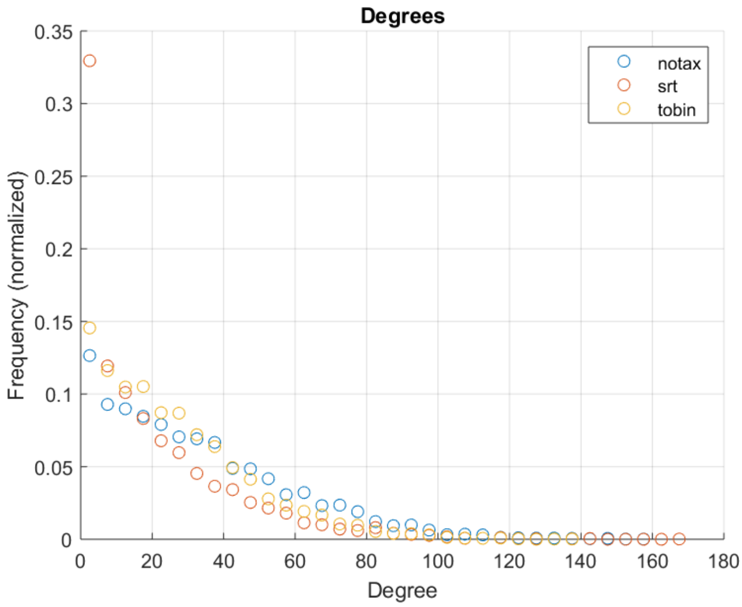


Figure 20. Degrees for SRT check in linear scales

We can notice, that both taxes slightly decrease the number of high-degree nodes in approximately the same quantities.

iv. SinkRank parameter search

Taxation scheme	Mean loss	Mean time	Mean bad debts	Mean taxes	Mean defaults	Mean credits volumes
Acyclic DebtRank, $\zeta = 0.02$	42.3122	89.14	65.5409	0.0235	2.7433	43.4818

SinkRank, $\zeta = 0.08$	210.6515	86.51	187.1199	8.7139e-04	5.8800	**41.0362**
SinkRank, $\zeta = 0.16$	**172.6679**	**84.24**	**159.4879**	0.0017	**5.2400**	40.3624
SinkRank, $\zeta = 0.24$	200.3017	85.74	183.7537	0.0023	5.9600	39.9972
SinkRank, $\zeta = 0.32$	193.3747	84.57	166.2605	0.0039	5.2800	40.5305
Tobin, $\zeta = 0.002$	142.4003	91.8367	121.7794	0.0592	4.3933	34.4758
No tax, $\zeta = 0$	234.9117	83.7700	207.1787	0	6.3467	44.0656

Table 10. SinkRank parameter search average results

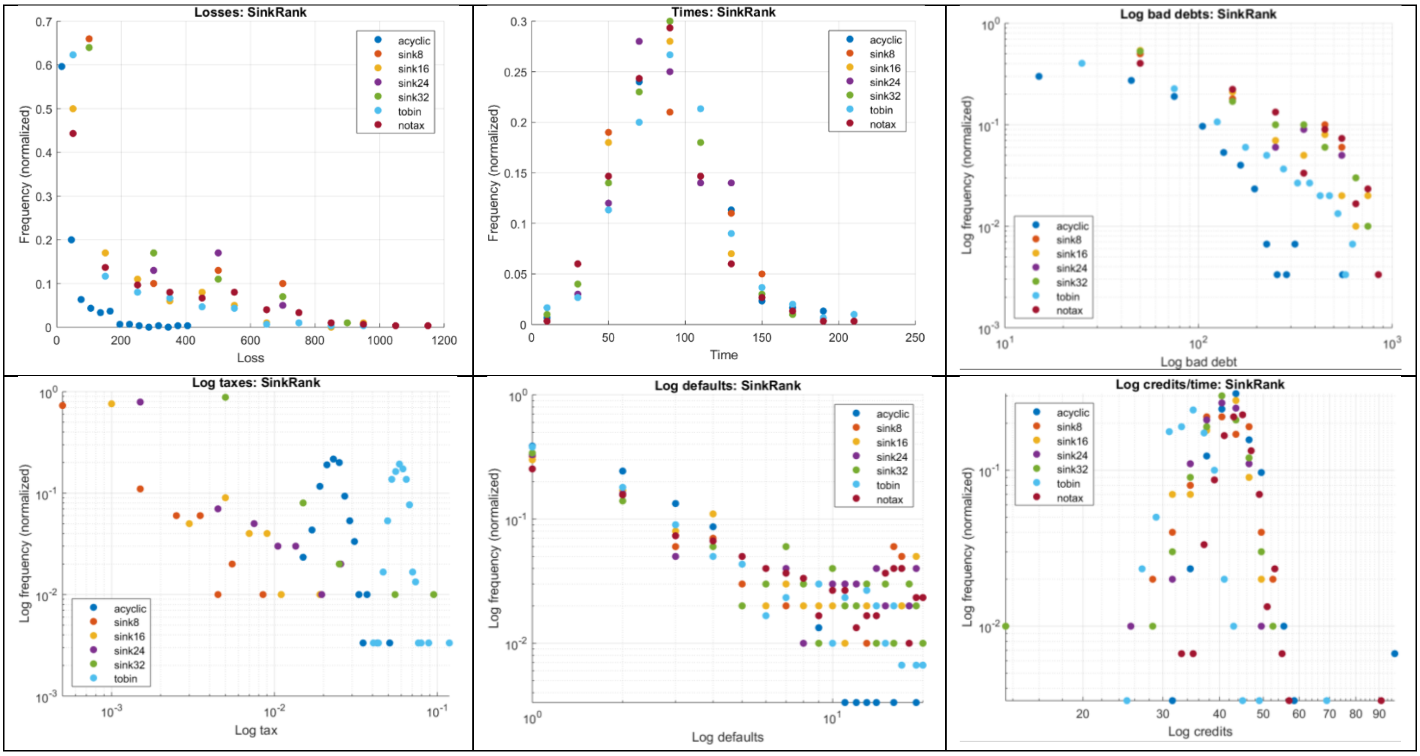


Table 11. SinkRank parameter search main plots

At $\zeta = 0.16$ losses are slightly decreased by SinkRank taxation.

Based on living times, we cannot conclude on the SinkRank tax impact. Even DebtRank with this ζ does not show much greater mean value, however, it changes the distribution much.

Bad debts distributions are not affected much by SinkRank tax, which slightly cuts tail; unlike Tobin and DebtRank tax, which significantly decrease bad debts. Mean values are still decreased by SinkRank tax, $\zeta = 0.16$ is the best performing one.

Taxes distributions of SinkRank are of a significantly different shape compared to Tobin and DebtRank tax. On average, they are much less than both Tobin and DebtRank taxes.

Defaults distributions are neither much affected by the SinkRank nor Tobin tax unlike DebtRank tax cutting the tail of the number of defaults distribution. On average, SinkRank also performs worse than even Tobin tax ($\zeta = 0.16$ also is the best selection for the SinkRank):

The economic activity being measured by average credits given per step is almost not affected by SinkRank taxes. On average, SinkRank also performs much better than Tobin tax and slightly worse than DebtRank.

Overall, we select $\zeta = 0.16$ for the further comparison of the SinkRank tax. However, based on a "peak" point in the performance – for greater ζ we can obtain more bad debts – we can already conclude about the inefficiency of the SinkRank for the systemic risk tax purpose.

v. 2-step DebtRank parameter search

Taxation scheme	Mean loss	Mean time	Mean bad debts	Mean taxes	Mean defaults	Mean credits volumes
Acyclic DebtRank, $\zeta = 0.02$	42.3122	89.1433	65.5409	0.0235	2.7433	43.4818
2-step DebtRank, $\zeta = 0.015$	44.5122	92.9850	50.9328	0.0358	2.3050	**34.5564**
2-step DebtRank, $\zeta = 0.02$	49.4606	90.7900	48.7891	0.0399	2.4750	32.9865
2-step DebtRank, $\zeta = 0.04$	26.4562	95.8750	27.7202	0.0482	1.6800	29.4462
2-step DebtRank, $\zeta = 0.08$	15.8857	111.0550	22.4461	0.0549	1.7050	25.8669
Tobin, $\zeta = 0.002$	142.4003	91.8367	121.7794	0.0592	4.3933	34.4758
No tax, $\zeta = 0$	234.9117	83.7700	207.1787	0	6.3467	44.0656

Table 12. 2-step DebtRank parameter search average results

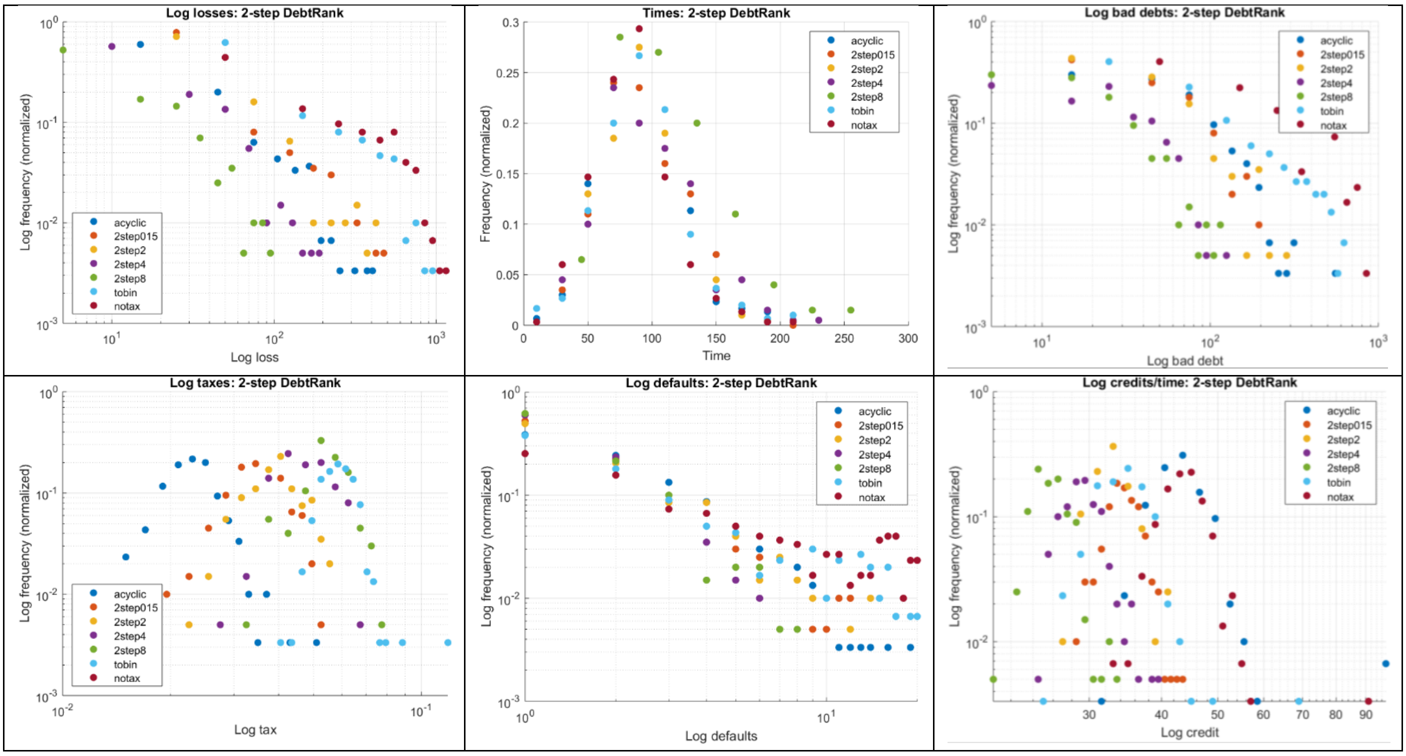


Table 13. 2-step DebtRank parameter search main plots

Unlike SinkRank, 2-step DebtRank decreases losses, bad debts and defaults numbers with ζ increase. Overall, 2-step DebtRank behaviour is similar to the original DebtRank, so, based on the credit volumes being close to the Tobin tax we select $\zeta = 0.015$ for the further comparison.

Surprisingly, with smaller ζ 2-step DebtRank average tax is greater, than with the original DebtRank, however, having a complex evolving system, there might be different less straightforward explanations for this phenomenon.

vi. Acyclic DebtRank vs. Cyclic DebtRank

Taxation scheme	Mean loss	Mean time	Mean bad debts	Mean taxes	Mean defaults	Mean credits volumes
Acyclic DebtRank, $\zeta = 1$	4.5638	129.2000	4.9706	0.0150	1.3733	13.2938
Cyclic DebtRank, $\zeta = 1$	4.6630	122.5767	4.3138	0.0149	1.3300	13.4829

Table 14. Acylic DebtRank vs. Cyclic Debtrank average results

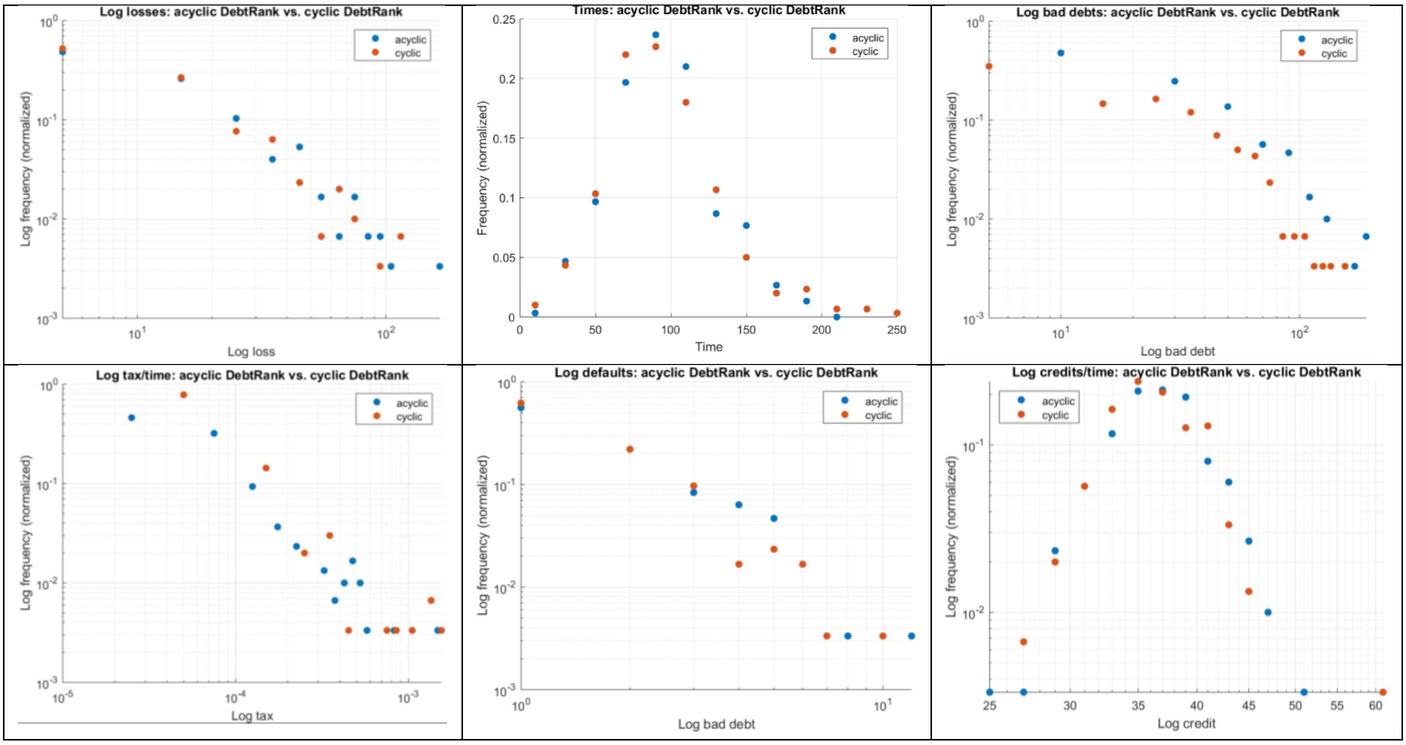


Table 15. Acyclic DebtRank vs. Cyclic DebtRank comparison main plots

In theory, we would suppose better performance of the cyclic DebtRank; in practice, the difference is vague, and the average losses amount is slightly less and the living time is greater for the acyclic DebtRank. For other mean values, cyclic DebtRank is slightly better, especially for the bad debts we can even visually notice the dominance of the cyclic DebtRank.

vii. Results

Taxation scheme	Mean loss	Mean time	Mean bad debts	Mean taxes	Mean defaults	Mean credits volumes
Acyclic DebtRank, $\zeta = 0.02$	42.3122	89.1433	65.5409	0.0235	2.7433	43.4818
2-step DebtRank, $\zeta = 0.015$	44.5122	92.9850	50.9328	0.0358	2.3050	34.5564
SinkRank, $\zeta = 0.16$	172.6679	84.24	159.4879	0.0017	5.2400	40.3624
Tobin, $\zeta = 0.002$	142.4003	91.8367	121.7794	0.0592	4.3933	34.4758
No tax, $\zeta = 0$	234.9117	83.7700	207.1787	0	6.3467	44.0656

Table 16. Final taxation schemes comparison average results

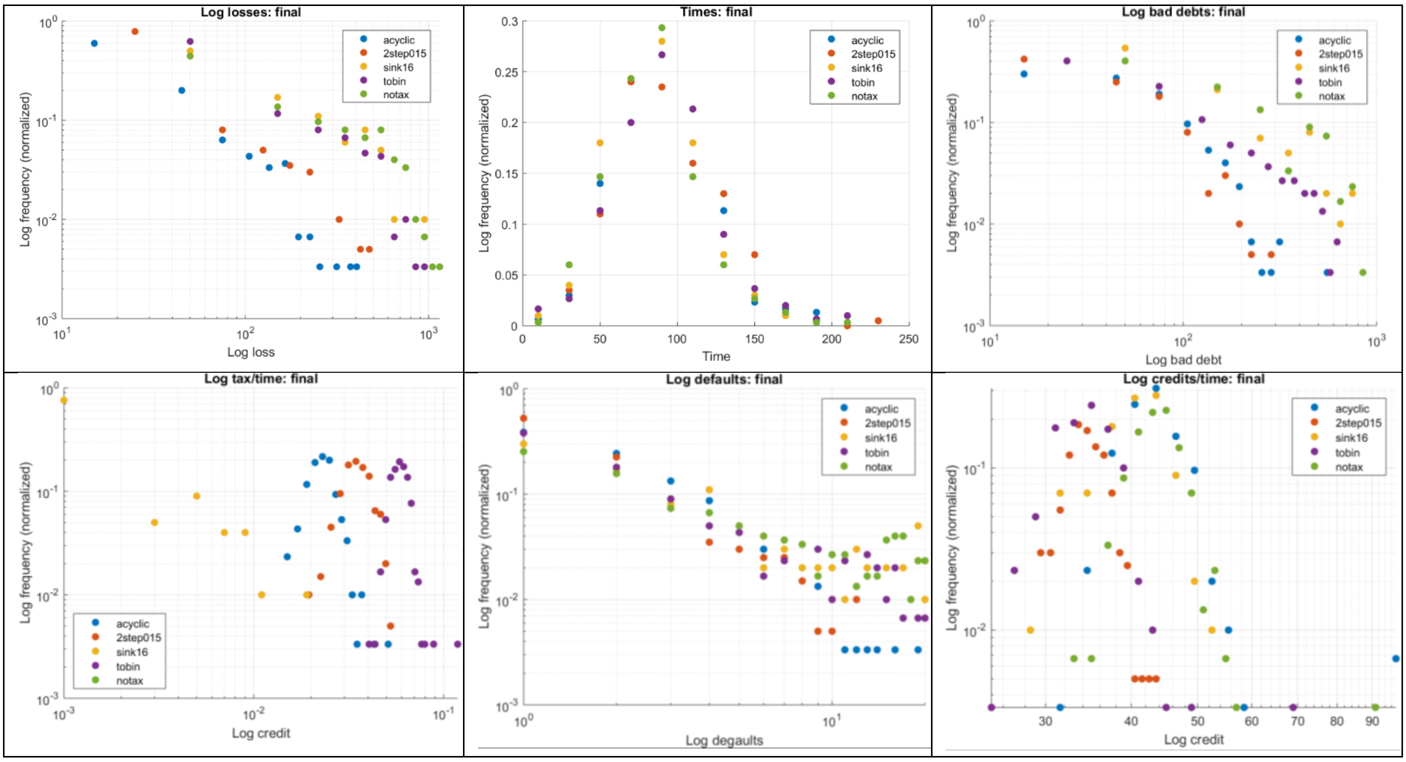


Table 17. Final taxation schemes comparison main plots

2-step DebtRank performs better than SinkRank for all the distributions (except tax/time, but we should consider more credits/time: if SinkRank gathers less tax, but affects the economic activity more, than it is ineffective). In addition, SinkRank's non-monotonicity sets certain limits on its applicability. Our primary investigation of SinkRank for the systemic risk taxation shows its inappropriateness.

2-step DebtRank also outperforms Tobin tax and is close to the original DebtRank. We suppose it is worth investigating further.

B. Equity layer investigation

Variable	Initial value
μ	0.2
σ	1

μB	0.2
σB	0.5
$market\_prices_i(1)$	10
$market\_pricesB_j(1)$	5
μ_{roe}	0.1
σ_{roe}	0.05
μB_{roe}	0.05
σB_{roe}	0.025
$eq_{ij}(1)$	0
$eqB_{ij}(1)$	0
E_f	**0**
E_b	**0**
eq_sold_i	0
eq_soldB_i	0
κ	0.2

Table 18. Initial values for the equity layer modelling

When introducing equity layer, we did not expect any certain results. We obtained the less-living system because of the new risky asset (however, this can be tuned by the change in the underlying stochastic processes parameters) and greater impact of the systemic risk tax. We also can plot here the degree distribution for the equity layer tax (also including bank-firm equity relations). We can notice, that here taxes (both Pigouvian and Tobin) increase degrees in opposite of what we observed in (IV.A.iii).

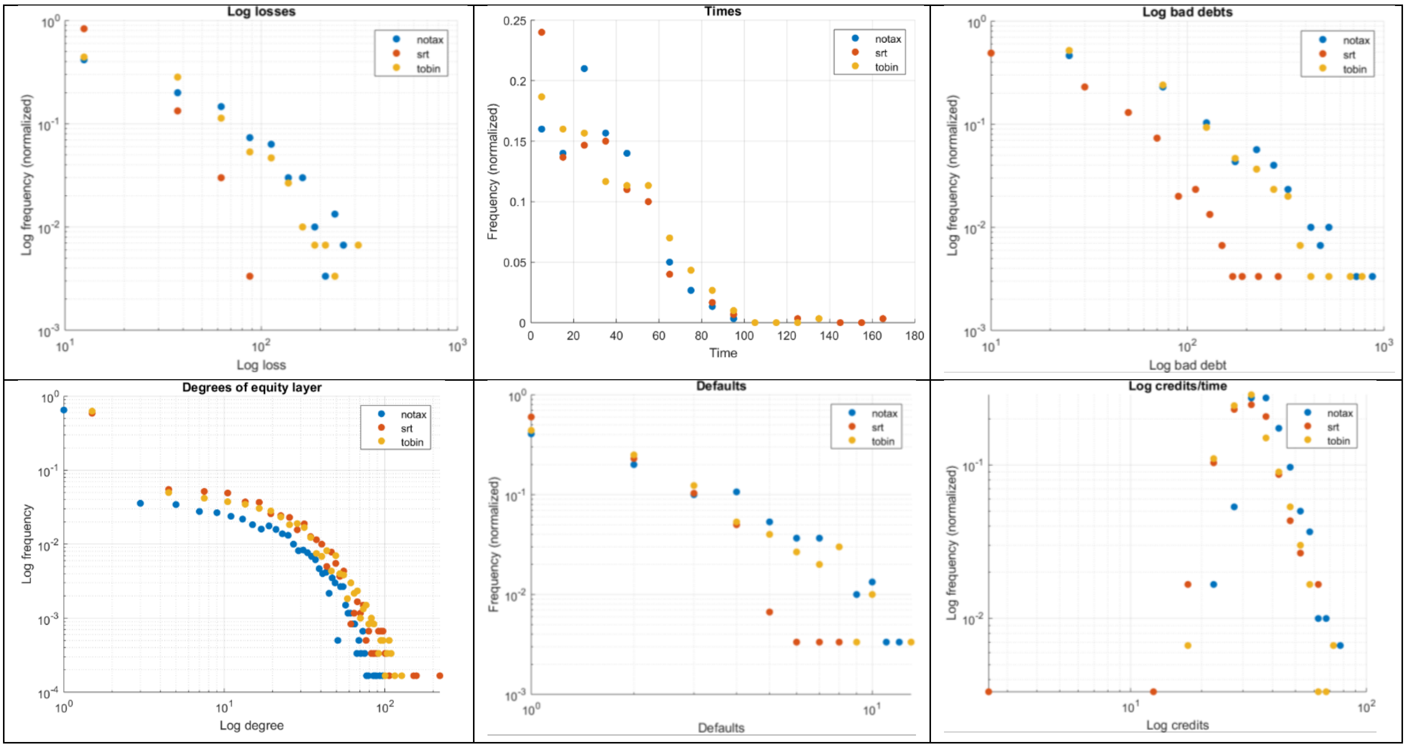


Table 19. Equity layer simulation main plots

An effect discussed before the implementation concerns the interaction of the layers. On the one hand, systemic risk in both layers should be correlated because of the economic activity oscillations:

when the economic activity is higher, both layers evolve more thus increasing the systemic risk. On the other hand, higher risk in one layer implies higher taxes compared to the other layer thus leading to more transactions in the layer with lower tax. Both theories appeared to be correct: average DebtRanks sum correlation between layers for 300 simulations with $\zeta = 0.02$ equals 0.662, with a minimal value of -0.21; and for the $\zeta = 1$ average correlation equals 0.159 with the minimal correlation value of -0.82. Examples of the DebtRanks time series from single simulations are presented in the following table:

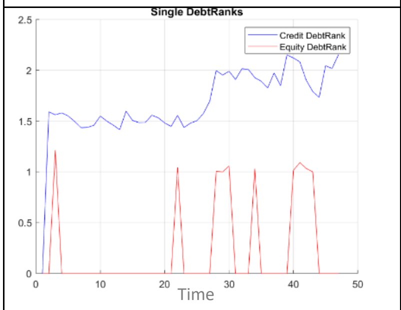
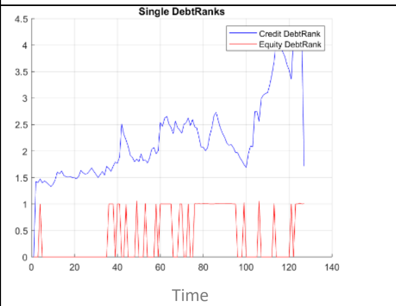
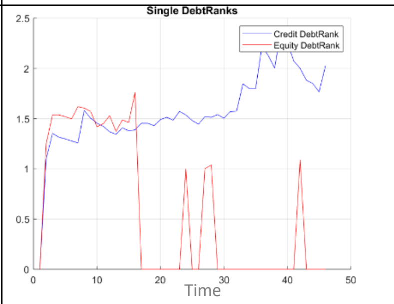
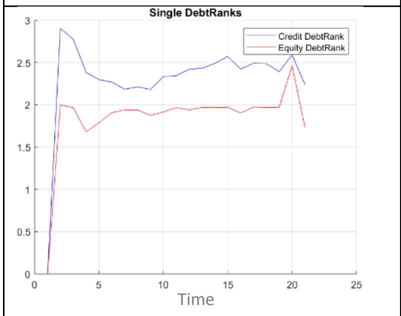
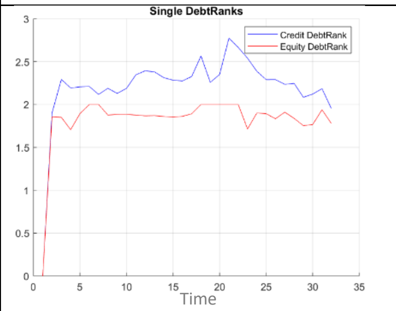
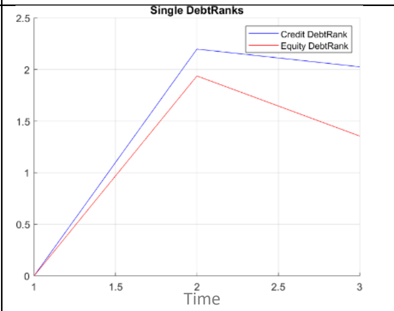
DebtRanks vs. time example	DebtRanks vs. time example	DebtRanks vs. time example	ζ
			1
			0.02

Table 20. Examples of DebtRanks time series for 2 different taxation parameter values

V. Discussion

We have replicated the results of the basic paper [6] and brought changes in two directions: 1) tried different metrics and 2) implemented a new layer. We have confirmed that systemic risk tax, based on DebtRank, can efficiently create stimuli for the agents to restructure the network in a more robust way without a significant decrease in transaction volumes.

The investigated virtual economic system appeared to be very complex, and the result of the direct impact of the tax on the metrics it is based on is very impressive. The economy does not always response to the external stimuli the way it is expected. For instance, tax increase under certain conditions may decrease the volume of taxes delivered to the government because of the Laffer curve underlying reasons [25] or because of the squeezing businesses to the shadow sector of the economy. In the light of these considerations, the result of really working (at least, in simulations) systemic risk tax pointwise influencing the complex system, is something worth attention.

We have come to the conclusion that metrics calculating expected losses in a distressed network are suited better for the purpose of the systemic risk tax. Moreover, we can also suppose, that the tax based on a certain metric will first improve this metric in the system. So, if instead of bad debts measured by DebtRanks, one would propose a routine that would calculate losses (in a sense of chapter IV) or the number of consequent defaults, then the tax based on this metric would better perform for this metric maybe conceding to, e.g., DebtRank in bad debts. In this case, it might be worth considering, for instance, linear combinations of taxes aimed at certain metrics.

We also have tested a very different systemic risk metric, SinkRank. It appeared to show a non-monotonicity in response to the increase of the fraction of it to be taxed. So, this could be an example of a totally inappropriate systemic risk metric: if we consider a degree centrality (which leads to the Tobin taxation scheme), we do not obtain this result. Instead, for the degree centrality, we have a trade-off between economic activity and the system's robustness.

We have considered the illegality of such taxation and proposed the truncated version of the DebtRank, involving only neighbours and neighbours of the neighbours of the counterparties. The tax, based on these values, has shown great performance, being close to the original DebtRank. This result appears to be intuitively surprising, and we suppose that this metric performance may depend too much on the network topology. We assume that for the power-law degree distribution of the credit network results may change dramatically. Anyway, we have shown that there might be fruitful results in this area.

The agent-based model used, despite having fair and reasonable assumptions about agents behaviour, produces network without power law in degrees distribution, what might be crucial for the examination of critical events. We suppose that it worth adjusting this model to produce scale-free credit networks. While this model is quite complex and temporal, one cannot explicitly define the desired network to be produced, however, other topology metrics influence might also be interesting to investigate. For example, higher clustering may lead to the dominance of cyclic DebtRank over the acyclic one, and smaller average shortest paths may lead to a greater number of defaults. We would propose to start with the degree distribution, implicitly inserting a preferential attachment mechanism at the stage of firm-bank loans: banks select their counterparties among all of the existing banks, while firms send applications only to a certain number of randomly chosen banks. Instead of the uniform distribution, one could try the probabilities proportional to some balance sheet quantities of the banks.

When implementing an equity layer, the biggest problem was the modelling of market values. We tried to imitate the real business valuation processes via DCF's: we know all the book and historical values about all the banks and firms to evaluate them, however, this method did not help us to create a developing system. Anyway, we do not deny the possibility of such modelling fashion. In addition, such an implicit valuation would be harder to tune, for instance, in case of attempts to fit this model to the real data.

Instead, we have chosen an easier way to model market values as stochastic processes. We did not produce any substantially new results except the multilayer risk oscillation under the systemic risk tax. For the more detailed investigation, one should measure systemic risk not after every simulation cycle, but after every transaction in order to consider correlations with unitary lag.

VI. Acknowledgements

I thank Prof. Jari Saramäki, my supervisor, for creating very comfortable conditions for conducting this research and for wise advice. I am also very grateful to PhD Sebastian Poledna for the important impact in this research and productive collaboration.

VII. References

[1] Volker Wieland, Maik Wolters, 13 February 2012 (http://voxeu.org)
[2] N.N. Taleb, *Antifragile: Things That Gain from Disorder*, 2012 (Random House, ISBN 978-1-4000-6782-4)
[3] J. Buchanan, Wm. Craig Stubblebine, *Externality*, November 1962 (Economica. 29 (116): 371–84. doi:10.2307/2551386. JSTOR 2551386)
[4] J. Tobin, *A Proposal for International Monetary Reform*, July–October 1978 (Eastern Economic Journal. 4 (3–4): 153–159)
[5] S. Battiston, M. Puliga, R. Kaushik, P. Tasca & G. Caldarelli, *DebtRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk,* 2 August 2012 (Scientific Reports 2. 541. doi:10.1038/srep00541)
[6] S. Poledna, S. Thurner, *Elimination of systemic risk in financial networks by means of a systemic risk transaction tax*, 30 January 2014 (Quantitative Finance 16 1469-7696 2016)
[7] Basel III: international regulatory framework for banks (https://www.bis.org/bcbs/basel3.htm)
[8] G. Visentin, S. Battiston, M. D'Errico, *Rethinking financial contagion*, 28 August 2016 (arXiv:1608.07831v1)
[9] P. Barucca, M. Bardoscia, F. Caccioli, M. D'Errico, G. Visentin, S. Battiston, and G. Caldarelli, *Network valuation in financial systems*, 2016 (Available at SSRN 2795583)
[10] P. Glasserman, H.P. Young, *How likely is contagion in financial networks?*, 2015 (Journal of Banking & Finance)
[11] K. Soramäki, S. R. Cook, *SinkRank: An Algorithm for Identifying Systemically Important Banks in Payment Systems*, June 2013 (Economics E-Journal 7)
[12] P. W. Anderson, K. Arrow, D. Pines, *The Economy As An Evolving Complex System*, January 1988 (Santa Fe Institute Series, 1st Edition)
[13] R. Lucas, *Econometric Policy Evaluation: A Critique*, 1976 (Carnegie-Rochester Conference Series on Public Policy, vol. 1)
[14] D.D. Gatti, S. Desiderio, E. Gaffeo, P. Cirillo, M. Gallegati, *Macroeconomics from the Bottom-up*, January 2011 (Springer, New Economic Windows series)
[15] L.J. Christiano, M.S. Eichenbaum, M. Traband, *On DSGE Models*, Summer 2018 (Journal of Economic Perspectives, vol. 32, no. 3)
[16] E. Chappin, A. Chmieliauskas, L.J. De Vries, *Agent-based models for policy makers*, January 2012 (Conference: Salzburger Geographische Arbeiten)
[17] Institute of Medicine, *Assessing the Use of Agent-Based Models for Tobacco Regulation*, 2015 (Washington, DC: The National Academies Press)
[18] M. Bardoscia, S. Battiston, F. Caccioli, & G. Caldarelli, *DebtRank: A microscopic foundation for shock propagation*, 19 June 2015 (PLoS One; 10(6): e0130406)
[19] J.C. Hull, A.D. White, *Valuing credit default swaps I: No counterparty default risk*, 2000 (J. Derivatives, 2000, 8(1), 29–40.)
[20] J.C. Hull, A.D. White, *Valuing credit default swaps II: Modeling default correlations*, 2001 (J. Derivatives, 2001, 8(3), 12–21.)
[21] I. Fisher, *The theory of interest*, 1930 (New York 43)
[22] D. Hughes, *The Business Value Myth*, 2012 (Canopy Law Books. ASIN B009XB91CU)
[23] A. Kraus, R.H. Litzenberger, *A State-Preference Model of Optimal Financial Leverage*, 1973 (Journal of Finance. 28: 911–922.)
[24] S. C. Myers, N.S. Majluf, *Corporate financing and investment decisions when firms have information that investors do not have*, 1984 (Journal of Financial Economics)
[25] A. Laffer, *The Laffer Curve: Past, Present, and Future*, 1 June 2004 (The Heritage Foundation)