



**Aalto-yliopisto**  
Insinööritieteiden  
korkeakoulu

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**Granger Causality of interest rate on REIT's return**

Master's thesis, which has been submitted as a thesis for examination for a Master's degree

Espoo 23.2.2019

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<b>Title of thesis</b> Granger Causality of interest rate on REIT's return		
<b>Master programme</b> Real Estate Economics	<b>Code</b> ENG24	
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<b>Date</b> 23.2.2019	<b>Number of pages</b> 46	<b>Language</b> English

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**Abstract:**

By testing Granger causality of interest rates on REIT's total return index, the study found out this indirect causal relationship have some similarities with the direct relationship between REIT's total return index and interest rate concluded by earlier studies. The first similarity is that the relationship does exist but is time-specific. For the whole tested time frame of 16 years, REITs is sensitive to all 11 maturities of CMT. However, when testing on the sub-periods, the long-term CMTs are found to "Granger" cause REIT's total return during the 2008 financial crisis while none of the CMTs have explanatory power on REITs during stable environment. The result is in coherence with hypothesis of REIT's high sensitivity to long-term interest rate. It also support an earlier finding that REIT takes more effect from interest rate during shocks than during stable times. In explaining the results, the thesis analyses the three main channels through which REIT takes effect from interest rate changes. They are through financial market forces (supply and demand), real estate industry and operating activity.

This model of testing for Granger causality, which is also time-specific, should be done as a prerequisite for building time series model as it helps to decide which variable should be included. The thesis practices Toda-Yamamoto procedure, which is recommended for Granger causality test as it allows better data flexibility.

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**Keywords** REITs, interest rate, US CMT, Granger Causality, Toda-Yamamoto

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## Foreword

I would like to thank my advisor Alexey Zhukovskiy and my supervisor Heidi Falkenbach for guiding and supporting me not only on completing this thesis but also through two years of master studying. I also want to thank my mentor, Kristian Kullberg, and my colleague, Simo Heliövaara for their assistances with this study. Last but not least, I am grateful for my co-workers and my team in NIB. I am blessed to learn so much about finance as well as programming, which greatly contributed to my final paper.

Espoo 23.2.2019

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## Abbreviations

AIC: Akaike Information Criterion

CMT: Constant Maturity Treasury

CPI: Consumer Pricing Index

DF test: Dickey-Fuller test

HQC: Hannan-Quinn Information Criterion

IPO: Initial Public Offering

IR: Interest rate

I(n): order of integration of n

KPSS test: Kwiatkowski-Phillips-Schmidt-Shin test

REITs: Real Estate Investment Trusts

T-Y procedure: Toda-Yamamoto procedure

VAR: Vector Autoregressive Model

# 1 Introduction

Real estate investment trust (REIT) has become a more and more popular investment option as it allows investors to get involved into real estate industry at convenience of the stock market. This makes REITs the best of both worlds when they are as liquid as stocks, as steadily and high dividend as fixed income instruments while providing as comfortable risk level as real estate does. Studies on REITs and its relationship with other economic variables, especially interest rates have been carried out since the 80s, but none has taken a look at the existence of Granger Causality between them.

The thesis will answer two main questions: whether interest rate Granger causes the US REIT's total return, if the Granger Causality exist, which term interest rates REITs are sensitive to and how stable the relationship is. By answering the research questions, this study serves several purposes. First, it will fill in the gap of the existing literature on Granger Causality between REIT's return and interest rate which has been overcrowded by studies on direct relationship between the two variables.

Second, the study will test if some of the findings from existing studies on direct relationship stay true for Granger Causality. The first hypothesis is that the connection between REIT's return and interest rate does exist but unstable through time. Another theory to be tested is that REITs are sensitive to long-term interest rate. And finally, REIT's interest rate sensitivity is more intense during shocks than during stable environment. The paper attempt to explain the result based on 3 channels of sensitivity that also identified by earlier literature. They are financial market forces, real estate industry and operating activities.

Third, the study is done on different REIT's and interest rate's proxies, i.e. MSCI and CMTs comparing to single stock or NAREITs and the T-bill, corporate bonds or high-yield bonds used in earlier researches. These proxies will give a different perspective on REITs and interest rate, which will complement the existing literature on this relationship.

Last but not least, this thesis is an example of how Toda-Yamamoto procedure, a more superior and versatile approach to Granger causality than the traditional Granger test, is applied. Additionally, the Python code created in this thesis is a compact and well-structured 4-step tool for T-Y application that can be used in different researches and dataset. The practice done in this study can be applied as a prerequisite step for building a time series models, especially VAR because it helps determining which variables to be included.

The thesis is divided into three main parts. The first section reviews earlier researches regarding REITs and interest rate. It touches on definitions, characteristics, findings on the relationship between the two variables as well as a recap of their movements in the past in the US market. The next part describes the methodology and data selection used for this study. It introduces the T-Y model for Granger causality test, breaks the procedure into 4 steps and explains the choice of data. The last section summarizes the study's empirical result, interprets findings and ends with limitations and conclusions.



## 2 Theoretical Background

### 2.1 REITs definition and characteristics

#### 2.1.1 REIT's definition

Real Estate Investment Trusts (REITs) are companies that follow real property investment strategies to directly and purposely invest in multiple tradable real estate assets. They can be property, land, mortgages, debt or shares of other REITs. REITs are often established as corporation, public limited-company or trusts whose stocks are traded over stock exchanges. (Glickman A., et.al, 2014, p.361). In the nutshell, REITs can be considered as a pool of real estates that are tradable in the stock exchange. More specified definition of REITs can vary from market to market, depending on each country's regulations and requirements. (Goddard, et.al, 2012, p.254). This paper only looks at characteristics and requirements of companies qualified to be REITs in the US market under the US regulations.

#### 2.1.2 The US REIT's characteristics

In order to be qualified as a REIT, a trust needs to fulfil some certain requirements, which also define its characteristics. First, REITs are required to have minimum of 75% of their assets as real property, property's securities or share of other REITs. The rest can be other types of securities or cash coming from its taxable subsidiaries. Because of the asset's requirement, 75% of REIT's income must come from real estate rental activity. (Glickman A., et.al. 2014, p.364)

The most signature attribution of REITs is that they must return at least 90% of their profit to their shareholder as dividend. If this condition is fulfilled, REITs are exempted from paying ordinary income tax. This makes reinvesting from retained earning difficult as majority must be paid out. Therefore, REITs are frequent customers of capital market and actively raising fund from investors. (Glickman A., et.al. 2014, p.364)

Because of the low retain earnings, REIT's capital structures rely mostly on public equity, unsecured or secured public debts and line of credit with commercial banks. REITs can obtain public debt from mortgaging their properties. Thanks to the direct ownership of real properties, there are great leverage opportunities for REITs. However, most REITs are more conservative than private real estate firms in leverage strategy with below 50% of debt to market capitalization ratio. REITs can take advantage of both long and short-term mortgage debts from commercial banks. Long-term debt normally has fixed rate with tenor of more than 5 years. Short-term debts are mostly under floating rate. In order to stabilize short-term debts, REITs often enter interest swap contracts where REITs swap their floating obligation for a fixed interest rate. (Glickman A., et.al. 2014, p.368)

In the US, REITs are generally divided into four groups, equity REITs, mortgage REITs, mutual fund REITs and hybrid REITs. This study will only focus on reviewing equity REITs. Certified equity REITs need minimum 75% assets as direct investment on property. The National Association of Real Estate Investment Trust (NAREIT) categorized equity REITs according to their assets' types: apartment, office, retails

including shopping centers and local malls, logistics consisting storage centers, warehouse, industrial parks, lodging properties such as hotels, health care facilities and natural resources like farm. Nowadays, REIT's market is dominated by equity REITs even though during the 70s, it was mortgage REITs that dominated the market. (Goddard, et.al, 2012, p.254).

## **2.2 The US REIT's market**

This section takes a look at how the US REIT's market evolved in the past (end of 90s to after the 2008 financial crisis) as the US's economy as well as interest rate changes. REITs have been through many milestones of the US economy. After the slowdown in the 90s, when the blooming technology pushed hard-asset industry to a downshift, REITs market had stayed modest through the stock-bubble in year 2000 as well as the September 11 attack. After these shocks, the economy requires a reset with a low interest rate. Along with that, investors, after experiencing the highly risky virtual asset market, turn their reference back to hard-asset and created a favorable environment for real estate industry in general and REITs in particular to grow. Private equity funds sought to arbitrage the asset value of REITs against their stock prices by buying REITs and liquidating their assets. This development led to a flurry of consolidation as REITs were liquidated into a private real estate market fueled by the availability of high leverage and loose credit standards. (Glickman A., et.al. 2014, p.393). The cheap credit, overvalued assets and high leverage during this period are the main drivers for REIT's failure when the financial crisis hit the market later in 2008. (The New York Times, 2008)

This robust expansion of REITs was interrupted by financial crisis with huge losses of average 42.16% as the investment flowing into real estate industry inclining. Assets lost their value, income from properties dropped and leverage levels were thus dragged up. Prior to 2008, REITs used to be a good portfolio diversification and outperformed stocks for couple of consecutive years but got hit harder by the recession. (The New York Times, 2008). This phenomenon is analyzed in Basse's (2009) paper, which found that REITs, even though behave similarly to stocks, become more risky than stocks during crisis. In order to keep leverage rate at level and to deal with financial distresses from maturing liabilities, REITs had to aggregate funding through equity, which is more expensive than loans. Many of them were also actively buying distressed properties as a mean to stabilize real estate market and keep the attractiveness of securities. (Glickman A., et.al. 2014, p.393).

In an interview for NAREIT (2015), professor Wachter, University of Pennsylvania pointed out that REITs having the least loss from the recession are the ones who adjusted their debt and lowered their leverage level in advance. This supports what found in Pavlov's study one year later that REITs who extended their loan maturity before the crisis outperformed the others during financial crisis. The more conservative REITs were well rewarded by the investors while ones with riskier capital structure suffered. (NAREIT 2015). This is one of the reasons explaining why after the crisis, debt proportion in REIT's capital structure shrunk to the lowest level in 20 years. Both debt-to-market capitalization and debt-to-total book-assets ratios of Equity REIT dropped to a range of 40% to 50%, approximately 10% reduce since post-crisis period. On top of that, many REITs prolonged the maturity of their debts, allowing longer-term loan in their portfolio. This also happened as a result of the blooming capital market, which allowed REITs to raise a large sum of capital from equity. (NAREIT 2017)

## **2.3 Interest Rate and Interest Rate proxies**

### **2.3.1 Interest Rate**

Interest rate in its nutshell is the price of borrowing or lending money. However, there are a lot more of a price than what interest rate is presenting. Interest rate is one of the indicators of demand vs supply of funding as well as saving vs consumption in the economy. It fluctuates as the demand of loanable money moves away from its intersection with the supply. Interest rate also reflects the economy's activity level and acts as a tool for economic policy making. (Naghshpour, S. 2013, p.57)

There are various types of interest rate as a result of many forms of loans and borrowers. These rates indicate different tenors ranging from overnight to decades. They also vary depending on the creditworthiness of the borrowers and the collateral securing the loan. On a broader level, the economy's well-being also has its impacts on interest rate via inflation and vice versa. When inflation rate increases, money power decrease resulting in a rise of all prices, not excluding money's price. Because of this, inflation rate is included inside interest rate as a compensation for lenders. The interest rate observed from the market already taken inflation into account and is called the nominal interest rate. Real interest rate equals nominal interest rate divided by inflation rate. (Stengel, DN 2014, p.70)

### **2.3.2 Interest Rate proxies**

Like most of the countries, in the US, the most important interest rate benchmarks are coming from the State's and other major financial entities' borrowing and lending activities. They are Federal Funds Rate, the Prime Rate, Mortgage rate, some Bonds Rate and Treasury Debt Rates. (Stengel, DN 2014, p.70)

The Treasury Debt rate is the general term for the interest rate the US government pays for their debt. The US Department of Treasury borrow capital on behalf of the US government by issuing a number of financial instruments. They are Treasury bills, Treasury notes, Treasury bonds, Treasury Inflation-Protected Securities (TIPS) and saving bonds. All of them have fixed rate but with different terms. Treasury bills have the shortest tenor which ranges from couple days to 52 weeks. Treasury notes have slightly longer maturity of 2 to 10 years and Treasury bonds have the longest terms of up to 30 years. TIPS are medium-long term instruments where interest are adjusted according to the US consumer price index (CPI). Saving bonds can provide interest at either CPI adjusted or fixed interest rate. The rate of these instruments is set after auction before selling for the first time in primary market. However, the key indicator is the yield, or the effective interest earned, of these instruments when traded in the secondary market. These rates are generally close to the Fed fund rate. (Stengel, DN 2014, p.73)

Majority of the above mentioned Treasury products are embedded in another proxy for interest rate, the US Treasury interpolated yield curve rates. The official name of this indicator is Constant Maturity Treasury or CMTs. This rate is available in 12 terms: 1-month, 2-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year rates. The 2-month CMT is recently added in 2018. The rates available for above mention terms are calculated from the daily Treasury yield curve, which

derived from the closing bid yield of different securities offered over-the-counter by the US Treasury in secondary market. CMT represents the yield of all bond-equivalent instruments that pays out dividend twice a year. The rate is simply annualized but is not effectively annualized or in form of Annualize Percentage Yield (APY). CMT only reflects the current and past bond market as well as economic activities. It also replicates the market volatility and investor's perspectives to the trend of future interest rates. The idea behind constant maturity is that even if there are no securities offered with that exact term, the curve still provides a yield for that particular maturity. This made it a convenient indicator to look up as it can provide investment benchmarks with just a single indicator for a much wider range of structure term than the rate obtained from single specific instruments. For example, the T-bill would only suitable to compare against short-term investment, while the Treasury bonds are more reliable for long-term ones. CMTs acts as an all-in-one benchmark of risk-free interest rate. (U.S Department of Treasury, 2018)

## **2.4 REITs sensitivity on interest rate**

### **2.4.1 Some characteristics of REIT's interest rate sensitivity**

Studies done on the relationship between REITs and interest rate have been fruitful. All these studies acknowledge and confirm that the movements in interest rate do have effects on REITs. However, many showed that this relationship is not always consistent. Chan (1990) found changes in unexpected inflation, the risk and structure of interest rate has great impact on REITs consistently through the two sub-periods in the 70s and 80s. Likewise, Chen's work in 1988 described the sensitivity level of different REITs on interest rate to be varied over time. Specifically, only long-term interest rate has explanatory power on REITs during period of 1973 – 1979. However, years from 1980 to 1985, REITs were found to be sensitive to both long and short-term interest rate. More recent literature like one done by Ling T. He (2003) also confirmed the time-specific characteristic of the effect of interest on REITs found in Chen's work. The paper used different interest rate indicators and analyze their influences on REITs and noted out how this relationship changes over time. The interest rate risk sensitivity existed only between 01/1975 and 06/1984 for equity REITs.

Most research found the correlation between REIT's return and interest rate to be slightly negative, but it is also inconsistent through time, meaning the correlation occurring in one time period can be stronger than in others. In fact, Muller's study in 1995 showed a less powerful negative correlation during rising interest rate period comparing to what happened when interest rates were decreasing. This is consistent with what found in McCue's 1994 paper where in case of shocks, nominal rate has a huge adverse influence on the REIT's return. This makes sense because even though interest rate spikes in the shocks, it is quickly interfered by the government and stays at a low level for the whole recession period. Thus, we hardly see an increasing but rather a declining interest rate during a recession. (Depersio 2018).

Results of these studies also depend greatly on what indicators used for REITs and interest rate as well as the models fitting them. In term of data, for the US market, most of the data used on above mentioned studies are before the 2008 crisis. The more recent studies such as Weis (2017) and EPRA (2015) has newer data from the US but they studied on REITs worldwide and include a lot more data from different countries. The choices of indicators for REITs are not hugely varied, it is either looking at individual

stocks or indices. For example, Chan's work in 1990 used both equally-weighted and value-weighted return of REITs during period of 1973-1987. McCue (1994) used monthly NAREIT index or value-weighted return of REITs collected by the National Association of Real Estate Investment Trust.

Interest rate proxy is where it gets prolific. Chen (1988) used 20-year US government bonds, three-month, six-month and one-year T-bill. On top of that, the paper also included the expected real interest rate, which is the average of difference between the most recent 12-month collected interest rate and processed CPI. Chan (1994) took the one-month T-bill along with 5 other macroeconomics factors into his model and discovered that a rise in long-term interest rate has negative impact on both common stocks and REITs. McCue (1994) studies on the effects of macro-economy on equity REITs data showed that short-term nominal interest rate explains majority of changes in REIT's series. T-bill and government bond rate seem to be the most popular indicators. As a long-term investment, real estate seems to have higher correlation with longer-term rates but it does not make a considerable difference in correlation with short or medium interest rates (Muller 1995). In the same paper, Muller claimed that conservatively leveraged REITs with long-term fixed rate loan enjoyed the highest growth while ones with short-term suffered from falling operating profit. Also, because REITs can use long-term loan to hedge against their fixed long-term lease (Chan, 1990), it can be intuitive to take long-term interest rate as the interest rate proxy for the test. However, the result in Chan (1990) study did not suggest the use of long-term debt in balancing out long-term lease. In addition to the risk-free rates, many authors also look at other indicators with higher risk levels like corporate bonds, high-grade bonds, and high-yield bonds. Ling T. He (2003) claimed that high-yield bond is the most effective indicator to explain the changes of return on both equity and mortgage REITs over period of 1972 - 1998.

Another factor affecting the sensitivity is debt structure of REITs. Research done by EPRA (2015) showed an increase in interest rate sensitivity REITs have with bigger proportion of short-term debt in their capital structure. It also pointed out REIT's interest rate sensitivity increases as the return of REITs decreasing or getting riskier.

## **2.4.2 Channels of sensitivity**

It boils down to 3 main channels through which REIT's take the impact from interest rate. These were all touched on by Weis (2017) while other researchers often pick and choose the ones they need to base their models on. The first one coming from the financial market forces (supply and demand). REITs, when considered as either a stock-like or bond-like instrument, are influenced by interest rate in the similar way when interest rate affects the market. Interest rate can act as a performance benchmark that pressures the yields of other financial investments to move accordingly, or it can be a component of discount rate that changes the present value of the financial products. For the stock-like case, interest rate's fluctuation changes the attractiveness of different equities and thus their returns. This was Chan's stand point when taking equity REITs and compare their behaviors on interest rate changes with common stock indexes. Chen (1988) regarded REITs as a type of high-yield stocks when explaining their decreasing return on a rising interest rate. Like stocks with high dividends, high interest rate reduces the present value of the future pay-out which lowering the return.

REIT is also often compared to bond due to its steady dividend streams. This is explained more clearly in EPRA's study in 2015 using the concept of equity duration, a measure of bond or equity price sensitivity to interest rate changes. It calculates how much time it takes for the accumulated future cash flows (can be dividend in case of equities or coupon in case of bonds) to pay off the initial investment by discounting weighted future payouts. Interest rate is one of the factors that triggers changes in the discount rate. The higher duration the instrument has, the more likely its value fluctuates with interest rate changes. Muller (1995) also browsed on this channel and pointed out the bond-like characteristic cash flow REITs have make them react to interest rate changes like bonds do. In case of bonds, because coupon amount stays constant once the bond is issued, fluctuations of interest rate would alter the attractiveness of the bonds which then forcing bond price to change according to the market's expected coupon rate. Therefore, a rise in interest rate pushes the existing bond price as well as capital gain from bonds down and vice versa. Since REIT's dividend stream is highly stable, it replicates this effect interest rate has on bonds.

The second channel is inherited from the real estate industry, where interest rate directly affects the prices of real properties (Weis, 2017). Muller (1995) claims rising interest rate has minor effect on real estate. However, McCue (1987) looked into this channel under the perspective of real estate development and claimed that nominal interest rate is key in forecasting the return of real estate. This is due to the nature of real estate as a long-term investment that requires high commitment from the start and is hard to reverse in case of unfavorable environment. The process of appraising, anticipating and planning output of a real estate project takes place heavily at the beginning. Discounted Cash Flow is a popular method used in real estate valuation where the value of the property is the sum of the present value of future cash flows and terminal value of the asset. Real estate is a long-term investment with long business cycle and in order to predict the cash flows coming from the far future, the discount rate used to discount these future incomes is commonly referenced from the long-term interest rate. (Glickman A., et.al, 2014, p.139)

The last way interest rate influencing REITs is through the firms' operating activities, i.e. costs of loans. This is well explored by most studies. Generally, the more leveraged the firm is, the greater effect it takes from interest rate's movement. This was proven to be true for Wilshire Index as well as NAREITs Equity index in Muller's study (1995). The excessive level of leverage for high-risk constructions along with high dependence on short-term loans, which often uses floating rates, of many REITs was the explanation for their failure in 1973 and 1974). In the same paper, Muller (1995) also indicated that unleveraged real estate is quite insensitive to interest rate. However, on the sector level, expected increase in cost for debt should be compensated by the rise of operating income from rental activities (Muller, 1995). In Chan (1990) paper, equity REITs chosen were considerably levered with more than 60% of debt to asset ratio and testing the sensitivity level of high and moderate level of leverage. The highly-levered REITs had stronger correlation with interest rate's risk and term structure. However, EPRA research group concluded an interesting result in 2015. The leverage level does not make REITs become more sensitive to interest rate but the debt structure does. The combination of floating-rate and short-term loan is the formula for the highest REIT's vulnerability on interest rate changes.

## **2.5 Hypothesis Development**

The hypotheses in this study is conducted base on the findings repeatedly showed from the earlier studies on REIT's return and interest rate relationship. First of all, since majority of earlier studies found a direct causation between REIT's return and interest rate, but none has explored if Granger Causality also exist. This paper is interested in finding one-way indirect effect from interest rate to REIT, in other words, to answer if including interest rate series and its past data helps predicting the future REIT's time series.

Second, as many studies found this relationship to be unstable and time-specific, we expect Granger Causality of interest rate on REIT will have inconsistency through time. In other words, it can show up in one period but not necessary in the others.

Thirdly, the negative correlation between REIT and interest rate is found to be stronger during interest rate shocks or falling movement comparing to rising interest rate period. This could also be the case for indirect relationship that Granger Causality of interest rate on REIT can appear more dominantly during decreasing interest rate environment or shocks (crisis sub-period) than in the more stable time with upward trend.

Next, given that REITs are exposed heavily to long-term interest rate in all channels of sensitivity, longer maturity interest rates are more likely to "Granger" causes REIT's return.

Last but not least, due to the changes in debt structure and capital strategy between pre and post crisis that REITs have, from higher leverage with a higher proportion of short-term floating debt to lower leverage with more shares of long-term fixed-rate loan, the result should indicate this shift. For example, short-term interest rate might "Granger" causes REITs for the pre-crisis and crisis sub-period but no longer have any or having less of an effect on REIT for the post-crisis time. If however the result does not imply this change, it will support the argument made by Muller (1995) that when looking at REITs as a whole sector, expected changes in interest rate are offset by adjustment in operating income from existing rental activities, thus will not make an impact on REIT's return. This scenario is more likely to happen because the data collected for this study is Equity REIT index, which represents the whole sector instead of looking at individual companies. Even for the scope of individual REIT, if the organizations are able to anticipate the movement of interest rate and promptly react to it, they do not get affected through operational activity channel of sensitivity.

This thesis's main purpose is not to test on the prominence of each channel of sensitivity given the complexity among them. There is no fine line separating the effects of these channels and they overlap each other at certain points. For instance, the way interest rate affecting REITs as a stock-like financial products is similar to how it moves REITs as real estate investment. In both cases, interest rate acts as the discount rate, which is used in discounting stocks' future dividend as well as real estate's future cash flows. With just observing interest rate and the movement of REIT's return, it is hard to identify which channels of sensitivity are showing its effects. For this reason, it requires a more complex methodology that involves different variables, such as leverage rate, debt structure, etc., to test which channels of sensitivity are more prominent.

## 3 Data and Methodology

### 3.1 Data

#### 3.1.1 Interest Rate

The interest rate for testing against REIT's price will be the risk-free rate. Even though, Ling T. He (2003) claimed that the high-yield bond rates, which includes a certain risk levels on top of the risk-free rate, is the most effective proxy for explaining REITs movement, their study looked at a more micro-level of different individual REITs. Higher-risk rates worked in the sense that it reflects the company's specific risk of REITs. This paper will take a look at a boarder level by taking more general and standardized proxies of both interest rate and REITs.

The proxy for risk-free interest rate will be the US CMTs. The reason is that the curve includes information of all of the most actively-traded securities provided by the US Treasury. Earlier researches mostly used single-instrument yield maturities either the US T-bill or the US Treasury bond but neglecting other risk-free instruments also sold by the US government. Using the CMTs will give a boarder and less bias view of the risk-free interest rate in the US. It will also cover a wider selection of maturities, which makes data collecting much more convenient.

Another reason for using CMTs is that it does not contain negative yields. Negative returns can be found in multiple securities of the US Treasury especially in this exceptionally low borrowing rates. However, these are due to technical factors of the Treasury market rather than reflecting the time value of money. As a result, having rates which are below-zero in the series would not be accurate. CMTs is more superior to the single security's rate by setting the negative rate to zero. (U.S Department of Treasury 2018).

Thanks to its conveniences, CMT is widely used as the index to set the regulatory and credit programs as well as securities. (U.S Department of Treasury 2018). Considering CMT's popularity in real estate as well as its advantages over the single securities indices, it is surprising to see few to no earlier researches on REITs and interest rate have taken all CMT maturities into consideration. By using the whole set of CMT maturities as the interest proxy, this thesis will fill in that missing gap.

Daily CMTs with all available terms, i.e. 1-month (1MO), 3-month (3MO), 6-month (6MO), 1-year (1YR), 2-year (2YR), 3-year (3YR), 5-year (5YR), 7-year (7YR), 10-year (10YR), 20-year (20YR) and 30-year (30YR) are collected from Quandl database. (Quandl, 2018). It is worth mentioning that the 30YR CMT rate was discontinued between February 18, 2002 and February 9, 2006. (U.S Department of Treasury, 2018).

Figure 1 shows the movement of all 11 CMT maturities from 01/01/2001 - 31/12/2016. The spread between shorter-term and longer-term maturity represent the maturity premium between them. The longer the term is, the more risky the loan is. Therefore, longer-term contract requires higher interest rate to compensate for the higher risk it has. This spread has been relatively constant during the stable economy, which keep the curves moving in tandem. However, during the 2008 financial crisis, this spread disappeared, the cost of money for short-term borrowing is as high as that for long-term loan. This happened when credit gets tight, the demand for borrowing is there but the



money supply drops drastically. Shortly after that, the FED adjusted the short-term interest rate and keep it at low level, which brings back the spread between long and short-term rate. This is a strategy to allow more cash lent out and thus stimulate economy's growth and recovery. Therefore, during the crisis sub-period, the CMTs, especially the short-term ones, has a big downward trend while during the other two sub-periods, they are rather stable and follow an upward trend.

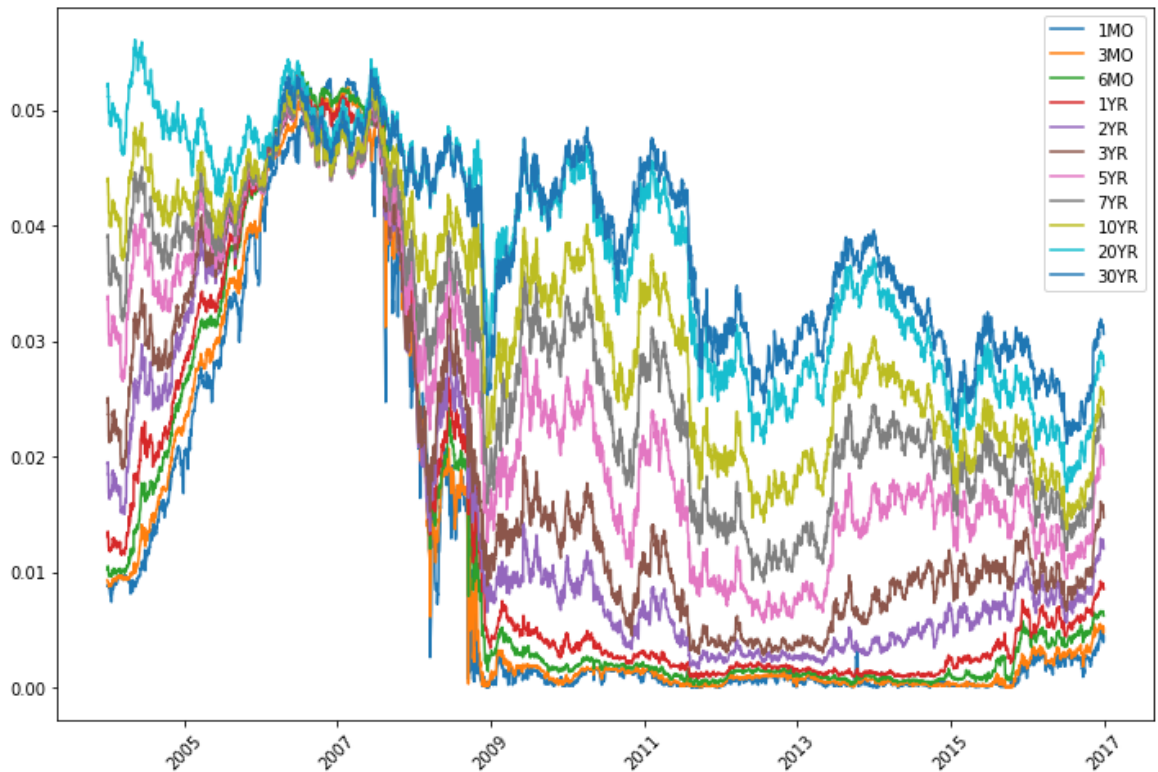


Figure 1: 11 CMT maturities (Source: Author's Calculation)

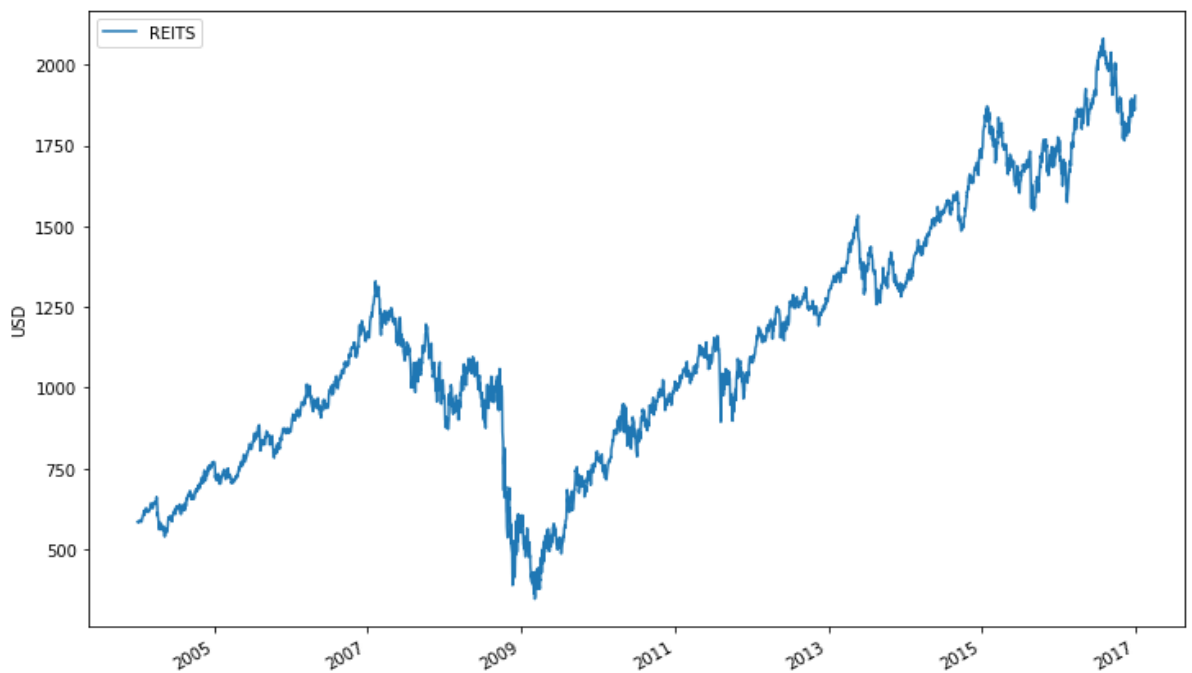


Figure 2: MSCI REIT index (Source: Author's Calculation)

### 3.1.2 REITs data

REITs are indicated by the daily MSCI REIT (RMS) index collected from SNL database. The collected data is from 01/01/2001 to 31/12/2016. The index is the weighted capitalization of US REITs Equity which is calculated by MSCI Global Investable Index (GIMI) methodology. (MSCI, 2016). Eligible stocks must be qualified as REITs according to the Internal Revenue Code, i.e. they must pay out at minimum 90% of taxable income to their shareholders and get at least 75% income from real estate – related activities. These REITs must have REIT tax status and is based on MSCI USA Investable Market Index (IMI). MSCI REIT companies are belong to the Equity Real Estate Investment Trust classification under the Global Industry Classification Standard (GICS). These do not include Mortgage REITs or Real Estate Operating companies. (MSCI, 2016).

The index is reviewed and reassessed every four months and on related-events occurring such as redemption, corporate events or eligibility changes to make sure it correctly represents the US REITs. This process includes simultaneously adding new eligible and removing illegible shares. (MSCI, 2016).

Figure 2 shows the MSCI REIT index's movement during this study's tested time frame. REIT as a sector has been evolving with an upward trend through 16 years. It has been increasing in size since the early 2000s before having a drop during the financial crisis but then has been quickly back on track for the next 7-year period.

The data selection is inspired by the choice of proxy in Muller's 1995 study, which used REIT index (NAREIT). The reason being that most of the existing literature already saturated with using data from individual REIT where the paper done by Muller (1995) is the only study that utilized an index. Even though looking at individual REIT allows more accurate return rates that account for both capital and dividend return, it does not provide the big picture of REIT as a whole sector. Especially when one of the objective of this thesis is to examine whether expected changes in interest rate affect the return of REIT, as a whole sector, through operating level (Muller's theory, 1995). With that being said, the return calculated from MSCI Equity REIT (RMS) index is total return index (TRI), which mean it will reflect REIT's income from both capital gain/loss and dividend.

## 3.2 Methodology

The testing strategy in this thesis is to pair-by-pair find out the Granger-causality relationship between REITs and each of CMT maturity using the Toda-Yamamoto procedure. The results from those single tests are then compared with each other to see if different CMT maturities have different explanatory power over REIT's return. This approach is easier to implement and to be interpreted than including all 11 maturities into one model. On top of that, by regressing REIT on each CMT maturity, we eliminate the possibility of autocorrelation among the maturities which is highly prone to occur due to how CMT is calculated.

The data is divided into periods: from 01/01/2001 until before the 2008 financial crisis (31/01/2007), during financial crisis (01/08/2007 – 31/07/2009) and after the crisis (01/08/2009 – 01/01/2016). Data slicing is based around the shock in 2008-2009, when

there are drastically shift in both REITs and interest rate data. The Crisis is defined by the two milestones that happened in the US. The first one is at the beginning of July 2007 when HSBC revealed its losses from its subprime mortgage in the US. This event started the subprime mortgages meltdown in the States' real estate market, which then led to a series of bank's failures. The crisis marked its end in the US when Barack Obama, US President at the time, reported new improvement in the country's economy performance on the last day of July 2009. The US Federal Reserve several days later announced the worst period of the Crisis has passed for the US economy when consumer spending creeping up starting August 2009. (Guillen, n.d.)

The reason behind slicing data into periods is to examine the changes of Granger relationship between REITs and interest rate during different time frames. This is essential to examine the inconsistency and time-specific characteristics of the relationship between REITs and interest rate suggested by many earlier researches.

### 3.2.1 Toda and Yamamoto (1995) procedure

Granger Causality Test is one of the methods in interpreting the Vector Autoregression (VAR) model developed by Sims (1980). The model displays the dynamic relationship among time series. Granger Causality answers if adding one time series into another's forecast model would improve the model's accuracy than just using the other's own past series. Improved accuracy of the model is measured by the smaller prediction error when adding the past value of the extra series. The coefficients are estimated with the simple Vector Autoregression (VAR) between two variables, one is endogenous or dependent and the other is exogenous or independent and vice versa. These coefficients are then tested for significance with Wald test in which the null hypothesis is all the coefficients of added series and its lags is equal to zero, in other words, the added series' past value has no explanatory power over the endogenous series. Rejecting the null means the exogenous variable Granger causes the endogenous one. However, the model requires the tested series to be non-stationary and non-cointegrated to each other as when these conditions not met, the F-test statistic do not follow standard distribution and thus the result is not valid. (Gujarati 1995). In order to fulfill these requirements, users must transform their data and test for the series' cointegration. The process of data transforming is often done by differencing the series. This then requires an extra step of re-transform the data later on to interpret the result. (Granger 1987). This is the biggest disadvantage of the ordinary Granger Causality Test as most time series are non-stationary at level.

Toda and Yamamoto (1995) is a more superior procedure of the traditional Granger Causality Test. It overcomes the strict requirements the ordinary model has on the data. Toda and Yamamoto model allows to pass in non-stationary and cointegrated time series. The differences lie in the extra lags of both endogenous and exogenous variables included in the VAR. This number of extra lags is determined as the maximum order of integration of imputed series. However, when testing for Wald test on the exogenous series, the extra lags are excluded out of the null hypothesis. This is called the Modified Wald Test and it is used to ensure the asymptotic distribution of the Wald statistic. Thus, the test result is valid even with non-stationary or cointegrated data. The procedure can be summarized into the following steps: (Toda and Yamamoto, 1995).

Step number one is to determine the maximum order of integration ( $d_{max}$ ) of series fed into the model. This can be done on each series based on the combination of Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (Kwiatkowski, 1992) and the Dickey-

Fuller (DF) test. The highest result found in each pair of time series with these criteria is then picked for the next steps. (Toda, 1995).

Step number two is to measure the optimal number of lag ( $op\_lag$ ) when fitting the data into VAR. The VAR model here uses the level instead of transformed data. The initial optimal lag length can be derived with multiple methods. The most popular are Akaike Information Criterion (AIC) (Akaike, 1981), Schwarz Information Criterion (SIC) or Hannan-Quinn (HQC). The data is fit again in VAR with the initial optimal lag length. After that, the model's residual is tested for serial correlation. The optimal order length is number of lags at which result in insignificant residual serial correlation. (Toda, 1995).

Step number three is create VAR with lag number ( $lag$ ) equal sum of  $dmax$  and  $op\_lag$ . The first  $op\_lag$ -th lag of exogenous series' coefficient are then tested for statistical significance with Wald test with null hypothesis of all of them are equal to zero. The p-value ( $pvalue$ ) should be less than 0.05 (5%) in order to reject the null or to acknowledge the one-way Granger causality. (Toda, 1995).

The last step is to double check if the result found in the third step conflict with a cointegration test between series using Johansen's methodology (Johansen, 1991). If two series are cointegrated, there is either two-way or one-way Granger causality between them. However, if there is Granger causality exist, the cointegration is not necessarily true. (Toda, 1995).

### 3.2.2 Stationarity and Order of Integration

There are three conditions for a time series, or a stochastic process with time index, to be weakly stationary. The first one is the expectation of the process is a constant, which is not a function of time. The second assumption is that the variance of the series must also be a constant. The last requirement is that the covariance the data point in the time series depends only on their lag but is not a function of actual time. This means the further two data points in the series are apart, the closer to zero their covariance is. Generally, mean, variance and covariance of stationary series do not change with time. (Gujariti, 2004, p.797)

The intuition behind stationary requirement for time series is to gain stable coefficients, or relationship among series through time. (Gujariti, 2004, p.798). It is also required to eliminate the chance of getting spurious regression. Spurious regression happens when two totally independent series appear to have a statistically significant correlation to each other with high R-square. (Gujariti, 2004, p.806)

Testing stationarity of series should be done with both graphical and numerical analysis. The stationarity of series in question can be diagnosed with its plot. If the plot looks like random noise where the line always returns to mean without substantial positive or negative run. On contrary, non-stationary series graphs to long run without returning to mean. (Gujariti, 2004, p.797).

Unit root tests are useful for testing stationarity. One of them is the Dickey-Fuller (DF) test (Dickey, 1979), which is testing for null hypothesis of the time series is non-stationary or the series has a unit root. The test can be applied for both random walk with (with trend or  $\alpha \neq 0$ ) and without drift (no trend or  $\alpha = 0$ ). With  $t$  being time, the DF test on null hypothesis of  $\rho = 1$ , and the alternative hypothesis is  $\rho < 1$  by

calculating an ordinary t-statistic on  $(\rho - 1)$  and compare it to the critical values of DF distribution. We can reject the null hypothesis if the t-statistic is less than the critical values of DF distribution. (Gujariti, 2004, p.815).

$$X_t = \alpha + \rho X_{t-1} + \varepsilon$$

Or after taking out  $X_{t-1}$  in both side:

$$\Delta X_t = \alpha + (\rho - 1)X_{t-1} + \varepsilon$$

Another stationary test is the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, which is often used to complement the unit roots test by having the opposite hypothesis setting than the DF test. The null here is the time series is trend-stationary, meaning the series is stationary around a deterministic trend. A series without unit root is trend-stationary where the time has convergence impact on the mean. The idea behind KPSS test is that a series can be seen as the sum of the deterministic trend, random walk and a stationary process. It tests if the coefficient of the random walk is zero, otherwise the series is integrated of order one. The null is rejected for test statistic with large value than KPSS critical values. (Neusser K., 2016, p.157).

Order of integration of a series is the number of times the process has to differentiate to reach stationarity. Level data, or stationary series without the need of differentiate has order of integration of zero. The process of finding order of integration of time series starts with checking if the level series is stationary. If not, we take the first difference of the series  $(X_t - X_{t-1})$  and then test for stationarity on the first-difference series. This process repeats until the kth-difference series is stationary. The series is then having k order of integration and is denoted as I(k). (Neusser K., 2016, p.134).

In this paper, order of integration of each series are determined by cross-checking between DF and KPSS test to deliver the order of integration (ooi) of each series. Each of the interest maturity's ooi are then compared to that of REITs. Whichever is higher will be dmax.

### 3.2.3 Optimal Lag Length and Residual Serial Correlation

The number of regressor or the optimal lag length in the case of this model is determined by two information criteria. The basic notion of model selection criterion derives from the fact that by continuously adding parameters into model, data will always fit a bit better. However, this also trades off against over fitting which result in losing information of the real patent. So, it is the balance between the number of added lags and the incremental amount of error by doing so. Akaike Information Criterion (AIC) is the most widely applied approach, along with Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQC) (Hannan, 1979), to decide model fit that named after the famous statistician Hirotugu Akaike. The formula of AIC is below for  $k$  as the number of parameter and  $n$  is number of observations. Its idea is to create a penalty for expanding the model which is harsher than that of R-Square. Adding more parameters will lower RSS but at the same time increases  $k$ , which is the cost of expanding the model. The goal is achieving the as lowest AIC as possible. (Gujariti, 2004, p.537).

$$AIC = e^{2k/n} \frac{RSS}{n}$$

However, out of the most used methods, AIC is the most forgiving for larger model, which tends to cause overfitting. Therefore, it makes sense to accompany it with a stricter criterion. The reason of choosing HQC is that the method has a more asymptotically consistent result than AIC does because it trades off the efficiency by at minimum  $\ln \ln n$  factor to follow the law of iterated logarithm. The formula of HQC is follow for  $k$  as the number of parameter and  $n$  is number of observation (Neusser. K, 2016, p.101):

$$HQC = \ln \sigma^2 + (k) \frac{2 \ln(n)}{n}$$

Serial correlation or autocorrelation in errors happens when there are some common relations among residual within the population. In other words, covariance of errors is not zero and there are other estimators resulting in lower variance among residuals. (Gujariti, 2004, p.203). No serial correlation is one of the Gauss-Markov assumptions of regression model on the least-square estimators. For the model to be specified correctly, the least-squares estimators need to be BLUE, i.e. to be linear, unbiased and to have the minimum variance. (Gujariti, 2004, p.79). Serial correlation of error can happen when an important variable is omitted in the model, functional miss-specification or measurement errors in independent variables.

Testing for serial correlation in error terms makes sure the model's least-squares estimator is BLUE or there is no other model would explain the data better. Ljung-Box Test is a commonly used test in statistic to determine if residual's autocorrelation exists in the model (G. M. Ljung, 1978). The test calculates the Q-statistic that includes the weighted sum of the squared autocorrelation ( $\rho^2$ ) from lag 1 to  $k$ . In order to determine the statistically significant of Q, it is compared to a chi-square distribution at  $k$  degree of freedom. The null hypothesis is no serial correlation in the model, hence small p-values means significant autocorrelation in the time series. Formula of Q-statistics is written as: (Gujariti, 2004, p.813).

$$Q = n(n+2) \sum_{i=1}^k \frac{\rho^2}{n-k}$$

In this paper, raw data is fitted into VAR that allows for trial maximum lags up to 100 lags because it is a large number of daily data point. Both AIC and HQC are considered but result from the stricter (HQC) criterion, which allows including fewer lags, is selected for VAR to test for autocorrelation. The number of lag (`op_lag`) will then increases until the result from Ljung-box test shows no serial correlation among the model's residuals.

### 3.2.4 VAR, Modified Wald Test and Granger-Causality

Vector Autoregressive (VAR) is fundamentally multivariate linear time model designed to capture the joined dynamics of multiple time series. It treats each endogenous variable as functions of lagged or passed values of all endogenous variables. (Gujariti, 2004, p.848).

In a simple case of two economic variables  $X_t$  and  $Y_t$ , VAR with  $k$  order is estimated in the following general formulas (Kirchgssner, G. 2014, p.):

$$\begin{aligned} X_t &= \alpha + \sum_{i=1}^k \beta_i X_{t-i} + \sum_{i=0}^k \gamma_i Y_{t-i} + u_t \\ Y_t &= \alpha + \sum_{i=1}^k \theta_i Y_{t-i} + \sum_{i=0}^k \delta_i X_{t-i} + u_t \end{aligned}$$

Granger Causality is one of the approaches in interpreting VAR processes.  $X_t$  is said to have Granger Causality on  $Y_t$  when including  $X_t$  and its past as endogenous variables in  $Y_t$  model helps reduce mean-squared forecast error. In other words, it improves the forecast outcome for  $Y_t$  than just using  $Y_t$ 's past series itself. This concept is based on the fundamental of only the past has effect on the future but not the other way around. The method of detecting Granger Causality is rather straightforward. The relationship exists if R-Squared declines when adding  $X_t$  and its lags in addition to  $Y_t$ 's past series to the model. This is done by a simple Wald Test (F-test) on the null hypothesis of coefficients of  $X_t$  and its past series are all equal to zero, or  $X_t$  does not Granger cause  $Y_t$ . For example, for testing if  $Y_t$  Granger causes  $X_t$ , the null hypothesis of Wald test would be:

$$H_0: \gamma_i = 0 \text{ for } i = 0 \text{ to } k$$

At a certain significant level, if the test result is greater than the F critical value then the null can be rejected and the relationship exist. For cases of more than two variables, the process of Granger Causality test is no longer straightforward because the null hypothesis of Wald test can get complicated with grouping variables. (Neusser, K., 2016, p.257-258). Another important assumption for Granger Causality test to be valid is that all the series are stationary and not cointegrated to each other. There is no serial correlation existing among error terms of the model. It does not require to present the estimated coefficients of the VAR model given that F-test result is satisfied. (Gujariti, 2004, p.689). This along with the complication of implementing on more than 2 variable models are the biggest drawbacks of the conventional Granger-Causality test.

The T-Y procedure is more superior to the original Granger test in the sense that it does not require the series to be stationary or cointegrated, which makes testing opportunities for many processes possible because non-stationarity and cointegration in time series are very common. The main difference lies in the VAR model as well as the modified Wald test the T-Y method implements. Instead of including  $k$  lag, which is often estimated based on AIC, SIC, etc., T-Y adds extra  $dmax$  lag of both exogenous and endogenous variables to the model. VAR now would look like:

$$\begin{aligned} X_t &= \alpha + \sum_{i=1}^k \beta_i X_{t-i} + \sum_{i=1}^k \gamma_i Y_{t-i} + \sum_{i=1}^{dmax} \beta_i X_{t-i} + \sum_{i=1}^{dmax} \gamma_i Y_{t-i} + u_t \\ Y_t &= \alpha + \sum_{i=1}^k \theta_i Y_{t-i} + \sum_{i=1}^k \delta_i X_{t-i} + \sum_{i=1}^{dmax} \theta_i Y_{t-i} + \sum_{i=1}^{dmax} \delta_i X_{t-i} + u_t \end{aligned}$$

Instead of testing all of the estimated coefficients of the added variables, T-Y procedure only employs Wald T on the first  $k$  lags of exogenous variables. Their last  $dmax$  lags

are used to keep Wald test statistics asymptotically follow chi-square distribution with  $k$  degree of freedom (d.o.f). The null would be for  $X_t$  model:

$$H_0: \gamma_i = 0 \text{ for } i = 0 \text{ to } k$$

For the purpose of this thesis, we will only look at one-way Granger causality of interest rate (IR) on REIT's return but not the other way around. Therefore, the VAR model would be:

$$REIT_t = \alpha + \sum_{i=1}^k \beta_i REIT_{t-i} + \sum_{i=1}^k \gamma_i IR_{t-i} + \sum_{i=1}^{dmax} \beta_i REIT_{t-i} + \sum_{i=1}^{dmax} \gamma_i IR_{t-i} + u_t$$

With  $REIT$  is time series of the MSCI Index,  $IR$  are 11 CMT maturities singly added. So, in total there are 11 equations of REITs and each of the CMT time series.  $dmax$  is the maximum order of integration of REITs and each of CMT got from Step 1,  $k$  is the optimal lag-length of each equation determined in Step 2.

The Wald test is implemented on each model on hypothesis as follow:

$$H_0: IR_i = 0 \text{ for } i = 0 \text{ to } k$$

### 3.2.5 Cointegration

Two non-stationary series are cointegrated to each other when their linear combination is stationary. This happens when the linear combination of the two series cancel out their trend. The residual of the regression of one series to the other is stationary or  $I(0)$ . Cointegration of two-time series shows that there is a long-term equilibrium relationship between them. (Gujariti 2004, p.822).

Pre-testing for cointegration is a good way to check for spurious regression problem. There are a number of methods used for cointegration test. The principle is to examine the unit roots of the residual from the bivariate regression two series in questions. However, this only work best for a pair of processes. With more than two series, the choice of regressand and regressor can affect the result and not all of them would have cointegration relationship to each other. For this reason, Johansen method of testing is useful because it does not isolate any single variable.

According to T-Y procedure, the cointegration test result will not have influence on the procedure. It is used for cross-testing with the T-Y Granger test result. If the data found to be cointegrated, there must be Granger causality relationship between them, either one direction or both. However, Granger causality does not necessary means cointegration. (Toda and Yamamoto, 1995). Therefore, there is conflict if cointegration exist but no Granger causality in any direction is found. Since only one-direction Granger causality is tested in this model, if there cointegration without Granger Causality of interest rate on REITs, it is necessary to do an extra step of testing if the relationship exist in the other way, i.e. checking if REITs Granger causes interest rate. In case of the Granger relation is not exist in the other direction, the conclusion is there is a conflict in result and the validity of the model is in question.



## 4 Empirical Result

This part presents the results of each step of the T-Y procedure implemented on REIT index and each of the collected CMT maturities the entire period from 01/01/2004 to 31/12/2016 and its 3 sub-periods.

### 4.1 Step 1

The order of integration results is collected into Table 1, Table 2, Table 3 and Table 4 respectively for the entire time frame and its sub-periods. Each table show the order of integration ( $d$ ) of individual time series, i.e. REITs and 11 CMTs. The first row of each of the below result table is the order of integration of REITs compared to itself, that is why the first row will have  $d = d_{max}$ . From the second row,  $d_{max}$  is the maximum order of integration when comparing REIT's order of integration and that of each CMT maturity.

For example: for a specific period, if REITs process has order of integration equals 1, but it takes 2 times differencing for 6-month CMTs to become stationary, the max order of integration ( $d_{max}$ ) for the regression model of REITs and 6-month CMTs will be 2.

#### 4.1.1 The entire period (01/01/2004 – 31/12/2016)

All time processes of entire testing period have order of integration of 1, except for the 6-month CMT. This series required 2 times of difference process to be de-trended. Therefore, the maximum order of integration for the model consisting of REIT and the 6-month CMT is 2. The regression of REITs and the rest of the CMTs has  $d_{max}$  of 1.

Series	$d$	$d_{max}$
REITs	1	1
1MO	1	1
3MO	1	1
6MO	2	2
1YR	1	1
2YR	1	1
3YR	1	1
5YR	1	1
7YR	1	1
10YR	1	1
20YR	1	1
30YR	1	1

Table 1: Step 1 Entire Period (Source: Author's calculation)

#### 4.1.2 Pre-crisis (01/01/2001 – 31/01/2007)

For the period before the Financial Crisis, all of the time series are stationary after one-time difference. As a result, the max order of integration for each pair of REIT and CMT is 1.

<b>Series</b>	<b>d</b>	<b>dmax</b>
REITs	1	1
1MO	1	1
3MO	1	1
6MO	1	1
1YR	1	1
2YR	1	1
3YR	1	1
5YR	1	1
7YR	1	1
10YR	1	1
20YR	1	1
30YR	1	1

**Table 2: Step 1 Result SubPeriod: Pre-crisis (Source: Author's calculation)**

Trend analysis wise, most of the CMTs follow the same pattern of general upward trend that peaks around July 2006 and followed by a slight fall. The odd one is the 30-year maturity which peaked twice in April and June 2006. It is also shorter than the others as the 30-year CMT was only re-introduced earlier that year. Additionally, the longer term the CMTs are, the more fluctuated they are. REIT's returns on the other hand mirrored the changes in long-term IRs with a downward trend and hitting bottom during July 2007.

### 4.1.3 During crisis (01/02/2007 – 31/07/2009)

Table 3: Step 1 Result Sub Period: During crisis (Source: Author's calculation)

Series	d	dmax
REITs	1	1
1MO	1	1
3MO	1	1
6MO	1	1
1YR	1	1
2YR	1	1
3YR	1	1
5YR	1	1
7YR	1	1
10YR	1	1
20YR	1	1
30YR	1	1

The order of integration of the data during crisis is similar to the previous period with d and dmax of all processes are 1.

However, it also shows the drastic changes of REITs and IR during this period.

Comparing to the previous period, with approximately the same number of observation and length of data (2 years), the volatility in this time frame is dramatically larger than that during the previous 2 years. REITs was no longer mirroring CMTs. Instead, both REITs and all CMTs went stiffly downhill, corrected slightly during mid-year before hitting the lowest points by the last quarter of 2008. REITs plunged the second time 3 months later. After 2008, short-term term CMTs (1-year or less) remained the same low level while long-term maturities (10-year or longer) immediately spiked up. Mid-term IRs reflects the transitional movement between the short and long-term maturities.

#### 4.1.4 Post-crisis (01/08/2009 – 31/12/2016)

For the post-crisis years, the 3-month and the 1-year CMTs both have order of integration of 2. Their model with REITs will have the maximum order of integration of 2. The rest of time series is transformed to be stationary after 1 time differencing and the maximum order of integration of their models take value of 1.

Series	d	dmax
REITs	1	1
1MO	1	1
3MO	2	2
6MO	1	1
1YR	2	2
2YR	1	1
3YR	1	1
5YR	1	1
7YR	1	1
10YR	1	1
20YR	1	1
30YR	1	1

**Table 4: Step 1 Result Sub Period: Post-crisis (Source: Author's calculation)**

REITs index after crisis took a year and a half to recover to its level before crisis and stays stable for most of the time, except for a considerable dip in 2013-2014. The short-term IR's maturities are not comparable to REIT's when most of them has not reach their level before crisis given a drastic rise starting 2016. On contrary, the mid and long-term CMTs have been strongly fluctuated, quickly climbed half way back during the first year before have a similar dip but half to one year earlier than what REITs had. Generally, even though the trend is upward, interest rates were still kept at low levels.

## 4.2 Step 2

The optimal lag length for equations of REITs and each CMTs is listed in the following 4 tables. Each of them orderly summarizes the result of the entire examined time period (01/01/2004 – 31/12/2016) and the sub-periods: pre-crisis (0/0/2005 – 31/01/2007), during crisis (01/02/2007 – 31/07/2009) to post-crisis (01/08/2009 – 01/01/2016). For example, in Table 5. The first row after table headers represents the lag length result for REIT's regression model with 1-month CMTs as independent variable. AIC suggested that the model can include 50 lags while HQC only allows 13 lags of both REIT and 1-month CMT time series into the model as exogenous factors. The optimal number of lag (Op\_Lag) will be included in REIT's model is 17. This means that with lag-time equal 1 day (daily data), in the next step, REIT data will be regressed against its 17 lagged series, 1-month CMT's 17 lags and the 1-month CMT series.

Generally, lag length suggested by AIC results in the highest possible orders of series can be included in the model while ones produced from HQC are much more conservative. The final optimal lag length, from which there is no statistically significant serial correlation in the model, somewhat lies between AIC and HQC but closer to the HQC result for most of the cases.

Series	AIC	HQC	Op_Lag
REITs & 1MO	50	13	17
REITs & 3MO	49	31	31
REITs & 6MO	50	29	30
REITs & 1YR	50	21	21
REITs & 2YR	49	3	5
REITs & 3YR	49	4	5
REITs & 5YR	49	4	5
REITs & 7YR	49	4	5
REITs & 10YR	49	4	5
REITs & 20YR	49	1	5
REITs & 30YR	49	1	5

**Table 5: Step 2 Result Entire Period (Source: Author's calculation)**

Series	AIC	HQC	Op_Lag
REITs & 1MO	6	6	6
REITs & 3MO	11	6	7
REITs & 6MO	11	6	6
REITs & 1YR	6	1	2
REITs & 2YR	3	1	2
REITs & 3YR	2	1	2
REITs & 5YR	2	1	2
REITs & 7YR	3	1	2
REITs & 10YR	1	1	2
REITs & 20YR	2	1	2

REITs & 30YR	2	1	3
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**Table 6: Step 2 Result SubPeriod: Pre-crisis (Source: Author's calculation)**

Series	AIC	HQC	Op_Lag
REITs & 1MO	4	4	5
REITs & 3MO	4	4	6
REITs & 6MO	5	2	2
REITs & 1YR	2	2	2
REITs & 2YR	2	1	5
REITs & 3YR	2	1	6
REITs & 5YR	10	1	5
REITs & 7YR	4	1	3
REITs & 10YR	9	1	5
REITs & 20YR	2	1	3
REITs & 30YR	9	1	3

**Table 7: Step 2 Result SubPeriod: During-crisis (Source: Author's calculation)**

Series	AIC	HQC	Op_Lag
REITs & 1MO	9	4	4
REITs & 3MO	7	4	4
REITs & 6MO	6	1	1
REITs & 1YR	1	1	1
REITs & 2YR	3	1	1
REITs & 3YR	1	1	1
REITs & 5YR	1	1	1
REITs & 7YR	1	1	1
REITs & 10YR	1	1	1
REITs & 20YR	1	1	1
REITs & 30YR	1	1	1

**Table 8: Step 2 Result SubPeriod: Post-crisis (Source: Author's calculation)**

### 4.3 Step 3

This part presents not only the Granger causality test output but also summarizes the results from previous steps along with the size of the data in each period. Granger causality test result includes the null hypothesis of the modified Wald test, its p-values, statistics and degree of freedom.

The degree of freedom of each Wald test are equal to the optimal lag length (op\_lag) calculated from step 2. The actual number of lag fed into VAR are the sum of maximum order of integration (max\_ooi) and op\_lag. The actual lag number is in the below table at column number 5 (lag). The null hypothesis complies with the modified Wald test: excluding the last max\_ooi lags' coefficients out of the test to keep its statistic following asymptotic chi-square distribution so that its results stay valid. This indicates that the model has follow the T-Y procedure correctly. For example, the result for the model of REITs regressed against the 1-month CMTs (Table 9, first row after header) will be interpreted as follow:

no\_data\_point: each of the REIT and the 1-month CMTs process has 3248 data points

max\_ooi: the maximum order of integration is 1, this is result from step 1

lag: the actual lag length fed into the model, this number equals the maximum order of integration from step 1 plus the optimal lag length from step 2. The number of lags for this model is 18, meaning we include the first 18 lags of 1-month CMT and the first 18 lags of REITs as exogenous variables into the model.

Statistic: Modified Wald statistic is 103,674

pvalue: Modified Wald test's p-value is 1.846e-14 (0.000)

H0: The null hypothesis of Modified Wald test: the coefficients of the first 17 lags of the 1-month CMTs are 0. The 18th lag is not included in the Wald test as instructed in the T-Y model.

d.o.f: degree of freedom of the Modified Wald test

Table 9 represents the Granger Causality test result of the entire time frame (01/01/2001 – 31/12/2016). All of the models of REITs and 11 CMTs has p-values much smaller than 0.05. This means at 95% confidence interval, we can reject the null hypothesis of no Granger Causality. In other words, all the interest rate maturities in this period “Granger” cause REITs.

On to the sub-periods, Table 10 summarizes the result from the pre-crisis sub-period, which starts on 01/01/2001 and ends on 31/01/2007. Under 95% confidence level, there is no sign of Granger Causality found of interest rates on REITs. All 11 equations have p-value greater than 0.05. Under 10% confidence level, the only maturity would have Granger causes REITs is the 3-month CMTs at p-value of 0.07.

The result of the 2-year of crisis is reported in Table 11. The only two equations with which we can reject the null hypothesis of no Granger Causality is the one with the 20-year and the 30-year CMTs. These two have p-value of respectively of 0.034 and 0.038. REIT's model with the 7-year and the 10-years CMTs also have low p-value of 0.058 and 0.089 but we can only reject the null hypothesis with 90% confidence level.

Last but not least, Table 12 shows the result for post-crisis sub period (01/08/2009-31/12/2016). This 7-year sub-period indicates there is no Granger Causality relationship found for any of the CMTs on REITs with all p-values are greater than 0.05. This means interest rate for this sub-period does not contribute to better explanation of REIT time series.

IR	no_data_point	max_ooi	lag	statistic	pvalue	H0	d.o.f
1MO	3248	1	18	103.674	1.8468e-14	(L1.1MO.REITs=0),(L2.1MO.REITs=0),(L3.1MO.REITs=0),(L4.1MO.REITs=0),(L5.1MO.REITs=0),(L6.1MO.REITs=0),(L7.1MO.REITs=0),(L8.1MO.REITs=0),(L9.1MO.REITs=0),(L10.1MO.REITs=0),(L11.1MO.REITs=0),(L12.1MO.REITs=0),(L13.1MO.REITs=0),(L14.1MO.REITs=0),(L15.1MO.REITs=0),(L16.1MO.REITs=0),(L17.1MO.REITs=0)	17
3MO	3245	1	32	434.811	9.492e-73	(L1.3MO.REITs=0),(L2.3MO.REITs=0),(L3.3MO.REITs=0),(L4.3MO.REITs=0),(L5.3MO.REITs=0),(L6.3MO.REITs=0),(L7.3MO.REITs=0),(L8.3MO.REITs=0),(L9.3MO.REITs=0),(L10.3MO.REITs=0),(L11.3MO.REITs=0),(L12.3MO.REITs=0),(L13.3MO.REITs=0),(L14.3MO.REITs=0),(L15.3MO.REITs=0),(L16.3MO.REITs=0),(L17.3MO.REITs=0),(L18.3MO.REITs=0),(L19.3MO.REITs=0),(L20.3MO.REITs=0),(L21.3MO.REITs=0),(L22.3MO.REITs=0),(L23.3MO.REITs=0),(L24.3MO.REITs=0),(L25.3MO.REITs=0),(L26.3MO.REITs=0),(L27.3MO.REITs=0),(L28.3MO.REITs=0),(L29.3MO.REITs=0),(L30.3MO.REITs=0),(L31.3MO.REITs=0)	31
6MO	3248	2	32	364.657	3.644e-59	(L1.6MO.REITs=0),(L2.6MO.REITs=0),(L3.6MO.REITs=0),(L4.6MO.REITs=0),(L5.6MO.REITs=0),(L6.6MO.REITs=0),(L7.6MO.REITs=0),(L8.6MO.REITs=0),(L9.6MO.REITs=0),(L10.6MO.REITs=0),(L11.6MO.REITs=0),(L12.6MO.REITs=0),(L13.6MO.REITs=0),(L14.6MO.REITs=0),(L15.6MO.REITs=0),(L16.6MO.REITs=0),(L17.6MO.REITs=0),(L18.6MO.REITs=0),(L19.6MO.REITs=0),(L20.6MO.REITs=0),(L21.6MO.REITs=0),(L22.6MO.REITs=0),(L23.6MO.REITs=0),(L24.6MO.REITs=0),(L25.6MO.REITs=0),(L26.6MO.REITs=0),(L27.6MO.REITs=0),(L28.6MO.REITs=0),(L29.6MO.REITs=0),(L30.6MO.REITs=0)	30



1YR	3248	1	22	112.780	1.480e-14	(L1.1YR.REITs=0),(L2.1YR.REITs=0),(L3.1YR.REITs=0),(L4.1YR.REITs=0),(L5.1YR.REITs=0),(L6.1YR.REITs=0),(L7.1YR.REITs=0),(L8.1YR.REITs=0),(L9.1YR.REITs=0),(L10.1YR.REITs=0),(L11.1YR.REITs=0),(L12.1YR.REITs=0),(L13.1YR.REITs=0),(L14.1YR.REITs=0),(L15.1YR.REITs=0),(L16.1YR.REITs=0),(L17.1YR.REITs=0),(L18.1YR.REITs=0),(L19.1YR.REITs=0),(L20.1YR.REITs=0),(L21.1YR.REITs=0)	21
2YR	3248	1	6	12.947	0.023	(L1.2YR.REITs=0),(L2.2YR.REITs=0),(L3.2YR.REITs=0),(L4.2YR.REITs=0),(L5.2YR.REITs=0)	5
3YR	3248	1	6	33.229	3.388e-06	(L1.3YR.REITs=0),(L2.3YR.REITs=0),(L3.3YR.REITs=0),(L4.3YR.REITs=0),(L5.3YR.REITs=0)	5
5YR	3248	1	6	35.973	9.613e-07	(L1.5YR.REITs=0),(L2.5YR.REITs=0),(L3.5YR.REITs=0),(L4.5YR.REITs=0),(L5.5YR.REITs=0)	5
7YR	3248	1	6	35.165	1.394e-06	(L1.7YR.REITs=0),(L2.7YR.REITs=0),(L3.7YR.REITs=0),(L4.7YR.REITs=0),(L5.7YR.REITs=0)	5
10YR	3248	1	6	37.878	3.991e-07	(L1.10YR.REITs=0),(L2.10YR.REITs=0),(L3.10YR.REITs=0),(L4.10YR.REITs=0),(L5.10YR.REITs=0)	5
20YR	3248	1	6	31.151	8.743e-06	(L1.20YR.REITs=0),(L2.20YR.REITs=0),(L3.20YR.REITs=0),(L4.20YR.REITs=0),(L5.20YR.REITs=0)	5
30YR	2722	1	6	29.096	2.219e-05	(L1.30YR.REITs=0),(L2.30YR.REITs=0),(L3.30YR.REITs=0),(L4.30YR.REITs=0),(L5.30YR.REITs=0)	5

**Table 9: Step 3 Result Entire Period (Source: Author's calculation)**

IR	no_data_point	max_ooi	lag	statistic	pvalue	H0	d.o.f
1MO	770	1	7	2.214	0.898	(L1.1MO.REITs=0),(L2.1MO.REITs=0),(L3.1MO.REITs=0),(L4.1MO.REITs=0),(L5.1MO.REITs=0),(L6.1MO.REITs=0)	6
3MO	770	1	8	7.246	0.403	(L1.3MO.REITs=0),(L2.3MO.REITs=0),(L3.3MO.REITs=0),(L4.3MO.REITs=0),(L5.3MO.REITs=0),(L6.3MO.REITs=0),(L7.3MO.REITs=0)	7
6MO	770	1	7	2.775	0.836	(L1.6MO.REITs=0),(L2.6MO.REITs=0),(L3.6MO.REITs=0),(L4.6MO.REITs=0),(L5.6MO.REITs=0),(L6.6MO.REITs=0)	6
1YR	770	1	3	1.383	0.500	(L1.1YR.REITs=0),(L2.1YR.REITs=0)	2
2YR	770	1	3	3.034	0.219	(L1.2YR.REITs=0),(L2.2YR.REITs=0)	2
3YR	770	1	3	5.123	0.077	(L1.3YR.REITs=0),(L2.3YR.REITs=0)	2
5YR	770	1	3	4.362	0.112	(L1.5YR.REITs=0),(L2.5YR.REITs=0)	2
7YR	770	1	3	4.093	0.129	(L1.7YR.REITs=0),(L2.7YR.REITs=0)	2
10YR	770	1	3	3.004	0.222	(L1.10YR.REITs=0),(L2.10YR.REITs=0)	2
20YR	770	1	3	1.477	0.477	(L1.20YR.REITs=0),(L2.20YR.REITs=0)	2
30YR	244	1	4	0.858	0.835	(L1.30YR.REITs=0),(L2.30YR.REITs=0),(L3.30YR.REITs=0)	3

**Table 10: Step3 Result SubPeriod: Pre-crisis (Source: Author's calculation)**

IR	no_data_point	max_ooi	lag	statistic	pvalue	H0	d.o.f
1MO	623	1	6	6.087	0.297	(L1.1MO.REITs=0),(L2.1MO.REITs=0),(L3.1MO.REITs=0),(L4.1MO.REITs=0),(L5.1MO.REITs=0)	5
3MO	623	1	7	6.354	0.384	(L1.3MO.REITs=0),(L2.3MO.REITs=0),(L3.3MO.REITs=0),(L4.3MO.REITs=0),(L5.3MO.REITs=0),(L6.3MO.REITs=0)	6
6MO	623	1	3	5.529	0.063	(L1.6MO.REITs=0),(L2.6MO.REITs=0)	2
1YR	623	1	3	4.399	0.110	(L1.1YR.REITs=0),(L2.1YR.REITs=0)	2
2YR	623	1	6	2.381	0.794	(L1.2YR.REITs=0),(L2.2YR.REITs=0),(L3.2YR.REITs=0),(L4.2YR.REITs=0),(L5.2YR.REITs=0)	5
3YR	623	1	7	5.897	0.434	(L1.3YR.REITs=0),(L2.3YR.REITs=0),(L3.3YR.REITs=0),(L4.3YR.REITs=0),(L5.3YR.REITs=0),(L6.3YR.REITs=0)	6
5YR	623	1	6	5.790	0.327	(L1.5YR.REITs=0),(L2.5YR.REITs=0),(L3.5YR.REITs=0),(L4.5YR.REITs=0),(L5.5YR.REITs=0)	5
7YR	623	1	4	7.462	0.058	(L1.7YR.REITs=0),(L2.7YR.REITs=0),(L3.7YR.REITs=0)	3
10YR	623	1	6	9.533	0.089	(L1.10YR.REITs=0),(L2.10YR.REITs=0),(L3.10YR.REITs=0),(L4.10YR.REITs=0),(L5.10YR.REITs=0)	5
20YR	623	1	4	8.648	0.034	(L1.20YR.REITs=0),(L2.20YR.REITs=0),(L3.20YR.REITs=0)	3
30YR	623	1	4	9.071	0.028	(L1.30YR.REITs=0),(L2.30YR.REITs=0),(L3.30YR.REITs=0)	3

**Table 11: Step3 Result Sub Period: During-crisis (Source: Author's calculation)**

IR	no_data_point	max_ooi	lag	statistic	pvalue	H0	d.o.f
1MO	1852	1	5	3.667	0.452	(L1.1MO.REITs=0),(L2.1MO.REITs=0),(L3.1MO.REITs=0),(L4.1MO.REITs=0)	4
3MO	1852	2	6	5.561	0.234	(L1.3MO.REITs=0),(L2.3MO.REITs=0),(L3.3MO.REITs=0),(L4.3MO.REITs=0)	4
6MO	1852	1	2	0.001	0.969	(L1.6MO.REITs=0)	1
1YR	1852	2	3	0.345	0.556	(L1.1YR.REITs=0)	1
2YR	1852	1	2	0.125	0.722	(L1.2YR.REITs=0)	1
3YR	1852	1	2	0.303	0.581	(L1.3YR.REITs=0)	1
5YR	1852	1	2	1.253	0.262	(L1.5YR.REITs=0)	1
7YR	1852	1	2	1.995	0.157	(L1.7YR.REITs=0)	1
10YR	1852	1	2	1.911	0.166	(L1.10YR.REITs=0)	1
20YR	1852	1	2	2.313	0.128	(L1.20YR.REITs=0)	1
30YR	1852	1	2	0.713	0.398	(L1.30YR.REITs=0)	1

**Table 12: Step3 Result Sub Period: Post-crisis (Source: Author's calculation)**

#### 4.4 Step 4

The last step result cross-checks if there is conflict between Granger causality test and cointegration test. Table 13 to 15 shows the cointegration result of each pairs of variables in 3 studied time frames. None has p-value low enough (<0.05 for 95% confidence interval) to reject the null hypothesis of no cointegration. The conclusion is cointegration does not exist between REITs and each IRs. Therefore, there is no conflict to the evidence of Granger causality found in step 3.

Series	p-value	coint t	critical value (5%)
REITs & 1MO	0.928	-0.844	-3.899
REITs & 3MO	0.929	-0.836	-3.899
REITs & 6MO	0.927	-0.8501	-3.899
REITs & 1YR	0.919	-0.907	-3.899
REITs & 2YR	0.925	-0.867	-3.899
REITs & 3YR	0.907	-0.976	-3.899
REITs & 5YR	0.851	-1.223	-3.899
REITs & 7YR	0.786	-1.430	-3.899
REITs & 10YR	0.747	-1.534	-3.899
REITs & 20YR	0.716	-1.610	-3.899
REITs & 30YR	0.674	-1.704	-3.899

**Table 13: Step4 Result Entire Period(Source: Author's calculation)**

Series	p-value	coint t	critical value (5%)
REITs & 1MO	0.932	-0.814	-3.910
REITs & 3MO	0.921	-0.891	-3.910
REITs & 6MO	0.834	-1.283	-3.910
REITs & 1YR	0.793	-1.410	-3.910
REITs & 2YR	0.693	-1.662	-3.910
REITs & 3YR	0.697	-1.653	-3.910
REITs & 5YR	0.860	-1.190	-3.910
REITs & 7YR	0.771	-1.471	-3.910
REITs & 10YR	0.653	-1.750	-3.910
REITs & 20YR	0.339	-2.369	-3.910
REITs & 30YR	0.514	-2.026	-3.910

**Table 14: Step4 Result SubPeriod: Pre-crisis (Source: Author's calculation)**

Series	p-value	coint t	critical value (5%)
REITs & 1MO	0.849	-1.232	-3.914
REITs & 3MO	0.873	-1.137	-3.914
REITs & 6MO	0.913	-0.939	-3.914
REITs & 1YR	0.910	-0.958	-3.914
REITs & 2YR	0.851	-1.222	-3.914
REITs & 3YR	0.828	-1.305	-3.914
REITs & 5YR	0.785	-1.435	-3.914
REITs & 7YR	0.741	-1.549	-3.914

REITs & 10YR	0.732	-1.571	-3.914
REITs & 20YR	0.594	-1.871	-3.914
REITs & 30YR	0.609	-1.841	-3.914

**Table 15: Step4 Result SubPeriod: During Crisis (Source: Author's calculation)**

Series	p-value	coint t	critical value (5%)
REITs & 1MO	0.981	-0.165	-3.902
REITs & 3MO	0.991	0.362	-3.902
REITs & 6MO	0.958	-0.571	-3.902
REITs & 1YR	0.961	-0.536	-3.902
REITs & 2YR	0.450	-2.148	-3.902
REITs & 3YR	0.278	-2.500	-3.902
REITs & 5YR	0.246	-2.577	-3.902
REITs & 7YR	0.275	-2.507	-3.902
REITs & 10YR	0.379	-2.288	-3.902
REITs & 20YR	0.525	-2.006	-3.902
REITs & 30YR	0.552	-1.953	-3.902

**Table 16: Step4 Result SubPeriod: Post-crisis (Source: Author's calculation)**

## 5 Discussion

The study result confirms multiple conclusions drawn from earlier researches. First, the relationship between REIT and interest rate does exist. However, like what found by Ling T. He (2003) and Chen (1988), this connection is not stable and is time-specific. In fact, when examining the sub periods, Granger Causality is only found between REIT's return and 20-year and 30-year CMTs during the 2008 crisis but not before or after that. However, when looking at the result of the whole 13-year period, in which REIT's return is sensitive to all the 11 CMTs, it is possible that the relationship is not only point-in-time specific. What caused this drastic variance between the results of the entire tested time frame and its sub periods can come from the difference in the length of time frame. The entire period is 13 years while the sub periods are respectively 3, 3 and 7 years. When looking at the pre-crisis and crisis periods, both of them have relatively the same time frame's length. However, their results are not the same. This indicates that the relationship is found in crisis period is more point-in-time specific rather than depending on the length of time.

The second drawdown from the result that is consistent with earlier studies is that for shorter time frame, REIT's return is more sensitive to interest rate during shocks than in stable market. Similar conclusion was made by McCue (1994), who pointed out shock in 3-month T-bill has a massive impact on the return of NAREIT Equity REIT index. Muller (1995) also found that REITs react more strongly to falling interest rate than to the rising rate. Interestingly, the crisis period (01/02/2007-31/07/2009) witnessed the biggest and fastest fall for all 11 CMTs out of the total 13-year time frame. Interest rates during the other two sub-periods are much more stable and follow an upward trend. The fact that Granger Causality of interest rate projected on REITs exist only in this sub period but not in the others where interest rate is less volatile is in-line with Muller (1995)'s finding. The time-specific characteristic of this relationship calls for a prerequisite for building REIT's time series model. The prerequisite test should be

tailored for the specific data and time frame. Its result will then help to decide which variables are meaningful in forecasting REIT's return and thus, to be included to the model.

Granger Causality found only during crisis reflects how prominent some of the channels of sensitivity between REITs and interest rate are. Firstly, the result indicates trace of the third channel of sensitivity or REIT's operative activity. Even though REITs considerably expose to interest rate risk due to their dependence on loans as a major source of capital, their operative profit is unlikely to take a huge effect from expected interest rates' change. REITs are well-known as an inflation-hedging tool. One of the reason for this characteristic comes from how rental income, which account for at least 75% of total income, are often fixed against inflation rate. As a result, anticipated changes in inflation, if well hedged, do not significantly affect REIT's operating profit. That does not apply for unexpected inflation. Chan (1990) found changes in unexpected inflation has consistent and significant impacts on REITs. Since inflation rate is embedded inside interest rate, adjusted rent levels will offset changes in public capital cost, provided that these fluctuations are within the firms' expectation. Since rent levels are commonly adjusted on yearly basis, income cannot react quickly enough to sudden changes in capital cost. This is when REITs take the hit from the money market's shocks through their operating activity.

It is logical to raise a question on the prominence of this channel when the result does not reflect the clear switch in REIT's loan composition before and after crisis, from highly leveraged positions with high proportion of short-term loans to a more conservative structure with favor for longer-term debts. This is again can be explained by the fact that this change in capital structure is not an unexpected shock but initiated by REITs themselves and thus, it should be well balanced out by adjusting operating income.

Another explanation could be that even though most short-term loans have floating rate, they are often paired with an interest rate swap, where REITs pay fixed interest rate. This will help REITs in planning and stabiling their operating cash flow but not necessary save them from losses from interest rate shocks. Hedging a floating loan can get expensive when credits are tight, REITs might end up much out of the money when the short-term rate is adjusted shortly after the spike. However, this again depends a lot on individual company's cases.

The result indicates that REIT's return tend to be sensitive with long-term than with short-term interest rates during shocks. Only one short-term rate, the 6-month CMTs, "Granger" causes REITs while all the long-term CMTs do. This conclusion is only valid under 10% of confidence level. However, when looking at the entire period, REITs are sensitive to all the interest rates. There could be many explanations behind this outcome that calls for further study in the future. One reason could be that even though short-term debts were more popular before crisis, long-term debts in REIT's debt structure possibly are still proportionally dominant and REIT's operational activity are thus, more exposed to long-term rate. Not only in the third but also in the second channel of sensitivity REIT's return are more prone to take effects from long-term interest rate. These rates play an important role in real estate valuation when they are often used for referencing discount rate for both future periodic cash flows as well as properties' terminal value. The same token applied when considering REIT as high-yield stocks, whose values changes as how much future dividend payments got discounted. The

discount rate for future dividend flow is also often based on interest rates. Because when considered REIT's stocks as financial instruments, serves as long-term holdings, it makes sense to use long-term interest rate for referencing the future payout's discount rate. Similarly, when comparing REITs to long-term bonds, their values will be benchmarked against other similar-term bonds or similar-term debt rate. Another explanation could come from the fact that short-term interest rate is commonly administered or interfered by the US central bank in case of meltdowns, but long-term interest rates are always moved by the market forces.

This thesis runs into a similar problem as the earlier studies, that is, it does not clearly identify or separate the channels through which the effects of interest rate transferred to REIT's return. Rather, the result just indicates possible evidences from those channels. However, the model did serve its main purpose of examining the indirect causality between REIT's return and interest rate. It identifies for given time period, which interest rate maturities are useful to predict REIT's return and should be included as a variable in REIT's return regression model. On top of that, the result also confirmed some of the earlier studies' conclusion regarding REIT's return and interest rate relationship.

Regarding the chosen methodology, implementing the T-Y procedure instead of the ordinary Granger test showed its advantage when non-stationary data and data with greater-than-1 order of integration was statistically explained by the model. While most of the fed series becomes stationary at 1 time of differencing  $I(1)$  and would be able to be tested by the normal Granger test, some processes required 2 times or  $I(2)$  differencing to be transformed such as the 6MO CMTs of the entire period as well as the 3MO and 1YR CMTs post crisis. Additionally, the result comes out of the T-Y is straight forward and easy to interpret. Therefore, T-Y is clearly a better solution for the test than traditional Granger test.

Since different CMTs maturities are highly correlated to each other, the model in this thesis includes only one interest rate time series at a time as the endogenous variable. However, T-Y procedure can be used to test if a matrix of multiple endogenous variables Granger causes an exogenous one. This application of T-Y procedure can be seen in many studies such as Mohammad's in 2011 when he tested the Granger causality of different macroeconomic variables on stock prices. Including a combination of endogenous into the ordinary Granger causality test can get complicated and is hard to translate the result. Therefore T-Y test opens up the possibility of including several of variables that are not that correlated, for testing their Granger causality on REIT's return in the future.



## 6 Evaluation and Limitation

The study was done on a large data sample with more than 3200 data points. For Granger Causality it used the T-Y methodology, which is a more advanced approach than the original procedure. The approach crossed out multiple unrealistic assumptions that are required by the traditional Granger test. They are non-cointegration and stationary data. Although the collected data set is not cointegrated, it is clearly non-stationary and some series needed more than 1 time of differencing to become de-trended, which makes it not eligible for the ordinary Granger test. Order of integration is cross-checked by two test DF and KPSS. Optimal lag length is also delivered by two information criterion for references with a serial correlation check on top. The final result is also tested against cointegration test for possible conflicts in outcome.

Limitations of this thesis lie in a couple of aspects. First, the Granger causality result from this particular thesis does not allow great applicability or embedded that meaningful interpretation. For example, the 1-year rate Granger causes REIT's return means it will improve the accuracy of REIT's return VAR comparing to if the model using only REIT's return past values. This interpretation does not necessary hold when VAR already includes other series rather than return on REITs, which is normally the case when REIT' return can take effects from many economics variable other than just interest rate. The Granger causality relationship itself also does not indicates a direct causal relationship, which would be more meaningful for forecasting or studying variable explanatory power.

Last but not least, the data selection might not be a comprehensive representative of studied economic variables, i.e. the MSCI REIT index not necessarily carries all the characteristic of REIT's return and CMT, by excluding the risks added on top of risk-free interest rates, cannot completely describe the cost of money.

Another limitation is that the data slicing can result in different outcomes. Even though, data sectioning is built around the shock happened in the dataset, exactly when the crisis starts and ends are not set in stone but rather estimated. This study did not take into account the fact that the real estate industry generally reacts a step slower than others to the financial market. Although it is still controversial if return on REITs inherit more from stock or real estate market, the latter still surely has some effect on how REIT's return react to market's changes.

## 8 Conclusion

The study found there is a one-way Granger Causality relationship of interest rate (all 11 maturities of CMT) on the REIT's total return for the tested period of 01/01/2001 – 31/12/2016. This means that time series models of REIT for this particular time frame should include all the CMTs because they help to improve the accuracy of the model's prediction. However this relationship is not consistent through the sub-periods: pre-crisis (01/01/2001 – 31/01/2007), during crisis (01/02/2007 – 31/08/2009) and post crisis (01/08/2009 – 31/12/2016). Specifically, we found no Granger Causality from any of the CMTs projected on REIT's return for the sub period of pre-crisis and post-crisis. The long-term CMTs (20-year and 30-year CMTs) have the best explanatory power over REIT's return during the 2-year crisis sub period. This not only suggests the use of long-term interest rate in REIT's time series models for this particular period but also supports earlier studies' findings that REIT, as a sector, takes more effect from interest rate during shocks than during stable times. This result is in line with how REITs can reduce their exposure to interest rate risk through operating activity by adjusting their operating income to balance out the expected changes in the cost of loan. So during shocks, since interest rate changes are drastic, firms could not predict or react to it promptly enough to offset the sudden increase in the cost of debt. Thus, the jump in interest rate shown up in REIT total return. The result can be seen as an evidence of a more exposure toward long-term interest rate REITs having, which was explained by REIT's exposure to long-term interest rate in all the discussed channels of sensitivity.

The relationship found in the entire time frame does not necessary means it will be found in all of its sub periods. This time-specific characteristic and inconsistency of Granger Causality relationship suggest that this model should be used as a prerequisite for REIT's time series regression because it helps users to pick which variables are meaningful to be included into the regression.

This study fills a gap in the earlier researches on the relationship between interest rate and REIT's return. Majority of earlier studies have been focusing on the direct causality between them but none have touched on Granger Causality. In term of data selection, CMTs were used as interest rate indications, which gives a more comprehensive view of risk-free than using instruments like T-bill or Treasury note alone. This is also one of the few studies that use a REIT index instead of collecting individual REIT. REIT index return also provides a broader view of REIT as a sector, which is more suitable for testing some of the earlier studies' findings.

In addition, the thesis shown an example of how T-Y procedure is applied and run with Python program language and Statsmodels module. The model can be helpful to decide which interest rate maturities should be included into a VAR model of REIT's return. The model should be run on a tailored time period for VAR.

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## Appendix

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
```

Created on Fri Aug 3 17:18:54 2018

```
@author: VanVi
"""
```

```
import pandas as pd
import datetime as dt
import matplotlib.pyplot as plt
import numpy as np
import quandl
import statsmodels.tsa.api as smt
import statsmodels.api as sm
import statsmodels.tsa.stattools as ts
import statsmodels.tsa
from arch.unitroot import KPSS, ADF
import seaborn as sns
from statsmodels.tsa.api import VAR
import docx
import matplotlib.dates as mdates
quandl.ApiConfig.api_key = "T5YU_YayHvRq7VsrMWes"
```

```
pd.options.display.float_format = '{:.3f}'.format
```

```
# Stationarity Function to check if the passed series is stationary.
```

```
def Stationarity (X, cutoff = 0.05):
```

```
    pvalueADF = ADF(X).pvalue
```

```
    pvalueKPSS = KPSS(X).pvalue
```

```
    cvalueADF = ADF(X).critical_values[str("{0:.0%}".format(cutoff))]
```

```
    statADF = ADF(X).stat
```

```
    cvalueKPSS = KPSS(X).critical_values[str("{0:.0%}".format(cutoff))]
```

```
    statKPSS = KPSS(X).stat
```

```
    if pvalueADF < cutoff and pvalueKPSS > cutoff and statADF < cvalueADF and statKPSS <
cvalueKPSS:
```

```
        return True
```

```
    else:
```

```
        return False
```

```
# Function finds the order of integration of a series.
```

```
def order_of_integration(series, cutoff=0.05):
```

```
    if Stationarity(series, cutoff=cutoff) == True:
```

```
        return 0
```

```
    else:
```

```

    i = 1
    s = series.diff( periods=i ).dropna()
    while Stationarity(s, cutoff=cutoff) == False:
        s = s.diff( periods=i ).dropna()
        i += 1
    return i

# Function to difference the series based on number of differencing times passed in.

def data(series, time):

    for i in range(1, time + 1):
        series = series.diff( periods=i ).dropna()

    return series

# Function to create Toda-Yamamoto model's report.

def TY_Report (start, end, step):

    # Reading in MSCI REIT data, which was collected into Book1.xlsx
    df = pd.read_excel(r'Book1.xlsx', 2, header = 0, index_col = 0, usecols = [0,3], parse_dates =
True, convert_float = True, na_values = 'NA')
    df.columns = ['REIT's']
    df.dropna(inplace=True)
    df.REITs = df.REITs.pct_change( periods=1)

    # Calculate REIT's total return index

    #df.REITs=df.REITs.pct_change( periods=1)
    df.dropna(inplace=True)

    df.to_pickle('reits.pkl')

    # Reading in CMT data straight from Quandl

    rate = quandl.get("USTREASURY/YIELD", end_date="2016-12-31", start_date='2001-01-
01', collapse='daily')
    rate.sort_index(axis = 0, inplace=True, ascending = False)
    rate.columns = rate.columns.str.replace(',', '')

    # Step 1 of T-Y process: find order of integration and the maximum of order of integration
for each of the equation

def Step1_Report ():

    table = pd.DataFrame()
    max_ooi = []
    for col in df.columns:
        ooi = order_of_integration(df[col])
        max_ooi = int(np.amax(np.append(max_ooi, ooi)))
        result = pd.DataFrame( {'Series':col, 'd':ooi, 'dmax':max_ooi}, index = [0])
        table = table.append(result)

```



```
return table
```

# Step 2 of T-Y process: calculate lag AIC and HQC lag recommendations, check for serial correlation at recommended lag length, from there give out the optimal lag length at which there is no serial correlation.

```
def Step2_Report():
    inn_model = VAR(df)
    aic = inn_model.select_order(maxlags=50).aic
    hqc = inn_model.select_order(maxlags=50).hqc
    inn_lag = min(aic, hqc)
    op_lag = inn_lag
    model = VAR(df)
    result = model.fit(maxlags=op_lag)
    resid = result.resid.iloc[:,0].values
    ljungbox = statsmodels.stats.diagnostic.acorr_ljungbox(resid)
    op_lag_pvalue = ljungbox[1][0]

    while op_lag_pvalue<=0.95:
        op_lag += 1

        model = VAR(df)

        result = model.fit(maxlags=op_lag)
        resid = result.resid.iloc[:, 0].values
        ljungbox = statsmodels.stats.diagnostic.acorr_ljungbox(resid)
        op_lag_pvalue = ljungbox[1][0]

    return {'Series': '%s & %s' %(df.columns[0], df.columns[1]), 'AIC':aic, 'HQC':hqc,
            'Op_Lag':op_lag}
```

# Step 3 of T-Y process: run the model using lag length equal the optimal lag length suggested from Step 2 added to the maximum order of integration. The model then tested for Modified Wald Test

```
def Step3_Report():
    op_lag = Step2_Report()['Op_Lag']
    max_ooi = Step1_Report().dmax.values[1]
    lag = int(op_lag + max_ooi)
    try:
        model = sm.tsa.VARMAX(df, order = (lag,0), trend = 'c')
        result = model.fit()
        string = interest_rate+'.REIT's
        exog = [i for i in model.param_names if string in i][:-int(max_ooi)]
        hypo = ".join(['+ i +=0),' for i in exog[:-1]]) + '(+ exog[-1] +=0)'
        wald = result.wald_test(hypo)
        return {'interest_rate':interest_rate, 'no_data_point':df.REITs.count(),
                'max_ooi':max_ooi, 'lag':lag, 'statistic':wald.statistic[0][0], 'pvalue':wald.pvalue, 'H0': hypo,
                'df_demoms': wald.df_denom}
    except:
        pass
    return None
```

```

# Step 4 of T-Y process: Testing if there is cointegration between REIT and CMT

def Step4_Report ():

    test = ts.coint(df.iloc[:,1], df.iloc[:,0])
    return {'Series': '%s & %s' %(df.columns[0], df.columns[1]), 'p-value': test[1],
'coint_t': test[0], 'critical value (0%)': test[2][0]}

rate = pd.read_pickle('rate.pkl')
rate = rate.drop(['2MO'], axis = 1)
rate = rate/100

table = pd.DataFrame()

# Paring REIT with each CMT into one dataframe
for interest_rate in rate.columns.values.tolist():
    df = pd.read_pickle('reits.pkl')

    df = df.join(rate[interest_rate], how = 'inner')
    df.dropna(inplace=True)
    df.index = pd.DatetimeIndex(df.index).date
    df = df.loc[start:end]

    if step == 1:
        test = Step1_Report()
        table = table.append(test, ignore_index = True)
        table = table.loc[table.Series != 'REIT's']

    elif step == 2:
        test = pd.DataFrame(Step2_Report(), index = [0])
        table = table.append(test)

    elif step == 3:
        test = pd.DataFrame(Step3_Report(), index = [0])
        table = table.append(test)

    elif step == 4:
        test = pd.DataFrame(Step4_Report(), index = [0])
        table = table.append(test)

table.set_index(table.columns[0], inplace=True)

return table

# Save T-Y report to Word file
def save (table, start, end, step):

    table.to_pickle('step %s %s - %s.pkl' %(str(step), str(start), str(end)))
    doc = docx.Document(r'Data/Tables.docx')
    table.reset_index(inplace=True)

```

```
# add a table to the end and create a reference variable
# extra row is so we can add the header row

doc.add_paragraph(''.join([str(start), str(end), 'Step:', str(step)]))
t = doc.add_table(table.shape[0]+1, table.shape[1])

# add the header rows.
for j in range(table.shape[-1]):
    t.cell(0,j).text = table.columns[j]

# add the rest of the data frame
for i in range(table.shape[0]):
    for j in range(table.shape[-1]):
        t.cell(i+1,j).text = str(table.values[i,j])

# save the doc
doc.save(r'Data/Tables.docx')

# list of all the tested sub periods
#2004,1,1
#2007,1,31
#2007,2,1
#2009,7,31
#2009,8,1
#2016,12,31

#Implementation

step = 4
start = dt.date(2004,1,1)
end = dt.date(2007,1,31)
table = TY_Report(start, end, step = step)
save(table, start, end, step=step)
```