MULTIOBJECTIVE AND ROBUST OPTIMIZATION IN PHARMACY DELIVERY AND EMERGENCY DEPARTMENT NURSE STAFFING

by

Anna Camille Svirsko

B.S. in Mathematical Sciences, Carnegie Mellon University, 2012

M.S. in Statistics and Operations Research, University of North Carolina at Chapel Hill, 2014

Submitted to the Graduate Faculty of

the Swanson School of Engineering in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

University of Pittsburgh

2019

UNIVERSITY OF PITTSBURGH

SWANSON SCHOOL OF ENGINEERING

This dissertation was presented

by

Anna Camille Svirsko

It was defended on

March 18, 2019

and approved by

Bryan A. Norman, Ph.D., Professor

Department of Industrial, Manufacturing, & Systems Engineering, Texas Tech University

Bo Zeng, Ph.D., Assistant Professor, Department of Industrial Engineering

Oleg A. Prokopyev, Ph.D., Professor, Department of Industrial Engineering

Jerrold H. May, Ph.D., Professor, Joseph M. Katz Graduate School of Business

Seth Hostetler, Lead Process Engineer, Geisinger

Dissertation Director: Bryan A. Norman, Ph.D., Professor

Department of Industrial, Manufacturing, & Systems Engineering, Texas Tech University

MULTIOBJECTIVE AND ROBUST OPTIMIZATION IN PHARMACY DELIVERY AND EMERGENCY DEPARTMENT NURSE STAFFING

Anna Camille Svirsko, PhD

University of Pittsburgh, 2019

Over the past 20 years, hospitals have seen a drastic improvement in patient service with improved patient recovery times, faster delivery of care, and an increased focus on patient safety. In addition, healthcare has also began to focus on its staffing, making efforts to improve staff satisfaction. Despite these advancements, healthcare spending has continued increasing. The objectives of cost, staff satisfaction, and patient safety do not always align. In addition, in the healthcare sector demand is frequently unknown, compounding the difficulty in solving these problems. This dissertation provides deterministic and robust optimization models to solve the pharmacy distribution problem and the emergency department nurse staffing problem.

Pharmacy distribution is one area of the healthcare system which has seen an increase in technology to improve patient safety and help facilitate distributing medication from the central pharmacy to the patient. Pharmacy robots are used to pick medication cost effectively in the central pharmacy while automated dispensing cabinets (ADCs) are point-of-use storage maintained on the inpatient units. These technologies have provided the ability to deliver medication more cost effectively and improve patient safety. However, the two technologies are often implemented independently of each other and therefore may not work cohesively within the distribution process. This dissertation presents a model which focuses on the trade-off between cost and patient safety. The model is solved to minimize the total distribution system cost, including purchasing and maintenance of technology, as well as minimizing the total distribution workload cost. A robust model is formulated and solved to account for the variation in medication demand and determine the effect variation has on planning decisions.

Emergency departments treat patients with various illnesses which results in a wide range of complexities in determining staffing. Patient demand varies by the day of the week and the hour of the day. This results in difficulty determining the daily staffing levels necessary to treat patients efficiently while also considering available staffing resources. This dissertation provides a basic model which determines the daily staffing levels for an emergency department based on predetermined staffing levels. This model is expanded by determining staffing levels in the presence of understaffing by assessing nurse to patient ratios. Several additional models are introduced which consider the variation in patient demand that occurs daily. The performance of these models is considered across a calendar year by comparing the results with current patient safety standards and in comparison with each other to determine the best method to determine nurse staffing levels in the presence of variable demand and understaffing.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

PREFACE

I would like to take the time to thank all the people who have helped me during the past 5 years. First, I would like to thank my advisor Dr. Bryan Norman for guiding and supporting me for the past several years. Your support and advice while I was on the job market was invaluable. Thank you for caring about what I wanted, encouraging me to look for positions that aligned with my skills, and letting me practice both my research talk and student engagement talks for you. I will miss our weekly meetings where we spend at least the first 10 minutes talking about life. I not only found an incredible advisor, but also an incredible friend. Thank you for not just caring about me as your student, but also as a person. I hope that one day I can be as influential on my students' lives as your were on mine.

Thank you to my committee: Dr. Bo Zeng, Dr. Oleg Prokopyev, Dr. Jerry May, and Seth Hostetler. While preparing for my proposal I was convinced that I could never make it in academia, let alone get a job. Your support during my proposal and after encouraged me to enter the academic job market and actually succeed in getting a job. Thank you for taking the time to be on my committee, reading my dissertation, and giving me valuable feedback.

Dr. Bidanda, thank you for always being supportive of me during my time pursuing my PhD. Dr. Rajgopal and Dr. Luangkesorn, thank you for taking on the project with Children's Hospital of Pittsburgh and letting me stay on the project. This project meant so much to me and I enjoyed mentoring the new students on the project. Michael and Minerva, thank you for always answering an endless list of questions, processing refunds, and the warm front office environment you have created.

Special thanks to Children's Hospital of Pittsburgh for funding my PhD and Geisinger for collaborating with us and providing problems that motivated my research. In particular, I would like to thank Dr. Richard Saladino, Jennifer Woodring RN, and David Rausch RN from Children's Hospital of Pittsburgh and Dean Parry along with Seth Hostetler from Geisinger.

To my fellow graduate students, thank you for making a great community to belong to. Colin and Hamdy, thank you for studying for qualifying exams with me and letting me yell at you when you challenged me so I wouldn't do that in the real exam (I don't think that would have gone over as well).

To my friends, Jackie, Megan, and Neda, thank you for being the best group of girlfriends anyone could ask for. Jackie, I love dancing with you and cannot wait to go out dancing once all this is done. Megan, thank you for always agreeing to run with me and keeping me in shape throughout grad school. I hoping the marathon in May is easier than this dissertation was. Neda, thank you for always letting me practice my presentations, listening to my research problems, and cheering me on during the interview process. I'm not sure how I'll survive without our nightly chats before bed, you fixing my makeup every time we go out, or the fashion shows after we do a little too much shopping.

Above all, I would like to thank my family: my parents, Dawn and Alex, my sister Clare, my unofficial sister, Laura, and my grandparents. Mom, after every visit home you always sent me back to school with food so I wouldn't have to worry about cooking for the week. Dad, you always wanted me to study drones and I'm sorry I didn't but I did get a few ideas from you that I hope I can study in the future. Memaw and Bepop, I look forward to our weekly phone calls where you cheer me on, and ask about my papers and my research. Clare and Laura, thank you for calling me at 7am so many mornings when I was at the office working late the night before. With your tag team I always got to where I needed to be on time. Thank you all for your constant love and support over the past several years. I wouldn't be where I am today without you all.

1.0 INTRODUCTION

Basic linear optimization focuses on solving problems which have a single objective and known parameters. However, in the real world this is rarely the case. Many times, problems have multiple conflicting objectives and the model parameters are often not known. Multiobjective optimization techniques can be used to determine a Pareto frontier which demonstrates the optimal solutions that exist for the problem, depending on the perspective of the decision maker. Robust optimization techniques can be applied to situations where the model parameters are uncertain in order to obtain a solution which performs the best in the worst case scenario.

In the field of healthcare, most operational problems have conflicting objectives. Patient safety, staff satisfaction, and cost are factors which play important roles in decisions made by leadership. However, these three factors often conflict with each other. This dissertation focuses on two problems-the pharmacy distribution problem and the emergency department nurse staffing problem, that demonstrate these conflicts. Both of these problems exhibit daily variation in demand, which can make it difficult to determine an optimal solution that performs well across all days. In addition, these problems have multiple objectives which do not align with each other.

The in-hospital pharmacy is responsible for ensuring that all patients receive the correct medication at the correct time. Each day, thousands of medications are delivered from the in-hospital pharmacy to the in-patient units, requiring coordination between staff in the central pharmacy and on the units. The pharmacy distribution process can result in either a centralized model, where the medication is prepared in a central location and then distributed to the inpatient units, or a decentralized model, where the medication is stored and prepared at the inpatient units. The centralized system requires lower inventory levels and fewer labor hours whereas the decentralized system results in fewer missing medications. Automation is available for both the central pharmacy, in the form of a central pharmacy robot, and on the unit, in the form of an Automated Dispensing Cabinet (ADC), to automate the process and reduce the number of missing medications and medication errors. However, both pieces of technology require a large upfront cost. The intricacies regarding the costs of centralizing or decentralizing pharmacy operations present a difficult decision for health systems seeking to provide the best patient care possible while also reducing their costs. The decision to centralize or decentralize pharmacy operations is often impacted by the average medication demand for the hospital. However, this demand varies daily and depends on the set of patients in the hospital adding further complexity to the problem. Chapters 2-4 of this dissertation demonstrate how centralization/decentralization of the pharmacy process affects cost, missing doses, and staff workload in both the average and uncertain cases.

Emergency department nurses are on the front line of providing care to patients experiencing heath issues that require prompt attention and result in a visit to the emergency department. The chief complaint, complexity, and amount of service vary according to every patient, leading to variable demand across every hour of each day. Nursing managers typically determine staffing levels based off room ratios and feelings of "busyness" in the emergency department. While this method provides very basic staffing levels, it frequently results in staffing that does not align with patient volume nor does it take into consideration the available nursing staff, leading to high patient to nurse ratios during times of high volume, low patient to nurse ratios during times of low volume, and a need for nurse overtime to reach the preferred staffing level. These complications coupled with the variation in demand lead to frequent instances of inadequate staffing. Chapters 5 and 6 of this dissertation present models that use current patient safety standards with regards to emergency department staffing along with nurse:patient ratios to determine the optimal staffing levels for a given available nurse staffing levels.

This dissertation is separated into five chapters, with Chapters 2-4 focusing on the pharmacy distribution problem and Chapters 5-6 focusing on the emergency department nurse staffing problem. Chapter 2 of this dissertation will introduce the pharmacy distribution problem, formulate a deterministic mixed integer linear program which minimizes the pharmacy distribution system cost, and solve the problem for a subset of units at Geisinger Hospital for the average daily demand. Chapter 3 analyzes the pharmacy distribution process when minimizing the average workload cost across the distribution system. This chapter analyzes the effects of centralization/decentralization on hospital staff, in particular pharmacy technicians and nurses. In Chapter 4 we consider the daily variation that can occur in the pharmacy distribution process by formulating the robust counterpart for the deterministic model presented in Chapter 2. We solve the robust counterpart to determine the best distribution process under the uncertain demand. The emergency department nurse staffing problem is introduced in Chapter 5 with a basic linear programming formulation to determine a daily schedule. The work from Chapter 5 was implemented successfully at a local hospital. This initial emergency department nurse staffing problem is extended in Chapter 6 by formulating multiple models that determines the daily schedule based on patient deterministic and uncertain patient demand.

2.0 STANDARDIZING PHARMACEUTICAL DELIVERY TO REDUCE PHARMACY COSTS WHILE SIMULTANEOUSLY REDUCING MISSING DOSES

2.1 BACKGROUND

The goal of a hospital pharmacy is to provide all patients with the medications ordered at the time the medication is scheduled to be administered. This is a large logistics problem because the pharmacy provides medication to all the inpatient units through different dispensing methods and delivery routes. Patient safety is a top priority and patient care suffers when medication doses are either missing or not delivered and the patient does not receive the dose at the correct time, if at all.

Within a hospital pharmacy system, there are several different types of technology available to increase patient safety by reducing medication errors and missing medications such as automated dispensing cabinets (ADCs) and robots. ADCs, like the unit in Figure 2.1a, are utilized on inpatient units at a point-of-use storage location. These cabinets require nurses to select a patient profile and the specific medication in order for the medication to become accessible. When the drawer is opened, the patient's medication is highlighted by lights within the drawer that point to the correct compartment, as demonstrated in Figure 2.1b. While ADCs can significantly reduce medication errors and missing medications, ADCs increase hospital inventory. Storing a large portion of demand in ADCs can lead to an increase in nurse queueing time required to obtain doses during major medication administration times.

In the hospital pharmacy, a robot, as shown in Figure 2.2, can be used to pick unit dose medications for patients on specific floors. Unit dose medications are any medication that is in solid form and comes in individual unitized packages, such as a tablet or a capsule. The

(a) Sample ADC Unit

(b) ADC Drawer with guiding light

Figure 2.1: Examples of an ADC and how it functions

unitized packages are placed on rods inside the robot and the robot picks the medication off the rods for an individual patient and places the medication into the patient envelope. There is a limit on the number of medications that a single rod can hold and often times a single medication with high demand will require multiple rods within the robot.

Use of a robot ensures a significant reduction in both the time to fill the order and the probability of an order fulfillment error. However, robots have a large upfront cost and there is a limit on the number of medications the robot can hold and the number of doses that can be processed by the robot in a single day.

While ADCs and central pharmacy robots have improved a hospital pharmacy's ability to safely distribute medications to patients, they are costly to purchase and maintain. According to the Association for Healthcare Resource Materials Management, supply chain activities account for more than 40% of hospital expenses [2]. By ensuring efficient supply chain operations, hospitals can decrease expenses while maintaining high quality patient care.

Considering the technology described above, we can identify three distinct paths through which unit dose medication can be delivered to a patient in a hospital. The pathways to

Figure 2.2: Example of a hospital pharmacy robot

each unit are depicted in Figure 2.3. The medication is either picked by the hospital robot or the pharmacy technician and delivered to either the cart fill or ADC of each unit. In the first pathway the medication for each patient is picked by the pharmacy technician, placed in an envelope, and delivered to the floor through a process called cart fill. In a similar pathway, the medication is picked by the robot in the hospital central pharmacy and placed in the envelope and delivered via cart fill. In the cart fill process all medications sent from the central pharmacy are stored in a cart which has a compartment designated for each patient until it is time for the medication to be administered. The patient envelope filled by the robot and the pharmacy technician is placed into the patient's assigned compartment. The cart is usually filled once every day at the beginning of the day with all the patient's medications that are supplied from the inpatient pharmacy. The last pathway routes the medication to the patient via the ADC on the unit. In this process, the medication is picked in bulk by the pharmacy technician at the central pharmacy and delivered to the unit ADC where it is stored until a nurse retrieves the medication to administer to the patient. Note that while there are other pathways that can be taken to deliver medication to the patient, these three are the most commonly utilized pathways.

There are advantages and disadvantages of every pathway. The pharmacy robot has the lowest picking cost, but a large upfront purchase and maintenance cost. The robot makes very few picking errors; however, missing doses still occur because the medication can get lost during the delivery process. The pharmacy technician has the highest picking cost and

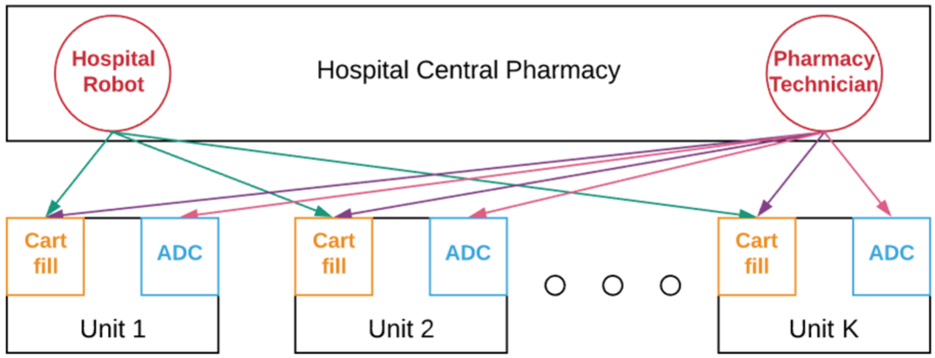

Figure 2.3: Diagram of pharmacy distribution pathways for a single hospital

requires no additional investments, but the pharmacy technician has the highest error rate out of all three pathways. The ADC has the lowest error rate due to the point-of-use storage. However, due to its purchase cost and nurse queueing cost it is the most expensive pathway to deliver medication to the patient.

Every dose given to a patient is classified as either a “STAT” dose, first dose, maintenance dose, or “pro re nata” (PRN) dose. A “STAT” dose must be given to the patient immediately and is classified as any dose whose desired first administration time is within a half hour of the order time and does not include any patient requested medications. A first dose is defined as the first administration of any medication whose desired first administration time exceeds the order time by thirty minutes. A maintenance dose is any recurring dose that is scheduled to be administered at a specific time, for example when a patient receives a hypertension medication every morning at 8am. The PRN doses are administered upon patient request and do not necessarily have scheduled administration times. The dose type plays a role in the importance of the medication and the delivery routes it can take. For instance, a STAT dose cannot be filled via a robot because the order would need to be programmed within the robot, picked, and delivered immediately, which is not feasible. For every medication delivered to the patient, there is a probability that the medication could be the wrong medication, the medication is not administered to the patient, or administered outside the scheduled administration time range. We classify these three instances as missing doses.

We develop an optimization model to improve patient safety by minimizing the total cost to pick and deliver medication to the patient while limiting the number of missing doses that occur. For each unit in a hospital we establish pathways to deliver every medication and dose type to the patient.

2.2 LITERATURE REVIEW

There have been several studies focusing on improving pharmacy logistics and patient safety which have analyzed the effect of different systems on cost, labor effort, and missing medications and medication errors. The literature originally focused on decentralization of pharmacy operations, particularly with the implementation of ADCs, due to the decrease in both missing medication and medications errors. Meanwhile, the implementation of robots within the central pharmacy has produced research studying the role of a centralized pharmacy and the effect on the labor effort of nurses and pharmacy technicians.

In 1965, Black and Tester claimed improved patient safety with a decentralized pharmacy model which increased pharmacist availability to both the patient and physician [26]. One modern goal of hospital pharmacists pertains to the Pharmacy Practice Model Initiative (PPMI). PPMI is a subset of the American Society of Health-System Pharmacists whose goal is to increase pharmacist clinical involvement. By making the pharmacist available to physicians for consulting and to patients for instruction, hospitals can reduce errors and save money [27]. One hospital system (Providence Health and Services) in Montana increased clinical duties of their pharmacists by assigning their distribution-related responsibilities to pharmacy technicians and optimizing the use of technology. The system saw annual pharmacist interventions increase, in turn reducing overall estimated error cost [47]. Literature continues to focus on pharmacy decentralization as a means of improving patient safety; in fact, a recent survey shows that American pharmacy practice is trending in the decentralized direction where frequently used drugs are mainly stored on the unit and use of the main hospital pharmacy is kept to a minimum [50].

New technology like ADCs has further increased the practicality of decentralized systems by providing automatic inventory tracking in secure and geographically convenient locations. A Canadian study recently found that, although success is highly dependent on the effectiveness of implementation, ADC use yields a reduction in missing doses, technician fill errors, and medication storage errors [55]. Similarly, a study conducted by Cardinal Health compared multiple hospitals with multiple distribution systems and found that the time to initial dose and the frequency of missing doses is significantly reduced when medication is routed primarily through ADCs [4]. In the South Jersey Health System the pharmacy reduced their average medication delivery time to patients from about 2 hours to 18 minutes through the implementation of ADCs [43].

When utilizing ADCs, inventory management is an important factor in the effectiveness of the implementation. The cabinets should have available inventory streamlined for items with multiple strength prescriptions and order up to levels should be reduced to create space for bulkier items. Using these methods to optimize the inventory kept in the ADC, Hussey et al. showed that the number of medications in the cabinet will increase leading to a reduction in the number of dispenses from the central pharmacy and the cost of operating the ADCs. However, this could result in more stockouts per day [28]. Similarly, McCarthy and Ferker studied prescription frequency to determine which medications should be stored in the ADC and the appropriate order up to and reorder levels to result in a reduction in cost, weekly stockouts, and improved medication turnaround time [41]. In [21, 22], Esmaili et al. determine how pharmaceutical products should be stored within an ADC with regards to an effective layout and efficient inventory policies. Similarly, Kelle et al. [32] create a decision support tool to help determine what items to hold in an ADC based on two models: (1) optimal ordering and holding costs and (2) optimizing space allocation based on ordering cost. Dobson et al. [16] determine medication assignments to an ADC and par levels in order to minimize the total costs related to refilling and storing medications. Dobson et al. [17] performs analysis regarding set up time and holding cost with regards to compounded sterile products to determine the batching process.

Some studies have tried to quantify a cost savings associated with using ADCs. Financial analysis for a single ADC by Chapuis et al. showed that when implementing ADCs within a hospital intensive care unit, that in addition to fewer missing medications, there were more pharmacy technician hours to complete floor stocking activities, and a reduction in total cost. This study did operate the ADCs differently than the traditional model by also implementing their cart fill and ADC design together by including a bin for each patient within the ADC [12]. A study of hospitals in France which looked at implementing ADCs over a seven year period also found cost savings; however, this work did not consider the change in nurse time for pharmacy activities [5]. The findings of these papers are not consistent across all ADC studies in the literature, in particular [25] shows the opposite, and could be a result of the particular unit and hospital system.

A review of current practice shows that pharmacists generally favor the decentralized model as a result of its ability to reduce errors and improve the patient experience; however, classic supply chain literature shows that when demand is unknown, as with the hospital pharmacy, and inventory is consolidated in a central location, demand and supply risk can be lowered to reduce the overall system cost [13]. One study performed at a 561 bed hospital in Wisconsin weighed the impact of shifting different amounts of unit dose prescription fills from the ADC to cart fill. Deviating from the original layout of 64% cart fill and 36% ADC storage, the team ran simulations to test a 100% ADC-use model, a 100% cart fill model, and a model with 89% ADC and 11% cart fill. Since nursing time related to pharmacy activities increased unfavorably with increased ADC use, it was determined that complete decentralization was not ideal and the current level of decentralization, 36% ADC storage, was maintained [25]. One study conducted in the Netherlands compared the efficiency of central prescription fill with decentralized prescription fill at a childrens hospital. Management saw consistently lower prescription-fill nursing labor requirements by filling prescriptions with pharmacy technicians in the central pharmacy compared to nurses retrieving prescriptions from the ADCs on the unit. When using the central pharmacy total cost remained the same and total labor slightly increased but the personnel mix was considered more favorable due to the increase in direct patient care time for nurses. The study concluded that economies of scale, as a result of inventory consolidation in the central pharmacy, coupled with pharmacy technician expertise were likely the causes of this reduction [48].

In the 1990s, hospitals began utilizing robots within the central pharmacy to pick medications to fill inpatient orders. With robotics, hospitals have seen a decrease in medication error rates for picking in the central pharmacy. Compared to the robots of the 1990s, the newer robots are even more efficient. For example, the robot implemented in the central pharmacy at UCSFs Mission Bay hospital had zero incorrect picks over the period from its installation in 2011 until 2015 [7]. Most of the dispensing robots currently utilized focus on picking unit dose items. However, some relatively new dispensing robots can also be equipped to handle oddly shaped items other than pills. For example, a robot at the Brigham and Womens/Dana-Farber Cancer Care Center was used for IV preparation of toxic chemicals. Use of the robot, however, did not alter the frequency of medication errors. Despite its current inability to reduce errors, the robot was able to protect the staff from coming in contact with dangerous chemicals and, as technology advances, IV robots should eventually be able to prepare IV bags more efficiently than people. With the growth of automation, there has been a growth of literature evaluating using a central pharmacy in a hospital versus a decentralized model.

According to the American Society of Health-System Pharmacists national survey, only 8.4% of all hospitals use robots to pick maintenance doses. However, larger hospitals fulfilling maintenance doses from the central pharmacy are more likely to use a robot [50]. Another study showed consistent decreases in labor times for pharmacists and technicians after the implementation of an automated dispensing robot in a large pharmacy [39]. At the University of Wisconsin Hospital and Clinics, the central pharmacy used a daily cart fill process to deliver maintenance doses to patients. In 2014, the pharmacy switched from a once daily cart fill to thrice-daily cart fill which reduced the lead time for three of the four peak medication administration times. Evaluating both systems, they showed that with thrice-daily cart fill the average number of cart fill doses per day increased while the average number of first doses dispensed from the central pharmacy, the average number of doses returned to the central pharmacy, and the number of missing medications all decreased compared to the once daily cart fill [37].

The literature we have found concerning hospital pharmacies focuses on either decentralizing or centralizing a single hospital with no description of how the individual hospital operations work. In comparison to the previous hospital pharmacy literature, our model focuses on determining how specific medications should be routed through the hospital system to reach the patient both safely and cost-effectively. From the results of our model, we look to determine if the pharmaceutical operations for a single hospital should be centralized. In addition, most of the literature rationalizes whether ADCs or a robot is worth the expense, depending on which technology the hospital implemented. Although in reality most pharmacies use a mix of these technologies, only [25] considers a mix of technology in the analysis. In contrast to the articles cited above, we focus on determining if these technologies can be used to complement each other within the same system and what degree of utilization of each delivery method provides the best reduction in cost and improved patient care.

2.3 PHARMACY DISTRIBUTION MODEL FOR A SINGLE HOSPITAL

The goal of our model is to minimize the total unit dose distribution costs in the hospital by determining the most cost-effective manner to route unit dose medications from the central hospital pharmacy to the units while also reducing the number of missing doses. Our main decision is to determine a pathway to route each medication and dose type to each unit when we consider the average daily demand. We limit the number of missing doses by constraining the average number of missing doses allowed on each unit to an acceptable threshold.

Within pharmacy distribution, the delivery costs include unit picking and delivery costs, robot purchasing and maintenance, ADC purchasing and maintenance, and robot and ADC restocking. In addition, more complicated costs that fluctuate based on pathway interactions include a cart fill preparation cost and the ADC nurse queueing costs. Note we do not consider costs resulting from inventory holding as the total amount of medication in the system does not vary significantly between the different pathway options. We also do not consider perishability as this model is for unit dose medications which have a relatively long shelf life, typically longer than one year. As more medication doses are dispensed through an ADC, the percentage of patients that require cart fill decreases while the time nurses are required to queue at the ADC increases. Figure 2.4 depicts the piecewise linear function we will use to approximate the fraction of patients that will be using cart fill as a function of the number of doses that have a primary pathway through the ADC. The dots on the graph represent the average fraction of patients who would receive medication via cart fill based on the number of doses stored in the ADC. These points were generated by analyzing a year's worth of data and randomly assigning a percentage of the medications to be delivered through the ADC. At each percentage, there were 20 random samples generated that were used to determine the average percentage of patients receiving doses via cart fill was calculated. The bold line is the piecewise function that is used within the linear model to determine the cost of routing medications to the patient via cart fill. Note that the fraction of patients receiving doses via cart fill and the fraction of doses stored in the ADC will not equal one because patients can receive doses from both the cart fill and the ADC.

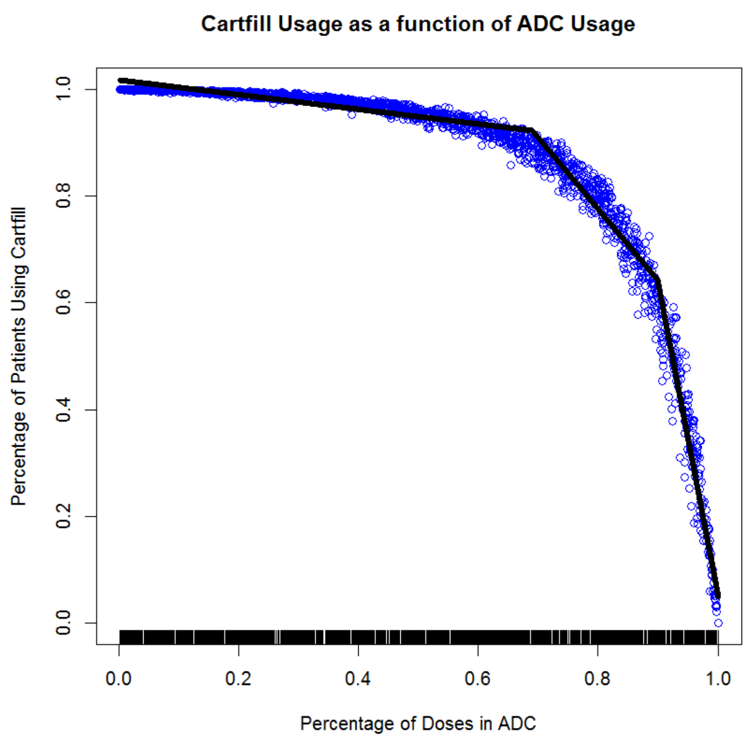

Figure 2.4: The percentage of patients using cart fill as a function of the total doses distributed from the ADC

Throughout the day, there are four major medication administration times: 8a, 12p, 4p, and 8p. Note, these times are specific to the hospital we analyze in our problem so other hospitals may have a different number of major medication administrations that can occur at different times of the day. We will assume that during every major medication administration time, the nurse visits the ADC once to get all of their patients medications. Note, that this assumption represents what often occurs in practice, even though ideally a nurse would only handle one patient's medications at a time. Though the nurses care for multiple patients, the use of bar code scanning, which requires nurses to scan the medication and the patient's wristband at the bedside to ensure that nurses are administering the correct medication to the correct patient, allows them to pick medications for multiple patients at a single time. Each time the nurse visits the ADC, they wait for some amount of time which is determined based on the function in Figure 2.5. Figure 2.5 uses the two points from two separate hospitals in [25] which are the total percentage of doses routed through the ADC in each hospital and the average nurse queueing time at each hospital to create a linear function to approximate the number of seconds the nurse will wait until it is his/her turn to retrieve medication from the ADC. We assume that time will grow linearly with the percentage of doses routed through the ADC. Although these time measurements were given as averages across the hospital, we assume that the average hospital behavior can be extended to the individual units as well. We multiply the average nurse waiting time by the number of nurses on the unit, the number of large medication administrations, and the average nurse salary to determine the nurse queueing time cost. Note that a large medication administration refers to a time of day when a large portion of maintenance medications are scheduled to be administered.

In the following model we focus on defining a standard pathway to every unit within the hospital for delivering each medication and dose type to the unit. While we note that there will be circumstances where other routes are required to be taken, we want to define a pathway for the standard case. By defining these pathways, we can reduce costs and later determine improvements that can lead to a more cost effective delivery route with fewer medication errors. The model indices, variables, and parameters are given in Tables 2.1, 2.2, and 2.3.

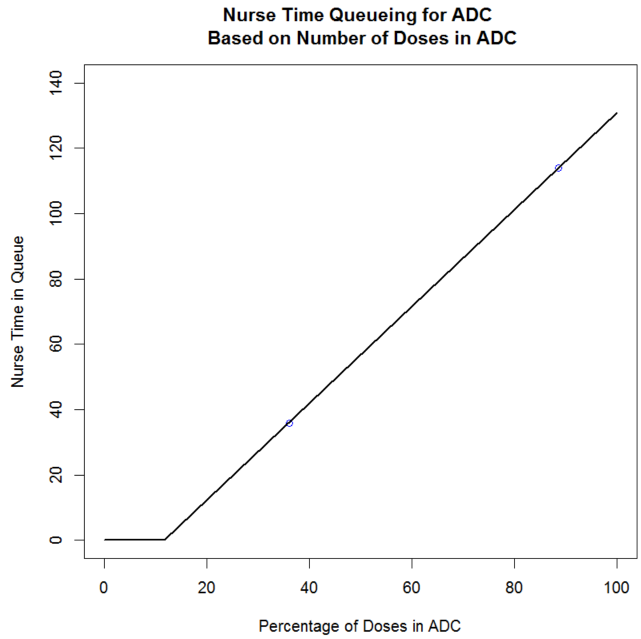

Figure 2.5: The time nurses spend in the ADC Queue as a function of the percentage of total doses distributed from the ADC

Table 2.1: Index notation for the pharmacy pathway model

Index	Description
i	Medication Type
j	Dose Type
k	Unit
l	Pathway
q	Medication dose type combination that is distributed through the ADC
$Tech$	Pharmacy Technician
$Nurse$	Nurse

Table 2.2: Variable notation for the pharmacy pathway model

Variable	Description
u_{ijkl}	Indicator that medication i with dose type j is delivered to unit k via pathway l
x_{ikq}	Indicator that dose type combination q is stored in the ADC on unit k for medication i
w_k	Number of ADCs required on unit k
γ	Indicator that a robot is needed in the hospital pharmacy
β_i	Indicator that medication i is routed through the hospital robot
ϕ_k	Average time nurses queue at the ADC on unit k
χ	Variable representing the percent of patients using cart fill
Υ	Binary variable determining the segment of the piecewise function for cartfill delivery cost to use based on the percentage of doses routed through the ADC

Table 2.3: Parameter notation for the pharmacy pathway model

Parameter	Description
ξ_{Tech}, ξ_{Nurse}	Average salary of pharmacy technician and nurse respectively
Λ_l	Inventory replenishment cost for technology used in pathway l
Δ	Average number of medications that can be stored on a single rod in the robot
Ξ	Average census of the hospital
e	Cost to label patient envelope
Ω	Number of large medication administrations that occur in the hospital
N_k	Number of nurses on unit k
p_{ikq}	The number of bins in the ADC required to hold the par level of medication i on unit k when dose combination q is held in the ADC
r_q	The number of dose types stored in the ADC under dose combination q
B	The total number of bins in an ADC
s_k	The maximum number of ADCs that can be used in unit k
c_{ijkl}	The delivery cost of medication i to unit k via pathway l for dose type j
d_{ijk}	The average daily demand of medication i with dose type j on unit k
g	Cost to buy and maintain a robot
h	Cost to buy and maintain an ADC
n	Throughput capacity of the robot
ν	Unique medication capacity of the robot
μ_{jl}	The percentage of missing doses resulting from dose type and delivery pathway l
MMD	The maximum number of missing doses allowed in the system

Using the notation given in Tables 2.1, 2.2, and 2.3 we formulate the linear program given in equations (2.1 - 2.23). Note, that for the dose type we define $j = 1$ as “STAT Dose,” $j = 2$ as “First Dose,” $j = 3$ as “Maintenance Dose,” and $j = 4$ as “PRN Dose.” For the pathways, we define $l = 1$ to use the robot to pick the dose, $l = 2$ to use the pharmacy technician to pick the dose, and $l = 3$ to store the medication in the ADC on the unit.

$$\begin{aligned} \text{Min} \quad & \sum_{i,j,k,l} c_{ijkl} d_{ijk} u_{ijkl} + g\gamma + \sum_k h w_k + \Lambda_1 \sum_{i,j,k} d_{ijk} u_{ijk1}/\Delta \\ & + \Lambda_3 \sum_q x_{ikq} + \Xi e \left(\chi_0 + 0.91\chi_1 + 0.65\chi_2\right) + \sum_k \xi_{Nurse} \Omega N_k \phi_k \end{aligned} \tag{2.1}$$

$$\text{s.t.} \quad \sum_l u_{ijkl} = 1 \ \forall i, j, k \tag{2.2}$$

$$u_{i0k1} = 0 \ \forall i, k \tag{2.3}$$

$$u_{i1k3} \geq u_{i2k3} \ \forall i, k \tag{2.4}$$

$$u_{i2k3} \geq u_{i3k3} \ \forall i, k \tag{2.5}$$

$$u_{i1k3} \geq u_{i4k3} \ \forall i, k \tag{2.6}$$

$$\sum_q r_q x_{ikq} = \sum_j u_{ijk3} \ \forall i, k \tag{2.7}$$

$$\sum_{q=4}^{6} x_{ikq} = u_{i4k3} \ \forall i, k \tag{2.8}$$

$$\sum_q x_{ikq} \leq 1 \ \forall i, k \tag{2.9}$$

$$\sum_{i,q} p_{ikq} x_{ikq} \leq B w_k \ \forall k \tag{2.10}$$

$$w_k \leq s_k \ \forall k \tag{2.11}$$

$$\sum_{i,j,k} d_{ijk} u_{ijk1} \leq n\gamma \tag{2.12}$$

$$\sum_{j,k} u_{ijk1} \leq JK\beta_i \ \forall i \tag{2.13}$$

$$\sum_i \beta_i \leq \nu\gamma \tag{2.14}$$

$$\sum_{i,j,l} \mu_{jl} d_{ijkl} u_{ijkl} \leq MMD \ \forall k \tag{2.15}$$

$$\chi_0 + \chi_1 + \chi_2 + \chi_3 = 1 \tag{2.16}$$

$$\chi_0 \leq \Upsilon_0 \tag{2.17}$$

$$\chi_1 \leq \Upsilon_0 + \Upsilon_1 \tag{2.18}$$

$$\chi_2 \leq \Upsilon_1 + \Upsilon_2 \tag{2.19}$$

$$\chi_3 \leq \Upsilon_2 \tag{2.20}$$

$$\Upsilon_0 + \Upsilon_1 + \Upsilon_2 = 1 \tag{2.21}$$

$$\frac{\sum\limits_{i,j,k} d_{ijk} u_{ijk3}}{\sum\limits_{i,j,k} d_{ijk}} \geq 0.6903\chi_1 + 0.8999\chi_2 + \chi_3 \tag{2.22}$$

$$\phi_k \geq 148 \frac{\sum\limits_{i,j} d_{ijk} u_{ijk3}}{\sum\limits_{i,j} d_{ijk}} - 17.32 \;\forall k \tag{2.23}$$

Equation (2.1) is the objective function which minimizes the average daily cost to deliver medication from the pharmacy to the patient. These costs include the average demand picking and administration costs, the robot purchase and maintenance cost, the ADC purchase and maintenance cost, restocking costs, cart fill patient costs, and nurse queueing costs. The constraints for the linear program are given by equations (2.2-2.23). Equation (2.2) requires each medication, dose type, and unit to have a standard pathway while equation (2.3) prevents STAT doses from being routed through the robot. Equations (2.4-2.6) ensure that if first doses are kept in the ADC then STAT doses are kept in the ADC, if maintenance doses are kept in the ADC then first doses are kept in the ADC, and if PRN doses are kept in the ADC then STAT doses are kept in the ADC. Equations (2.7-2.9) determine which dose types are held in the ADC on the unit for each medication and allow only one combination of dose types to be stored in the ADC. Equation (2.10) determines the number of ADCs necessary to store the medications for the unit while equation (2.11) limits the number of ADCs that can be on the unit. Equation (2.12) determines if a robot is needed for the hospital and limits the number of medications distributed through the robot based upon its throughput capacity. The medication routed through the robot is limited by the unique medication capacity in equations (2.13-2.14). Equation (2.15) ensures that the expected number of missing doses is lower than the missing dose threshold on every unit of the hospital. The constraints given in

Equations (2.16-2.21) determine the value of the piecewise linear function given in Figure 2.4 representing the cartfill delivery costs. Equation (2.22) determines the percentage of total doses delivered by the ADC and equations (2.17-2.21) determine the corresponding segment of the piecewise function that is used to determine the cartfill delivery cost. Equation (2.23) is the corresponding constraint for the nurse queueing time at the ADC for each unit based on Figure 2.5.

Note that in this problem we seek to minimize cost while maintaining a particular level of patient safety, as denoted by the missing dose threshold. However, this linear program can be rewritten with the objective of minimizing the number of missing doses while preventing cost from exceeding a particular budget. In addition, this problem can be considered as a multiobjective optimization problem where our first objective is minimizing cost and our second objective is minimizing missing doses.

2.3.1 Data Summary

The Geisinger Health System (GHS) is an integrated health services organization which serves more than 3 million residents throughout 45 counties in central, south-central, north-east Pennsylvania, and southern New Jersey. For this study we choose to specifically focus on all inpatient units at the Geisinger Medical Center (GMC), the primary hospital for GHS. GMC has 24 inpatient units with 560 beds. For inpatient units, the pharmacy manages 3,350 medications and has to process roughly 2,300 medication orders every day, dispense 8,800 medications per day, and nurses administer 8,600 medications every day.

In our analysis we will focus on analyzing the data for all medications that are ordered, dispensed, and administered to the inpatient units. For purposes of our problem, we focus on uncontrolled medications which can be delivered via a unit dose form. There are 1,105 uncontrolled unit dose medications, which is roughly one third of all medications dispensed to patients on the inpatient units. Unlike controlled medications which have restrictions on how they can be filled, these types of medications can be delivered to the patient via the ADC or cart fill. Furthermore, unit dose medications have a longer shelf life than other medications, typically up to a year or more; therefore, in this context perishability can be omitted from the analysis. The five most commonly ordered medications are given in Table 2.4 along with the number of orders, dispenses, and administrations on all inpatient units. Note, these medications are only a small percentage of pharmacy operations, accounting for 7.5% of all medications ordered, 11.2% of medications dispensed, and 10.0% of medications administered.

Currently each of these medications can be prepared by a pharmacy technician in the hospital pharmacy, picked by the robot and delivered via cart fill, or picked from the ADC by the nurse. Looking at all medications, Geisinger currently has five primary locations which the medication can come from: the pharmacy robot, inpatient pharmacy, ADCs, the pediatric pharmacy, and the operating room pharmacy. Table 2.5 shows the percentage of all medication volume that is dispensed from each pharmacy and the percentage of unit dose medications dispensed from each pharmacy. As the table shows, under current operations, the unit doses are very close to completely centralized with almost 90% of all unit doses being dispensed using the pharmacy robot.

Table 2.6 shows the volume of dispenses from each pharmacy for the most commonly ordered uncontrolled unit medications. The percentage of each dose type is given in Table 2.7 and the percentage of each dose type for the five most commonly ordered medications is given in Table 2.8. Within the five most commonly ordered medications there are a variety of demand profiles. Acetaminophen, in particular, has a high number of PRN doses and potassium chloride has a high number of STAT doses dispensed. The other three drugs have a similar demand profile with regards to the dose type.

Table 2.4: The five most commonly ordered medications on inpatient units

Medication	Acetaminophen 325 MG PO Tabs	Docusate Sodium 100 MG PO Caps	Potassium Chloride CRYS ER 10 MEQ PO TBCR	Omeprazole 20 MG PO CPDR	Sennosides 8.6 MG PO Tabs
Yearly Orders	20,050	14,339	13,202	12,617	9,751
Yearly Dispenses	117,400	100,151	23,015	61,900	50,196
Yearly Administrations	103,360	86,940	19,899	57,624	45,991

Table 2.5: Pharmacy dispenses for all medications for all GMC inpatient units

Pharmacy	Percentage of All Medications	Percentage of Unit Doses
GMC Robot	49.6%	89.4%
GMC Inpatient Rx	24.9%	3.9%
ADC	21.4%	5.8%
GMC Pediatric Pharmacy	2.3%	0.9%
GMC OR Pharmacy	1.9%	0.0%

Table 2.6: Pharmacy dispenses to GMC inpatient units for the most commonly ordered medications

Pharmacy	Acetaminophen 325 MG PO Tabs	Docusate Sodium 100 MG PO Caps	Potassium Chloride CRYS ER 10 MEQ PO TBCR	Omeprazole 20 MG PO CPDR	Sennosides 8.6 MG PO Tabs
GMC Robot	61.1%	96.1%	57.8%	99.3%	99.4%
GMC Inpatient Rx	0.4%	0.4%	0.7%	0.4%	0.3%
ADC	36.9%	2.3%	41.2%	0.0%	0.0%
GMC Pediatric Pharmacy	1.6%	1.2%	0.3%	0.4%	0.3%
GMC OR Pharmacy	0.0%	0.0%	0.0%	0.0%	0.0%

Table 2.7: Pharmacy dispenses for all medications across GMC inpatient units

Dose Type	Percentage of All Medications	Percentage of Unit Doses
STAT	7.5%	4.6%
First Dose	11.9%	13.4%
Maintenance Dose	66.4%	76.4%
PRN Dose	14.3%	5.6%

Table 2.8: Pharmacy dispenses to GMC inpatient units for the most commonly ordered medications

Dose Type	Acetaminophen 325 MG PO Tabs	Docusate Sodium 100 MG PO Caps	Potassium Chloride CRYS ER 10 MEQ PO TBCR	Omeprazole 20 MG PO CPDR	Sennosides 8.6 MG PO Tabs
STAT	3.0%	2.5%	39.1%	3.3%	4.6%
First Dose	4.7%	10.0%	17.2%	15.4%	12.7%
Maintenance Dose	62.1%	85.8%	41.1%	81.1%	80.6%
PRN Dose	30.3%	1.7%	2.6%	0.1%	2.1%

2.3.2 Results

We focus on determining the standard pathway for all medications which are in a unit dose form. These medications can be dispensed by the robot, through the ADC, or by a pharmacy technician in the central pharmacy. Therefore, they can utilize any of the three pathways available. The parameters used to solve model (2.1) are given in Table 2.9. Note, for ADC capacity we assume that only 8 drawers within the ADC can be used to hold unit dose medication in order to leave space for controlled medications which must be delivered via the ADC. We solve the model for the current ADC allocation on each floor. Several other parameters that were used within the linear program were derived from the results of the time study and findings of [25] and the knowledge of pharmacy experts at Geisinger Medical Center. Due to the size of the problem, we solve the model for the five inpatient units that have the highest unit dose medication demand among all inpatient units in the hospital.

Table 2.9: Parameter values

Parameter	Description	Value
B	The total number of bins in an ADC	192
c_{ijk1}	The delivery cost per dose of medication i dose type j to unit k when a maintenance dose is picked by the robot	\$0.21
c_{ijk2}	The delivery cost per dose of medication i dose type j to unit k when a maintenance dose is picked by the pharmacy technician	\$0.57
c_{ijk3}	The delivery cost per dose of medication i dose type j to unit k when a maintenance dose is picked via the ADC	\$0.38
g	Daily cost to buy and maintain a robot	\$700.84
h	Daily cost to buy and maintain an ADC	\$17.70
n	Throughput capacity of the robot	4,200
ν	Unique medication capacity of the robot	500
Λ_1	Robot dose restocking cost	\$0.0043
Λ_3	ADC medication restocking cost	\$0.45
e	Cost to label patient envelope	\$0.033
$\mu_{2,1}$	The percentage of missing doses resulting from first doses delivered by the robot	25.0%
$\mu_{3,1}$,$\mu_{4,1}$	The percentage of missing doses resulting from maintenance and PRN doses delivered by the robot	0.75%
$\mu_{1,2}$,$\mu_{2,2}$	The percentage of missing doses resulting from STAT and first doses delivered by the pharmacy technician	25.0%
$\mu_{3,2}$,$\mu_{4,2}$	The percentage of missing doses resulting from maintenance and PRN doses delivered by the pharmacy technician	1.5%
μ_{j3}	The percentage of missing doses resulting from the ADC pathway for all doses	0.35%

The values for d_{ijk} are calculated based on the average number of dispenses for each medication to each unit of the hospital over the course of a year. While the number of doses of each medication dispensed to the units varies day to day, there is not high seasonality experienced on the units. Therefore, we only use the average values within this analysis. However, future analysis of the problem will include considering the variation that exists in our demand. In this scenario, we only consider the current inventory policy at Geisinger Medical Center for supplying the ADC, which has a max par level of three times the average daily demand and a min par level of one and half times the average daily demand. The values of p_{ik}, q_{ik}, and r_{ik} are calculated based on the values of d_{ijk} and the number of doses that can be placed in a single bin of a specific medication.

While pharmacy technology can greatly improve patient safety, it can also be very costly. Therefore, it is important that the hospital has the correct pharmacy technology to enable them to deliver medication to the patient in a safe and cost effective manner. The graph in Figure 2.6 shows the tradeoff curve for the optimal average daily cost of pharmacy operations across the five units we analyzed versus the value of the missing dose threshold. There is also a comparison with the current missing dose rate and average daily delivery cost at the hospital setting. The hospital is currently distributing medication to the inpatient units for approximately an extra \$300 compared to the optimal solution; therefore, there is a significant potential cost savings from using the proposed model.

For every missing dose threshold, the volume of medication dispenses that are processed by the robot, pharmacy technicians, and ADCs for each optimal solution are shown in Figure 2.7. When our missing dose rate is unconstrained, every unit has a missing dose rate less than 6.22%. For the demand at GMC, the robot is always the most cost effective option and we route the 500 medications with the highest demand through the robot and all remaining doses are routed through the pharmacy technician. Without considering a missing dose threshold, the ADC option is not cost effective.

In order to determine how changing the missing dose threshold affects the optimal solution, we decrease the missing dose threshold by increments of 0.05%. As we begin to limit the number of missing doses, we must use an ADC in order to reach the desired missing dose level. However, we store a minimal amount in the ADC with most doses being processed

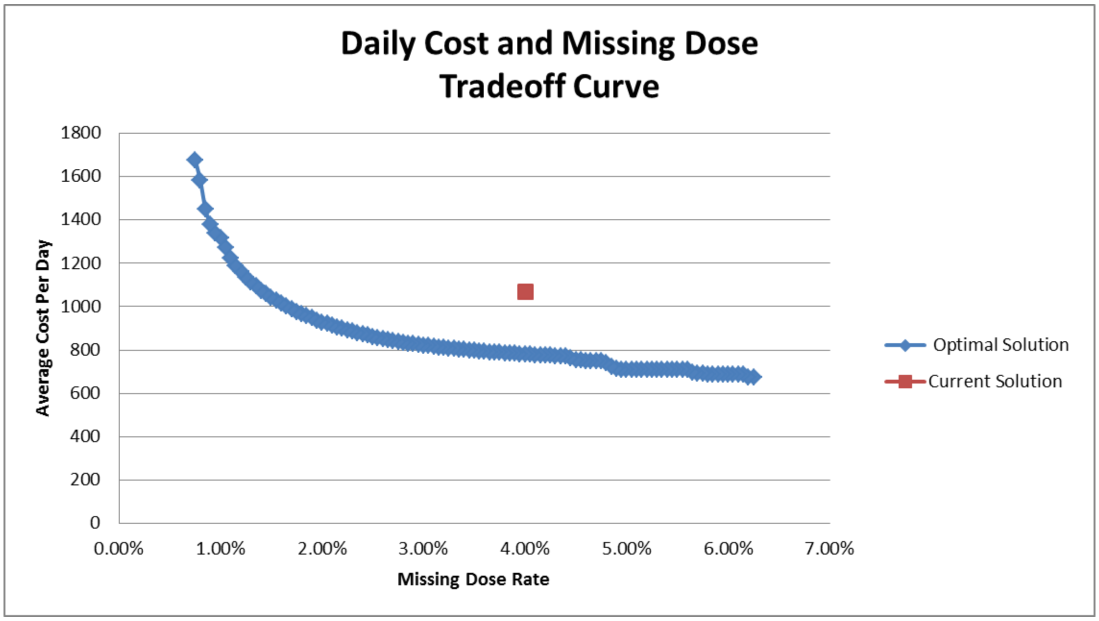

Figure 2.6: The optimal average daily cost of pharmacy operations based on the missing doses constraint

through the robot. When we reach a missing dose threshold of 1.00% the robot is no longer cost effective because it is cheaper to route doses through the tech, rather than the robot and all doses are routed through the ADC and the tech. When the missing dose rate decreases to 0.80% utilizing only the ADC and the pharmacy technician no longer enables us to reach the missing dose threshold, so we utilize both the robot and the ADC again. When we begin utilizing both technologies, we see a sharp increase in total cost because we are routing a very small amount of medications through the robot despite the high set up cost associated with the robot. However, in order to reach the missing dose threshold, the ADC is the most highly utilized delivery pathway while the robot is only processing a small percentage of the doses. When the missing doses threshold decreases to less than 0.75% the problem is no longer feasible and we cannot reach the missing dose threshold.

Figure 2.7 clearly displays that the pharmacy technology the hospital needs to purchase depends on the missing dose rate the hospital wishes to attain. For instance, if a hospital wishes to maintain a missing dose level higher than 6.22%, then the entire system can be

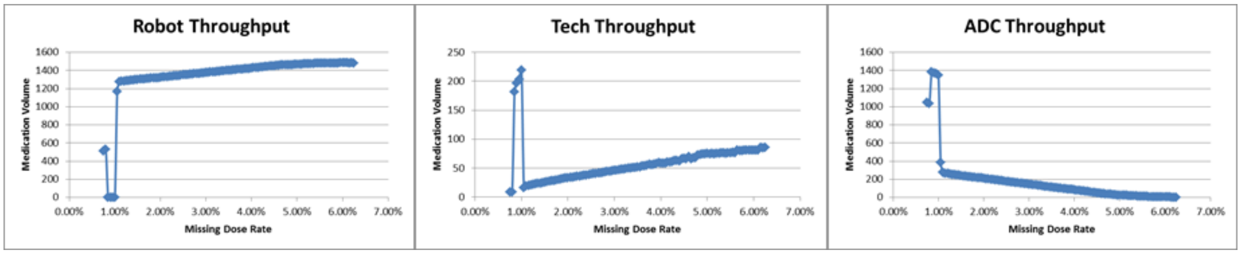

Figure 2.7: Volume of dispenses through each pathway as a function of the missing doses constraint

centralized and require no ADCs. In this scenario, all first, maintenance, and PRN doses are processed through the robot for the 500 medications with the highest demand and the pharmacy technician processes all STAT doses and remaining medications not held in the robot. Similarly, if the goal is to have a missing dose level between 0.80% and 1.00%, then the most cost effective option does not require the use of a robot. However, there are intervals during which both technologies are utilized to limit the number of missing doses. When both technologies are used, the level of centralization is based on the unit and the number of doses being processed through the ADC compared to the central pharmacy.

One consideration we made in our linear program is that a medication could be routed through different pathways based on the dose type. The linear program demonstrates that determining the primary pathway of a dose type for a medication relies not only on the missing dose rate but also on the volume of doses given to patients on the unit. Note that the missing dose rate varies upon dose type; however, the costs are constant regardless of dose type and only vary depending on the unit demand. Focusing solely on the number of each dose type delivered via each pathway, we find that STAT doses and first doses are added to the cabinet first as they have the highest missing dose rate. Meanwhile, maintenance doses and PRN doses are delivered by the robot for as long as possible since it is the most cost effective pathway. When the missing dose rate is less than 0.80% and both the robot and ADC are required to reach the missing dose tolerance goal, the robot will process mostly maintenance and PRN doses as before, but in smaller quantities with the remainder of doses processed through the ADC to reduce the missing dose rate.

2.4 HEURISTIC

Due to the complexity and size of the MIP along with the practical constraint of having limited computer resources, we were only able to run the linear program for five units. When solving the problem, we set an optimality gap of 0.25% and the problem solves for each missing dose rate from 6.25% to 0.40% in 10.75 hours. (We intentionally used a typical personal computer for processing to determine generalizability to industry users.) However, Geisinger Medical Center has twenty-four inpatient units that we would like to solve this problem for. In order to do this, we have created a heuristic which can be used to solve the problem for all twenty-four inpatient units when there are no space limitations on the number of ADCs that can fit on a floor. The heuristic is programmed in C++ and generates a solution for each missing dose rate from 6.25% to 0.40% in 102 seconds.

The heuristic begins by finding the solution for the minimum cost without constraining the number of missing doses, which we will call the base case. In order to determine the base case, we need to determine the conditions under which each pathway is the best delivery method. We use the parameters given in Table 2.9 and the average quantities for bins per medication and doses per medication to determine the conditions under which each pathway is most cost effective. Based on the solutions from these equations, Heuristic 1 determines the best delivery pathway for each medication, dose type, and unit.

The base case begins by assigning all doses to be delivered via the pharmacy technician. The first step is to sum the total number of doses given of each medication that can be routed through the robot, $PossibleRobotDoses$, and the number of doses that can be routed through an ADC on unit k, $PossibleADCDoses$. Note, the robot cannot process STAT doses. These values are then sorted from greatest to least. If the number of doses for the first 500 medications is larger than the breakeven point, a function of the picking and administrative costs for the robot and the pharmacy technician and the cost of the robot, then the robot is the most cost effective pathway to deliver the medications and they are routed through the robot. We check if the ADC is a more cost effective form of delivery than the pharmacy technician in lines 20-35 by adding medications that are delivered by the pharmacy technician to the ADC and testing the cost effectiveness of the ADC. In the event the ADC is more cost effective, then we reroute the medications to be delivered via the ADC.

Heuristic 1 Base Case Delivery Method

for all i, j, k **do**
    $DeliveryMethod_{ijk} = 2$
end for
for all i **do**
    $PossibleRobotDoses_i = \sum_{j=2}^{4} \sum_{k} d_{ijk}$
    for all k **do**
        $PossibleADCDoses_{ik} = \sum_{j} d_{ijk}$
    end for
end for
Sort $PossibleRobotDoses$ from greatest to least
for all k **do**
    Sort $PossibleADCDoses$ from greatest to least for every k
end for
if $\sum_{i}^{500} PossibleRobotDoses_i \geq g/0.35633$ **then**
    $DeliveryMethod_{ijk} = 1$
end if
$ADCDoses_k = 0$
$ADCMeds_k = 0$
$ADCComp_k = 0$
for all k **do**
    for all i **do**
        if $ADCComp_k + ceil(3 * PossibleADCDoses_{ik}/medSize_i) \leq 192$ and $DeliveryMethod_{ijk} == 2$ **then**
            $ADCDoses_k = ADCDoses_k + PossibleADCDoses'_{ik}$
            $ADCMeds_k = ADCMeds_k + 1$

$ADCComp_k = ADCComp_k + ceil(3 * r'_{ik}/medSize_i)$
end if
end for
if $(17.70+0.45*ADCMeds_k)/0.18661 \leq ADCDoses_k$ and $DeliveryMethod_{ijk} == 2$ **then**
for all i **do**
if $ADCComp_k + ceil(3 * PossibleADCDoses_{ik}/medSize_i) \leq 192$ **then**
$DeliveryMethod_{ijk} = 3$
end if
end for
end if
end for

Once the base case is established, we use the results from the base case to iteratively add medications to the ADC until we reach our missing dose rate target. Let $RobotThroughput_0$ be the number of doses routed through the robot in the base case. To decide which doses should be moved from the robot and the pharmacy technician to the ADC, we construct a ratio which captures the reduction in missing doses compared to the cost increase incurred. The ratio is used in two different methods to assign medications and dose types to the ADC. The method which gives the minimum cost is used to determine the best allocation of medications to each pathway. The first method considers each medication dose type independently when deciding which specific items to store within the ADC while the second method adds all dose types of a single medication to the ADC. Our third method takes the minimum cost of methods one and two to reach the final allocations. Both of these methods utilize the concept of putting items into the ADC that generate the largest reduction in MMD for each additional dollar spent.

The first method is given in Heuristic 2. The method first determines the MMD Reduction to Cost Increase Ratio (the MMD to Cost Ratio) which captures the missing dose reduction indicated by the additional cost of switching the delivery pathway for every medication, dose type, and unit from the original pathway identified in the base case to the ADC. We sort the MMD to Cost Ratio from greatest to least for every medication and dose type pair. Then, while we have not met the missing dose goal for an individual unit, we iteratively add a medication and dose type pair to the ADC based on the sorted MMD to Cost ratios. After an addition is made to the ADC for all units, if needed, we check if the robot has any additional unique medication capacity or dose capacity and add medications and dose types accordingly. After we have filled any additional robot capacity, we check if the robot is still cost effective based on the same set of conditions used in Heuristic 1. In the event the robot is not cost effective, then all doses routed through the robot are routed through the pharmacy technician instead. Taking into account all doses that are currently in the ADC and the total number of doses delivered to the unit, we consider each medication delivered through the ADC for each unit and determine if the ADC has available space and if any additional dose types can delivered more cost effectively by the ADC compared to the pharmacy technician. The last step is to recalculate the MMD and iterate through the steps above until the recalculated MMD is lower than the Goal MMD for every unit.

Heuristic 2 Method 1: Considering Each Dose Type

for all i, j, k **do**
    $Cost_{ijk1} = (g/RobotThroughput_0 + c_{ijk1} + \Lambda_1/\Delta)\, d_{ijk}$
    $Cost_{ijk2} = c_{ijk2} d_{ijk}$
    $Cost_{ijk3} = c_{ijk3} d_{ijk} + h/192\lceil 3d_{ijk}/medSize_i \rceil + 0.45/4$
    $MMDCostRatio_{ijkll'} = MMDReduction_{ijkll'} / \left(Cost_{ijkl'} - Cost_{ijkl}\right)$
    for all l, l' **do**
        $MMDReduction_{ijkll'} = \left(\mu_{jl} - \mu_{jl'}\right) d_{ijk}$
    end for
end for
for all k **do**
    Sort $MMDCostRatio_{ijkll'}$ from greatest to least where $l = DeliveryMethod_{ijk}$ and $l' = 3$

end for

$RobotOptimal = 0$

$Count_k = 1$

while $GoalMMD < MMD$ **do**

  for all k where $GoalMMD_k < MMD_k$ **do**

    if $DeliveryMethod_{ijk} \neq 3$ for $i, j = Count_k$ **then**

      $DeliveryMethod_{ijk} = 3$ for $i, j = Count_k$ in the sorted $MMDCostRatio$ vector

      Set $DeliveryMethod_{ij'k} = 3$ for all lower dose types, j', not already delivered by ADC

    end if

    $Count_k = Count_k + 1$

  end for

  Count $RobotMeds$, the number of unique medications delivered by the robot

  Sum $RobotDoses$, the total number of doses delivered by the robot

  for all $i, j > 1, k$ **do**

    if $RobotOptimal == 1$ and $RobotMeds < \nu$ and $RobotDoses < n$ **then**

      $DeliveryMethod_{ijk} = 1$

      Increment $RobotMeds$ and $RobotDoses$ accordingly

    end if

  end for

  if $RobotDoses < 897.5$ **then**

    for all i, j, k **do**

      if $DeliveryMethod_{ijk} == 2$ **then**

        $DeliveryMethod_{ijk} = 1$

      end if

    end for

    $RobotOptimal = 0$

  end if

```
39:     Recalculate MMD
40:     Calculate ADCPercentage_k, the total percentage of doses of all medications routed through the ADC on Unit k
41:     Calculate Total_k, the total number of doses delivered to Unit k of all medications
42:     for all i, j, k do
43:         Calculate PathwayDoses_ikl, the total doses of medication i delivered to unit k by pathway l
44:         if Medication i is delivered for through the ADC on unit k and 0.56713PathwayDoses_ik2 > 0.38052PathwayDoses_ik2 + 0.114 * 4 * N_k (1.48 * (ADCPercentage_k + PathwayDoses_ik2 * 100/TotalDoses_k)) then
45:             DeliveryMethod_ijk = 3
46:         end if
47:     end for
48: end while
```

The second method, Heuristic 3, does not consider each dose type but instead considers assigning every dose type of a single medication to the ADC. The medications are still placed into the ADC in such a way that we look to achieve the maximum missing dose reduction for each additional dollar spent to deliver the doses. This method follows the same set of steps as Heuristic 2 however, it does not need to consider adding additional doses types to the ADC as in lines 42-47 of Heuristic 2.

Heuristic 3 Method 2: Combined Dose Type

```
for all i, j, k do
    DeliveryMethod'_ijk = DeliveryMethod_ijk
end for
for all j do
    MMDCostRatio'_ikll' = Σ_j MMDCostRatio_ijkll'
end for
for all k do
```

Sort $MMDCostRatio'_{ikll'}$ from greatest to least where $l = DeliveryMethod_{i3k}$ and $l' = 3$
end for
$RobotOptimal \leftarrow 0$
$Count'_k \leftarrow 1$
while $GoalMMD < MMD$ **do**
  for all k where $GoalMMD_k < MMD_k$ **do**
    if $DeliveryMethod'_{ijk}! = 3$ for $i, j = Count'_k$ **then**
      $DeliveryMethod_{ijk} = 3$ for $i, j = Count_k$ in the sorted $MMDCostRatio$ vector
      Set $DeliveryMethod_{ij'k} = 3$ for all lower dose types, j', not already delivered by ADC
    end if
    $Count'_k \leftarrow Count'_k + 1$
  end for
  Count $RobotMeds$, the number of unique medications delivered by the robot
  Sum $RobotDoses$, the total number of doses delivered by the robot
  for all $i, j > 1, k$ **do**
    if $RobotOptimal == 1$ and $RobotMeds < \nu$ and $RobotDoses < n$ **then**
      $DeliveryMethod_{ijk} \leftarrow 1$
      Increment $RobotMeds$ and $RobotDoses$ accordingly
    end if
  end for
  if $RobotDoses < 897.5$ **then**
    for all i, j, k **do**
      if $DeliveryMethod_{ijk} == 2$ **then**
        $DeliveryMethod_{ijk} \leftarrow 1$
      end if
    end for

$RobotOptimal \leftarrow 0$
    end if
    Recalculate MMD
end while

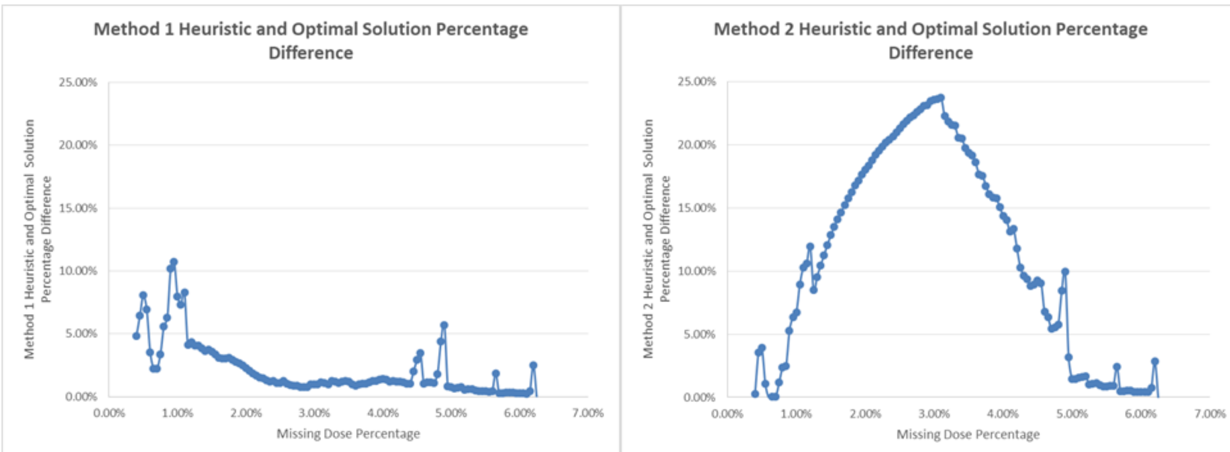

Figure 2.8: Percentage difference between optimal solution and heuristic solution for Method 1 and Method 2 separately.

Figure 2.8 shows the performance of Method 1 and Method 2 in comparison to the optimal solution for different missing dose rates. Figure 2.9 shows the performance of the combined method which takes the minimum solution from Method 1 and Method 2. The heuristic finds solutions that are very close to the optimal solution found by CPLEX. The overall performance of the heuristic is less than 9% above optimality for every missing dose rate that we tested and the average performance is within 1.74% of optimality. The four peaks that occur in Figure 2.9 between 4% and 6.25% in the missing dose rate range are a result of the purchase of an ADC. The highest variation between the heuristic and optimal solution occurs when we decide to no longer use the robot and begin purchasing more ADCs. This indicates that while the MMD to Cost Ratio is a good predictor when we are not considering space, the decision becomes more difficult when we take space limitations into consideration. The decision to no longer use the robot coincides with the use of Method 2 to allocated

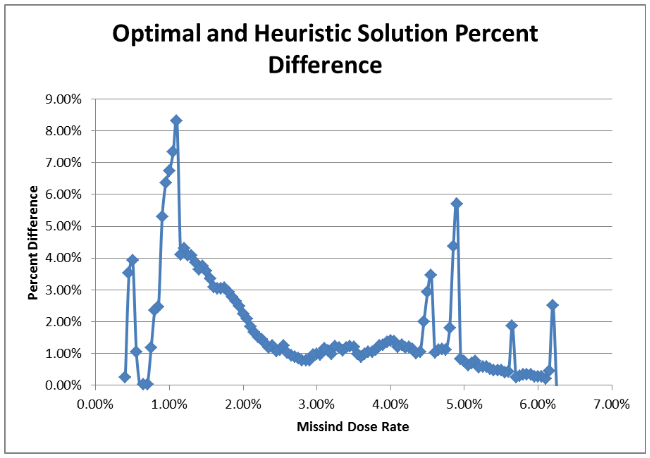

Figure 2.9: Percentage difference between optimal solution and heuristic solution

medications to the ADC. This indicates that once the units are distributing solely through ADCs and pharmacy technicians, space considerations are even more important in deciding which items to store in the ADC. Even though this heuristic performs relatively well, there are still peaks resulting from purchases of additional ADCs that are required by the heuristic but not by the optimal solution. These peaks indicate the complexity that exists with the space constraints in the problem.

2.5 CONCLUSIONS AND EXTENSIONS

In this paper, we constructed a novel linear program that takes a new approach to determining the effectiveness of pharmacy technology by capturing the entire inpatient pharmaceutical delivery process. This includes the time from when the medication enters the hospital pharmacy until it is received by the nurse on the inpatient unit. Compared to previous work, this model takes a system wide view of pharmacy distribution whereas many of the previous works had a myopic view, considering only the pharmacy or the unit. When we solved the model for GMC, we created a tradeoff curve which shows the minimum cost at multiple missing dose levels. We see that in order to attain the minimum cost, we begin by adding in medications which have a large number of STAT and first doses to the ADC and then slowly adding in maintenance doses. Using the knowledge of how items are added into the ADC, we create a heuristic which uses a ratio between the missing dose reduction and cost increase to add medications to the ADC. For our example, the heuristic solves the problem within 9% of optimality and overall performance is within 1.74% of optimality.

Our current model has several limitations. First, we only consider uncontrolled unit dose medications, as these are the only types of medications that can be processed by the robot and assume all other medications needed on the floor can fit into 5 drawers of an ADC. In addition, the inventory policies we use to fill the ADC are not necessarily optimal policies as they are the current inventory policy the hospital uses. Within our model, all costs are based on average measurements such as the daily average number of doses and average costs. Our current formulation considers each medication of equal importance with respect to missing doses; however, some missing doses have more consequences with regards to patient safety than others. Finally, while we consider all the tasks that are required in the dispensing and administration of the medication among the three pathways, we do not consider the additional cost of nurse time used to report and replace a missing dose. Taking all these limitations in mind we can make improvements on the model by considering the following: including uncontrolled unit doses and non-unit dose medications within the model, considering alternate inventory policies, considering uncertainty within demand, adding a weighting factor to the objective and/or missing dose constraints to indicate the importance of the missing dose based on medication type, and considering nursing effort with regards to missing doses.

By considering controlled unit dose medications and all other medications types, we can better assign medications to the ADC based on the space these types of medication will take. In addition, we can improve the costs by using better inventory policies that are tailored to each medication compared to the standard policy employed by the hospital. While the averages do show policies that would help us better stock the ADC, we know that the medication demanded on the floor varies based upon the patient population. To ensure that no day performs significantly worse than all other days, we can formulate a robust model where our uncertainty sets will be based on historical demand on the units. In addition, we can consider different inventory policies with the goal of reducing the cost as a result of better managed inventory. The model we created accurately portrays the delivery of medication within a single hospital. We can extended our formulation to a hospital system to aid in system wide decisions such as a centrally located pharmacy warehouse and system wide delivery pathways.

3.0 OPTIMIZING THE MEDICATION DISTRIBUTION PROCESS FOR INPATIENT UNITS

3.1 INTRODUCTION

The hospital pharmacy is responsible for providing patients their medications at scheduled administration times. There are currently multiple pathways, including cart fill and automated dispensing cabinets (ADCs), for medication to travel from the pharmacy to the inpatient unit, creating a large and challenging logistics problem. When determining the pathways to distribute medication to the patient, one measure of quality that is often evaluated is the percentage of missing doses. We define a missing dose as any dose that is not given to the patient within an hour of the scheduled administration time.

To reduce the number of missing medications and medication errors, hospitals often utilize a robot within the central pharmacy or ADCs on the units to dispense medications. The robot picks with high accuracy; however, missing doses can result while transporting the dose from the central pharmacy to the unit. When medication is kept in an ADC it is stored at the point of use and missing doses only occur when there is a stock out or the nurse does not administer the medication. While both of these technologies are effective at reducing the number of missing doses, they are rarely employed within the same pathway.

The probability of experiencing a missing dose depends on the pathway through which the medication is distributed and the technology utilized within the pathway. We focus on three standard pathways for routing medication from the pharmacy to inpatient units: cart fill via robot, cart fill via pharmacy technician, and ADCs. While we acknowledge that there are other pathways used to distribute medication to inpatient units, these three pathways are the standard method used for most distributing medications.

In addition to the distribution pathway, the probability of a missing dose also depends on the medication type. For instance, a STAT dose that is distributed via cart fill by the pharmacy technician is much more likely to be a missing dose than a maintenance dose that is distributed via cart fill due to the difference in lead time for the medication to reach the unit. Therefore, in our consideration of missing doses we consider both the distribution pathway and the dose type.

In industrial engineering, consistency and standardization are vital to reducing errors and increasing efficiency. We employ these two ideas to improve the distribution of medication to inpatients. In particular, we are interested in standardizing the distribution of each medication and dose type to each unit and efficiently utilizing the limited space available for a robot or ADCs. Our goal is to best utilize these two technologies simultaneously to reduce the average number of missing doses and the average daily medication distribution cost for the entire hospital.

3.2 LITERATURE REVIEW

In the pharmacy literature, multiple studies have examined the effects of utilizing either a pharmacy robot or ADCs with regards to cost, labor, and missing medication and medication errors. The literature shows that missing doses and medication errors can be reduced through the implementation of ADCs but nurse workload increases. Alternatively, the use of pharmacy robots can reduce pharmacy technician and nurse labor required, but the logistics of distributing the medication to the patient becomes more complicated. In 2014, the American Society of Health-Systems Pharmacists surveyed dispensing and administration practices in hospital pharmacies. The survey results showed larger hospitals were more likely to use a robot and most hospitals use ADCs in some form. Furthermore, the primary distribution method for approximately 66% of hospitals was ADCs, while 26% used a centralized manual system and 6% used robots. Hospital size correlated with the distribution method for a centralized system, as smaller hospitals were more likely to use a manual system and larger hospital used robots[46].

When using a decentralized model, hospitals can employ pharmacists on inpatient units to improve patients quality of service. Using a decentralized pharmacy model was first studied by Black and Tester[26] who noted that the effects of using a decentralized model within their hospital included more pharmacist time for physicians and patients and fewer medication errors. The Pharmacy Practice Model Initiative (PPMI)[27] and Providence Health Services in Montana[47] documented similar results by increasing the clinical duties of the hospital pharmacists.

The use of ADCs in decentralization can also lead to a significant decrease in missing doses, medication errors, and the time to initial dose[4, 43, 55]. However, multiple studies noted that proper inventory management and restocking are important for effective ADC implementation. Hussey et al. focused on streamlining prescriptions with multiple strengths and reducing space occupied by large items[28] while McCarthy and Ferker focused on prescription frequency[41]. Esmaili[22] focused on ADC layout and inventory policies to effectively use the ADCs limited space while decreasing the likelihood of selection errors. Berdot et al. analyzed ADC use within a hospital over a seven year period and found a decrease in pharmacy spending; however, this study did not consider nurse staffing costs[5].

Although the decentralized model provides high quality of service, these systems can experience high costs due to the increased nursing time and inventory required to sustain ADC operations. Classic supply chain studies have demonstrated that by consolidating inventory in a central location, such as the central pharmacy, total inventory can be reduced, which decreases the system cost[13]. In Wisconsin, a simulation study was performed to compare an existing system which filled 64% of medications via cart fill and 36% through ADCs with three models: 100% cart fill, 100% ADC, and 89% ADC and 11% cart fill. The original system was retained after the study found that increasing ADC usage led to increased nursing time for pharmacy related activities which their current staffing could not support[25]. Similarly, a hospital in the Netherlands used a cart fill system after they found the nursing labor associated with the central fill system was much less while the system cost remained the same[48].

Robots were implemented in central pharmacies in the 1990s to help decrease picking errors and the labor necessary to sustain a central fill. Robots have become increasingly more efficient with one recent implementation having recorded zero incorrect picks from 2011 until 2015[7]. Most robots focus on picking unit dose items; however, new dispensing robots can also handle oddly shaped items. Lin et al. found that using a robot[39] decreased pharmacy staff labor requirements for filling prescriptions. Lathrop et al. found that a thrice-daily cart fill system could reduce the lead-time for most peak medication administration times, increasing the number of first doses dispensed from the pharmacy while reducing the number of doses returned to the central pharmacy and the number of missing medications[37].

While there have been many studies focusing on how to improve pharmaceutical distribution within a hospital, very few studies have provided a system wide approach to solve the problem. We propose a system wide approach with wide applicability across an array of hospitals. We understand that medication demand and hospital size vary significantly and therefore the best solution for one system may not apply to another. However, each hospital can use their own data to populate the parameters of our model and determine the best solution for their individual system. Our model evaluates the workload of employees interacting with the medication distribution system while considering the quality of service provided by the system.

3.3 METHODOLOGY

In process engineering, standardization is one key to reducing errors and improving safety. Therefore, in the pharmacy system, it is logical to reduce the number of missing doses by identifying the standard pathway to distribute each medication and dose type to each unit. In making these decisions, we focus on minimizing the average staff distribution costs based on a target missing dose rate across the system. This corresponds to minimizing the weighted time spent on pharmacy distribution activities for the pharmacy technician, nurse, and pharmacist. This weighting has higher costs for nurses and pharmacists due to their higher salary, indicating that they are more important resources than the pharmacy technician.

We formulate and solve a mathematical model to determine the optimal distribution pathways. The mathematical model minimizes the average daily cost of distributing medication from the inpatient pharmacy to the inpatient units. The costs captured within the model are average medication picking costs, robot restocking costs, ADC restocking costs, cart fill preparation costs, and nurse queueing costs.

Each pathways workload includes the activities presented in the flow chart in Figure 3.1. We assume that the labor to transport medication from the central pharmacy to a unit and the nursing effort to administer a patients medication is the same, regardless of the medications primary pathway. Therefore, these activities are not included in our analysis.

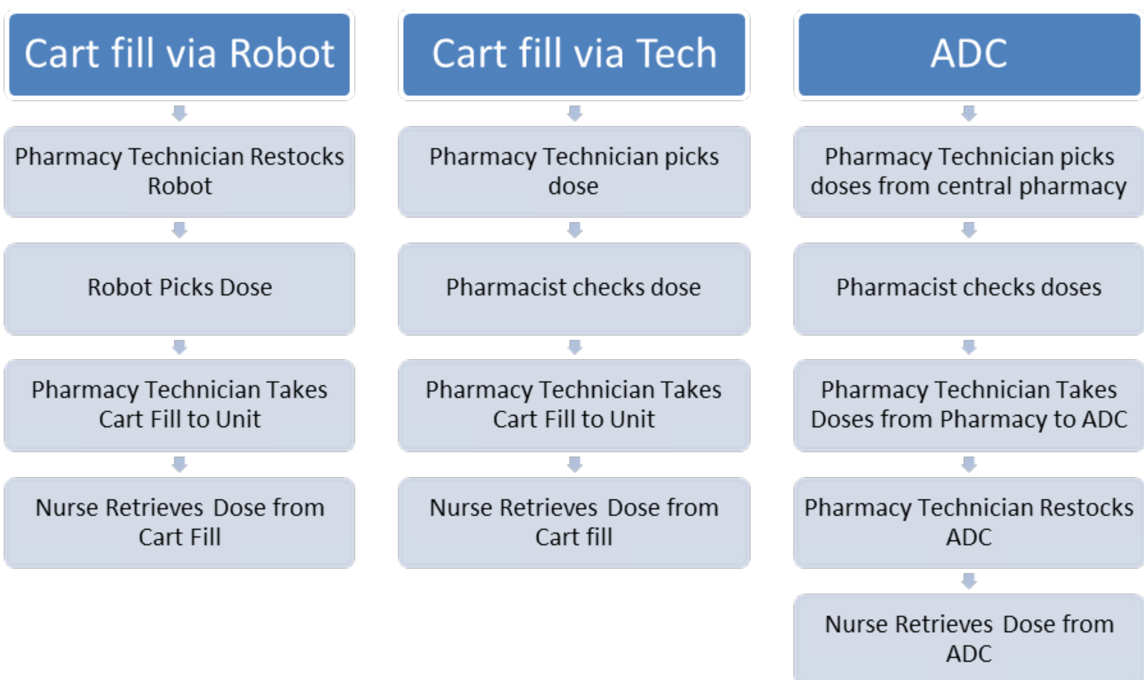

Figure 3.1: The staff activities that are included within the cost calculation of each pathway.

The average medication picking costs capture the average labor costs associated with the pharmacy technician, the pharmacist, and the nurse. The picking activity times are based on the observations conducted in Gray et al. [25] and consultation with pharmacy experts from

Geisinger. For medication picked by the robot, the workload stems from the nurse picking the medication from the patients envelope. In the cart fill via pharmacy technician pathway, the workload includes the pharmacy technician picking the medication, the pharmacist checking the medication, and the nurse picking the medication from the patients envelope. Lastly, for medications distributed via the ADC, the only picking costs stem from the time the nurse spends picking the medication from the ADC.

We base the robot restocking cost on the time required to restock a single rod within the robot and the number of doses per rod to estimate the restocking cost per dose. We calculate ADC daily restocking cost using the cost to restock a pocket of the ADC and the frequency of restocking, which we assume is every other day. The cart fill preparation costs comprise the costs to label and prepare the envelope for each patient. Since patients can receive medications from both cart fill and ADCs, we simulated 2,000 instances where a different percentage of medications were stored in the ADCs and calculated the number of patients receiving cart fill doses under each instance. Based on the simulation, we constructed a piecewise linear function that accurately depicted the number of patients receiving cart fill medications as a function of the percentage of doses stored in the ADC. The nurse queueing cost is a linear function which captures the time nurses spend waiting to retrieve doses from the ADC based on the percentage of doses stored in the ADC. In our model, we assume that every patient has medication in the ADC when more than 11% of doses are stored in the ADC and nurses visit the ADC one time for each patient during the major medication administration times.

The following constraints define the set of feasible solutions:

- Every medication and dose combination has one distribution pathway to each unit.
- STAT doses may not be routed through the robot due to time constraints.
- If First Doses, Maintenance Doses, or PRN doses of a medication are distributed to a unit via the ADC then STAT doses of the medication are also distributed via the ADC.
- If maintenance doses are distributed to a unit via the ADC then first doses of the medication are also distributed via the ADC.
- The number of ADCs needed for a unit cannot exceed the space limitations of the unit.

- The number of unique medications dispensed by the robot cannot exceed the medication capacity of the robot.
- The number of doses dispensed by the robot cannot exceed the robots throughput.
- The number of doses dispensed by the robot must exceed the minimum cost effectiveness throughput of the robot.
- The average number of missing doses on each unit cannot exceed the missing dose tolerance.

Additionally, there are constraints used to formulate the cart fill and nurse queueing costs based on the percentage of doses routed through the ADC. Although the objective in our problem is to minimize employee costs by setting a missing dose tolerance, our model also considers quality of service. Alternatively, we could create an objective which would minimize the number of missing doses and add a constraint that limits the total allowable budget.

3.4 DATA SUMMARY

We solve the mathematical model using data from Geisinger Medical Center (GMC), the flagship hospital for Geisinger. Geisinger is an integrated health services organization which serves more than 3 million residents throughout 45 countries in central, south-central, northeast Pennsylvania, and southern New Jersey. During the year of data analyzed, the pharmacy processed approximately 2,300 medication orders every day which translated to 8,800 medications dispensed from the central pharmacy, of which an average of 8,600 were administered to patients. There were 3,350 unique medication types and 1,105 were non-controlled unit dose medications. For our problem, we focus solely on non-controlled unit dose medication types because they can be distributed through all three pathways.

When fulfilling orders, there are five primary locations within Geisinger Medical Center that can dispense doses: the unit dose robot, the inpatient pharmacy, ADCs, the pediatric pharmacy, and the operating room pharmacy. Figure 3.2 shows their relative frequency of use. Note, that the inpatient pharmacy, pediatric pharmacy, and operating room pharmacy

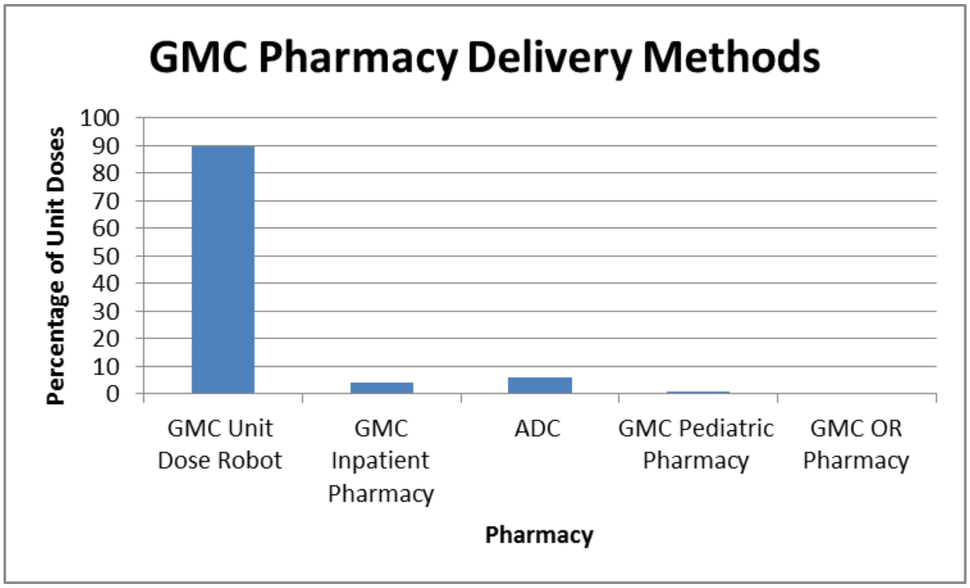

Figure 3.2: Current methods used to distribute uncontrolled unit dose medication to inpatient units from the central pharmacy at GMC.

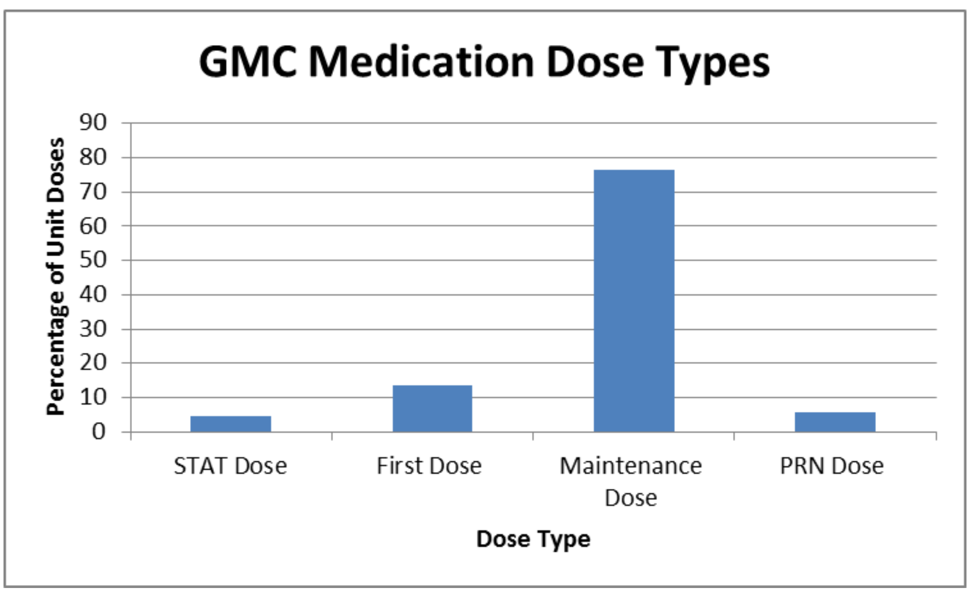

Figure 3.3: Percentage of uncontrolled unit dose medications in each dose type category.

correspond to the pharmacy technician pathway. Currently, Geisinger distributes most of their unit dose medications through the pharmacy robot and uses the ADCs and pharmacy technicians to distribute other medication types.

One factor we consider in determining the distribution pathway is dose type. Figure 3.3 shows the portion of all medications that are STAT, first, maintenance, and PRN doses. We can see here that most uncontrolled unit doses are maintenance doses, although there are some exceptions.

3.5 RESULTS

Due to the large problem size, we solve the model for the five inpatient units with the highest dose volume. These units include one orthopedic unit, two med/surg units, one cardiac med/surg unit, and an intensive care unit. We permit an unconstrained number of ADCs on each unit to examine the fully decentralized model.

When the missing dose level is unconstrained, each unit has a missing dose rate less than 5.95%. In this scenario, the robot is always the most cost effective route and we route as many doses as possible through the robot and the remaining doses are routed through the pharmacy technician and the ADC. As the missing dose rate decreases, the ADCs process more doses. We solve the model with an initial missing dose rate of 5.95% and decrement by steps of 0.05% until the model is infeasible. The bar chart in Figure 3.4 shows the optimal medication distribution labor costs versus the missing dose rate. The stacked bars demonstrate the amount of the daily labor costs comprised of the five cost categories in the objective function: cart fill preparation cost, robot restocking cost, picking cost, ADC restocking cost, and nurse queueing cost.

Figure 3.4 shows that as the missing dose rate is lowered, primarily by storing more medications in the ADCs, then the minimum daily cost increases steadily. Figure 3.5 indicates the dose routing pathways as a function of the missing dose rate. The ADC and pharmacy technician both process very few doses initially and we finish with the ADC processing all doses. Figure 3.4 shows that the ADC requires more staffing resources for restocking and

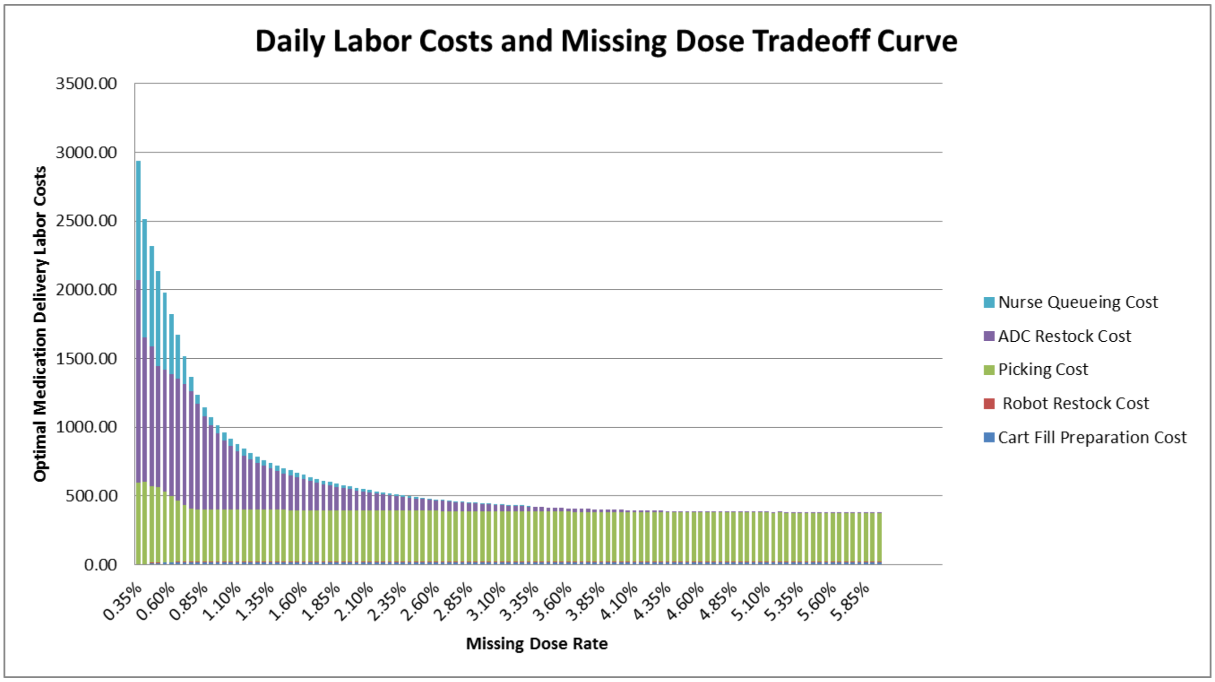

Figure 3.4: The minimum cost of the medication distribution process by missing dose rate.

obtaining medication compared to the robot which requires minimal restocking effort and cart fill preparation. The picking costs remain relatively constant with a slight increase as more doses are processed through the ADC indicating the increase in time to pick each individual dose.

Our results demonstrate that the volume of STAT and first doses play an important role in deciding which medications are added to the ADC. This can be seen in the shape of the graphs in Figure 3.5. We add minimal doses to the ADC for missing dose rates of 1.00%-5.95 because we can reach the missing dose targets through only adding STAT and first doses since they have a significantly higher missing dose rate when distributed via cart fill. In comparison to the STAT and first doses, maintenance doses and PRN doses have a relatively small difference in the missing dose rate between the robot and the ADC. Thus, we see a dramatic increase in the number of doses added to the ADC after 1.00% because we require more than just STAT and first doses in the ADC to meet our missing dose rate goal.

While we have focused on the cost of the medication distribution process, it is also important to consider the pharmacy technician and nursing time required in this process.

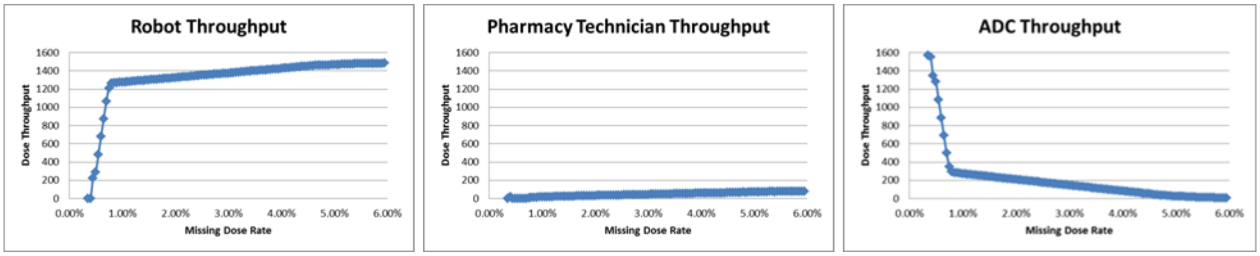

Figure 3.5: The optimal throughput of each pathways based on the missing dose rate.

The time spent by both roles within the process is an important consideration in the decision to centralize or decentralize the pharmacy and is now explored in more detail. Due to the small number of doses processed manually by a pharmacy technician, the amount of time the pharmacist spends on dose distribution activities (primarily validating the pharmacy technicians work) is very small and therefore is not considered.

Figure 3.6 illustrates the total pharmacy technician time that is required for the minimum cost at each missing dose rate. As the missing dose rate is restricted and the pharmacy begins distributing more medication through the ADC, the amount of pharmacy technician labor required for the medication distribution process grows exponentially. This can be attributed to the increase in ADC stocking activities which often require more time than picking an order in the central pharmacy or restocking the robot. The cost plateau that occurs at a missing dose rate of 0.70% is a result of the significant increase in the number of doses that are distributed via the ADC. These doses added to the ADC are maintenance doses and PRN doses for medication which is already stored in the ADC, so the restocking cost will not change as these doses are added. As a result, we see a decrease in the number of doses processed through the robot and pharmacy technician which in turn decreases the time the pharmacist technician spends on restocking the robot and preparing the cart fill. These factors all lead to a decrease in the total labor costs.

Figure 3.7 plots the total time an individual nurse spends on medication retrieval, the waiting time for the ADC and the time to pick the medication. Similar to the pharmacy technician, with increased ADC usage, the average nurse time required to perform medication retrieval tasks increases. When we store minimal doses in the ADC, the average nurse spends a total of 10 minutes a day retrieving medications. However, as we route more doses through the ADC the nurse time increases due to the increase in picking and waiting time. In the maximum case the average time an individual nurse will spend on medication retrieval activities can reach over 50 minutes. Despite the increase in nurse queueing and retrieval time, utilizing the ADC decreases the frequency of missing doses, in turn reducing the nurse workload resulting from replacing missing medications. Our missing medication rate is less than 6% for all doses indicating less than 3 missing medications per nurse each day. This results in minimal nurse labor to replace missing doses compared to the queueing and retrieval costs resulting from the increased ADC usage. Therefore, we do not explicitly account for the nursing time required to replace missing medications but consider the missing dose rate as a quality measure.

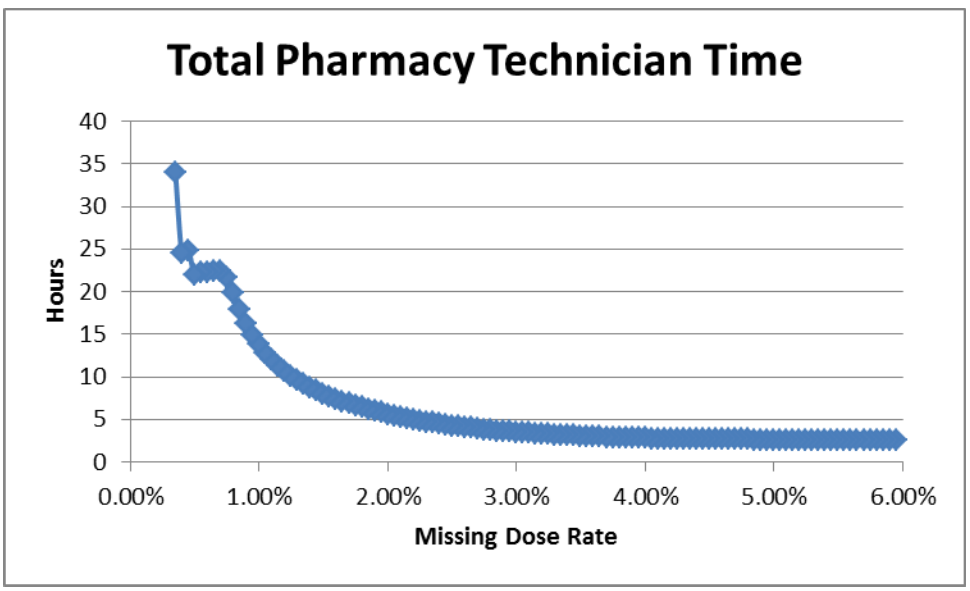

Figure 3.6: Total pharmacy technician time spent on medication distribution activities.

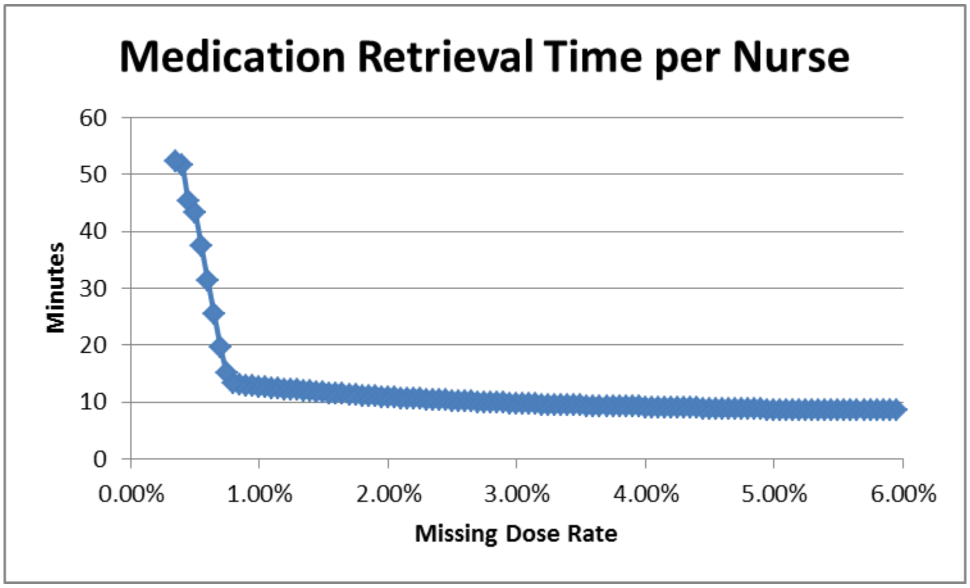

Figure 3.7: Medication retrieval time for each nurse within the medication distribution process.

3.6 CONCLUSION

We formulate a linear programming model which captures key aspects of the pharmacy distribution system that can find the most cost effective method of distribution for unit dose items given a set missing dose rate. The results show an increase in pharmacy technician and nurse labor effort as more items are distributed via ADCs but the missing dose rate can be decreased significantly through increased ADC usage. The results demonstrate that missing dose rates can be decreased considerably by dispensing most STAT and first doses through the ADC while maintenance and PRN doses are routed through the robot and pharmacy technician at considerably lower costs.

Our model expands the current pharmacy literature by creating a generalized model which can be implemented by any hospital. The model includes key activities that occur within the medication distribution process and makes decisions which are best for the system as a whole. Our model can be improved by considering more doses than just unit dose items, considering the variation in medication demand, and considering alternative inventory policies. Limitations of our model include that our model does not consider the labor effort required to replace missing doses and our ADC inventory replenishment rate is assumed to be every other day.

4.0 ROBUST PHARMACY

4.1 INTRODUCTION

The in-hospital pharmacy distribution process requires several resources and a substantial amount of staff time. A hospital pharmacy inventory system can store anywhere from 6,000-10,000 types of medication. Hospitals need to distribute medication from the central pharmacy to inpatient units in a cost effective manner that does not interfere with patient safety. Several different technologies can be implemented to ensure patients are getting the right dose at the right time. Automated dispensing cabinets (ADCs) are located in each unit as point-of-use inventory. The ADC enables the pharmacy to remotely track their inventory levels using an (S,s) system to decrease the number of stockouts that occur. The features within the ADC guide the nurse to select the correct medication for each patient and nurses can retrieve medications for the patient at any time throughout the day with this system. Within the central pharmacy, a robot can be used each night to complete the batch picking for each patient for the following day. The robot has a very low error rate that allows it to pick medication in a cost efficient and safe manner. In addition to the safety features of ADCs and the pharmacy robot, hospitals have recently implemented bedside scanning during medication administration to ensure that the patient is receiving the correct medication in the correct dosage. Although many hospitals have implemented some of these safety features, a patient's safety is affected when they miss a dose of their medication. While there are many reasons a missing dose can occur, the primary reasons are that the medication was never delivered to the floor, the MD changed the patient's prescription and the new prescription has not been delivered, the patient has moved units recently but their medication did not move with them, or a stockout has occurred in the ADC making the medication unavailable through the ADC.

Although both technologies help create a safer patient experience, there are difficulties that arise with the implementation of these technologies. While ADCs allow medications to reach the patient faster, they are typically more expensive to operate due to the labor required for pharmacy technicians to maintain the inventory levels and for nurses to pick medication from the ADC. As ADC use increases, it requires more nurse time for pharmacy related activities, taking away from patient care time. In comparison, while the robot is very cost-efficient, it requires a large initial investment to purchase the robot. In addition, the robot is used to pick large batches, therefore it requires a large lead time to be able to pick orders. There is also the challenge that medication picked by the robot must be delivered to the unit in order to reach the patient, and this delivery process increases the chance that medications go missing due to breakdowns in the transport process. Overall, the ADC is more costly to implement in comparison to the robot but the rate of missing medications decreases significantly with ADC use.

While these changes in technology have increased the level of patient safety in the hospital, they have also led to new questions regarding pharmacy operations. These technologies are generally not used together to distribute medications, leading to many hospitals choosing to implement one or the other. Therefore, an important question is "which technologies the hospital should invest in?" There are three main pathways that are frequently used to distribute medications from the central pharmacy in the hospital to the patient, pictured in Figure 4.1 and explained in detail below. The figure demonstrates the transportation of medication from the central pharmacy to the inpatient unit where the nurse administers the medication to the patient. By determining the pharmacy distribution pathway for each medication, we can determine how the hospital can best allocate their resources to increase patient safety and distribute medication cost effectively.

In the central pharmacy, there are two standard places where medication is stored, either in unit dose quantities in the hospital robot or in bulk containers. The robot dispenses medication in a patient-wise manner, meaning that all medications for patient 1 are picked and put into the envelope for patient 1, then the robot begins picking for patient 2, etc.

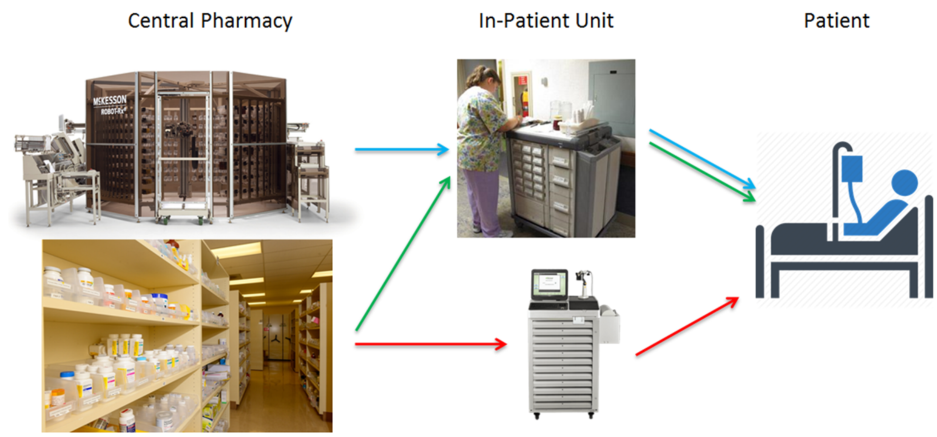

Figure 4.1: Pathways to distribute medication from the central pharamcy to patients.

The medication in bulk containers is either picked by a pharmacy technician for a specific patient and placed in that patient's envelope or picked in bulk to be put into the ADC. Once the medication is picked in the central pharmacy, a pharmacy technician transports it to the inpatient units where it is put into either the cart or the ADC. The cart has a drawer for each patient, and the envelopes with medications picked by the robot and pharmacy technician are placed into the cart. When the nurse retrieves a medication for the patient from the cart he/she opens the patient's drawer, sorts through the envelope for the correct medication, and takes the medication to scan and administer to the patient. Medication that is delivered in bulk to the inpatient unit for the ADC is loaded into the ADC by a pharmacy technician. Whenever a nurse needs to retrieve medication from the ADC, he/she scans into the computer to access the patient's medication orders. Upon choosing a patient's name, all the medications prescribed for the patient appear on the computer. The nurse selects the medication he/she is going to administer and the drawer containing the medication opens with a guiding light illuminated to direct the nurse to the specific compartment containing the medication. Upon retrieval, the nurse updates the system with the number of remaining doses for inventory control purposes and delivers the dose to the patient. These steps form three unique pathways identified in Figure 4.1 where the medication is picked by the robot and delivered via cartfill, picked by the tech and delivered via cartfill, or delivered via the ADC.

By identifying standard pathways to deliver unit doses from the hospital pharmacy to the inpatient units, we can increase efficiency in the medication administration process. With standard pathways, nurses know whether a certain medication and dose type will be delivered via the cartfill or ADC and this will reduce time spent searching for medications. In addition, we can decrease the risk of missing doses by identifying steps within the delivery process where there is potential for doses to become missing and create solutions that decrease these risks.

While standardization can help us decrease the risk of missing doses, there is also risk associated with the individual order when it is being delivered via the cartfill process. Within the cartfill process, the medication is picked for each individual patient and the time frame for picking and delivering the dose to the unit will vary based on the dose type. We identify four medication dose types: STAT, first, maintenance, and PRN. STAT doses are required to be given to the patient immediately and the time difference between when the medication is ordered and when it is scheduled to be administered is less than 30 minutes. First doses are the first dose of any medication given to the patient where the difference between the time when the medication is ordered and when it is to be administered exceeds 30 minutes. Maintenance doses are regularly scheduled doses, such as an 8AM blood pressure medication. PRN doses are any dose which is given upon patient request, such as Tylenol for a headache. When we solve this problem, we take the dose type into account in order to reflect the risk that exists, not only within the individual pathways, but also within the specific dose types.

In this paper, we look to identify the standardized pathway to distribute each medication and dose type to each unit. Our model considers the cost of the technology and the workload involved in the medication distribution process along with the probability of missing doses. In addition, we account for the intrinsic variation that occurs daily in medication demand by formulating and solving the robust counterpart to identify the delivery pathways. We analyze the results of the robust model for the missing dose rate, the total cost, and the individual workloads for the pharmacy technicians and the nursing staff.

4.2 LITERATURE REVIEW

4.2.1 Pharmacy Literature

In 1965, Black and Tester [26] claimed improved patient safety with a decentralized pharmacy model which increased pharmacist availability to both the patient and physician. One modern goal of hospital pharmacists pertains to the Pharmacy Practice Model Initiative (PPMI). PPMI is a subset of the American Society of Health-System Pharmacists whose goal is to increase pharmacist clinical involvement. By making the pharmacist available to physicians for consulting and to patients for instruction, hospitals can reduce errors and save money [27]. One hospital system (Providence Health and Services) in Montana increased clinical duties of their pharmacists by assigning their distribution-related responsibilities to techs and optimizing the use of technology. The system saw annual pharmacist interventions increase, in turn reducing overall estimated error cost [47]. Literature continues to focus on pharmacy decentralization as a means of improving patient safety; in fact, a recent survey shows that American pharmacy practice is trending in the decentralized direction where frequently used drugs are mainly stored on the unit and use of the main hospital pharmacy is kept to a minimum [46].

New technologies like automated dispensing cabinets have further increased the practicality of these systems by providing automatic inventory tracking in secure and geographically convenient locations. When utilizing ADC cabinets, the cabinets should have available inventory streamlined for items with multiple strength prescriptions and par levels should be reduced to create space for bulkier items. Using these methods to optimize the inventory kept in the ADC, [28] showed that the number of medications in the cabinet will increase leading to a reduction in the number of dispenses from the central pharmacy and the cost of the ADCs. However, this could result in more stock outs per day and require a greater replenishment frequency. A Canadian study recently found that ADC use yields a reduction in missing doses, technician fill errors, and medication storage errors; however, success was highly dependent on proper use of the devices [55]. One hospital evaluated nurse perception of recently implemented ADCs. Although nurses agreed that they felt the devices improved patient safety, study observers noted that there was only a 66% compliance rate with standard ADC practice when the devices were new. Time-consuming processes included to improve safety were sometimes ignored by nurses who were in a rush. Standard procedure must be followed for the realization of ADC success; this must be considered. A review of current practice showed that popular opinion generally favors the decentralized model as a result of its ability to reduce errors and improve the patient-experience. [50]

On the other hand, classic supply chain literature shows that when demand is unknown, as with the hospital pharmacy, and inventory is consolidated in a central location demand and supply risk can be lowered to reduce the overall system cost [13]. One study performed at a 561 bed hospital in Wisconsin weighed the impact of shifting different amounts of unit-dose prescription fills from unit-fill to cart-fill. Deviating from the original layout of 64% cartfill and 36% ADC storage, the team ran simulations to test a 100% ADC-use model, a 100% cartfill model, and a model with 89% ADC and 11% cartfill. Since staff labor increased unfavorably with increased ADC use, it was determined that complete decentralization was not ideal [25]. One study conducted in the Netherlands compared the efficiency of central prescription fill with decentralized prescription fill at a childrens hospital. Management saw consistently lower prescription-fill labor requirements with pharmacy technicians in the central pharmacy than with nurses on the unit. The study concluded that economies of scale, as a result of inventory consolidation in the central pharmacy, coupled with pharmacy technician expertise was likely the cause of this reduction [48].

Hospitals with central pharmacies began implementing robots to do picking in the 1990s. With robotics, hospitals have seen a decrease in medication error rates as a result of picking in the central pharmacy. Compared to the robots of the 1990s, the newer robots are even more efficient. For example, the robot implemented in the central pharmacy at UCSFs Mission Bay hospital has had zero incorrect picks since its installation in 2011 [7].

According to the ASHP national survey, only 5.7% of all hospitals use robots to pick maintenance doses [50]. However, larger hospitals fulfilling maintenance doses from the central pharmacy are more likely to use a robot. South Jersey Health System, for example, reduced their average med delivery time to patients from about 2 hours to 18 minutes [43].

Another study showed consistent decreases in labor times for pharmacists and technicians after the implementation of an automated dispensing robot in a large pharmacy [39].

At the University of Wisconsin Hospital and Clinics, the central pharmacy used a daily cartfill process to deliver maintenance doses to patients. In 2014, the pharmacy switched from a once daily cartfill to thrice-daily cartfill which reduced the lead time for three of the four peak medication administration times. Evaluating both systems, they showed that with thrice-daily cartfill there was a 44.1% decrease in first doses dispensed, a 42.9% decrease in the number of returns to the central pharmacy, and a 2.1% increase in the number of doses dispensed by cartfill. These improvements reduced waste and allowed deployment of 2 pharmacist FTEs to clinical roles [37].

In addition to hospital pharmacy centralization and decentralization research, previous literature also focuses on centralizing pharmacy operations for a health care system. This literature demonstrates the advantages of centralization for a hospital pharmacy on a large scale. A study at an Italian public hospital with three hospital centers examines the effect of consolidating the three individual pharmacies at each hospital into one pharmacy at the largest location. Upon centralization, the hospital also installed a robot for pharmacy warehouse management and a robot to prepare the drug baskets for each department all every hospital. From centralization the hospital sees a 30% reduction in stock and a 40% reduction in expired medications. According to the paper projections, the system will see profit from the implementation of centralization and automation after three years [23]. In [3] a centralized and decentralized hospital system are compared to determine the best replenishment strategy for all hospitals when the total cost in the system is minimized and there are no shortages allowed. These models were implemented as a case study in France and results showed that pharmaceuticals should be centralized. These results are further confirmed in a simulation case study [34] based on a hospital system in Singapore where upon simulating a central pharmacy results showed a small reduction in staff and a large reduction in inventory.

4.2.2 Robust Optimization Literature

Uncertainty in health care is dependent on the type of problem to be addressed. Mousazadeh et al. [44] determine how to distribute medications along the pharmaceutical supply chain from secondary manufacturing sites to distribution centers to wholesalers to hospitals and clinics. They solve the model for an example medication to determine the minimal cost and minimal unfulfilled demand. Franco et al. [24] formulate a model to determine how different pharmaceutical distribution technologies should be allocated across a network of hospitals to deliver medications in spite of disruptions that can occur along the supply chain. They formulate two models, one with deterministic demand and one with stochastic demand and their results show that the optimal solution for the first model cannot satisfy the demand when it exceeds the deterministic demand while the second model is capable of handling the natural variation that occurs.

While there are few applications of robust optimization in pharmacy distribution, there have been multiple studies applying robust optimization to healthcare. For example, Denton 2010 discusses an operation room scheduling problem where uncertainty is found in surgery procedures. Denton uses two models to deal with uncertainty. The first model is a two stage stochastic model that includes a fixed cost for opening operations rooms and a variable cost that depends on over time. The second model is the robust counterpart for the two stage optimization model. The counterpart minimizes the maximum cost associated with uncertainty set for surgery durations [15]. Another use of uncertainty in health care is shown in an emergency vehicle location problem where uncertainty could be found in the availability of vehicle [6].

Home Care is another healthcare domain that encompasses a lot of uncertainty but is doesnt get much attention in the research community according to [9]. In [9] a nurse to patient assignment problem is discussed where uncertain transportation and service times are taken into consideration. Lanzarone discussed a robust nurse to patient assignment problem that minimizes overtimes under the continuity of care which is treated as a soft constraint in most previous home care applications [36].

Healthcare optimization models with uncertain demands are quit common and different due to the fact that there are several healthcare domains and several types of applications in each domain. Zhang focused on patient flow scheduling study in emergency department with targeted deadlines. The goal was to optimize operations in the emergency department by focusing on doctor response time for different loads and examining historical data as well as hospital key performance indicators. Two main key performance measures are used extensively in this study which are Length of Stay (LoS) and First Waiting time (FW). The model maximizes the number of patients whose LoS and FW are within the allowed limits. The main uncertain parameter in this study was the service time which can be controlled by the doctors effort [42].

Our paper is focused on a specific healthcare domain which is pharmacy operations. The literature we have found concerning hospital pharmacies focuses on either decentralizing or centralizing a single hospital or centralizing a hospital pharmacy system with no description of how the individual hospital operations work. Current work on optimization applications to healthcare literature which focus on hospital pharmacy operations do not consider demand uncertainties. Our paper will determine how the pharmacy should deliver medications to patients in a cost effective manner without sacrificing patient safety under uncertain demand. In addition, we will determine if a hospital pharmacy system should be centralized, the location of a centralized facility, and the logistics of the individual hospitals in the system taking into consideration that the demands are uncertain.

Most of the previously literature on distributing medication from the hospital central pharmacy to the inpatient unit has focused on the idea of either a completely centralized or decentralized model. Most of the recommendations result from case studies where either decentralized or centralized distribution was implemented and the findings for the specific case study were reported. The high costs involved in this decision make it difficult to test both scenarios and determine which one is truly optimal for the individual hospital. In this paper, we formulate a linear program which is representative of the in-hospital pharmacy distribution problem, formulate the robust counterpart for the nominal problem, and solve the robust counterpart based on a year of data from a local hospital. Our model demonstrates the effect of a missing dose threshold on the optimal cost, the decision to decentralize or centralize, and staffing time. In Section 3 we give the complete pharmacy distribution mixed integer linear programming problem, in Section 4 we identify our uncertainty sets and formulate the robust counterpart, and Section 5 displays the results when we implement the model using data from a local hospital.

4.3 MODEL FORMULATION

In identifying the distribution pathways, the two primary concerns we address are the missing dose rate and distribution cost. The missing dose rate is based on the standardized pathway and the dose type. The costs to distribute medication are the dose picking cost, the robot purchase price and maintenance cost, the ADC purchase price and maintenance cost, the robot restock cost, the ADC restock cost, the cartfill preparation cost, and the nurse queueing cost. The picking cost calculates the workload required of the pharmacists, pharmacy technicians, and nurses to pick a single dose and deliver it by each pathway. The robot and ADC purchasing and maintenance costs include the cost to purchase and maintain the technology, amortized over the lifetime of the technology. The robot restock cost calculates the cost to restock one dose in the robot by determining the workload cost for the pharmacy technician to restock one individual rod within the robot and dividing this cost by the average number of doses per rod. The ADC restocking uses the pharmacy technician workload cost to restock one individual compartment and the restocking frequency based on demand to calculate the average daily ADC restocking cost. The cartfill preparation costs and the nurse queueing costs are more complicated because these costs decrease and increase accordingly as more doses are kept in the ADC. The cartfill preparation cost captures the cost required to prepare an envelope for each patient whose medication will be delivered via the cartfill process. Therefore, this is not necessarily a linear function because adding medication to the ADC does not necessarily mean that a patient does not receive medication via the cartfill anymore. We use simulation to create a link between the percentage of total doses stored in the ADC and the percentage of patients receiving medication via cartfill. We randomly choose to store a percentage of medications in the ADC and determine the volume of doses stored in the ADC and the number of patients who receive cartfill. We repeat this 20 times for storing 1% to 99% of medication in the ADC. Upon plotting the points, we find the piecewise linear function of best fit, given in Equation (4.1), where x is the fraction of total doses stored in the ADC.

$$\text{Percentage of Patients Receiving Cartfill} = \begin{cases} 1.00 - 0.13x & 0 \leq x \leq 0.69 \\ 1.83 - 1.31x & 0.69 \leq x \leq 0.90 \\ 5.99 - 5.94x & 0.9 \leq x \leq 1 \end{cases} \tag{4.1}$$

The nurse queueing cost captures the time that nurses spending waiting to access the medications in the ADC for their patients. There are regularly scheduled large medication administration times where the majority of patients are scheduled to receive maintenance medications. The hospital we solve this problem for has 4 major medication administration times at 8am, 12pm, 4pm, and 8pm. Due to the large number of patients receiving medications during this time, the nurses all need to access the ADC, resulting in time spent waiting in line. As more medications are distributed through the ADC, the time nurses spend waiting increases. We use a hospital wide time study by Grey et al. [cite] to determine Equation (4.2), the nurse queuing time on each unit, where x is the fraction of total doses stored in the ADC. We assume that the waiting time increases linearly and that the performance across the hospital is representative of the individual units.

$$\text{Nurse Queueing Time}_k = \max\{147.547x - 17.317\} \tag{4.2}$$

The indices, variables, and parameters definitions for the model are given in Tables 4.1, 4.2, and 4.3.

Table 4.1: Index notation for the pharmacy pathway model

Index	Description
i	Medication Type
j	Dose Type
k	Unit
l	Pathway
q	Dose type combination that is distributed through the ADC
$Tech$	Pharmacy Technician
$Nurse$	Nurse

Table 4.2: Variable notation for the pharmacy pathway model

Variable	Description
u_{ijkl}	Indicator that medication i with dose type j is delivered to unit k via pathway l
x_{ikq}	Indicator that dose type combination q is stored in the ADC on unit k for medication i
w_k	Number of ADCs required on unit k
γ	Indicator that a robot is needed in the hospital pharmacy
β_i	Indicator that medication i is routed through the hospital robot
ϕ_k	Average time nurses queue at the ADC on unit k
χ	Variable representing the percent of patients using cartfill
Υ	Binary variable determining the segment of the piecewise function for cartfill delivery cost to use based on the percentage of doses routed through the ADC

Table 4.3: Parameter notation for the pharmacy pathway model

Parameter	Description
ξ_{Tech}, ξ_{Nurse}	Average salary of pharmacy technician and nurse respectively
Λ_l	Inventory replenishment cost for technology used in pathway l
Δ	Average number of medications that can be stored on a single rod in the robot
Ξ	Average census of the hospital
e	Cost to label patient envelope
Ω	Number of large medication administrations that occur in the hospital
N_k	Number of nurses on unit k
ρ_{ikq}	The refill rate for medication i on unit k when dose combination q is stored in the ADC
p_{ikq}	The number of bins in the ADC required to hold the par level of medication i on unit k when dose combination q is held in the ADC
r_q	The number of dose types stored in the ADC under dose combination q
B	The total number of bins in an ADC
s_k	The maximum number of ADCs that can be used in unit k
c_{ijkl}	The delivery cost of medication i to unit k via pathway l for dose type j
d_{ijk}	The average daily demand of medication i with dose type j on unit k
g	Cost to buy and maintain a robot
h	Cost to buy and maintain an ADC
n	Throughput capacity of the robot
ν	Unique medication capacity of the robot
μ_{jl}	The percentage of missing doses resulting from dose type and delivery pathway l
MMD	The maximum number of missing doses allowed in the system

Using the notation given in Tables 4.1, 4.2, and 4.3 we formulate the linear program given in Equations (4.3)-(4.25). Note, that for the dose type we define $j = 1$ as "STAT Dose," $j = 2$ as "First Dose," $j = 3$ as "Maintenance Dose," and $j = 4$ as "PRN Dose." For the pathways, we define $l = 1$ for the robot, $l = 2$ for the pharmacy technician, and $l = 3$ for the ADC. There are 6 different feasible medication dose combinations, q, which are determined based on the logical constraints regarding which dose types can be stored in the ADC at the same time. We define $q = 1$ as only STAT doses are stored in the ADC, $q = 2$ as STAT and first doses are stored in the ADC, $q = 3$ as STAT, first, and maintenance doses are stored in the ADC, $q = 4$ as STAT and PRN doses are stored in the ADC, $q = 5$ as STAT, first doses, and PRN doses are stored in the ADC, and $q = 6$ as all doses are stored in the ADC.

$$\begin{aligned} \min \ & \sum_{i,j,k,l} c_{ijkl} d_{ijk} u_{ijkl} + g\gamma + \sum_k h w_k + \Lambda_1 \sum_{i,j,k} d_{ijk} u_{ijk1} / \Delta \\ & + \sum_{i,k,q} \Lambda_3 \rho_{ikq} x_{ikq} + \Xi e \left(\chi_0 + 0.91\chi_1 + 0.65\chi_2\right) + \sum_k \xi_{Nurse} \Omega N_k \phi_k \end{aligned} \tag{4.3}$$

$$\text{s.t.} \sum_l u_{ijkl} = 1 \ \forall i, j, k \tag{4.4}$$

$$u_{i0k1} = 0 \ \forall i, k \tag{4.5}$$

$$u_{i1k3} \geq u_{i2k3} \ \forall i, k \tag{4.6}$$

$$u_{i2k3} \geq u_{i3k3} \ \forall i, k \tag{4.7}$$

$$u_{i1k3} \geq u_{i4k3} \ \forall i, k \tag{4.8}$$

$$\sum_q r_q x_{ikq} = \sum_j u_{ijk3} \ \forall i, k \tag{4.9}$$

$$\sum_q x_{ikq} \leq 1 \ \forall i, k \tag{4.10}$$

$$\sum_{q=4}^{6} x_{ikq} = u_{i4k3} \ \forall i, k \tag{4.11}$$

$$\sum_{i,q} p_{ikq} x_{ikq} \leq B w_k \ \forall k \tag{4.12}$$

$$w_k \leq s_k \ \forall k \tag{4.13}$$

$$\sum_{i,j,k} d_{ijk} u_{ijk1} \leq n\gamma \tag{4.14}$$

$$\sum_{j,k} u_{ijk1} \leq JK\beta_i \ \forall i \tag{4.15}$$

$$\sum_{i} \beta_i \leq \nu\gamma \tag{4.16}$$

$$\sum_{i,j,l} \mu_{jl} d_{ijk} u_{ijkl} \leq MMD \ \forall k \tag{4.17}$$

$$\chi_0 + \chi_1 + \chi_2 + \chi_3 = 1 \tag{4.18}$$

$$\chi_0 \leq \Upsilon_0 \tag{4.19}$$

$$\chi_1 \leq \Upsilon_0 + \Upsilon_1 \tag{4.20}$$

$$\chi_2 \leq \Upsilon_1 + \Upsilon_2 \tag{4.21}$$

$$\chi_3 \leq \Upsilon_2 \tag{4.22}$$

$$\Upsilon_0 + \Upsilon_1 + \Upsilon_2 = 1 \tag{4.23}$$

$$\frac{\sum\limits_{i,j,k} d_{ijk} u_{ijk3}}{\sum\limits_{i,j,k} d_{ijk}} = 0.6903\chi_1 + 0.8999\chi_2 + \chi_3 \tag{4.24}$$

$$\phi_k \geq 148 \frac{\sum\limits_{i,j} d_{ijk} u_{ijk3}}{\sum\limits_{i,j} d_{ijk}} - 17.32 \ \forall k \tag{4.25}$$

Our objective in Equation (4.3) is to minimize the average daily cost to distribute medication from the central pharmacy to the patient, including picking costs, robot and ADC purchase and maintenance costs, restocking costs, cartfill preparation costs, and nurse queueing costs. The constraints for the linear program are given by equations (4.4)-(4.25). Equation (4.4) states that there is a standardized pathway for every medication and dose type to each unit. Equations (4.5-4.8) define the logical constraints that result from the standardized pathways. First, equation (4.5) prevents STAT doses from being routed through the robot due to the short turnaround time. Equations (4.6) and (4.8) ensure that if either first doses or PRN doses are kept in the ADC then STAT doses are kept in the ADC. This is based on the understanding that if a medication is stored in the ADC the nurse will get it from the ADC upon placement of a STAT order regardless of the standardized pathway for STAT doses. We use similar logic in equation (4.7) between first doses and maintenance doses.

Equations (4.9-4.11) determine which dose types are held in the ADC. Based off which dose types are stored in the ADC, equation (4.12) determines the total number of ADC compartments needed and equation (4.13) ensures that we do not exceed the ADC capacity on each unit. Equation (4.14) determines if the central pharmacy uses a pharmacy robot for the distribution process and ensures the total number of doses distributed through the robot does not exceed the robot's throughput capacity. Equation (4.15) determines if the robot is holding a particular medication type and equation (4.16) limits the total number of unique medications that the robot can hold. Equation (4.17) ensures that the expected number of missing doses on each unit does not exceed the target missing dose rate. The constraints given in Equations (4.18-4.24) determine the value of the piecewise linear function representing the cartfill preparation costs. Equation (4.25) determines the average nurse queueing time on each unit each time the nurse accesses the ADC during major medication administrations.

4.4 FORMULATING A ROBUST PHARMACY DISTRIBUTION MODEL

The model described in the previous section was developed to provide various decisions on whether to use centralized or decentralized pharmacies, ADCs, etc. These decisions are highly dependent on the actual demand of medications which depends on the patient population and their diagnosis. Fluctuations in demand could cause decisions that are based on the average demand to perform poorly and in some cases could lead to a solution that is infeasible under variation. Robust optimization tools are used to make decisions robust when there is high fluctuation in demand and hence improve patient safety in a more secure way.

The objective function and four constraints contain d_{ijk}. We rewrite these constraints with the uncertainty denoted by $\tilde{d}_{ijk}$ and for an uncertainty set which we denote as U_d. Note that constraints ($4.24'$) and ($4.25'$) both have fractional expressions with d_{ijk} in both the numerator and denominator. However, for both these functions we only allow the values of d_{ijk} in the numerator to vary. This is because our original functions are based on the average number of patients and medication distribution. Therefore, if there is more medication than average passing through the cartfill process, then it follows that we may have more patients in the hospital than the average census, and thus see a higher cartfill preparation cost. Similarly, if more medication is delivered via the ADC, it suggests that the time it requires nurses to access medication will increase, in turn increasing the nurse queueing time.

$$\begin{aligned}\min \ & \sum_{i,j,k,l} c_{ijkl}\tilde{d}_{ijk}u_{ijkl} + g\gamma + \sum_{k} hw_k + \Lambda_1 \sum_{i,j,k} \tilde{d}_{ijk}u_{ijk1}/\Delta + \sum_{i,k,q} \Lambda_3 \rho_{ikq} x_{ikq} \\ & + \Xi e\left(\chi_0 + 0.91\chi_1 + 0.65\chi_2\right) + \sum_{k} \xi_{Nurse}\Omega N_k \phi_k, \ \ \tilde{d} \in \mathcal{U}_d\end{aligned} \tag{4.3'}$$

$$\sum_{i,j,k} \tilde{d}_{ijk}u_{ijk1} \leq n\gamma, \ \ \tilde{d} \in \mathcal{U}_d \tag{4.14'}$$

$$\sum_{i,j,l} \mu_{jl}\tilde{d}_{ijkl}u_{ijkl} \leq MMD \ \forall k, \ \ \tilde{d} \in \mathcal{U}_d \tag{4.17'}$$

$$\frac{\sum_{i,j,k} \tilde{d}_{ijk}u_{ijk3}}{\sum_{i,j,k} d_{ijk}} = 0.6903\chi_1 + 0.8999\chi_2 + \chi_3, \ \ \tilde{d} \in \mathcal{U}_d \tag{4.24'}$$

$$\phi_k \geq 148\frac{\sum_{i,j} \tilde{d}_{ijk}u_{ijk3}}{\sum_{i,j} d_{ijk}} - 17.32 \ \forall k, \tilde{d} \in \mathcal{U}_d \tag{4.25'}$$

Our robust problem is found by replacing equations (4.3), (4.14), (4.17), (4.24), and (4.25) by the new equations (4.3′), (4.14′), (4.17′), (4.24′), and (4.25′). In the next several sections we will define our uncertainty sets which we will use when solving the problem. Based on our uncertainty sets, we will discuss how these constraints are transformed and rewritten in the robust counterpart so we are able to solve the new model. Note that the nominal problem is a large scale MIP that has thousands of variables and constraints. The robust counterpart size is dependent upon which uncertainty set is used. Hence, the challenge is to select sets that do not increase the size of the problem greatly. Our choice of uncertainty sets is dependent upon the assumption that the demands are independent and identically distributed random variables.

4.4.1 Uncertainty Sets

We will analyze this problem with the following uncertainty set, denoted $\mathcal{U}_d$. We will use the average and standard deviation of d_{ijk} to define the uncertainty. We choose to focus on the individual uncertainty for each medication and dose type on each unit.

$$U_d = \left\{ d_{ijk} | -\Gamma \leq \frac{d_{ijk} - \bar{d}_{ijk}}{\sigma_{d_{ijk}}} \leq \Gamma, d_{ijk} \geq 0 \right\}$$

We can rewrite U_d as

$$U_d = \left\{ d_{ijk} | -\Gamma\sigma_{d_{ijk}} + \bar{d}_{ijk} \leq d_{ijk} \leq \Gamma\sigma_{d_{ijk}} + \bar{d}_{ijk}, d_{ijk} \geq 0 \right\}.$$

This uncertainty set creates an individual range of uncertainty for every individual observation of d_{ijk}.

4.4.2 Constructing the Robust Counterpart

Our objective function and the uncertain constraints, equations (4.3′), (4.14′), (4.17′), (4.24′), and (4.25′), cannot be solved in their current form. Using the properties of $\mathcal{U}_d$ we will formulate the corresponding robust counterpart for each equation. Note that equation 3′ has two specific terms which include $\tilde{d}_{ijk}$ so we focus on these two terms when creating the robust counterpart. Since our objective is to minimize the total cost, we can rewrite equation

$$\min \sum_{i,j,k,l} c_{ijkl}\tilde{d}_{ijk}u_{ijkl} + \Lambda_1 \sum_{i,j,k} \tilde{d}_{ijk}u_{ijk1}/\Delta + ..., \;\; \tilde{d} \in \mathcal{U}_d$$

as

$$\min \max_{\tilde{d}_{ijk} \in U_d} \left(\sum_{i,j,k,l} c_{ijkl}\tilde{d}_{ijk}u_{ijkl} + \Lambda_1 \sum_{i,j,k} \tilde{d}_{ijk}u_{ijk1}/\Delta \right) + ..., \;\; \tilde{d} \in \mathcal{U}_d.$$

Then the inner maximization problem can be rewritten as the linear optimization problem.

$$\max \quad \sum_{i,j,k,l} c_{ijkl}\tilde{d}_{ijk}u_{ijkl} + \Lambda_1 \sum_{i,j,k} \tilde{d}_{ijk}u_{ijk1}/\Delta$$

$$
\begin{aligned}
\text{s.t.} \quad & \tilde{d}_{ijk} \geq -\Gamma\sigma_{d_{ijk}} + \bar{d}_{ijk} \quad \forall i,j,k \\
& \tilde{d}_{ijk} \leq \Gamma\sigma_{d_{ijk}} + \bar{d}_{ijk} \quad \forall i,j,k \\
& d_{ijk} \geq 0 \quad \forall i,j,k
\end{aligned}
$$

We take the dual of the linear program.

$$
\begin{aligned}
\min \quad & \sum_{i,j,k} \left(-\Gamma\sigma_{d_{ijk}} + \bar{d}_{ijk}\right)\pi_{1ijk} + \sum_{i,j,k} \left(\Gamma\sigma_{d_{ijk}} + \bar{d}_{ijk}\right)\pi_{2ijk} \\
\text{s.t.} \quad & \pi_{1ijk} + \pi_{2ijk} \geq \sum_{l} c_{ijkl}u_{ijkl} + \Lambda_1 u_{ijk1}/\Delta \quad \forall i,j,k \\
& \pi_{1ijk} \leq 0, \pi_{2ijk} \geq 0 \quad \forall i,j,k
\end{aligned}
$$

Then using duality, we can adjust equation (4.3′) to equation (4.3*) to form the robust counterpart. In addition, the constraints in equations (4.26*) and (4.27*) are added to the robust model for uncertainty set 1.

$$
\begin{aligned}
\min \quad & \sum_{i,j,k} \left(-\Gamma\sigma_{d_{ijk}} + \bar{d}_{ijk}\right)\pi_{1ijk} + \sum_{i,j,k} \left(\Gamma\sigma_{d_{ijk}} + \bar{d}_{ijk}\right)\pi_{2ijk} + g\gamma + \sum_{k} hw_k + \sum_{i,k,q} \Lambda_3\rho_{ikq}x_{ikq} \\
& + \Xi e\left(\chi_0 + 0.91\chi_1 + 0.65\chi_2\right) + \sum_{k} \xi_{Nurse}\Omega N_k\phi_k
\end{aligned} \tag{4.3*}
$$

$$
\pi_{1ijk} + \pi_{2ijk} \geq \sum_{l} c_{ijkl}u_{ijkl} + \Lambda_1 u_{ijk1}/\Delta \quad \forall i,j,k \tag{4.26*}
$$

$$
\pi_{1ijk} \leq 0, \pi_{2ijk} \geq 0 \quad \forall i,j,k \tag{4.27*}
$$

Formulating the robust counterpart for the constraints is very similar. We will demonstrate first for constraint (4.14’) below and then show the additional robust counterpart constraints for the additional constraints.

$$
\sum_{i,j,k} \tilde{d}_{ijk}u_{ijk1} \leq n\gamma, \;\; \tilde{d} \in \mathcal{U}_d
$$

We can rewrite this equation as

$$\max_{\tilde{d}\in\mathcal{U}_d}\sum_{i,j,k}\tilde{d}_{ijk}u_{ijk1}\leq n\gamma$$

The left side of the constraint can be rewritten as the linear program

$$\begin{aligned}
\max \quad & \sum_{i,j,k} d_{ijk}u_{ijk1} \\
\text{s.t.} \quad & \tilde{d}_{ijk}\geq -\Gamma\sigma_{d_{ijk}}+\bar{d}_{ijk} \quad \forall i,j,k \\
& \tilde{d}_{ijk}\leq \Gamma\sigma_{d_{ijk}}+\bar{d}_{ijk} \quad \forall i,j,k \\
& d_{ijk}\geq 0 \quad \forall i,j,k
\end{aligned}$$

The dual of the linear program is

$$\begin{aligned}
\min \quad & \sum_{i,j,k}\left(-\Gamma\sigma_{d_{ijk}}+\bar{d}_{ijk}\right)\pi_{3ijk}+\sum_{i,j,k}\left(\Gamma\sigma_{d_{ijk}}+\bar{d}_{ijk}\right)\pi_{4ijk} \\
\text{s.t.} \quad & \pi_{3ijk}+\pi_{4ijk}\geq u_{ijk1}\forall i,j,k \\
& \pi_{3ijk}\leq 0,\pi_{4ijk}\geq 0 \quad \forall i,j,k
\end{aligned}$$

We use duality, similar to the earlier example with the objective function, to construct the robust counterpart constraints below.

$$\sum_{i,j,k}\left(-\Gamma\sigma_{d_{ijk}}+\bar{d}_{ijk}\right)\pi_{3ijk}+\sum_{i,j,k}\left(\Gamma\sigma_{d_{ijk}}+\bar{d}_{ijk}\right)\pi_{4ijk}\leq n\gamma \tag{4.14*}$$

$$\pi_{3ijk}+\pi_{4ijk}\geq u_{ijk1} \tag{4.28*}$$

$$\pi_{3ijk}\leq 0,\pi_{4ijk}\geq 0 \quad \forall i,j,k \tag{4.29*}$$

The following set of equations are added to the robust counterpart for equations (4.17$'$), (4.24$'$), and (4.25$'$).

$$\sum_{i,j,k}\left(-\Gamma\sigma_{d_{ijk}}+\bar{d}_{ijk}\right)\pi_{5ijk}+\sum_{i,j,k}\left(\Gamma\sigma_{d_{ijk}}+\bar{d}_{ijk}\right)\pi_{6ijk}\leq MMD \quad \forall k \tag{4.17*}$$

$$\pi_{5ijk} + \pi_{6ijk} \geq \sum_{l} \mu_{jl} u_{ijkl} \quad \forall i, j, k \tag{4.30*}$$

$$\pi_{5ijk} \leq 0, \pi_{6ijk} \geq 0 \quad \forall i, j, k \tag{4.31*}$$

$$\sum_{i,j,k} \left(-\Gamma \sigma_{d_{ijk}} + \bar{d}_{ijk}\right) \pi_{7ijk} + \sum_{i,j,k} \left(\Gamma \sigma_{d_{ijk}} + \bar{d}_{ijk}\right) \pi_{8ijk} \geq 0.6903\chi_1 + 0.8999\chi_2 + \chi_3 \tag{4.24*}$$

$$\pi_{7ijk} + \pi_{8ijk} \leq \frac{u_{ijk3}}{\sum\limits_{i,j,k} d_{ijk}} \quad \forall i, j, k \tag{4.32*}$$

$$\pi_{7ijk} \geq 0, \pi_{8ijk} \leq 0 \quad \forall i, j, k \tag{4.33*}$$

$$\sum_{i,j} \left(-\Gamma \sigma_{d_{ijk}} + \bar{d}_{ijk}\right) \pi_{9ijk} + \sum_{i,j} \left(\Gamma \sigma_{d_{ijk}} + \bar{d}_{ijk}\right) \pi_{10ijk} \leq \phi_k + 17.32 \quad \forall k \tag{4.25*}$$

$$\pi_{9ijk} + \pi_{10ijk} \geq \frac{148 u_{ijk3}}{\sum\limits_{i,j} d_{ijk}} \quad \forall i, j, k \tag{4.34*}$$

$$\pi_{9ijk} \leq 0, \pi_{10ijk} \geq 0 \quad \forall i, j, k \tag{4.35*}$$

4.5 RESULTS

We solve the robust counterpart for 5 units from Geisinger Medical Center which comprise 42% of the total demand across the hospital. We use the same parameter values as in Chapters 2 and 3 to solve the model. The model requires 11.45 hours to solve for the complete efficiency frontier for 295 different missing dose rates. The throughput of each pathway is depicted in Figure 4.2. When we solve the robust optimization model, the robot becomes the most utilized pathway while the tech sees fairly low utilization and we slowly add medications to the ADC. Note that the missing dose rate in the robust problem corresponds to the missing dose rate on the worst day, not the average day. Therefore, we see a much higher missing dose rate compared to the deterministic model. As we constrain our missing dose rate more, we continue adding medications to the ADC until we reach the ADC capacity. When determining which medication types to add to the ADC, we find that initially as there is a steady increase in doses sent through the ADC, we are adding STAT

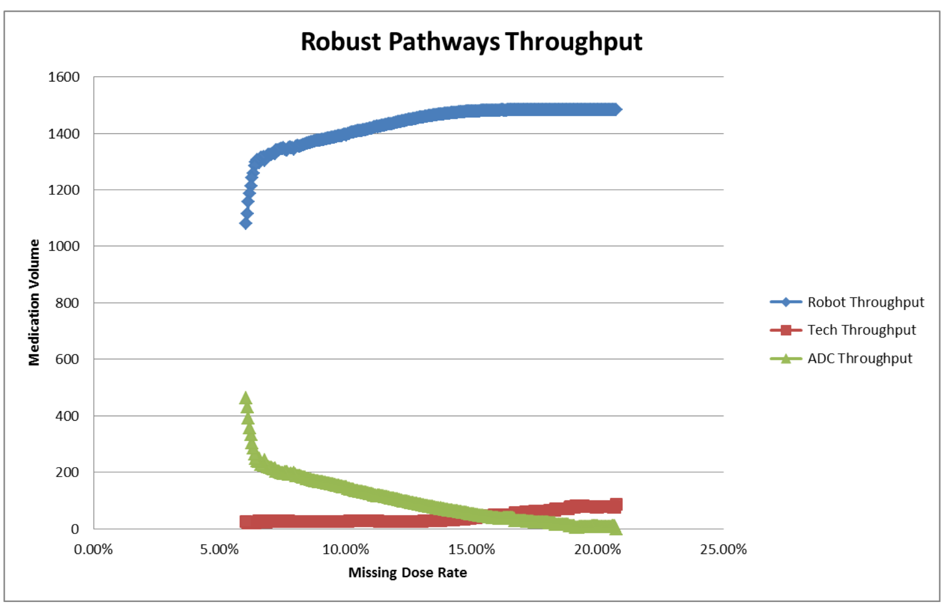

Figure 4.2: The optimal delivery pathways for the robust counterpart.

and first dose medications to the ADC. Later, when we see a sharp increase in the number of medications that are distributed through the ADC, this aligns with the addition of maintenance and PRN doses to the the ADC.

In Figure 4.3 we observe that the cost of the optimal solution steadily increases as we reduce our missing dose rate. However, the increase in cost is not as large as in the previous model from Chapters 2 and 3 because this is a robust model and because we changed the ADC restock cost to more accurately reflect the restock rate. We can see that in the worst case, the cost to deliver medication for the day will exceed $2,000.

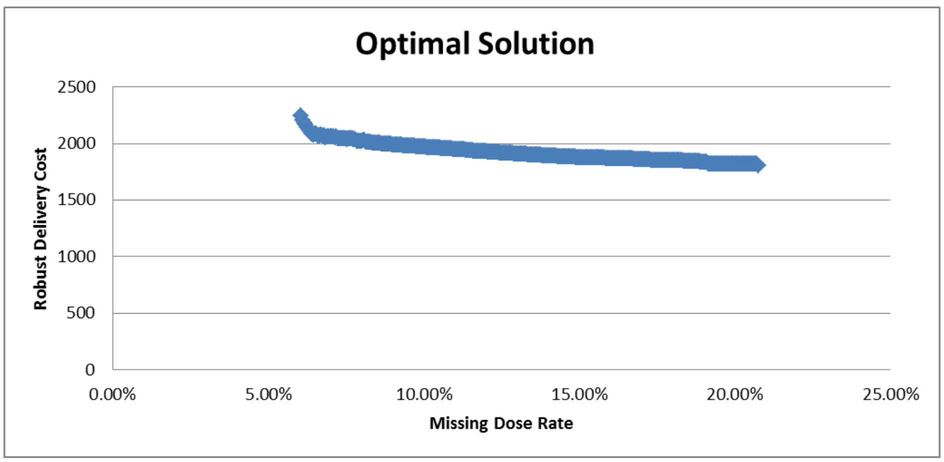

Figure 4.3: The minimal cost to deliver medication for the robust counterpart.

While we have focused on the robust case, we also want to observe how this affects the average missing dose rate. Figure 4.4 demonstrates that when using the robust model, our average missing dose rate will never exceed 5.25%. However, as we restrict the worst case missing dose rate more and more we see a significant reduction in the average missing dose rate. We see that while in the worst case, the smallest missing dose rate we can reach is 6%, this actually corresponds to an average missing dose rate of 1.15%, indicating that the robust decision performs well in the average case.

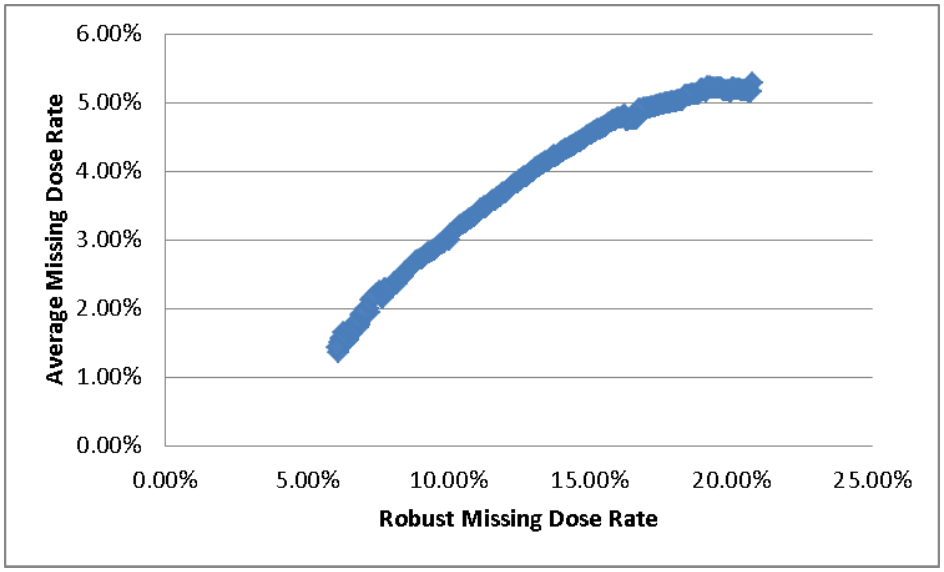

Figure 4.4: Comparison of the allowable missing dose rate to the average missing dose rate under the robust solution.

4.6 CONCLUSION

We expand on our pharmacy distribution model to capture that variation that occurs in the daily pharmacy demand. We construct the uncertainty set based on a year of data from the Geisinger Medical Center. We formulate and solve the robust counterpart to determine the best method to deliver medications from the inpatient pharmacy to the patient in the worst case scenario. Our results demonstrated that using the ADC and the robot together results in the minimal cost for the worst case demand. While the deterministic problem follows a similar solution, in the deterministic problem we do not use the robot for some lower missing dose rates because of the cost. However, in this scenario, the cost combined with the missing dose rate force us to always use the robot and the ADC together. Furthermore, the results demonstrate that this solution performs well in the average case as well. Similarly to the deterministic model, the robust model first distributes STAT and first doses through the ADC and then slowly adds maintenance and PRN doses.

In addition to some model limitations which were discussed in Chapter 2, our current formulation for the robust counterpart has limitations. For our robust counterpart, we chose an uncertainty set that allowed the demand for every medication and dose type on each unit to vary. However, in reality we know that while the demand does vary, on average there are only 245 medications distributed each day with a maximum of 287 medications distributed and a minimum of 211 medications distributed each day over the course of 1 year. In our current model we have 839 medications with each medication varying in demand in the robust model. So while our model clearly demonstrates the advantages of using robust optimization and has formulated the robust model, it does not necessarily capture the worst-case scenario that the hospital would actually experience. We plan to extend the current formulation for the robust model by using a new uncertainty set which will limit the number of individual medications that can vary based on our medication distribution set. Since we know that not all medications are distributed every day and some vary only a small degree, this change will better capture the worst-case scenario that the hospital actually encounters. In addition, our current robust formulation uses the same curve for the cartfill preparation costs and nurse queueing costs as we found when creating our model in Chapter 2. While we have kept the denominator constant to allow these equations to scale in comparison to the average scenario, it is also possible to consider a robust version of these equations. The robust curve for the cartfill preparation cost may not follow the same curve, and in fact may be much higher. Similarly for the nurse queueing curve, we may see that on a worse day the nurses may begin queueing for a smaller percentage of medications or may have a higher queueing time than that of the curve we currently have. However, we expect the changes for both of these to be small in comparison to the overall cost, having minimal effect on the solution of the problem.

5.0 USING MATHEMATICAL MODELING TO IMPROVE THE EMERGENCY DEPARTMENT NURSE SCHEDULING PROCESS

5.1 INTRODUCTION

In emergency departments, nurses are a vital resource in treating patients; therefore, it is important that nurses are scheduled in a manner that provides enough nurses during critical times of the day. The emergency department is unique from other areas of the hospital in that the number of patients visiting the emergency department varies every day and across the day. There is no limit on the number of patients that can visit the emergency department and all patients require a different level of care. This creates complications for nurse scheduling, particularly because nurses typically work 8- or 12-hour shifts and the number of nurses needed at each time of day may not easily align with the necessary set of 8- and 12-hour shifts.

In this paper, we examine the scheduling process that was used at our partner hospital, UPMCs Childrens Hospital of Pittsburgh (CHP), to improve nurse scheduling in an effort to improve service in the emergency department. Based on the scheduling problems in the current process we determined a set of daily shifts that can be used to create a standard daily schedule. We formulated a mathematical model that minimized the number of shifts required to reach the target nurse staffing level at each hour. Two key aspects captured by our model include the consideration of the length of nursing shifts and meal coverage. Our results demonstrate the benefits CHP has experienced with the implementation of the new scheduling process.

5.2 LITERATURE REVIEW

Nurse scheduling is a widely studied topic in the operations research and nursing literature, particularly in the inpatient unit setting. These studies adjust nursing schedules to increase staff participation and include preferences as a motivator to boost morale and reduce nurse call-offs. Despite the work done in the academic setting, it is rarely applied in practice with only 30% of systems discussed in research articles actually implemented[33]. Table 5.1 summarizes the different topics and fields studied in emergency department nursing literature.

Table 5.1: Literature Review Summary

Operations Research		Nursing Literature	
Topic	Papers	Topic	Papers
Daily Nursing Schedule	[18],[35]	Staffing Levels	[1], [?]
Nurse Staffing Levels	[10],[51],[31],[29]	Self-Scheduling	[?],[54],[45],[30]
Monthly Schedule	[14], [59]		

In the operations research field, most of the nurse staffing literature comprises a number of different optimization strategies to solve the nurse staffing problem. Simulation was used to compare different nursing schedules and analyze which was best for patient care[18]. Goal programming was used to consider multiple goals, including a nurse-to-physician ratio, to determine the weekly 8-hour shifts while accounting for over and understaffing[35]. Additionally, multiple papers have tackled the problem by determining staffing levels and then the aligning shift schedules[10, 51, 31, 29]. In addition to shift schedules, mathematical models were used to create monthly schedules which take into account both emergency department staffing requirements, legal regulations, and nurse preferences to improve nurse satisfaction while significantly decreasing the amount of time required to compile a monthly schedule[14, 59].

The nursing literature presents successful scheduling processes that have been implemented in emergency departments. A common theme within the nursing literature is the idea of self-scheduling where nurses select their work schedule by signing up for specific shifts available throughout the day. A study exploring work-life balance for shift workers showed improved work-life balance when self-scheduling was an option for workers[1]. The Bureau of Labor Statistics expects a nursing shortage as growth in the sector increases and nurses decrease. Many solutions to the nursing shortage have been discussed, including improving staffing by accounting for the patient workload instead of staffing based on overall volume[**?**]. This is difficult as emergency departments across the United States vary in size and staffing profiles, which affects how they employ nurse scheduling within their unit. A benefit reiterated in every scenario is that nurses learn to work together to schedule as a group. However, in multiple hospitals, they found that overstaffing can result from the 8- and 12-hour shifts nurses decide to work or sometimes there can be 4 hour holes which are difficult to fill[**?**]. Another method used a patient-to-nurse ratio in order to have scheduling patterns reflect the expected patient census pattern[54]. In line with self-scheduling, shared scheduling, requires all the nurses in the emergency department to work together to complete the schedule. They found nurses preferred the flexibility and control over the schedule, which led to improved morale and less staff turnover[45]. The idea of self-scheduling was also employed at Novant Health where they used Microsoft Excel to help guide the idea of self-scheduling by coding the schedule into Excel and having staff sign up in the Excel model based on commonly filled 8- and 12-hour shifts[30].

Most of the previous mathematical models have determined specific shifts by considering over- and understaffing or have created monthly schedules with nurse preferences. In the nurse staffing literature, most of the systems use self-scheduling methods, which allow nurses to sign up for any shift while the department attempts to schedule enough nurses to meet the staffing levels set by nursing management. In this paper we use operations research to develop a method for self-scheduling, which has reduced the total time necessary to reconcile the final monthly schedule while ensuring that the emergency department has adequate staffing at all times.

5.3 BACKGROUND

CHP is a Magnet-recognized tertiary care facility with a level 1 trauma program and has made the list of U.S. News and World Report Honor Roll of Americas Best Childrens Hospitals. The emergency department treats approximately 80,000 patients annually. The average patient arrivals and average number of patients at each hour of the day are given in Figure 5.1(a) and (b), respectively.

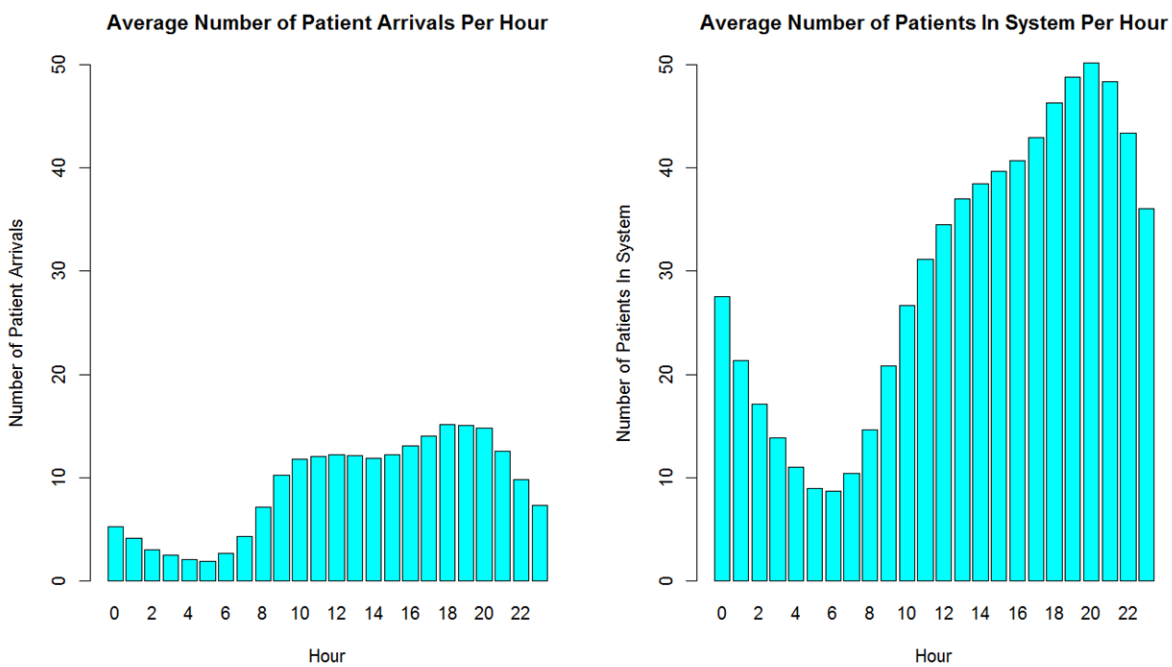

Figure 5.1: Average emergency department (a) patient arrivals (b) number of patients by hour.

The patient arrival pattern can be seen in Figure 5.1(a). We see that there are very few patients arriving to the emergency department between the hours of 1 AM and 6 AM. However, the number of patient arrivals increases from 7AM until approximately 8 PM with the highest number of patients arriving typically around 6 PM– 8 PM. In comparison, Figure 5.1(b) shows the number of patients in the system at every hour of the day. While we see similar patterns between the two, the average number of patients in the system is much higher than the average number of patients arriving to the system. In particular, we see large increases in the number of patients in the system starting at 8 AM and the number of patients continues to increase until approximately 9 PM after which the number in system begins to decrease.

Based on patient arrivals, volume, and the number of nursing hours available each week, the nursing leadership identified nurse staffing targets for the number of nurses they would like in the emergency department at each hour of the day. The number of nurses was based on the sections of the emergency department that were open, the nurse-to-room ratio for each section, and the additional positions such as rapid triage, full triage, and sedation that were staffed at each time. The staffing targets, given in Table 5.2, were assigned in 4-hour increments beginning at 7 AM and increasing through the day. The maximum number of nurses staffing the ED occurs from 3 PM–11 PM when the maximum number of patients are in the system.

Table 5.2: Nurse staffing target by time of day

Time	11p-3a	3a-7a	7a-11a	11a-3p	3p-7p	7p-11p	11p-12a
Nurse Staffing Target	15	8	8	15	19	19	15

While the administration was able to identify the ideal number of nurses that should be staffing the emergency department, there were no predetermined shifts within the daily staffing schedule. Instead, the process used a grid of 4-hour blocks and nurses would self-select their shift by signing up for 2 or 3 4-hour blocks to create an 8- or 12-hour shift. An example of the grid can be seen in Figure 5.2.

As shown in Figure 5.2, Nurse A signs up for a 12-hour shift from 7 AM–7 PM. After the grid is completely filled, the two nurses that comprise the scheduling team meet to finalize the schedule. This can be a tedious process as they may have to search through the schedule to determine if a nurse is working an 8- or 12-hour shift. For instance, the highlighted nurses in orange, red, and yellow are scattered throughout the signup sheet. In addition, in this

7 am	11 am	3 pm	7 pm	11 pm	3 am
Nurse A	Nurse A	Nurse A	Nurse G	Nurse L	Nurse L
Nurse B	Nurse B	Nurse B	Nurse H	Nurse M	Nurse M
Nurse C	Nurse C	Nurse C	Nurse I	Nurse N	Nurse N
Nurse D	Nurse D	Nurse D	Nurse K	Nurse O	Nurse O
Nurse E	Nurse E	Nurse E	Nurse L	Nurse P	Nurse P
Nurse F	Nurse F	Nurse F	Nurse M	Nurse Q	Nurse X
Nurse J	Nurse G	Nurse G	Nurse N	Nurse R	
	Nurse H	Nurse H	Nurse O	Nurse S	
	Nurse I	Nurse I	Nurse P	Nurse T	
	Nurse J	Nurse K	Nurse Q	Nurse U	
	Nurse K	Nurse Q	Nurse R	Nurse V	
	Nurse W	Nurse R	Nurse S	Nurse X	
		Nurse S	Nurse T	Nurse Y	
		Nurse T	Nurse U		
		Nurse U	Nurse V		
		Nurse V	Nurse Y		
		Nurse W			
		Nurse Y			

Figure 5.2: Example of former scheduling process.

schedule there are 8 4-hour blocks, indicated with bold outlines, which no one has selected. If Nurse Y decides to work a 3 PM–11 PM shift, we see that there is an isolated block, denoted in pink, at 11 AM. This four hour block is an undesirable shift so the scheduling group will either try to move Nurse Ys time to 11 AM–11 PM, add another nurse from 11 AM–11 PM, or leave the spot unstaffed. In the first scenario, if Nurse Y moves, they can make the remaining blocks into one 12-hour block and one 8-hour block and reach their ideal staffing. In the second scenario, the emergency department will be overstaffed from 3 PM–3 AM. While overstaffing is not a problem for the day it occurs, it could lead to understaffing on other days of the week. Finally, in the last scenario, the emergency department is understaffed from 11 AM–3 PM, where we typically see a rapid increase in the number of patients arriving to the emergency department.

One way to prevent overstaffing and understaffing problems from occurring is to have a set of 8- and 12-hour shifts that allow the emergency department to meet their target nurse staffing, as demonstrated in Figure 5.3. Figure 5.3 organizes Figure 5.2 into a set of 8- and 12-hour shifts which gives us the correct number of nurses at every hour to meet our staffing targets. Ideally, the schedule in Figure 5.3 could be used each day and nurses would sign up for an 8- or 12-hour shift. Use of a daily schedule, like this example, would reduce the number of shifts that the scheduling team must adjust to get full coverage throughout the day and eliminate instances of overstaffing and understaffing that can occur as a result of using the 4-hour grid in Figure 5.2.

Some key features regarding the current process include:

- Every nurse signs up for their own shifts so they set their schedule.
- Every nurse works a combination of 8- and 12-hour shifts a week based on their contract.
- The day begins and ends at 7 AM, so no shift spans 7 AM.

We preserved these features when we created the daily schedule.

7 am	11 am	3 pm	7 pm	11 pm	3 am
Nurse A	Nurse A	Nurse A			
Nurse B	Nurse B	Nurse B			
Nurse C	Nurse C	Nurse C			
Nurse D	Nurse D	Nurse D			
Nurse E	Nurse E	Nurse E			
Nurse F	Nurse F	Nurse F			
Nurse J	Nurse J				
	Nurse G	Nurse G	Nurse G		
	Nurse H	Nurse H	Nurse H		
	Nurse I	Nurse I	Nurse I		
	Nurse K	Nurse K	Nurse K		
	Nurse W	Nurse W			
		Nurse Q	Nurse Q	Nurse Q	
		Nurse R	Nurse R	Nurse R	
		Nurse S	Nurse S	Nurse S	
		Nurse T	Nurse T	Nurse T	
		Nurse U	Nurse U	Nurse U	
		Nurse V	Nurse V	Nurse V	
			Nurse L	Nurse L	Nurse L
			Nurse M	Nurse M	Nurse M
			Nurse N	Nurse N	Nurse N
			Nurse O	Nurse O	Nurse O
			Nurse P	Nurse P	Nurse P
				Nurse X	Nurse X

Figure 5.3: Schedule in Figure 5.2 with a set of 8 and 12 hour shifts.

5.4 METHODS

Using mathematical modeling, we solved the problems that resulted from scheduling with the 4-hour grid by creating a schedule, which can be used every day, consisting of 8- and 12-hour blocks. Within this model we considered meal breaks as well as the types of nursing shifts available and nurse preferences. We defined the following variables:

- x_j: The number of 12-hour shifts beginning at block j
- y_j: The number of 8-hour shifts beginning at block j

We assumed that all nurses working a 12-hour shift take their half-hour meal break during the middle 4-hour block of their shift and that all nurses working an 8-hour shift take their meal break during the second block of their shift, except any 8-hour nurses who work at 11 PM who take their meal during the first block of their shift.

Using these variables, we created the following linear program:

$$\begin{aligned}
\text{Minimize} \quad & \sum_{j=1}^{4} x_j + \sum_{j=1}^{5} y_j && (5.1)\\
\text{Subject to} \quad & x_1 + y_1 \geq 8 && (5.2)\\
& \frac{7}{8}x_1 + \frac{7}{8}y_1 + x_2 + y_2 \geq 15 && (5.3)\\
& x_1 + \frac{7}{8}x_2 + \frac{7}{8}y_2 + x_3 + y_3 \geq 19 && (5.4)\\
& x_2 + \frac{7}{8}x_3 + \frac{7}{8}y_3 + x_4 + y_4 \geq 19 && (5.5)\\
& x_3 + \frac{7}{8}x_4 + \frac{7}{8}y_4 + \frac{7}{8}y_5 \geq 15 && (5.6)\\
& x_4 + y_5 \geq 8 && (5.7)\\
& \sum_{j=1}^{4} x_j \geq 0.80 \left(\sum_{j=1}^{4} x_j + \sum_{j=1}^{5} y_j \right) && (5.8)
\end{aligned}$$

In this linear program the objective function (5.1) minimized the total number of shifts needed to reach the target nurse staffing levels. Equations (5.2-5.7) ensured that the number of nurses available at each block is at least the target nurse staffing levels. Since the nurse meal break is one-half hour of a 4-hour block we assumed that a nurse works 7/8 of the block that included their meal break. We ensured that we had the appropriate balance of 8- and 12-hour shifts in equation (5.8). Note that the target value for this balance is determined by the nursing contracts which state how many 8- and 12-hour shifts a nurse is expected to work each week.

In addition to these constraints, we included constraints to accommodate nursing preferences for the 8-hour shifts. For instance, if a nurse works five 8-hour shifts each week and they prefer working from 11p-7a, we accommodated this nurse by requiring at least one 8-hour shift at night. Based on our work force, we needed two 8-hour shifts from 7AM-3PM and one from 11AM-7PM, 7PM-3AM, and 11PM-7AM. So the model had the following constraints:

$$y_1 \geq 2 \tag{5.9}$$

$$y_2 \geq 1 \tag{5.10}$$

$$y_4 \geq 1 \tag{5.11}$$

$$y_5 \geq 1 \tag{5.12}$$

5.5 RESULTS

When we solved the linear program given in equations (5.1-5.12) we generated multiple solutions. The one which aligned best with the nurses for CHP is listed in Table 5.3.

Table 5.3: Optimal Solution to the linear program.

Optimal Solution	
$x_1 = 6$	$y_1 = 2$
$x_2 = 7$	$y_2 = 1$
$x_3 = 7$	$y_3 = 0$
$x_4 = 7$	$y_4 = 1$
	$y_5 = 1$

This solution required 28 additional nursing hours compared to the current schedule to cover meal breaks and meet the nurse staffing targets. This breaks down to 16 hours to cover meal breaks and an additional 12 hours which supplements the staffing requirements. Table 5.4 compares the number of hours between the nurse staffing targets set by management and the nurse staffing levels throughout the day resulting from the optimal solution. Note we have modeled the problem to account for meal breaks, so the nurse staffing levels are adjusted so that nurses who are taking a meal break account for 78 nurses during the meal break period. The majority of additional nursing resources are allotted during 3PM-11PM when we have the highest number of patients in the ED. In addition to adding nursing coverage, these nurses help increase the throughput of patients in the ED when they are not required to cover meal breaks.

Table 5.4: Comparison of nurse staffing targets and levels from the optimal solution.

Time	7a-11a	11a-3p	3p-7p	7p-11p	11p-3a	3a-7a
Nurse Staffing Target	8	15	19	1	15	8
Nurse Staffing Levels	8	15	20.125	21	15	8

From the optimal solution we generated the daily schedule depicted in Figure 5.4. Note that there are 6 blocks from 7AM–7PM which corresponds to 6 12-hour shifts beginning at block 1 (7 AM) and 2 blocks from 7AM–3PM which correspond to 2 8-hour shifts. In each shift there is one block, which is highlighted orange, indicating when the nurse should take their meal break in order to have full coverage.

In comparison to the original sign up method, the daily schedule improves the scheduling process. First, nurses have fewer options to consider when they are signing up for shifts. When there are empty shifts, nurses can sign up for overtime and work a full 8- or 12-hour shift compared to the original schedule where they needed nurses to work overtime for only 4 hours. Because there are no 4-hour blocks, the scheduling team does not need to change the shifts nurses originally signed up for to ensure coverage. In addition, because there are

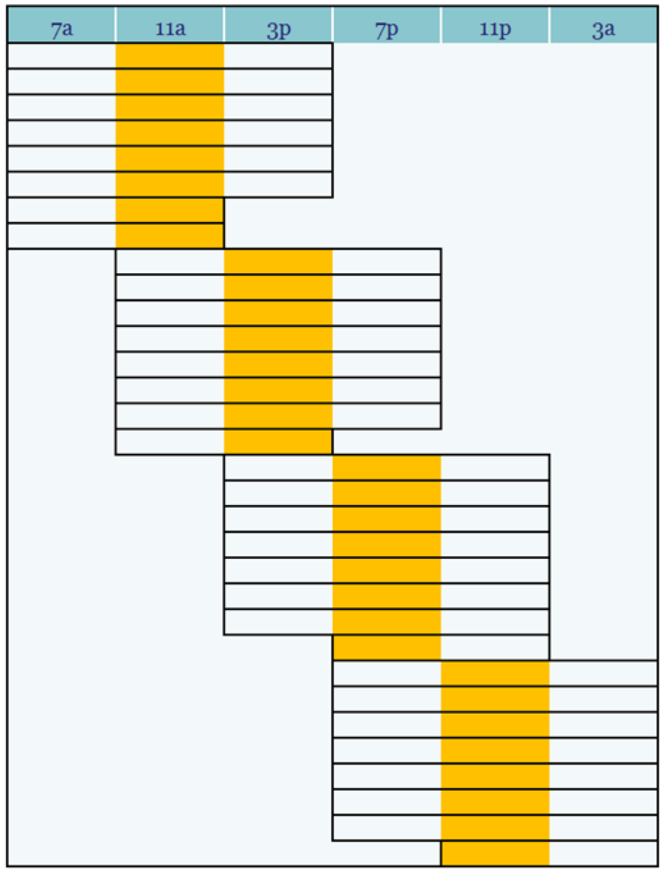

Figure 5.4: Schedule grid that results from solving the linear program.

no 4-hour blocks, understaffing cannot occur at only one time slot unless an entire shift is not covered. Because the shift is either an 8- or 12-hour shift, it is more desirable as an overtime shift.

5.6 DISCUSSION

The increased nurse coverage resulting from the new schedule has the potential to increase nurse satisfaction. Under the new schedule, nurses are guaranteed to work the shift that they signed up for on the original schedule. Furthermore, understaffing happens less frequently so the number of nurses in the emergency department is more reliable. With the new schedule, there are enough nurses in the emergency department that each nurse can have a fully covered meal break without increasing their patient load to cover a fellow nurse during their meal break.

The new schedule is also beneficial to the scheduling team as the scheduling team found a significant decrease in the total time to reconcile the schedule. Under the original process, reconciling the schedule occurred over a two-week period and required 40 hours total, or 20 hours per nurse. When the new daily scheduling method was implemented, the nurses were able to reconcile the schedule during a single week, requiring approximately 18 hours total.

Despite the advantages, there are disadvantages and limitations of the model. In this instance, the model determined a schedule that aligned with the staffing at CHP but required 12 additional nursing hours each day. While we demonstrated that these 12 additional hours occur during the busiest time of day, which improve the staffing levels, the additional annual cost is significant. In addition, the model requires the user to input previously determined staffing levels. While the model can determine the best schedule for a given users input, if the staffing levels are not appropriate (for example, far too low for the demand) then the model is limited in its capability to improve nurse staffing. Finally, the model requires nurses to be able to make changes to the model and solve it whenever they adjust staffing requirements.

In this paper, we demonstrated how mathematical modelling can be combined with self-scheduling to result in an implementable schedule within an emergency department. A unique aspect of our model is that our staffing results in each nurse having time for a fully covered meal break while the department will still maintain its target staffing levels. Through using this staffing approach, the emergency department we worked with improved its nurse staffing, increased staff morale, and reduced the total time necessary to complete the monthly schedule. Although the model has limitations, it is applicable to a wide variety of hospitals and shift work settings that use self-scheduling to complete a schedule.

6.0 CAPTURING UNCERTAINTY IN EMERGENCY DEPARTMENT NURSE SCHEDULING

6.1 INTRODUCTION

Nurses scheduling is a complicated process that requires collaboration between nursing management and their staff. The emergency department has several factors which influence nurse staffing that do not exist within other settings of the hospital. Patient arrivals to the system are random by time of day and treatment times depend on the patient's diagnosis and the supporting services required by the patient. Therefore, the number of patients and the nursing workload varies drastically across the day every day. In addition, there has been a push for regulation to set minimum nurse-patient ratios for hospitals. The emergency department requires a nursing schedule which aligns with patient arrivals and volume in order to treat patients in a timely but safe manner and meets the minimum nurse-patient ratio across the day.

The final nursing schedule can be a source of staff dissatisfaction due to unfavorable shifts and understaffing. In order to take nurse scheduling preferences into account, self-scheduling, where nurses sign up for the shift they want to work, has become a preferred method within many hospitals. In Chapter 5, we provide a basic linear program to help nursing leadership identify a template to use within the self-scheduling process. The suggested template was used in the emergency department at a large regional children's hospital and resulted in improved nurse staffing levels and decreased the time required to finalize the monthly nursing schedule. While creating the nursing template and setting up the self-scheduling process are relatively easy tasks, the difficulty arises in trying to create a template that aligns the nursing schedule and patient volume.

While hours per patient visit is a commonly used metric to determine emergency department staffing, it cannot necessarily be used to determine what the hourly staffing should be [49]. In this paper, we consider two separate models to determine the staffing level at each hour of the day based on the number of available nurses while creating a schedule of 8 and 12 hour shifts that can be used in the self-scheduling process. The first model minimizes the difference in nurse time per patient across the day based on historical data and the second model minimizes the amount of time we exceed the minimum nurse-patient ratio given specified nurse staffing resources. We also analyze both models in the event of uncertainty to capture variation which exists in the patient volume of the hospital by creating a robust model. The models are tested on data from our partner hospital to compare performance. We demonstrate how this method can be used within the self-scheduling process to find the best set of shifts to provide enough staffing coverage throughout the day.

6.2 LITERATURE REVIEW

Nurse scheduling is a problem that has been widely studied within the operations research and nursing literature due to the complex nature of the scheduling process and the dissatisfaction that can occur amongst nurses after the schedule is released. Dissatisfaction results from unfavorable shifts, such as night or weekend shifts, and can lead to poor morale amongst staff, high nurse turnover and increased call-offs. One method frequently used to combat dissatisfaction is self-scheduling of nurse schedules. There have been multiple literature reviews that have examined the previous work completed in both the operations research [56] and [20] and nursing literature [8]. Most of the literature discussed here focuses on nurse staffing on inpatient units; however, inpatient staffing methods typically do not extend well to emergency department staffing, therefore we will only review literature which specifically considers ED staffing.

In 1999, the California legislature passed nurse-patient ratios which were mandated in 2004, requiring a minimum of 1 nurse: 4 patients in the emergency department. Chan et al. [11] analyzed the effect of the mandated nurse-patient ratios and found that when a nurse exceeded the 4 patient maximum, the care time was longer and wait time in the emergency department increased. A study by Weichenthal and Hendey [57] sought to determine if there is an association between nurse-patient ratios and quality of care. The study found that after implementation of mandated nurse-patient ratios, the number of patients who left without receiving care decreased slightly, however the average waiting time to room, length of stay, and admission time all increased significantly. Based on their analysis the authors did not find that nurse-patient ratios had an impact on the quality of care patients receive. When nursing unions in Australia were pursuing mandated nurse-patient ratios, Wise et al. [58] completed a study to evaluate the staffing levels at emergency departments in New South Wales. The study found the average staffing ratios were below 4 patients per nurse for the morning and evening shifts but slightly over 5 patients per nurse for the night shift. However, ratios were variable and sometimes as high as 8 patients per nurse during the morning and evening and up to 16 patients per nurse during the night shift. This study demonstrated that current staffing practices do not always reflect patient volume and the ratios can be used to identify poor staffing. Spaite et al. [53] looks at using process redesign to decrease patient waiting time and increase patient satisfaction. One process improvement suggested was decreasing nurse-patient ratios from 5 or 6 patients per nurse to 4 patients per nurse. While this change required hiring new nurses, there was an increase in nursing availability and patients could be treated more efficiently.

In the operations research field, most of the nurse staffing literature comprises a number of different optimization strategies to solve the nurse staffing problem. Draeger [18] used simulation to compare the current nursing schedule with two alternative schedules to determine which one is better for patient care. Kwak and Lee [35] apply goal programming to the health care staffing problem by considering multiple goals such as budgeting, physician utilization, nurse-to-physician ratio, and tech-to-physician ratio. Using this ratio, they determine the weekly 8-hour shifts that nurses should be scheduled for by penalizing for over and understaffing.

Additionally, multiple authors have tackled the problem by determining staffing allocations and then shift schedules. Centeno et al. [10] used simulation to identify the nursing requirements at every hour of the day. They then constructed a mathematical model which determined a shift schedule that had at least as many nurses as the requirements from the simulation for every hour of the day. Sinreich and Jabali [51] apply simulation to determine the total number of resources needed in the emergency department. Using the output of the simulation, they determined the set of shifts for the different staff resources throughout the day to minimize the penalty of overstaffing and understaffing. They show that using their simulation results they are able to reduce the total number of nurse staffing hours while only slightly increasing the nurse workload by establishing a set of shifts that are anywhere from 8 to 12 hours long. Izady and Worthington [31] used simulation to determine the number of nurses that should staff the emergency department throughout the day. After determining the necessary staffing levels, they used the optimization problem from Sinreich and Jabali to determine the set of shifts that nurses should work to best meet the staffing demand when minimizing the total penalty for over and understaffing. The two stage process used by Sinreich and Jabali and Izady and Worthington can result in shift schedules which can be either infeasible or suboptimal, so Ingolfsson et al. [29] creates a method that iterates between two processes to find shift schedules in a system where the required service level is variable. They use a schedule evaluator to identify areas in the schedule with low service levels and then generate a new schedule through integer programming. The method iterates between these two steps until a feasible solution is reached.

In addition to the work done on establishing nursing shifts, some authors have focused on creating a model that can aid in the creation of the monthly schedule. De Grano et al. [14] helped the emergency department at York hospital complete their monthly schedule through the use of an auction and optimization. In this paper they a use a two stage process where nurses first fill out their bids and they optimize the schedule to maximize the total bids. Then, nurses who havent met their required work hours are added to additional shifts. Their method decreased the time to schedule from approximately 8 hours a month to a few minutes of running the linear program and the final schedule satisfied more nurses preferences. Wong et al. [59] use a two-stage heuristic approach to figure out the monthly schedule for nurses in an emergency department. The first step generates schedules by assigning nurses to shifts while satisfying all the constraints regarding labor regulations and emergency department staffing. After a schedule is generated they use a sequential local search to improve the schedule by looking to include soft constraints which reflect the nurses preferences. The algorithm solutions are comparable to the results of the optimization model while running significantly faster.

While no emergency department nurse staffing literature has used robust optimization to determine nursing schedules, there have been articles which use different methods to evaluate the robustness of a proposed solution. Wong et al. [59] determines the robustness of the generated schedule by showing that different constraint weightings result in similar fulfillment rates of the soft constraints. El-Rifai, et al. [19] used a stochastic optimization model to determine the optimal set of nursing shifts to staff by minimizing the expected patient waiting time. They find multiple schedules based on different constraints regarding shift times and start times. The schedules found by the stochastic optimization model were tested against the current staffing levels in the emergency department using a simulation model. To test the robustness of the schedules, the schedules were compared to each other when the patient arrivals were increased by 15%. They found that the stochastic models still outperformed the current schedule, demonstrating that the schedules were robust, even under high patient volume.

Despite the lack of research in the emergency department application, robust optimization has been applied in similar areas. Soteriou and Chase [52] have proposed a framework for the use of robust optimization within the context of service quality. They form a three step method that links the operation variables to the service quality obtained through questionnaires completed by customers. Through robust optimization, managers can make decisions that lead to the best service quality while accounting for the uncertainty that exists in the service quality due to the variation in customer opinions.

Call centers are comparable to emergency departments as they have variable demand that can fluctuate by time of day, day of week, and season. Similar to Emergency Departments, the uncertainty in the number of callers can lead to long waiting times for callers to receive help. Some papers which determine call center staffing through robust optimization inclue Mattia et al. [40] and Liao et al. [38]. Mattia et al. formulates a two-stage robust integer program which has uncertain staffing levels and looks to determine feasible shift schedules along with the minimum reallocation cost for a fixed schedule. The robust model results in lower staffing costs in comparison to the traditional staffing model. Liao et al. models the call center staffing problems using a stochastic programming approach and a robust programming formulation with uncertainty existing in the mean arrival rate of calls. They conduct a numerical study with data from a Dutch hospital to compare the models. As the uncertainty increases, the robust approach total cost increases, however the under staffing penalties decrease significantly as more staff is added to meet the uncertain demand, eliminating under staffing. For low uncertainty, the robust model outperforms the stochastic programming approach. However, as the uncertainty increases, the stochastic programming approach begins outperforming the robust model.

In the nurse staffing literature, there has been working analyzing the effects of the implementation of minimum nurse-patient ratios and work to determine ideal staffing levels. Compared to the current literature, we focus on determining staffing for our current level of available nursing hours. We determine staffing by considering two problems: (1) what are the correct staffing levels to meet the minimum nurse-patient ratios as frequently as possible and (2) how can we create equal workloads for all nurses throughout the day. We compare the individual models along with a multiobjective model containing both objectives. We consider the deterministic version of the models as well as the robust models to account for daily variation.

6.3 BACKGROUND

In the ED, nursing leadership is tasked with creating a staffing schedule that meets patient demand. The schedule is frequently based on a combination of feelings regarding "busyness", patient volume, and nurse to room ratios. We worked with a large regional children's hospital to create a schedule which accurately reflected patient volume throughout the day. The daily patient volume in the emergency department during 2015 is given in Figure 6.1.

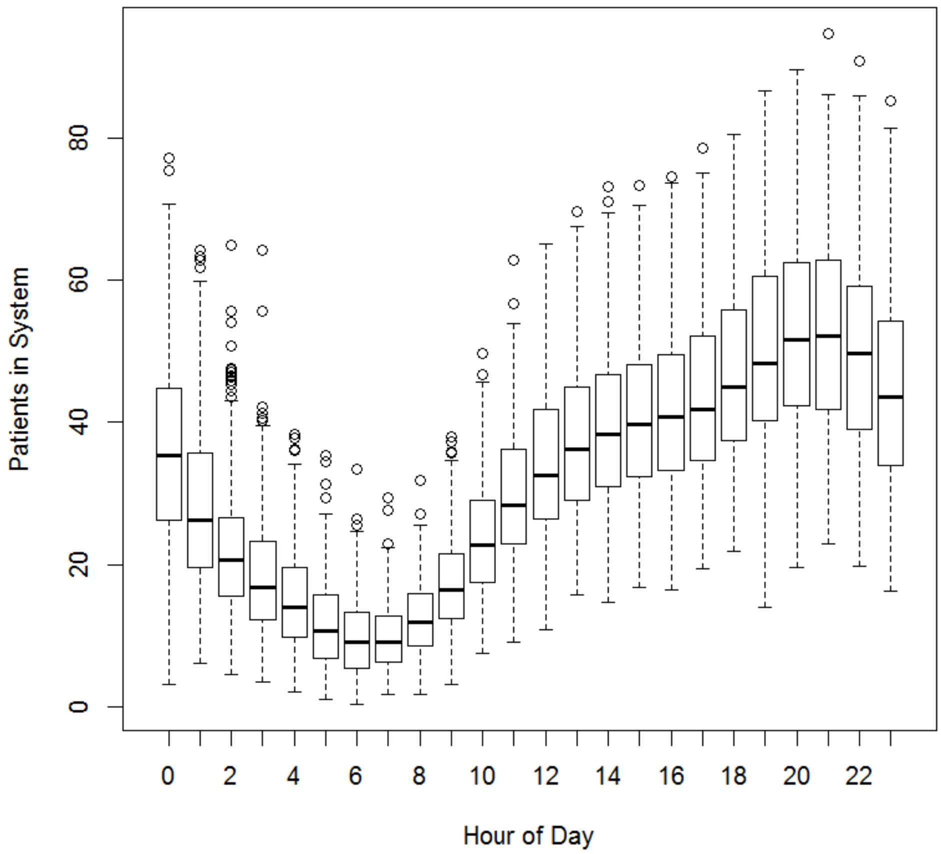

Figure 6.1: Box plot of patient volume in the emergency department based on the time of day.

Table 6.1: ED nurse staffing levels

Block	7a-11a	11a-3p	3p-7p	7p-11p	11p-3a	3a-7a
Nurses	8	15	19	19	15	8

As shown in the boxplot, most days follow a distinct pattern where beginning around 7AM we have very few patients and see an increase in patients until almost 11PM when the patient load begins to decrease with the highest number of patients occurring between the hours of noon and midnight. However, there is also drastic variability for the number of patients in the emergency department at every hour. Considering emergency department leadership's knowledge of patient volume and using room ratios, the emergency department leadership determined nurse staffing numbers for 4-hour blocks throughout the day which are given in Table 6.1. Part of the logic behind the four hour blocks is that it is easier to have large onboarding and offboarding times rather than having new nurses arriving for work every hour. In addition, having a stationary number of nurses over a 4 hour interval creates stability within the schedule and allows for easy transitions when nurses are leaving their shift for the day. For instance, if one nurse is leaving and no one is starting at the same time then the patient assignments may need to be reconfigured. The block assignments also allow for the nurses to switch their task once during their shift to combat fatigue. Figure 6.2 compares patient volume to the nurse staffing levels, indicated by the pink blocks. As the figure demonstrates, although the nurse staffing levels increase and decrease accordingly with the overall patient volume trends, the nursing workload is not even at the early hours of the day compared with the peak hours of the day.

While creating the staffing numbers, there are multiple considerations that leadership must take into effect. First, they want to create a scheduling template that they can use in the self-scheduling process they currently employ. Each day starts at 7AM and no nurse shifts should span 7AM, so all shifts scheduled must either begin after 7AM or end before 7AM. Nurses are contracted to work a certain number of 8 and 12-hour shifts, so we want to ensure that all possible shifts are in increments of 8 and 12 hours. We also need to ensure that the correct percentage of nurses are scheduled for each shift type. So in our example, at least 80% of the shifts nurses are contracted to work must be 12 hour shifts and at least 10% must be 8 hour shifts. All nurses who work 8-hour shifts do not work at the same time, so we require that our 8-hour shifts are spread across the day, by not allowing more than half of the 8-hour shifts to be scheduled for the same time slot. We want to distribute the nurse workload across the day according to patient volume, so we will consider the measure

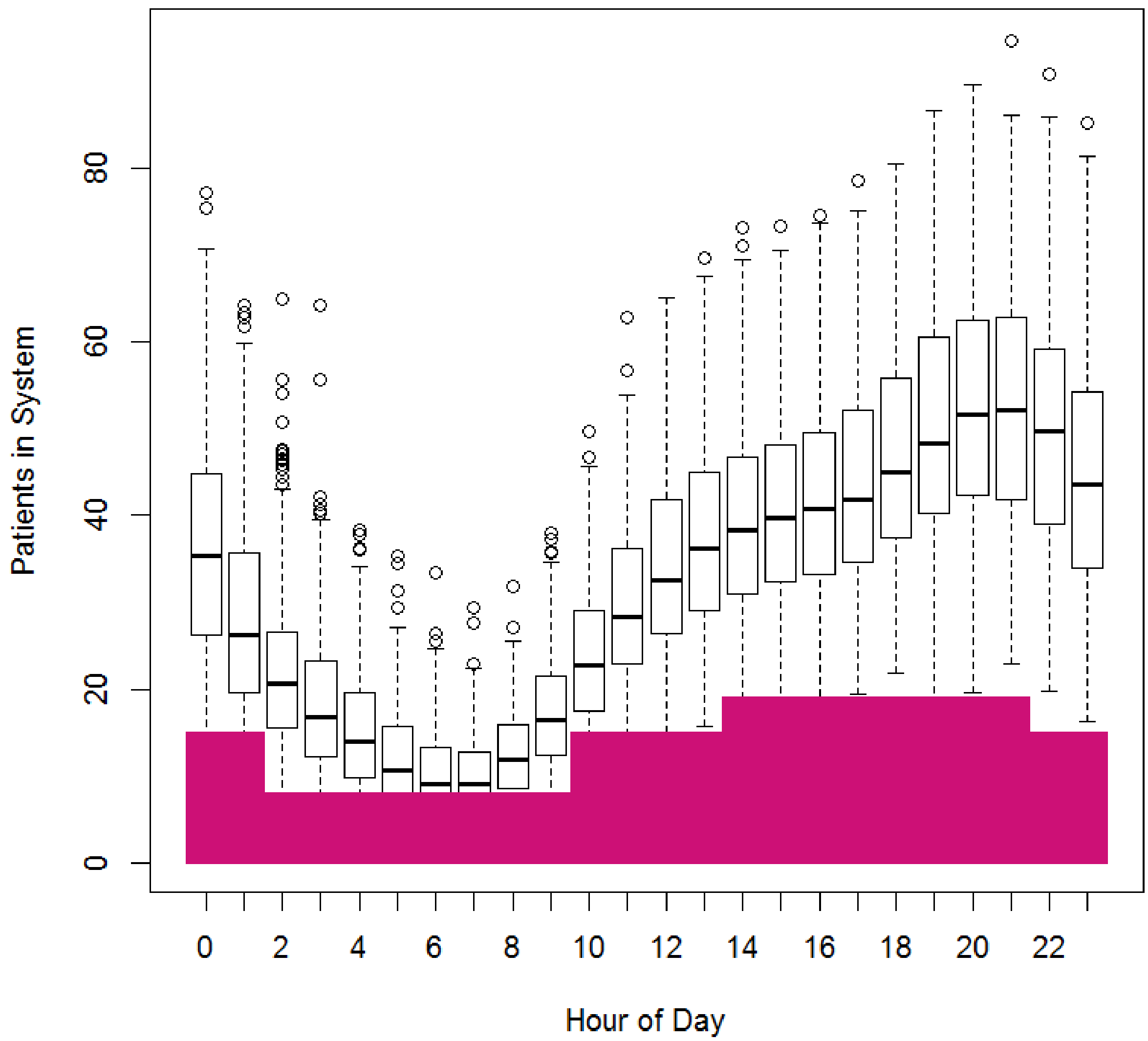

Figure 6.2: Comparison of emergency department nurse staffing levels with patient volume

of available nurse time per patient which is given by the ratio of nurses to patients. In addition to considering the nurse-patient ratio, we also want to ensure that we have the schedule time blocks beginning at the best times so that the blocks align well with patient volume.

6.4 MATHEMATICAL MODEL

A complication within the nurse staffing process is that the number of nurses the department has may not necessarily align with the number of nurses that are needed and nurses cannot be added into the staffing plan instantaneously. In this section we look to determine models which use our current available staff to determine the staffing levels across a single day. The first model maximizes patient safety by considering nurse-patient ratios while the second model minimizes the differences in nursing workload using nurse-patient ratios as a measure of workload. We also consider a multiobjective model which weights the two objectives equally.

In Chapter 5, the model is based on block scheduling, where we assume that nurses can only start or end their shifts at the hours of 7AM, 11AM, 3PM, 7PM, 11PM, or 3AM. We recognize the block scheduling feature is important within the hospital to provide easy shift changes, assignment transitions, and a stable staffing. However, as shown in Figure 6.1, the patient volume at 7AM does not necessarily align with the patient volume at 10AM. But, in the block scheduling method, we have the same number of nurses at both times of the day. Instead, we want to consider a method where we allow for nurses to start at any hour of the day, as long as their shift begins after 4AM and ends before 8AM. In this context, we allow flexibility in the shift start and end time to better capture the patient volume changes that are occurring hourly.

6.4.1 Maximizing Patient Safety Using Nurse-Patient Ratios

While nurse-patient ratios are debated, the literature has demonstrated that they can be indicators of poor staffing. Furthermore, we use the nurse-patient ratio as a way to evaluate the nurse workload. Our goal is to maximize patient safety, which in this respect corresponds to minimizing the frequency with which our nurse-patient ratio is smaller than $\frac{1}{4}$. We aim to determine a set of 8- and 12-hour shifts for nurses to work during the day. These shifts serve as the minimum set that should be fully staffed each day. The model does not account for overtime scheduling.

The model parameters are defined in Table 6.2.

Table 6.2: Nurse Staffing Model Notation

Index	Description
j	Hour
Variable	**Description**
u_j	Number of nurses working during hour j
r_j	Number of nurses working a 12-hour shift which begins during hour j
s_j	Number of nurses working an 8-hour shift which begins during hour j
v_j	Indicator variable that nurses are starting a shift at hour j
z_j	Understaffing difference for hour j
Parameter	**Description**
m	Minimum number of nurses required in the ED
M	Maximum number of nurses allowed in the ED
b	Number of nursing blocks available daily
p_j	50th percentile for the number of patients in the system at hour j

Using the parameters defined in Table 6.2, the linear program to determine the daily shift schedule to maximize patient safety is given in Equations (6.1)-(6.12).

$$\min \sum_{j} z_j \tag{6.1}$$

$$\text{subject to } u_j \geq m \ \forall j \tag{6.2}$$

$$u_j \leq M \ \forall j \tag{6.3}$$

$$\sum_{j=1}^{24} u_j = 4b \tag{6.4}$$

$$z_j \geq 0 \ \forall j \tag{6.5}$$

$$z_j \geq \frac{1}{4} - \frac{u_j}{p_j} \ \forall j \tag{6.6}$$

$$u_j = \sum_{\min\{j-12,0\}}^{j} r_j + \sum_{j+13}^{24} r_j + \sum_{\min\{j-8,0\}}^{j} s_j + \sum_{j+17}^{24} s_j \ \forall j \tag{6.7}$$

$$\sum_{j=1}^{24} r_j \geq 0.8 \left(\sum_{j=1}^{24} r_j + \sum_{j=1}^{24} s_j \right) \tag{6.8}$$

$$\sum_{j=1}^{24} s_j \geq 0.1 \left(\sum_{j=1}^{24} r_j + \sum_{j=1}^{24} s_j \right) \tag{6.9}$$

$$s_j \leq 0.5 \sum_{i=1}^{24} s_i \ \forall j \tag{6.10}$$

$$M v_j \geq r_j + s_j \ \forall j \tag{6.11}$$

$$\sum_{i=j}^{j+4} v_i = 1 \ \forall j \tag{6.12}$$

The objective function (6.1) in the linear program minimizes the frequency with which we are understaffed and do not meet the ideal nurse-patient ratio. Constraints (6.2) and (6.3) ensure that the number of nurses staffed at every hour is greater than the minimum threshold, m, and less than the maximum threshold, M. Equation (6.4) requires that all available nursing hours are scheduled throughout the day. The understaffing ratio is determined in constraints (6.5) and (6.6). Constraint (6.7) assigns the number of nurses working during each 4 hour block, based on the number of hours they work a day and their initial start time. Constraints (6.8) and (6.9) ensure that at least 80% of the shifts in the daily schedule are 12 hours long and at least 10% of the shifts are 8 hour shifts. These numbers are placed to correspond with the number of 8 and 12 hour shifts nurses are contracted to work for. Constraint (6.10) limits the 8 hour shifts that can occur at the same time because not all nurses who work 8 hour shifts work at the same time. Constraints (6.11) and (6.12) determine the initial start times and ensure that shifts can only start in 4 hour increments.

However, the deterministic model only considers a mean or median value of patient demand and does not consider any of the variation present in Figure 6.1. In order to capture the variability within the process, we identify multiple uncertainty sets, formulate a robust model, and evaluate the solution of the robust model under each uncertainty set against the the solution found by the deterministic model.

The volume of patients at each hour in the emergency department varies daily. While we originally solve the model with p_j representing the median patient volume at hour j, we now consider p_j as an uncertian parameter. For the robust model, we consider an uncertainty set, $\mathcal{U}_j = [g_j, h_j]$ which contains p_j. In the robust model, constraint (6.6) becomes

$$z_j \geq \frac{1}{4} - \frac{u_j}{\tilde{p}_j}, \forall j, \tilde{p}_j \in \mathcal{U}_j.$$

In order to solve the model, we must construct the robust counterpart. The robust counterpart is given by replacing equation (6.6) with equation (6.6*).

$$z_j \geq \frac{1}{4} - \frac{u_k}{h_j}, \forall j. \tag{6.6*}$$

6.4.2 Minimizing Differences in Nurse Workload

The objective in this model variant is to balance nurse workload by minimizing the difference between the maximum and minimum nurse-patient ratios across the day. In addition to the parameters defined in Table 6.2, we need the new variables defined in Table 6.3.

Table 6.3: Additional Nurse Staffing Model Notation

Variable	Description
w	Minimum nurse-patient ratio across all hours in the day
x	Maximum nurse-patient ratio across all hours in the day

We replace our objective (6.1) and constraints (6.5) and (6.6) with objective (6.1′) and constraints (6.5′) and (6.6′).

$$\min \quad x - w \tag{6.1$'$}$$

$$w \leq \frac{u_j}{p_j}, \forall j \tag{6.5$'$}$$

$$x \geq \frac{u_j}{p_j}, \forall j \tag{6.6$'$}$$

For the robust model, we consider an uncertainty set, $\mathcal{U}_j = [g_j, h_j]$ which contains p_j. In the uncertain model, constraints (6.5′) and (6.6′) become

$$w \leq \frac{u_j}{\tilde{p}_j}, \forall j, \tilde{p}_j \in \mathcal{U}_j$$

$$x \geq \frac{u_j}{\tilde{p}_j}, \forall j, \tilde{p}_j \in \mathcal{U}_j$$

In order to solve the model with the uncertain equations, we formulate the robust counterpart, given in equations (6.5′*) and (6.6′*).

$$w \leq \frac{u_j}{h_j}, \forall j \tag{6.5$'^*$}$$

$$x \geq \frac{u_j}{g_j}, \forall j \tag{6.6$'^*$}$$

6.4.3 Multiobjective Model

While these two models both use nurse-patient ratios to determine the daily schedule, they both have different objectives which limit their performance with regards to each other. When we maximize our patient safety, our objective is to have the nurse-patient ratio at each hour of the day under 4; however, we do not ensure that the nurse-patient ratio is distributed equally throughout the day. Similarly, when we look to minimize the difference in nurse workload, we do not necessarily ensure that we also maintain a nurse-patient ratio of 1:4. Therefore, we also create the multiobjective model given in equations (6.13)-(6.26) which combines the two individual models to find a solution which works to satisfy both objectives. We let c_1 and c_2 indicate the weights of the individual objectives.

$$\min \; c_1 \sum_j z_j + c_2(x - w) \tag{6.13}$$

$$\text{subject to } u_j \geq m \; \forall j \tag{6.14}$$

$$u_j \leq M \; \forall j \tag{6.15}$$

$$\sum_{j=1}^{24} u_j = 4b \tag{6.16}$$

$$z_j \geq 0 \; \forall j \tag{6.17}$$

$$z_j \geq \frac{1}{4} - \frac{u_j}{p_j} \; \forall j \tag{6.18}$$

$$w \leq \frac{u_j}{p_j} \; \forall j \tag{6.19}$$

$$x \geq \frac{u_j}{p_j} \; \forall j \tag{6.20}$$

$$u_j = \sum_{\min\{j-12,0\}}^{j} r_j + \sum_{j+13}^{24} r_j + \sum_{\min\{j-8,0\}}^{j} s_j + \sum_{j+17}^{24} s_j \; \forall j \tag{6.21}$$

$$\sum_{j=1}^{24} r_j \geq 0.8 \left(\sum_{j=1}^{24} r_j + \sum_{j=1}^{24} s_j \right) \tag{6.22}$$

$$\sum_{j=1}^{24} s_j \geq 0.1 \left(\sum_{j=1}^{24} r_j + \sum_{j=1}^{24} s_j \right) \tag{6.23}$$

$$s_j \leq 0.5 \sum_{i=1}^{24} s_i \ \forall j \tag{6.24}$$

$$Mv_j \geq r_j + s_j \ \forall j \tag{6.25}$$

$$\sum_{i=j}^{j+4} v_i = 1 \ \forall j \tag{6.26}$$

As with the previous two models, we wish to account for the variability that occurs in the patient volume. Our uncertain constraints are given by equations (6.18), (6.19), and (6.20). To formulate the robust model, we replace equations (6.18), (6.19), and (6.20) with equations (6.6^*)($6.5'^*$), and ($6.6'^*$), respectively.

6.5 RESULTS

We solve the deterministic and robust models for the patient safety, workload difference, and multiobjective problems presented in Section 4. We use current emergency department staffing guidelines which set the minimum number of nurses required in the emergency department to 6. Due to the number of rooms and auxiliary positions in our ED, we set the maximum number of nurses in the emergency department to 28. Based on current filled emergency department nursing positions, we have 84 nursing blocks available each day. We use the data presented in Figure 6.1 to create the patient volume parameters for our model. In the deterministic model, we use the median number of patients at each hour as our patient volume. Because the deterministic models only use the median number of patients, they do not acknowledge that variation occurs throughout the day. To account for this variation, we introduce the robust model. The robust model allows us to consider the daily variation when finding our optimal solution and find the best solution for the worst case scenario. We consider multiple uncertainty sets of the form $[p_q, p_{1-q}]$ where p_q denotes the q^{th} percentile of patient volume.

6.5.1 Maximizing Patient Safety Using Nurse-Patient Ratios

Lower nurse-patient ratios have been associated with improving patient safety. We solve this problem to minimize the frequency and degree to which we exceed the mandated ratio of 4 patients to 1 nurse. Figure 6.3 plots the optimal nurse staffing levels at each hour of the day when we solve the patient safety model for both the deterministic and robust models. We see that for the uncertainty sets based on higher percentiles of patient volume, [0%, 100%] and [5%, 95%], we start the day at 7AM. However, for all other times, we begin the day at 4AM. This earlier start time allows us to have lower nurse staffing levels during the early morning when the emergency department is least busy and steadily build up our staffing levels as patient volume increases.

To evaluate the effectiveness of the solutions, we will compare the nurse-patient ratios resulting from the optimal staffing levels. In Figure 6.4 we can see the nurse-patient ratio at every hour of the day for the q^{th} percentile of patients. The black horizontal line at 4 indicates where the nurse-patient ratio exceeds 4:1. We can see in the deterministic model, we are able to be under the 4:1 ratio for every hour of the day. However, for days with higher patient volume, we may exceed this value. This indicates that daily variation needs to be considered when determining staffing levels to meet a set nurse-patient ratio.

When solving the robust model, we consider 6 different uncertainty sets based off historical patient percentiles: [25%, 75%], [20%, 80%], [15%, 85%], [10%, 90%], [5%, 95%], and [0%, 100%]. We can see that we can maintain the 4:1 nurse-patient ratio for up to the 90th percentile of patients.

One shortcoming of this model is that there is significant variation across the day in the nurse-patient ratios. This can lead to uneven workloads in our staffing, where nurses who work during the hours from 11AM-11PM have a higher workload in comparison to nurses who work earlier in the day or overnight.

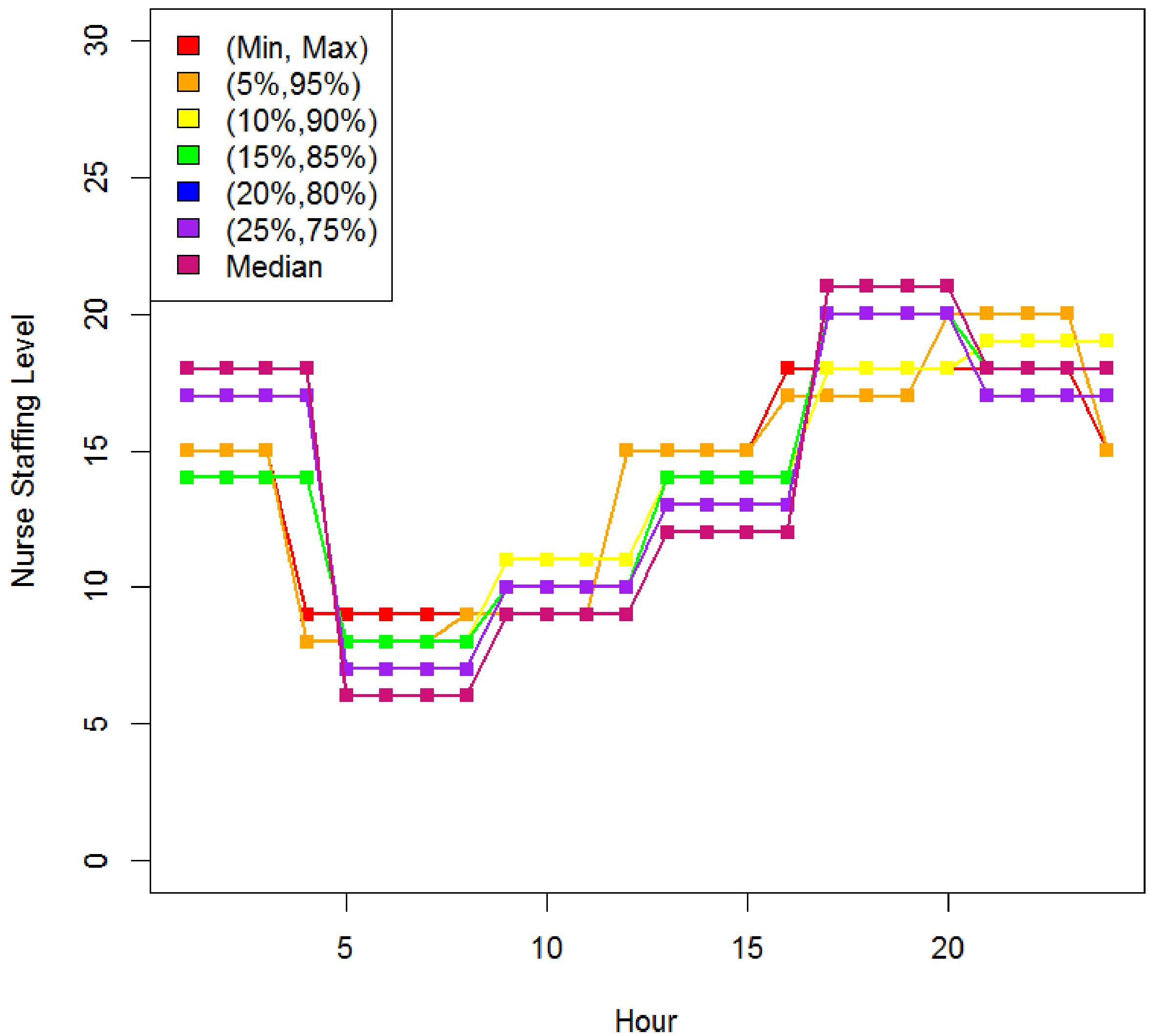

Figure 6.3: Optimal staffing levels under the patient safety deterministic and robust models.

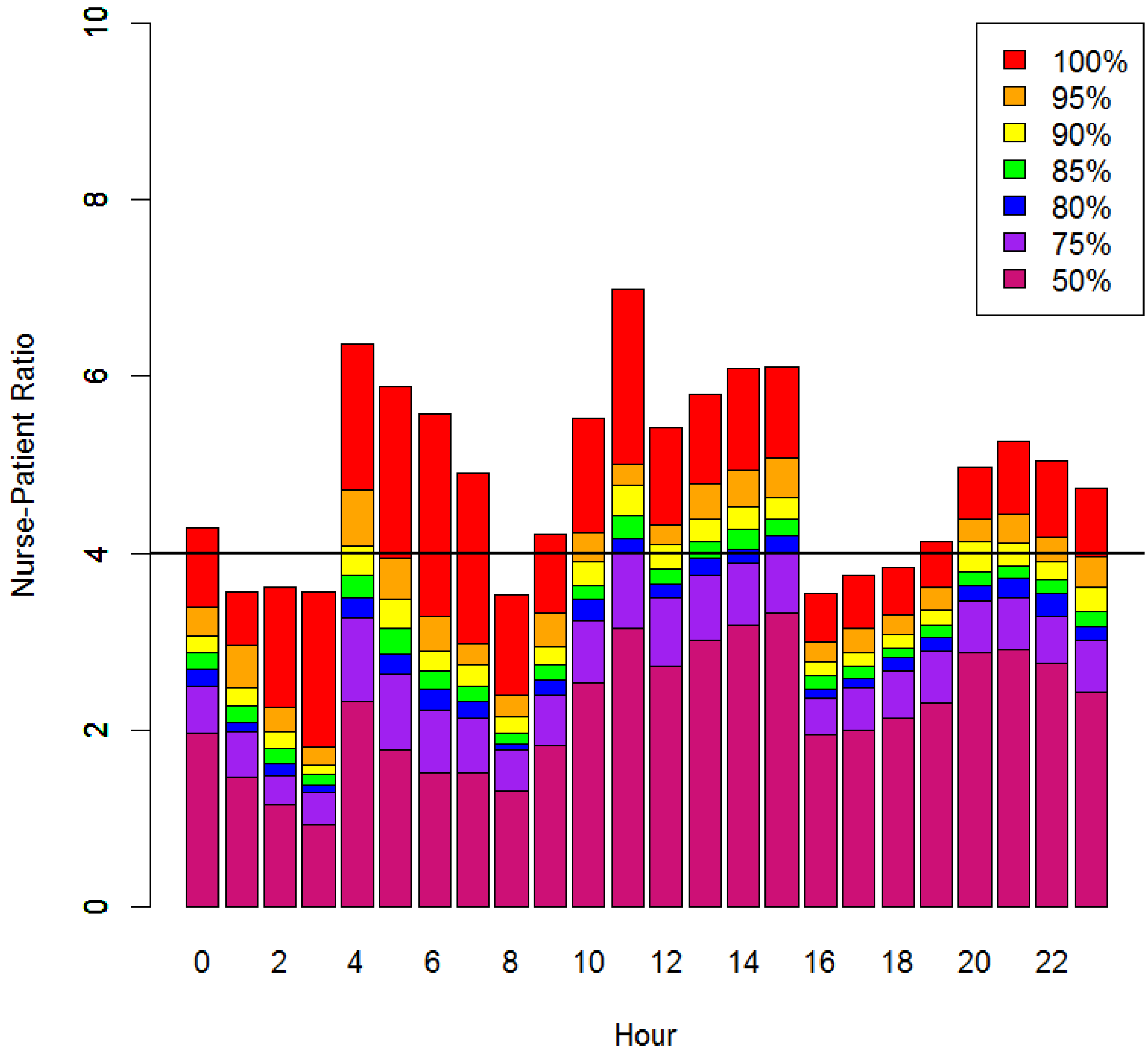

Figure 6.4: Nurse-Patient Ratios for the deterministic patient safety model.

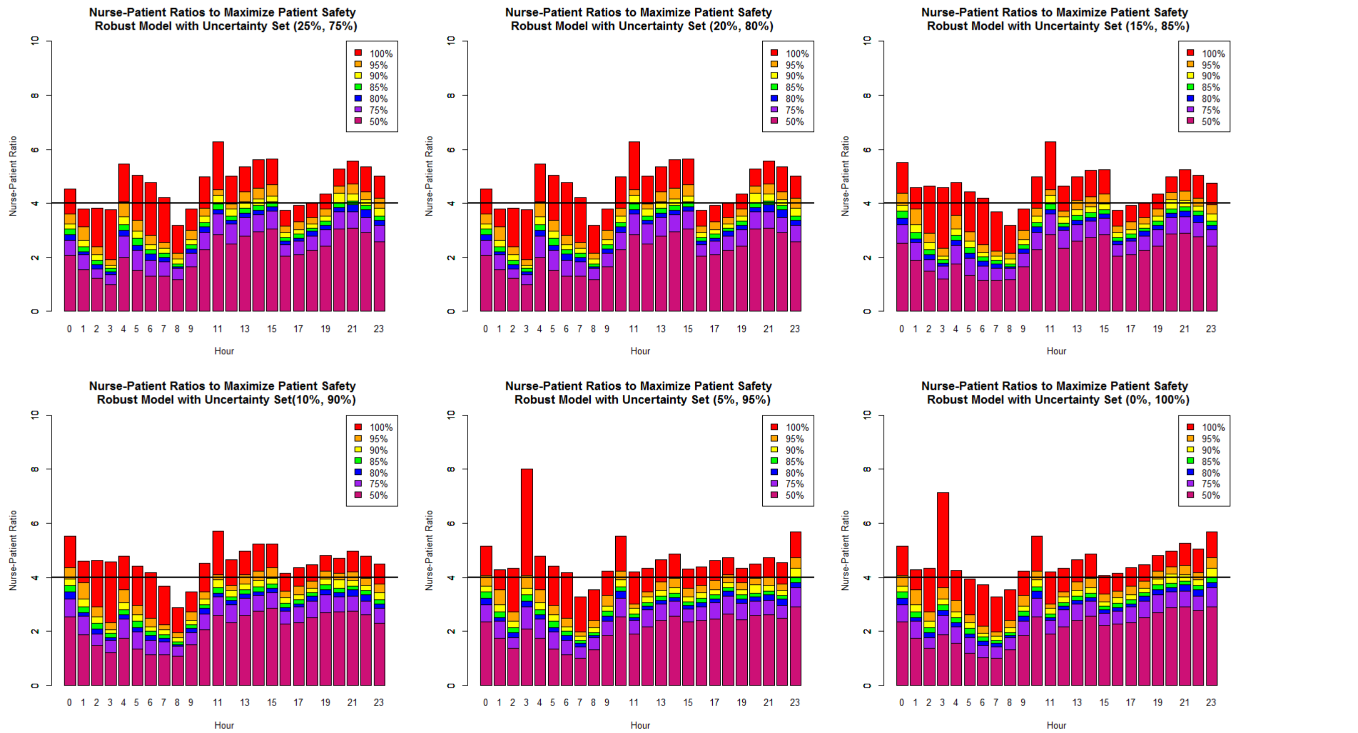

Figure 6.5: Nurse-Patient Ratios for the robust patient safety models.

6.5.2 Minimizing Differences in Nursing Workload

As noted in the previous results, the variation that occurs across the day can lead to differences in nursing workload based on the time of day a nurse is working. In order to eliminate differences in workload that occur depending on time of day, we solve the model to minimize the difference between the minimum and maximum nurse workload. The solutions for the workload differences models are given in Figure 6.6. In this problem, the day begins at 5AM in all robust models but 6AM in the deterministic model. Again, we see that the earlier start time is optimal in all instances to prevent uneven workload allocation, indicating that the nursing levels required at 7AM are not sufficient at 10AM.

Figure 6.7 shows the nurse-patient ratio for the deterministic solution for every hour of the day against various percentiles of patient volume. We can see in the deterministic solution that although the workload is fairly even, we frequently exceed the goal of 4:1 nurse patient for higher instances of patient volume.

The nurse-patient ratios for the robust solutions are given in Figure 6.8. Similar to the deterministic solution, the workload in this model is fairly even, but we exceed the goal of 4:1 nurse-patient ratio frequently for higher levels of patient volume. While we are able to minimize the workload difference, there are several shortcomings of this model. We are limited in the degree to which we can truly minimize the difference in workload with our model due to the minimum nurse staffing level coupled with the large fluctuations that occur in the early morning hours which can have either few or many patients. In addition, while the model works to distribute the workload, under every scenario we exceed the 4:1 ratio for high patient volumes for a substantial portion of the day.

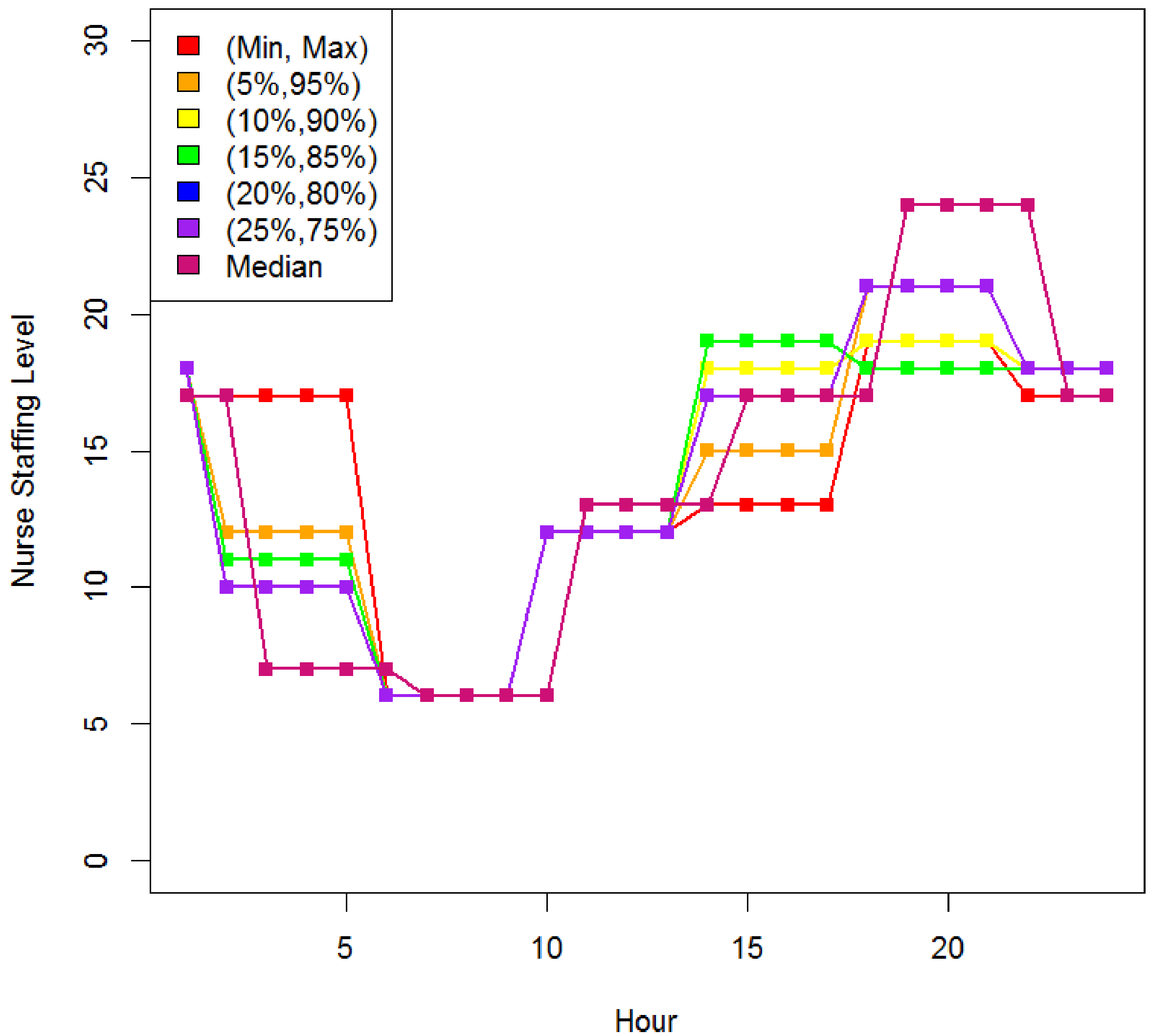

Figure 6.6: Optimal staffing levels under the workload difference deterministic and robust models.

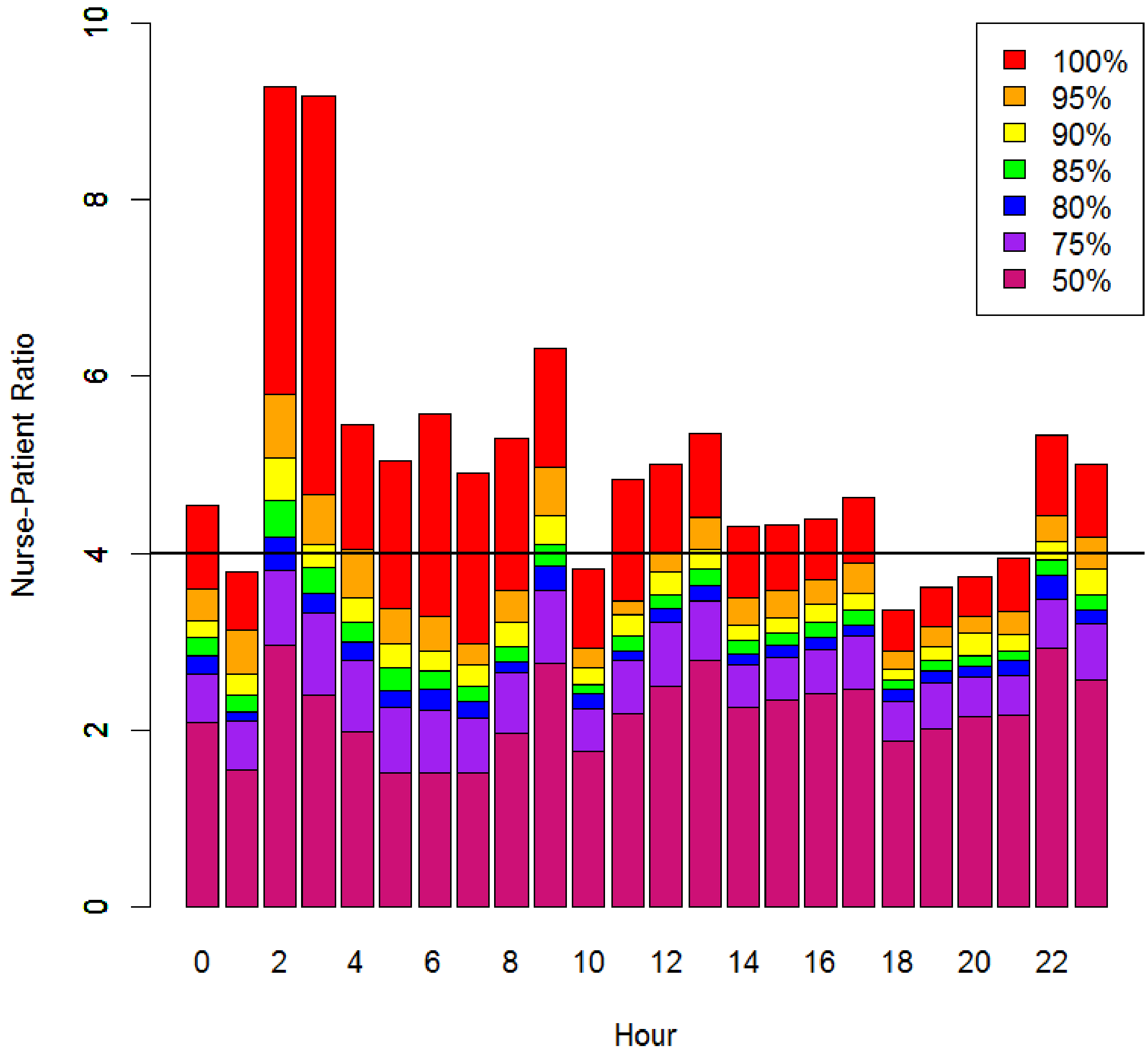

Figure 6.7: Nurse-Patient Ratios for the deterministic workload difference model.

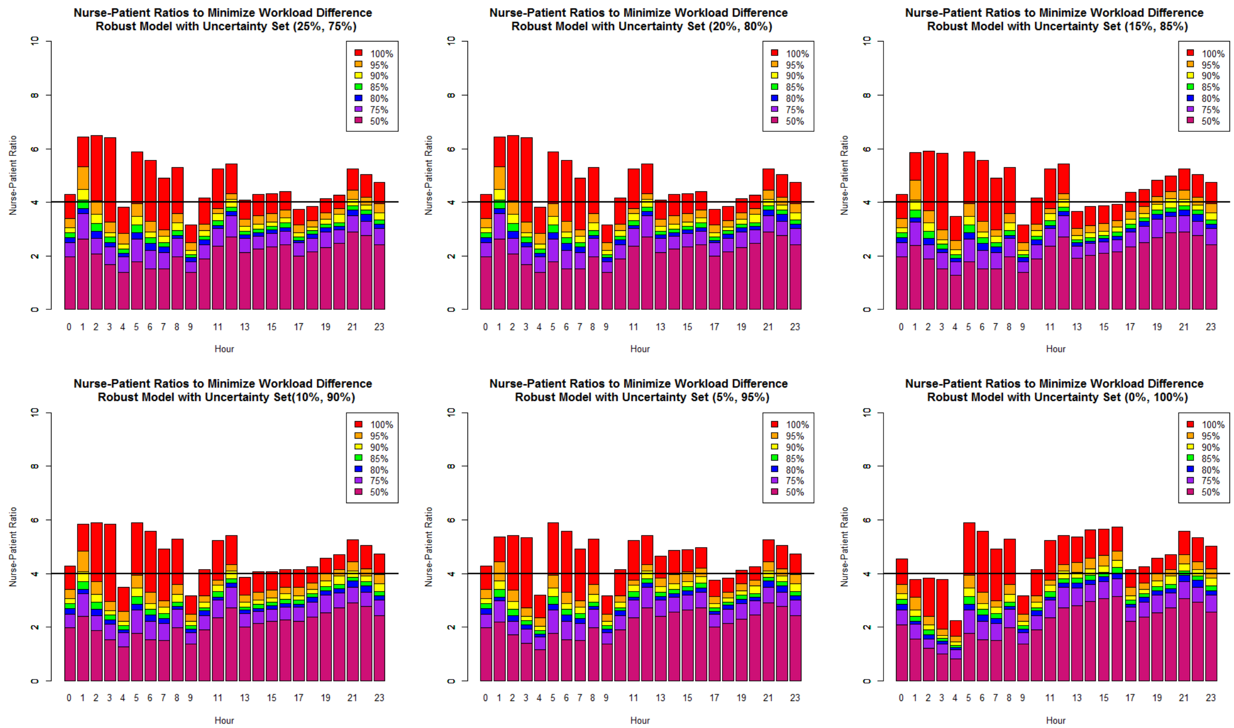

Figure 6.8: Nurse-Patient Ratios for the robust workload difference models.

6.5.3 Multiobjective Model

Comparing the patient safety and workload difference models, we see that the patient safety model frequently chooses earlier start times for the day and has a higher staffing levels at the beginning and end of the day while the workload model has higher staffing levels in the middle of the day. This can be a result of the larger variations in patient volume that occur during the late night and early morning hours of the day. Since the patient safety model looks to keep a ratio of 4:1, the late evening and early morning hours require more staffing to meet this ratio. In comparison, the workload difference model aims to create an even workload regardless of the time of day so it staffs higher during times with higher patient volume.

Table 6.4: Optimal solutions for the mulitobjecitve model with the patient safety and workload difference values for the deterministic and robust scenarios

Uncertainty Set	Deterministic	[25%, 75%]	[20%, 80%]	[15%, 85%]	[10%, 90%]	[5%, 95%]	[0%, 0%]
Patient Safety Model	0	0	0	0	0	0.068	0.746
Workload Difference Model	0.32	0.82	0.94	1.13	1.38	1.95	15.48

The disadvantages of each model make the multiobjective model necessary to achieve both goals. Table 6.4 shows the optimal solutions for both the patient safety and workload difference models when we solve both the deterministic and robust models. For the patient safety model, if the objective value is 0, it indicates that we meet the 4:1 nurse-patient ratio at every hour. The patient safety model shows that we can have staffing which meets the 4:1 nurse-patient ratio up to the 90th percentile of patient volume. As the value of the optimal solution to the patient safety model increases, it indicates that we are meeting the

4:1 nurse-patient ratio less often. When we have the uncertainty set, [0%, 100%], 7.1:1 is the highest nurse-patient ratio and we see a nurse-patient ratio that exceeds 4:1 frequently. However, for the [5%,95%] uncertainty set, the nurse-patient ratio never exceeds 5:1 and only exceeds 4:1 five hours of the day. The optimal values of the workload difference model show the difference in the maximum and minimum nurse-patient ratio across all hours of the day. So, as the value of the optimal solution for the workload difference model increases, this indicates that the difference between the maximum and minimum workload is growing. The workload difference model has a significant increase in the optimal solution as our uncertainty set becomes more extreme, due to the minimum staffing levels and the high variations that occur in our patient volume during the early hours of the morning when there can be very few patients. However when we look at the nurse-patient ratios, we see that for the uncertainty set [0%,100%] the maximum nurse patient ratio is close to 6:1 while the minimum ratio is 0.05:1. For the ratio of 0.05:1, we see that in the early morning hours we had one patient in the emergency department for 3 minutes during an entire hour. However, regardless of how few patients there are, we are required to maintain 6 nurses in the emergency department at all times, in spite of how few patients there may be during certain hours of the day.

There is a significant disparity in the optimal solution of the two models, indicating that we need to carefully select the weighting we choose in solving the models. From the optimal solutions for the uncertainty sets [5%, 95%] and [0%, 100%] we can see that the solution for the workload difference model is 20-30 times larger than the patient safety solution. We want the two objectives to have equal importance within our model, so we choose weights of $w_1 = 20$ and $w_2 = 1$ to prevent the workload difference model from usurping the patient safety model. Note, we also tested the model for weights of $w_1 = 25$ and $w_1 = 30$ while $w_2 = 1$ and got the same solutions as when $w_1 = 20$ and $w_2 = 1$, indicating our weightings are relatively robust.

The solutions given in Figure 6.9 demonstrate the wide range of solutions resulting from solving the multiobjective model. The optimal solution when solving the weighted multiobjective problem is given in Table 6.5. In addition, we give corresponding optimal solutions from the patient safety model and the workload difference model. We can see that while our actual staffing levels differ from the original models, the objective values of both models stay the same for the deterministic model as well as for uncertainty sets $[25\%, 75\%]$, $[20\%, 80\%]$, and $[15\%, 85\%]$. Four of the seven models determine a daily start at 5AM. In comparison to the patient safety model, the multiobjective model starts the day one hour later and has higher staffing levels in the later evening. The results of the multiobjective model closely align with the workload difference model for the deterministic model and for the robust model with less extreme uncertainty sets. However, once it becomes difficult to meet the objective of a 4:1 nurse-patient ratio we begin to see more deviation from the workload difference model.

The resulting nurse-patient ratios for the multiobjective deterministic model are given in Figure 6.10. From the deterministic model, we see that when we only consider the median patient volume, we will exceed the 4:1 patient ratio during the early hours of the day and during the evening.

In comparison to the deterministic model, the nurse-patient ratios have decreased, especially when we have high patient volume in the early morning hours. For the multiobjective model, we can set our nurse staffing levels such that we meet the nurse-patient ratio of 4:1 for up to the 85th percentile of patient volume.

Table 6.5: Optimal solutions for the mulitobjecitve model with the patient safety and workload difference values for the deterministic and robust scenarios

Uncertainty Set	Deterministic	[25%, 75%]	[20%, 80%]	[15%, 85%]	[10%, 90%]	[5%, 95%]	[0%, 0%]
Multiobective Solution	0.32	0.82	0.94	1.13	1.59	3.45	33.02
Patient Safety Model	0	0	0	0	0.005	0.070	0.876
Workload Difference Model	0.32	0.82	0.94	1.13	1.49	2.06	15.50

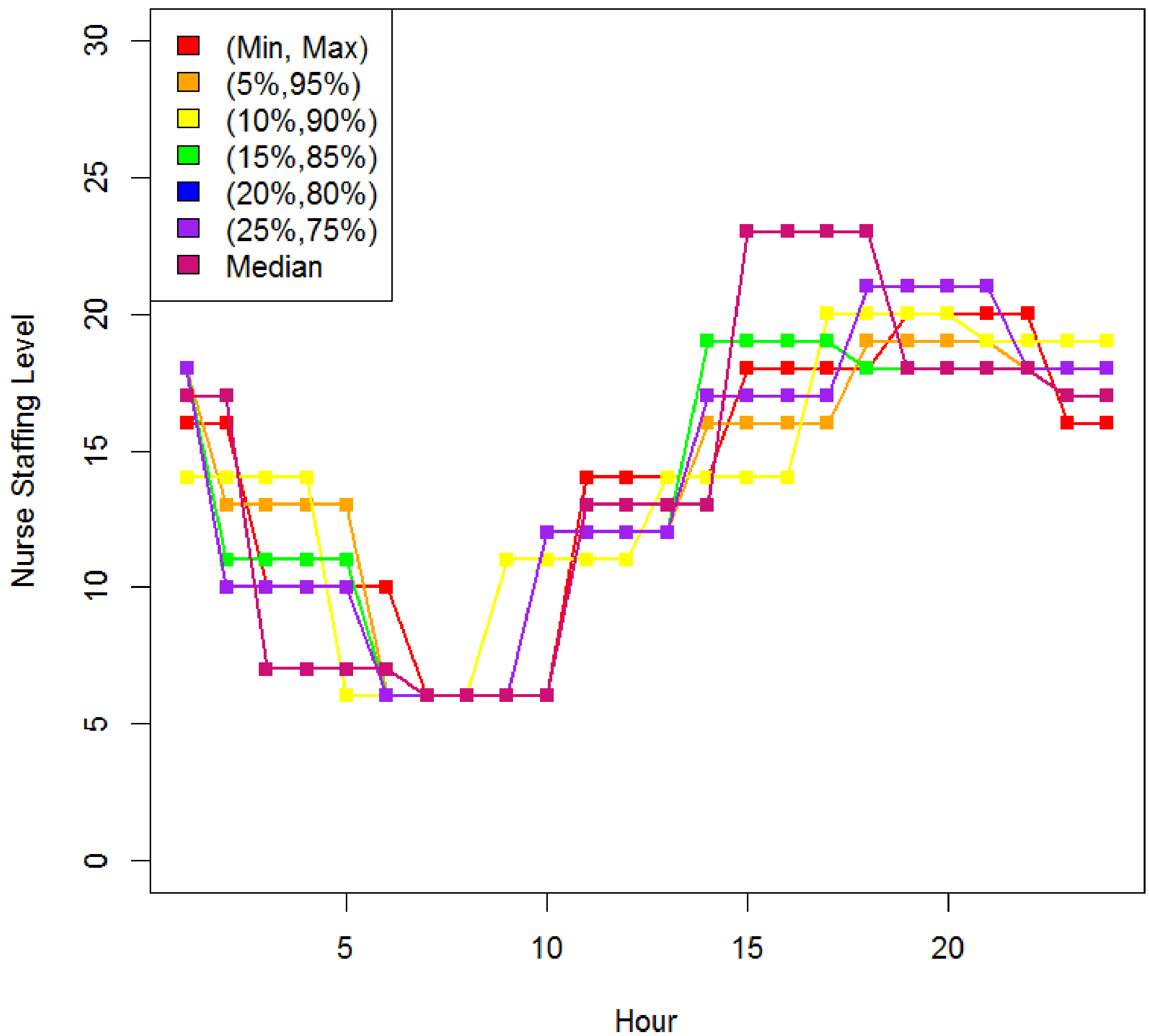

Figure 6.9: Optimal staffing levels under the patient safety deterministic and robust models.

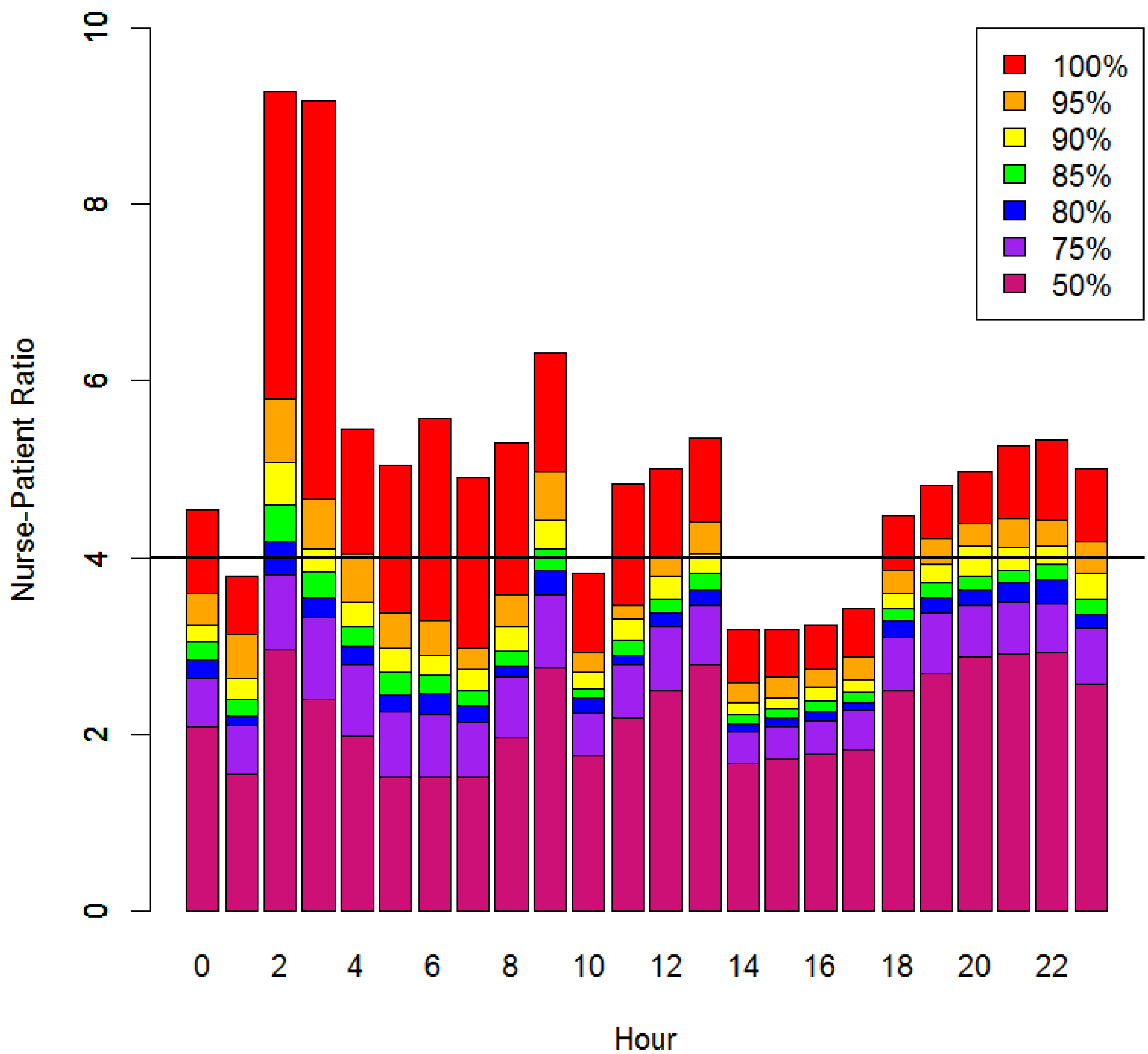

Figure 6.10: Nurse-Patient Ratios for the deterministic multiobjective model.

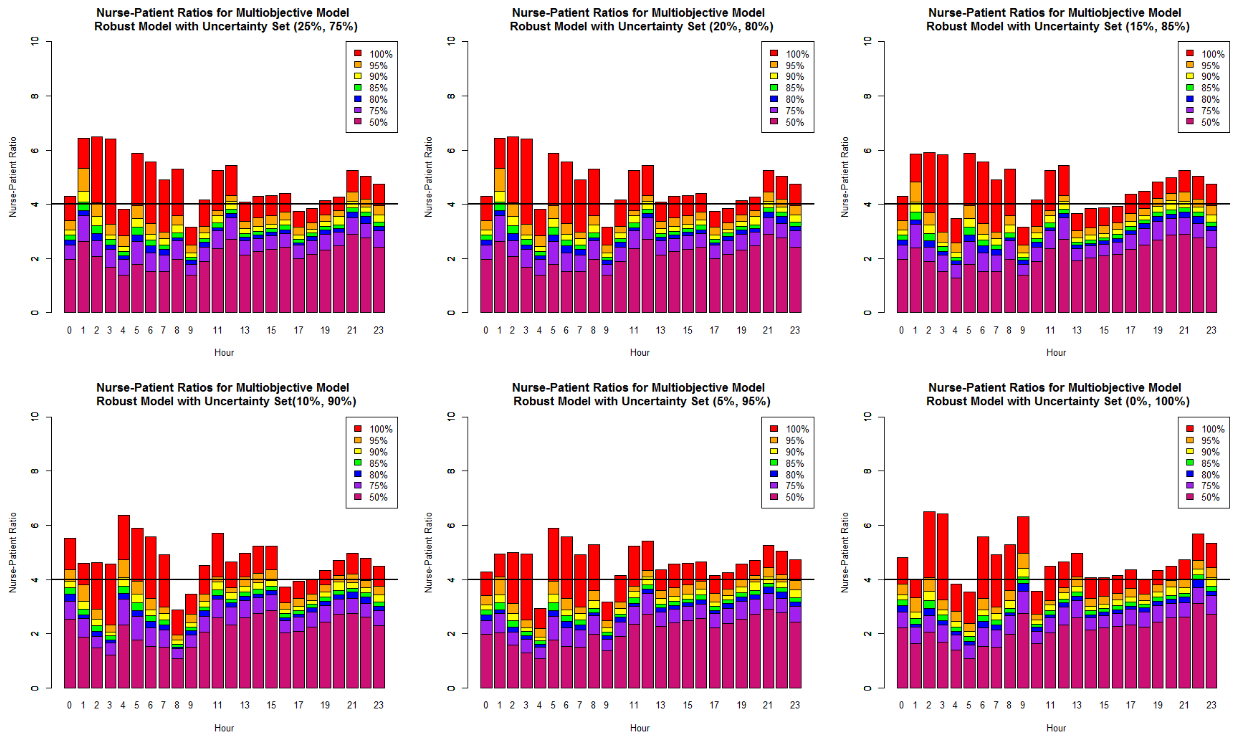

Figure 6.11: Nurse-Patient Ratios for the robust multiobjective models.

Although the 7 models (deterministic, [25%, 75%], [20%, 80%], [15%, 85%], [10%, 90%], [5%, 95%], and [0%, 100%]) have various solutions, each has valuable input. In order to determine the final staffing recommendation, we combine the models using the mode, median, and mean staffing values based off all the models, as in Table 6.6. Based on the 5AM start time for 4 of the 7 models, we decide to use time blocks starting at 5AM to base our staffing on.

Looking at the table, we can see that for many instances, the mode, median, and mean give a clear indication of how many nurses should be working during each block. Focusing on the nurse staffing levels of the multiobjective function, we find that the uncertainty sets [25%, 75%] and [20%, 80%] produce the same optimal staffing levels given in Table 6.7, which aligns well with the central tendencies found in Table 6.6. In addition, the optimal staffing level for the multiobjective model is optimal under each individual problem as well. When we compare the performance of this solution for the patient safety model, we find that we

Table 6.6: The mode, median, and mean number of nurses needed at each hour based on solutions to the deterministic and robust multiobjective models.

	Mode	Median	Mean		Mode	Median	Mean
5A	6	6	6.7	5P	18	20	20
6A	6	6	6	6P	20	20	19.6
7A	6	6	6	7P	20	20	19.6
8A	6	6	6.7	8P	19	19	19.4
9A	12	12	10.1	9P	18	18	18.4
10A	12	12	12.3	10P	18	18	17.7
11A	12	12	12.3	11P	18	18	17.7
12P	12	12	12.7	12A	18	18	17
1P	14	16	15.7	1A	10	13	13
2P	17	17	17.7	2A	10	10	10.7
3P	17	17	17.7	3A	10	10	10.7
4P	17	18	18.6	4A	10	10	9.6

Table 6.7: Recommended schedule for the multiobjective model.

Time Block	5A-9A	9A-1P	1P-5P	5P-9P	9P-1A	1A -5A
Recommendation	6	12	17	21	18	10

exceed the 4:1 nurse-patient ratio at 85% instead of 95%. However, we have a surge nurse available from 6PM-6AM which is the time period during which we are most likely to exceed the ratio. Compared to the workload difference model, we find that there is less than a 0.05 difference between the optimal solution for each uncertainty set and the solution in Table 6.7. The availability of the surge nurse justifies our focus on the less extreme uncertainty sets in determining our model, since the surge nurse would help us reach the 4:1 nurse-patient ratio and decrease the difference in workload.

6.6 CONCLUSION

In this paper, we provide a framework to determine daily staffing levels for an emergency department based on their current workforce. We formulate three models, one which considers patient safety in terms of nurse-patient ratios, one which considers staff satisfaction by minimizing the difference in nursing workload, and a multiobjective model which considers both objectives. We solve the deterministic model as well as a robust model for several uncertainty sets for all three models. We use historic patient volume to align the staffing levels to the patient volume and determine the daily start time.

Within the paper, we have used multiple mathematical models and uncertainty sets to determine the nurse staffing levels. We have demonstrated the the multiobjective model captures aspects of the nursing workload and patient safety when determining the staffing levels. We have selected the best nurse staffing levels based on the optimal staffing levels for the uncertainty sets [25%, 75%] and [20%, 80%] because this solution performs close to optimal for all the other uncertainty sets. In addition, our ability to bring in additional nursing staff during a patient surge allows us to focus less on the extreme uncertainty sets.

The purpose of our model is to aid emergency department leadership in determining staffing levels which align with their daily volume. However, the model does have several limitations. While we have demonstrated one way to make a decision regarding the best staffing levels based on the solutions for our partner hospital, we cannot guarantee that the recommendation using all models will be as clearly indicated as we saw within our results. In addition, our model is solely based on historic patient volume, which is highly dependent on the state of the emergency department. An increase in arrivals, a process change within the emergency department such as adding a fast track option, or staffing changes can all affect patient volume, in turn affecting the effectiveness of the suggested staffing levels. Another limitation is that the minimum staffing numbers along with times of low patient volume can limit the ability of the model to fairly distribute workload during busier times of day. This leads to an alternative formulation which would not include hours with patient volume less than the minimum staffing levels when considering the difference in workload distribution. This work can be extended by connecting the patient volume with the nursing levels to account for the expected changes in patient volume that will result from improved staffing levels.

7.0 CONCLUSIONS

Through our modeling of pharmacy operations in Chapters 2-4, we contribute to the existing literature by formulating a model that captures the medication delivery process and can be used to determine the best way for a hospital pharmacy to deliver medication to patients. In Chapter 2, we focus on the cost of the entire delivery process, factoring in automation and workload costs. The resulting formulation was solved to determine the optimal pathways to distribute medication and demonstrated how the model can be used to reduce both costs and missing doses in comparison to the current delivery strategy. An algorithm was written that uses a cost increase to missing doses saved ratio to determine which medications to add to the ADC. In 3, we supplement 2 by considering the effect of centralization/decentralization on the workload of pharmacy technicians and nursing staff. Therefore, it is important to not only consider costs and missing doses when determining how to distribute medication, but also the effect on staff workload. In particular, more decentralization results in increased hours of pharmacy technician time needed to restock the ADC and increased time nurses spend retrieving medication from the ADC. The model demonstrates that as decentralization increases, the drastic increase in cost results from the increase in the workload of pharmacy technicians and nurses. In Chapter 4 we extend the model given in Chapter 2 by formulating and solving the robust counterpart. Our results demonstrate the importance of considering variation whenever determining how to deliver medication from the hospital central pharmacy to the patient. We observe that the ADC and robot are used in conjunction to maintain low missing dose rates in the worst case scenario. In addition to the better performance on the worst case day, the solution performs well on the average day further demonstrating the benefits of using the robust model.

Chapters 5 and 6 contribute to the existing literature regarding nurse staffing models for emergency departments. In Chapter 5, we contribute to the current emergency nursing literature by providing a basic framework for determining a set of daily shifts to meet predetermined staffing levels for an emergency department. The purpose of the paper is to provide a new template for self-scheduling that decreases the time necessary to finalize the monthly schedule, prevents understaffing from occurring, unless an entire shift is not filled, and reduces staff dissatisfaction by ensuring that nurses are working the preferred shift they originally signed up to work. In addition, we account for meal breaks, allowing the emergency department to maintain their staffing levels throughout the 4-hour block while nurses can have a fully covered meal break. In Chapter 6 we contribute to the current literature by determining a nurse staffing schedule to meet patient volume utilizing the current staff available. The model we provide solves three different models: (1) staffing for mandated nurse-patient ratios and (2) minimizing difference in workload and (3) a multiobjective problem that considers both objectives simultaneously. We formulate the robust counterparts to all three models to account for the daily variation that occurs in patient volume.

BIBLIOGRAPHY

[1] K. Albertsen, A. H. Garde, K. Nabe-Nielsen, A. M. Hansen, H. Lund, and H. Hvid. Work-life balance among shift workers: results from an intervention study about self-rostering. *International Archives of Occupational and Environmental Health*, 87:265–274, 2014.

[2] Association for Healthcare Resource & Materials Management. *Healthcare supply chain management*. Association for Healthcare Resource & Materials Management, 2010.

[3] A. Baboli, J. Fondrevelle, R. Tavakkoli-Moghaddam, and A. Mehrabi. A replenishment policy based on joint optimization in a downstream pharmaceutical supply chain: centralized vs. decentralized replenishment. *The International Journal of Advanced Manufacturing Technology*, 57:367–378, 2011.

[4] J. Baker, M. Draves, and A. Ramudhin. Analysis of the medication management system in seven hospitals. Technical report, San Diego, CA, 2010.

[5] S. Berdot, V. Savoldelli, V. Zaugg, E. Jaccoulet, P. Prognon, L. Minh, M. Lê, and B. Sabatier. Return on Investment after Implementation of a Centralized Automated Storage System in a Hospital Pharmacy. *Journal of Pharmacy and Pharmacology*, 4:526–532, 2016.

[6] L. Brotcorne, G. Laporte, and F. Semet. Ambulance location and relocation models. *European Journal of Operational Research*, 147(3):451–463, 6 2003.

[7] Q. Bui. Watch Robots Transform A California Hospital, 2015.

[8] E. K. Burke, P. De Causmaecker, G. V. Berghe, and H. Landeghem. The State of the Art of Nurse Rostering. *Journal of Scheduling*, 7:441–499, 2004.

[9] G. Carello and E. Lanzarone. A cardinality-constrained robust model for the assignment problem in Home Care services. *European Journal of Operational Research*, 236(2):748–762, 7 2014.

[10] M. A. Centeno, R. Giachetti, L. Richard, and A. M. Ismail. A Simulation-ILP Based Tool for Scheduling ER Staff. In S. Chick, P.J. Sanchez, D. Ferrin, and D.J. Morrice, editors, *Proceedings of the 2003 Winter Simulation Conference*, pages 1930–1938, 2003.

[11] T. C. Chan, J. P. Killeen, G. M. Vilke, J. B. Marshall, and E. M. Castillo. Effect of Mandated Nurse-Patient Ratios on Patient Wait Time and Care Time in the Emergency Department. *Academic Emergency Medicine*, 17(5):545–552, 5 2010.

[12] C. Chapuis, P. Bedouch, M. Detavernier, M. Durand, G. Francony, P. Lavagne, L. Foroni, P. Albaladejo, B. Allenet, and J.-F. Payen. Automated drug dispensing systems in the intensive care unit: a financial analysis. *Critical care (London, England)*, 19(1):318, 9 2015.

[13] S. Chopra and P. Meindl. *Supply Chain Management: Strategy, Planning, and Operation*. Pearson, 6 edition, 2016.

[14] M. L. De Grano, D. J. Medeiros, and D. Eitel. Accommodating individual preferences in nurse scheduling via auctions and optimization. *Health Care Management Science*, 12:228–242, 2009.

[15] B. T. Denton, A. J. Miller, H. J. Balasubramanian, T. R. Huschka, O. Research, and B. T. Denton Edward P Fitts. Optimal Allocation of Surgery Blocks to Operating Rooms Under Uncertainty. *Operations Research*, 58(4):802–816, 2010.

[16] G. Dobson, S. Sullivan, V. Tilson, and D. Webster. Reducing Costs of Managing Medication Inventory in Automated Dispensing System in Hospital Units. *SSRN Electronic Journal*, 6 2018.

[17] G. Dobson, D. Tilson, and V. Tilson. Optimizing the timing and number of batches for compounded sterile products in an in-hospital pharmacy. *Decision Support Systems*, 76:53–62, 2015.

[18] M. A. Draeger. An Emergency Department Simulation Model Used To Evaluate Alternative Nurse Staffing and Patient Population Scenarios. In J.J. Swain, D. Goldsman, R.C. Crain, and J.R. Wilson, editors, *Proceedings of the 1992 Winter Simulation Conference*, pages 1057–1064, 1992.

[19] O. El-Rifai, T. Garaix, V. Augusto, and X. Xie. A stochastic optimization model for shift scheduling in emergency departments. *Health Care Management Science*, 18(3):289–302, 2015.

[20] A. T. Ernst, H. Jiang, M. Krishnamoorthy, and D. Sier. Staff scheduling and rostering: A review of applications, methods and models. *European Journal of Operations Research*, 153(1):3–27, 2004.

[21] N. Esmaili. *Optimizing Multi-Item Inventory Management Decisions in Healthcare Facilities*. PhD thesis, University of Pittsburgh, 2016.

[22] N. Esmaili, B. A. Norman, and J. Rajgopal. Shelf-space optimization models in decentralized automated dispensing cabinets. *Operations Research for Health Care*, 2018.

[23] M. Ferretti, F. Favalli, and A. Zangrandi. Impact of a logistic improvement in an hospital pharmacy: effects on the economics of a healthcare organization. *International Journal of Engineering, Science and Technology*, 6(3):85–95, 2014.

[24] C. Franco, V. Augusto, T. Garaix, E. Alfonso-Lizarazo, M. Bourdelin, and H. Bontemps. Strategic territorial deployment of hospital pharmacy robots using a stochastic p-robust optimization approach. In *2018 IEEE 14th International Conference on Automation Science and Engineering*, pages 390–395, Munich, 2018.

[25] J. P. Gray, B. Ludwig, J. Temple, M. Melby, and S. Rough. Comparison of a hybrid medication distribution system to simulated decentralized distribution models. *American Journal of Health-Systems Pharmacy*, 70(15):1322–35, 8 2013.

[26] P. Greth, W. Tester, and H. Black. Decentralized pharmacy operations utilizing the unit dose concept. II. Drug information services and utilization in a decentralized pharmacy substation. *American Journal of Health-System Pharmacy*, 22(10), 1965.

[27] J. B. Hertig, M. Jenkins, S. M. Mark, and R. J. Weber. Developing Patient-Centered Services, Part 1: A Primer on Pharmacy Practice Models. *Hospital Pharmacy*, 46(1):61–65, 1 2011.

[28] A. P. Hussey, B. M. Cook, A. D. Quane, S. R. Sweet, and P. M. Szumita. Implementation of an Automated Dispensing Cabinet Stewardship Program at a Tertiary Academic Medical Center. *Journal of Pharmacy Technology*, 30(6):191–194, 12 2014.

[29] A. Ingolfsson, F. Campello, X. Wu, and E. Cabral. Combining integer programming and the randomization method to schedule employees. *European Journal of Operational Research*, 202:153–163, 2010.

[30] S. A. Irvin and H. N. Brown. Self-Scheduling with Microsoft Excel. *Nursing Economics*, 17(4):201–201, 7 1999.

[31] N. Izady and D. Worthington. Setting staffing requirements for time dependent queueing networks: The case of accident and emergency departments. *European Journal of Operational Research*, 219(3):531–540, 6 2012.

[32] P. Kelle, J. Woosley, and H. Schneider. Pharmaceutical supply chain specifics and inventory solutions for a hospital case. *Operations Research for Health Care*, 1:54–63, 2012.

[33] D. L. Kellogg and S. Walczak. Nurse Scheduling: From Academia to Implementation or Not? *Interfaces*, 37(4):355–369, 2007.

[34] A. Kumar, L. Ozdamar, and C. N. Zhang. Supply chain redesign in the healthcare industry of Singapore. *International Journal of Operations &amp*, 13(2):95–103, 2008.

[35] N. K. Kwak and C. Lee. A Linear Goal Programming Model for Human Resource Allocation in a Health-Care Organization. *Journal of Medical Systems*, 21(3):129–140, 1997.

[36] E. Lanzarone and A. Matta. Robust nurse-to-patient assignment in home care services tominimize overtimes under continuity of care. *Operations Research for Health Care*, 3(2):48–58, 6 2014.

[37] K. Lathrop, J. Lund, B. Ludwig, and S. Rough. Design, implementation, and evaluation of a thrice-daily cartfill process. *American Journal of Health-Systems Pharmacy*, 71(13):1112–9, 7 2014.

[38] S. Liao, . Ger, K. . Christian Van Delft, O. Jouini, S. Liao, . O. Jouini, G. Koole, C. Van Delft, and O. Jouini. Staffing a call center with uncertain non-stationary arrival rate and flexibility. *OR Spectrum*, 34:691–721, 2012.

[39] A. C. Lin, Y.-C. Huang, G. Punches, and Y. Chen. Effect of a robotic prescription-filling system on pharmacy staff activities and prescription-filling time. *American Journal of Health-Systems Pharmacy*, 64(17):1832–9, 9 2007.

[40] S. Mattia, F. Rossi, M. Servilio, and S. Smriglio. Staffing and scheduling flexible call centers by two-stage robust optimization. *Omega*, 72:25–37, 10 2017.

[41] B. C. McCarthy and M. Ferker. Implementation and optimization of automated dispensing cabinet technology. *American Journal of Health-Systems Pharmacy*, 73(19):1531–6, 10 2016.

[42] Z. MEILIN. *Robust Optimization With Applications In Healthcare Operations Management*. PhD thesis, National University of Singapore, 2014.

[43] Microsoft & Omnicell. Healthcare System Cuts Medication- Distribution Time by 85 Percent, 2012.

[44] M. Mousazadeh, S. A. Torabi, and B. Zahiri. A robust possibilistic programming approach for pharmaceutical supply chain network design. *Computers and Chemical Engineering*, 82:115–128, 2015.

[45] T. Neal and P. A. Shumate. Shared Scheduling in the Pediatric Emergency Department. *Journal of Emergency Nursing*, 29(5):418–419, 2003.

[46] C. A. Pedersen, P. J. Schneider, and D. J. Scheckelhoff. Dispensing and administration ASHP national survey of pharmacy practice in hospital settings: Dispensing and administration2014. *American Journal of Health-Systems Pharmacy*, 72, 2015.

[47] S. G. Pickette, L. Muncey, and D. Wham. Implementation of a standard pharmacy clinical practice model in a multihospital system. *American Journal of Health-Systems Pharmacy*, 67(9):751–6, 5 2010.

[48] M. J. Poley, C. A. M. Bouwmans, L. M. Hanff, P. J. Roos, and B. M. van Ineveld. Efficiency of different systems for medication distribution in an academic children's hospital in The Netherlands. *Pharmacy world & science : PWS*, 26(2):83–9, 4 2004.

[49] C. E. Ray, J. Agnew, M. Jagim, J. McKay, and S. Sheehy. ENA's New Guidelines for Determining Emergency Department Nurse Staffing. *Journal of Emergency Nursing*, 29(3):245–253, 2003.

[50] P. J. Schneider, C. A. Pedersen, and D. J. Scheckelhoff. ASHP national survey of pharmacy practice in hospital settings: Dispensing and administration-2017. *American Journal of Health-System Pharmacy*, 75(16):1203–1226, 8 2018.

[51] D. Sinreich and O. Jabali. Staggered work shifts: a way to downsize and restructure an emergency department workforce yet maintain current operational performance. *Health Care Management Science*, 10(3):293–308, 2007.

[52] A. C. Soteriou and R. B. Chase. A Robust Optimization Approach for Improving Service Quality. *Manufacturing & Service Operations Management*, 2(3):264–286, 2000.

[53] D. W. Spaite, F. Bartholomeaux, J. Guisto, E. Lindberg, B. Hull, A. Eyherabide, S. Lanyon, E. Criss, T. Valenzuela, and C. Conroy. Rapid Process Redesign in a University-Based Emergency Department: Decreasing Waiting Time Interval and Improving Patient Satisfaction. *Annals of Emergency Medicine*, 39(2):168–177, 2002.

[54] N. Stephens. One emergency department's responses to the increasingly complex challenges of patient care at century. *Journal of Emergency Nursing*, 26(4):318–328, 2000.

[55] N. W. Tsao, C. Lo, M. Babich, K. Shah, and N. J. Bansback. Decentralized automated dispensing devices: systematic review of clinical and economic impacts in hospitals. *The Canadian journal of hospital pharmacy*, 67(2):138–48, 3 2014.

[56] J. Van Den Bergh, J. Beliën, P. De Bruecker, E. Demeulemeester, and L. De Boeck. Personnel scheduling: A literature review. *European Journal of Operational Research*, 226:367–385, 2013.

[57] L. Weichenthal and G. W. Hendey. The Effect of Mandatory Nurse Ratios on Patient Care in an Emergency Department. *The Journal of Emergency Medicine*, 40(1):76–81, 1 2011.

[58] S. Wise, M. Fry, C. Duffield, M. Roche, and J. Buchanan. Ratios and nurse staffing: The vexed case of emergency departments. *Australasian Emergency Nursing Journal*, 18(1):49–55, 2 2015.

[59] T. Wong, M. Xu, and K. Chin. A two-stage heuristic approach for nurse scheduling problem: A case study in an emergency department. *Computers and Operations Research*, 51:99–110, 2014.