Hydrodynamic forces on circular cylinders oscillating with small amplitude in still fluid or transverse to a free stream

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Summary: In the present study we compare the hydrodynamic forces acting on circular cylinders oscillating in still fluid with corresponding oscillations transverse to a free stream. We find that at small amplitudes of motion the time history of the total force acting in the direction of oscillation in the presence of a free stream is virtually the same as in still fluid and in very good agreement with Stokes–Wang analytical solutions. However, the flow patterns around the cylinders that generate the consistent force history are remarkably disparate.

Introduction

Bluff-body flows present many challenges as they become separated and unsteady even at low Reynolds numbers. As a consequence, the hydrodynamic forces that act on oscillating bluff bodies are difficult to analyze, model and predict. In this context, a key point is whether the total force acting on an oscillating body can be appropriately segregated into contributions due to drag and inertia. In particular, the decomposition of the total force into an inviscid potential force, a.k.a. added-mass or virtual-mass force, and a vortex force has been the subject of considerable discussion in the literature [1-3]. The motivation for the present work is to study flows generated by oscillating bluff bodies in a regime for which some theory exists so as to contribute to the understanding of the above issue in view of recent theoretical work [4].

The problem of a circular cylinder oscillating with small amplitude perpendicular to its axis in a fluid at rest was first considered by Stokes [5] and later by Wang [6], who both obtained analytical solutions for the viscous flow. Both solutions, which will be collectively referred to as S–W theory, yield the normalized force acting on the cylinder that can be expressed as a series expansion in powers of $\beta^{-1/2}$, where β is the dimensionless frequency, or the Stokes number. The sectional force normalized by $\frac{1}{2}\rho D U_{\max}^2$, where ρ is the fluid density, D is the diameter of the cylinder, and $U_{\max}$ is the maximum velocity of the oscillating cylinder, is inversely proportional with *KC*, i.e. the Keulegan–Carpenter number [7], which represents the dimensionless amplitude in our terminology. Theoretically, the analytical solution holds in the limit of $KC \ll 1$, $\beta \gg 1$, and $\beta \cdot KC^2 \ll 1$ but good agreement with S–W theory up to $\beta \cdot KC^2 \approx O(1000)$ has been reported in experiments [8-9].

In the present work, we investigate the effect of superposing a free stream transverse to the direction of oscillation at *KC* and β numbers for which S–W theory is valid. For this purpose, we employ a numerical method to solve the governing equations in two dimensions. The Reynolds numbers based on both the oscillation and free-stream velocities are restricted to low values in order to avoid regimes in which three-dimensional instabilities have been found in related studies.

Methodology

The full Navier–Stokes equations in two dimensions are solved using an in-house CFD code based on the finite-difference method [10-11]. The equations are expressed in the velocity-pressure scheme in dimensionless form. The flow domain is bounded by two concentric circles with logarithmically-spaced cells in the radial direction that are mapped into rectangular computational domain with equispaced cells. In the transformed plane, spatial derivatives are approximated by 4th order central differences but for the convective terms for which a 3rd order modified upwind scheme is used. The pressure Poisson equation is solved by the successive over-relaxation (SOR) method. The Navier–Stokes equations are integrated explicitly and continuity is enforced. The instantaneous force acting on the cylinder is computed at each time step by suitable integration of pressure and skin friction around the cylinder walls.

Results and discussion

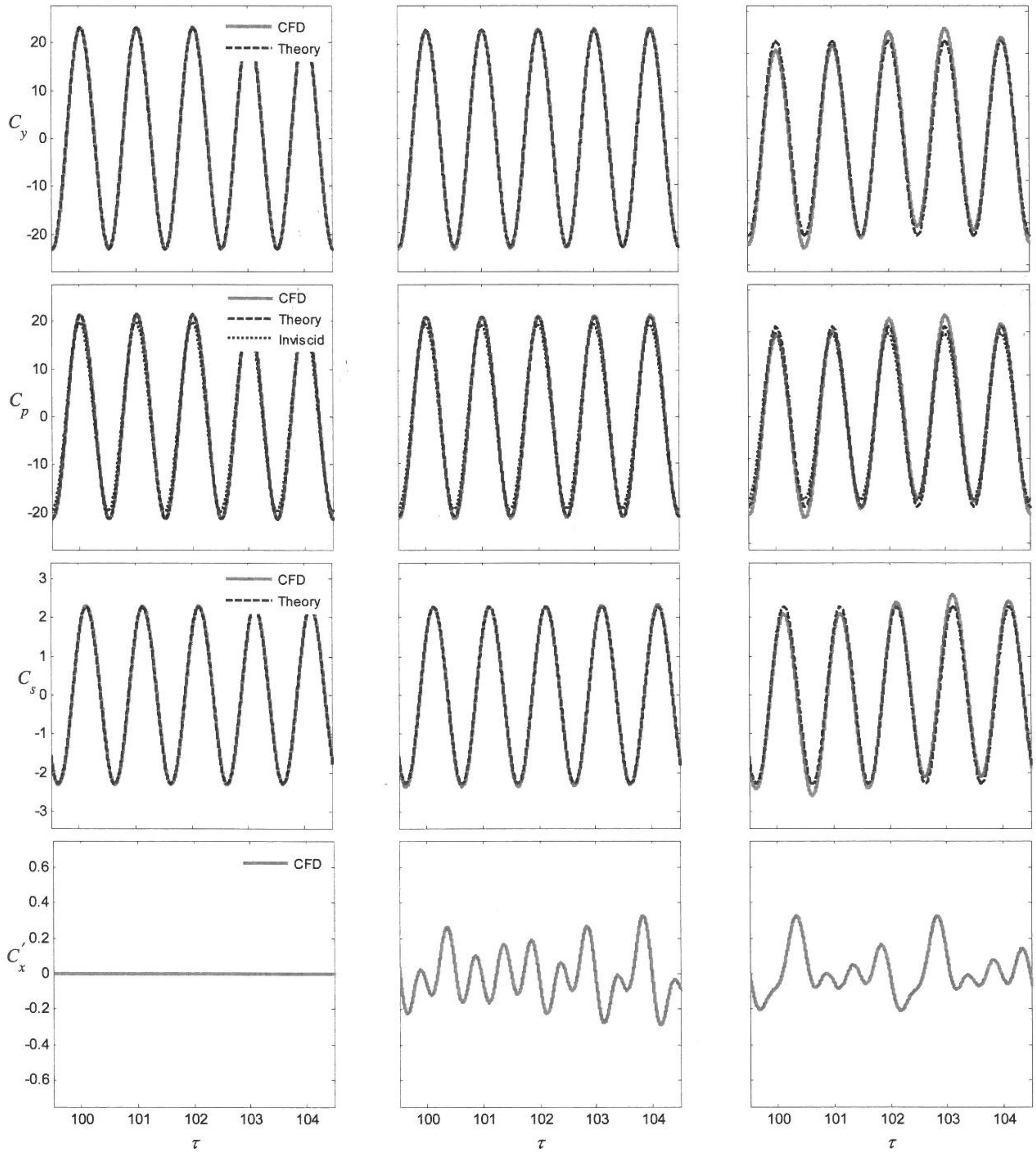
Initially, we studied cylinders oscillating in still fluid within the ranges of KC = 0.1 – 5 and β = 10 – 10^4. This initial study showed that the time-history of the force acting in the direction of oscillation obtained by the simulations and that predicted by S–W analytical solution match very well and that the agreement improves as the KC number is decreased. Based on these results, a value of KC = 0.5 was selected to investigate the influence of superposing a free stream transverse to the direction of oscillation.

Subsequently, we examined the effect of superposing a free stream transverse to the direction of cylinder oscillation as a function of the dimensionless parameter, γ, which is the ratio of the free-stream velocity to the maximum velocity of cylinders mechanically oscillated at KC = 0.5 and β = 100–600. The results obtained within the range γ = 0–5 showed that the force on cylinders oscillating transverse to a free stream remains similar to that for corresponding oscillations in still fluid and again can be well predicted by S–W theory, except for the higher γ values, for which $\gamma \cdot KC$ becomes higher than some value, approximately $\gamma \cdot KC > 2$. The exact value increases with β . Note that the product $\gamma \cdot KC$ corresponds to the reduced velocity, which is another dimensionless parameter customarily employed in related studies. The present results are in good agreement with the experimental study of Morse and Williamson [12], who have directly measured the force and have found that the effective added mass (inertia) coefficient remains close to unity at reduced velocities below 4, albeit at a Reynolds number based on the free-stream velocity of 4000, i.e. at least an order of magnitude higher than the Reynolds numbers considered in the present numerical study. Below, we present some selected results in support of the above discussion.

Figure 1 shows the time history of the forces acting on cylinders oscillating in still fluid and transverse to a free stream with KC = 0.5 and β = 200. It can be seen that the results obtained by the CFD simulations for total transverse force as well as the force due to pressure and shear stresses acting on the cylinder are in excellent agreement with S–W theory even under the influence of a free stream. However, minor modulations in the magnitude can be observed for simulations at γ = 2; it is interesting to note that both force due to pressure and force due to shear stress have modulations of similar magnitude, in conformity with S–W theory. We have also included in the second row in Figure 1 the force predicted by inviscid potential flow; it can be seen that this 'potential' force makes most of the contribution to the total force, i.e. the influence of viscosity appears to be very minor for very low KC numbers. Finally, it can be seen that the streamwise force contains modulated fluctuations at twice the oscillation frequency (middle and right columns in Figure 1). This contrasts with the case of oscillations in still fluid, for which the force in the direction transverse to the oscillation is zero (left column in Figure 1). It is interesting to note that the fluctuations in the streamwise force cannot be associated with vortex shedding in the wake of a cylinder oscillating in the free stream, since vortex shedding takes place at a frequency that is approximately an order of magnitude lower than the oscillation frequency. On the other hand, there seems to be some correlation between modulations in the streamwise and transverse forces.

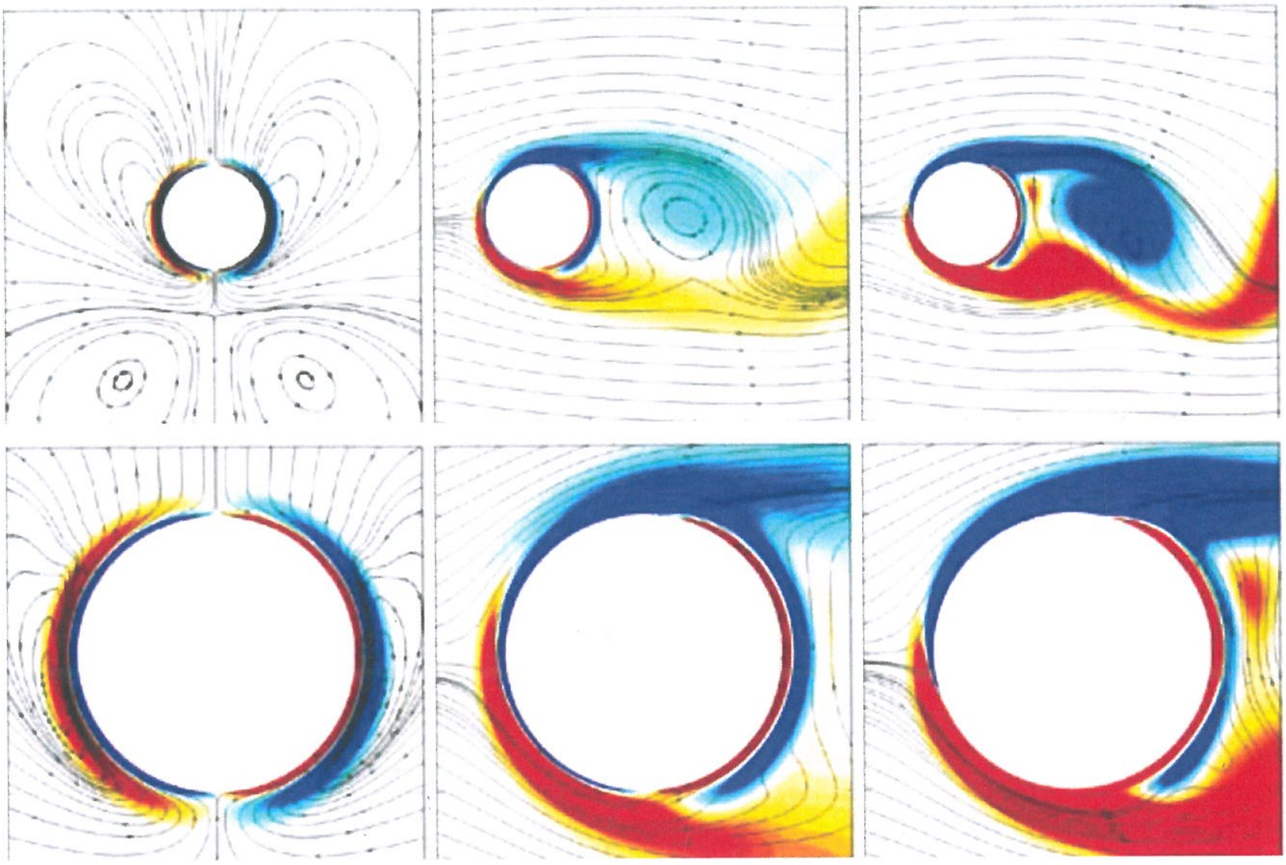
Although the pressure and shear-stress forces acting in the direction of cylinder oscillation are very similar in both still fluid and in a free stream, as discussed above, the flows that produce those forces are remarkably disparate, as can be seen in Figure 2, which shows the distributions of vorticity and streamlines around the cylinders for the same three cases as in Figure 1. All instants of the flow field in Figure 2 correspond to the top-most position of the cylinder during its oscillation. The plots at the bottom show a close-up view of the flow fields shown in the top plots. It can be seen that the flow remains attached for a cylinder oscillating in still fluid, whereas the flow separates and a vortex wake is formed for corresponding oscillations transverse to a free stream. However, it may be noted in the bottom plots that the streamlines closest to the cylinder follow its surface in the latter cases (γ=1 and 2), much like in the case of the cylinder oscillating in still fluid (γ=0), although they generally are quite different in the outer flow. For oscillations in still fluid, vorticity is concentrated in two thin layers of opposite-sign vorticity around the cylinder while the rest of the flow is almost irrotational. The superposition of a free stream alters the distribution of vorticity and as γ, i.e. the free-stream velocity, gradually increases the vorticity layer around the cylinder thickens due to additional generation of vorticity at the wall induced by the free stream. As another comparison of the flow in still fluid and free stream, we note that the thin layer of negative vorticity in the bottom half on the left-hand side of the cylinder seen at γ=0 becomes even thinner at γ=1 and disappears at γ=2. A full discussion of the flow patterns requires more length than available here. We hypothesize that the large difference in the time scales of vorticity convection by the free stream and the vorticity generation due to oscillation are responsible for virtually independent actions.

Similar results to those presented here for β = 200 have been obtained for β = 100 and 600. We cannot readily explain how the time history of the force is so similar for oscillations both in still fluid and transverse to a free stream, given that the forces due pressure and shear stresses are directly linked to the wall distributions of vorticity and wall-normal vorticity gradient, respectively. Our plan is to examine the wall distributions in more detail and extend this study to even lower *KC* numbers.



γ=0 γ=1 γ=2

Figure 1. Comparison of CFD simulations and S–W theory for the total force C_y, force due to pressure C_p, force due to skin friction C_s, and the fluctuating drag force C'_x on a cylinder oscillating in still fluid (γ=0) and transverse to a free stream (γ=1,2) at *KC* = 0.5 and β = 200.



$\gamma = 0$ $\gamma = 1$ $\gamma = 2$

Figure 2. Flow patterns revealed by streamlines and vorticity distributions around cylinders oscillating in still fluid (γ=0) and transverse to a free stream (γ=1 and 2) at *KC*=0.5 and β =200.

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