

**DEVELOPMENT OF MOIRÉ FRINGE
RECOGNITION SYSTEM USING ARTIFICIAL
NEURAL NETWORK FOR 2-D DISPLACEMENT
MEASUREMENT**

WOO WING HON

UNIVERSITI SAINS MALAYSIA

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**DEVELOPMENT OF MOIRÉ FRINGE RECOGNITION SYSTEM USING
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MEASUREMENT**

by

WOO WING HON

**Thesis submitted in fulfilment of the
requirements for the degree of
Master of Science**

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DECLARATION

I hereby declare that the work reported in this thesis is the result of my own investigation and that no part of the thesis has been plagiarized from external sources. Materials taken from the sources are duly acknowledged by giving explicit references.

Signature:

Name of student: WOO WING HON

Matrix number: P-CM0003/14(R)

Date: 06/04/2018

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LIST OF ABBREVIATIONS

ANN	Artificial neural network
CWT	Continuous wavelet transformation
FFBGNN	Feed forward back propagation neural network
FFT	Fast Fourier transformation
FTM	Fourier transformation method
GAM	Graphical analysis method
ROI	Region of interest
WTM	Wavelet transformation method

LIST OF SYMBOLS

A	Amplitude of the sine curve profile
$\phi(x, y)$	Angular phase of the fringe pattern
y_i	ANN output
t_i	ANN target
I_o	Background intensity
$S(a, b)$	CWT coefficient
$U(x, y)$	Displacement field in x -direction
$V(x, y)$	Displacement field in y -direction
ε	Eccentricity magnitude
θ	Eccentricity direction
h	First indexed family of gratings
$f_c(\theta)$	Fitted sine function
I_n	Harmonic components
z	Height of reference grating from specimen
α	Incident angle of light source
$I_{local}(r, \theta)$	Intensity at a local pixel (polar coordinates)
$I_m(x, y)$	Mean intensity

$I_a(x, y)$	Modulation of interference fringe
$N(x, y)$	Moiré field under investigation
$M(t)$	Mother wavelet
n	Number of circles in circular grating
N	Number of samples
N	Order of moiré fringe
c	Offset
ϕ	Phase shift of the sine curve profile
g	Pitch of reference grating
σ_E	Propagation of error
r	Radius of circular moiré patterns
p	Resultant moiré fringes
k	Second indexed family of gratings
a	Scaling parameter
b	Shifting parameter
$s(t)$	Wavelet signal
σ_{ANN1}	Variance of mean error in ANN 1
σ_{ANN2}	Variance of mean error in ANN 2

**PEMBANGUNAN SISTEM PENGECAMAN PINGGIR MOIRÉ DENGAN
MENGUNAKAN RANGKAIAN NEURAL TIRUAN UNTUK
PENGUKURAN ANJAKAN 2-D**

ABSTRAK

Pelbagai kaedah telah dicadangkan untuk mendapatkan maklumat anjakan dalam analisis corak moiré. Kaedah-kaedah ini boleh dikategorikan kepada analisis manual oleh inspektor manusia, kaedah komputasi dan kaedah analisis berasaskan imej. Analisa manual terdedah kepada ralat manusia kerana ia bergantung kepada keputusan manusia dalam analisa corak moiré. Penggunaan kaedah pengiraan dalam analisa corak moiré adalah terhad kepada corak moiré yang dihasil daripada parutan berfrekuensi tinggi yang sinusoid. Dalam kaedah berasaskan analisis imej, Algoritma yang kompleks menyebabkan butir-butir halus dalam corak moiré terhilang dalam operasi pra-proses imej. Situasi ini menyebabkan ketidakpastian dalam analisa corak moiré. Untuk mengatasi kelemahan yang disebut di atas, kaedah rangkaian saraf buatan (ANN) dicadangkan untuk sistem pengenalan corak moiré dalam pengukuran anjakan 2-D. Sistem pengenalan corak moiré terdiri daripada dua ANN dengan dua tugas yang berbeza iaitu (i) penentuan pusat pinggiran moiré dan (ii) penentuan kesipian berdasarkan corak moiré. Kaedah ANN dibandingkan dengan kaedah analisa grafik (GAM), sejenis kaedah analisa berasaskan imej, dari segi ketepatan dan masa pengiraan untuk pengukuran anjakan 2-D pola moiré. The experiments prove that ANN approach has a higher accuracy to GAM with mean errors with 95% confidence of 0.068 ± 0.013 mm for eccentric magnitudes and $1.85 \pm 0.465^\circ$. An improvement of 66.18% in the computation time is also reported in the comparison. A straightforward solution for the moire fringe recognition system of circular grating moire pattern is achieved using ANN approach.

DEVELOPMENT OF MOIRÉ FRINGE RECOGNITION SYSTEM USING ARTIFICIAL NEURAL NETWORK FOR 2-D DISPLACEMENT MEASUREMENT

ABSTRACT

Various methods have been proposed in the analysis of moiré pattern. These methods can be categorized into manual inspection by human inspector, computational methods and image analysis based methods. Manual interpretation of moiré patterns is prone to human errors as it is highly dependent on the decision of the human inspector. The computational methods are lack of flexibility as they are limited to high frequency gratings which are sinusoidal in the transmittance of grating. As for the image analysis based methods, complex algorithms can unintentionally remove the fine details in the moiré patterns and cause uncertainty in the analysis. To overcome the above mentioned drawbacks, an artificial neural network (ANN) approach is proposed for a moiré fringe recognition system in 2-D displacement measurement. The moiré fringe recognition system consists of two ANNs with two different tasks : (i) the determination of moiré fringe centers of the circular grating moiré patterns and (ii) the determination of eccentricity magnitudes and eccentricity directions of the circular grating moiré patterns. The ANN approach is compared to graphical analysis method (GAM), an image analysis based method, in terms of accuracy and computational time for 2-D displacement measurement of circular grating moiré patterns. The experiments prove that ANN approach has a higher accuracy to GAM with mean errors with 95% confidence of 0.068 ± 0.013 mm for eccentric magnitudes and $1.85 \pm 0.465^\circ$. An improvement of 66.18% in the computation time is also reported in the comparison. A straightforward solution for the moire fringe recognition system of circular grating moire pattern is achieved using ANN approach.

CHAPTER ONE

INTRODUCTION

1.1 Background of research

Moiré pattern is a complex map of intersections of lines comprising of two overlapped gratings. The broad dark lines that are observed after the overlapping of two gratings are called moiré fringes. The advantage of moiré pattern is the amplification effect of displacement change between two gratings. A small change in displacement between two fine gratings will cause the moiré pattern to change. Displacement components of two gratings can be determined by analyzing the changes in moiré pattern (Sciammarella & Piroozan, 2007). Besides that, the moiré patterns can be reproduced using the same gratings set and with the same in-plane or out-of-plane displacement. The reproducibility of the moiré pattern enables it to become a useful tool in metrology (Chiang, 1979; Sciammarella, 1982).

The application of moiré patterns can be found in many fields of engineering metrology which includes full field displacement measurements, positioning and alignment systems, strain analysis, surface topography etc. The utilization of the moiré patterns to measure displacements is known as the moiré methods. The moiré methods can be categorized into geometric moiré, shadow moiré, projection moiré and moiré interferometry. These moiré methods provide full contour maps of in-plane displacement fields and out of plane displacement fields with high sensitivity and high spatial resolution (Post & Han, 2008).

In the early development of moiré methods, the analysis of the moiré patterns was performed manually by a human inspector using fringe sign determination, fringe ordering, fringe counting and fringe interpolation (Han et al., 2001; Lay & Chen, 1998; Lee et al., 1988). These methods required human inspectors to have the knowledge of

moiré pattern for the measurement of displacement. The accuracy of the manual inspection was limited by the human errors. The method was ineffective due to low repeatability and slow processing speed in the procedures of analyzing the moiré patterns by human inspectors.

Computational methods, such as Fourier transformation methods (de Oliveira et al., 2012; Nicola & Ferraro, 2000; Park & Kim, 1994; Wang et al., 1999) and phase shifting methods (Poon et al., 1993; Cordero & Lira, 2004; Du et al., 2014; Liu & Chen, 2005; Trivedi et al., 2013; Zhu et al., 2014) had been proposed to address the issue of ineffectiveness in manual inspection methods. Fast computational algorithms were used to automate the analysis of moiré patterns. The displacement information was extracted mathematically from the moiré patterns. These computational methods give a fast and accurate measurement by eliminating the laborious and subjective procedures in manual inspections methods. However, the application of computational methods is limited to the moiré patterns with sinusoidal intensity distribution.

Image analysis based methods had also been proposed for the automated analysis of moiré pattern. Image processing techniques were applied to the images of moiré patterns to extract the moiré fringes from the moiré patterns (Agarwal & Shakher, 2015; Lay et al., 2012; Yen & Ratnam, 2011, 2012a). The displacement of the moiré patterns could be obtained graphically from the information of moiré fringes such as the profile of moiré fringes and the intensity distribution of the moiré fringes. The drawback of the image analysis based methods is the uncertainty that is caused by the preprocessing operations to remove the residual gratings in the background.

Artificial neural networks (ANN), which have been proposed as tools for solving image processing and pattern recognition tasks (Egmont-Petersen et al., 2002; Mah & Chakravarthy, 1992), constitute a typical soft computing approach that is

capable of learning and classifying patterns using a set of learning algorithms that are tolerant of uncertainty and approximation (Cristea, 2009; Mah & Chakravarthy, 1992; Senthilkumaran & Rajesh, 2009). ANNs mimic the human-like decision making and have a consistent and repeatable machine-like performance. ANN approach has the potential to replace the conventional image processing techniques that can cause uncertainty in the 2-D displacement measurement of moiré pattern. With proper feature selection and training, an ANN can determine the displacement of moiré patterns regardless of the background residual gratings and unevenness of the images of moiré patterns. However, no study has reported the use of this ANN approach for moiré fringe recognition to obtain the displacement of moiré pattern for measurement purposes. The current applications of ANN approach in moiré pattern analyses are limited to classification problems based on the features of moiré patterns (Chiang et al., 2014; Sciammarella & Piroozan, 2007).

This work proposes an ANN approach for moiré fringe recognition system in 2-D displacement measurement of circular grating moiré patterns. In this study, two ANNs were developed for the moiré fringe recognition system in 2-D displacement measurement of circular grating moiré pattern. The ANNs were designed for two different tasks which are (i) to determine the centers of the moiré fringes and (ii) to determine the displacement components (eccentricity magnitude and eccentricity direction) of the moiré patterns. The advantages of using two ANNs for different tasks instead of single ANN with two outputs are the simplification of the feature selection and training stage of the ANN as well as reduce the requirement of computational power by using simple ANN architectures for the training. The input of ANN1 (moiré fringe center determination) is the column pixel value of the the polar transformed circular grating moiré pattern. The inputs of ANN2 (2-D displacement determination)

are number of discontinuities in fitted curve from ANN 1 and the peak value at the gradient change of the fitted curve. The accuracy of the ANN approach was compared with the theoretical results from mathematically generated moiré patterns and the results of graphical analysis method (GAM) which is one of the conventional image analysis based methods. The mean errors with confidence interval of 90% was measured for the determination of 2-D displacement measurement. The plot of graphs on outputs of ANN approach and GAM are presented by comparing to the targets of 2-D displacement components that were recorded on micrometer readings. The correlation factor of outputs and targets were calculated for ANN approach and GAM to show the accuracy of respective methods.

1.2 Problem statement

The accuracy of manual interpretation techniques is strongly dependent on the decisions of the human inspectors who perform moiré fringe recognition. Therefore, such techniques are prone to human error, resulting in uncertainty in moiré pattern analyses. Manual interpretation techniques have poor repeatability and reproducibility. It is ineffective for human inspector to monitor the change in the moiré patterns repeatedly in a large number of samples.

Computational methods are limited by moiré patterns with sinusoidal intensity distribution. High frequency gratings with sinusoidal intensity variation are used in the generation of moiré patterns for the applications of computational methods. Computational methods are not readily applied to low frequency gratings with grating pit. Low frequency gratings are more favorable than high frequency gratings in the applications of measurement due to the simplicity and the cost of producing low frequency gratings (Piro & Grediac, 2004). The intensity of moiré fringes formed by