AN EXPLORATION INTO STRUCTURALLY RATIONAL ARCHITECTURAL FORMS

-A SERIES OF SMALL PROJECTS-

ARCE 453 - SENIOR PROJECT - Spring 2019
California Polytechnic State University – San Luis Obispo

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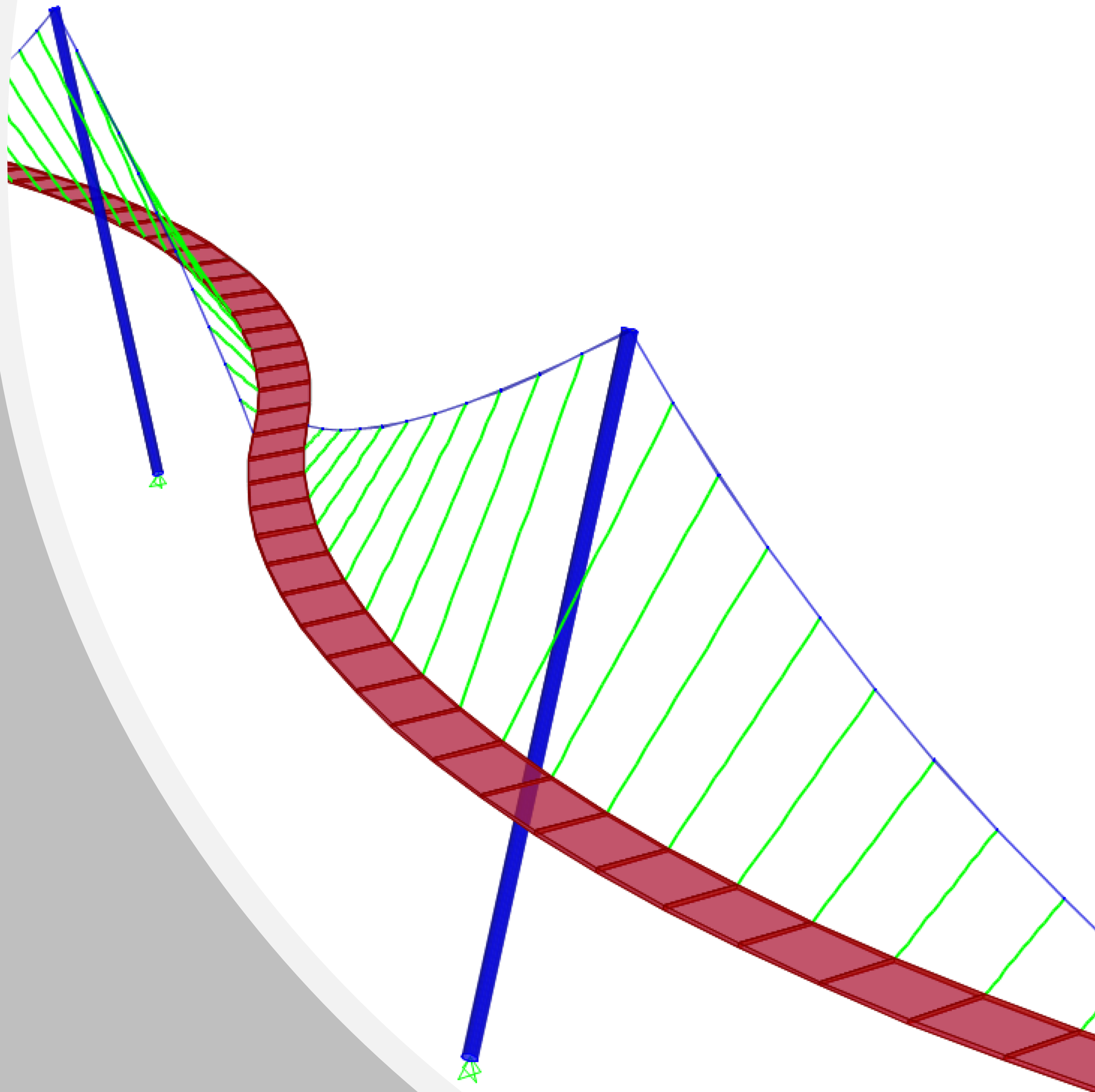
For: Edmond Saliklis

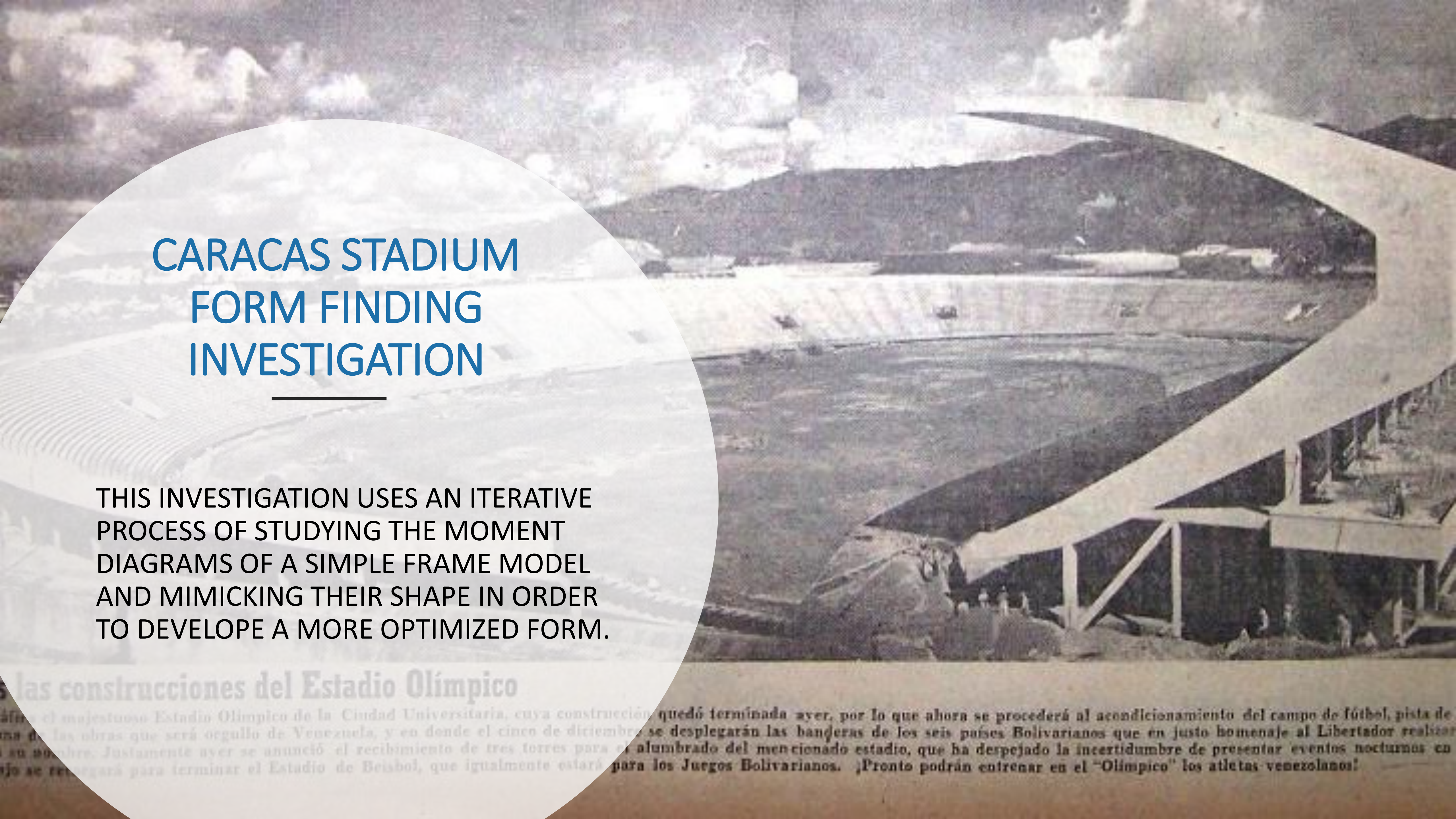
ARCE 453 PROJECTS:

- Caracas Stadium Form Finding Investigation
- Muller Breslau Method Exploration
- Vierendeel Girder/Truss Investigation
- Earth Pavilion/Arch Investigation
- Erzbahnschwinge Bochum Investigation



CARACAS STADIUM FORM FINDING INVESTIGATION

THIS INVESTIGATION USES AN ITERATIVE PROCESS OF STUDYING THE MOMENT DIAGRAMS OF A SIMPLE FRAME MODEL AND MIMICKING THEIR SHAPE IN ORDER TO DEVELOPE A MORE OPTIMIZED FORM.

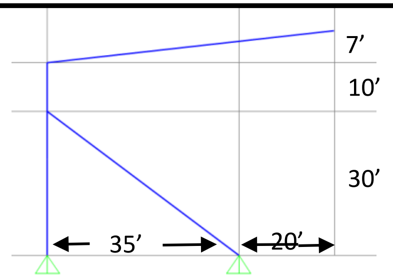
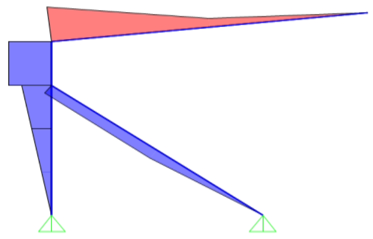
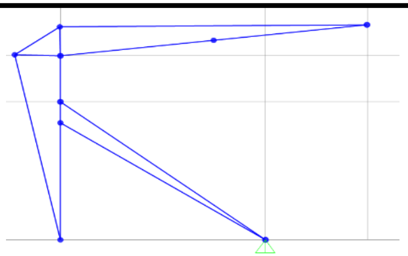
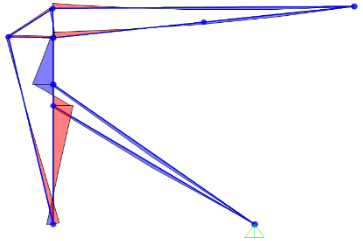
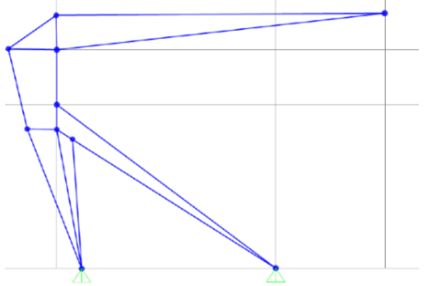
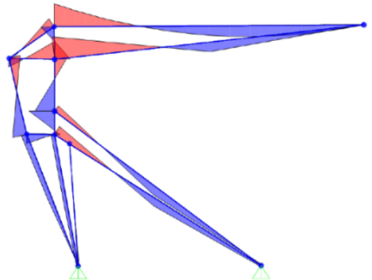
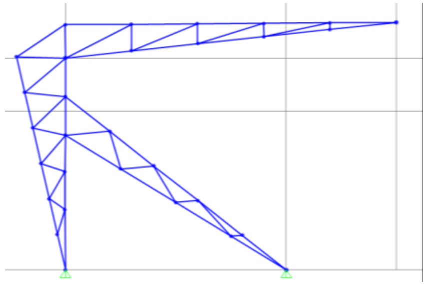
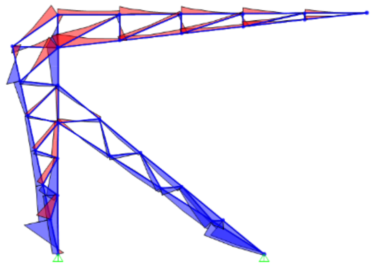


FORM FINDING INVESTIGATION

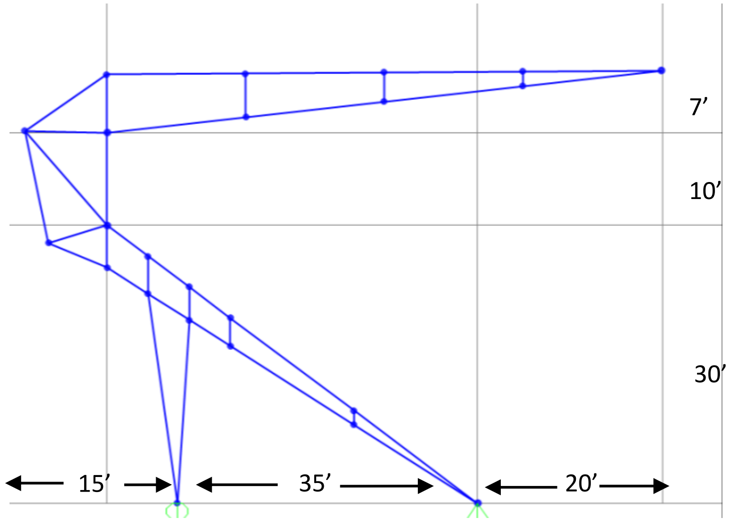
The Project

The inspiration for this frame was the large cantilevered shape of Carlos Villanueva's 1951 Olympic Stadium in Caracas, Venezuela. The goal of the investigation is to gain an understanding of the structure and to explore possible adjustments by running analysis on a frame model in SAP2000.

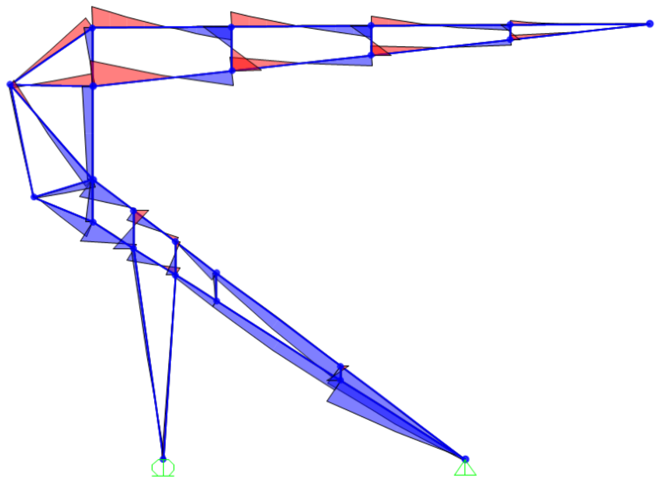
The shape of the initial frame is loosely based on the stadium, and its moment diagram is then used to find the desired adjustments to the model. After running analysis on the new model its moment diagram can be used to recommend further modifications. Each iteration looks at something new while still keeping in mind the results of previous models.

	FRAME MODEL	MOMENT DIAGRAM
INITIAL FRAME		
FIRST ITERATION		
SECOND ITERATION		
THIRD ITERATION		

FORM FINDING INVESTIGATION



FOURTH ITERATION

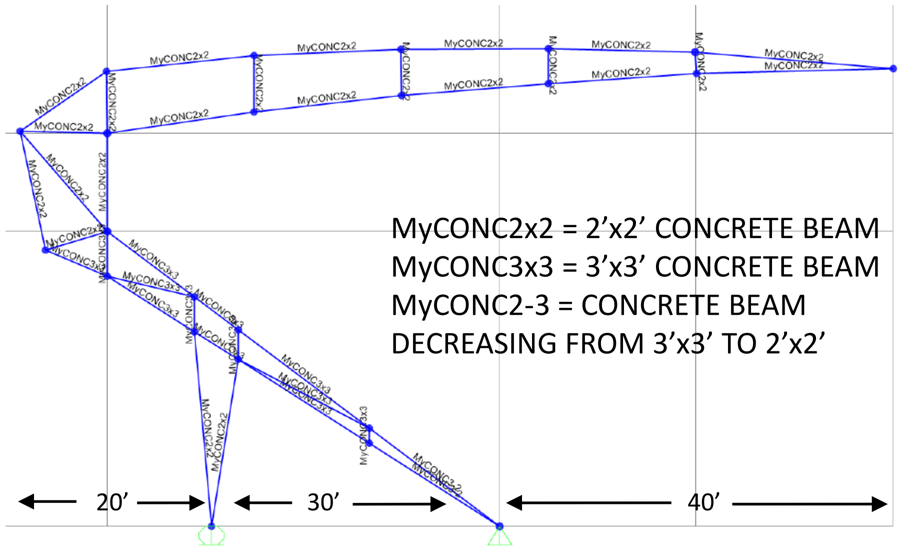


MOMENT DIAGRAM

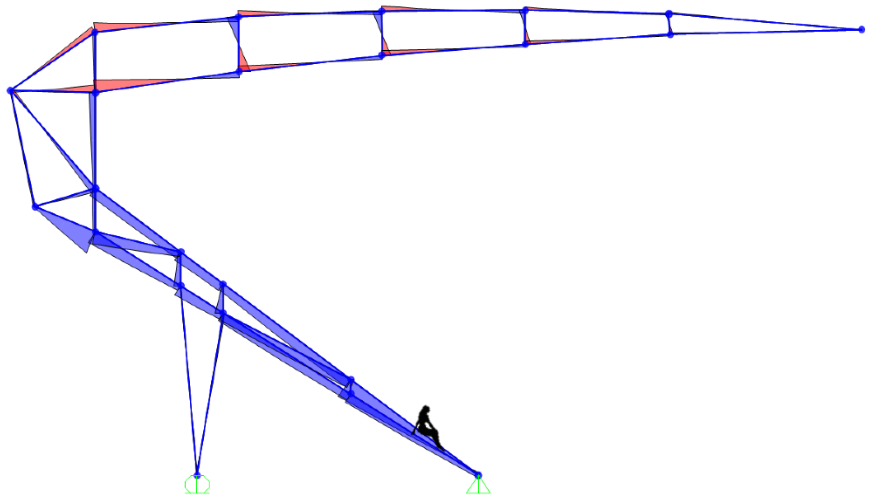
The Resulting Model

The later iterations of the model combine elements from the previous ones and minimize the moment forces on most members.

The final frame model also adjusts the element sizes, utilizing the SAP tool which allows member sizes to be adjusted so they taper from one thickness to another. This technique is used to maintain the pinned base from the models even when including member width. This model uses large concrete beams in the area under the seating to increase the strength in this crucial section. However, where these are not necessary – in the cantilever – it uses smaller beams to minimize the structure’s weight. Smaller members are also used in the leg under the seating because the member is angled towards the center of mass and is almost entirely in compression without any moment forces.



FINAL MODEL



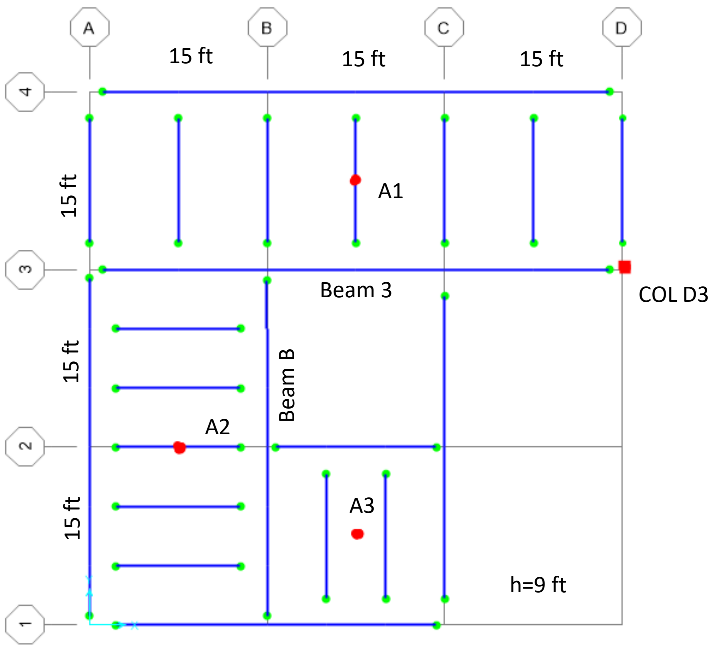
MOMENT DIAGRAM

MULLER BRESLAU METHOD INVESTIGATION

THE MULLER BRESLAU METHOD USES INFLUENCE LINES TO FIND THE SCALED DEFLECTION - OR LIFT - IN ORDER TO EASILY DETERMINE THE EFFECT OF A LOAD ON THE TOTAL AXIAL FORCE EXPERIENCED BY COLUMNS OR THE SHEAR AND MOMENT FORCES OF BEAMS.

MULLER BRESLAU INVESTIGATION

FRAMING LAYOUT:



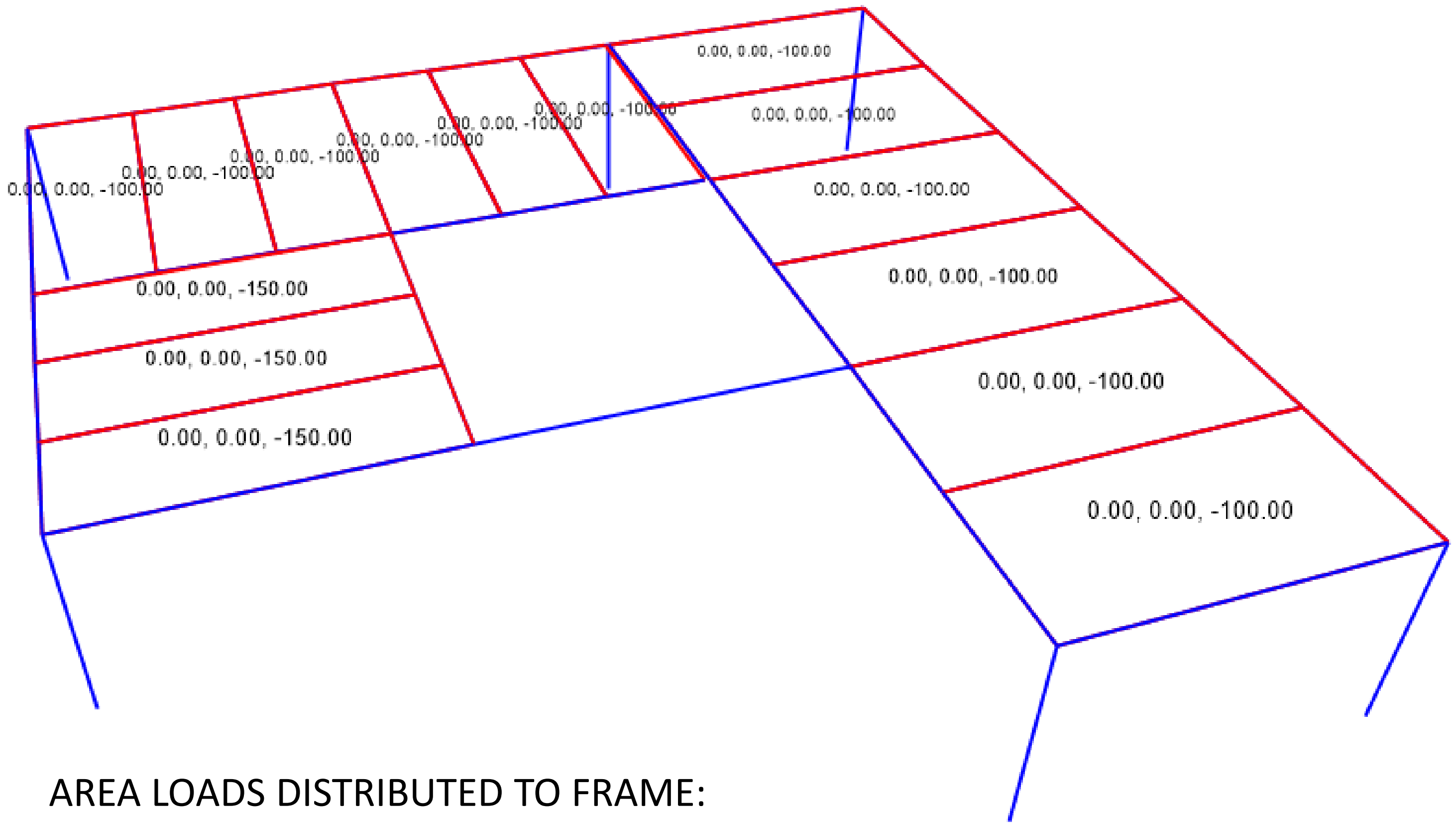
Introduction

The Muller Breslau Method of finding member forces is beneficial because it can be used to quickly and accurately identify the effect of any load on any particular member, even in an indeterminate structure. Because it does not require an understanding of statics, but rather uses geometry to determine a load's influence, it can be used by people who have little or no engineering background.

This method works by imposing an artificial displacement, or loft, at the point of interest and then using the resulting line of the beams to determine the loft at the point of the loads.

The Project

This investigation looks at how the Muller Breslau method can be used to find the axial force in Column D3, the shear in Beam 3 from Beam B, and the moment at the center of Beam B. The frame model has a 100 psf distributed load in areas A1 and A2, but a 150 psf load in A3, and an opening in the center. Despite these complications, this method makes the calculation of these forces easy.



AREA LOADS DISTRIBUTED TO FRAME:

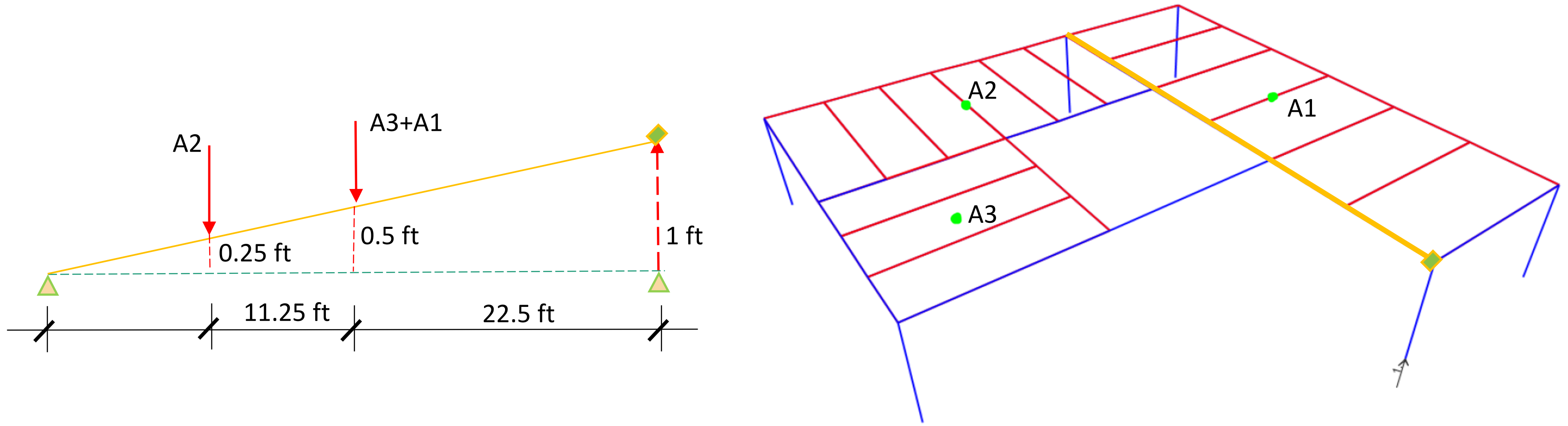
MULLER BRESLAU INVESTIGATION

Calculations:

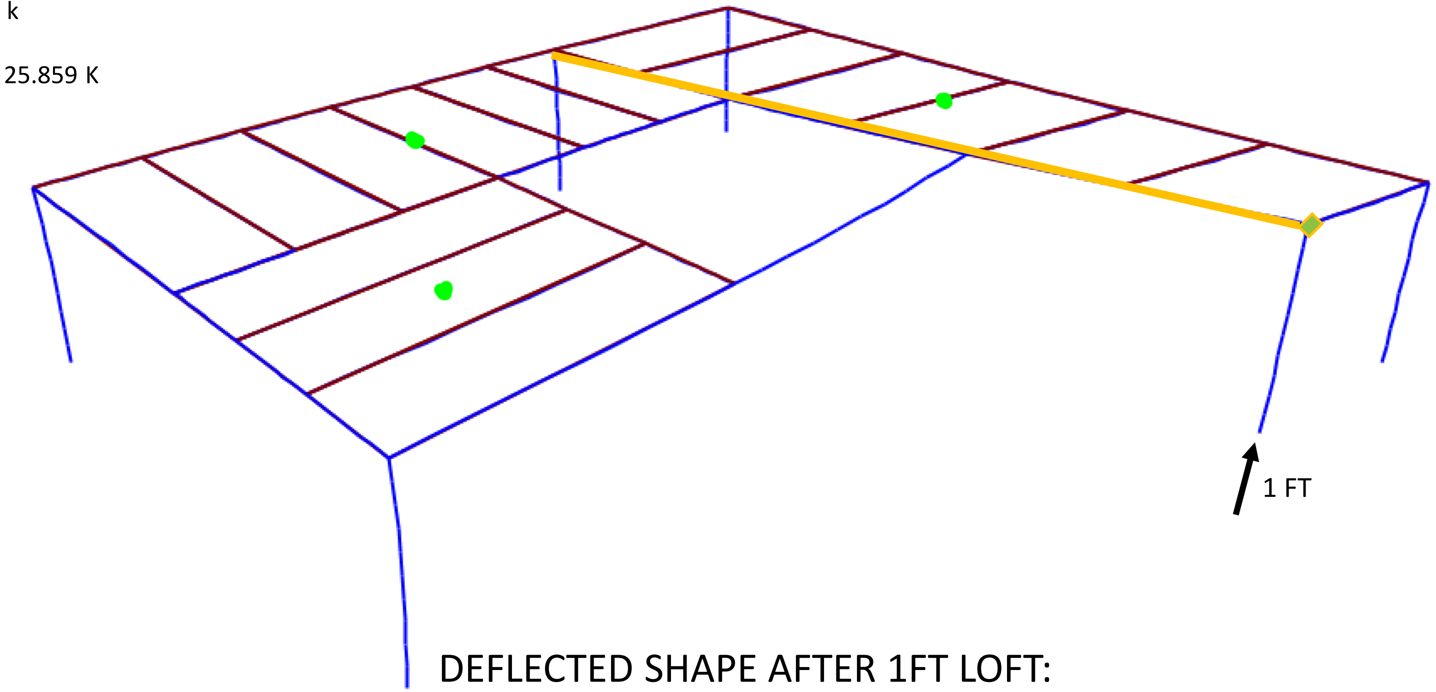
```
clear all; close all; clc;
syms ft lb k

% FORCE WHEN COLUMN D3 LOFTED 1FT:

W1= 100*(lb/ft^2);       % A1 & A2 AREA LOAD
W3= 150*(lb/ft^2);       % A3 AREA LOAD

FA1= 0.5*0.5*(15*ft*45*ft)*W1;
FA2= (1/6)*0.5*(30*ft*15*ft)*W1;
FA3= 0.25*0.5*(15*ft*15*ft)*W3;

FCol=FA1+FA2+FA3
```

FCol = (99375*lb)/4
= 24.84 k

SAP VALUE: 25.859 K



APPLIED LOFT:



DEFLECTED SHAPE AFTER 1FT LOFT:

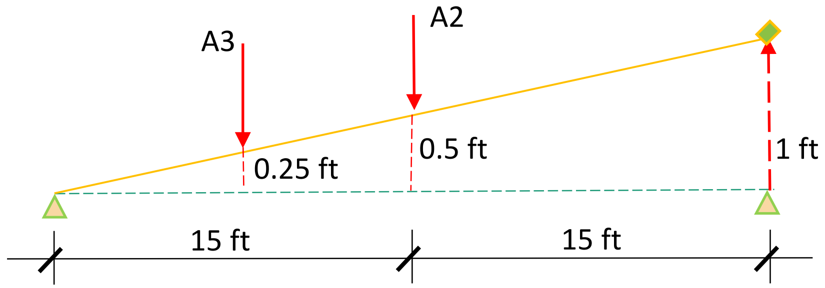
COLUMN AXIAL FORCE

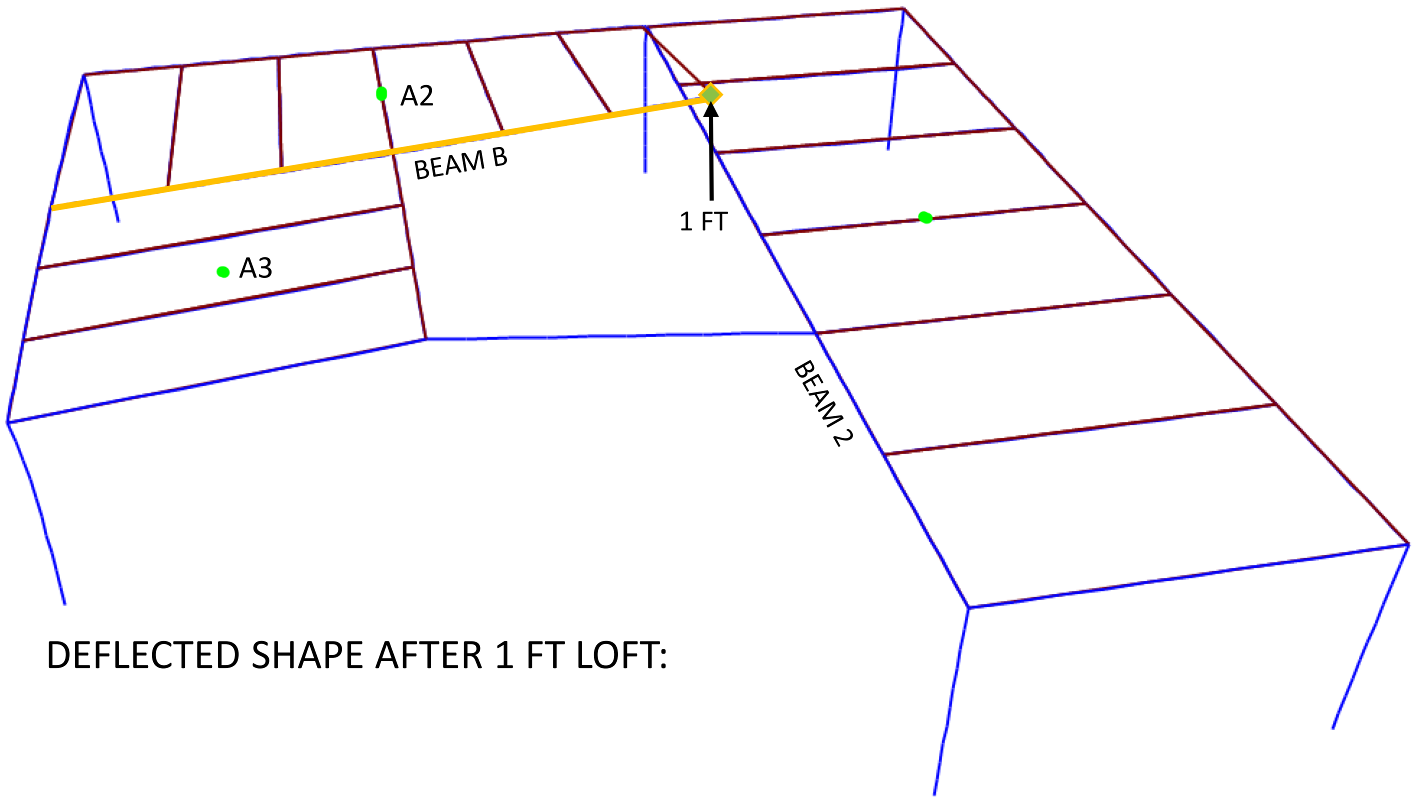
By following the resulting line of the beam when one end is lofted, as well as the displacement of the joists attached, we can easily determine the lofts of the center of mass for each separate area. The product of these lofts and the forces applied at those points give the axial force applied on the column from that area. The sum of these forces produces the total axial force experienced by the lofted column.

MULLER BRESLAU INVESTIGATION

BEAM SHEAR

The process for finding the shear in a beam is very similar to finding the axial forces in a column. To find the shear on Beam 2 where it supports Beam B, loft Beam B 1 ft at the end. Because the resulting deflections along this line vary linearly, we can easily determine the resulting loft of the center of mass for the areas it supports. The product of these lofts and the forces applied at those points give the shear applied on beam 2 from that area. The sum of these shear forces gives the total shear experienced by Beam 2 at this point.





DEFLECTED SHAPE AFTER 1 FT LOFT:

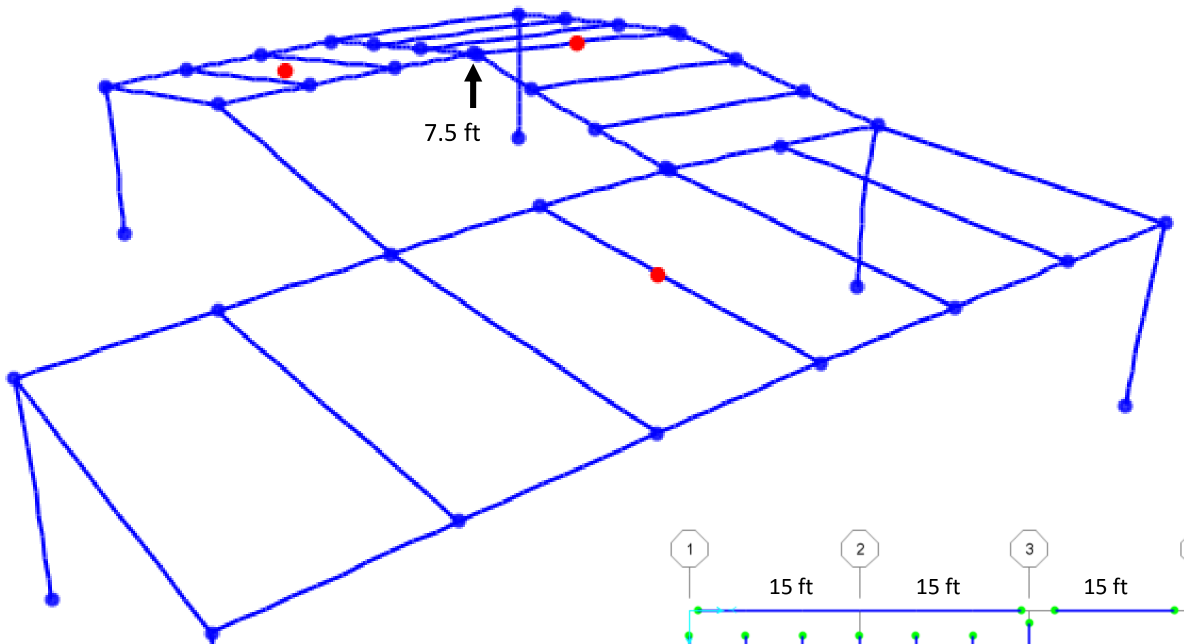
CALCULATIONS:

```
% SHEAR WHEN BEAM B LOFTED 1FT:
syms ft lb k

W1= 100*(lb/ft^2);                    % A1 & A2 AREA LOAD
W3= 150*(lb/ft^2);                    % A3 AREA LOAD

VA2= 0.5*0.5*(30*ft*15*ft)*W1;
VA3= 0.5*0.25*(15*ft*15*ft)*W3;

VToT = (W1*(30*ft*15*ft))+(W3*(15*ft*15*ft))
VB2 = VA2+VA3                         %SHEAR TO BEAM 2
```

VToT = 78750*lb
VB2 = (61875*lb)/4
= 15.47 lb

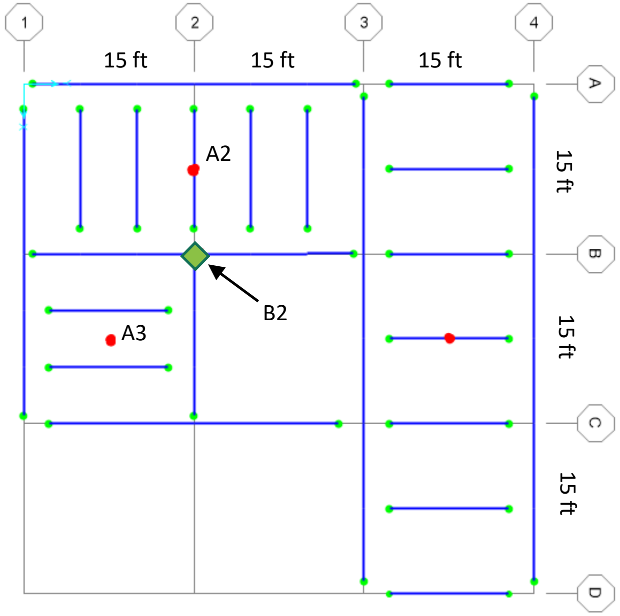
SAP VALUE: 12.896 k

MULLER BRESLAU INVESTIGATION



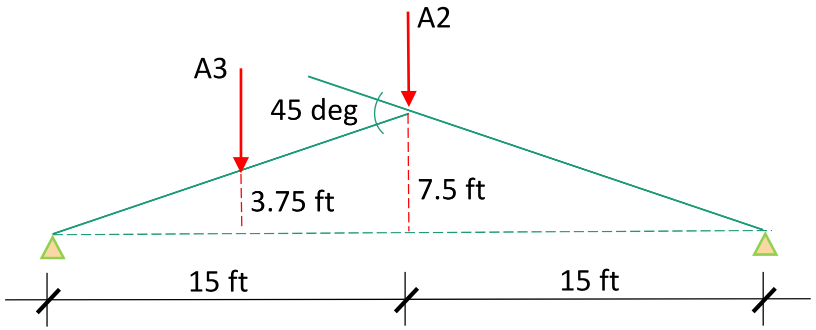
DEFLECTED SHAPE AFTER 45 DEG. ROTATION (OR 7.5 FT LOFT):

LOFT APPLIED AT POINT B2:



BEAM MOMENT

To find the moment at the center of Beam B we apply a 1 unit rotation (or mimic a rotation by lofting). Because the resulting deflections along the line vary linearly, we can easily determine the resulting loft of the center of mass for the areas it supports. The product of these lofts and the area point loads give the total moment at the center of Beam B.



LOFT APPLIED AT CENTER OF BEAM 'B':

CALCULATIONS:

% MOMENT AT PT. B2 WHEN LOFTED 7.5 FT (45 DEG. ROTATION)

W1= 100*(lb/ft^2); % A1 & A2 AREA LOAD
W3= 150*(lb/ft^2); % A3 AREA LOAD

M2= 1.0*0.5*(30*ft*15*ft)*0.100*(k/ft^2)*(7.5*ft);
M3= 0.5*0.25*(15*ft*15*ft)*0.150*(k/ft^2)*(7.5*ft/2);

MTOT= M2+M3

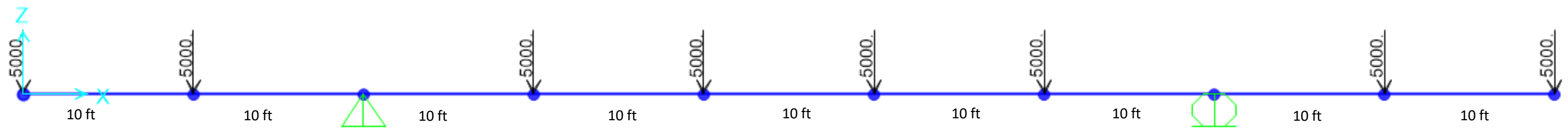
MTOT = (23625*ft*k)/128
= 184.57 ft*k

VIERENDEEL GIRDER/TRUSS INVESTIGATION

THIS INVESTIGATION LOOKS AT DEVELOPING AN OPTIMUM GIRDER AND TRUSS SIZE FOR A GIVEN BEAM CONFIGURATION BY LOOKING AT THE RESULTANT MOMENTS.

VIERENDEEL GIRDER/TRUSS INVESTIGATION

BEAM LOADING:



BEAM MOMENT DISTRIBUTION:

Project Description

A beam spanning 90 ft with a tributary width of 20 ft and a load of 20 psf is cantilevering on both ends. The purpose of this investigation is to find the ideal shape of the girder and truss that would minimize and equalize member loading. The moment diagram gives a base for developing this optimized shape. By dividing the peak moment by the desired T-C Couple (arbitrarily chosen for this example) you can find the required moment arm. This is the ideal height for the girder.

```
% BRIDGET HEALEY
% ARCE453 - TRUSS AXIAL FORCE MINIMIZING
INVESTIGATION
% 04/24/2019

    clear all; close all; clc

    syms FT PSF PLF LB
    PSF=LB/(FT^2);
    PLF=LB/FT;

    SPAN1=50*FT;
    SPAN2=20*FT;
    WIDTH=20*FT;
    psf=20*PSF;

    plf=psf*WIDTH;
    LOAD=plf*(SPAN1+2*SPAN2);
    NUM_POINTS=8;
    PT_LOAD=LOAD/NUM_POINTS
    PK_MOMT=150000*FT*LB;
    MOMT_2=50000*FT*LB;
    TC_COUPLE=26000*LB;

    ARM1=PK_MOMT/TC_COUPLE
    ARM2=MOMT_2/TC_COUPLE
```

PT_LOAD =
5000*LB

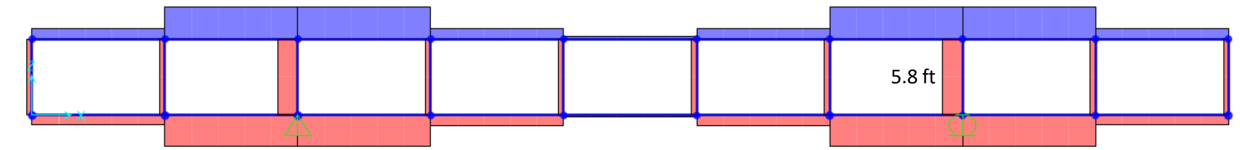
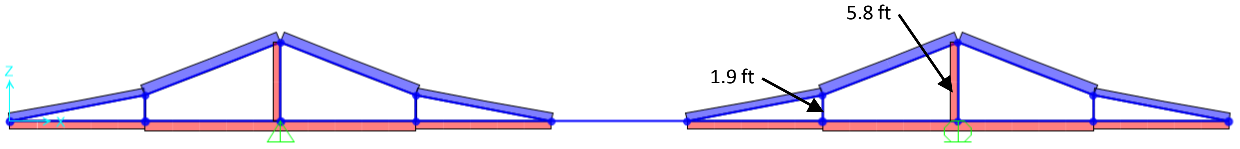
ARM1 =
(75*FT)/13
=5.8 FT

ARM2 =
(25*FT)/13
=1.9 FT

VIERENDEEL GIRDER/TRUSS INVESTIGATION

Developing Vierendeel Trusses

By dividing the moment at any position along the length of the beam by the T-C couple you can find the ideal height for the truss at that position. Once this height for each section of the truss is determined, it is found that the orientation of these sections does not significantly affect the axial load on the members. This allows aesthetic design to determine the final shape of the truss.

<table>
<tr><th></th><th>TRUSS AXIAL FORCES:</th></tr>
<tr><td>IDEAL VIERENDEEL GIRDER</td><td>
</td></tr>
<tr><td>FIRST ITERATION TRUSS</td><td>
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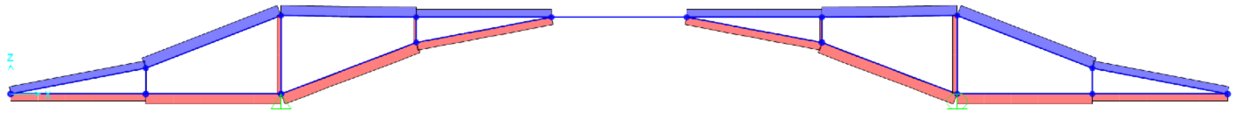
<tr><td>SECOND ITERATION TRUSS</td><td></td></tr>
<tr><td>THIRD ITERATION TRUSS</td><td></td></tr>
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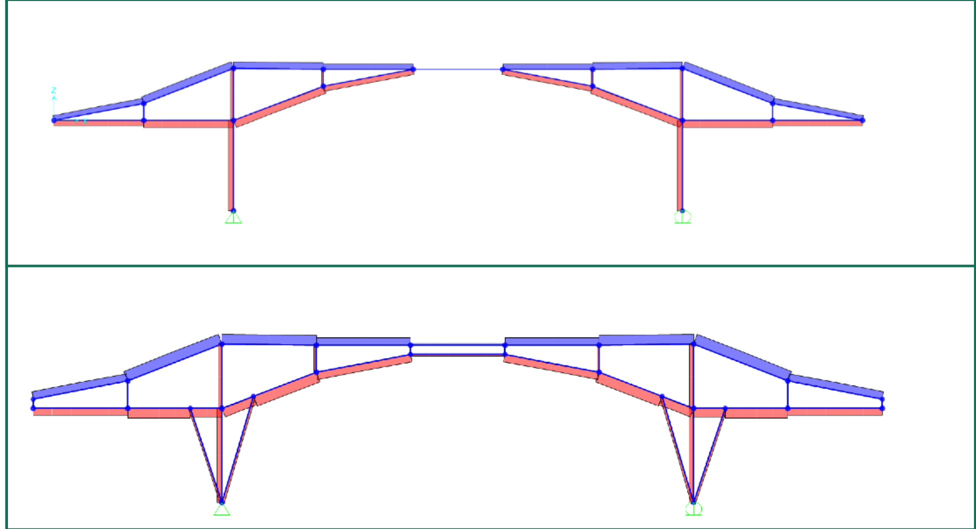
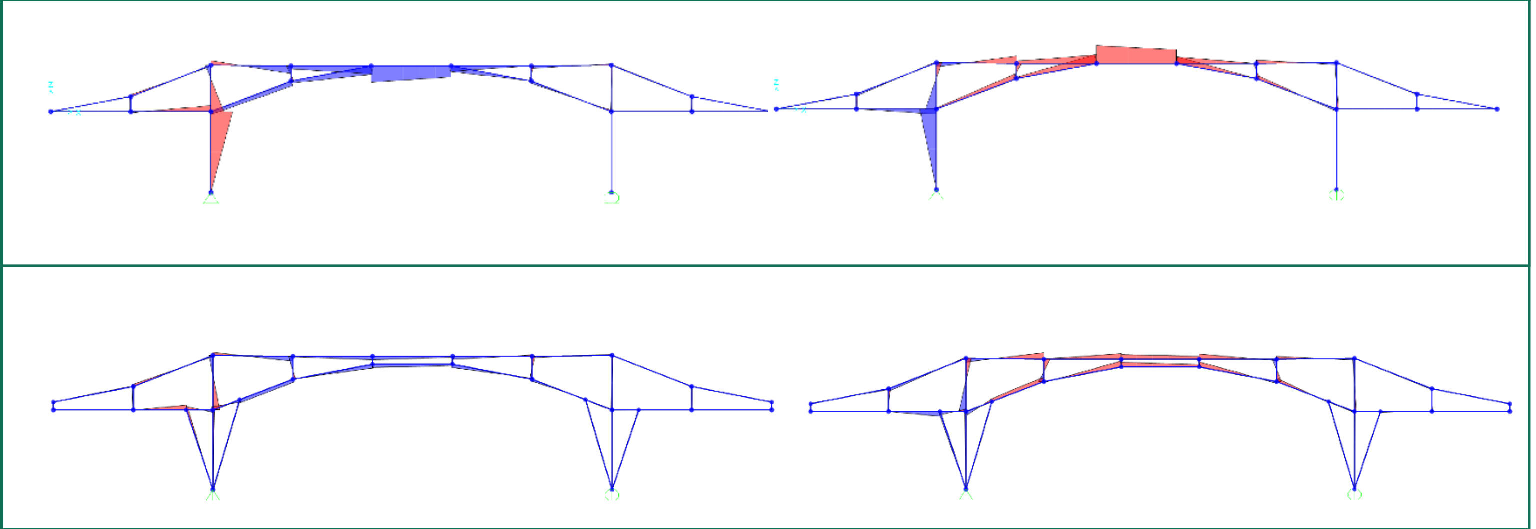
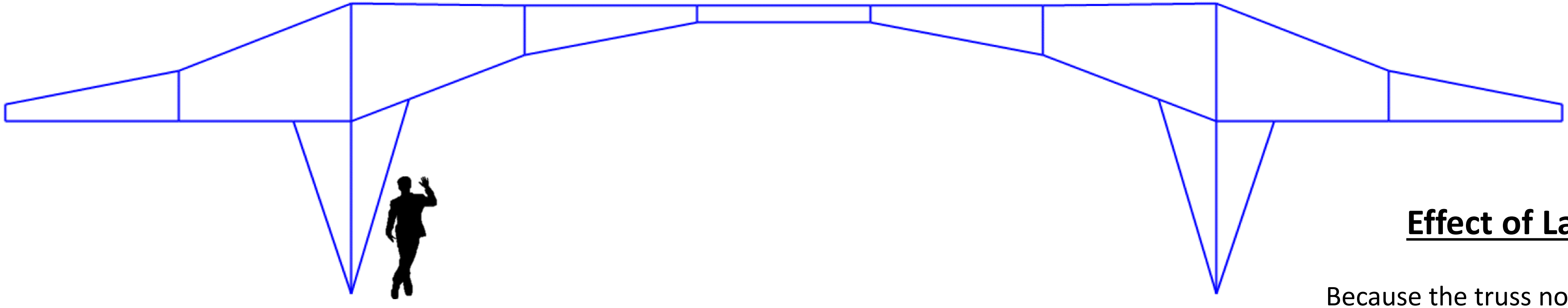
VIERENDEEL GIRDER/TRUSS INVESTIGATION

TRUSS AXIAL FORCES:	TRUSS MOMENTS FROM LATERAL FORCES:
	



FINAL OPTIMIZED TRUSS:

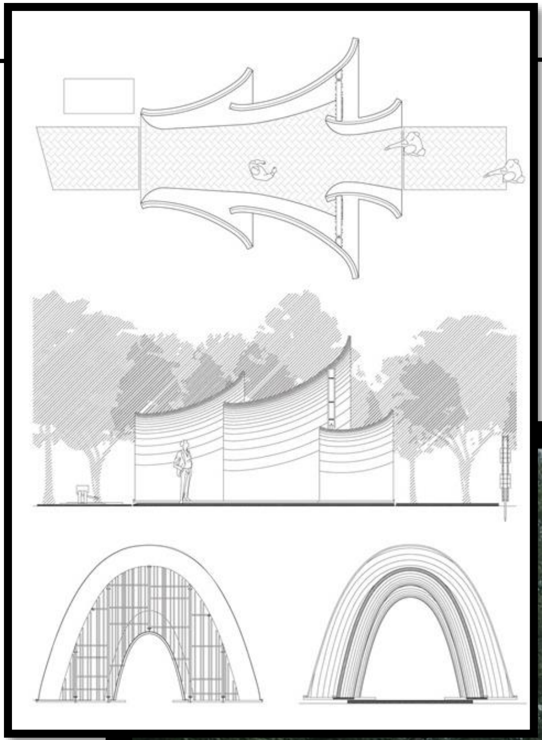
Effect of Lateral Force

Because the truss now has height, lateral loads should be considered. The effect of these loads on the structure’s legs inspires their new tapering shape. A few iterations results in a final optimized truss shape.

EARTH PAVILION/ARCH INVESTIGATION

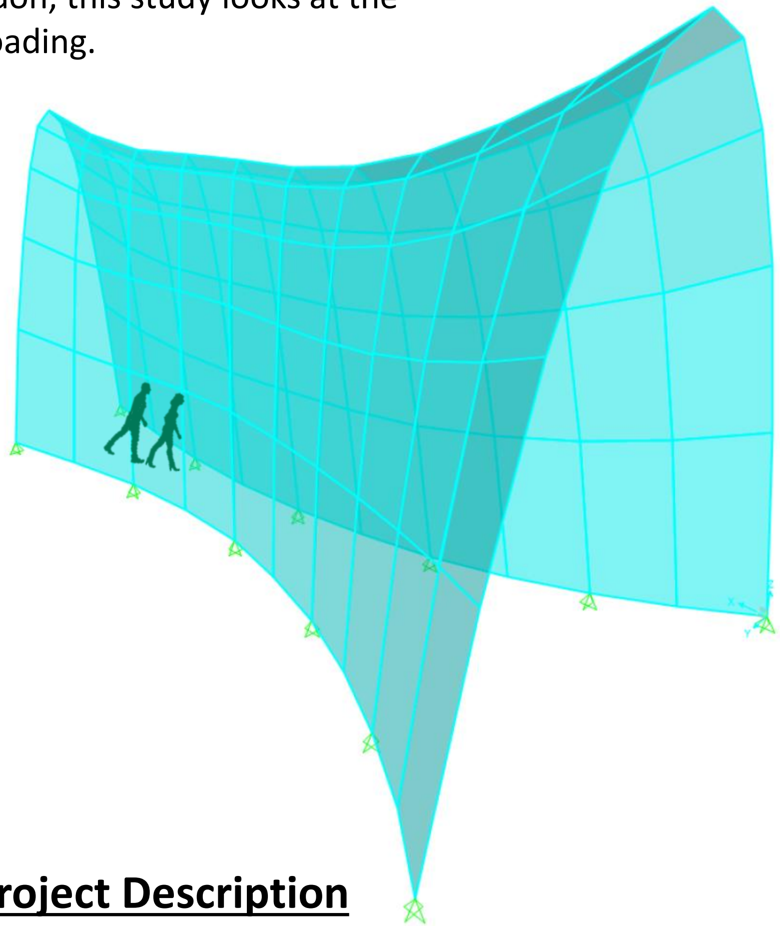
THIS INVESTIGATION LOOKS AT THE LOAD FLOW AND BUCKLING ANALYSIS OF A THIN SHELL PARABOLIC ARCH FORM.

EARTH PAVILION/ARCH INVESTIGATION



Inspired by Peter Rich & Michael Ramage's Earth Pavilion in London, this study looks at the reactions of the thin shell arches due to gravity and horizontal loading.

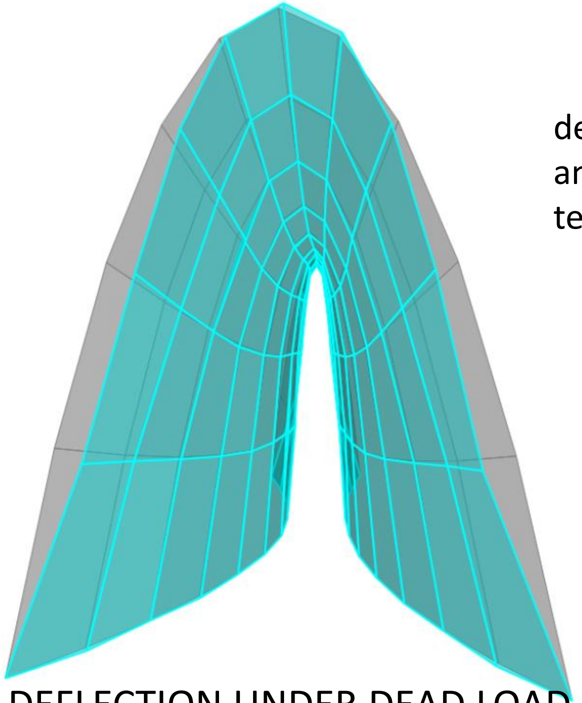
- Built: 2010
- Materials: two layers
  1" cement tile with light gage geotextile membrane
- Slatted timber screen is architectural



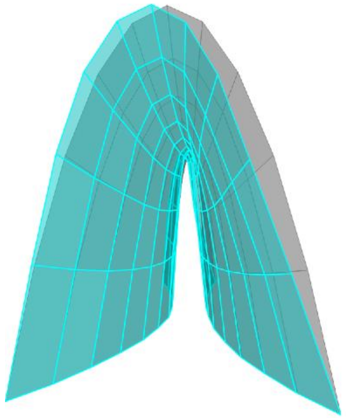
Project Description

For this investigation, the parabolic arch model accentuates the dimensions of the earth pavilion. Instead of reaching from 10 ft to 20 ft tall, its height ranges from 35 ft to 70 ft.. To understand the strengths of the shape requires load flow and buckling analysis of the arches under vertical and horizontal loading.

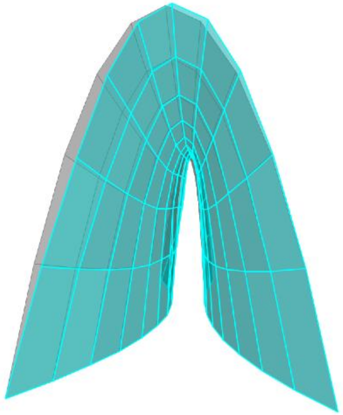
EARTH PAVILION/ARCH INVESTIGATION



DEFLECTION UNDER DEAD LOAD



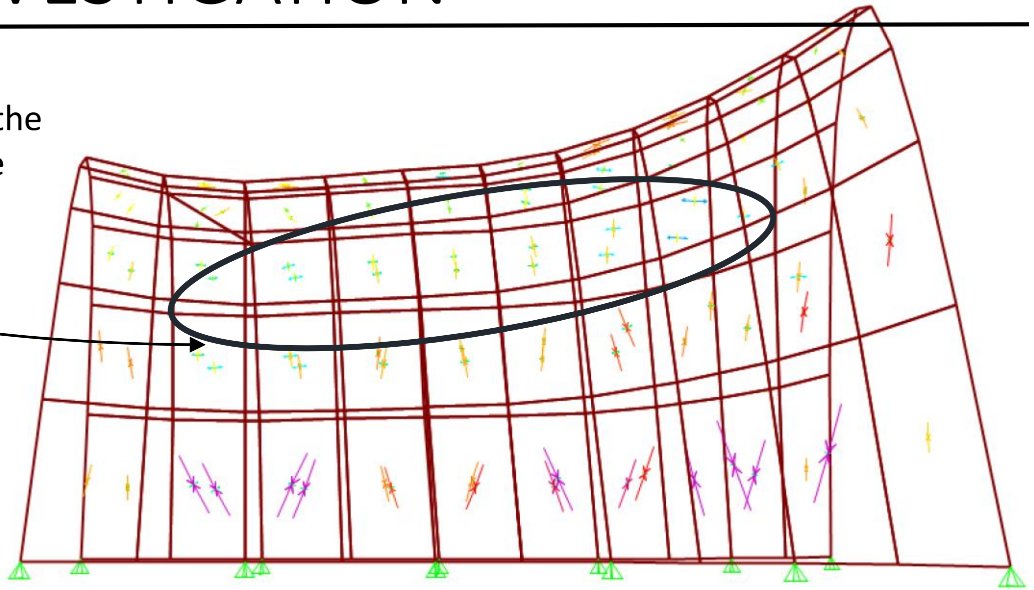
DEFLECTION UNDER LOAD FROM LEFT

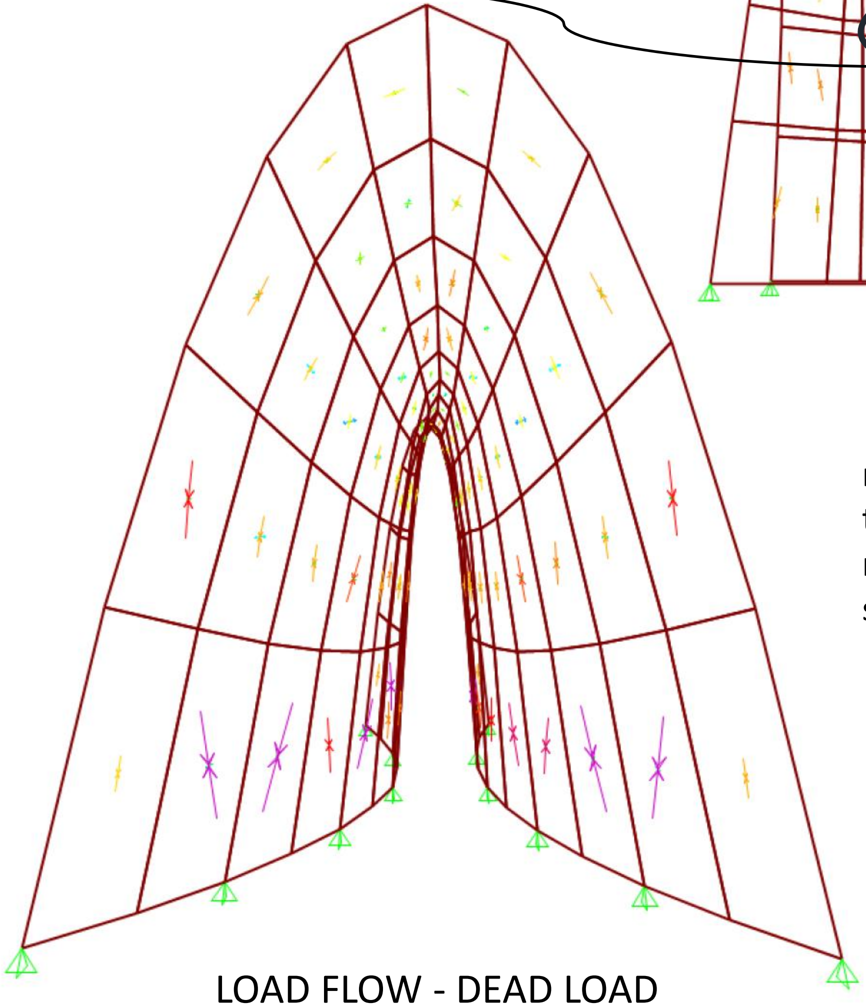


DEFLECTION UNDER LOAD FROM RIGHT

Load Flow

The load flow analysis of the arches shows how the dead load forces (Fmax) travel through the system. The analysis also shows this shape is not funicular; there is tension in the middle.

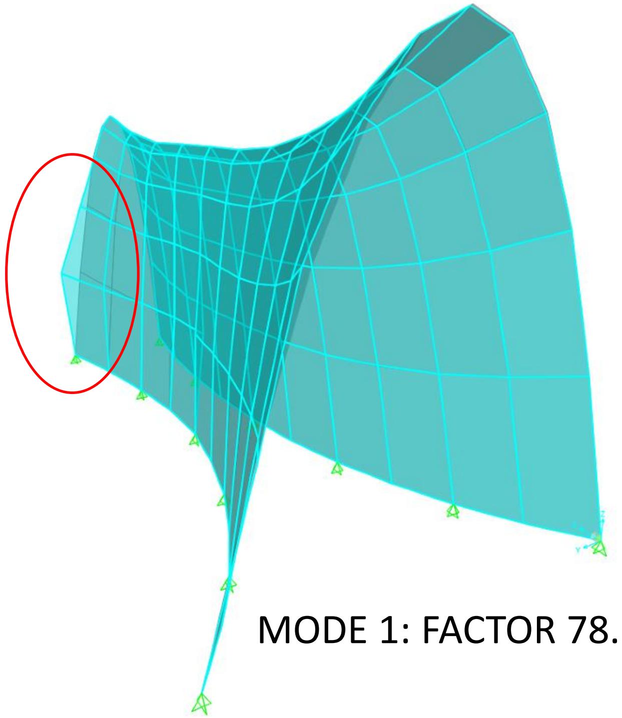




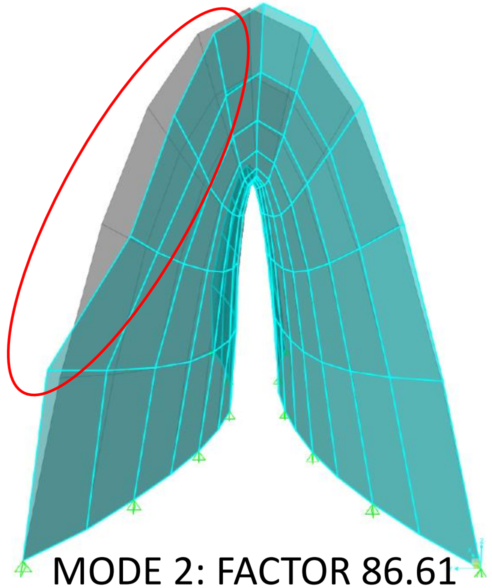
LOAD FLOW - DEAD LOAD

The arrows show that the forces along the ridge and in the middle are small, while those at the base are the largest. Because the forces in the middle are so small, adding an opening in this area should not compromise the strength of the shell.

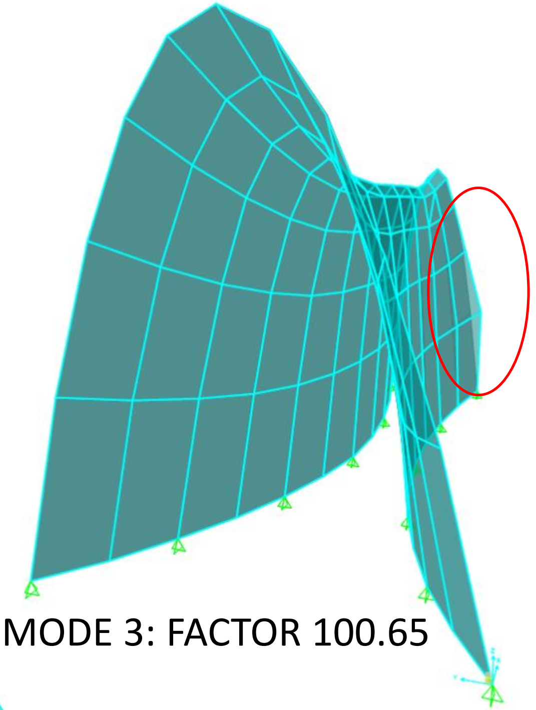
EARTH PAVILION/ARCH INVESTIGATION



MODE 1: FACTOR 78.74



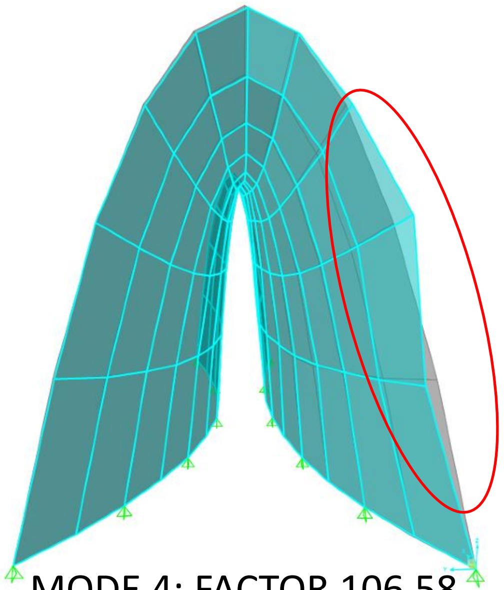
MODE 2: FACTOR 86.61



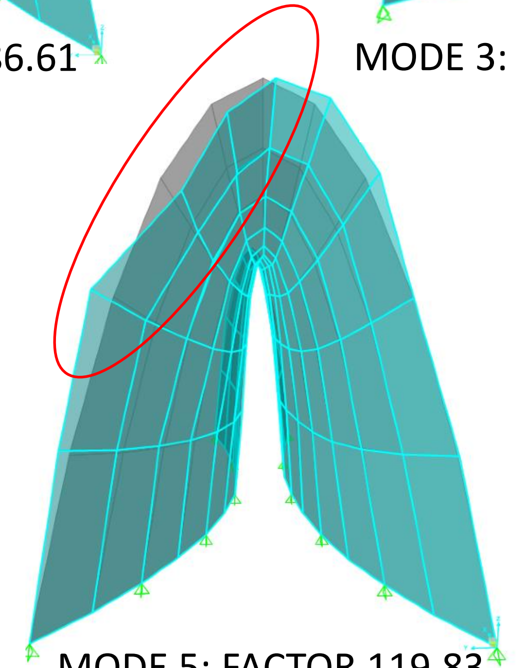
MODE 3: FACTOR 100.65

Buckling Analysis

The mode factor shown represents the factor that could be applied to the current loading before the corresponding failure mode would be reached. This number is a good representation of the strength of the shape. It would need to be loaded by almost 79 times its own weight before its first failure.



MODE 4: FACTOR 106.58

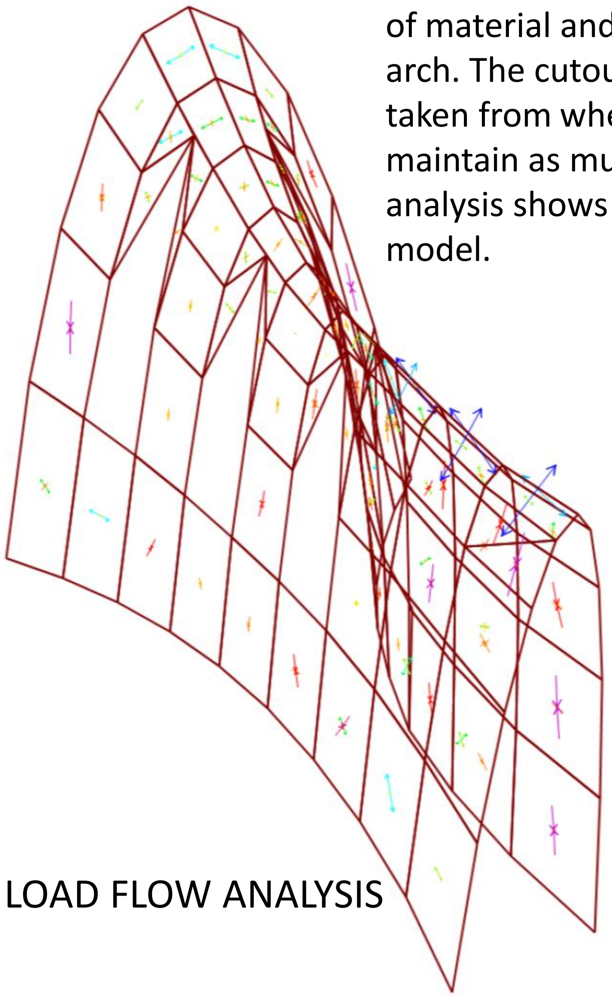


MODE 5: FACTOR 119.83

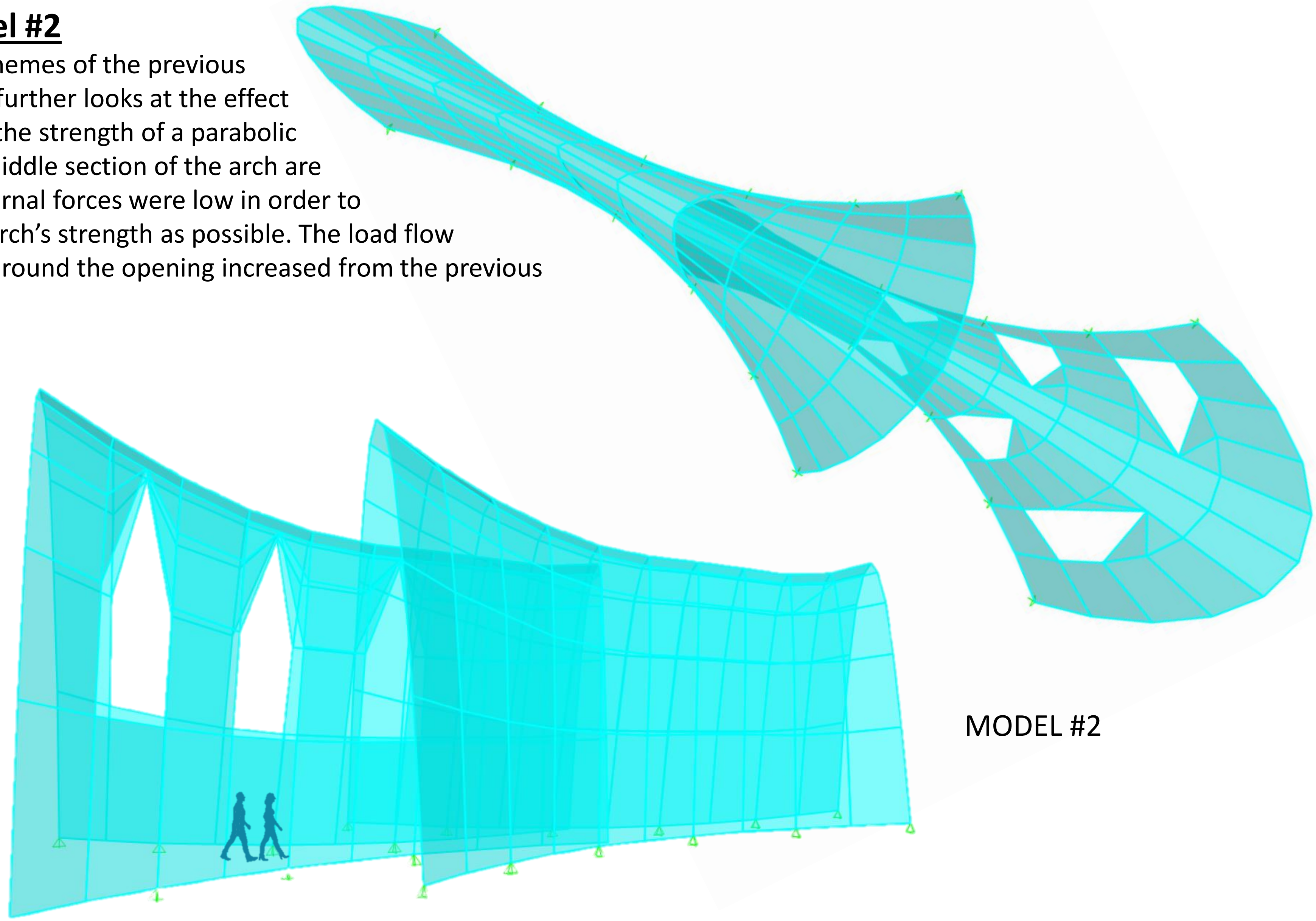
EARTH PAVILION/ARCH INVESTIGATION

Model #2

Expanding on the themes of the previous model, this investigation further looks at the effect of material and shape on the strength of a parabolic arch. The cutouts in the middle section of the arch are taken from where the internal forces were low in order to maintain as much of the arch's strength as possible. The load flow analysis shows the loads around the opening increased from the previous model.

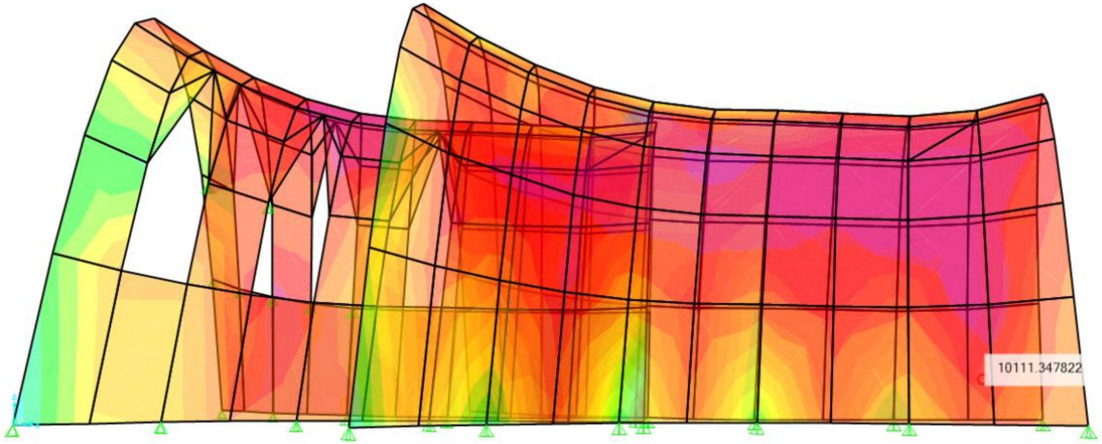


LOAD FLOW ANALYSIS

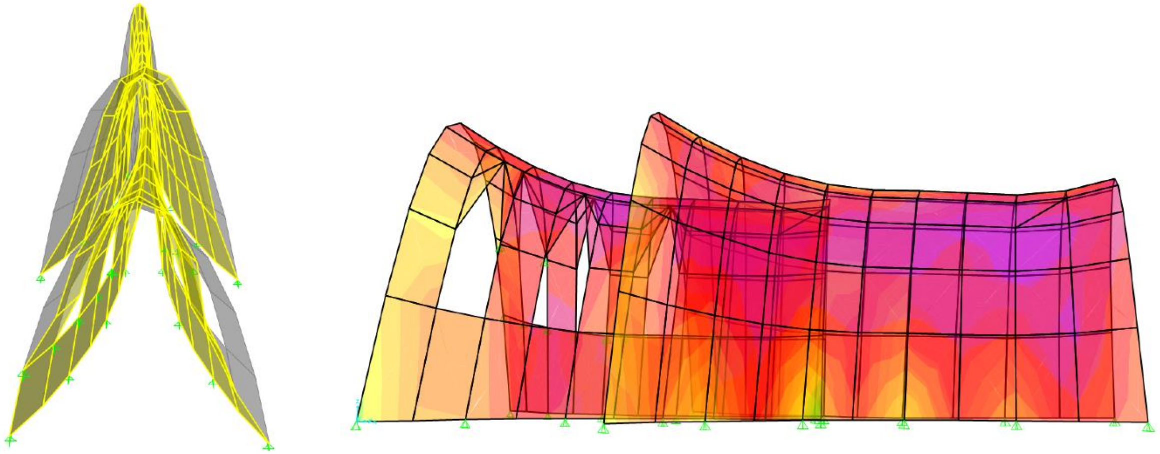


MODEL #2

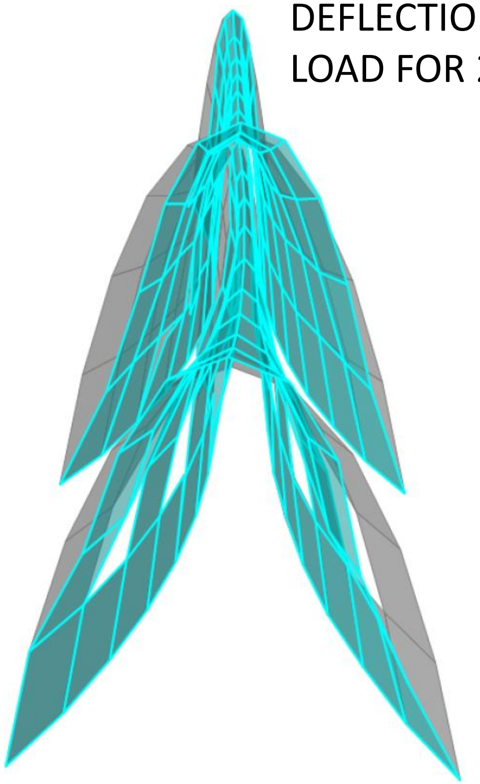
EARTH PAVILION/ARCH INVESTIGATION



DEFLECTION AND SHELL STRESS (SMV) UNDER DEAD LOAD FOR 2” THICK 4000 PSI CONCRETE SHELL

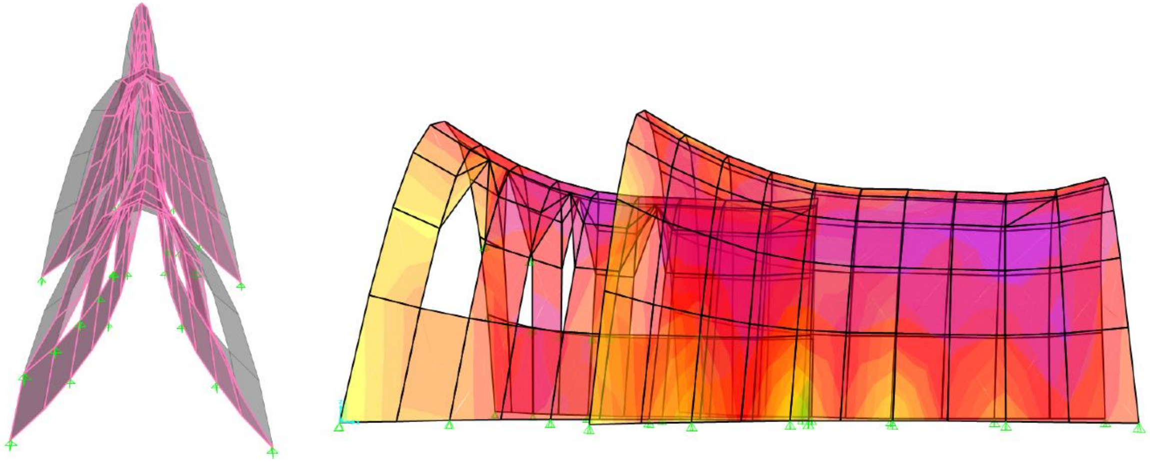


DEFLECTION AND SHELL STRESS (SMV) UNDER DEAD LOAD FOR 1.5” THICK 3000 PSI LW CONCRETE SHELL



Material Effect

The strength of the shell comes from its shape. This is demonstrated by changing the materials and thicknesses used and examining the results. After varying the concrete strength and thickness the shell’s stresses and deflection do not show significant change. This upholds the notion that shape is the most important factor in thin shell arches. This idea is used by Peter Rich & Michael Ramage in another of their thin shell structures, the Mapungubwe Interpretive Center in South Africa, where the tiles used to construct their arches are made by laypersons from the clay on site.

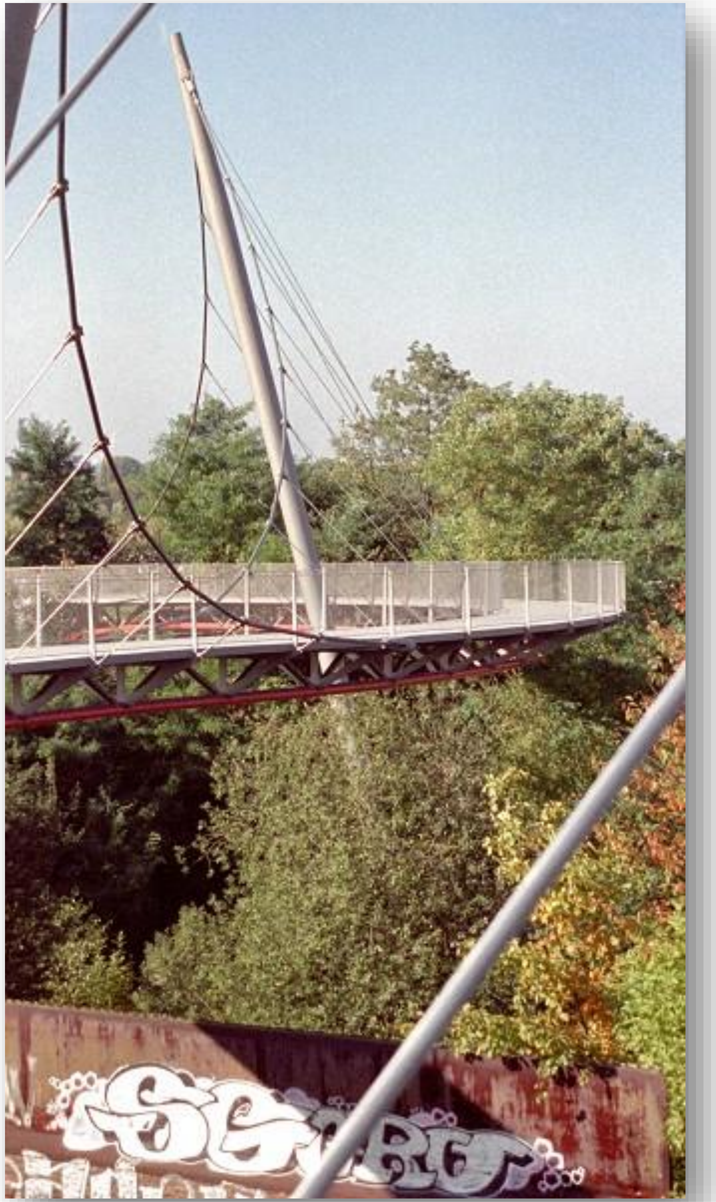


DEFLECTION AND SHELL STRESS (SMV) UNDER DEAD LOAD FOR 3” THICK 6000 PSI LW CONCRETE SHELL

ERZBAHNSCHWINGE BOCHUM INVESTIGATION

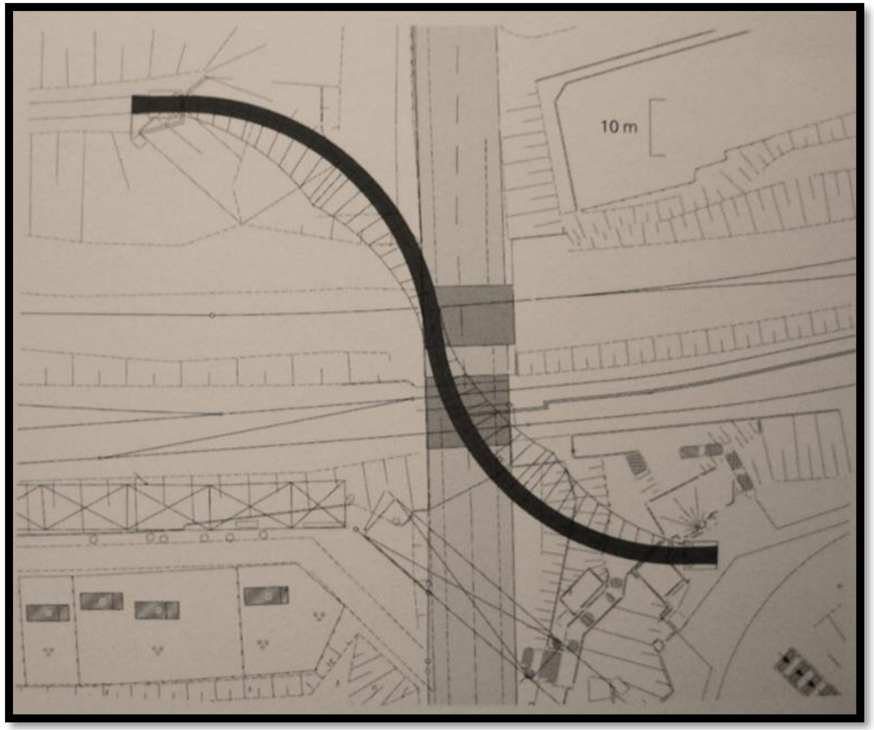
THIS INVESTIGATION LOOKS INTO THE STATICS OF SCHLAICH'S S-SHAPED PEDESTRIAN SUSPENSION BRIDGE IN BOCHUM, GERMANY, FOCUSING ON THE VERTICAL PYLONS AND THE STABILITY OF THE DECK.

ERZBAHNSCHWINGE BOCHUM INVESTIGATION



Structure Description

- Footbridge over Gahlensche Strasse in Bochum, Germany
- Built: 2003
- Structure: Mono-cable suspension bridge
- Materials: Reinforced Concrete, Steel, Cast Steel, Cables
- Length: approx. 420 ft
- Deck width: approx. 10 ft
- Pylon height: approx. 100 ft
- Pylon diameter: 2 ft



BRIDGE: PLAN VIEW

Project Description

This investigation models Jörg Schlaich’s Pedestrian Bridge in Bochum in order to gain some understanding of the statics of the structure.

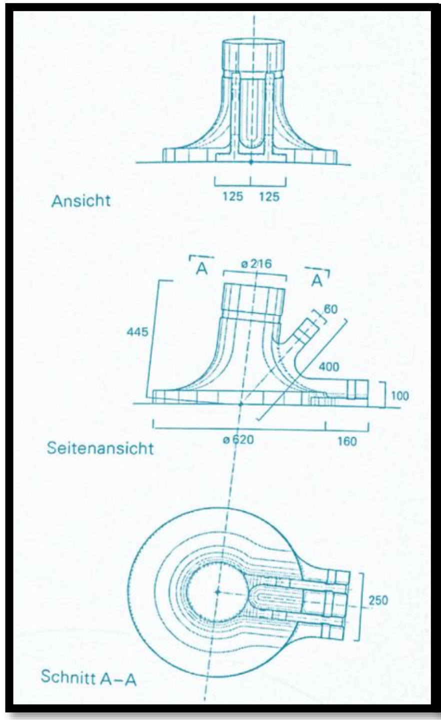
The primary points of interest for this investigation are the tilted, un-guyed pylons, as well as the stability of the cantilevered deck. The item that initially drew my interest to this structure was that it appears that the curves of the pathway defy gravity by cantilevering out from the cables supporting their inner radii.

ERZBAHNSCHWINGE BOCHUM INVESTIGATION

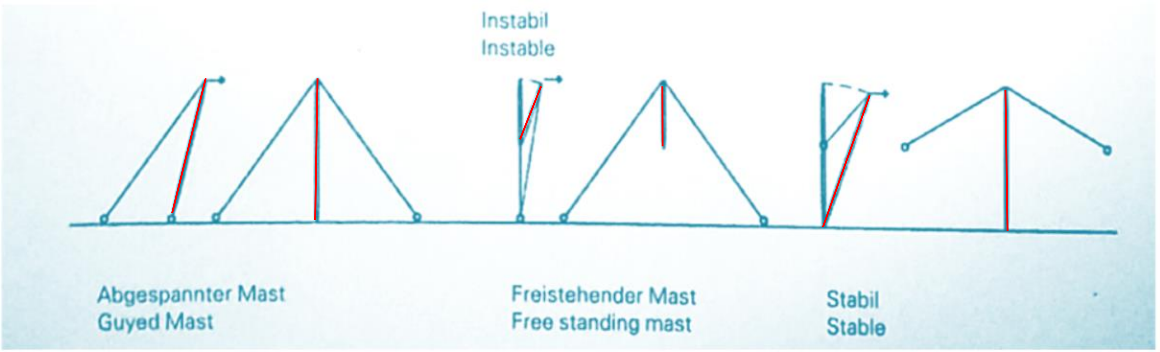
The Pylons

A quick study looks at the stability of the two pylons supporting the deck. Each one has a true pin connection at the base which allows its rotation. This was done in order to allow the tip of the pylon to find the necessary point of balance for each new live load.

It is because of this balance that the pylons do not experience moment or shear forces, nor do they require guy lines.



Although the pylons there are free standing, they are still stable. With his drawings, Schlaich explains how it's possible to use the main cables to stabilize the pylons because they are anchored above the base connection. This would not be the case if they were anchored level with or below the pylon base.



Unstable Pylon Unstable Pylon Stable Pylon

The tilt of the pylon away from the deck enables the system to balance. The pin connection at the base allows the pylon and deck to move, which is necessary in order to maintain balance when the pedestrian live load is constantly changing.

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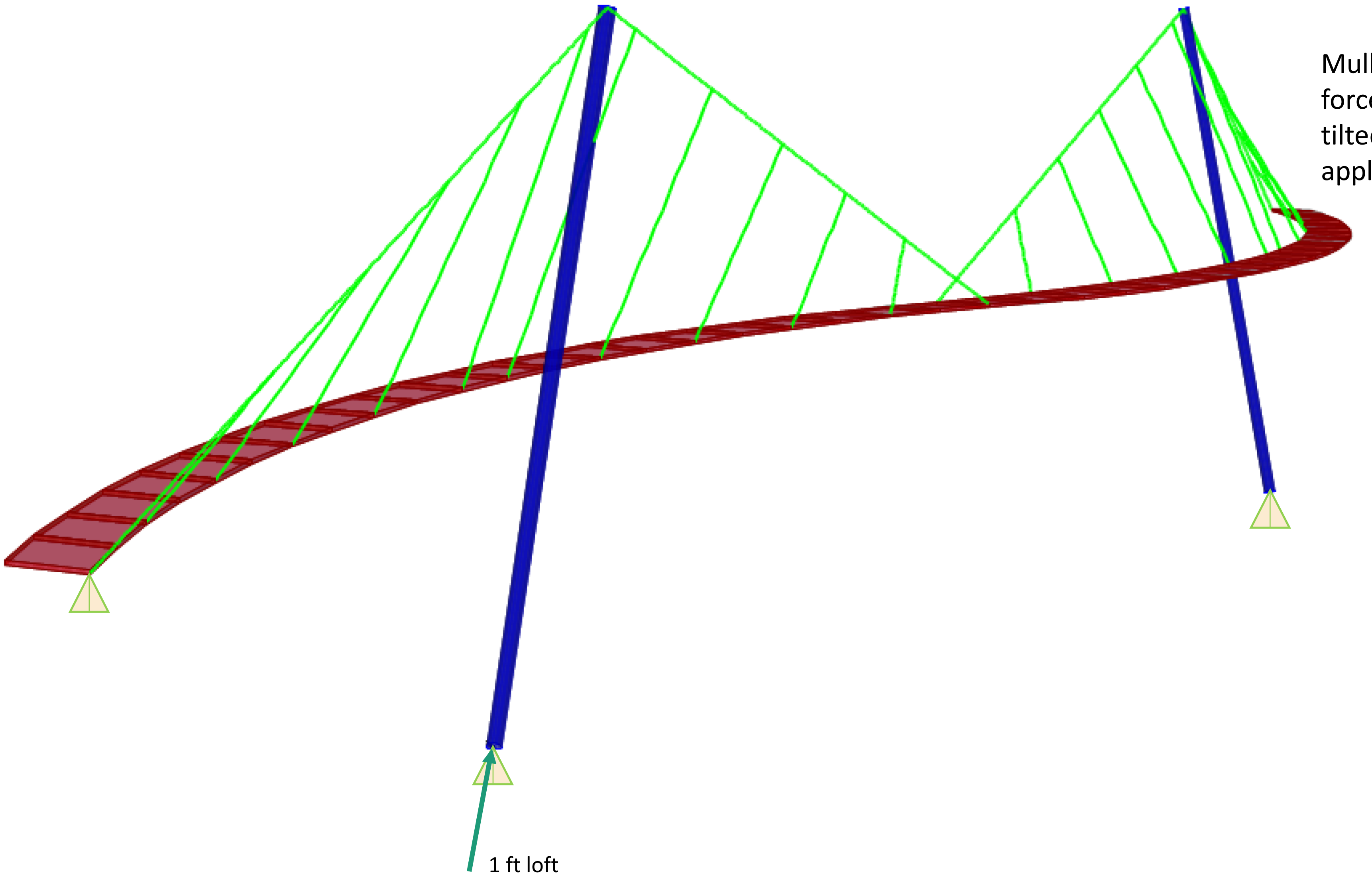
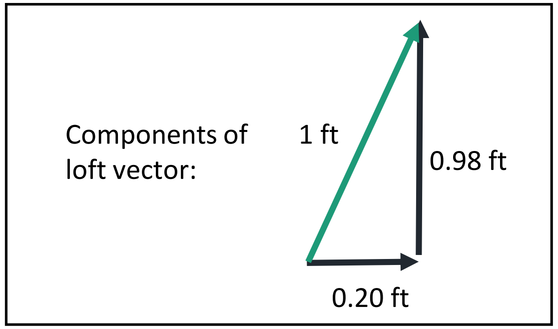


Column Axial Force

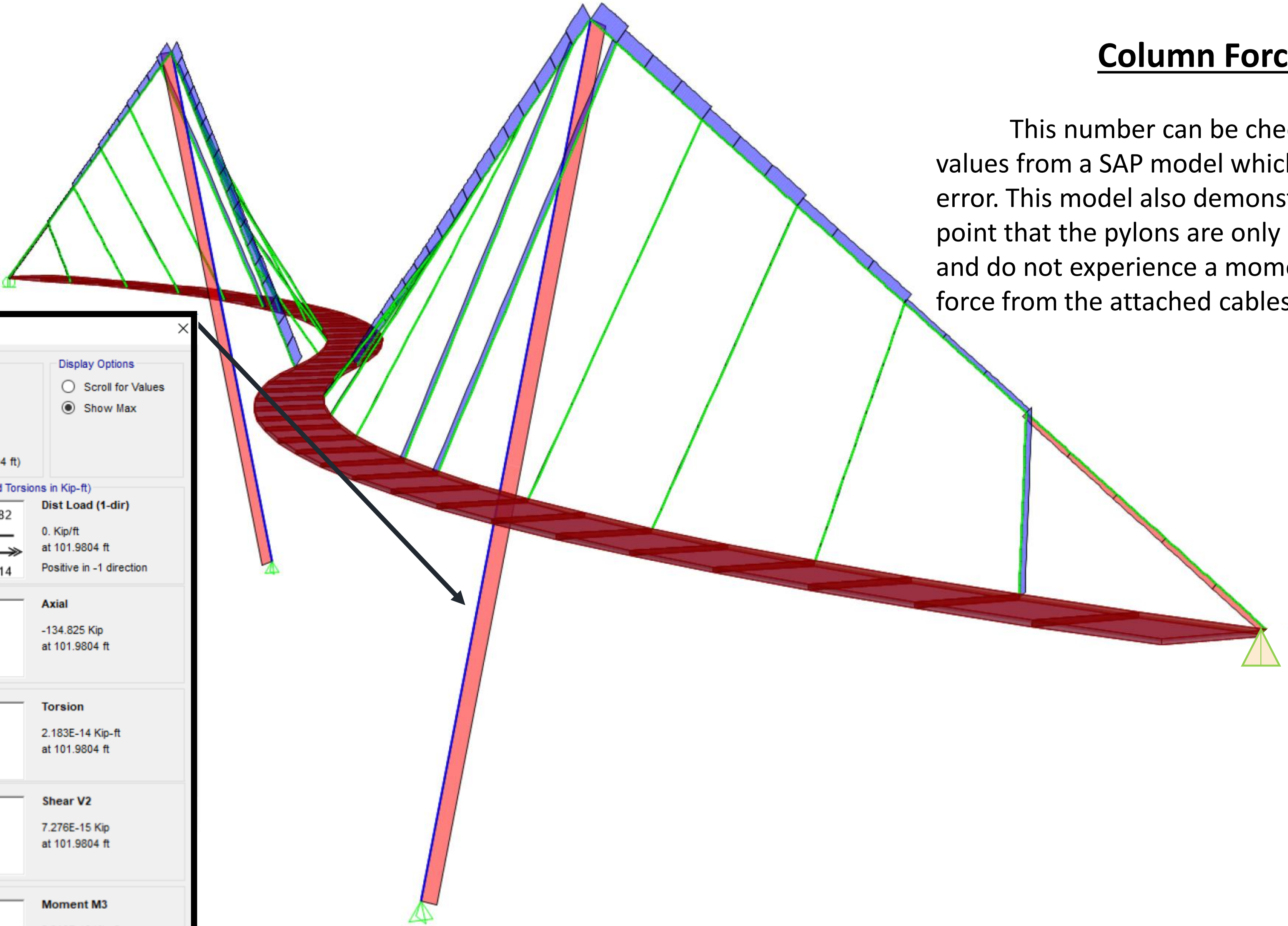
Because the structure is indeterminate, the Muller Breslau method is helpful in quickly finding the forces in the pylons. However, because the pylons are tilted, instead of applying the loft vertically it is applied along the column axis.

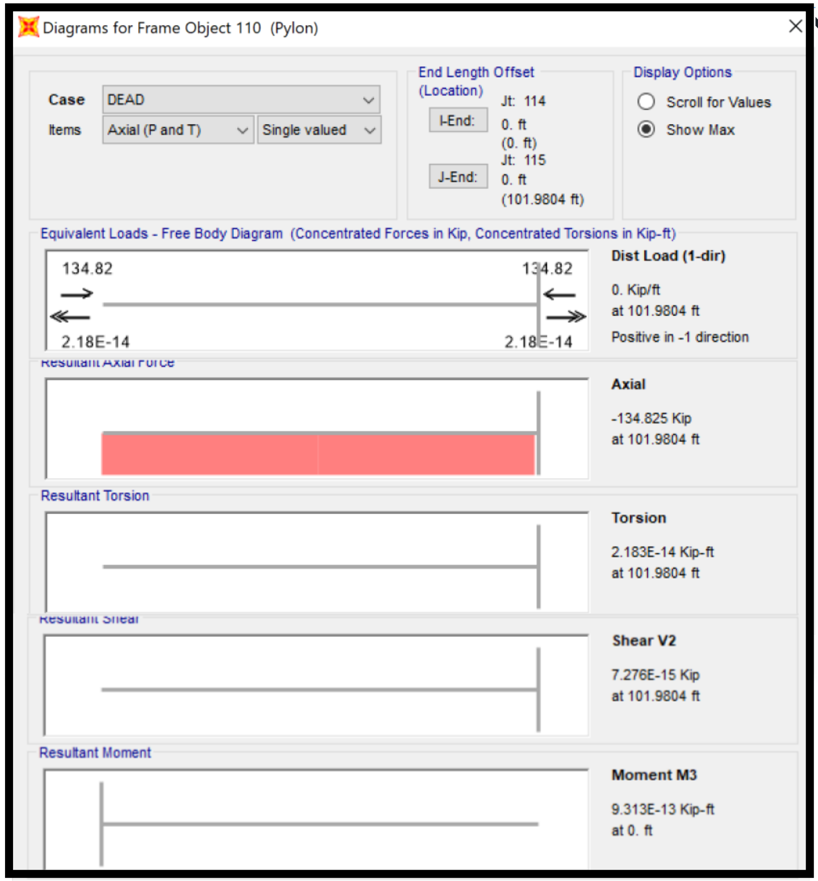


Calculations:

```
syms ft lb k

W = 115.72 * k;            % WEIGHT OF HALF OF
                             DECK (ONE CURVE)
LOFT = 0.25 * 4.46;        % LOFT AT Cm AFTER
                             COLUMN LOFT
AXIAL = (100/98)* LOFT*W

AXIAL =
131.66 * K

SAP: AXIAL = 134.82 * K
```

ERZBAHNSCHWINGE BOCHUM INVESTIGATION





Column Forces

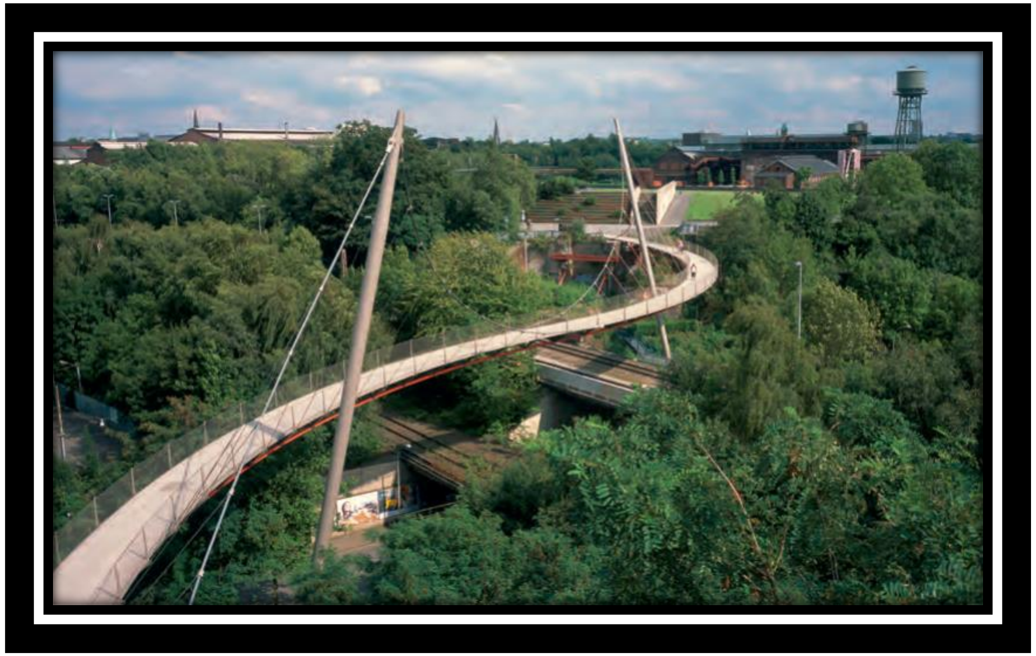
This number can be checked against values from a SAP model which show a 2.3% error. This model also demonstrates the earlier point that the pylons are only under axial load and do not experience a moment or shear force from the attached cables.

* This model neglects self weight of the pylon.

ERZBAHNSCHWINGE BOCHUM INVESTIGATION

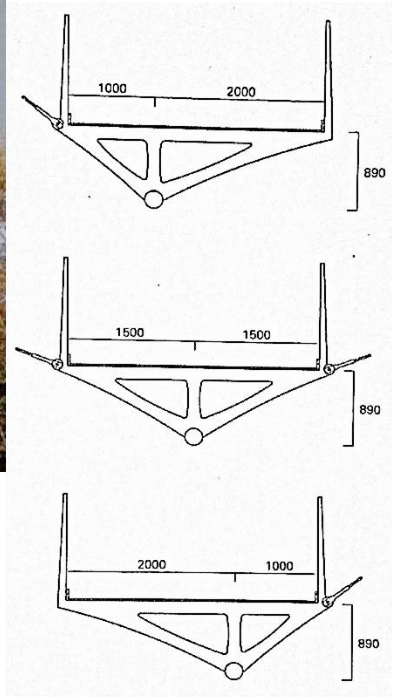
The Curved Deck

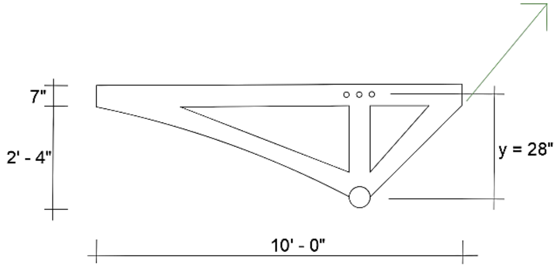
Schlaich refers to the curved deck as a circular ring girder. This investigation looks at the forces in these girders that result from having support only along the inner radius, and then explains how this problem is dealt with.



The Solution

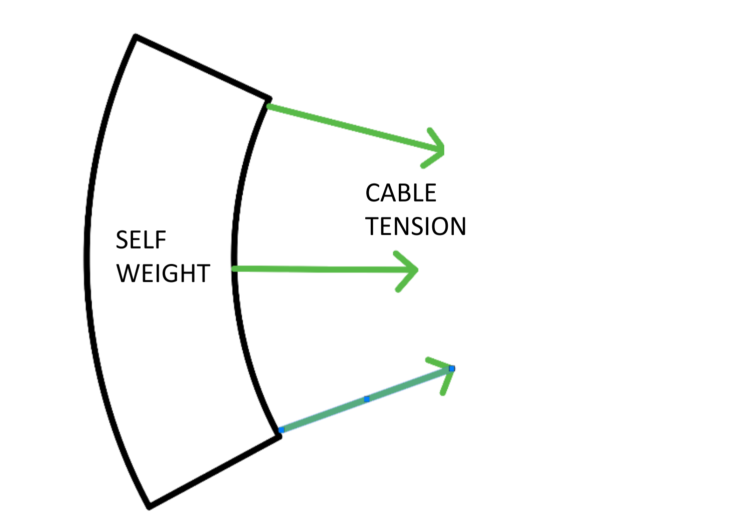
The bridge in Bochum uses the idea of circular ring girders to stabilize the deck. What this means is that the bridge combines a cable sheet in the deck, which creates tensile stress, with a compression chord under the deck in order to create the force couple needed.



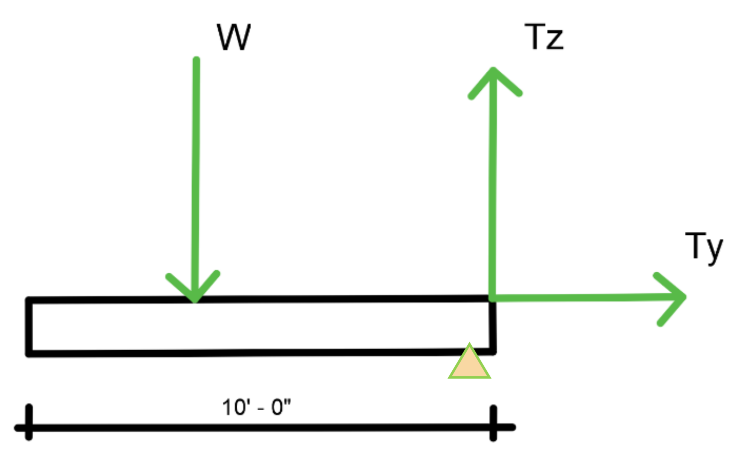


DECK SECTION VARIES ALONG BRIDGE SPAN

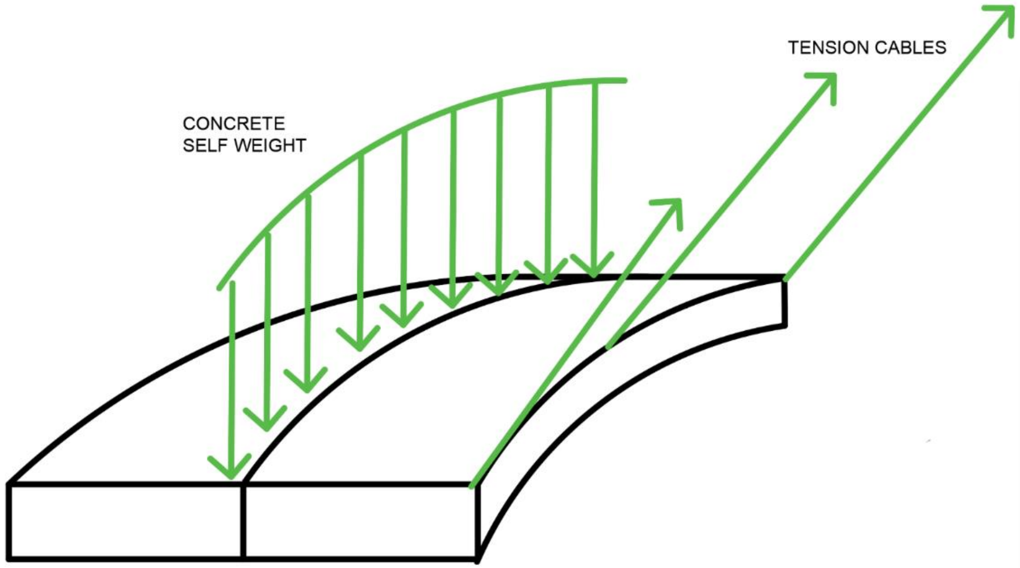
ERZBAHNSCHWINGE BOCHUM INVESTIGATION



PLAN VIEW OF DECK SECTION

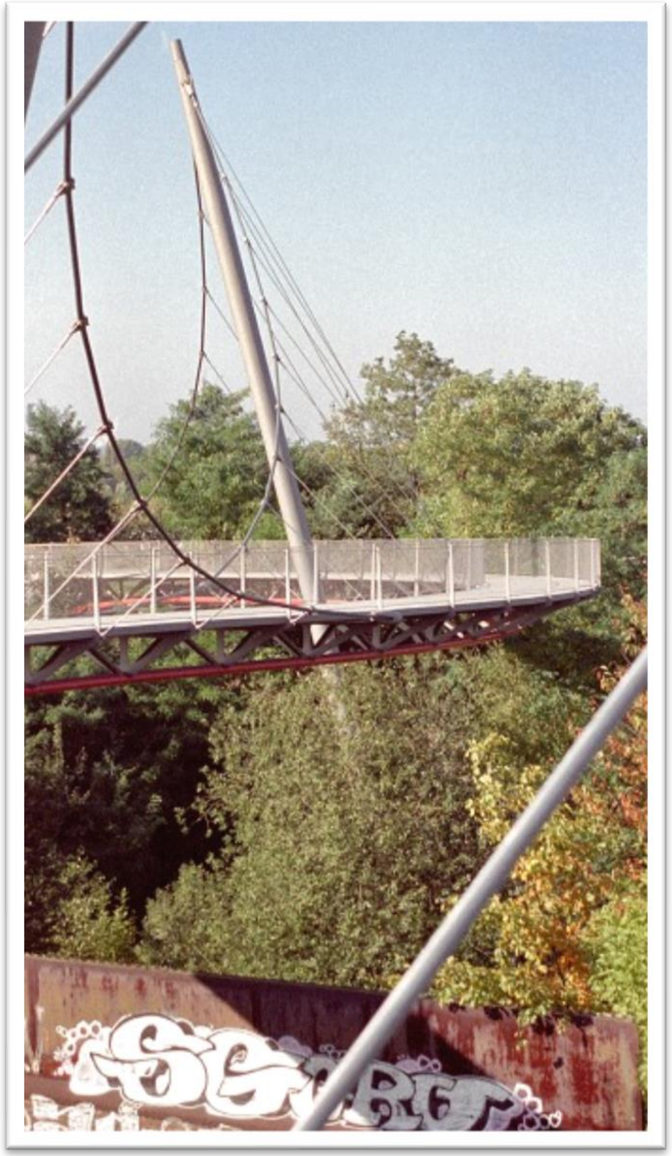


CANTILEVERING DECK WIDTH



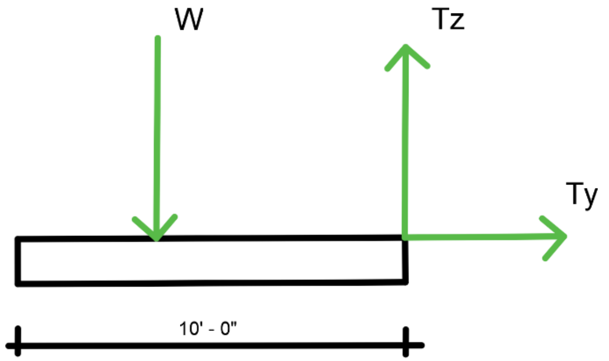
<u>Unstable Deck Cantilever</u>

When walking across the deck of the bridge the illusion is that the outer radius is simply cantilevering away from the tension cables. And because the pylons are designed to move visibly to adjust the balance point, pedestrians may assume there is some instability in the bridge. But while the cables are an important support system they are not the only one. The ring girder system is in place to counter the moment caused by the deck's self weight.

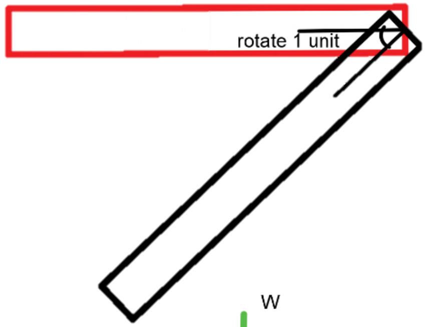


ERZBAHNSCHWINGE BOCHUM INVESTIGATION

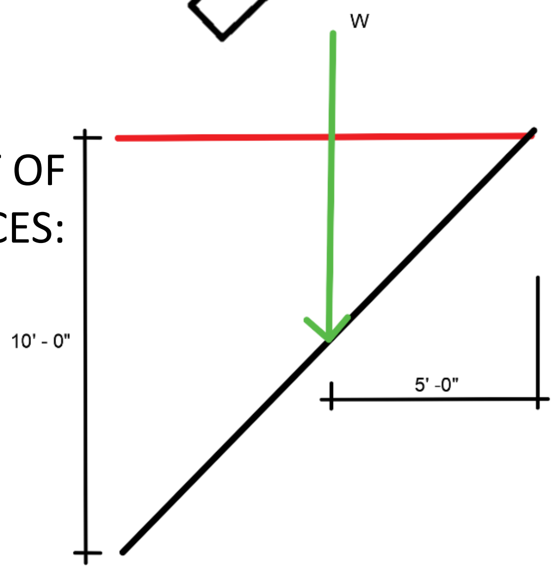
FBD:



ROTATE ABOUT SUPPORT:



LOFT OF FORCES:

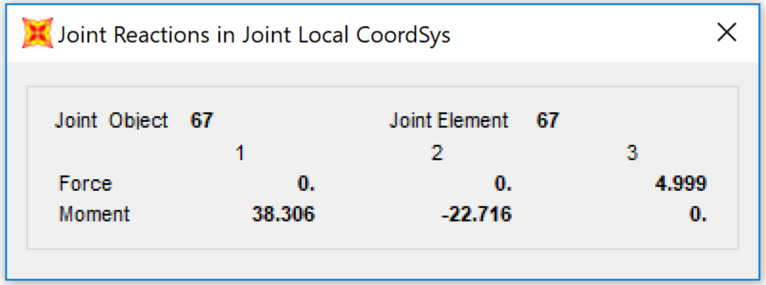


Muller Breslau-Deck Moment

The self weight of the deck would cause a rotation about the cable supports. The Muller Breslau Method can be used to provide the moment we need to counteract in order to prevent this rotation.

```
CALCULATIONS:
syms ft lb k

W = 5.22 * k;            % WEIGHT OF HALF OF
                          DECK (ONE CURVE)
LOFT = 5 * ft;          % LOFT AT Cm AFTER
                          ROTATION
M = LOFT*W

M =
26.1*ft*k

SAP: M = 22.7 ft*K
```

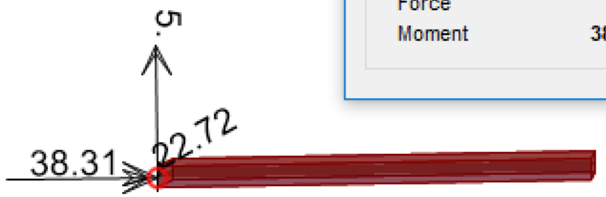
SAP Investigation:



Joint Reactions in Joint Local CoordSys

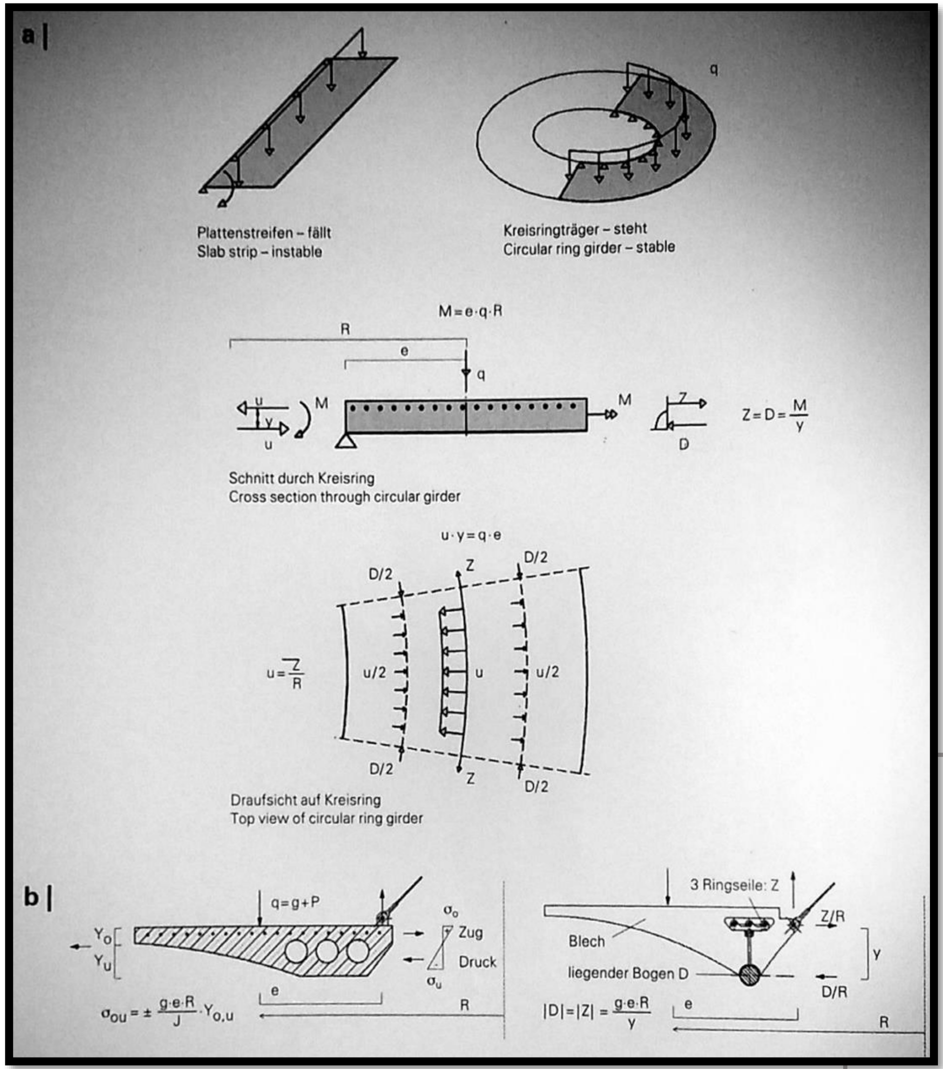
Joint Object 67 Joint Element 67

	1	2	3
Force	0.	0.	4.999
Moment	38.306	-22.716	0.

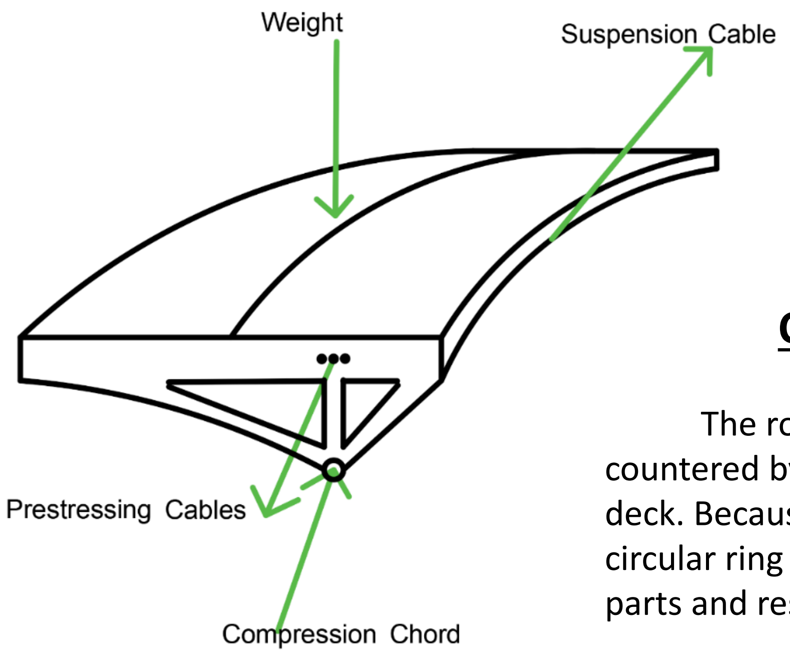


Units of: K, ft, F

ERZBAHNSCHWINGE BOCHUM INVESTIGATION



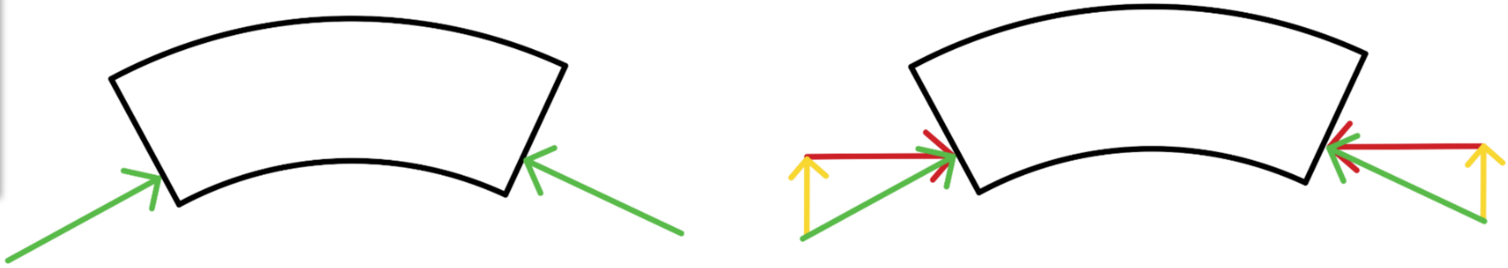
SCHLAICH'S RING GIRDER DRAWINGS



Circular Ring Girders

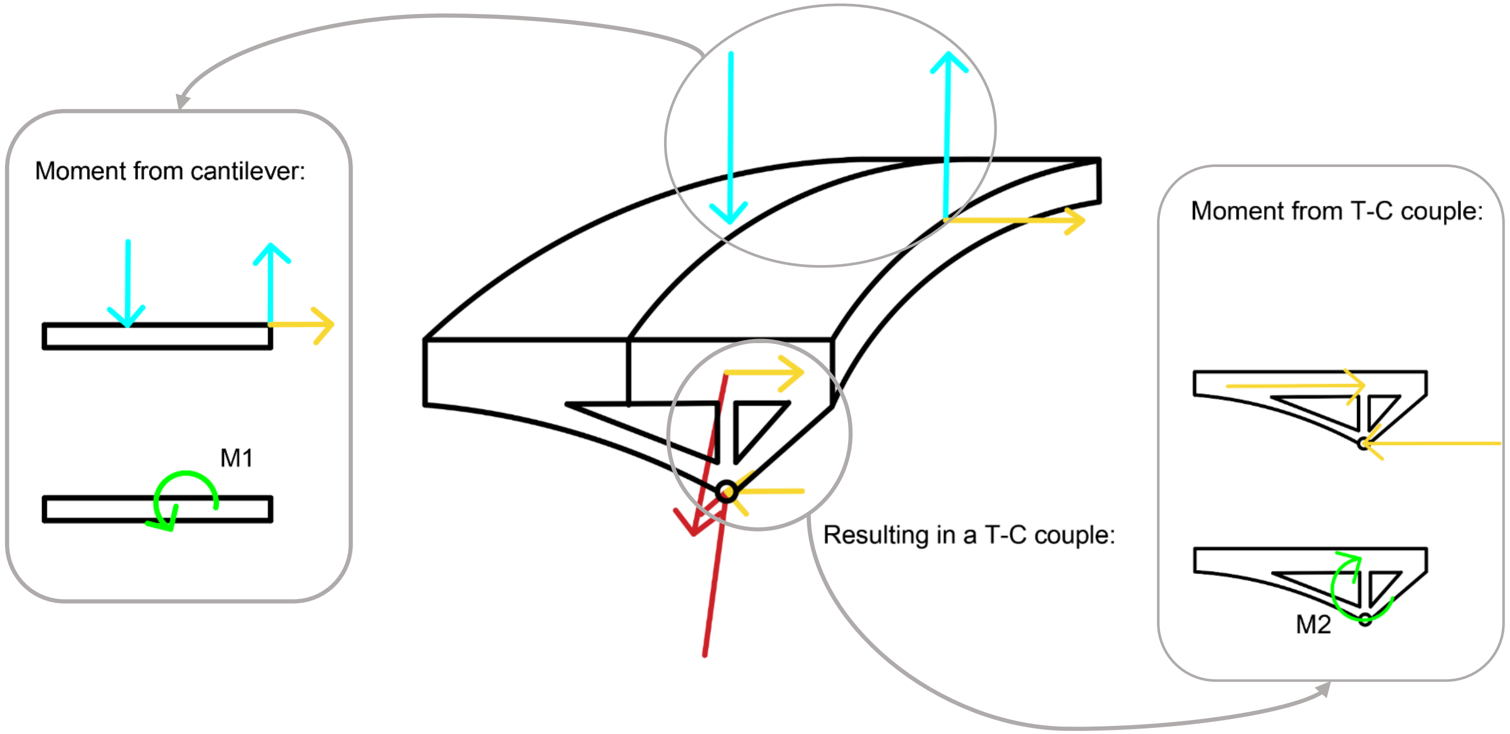
The rotation caused by self weight is countered by imposing forces into and below the deck. Because of their shape, the forces in circular ring girders break into their component parts and result in force couples.

PLAN VIEW OF DECK SECTION

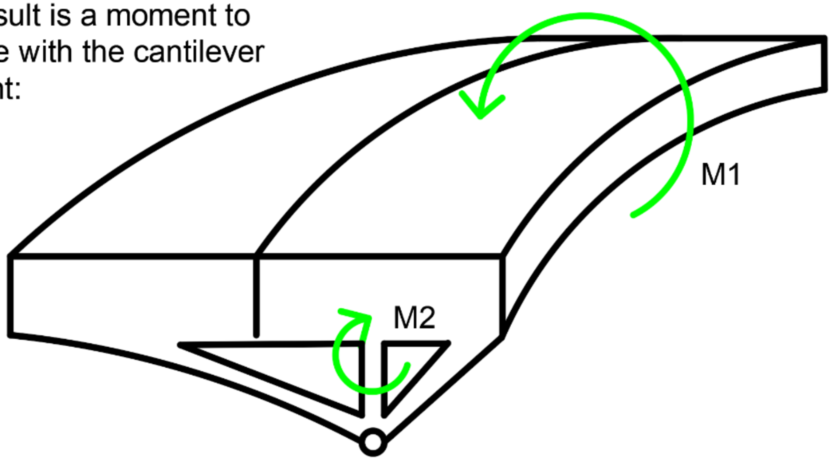


It is only because of the circular shape of the deck that the tension and compression forces can be broken up into the components necessary to create a force couple.

ERZBAHNSCHWINGE BOCHUM INVESTIGATION

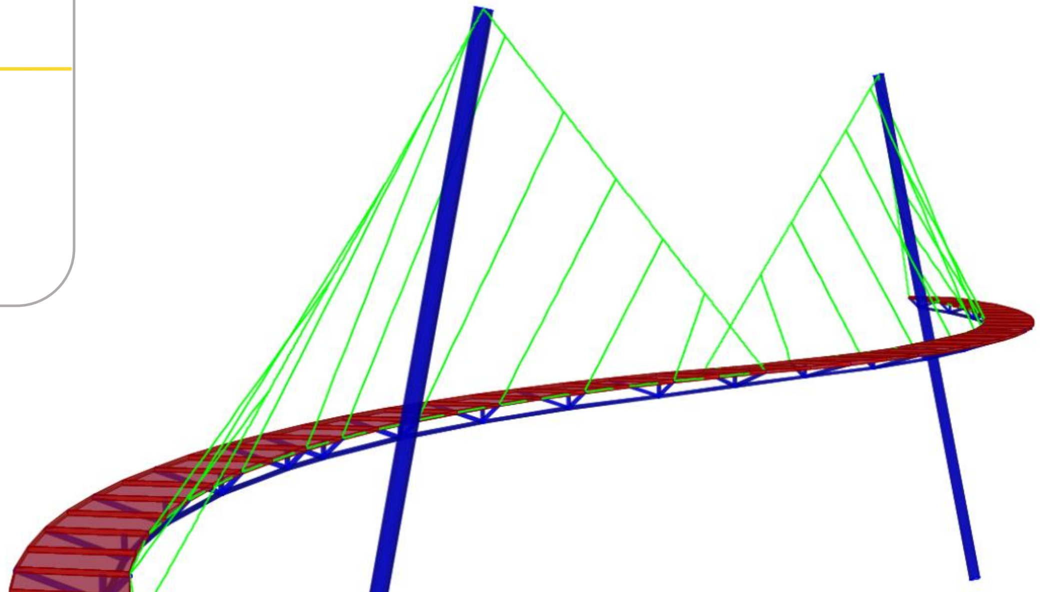


The result is a moment to balance with the cantilever moment:



Circular Ring Girders

When the forces from the tension and compression chords are broken into their component parts (in yellow) they form a moment to balance the rotation from self weight. However, this solution only works on decks or beams that curve. A straight deck would still require a single fixed support or a second pin connection.



By adding the pre-tensioning (along the inner radius) and the compression chord into the model, the stability of the deck is significantly increased.

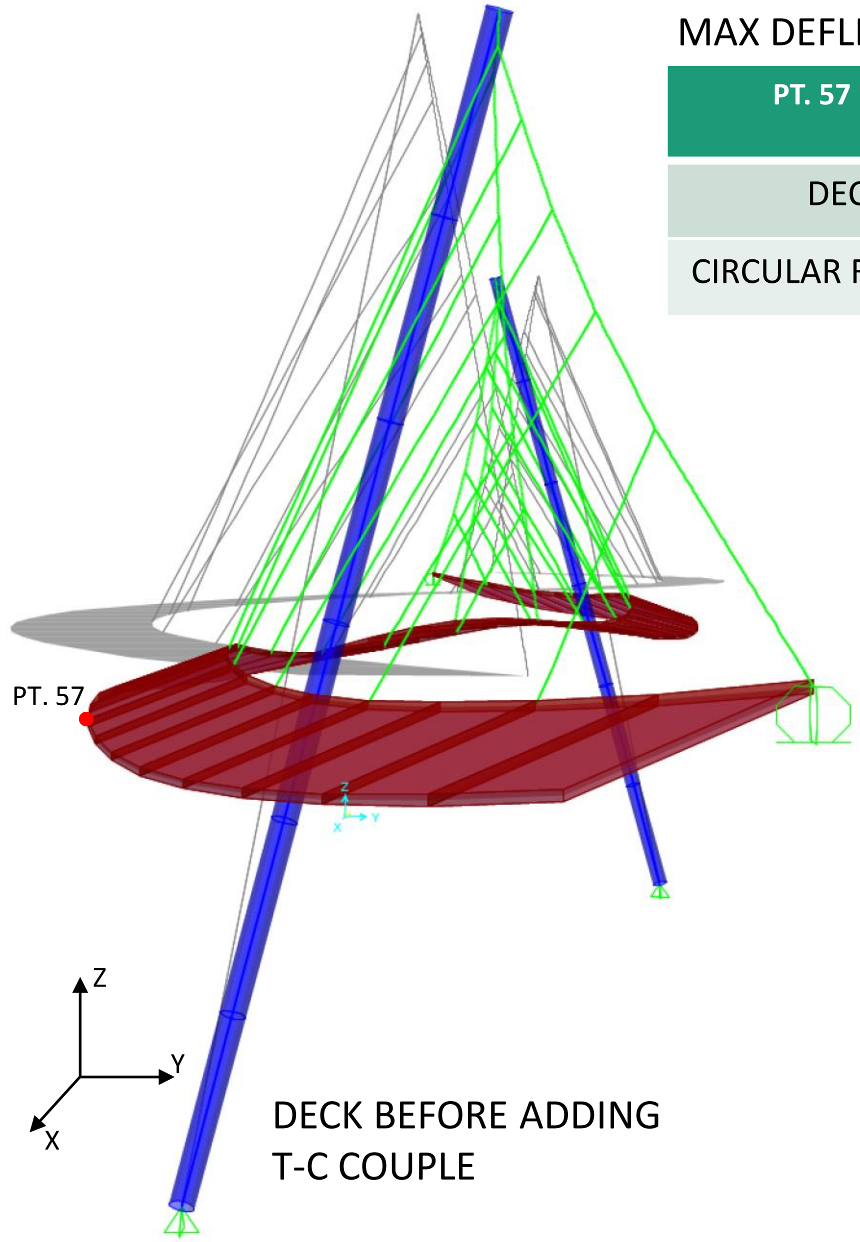
Pretension Cable : 6000 lbs

Compression Chord: 2” deformation

ERZBAHNSCHWINGE BOCHUM INVESTIGATION

MAX DEFLECTION RESULTS:

PT. 57 FOR:	X TRANSLATION (FT)	Y TRANSLATION (FT)	Z TRANSLATION (FT)
DECK	21.7	12.0	-12.2
CIRCULAR RING GDR	10.7	14.0	-1.7

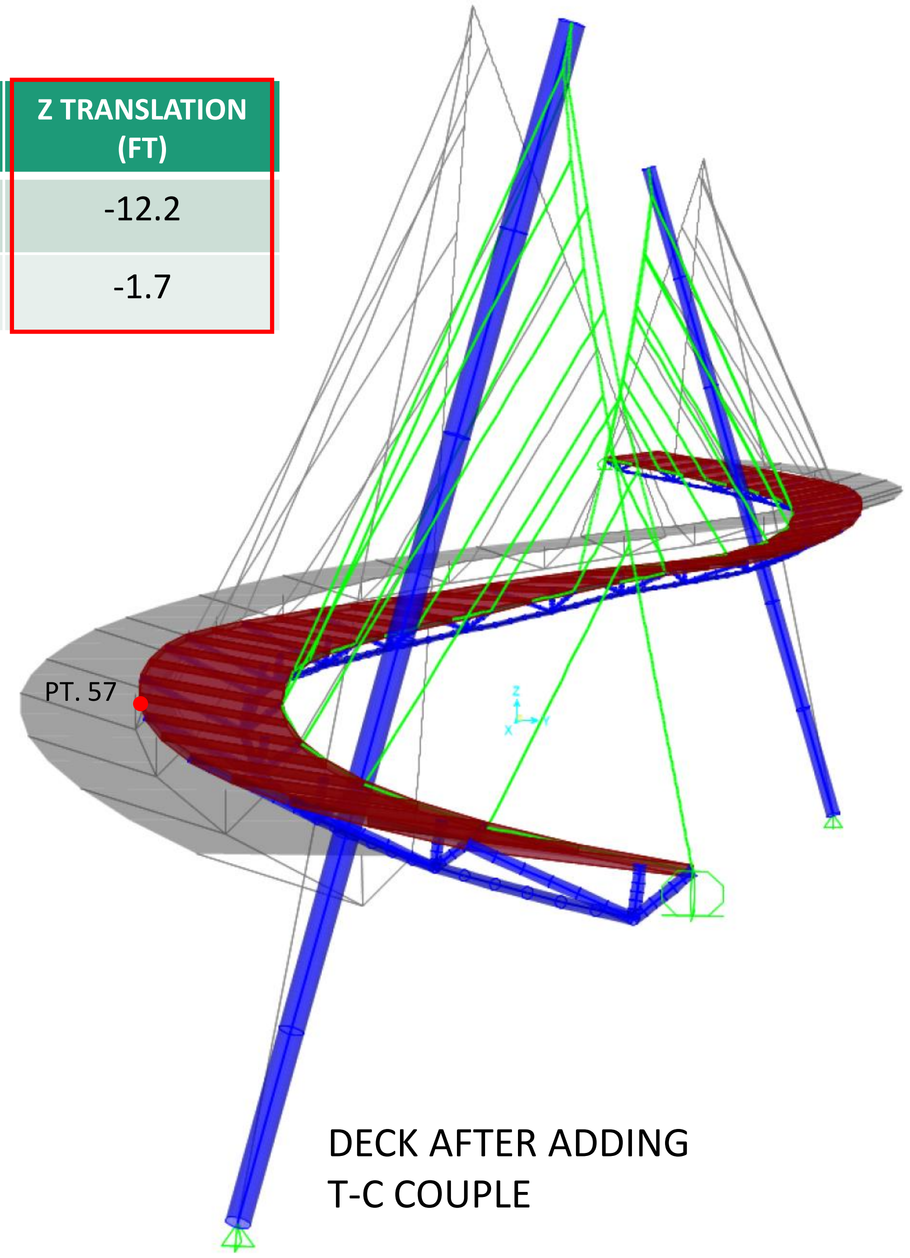


DECK BEFORE ADDING
T-C COUPLE

An Elegant Solution

By including the ideas of the circular ring girder into the SAP model the stability of the deck is significantly increased. The deck in the original model deflected over 12 ft in the z-direction, but after adding the tension and compression chords it only deflects 1.7 ft.

This bridge gracefully demonstrates the idea that the best structures synthesize form and function rather than attempting to make an architectural idea work simply using the brute strength of member size or material.



DECK AFTER ADDING
T-C COUPLE

PROJECT CONSIDERATIONS

Global Influence

The structures reviewed in this project take you all over the world. By studying the work of architects and engineers from different cultures and with different specialties, we can learn about the old and the new solutions to any structural quandary, we can design more innovative and economic structures, and, by integrating these ideas, we can widen people's worldviews.

Social Impact

Beautiful and structurally rational architecture, like the Bochum Suspension Bridge and the Earth Pavilion, can inspire people and make them question the world around them. Projects that are both aesthetic and efficient demonstrate the endless potential of architecture and motivate people to explore the possibilities.

Cultural Impact

These innovative designs bring travelers from all over the world who then take the ideas back with them. This encourages efficient design in other areas.

Environmental Considerations

There are many environmental concerns that affect the design of a building. Foundation excavation and transportation of materials can have a significant impact on the environment. But the largest environmental concern comes from the process of creating the materials. Forming either steel or concrete members creates significant carbon emissions, so knowing how to find their optimum shape and orientation is important. All of these projects can be used to either study or design an efficient shape.

Economic Considerations

The efficiency of a design is also important in the economic considerations. Using fewer materials means lower materials cost, lower transportation cost, and lower labor cost. But these innovative designs do not just save money; they also attract travelers, which brings in revenue as well as other economic benefits for the surrounding areas.

Constructability Factors

Some of the structures studied are not at all labor intensive, like the Earth Pavilion which could be constructed from local materials and by amateur builders. But some of the projects require more planning and care. The prestressed suspension bridge would be constructed on the ground then lifted into place. And the Caracas stadium uses enormous cantilevered members that would require special transportation and installation.

CITATIONS:

Schlaich, Jorg and Bergermann, Rudolf. *Light Structures.* Prestel. 2004. Print

And a special thank you to Professors Ed Saliklis and Mike Botwin for their time and their much appreciated knowledge.