ORBITAL CONSTELLATION DESIGN AND ANALYSIS USING SPHERICAL TRIGONOMETRY AND GENETIC ALGORITHMS: A MISSION LEVEL DESIGN TOOL FOR SINGLE POINT COVERAGE ON ANY PLANET

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ABSTRACT

Orbital Constellation Design and Analysis Using Spherical Trigonometry and Genetic Algorithms: A Mission Level Design Tool for Single Point Coverage on Any Planet

Joseph Ratcliffe Gagliano

Recent interest surrounding large scale satellite constellations has increased analysis efforts to create the most efficient designs. Multiple studies have successfully optimized constellation patterns using equations of motion propagation methods and genetic algorithms to arrive at optimal solutions. However, these approaches are computationally expensive for large scale constellations, making them impractical for quick iterative design analysis. Therefore, a minimalist algorithm and efficient computational method could be used to improve solution times. This thesis will provide a tool for single target constellation optimization using spherical trigonometry propagation, and an evolutionary genetic algorithm based on a multi-objective optimization function. Each constellation will be evaluated on a normalized fitness scale to determine optimization. The performance objective functions are based on average coverage time, average revisits, and a minimized number of satellites. To adhere to a wider audience, this design tool was written using traditional Matlab, and does not require any additional toolboxes.

To create an efficient design tool, spherical trigonometry propagation will be utilized to evaluate constellations for both coverage time and revisits over a single target. This approach was chosen to avoid solving complex ordinary differential equations for each satellite over a long period of time. By converting the satellite and planetary target into vectors of latitude and longitude in a common celestial sphere (i.e. ECI), the angle A can be calculated between each set of vectors in three-dimensional space. A comparison of angle A against a maximum view angle, A_{max} , controlled by the elevation angle of the target and the satellite's altitude, will determine coverage time and number of revisits during a single orbital period.

Traditional constellations are defined by an altitude (a), inclination (I), and Walker Delta Pattern notation: *T/P/F*. Where T represents the number of satellites, P is the number of orbital planes, and F indirectly defines the number of adjacent planes with satellite offsets. Assuming circular orbits, these five parameters outline any possible constellation design. The optimization algorithm will use these parameters as evolutionary traits to iterate through the solutions space. This process will pass down the best traits from one generation to the next, slowly evolving and converging the population towards an optimal solution. Utilizing tournament style selection, multi-parent recombination, and mutation techniques, each generation of children will improve on the last by evaluating the three performance objectives listed. The evolutionary algorithm will iterate through 100 generations (G) with a population (n) of 100.

The results of this study explore optimal constellation designs for seven targets evenly spaced from 0° to 90° latitude on Earth, Mars and Jupiter. Each test case reports the top ten constellations found based on optimal fitness. Scatterplots of the constellation design solution

space and the multi-objective fitness function breakdown are provided to showcase convergence of the evolutionary genetic algorithm. The results highlight the ratio between constellation altitude and planetary radius as the most influential aspects for achieving optimal constellations due to the increased field of view ratio achievable on smaller planetary bodies. The multi-objective fitness function however, influences constellation design the most because it is the main optimization driver. All future constellation optimization problems should critically determine the best multi-objective fitness function needed for a specific study or mission.

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"To give anything less than your best is to sacrifice the gift"
- Steve Prefontaine -

47.335598 -122.619563	47.292412 -122.619608	35.297441 -120.661365
42.313513 -122.878368	37.421616 -122.109241	35.303146 -120.664616

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CHAPTER

1. INTRODUCTION

Satellite constellations are space based systems of two or more spacecraft orbiting one celestial body in a designed pattern or scheme. Utilizing multiple satellites provides more frequent passes per day, and can provide greater or continuous coverage over a planet's surface. A few notable constellation types are; Walker Delta Patterns (WDP), elliptical, streets of coverage, satellite trains, and geosynchronous constellations, which will all be defined in more depth. The most common among them are WDP's, which consist of circular orbits evenly spaced around a celestial body.

Constellations have been in use since the dawn of spaceflight with the inaugural system TRANSIT. First used for global positioning by the United States Navy in 1964, this constellation system consisted of only five satellites in polar orbits, relaying global positioning of military units around the globe roughly once per hour [1]. Since then, TRANSIT has been eclipsed by multiple military programs, one of which is widely known as the Global Positioning System (GPS), which provides millions of people with instant access to their current location on Earth at any time. This constellation, while small, has become essential to human life and could be considered the most important constellation to date. It's comprised of twenty-four satellites which are evenly divided into six equally spaced orbital planes creating a WDP, providing constant coverage around Earth [2].

1.1. Problem Statement

Over the last twenty years satellites have adopted smaller, more advanced components leading to size and mass reductions. To compliment this, launch vehicles have become more accommodating by allowing multiple payloads on a single launch, able to carry tens or even hundreds of payloads into a single orbit. This allows mission designers to expand their solutions to include larger satellite constellations. However, the design and analysis of hundreds or even thousands of satellites can slow down the mission design process due to extra computational time needed for trade evaluations. So, to combat this issue, the development of a simple, efficient constellation design tool is needed. This thesis provides the background, methodology, and implementation of a constellation optimization design tool using spherical trigonometry, and an evolutionary genetic algorithm based on a multi-objective function.

This work is a spin-off of Christopher Hind's thesis, "A Pareto-Frontier Analysis of Performance Trends for Small Regional Coverage LEO Constellation Systems" [3]. Aiming to extend the constellation size and propagation limitations in Hind's work, while retaining the evolutionary genetic algorithm aspects. To diversify this work further, the results and discussion section will explore the performance of constellations on multiple planetary bodies.

1.2. Background

A comprehensive overview of constellation types will be covered with a list of example missions to date. Afterwards, Walker Delta Patterns will be introduced and explained in detail followed by a brief history of genetic algorithms. Finally, a review of previous works and their findings will provide a basis of constellation design knowledge before diving into the methodology.

1.2.1. Constellation Classifications

A constellation is a set of satellites distributed around a central body working together to accomplish a common goal. There are an infinite number of constellation solutions to a given mission, whether it be global navigation, constant communication, or real-time weather tracking, so selecting a final design relies on the pros and cons of each system's performance. Constellations can be organized in various ways based on orbital parameters such as altitude, inclination, eccentricity and pattern formation. [4]

Most constellations can be classified by one of the following categories:

- Geosynchronous constellations
- Streets-of-Coverage constellations
- Walker-Delta pattern constellations
- Elliptical Orbit pattern constellations
- Other constellations

Satellites in geosynchronous constellations are located approximately 35,786 km above the Earth's equator experiencing an orbital period of twenty-four hours, matching the rotation of a single point on the surface. These systems are simple to analyze, since they do not move relative to Earth's orientation, and they can sufficiently cover surfaces of interest with small constellations of two or three satellites.

Streets-of-Coverage constellations use satellites oriented in multiple polar orbit planes to evenly cover the equator's various longitude bins. This setup is advantageous when coverage of the entire planet is required, however, these constellations require satellites to spend most of their life span over the poles, and can end up in excessive coverage scenarios [5]. Figure 1.5 shows how each polar orbit accounts for a certain longitude bin around the constellations host planet.

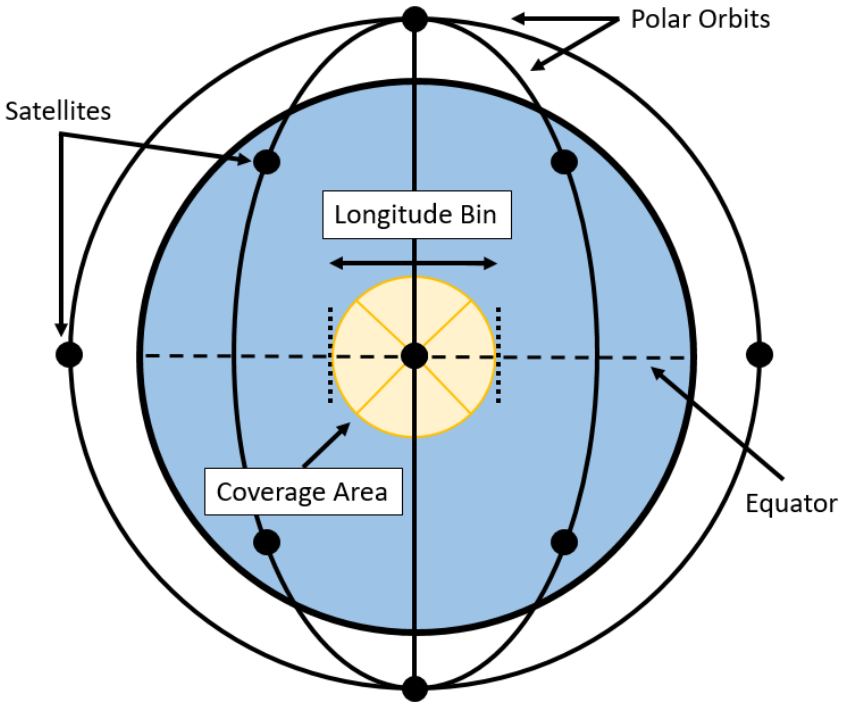

Figure 1.1 *STREETS-OF-COVERAGE AND LONGITUDE BIN DIVISION*

Walker Delta Pattern (WDP) constellations consist of satellites with circular orbits, along with a set altitude and inclination. These constellations rely heavily on patterns to accomplish coverage and revisit goals. WDP constellation design parameters will be described in more detail in the next section.

Elliptical orbit pattern constellations are systems utilizing highly eccentric orbiters. Historically most of these constellations have been made up of Russian communications satellites in Molniya orbits. These orbital planes are highly inclined allowing for the satellite's apogee to sit above high latitude sites for long periods of time. Due to varying altitudes, elliptical orbiters are affected more severely by perturbations such as J2, drag, and SRP.

Other constellations are mixtures of the aforementioned categories, or constellation systems that follow no noticeable pattern. For example, the proposed NISAR mission in 2020 could utilize a constellation of twelve satellites clustered together on the same orbital trajectory. The organization of multiple satellites orbiting in unison create a train of observation units that cover the same path or coverage area within minutes of one another, allowing for rapid-revisits. These tight flying constellations can reduce launch vehicle costs by using rideshare opportunities on launch vehicles. [6]

1.2.2. Walker Delta Patterns

In 1970, J.G. Walker of the Royal Aircraft Establishment published a technical study exploring how many satellites would be needed to continuously cover any geographical point on the surface of the Earth. Walker's work provided various solutions to the global coverage problem by arranging satellites evenly around a celestial body in different patterns and orbital spacing. In doing so, Walker developed a reference nomenclature to track each scenario variation.

Walker Delta Patterns (WDP) are traditionally defined by three parameters in the following syntax, *T/P/F*. Where *T* indicates the total number of satellites in the entire constellation. Then *P* defines the number of equally spaced orbital planes around 360° of radial space with respect to a defined reference plane (i.e. Earth's equator). The total number of satellites, *T*, must be evenly divided to provide *S* number of satellites per orbital plane, $T = S*P$. Lastly, *F* is an integer that infers the phase angle offset of satellites found in neighboring orbital planes. *F* can be any value between zero and *P*-1, giving a phase angle of $\gamma = F*(360°/T)$, which again defines the ascending node offset in neighboring orbital planes. [7]

Each satellite in a WDP maintains a circular orbit ($e = 0$) along with a set altitude (a) and inclination (I) for each constellation. In terms of classical orbital elements, all satellites in a WDP are comprised of the exact same elements, except for the RAAN, Ω , and True Anomaly, θ. These two unique parameters define the orbital plane orientation, and the satellite spacing around each plane's orbit. With such similar orbits, each unit in a constellation undergoes approximately the same minimal perturbations over any given period. So the effects of J2, drag, SPR and other perturbations will not be included in this study.

1.2.3. Genetic Algorithms

A genetic algorithm is a metaheuristic solver that exploits Darwin's theory of natural selection to diversify and evolve a population's gene pool, first introduced by John Holland in the mid-1960s [8]. Based heavily on survival of the fittest selection techniques, genetic algorithms provide quick and robust processes for obtaining optimal solutions in large non-linear, multivariable problems. Genetic algorithms have these five basic components:

1) A genetic representation of solutions to the problem
2) A way to create an initial population of solutions
3) An evaluation function rating solutions in terms of their fitness
4) Genetic operators that alter the genetic composition of children during reproduction
5) Values for the parameters of genetic algorithms

Based on these principles, genetic algorithms have spread into many scientific fields of study including economics, machine learning, energy efficiency, social systems, and of course optimization [9]. This study will rely on an evolutionary genetic algorithm to iterate and optimize constellation designs.

Evolutionary genetic algorithms utilize biological evolution techniques to mix and vary a given gene pool until an optimal solution is found. Starting with an initial random population, each solution is evaluated based on an objective fitness function and assigned a fitness value. The standing population is then reduced to a few parents that have endured a "survival of the fittest" selection process. These parents' genes are then mixed to create an enhanced population of children for the next generation in the recombination phase. Additionally, implementing randomized mutations will diversify the gene pool, reducing the risk of finding a localized minimum or maximum. Over multiple generations, the population will slowly cross-breed until the highest performing genes achieve the greatest fitness value within a solution space are found [8] [9] [3]. For more information regarding evolutionary genetic algorithm, please reference the book *Introduction to Evolutionary Computing* by Eiben and Smith [10].

1.2.4. Previous Studies

Constellation design has been studied since the 1960's, with publications from labs and research institutes around the world. With the Space Race at hand, this field of study grew quickly to advance technologies and improve military capabilities. Trade studies at the time relied heavily on trial and error to evaluate fitness performance of multiple constellations. In the 1970's J. G. Walker published two reports on analyzing Walker Delta Pattern systems using spherical trigonometry to determine coverage time. His research, relying on a personally developed program deemed COCO (Circular Orbit COverage), was written in FORTRAN and could only evaluate a maximum of 25 satellites at a time via punch cards with terms *T/P/F* designated [7] [11]. Computers soon allowed for faster evaluation turnaround and extensive constellation variations for larger trade studies. Around the turn of the century, a comprehensive study by Lang and Adams developed an extensive list of constellations between 5 and 100 satellites that would provide 1-to-4 fold full Earth coverage performance analysis. Their study provided mission architects with a list of constellation options that would optimize overall system cost of a mission by accounting for the number of orbital planes, or considering the best constellation to deploy from minimal launch vehicles [12]. Further studies, like Yuri Ulybyshev's *Satellite Constellation Design for Complex Coverage*, provide alternative geometric analytical methods to evaluate constellations. For example, continuous and dis-continuous coverage can be determined based on the location of a target grid within a two-dimensional polygon mapping of satellite positioning [13].

Genetic algorithms, on the other hand, were slower to be adopted because of the limited computational power available at the time for massive iteration studies. Driven mainly by theory and small-scale demonstrations, limited research or advances were made before the 1980's. Holland's 1975 book *Adaptation in Natural and Artificial Systems* popularized the theory of computational evolution [14], but it wasn't until 1989 when David E. Goldberg published his book, *Genetic Algorithms in Search, Optimization, and Machine Learning* that genetic algorithms were widely implemented into various fields of study. His book provided source code and clear examples of researchers applying genetic algorithms to solve various problems [9] [15]. Over the next decade genetic algorithms would integrate new optimization schemes to a variety of fields. So, problems that normally posed large, discontinuous, or traditionally unsolvable solution spaces began to rely on these genetic algorithms.

As genetic algorithms became more well-known, they began to be used in constellation optimization problems. One of the first studies completed by Eric Frayssinhes in 1996 implemented a binary encoding mechanism to iterate on circular orbits characteristics. The resulting optimal constellations from his work broke from the norm of tradition Walker Delta Patterns, expanding the constellation solution space [16]. Another early report by William Mason utilized a Pareto genetic algorithm to get a hand-full of optimized constellation solutions. These options created an optimal frontier, where no performance metric could be improved without degrading another. The scheme was called the Modified Illinois Non-dominated Sorting Genetic Algorithm (MINSGA). Mason also implemented STK (Satellite Tool Kit, at that time) to help evaluate each constellation design for global coverage [17]. Continued studies explored multi-objective optimization fitness functions along with more advanced Pareto-Frontiers. For example, M. Asvial's work relied on a rank fitness assignment method for multi-objective optimization, where another non-dominated genetic algorithm was used to find multiple Pareto-optimal constellation design solutions [18]. Similarly, Matthew Ferringer's article in 2006, *Satellite Constellation Design Tradeoffs Using Multiple-Objective Evolutionary Computation,* provides a constellation designer with an optimization tool for a discontinuous solution space. His non-dominating sorting genetic algorithm (NSGA-II) generates sets of constellations on Pareto-Frontiers, which again highlights the tradeoffs between a set of conflicting metrics [19].

1.3. Current Field Status

Following a proven history of successfully optimizing constellations with genetic algorithms, researchers have continued to advance their evaluation and iteration methods over the past two decades. In attempts to reduce computation time and improve solution reliability, recent studies have provided a series of useful methodologies for optimizing constellation designs. Current approaches for design algorithms usually contain the following three main components.

1) Orbital propagator – To evaluate constellations and obtain performance metrics
2) Genetic algorithm – To iterate a population through a non-linear solution space
3) Objective fitness function – To evaluate fitness and arrive at an optimal solution

New studies focus on improving one or more of these components by trying alternative orbital propagation methods, advancing or refining GA's, or varying multi-objective functions to arrive at Pareto-Frontiers quicker.

Tengyue Mao's report on *Efficient Constellation Design Based on Improved Non-dominated Sorting Genetic Algorithm-II* increased the convergence accuracy of his genetic algorithm by implementing alternative multi-parent and SBX crossover operators to improve searching capabilities. He further improved the process by introducing Gaussian and Cauchy mutation methods to arrive at a Pareto-Frontier. Relying on the trusted propagation program STK to evaluate orbital performance, Mao only explored the effects of a modified genetic algorithm [20].

A recent study by Tania Savitri at the *Department of Aerospace Engineering, Korea Advanced Institute of Science and Technology (KAIST)* explored how the implementation of a semi-analytical initial guess could reduce the overall computational load of his solver when applied to the orbital propagator. By avoiding the integration of satellite positioning over time, the computational load was reduced nine-fold with a recorded error increase of 0.5%. To prove his new propagation method, Savitri used the common NSGA-II genetic algorithm scheme to arrive at Pareto-Frontiers [8].

Christopher Hind's dissertation *A Pareto-Frontier Analysis of Performance Trends for Small Regional Coverage LEO Constellation Systems* focuses on a variation of the εNSGA-II algorithm to explore its effects on Pareto-Frontiers. This multi-objective optimization scheme prioritized epsilon dominated Pareto solutions within a pre-determined volume of the solution space. This technique thinned out the solution archive, reducing the computational storage required to save every optimal solution, while allowing a user to define the resolution of the Pareto-Frontier. Hind's work also relied on STK for orbital propagation [3].

1.4. Proposed Solution

As shown in the previous sections, multiple research papers have recreated and proven the effectiveness of constellation design algorithms. The proposed solution of this thesis will aim to further improve these studies by utilizing an alternative orbital propagation method based in spherical trigonometry, along with a unique evolutionary genetic algorithm and multi-objective function.

By limiting constellations to circular trajectories in idealized environments (i.e. no perturbations), satellites will exhibit predictable and repeatable orbits in inertial coordinate frames. Similarly, any fixed target on the surface of a planet will follow an exact path along its given latitude line. Expressing both the satellite and target as points in a common celestial sphere, allows the latitude and longitude of each object to be found based solely on a time step such as True Anomaly, θ, and its initial position. No state vectors or iterative propagation will be required. This process will then be scaled to include every satellite in a constellation.

To evaluate each constellation, a common multi-objective function will evaluate each solution based on various performance objectives to provide a unique fitness value. The performance objectives coverage time, total revisits, and number of satellites will be assessed over a single orbital period. However, to summarize the long-term performance metrics of a constellation, each solution must be analyzed multiple times to simulate the target's rotation on a planet. This can be accomplished by progressing the target locations initial position along its latitude line, evaluating an entire constellation for a single orbit at each point, and then averaging the resulting performance objectives. Depending on the step size, this approach can account for nearly any scenario the constellation could encounter as the host planet rotates about its spin axis. To find the fitness value of a constellation, each averaged performance objective evaluated will be equally weighted and summed.

Once assigned a fitness value, an evolutionary genetic algorithm will iterate on the solutions by selecting the most fit individuals and recombining their traits into the next generation of children solutions. The selection process will be tournament based to reinforce Darwin's theory of natural selection. The surviving parents will have their traits broken down and shuffled to create a child population. However, all traits have a probability to be mutated to help diversify and expand the solution space evaluated. Over multiple iterations the population will slowly approach an optimal constellation design.

2. METHODOLOGY

In the methodology section each equation, tool, and analytic method used to arrive at the results will be covered in detail. Many subjects in this section will build off topics introduced in previous introduction sections. Review of the methodology will provide a full understanding of the numerical processes and workflow needed to arrive at the results.

2.1. Reference Frames

When describing orbits, it is important to introduce a consistent and relevant coordinate frame. Constellations defined in this study will rely on an inertial coordinate frame with its origin fixed at a planet's center. Similar to the Earth Centered Inertial frame (ECI), these coordinate systems are fixed in space, with the X axis pointing towards the first point of Aries on the Vernal Equinox, while the Z axis is normal to the equatorial plane. These coordinate systems are useful when assessing the repeatability of satellites on the same orbital trajectory, since they will repeat indefinitely without perturbations. Figure 2.1 shows the ECI coordinate frame.

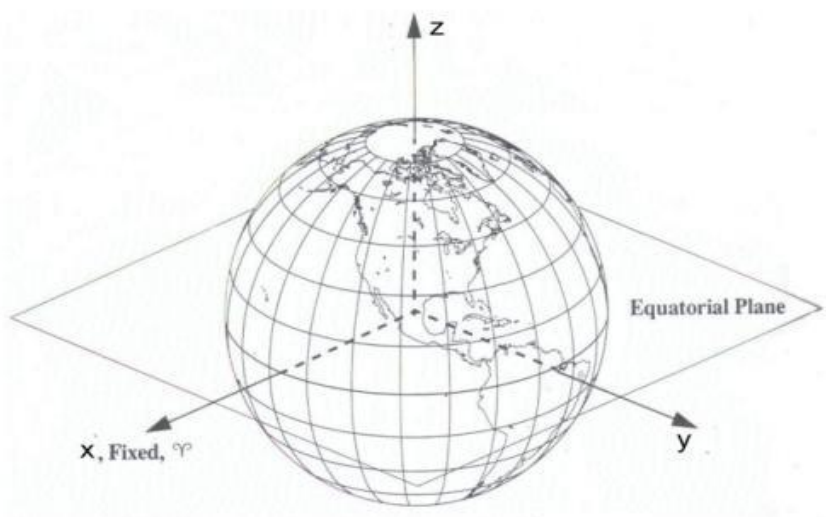

Figure 2.1 *ECI COORDINATE FRAME [21]*

Inertial coordinate frames were chosen to express positioning vectors in a common celestial sphere via latitudes and longitudes. Both the satellite and surface target will be defined in this manner to exploit spherical trigonometry propagation techniques.

2.2. Spherical Trigonometry Propagation

2.2.1. Intro

Relying on simplified circular orbits and idealized propagation, spherical trigonometry provides quick and efficient predictions of satellite and target positioning at any time step.

Without requiring traditional ordinary differential equation solvers such as ode45, spherical trigonometry reduces the computational time needed to fully evaluate large constellations in various scenarios. Expressed by latitudes and longitudes in a common celestial sphere, the position of a given satellite and target can always be expressed as a function of time or true anomaly with the use of basic trigonometric function.

Spherical trigonometry propagation is being used in this application to determine the angle A between a satellite's position vector and the target's position vector in a common celestial sphere. Since WDP constellation's exhibit fixed altitudes, there exists a maximum angle A_{max} where the target loses line of sight with a satellite over the horizon. This limit will be the primary metric used to determine a constellation's evaluation of coverage. A satellite will have a successful coverage pass if it maintains an angle A in the range, $0° \leq A \leq A_{max}$, where a 0° angle represents when a satellite is directly above the target.

2.2.2. Analysis

To reduce computational time, orbits will be evaluated using spherical trigonometry to determine the angle between two points at any given time between a satellite and target. Equivalent to the angle extracted from the dot product of two vectors, A will represent the angle between two vectors attached at their origin, see Figure 2.2.

$$\boldsymbol{r_{sat}} \cdot \boldsymbol{r_{target}} = r_{sat} r_{target} cos(A) \tag{2.1}$$

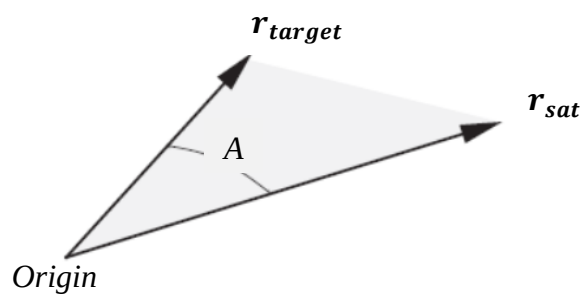

Figure 2.2 *EXAMPLE DOT PRODUCT SHOWING THE ANGLE BETWEEN TWO VECTORS [27]*

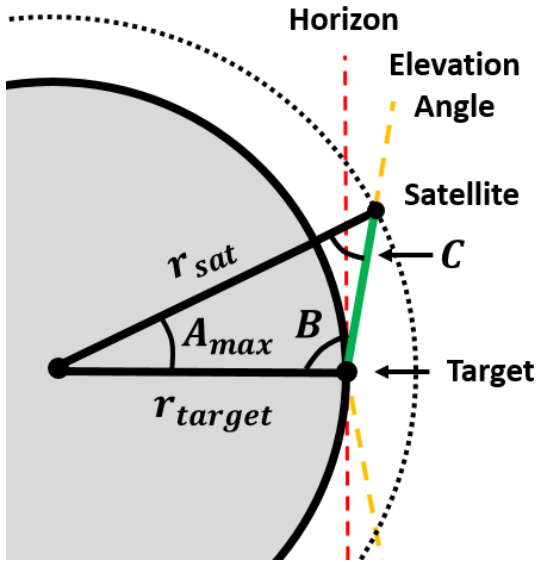

Figure 2.3 *MAXIMUM ANGLE BETWEEN TARGET AND SATELLITE*

A target's minimum elevation angle and satellite altitude will bound the maximum angle, A_{max}, between a satellite's and target's position as shown in Figure 2.3. Using the law of sines, angles A_{max} and C can be found using the position vectors $\boldsymbol{r_{target}}$ and $\boldsymbol{r_{sat}}$ which are defined by planet radius and satellite altitude. In this instance angle $B = 90° + Elevation\ Angle$. Alternatively, the A_{max} value can be determined based on maximum satellite field of view angles by changing the elevation angle variable.

To evaluate an orbit using spherical trigonometry, both the target and satellite must first be defined in the celestial sphere with a respective latitude and longitude. A satellite's latitude and longitude will be represented by variables Ψ, and Φ respectively, while the target's latitude and longitude will be Ψ_E and Φ_E respectively. Since each constellation has a pre-set inclination, the latitude of a satellite is bounded by a special case of Napier's rules for right spherical triangles,

$$\sin(\Psi) = \sin(\theta)\sin(I) \tag{2.2}$$

Similarly, a satellites longitude can be found using

$$\tan(\Phi) = \tan(\theta)\cos(I) \tag{2.3}$$

where θ is true anomaly and I represents the inclination. Exploiting circular orbits, θ will always increase linearly over time, meaning both Ψ and Φ will repeat indefinitely. This is expected for orbits defined in inertial space without perturbations. Also, Equation 2.2 infers that a satellite's celestial latitude will never exceed its inclination. Similarly, the longitude of a planetary target

will repeat in inertial coordinates, while its latitude will remain fixed. Equations 2.4 and 2.5 define the target's latitude and longitude in the celestial sphere.

$$\Psi_E = \Psi_0 \tag{2.4}$$

$$\Phi_E = \Phi_0 + \theta/Q \tag{2.5}$$

where Q represents the number of orbits over a single rotation of the host planet. For example, $Q = 16$ for an orbital period of 90 minutes over a 24-hour day on Earth. The θ/Q term in Equation 2.5 continually advances the target's longitude with respect to the celestial sphere, and allows synced linear progressions of both the satellite and target.

With both the target and satellites well defined within the celestial sphere, the angle A can now be determined between any latitude and longitude vectors using the following relationship.

$$\cos(A) = \cos(\Psi_E)\cos(\Psi)\cos(\Phi_E - \Phi) + \sin(\Psi_E)\sin(\Psi) \tag{2.6}$$

By expanding $\cos(\Phi_E - \Phi)$ and utilizing Equation 2.2, this can be rewritten as Equation 2.7 below which avoids tangential singularities and directly correlates angle A with the propagation steps of true anomaly, θ.

$$\cos(A) = Bcos(\theta) + Ccos(\theta) \tag{2.7}$$

where

$$B = \cos(\Psi_E)\cos(\Phi_E) \tag{2.8}$$

$$C = \cos(\Psi_E)\sin(\Phi_E)\cos(I) + \sin(\Psi_E)\sin(I) \tag{2.9}$$

In their simplest relationships, the system of equations presented above assumes that the satellite and target begin propagation when their vectors of latitude and longitude are aligned. So to evaluate a constellation, offsets must be implemented between adjacent satellites for accurate evaluations [22]. Figure 2.4 depicts angle A results between a single satellite at 61° and 800km and a given target at 35° latitude over five days. Figure 2.5 shows the same system but with two satellites in orbit separated by 180° in Ω (RAAN). The red line in both graphs represents A_{max}, meaning any time the oscillating lines spend below A_{max} the target and satellite will maintain line of sight.

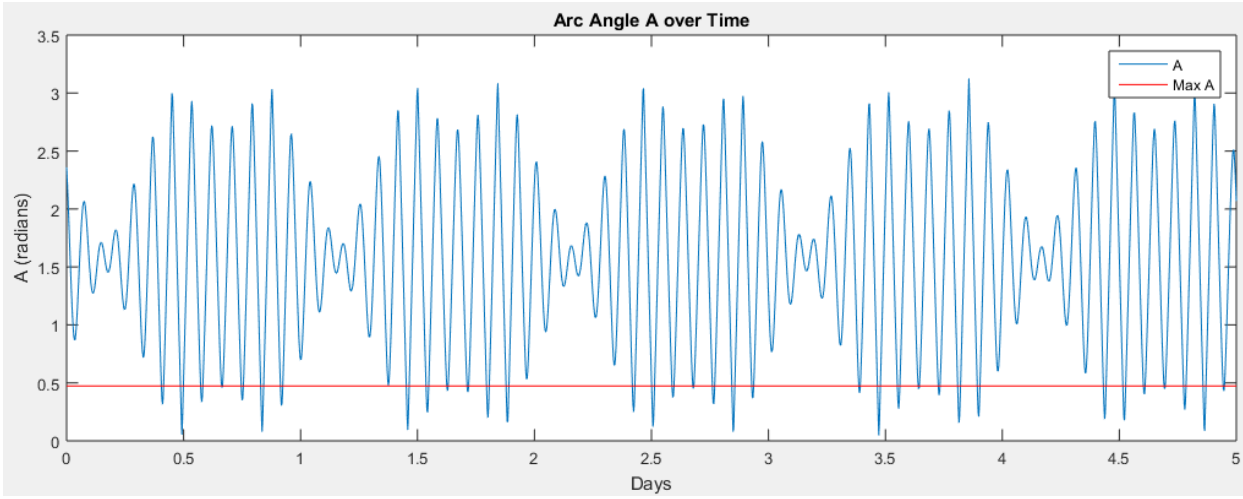

Figure 2.4 *ANGLE A BETWEEN SATELLITE AND TARGET OVER FIVE DAYS. RED LINE REPRESENTS THE ANGLE* A_{max}

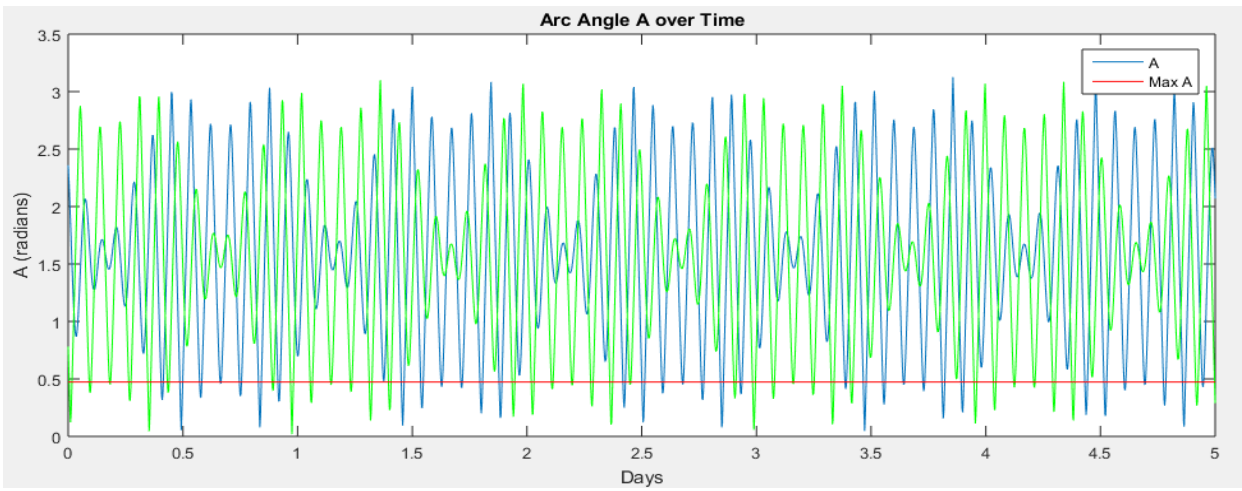

Figure 2.5 *ANGLE A BETWEEN TWO SATELLITES AND A TARGET OVER FIVE DAYS. SATELLITE ORBITS ARE ROTATED 180° IN RAAN. RED LINE REPRESENTS THE ANGLE* A_{max}

As outlined in Section 1.2.2, there are five main parameters of a constellation that are considered for modeling any given scenario.

1. Altitude: *a*
2. Inclination: *I*
3. Number of Satellites: *T*
4. Number of Planes: *P*
5. Non-Dimensional Measure of Relative Spacing: *F*

Altitude and inclination determine the location and propagation metrics of a standard satellite, while T/P/F describe the relative spacing and offsets of a constellation. These offsets will be expressed in angles relative to either the celestial sphere or along a given orbital plane.

Angle α (alpha) will represent the radial spacing from plane to plane in the celestial sphere. β (beta) will represent the angle between two adjacent satellites who share the same orbital plane. And γ (gamma) will represent the angular offset between satellites from one plane to the next. See Figure 8 for a depiction of each angle offset.

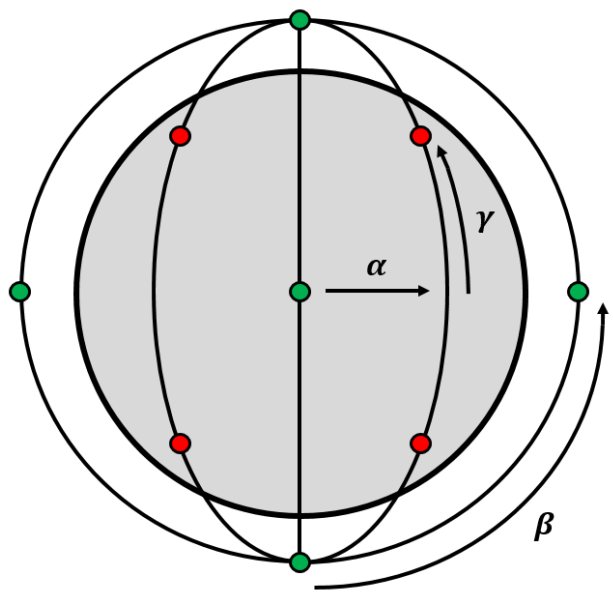

Figure 2.6 *ANGLES ALPHA BETA AND GAMMA IN A 16/4/2 WALKER DELTA PATTERN CONSTELLATION*

In the illustration above there are four orbital planes, each containing four satellites. These sixteen satellites are spread out evenly due to the offset angles expressed in Equations 2.10 through 2.12. Each angle is determined from the necessary satellites, planes, or adjacent phase angle offsets required to satisfy the constellation parameters 16/4/2 (T/P/F) [7].

$$\alpha = 360/P \qquad \beta = \frac{360}{(\frac{T}{P})} \qquad \gamma = 360 * F/T \tag{2.10-12}$$

Starting with Equation 2.7, the true anomaly term needs to be expanded to include satellite offsets within a single plane. Adding in both β and γ angles will force each respective satellite to take on a different starting position for evaluation.

$$\cos(A) = Bcos(\theta + \beta + \gamma) + Ccos(\theta + \beta + \gamma) \tag{2.13}$$

To account for Ω (RAAN) offsets between planes, angle α is added to Equation 2.5.

$$\Phi_E = \Phi_0 + \alpha + \theta/Q \tag{2.14}$$

With each satellite clearly defined in the celestial sphere, a full constellation can be evaluated by stepping through a progression of true anomaly.

To account for initial conditions effecting constellation performance, each constellation will be evaluated over a single orbit period at multiple planetary start points. In other words, to evaluate a constellation over its lifespan, multiple initial conditions need to be looked at as the planet rotates in the celestial sphere. This approach will reduce computational time and allow for averaged performance evaluations of ever constellation.

2.2.3. Verification Examples

Since spherical trigonometry is an uncommon solution method for orbit propagation, it must be verified and compared against traditional methods. Conveniently, Matlab provides an ordinary differential equation solver called ode45, which has become a standard for accurately propagating satellites from initial position and velocity vectors. In its simplest terms the ode45 solver takes these two state vectors and iterates them over time via the equations of motion.

To validate spherical trigonometry propagation, the following two scenarios were setup for comparison.

1. Propagate a single satellite in orbit with the same starting position in ECI. The target will maintain a latitude of 55 degrees. The satellite will have an altitude of 800km and an inclination of 60°.
    a. Matlab ode45 propagator (Figure 2.7)
    b. Spherical Trigonometry propagator (Figure 2.8)
    c. Overlay of both propagators (Figure 2.9)
2. Propagate an 800/60°/12/3/1 WDP constellation. The target will maintain a latitude of 55 degrees.
    a. Matlab ode45 propagator (Figure 2.10)
    b. Spherical Trigonometry propagator (Figure 2.11)

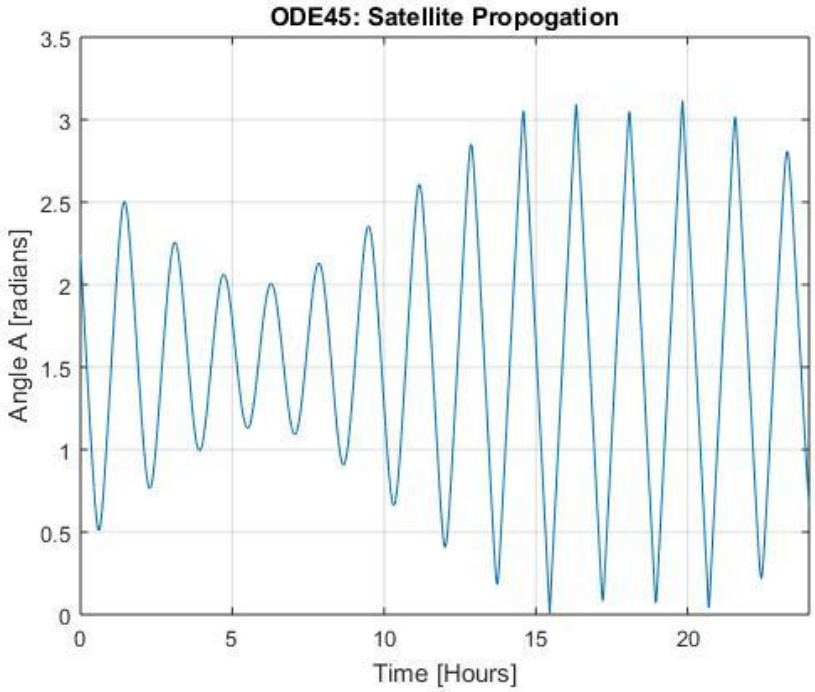

Figure 2.7 *ONE SATELLITE AT 800KM ALTITUDE AND 60-DEGREE INCLINATION (ODE45)*

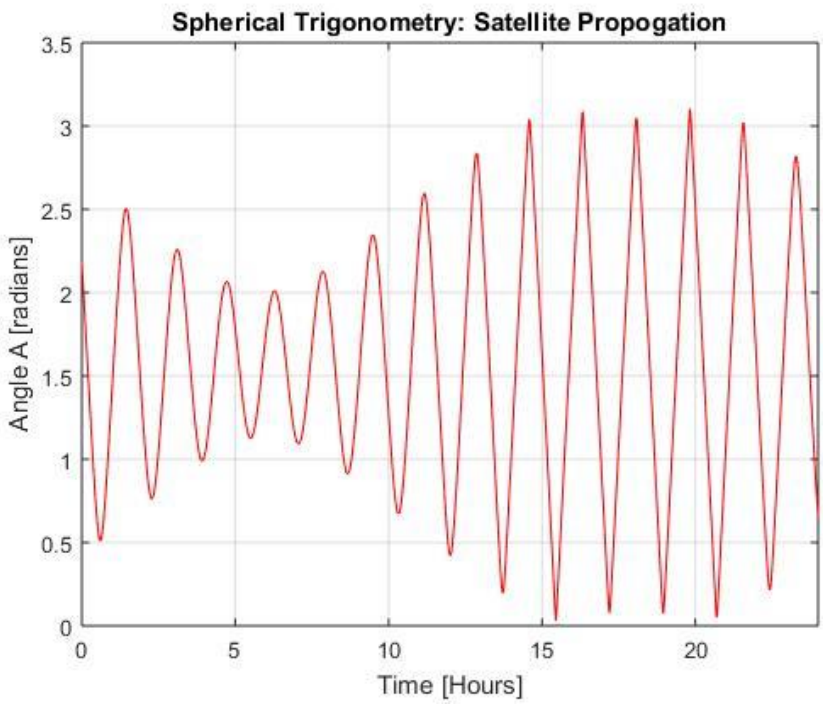

Figure 2.8 *ONE SATELLITE AT 800KM ALTITUDE AND 60-DEGREE INCLINATION (SPHERICAL TRIGONOMETRY)*

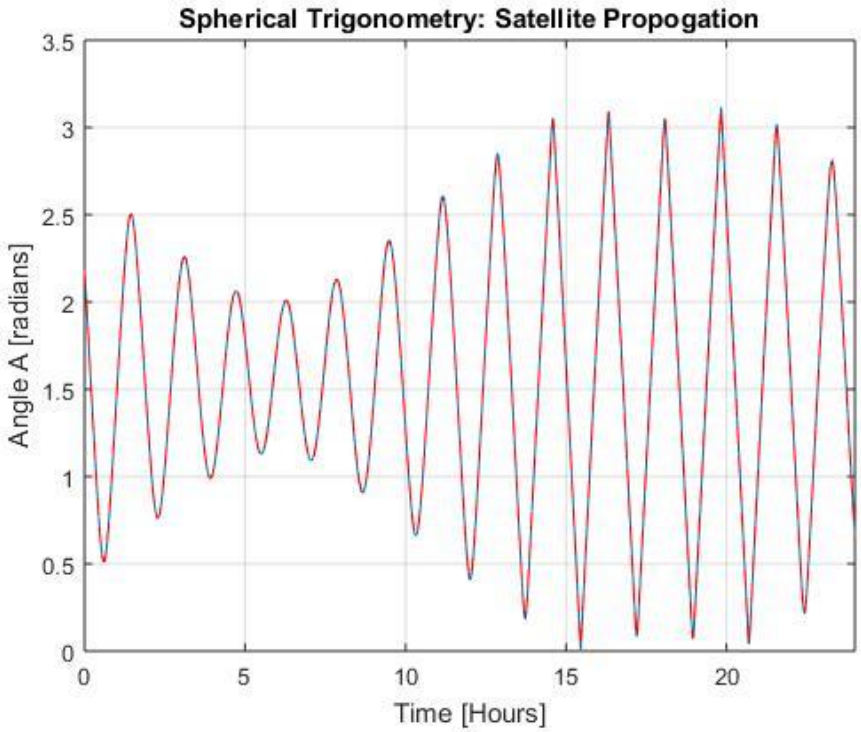

Figure 2.9 *OVERLAID COMPARISON OF ODE45 AND SPHERICAL TRIGONOMETRY PROPAGATIONS (BLUE: ODE45, RED: SPHERICAL TRIGONOMETRY)*

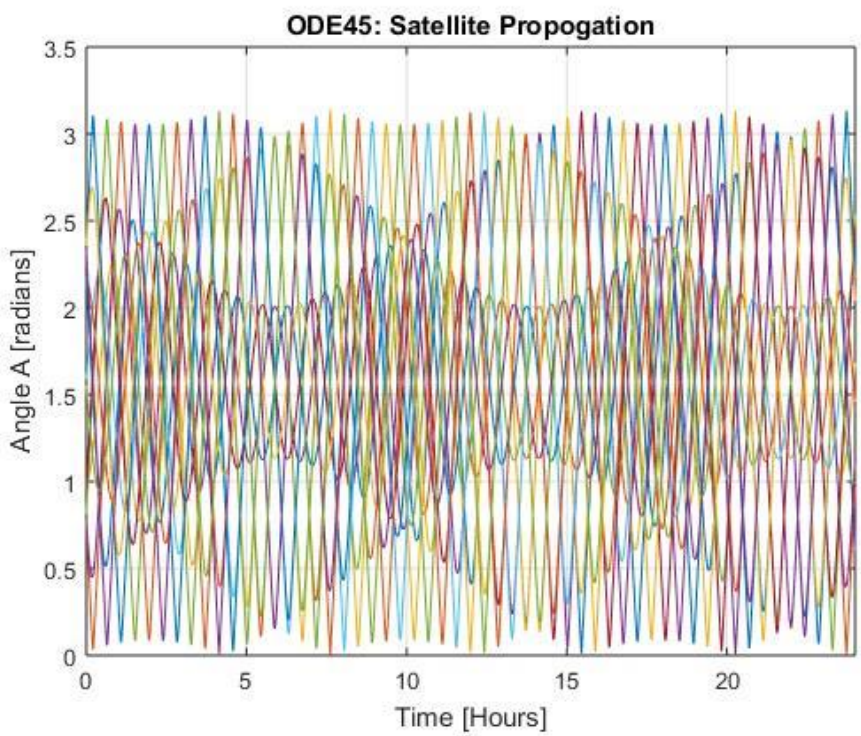

Figure 2.10 *ODE45 PROPAGATOR OF AN 800/60°/12/3/1 CONSTELLATION*

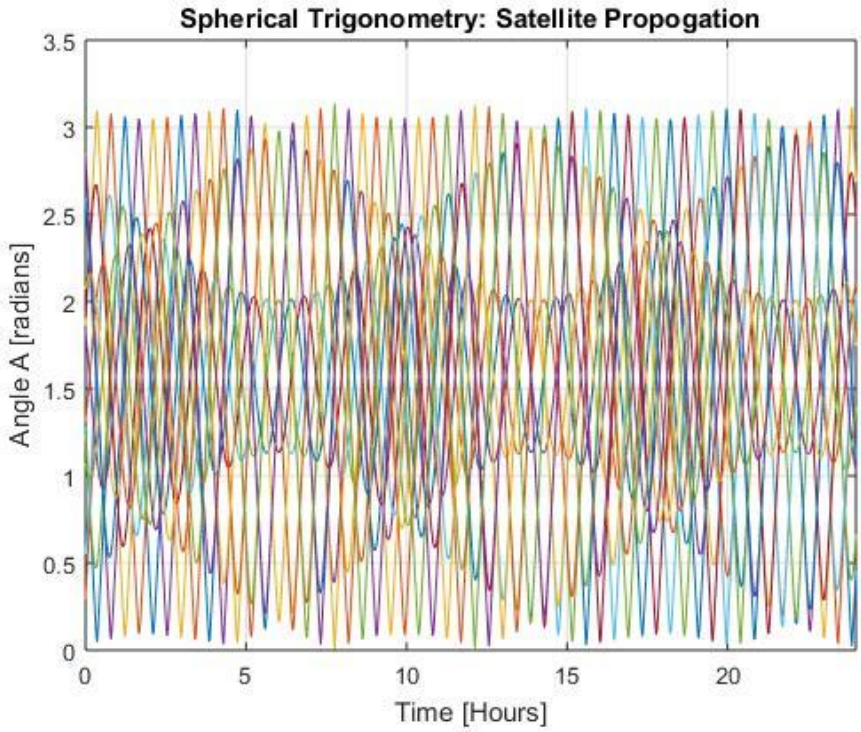

Figure 2.11 *SPHERICAL TRIGONOMETRY PROPAGATOR OF AN 800/60°/12/3/1 CONSTELLATION*

Figures 2.7 through 2.9 compare a single satellite using both ode45 and spherical trigonometry propagation. Looking closely at the peaks and valleys of Figures 2.7 and 2.8, smoother curves and greater fidelity can be seen in the ode45 solver since it accounts for more iterative time steps verses the set number of time steps for spherical trigonometry.

Figures 2.10 and 2.11 depict a similar comparison with twelve satellites being propagated over twenty-four hours. Three distinct planes of four satellites can be seen in each of these graphical representations. From these examples it can be concluded that spherical trigonometry is a suitable substitute for propagating simple circular orbital constellations.

Comparing the two, the spherical trigonometry propagator for a single orbiter took 0.003 seconds to run while the ode45 propagator took 0.1 seconds. This difference begins to significantly balloon solution times with ode45 solvers.

2.3. Multi-Objective Optimization

Multi-objective optimization is the process by which an algorithm evaluates multiple objective functions to assign a fitness value used to converge on an optimal solution. This study will rely on the weighting and summation of multiple objective functions to produce a singular fitness value. This approach allows for quicker computations and simple selection of final

optimization solutions. To begin this process, one must decide which objectives should be used for optimization.

For constellation design there are many objectives that can be used for optimization. Below is a short list of common options.

1. Number of Spacecraft
2. Coverage Time
3. Gap Time
4. Number of Revisits
5. Constellation Altitude
6. Constellation Inclination
7. Number of Planes
8. Excess Coverage Time

This study will use the following objectives for evaluation: *Number of Spacecraft*, *Coverage Time*, and *Number of Revisits*. These three objectives provide a diverse look at a constellation's performance and efficiency, and would be among the most important aspects for a mission architect to consider. The objectives chosen for this study were determined by the author, and were based heavily on the performance metrics considered in Hinds' previous study. Also simplifying the evaluation method to compare a single fitness value verses multiple fronts (Pareto-Frontier) leads to quicker computational solution times.

Equation 2.15 shows how multiple objective functions are weighted and summed together to create a standardized fitness value F.

$$F = \sum_{i=1}^{M} w_i f_i(x) \tag{2.15}$$

where w_i and f_i represent the weight and objective function of the ith objective, x, respectively for M number of objectives. The sum of the weights must equal one [3]. This technique is useful when the influence of an objective is well known relative to a constellation's design characteristic. Thus, this approach is applicable and reliable for determining optimized constellation designs, but it can limit the knowledge of objective trade-offs within a solution space. However, since all mission requirements are unique, there are no perfect multi-objective functions for solving every constellation design problem. Therefore, each objective and weight of the multi-objective function should be reevaluated before attempting optimization. For the sake of demonstration and consistency, the case studies in this paper will all rely on the following objective weight distribution for constellation analysis.

$$F = \frac{1}{3}S + \frac{1}{3}C + \frac{1}{3}R \tag{2.16}$$

where S, C, and R represent the objective functions for *Number of Spacecraft*, *Coverage Time*, and *Number of Revisits* respectively. These objective functions are further defined below.

$$S = \left(\frac{Sats}{Sats_{max}}\right) \qquad C = 1 - \frac{t_{cov}}{T_p} \qquad R = 1 - \frac{Revs}{Sats_{max}} \tag{2.17-19}$$

where $Sats$ represents the current number of satellites in the constellation being evaluated, $Sats_{max}$ represents the maximum number of satellites available for evaluation in a single constellation, t_{cov} is the total coverage time of the constellation during one orbital period, T_p, and $Revs$ represents the number of revisits that occur over a single orbital period.

These objective functions aim to normalize the performance of any constellation with three metrics. For this study S, C, and R will range from 0 to 1, where smaller numbers indicate better constellation performance. Therefore, the algorithm defined in the next section will optimize the solution space by minimizing the multi-objective fitness function. A roadmap of each objective function can be found below.

$$\text{(Minimum Satellites) } 0 \leq S < 1 \text{ (Maximum Satellites)} \tag{2.20}$$

$$\text{(100\% Coverage) } 0 \leq C \leq 1 \text{ (No Coverage)} \tag{2.21}$$

$$\text{(Max Revisits) } 0 \leq R \leq 1 \text{ (No Revisits)} \tag{2.22}$$

Once normalized and weighted, these terms are summed to find F, the total fitness value of a constellation solution, which should be bounded from 0 to 1 as well. With a clearly defined fitness scale, each constellation design can be ranked and compared side by side to clearly identify an optimal solution.

2.4. Evolutionary Genetic Algorithm

2.4.1. Intro

Evolutionary genetic algorithms, as defined in Section 1.2.3, are iterative processes based on biological evolution principles that select, mix, and diversify traits in search of an optimal solution. This section will dive into the details and techniques of the evolutionary genetic algorithm used in this study. Emphasis will be put on the selection, recombination, and mutation aspects of the algorithm.

By using the five main constellation parameters *a/I/T/P/F* defined in Section 1.2.2 as traits, an evolutionary algorithm can be implemented to find an optimal constellation design. To determine the most optimal solution each constellation will be evaluated and assigned a normalized fitness value based on the multi-objective function defined in the previous section.

2.4.2. Process

Starting with an initial random population of n solutions, the evolutionary genetic algorithm will iterate through *G* generations, gradually enhancing each of the five constellation traits *a/I/T/P/F* until an optimal solution is achieved. To iterate through each generation there will be multiple steps employed to simulate natural selection. Figure 2.12 gives an overview of the evolutionary genetic algorithm steps.

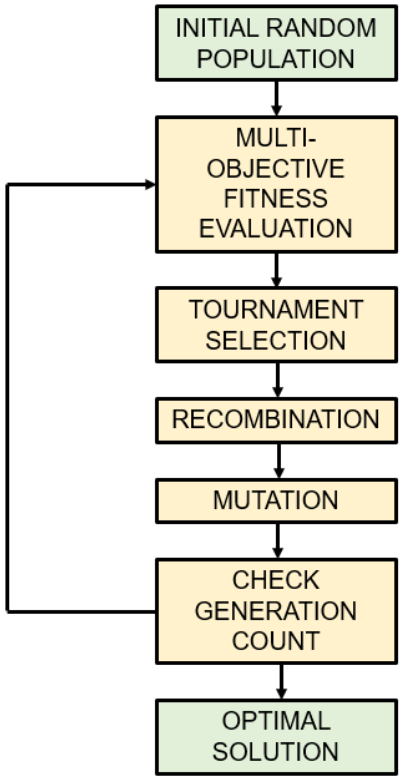

Figure 2.12 *EVOLUTIONARY GENETIC ALGORITHM FLOW CHART*

First, the algorithm will determine each constellation's fitness by evaluating individual performance with the multi-objective function outlined in Section 2.3. Each constellation will then have an equally random chance of being chosen to compete in a tournament based selection process. The randomly selected individuals will face off in n number of groups. The selection pressure factor was set as $k = n * 0.1$ to maintain a ratio that would not mis-represent the diversity of any population. This aspect will also restrict the number of high performing constellations per tournament, reducing the risk of premature convergence [23]. The individual with the greatest fitness from each pair will become a parent for the next generation. The traits from each winning parent will be pooled together in preparation for step two.

The second step of this process is recombination, which mixes parent traits together to produce child constellations for subsequent generations. Drawing from the parent trait pool, each

child has an equal chance to be randomly assigned all five traits, *a/I/T/P/F*. However, since the number of satellites (*T*), planes (*P*) and offsets (*F*) are not always compatible between constellations (i.e. 15 satellites cannot evenly distribute across 10 planes), *P* and *F* will be randomly regenerated based on the value of *T* assigned to a child. This step aims to uniformly mix constellation traits by using the multi-parent recombination method [24].

The third step involves a mutation process in which each child has an equal opportunity to have one or more of its traits randomly regenerated. The mutation parameter, m, represents the percentage of traits that will be altered among the current generation of children. This study uses a 5% mutation chance to promote constellation diversity. Past studies have explored mutation rates from 1-5% which influence convergence rates based on population and generation sizes [25]. Mutations are necessary to ensure that pre-mature convergence does not occur within the solution space since localized maxima can be found. By introducing a mutation rate, traits diversify and expand constellation capabilities, opening new portions of the solution space previously unexplored.

Once the child population has been recombined and mutated appropriately, the process repeats itself, starting with fitness evaluations of the new generation. This sequence will continue until *G* generations have been evaluated, at which time the algorithm will identify an optimal constellation design.

2.4.3. Verification

To verify the evolutionary genetic algorithm functionality outlined in the previous section, two industry standard fitness models were optimized using the same algorithm used in this study. The first function was *Rosenbrocks' Valley* which demonstrates a smooth contour solution space with three distinct and easily found maxima surrounding a valley with one distinct minima, which is the optimization goal. This function can be visualized in Figure 2.13. The second optimization problem was *Ackley's* function which presents a rippled solution space spotted with local minima and maxima. The optimal solution however is clearly a distinct divot in the center of the contour which can be seen in Figure 2.14. These two examples were chosen to demonstrate how the evolutionary algorithm design chosen can optimize a variety of solution spaces. The following equations define these two fitness functions. Variables x and y in the following functions will act as the evolutionary traits that will be iterated by the genetic algorithm, and f will represent the fitness function to minimize. [26]

$$f(x,y) = 100(y - x^2)^2 + (1 - x)^2 \tag{2.23}$$

$$f(x,y) = -20 * e^{-.2*\sqrt{\frac{1}{2}(x^2+y^2)}} - e^{\frac{1}{2}(\cos(2*pi*x)+\cos(2*pi*y))} + 20 + e \tag{2.24}$$

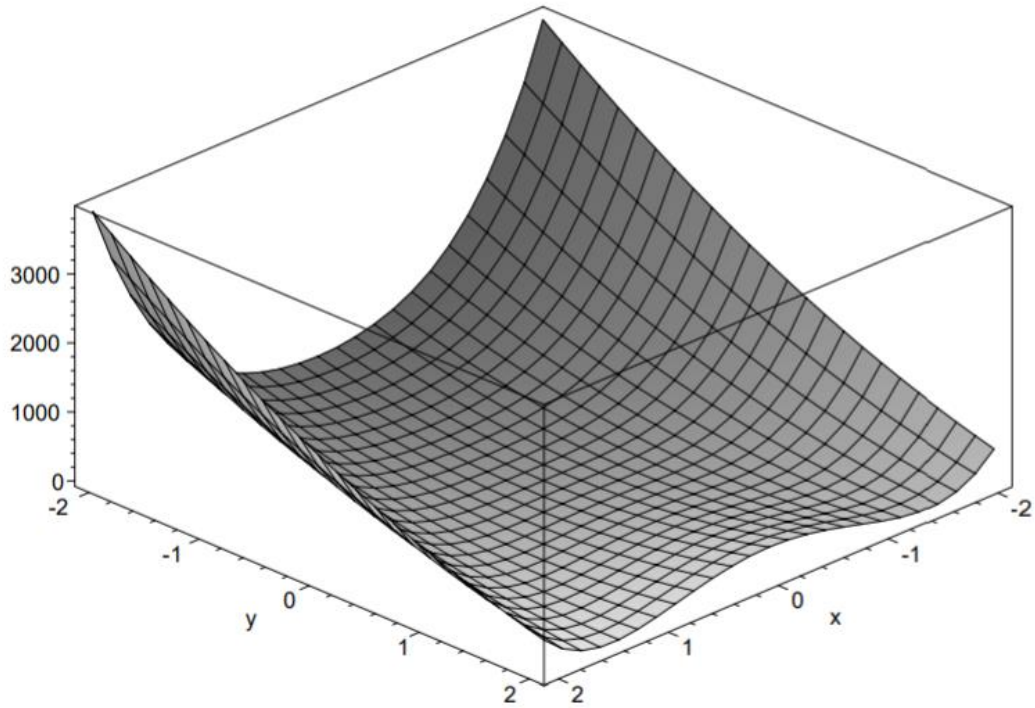

Figure 2.13 *ROSENBROCKS' VALLEY FUNCTION FOR OPTIMIZATION VERIFICATION, REF EQUATION 2.23 [26]*

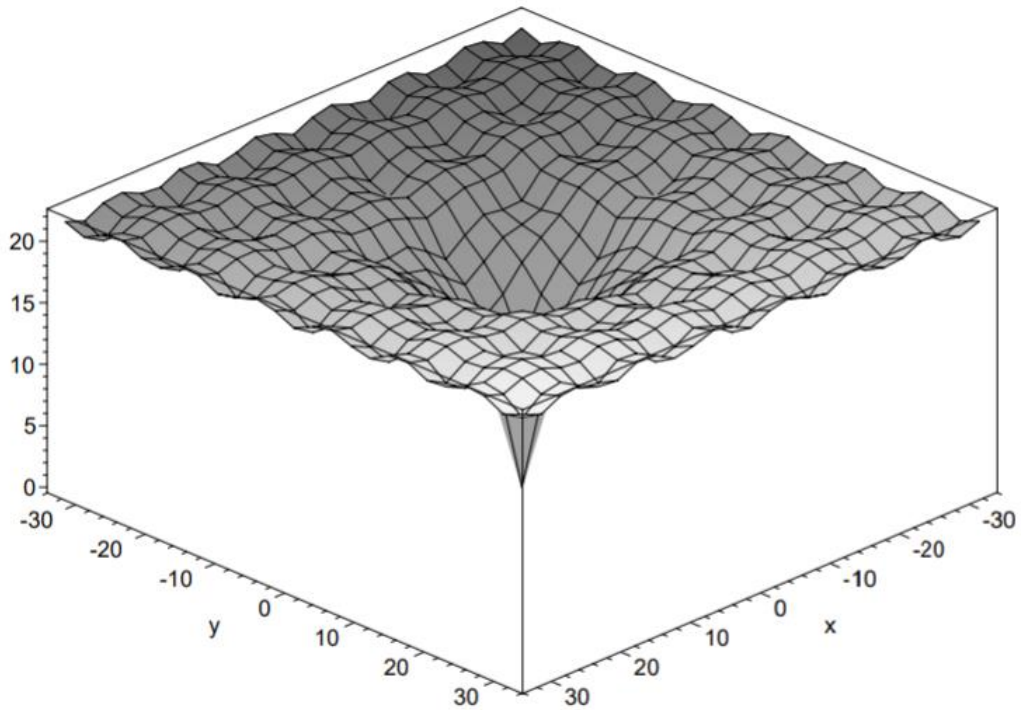

Figure 2.14 *ACKLEY'S FUNCTION FOR OPTIMIZATION VERIFICATION, REF EQUATION 2.24 [26]*

Similar to the multi-objective fitness function defined in Section 2.3, Equations 2.23 and 2.24 use x and y to represent two traits that undergo iterative evolutions to optimize each function to its minimum value. Again, the evolutionary genetic algorithm will set the tournament selection pressure to $k = n * 0.1$ and mutation rate as $m = 5\%$. The population and generation sizes however, were varied to explore the performance of the evolutionary genetic algorithm. Both n and G stepped through the following sizes [10,20,30,50,100,200,500,1000]. The algorithm's performance for each system is given below in Tables 2.1 and 2.2.

Table 2.1 *RESULTS FROM EVOLUTIONARY GENETIC ALGORITHM PERFORMANCE TRIALS ON ROSENBROCK'S VALLEY FUNCTION*

		Generations (*G*)							
	Rosenbrock's	10	20	30	50	100	200	500	1000
Population (*n*)	10	65.113	3.371	29.606	12.962	3.948	1.670	2.522	0.895
	20	61.155	0.722	1.701	0.036	5.378	3.618	1.357	0.370
	30	7.970	0.120	4.670	0.471	0.947	0.053	1.345	0.139
	50	2.861	0.100	2.443	0.051	0.854	0.007	0.028	0.392
	100	0.890	2.138	0.205	3.310	0.190	0.010	0.233	0.556
	200	0.002	0.306	0.037	0.113	0.306	0.069	0.114	0.469
	500	0.166	0.230	0.203	0.176	0.036	0.264	0.273	0.328
	1000	0.091	0.069	0.135	0.081	0.171	0.062	0.079	0.250

Table 2.2 *RESULTS FROM EVOLUTIONARY GENETIC ALGORITHM PERFORMANCE TRIALS ON ACKLEY'S FUNCTION*

		Generations (*G*)							
	Ackley's	10	20	30	50	100	200	500	1000
Population (*n*)	10	19.735	5.666	8.321	5.559	2.208	4.436	2.684	1.725
	20	15.809	7.237	9.133	4.324	4.301	4.322	0.182	0.497
	30	7.771	5.664	3.634	4.562	2.666	2.089	0.271	0.559
	50	5.983	9.835	3.438	3.892	2.073	1.512	0.166	0.025
	100	3.627	0.354	2.456	1.525	0.135	0.077	0.057	0.043
	200	1.844	3.518	0.462	0.855	0.156	0.083	0.038	0.042
	500	2.729	0.238	0.415	0.098	0.037	0.041	0.016	0.008
	1000	0.272	0.175	0.175	0.217	0.024	0.042	0.002	0.006

By inspecting the two tables above and comparing multiple trials of this same verification technique to allow for various permutations to occur, it was found that increasing generation size (*G*) and population size (*n*) will minimize fitness. However, maximizing *G* and *n* to 1000 increased solution times to an unpractical length. So to reduce overall solution time and achieve function convergence, the generation and population size of 100 was selected for this study. The verification trials above show minimization at these levels for both functions.

2.5. MATLAB Code and Implementation

The intent of this thesis is to provide a tool for students and researchers to use when designing orbital constellations. All functions and scripts were written in MATLAB to provide a tool that that can be utilized and altered in both educational and commercial settings. These functions and scripts will accompany this thesis for both reference and implementation purposes.

The following flow chart outlines the inputs, outputs, and flow of the constellation optimization process detailed in the previous sections. This will provide the audience a clear progression of the steps and processes followed to achieve the results presented. The full code in Appendix A will be complemented with comments to describe the terms or process occurring in each section.

2.5.1. Flow Chart

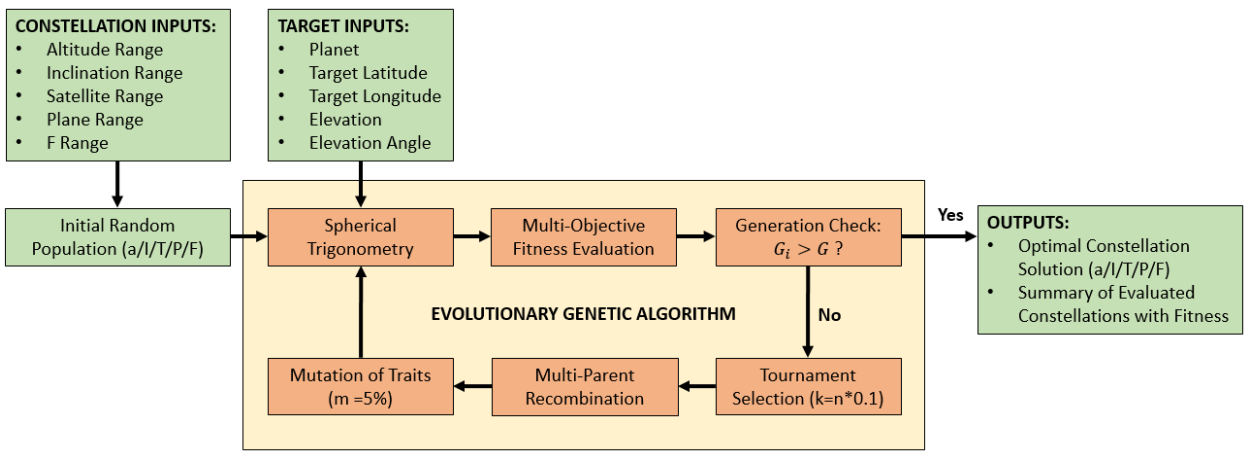

Figure 2.15 *FLOWCHART OF CONSTELLATION OPTIMIZATION EVOLUTIONARY GENETIC ALGORITHM*

3. RESULTS AND DISCUSSION

This section will reveal the results from various test cases while elaborating on the effects of the evolutionary genetic algorithm and multi-objective fitness function used. The test cases chosen provide a varied look at optimal constellation designs for multiple planets with targets at different latitudes. The planets Earth, Mars and Jupiter were evaluated to highlight the design differences between smaller and larger bodies for constellation optimization.

3.1. Performance Objectives

The multi-objective optimization function aims to minimize the fitness of a constellation. Each of the three objectives in this study will be scales on a normalized factor from zero to one, with better performance being indicated by a smaller value. These objectives will then be weighted and summed together to arrive at a final fitness value used to compare constellations for selection purposes.

Satellite minimization is the objective that observes how many satellites are in a constellation compared to the maximum satellite limit set by the user. For example, if the user wants to evaluate constellations with 20-50 satellites, the solution with twenty satellites would be given a value of zero, and a solution with fifty satellites would be given a value of one.

Coverage maximization is the objective that evaluates the fraction of time a constellation spends over a target during one orbital period on average. Limited to a scale from zero to one, no coverage of the target is assigned a value of one, while full coverage is given a value of zero.

Revisit maximization is the objective that evaluates the average number of satellites that pass within view of the target during an orbital period. A value of zero means that a constellation used the maximum number of satellites available, and each satellite passed over the target at least once during its orbit. A value of one infers that no satellites passed over the target during one orbital period. This objective relies on parameters. One, how many satellites were used compared to the maximum allowed for a constellation, and two, how many of those satellites passed the target during one orbital period on average.

The objective weights are as follows:

1) Satellite Minimization = 1/3
2) Coverage Maximization = 1/3
3) Revisits Maximization = 1/3

3.2. Constellation Design Solutions

The following sets of results reflect the outcomes of the evolutionary genetic algorithm attempting to optimize constellation designs using the multi-objective fitness function outlined in previous sections. The top ten optimal constellation solutions are given to show diversity and

convergence in the solution space. These results do not reflect perfect constellations for the given targets and are very subject to the multi-objective fitness function used.

Each of the following tables, graphs, and discussion points will refer to a constellation optimization problem based around the following planets at each of the following latitudes:

Planets:

1) Earth at [0°, 15°, 30°, 45°, 60°, 75°, 90°]
2) Mars at [0°, 15°, 30°, 45°, 60°, 75°, 90°]
3) Jupiter at [0°, 15°, 30°, 45°, 60°, 75°, 90°]

The solution space was kept constant for each test case listed and intentionally included a wide range for each design parameter. These relatively unbounded design parameters allowed each of these test cases to demonstrate the convergence performance of the evolutionary genetic algorithm around each of the host planets. For consistency the constellation solution space was limited by the following bounds for each test case.

1) Altitude Range: [200, 1000] (km)
2) Inclination Range: [0, 90] (degrees)
3) T Range: [5, 100]
4) P Range: [1, 100]
5) F Range: [0, 99]

3.2.1. Earth Constellations

Constellations covering targets on Earth's equator are relatively simple in nature and do not require strict altitudes to achieve peak performance. As shown in Table 3.1 below, constellations with varying low inclinations and one plane will provide optimal fitness based on the multi-objective function defined. The altitude for each constellation can range from 200 km to 1000 km because there is no benefit or drawback from higher or lower altitudes when nearly 100 satellites can continually cover an equatorial target. Constellations only need to ensure full coverage, max revisits, and minimal satellites to arrive at an optimal solution. However, the results below highlight an important characteristic of the multi-objective fitness function being used in this study. The revisits fitness value and satellite fitness value are inversely related since the maximum number of revisits can only be achieved by using all the satellites available. So the sum of these two fitness values will never drop below 0.33.

Table 3.1 OPTIMAL CONSTELLATION DESIGNS FOUND FOR EARTH TARGET AT 0° LATITUDE

Earth: Target 0° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
247.43	1.72	93.00	1.00	0.00	0.33	0.00	0.02	0.31
727.59	12.23	93.00	1.00	0.00	0.33	0.00	0.02	0.31
739.99	7.45	93.00	1.00	0.00	0.33	0.00	0.02	0.31
615.94	1.72	93.00	1.00	0.00	0.33	0.00	0.02	0.31
754.88	7.45	93.00	1.00	0.00	0.33	0.00	0.02	0.31
320.27	1.72	84.00	1.00	0.00	0.33	0.00	0.05	0.28
320.27	2.21	84.00	42.00	0.00	0.33	0.00	0.05	0.28
366.30	7.45	93.00	1.00	0.00	0.33	0.00	0.02	0.31
923.58	2.21	84.00	1.00	0.00	0.33	0.00	0.05	0.28
310.85	1.72	84.00	1.00	0.00	0.33	0.00	0.05	0.28

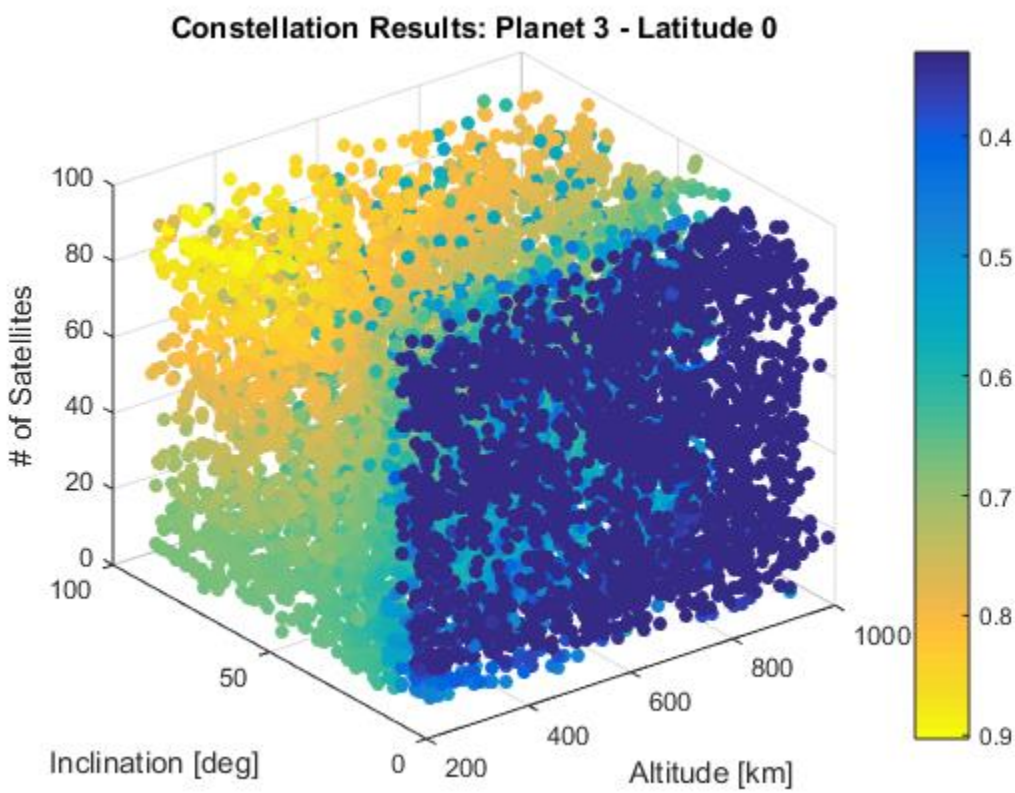

Figure 3.1 CONSTELLATION DESIGNS SOLUTION SPACE FOR EARTH TARGET AT 0° LATITUDE

Figure 3.1 above depicts the three-dimensional constellation solution space populated with every solution found over 100 generations, each with 100 individuals. The fitness values shown represent the multi-objective fitness of each solution. The dark blue points are the optimal

solutions with minimized fitness. Intuitively this scatter plot makes sense with lower inclinations providing optimal solutions for targets at 0° latitude and any altitude.

Figure 3.2 below provides another look at the solutions space in terms of each objective in the multi-objective fitness function. With the same fitness scale in place, this scatter plot highlights the linear correlation between the number of satellites and revisits achieved by each constellation. As the number of satellites reduces, dropping the overall fitness of a constellation solution, the revisit fitness value increases. Again, this occurs because the revisit fitness value is reliant on using the maximum number of satellites allowable. This correlation shows how the components of the multi-objective fitness function are intertwined and dependent on one another. For this reason, the fitness of any constellation will not drop below 0.33.

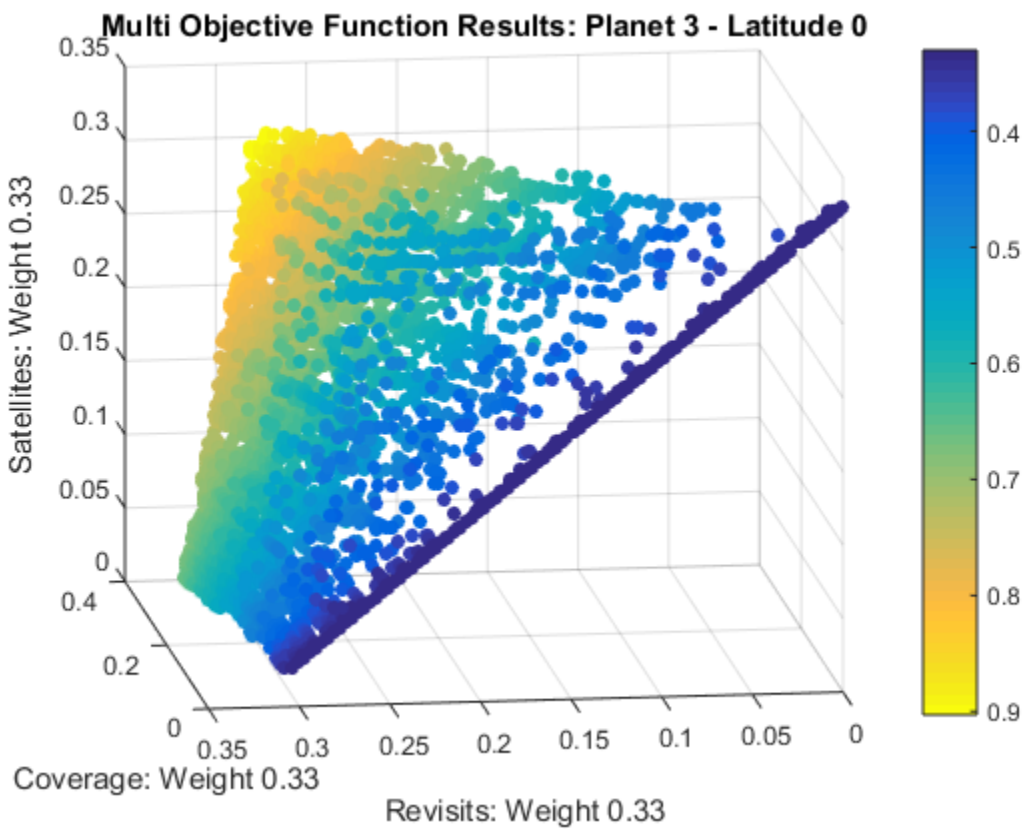

Figure 3.2 *MULTI-OBJECTIVE FITNESS SOLUTION SPACE FOR EARTH TARGET AT 0° LATITUDE*

Table 3.2 *Optimal Constellation Designs Found for Earth Target at 15° Latitude*

Earth: Target 15° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
989.10	1.66	93.00	1.00	0.00	0.33	0.00	0.02	0.31
914.00	3.57	84.00	7.00	0.00	0.33	0.00	0.05	0.28
684.24	0.02	93.00	1.00	0.00	0.33	0.00	0.02	0.31
940.33	0.02	93.00	31.00	0.00	0.33	0.00	0.02	0.31
940.33	2.31	93.00	3.00	0.00	0.33	0.00	0.02	0.31
684.24	0.02	93.00	1.00	0.00	0.33	0.00	0.02	0.31
915.95	5.25	93.00	1.00	0.00	0.33	0.00	0.02	0.31
940.33	5.25	93.00	1.00	0.00	0.33	0.00	0.02	0.31
907.52	0.02	84.00	1.00	0.00	0.33	0.00	0.05	0.28
940.33	2.91	93.00	1.00	0.00	0.33	0.00	0.02	0.31

Solutions for Earth targets at 15° latitude develop similar constellations to those found for equatorial targets. By using low inclinations, and maximum satellites, peak coverage and revisit performance can be achieved. However, to have line of sight with targets at 15° latitude each constellation needs to utilize higher altitudes to maintain a larger field of view. Increasing the inclinations for the constellations using a single plane would in fact degrade performance due to out of phase oscillations with the target.

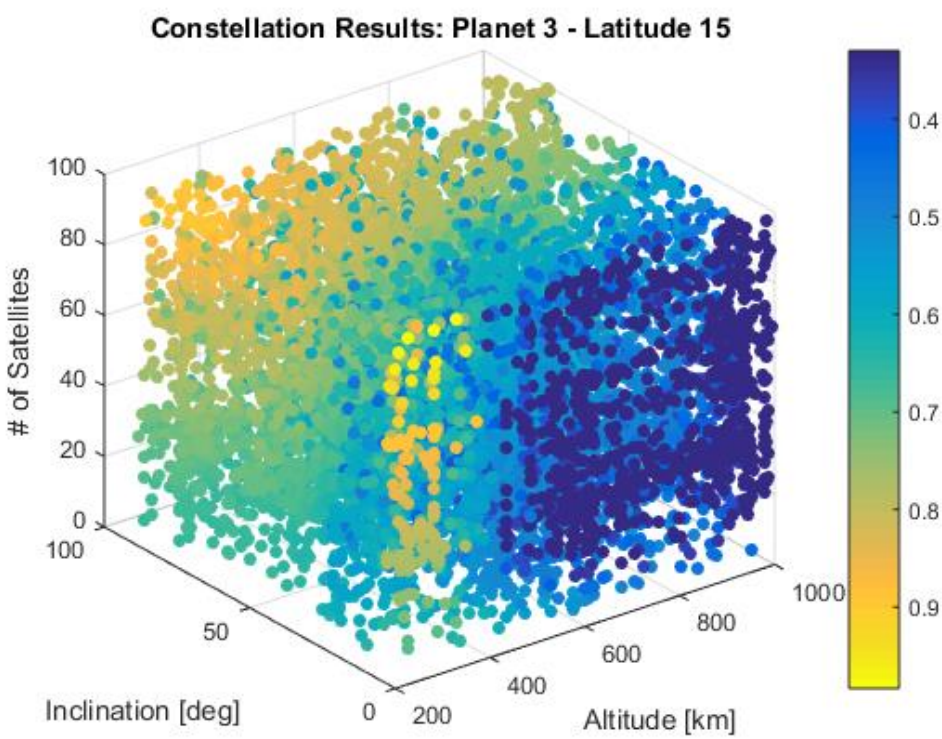

Figure 3.3 *Constellation Designs Solution Space for Earth Target at 15° Latitude*

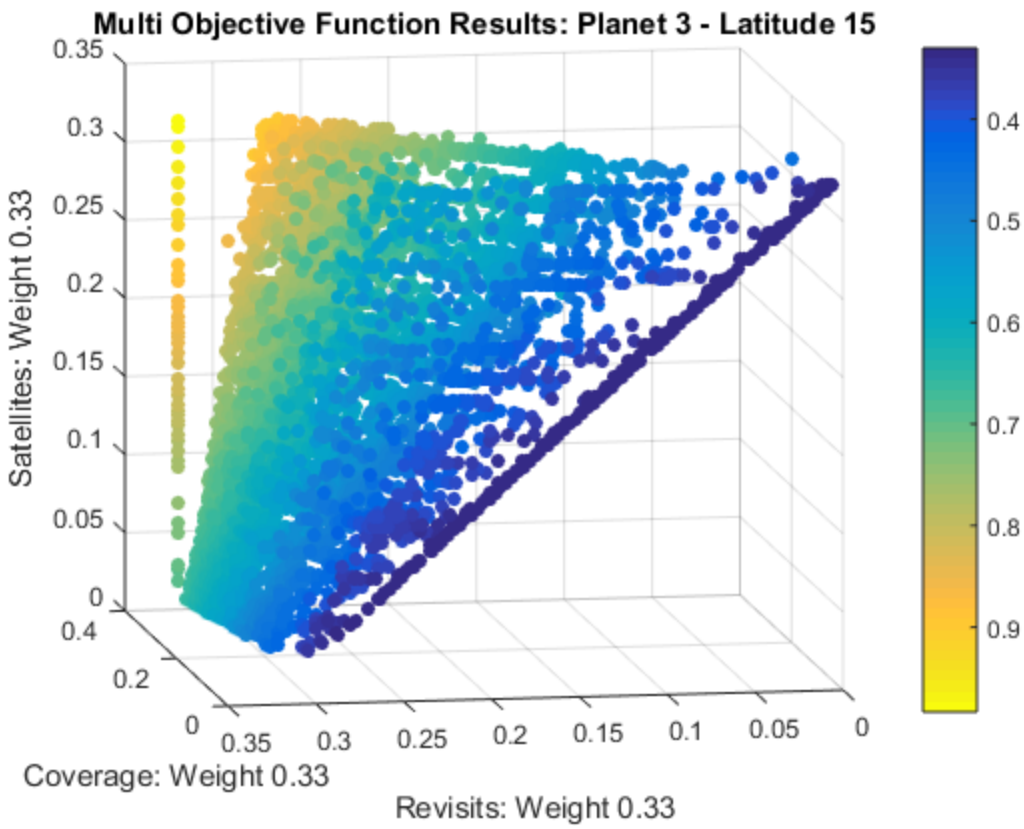

Figure 3.4 *MULTI-OBJECTIVE FITNESS SOLUTION SPACE FOR EARTH TARGET AT 15° LATITUDE*

Figure 3.3 and 3.4 above depicts the constellation design and multi-objective solution spaces for targets at 15° latitude. As explained, low inclinations and high altitudes play a key role in achieving peak performance in this scenario. This solution space is also clearly bounded by the inverse linear relationship between number of satellites and revisits. The vertical line of solutions on the left hand side indicate solutions that did not achieve any coverage or revisits.

Table 3.3 OPTIMAL CONSTELLATION DESIGNS FOUND FOR EARTH TARGET AT 30° LATITUDE

Earth: Target 30° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
956.72	47.32	20.00	2.00	0.00	0.36	0.00	0.29	0.07
961.20	43.44	20.00	2.00	0.00	0.36	0.00	0.29	0.07
932.96	41.96	25.00	25.00	13.00	0.37	0.00	0.29	0.08
961.20	37.63	20.00	2.00	0.00	0.37	0.01	0.30	0.07
961.20	40.17	26.00	2.00	1.00	0.37	0.00	0.28	0.09
899.58	40.17	26.00	2.00	0.00	0.37	0.00	0.29	0.09
995.41	24.83	27.00	9.00	0.00	0.38	0.00	0.29	0.09
669.23	42.34	28.00	28.00	10.00	0.38	0.00	0.28	0.09
899.58	46.58	30.00	30.00	4.00	0.38	0.00	0.28	0.10
826.22	39.97	31.00	31.00	13.00	0.38	0.00	0.28	0.10

Upon inspection, Table 3.3 shows that Earth targets at 30° latitude require constellations with multiple planes, high inclinations, and less satellites to minimize the multi-objective fitness function. With satellites spread-out on multiple planes more passes can be achieved over the target because of precession around the Earth's spin axis. The most common inclinations listed are above 30° to provide constellations with greater opportunities to supply total coverage with less satellites. By increasing constellation inclinations above a target's latitude each satellite can utilize both southern and northern passes. Notice how the one instance of a constellation with a lower inclination (24.83°) requires maximum altitude to maintain performance with the other constellations.

The two graphs below show the constellation and multi-objective solution spaces for Earth targets at 30° latitude. In Figure 3.5 solutions achieve better performance with inclinations near and above 30° and maximum altitudes. Figure 3.6 resembles the earlier multi-objective solution spaces, but with a less pronounced linear trend optimization line between satellites and revisits.

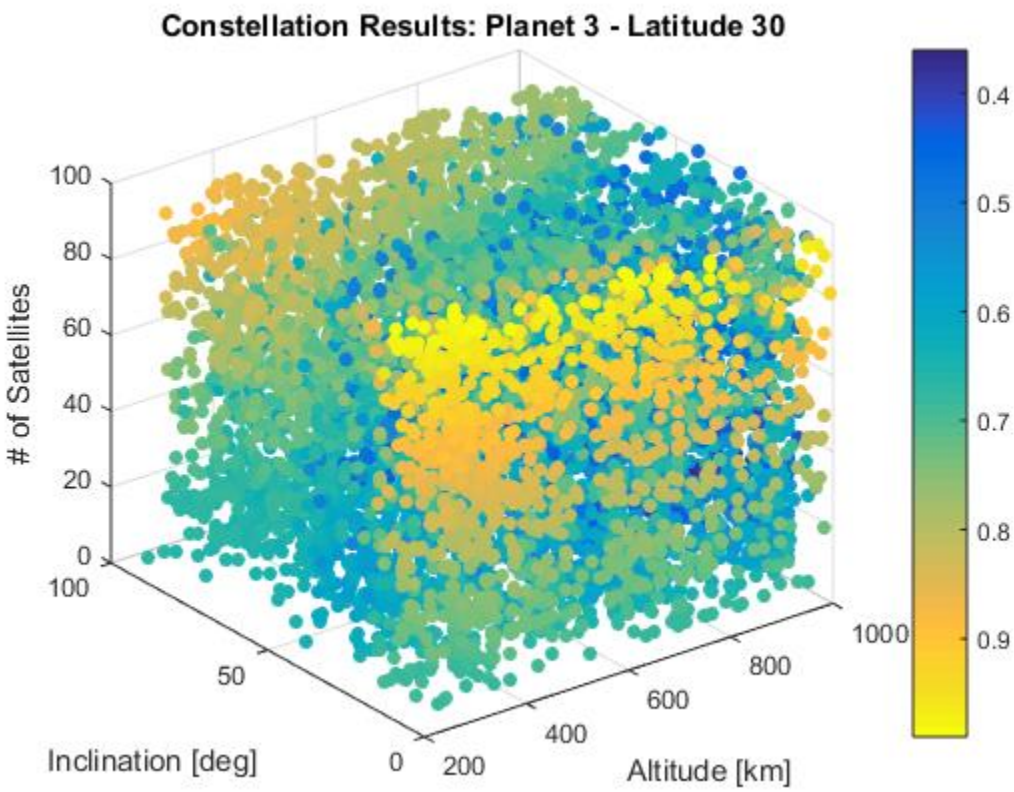

Figure 3.5 *CONSTELLATION DESIGN SOLUTION SPACE FOR EARTH TARGET AT 30° LATITUDE*

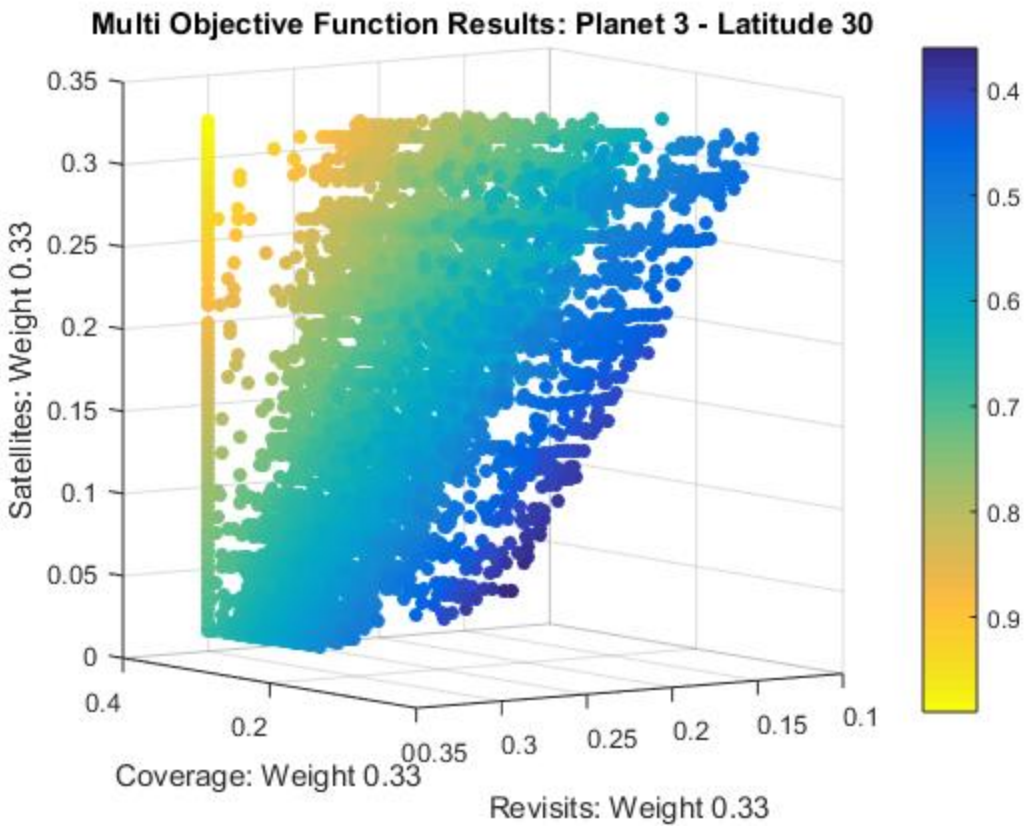

Figure 3.6 *MULTI-OBJECTIVE FITNESS SOLUTION SPACE FOR EARTH TARGET AT 30° LATITUDE*

Table 3.4 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR EARTH TARGET AT 45° LATITUDE*

Earth: Target 45° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
991.63	59.02	22.00	2.00	0.00	0.36	0.00	0.29	0.07
970.13	61.60	20.00	5.00	0.00	0.37	0.01	0.29	0.07
983.06	57.44	24.00	6.00	0.00	0.37	0.00	0.29	0.08
906.02	57.09	20.00	2.00	0.00	0.37	0.01	0.30	0.07
947.47	59.43	26.00	2.00	0.00	0.37	0.00	0.28	0.09
815.94	61.60	26.00	2.00	0.00	0.37	0.00	0.28	0.09
766.80	60.86	26.00	2.00	0.00	0.37	0.00	0.29	0.09
775.41	63.58	26.00	2.00	0.00	0.37	0.00	0.28	0.09
876.55	66.20	26.00	2.00	0.00	0.37	0.01	0.28	0.09
876.55	66.20	26.00	2.00	0.00	0.37	0.01	0.28	0.09

Similarly, targets at 45° latitude need constellations with inclinations greater than the target to maximize coverage during a given orbital period. Multiple planes are required to provide continuous coverage, but this induces a trade-off by reducing the number satellites and revisits. Figures 3.7 and 3.8 outline the constellation and multi-objective function solution spaces

for targets at 45° latitude (notice that the axes of inclination and altitude have been reversed to view the optimal constellations).

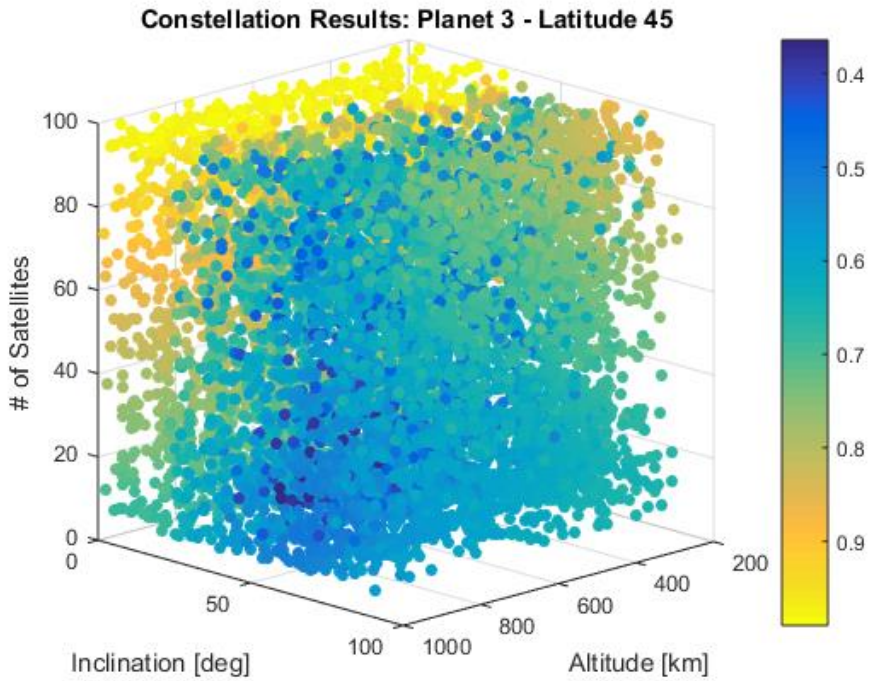

Figure 3.7 *CONSTELLATION DESIGN SOLUTION SPACE FOR EARTH TARGET AT 45° LATITUDE*

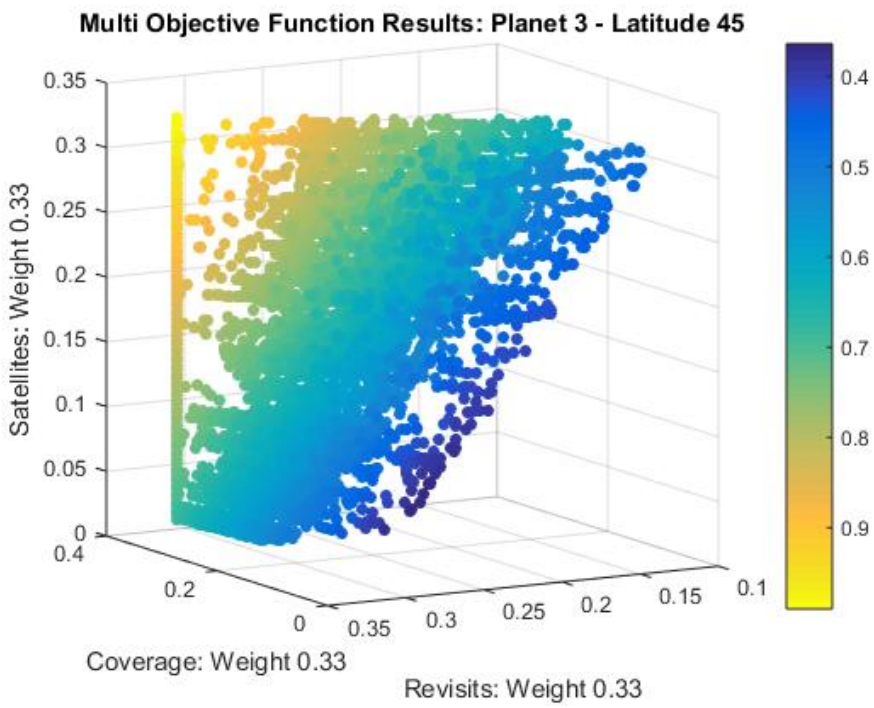

Figure 3.8 *MULTI-OBJECTIVE FITNESS SOLUTION SPACE FOR EARTH TARGET AT 45° LATITUDE*

Table 3.5 OPTIMAL CONSTELLATION DESIGNS FOUND FOR EARTH TARGET AT 60° LATITUDE

Earth: Target 60° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
790.26	80.16	20.00	5.00	0.00	0.35	0.00	0.29	0.07
880.33	77.81	24.00	4.00	0.00	0.36	0.00	0.28	0.08
892.05	77.78	26.00	2.00	0.00	0.36	0.00	0.27	0.09
731.21	77.81	24.00	6.00	0.00	0.36	0.00	0.28	0.08
962.06	71.69	24.00	6.00	0.00	0.36	0.00	0.28	0.08
960.35	79.10	34.00	2.00	0.00	0.37	0.00	0.25	0.11
953.18	78.59	35.00	5.00	0.00	0.37	0.00	0.25	0.12
853.38	80.16	35.00	5.00	0.00	0.37	0.00	0.25	0.12
880.33	77.81	34.00	2.00	1.00	0.37	0.00	0.26	0.11
897.41	78.63	35.00	7.00	0.00	0.37	0.00	0.25	0.12

For targets at 30°, 45°, and 60° latitude there are many similarities in the optimal constellation design found. Minimal satellites near 20-30 units, multiple planes, maximized altitudes, and inclinations larger than the target latitudes. The minimum fitness values also infer the difficulty of each target latitude. Lower latitudes could optimize to the minimal 0.33 values whereas higher latitude test cases require larger fitness values inferring more complex designs that require performance trade-offs. Figure 3.9 below shows the constellation design solution space. Figure 3.10 depicts the multi-objective function solution space

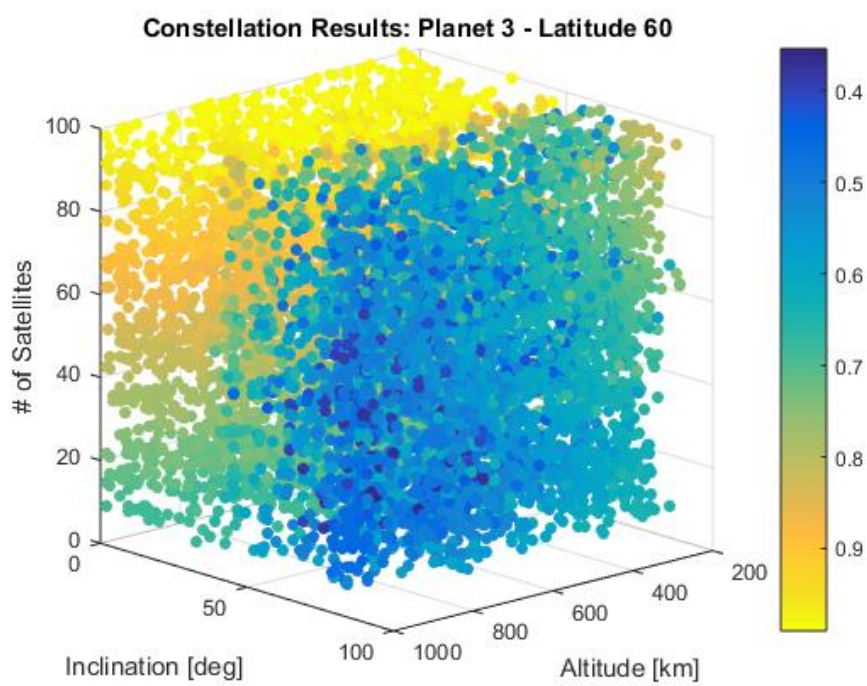

Figure 3.9 CONSTELLATION DESIGN SOLUTION SPACE FOR EARTH TARGET AT 60° LATITUDE

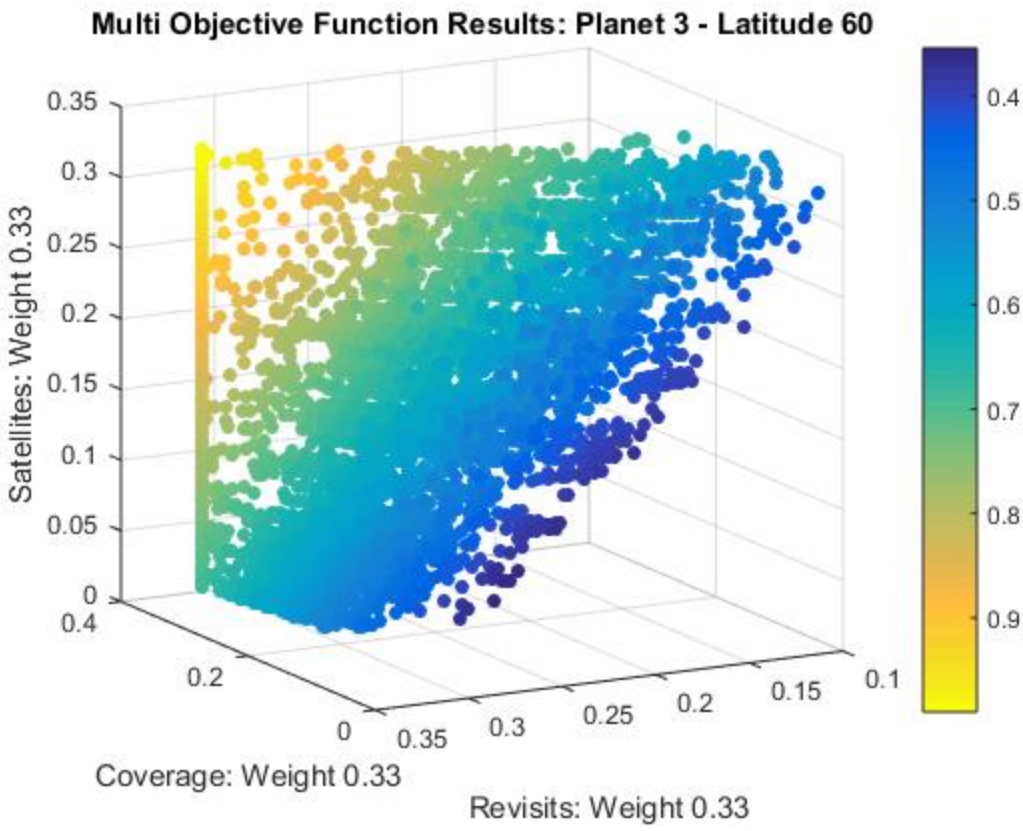

Figure 3.10 *MULTI-OBJECTIVE FITNESS SOLUTION SPACE FOR EARTH TARGET AT 60° LATITUDE*

Table 3.6 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR EARTH TARGET AT 75° LATITUDE*

Earth: Target 75° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
896.53	84.21	84.00	1.00	0.00	0.33	0.00	0.05	0.28
896.53	83.60	84.00	7.00	0.00	0.33	0.00	0.05	0.28
777.28	83.60	84.00	1.00	0.00	0.33	0.00	0.05	0.28
876.85	83.60	84.00	1.00	0.00	0.33	0.00	0.05	0.28
781.78	83.60	84.00	1.00	0.00	0.33	0.00	0.05	0.28
588.80	86.11	93.00	1.00	0.00	0.33	0.00	0.02	0.31
652.16	86.11	93.00	1.00	0.00	0.33	0.00	0.02	0.31
953.77	82.64	84.00	1.00	0.00	0.33	0.00	0.05	0.28
978.09	82.64	84.00	1.00	0.00	0.33	0.00	0.05	0.28
535.04	89.82	93.00	1.00	0.00	0.33	0.00	0.02	0.31

As target latitude rise the constellation inclination begins to approach 90°. This occurs for two reasons, first and foremost the inclination limit is bounded from 0° to 90° and secondly, targets travel less distance the closer they get to the poles allowing constellations to maintain coverage and revisits with high inclination satellites. Comparing Table 3.5 and Table 3.6 above

shows how constellations achieve minimum fitness again for targets at high latitudes with the use of maximum satellites on a single plane. The following graph shown in Figure 3.11 showcases this effect with higher inclinations and higher altitudes performing better than the rest of the constellation solution space. Figure 3.12 depicts a familiar multi-objective fitness function breakdown trend where optimal designs are found along the linear correlation between satellite and revisits.

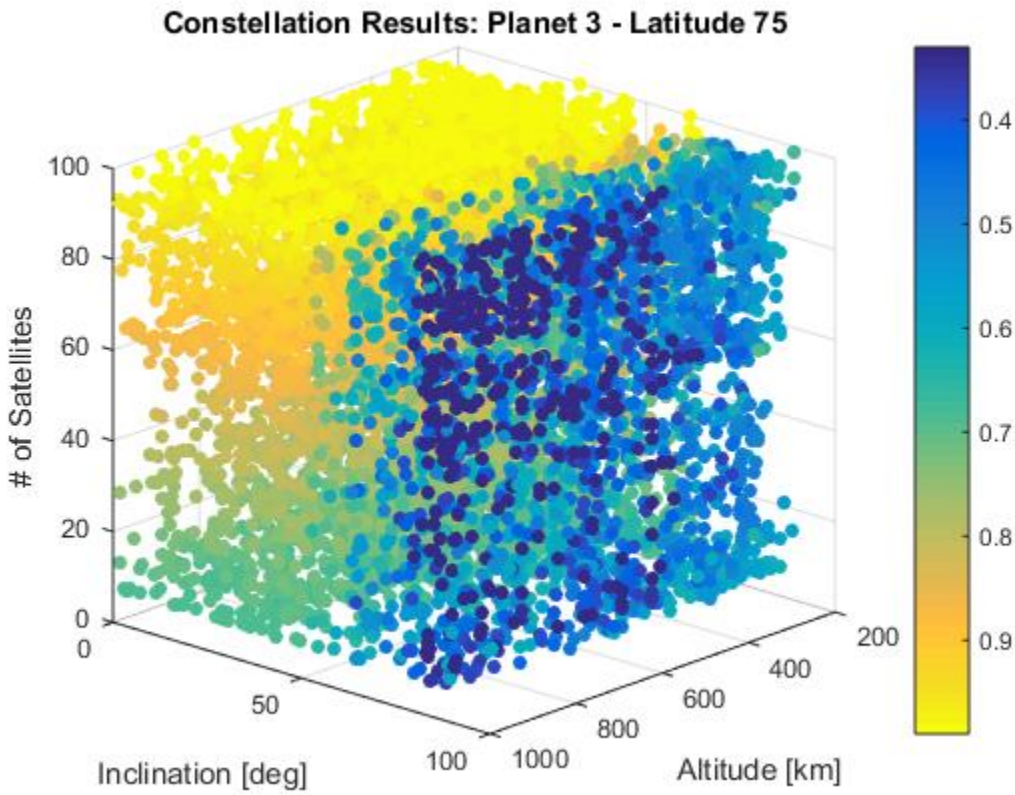

Figure 3.11 *CONSTELLATION DESIGN SOLUTION SPACE FOR EARTH TARGET AT 75° LATITUDE*

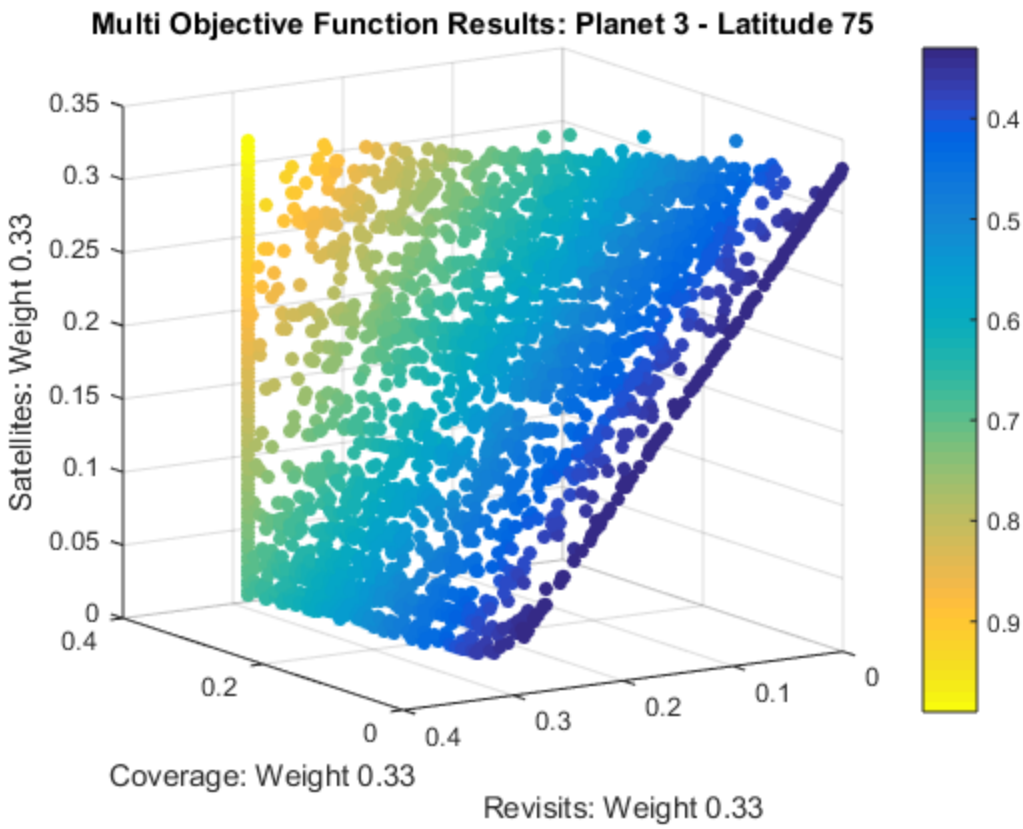

Figure 3.12 *MULTI-OBJECTIVE FITNESS SOLUTION SPACE FOR EARTH TARGET AT 75° LATITUDE*

Table 3.7 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR EARTH TARGET AT 90° LATITUDE*

Earth: Target 90° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
422.03	87.38	84.00	7.00	0.00	0.33	0.00	0.05	0.28
961.44	87.38	84.00	1.00	0.00	0.33	0.00	0.05	0.28
339.40	87.38	84.00	1.00	0.00	0.33	0.00	0.05	0.28
310.05	87.38	84.00	1.00	0.00	0.33	0.00	0.05	0.28
591.61	85.61	84.00	1.00	0.00	0.33	0.00	0.05	0.28
591.61	87.38	84.00	1.00	0.00	0.33	0.00	0.05	0.28
211.07	81.64	84.00	1.00	0.00	0.33	0.00	0.05	0.28
591.61	79.72	84.00	1.00	0.00	0.33	0.00	0.05	0.28
591.61	87.38	84.00	4.00	0.00	0.33	0.00	0.05	0.28
867.78	87.38	84.00	1.00	0.00	0.33	0.00	0.05	0.28

With a target on the Earth's north pole, the evolutionary genetic algorithm settled on multiple optimal constellations that vary greater with altitude. Inclinations stay near 90° to achieve sufficient pass time, and most constellations utilize a single plane to achieve full coverage. The number of satellites however seems to have oddly converged to 84 units in this

test case. This is a pure coincidence that occurred in the ordering of the results. Upon further inspection of all optimal constellations (fitness = .033), it was found that constellations with more and less satellites can achieve peak performance. Figure 3.13 shows the constellation solution space of this scenario where increased performance is clearly influenced by inclination. Figure 3.14 again shows the linear correlation between satellites and revisits in the multi-objective fitness function breakdown. The vertical line represent all the solutions that did not have sufficient inclinations to achieve target passes.

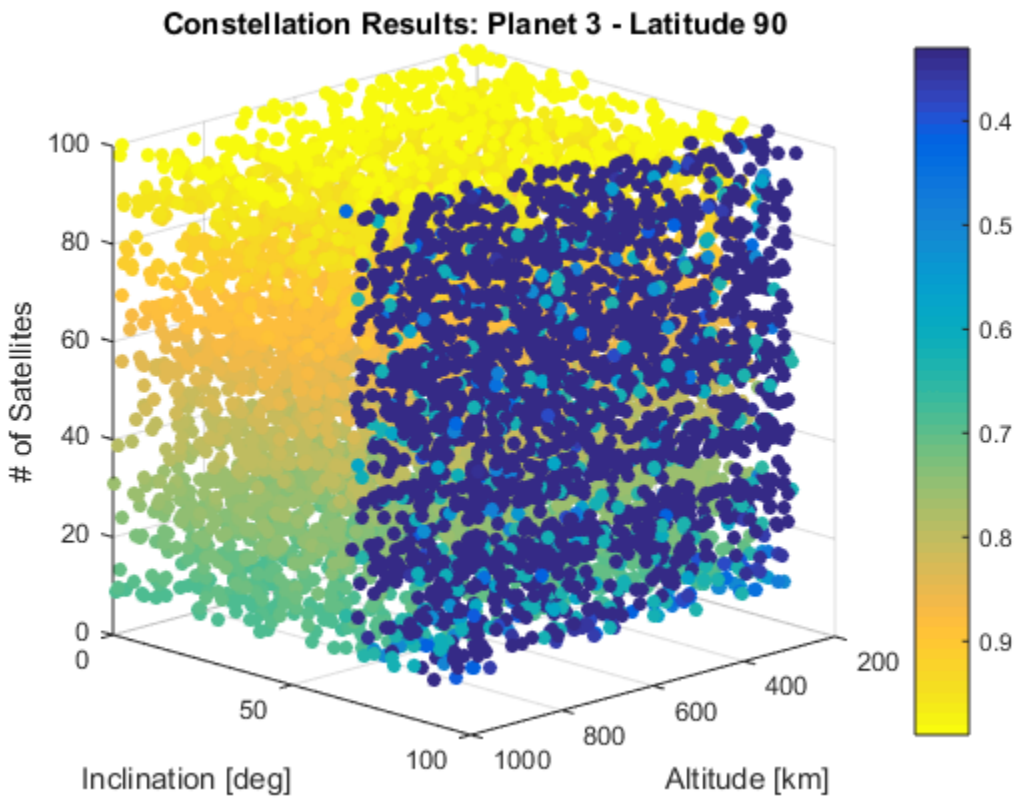

Figure 3.13 *CONSTELLATION DESIGN SOLUTION SPACE FOR EARTH TARGET AT 90° LATITUDE*

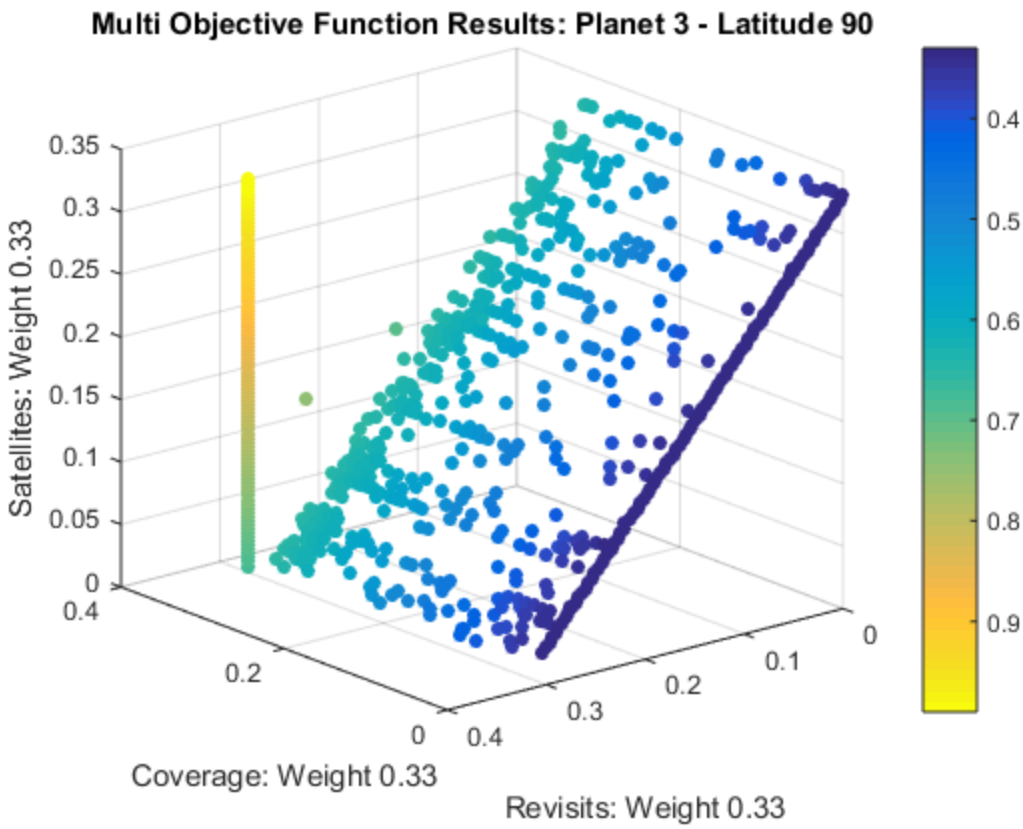

Figure 3.14 *MULTI-OBJECTIVE FITNESS SOLUTION SPACE FOR EARTH TARGET AT 90° LATITUDE*

3.2.2. Martian Constellations

With smaller planets like Mars, constellations will use different orbital parameters and organization schemes to achieve optimal performance compared to Earth constellation. Mainly attributed to the larger ratio between a planet's radius and the maximum constellation altitude available, Martian constellations can cover larger portions of the planet's surface by maximizing field of view. This effect allows for low inclination constellations around Mars to cover targets at higher latitudes compared to Earth. For instance, targets at 15° and 30° latitude can still be covered by near zero inclination constellations with maximum altitudes of 1000 km. Constellation designs in Tables 3.8 - 3.10 provide a closer look at some optimal solutions for Martian targets at 0°, 15°, and 30° latitude. Notice how the inclinations for each target latitude will converge towards 0° to rely on the coverage consistency with a single plane. Appendix B will contain all of the constellation design and multi-objective solution space scatter plots for Martian test cases to avoid redundancy and confusion with the previous section.

Table 3.8 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR MARS TARGET AT 0° LATITUDE*

Mars: Target 0° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
776.91	12.06	93.00	1.00	0.00	0.33	0.00	0.02	0.31
736.65	8.00	93.00	3.00	1.00	0.33	0.00	0.02	0.31
732.82	24.54	93.00	3.00	0.00	0.33	0.00	0.02	0.31
260.11	5.04	93.00	3.00	1.00	0.33	0.00	0.02	0.31
736.65	5.04	84.00	1.00	0.00	0.33	0.00	0.05	0.28
338.92	0.05	93.00	1.00	0.00	0.33	0.00	0.02	0.31
264.53	9.32	93.00	1.00	0.00	0.33	0.00	0.02	0.31
880.55	28.33	93.00	3.00	0.00	0.33	0.00	0.02	0.31
880.55	1.98	93.00	1.00	0.00	0.33	0.00	0.02	0.31
648.78	12.23	93.00	1.00	0.00	0.33	0.00	0.02	0.31

Table 3.9 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR MARS TARGET AT 15° LATITUDE*

Mars: Target 15° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
977.29	11.43	93.00	31.00	25.00	0.33	0.00	0.02	0.31
460.63	2.11	93.00	1.00	0.00	0.33	0.00	0.02	0.31
997.10	6.31	93.00	1.00	0.00	0.33	0.00	0.02	0.31
737.81	2.11	93.00	1.00	0.00	0.33	0.00	0.02	0.31
943.50	6.31	93.00	31.00	0.00	0.33	0.00	0.02	0.31
986.74	2.11	93.00	31.00	0.00	0.33	0.00	0.02	0.31
737.81	8.61	93.00	1.00	0.00	0.33	0.00	0.02	0.31
717.55	2.11	93.00	3.00	0.00	0.33	0.00	0.02	0.31
943.50	8.61	93.00	1.00	0.00	0.33	0.00	0.02	0.31
737.81	8.61	93.00	3.00	0.00	0.33	0.00	0.02	0.31

Table 3.10 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR MARS TARGET AT 30° LATITUDE*

Mars: Target 30° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
975.91	0.53	13.00	1.00	0.00	0.33	0.00	0.29	0.04
993.70	0.72	83.00	1.00	0.00	0.33	0.00	0.06	0.27
934.48	0.51	32.00	1.00	0.00	0.33	0.00	0.23	0.11
922.13	0.95	27.00	1.00	0.00	0.33	0.00	0.24	0.09
948.00	0.69	61.00	1.00	0.00	0.33	0.00	0.13	0.20
929.90	0.11	39.00	1.00	0.00	0.33	0.00	0.20	0.13
948.00	2.05	50.00	50.00	0.00	0.33	0.00	0.17	0.17
948.00	0.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28
903.33	2.25	34.00	1.00	0.00	0.33	0.00	0.22	0.11
908.88	0.61	67.00	1.00	0.00	0.33	0.00	0.11	0.22

As targets on Mars rise to 30° and 45° latitude the total number of satellites per constellation drops significantly due to the reduced number of revisits that cannot be achieved with low inclinations. This effect lowers the satellite fitness value while raising the revisit fitness value to achieve optimal solutions as seen in Tables 3.10 and 3.11.

Table 3.11 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR MARS TARGET AT 45° LATITUDE*

Mars: Target 45° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
929.22	63.36	14.00	2.00	0.00	0.35	0.00	0.30	0.05
930.55	63.36	14.00	2.00	0.00	0.35	0.00	0.30	0.05
908.05	63.36	14.00	2.00	0.00	0.35	0.00	0.30	0.05
828.81	46.45	16.00	16.00	2.00	0.36	0.00	0.30	0.05
898.15	62.29	20.00	5.00	0.00	0.36	0.00	0.29	0.07
908.05	43.78	14.00	2.00	0.00	0.36	0.01	0.31	0.05
722.55	53.95	22.00	2.00	0.00	0.36	0.00	0.29	0.07
908.05	42.84	14.00	2.00	1.00	0.36	0.01	0.31	0.05
866.33	58.87	25.00	5.00	0.00	0.36	0.00	0.28	0.08
881.14	56.42	25.00	5.00	0.00	0.36	0.00	0.28	0.08

For targets at 60°, 75°, and 90° latitude, however, the relationship between revisits and satellites becomes arbitrary. Each of the following three subcases can achieve optimal fitness by increasing or reducing the number of satellites in any given constellation since large satellites with max altitudes can exploit large fields of view to provide full coverage. Since Mars is small in comparison to Earth, fewer units are required to obtain full coverage, resulting in fluctuating constellation sizes. Reference Tables 3.12-3.14 below.

Table 3.12 OPTIMAL CONSTELLATION DESIGNS FOUND FOR MARS TARGET AT 60° LATITUDE

Mars: Target 60° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
926.52	88.13	70.00	35.00	1.00	0.33	0.00	0.10	0.23
863.95	88.13	70.00	1.00	0.00	0.33	0.00	0.10	0.23
803.13	89.77	52.00	13.00	0.00	0.33	0.00	0.16	0.17
952.17	89.26	52.00	1.00	0.00	0.33	0.00	0.16	0.17
965.63	87.57	52.00	1.00	0.00	0.33	0.00	0.16	0.17
995.54	87.57	52.00	1.00	0.00	0.33	0.00	0.16	0.17
952.17	88.74	52.00	1.00	0.00	0.33	0.00	0.16	0.17
926.52	88.13	52.00	2.00	0.00	0.33	0.00	0.16	0.17
997.28	88.13	52.00	2.00	0.00	0.33	0.00	0.16	0.17
997.28	85.97	52.00	1.00	0.00	0.33	0.00	0.16	0.17

Table 3.13 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR MARS TARGET AT 75° LATITUDE*

Mars: Target 75° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
557.66	85.77	84.00	7.00	0.00	0.33	0.00	0.05	0.28
550.81	85.77	84.00	1.00	0.00	0.33	0.00	0.05	0.28
550.81	88.11	84.00	1.00	0.00	0.33	0.00	0.05	0.28
541.54	88.11	93.00	1.00	0.00	0.33	0.00	0.02	0.31
374.48	85.77	84.00	1.00	0.00	0.33	0.00	0.05	0.28
353.21	88.11	84.00	1.00	0.00	0.33	0.00	0.05	0.28
648.79	85.77	93.00	1.00	0.00	0.33	0.00	0.02	0.31
911.58	74.42	84.00	1.00	0.00	0.33	0.00	0.05	0.28
550.81	85.77	84.00	1.00	0.00	0.33	0.00	0.05	0.28
798.78	80.41	93.00	3.00	0.00	0.33	0.00	0.02	0.31

Table 3.14 OPTIMAL CONSTELLATION DESIGNS FOUND FOR MARS TARGET AT 90° LATITUDE

Mars: Target 90° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
485.57	83.11	93.00	93.00	14.00	0.33	0.00	0.02	0.31
541.01	86.33	84.00	1.00	0.00	0.33	0.00	0.05	0.28
541.01	69.12	84.00	28.00	2.00	0.33	0.00	0.05	0.28
305.53	77.04	84.00	2.00	1.00	0.33	0.00	0.05	0.28
884.16	77.04	84.00	1.00	0.00	0.33	0.00	0.05	0.28
481.89	86.33	84.00	2.00	0.00	0.33	0.00	0.05	0.28
555.42	77.04	84.00	7.00	0.00	0.33	0.00	0.05	0.28
551.77	86.33	84.00	2.00	0.00	0.33	0.00	0.05	0.28
305.53	77.04	84.00	1.00	0.00	0.33	0.00	0.05	0.28
821.10	67.48	84.00	1.00	0.00	0.33	0.00	0.05	0.28

3.2.3. Jupiter Constellations

Switching to Jupiter, which is volumetrically massive compared to both Earth and Mars, optimal constellations were found to closely correlate latitude and inclination to achieve optimal coverage performance. Due to the increased ratio between planetary radius and maximum constellation altitude of 1000 km, a single satellite's field of view will cover a significantly smaller portion of Jupiter's surface area compared to Earth or Mars. Table 3.15 shows the tradeoff between minimizing number of satellites and obtaining full coverage for an equatorial target on Jupiter.

Table 3.15 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR JUPITER TARGET AT 0° LATITUDE*

Jupiter: Target 0° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
975.17	2.46	37.00	1.00	0.00	0.36	0.00	0.24	0.12
962.65	2.46	37.00	1.00	0.00	0.36	0.00	0.24	0.12
956.56	2.52	37.00	1.00	0.00	0.36	0.00	0.24	0.12
978.54	3.09	37.00	37.00	0.00	0.36	0.00	0.24	0.12
949.34	0.88	41.00	1.00	0.00	0.37	0.00	0.23	0.14
978.54	3.30	37.00	1.00	0.00	0.37	0.00	0.24	0.12
978.54	3.30	37.00	1.00	0.00	0.37	0.00	0.24	0.12
975.17	3.30	37.00	37.00	0.00	0.37	0.00	0.24	0.12
975.17	3.30	37.00	1.00	0.00	0.37	0.00	0.24	0.12
975.17	0.91	42.00	1.00	0.00	0.37	0.00	0.23	0.14

Table 3.16 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR JUPITER TARGET AT 15° LATITUDE*

Jupiter: Target 15° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
984.97	20.45	87.00	29.00	0.00	0.64	0.10	0.25	0.29
972.34	19.48	32.00	1.00	0.00	0.64	0.24	0.30	0.11
954.17	17.85	35.00	1.00	0.00	0.65	0.23	0.30	0.12
989.68	16.56	30.00	3.00	0.00	0.65	0.24	0.31	0.10
972.34	18.63	95.00	5.00	0.00	0.65	0.08	0.25	0.31
954.17	17.27	30.00	1.00	0.00	0.65	0.24	0.31	0.10
954.17	17.27	30.00	1.00	0.00	0.65	0.24	0.31	0.10
954.17	16.40	59.00	59.00	6.00	0.65	0.17	0.29	0.19
995.12	20.34	28.00	1.00	0.00	0.65	0.25	0.31	0.09
920.68	16.56	33.00	1.00	0.00	0.65	0.24	0.31	0.11

Once target latitudes rise above Jupiter's equator, full coverage is no longer achievable without increasing the number of satellites and simultaneously decreasing revisit performance. Since satellites will being to oscillate out of phase when planes are inclined continuous coverage requires numerous satellites which will in turn increase fitness values on two fronts. In Table 3.16 constellation inclination settles above 15° to allow for coverage opportunities before and after a satellite reaches its highest latitude in the celestial sphere. This effect can be seen throughout constellations covering targets at latitudes 15°, 30°, 45°, 60°, and 75°.

Table 3.17 OPTIMAL CONSTELLATION DESIGNS FOUND FOR JUPITER TARGET AT 30° LATITUDE

Jupiter: Target 30° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
994.59	33.69	14.00	1.00	0.00	0.66	0.30	0.32	0.05
934.50	34.89	7.00	7.00	1.00	0.66	0.31	0.32	0.02
994.59	32.80	16.00	1.00	0.00	0.66	0.29	0.32	0.05
992.78	33.07	19.00	1.00	0.00	0.66	0.28	0.32	0.06
994.59	32.30	14.00	1.00	0.00	0.66	0.30	0.32	0.05
939.31	30.56	6.00	1.00	0.00	0.66	0.32	0.33	0.02
946.64	30.56	6.00	1.00	0.00	0.66	0.32	0.33	0.02
994.59	29.64	6.00	1.00	0.00	0.66	0.32	0.33	0.02
962.35	34.43	18.00	1.00	0.00	0.67	0.29	0.32	0.06
939.31	34.43	15.00	1.00	0.00	0.67	0.30	0.32	0.05

Table 3.18 OPTIMAL CONSTELLATION DESIGNS FOUND FOR JUPITER TARGET AT 45° LATITUDE

Jupiter: Target 45° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
988.49	48.58	6.00	6.00	4.00	0.66	0.32	0.33	0.02
910.61	48.58	6.00	1.00	0.00	0.66	0.32	0.33	0.02
914.91	48.58	6.00	6.00	0.00	0.66	0.32	0.33	0.02
908.72	48.59	6.00	6.00	0.00	0.66	0.32	0.33	0.02
982.77	48.58	11.00	1.00	0.00	0.66	0.31	0.32	0.04
982.77	48.58	11.00	1.00	0.00	0.66	0.31	0.32	0.04
899.69	46.97	6.00	1.00	0.00	0.66	0.32	0.33	0.02
812.40	48.58	6.00	1.00	0.00	0.66	0.32	0.33	0.02
797.28	48.58	6.00	1.00	0.00	0.66	0.32	0.33	0.02
797.28	48.58	6.00	2.00	0.00	0.67	0.32	0.33	0.02

Table 3.19 OPTIMAL CONSTELLATION DESIGNS FOUND FOR JUPITER TARGET AT 60° LATITUDE

Jupiter: Target 60° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
986.99	64.58	77.00	11.00	0.00	0.66	0.14	0.26	0.25
966.88	64.58	8.00	1.00	0.00	0.66	0.31	0.32	0.03
974.78	62.99	8.00	1.00	0.00	0.66	0.31	0.32	0.03
963.57	64.58	20.00	1.00	0.00	0.66	0.28	0.31	0.07
995.95	62.49	16.00	1.00	0.00	0.66	0.29	0.32	0.05
980.99	62.99	29.00	1.00	0.00	0.66	0.26	0.31	0.10
980.99	62.99	24.00	1.00	0.00	0.66	0.27	0.31	0.08
939.31	62.99	8.00	1.00	0.00	0.66	0.31	0.32	0.03
939.31	62.99	8.00	1.00	0.00	0.66	0.31	0.32	0.03
974.78	62.99	24.00	1.00	0.00	0.66	0.27	0.31	0.08

Constellations shown in Tables 3.17 - 3.19 depict the most optimal constellations for targets on Jupiter's surface at latitudes 30°, 45°, and 60°. Each of these test cases present a unique solution that is unlike the Earth of Martian solutions presented. In these three test cases optimization is achieved by foregoing coverage and revisit maximization, and instead minimizing the total number of satellites used in the constellations. With a larger planet to orbit the number of passes becomes very infrequent at mid-range latitudes, which in turn results in reduced coverage. So the multi-objective algorithm compensates by reducing both the number of satellites and revisits since better fitness performance can be achieved with minimal satellites.

Table 3.20 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR EARTH TARGET AT 75° LATITUDE*

Jupiter: Target 75° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
987.32	79.86	99.00	9.00	0.00	0.56	0.03	0.20	0.33
964.58	77.97	96.00	3.00	0.00	0.56	0.02	0.22	0.32
995.01	79.57	78.00	39.00	10.00	0.57	0.08	0.23	0.26
987.32	77.85	64.00	8.00	0.00	0.58	0.12	0.26	0.21
987.32	77.85	64.00	2.00	0.00	0.58	0.12	0.26	0.21
968.21	79.86	54.00	6.00	0.00	0.59	0.15	0.26	0.18
987.32	77.85	60.00	3.00	0.00	0.59	0.13	0.26	0.20
987.32	79.86	49.00	7.00	0.00	0.59	0.17	0.27	0.16
984.14	77.71	56.00	14.00	2.00	0.59	0.14	0.27	0.18
987.32	77.85	80.00	2.00	0.00	0.60	0.10	0.24	0.26

Both 15° and 75° constellations exhibit an odd dichotomy of constellations that can be optimized with maximum and minimum satellites. Tables 3.16 and 3.20 show these constellation design assortments and their respective fitness break downs.

In Table 3.21 targets at Jupiter's north pole are optimized by constellations using inclinations of ~90°. By maximizing coverage, and increasing the number of revisits, polar orbit constellations should provide the most optimal constellation designs around Jupiter. It should also be noted that the top ten results listed below indicate that 84 satellites provide the most optimal constellation. However, upon further result inspection it was found that this test case could be optimized with any number of satellites if they provided continuous coverage. The evolutionary genetic algorithm was merely saturated with optimal results containing 84 satellites.

Table 3.21 *OPTIMAL CONSTELLATION DESIGNS FOUND FOR EARTH TARGET AT 90° LATITUDE*

Jupiter: Target 90° Latitude								
Alt (km)	Inc (deg)	T	P	F	Fitness	Cov. Fit	Rev. Fit	Sat. Fit
657.59	89.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28
947.62	85.15	84.00	1.00	0.00	0.33	0.00	0.05	0.28
287.72	89.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28
804.65	89.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28
947.62	85.15	84.00	1.00	0.00	0.33	0.00	0.05	0.28
720.14	89.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28
929.68	89.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28
804.65	89.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28
488.78	89.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28
657.59	89.63	84.00	1.00	0.00	0.33	0.00	0.05	0.28

4. CONCLUSION

4.1. Future Works

1) Explore mission sensitivity studies by varying constellation limitations and multi-objective fitness function scope. The results presented in the previous sections provide a narrow view into the capabilities of constellation optimization. By reporting on numerous mission types with various requirements one could develop a more comprehensive constellation guide or universal multi-objective fitness function.
2) Implement a Pareto-Frontier based optimization scheme with the NSGA-II method or similar. The optimization scheme used in this study relied on a pure fitness value comparison with three relatively simple objectives. Creating a more complex system that relies on multiple fronts could lead to more robust constellation results to be compared by the user.
3) Explore the effects of alternative mutation rates and methods in the evolutionary genetic algorithm. Perhaps introducing weighted probability selection for individuals or traits based on performance.
4) Explore the effects of different recombination techniques such as multiple point cross-over or simple arithmetic recombination with pairs of parent selected traits. The mixture of traits could also be changed to reward higher performing individuals of the populations.
5) Alter the spherical trigonometry propagator to evaluate full planetary coverage instead of regional coverage. The same calculation techniques could be changed to evaluate constellation coverage over an entire planet instead of just a small target or single latitude.
6) Expand the spherical trigonometry propagation capabilities to include elliptical constellations. With reduced computations necessary, the spherical trigonometry methods outlined in this paper could prove to be an efficient constellation evaluation tool for elliptical orbits. Be sure to implement perturbational effects.
7) Implementation of constellation perturbations. By considering how constellation patterns deform or maintain their shape and performance over time could provide insights for future constellation designers.
8) Conduct a gravitational parameter based constellation comparison between planets. The results in this study focused on constant constellation limitations to compare planetary systems, and the results were based heavily on the ratio between planetary radius and maximum altitude. By reworking these test cases to focus on the effects of gravity and orbital periods on constellation performance there may be significant conclusions to be drawn.
9) Expand the capabilities of the design tool to allow for multiple targets to be evaluated. Many missions rely on multiple ground stations, so determining optimal constellations for more complex systems could be of interest.

4.2. Conclusions

Overall, the results presented show a sub-optimal convergence of constellation design. The tables presented however do not represent the only optimal solutions. Upon further investigation, many of the test cases explored could achieve optimal solutions with a variety of satellites, inclinations, and altitudes, but these options were not listed due to MATLAB's standardized outputs. Earth at 90°, for example, does not require 84 satellites to be optimized, there were many optimal solutions found with any number of satellites per constellation. The genetic algorithm, however, archives all of the solutions to give the user a closer look at the performance metrics of each constellation. This in turn provides a partial Pareto-Frontier that can be interpreted through the solution spaces shown in the previous section. Users should refer to these constellation design solution spaces for insights regarding constellation design trades and performance.

4.3. Final Thoughts

The development and implementation of a mission level constellation design tool was a success. From the test cases explored, optimal constellations were achieved by using spherical trigonometry, an evolutionary genetic algorithm, and a multi-objective fitness function. The results however should not indicate optimal solutions for the given targets in all scenarios. The multi-objective function was the main driver for the solutions presented and any modifications or change of scope would drastically influence the results. It was found that the performance of two objectives used in this study were inversely related and closely tied to the number of satellites in each constellation. This aspect drove the solutions to unpredicted optimal convergence. Another influential design limitation highlighted was the ratio between planetary radius and maximum altitude. By restricting altitudes to less than 1000 km, optimal constellations varied quite a bit for each latitude test case on Earth, Mars and Jupiter.

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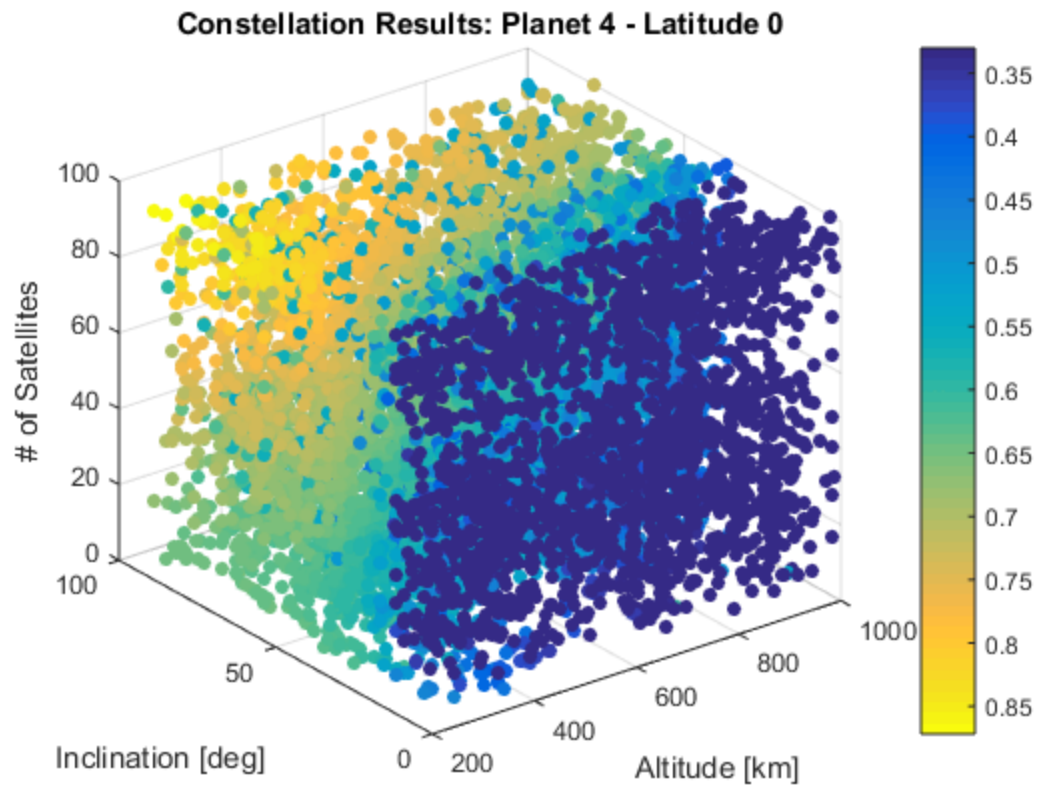
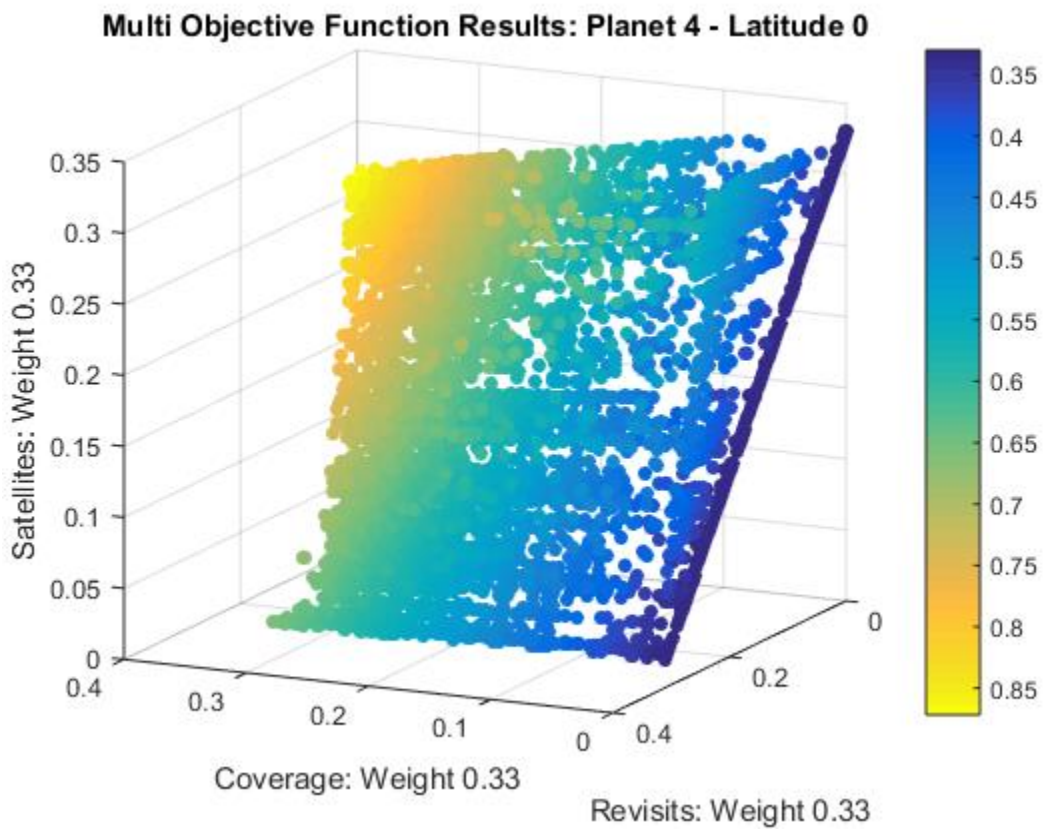
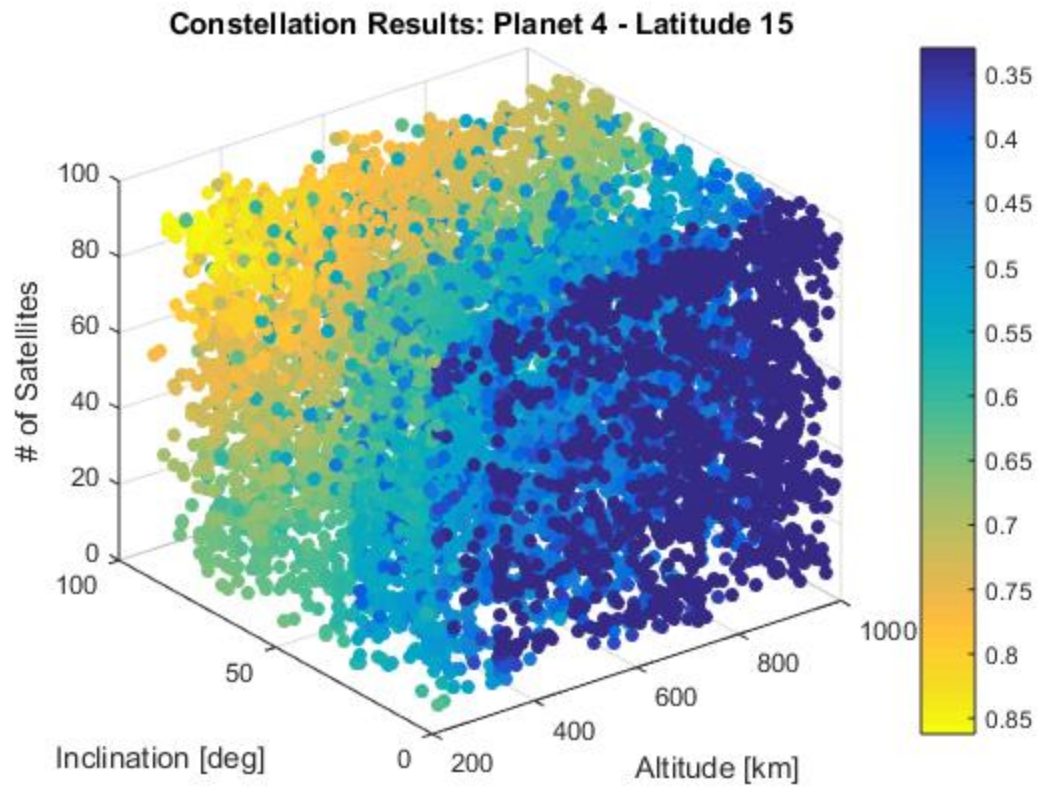
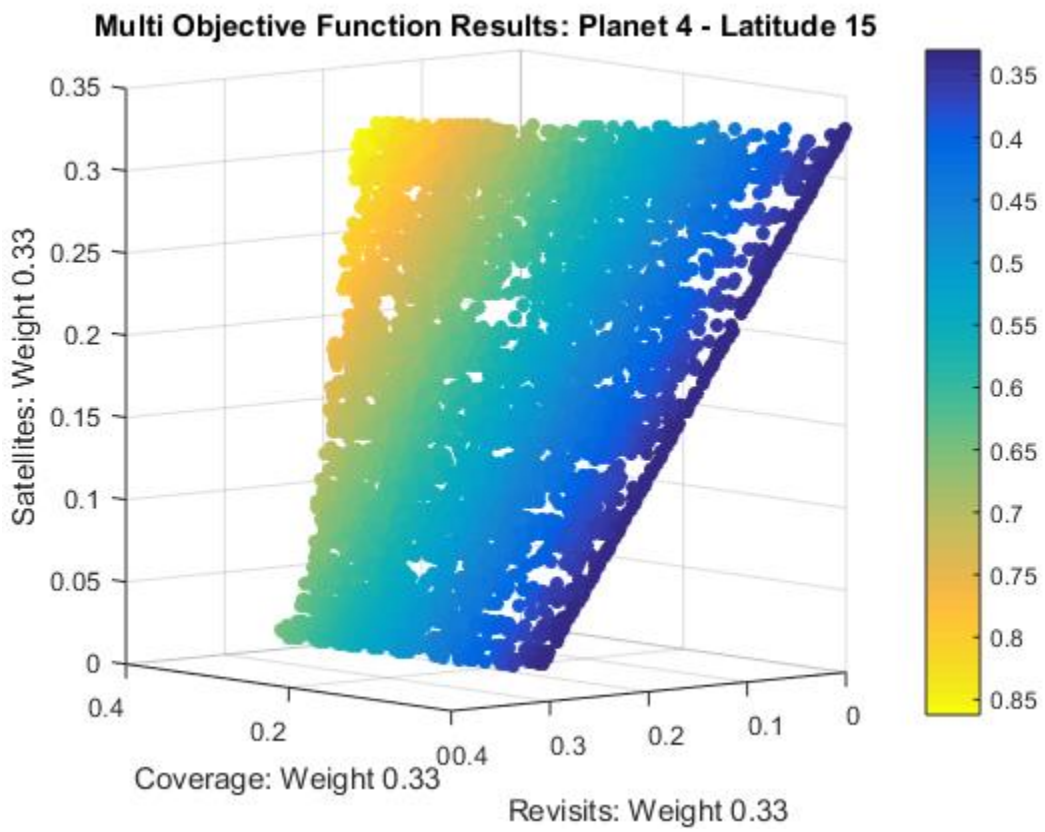
APPENDICES

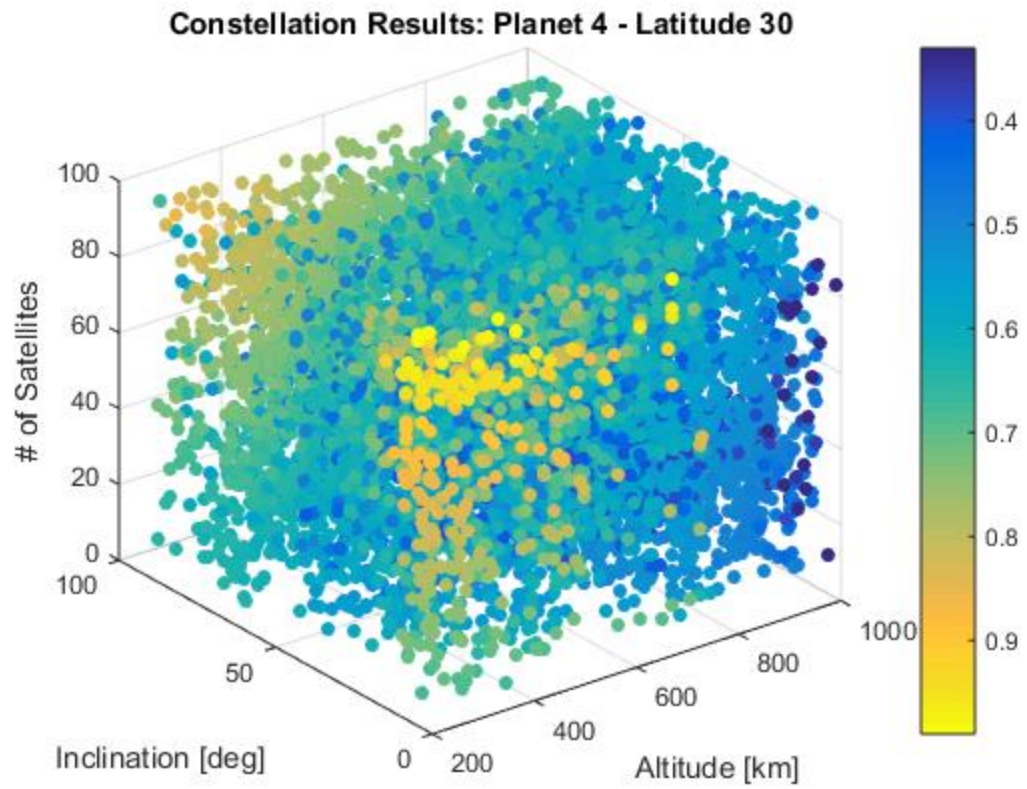
A. Result Scatter Plots

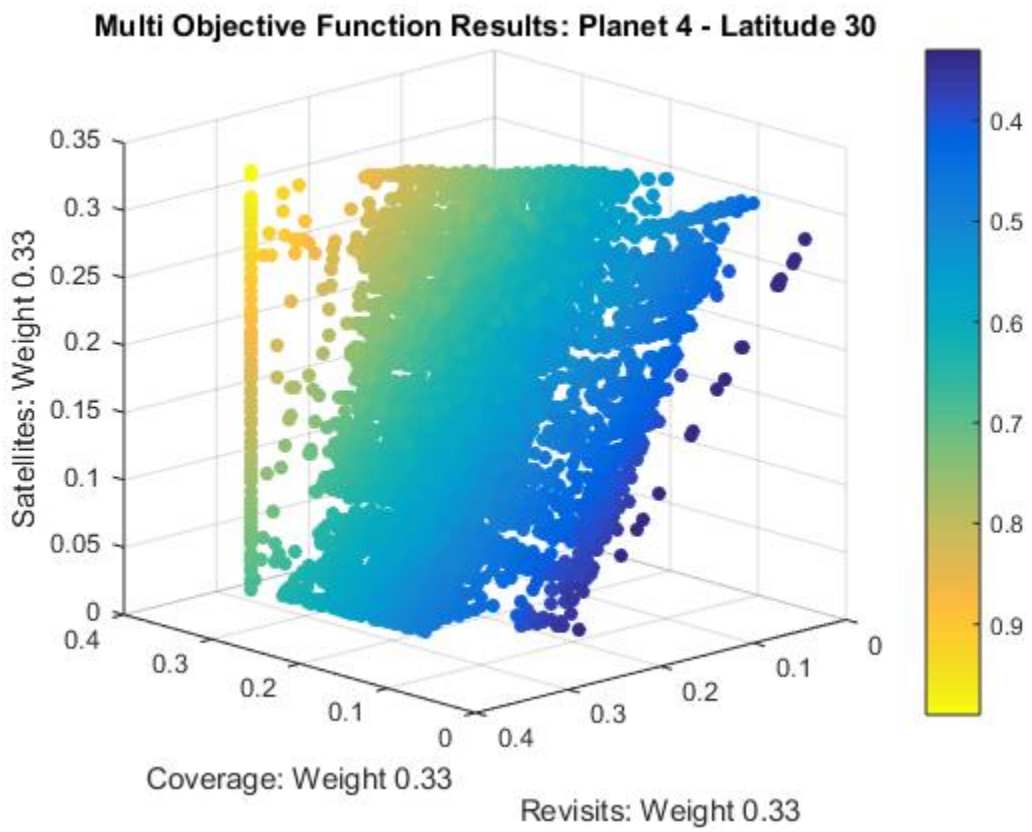

Constellation Results: Planet 4 - Latitude 0
# of Satellites
100
80
60
40
20
0
100
50
0
Inclination [deg]
200
400
600
800
1000
Altitude [km]
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85

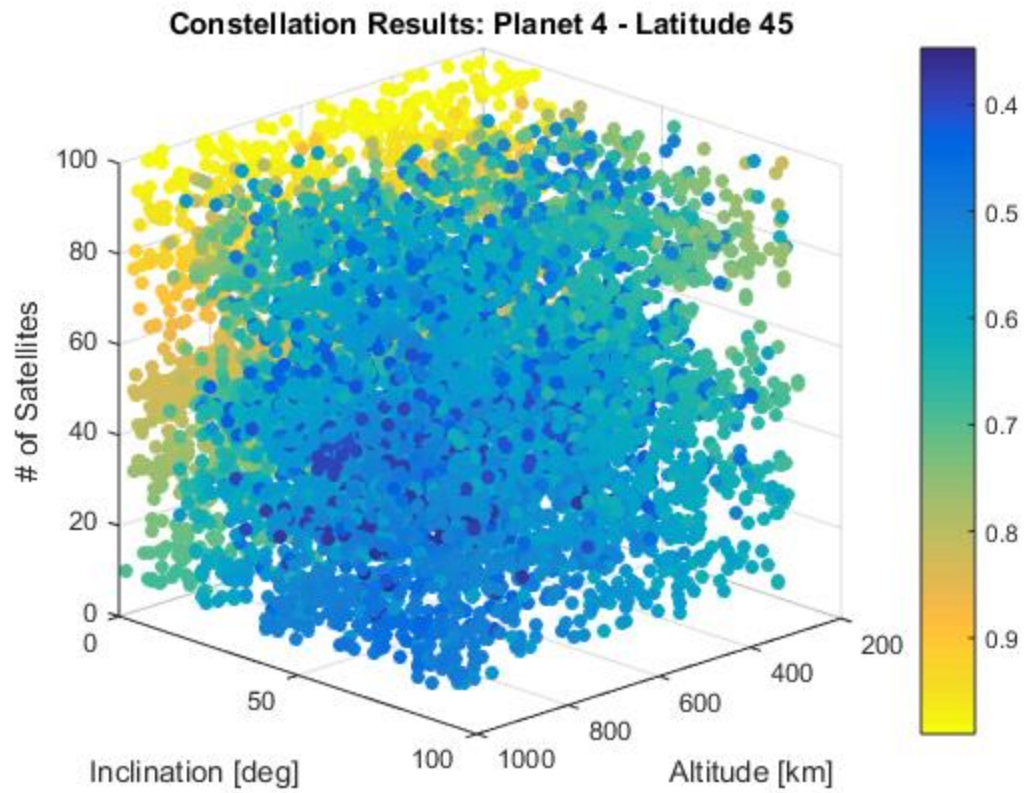

Multi Objective Function Results: Planet 4 - Latitude 0
Satellites: Weight 0.33
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.4
0.3
0.2
0.1
0
Coverage: Weight 0.33
0
0.2
0.4
Revisits: Weight 0.33
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85

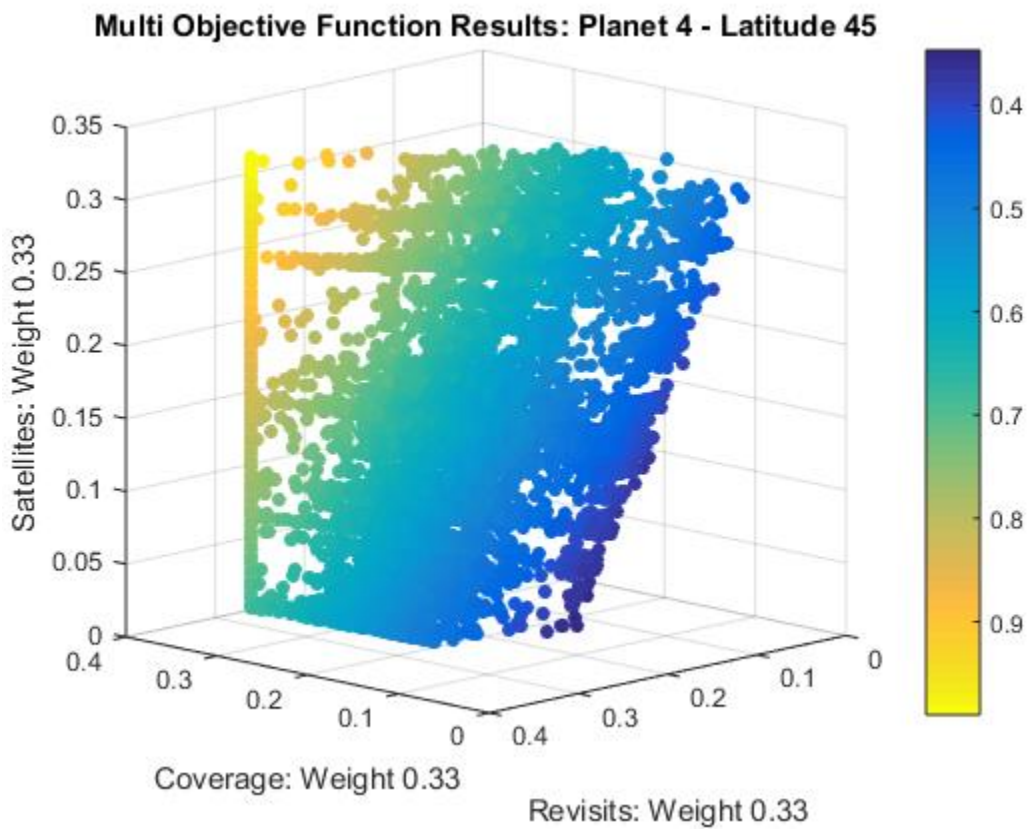

Constellation Results: Planet 4 - Latitude 15
# of Satellites
100
80
60
40
20
0
100
50
0
Inclination [deg]
200
400
600
800
1000
Altitude [km]
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85

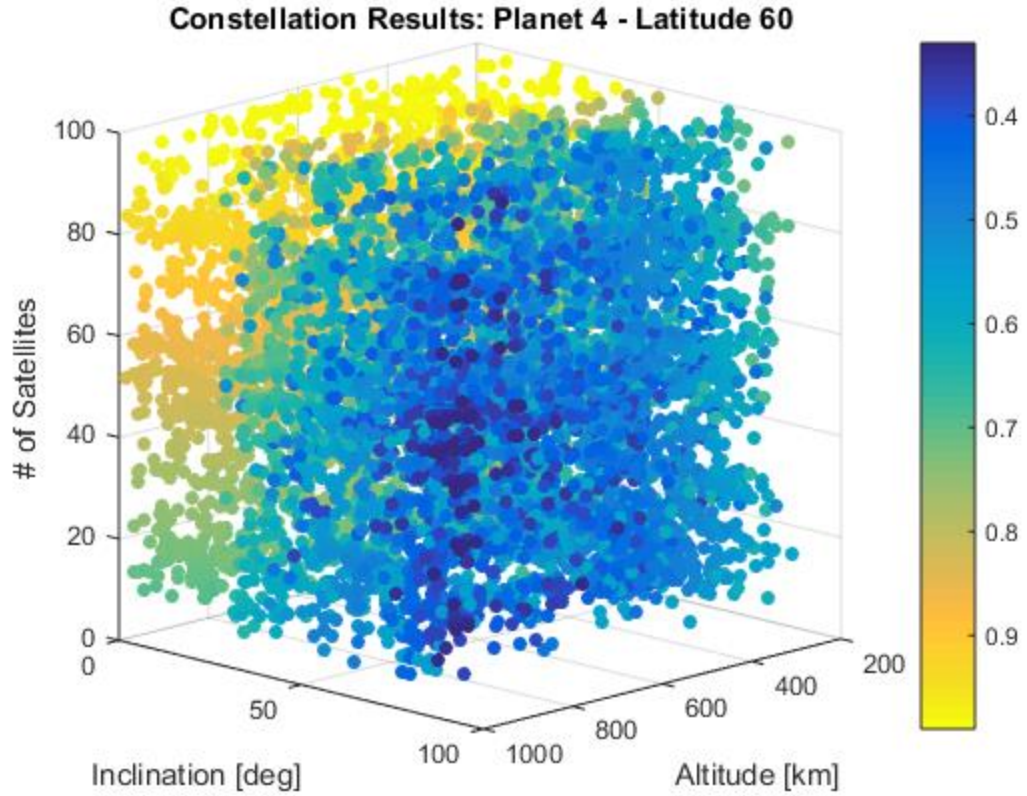

Multi Objective Function Results: Planet 4 - Latitude 15
Satellites: Weight 0.33
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.4
0.2
0
Coverage: Weight 0.33
0.4
0.3
0.2
0.1
0
Revisits: Weight 0.33
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85

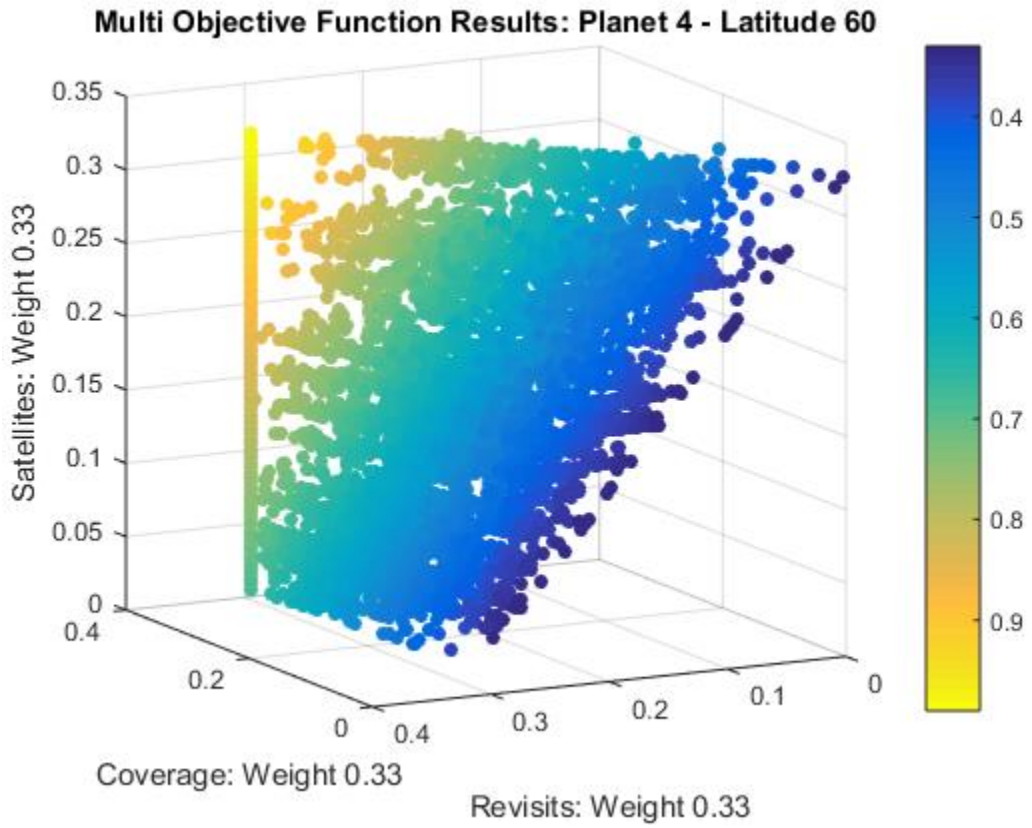

Constellation Results: Planet 4 - Latitude 30
# of Satellites
100
80
60
40
20
0
100
50
0
Inclination [deg]
200
400
600
800
1000
Altitude [km]
0.4
0.5
0.6
0.7
0.8
0.9

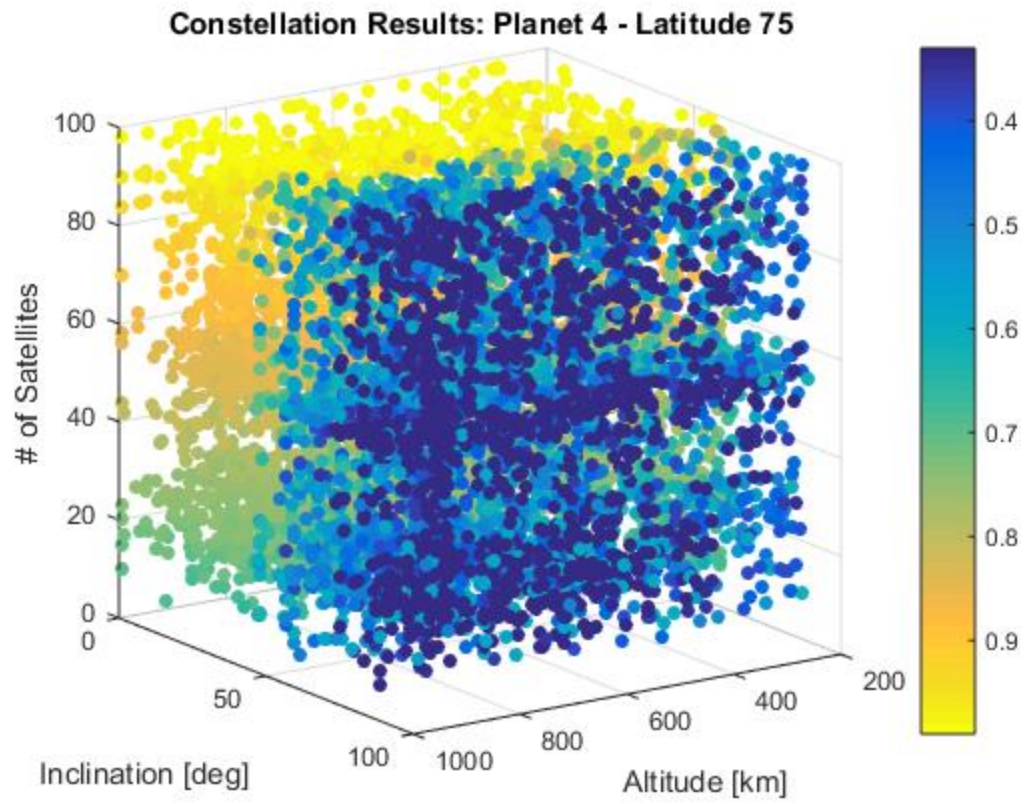

Multi Objective Function Results: Planet 4 - Latitude 30
Satellites: Weight 0.33
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.4
0.3
0.2
0.1
0
Coverage: Weight 0.33
0.4
0.3
0.2
0.1
0
Revisits: Weight 0.33
0.4
0.5
0.6
0.7
0.8
0.9

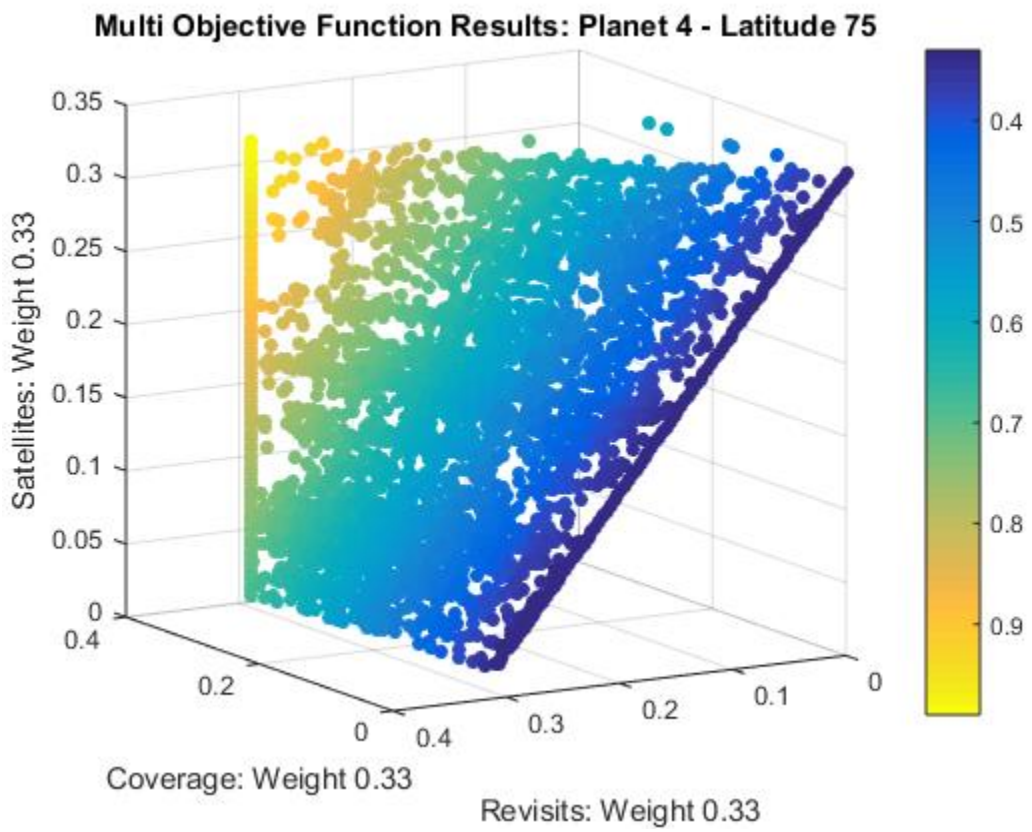

Constellation Results: Planet 4 - Latitude 45
# of Satellites
Inclination [deg]
Altitude [km]

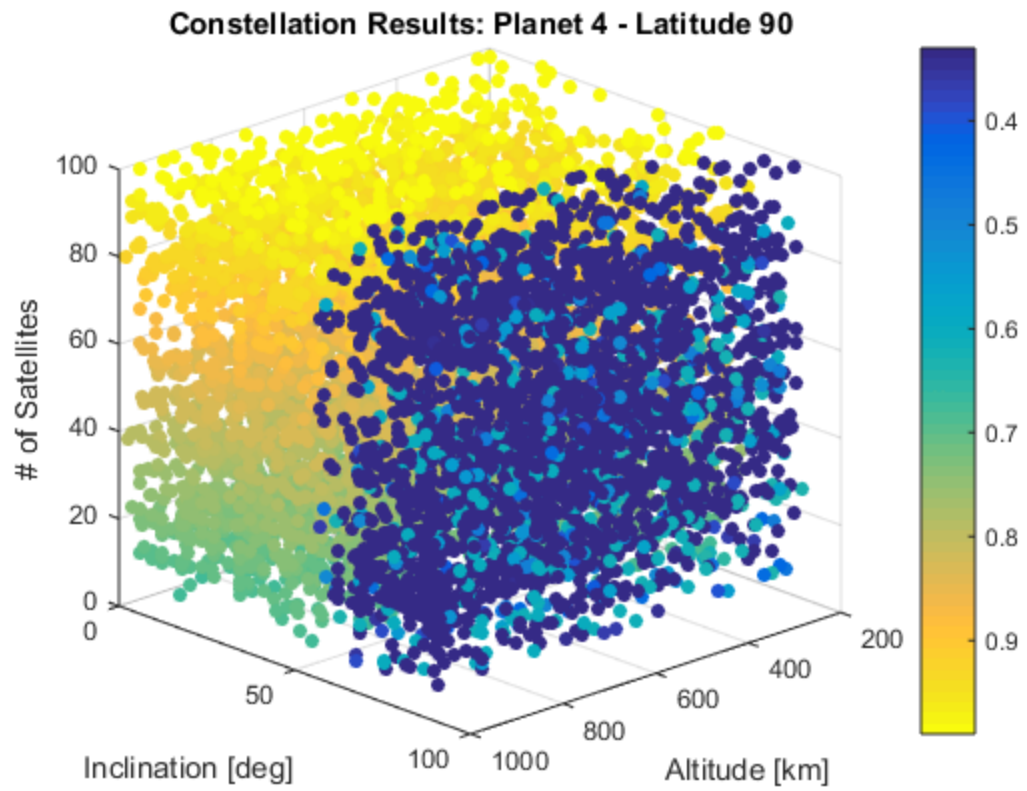

Multi Objective Function Results: Planet 4 - Latitude 45
Satellites: Weight 0.33
Coverage: Weight 0.33
Revisits: Weight 0.33

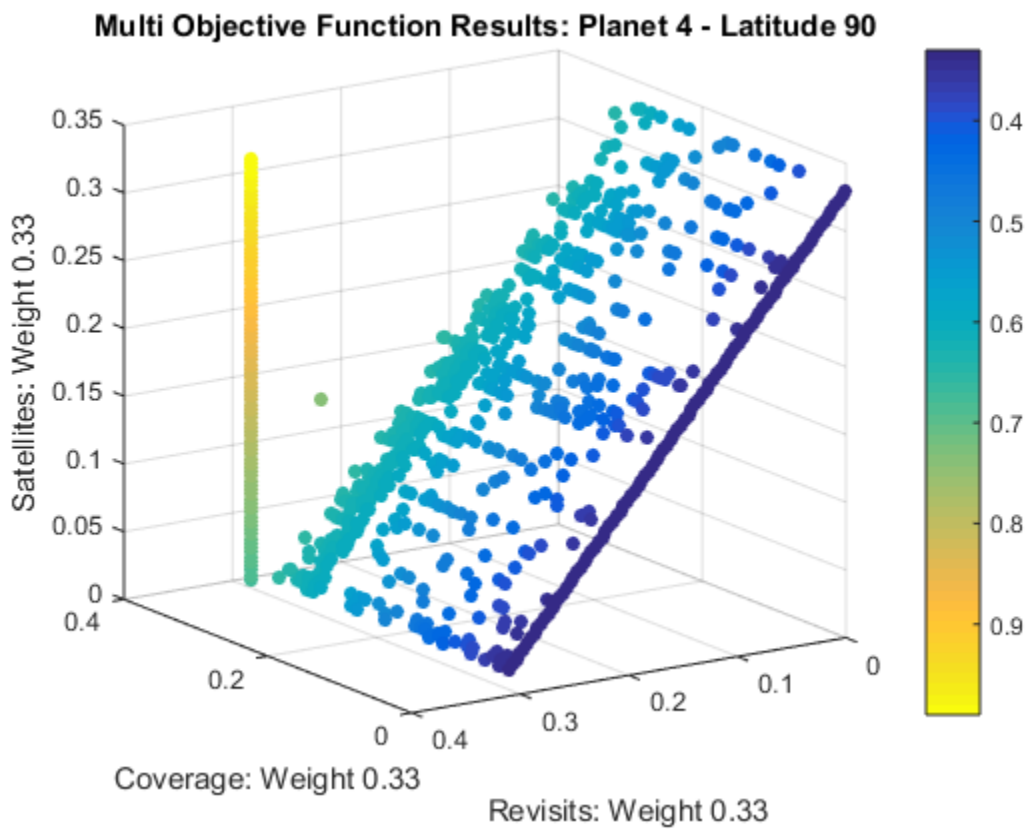

Constellation Results: Planet 4 - Latitude 60
# of Satellites
Inclination [deg]
Altitude [km]
0
20
40
60
80
100
0
50
100
1000
800
600
400
200
0.4
0.5
0.6
0.7
0.8
0.9

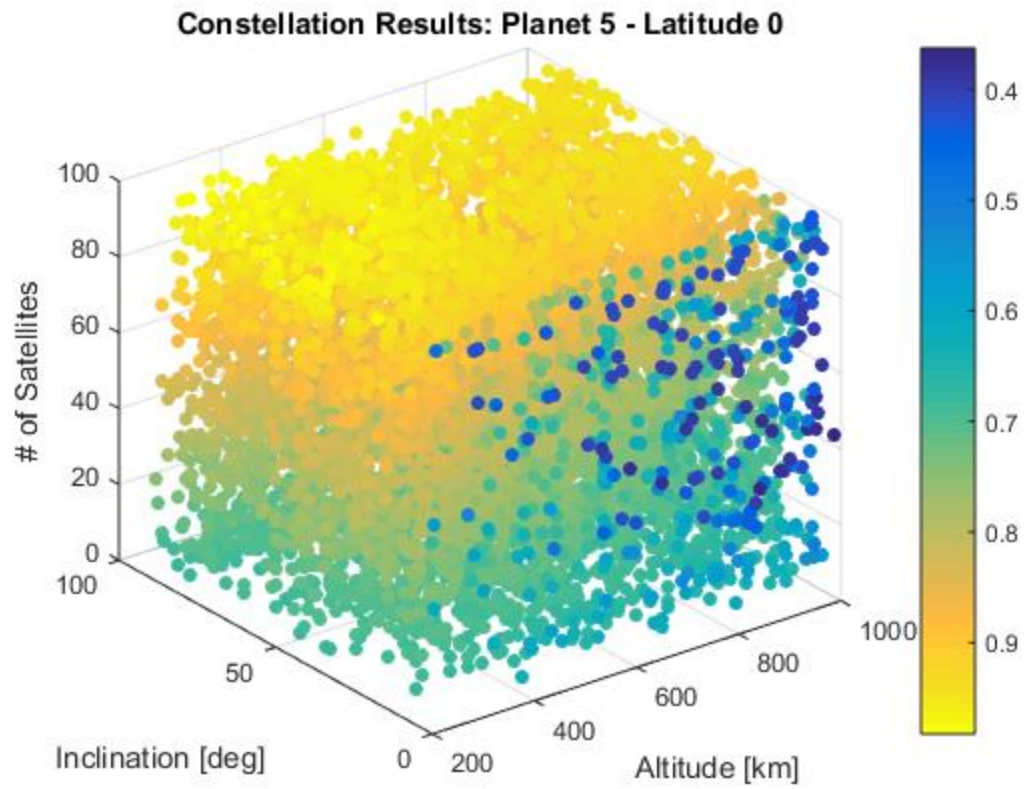

Multi Objective Function Results: Planet 4 - Latitude 60
Satellites: Weight 0.33
Coverage: Weight 0.33
Revisits: Weight 0.33
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.2
0
0.4
0.3
0.2
0.1
0
0.4
0.5
0.6
0.7
0.8
0.9

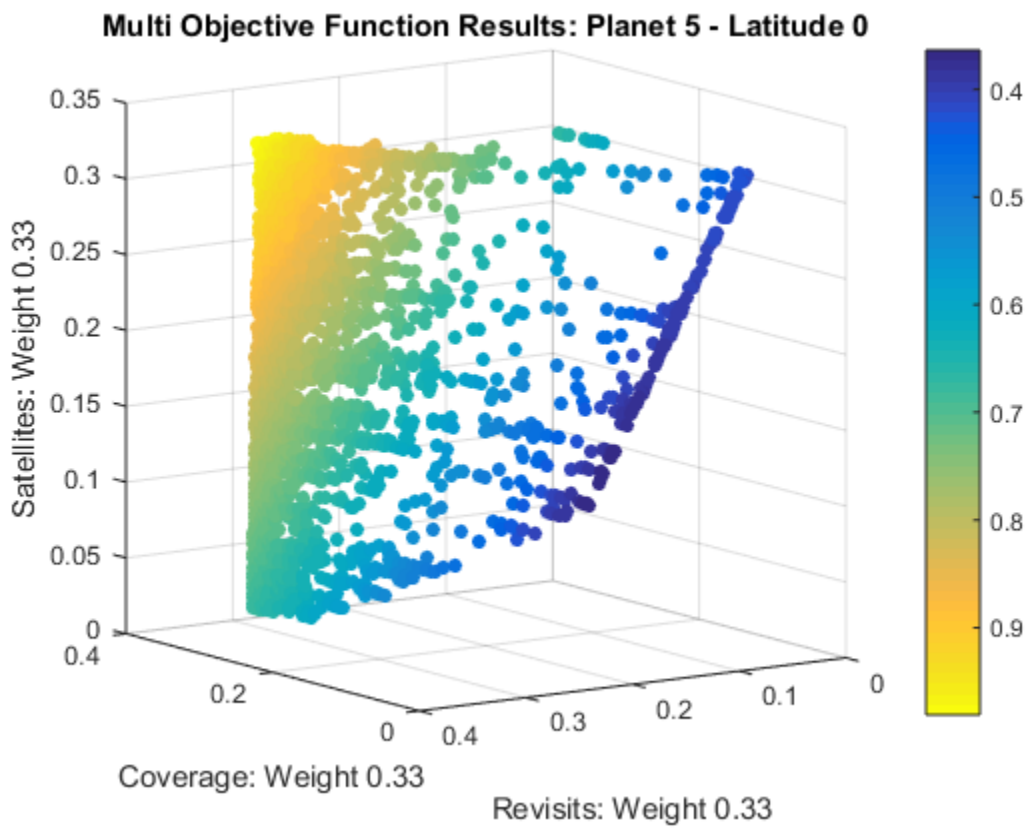

Constellation Results: Planet 4 - Latitude 75
# of Satellites
Inclination [deg]
Altitude [km]

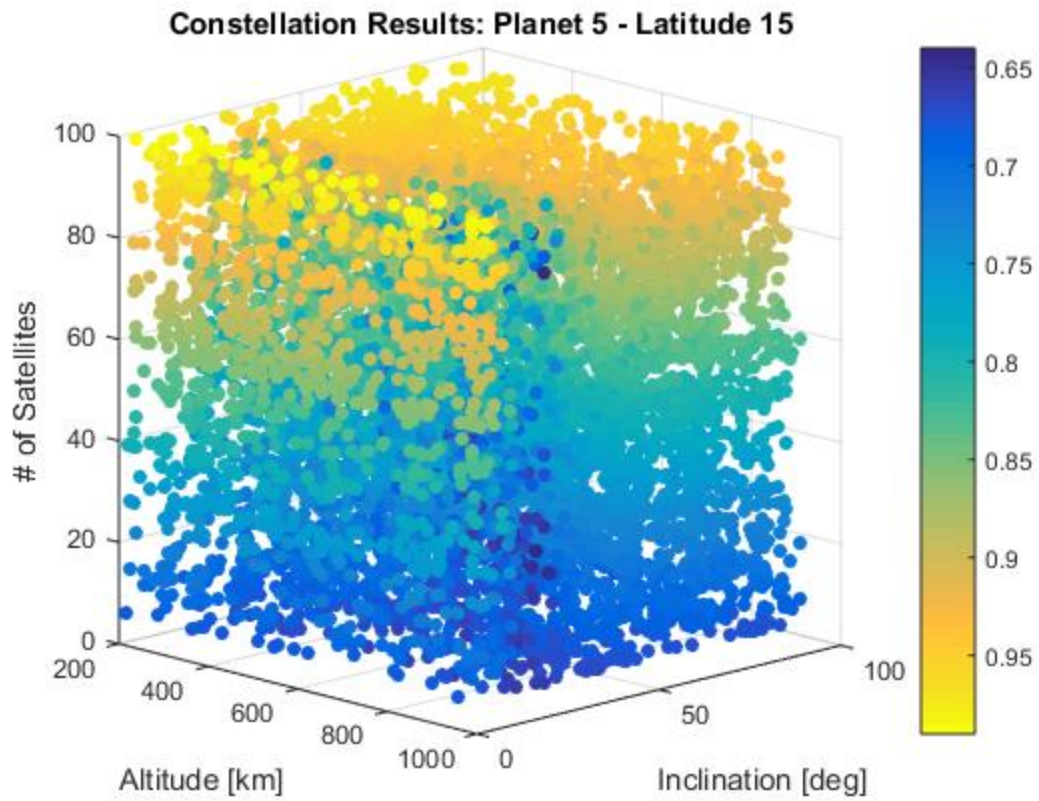

Multi Objective Function Results: Planet 4 - Latitude 75
Satellites: Weight 0.33
Coverage: Weight 0.33
Revisits: Weight 0.33

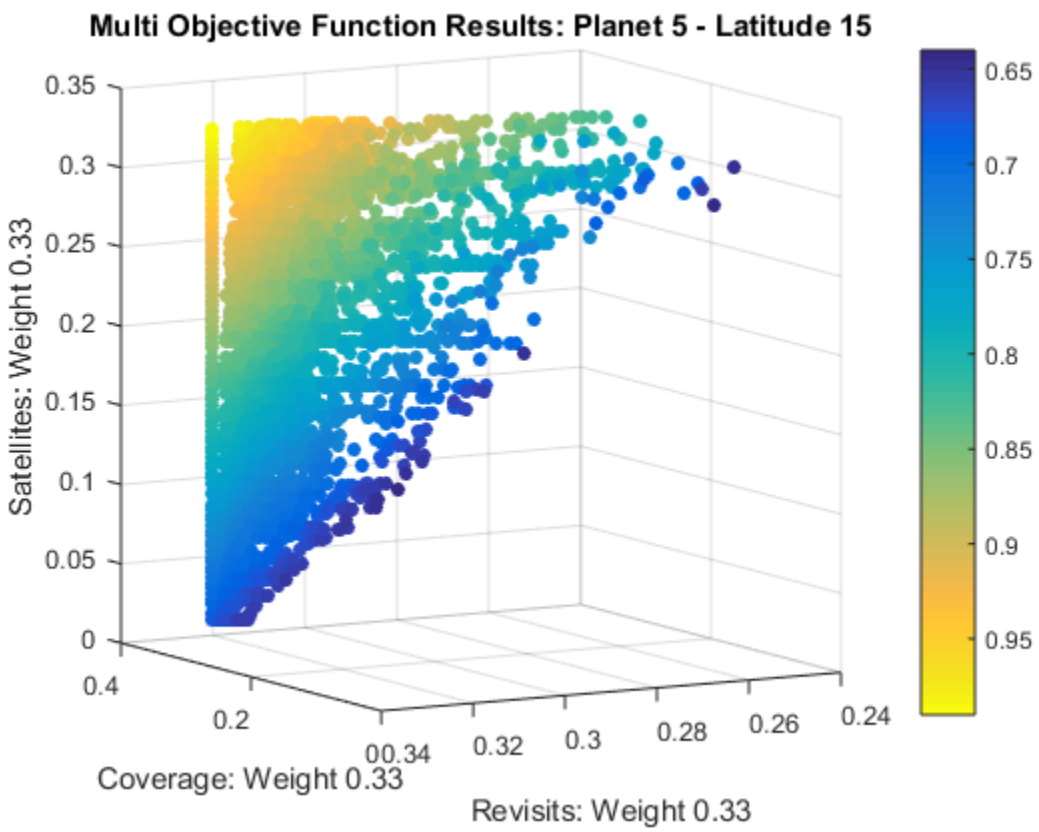

Constellation Results: Planet 4 - Latitude 90
# of Satellites
100
80
60
40
20
0
0
50
100
Inclination [deg]
1000
800
600
400
200
Altitude [km]
0.4
0.5
0.6
0.7
0.8
0.9

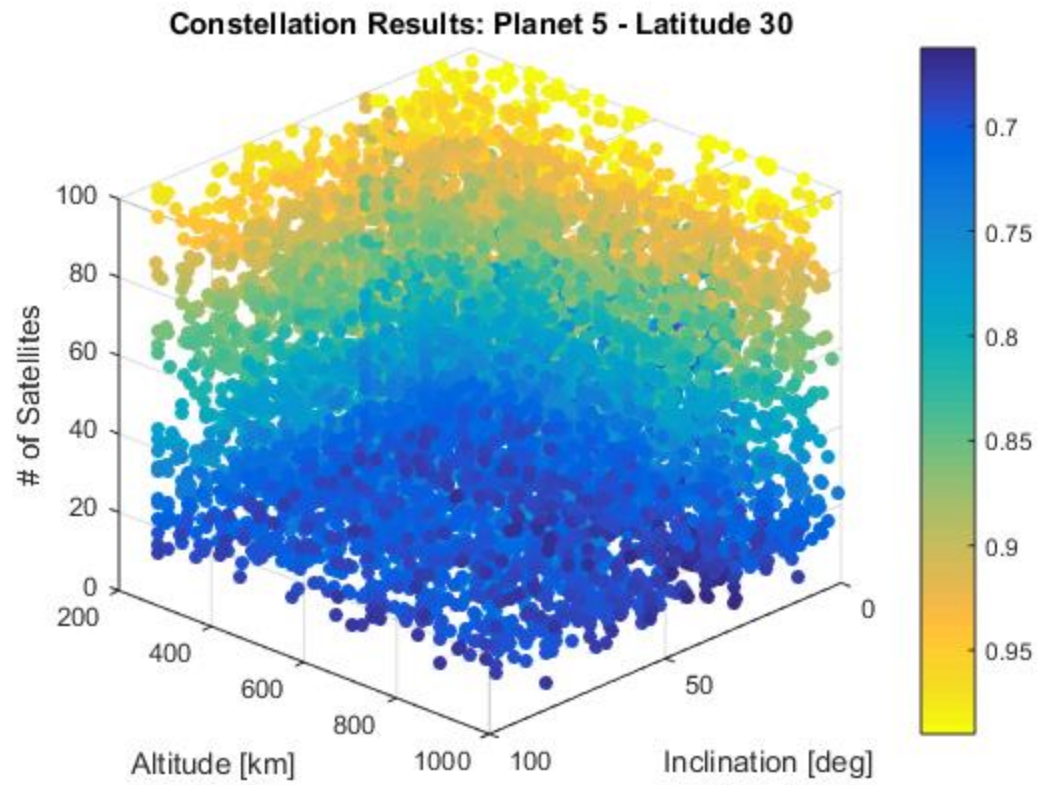

Multi Objective Function Results: Planet 4 - Latitude 90
Satellites: Weight 0.33
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.4
0.2
0
Coverage: Weight 0.33
0.4
0.3
0.2
0.1
0
Revisits: Weight 0.33
0.4
0.5
0.6
0.7
0.8
0.9

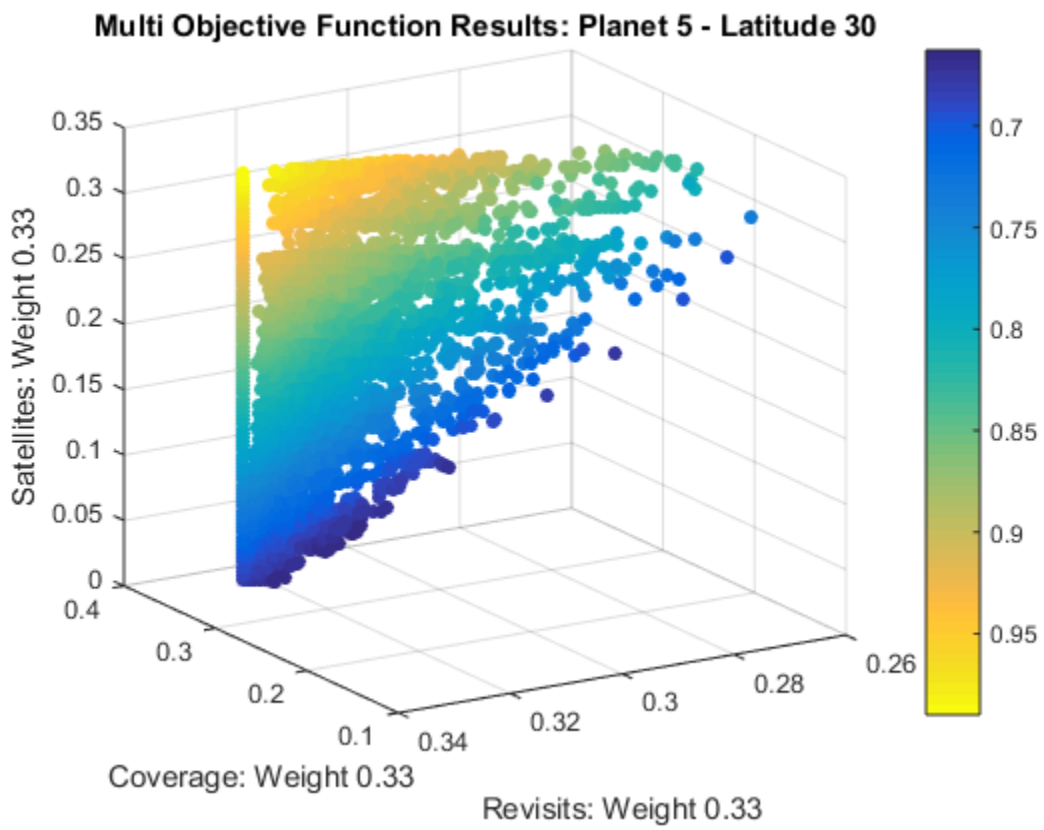

Constellation Results: Planet 5 - Latitude 0
# of Satellites
100
80
60
40
20
0
100
50
0
Inclination [deg]
200
400
600
800
1000
Altitude [km]
0.4
0.5
0.6
0.7
0.8
0.9

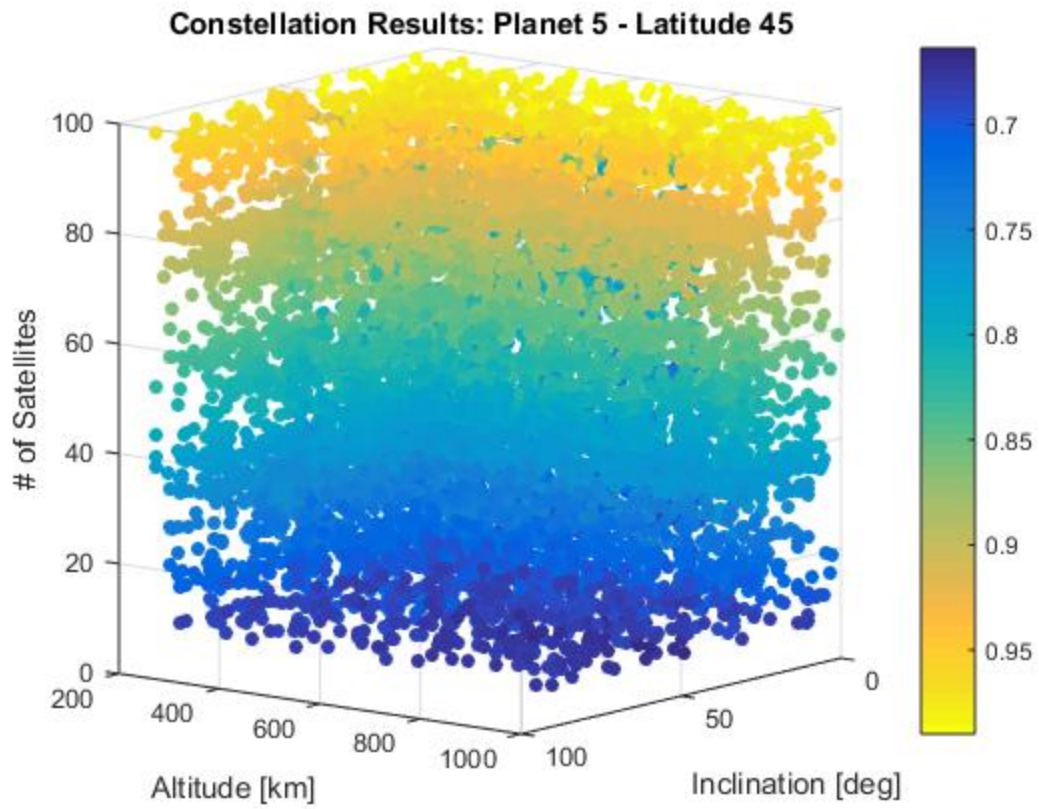

Multi Objective Function Results: Planet 5 - Latitude 0
Satellites: Weight 0.33
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.4
0.2
0
Coverage: Weight 0.33
0.4
0.3
0.2
0.1
0
Revisits: Weight 0.33
0.4
0.5
0.6
0.7
0.8
0.9

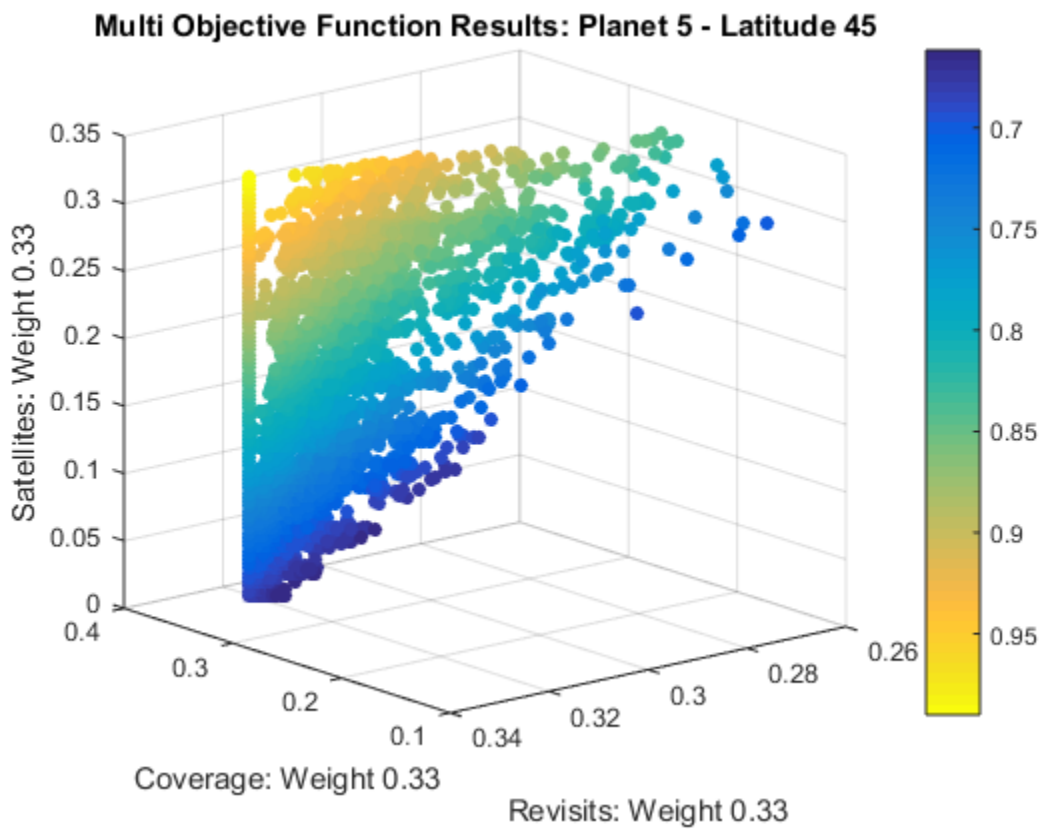

Constellation Results: Planet 5 - Latitude 15
# of Satellites
Altitude [km]
Inclination [deg]
0
20
40
60
80
100
200
400
600
800
1000
0
50
100
0.65
0.7
0.75
0.8
0.85
0.9
0.95

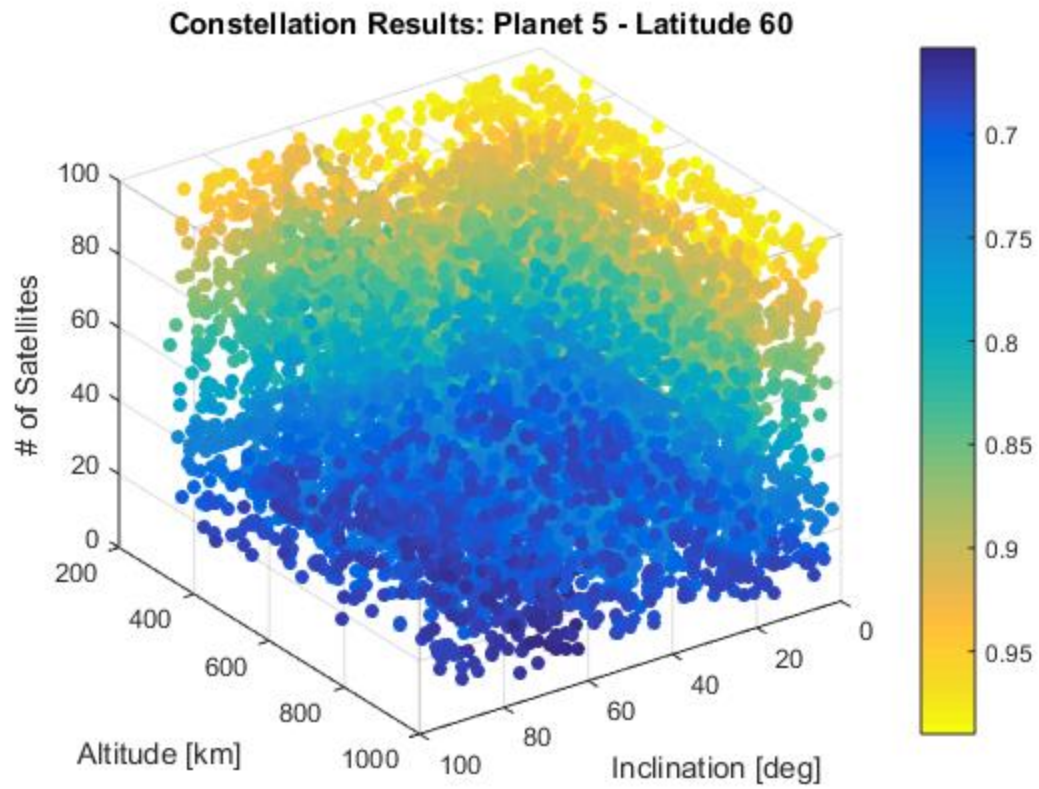

Multi Objective Function Results: Planet 5 - Latitude 15
Satellites: Weight 0.33
Coverage: Weight 0.33
Revisits: Weight 0.33
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.2
0
0.34
0.32
0.3
0.28
0.26
0.24
0.65
0.7
0.75
0.8
0.85
0.9
0.95

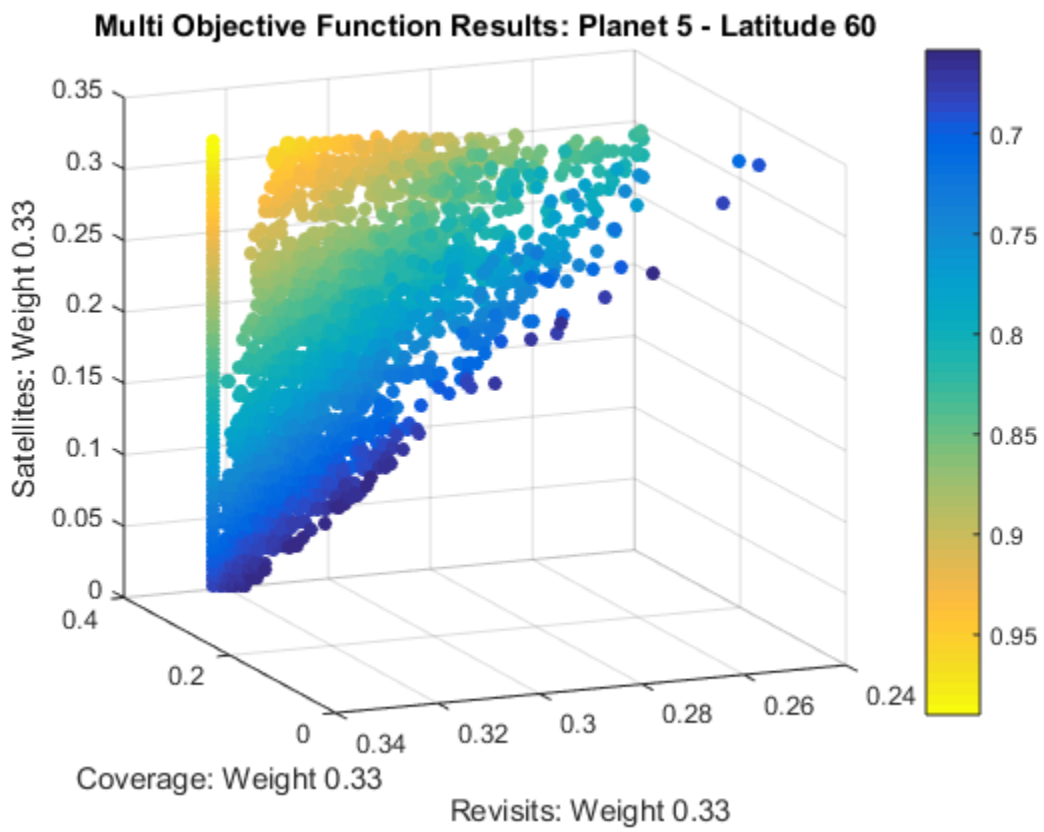

Constellation Results: Planet 5 - Latitude 30
# of Satellites
100
80
60
40
20
0
200
400
600
800
1000
Altitude [km]
100
50
0
Inclination [deg]
0.7
0.75
0.8
0.85
0.9
0.95

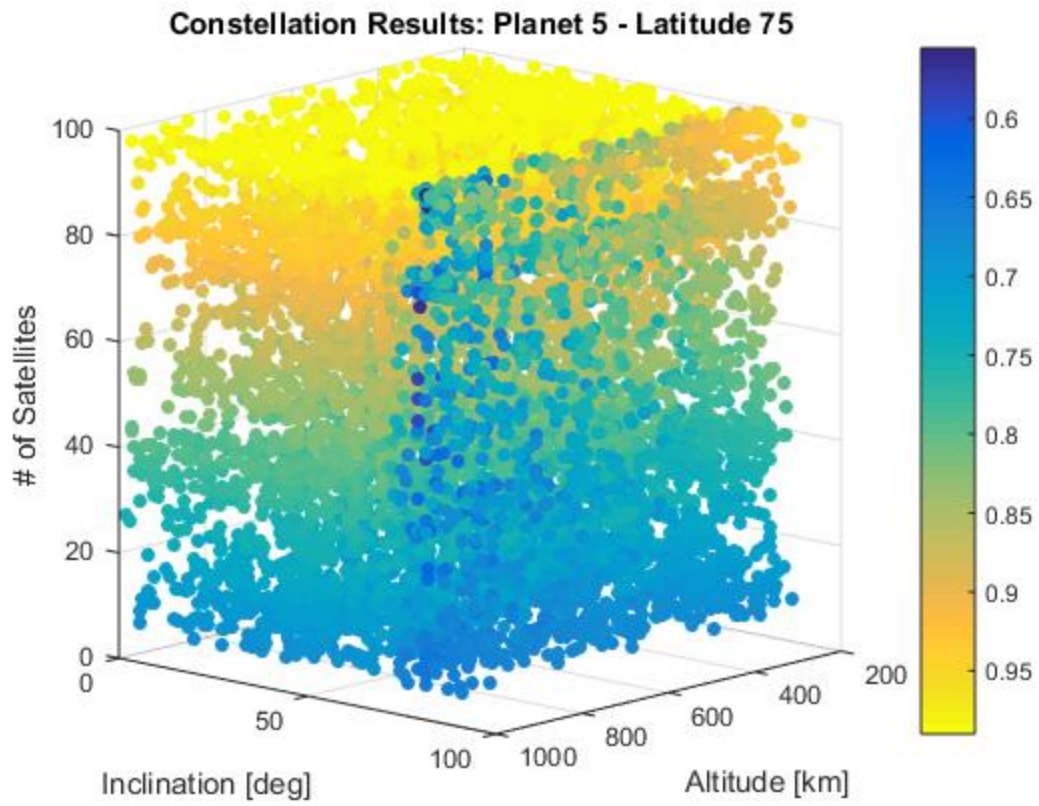

Multi Objective Function Results: Planet 5 - Latitude 30
Satellites: Weight 0.33
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.4
0.3
0.2
0.1
Coverage: Weight 0.33
0.34
0.32
0.3
0.28
0.26
Revisits: Weight 0.33
0.7
0.75
0.8
0.85
0.9
0.95

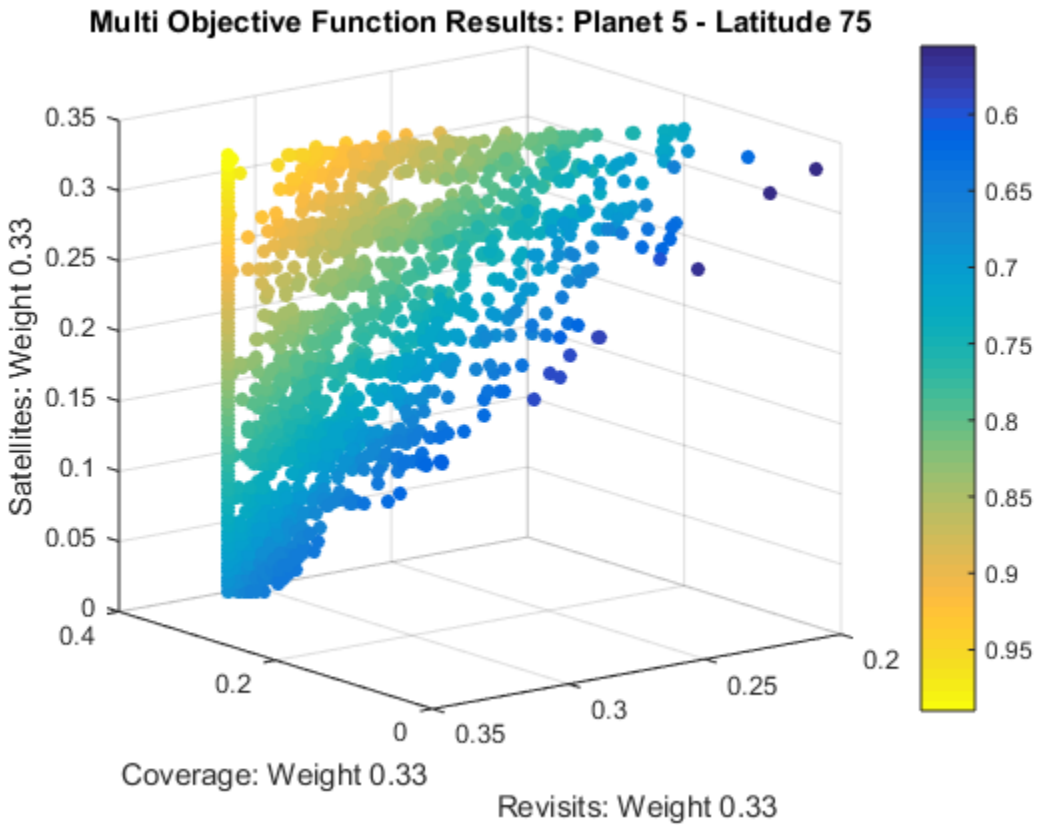

Constellation Results: Planet 5 - Latitude 45
# of Satellites
Altitude [km]
Inclination [deg]
0
20
40
60
80
100
200
400
600
800
1000
100
50
0
0.7
0.75
0.8
0.85
0.9
0.95

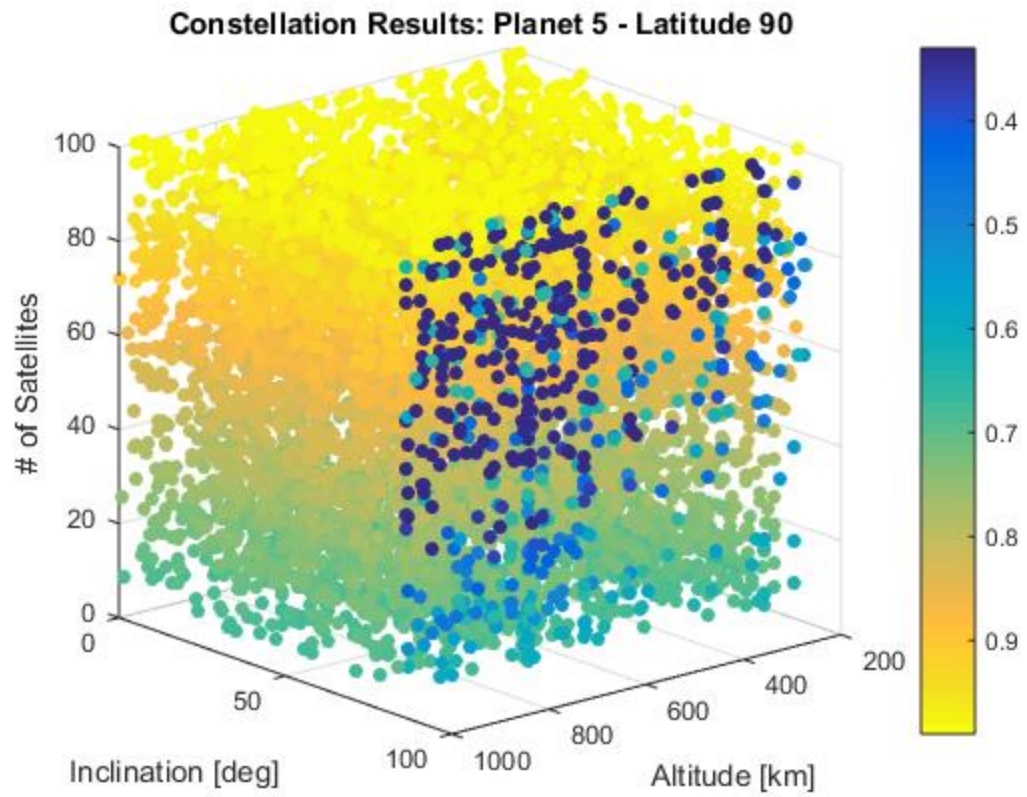

Multi Objective Function Results: Planet 5 - Latitude 45
Satellites: Weight 0.33
Coverage: Weight 0.33
Revisits: Weight 0.33
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.3
0.2
0.1
0.34
0.32
0.3
0.28
0.26
0.7
0.75
0.8
0.85
0.9
0.95

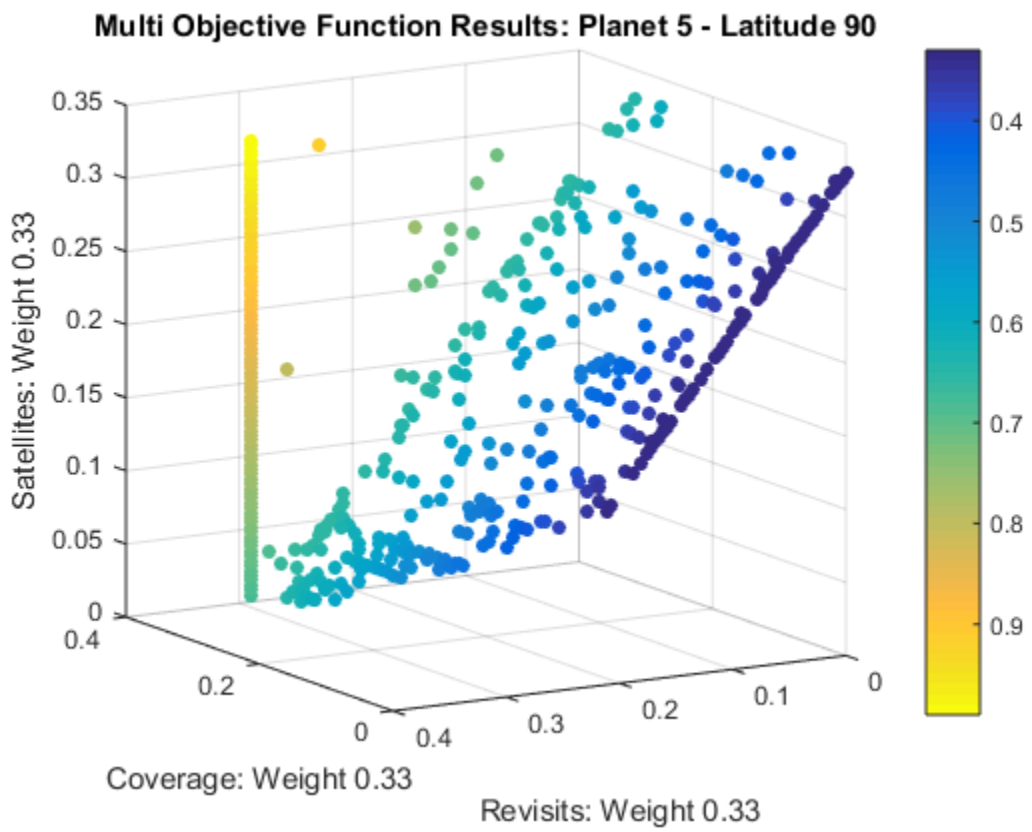

Constellation Results: Planet 5 - Latitude 60
# of Satellites
100
80
60
40
20
0
200
400
600
800
1000
Altitude [km]
100
80
60
40
20
0
Inclination [deg]
0.7
0.75
0.8
0.85
0.9
0.95


Multi Objective Function Results: Planet 5 - Latitude 60
Satellites: Weight 0.33
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.4
0.2
0
Coverage: Weight 0.33
0.34
0.32
0.3
0.28
0.26
0.24
Revisits: Weight 0.33
0.7
0.75
0.8
0.85
0.9
0.95


Constellation Results: Planet 5 - Latitude 75
# of Satellites
Inclination [deg]
Altitude [km]
0
20
40
60
80
100
0
50
100
1000
800
600
400
200
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95


Multi Objective Function Results: Planet 5 - Latitude 75
Satellites: Weight 0.33
Coverage: Weight 0.33
Revisits: Weight 0.33
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.2
0
0.35
0.3
0.25
0.2
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95


Constellation Results: Planet 5 - Latitude 90
# of Satellites
Inclination [deg]
Altitude [km]
0
20
40
60
80
100
0
50
100
1000
800
600
400
200
0.4
0.5
0.6
0.7
0.8
0.9


Multi Objective Function Results: Planet 5 - Latitude 90
Satellites: Weight 0.33
Coverage: Weight 0.33
Revisits: Weight 0.33
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.2
0
0.4
0.3
0.2
0.1
0
0.4
0.5
0.6
0.7
0.8
0.9