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Syndication and Bargaining With a Monopolist

Bruce Randolph Barnes

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Abstract
This dissertation consists of three related game-theoretic essays on bargaining.

The first essay develops a model of monopolistic economies in which total profit is determined by the set of traders that cooperate with the monopolist. The traders each bargain bilaterally with the monopolist for their profits and each bargaining outcome is determined by some bargaining solution. For any set of continuous bargaining solutions, there exists a general bargaining equilibrium, which is a profit distribution that is the fixed point of a system of bilateral bargaining outcome functions.

The second essay defines the class of strongly power sensitive bargaining solutions, which includes all bargaining solutions that are strongly individually rational and either independent of irrelevant alternatives or individually monotonic. Measures of bargaining power are introduced for generalized Nash bargaining solutions and generalized monotonic bargaining solutions.

The final essay analyzes the effects of syndication among traders bargaining with a monopolist with respect to the general bargaining equilibria associated with risk sensitive bargaining solutions and strongly power sensitive bargaining solutions. If the monopolist is risk neutral, the effects of syndication depend on the profit function: syndication is neutral if the profit function is additive for the set of traders, advantageous if it is submodular, and disadvantageous if it is supermodular.

A strictly risk averse monopolist creates opportunities for advantageous syndication among the traders. If the monopolist is strictly risk averse and the profit function is additive, then traders in larger syndicates receive greater profits, and syndicate merger is advantageous. The traders can maximize their profits by forming a single monopolistic syndicate. This confirms the conventional wisdom that traders faced with a monopolist should syndicate to form a bilateral monopoly.

In bargaining with a strictly risk averse opponent, size alone creates bargaining power. In the absence of any cost considerations, a strictly risk averse opponent may grant a larger player more favorable terms in bargaining. This may provide an explanation of volume discounts in the absence of price discrimination, economies of scale or transaction costs.

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ABSTRACT

SYNDICATION AND BARGAINING WITH A MONOPOLIST

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This dissertation consists of three related game-theoretic essays on bargaining.

The first essay develops a model of monopolistic economies in which total profit is determined by the set of traders that cooperate with the monopolist. The traders each bargain bilaterally with the monopolist for their profits and each bargaining outcome is determined by some bargaining solution. For any set of continuous bargaining solutions, there exists a general bargaining equilibrium, which is a profit distribution that is the fixed point of a system of bilateral bargaining outcome functions.

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profit function: syndication is neutral if the profit function is additive for the set of traders, advantageous if it is submodular, and disadvantageous if it is supermodular.

A strictly risk averse monopolist creates opportunities for advantageous syndication among the traders. If the monopolist is strictly risk averse and the profit function is additive, then traders in larger syndicates receive greater profits, and syndicate merger is advantageous. The traders can maximize their profits by forming a single monopolistic syndicate. This confirms the conventional wisdom that traders faced with a monopolist should syndicate to form a bilateral monopoly.

In bargaining with a strictly risk averse opponent, size alone creates bargaining power. In the absence of any cost considerations, a strictly risk averse opponent may grant a larger player more favorable terms in bargaining. This may provide an explanation of volume discounts in the absence of price discrimination, economies of scale or transactions costs.

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PREFACE

The phenomenon of volume discounting is pervasive in market economies. Most examples of volume discounting are readily explained by generally accepted theories of industrial organization. The explanations include price discrimination, economies of scale, transactions costs, and search costs. Nevertheless some examples of volume discounting are not adequately explained by traditional economic theories.

The research culminating in this dissertation was motivated by the inability of these traditional approaches to explain the pricing structure of the royalty agreements between motion picture studios and cable television networks during the early 1980s. Motion picture studios (Columbia, MGM, Paramount, Twentieth-Century Fox, United Artists, Universal, and Warner) produce feature films which they also distribute for theatrical release. The cable television networks (HBO/Cinemax and Showtime/The Movie Channel) purchase the rights to present these films to their subscribers. It has been observed that HBO, by far the largest cable network, historically extracted much more favorable royalty terms from the studios than the other, smaller networks; that is, HBO paid substantially lower fees per subscriber than its rivals, although it paid a higher total fee.

These discounts are not counter-intuitive: it is expected that larger customers will be granted discounts. My interest in the phenomenon was piqued by the vehement objections that the motion picture studios raised to the terms of their royalty agreements with HBO. Surprisingly, traditional economic theories offer no explanation of the more favorable royalty terms in a rigorous model.

The relationship between the motion picture studios and the cable television networks is best described as that between a producer of an intermediate good and a firm which uses the intermediate good as an input. The studio produces motion pictures which are used as an input by the cable network in the production of the programming package which is its final product.

The existence of volume discounts is commonly explained by economies of scale or transactions costs. This rationale for volume discounts holds that if unit costs are lower for larger volumes, then, in any market environment, profit maximization by the seller results in quantity discounts. This classical theory does not apply to the market for cable television rights. Both transactions costs and economies of scale are negligible relative to the value of the royalty fees. Neither would motivate a profit-maximizing studio to offer substantially better terms to the larger cable network.

Three types of costs can be identified in the motion picture and cable television industries. The first type of costs are the production costs incurred by the studio in the production of the motion pictures. The second type of costs are the transfer costs from the studio to the cable network. The final costs are the distribution costs incurred by the cable network in delivering programming to its subscribers.

The production costs are fixed and sunk. At the time of negotiation for cable film rights the motion picture has already been produced and released in theaters. There are no additional production costs which need be incurred for the sale of film rights to the cable networks. Generally the cable television royalties are based on the theatrical success of the film and are not substantially affected by the film's production costs.

Transactions costs between the studio and cable network are insubstantial. They consist only of the negotiation costs and the minimal costs of videotape production. But there is another form of transfer costs that are incurred by the studios. The sale of film rights to cable television diminishes the film's revenues from other sources such as videocassette sales and commercial television networks. Hence there are opportunity costs to the studio in the form of lower royalties in other media. Nevertheless, these opportunity costs should be proportional to the number of subscribers to the cable network, exhibiting neither economies nor diseconomies of scale, because the revenue loss results from lower demand by cable television subscribers.

Finally, there are substantial distribution costs incurred by the cable networks in order to deliver programming to its subscribers. These distribution costs may involve substantial economies of scale, but these savings are realized by the cable networks, rather than the studios. Therefore, such economies do not motivate volume discounting by the studios. In fact, in the context of cooperative models, the larger cable companies might be expected to transfer a portion of their cost savings to the studios in the form of higher royalty payments.

Nor is volume discounting an example of price discrimination by the studios. The discounts could be explained as such if the audiences of the cable networks were disparate. If, for example, HBO subscribers paid a lower subscription fee, one could infer that programming was less valuable to the average HBO subscriber, and HBO would be expected to pay a lower fee per subscriber for film rights.

All the evidence denies disparity in the compositions of the cable television audiences. There is little price discrimination within the cable market: subscription fees are almost identical and characteristic distinctions among programming at the major cable movie networks are insubstantial. Therefore, there is no means by which subscribers are separated to effect price discrimination.

This dissertation offers an alternative explanation for volume discounting, which is unrelated to price discrimination, economies of scale or transactions costs. A bargaining model is introduced in which, in the absence of any cost considerations, a player who is strictly risk averse with respect to its profits necessarily grants larger players more favorable terms in bargaining. Thus, if a firm is strictly risk averse, it may grant discounts to its larger customers simply because it is disproportionately more concerned with retaining them as customers.

This explanation would give credence to the movie studios' objections to the discounts. If HBO used its greater size to its advantage in bargaining with strictly risk averse movie studios, it is not surprising that the studios would be resentful despite the voluntary nature of the royalty agreements.

This role of risk aversion in volume discounting appears consistent with conventional wisdom in the business community, in which managers generally regard themselves as risk averse. Risk aversion has been extensively explored in the bargaining literature, but has not heretofore been postulated as an explanation for volume discounts. Nevertheless, in my discussions of this research, business managers have generally acknowledged that risk aversion offers a natural and compelling explanation of volume discounting.

The following dissertation examines the effects of syndication among a group of identical traders who bargain with a monopolist. Since the traders are identical, there is no incentive for price discrimination. The payoff function is assumed to have constant returns and there are no transactions costs. The more favorable terms received by the larger syndicates are a generalized form of volume discounts.

The framework introduced herein is unable to fully accommodate the market for cable television film rights, but it does demonstrate an important fundamental result: greater size can result in greater profits in the absence of economies of scale or transactions costs. In the absence of any cost considerations, a player who is strictly risk averse with respect to his profits may grant larger players more favorable terms in bargaining.

CHAPTER I: INTRODUCTION

The literature on disadvantageous monopolies has called into question the intrinsic value of a monopoly. A monopolistic position now appears to be valuable only when it is combined with another factor which gives the monopolist an advantage over its opponents. Otherwise, it may be worthless. As a simple example, if the demand for a good is perfectly elastic, then a monopoly is to no advantage because the monopolist is unable to affect the market price. Furthermore, many examples of disadvantageous monopoly have been presented.

Given these observations on monopoly, the intrinsic value of greater market share must also be uncertain. Greater size is advantageous only if other factors are present which give larger players an advantage. Economies of scale and transaction costs are commonly cited as allowing larger players to be disproportionately more profitable.

This dissertation points to a source of power for larger players which is unrelated to the profit function. In the presence of a strictly risk averse opponent, size creates bargaining power. An opponent who is strictly risk averse with respect to its profits may grant a larger player more favorable terms in bargaining.

These results may offer insight into the advantages of syndication in the absence of transactions costs or economies of scale that would generally motivate organizational cooperation to achieve greater efficiency. In bargaining economies, syndication may be advantageous simply as a means to exploit the strict risk aversion of one's opponent.

The results also may provide an explanation of volume discounts in the absence of price discrimination, economies of scale or transactions costs. If a firm is strictly risk averse, it may grant discounts to its larger customers simply because it is disproportionately more concerned with retaining them as customers.

This dissertation is divided into three essays. The first essay develops the framework for the analysis: a new model of a monopolistic bargaining economy, in which each of the several traders bargains with the monopolist, and an equilibrium for the set of bargaining games between each of the traders and the monopolist. The second essay introduces a new class of bargaining solutions to broaden the generality of the results: the class of strongly power sensitive bargaining solutions, which contains the Nash and monotonic solutions and is contained in the class of risk sensitive solutions. The final essay presents the conclusions: the effects of syndication among traders in a monopolistic bargaining economy on the general bargaining equilibria associated with risk sensitive and strongly power sensitive bargaining solutions.

The remainder of this introductory chapter is a summary of each of the three essays.

SECTION I: GENERAL BARGAINING EQUILIBRIUM

In the classical monopolistic economy consisting of a monopolist and a large number of buyers, only the monopolist has economic power. On the other hand, in a small economy that comprises a monopolist and several atomic traders, every player has economic power. In this context, the emergence of a single market price is most unlikely. The monopolist may attempt to fix uniform prices, but both the success and the advantages of such a strategy are suspect. Both the monopolist and the traders have power and they will use it. The monopolist and the traders are likely to arrange bilateral negotiations, and it is these negotiations that will determine the profit distribution of the monopolistic economy.

The first essay develops a model of monopolistic bargaining economies, which comprise a monopolist and several atomic traders. Profits can only be obtained through bilateral agreements between the monopolist and individual traders. The coalitional profit function determines the total profits of the economy based on the set of traders that cooperate with the monopolist. The profits are freely transferable and are shared by the monopolist and the traders that cooperate with the monopolist. Each trader's payoff is determined by its bilateral bargaining with the monopolist.

It is assumed that the outcome of bargaining between each trader and the monopolist is determined by a given bargaining solution. The solutions which determine the outcome of bargaining between each trader and the monopolist may be different. In the bargaining between each trader and the monopolist, the disagreement outcome for the monopolist is affected by the outcome of the monopolist's bargaining with the other traders. Therefore, the profit received by each trader as the outcome of its bargaining with the monopolist is a function of the outcomes of the bargaining between the monopolist and the other traders. A general bargaining equilibrium is a profit distribution that is the fixed point of these bargaining outcome functions.

A general bargaining equilibrium exists for any set of continuous bargaining solutions. General bargaining equilibria are applicable to both transferable and non-transferable utility economies.

SECTION II: BARGAINING SOLUTIONS AND BARGAINING POWER

Early research for this dissertation was limited to the effects of syndication on the general bargaining equilibria associated with the Nash and monotonic bargaining solutions. Those early results were then easily generalized to the class of bargaining solutions that are independent of irrelevant alternatives (generalized Nash solutions) and the class of bargaining solutions that are individually monotonic (generalized monotonic solutions).

To allow these results to be generalized to a broader class of bargaining solutions, the second essay introduces strongly power sensitive bargaining solutions. If the bargaining game is altered in favor of one of the players in the neighborhood of the bargaining solution and at either of the ideal points, then a strongly power sensitive bargaining solution assigns the player greater utility in the altered game.

Classical bargaining solutions are individually rational, Pareto-optimal, and independent of equivalent utility representations. Classical bargaining solutions that are strongly individually rational and independent of irrelevant alternatives, such as the Nash solution, are strongly power sensitive. Classical bargaining solutions that have restricted strong monotonicity, such as the monotonic solution, are also strongly power sensitive. If a strongly power sensitive bargaining solution is continuous, then it is twist sensitive and risk sensitive.

The second essay also introduces measures of a player's bargaining power, that is, its ability to demand a greater proportion of the profit contribution from agreement. Nash bargaining power measures a player's ability to demand a greater proportion of the profit contribution from agreement if the bargaining outcome is determined by a generalized Nash bargaining solution. Nash bargaining power equals the probability of disagreement per fractional increase in share that a player is willing to risk in order to increase its share of the profit contribution from agreement. Monotonic bargaining power is related to a player's ability to demand a greater proportion of the profit contribution from agreement if the bargaining outcome is determined by a bargaining solution that is individually monotonic. Monotonic bargaining power equals the inverse of the proportion of the player's ideal utility received by the player.

Bargaining power is defined as the pair consisting of a player's Nash and monotonic bargaining power. Bargaining power is related to a player's ability to demand a greater proportion of the profit contribution from agreement if the bargaining outcome is determined by any strongly power sensitive bargaining solution. Bargaining power has several properties which are used in the final chapter of this dissertation to demonstrate the effects of syndication on general bargaining equilibria associated with strongly power sensitive bargaining solutions.

SECTION III: SYNDICATION AND GENERAL BARGAINING EQUILIBRIA

Conventional wisdom in economics holds that if a group of traders is confronted with a monopolist, then the traders should cooperate to form a syndicate in order to establish a bilateral monopoly in the market, but this result has never been demonstrated. Although several examples exist in the literature of disadvantageous monopolies, none of these examples indicates that a monopoly position is disadvantageous when the other side of the market is controlled by a monopolist.

The third essay presents the principal results of this dissertation: the effects of syndication among several traders that bargain with a monopolist. The effects of syndication depend on the coalitional profit function and the risk aversion of the monopolist.

Limited results are presented for the general bargaining equilibria associated with strongly risk sensitive bargaining solutions (see chart on page 10). If the monopolist is strictly risk averse with decreasing absolute risk aversion and increasing relative risk aversion, then larger syndicates receive greater profits per trader and syndication to form a bilateral monopoly is advantageous.

The effects of syndication on the general bargaining equilibria associated with strongly power sensitive solutions is extensively analyzed. A brief summary of these results follows.

If the monopolist is risk neutral, the effects of syndication depend on the coalitional profit function (see chart on page 11). If the coalitional profit function is additive, then syndication is neutral, that is, it has no effect on the profits of the traders and the monopolist. If the coalitional profit function is submodular, syndication is advantageous for the traders. Finally, if the coalitional profit function is supermodular, syndication is disadvantageous.

If the coalitional profit function is additive, the effects of syndication among the traders depend on the utility function of the monopolist (see chart on page 12). If the monopolist is risk neutral, then all syndicates receive identical profits per trader regardless of the organization of the traders. On the other hand, if the monopolist is strictly risk averse, then the organization of the traders will affect the profits of the players. The introduction of a strictly risk averse monopolist creates opportunities for advantageous syndication. Syndication among traders which is neutral or disadvantageous in bargaining with a risk neutral monopolist may be advantageous if the monopolist is strictly risk averse. The strictly risk averse monopolist is weaker in bargaining for a share of the greater profit produced by cooperation with larger syndicates, while the risk neutral monopolist is not. Syndication can allow the traders to exploit the monopolist's strict risk aversion. If the monopolist is strictly risk averse, then traders in larger syndicates receive greater profits and syndication to form a bilateral monopoly is advantageous. Furthermore, if the monopolist has nonincreasing absolute risk aversion and nondecreasing relative risk aversion, then any syndication is advantageous.

If the economy comprises a strictly risk averse monopolist and a continuum of traders which form a finite number of identical syndicates, then the monopolist would prefer that the traders be distributed among many syndicates. As the traders are divided into an increasing number of smaller syndicates, the monopolist's profits increase and the traders' profits decrease. As the syndicates become arbitrarily small, the profits of the strictly risk averse monopolist approach the profits that would be received by a risk neutral monopolist.

The charts on the following three pages present a summary of the principal results concerning syndication and the general bargaining equilibria associated with strongly risk sensitive and strongly power sensitive bargaining solutions.

BARGAINING EQUILIBRIA FOR STRONGLY RISK SENSITIVE BARGAINING SOLUTIONS

SUMMARY OF RESULTS FOR ADDITIVE ECONOMIES

	ASSUMPTIONS ON THE MONOPOLIST'S UTILITY FUNCTION				
	RISK NEUTRAL	STRICTLY RISK AVERSE	STRICTLY RISK AVERSE NONINCREASING R	STRICTLY RISK AVERSE DECREASING R INCREASING R*	STRICTLY RISK AVERSE NONINCREASING R NONDECREASING R*
UNIQUENESS	There is at most one bargaining equilibrium.		There is at most one equilibrium with a given total profit for the monopolist.		
SYNDICATE SIZE	All syndicates receive the same profit per trader.			A larger syndicate receives greater profits per trader.	A larger syndicate receives at least as great profits per trader.
MERGER OF ALL SYNDICATES	All syndicates receive the same profit per trader regardless of their organization.			Increases profits per trader for every merging syndicate.	Does not decrease profits per trader for any merging syndicate.
MERGER OF SOME SYNDICATES	All syndicates receive the same profit per trader regardless of their organization.				

R is absolute risk aversion and R* is relative risk aversion.

BARGAINING EQUILIBRIA FOR STRONGLY POWER SENSITIVE BARGAINING SOLUTIONS

SUMMARY OF RESULTS FOR ECONOMIES WITH A RISK NEUTRAL MONOPOLIST

	ASSUMPTIONS ON THE COALITIONAL PROFIT FUNCTION		
	ADDITIVE	STRICTLY SUBMODULAR	STRICTLY SUPERMODULAR
UNIQUENESS	The equilibrium is unique.	The equilibrium is unique.	If an equilibrium exists, then it is unique.
SYNDICATE SIZE	All syndicates receive equal profits per trader.	Larger syndicates receive greater profits per trader.	Larger syndicates receive smaller profits per trader.
MERGER OF ALL SYNDICATES	Does not affect players' profits.	Maximizes every trader's profit. Minimizes monopolist's total profit.	Minimizes every trader's profit. Maximizes monopolist's total profit.
MERGER OF SOME SYNDICATES	Does not affect players' profits.	Increases profits of traders in merging syndicates. Decreases monopolist's total profit.	Decreases profits of traders in merging syndicates. Increases monopolist's total profit.
INDEPENDENCE OF ALL TRADERS	Does not affect players' profits.	Minimizes every trader's profit. Maximizes monopolist's total profit.	Maximizes every trader's profit. Minimizes monopolist's total profit.

BARGAINING EQUILIBRIA FOR STRONGLY POWER SENSITIVE BARGAINING SOLUTIONS

SUMMARY OF RESULTS FOR ADDITIVE ECONOMIES

	ASSUMPTIONS ON THE MONOPOLIST'S UTILITY FUNCTION				
	RISK AVERSE	STRICTLY RISK AVERSE	STRICTLY RISK AVERSE NONINCREASING R	STRICTLY RISK AVERSE NONDECREASING R*	STRICTLY RISK AVERSE NONINCREASING R NONDECREASING R*
UNIQUENESS	At most one equilibrium with a given total profit for monopolist.			The equilibrium is unique.	
SYNDICATE SIZE	Monopolist receives greater profit from larger syndicate.	A larger syndicate receives greater profits per trader.			
MERGER OF ALL SYNDICATES		Increases total traders profits; decreases monopolist's profit.	Increases profits per trader for every merging syndicate.		
MERGER OF SOME SYNDICATES				Increases total profit of merging syndicates. Decreases monopolist's total profits.	Increases profits of merging syndicates. Does not decrease profits of nonmerging syndicates.
DIVIDING CONTINUUM OF TRADERS INTO A GREATER NUMBER OF IDENTICAL SYNDICATES		As syndicates become arbitrarily small, monopolist's profit approaches that of risk neutral monopolist.		Lowers traders' profits; increases monopolist's profit.	

GENERAL BARGAINING EQUILIBRIUM

IN MONOPOLISTIC ECONOMIES

CHAPTER II: GENERAL BARGAINING EQUILIBRIUM IN MONOPOLISTIC ECONOMIES

This essay develops a new solution concept for monopolistic economies with few players in which cooperation is through negotiated bilateral agreements rather than coalitions. In a monopolistic bargaining economy, a monopolist engages in bilateral bargaining with the other players (call them traders) over the division of the profits from their cooperation and each of the pairwise negotiations concludes with an agreement associated with a given bargaining solution. The general bargaining equilibrium is a profit distribution among the monopolist and the traders such that the outcome of the bargaining between each trader and the monopolist is determined by a given bargaining solution. The solutions which determine the outcome of bargaining between each trader and the monopolist may be different. A general bargaining equilibrium exists for any set of continuous bargaining solutions. General bargaining equilibria are applicable to both transferable and non-transferable utility economies. The general bargaining equilibrium may provide insight into the role of bilateral transactions in small economies.

SECTION I: MARKET POWER IN SMALL MARKETS

Market power has been extensively studied for monopolistic and oligopolistic economies with many buyers, and bargaining power has been extensively studied for bilateral monopolies. The study of market power has been more limited with respect to "small" markets with only a few sellers and a few buyers, but such markets are important in modern capitalist economies. In many sectors of the economy, there are only a few major suppliers and consumers, especially of intermediate goods.

A game-theoretic model would appear to be most appropriate for studying small markets. Nevertheless, many of the game-theoretic models and solutions may not be sufficiently robust in their description of the economy. For example, the cooperative solutions -- the core, the value, the kernel, and the nucleolus -- are most easily interpreted in terms of the formation of large coalitions or the adoption of multilateral agreements. On the other hand, much economic activity is based on bilateral agreements between a buyer and a seller. The other players may affect the buyer's and seller's power in negotiating a bilateral agreement with one another, but these other players are not likely to be parties to the agreement.

This essay will be restricted to economies in which one side of the market is controlled by a monopolist and the other side of the market is composed of several traders.

SECTION II: TOWARD A BARGAINING EQUILIBRIUM

In the classical monopolistic economy consisting of a monopolist and a large number of buyers, only the monopolist has economic power. The market framework in such an economy is dictated by the monopolist, who may simply select a price at which it will execute transactions with the other players. The monopolist may also create a more complex system in order to price discriminate among different buyers.

On the other hand, in an economy that comprises a monopolist and several atomic traders who must cooperate with the monopolist in order to produce any positive payoff, every player has economic power. In this context, the emergence of a single market price is unlikely. The monopolist may attempt to fix uniform prices, but both the success and the advantages of such a strategy are suspect. Both the monopolist and the traders have power and they will use it. For example, a large trader may seek to obtain an advantage over the smaller traders.

Bilateral monopoly is the extreme example of such an economy. The distribution of the payoff in a bilateral monopoly has been analyzed in the literature on the two-person bargaining problem. Several bargaining solutions have been developed to satisfy different sets of axioms, most notably the seminal Nash solution (Nash [1950]), the monotonic solution (Kalai and Smorodinsky [1975]), and the superadditive solution (Perles and Maschler [1981]).

The generalization of some of these bargaining solutions to economies with several traders is impossible. There may be no solution for n-player economies that satisfies the axioms which generated the bargaining solution for two-person bargaining games. There is no n-person generalization of either the monotonic solution (see Roth [1979]) or the superadditive solution (see Perles [1982]).

Furthermore, it is inappropriate to use a generalization of a two-person bargaining solution to model a monopolistic economy with several traders. For example, if one applies the Nash bargaining solution to the n-player bargaining problem, all players are treated identically and there is no structural distinction between the monopolist and the traders. One could attempt to address this limitation by applying the generalized Nash solution, in which the players are assigned different weights, but there is no obvious method of determining such a weighting scheme. Moreover, any n-person bargaining game would imply that all of the players bargain together over the distribution of the payoff, and such multilateral bargaining among all of the players does not conform to observed reality.

The monopolist and the traders are more likely to arrange bilateral negotiations. This essay examines monopolistic economies in which each of the traders participates in bilateral bargaining with the monopolist. In the bargaining between each trader and the monopolist, the disagreement outcome for the monopolist is affected by the outcome of the monopolist's bargaining with the other traders.

Therefore, the final result of the bargaining between each trader and the monopolist may be affected by the outcomes of the bargaining between the monopolist and all of the other traders.

To focus on the interactions among the several bilateral bargaining games between the monopolist and each trader, the proposed general bargaining equilibrium is based on the assumption that the outcome of bargaining between each trader and the monopolist is determined by a given bargaining solution, and the dynamics of the individual bargaining games which result in that outcome are ignored. The bargaining solutions for the bilateral bargaining between each trader and the monopolist may be different. The selection of the bargaining solutions is assumed rather than justified. Notwithstanding the interdependence of the bargaining games, it is assumed that in their bargaining problem, the monopolist and a given trader do not consider the effect that their bargaining outcome will have on the monopolist's bargaining with other traders.

In a monopolistic bargaining economy, the profit received by each trader is the outcome of bilateral bargaining with the monopolist. Each bargaining outcome is a function of the outcomes of the bargaining between the monopolist and the other traders. A general bargaining equilibrium is a profit distribution that is the fixed point of these bargaining outcome functions.

This equilibrium is internally consistent. If a given bargaining solution necessarily determines the result of negotiations, then the general bargaining equilibrium is the most reasonable solution to the bargaining economy. It requires that each of the bargaining outcomes is consistent with all of the others.

The bargaining equilibrium can be interpreted in a rational expectations framework. Suppose that the bargaining between the monopolist and the traders occurs sequentially. Each trader is assumed to offer positive marginal contribution to total profits regardless of the set of other traders that cooperate with the monopolist. Therefore, the cooperation of all the traders with the monopolist is the only equilibrium that is Pareto-optimal and it is the only set of bargaining outcomes that is a Nash equilibrium in that there are no profitable agreements that are not executed.

In this context, it is reasonable to assume that each player expects all the profitable negotiations to culminate in agreement. Therefore, each player proceeds with the expectation that all players will cooperate with the monopolist. Each player also has an expectation of the profits that each other player will receive. In their bargaining, each trader and the monopolist considers their expectation of the monopolist's profits from agreement with other traders. The outcome of their bargaining depends on these expectations. The general bargaining equilibrium is a set of bargaining outcomes such that if all players expected the bargaining equilibrium, then the bargaining equilibrium would result. The order in which the negotiations are concluded is irrelevant because each player anticipates profits from subsequent negotiations.

Bennett [1988] independently developed a similar bargaining equilibrium for the system of bilateral bargains in a marriage economy, that is, an economy with several buyers and sellers in which economic gains are generated only from the exclusive pairing of a buyer and a seller. The rational expectations interpretation of the general bargaining equilibrium is similar to the consistent conjecture condition introduced therein.

SECTION III: BARGAINING GAMES

The theory of bargaining games was developed by Nash [1950]. A two-person *bargaining game* is a pair (d,S), where the disagreement utility pair is $d \in R^2$ and the set of feasible utility pairs is $S \subset R^2$, such that $d \in S$, S is compact and convex, and $\exists\, u \in S$ such that $u > d$. Let B* be the set of all bargaining games. A bargaining game is *comprehensive* iff $x \in S$ and $d \leq y \leq x \Rightarrow y \in S$. The *Pareto frontier* of S is $P[S] \equiv \{u \in S \mid u' \in S \text{ and } u' \geq u \Rightarrow u' = u\}$. A bargaining game (d,S) is *smooth* iff there is a unique supporting hyperplane for S at each point $u \in P[S]$, that is, if the implicit function of its Pareto frontier is differentiable at all points.

A *bargaining solution* is a function $f\colon B^* \to R^2$ such that $f(d,S) \in S$, $\forall\, (d,S) \in B^*$. Let F be the set of bargaining solutions. $f \in F$ is *continuous* iff for $(d,S) \in B^*$ and each sequence $\{(d_i,S_i)\}^\infty \in B^*$ with $d_i \to d$ and $S_i \to S$ in the Hausdorff metric, $f(d_i,S_i) \to f(d,S)$.

A bargaining solution f is *classical* ($f \in C$) iff, $\forall\, (d,S) \in B^*$:

(1) INDIVIDUAL RATIONALITY: $f(d,S) \geq d$;

(2) PARETO OPTIMALITY: $f(d,S) \in P[S]$;

(3) INDEPENDENCE OF EQUIVALENT UTILITY REPRESENTATIONS: for every affine transformation T of utility scales: $T[f(d,S)] = f[T(d),T(S)]$, where $T(u_1,u_2) = (a_1 \cdot u_1+b_1, a_2 \cdot u_2+b_2)$ for some $a_1,a_2 > 0$, $b_1,b_2 \in R$. Furthermore, $f \in F$ is *strongly individually rational* iff $f(d,S) > d$.

DEFINITION 2.1: A *profit division game* Γ is defined by $\Gamma = (w,t,u,v)$ where $w \in R^2$ is the disagreement profit pair, $t \in R^+$ is the profit contribution from agreement, and $u: R \rightarrow R$ and $v: R \rightarrow R$ are the players' utility functions which are each dependent only on the player's total profits ($x \in R$ or $y \in R$) and which satisfy, $\forall$ x,y:

TWICE DIFFERENTIABILITY: u and v are twice differentiable;

STRONG MONOTONICITY: $u'(x) > 0$ and $v'(y) > 0$;

CONCAVITY (RISK AVERSION): $u''(x) \leq 0$ and $v''(y) \leq 0$.

A profit division game is essentially a bargaining game recast in terms of profit rather than utility: an opportunity for two players to share a fixed profit if an agreement can be reached on the distribution of the profit. The combined profits of the players if they reach agreement in the profit division game $\Gamma = (w,t,u,v)$ is $(w_1 + w_2 + t)$.

The profit division game is introduced in order to analyze bargaining in terms of observable profit levels rather than utility levels. To apply bargaining solutions requires that the profit division game be converted into utility space. Let Γ^* be the set of all profit division games. Every profit division game, $\Gamma \in \Gamma^*$, is associated with a unique bargaining game $B(\Gamma) = (d(\Gamma),S(\Gamma))$ where:

$d(\Gamma) \equiv (u(w_1),v(w_2))$

$S(\Gamma) \equiv \{ (u(x),v(y)) \mid x \geq w_1,\ y \geq w_2,\ (x + y) \leq (w_1 + w_2 + t) \}$.

It can easily be demonstrated that the characteristics of the utility functions imply that $B(\Gamma)$ is a smooth and comprehensive bargaining game.

For every profit division game, the profit pair associated with a bargaining solution is determined by the *bargaining outcome function* Φ: $F \times \Gamma^* \rightarrow R^2$, where $\Phi(f,\Gamma) \equiv (u^{-1}(f_1(B(\Gamma))),v^{-1}(f_2(B(\Gamma))))$. The bargaining outcome associated with a bargaining solution retains the single-valuedness of the bargaining solution (because of the strong monotonicity of the utility functions). As the next result shows, the bargaining outcome associated with a bargaining solution also inherits its continuity.

LEMMA 2.1: If f: $B^* \rightarrow R^2$ is continuous on B^*, then the associated bargaining outcome $\Phi(f,\Gamma)$ is a continuous function of the disagreement profit pair of the profit division game Γ.

PROOF: The players' utilities u and v are continuous functions of the players' profits. Thus, the disagreement utility pair $d(\Gamma)$ is a continuous function of the disagreement profit pair, and the set of agreement utility pairs $S(\Gamma)$ is a continuous correspondence from the disagreement profit pair. By hypothesis, the bargaining solution f(d,S) is a continuous function of the bargaining game (d,S). Thus, the bargaining solution $f(B(\Gamma))$ is a continuous function of the disagreement profit pair.

Since u and v are continuous and strongly monotonic, u^{-1} and v^{-1} are continuous and single-valued functions of f_1 and f_2. Recall that $\Phi(f,\Gamma) = (u^{-1}(f_1(B(\Gamma))), v^{-1}(f_2(B(\Gamma))))$. Therefore, $\Phi(f,\Gamma)$ is a continuous function of the disagreement profit pair. ∎

SECTION IV: THE MONOPOLISTIC ECONOMY

This section describes an economic system in which there are several traders and a monopolist. Profit can only be obtained through bilateral agreements between the monopolist and the traders, and the total profits of the economy are determined by the set of traders that cooperate with the monopolist. The monopolist and at least one trader are necessary for all productive activity in the economy, that is, no positive profit can be obtained by agreement between two traders that excludes the monopolist nor can profits be made by monopolist alone. The following treatment of monopolistic economies is essentially a restriction of a game with (n+1) players (the monopolist and n traders) to a game with n players (the traders).

DEFINITION 2.2: A *monopolistic economy* Γ^N consists of a monopolist and n traders, $N = \{1,\ldots,n\}$, and is defined by $\Gamma^N = \{u,v_1,\ldots,v_n,\pi\}$, where u is the monopolist's utility function and v_i $(i \in N)$ are the traders' utility functions, all of which are strongly monotonic, concave (risk averse) and twice differentiable, and π: $2^N \to R$ is the coalitional profit function satisfying $\pi(\phi) = 0$.

For each $S \subseteq N$, the *coalitional profit* $\pi(S)$ is the total profit produced if the players in S cooperate with the monopolist.

π is *monotonic* iff $\pi(T) > \pi(S)$, $\forall$ $S \subseteq N$ and $T \subseteq N$ such that $S \subset T$.

Furthermore, π is *[submodular] additive [supermodular]* iff $\pi(T) - \pi(T\setminus i)$ $[\leq] = [\geq]$ $\pi(S) - \pi(S\setminus i)$, $\forall$ $S \subseteq N$ and $T \subseteq N$ such that $S \subset T$ and $\forall$ $i \in S$.

A *network* $S \subseteq N$ is the set of traders that cooperate with the monopolist. A *profit distribution* in a monopolistic economy is denoted $(y_1,\ldots,y_n) \in R^n$, where y_i is the profit of trader i. Since the total profit from each network is given, the players' profits can be fully defined by the set of the traders' profits. The monopolist receives the residual profit, that is, its profit is the total profit generated by the network less the profits received by the traders: $x \equiv \pi(S) - \Sigma_{i\in S}\, y_i$.

DEFINITION 2.3: An *agreement equilibrium* is a pair (S,y), where $S \subseteq N$ and $y \in R^n$ is a profit distribution, that satisfy:

(1) FEASIBILITY: $\Sigma_{i\in S}\, y_i \leq \pi(S)$;

(2) JOINT RATIONALITY: $\pi(S) \geq \pi(S \cup i)$, $\forall$ $i \in N\setminus S$;

(3) TRADER RATIONALITY: $y_i \geq 0$, $\forall$ $i \in S$;

(4) MONOPOLIST RATIONALITY: $\Sigma_{i\in T}\, y_i \leq \pi(S)-\pi(S\setminus T)$, $\forall$ $T \subseteq S$;

(5) EXCLUSION: $y_i = 0$, $\forall$ $i \in N\setminus S$.

An agreement equilibrium consists of a network and profit distribution that are both feasible and stable. A profit distribution is feasible if it is affordable given the network. A network is stable if, given the profit distribution, neither the monopolist nor any of the traders desire to change the network. The conditions of an agreement equilibrium have the following interpretation:

(1) Feasibility requires that the profit distribution is affordable given the network, that is, that the sum of the traders' profits is not greater than the total profits produced by the network.

(2) Joint rationality requires that there is no profitable bilateral agreement that is not executed. If there is a profitable agreement that has not been executed, then the monopolist and the trader would both act to reach an agreement and the network would be altered.

(3) Trader rationality requires that the profit distribution is individually rational for each trader.

(4) Monopolist rationality requires that no set of agreements collectively decreases the monopolist's profit. Note that the standard with respect to breaking agreements is stronger than the standard for executing agreements: bilateral agreements are executed individually, but many may be broken at once.

(5) Any trader who does not cooperate with the monopolist is excluded from the economy and receives zero profits.

If each trader makes a positive marginal contribution to every network to which it belongs, then an agreement equilibrium exists and its network will include all traders.

LEMMA 2.2: If π is monotonic, then (1) there exists an agreement equilibrium, and (2) (S^*,y^*) is an agreement equilibrium implies $S^* = N$.

PROOF: Since π is monotonic, the profit distribution $y = (0,\ldots,0)$ satisfies the conditions for an agreement equilibrium. (This generally is not the only agreement equilibrium.) Suppose $S^* \neq N$. By monotonicity, for $S \neq N$, there exists a profitable agreement that is unexecuted, that is, $\pi(S) < \pi(S \cup i)$, for each $i \in N\backslash S$, which violates the joint rationality condition of agreement equilibria. ∎

Note that if π is not monotonic, then an agreement equilibrium may not exist or its network may not be N.

The profit contribution of trader i to network S equals $\pi(S) - \pi(S\backslash i)$. Milnor [1952] described a payoff as *reasonable* if each player did not receive more than its profit contribution to the coalition. The profit allocation y is *reasonable for network S* iff $y_i \leq \pi(S) - \pi(S\backslash i)$, $\forall\ i \in S$. For an important class of monopolistic bargaining economies, the set of reasonable profit distributions for N is the set of agreement equilibria.

LEMMA 2.3: If π is monotonic and submodular, then (S^*, y^*) is an agreement equilibrium iff (1) $S^* = N$ and (2) y^* is a reasonable and individually-rational profit distribution for the network N.

PROOF: Since π is monotonic, $S^* = N$ [Lemma 2.2]. If the network is N, the set of reasonable and individually rational profits for player i is $Y_i \equiv \{y_i \mid 0 \leq y_i \leq [\pi(N) - \pi(N \setminus i)]\}$.

By submodularity, $[\pi(N) - \pi(N \setminus i)] \leq [\pi(S) - \pi(S \setminus i)]$, $\forall\, S \subset N$. Therefore, $y_i \leq [\pi(S) - \pi(S \setminus i)]$, $\forall\, S \subset N$.

By recursive application of this inequality, it can be shown that the monopolist cannot increase its profits by breaking agreements, that is, $\Sigma_{i \in T}\, y_i \leq [\pi(N) - \pi(N \setminus T)]$, $\forall\, T \subseteq N$. It is similarly demonstrated that the profit allocation is feasible, that is, that $\Sigma_{i \in N}\, y_i \leq \pi(N)$. The other conditions for agreement equilibria are trivially satisfied.

Thus, all reasonable and individually rational profit distributions are agreement equilibria. On the other hand, by definition, agreement equilibria are reasonable and individually rational. ∎

SECTION V: THE GENERAL BARGAINING EQUILIBRIUM

The conditions imposed on agreement equilibria eliminate the networks and profit distributions that are inconsistent with rational economic behavior. Nevertheless, there may be many profit distributions associated with agreement equilibria for a monopolistic economy, and we are interested in selecting from this set of agreement equilibria. This selection is constructed on two premises: (1) that the determination of each trader's profit is the result of bilateral bargaining between the trader and the monopolist, and (2) that in their bilateral bargaining the monopolist and the trader both accurately project the results of the monopolist's negotiations with the other traders and consider such results as given.

In a monopolistic economy, the negotiation between the monopolist and each trader is a profit division game. Let $\Gamma_i(\Gamma^N,S,y)$ be the profit division game between the monopolist and trader i in monopolistic economy Γ^N with network S and profit distribution y, where $\Gamma_i(\Gamma^N,S,y) \equiv ([(\pi(S\backslash i)-\Sigma_{k\epsilon S\backslash i}\ y_i],0),[\pi(S)-\pi(S\backslash i)],u,v_i)$.

In the bargaining between the monopolist and each trader, the monopolist's disagreement profit depends on the profits received by the other traders. Thus, the outcome of the bargaining between each trader and the monopolist is a function of the other bargaining outcomes. Let f^i be the bargaining solution for trader i. For a continuous classical bargaining solution, each player's bargaining outcome is a continuous function of the other bargaining outcomes.

LEMMA 2.4: If $f^i \in C$ is continuous, then the outcome $\Phi(f^i,\Gamma_i(\Gamma^N,S,y))$ of the profit division game $\Gamma_i(\Gamma^N,S,y)$ between the monopolist and trader i is a continuous function of the profits of all of the other traders (y).

PROOF: The monopolist's disagreement profit in $\Gamma_i(\Gamma^N,S,y)$ is a continuous function of the profits received by other traders in S, but $\Gamma_i(\Gamma^N,S,y)$ is otherwise unaffected by other traders' bargaining. Since f^i is continuous, $\Phi(f^i,\Gamma_i(\Gamma^N,S,y))$ is a continuous function of the disagreement profit pair [Lemma 2.1]. Thus, $\Phi(f^i,\Gamma_i(\Gamma^N,S,y))$ is a continuous function of y.

Since $f^i \in C$ is individually rational for both the monopolist and the trader, $y_i = \Phi_2(f^i,\Gamma_i(\Gamma^N,S,y))$ is reasonable and individually rational. The set of reasonable and individually-rational outcomes for $i \in S$ is $Y_i \equiv \{y_i \mid [\pi(S) - \pi(S\backslash i)] \geq y_i \geq 0\}$. Define $Y_i \equiv \{0\}$, for $i \in N\backslash S$. Define the set of profit distributions $Y \equiv \Pi_{i \in N} Y_i$.

Define $\beta_i: Y \rightarrow Y_i$ such that $\beta_i(y) \equiv \Phi_2(f^i,\Gamma_i(\Gamma^N,S,y))$. Then β_i $(i \in N)$ is a set of continuous functions that determine the outcomes of bargaining between the monopolist and each trader as a function of the profits received by the other traders (y). ∎

A general bargaining equilibrium is a profit distribution that is an agreement equilibrium and is the fixed point of the system of bargaining outcome functions. The profit distribution depends on the bargaining solution selected for each trader.

DEFINITION 2.4: For monopolistic economy Γ^N and $\{f^1,\ldots,f^n\} \subset F$: (S^*,y^*) is a *general bargaining equilibrium* iff

(1) (S^*,y^*) is an agreement equilibrium, and

(2) $y^*_i = \Phi_2(f^i,\Gamma_i(\Gamma^N,S^*,y^*))$, $\forall\ i \in S^*$.

Since the bargaining outcome functions are continuous and the set of profit distributions is non-empty, compact and convex, the existence of a general bargaining equilibrium is easily demonstrated.

THEOREM 2.1: If π is monotonic and submodular and $\{f^1,\ldots,f^n\} \subset C$ are continuous, then a general bargaining equilibrium exists.

PROOF: Since π is monotonic and submodular, any agreement equilibrium is such that the network is N and the profit distribution is reasonable and individually rational for N [Lemma 2.3].
The set of reasonable and individually rational profits for $i \in N$ is $Y_i \equiv \{y_i \mid 0 \le y_i \le [\pi(N) - \pi(N\backslash i)]\}$. The set of profit distributions $Y \equiv \Pi_{i\in N} Y_i$ is non-empty, compact and convex.

Since f^i $(i \in N)$ are continuous, there exist continuous bargaining outcome functions β_i: $Y \to Y_i$ $(i \in N)$ that determine the outcome of bargaining between the monopolist and each trader as a function of the other traders' profits [Lemma 2.4]. Define β: $Y \to Y$ such that $\beta(y) \equiv (\beta_1(y),\ldots,\beta_n(y))$. β is a continuous function from Y into itself. Thus, Brouwer's fixed point theorem applies, and $\exists\ y^* \in Y$ such that $y^* = \beta(y^*)$. Since π is monotonic and submodular, any $y \in Y$ is an agreement equilibrium [Lemma 2.3]. ∎

SECTION VI: NON-MONOPOLISTIC ECONOMIES

Economic activity is dominated by bilateral transactions. In markets with a large number of participants -- where none of the players has economic power -- these individual transactions can be ignored. They are subsumed in the market mechanism. On the other hand, in small markets -- where all of the players have economic power -- the emergence of a market with uniform prices at which any player can buy or sell should not be expected. Each transaction is affected by the buyer's and the seller's bargaining power. The general bargaining equilibrium is thus an attractive solution concept for any small economy, not only monopolistic economies.

Bennett [1988] independently developed a similar bargaining equilibrium for the system of bilateral bargaining in a marriage economy, that is, an economy with several buyers and sellers in which economic gains are generated only from the exclusive pairing of a buyer and a seller. Unfortunately, these equilibrium concepts may not be applicable to many non-monopolistic economies in which several of the players can cooperate with more than one other player. These equilibrium concepts as currently defined are not robust enough to accommodate a non-monopolistic player's decision to cooperate with several players and do not allow players to vary their degree of cooperation with one another. These generalizations will be essential if a form of general bargaining equilibrium is to be more broadly applicable in small economies.

POWER SENSITIVE BARGAINING SOLUTIONS

AND BARGAINING POWER

CHAPTER III: POWER SENSITIVE BARGAINING SOLUTIONS

This chapter examines several related classes of bargaining solutions. The class of power sensitive bargaining solutions and strongly power sensitive bargaining solutions is introduced. Each of these bargaining solutions is such that alterations in the bargaining set in the neighborhood of the point of the bargaining solution and at the ideal utility pair affect the outcome prescribed by the bargaining solution.

The class of power sensitive bargaining solutions includes, but is not limited to, all classical bargaining solutions which are independent of irrelevant alternatives and all classical bargaining solutions which are individually monotonic (restricted monotonicity). The class of strongly power sensitive bargaining solutions includes, but is not limited to, all classical bargaining solutions which are independent of irrelevant alternatives and strongly individually rational and all classical bargaining solutions which have restricted strong monotonicity. Classical power sensitive bargaining solutions are both risk sensitive and twist sensitive.

Throughout this chapter, let the two players be $\{1,2\}$, and as a notational convention, for any player $i \in \{1,2\}$, let $j = \{1,2\}\backslash i$. Recall that F is the set of all bargaining functions and C is the set of all classical bargaining functions.

SECTION I: A REVIEW OF BARGAINING SOLUTIONS

This section presents a brief review of bargaining solutions that are (1) independent of irrelevant alternatives, (2) individually monotonic, (3) twist sensitive and (4) risk sensitive.

Nash [1950] introduced the axiom of independence of irrelevant alternatives. $f \in F$ is *independent of irrelevant alternatives* ($f \in N$) iff $f(d,T) \in S \Rightarrow f(d,S) = f(d,T)$, $\forall$ (d,S) and (d,T) such that $S \subset T$. Independence of irrelevant alternatives has been extensively studied, notably in Luce and Raiffa [1957], Roth [1979a], and Thomson and Myerson [1980], among others.

Nash [1950] also introduced the unique classical solution that is symmetric and independent of irrelevant alternatives, which is now generally called the *Nash solution*. The family of all classical bargaining solutions that are independent of irrelevant alternatives was characterized by de Koster, Peters, Tijs, and Wakker [1983].

THEOREM 3.1 (de Koster, Peters, Tijs, and Wakker):
$(C \cap N) = \{F^\alpha\colon B^* \to R^2 \mid 0 \le \alpha \le 1\}$ where
$F^\alpha(d,S) \equiv \{u^* \in P[S] \mid u^* = \arg\max_{u \ge d} (u_1-d_1)^\alpha \cdot (u_2-d_2)^{1-\alpha}\}$, $0 < \alpha < 1$;
$F^0(d,S) \equiv \{u^* \in P[S] \mid u^*_2 > u_2,\ \forall\ u \in S \text{ such that } u \ge d\}$; and
$F^1(d,S) \equiv \{u^* \in P[S] \mid u^*_1 > u_1,\ \forall\ u \in S \text{ such that } u \ge d\}$.

F^0 and F^1 are eliminated if the bargaining solution is required to be strongly individually rational. Let N^* be the class of bargaining solutions that are independent of irrelevant alternatives and strongly individually rational.

THEOREM 3.2 (de Koster, Peters, Tijs, and Wakker):
$(\mathcal{C} \cap \mathcal{N}^*) = \{F^\alpha: B^* \rightarrow R^2 \mid 0 < \alpha < 1\}$.

Let δ_1 and δ_2 be set-valued functions such that $\delta_1(u,S)$ equals the slopes of the supporting hyperplanes of the bargaining set S at the point $u \in P[S]$ and $\delta_2(u,S) = \{ 1/\delta \mid \delta \in \delta_1(u,S)\}$. If S is smooth, then $\delta_1(u,S)$ and $\delta_2(u,S)$ are single-valued, $\forall\, u \in P[S]$.

Classical bargaining solutions that are independent of irrelevant alternatives and strongly individually rational are affected by alterations of the bargaining set only if the set of supporting hyperplanes at the bargaining solution is changed.

THEOREM 3.3: If $f \in (\mathcal{C} \cap \mathcal{N}^*)$, then for $(d,S) \in B^*$ and $(d,T) \in B^*$ such that $f(d,S) \in P[T]$:

(1) $\min \{-\delta_i(f(d,S),T)\} \leq \min \{-\delta_i(f(d,S),S)\}$
and $\max \{-\delta_i(f(d,S),T)\} \leq \max \{-\delta_i(f(d,S),S)\}$
imply $f_i(d,T) \geq f_i(d,S)$ and $f_j(d,T) \leq f_j(d,S)$.
(2) $\max \{-\delta_i(f(d,S),T)\} < \min \{-\delta_i(f(d,S),S)\}$
implies $f_i(d,T) > f_i(d,S)$ and $f_j(d,T) < f_j(d,S)$.

PROOF: Follows immediately from Theorems 3.1 and 3.2.

The *ideal utility* of a bargaining game for each player is the greatest utility that it can receive given that the other player receives at least its disagreement utility. Let the ideal utility for player i be $h_i(d,S) \equiv \max\{u_i \mid u \in S \text{ and } u \geq d\}$.

Kalai and Smorodinsky [1975] introduced the individual monotonicity property, which requires that if the individually rational bargaining set is expanded, but the disagreement point and the other player's ideal utility remain identical, then the bargaining solution assigns the player at least as great utility. $f \in F$ is *individually monotonic* ($f \in M$) iff $(S_d)^* \subset (T_d)^*$ and $h_j(d,S) = h_j(d,T) \Rightarrow f_i(d,S) \leq f_i(d,T)$, $\forall$ (d,S) and (d,T), where $(S_d)^* \equiv \{x \in R^2 \mid x \leq y \text{ for some } y \in S,\ y \geq d\}$. The axiom of individual monotonicity was first suggested by Luce and Raiffa [1957], and has also been studied by Roth [1979a], and Thomson and Myerson [1980], among others. Kalai and Smorodinsky [1975] characterized the unique classical solution that is symmetric and individually monotonic, which is commonly referred to as the *monotonic solution*.

Roth [1979a] introduced the property of restricted monotonicity which requires that if the bargaining set is expanded, but the disagreement and ideal points remain identical, then both players receive at least as great utilities under the bargaining solution. $f \in F$ has *restricted monotonicity* iff $S \subset T$ and $h(d,S) = h(d,T) \Rightarrow f(d,S) \leq f(d,T)$, $\forall$ (d,S) and (d,T). Peters and Tijs [1985] demonstrated that individual monotonicity is equivalent to restricted monotonicity.

Restricted strong monotonicity requires that if the bargaining set is expanded, but the disagreement and ideal points remain identical, then either the bargaining solution remains identical or both players receive higher utilities. $f \in F$ has *restricted strong monotonicity* ($f \in M^*$) iff $S \subset T$ and $h(d,S) = h(d,T) \Rightarrow f(d,S) = f(d,T)$ <u>or</u> $f(d,S) < f(d,T)$, $\forall$ (d,S) and (d,T). Clearly, restricted strong monotonicity implies restricted monotonicity.

The monotonic solution was generalized by Peters and Tijs [1985], who used monotonic curves to define the class of all classical bargaining solutions that are individually monotonic.

THEOREM 3.4 (Peters and Tijs): $(C \cap M) = \{\pi^\theta \mid \theta \in \Omega\}$ where $\Omega \equiv \{\theta: [1,2] \rightarrow \Delta \mid \theta_1(a) + \theta_2(a) = a \text{ and } \theta(a) \leq \theta(b),\ 1 \leq a < b \leq 2\}$ and $\pi^\theta(d,S) \equiv T^{-1}[\ P[T(d,S)] \cap \{\theta(a) \mid 1 \leq a \leq 2\}\ ]$.

For classical bargaining solutions with restricted monotonicity, if one player receives a greater proportion of its ideal utility gain in one bargaining game than in another, then the other player must receive at least as great a proportion of its ideal utility gain. All individually monotonic solutions are individually rational, but some are not be strongly individually rational.

The class of bargaining solutions is reduced if the bargaining solutions are required to have restricted strong monotonicity. The restricted strong monotonicity condition excludes any curve θ that is monotonic but not strongly monotonic, that is, any curve with any horizontal or vertical segments.

THEOREM 3.5: $(\mathcal{C} \cap \mathcal{M}^*) = \{\pi^\Theta \mid \Theta \in \Omega^*\}$, where
$\Omega^* \equiv \{\Theta \in \Omega \mid \Theta(a) < \Theta(b),\ 1 \le a < b \le 2\}$.

PROOF: See Peters and Tijs [1985], Theorems 2 and 3. ∎

For classical bargaining solutions with restricted strong monotonicity, if one player is to receive a greater proportion of its ideal utility gain in one bargaining game than in another, then the other player also must receive a greater proportion of its ideal utility gain. Furthermore, if one player receives a given proportion of its ideal utility gain, then the proportion that the other player receives of its ideal utility gain is uniquely determined. All bargaining solutions that satisfy restricted strong monotonicity are strongly individually rational.

Classical bargaining solutions that have restricted monotonicity are sensitive to alterations of the bargaining set at the ideal points. If the bargaining solution for a given bargaining set is Pareto-optimal in the altered bargaining set, then the bargaining solution is changed only if the ideal points are altered.

THEOREM 3.6: Given $(d,S) \in B^*$ and $(d,T) \in B^*$ with $f(d,S) \in P[T]$:

(1) For $f \in (\mathcal{C} \cap \mathcal{M})$: $h_i(d,T) \ge h_i(d,S)$ and $h_j(d,T) \le h_j(d,S)$

$\Rightarrow$ $f_i(d,T) \ge f_i(d,S)$ and $f_j(d,T) \le f_j(d,S)$;

(2) For $f \in (\mathcal{C} \cap \mathcal{M}^*)$: $h_i(d,T) >[\ge]\ h_i(d,S)$ and $h_j(d,T) \le[<]\ h_j(d,S)$

$\Rightarrow$ $f_i(d,T) > f_i(d,S)$ and $f_j(d,T) < f_j(d,S)$.

PROOF: Follows immediately from Theorems 3.4 and 3.5.

Tijs and Peters [1985] defined an operation on bargaining sets called *twisting*. T is a *[un]favorable twisting* of S for player i at $u \in P[S]$ iff (1) x_i >[<] u_i, $\forall x \in T \setminus S$, and (2) x_i <[>] u_i, $\forall x \in S \setminus T$. A twisting of a bargaining set affects the Pareto-frontier of the bargaining set. If a bargaining set is strongly twisted, then the only point on the Pareto-frontier of the original bargaining set that remains in the new bargaining set is the point at which the set is twisted. T is a *strongly [un]favorable twisting* of S for player i at $u \in P[S]$ iff (1) T is a [un]favorable twisting of S for player i at u and (2) $P[S] \cap P[T] = u$.

Tijs and Peters [1985] also introduced the class of twist sensitive bargaining solutions. $f \in \mathcal{F}$ is *twist sensitive* ($f \in \mathcal{T}$) iff for any $(d,S) \in B^*$ and $(d,T) \in B^*$: T is a [un]favorable twisting of S for player i at $f(d,S) \Rightarrow f_i(d,T) \geq [\leq] f_i(d,S)$. A stronger form of twist sensitivity can be defined. $f \in \mathcal{F}$ is *strongly twist sensitive* ($f \in \mathcal{T}^*$) iff (1) $f \in \mathcal{T}$ and (2) for smooth (d,S) and (d,T): T is a strongly [un]favorable twisting of S for player i at $f(d,S) \Rightarrow f_i(d,T) > [<] f_i(d,S)$.

Tijs and Peters [1985] demonstrated that classical bargaining solutions that are independent of irrelevant alternatives or have restricted monotonicity are twist sensitive. It can similarly be demonstrated that bargaining solutions that are independent of irrelevant alternatives and strongly individually rational or have restricted strong monotonicity are strongly twist sensitive.

THEOREM 3.7 (Tijs and Peters): $(\mathcal{C} \cap \mathcal{N}) \cup (\mathcal{C} \cap \mathcal{M}) \subset \mathcal{T}$ and $(\mathcal{C} \cap \mathcal{N}^*) \cup (\mathcal{C} \cap \mathcal{M}^*) \subset \mathcal{T}^*$.

Let S* be the set of all bargaining sets. There exist monotonically decreasing concave functions z_1: R × S* → R and z_2: R × S* → R such that $(u_1,u_2) \in P[S]$ iff $u_1 = z_1(u_2,S)$ and $u_2 = z_2(u_1,S)$. The Pareto-frontier P[S] consists of points of the form $(u_1,z_2(u_1,S))$ or $(z_1(u_2,S),u_2)$.

A player's utility function is at least as risk averse if it is transformed by an increasing, concave function and is more risk averse if it is transformed by an increasing, strictly concave function. A player's risk aversion in a bargaining game can similarly be transformed. For (d,S) and (d,T), player i is *at least as [more] risk averse* in T as in S iff $z_i(u_j,T) = k(z_i(u_j,S))$, $\forall\ u_j$, where k is some increasing, [strictly] concave utility function.

Kihlstrom, Roth and Schmeidler [1981] defined a class of bargaining solutions such that an opponent's greater risk aversion does not decrease the utility received by a player in bargaining. $f \in \mathcal{F}$ is *risk sensitive* ($f \in \mathcal{R}$) iff the utility it assigns to a player does not decrease when the player's opponent is replaced with one that is at least as risk averse. A strong form of risk sensitivity may be defined which requires that a more risk averse opponent increases a player's utility. $f \in \mathcal{F}$ is *strongly risk sensitive* ($f \in \mathcal{R}^*$) iff (1) $f \in \mathcal{R}$ and (2) f assigns a player a higher utility in smooth bargaining games when its opponent is replaced with one that is more risk averse.

Kihlstrom, Roth and Schmeidler [1981] demonstrated that the Nash solution, the symmetric monotonic solution and the superadditive solution are risk sensitive. Their results were generalized by de Koster, Peters, Tijs, and Wakker [1983], which demonstrated that if a bargaining solution is independent of irrelevant alternatives or individually monotonic, then it is risk sensitive. Tijs and Peters [1985] further demonstrated that twist sensitive classical bargaining solutions are risk sensitive. If a player's opponent is replaced with an opponent that is at least as risk averse, then the bargaining game has been favorably twisted for the player. It can similarly be demonstrated that strongly twist sensitive solutions are strongly risk sensitive. If a player's opponent is replaced with an opponent that is more risk averse, then the bargaining game has been strongly favorably twisted for the player.

THEOREM 3.8 (Tijs and Peters): $(C \cap T) \subset R$ and $(C \cap T^*) \subset R^*$.

Kihlstrom, Roth and Schmeidler [1981] demonstrated that if a bargaining solution is Pareto-optimal and risk sensitive, then it necessarily is independent of equivalent utility representations.

SECTION II: POWER SENSITIVE BARGAINING SOLUTIONS

This section introduces power sensitive bargaining solutions and strongly power sensitive bargaining solutions and examines their relationship with other classes of bargaining solutions. Each of these bargaining solutions is sensitive to alterations in the bargaining set that affect both the neighborhood of the point of the bargaining solution and the ideal points.

Power sensitive bargaining solutions require that if the bargaining game is not unfavorably altered for a player in the neighborhood of the bargaining solution or at either of the ideal points, then the player receives at least as great utility from the bargaining solution to the new bargaining game.

DEFINITION 3.1: $f \in F$ is *power sensitive* ($f \in P$) iff, $\forall$ $(d,S) \in B^*$ and $(d,T) \in B^*$ such that $f(d,S) \in P[T]$:

(1) $\min \{-\delta_i(f(d,S),T)\} \leq \min \{-\delta_i(f(d,S),S)\}$;

(2) $\max \{-\delta_i(f(d,S),T)\} \leq \max \{-\delta_i(f(d,S),S)\}$;

(3) $h_i(d,T) \geq h_i(d,S)$; and

(4) $h_j(d,T) \leq h_j(d,S)$

imply $f_i(d,T) \geq f_i(d,S)$ and $f_j(d,T) \leq f_j(d,S)$.

Strongly power sensitive bargaining solutions require that if the bargaining game is favorably altered for a player in the neighborhood of the bargaining solution and at the ideal points, then the player receives greater utility from the bargaining solution to the new bargaining game.

DEFINITION 3.2: $f \in F$ is *strongly power sensitive* ($f \in P^*$) iff $f \in P$ and $\forall$ $(d,S) \in B^*$ and $(d,T) \in B^*$ such that $f(d,S) \in P[T]$:

(1) $\max\{-\delta_i(f(d,S),T)\} < \min\{-\delta_i(f(d,S),S)\}$,

(2) $h_i(d,T) > [\geq]\ h_i(d,S)$, and

(3) $h_j(d,T) \leq [<]\ h_j(d,S)$,

with either (2) or (3) holding with strict inequality,

imply $f_i(d,T) > f_i(d,S)$ and $f_j(d,T) < f_j(d,S)$.

If a classical bargaining solution has restricted monotonicity or is independent of irrelevant alternatives, then it is power sensitive.

THEOREM 3.9: $(C \cap M) \subset P$ and $(C \cap N) \subset P$.

PROOF: $f \in (C \cap M)$ satisfies the weaker necessary and sufficient condition for $f \in P$ [Compare Theorem 3.6(1) and Definition 3.1].

Consider $f \in (C \cap N)$. Suppose $f = F^0$ or $f = F^1$. Then $f \in (C \cap M)$ (see Peters and Tijs [1985]), which implies $f \in P$. Otherwise, $f \in (C \cap N^*)$, which satisfies the weaker necessary and sufficient condition for $f \in P$ [Compare Theorem 3.3 and Definition 3.1]. ∎

Similarly, if a classical bargaining solution has restricted strong monotonicity or is independent of irrelevant alternatives and strongly individually rational, then it is strongly power sensitive.

THEOREM 3.10: $(C \cap M^*) \subset P^*$ and $(C \cap N^*) \subset P^*$.

PROOF: $f \in (C \cap M^*)$ satisfies the weaker necessary and sufficient condition for $f \in P^*$ [Compare Theorem 3.6(2) and Definition 3.2].

$f \in (C \cap N^*)$ satisfies the weaker necessary and sufficient condition for $f \in P^*$ [Compare Theorem 3.3 and Definition 3.2]. ∎

Note that there exist classical power sensitive bargaining solutions that are neither independent of irrelevant alternatives nor individually monotonic. An example of such a solution follows.

EXAMPLE 3.1: Define $f \in C$ such that $f(d,S) = u \in S$ which satisfies

$$\min \{-\delta_1(u,S)\} \cdot \frac{u_1 - d_1}{u_2 - d_2} \le \frac{\dfrac{h_1(d,S) - d_1}{u_1 - d_1}}{\dfrac{h_2(d,S) - d_2}{u_2 - d_2}} \le \max \{-\delta_1(u,S)\} \cdot \frac{u_1 - d_1}{u_2 - d_2}.$$

Recall that classical bargaining solutions that are independent of irrelevant alternatives and classical bargaining solutions that are individually monotonic are twist sensitive. For bargaining games that are comprehensive, power sensitive classical bargaining solutions are twist sensitive. Twisting a bargaining set in some direction at a given point on the Pareto frontier alters the entire Pareto frontier, which includes the ideal points and the Pareto frontier in the neighborhood of the bargaining solution that affect power sensitive bargaining solutions. If a comprehensive bargaining set is favorably [unfavorably] twisted for a player, then it necessarily is not unfavorably [favorably] altered for the player with respect to power sensitive bargaining solutions.

THEOREM 3.11: For comprehensive bargaining games, if $f \in (\mathcal{C} \cap \mathcal{P})$, then $f \in \mathcal{T}$.

PROOF: It will be shown that for favorable and unfavorable twistings, a power sensitive bargaining solution requires the same change in the utility pair as a twist sensitive solution. Without loss of generality, consider twistings that are favorable or unfavorable to player 1.

To examine favorable twistings, assume for $(d,S) \in B^*$ and $(d,T) \in B^*$ that $x_1 > f_1(d,S)$, $\forall\ x \in T\backslash S$, and $x_1 < f_1(d,S)$, $\forall\ x \in S\backslash T$.

It will be demonstrated that this assumption implies that:

(1) $\min \{-\delta_1(f(d,S),T)\} \leq \min \{-\delta_1(f(d,S),S)\}$,

(2) $\max \{-\delta_1(f(d,S),T)\} \leq \max \{-\delta_1(f(d,S),S)\}$,

(3) $h_1(d,T) \geq h_1(d,S)$, and

(4) $h_2(d,T) \leq h_2(d,S)$.

For $f \in P$, these conditions would imply that $f_1(d,T) \geq f_1(d,S)$, which satisfies the requirement of twist sensitive solutions with respect to favorable twistings.

Part I: Show that $\min \{-\delta_1(f(d,S),T)\} \leq \min \{-\delta_1(f(d,S),S)\}$ and $\max \{-\delta_1(f(d,S),T)\} \leq \max \{-\delta_1(f(d,S),S)\}$.

The value of $\delta_1(f(d,S),S)$ can be affected by changes in the bargaining set S in the neighborhood of $f(d,S)$.

If $x_1 > f_1(d,S)$, $\forall\, x \in T\backslash S$, and $x_1 < f_1(d,S)$, $\forall\, x \in S\backslash T$, then

(1) $\min \{-\delta_1(f(d,S),T)\} \leq \min \{-\delta_1(f(d,S),S)\}$ and

(2) $\max \{-\delta_1(f(d,S),T)\} \leq \max \{-\delta_1(f(d,S),S)\}$.

Part II: Show that $h_1(d,T) \geq h_1(d,S)$.

By individual rationality, $f_2(d,S) \geq d_2$. Recall that $h_k(d,S) \equiv \max \{u_k \mid u \in S \text{ and } u \geq d\}$. Thus, by definition, $h_1(d,S) \geq f_1(d,S)$.

Suppose $h_1(d,T) < h_1(d,S)$. Since (d,S) is comprehensive, $(h_1(d,S),d_2) \in S\backslash T$. By assumption, if $x \in S\backslash T$, then $x_1 < f_1(d,S)$. Thus, $h_1(d,S) < f_1(d,S)$, which would establish a contradiction. Therefore, $h_1(d,T) \geq h_1(d,S)$.

Part III: Show that $h_2(d,T) \leq h_2(d,S)$.

Suppose not. Since T is comprehensive, $(d_1,h_2(d,T)) \in T\backslash S$. By assumption, if $x \in T\backslash S$, then $x_1 > f_1(d,S)$. Thus, $d_1 > f_1(d,S)$, which would contradict individual rationality. Therefore, $h_2(d,T) \le h_2(d,S)$.

Since these three conditions have been shown to be satisfied, a power sensitive bargaining solution requires that $f_1(d,T) \ge f_1(d,S)$. Therefore, the requirement of twist sensitive bargaining solutions with respect to favorable twistings is satisfied.

On the other hand, if it is assumed for $(d,S) \in B^*$ and $(d,T) \in B^*$ that $x_1 < f_1(d,S)$, $\forall\ x \in T\backslash S$, and $x_1 > f_1(d,S)$, $\forall\ x \in S\backslash T$, then it can be demonstrated through a similar proof that power sensitive solutions would require that $f_1(d,T) \le f_1(d,S)$, which satisfies the requirement of twist sensitive bargaining solutions with respect to unfavorable twistings. ∎

Furthermore, for bargaining games that are comprehensive and smooth, strongly power sensitive classical bargaining solutions are strongly twist sensitive.

THEOREM 3.12: For comprehensive and smooth bargaining games, if $f \in (\mathcal{C} \cap \mathcal{P}^*)$, then $f \in \mathcal{T}^*$.

PROOF: By same method as Theorem 3.11.

On the other hand, twist sensitive solutions are not necessarily power sensitive. Tijs and Peters [1985] introduced the equal area split solution as an example of a twist sensitive solution that was neither independent of irrelevant alternatives nor individually monotonic. The equal area split solution is also strongly twist sensitive. Nevertheless, the equal area split solution is neither strongly power sensitive nor power sensitive.

EXAMPLE 3.2: Let $f\colon B^* \to R^2$ be the equal area split solution, that is, $f(d,S)$ is the point on the Pareto frontier $P[S]$ such that the area in S lying above the line through d and $f(d,S)$ equals the area in S lying below that line, for every $(d,S) \in B^*$.

CHAPTER IV: RISK AVERSION AND BARGAINING POWER

This chapter first reviews the measures of absolute and relative risk aversion that were developed by Pratt [1964] and Arrow [1965]. Absolute risk aversion is a measure of an individual's aversion to the risk of a given amount of his income or wealth. Relative risk aversion is a measure of an individual's aversion to the risk of a proportion of his income or wealth.

This chapter then introduces measures of a player's ability to demand a greater proportion of the profit contribution from agreement. Nash bargaining power is a measure of a player's power if the bargaining solution is independent of irrelevant alternatives and strongly individually rational. Monotonic bargaining power is a measure of a player's power if the bargaining solution has restricted strong monotonicity. Bargaining power is defined as the pair consisting of a player's Nash and monotonic bargaining power. These measures of bargaining power can assist in determining bargaining outcomes associated with power sensitive bargaining solutions.

Several properties of the bargaining power function are examined. These properties will be used in the final chapter of this dissertation to demonstrate the effects of syndication on general bargaining equilibria associated with strongly power sensitive bargaining solutions.

SECTION I: MEASURES OF RISK AVERSION

This section reviews several concepts in the study of risk aversion. Risk aversion requires that the players are not risk lovers. A risk averse player may be risk neutral or strictly risk averse. A utility function u is *risk neutral [strictly risk averse]* iff $u''(x) = [<] \ 0, \ \forall \ x$. If an individual is risk neutral, his absolute and relative risk aversion are both constant and zero.

Absolute risk aversion can be interpreted as a local measure of the aversion of an individual to the risk of a given amount of income or wealth. The measure of *absolute risk aversion* for the utility function u is $r(x) = - u''(x)/u'(x)$. Absolute risk aversion is useful in comparing the risk aversion of different utility functions, provided the utility functions are twice-differentiable. The utility function u_1 is *at least as risk averse* as u_2 iff $r_1(x) \geq r_2(x), \ \forall \ x$.

Absolute risk aversion was intended as a measure of local risk aversion, but it can also capture global properties of the utility function. If one utility function is more risk averse than another at all income levels, then the more risk averse individual would be willing to pay a greater premium for insurance against a risk of loss, and would be willing to accept a smaller cash equivalent for a risky gain.

If an individual's utility function has decreasing absolute risk aversion, then as his wealth increases, he is not willing to pay as much for insurance against a given risk. This assumption has substantial intuitive appeal. It is to be expected that the richer the individual, the smaller the premium which the individual would be willing to pay to insure against the loss of a given amount of income or wealth. The assumption of decreasing or nonincreasing absolute risk aversion is broadly supported in the economics literature. Arrow [1971] asserts compelling intuitive support for assuming decreasing absolute risk aversion. Epstein [1983] presents a justification of decreasing absolute risk aversion based on an infinite horizon consumption problem.

As a player's disagreement profit increases, if the player has decreasing [nonincreasing] absolute risk aversion, then the player is less [not more] risk averse in bargaining for a given incremental profit.[1]

THEOREM 4.1: If u has decreasing [nonincreasing] r, then for $u_1(q) \equiv u(q+a)$ and $u_2(q) \equiv u(q+b)$: $a > b > 0 \Rightarrow r_1(q) < [\leq]\ r_2(q)$.

PROOF: By definition: $r_1(q) = r(q+a)$ and $r_2(q) = r(q+b)$. If r is decreasing [nonincreasing], then $r(q+a) < [\leq]\ r(q+b)$, for $a > b > 0$. By substitution, $r_1(q) < [\leq]\ r_2(q)$, $\forall$ q. ∎

[1] To interpret Theorem 4.1, let a and b be disagreement profits such that a > b, let u be the player's utility function and r be its measure of risk aversion, and consider the player's bargaining over the incremental profit from agreement (q).

The measure of relative risk aversion can be interpreted as a local measure of aversion to the risk of a proportion of income or assets. The measure of *relative risk aversion* for the utility function u is $r^*(x) = - x \cdot u''(x)/u'(x)$.

Arrow [1971] claims that it should be expected that relative risk aversion is an increasing function of income or wealth. If relative risk aversion is increasing, then as an individual's income is increased, he is willing to pay a greater percentage of his income to insure against the risk of the loss of a given percentage of his income. Arrow [1971] cites time-series data for the United States which indicates that, as income has increased, the proportion of wealth held in the "riskless" assets of cash and cash equivalents has increased. Arrow [1971] also demonstrates that if u is bounded from above and below and r* is monotonic, then r* is increasing. It can be similarly demonstrated that if u is bounded and r* does not change direction, then r* is nondecreasing.

If a player has increasing [nondecreasing] relative risk aversion, then the player is more [at least as] risk averse in bargaining for a proportion of a larger profit contribution.[2]

THEOREM 4.2: If u has increasing [nondecreasing] r*, then for $u_1(k) \equiv u(ka + w)$ and $u_2(k) \equiv u(kb + w)$, where $w \geq 0$:
$a > b > 0 \Rightarrow r_1(k) > [\geq] r_2(k), \forall k > 0$.

[2] To interpret Theorem 4.2, let w be the player's disagreement profit, let a and b be the profit contributions from agreement such that a > b, and consider the player's bargaining for a proportion (k) of the profit contribution from agreement.

PROOF: If r^* is increasing [nondecreasing], then $a > b > 0 \Rightarrow r^*(ka + w) > [\geq]\ r^*(kb + w),\ \forall\ k > 0$.

$$\text{Thus,}\quad \frac{-(ka+w) \cdot u''(ka+w)}{u'(ka + w)} > [\geq] \frac{-(kb+w) \cdot u''(kb+w)}{u'(kb + w)}.$$

By hypothesis, $a > b > 0$ and $w \geq 0$, which implies $\dfrac{a}{ka+w} \geq \dfrac{b}{kb+w} > 0$.

$$\text{By multiplication,}\quad \frac{-a \cdot u''(ka+w)}{u'(ka + w)} > [\geq] \frac{-b \cdot u''(kb+w)}{u'(kb + w)}.$$

$$r_1(k) = \frac{-\dfrac{\partial^2[u_1(k)]}{\partial k^2}}{\dfrac{\partial[u_1(k)]}{\partial k}} = \frac{-a^2 \cdot u''(ka + w)}{a \cdot u'(ka + w)} = \frac{-a \cdot u''(ka + w)}{u'(ka + w)}.$$

$$r_2(k) = \frac{-\dfrac{\partial^2[u_2(k)]}{\partial k^2}}{\dfrac{\partial[u_2(k)]}{\partial k}} = \frac{-b^2 \cdot u''(kb + w)}{b \cdot u'(kb + w)} = \frac{-b \cdot u''(kb + w)}{u'(kb + w)}.$$

By substitution: $r_1(k) > [\geq]\ r_2(k),\ \forall\ k > 0$. ■

SECTION II: MEASURES OF BARGAINING POWER

This section introduces Nash and monotonic bargaining power, two measures of a player's ability to demand a greater proportion of the profit contribution from agreement.

A player's Nash bargaining power is related to its boldness in bargaining. Boldness is a measure of risk aversion which captures all of the information needed to determine the Nash outcome to a bargaining situation. Aumann and Kurz [1977] introduced boldness and its inverse, fear of ruin, and noted their application to the Nash solution of the bargaining problem. Roth [1979a] presented a more extensive study of boldness and bargaining. In a bargaining situation, each player should be more willing to accept the proposed outcome as his profit from agreement increases. A player's boldness is the maximum probability of disagreement per dollar of gain that the player is willing to accept in order to marginally increase its profit under the agreement. The following definition is a restatement of a definition presented by Roth [1979a].

DEFINITION 4.1: A player's *boldness* is $b(x,q) \equiv u'(x)/[u(x)-u(x-q)]$, where x is the player's agreement profit, and q is its incremental profit from agreement with the other player, and thus the player's disagreement profit is $(x-q)$.

Nash bargaining power measures a player's power to demand a greater proportion of the profit contribution from agreement if the bargaining solution is independent of irrelevant alternatives and strongly individually rational. Nash bargaining power is the probability of disagreement per fractional increase in share that a player is willing to accept in order to marginally increase its proportion of the profit contribution from agreement. A player's Nash bargaining power is equal to its boldness multiplied by the profit contribution from agreement.

DEFINITION 4.2: A player's *Nash bargaining power* is

$$p_N(u,x,q,t) \equiv \frac{t \cdot u'(x)}{u(x) - u(x - q)},$$

where u is the player's utility function, x is the player's total profit, q is the player's incremental profit from agreement in the bargaining situation, t is the profit contribution from agreement, and thus (x - q) is the player's disagreement profit.

REMARK 4.1: The Nash bargaining outcome is the unique Pareto-optimal outcome to a bargaining situation such that the two players' boldness measured from the disagreement point is equal. This characterization of the Nash bargaining outcome was introduced by Aumann and Kurz [1977] and formalized by Roth [1979a]. The Nash bargaining outcome is also the unique Pareto-optimal outcome to a bargaining situation such that the Nash bargaining power of the two players is equal.

Monotonic bargaining power measures a player's power to demand a greater proportion of the profit contribution from agreement if the bargaining solution has restricted strong monotonicity. A player's monotonic bargaining power is equal to the inverse of the proportion of the player's ideal utility that is received by the player.

DEFINITION 4.3: A player's *monotonic bargaining power* is

$$p_M(u,x,q,t) \equiv \frac{u(t + x - q) - u(x - q)}{u(x) - u(x - q)},$$

where u is the player's utility function, x is the player's total profit, q is the player's incremental profit from agreement in the bargaining situation, t is the profit contribution from agreement, and thus the player's disagreement profit is (x - q) and its ideal profit is (t + x - q), which equals its total profit if it received all of the profit contribution from agreement.

REMARK 4.2: The bargaining outcome associated with the symmetric monotonic solution proposed by Kalai and Smorodinsky [1975] is the unique Pareto-optimal outcome to a bargaining situation such that the monotonic bargaining power of the two players is necessarily equal.

For the remainder of this paper, a player's bargaining power will be defined as the pair consisting of its Nash and monotonic bargaining powers.

DEFINITION 4.4: A player's *bargaining power* is the pair $p(u,x,q,t) \equiv \{p_N(u,x,q,t), p_M(u,x,q,t)\}$.

If one player has greater Nash bargaining power and the other player has greater monotonic bargaining power, then the relative power of the players is indeterminate. Therefore, bargaining power is not a complete ordering for all bargaining outcomes and all bargaining situations.

REMARK 4.3: If the player is risk neutral, then its Nash and monotonic bargaining powers are both equal to the inverse of the proportion it receives of the profit contribution, that is, $p(u,x,q,t) = (t/q)$. If the player is strictly risk averse, then the player's Nash and monotonic bargaining powers are only equal for the unique bargaining outcome such that $u'(x) = ([u(t+x-q)-u(x-q)] / t)$, where x is the player's total profit, q is its incremental profit, and t is the profit contribution.

A result from Pratt [1964] can be applied to demonstrate the relation between risk aversion and bargaining power. If one player is more [at least as] risk averse, then the player is less [not more] powerful in bargaining.

THEOREM 4.3: $r_1(x) > [\geq]\ r_2(x) \Rightarrow p(u_1,x,q,t) < [\leq]\ p(u_2,x,q,t)$, $\forall\ x,q,t$.

PROOF: If $r_1(x) > [\geq]\ r_2(x)$, then:

$$\frac{u_1(a) - u_1(b)}{u'_1(a)} > [\geq] \frac{u_2(a) - u_2(b)}{u'_2(a)}, \quad \text{for } a > b \quad \text{[Pratt (1964)].}$$

Define $a \equiv x$ and $b \equiv (x - q)$.

By substitution, $$\frac{u_1(x) - u_1(x - q)}{u'_1(x)} > [\geq] \frac{u_2(x) - u_2(x - q)}{u'_2(x)},$$

which implies $$\frac{t \cdot u'_1(x)}{u_1(x) - u_1(x - q)} < [\leq] \frac{t \cdot u'_2(x)}{u_2(x) - u_2(x - q)}.$$

Equivalently, $p_N(u_1,x,q,t) < [\leq]\ p_N(u_2,x,q,t)$.

If $r_1(x) > [\geq]\ r_2(x)$, then:

$$\frac{u_1(a) - u_1(b)}{u_1(b) - u_1(c)} < [\leq] \frac{u_2(a) - u_2(b)}{u_2(b) - u_2(c)}, \quad \text{for } a > b > c \quad \text{[Pratt (1964)].}$$

Define $a \equiv (x + t - q)$, $b \equiv x$, and $c \equiv (x - q)$.

Thus, $$\frac{u_1(x + t - q) - u_1(x)}{u_1(x) - u_1(x - q)} < [\leq] \frac{u_2(x + t - q) - u_2(x)}{u_2(x) - u_2(x - q)},$$

which implies $$\frac{u_1(x+t-q) - u_1(x-q)}{u_1(x) - u_1(x-q)} < [\leq] \frac{u_2(x+t-q) - u_2(x-q)}{u_2(x) - u_2(x-q)}.$$

Equivalently, $p_M(u_1,x,q,t) < [\leq]\ p_M(u_2,x,q,t)$. ∎

SECTION III: BARGAINING POWER AND BARGAINING OUTCOMES

Measures of bargaining power can assist in determining bargaining outcomes associated with power sensitive bargaining solutions. All of the results of this section will be demonstrated by the same method. Two bargaining situations will be considered. One of the bargaining situations will have a known bargaining outcome. The bargaining outcome of the other bargaining situation will be unknown, and some Pareto-optimal profit pair will be proposed as its bargaining outcome. It will be demonstrated that if one player is more powerful for the proposed bargaining outcome in the latter bargaining situation than for the known bargaining outcome in the former bargaining situation and his opponent is less powerful, then the proposed bargaining outcome must be adjusted to increase the player's profit.

The identities of the players for all of these results are irrelevant. If the players' identities were reversed, then the restated theorems would remain valid. Recall that $\Phi(f,\Gamma)$ is the bargaining outcome function and that $B(\Gamma)$ is the bargaining game associated with the bargaining situation Γ.

Classical bargaining solutions that are strongly individually rational and independent of irrelevant alternatives require that if one player has greater Nash bargaining power, then the other player must also have greater Nash bargaining power. If this condition is not met, then the bargaining outcome must be adjusted.

THEOREM 4.4: For $f \in (C \cap N^*)$ and bargaining situations $\Gamma = (w,t,u,v)$ with known outcome $\Phi(f,\Gamma) = (x^*,y^*)$, and $\Gamma' = (w',t',u',v')$ with a proposed Pareto-optimal outcome of (x,y):

(1) $p_N(u',x,x-w'_1,t') > [\geq] \; p_N(u,x^*,x^*-w_1,t)$, and

(2) $p_N(v',y,y-w'_2,t') \leq [<] \; p_N(v,y^*,y^*-w_2,t)$,

imply $\Phi_1(f,\Gamma') > x$ and $\Phi_2(f,\Gamma') < y$.

PROOF: Let $(d,S) \equiv B(\Gamma)$, $(e,T) \equiv B(\Gamma')$, $\hat{u}_1 \equiv u'(x)$ and $\hat{u}_2 \equiv v'(y)$.

Define $\sigma_1(u,(d,S)) \equiv - \delta_1(u,S) \cdot \dfrac{u_1 - d_1}{u_2 - d_2}$.

It is easily demonstrated that σ_1 equals the ratio of the player's Nash bargaining powers:

$$\sigma_1(\hat{u},(e,T)) = \frac{p_N(v',y,y-w'_2,t')}{p_N(u',x,x-w'_1,t')};$$

$$\sigma_1(f(d,S),(d,S)) = \frac{p_N(v,y^*,y^*-w_2,t)}{p_N(u,x^*,x^*-w_1,t)}.$$

Thus, by hypothesis, $\sigma_1(\hat{u},(e,T)) < \sigma_1(f(d,S),(d,S))$.

Recall that $f \in (C \cap N^*)$ iff $f \in F^\alpha$ $(0 < \alpha < 1)$ [Theorem 3.2]. Then, by definition, f selects the utility pair $f(d,S) \in P[S]$ that maximizes $(f_1(d,S) - d_1)^\alpha \cdot (f_2(d,S) - d_2)^{1-\alpha}$.

Thus, f selects $u^* \in S$ such that $- \delta_1(u^*,S) \cdot \dfrac{u^*_1 - d_1}{u^*_2 - d_2} = \dfrac{\alpha}{1 - \alpha}$.

By substitution, $\sigma_1(f(d,S),(d,S)) = \alpha/(1 - \alpha)$.

Therefore, if $f \in (C \cap N^*)$, then $\sigma_1(f(d,S),(d,S)) = \alpha/(1 - \alpha)$ and $\sigma_1(f(e,T),(e,T)) = \alpha/(1 - \alpha)$

By hypothesis, $\sigma_1(\hat{u},(e,T)) < \sigma_1(f(d,S),(d,S))$. Thus, $\sigma_1(\hat{u},(e,T)) < \sigma_1(f(e,T),(e,T))$.

The function $\sigma_1(\hat{u},(e,T))$ is increasing in $\hat{u}_1$ and decreasing in $\hat{u}_2$. Thus, $f_1(e,T) > \hat{u}_1$ or $f_2(e,T) < \hat{u}_2$.

Since $\hat{u}$ and $f(e,T)$ are both Pareto-optimal utility pairs, if the inequality holds for either player, then it holds for the other. Therefore, $f_1(e,T) > \hat{u}_1 \equiv u'(x)$ and $f_2(e,T) < \hat{u}_2 \equiv v'(y)$.

By definition, $u'(\Phi_1(f,\Gamma')) = f_1(e,T)$ and $v'(\Phi_2(f,\Gamma')) = f_2(e,T)$. Thus, $u'(\Phi_1(f,\Gamma')) > u'(x)$ and $v'(\Phi_2(f,\Gamma')) < v'(y)$.

Both u' and v' are strongly monotonically increasing. Therefore, $\Phi_1(f,\Gamma') > x$ and $\Phi_2(f,\Gamma') < y$. ∎

Classical bargaining solutions with restricted strong monotonicity require that if one player has greater monotonic bargaining power, then the other player must also have greater monotonic bargaining power. If this condition is not met, then the bargaining outcome must be adjusted.

THEOREM 4.5: For $f \in (C \cap M^*)$ and bargaining situations $\Gamma = (w,t,u,v)$ with known outcome $\Phi(f,\Gamma) = (x^*,y^*)$, and $\Gamma' = (w',t',u',v')$ with a proposed Pareto-optimal outcome of (x,y):

(1) $p_M(u',x,x-w'_1,t') > [\geq] \; p_M(u,x^*,x^*-w_1,t)$, and

(2) $p_M(v',y,y-w'_2,t') \leq [<] \; p_M(v,y^*,y^*-w_2,t)$,

imply $\Phi_1(f,\Gamma') > x$ and $\Phi_2(f,\Gamma') < y$.

PROOF: Let $(d,S) \equiv B(\Gamma)$, $(e,T) \equiv B(\Gamma')$, $\hat{u}_1 \equiv u'(x)$ and $\hat{u}_2 \equiv v'(y)$.

Reinterpreting the hypothesis in terms of utility yields:

$$\frac{h_1(e,T) - e_1}{\hat{u}_1 - e_1} > [\geq] \frac{h_1(d,S) - d_1}{f_1(d,S) - d_1};$$

$$\frac{h_2(e,T) - e_2}{\hat{u}_2 - e_2} \leq [>] \frac{h_2(d,S) - d_2}{f_2(d,S) - d_2}.$$

Each of these terms is the inverse of the proportion of the ideal utility received by the player under the given bargaining outcome. For $f \in (C \cap N^*)$, if one player receives a greater proportion of its ideal utility, then its opponent must also receive a greater proportion of its ideal utility. Thus, $f \in \in (C \cap N^*)$ would require $f_1(e,T) > \hat{u}_1 \equiv u'(x)$ and $f_2(e,T) < \hat{u}_2 \equiv v'(y)$.

By definition, $u'(\Phi_1(f,\Gamma')) = f_1(e,T)$ and $v'(\Phi_2(f,\Gamma')) = f_2(e,T)$. Thus, $u'(\Phi_1(f,\Gamma')) > u'(x)$ and $v'(\Phi_2(f,\Gamma')) < v'(y)$.

Both u' and v' are strongly monotonically increasing. Therefore, $\Phi_1(f,\Gamma') > x$ and $\Phi_2(f,\Gamma') < y$. ▮

Classical bargaining solutions that are strongly power sensitive bargaining solutions require that if one player is more powerful, then the other player must also be more powerful. If this condition is not met, then the bargaining outcome must be adjusted.

THEOREM 4.6: For $f \in (C \cap P^*)$ and bargaining situations $\Gamma = (w,t,u,v)$ with known outcome $\Phi(f,\Gamma) = (x^*,y^*)$, and $\Gamma' = (w',t',u',v')$ with a proposed Pareto-optimal outcome of (x,y):

(1) $p(u',x,x-w'_1,t') > [\geq] \; p(u,x^*,x^*-w_1,t)$, and

(2) $p(v',y,y-w'_2,t') \leq [<] \; p(v,y^*,y^*-w_2,t)$,

imply $\Phi_1(f,\Gamma') > x$ and $\Phi_2(f,\Gamma') < y$.

PROOF: Let $(d,S) \equiv B(\Gamma)$, $(e,T) \equiv B(\Gamma^{\sim})$, $\hat{u}_1 \equiv u'(x)$ and $\hat{u}_2 \equiv v'(y)$.

By hypothesis, $\sigma_1(\hat{u},(e,T)) < \sigma_1(f(d,S),(d,S))$ [See Theorem 4.4].

Reinterpreting the hypothesis in terms of utility also yields:

$$\frac{h_1(e,T) - e_1}{\hat{u}_1 - e_1} > [\geq] \; \frac{h_1(d,S) - d_1}{f_1(d,S) - d_1};$$

$$\frac{h_2(e,T) - e_2}{\hat{u}_2 - e_2} \leq [<] \; \frac{h_2(d,S) - d_2}{f_2(d,S) - d_2}.$$

A classical bargaining solution is independent of equivalent utility representations. Define affine transformations of S and T:

$S^\circ \equiv \{(u^\circ_1,u^\circ_2) \mid u^\circ_k = ((u_k-d_k)/(f_k(d,S)-d_k)),\ u \in S,\ k = \{1,2\}\}$

$T^\circ \equiv \{(u^\circ_1,u^\circ_2) \mid u^\circ_k = ((u_k-e_k)/(\hat{u}_k-e_k)),\ u \in T,\ k = \{1,2\}\}$

$(0,S°) \in B^*$ and $(0,T°) \in B^*$ and $f(0,S°) = (1,1) \in P[T°]$ satisfy:

(1) $\max\{-\delta_1(f(0,S°),T°)\} < \min\{-\delta_1(f(0,S°),S°)\}$,

(2) $h_1(0,T°) \geq h_1(0,S°)$, and

(3) $h_2(0,T°) \leq h_2(0,S°)$,

with either (2) or (3) holding with strict inequality.

Since $f \in P^*$, these conditions imply $f_1(0,T°) > f_1(0,S°) = 1$ and $f_2(0,T°) < f_2(0,S°) = 1$.

Thus, since f is independent of equivalent representations, $f \in P^*$ implies $f_1(e,T) > \hat{u}_1 \equiv u'(x)$ and $f_2(e,T) < \hat{u}_2 \equiv v'(y)$.

By definition, $u'(\Phi_1(f,\Gamma')) = f_1(e,T)$ and $v'(\Phi_2(f,\Gamma')) = f_2(e,T)$. Thus, $u'(\Phi_1(f,\Gamma')) > u'(x)$ and $v'(\Phi_2(f,\Gamma')) < v'(y)$.

Both u' an v' are strongly monotonically increasing. Therefore, $\Phi_1(f,\Gamma') > x$ and $\Phi_2(f,\Gamma') < y$. $\blacksquare$

SECTION IV: PROPERTIES OF BARGAINING POWER

This section examines several properties of the power function which will be used in the final chapter of this dissertation to demonstrate the effects of syndication on general bargaining equilibria associated with power sensitive solutions. The following properties hold for utility function u with measures of absolute risk aversion r and relative risk aversion r^*, and $x > 0$, $q > 0$ and $t > 0$.

For a player whose total profits are at risk in bargaining, if its utility function is strongly monotonic and risk averse, then its bargaining power decreases as its profit from agreement increases.

PROPERTY 1: $u'(\cdot) > 0$ and $u''(\cdot) \leq 0 \Rightarrow p(u,x',x',t) < p(u,x,x,t)$, $\forall\ x' > x$.

PROOF: $u'(\cdot) > 0$ and $u''(\cdot) \leq 0 \Rightarrow \dfrac{t \cdot u'(x')}{u(x') - u(0)} < \dfrac{t \cdot u'(x)}{u(x) - u(0)}$.

$$u'(\cdot) > 0 \Rightarrow \frac{u(t) - u(0)}{u(x') - u(0)} < \frac{u(t) - u(0)}{u(x) - u(0)}.$$

Thus, $p_N(u,x',x',t) < p_N(u,x,x,t)$ and $p_M(u,x',x',t) < p_M(u,x,x,t)$. ∎

For a player who has less than all of its profits at risk in bargaining, if its utility function is strongly monotonic, then its bargaining power decreases as its incremental profit from agreement increases.

PROPERTY 2: $u'(\cdot) > 0 \Rightarrow p(u,x,q',t) < p(u,x,q,t), \ \forall\, q' > q.$

PROOF: $u'(\cdot) > 0 \Rightarrow \dfrac{t \cdot u'(x)}{u(x) - u(x - q')} < \dfrac{t \cdot u'(x)}{u(x) - u(x - q)}.$

$$u'(\cdot) > 0 \Rightarrow \frac{u(x+t-q') - u(x-q')}{u(x) - u(x-q')} < \frac{u(x+t-q) - u(x-q)}{u(x) - u(x-q)}.$$

Thus, $p_N(u,x,q',t) < p_N(u,x,q,t)$ and $p_M(u,x,q',t) < p_M(u,x,q,t)$. ∎

If its incremental profit from agreement remains constant, then the player with a strongly monotonic utility function is more powerful in demanding a given incremental profit as the profit contribution from agreement increases.

PROPERTY 3: $u'(\cdot) > 0 \Rightarrow p(u,x,q,t') > p(u,x,q,t), \ \forall\, t' > t.$

PROOF: $t > t' \Rightarrow \dfrac{t' \cdot u'(x)}{u(x) - u(x - q)} > \dfrac{t \cdot u'(x)}{u(x) - u(x - q)}.$

$$u'(\cdot) > 0 \Rightarrow \frac{u(x+t'-q) - u(x-q)}{u(x) - u(x-q)} > \frac{u(x+t-q) - u(x-q)}{u(x) - u(x-q)}.$$

Thus, $p_N(u,x,q,t') < p_N(u,x,q,t)$ and $p_M(u,x,q,t') < p_M(u,x,q,t)$. ∎

If a player is risk neutral, and its total profit and its proportion of the profit contribution from agreement are both fixed, then the player is equally powerful in demanding a given proportion of any profit contribution.

PROPERTY 4: $u''(\cdot) = 0 \Rightarrow p(u,x,kq,kt) = p(u,x,q,t), \ \forall\ k > 0.$

PROOF: $$u''(\cdot) = 0 \Rightarrow \frac{u(x) - u(x-kq)}{kq} = \frac{u(x) - u(x-q)}{q},$$

which implies $$\frac{kt \cdot u'(x)}{u(x) - u(x - kq)} = \frac{t \cdot u'(x)}{u(x) - u(x - q)}.$$

Equivalently, $p_N(u,x,kq,kt) = p_N(u,x,q,t)$.

$$u''(\cdot) = 0 \Rightarrow \frac{u(x + k[t-q]) - u(x - kq)}{u(x) - u(x - kq)} = \frac{u(x+t-q) - u(x-q)}{u(x) - u(x-q)}.$$

Equivalently, $p_M(u,x,kq,kt) = p_M(u,x,q,t)$. ∎

On the other hand, if a player is strictly risk averse, and its total profit and its proportion of the profit contribution from agreement are both fixed, then in bargaining for a given proportion of the profit contribution, the player's bargaining power decreases as the profit contribution from agreement increases.

PROPERTY 5: $u''(\cdot) < 0 \Rightarrow p(u,x,kq,kt) < p(u,x,q,t), \; \forall\, k > 1.$

PROOF: $u''(\cdot) < 0 \Rightarrow \dfrac{u(x) - u(x-kq)}{kq} > \dfrac{u(x) - u(x-q)}{q}, \; \forall\, k > 1.$

Thus, $\dfrac{kt \cdot u'(x)}{u(x) - u(x - kq)} < \dfrac{t \cdot u'(x)}{u(x) - u(x - q)}.$

Equivalently, $p_N(u,x,kq,kt) < p_N(u,x,q,t).$

$u''(\cdot) < 0 \Rightarrow [u(x + k[t-q]) - u(x)] < k \cdot [u(x+t-q) - u(x)]$ and $[u(x) - u(x-kq)] > k \cdot [u(x) - u(x-q)], \; \forall\, k > 1.$

By division: $\dfrac{u(x + k[t-q]) - u(x)}{u(x) - u(x - kq)} < \dfrac{u(x+t-q) - u(x)}{u(x) - u(x-q)}.$

Thus, $\dfrac{u(x + k[t-q]) - u(x-kq)}{u(x) - u(x - kq)} < \dfrac{u(x+t-q) - u(x-q)}{u(x) - u(x - q)}.$

Equivalently, $p_M(u,x,kq,kt) < p_M(u,x,q,t).$ ∎

As the profit contribution from agreement becomes arbitrarily small and the player's proportion of the profit contribution remains constant, the bargaining power of the strictly risk averse player approaches the bargaining power of a risk neutral player.

PROPERTY 6: $u''(\cdot) < 0$ and $\upsilon''(\cdot) = 0 \Rightarrow$

$\lim_{k\to 0} p(u,x,kq,kt) = p(\upsilon,x,q,t)$.

PROOF: $\upsilon''(\cdot) = 0 \Rightarrow p_N(\upsilon,x,q,t) = \dfrac{t \cdot \upsilon'(x)}{q \cdot \upsilon'(x)} = \dfrac{t}{q}$.

$$\lim_{k\to 0} p_N(u,x,kq,kt) = \lim_{k\to 0} \frac{kt \cdot u'(x)}{u(x) - u(x - kq)}.$$

By L'Hopital's Rule, $\lim_{k\to 0} p_N(u,x,kq,kt) = \lim_{k\to 0} \dfrac{t \cdot u'(x)}{q \cdot u'(x - kq)}$.

Therefore, $\lim_{k\to 0} p_N(u,x,kq,kt) = (t/q) = p_N(\upsilon,x,q,t)$.

$$\upsilon''(\cdot) = 0 \Rightarrow p_M(\upsilon,x,q,t) = \frac{t \cdot \upsilon'(x)}{q \cdot \upsilon'(x)} = \frac{t}{q}.$$

$$\lim_{k\to 0} p_M(u,x,kq,kt) = \lim_{k\to 0} \frac{u(x + k[t-q]) - u(x-kq)}{u(x) - u(x-kq)}.$$

By L'Hopital's Rule:

$$\lim_{k\to 0} p_M(u,x,kq,kt) = \lim_{k\to 0} \frac{[(t-q)\cdot u'(x+k[t-q])] - [-q\cdot u'(x-kq)]}{q \cdot u'(x - kq)}.$$

Therefore, $\lim_{k\to 0} p_M(u,x,kq,kt) = (t/q) = p_M(\upsilon,x,q,t)$. ∎

Recall that if one player is more [at least as] risk averse, then the player is less [not more] powerful in bargaining.

PROPERTY 7: $r_1(x) > [\geq]\ r_2(x),\ \forall\ x \Rightarrow p(u_1,x,q,t) < [\leq]\ p(u_2,x,q,t)$.

If the player's incremental profit from agreement and the profit contribution from agreement are held constant, then as its total income is increased, a player with nonincreasing absolute risk aversion does not become less powerful.

PROPERTY 8: r nonincreasing $\Rightarrow p(u,x',q,t) \geq p(u,x,q,t),\ \forall\ x' > x$.

PROOF: If the player has a greater total agreement profit and an equal incremental profit from agreement, then its disagreement profit is greater. If r is nonincreasing, then as the player's disagreement profit increases, the player is not more risk averse in any given bargaining situation [Theorem 4.1]. Therefore, the player's bargaining power is at least as great [Property 7]. ∎

If a player is bargaining for a given proportion of its total profit, then as its total profit is increased and its incremental profit from agreement is increased proportionately, the player with a strongly monotonic utility function with nondecreasing relative risk aversion does not become more powerful.

PROPERTY 9: $u'(\cdot) > 0$ and r^* nondecreasing $\Rightarrow$
$p(u,kx,kq,t) \leq p(u,x,q,t),\ \forall\ k \geq 1$.

PROOF: Define $u_1(x) \equiv u(kx)$ and $u_2(x) \equiv u(x)$.

By r* nondecreasing, $r_1(x) \geq r_2(x)$ [Theorem 4.2].

Thus, $$\frac{u'_1(x)}{u_1(x) - u_1(x - q)} \leq \frac{u'_2(x)}{u_2(x) - u_2(x - q)}.$$ [See Theorem 4.3]

Substitution yields $$\frac{k \cdot u'(kx)}{u(kx) - u(kx - kq)} \leq \frac{u'(x)}{u(x) - u(x - q)}.$$

Thus, $$\frac{t \cdot u'(kx)}{u(kx) - u(kx - kq)} \leq \frac{t \cdot u'(x)}{u(x) - u(x - q)}.$$

Equivalently, $p_N(u,kx,kq,t) \leq p_N(u,x,q,t)$.

Since $r_1(x) \geq r_2(x)$:

$$\frac{u_1(x+t-q) - u_1(x-q)}{u_1(x) - u_1(x-q)} \leq \frac{u_2(x+t-q) - u_2(x-q)}{u_2(x) - u_2(x-q)}$$ [See Theorem 4.3].

By substitution: $$\frac{u(kx+kt-kq) - u(kx-kq)}{u(kx) - u(kx-kq)} \leq \frac{u(x+t-q) - u(x-q)}{u(x) - u(x-q)}.$$

By hypothesis, $k \geq 1$. Since $u'(\cdot) > 0$, $u(kx+t-kq) \leq u(kx+kt-kq)$.

Thus, $$\frac{u(kx+t-kq) - u(kx-kq)}{u(kx) - u(kx-kq)} \leq \frac{u(x+t-q) - u(x-q)}{u(x) - u(x-q)}.$$

Equivalently, $p_M(u,kx,kq,t) \leq p_M(u,x,q,t)$. ∎

Bargaining power is independent of the scale measuring profits: if the scale measuring profits was changed and the player's utility function was modified to reflect this change, then the player's bargaining power would remain identical.

PROPERTY 10: $\exists\ k > 0 \mid u_1(x) = u_2(kx),\ \forall\ x \Rightarrow$
$p(u_1,x,q,t) = p(u_2,kx,kq,kt)$.

PROOF: By hypothesis, $u_1(x) = u_2(kx)$ and $u_1(x-q) = u_2(kx-kq)$.

Differentiating, $u'_1(x) = k \cdot u'_2(kx)$.

Thus, $$\frac{u'_1(x)}{u_1(x) - u_1(x-q)} = \frac{k \cdot u'_2(kx)}{u_2(kx) - u_2(kx-kq)},$$

which implies $$\frac{t \cdot u'_1(x)}{u_1(x) - u_1(x-q)} = \frac{kt \cdot u'_2(kx)}{u_2(kx) - u_2(kx-kq)}.$$

Equivalently, $p_N(u_1,x,q,t) = p_N(u_2,kx,kq,kt)$.

By hypothesis, $u_1(x+t-q) = u_2(kx+kt-kq)$.

Thus, $$\frac{u_1(x+t-q) - u_1(x-q)}{u_1(x) - u_1(x-q)} = \frac{u_2(kx+kt-kq) - u_2(kx-kq)}{u_2(kx) - u_2(kx-kq)}.$$

Equivalently, $p_M(u_1,x,q,t) = p_M(u_2,kx,kq,kt)$. ∎

SYNDICATION AND GENERAL BARGAINING EQUILIBRIA

CHAPTER V: SYNDICATION

Conventional wisdom in economics holds that if a group of traders is confronted with a monopolist, then the traders should form a syndicate in order to establish a bilateral monopoly in the market, but this result has never been demonstrated. On the other hand, several examples exist of disadvantageous monopolies, where absolute control of a market results in lower profit for a firm.

This essay analyzes syndication among several traders that bargain with a monopolist with respect to the general bargaining equilibria associated with risk sensitive and strongly power sensitive bargaining solutions. The effects of syndication among the traders depend on the coalitional profit function and the risk aversion of the monopolist.

If the monopolist is strictly risk averse and the coalitional profit function is additive or submodular, then syndication is advantageous. Traders in larger syndicates receive greater profits. The merger of syndicates results in greater profits for the traders in the merging syndicates. The traders receive the greatest profits if they all syndicate to create a bilateral monopoly.

If the monopolist is risk neutral, the effects of syndication depend on the coalitional profit function: syndication is neutral if the profit function is additive for the set of traders, advantageous if it is submodular, and disadvantageous if it is supermodular.

SECTION I: VOLUME DISCOUNTING

The phenomenon of volume discounting is pervasive in market economies. Volume discounts are not counter-intuitive: it is expected that larger customers will be granted discounts. Most examples of volume discounting are readily explained by generally accepted theories of industrial organization. The explanations include price discrimination, economies of scale, transactions costs, and search costs. Nevertheless some examples of volume discounting are not adequately explained by traditional economic theories.

This essay offers an alternative explanation for volume discounting. Monopolistic bargaining economies are studied in which the traders are identical, the coalitional profit function is additive (constant returns), and there are no transactions costs. Thus, there is no incentive for price discrimination and no cost considerations which would motivate volume discounting. Nevertheless, in any general bargaining equilibrium associated with a strongly risk sensitive or strongly power sensitive bargaining solution, a strictly risk averse monopolist necessarily grants larger traders more favorable terms in bargaining. The more favorable terms received by the larger syndicates are a generalized form of volume discounts. This indicates that if a firm is strictly risk averse, it may grant discounts to its larger customers simply because it is disproportionately more concerned with retaining them as customers.

This motivation for volume discounting appears consistent with conventional wisdom in the business community, in which managers generally regard themselves as risk averse. In my discussions of this research, business managers have generally acknowledged that risk aversion offers a natural and compelling explanation of volume discounting.

Risk aversion has been extensively explored in the bargaining literature, but has not heretofore been postulated as an explanation for volume discounts. Kohli and Park [1989] explored quantity discounts offered by a monopolist using a different application of bargaining theory, but focused on the transactions-efficiency rationale for quantity discounts.

SECTION II: A REVIEW OF THE SYNDICATION LITERATURE

A syndicate is a coalition of players who have agreed to act as a single entity (see Dreze and Gabszewicz [1971]). A syndicate is treated as a single player in game theoretic models. No coalition is valid if it contains some but not all of the members of a syndicate. Syndication is an agreement among players to form a syndicate.

The effects of syndication have been studied with respect to many cooperative solution concepts. The subject was first examined in terms of the core in Aumann [1973]. Further results with respect to the core were presented by Postlewaite and Rosenthal [1974] and Greenberg and Shitovitz [1977]. The effect of syndication on value allocations was investigated in Guesnerie [1977], Gardner [1977], and Legros [1984]. Finally, the effects of syndication on the nucleolus have recently been analyzed by Legros [1987] and Barnes [1990]. In a more specialized context, the effect of syndication on the core of a game with a communications graph was examined by Kalai, Postlewaite and Roberts [1978].

The effects of syndication on noncooperative solutions has also been analyzed. Okuno, Postlewaite, and Roberts [1980] examined the effect of syndication on the Nash equilibria of economies with noncooperative exchange. Szidarovszky and Yakowitz [1982] and Salant, Switzer and Reynolds [1983] evaluated syndication with respect to the Cournot equilibrium of an oligopolistic economy.

Throughout the syndication literature, the economies are two-commodity markets that consist of two sets of players, each of which has complete control over the endowment of one commodity. For simplicity, those players endowed with one commodity are referred to as sellers, and the players endowed with the other commodity are called buyers. The naming of the sellers and buyers is arbitrary and has no effect on the results.

The seminal work on disadvantageous monopolies is presented by Aumann [1973], who studies economies in which there is a non-atomic continuum of identical sellers and a non-atomic continuum of two types of buyers. Two examples are presented in which syndication among the sellers to form a monopoly results in an expansion of the core such that all the new core allocations are worse for the sellers than the original competitive core allocation. Therefore, in these economies syndication among the sellers is disadvantageous.

Postlewaite and Rosenthal [1974] study an economy in which there are a finite set of buyers and a finite set of sellers. An example is presented in which syndication of the sellers to form a monopoly results in an expansion of the core such that all the new core allocations are worse for the syndicate members than the original core allocation. Therefore, in this economy syndication among the sellers is disadvantageous. On the other hand, were the buyers in this economy syndicated to form a monopoly, then syndication among the sellers would not be disadvantageous -- it would result in the same set of core allocations.

Aumann [1973] suggests that the value may be a more appropriate solution concept with respect to the effects of syndication, but the value fails to rehabilitate the belief that market power is necessarily advantageous. Guesnerie [1977] studies syndication in replica economies and presents examples of disadvantageous monopolies. Gardner [1977] demonstrates that monopoly must be advantageous for at least one side of the market but provides an example in which a monopoly is disadvantageous for the other side of the market. Legros [1984] describes conditions for advantageous or disadvantageous monopolies in an economy with perfectly complementary commodities.

Syndication can also be disadvantageous with respect to the nucleolus. Legros [1987] presents examples of disadvantageous syndicates in economies with perfectly complementary commodities. Barnes [1990] presents examples of disadvantageous syndicates in economies with increasing returns.

Myerson [1977] introduces games with communications graphs in which each node represents a player and a link between two nodes represents the ability of the two players to cooperate directly with one another. Kalai, Postlewaite and Roberts [1978] study the effect of syndication on the core in game with a communications graph and present examples of games in which a monopolistic position in the communications graph is disadvantageous.

Okuno, Postlewaite, and Roberts [1980] study the Nash equilibria of a non-cooperative exchange economy. Syndicates with market power (atoms) adopt imperfectly competitive behavior and restrict their exchange of goods. Larger syndicates restrict their level of exchange further than smaller syndicates. Individual players gain by defecting from syndicates. Finally, there are examples of disadvantageous syndicates.

Szidarovszky and Yakowitz [1982] study the Cournot equilibria of oligopolistic economies and construct an example in which syndication among some of the oligopolists is disadvantageous for the firms in the syndicate and advantageous for the firms that are not in the syndicate. Salant, Switzer and Reynolds [1983] also present an example in which merger is disadvantageous with respect to the Cournot equilibrium.

SECTION III: THE EFFECTS OF SYNDICATION

Although syndication has been studied with respect to many game theoretic solution concepts, each solution appears limited in its ability to illuminate the effects of syndication. For example, in his seminal work on syndication, Aumann [1973] questions the appropriateness of the core in analyzing the effects of syndication:

> The concept of the core is based on what a coalition can guarantee for itself. Monopoly power is probably not based on this at all, but rather on what the monopolist can *prevent* other coalitions from getting. His strength lies in his threat possibilities, in the bargaining power engendered by the harm he can cause by refusing to trade. Put differently, the monopolist's power -- and for that matter, that of any other trader -- is measured by the difference between what others can get with him and what they can get without him. This line of reasoning is entirely different from that used in the definition of the core.

The general bargaining equilibrium may more effectively capture the nature of monopoly power. The monopolist's payoff in a general bargaining equilibrium is the result of its bilateral bargaining with the traders. The bargaining power of each player in a bilateral bargaining game is based on the difference between the players' opportunities if they agree and their disagreement outcome if they fail to reach an agreement. The general bargaining equilibrium is generated by a system of bilateral bargaining games and thus should reflect each player's bargaining power.

For the general bargaining equilibria associated with risk sensitive and power sensitive bargaining solutions, the effects of syndication among the traders depend on the profit function and the risk aversion of the monopolist. The effects of syndication are identical whether there are a finite number of atomic traders or there is a continuum of nonatomic traders that form a finite number of atomic syndicates.

Syndication is first analyzed with respect to general bargaining equilibria associated with risk sensitive and strongly risk sensitive bargaining solutions (see chart on page 87). For all bargaining equilibria associated with risk sensitive bargaining solutions, if the monopolist is risk neutral and the coalitional profit function is additive, then all syndicates receive equal profits per trader and merger among the syndicates does not affect the traders' profits. If the monopolist is strictly risk averse with decreasing absolute risk aversion and increasing relative risk aversion, then syndication is advantageous with respect to the bargaining equilibria associated with strongly risk sensitive solutions. Traders in larger syndicates receive greater profits, and the merger of all syndicates is advantageous.

Power equilibria are the general bargaining equilibria associated with strongly power sensitive bargaining solutions. All of the results for general bargaining equilibria associated with strongly risk sensitive bargaining solutions can be strengthened for power equilibria. The following results are presented for power equilibria.

The monopolist receives the greatest profits if it is risk neutral. If the monopolist is risk neutral, then the effects of syndication depend on the coalitional profit function (see chart on page 88). If the coalitional profit function is additive, then syndication is neutral, that is, it has no effect on the profits of the traders and the monopolist. If the coalitional profit function is strictly submodular, then syndication is advantageous for the traders. Finally, if the coalitional profit function is strictly supermodular, syndication is disadvantageous. If all of the players are risk neutral, these effects of syndication are identical to those for the value (McLean [1984]) and the nucleolus (Barnes [1990]) in the associated characteristic function game with transferable utility.

The principal results of this essay concern economies in which the coalitional profit function is additive. If the coalitional profit function is additive, the effects of syndication among the traders depend on the utility function of the monopolist (see chart on page 89). If the monopolist is risk neutral, then all syndicates receive identical profits per trader regardless of the organization of the traders. On the other hand, if the monopolist is strictly risk averse, the organization of the traders affects the profits of the players.

The introduction of a strictly risk averse monopolist creates opportunities for advantageous syndication. Syndication among traders which is neutral or disadvantageous in bargaining with a risk neutral monopolist may be advantageous if the monopolist is strictly risk averse. The strictly risk averse monopolist is weaker in bargaining for a share of the greater profit produced by cooperation with larger syndicates, while the risk neutral monopolist is not. Syndication can allow the traders to exploit the monopolist's strict risk aversion. If the monopolist is strictly risk averse, then traders in larger syndicates receive greater profits and the merger of all syndicates to form a bilateral monopoly is advantageous. Furthermore, if the monopolist is strictly risk averse with decreasing absolute risk aversion and increasing relative risk aversion, then the merger of any set of syndicates is advantageous.

If the economy comprises a strictly risk averse monopolist and a continuum of traders which form a finite number of identical syndicates, then the monopolist would prefer that the traders be distributed among many syndicates. As the traders are divided into an increasing number of smaller syndicates, the monopolist's profits increase and the traders' profits decrease. As the syndicates become arbitrarily small, the profits of the strictly risk averse monopolist approach the profits that would be received by a risk neutral monopolist.

These results may offer insight into the advantages of syndication in the absence of transactions costs or economies of scale that would generally motivate organizational cooperation to achieve greater efficiency. In bargaining economies, syndication may be advantageous simply as a means to exploit the strict risk aversion of one's opponent.

The following three charts present a summary of the principal results concerning syndication and the general bargaining equilibria associated with strongly risk sensitive and strongly power sensitive bargaining solutions.

BARGAINING EQUILIBRIA FOR STRONGLY RISK SENSITIVE BARGAINING SOLUTIONS

SUMMARY OF RESULTS FOR ADDITIVE ECONOMIES

	ASSUMPTIONS ON THE MONOPOLIST'S UTILITY FUNCTION				
	RISK NEUTRAL	STRICTLY RISK AVERSE	STRICTLY RISK AVERSE NONINCREASING R	STRICTLY RISK AVERSE DECREASING R INCREASING R*	STRICTLY RISK AVERSE NONINCREASING R NONDECREASING R*
UNIQUENESS	There is at most one bargaining equilibrium.		There is at most one equilibrium with a given total profit for the monopolist.		
SYNDICATE SIZE	All syndicates receive the same profit per trader.			A larger syndicate receives greater profits per trader.	A larger syndicate receives at least as great profits per trader.
MERGER OF ALL SYNDICATES	All syndicates receive the same profit per trader regardless of their organization.			Increases profits per trader for every merging syndicate.	Does not decrease profits per trader for any merging syndicate.
MERGER OF SOME SYNDICATES	All syndicates receive the same profit per trader regardless of their organization.				

R is absolute risk aversion and R* is relative risk aversion.

BARGAINING EQUILIBRIA FOR STRONGLY POWER SENSITIVE BARGAINING SOLUTIONS

SUMMARY OF RESULTS FOR ECONOMIES WITH A RISK NEUTRAL MONOPOLIST

	ASSUMPTIONS ON THE COALITIONAL PROFIT FUNCTION		
	ADDITIVE	STRICTLY SUBMODULAR	STRICTLY SUPERMODULAR
UNIQUENESS	The equilibrium is unique.	The equilibrium is unique.	If an equilibrium exists, then it is unique.
SYNDICATE SIZE	All syndicates receive equal profits per trader.	Larger syndicates receive greater profits per trader.	Larger syndicates receive smaller profits per trader.
MERGER OF ALL SYNDICATES	Does not affect players' profits.	Maximizes every trader's profit. Minimizes monopolist's total profit.	Minimizes every trader's profit. Maximizes monopolist's total profit.
MERGER OF SOME SYNDICATES	Does not affect players' profits.	Increases profits of traders in merging syndicates. Decreases monopolist's total profit.	Decreases profits of traders in merging syndicates. Increases monopolist's total profit.
INDEPENDENCE OF ALL TRADERS	Does not affect players' profits.	Minimizes every trader's profit. Maximizes monopolist's total profit.	Maximizes every trader's profit. Minimizes monopolist's total profit.

BARGAINING EQUILIBRIA FOR STRONGLY POWER SENSITIVE BARGAINING SOLUTIONS

SUMMARY OF RESULTS FOR ADDITIVE ECONOMIES

	ASSUMPTIONS ON THE MONOPOLIST'S UTILITY FUNCTION				
	RISK AVERSE	STRICTLY RISK AVERSE	STRICTLY RISK AVERSE NONINCREASING R	STRICTLY RISK AVERSE NONDECREASING R*	STRICTLY RISK AVERSE NONINCREASING R NONDECREASING R*
UNIQUENESS	At most one equilibrium with a given total profit for monopolist.			The equilibrium is unique.	
SYNDICATE SIZE	Monopolist receives greater profit from larger syndicate.	A larger syndicate receives greater profits per trader.			
MERGER OF ALL SYNDICATES		Increases total traders profits; decreases monopolist's profit.	Increases profits per trader for every merging syndicate.		
MERGER OF SOME SYNDICATES				Increases total profit of merging syndicates. Decreases monopolist's total profits.	Increases profits of merging syndicates. Does not decrease profits of nonmerging syndicates.
DIVIDING CONTINUUM OF TRADERS INTO A GREATER NUMBER OF IDENTICAL SYNDICATES		As syndicates become arbitrarily small, monopolist's profit approaches that of risk neutral monopolist.		Lowers traders' profits; increases monopolist's profit.	

SECTION IV: THE STABILITY OF A SYNDICATE

That a syndicate is advantageous is a necessary but not a sufficient condition for its stability. A disadvantageous syndicate is clearly not stable: its members can increase their payoffs by disintegrating the syndicate. On the other hand, an advantageous syndicate may be unstable. Advantageousness compares two extreme situations, one in which the players in the syndicate act in unison and the other in which they each act individually. There may be other forms of organization that would result in greater payoffs for some or all of the players.

In studying stability, it should be assumed that players expect that the organization of the other players will not change. Thus, the stable organization of the players is the Nash equilibrium of a process in which traders have some freedom to organize themselves as individuals or syndicates. Legros [1987] presents a formal definition of stability in an economy with two classes of players.

A player may be able to increase its payoff by defecting from a syndicate and acting as an individual. This is the simplest threat to the stability of a syndicate: no cooperation among players is required; a player simply chooses not to cooperate in maintaining a syndicate. Some of the players in a syndicate also may be able increase their payoffs by leaving the syndicate to form one or more new syndicates. This breakup may benefit all of the players in the original syndicate or only those that are defecting. This threat to syndicates requires some cooperation, but only among players that had already cooperated to form the original syndicate. Finally, players from several syndicates may conspire to form a new syndicate or set of syndicates. This threat to syndicates is the weakest since it requires the cooperation of the players from several syndicates. Thus, if the organization of the economy is stable against such drastic reorganization, then it satisfies the strongest form of Nash equilibrium.

SECTION V: SYNDICATES AND MONOPOLISTIC BARGAINING ECONOMIES

The results of this essay concern syndicates of traders that bargain with a monopolist. A syndicate will be defined as any trader or group of traders that bargains with the monopolist as a single entity. In the context of this essay, an individual trader that is not a member of a group syndicate will itself be referred to as a syndicate.

If an agreement is reached between the monopolist and a syndicate, then all of the traders that are members of the syndicate have reached an agreement with the monopolist. If the syndicate fails to reach an agreement with the monopolist, then none of the traders that are members of the syndicate can reach an agreement with the monopolist. Furthermore, none of the traders that are members of a syndicate can reach supplemental agreements with the monopolist.

For the class of economies with a finite set of traders, let the n identical traders be represented by $N = \{1,\ldots,n\}$. For the class of economies with a continuum of identical non-atomic traders, let the traders be represented by $T = [0,1]$. For both finite and continuous economies, let the traders be organized into a finite set of m syndicates, $S = \{S_1,\ldots,S_m\}$, that is a partition of the set of traders N or T. For finite economies, let the number of traders in the syndicate S_k be $n_k \equiv |S_k|$. For non-atomic economies, let the measure of the traders in syndicate S_k be $n_k \equiv \mu(S_k)$.

To analyze the effect of syndication, the model of the monopolistic bargaining economy will be simplified by assuming that all traders are identical.

ASSUMPTION 1: All traders have the identical utility function v and all traders are identical with respect to the coalitional profit function, that is, $|S| = |T| \Rightarrow \pi(S) = \pi(T)$.

Since all traders are identical, each member of a syndicate should receive an identical share of the syndicate's profits. If syndicate S_k receives total profits of y_k, then each member of the syndicate receives (y_k/n_k). Each trader's utility from its profit share is $v(y_k/n_k)$. Since all traders are identical and all syndicate members receive an equal share of a syndicate's profits, the utility function for a syndicate can be expressed as the utility of any one of its members.

ASSUMPTION 2: The utility function $v_k: R \rightarrow R$ of syndicate S_k is $v_k(y_k) \equiv v(y_k/n_k)$.

The *coalitional profit function* π determines the total profit produced in a bargaining economy, which is a function of the set of traders that cooperate with the monopolist. The model of monopolistic economies will be simplified by assuming that π is monotonic.

ASSUMPTION 3: π is monotonic, that is, $|S| > |T| \Rightarrow \pi(S) > \pi(T)$.

Since π is monotonic, all of the traders cooperate with the monopolist, and the profit contribution of syndicate S_k is $t_k \equiv \pi(N) - \pi(N \backslash S_k) > 0$. If the bargaining solution is strongly individually rational, then both the syndicate and the monopolist receive positive incremental profits from agreement. For syndicate S_k, the incremental profit from agreement with the monopolist is equal to the syndicate's total profit y_k. For the monopolist, the incremental profit from agreement with syndicate S_k is the excess of the profit contribution of the syndicate over the profit received by the syndicate: $x_k \equiv t_k - y_k$.

The effects of syndication on bargaining equilibria will depend on the profit function. π is *[strictly submodular] additive [strictly supermodular]* iff $\pi(T) - \pi(T \backslash i)$ [<] = [>] $\pi(S) - \pi(S \backslash i)$, $\forall$ $S \subseteq N$ and $T \subseteq N$ such that $S \subset T$ and $\forall$ $i \in S$. Additivity may be interpreted as constant returns to the cooperation of the traders, strict submodularity as decreasing returns and strict supermodularity as increasing returns.

Finally, in order to study the effects of syndication on bargaining equilibria, it is necessary to assume that the outcomes of bargaining between the monopolist and each of the syndicates are all determined by the same bargaining solution.

CHAPTER VI: RISK SENSITIVE BARGAINING EQUILIBRIA

This chapter analyzes the effects of syndication on bargaining equilibria associated with risk sensitive and strongly risk sensitive bargaining solutions. As a benchmark result, it is demonstrated that if the coalitional profit function π is additive and the monopolist is risk neutral, then all syndicates receive equal profits per trader and merger among the syndicates does not affect the traders' profits for risk sensitive bargaining equilibria.

If π is additive and the monopolist is strictly risk averse with decreasing r and increasing r*, then syndication is advantageous for <u>strongly</u> risk sensitive bargaining equilibria. Larger syndicates receive greater profits per trader. The syndicate of all traders receives greater profits per trader than any syndicate in any other organization of the traders. The merger of all syndicates thus increases traders' profits. The syndicate of all traders is stable against defection by an individual trader or a group of traders.

A weaker form of these results can be demonstrated for risk sensitive bargaining equilibria. If the monopolist is risk averse with nonincreasing r and nondecreasing r*, then syndication is not disadvantageous. Larger syndicates receive at least as great profits per trader. The syndicate of all traders receives at least as great profits per trader as any syndicate in any other organization of the traders. The merger of all syndicates thus does not decrease the traders' profits. Therefore, the syndicate of all traders is stable.

SECTION I: RISK AVERSION AND BARGAINING

There has been substantial study of the effects of risk aversion on the outcome of bargaining games, which has compared the utilities received under bargaining solutions by players with varying risk aversion. Kihlstrom, Roth and Schmeidler [1981] demonstrated that if two risk averse players bargain to select a single outcome from a set of riskless outcomes on which the players each have concave utility functions, then if one player becomes more risk averse, it is advantageous to the other player with respect to the Nash, monotonic, and superadditive solutions. Sobel [1981] presented a similar result for bargaining over the division of several divisible commodities. Roth [1985] showed that greater risk aversion in each period is advantageous to the other player in noncooperative multi-period bargaining. These results unanimously indicate that greater risk aversion yields an advantage to one's opponent in bargaining.

A more general bargaining model produced a more ambiguous result. Roth and Rothblum [1982] concluded that greater risk aversion can be either advantageous or disadvantageous to one's opponent in bargaining games with risky outcomes. Notwithstanding this result, greater risk aversion has uniformly been found to the other player's advantage in bargaining games with riskless outcomes. The model constructed herein encompasses only riskless bargaining outcomes, that is, distributions of a fixed profit contribution.

If a player's opponent becomes more risk averse, then risk sensitive solutions require that the player's utility is not decreased, and strongly risk sensitive solutions require that the player's utility is increased. Players in monopolistic bargaining economies are assumed to have utility functions that are monotonically increasing with respect to the player's profit. Therefore, in a profit division game, if a player's opponent is more risk averse, then the player's profit is not decreased for the bargaining outcome associated with a risk sensitive solution and is increased for the bargaining outcome associated with a strongly risk sensitive solution.

The effect of the monopolist's risk aversion on the traders' profits is the key to the principal results of this chapter. For the remainder of this essay, let r be the absolute risk aversion of the monopolist's utility function and let r* be the relative risk aversion of the monopolist's utility function. As the size of the syndicate increases, the monopolist becomes more risk averse in bargaining for a proportion of the profit contribution from agreement. A larger syndicate thus may have an advantage in bargaining with the monopolist and receive a greater proportion of the profit contribution from agreement.

On the other hand, if π is additive, then all syndicates are identically risk averse in bargaining for a proportion of their profit contribution because bargaining for a proportion of a syndicate's profit contribution is equivalent to bargaining for profits per trader.

LEMMA 6.1: If π is additive, then all syndicates are identically risk averse in bargaining for a proportion of their profit contribution.

PROOF: Consider syndicates S_i and S_j. Their utility functions are $v_i(y_i) = v(y_i/|S_i|)$ and $v_i(y_i) = v(y_j/|S_j|)$, where y_i and y_j are the syndicates' profits. These utility functions are identically risk averse in bargaining for profits per trader.

Suppose each syndicate bargains with the monopolist over its proportion of its profit contribution. Let V_i: $[0,1] \rightarrow R$ and V_j: $[0,1] \rightarrow R$ be functions that determine the utility of the syndicates if they receive a proportion $k \in [0,1]$ of their respective profit contributions, t_i and t_j. Define $V_i(k) \equiv v_i(k \cdot t_i)$ and $V_j(k) \equiv v_j(k \cdot t_j)$, for $k \in [0,1]$. Thus, $V_i(k) = v(k \cdot t_i/|S_i|)$ and $V_j(k) = v(k \cdot t_j/|S_j|)$.

Since π is additive, each syndicate's profit contribution is proportional to the number of traders in the syndicate: $(t_i/|S_i|) = (t_j/|S_j|)$. Thus, syndicates that receive an equal proportion of their profit contributions receive identical profits per trader, that is, $(k \cdot t_i/|S_i|) = (k \cdot t_j/|S_j|)$, which implies $V_i(k) = V_j(k)$. Since these functions are identical, they are identically risk averse. ∎

SECTION II: THE EFFECTS OF SYNDICATION

If the monopolist is risk neutral, then there is at most one general bargaining equilibrium associated with any risk sensitive classical bargaining solution. A bargaining equilibrium may not exist for some supermodular economies.

THEOREM 6.1: If $u''(x) = 0$, then for $f \in (\mathcal{C} \cap \mathcal{R})$ there is at most one bargaining equilibrium.

PROOF: Since $u''(x) = 0$, varying the monopolist's disagreement profit only results in an affine transformation of its utility function. Classical bargaining solutions are independent of affine transformations in utility functions. Thus, the outcome of bargaining between the monopolist and each syndicate is independent of the monopolist's disagreement profit. Therefore, since a bargaining solution selects a single outcome, the outcome of bargaining between the monopolist and each syndicate is unique, which implies that the bargaining equilibrium is unique. ∎

Regardless of the monopolist's attitude toward risk, if r is nonincreasing, then there is at most one general bargaining equilibrium that yields the monopolist a given total profit.

THEOREM 6.2: If r is nonincreasing, then for $f \in (\mathcal{C} \cap \mathcal{R})$, there is at most one general bargaining equilibrium such that the monopolist has a given level of total profit.

PROOF: Suppose that there are more than one bargaining equilibria such that the monopolist has a given total profit. Compare two of these bargaining equilibria. Select a syndicate such that the bargaining outcome between the monopolist and the syndicate is different in the two bargaining equilibria. By Pareto-optimality, the monopolist's incremental profit from agreement with the syndicate is greater in one outcome than in the other. Since the monopolist's total profit is equal in both bargaining equilibria, its disagreement outcome is smaller if its incremental profit is greater. By nonincreasing r, if the monopolist's disagreement profit is smaller, then it is at least as risk averse and does not receive a greater incremental profit in bargaining [Theorem 4.1]. This contradicts the supposition that the monopolist's incremental profit from bargaining is greater. Therefore, there is at most one bargaining equilibrium such that the monopolist receives a given total profit. ∎

If π is additive and the monopolist is risk neutral, then syndication does not affect the traders' profits. All syndicates receive identical profits per trader, and the merger of syndicates is neutral.

THEOREM 6.3: If $u''(x) = 0$ and π is additive, then for $f \in (C \cap R)$, all traders receive identical profits regardless of their organization.

PROOF: Let the monopolist and the syndicates bargain over their proportion of the profit contribution from agreement. Since π is additive, all syndicates are identically risk averse in bargaining for a proportion of their profit contribution [Lemma 6.1]. Since $u''(x) = 0$, varying the monopolist's disagreement profit and the profit contribution is equivalent to an affine transformation in the monopolist's utility function. The bargaining game for a proportion of the profit contribution is thus identical for all syndicates. Therefore, all syndicates receive an identical proportion of their profit contribution. By additivity of π, all syndicates receive equal profits per trader. ∎

On the other hand, if π is additive and the monopolist is strictly risk averse with decreasing r and increasing r*, then greater size is advantageous. Traders in a larger syndicate receive greater profits.

THEOREM 6.4: If π is additive and r is decreasing and r* is increasing, then for $f \in (\mathcal{C} \cap \mathcal{R}^*)$, a larger syndicate receives greater profits per trader.

PROOF: Let the monopolist and the syndicates bargain over their proportion of the profit contribution from agreement. Since π is additive, syndicates that receive an identical proportion of their profit contributions receive identical profits per trader. The conclusion thus is demonstrated if a larger syndicate receives a greater proportion of its profit contribution. Since π is additive, all syndicates are identically risk averse in bargaining for a proportion of their profit contribution [Lemma 6.1].

Suppose the larger syndicate did not receive a greater proportion of its profit contribution. Then the monopolist would receive at least as great a proportion of the greater profit contribution. Since the monopolist's total profits are equal in bargaining with all of the syndicates, the monopolist's disagreement profit would thus be smaller in bargaining with the larger syndicate. By decreasing r, as a result of its lower disagreement profit, the monopolist would be more risk averse in bargaining with the larger syndicate [Theorem 4.1]. By increasing r^*, as a result of the larger profit contribution, the monopolist would be more risk averse in bargaining for a proportion of the profit contribution of the larger syndicate [Theorem 4.2]. Therefore, in the bargaining equilibrium, if the monopolist received a greater proportion of the profit contribution of the larger syndicate, it would be more risk averse in bargaining for a proportion of the greater profit contribution.

On the other hand, since $f \in R^*$ and the bargaining game is smooth, if the monopolist is more risk averse in bargaining for a proportion of the profit contribution of the larger syndicate, then the larger syndicate receives a greater proportion of its profit contribution. This contradicts the supposition that the larger syndicate did not receive a greater proportion of its profit contribution. ∎

Furthermore, the merger of all syndicates to form a syndicate of all traders is advantageous. The profits per trader received by the syndicate of all traders would be greater than the profits per trader received by any of the merging syndicates. The syndicate of all traders is thus stable. The monopolistic syndicate could distribute its profits equally among all of its traders and assure its stability. No other syndicate could offer all of its traders greater profits than they received in the monopolistic syndicate. Therefore, no trader or group of traders would have the incentive to defect from the monopolistic syndicate.

THEOREM 6.5: If π is additive and r is decreasing and r* is increasing, then, for f ϵ $(C \cap R^*)$, the merger of all syndicates results in greater profits per trader for the merged syndicate than were received by any of the merging syndicates.

PROOF: Let the monopolist and the syndicates bargain over their proportion of the profit contribution from agreement. Since π is additive, the merged syndicate and each of the merging syndicates are identically risk averse in bargaining for a proportion of their profit contributions. [Lemma 6.1]

The merger of all syndicates decreases the monopolist's disagreement profit to zero, which implies, by decreasing r, that the monopolist is more risk averse in bargaining [Theorem 4.1]. The merger of all syndicates increases the profit contribution for which the monopolist is bargaining, which implies, by increasing r*, that the monopolist is more risk averse in bargaining for a proportion of the profit contribution [Theorem 4.2]. The monopolist is thus more risk averse in bargaining for a proportion of the profit contribution of the merged syndicate of all traders than for a proportion of the profit contribution of any of the merging syndicates.

Thus, since $f \in R^*$ and the bargaining game is smooth, the merged syndicate receives a greater proportion of its profit contribution than any of the merging syndicates. Therefore, by additivity of π, the syndicate of all traders receives greater profits per trader than any syndicate under any other organization of the syndicates. ∎

Similarly, syndication can be shown not to be disadvantageous with respect to the general bargaining equilibria associated with the broader class of risk sensitive bargaining solutions. To demonstrate these results, it is necessary to assume only that r is nonincreasing and r* is nondecreasing. The proofs of these results closely follow those for bargaining equilibria associated with strongly risk sensitive solutions.

THEOREM 6.6: If π is additive and r is nonincreasing and r* is nondecreasing, then for $f \in (C \cap R)$, a larger syndicate receives at least as great profits per trader.

THEOREM 6.7: If π is additive and r is nonincreasing and r* is nondecreasing, then, for f ϵ ($C \cap R$), the merger of all syndicates results in at least as great profits per trader for the merged syndicate as were received by any of the merging syndicates.

For the class of strongly power sensitive bargaining solutions, it can be demonstrated that syndication among the traders is advantageous if the monopolist is strictly risk averse, without any assumptions on its absolute or relative risk aversion. The final chapter presents a battery of results for the general bargaining equilibria associated with strongly power sensitive bargaining solutions.

CHAPTER VII: POWER SENSITIVE BARGAINING EQUILIBRIA

Power equilibria are the general bargaining equilibria associated with strongly power sensitive classical bargaining solutions. This chapter presents a comprehensive analysis of the effects of syndication on power equilibria. The results concerning risk sensitive classical bargaining equilibria are strengthened and extended for power equilibria.

Power equilibria are first analyzed for economies with nonadditive coalitional profit functions and a risk neutral monopolist. If π is strictly submodular, then syndication is advantageous with respect to power equilibria. On the other hand, if π is strictly supermodular, then syndication is disadvantageous.

The principal results of this chapter focus on economies in which the monopolist is strictly risk averse and the coalitional profit function π is additive. Syndication among traders in such economies is advantageous. Larger syndicates receive greater profits per trader. Syndicate merger is advantageous. The traders can maximize their profits by forming a single monopolistic syndicate.

The bargaining solution must be continuous if the existence of a power equilibrium is to be assured, but continuity is not required to demonstrate the properties of any such equilibrium.

SECTION I: NOTATION

The following notation is used throughout this chapter. The traders are partitioned into a finite set of m syndicates, $S = \{S_1,\ldots,S_m\}$. The number of traders in the syndicate S_k is n_k.

The profit contributions from agreement between the monopolist and the respective syndicates are denoted as $\{t_1,\ldots,t_m\}$. The profits of the syndicates are represented by $\{y_1,\ldots,y_m\}$. The monopolist's incremental profits from agreement with the syndicates are denoted as $\{x_1,\ldots,x_m\}$, where $x_k = t_k - y_k$. The total profit of the monopolist is represented by x, but it does not in general equal the sum of its incremental profits unless π is additive.

Several results concern the effects of the merger of syndicates. The above notation is used for the profits before the merger. Suppose S_i and S_j merge to form a new syndicate, S_o. The profit contribution of the merged syndicate is denoted t_o. The profits of the syndicates after merger are represented by $\{y'_o,\ldots,y'_m\}$. The monopolist's incremental profits from agreement with the syndicates are denoted as $\{x'_o,\ldots,x'_m\}$. The monopolist's total profit after merger is represented by x'.

The utility function of the monopolist is represented by u(x). The utility function for each of the identical individual traders is v(y), where y is the profit of the individual trader. The utility function of any syndicate S_k is represented by $v_k(y_k)$. Recall that $v_k(y_k) \equiv v(y_k/n_k)$.

SECTION II: METHOD OF PROOF

The method of proof for all the theorems in this chapter is similar. Each proof proceeds in two steps. The first step of each proof compares the power of two syndicates in bargaining with the monopolist. The second step compares the power of the monopolist in bargaining with each of the two syndicates. It is shown that if the power equilibrium did not satisfy the conclusion of the theorem, then the monopolist would be more powerful in bargaining with one syndicate, but the other syndicate would be more powerful in bargaining with the monopolist. Power equilibria require that if one player is more powerful under the bargaining outcome to one bargaining situation than under the outcome to another bargaining situation, then the other player must also be more powerful [See Theorem 4.6]. Therefore, the power imbalance demonstrates that if the conclusion of the theorem is not satisfied, then the profit distribution is not a power equilibrium.

SECTION III: SYNDICATE BARGAINING POWER

If two syndicates are identical in terms of profits per trader, then the syndicates will be identically powerful in bargaining. On the other hand, if one syndicate produces at least as great profit contribution per trader but does not receive greater incremental profit per trader, then the syndicate will be at least as powerful.

LEMMA 7.1: If $(t_i/n_i) \geq (t_j/n_j)$ and $(y_i/n_i) \leq (y_j/n_j)$, then $p(v_i,y_i,y_i,t_i) \geq p(v_j,y_j,y_j,t_j)$.

PROOF: $(t_i/n_i) = (t_j/n_j)$ and $(y_i/n_i) = (y_j/n_j)$ imply $p(v_i,y_i,y_i,t_i) = p(v_j,y_i[n_j/n_i],y_i[n_j/n_i],t_i[n_j/n_i])$ [Property 10].

By hypothesis, $(y_i/n_i) \leq (y_j/n_j)$, or $y_i \cdot (n_j/n_i) \leq y_j$, which implies $p(v_j,y_i[n_j/n_i],y_i[n_j/n_i],t_i[n_j/n_i]) \geq p(v_j,y_j,y_j,t_i[n_j/n_i])$ [Property 1].

By hypothesis, $(t_i/n_i) \geq (t_j/n_j)$, or $t_i \cdot (n_j/n_i) \geq t_j$, which implies $p(v_j,y_j,y_j,t_i[n_j/n_i]) \geq p(v_j,y_j,y_j,t_j)$ [Property 3].

Combining inequalities, $p(v_i,y_i,y_i,t_i) \geq p(v_j,y_j,y_j,t_j)$ ∎

SECTION IV: STRICTLY SUBMODULAR ECONOMIES

If π is strictly submodular, then the profit contribution of larger syndicates is more than proportionately larger. Therefore, if the monopolist is risk neutral and π is strictly submodular, then traders in larger syndicates receive greater profits and syndicate merger is advantageous.

THEOREM 7.1: If $u''(x) = 0$, $\forall$ x, and π is strictly submodular, then

(1) a larger syndicate receives greater profit per trader;

(2) the merger of any set of syndicates results in greater profit per trader for the merged syndicate than was received by any of the merging syndicates; and

(3) all traders receive the greatest profits if a syndicate of all traders is formed.

PROOF: Suppose that a given syndicate does not receive greater profit per trader than some smaller syndicate. Let syndicates S_i and S_j be such that $n_i > n_j$ and $(y_i/n_i) \leq (y_j/n_j)$.

<u>Step 1</u>: By strictly submodularity, $n_i > n_j$ implies $(t_i/n_i) > (t_j/n_j)$. By supposition, $(y_i/n_i) \leq (y_j/n_j)$, which with $(t_i/n_i) > (t_j/n_j)$ implies $p(v_i, y_i, y_i, t_i) \geq p(v_j, y_j, y_j, t_j)$ [Lemma 7.1].

<u>Step 2</u>: Since $u''(x) = 0$, $p(u,x,x_i,t_i) = p(u,x,x_i[t_j/t_i],t_j)$ [Property 4].

By supposition, $(y_i/t_i) < (y_j/t_j)$, and by Pareto optimality, $t_i = x_i + y_i$ and $t_j = x_j + y_j$. Thus, $x_i/t_i > x_j/t_j$, or $x_i \cdot (t_j/t_i) > x_j$, which implies $p(u,x,x_i[t_j/t_i],t_j) < p(u,x,x_j,t_j)$ [Property 2].

Combining the inequalities, $p(u,x,x_i,t_i) < p(u,x,x_j,t_j)$.

The power imbalance contradicts the conditions of a power equilibrium. Therefore, a larger syndicate must receive greater profit per trader for any organization of the traders.

Since the monopolist is risk neutral, varying its disagreement profit only results in an affine transformation of its utility function on the set of bargaining outcomes. Classical bargaining solutions are independent of affine transformations in utility functions. Thus, the power of the monopolist is not affected by changes in its disagreement profit. Therefore, a larger syndicate receives greater profit per trader regardless of the monopolist's disagreement profit.

Since a merged syndicate is larger than any of the merging syndicates, it receives greater profit per trader than any of the merging syndicates. The greatest profit per trader would be received by the largest possible syndicate, the syndicate of all traders. ∎

Therefore, if π is strictly submodular, then the syndicate of all traders is the only stable organization of the traders. If the traders were organized into several syndicates, they could all increase their profits by merging the syndicates. If the traders are organized into a single syndicate, no trader or group of traders can increase its profits by defecting from the syndicate.

There are excess traders if the incremental contribution of each trader to total profit in the economy is zero. The advantages of syndication with respect to core allocations in markets with excess traders can be observed in an example constructed by Postlewaite and Rosenthal [1974], which was designed to demonstrate disadvantageous syndication among a type of players when there was an excess of the other type of player. Syndication among the relatively abundant players in the example would have been advantageous. Syndication among relatively abundant traders has also been shown to be advantageous with respect to the value [Legros (1984)] and the nucleolus [Legros (1987)]. Similarly, in economies with excess traders, syndication of traders is advantageous with respect to power equilibria.

Excess traders are an extreme form of submodularity. Syndication has the advantage of preventing the traders from competing with one another to deal with the monopolist. The syndicate allocates the scarce demand or supply among its members and thus distributes the profit among its members.

In a market with excess traders, it is not obvious if all of the traders should syndicate or if a syndicate should form that admits only as many traders as are necessary to maximize the coalitional profit. It is clearly advantageous for a syndicate with complete control over a market to exclude additional members from participating in its profits. On the other hand, to maintain complete control of the market, the syndicate may have to admit all of the traders in order to prevent others from undercutting.

The assumption that the monopolistic economy is monotonic, which requires that the profit contribution of each trader is positive for all cooperation structures, excludes the case of excess traders. Suppose the model were augmented with the assumption that if there is zero profit contribution obtained by agreement, then the monopolist and the trader will cooperate, but the monopolist and the trader will each receive zero incremental profit from agreement. Under this additional assumption, if π is submodular until the marginal profit contribution of a trader drops to zero, then all traders will form a syndicate to bargain with the monopolist. The advantages of such syndication are easily demonstrated as a consequence of the advantages of syndication if π is strictly submodular.

In the example of disadvantageous syndication constructed by Postlewaite and Rosenthal [1974], the relatively abundant traders undercut one another, and the relatively scarce traders can better exploit this competition if they remain unsyndicated. Syndication among the scarce traders would not have been disadvantageous with respect to the core had the abundant traders formed a monopolistic syndicate. A monopolist will never undercut itself. Thus, syndication among traders bargaining with a monopolist would not be disadvantageous even if the traders are not overly abundant.

SECTION V: STRICTLY SUPERMODULAR ECONOMIES

Economies with strictly supermodular coalitional profit functions may not have a bargaining equilibrium because the bargaining outcome functions could require the monopolist to have negative profits, which would violate a condition of agreement equilibria. If π is strictly submodular, then the profit contribution of larger syndicates is less than proportionately larger. Therefore, if the monopolist is risk neutral and π is strictly supermodular, and a power equilibrium does exist, then the effects of syndication are precisely the reverse of those for strictly submodular coalitional profit functions: traders in larger syndicates receive smaller profits and the merger of syndicates is disadvantageous.

THEOREM 7.2: If $u''(x) = 0$, $\forall$ x, and π is strictly supermodular, then

(1) a larger syndicate receives smaller profit per trader;

(2) the merger of any set of syndicates results in smaller profit per trader for the merged syndicate than was received by any of the merging syndicates;

(3) all traders receive the greatest profits if there is no syndication among traders, that is, if all traders bargain as individuals.

PROOF: Follows same steps as Theorem 7.1. ∎

Therefore, if π is strictly supermodular, then the independence of all traders is the only stable organization of the traders. If any traders were organized in a syndicate with several members, each trader could increase its profits by defecting and bargaining as an individual, and they could all increase their profits by dissolving the syndicate. If the traders each bargain independently with the monopolist, then no traders can increase their profits by forming a syndicate of several traders.

This model can also provide results for economies in which several types of input are each essential for the production of the final product and each player has a monopoly in one of the inputs. If there is a set of players whose cooperation is necessary to obtain positive profit, then each of these players has veto power in the bargaining economy. This is the extreme case of supermodularity.

In general, a bargaining equilibrium would not exist in such an economy. If a bargaining equilibrium did exist, then, as the analysis of strictly supermodular economies indicates, syndication would be disadvantageous. This result has strong intuitive appeal. An individual who possesses veto power would not join a syndicate if the result would only be to obtain veto power for the coalition.

SECTION VI: ADDITIVE ECONOMIES

Several results have been presented for risk sensitive bargaining equilibria in economies in which π is additive. This section presents a more robust and stronger set of results for power equilibria in additive economies. If the monopolist is strictly risk averse, then larger syndicates receive greater profits per trader and syndicate merger is advantageous. All of the advantages of syndication among traders also apply if π is submodular rather than additive, and are magnified if π is strictly submodular.

The following elementary lemma is a fulcrum of the results for additive economies. Comparing the profits received by syndicates that reach agreement with the monopolist, a given syndicate receives equal [greater] profit per trader if and only if the monopolist receives equal [smaller] incremental profit per trader.

LEMMA 7.2: If π is additive, then for Pareto-optimal profit distributions (x_i,y_i) and (x_j,y_j):

(y_i/n_i) = [>] (y_j/n_j) iff (x_i/n_i) = [<] (x_j/n_j); and

(y_i/n_i) = [>] (y_j/n_j) iff (x_i/t_i) = [<] (x_j/t_j).

PROOF: By Pareto optimality: $t_i = x_i + y_i$ and $t_j = x_j + y_j$.

Thus, $(x_i/t_i) + (y_i/t_i) = (x_j/t_j) + (y_j/t_j)$.

By additivity of π: $(t_i/n_i) = (t_j/n_j)$.

Thus, $(x_i/n_i) + (y_i/n_i) = (x_j/n_j) + (y_j/n_j)$. ∎

Recall that for any set of risk sensitive solutions, if r is nonincreasing, there is no more than one bargaining equilibrium that yields the monopolist a given profit. For power equilibria, the identical result can be obtained without imposing the assumption on the monopolist's absolute risk aversion.

THEOREM 7.3: If π is additive, for any organization of the traders, there is at most one power equilibrium such that the monopolist has a given total profit.

PROOF: Suppose that the monopolist's total profit is fixed at x and that the outcome of bargaining between the monopolist and some syndicate S_k is not unique.

Select and compare two bargaining outcomes. Let the syndicate's profits be y_k and y'_k, such that $y'_k > y_k$, and let the monopolist's incremental profits from agreement with the syndicate be denoted x_k and x'_k.

Step 1: $y'_k > y_k$ implies $p(v_k,y'_k,y'_k,t_k) < p(v_k,y_k,y_k,t_k)$ [Property 1].

Step 2: By Pareto optimality: $t_k = x_k + y_k = x'_k + y'_k$.
Thus, $y'_k > y_k$ implies $x'_k < x_k$.

$x'_k < x_k$ implies $p(u,x,x'_k,t_k) > p(u,x,x_k,t_k)$ [Property 2]. ∎

The incremental profit received by the monopolist from agreement with a larger syndicate is greater than from a smaller syndicate. This confirms common sense. The monopolist should not be willing to accept a smaller incremental profit from agreement with a larger syndicate. For this result, it is not necessary to assume that the monopolist is strictly risk averse.

THEOREM 7.4: If π is additive, the monopolist receives greater incremental profit from agreement with a larger syndicate, that is, for any syndicates S_i and S_j: $n_i > n_j \Rightarrow x_i > x_j$.

PROOF: Suppose not. Then $n_i > n_j$ and $x_i \leq x_j$.

Step 1: By supposition, $(x_i/n_i) < (x_j/n_j)$. Thus, by Pareto optimality: $(y_i/n_i) > (y_j/n_j)$ [Lemma 7.2].

By additivity, $(t_i/n_i) = (t_j/n_j)$.

$(t_i/n_i) = (t_j/n_j)$ and $(y_i/n_i) > (y_j/n_j)$ imply $p(v_i,y_i,y_i,t_i) < p(v_j,y_j,y_j,t_j)$ [Lemma 7.1].

Step 2: By hypothesis, $n_i > n_j$, which implies $t_i > t_j$. $t_i > t_j$ implies $p(u,x,x_i,t_i) > p(u,x,x_i,t_j)$ [Property 3].

By supposition, $x_i \leq x_j$, which implies $p(u,x,x_i,t_j) \geq p(u,x,x_j,t_j)$ [Property 2].

Combining the inequalities, $p(u,x,x_i,t_i) > p(u,x,x_j,t_j)$. ∎

The central results of this essay concern the ability of a larger syndicate to obtain more favorable terms in bargaining with a strictly risk averse monopolist. A larger syndicate is able to demand more favorable terms because strict risk aversion makes the monopolist less powerful in bargaining with a larger syndicate.

LEMMA 7.3: If π is additive and $u''(x) < 0$, $\forall$ x, then $n_i > n_j$ and $(y_i/n_i) \le (y_i/n_i)$ imply $p(u,x,x_i,t_i) < p(u,x,x_j,t_j)$.

PROOF: By Pareto-optimality, $(y_i/n_i) \le (y_j/n_j)$ implies $(x_i/t_i) \ge (x_j/t_j)$, or $x_i \cdot (t_j/t_i) \ge x_j$ [Lemma 7.2].

$x_i \cdot (t_j/t_i) \ge x_j$ implies $p(u,x,x_i[t_j/t_i],t_j) \le p(u,x,x_j,t_j)$ [Property 2].

By hypothesis, $n_i > n_j$, which implies $t_i > t_j$.

Since $u''(x) < 0$, $t_i > t_j$ implies $p(u,x,x_i,t_i) < p(u,x,x_i[t_j/t_i],t_j)$ [Property 5].

Combining inequalities, $p(u,x,x_i,t_i) < p(u,x,x_j,t_j)$. ∎

Recall that if the monopolist is risk neutral and π is additive, then all syndicates receive equal profits per trader in risk sensitive bargaining equilibria. On the other hand, for strongly risk sensitive bargaining equilibria, if r is decreasing and r* is increasing, then a larger syndicate receives greater profit per trader. This result can be obtained for power equilibria without assumptions on absolute and relative risk aversion: if the monopolist is strictly risk averse, then a larger syndicate receives greater profit per trader.

The greater profit received by traders in the larger syndicates is not counter-intuitive, but is significant because the bargaining equilibrium grants more favorable terms to larger agents in the absence of any cost considerations. The economics literature is rife with examples of volume discounts generated by economies of scale or transactions costs, but these conditions are not present in a monopolistic bargaining economy with an additive coalitional profit function.

Strict risk aversion compels the monopolist to offer greater profit per trader to a larger syndicate, much as a strictly submodular coalitional profit function causes a risk neutral monopolist to offer greater profit per trader to a larger syndicate. Since there exists a bargaining equilibrium, consider the monopolist's total profit as given. Then the monopolist views its bargaining with each syndicate as a threat to its incremental profit from agreement with that syndicate. The incremental profit at risk in bargaining with the larger syndicate is greater than that at risk in bargaining with the smaller syndicate. As a result of its strict risk aversion, the monopolist is less powerful in bargaining with the larger syndicate for a proportion of the profit contribution. Thus, the larger syndicate can obtain a greater share of its profit contribution.

THEOREM 7.5: If π is additive and $u''(x) < 0$, $\forall\, x$, then, in any power equilibrium, a larger syndicate receives greater profit per trader, that is, for any syndicates S_i and S_j: $n_i > n_j \Rightarrow (y_i/n_i) > (y_j/n_j)$.

PROOF: Suppose not. Then $n_i > n_j$ and $(y_i/n_i) \leq (y_j/n_j)$.

<u>Step 1</u>: Since π is additive, $(t_i/n_i) = (t_j/n_j)$.

$(t_i/n_i) = (t_j/n_j)$ and $(y_i/n_i) \leq (y_j/n_j)$ imply
$p(v_i,y_i,y_i,t_i) \geq p(v_j,y_j,y_j,t_j)$ [Lemma 7.1].

<u>Step 2</u>: Since π is additive and $u''(x) < 0$, $n_i > n_j$ and
$(y_i/n_i) \leq (y_i/n_i)$ imply $p(u,x,x_i,t_i) < p(u,x,x_j,t_j)$ [Lemma 7.3]. ∎

If the monopolist is strictly risk averse, the merger of all of the syndicates to form a syndicate of all traders would increase the combined profits of the set of traders and decrease the total profit of the monopolist. The merger is thus advantageous.

Since the merger of all syndicates increases the combined profits of the traders regardless of the original organization of the traders, any other organization of the syndicates is not stable. If the traders were not organized as a monopoly, they could increase their profits by syndicate merger. The syndicate of all traders could afford to offer each trader more than it received under any other syndicate structure. Note, however, that such an offer might require that some traders receive greater profits than others.

THEOREM 7.6: If π is additive and $u''(x) < 0$, $\forall$ x, then, for any organization, the merger of all syndicates:

(1) increases the combined profits of the traders;

(2) decreases the total profit of the monopolist.

PROOF: Let the syndicate of all traders be N. Let the sum of the profits of the unmerged syndicates be y. Let the profit of the syndicate of all traders be y'. The conclusion demands that $y' > y$. Suppose $y' \leq y$.

Let the largest syndicate be S_1. By strict risk aversion, the largest syndicate receives at least as great profit per trader as any other syndicate [Theorem 7.5]. Therefore, among the unmerged syndicates, the average profit per trader is not greater than the profit per trader in the largest syndicate: $(y/n) \leq (y_1/n_1)$.

<u>Step 1</u>: By supposition, $y' \leq y$. Thus, $(y'/n) \leq (y_1/n_1)$.
Since π is additive, $(t/n) = (t_1/n_1)$.

$(t/n) = (t_1/n_1)$ and $(y'/n) \leq (y_1/n_1)$ imply
$p(v_N, y', y', t) \geq p(v_1, y_1, y_1, t_1)$ [Lemma 7.1].

<u>Step 2</u>: By supposition, $y' \leq y$. Therefore, by Pareto optimality, $x' \geq x$, which implies $p(u,x',x',t) \leq p(u,x,x,t)$ [Property 1].

Since $u''(x) < 0$, $n > n_1$ and $(y/n) \leq (y_1/n_1)$ imply
$p(u,x,x,t) < p(u,x,x_1,t_1)$ [Lemma 7.3].

Combining the inequalities, $p(u,x',x',t) < p(u,x,x_1,t_1)$. ∎

On the other hand, the syndicate of all traders may itself be unstable. Suppose that the profits of the monopolistic syndicate are distributed equally among all of the traders. A trader or a group of traders may be able to defect from the monopolistic syndicate and obtain greater profit. As a result, the other traders would receive lower profits.

The monopolistic syndicate could lure the defectors to rejoin by offering them greater profits. Furthermore, the other traders would find it advantageous to offer a reward for the defectors' return. Nevertheless, the inequitable distribution of profits that would ensue might induce those traders who received below average profits to defect from the monopolistic syndicate.

The monopolistic syndicate is stable if it receives greater profit per trader than any syndicate under any other organization of the traders. The monopolistic syndicate could thus distribute its profits equally among all of its traders and assure its stability. No other syndicate could offer all of its traders greater profits than they received in the monopolistic syndicate.

For strongly risk sensitive bargaining equilibria, if r is decreasing and r* is increasing, then the merger of all syndicates results in greater profit per trader than was received by any of the merging syndicates. For power equilibria, if the monopolist is strictly risk averse with r nonincreasing, then the syndicate of all traders receives greater profit per trader than any syndicate under any other organization. For any organization of traders, the merger of all syndicates would result in greater profits per trader for all merging syndicates. The stability of the monopolistic syndicate is assured. No group of traders could profitably defect. The syndicate of all traders is the unique stable organization of traders.

THEOREM 7.7: If π is additive and $u''(x) < 0$ and r is nonincreasing, $\forall x$, then the syndicate of all traders receives greater profit per trader than any syndicate under any other organization of the traders.

PROOF: By $u''(x) < 0$, since the syndicate of all traders is larger than any other syndicate, if the monopolist's total profit was fixed, then the syndicate of all traders would receive greater profit per trader than any other syndicate would in bargaining with the monopolist with the same total profit. [See Theorem 7.5]

By $u''(x) < 0$, the merger of all syndicates decreases the monopolist's total profit [Theorem 7.6]. By r nonincreasing, if the monopolist's total profit is smaller, it is no more powerful in bargaining with any given syndicate [Property 8]. Thus, if the monopolist's total profit was at a lower level, it would not receive a greater incremental profit in bargaining with any given syndicate, including the syndicate of all traders.

Therefore, combining the effects, the syndicate of all traders receives greater profit per trader than any syndicate under any other organization of the traders. ∎

The following results study economies in which the traders are organized into several syndicates. The effects of a merger that does not include all of the syndicates is analyzed. For these results, it must be assumed that r* is nondecreasing.

The following theorem will be critical to proving subsequent results. If r* is nondecreasing, then given larger total profit, the monopolist would generate a smaller proportion of its total profit from agreement with any syndicate.

THEOREM 7.8: If r* is nondecreasing, then if the monopolist's total profit increases, the share of its total profit from a given syndicate decreases, that is, if $x' > x$, then $(x'_k/x') < (x_k/x)$, $\forall$ k.

PROOF: Suppose $x' > x$ and $\exists$ k, $(x'_k/x') \geq (x_k/x)$. Then $x'_k > x_k$.

<u>Step 1</u>: By Pareto optimality: $t_k = x_k + y_k = x'_k + y'_k$.
Thus, $x'_k > x_k$ implies $y'_k < y_k$.

$y'_k < y_k$ implies $p(v_k,y'_k,y'_k,t_k) > p(v_k,y_k,y_k,t_k)$ [Property 1].

<u>Step 2</u>: By hypothesis, $x' > x$. Since r* is nondecreasing, $x' > x$ implies $p(u,x',x_k[x'/x],t_k) \leq p(u,x,x_k,t_k)$ [Property 9].

By supposition, $(x'_k/x') \geq (x_k/x)$, or $x'_k \geq x_k \cdot (x'/x)$, which implies $p(u,x',x'_k,t_k) \leq p(u,x',x_k[x'/x],t_k)$ [Property 2].

Combining the inequalities, $p(u,x',x'_k,t_k) \leq p(u,x,x_k,t_k)$. ∎

If r^* is nondecreasing, then for any organization of the traders, any power equilibrium is unique.

THEOREM 7.9: If π is additive and r^* is nondecreasing, then the power equilibrium is unique, for any organization of the traders.

PROOF: Suppose that there are several power equilibria. Select and compare two of the power equilibria.

The total profit of the monopolist must be greater in one power equilibrium than the other [Theorem 7.3]. By r^* nondecreasing, if the monopolist's total profit is larger, then the proportion of its total profit from agreement with any given syndicate is decreased [Theorem 7.8].

Since the set of syndicates is identical, if the monopolist's total profit is larger, then the proportion of the monopolist's total profit from agreement with the set of syndicates decreases. This contradicts the closed structure of the economy: the monopolist's profits are wholly generated by agreement with the syndicates. ∎

If the monopolist is strictly risk averse with r^* nondecreasing, then the merger of any set of syndicates decreases the monopolist's total profit. Therefore, it is in the interest of the monopolist to minimize concentration among the traders.

THEOREM 7.10: If π is additive and $u''(x) < 0$ and r^* is nondecreasing, $\forall$ x, then the merger of any set of syndicates to form a new syndicate results in lower total profit for the monopolist.

PROOF: Suppose that the merger of several syndicates does not result in lower total profit for the monopolist.

Case 1: Suppose the monopolist's total profit remains constant. The outcome of bargaining between the monopolist and each of the nonmerging syndicates would remain identical [See Theorem 7.3]. By $u''(x) < 0$, since the merged syndicate would be larger than any of the merging syndicates, it would receive greater profit per trader [See Theorem 7.5]. The monopolist's profits are wholly derived from agreement with the syndicates. Therefore, the monopolist's total profit must decrease. This contradicts the supposition that the monopolist's total profit remains constant.

Case 2: Suppose the monopolist's total profit increases.

By r^* nondecreasing, if the monopolist's total profit increases, then the proportion of its total profit from agreement with a given syndicate decreases [Theorem 7.8]. Therefore, the proportion of the monopolist's total profit from bargaining with each nonmerging syndicate decreases. By $u''(x) < 0$, since the merged syndicate is larger, it receives greater profit per trader than any of the merging syndicates.

Thus, if the monopolist's total profit is given, the monopolist's incremental profit from agreement with the merged syndicate is less than the sum of its incremental profits from agreements with the merging syndicates, which would imply that, if the monopolist's total profit increased, the proportion of the monopolist's total profit from agreement with the traders in the merging syndicates would decrease.

Thus, if the monopolist's total profit increased, the proportion of the monopolist's total profit from agreement with the syndicates would decrease. This contradicts the closed structure of the economy: the monopolist's profits are wholly derived from agreement with syndicates.

Therefore, the merger decreases the monopolist's total profit. ∎

The merger of syndicates results in a larger proportion of the monopolist's total profit being obtained from each of the syndicates not involved in the merger. On the other hand, the traders in the merged syndicate contribute a smaller portion of the monopolist's total profit than they contributed as unmerged syndicates.

THEOREM 7.11: If π is additive and $u''(x) < 0$ and r^* is nondecreasing, $\forall$ x, then the merger of syndicates:

(1) increases the proportion of the monopolist's total profit received from agreement with each of the nonmerging syndicates;

(2) decreases the proportion of the monopolist's total profit received from agreement with the traders in the merging syndicates.

PROOF: The merger of the syndicates decreases the monopolist's total profit [Theorem 7.10]. By r^* nondecreasing, if the monopolist's total profit decreases, then the proportion of its total profit from agreement with a given syndicate increases [Theorem 7.8]. Thus, the proportion of the monopolist's total profit from agreement with each of the nonmerging syndicates increases. Since the monopolist receives all of its profits from agreement with syndicates, the proportion of its total profit from agreement with traders in the merging syndicates necessarily decreases. ∎

Syndicate merger decreases the monopolist's incremental profits from agreement with the traders in the merging syndicates and increases the combined profits of the traders in the merging syndicates. Therefore, any merger among syndicates is advantageous.

Suppose that some organizations of the syndicates, especially the syndicate of all traders, are not possible. Antitrust laws may limit concentration among the traders. Consider an economy with an established organization of the traders into several syndicates. Then the syndicates will find it advantageous to merge as much as possible. If a coarser partition of the traders into syndicates is possible, then the organization of the syndicates is necessarily unstable because some of the syndicates can profitably merge.

On the other hand, suppose that the laws did not prohibit any organization of the traders, but rather limited each merger. For example, suppose regulations limited the number of syndicates that could merge at a given time. If the traders have not formed a monopolistic syndicate, then their organization is not stable. The existing syndicates will execute whatever mergers are feasible.

THEOREM 7.12: If π is additive and $u''(x) < 0$ and $r*$ is nondecreasing, $\forall$ x, then the merger of syndicates:

(1) decreases the monopolist's incremental profits from agreement with the traders in the merging syndicates;

(2) increases the combined profits of the merging syndicates.

PROOF: Syndicate merger decreases the monopolist's total profit [Theorem 7.10]. The merger decreases the proportion of the monopolist's total profit from the traders in the merging syndicates [Theorem 7.11]. Thus, the merger decreases the monopolist's incremental profit from agreement with the traders in the merging syndicate. By additivity, the profit contribution from agreement with a merged syndicate equals the sum of the profit contributions from agreement with the merging syndicates. Therefore, by Pareto-optimality, the merger increases the combined profits of the traders in the merging syndicates. ∎

The merged syndicate may be vulnerable to defection. Recall the susceptibility of the syndicate of all traders to defection and dissolution. Similarly, if the traders are organized into several syndicates, traders in some of the syndicates may be able to increase their profits by defecting from their syndicates.

Suppose that all of the traders in a syndicate consider the organization of the traders outside of the syndicate as given. Then a syndicate is stable against defection and dissolution if and only if the syndicate receives greater profit per trader than any traders could obtain by defecting from the syndicate and perhaps forming smaller syndicates among themselves. The syndicate could thus distribute its profits equally among all of its traders and assure its stability. If the organization of the traders outside the syndicate was taken as fixed, then none of its traders could defect and expect to obtain greater profits than they received in the original syndicate.

To demonstrate that the syndicate of all traders is stable, it was assumed that r was nonincreasing. Under this assumption, the merger of all of the syndicates was shown to result in greater profit per trader for all of the merging syndicates.

The assumption of r nonincreasing also assures that any nonmonopolistic syndicate is stable against dissolution and defections to form syndicates composed only of members of the syndicate. If r* is nondecreasing and r is nonincreasing, then any merger of syndicates results in greater profit per trader for the merged syndicate than was received by any of the merging syndicates. Since it is larger, the merged syndicate is able to exploit the strict risk aversion of the monopolist and thus obtain greater profits per trader than the merging syndicates. The greater profit per trader will allow the syndicate to discourage traders within the syndicate from dissolving the syndicate or defecting to form a syndicate among themselves. Nevertheless, traders may defect to join another syndicate or to form a new syndicate with traders from other syndicates.

Furthermore, if r is nonincreasing, then nonmerging syndicates are not adversely affected by a merger. It has been shown that if r* is nondecreasing, then its total profit is decreased by merger. Further, if r is nonincreasing, then as a result of its lower total profit, the monopolist will be at least as risk averse in bargaining. The merger of some of the syndicates thus does not strengthen the monopolist in bargaining with the other syndicates. Therefore, a given syndicate would receive at least as great incremental profit from agreement with the monopolist.

THEOREM 7.13: If π is additive and $u''(x) < 0$, r is nonincreasing and r* is nondecreasing, $\forall$ x, then the merger of syndicates results in:
(1) at least as large profits for the nonmerging syndicates;
(2) greater profit per trader for the merged syndicate than was received by any of the merging syndicates.

PROOF: By r^* nondecreasing, a merger decreases the monopolist's total profit [Theorem 7.10]. By r nonincreasing, if the monopolist's total profit is smaller, it is no more powerful in bargaining. The monopolist with smaller total profit thus would not receive a larger incremental profit in bargaining with any given syndicate. Therefore, the nonmerging syndicates receive at least as large profits after the merger.

By $u''(x) < 0$, if the monopolist's total profit were fixed, since the merged syndicate is larger, it would receive greater profit per trader than each of the merging syndicates [See Theorem 7.5].

Therefore, combining the effects, the merged syndicate receives greater profit per trader than any of the merging syndicates. ∎

The previous results study economies with finitely many traders. The seminal work of Aumann [1973] on disadvantageous monopolies concerned economies with a continuum of traders. If the continuum of traders are organized into a finite set of syndicates that have positive measure, then all of the results for the case of finite syndicates apply equally well to economies with a continuum of traders.

The following two results concern additive economies which comprise a strictly risk averse monopolist and a continuum of traders that are organized into a finite set of identical syndicates. Consider the power equilibria as the traders are divided among a greater number of identical syndicates and each syndicate becomes arbitrarily insignificant in the market.

If the monopolist is strictly risk averse with r^* nondecreasing, then as the traders are divided into more syndicates, each of which is smaller, the total profit of the monopolist increases. Therefore, the monopolist again has an interest in minimizing the concentration of the traders.

THEOREM 7.14: Suppose there is a continuum of non-atomic traders that are divided into a finite set of syndicates of identical measure. If π is additive and $u''(x) < 0$ and r^* is nondecreasing, $\forall$ x, then as the number of syndicates into which the traders are divided increases:

(1) the traders' profits decrease; and

(2) the monopolist's total profit increases.

PROOF: Suppose the monopolist's total profit did not increase when the traders reorganized into more syndicates. By r^* nondecreasing, if the monopolist's total profit did not increase, then the monopolist would receive at least as large a share of its total profit from agreement with a syndicate with a given measure [Theorem 7.8]. By $u''(x) < 0$, since each syndicate is smaller, the monopolist would receive greater incremental profit per trader from agreement with each of the syndicates [Theorem 7.5]. Therefore, the monopolist would receive greater total profit from agreement with the syndicates, which would establish a contradiction. ∎

Nevertheless, as the number of syndicates into which the non-atomic traders are divided increases, the profits of the traders remain positive. The monopolist is not able to drive the profits of the traders to zero, even if they are divided into arbitrarily small syndicates. As the non-atomic traders are divided into arbitrarily small syndicates, the strictly risk averse monopolist is able to obtain a total profit approaching that which would be received by a risk neutral monopolist.

THEOREM 7.15: Suppose there is a continuum of non-atomic traders that are divided into a finite set of syndicates of identical measure. If π is additive and $u''(x) < 0$, $\forall$ x, then as the number of identical syndicates increases without limit, the profits of the set of traders and the monopolist approach the profits that would result if the monopolist were risk neutral.

PROOF: Let m be the number of identical syndicates into which the traders are divided. Define the measure of each syndicate as $k \equiv (1/m)$. As the number of syndicates increases without limit, k approaches zero.

Let (t) be the profit contribution from agreement between the monopolist and the set of all traders. Then, by additivity, a syndicate's profit contribution equals $(k \cdot t)$.

Let υ be the utility function of a risk neutral monopolist. Recall that if the monopolist is risk neutral and π is additive, then the monopolist's total profit is not affected by syndication among the traders.

As k approaches zero, the power of the strictly risk averse monopolist approaches the power of a risk neutral monopolist: $\lim_{k \to 0} p(u,x,kx,kt) = p(\upsilon,x,x,t)$ [Property 6].

Let v_o be the utility function of the syndicate of all traders, and let v_k be the utility function of a syndicate of measure k. The power of the syndicate is identical regardless of its size: $p(v_k,k(t-x),k(t-x),kt) = p(v_o,(t-x),(t-x),t)$ [Property 10]

Therefore, as the trader syndicates become arbitrarily small, the equilibrium condition for an economy with a strictly risk averse monopolist approaches the equilibrium condition for an economy with a risk neutral monopolist. ∎

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