2018

Essays On Economic Growth And The Economics Of Innovation

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Abstract

In my dissertation, I study how legal institutions and financial system affect innovation and their impact on economic growth. This dissertation consists of two chapters. The themes of chapter 1 and 2 are intellectual property rights and the venture capital system, respectively.

Chapter 1 studies the impact of intellectual property rights on the business scope of firms. Stronger intellectual property rights induce specialization and contribute to economic growth. In the United States, a sweeping legal reform in 1982 created a more pro-patent legal environment. This legal reform fostered specialization and enhanced firm performance. Around the world, countries experience faster economic growth when their innovating sectors are characterized by a higher level of specialization. An endogenous growth model with endogenous firm boundaries is developed to disentangle the relationship between legal institutions, firm boundary decisions, and economic growth. I characterize the optimal strength of patent rights and evaluate the actual patent law enforcement in the United States. The pro-patent legal reform in 1982 was welfare-enhancing, but it was too extreme. Swinging back the legal pendulum and weakening patent rights can improve welfare.

Chapter 2 evaluates the contribution of venture capital (VC) to promoting entrepreneurship and spawning innovation. We assemble the stylized facts of venture capital, innovation, and economic growth. Funding by venture capitalists is positively associated with patenting activity. VC-backed firms have higher IPO values when they are floated. Following flotation, they have higher R&D-to-sales ratios and grow faster in terms of employment and sales. At the country level, VC investment is positively linked with economic growth. The relationship between venture capital and growth is examined using an endogenous growth model incorporating dynamic contracts between entrepreneurs and venture capitalists. The model is matched with stylized facts about venture capital; viz., statistics by funding round concerning the success rate, failure rate, investment rate, equity shares, and the value of an IPO. We examine how the innovative activity is affected by the capital gains tax rate. Raising capital gains taxation reduces growth and welfare.

Subject Categories
Economics

ESSAYS ON ECONOMIC GROWTH AND THE ECONOMICS OF INNOVATION

Pengfei Han

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2018

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ESSAYS ON ECONOMIC GROWTH AND THE ECONOMICS OF INNOVATION

ACKNOWLEDGEMENT

ABSTRACT

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Jeremy Greenwood

In my dissertation, I study how legal institutions and financial system affect innovation and their impact on economic growth. This dissertation consists of two chapters. The themes of chapter 1 and 2 are intellectual property rights and the venture capital system, respectively.

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1.1. Introduction

Stronger patent rights induce specialization by deterring infringement. To illustrate, consider the production of smartphone depicted in Figure 1. All the components of a smartphone can be produced in one single integrated company. This incurs management costs because the firm must coordinate operations across multiple business lines. Alternatively, each component of the smartphone can be produced by one specializing firm. For instance, Apple focuses on designing iPhone and outsources the other tasks to its suppliers. Apple has more than 200 suppliers (e.g., the display of iPhone is outsourced to Samsung and the modem is outsourced to Qualcomm). Every firm specializes in a niche of the market, and they access to each other's technologies through market transactions. However, these firms may infringe on the intellectual property of others. There is a war for patent infringement in the current smartphone industry. All major players are involved in high-profile litigations, with hundreds of millions of dollars at stake. From this perspective, there is a trade-off between "infringement costs" in the specialization scenario versus "management costs" in the integration scenario. This is the key trade-off for firms' business scope decisions, and it pins down the firm boundaries in this paper.

FIGURE 1: SPECIALIZATION VS. INTEGRATION
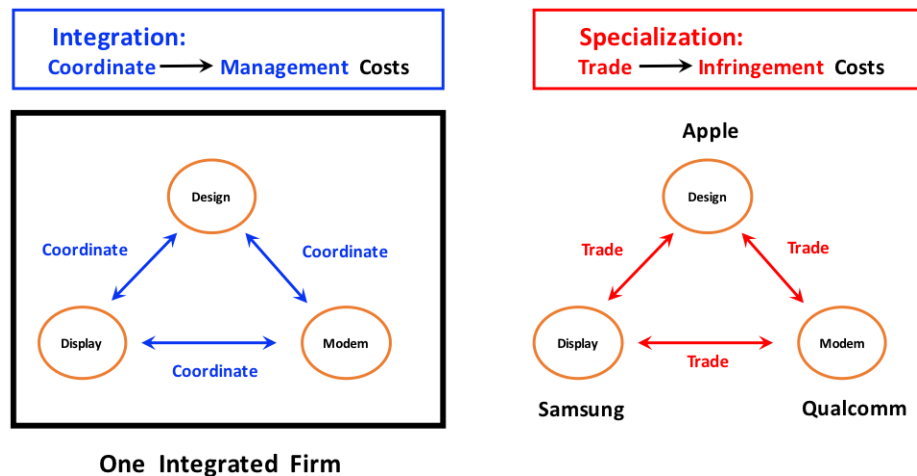
In light of this trade-off, stronger patent rights can induce specialization by deterring infringement. This is because an innovating firm can sue the violator if its intellectual property are infringed in the specialization scenario. Stronger patent rights increase the likelihood this innovating firm can win its lawsuit. Anticipating the costs of infringement, the potential violator can be deterred from infringing, and both firms can specialize in their core competencies.

This thought experiment sheds light on the theory of the firm. Coase (1937) posits some activities can be performed more efficiently within a firm's boundaries, because there will be "transaction costs" if these activities are conducted through market transactions. Coase posits the firm exists to internalize these "transaction costs." However, what exactly are these "transaction costs"? From this point of view, the theory of the firm in this paper builds on Coase (1937) by uncovering this black box of "transaction costs," in the specific context of firm innovation. As underscored in this thought experiment, there are "transaction costs" in the innovating sectors because the firms may infringe on each others' intellectual property. In addition, this is a unique problem for the innovating sectors, because the core assets of the innovating firms are intellectual property. These assets are essentially ideas. It can be difficult for courts to verify and enforce the ownership of ideas. In light of this, the infringement problems are substantially more severe for intellectual property than tangible assets. Therefore, the business scope decisions in the innovating sectors require special consideration. To fill this gap, this paper contributes to building a theory of the *innovating* firm.

In addition, this paper also contributes to the literature on the optimal strength of patent rights. As illustrated in Figure 2, there is a classic trade-off for optimal patent rights in the literature: stronger patent rights encourage R&D, but spur monopoly pricing by patent owners. As a contribution to this literature, this paper incorporates the impact of patent rights on firms' business scope decisions. In response to stronger patent rights, firms shrink their scope of business and specialize. This enhances firm performance and constitutes an

additional benefit of patent rights. Hence, the optimal patent rights will be stronger when its impact on firm boundaries is taken into account.

FIGURE 2: OPTIMAL PATENT RIGHTS: TRADE-OFF
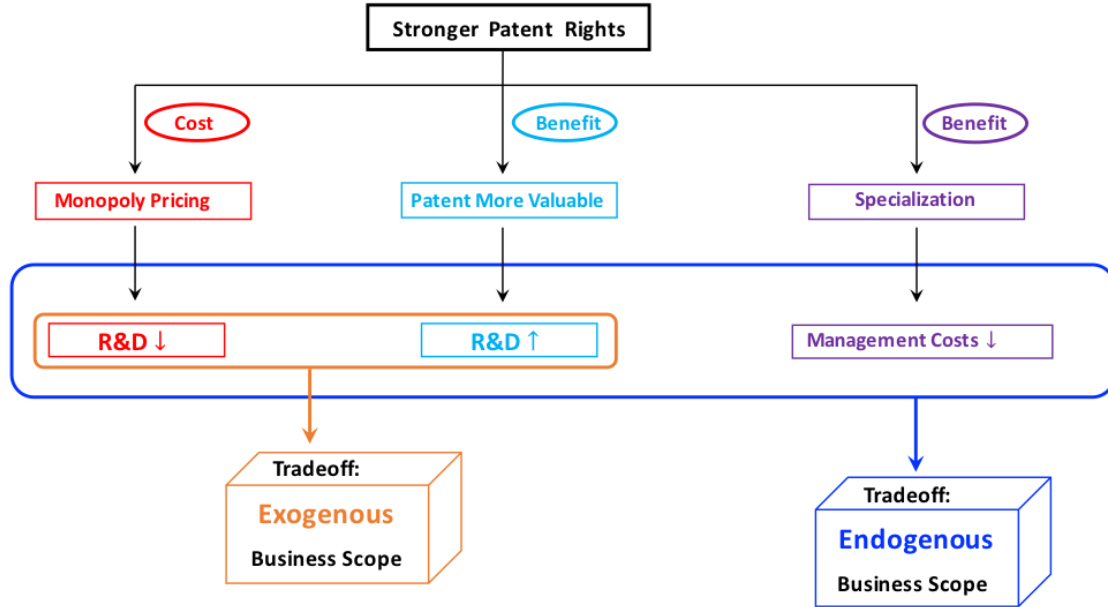
What is the optimal strength of patent rights? To address this issue, an endogenous growth model with endogenous firm boundaries is developed. Firm boundaries are identified by the number of intermediate inputs produced in-house. The production technology of every intermediate input is embodied in a patent. To perform R&D, a firm needs both the intermediate inputs based on its own patents and the inputs associated with others' patents. Each firm has two options to access to the inputs based on others' patents: it can buy their products or infringe on their patents by imitating their products. An infringer has to pay a legal settlement if it is sued and loses the lawsuit. To fend off infringement, a firm can expand its business scope and produce more intermediate inputs in-house. With stronger patent rights, the infringing problem becomes less severe, so the firm has weaker incentives to expand its business scope. The model is matched with stylized facts of firm boundaries and patent litigation, and it delivers three major implications: (1) stronger patent rights induce specialization, (2) specialization enhances firm performance, and (3) specialization

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contributes to economic growth.

These model implications are supported by the empirical analysis. In the United States, a sweeping pro-patent legal reform in 1982 strengthened patent rights. After this reform, the average U.S. innovator invents in more closely related technological fields, which implies shrinking business scope and increasing specialization. This legal reform fostered specialization and enhanced firm performance. Around the world, countries experience faster economic growth when their innovating sectors are characterized by higher level of specialization.

To be specific, the empirical findings suggest stronger patent rights can induce specialization. Geographically, the U.S. court system is divided into 12 circuit courts. There is regional variation in the strength of patent rights[1] across circuits. When the patent rights are strengthened in a circuit court, firms located in this circuit tend to innovate in more closely related technological fields. In addition, the impact of patent rights is more pronounced for firms facing higher exposure to patent litigation. This is suggestive evidence that stronger patent rights can induce specialization. The second section of the empirical analysis evaluates how the business scope strategy of a firm affects its performance. When a firm invents in technological fields that are closer to its existing patent portfolio, it will harvest more patents from the same R&D dollar, and its patent stock will have a stronger boost to its TFP and market value. Moreover, this analysis is extended to a cross-country study. At the country level, a nation will experience faster economic growth when its innovators invent in more closely related technological fields. The impact of specialization on growth is economically large. If the level of specialization is enhanced from the Japanese level to the U.S. level, the annual growth rate in Japan would have been higher by 62 basis point. Japan's GDP per capita would have been 13% higher after two decades.

Is the actual patent law enforcement optimal? To address this issue, the optimal strength of

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[1]The strength of patent rights is measured by the likelihood for a patent to be invalidated by the court. When a patent is involved in litigation, it can be adjudicated to be valid or invalid by the court. If a patent is less likely to be invalidated, the implied patent rights are stronger.

patent rights is characterized by quantitative analysis, and it is contrasted with the actual patent law enforcement in practice. Through the lends of the model, the pro-patent legal reform in 1982 was welfare-enhancing, but it was too extreme. Swinging back the legal pendulum and weakening patent rights can improve welfare.

Relationship to the Literature. This paper contributes to two branches of literature: the theory of the firm and the optimal strength of patent rights.

The theory of the firm originates from Coase (1937). In this seminal paper, Coase conceptualizes a trade-off between "transaction costs" and "coordination costs" for the firm boundary decision. Hence, the firms exist to internalize the "transaction costs." Though studies in this field halted for several decades, it was revived by Williamson (1971) and has become a fertile field since then. Williamson (1971) attributes the "transaction costs" to socially inefficient "haggling" for "appropriable quasi-rents." A classic example is the General Motors – Fisher Body relationship, as analyzed by Klein (1988, 2000b). This insight has triggered a large empirical literature on "transaction costs economics," such as Monteverde and Teece (1982), Anderson and Schmittlein (1984), Masten (1984), and Joskow (1985). This literature, however, focuses on the benefits of integration and is largely silent on its costs. In contrast, an alternative approach is taken in the "property-rights theory" pioneered by Grossman and Hart (1986), Hart and Moore (1990), and Hart (1995). In this literature, bargaining is efficient, and, thus, the parties in a contract can share the surplus from their investment via bargaining. A larger share of asset ownership implies larger share of the surplus, and, hence, stronger incentive to invest. In this framework, one party in a contract should own all the assets if it is socially optimal to maximize the investment of this party. On the other hand, the asset ownership should be divided if the investment incentives for both parties are important. In both approaches (i.e., the "transaction costs economics" and "property-rights theory"), the source of "transaction costs" is the hold-up problem in various forms. In contrast, the "transaction costs" in this paper originate from

the infringement problem (i.e., firms may infringe on each others' intellectual property). This is a unique problem for the innovating sectors, because the core assets of the innovating firms are intellectual property. This insight entails special consideration for the business scope decisions in the innovating sectors.

As epitomized in Arrow (1962), there is a classic trade-off for the optimal strength of patent rights: stronger patent rights encourage R&D, but spur monopoly pricing by patent owners. This is reminiscent in Tirole (1988), Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Based on this trade-off, one branch of this literature studies the optimal length and breadth of patent protection. In Klemperer (1990) and Gilbert and Shapiro (1990), the optimal patent protection features long duration to encourage R&D, but a narrow breadth to discourage monopoly pricing. Acemoglu and Akcigit (2012) studies the state-dependent policy and proposes a "trickle-down" optimal patent protection scheme (i.e., greater protection to technology leaders that are further ahead than those that are close to their followers). The second branch of this literature follows a mechanism design approach for the optimal strength of patent rights. As a classic example, Scotchmer (1999) characterized the patent renewal system as an optimal mechanism when the quality and costs of projects are unknown. Llobet, Hopenhayn and Mitchell (2006) study optimal patent policy with sequential innovation and heterogeneous quality. None of these papers considers the impact of patent rights on firms' business scope decisions. This gap is filled in this paper. As demonstrated in this paper, firms shrink their scope of business and specialize, in response to stronger patent rights. This enhances firm performance and constitutes an additional benefit of patent rights. Hence, the optimal patent rights will be stronger when its impact on firm boundaries is taken into account.

The rest of the paper is organized as follows. Section 1.2 reviews the background of patent law enforcement and specialization in the innovating sectors. The model is built in section 1.3 and matched with the stylized facts in section 1.4. The implications of the model are derived by the quantitative analysis in section 1.4, and these implications are tested by the

empirical analysis in section 1.5. The optimal strength of patent rights is characterized in section 1.6, and it is compared with the actual patent law enforcement in practice. Section 1.7 concludes.

1.2. Background

1.2.1. Patent Law Enforcement

As the first step to investigate the impact of patent law enforcement on specialization, this section outlines the legal background of patent litigation in the United States. In particular, this section highlights how a major legal reform during the 1980s created a more pro-patent legal environment.

Patent Litigation Process

The intellectual property rights of an invention are embodied in a patent, which provides the potential (but not the guarantee) to exclude others from using the patented technology. In the United States, patents are granted at the United States Patent and Trademark Office (USPTO). In contrast, the patent law is interpreted and enforced via the court system, and the court has a final say on the strength of the patent.

When a patentee identifies a potential infringement, she may bring the alleged violators to the court and sue them for infringement. In response, the defendant will typically counter sue by arguing that the patent is actually not valid in the very first place, and in consequence the patents can be invalidated by the court[2]. Therefore, the fate of the patent eventually depends on the judgment and the attitude of the court. From this perspective, the fraction of cases in which the patents are invalidated can be a proxy of the strength of patent rights. Based on this proxy of patent rights, the evolution of patent law enforcement in the United

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[2]A patents can be invalidated for a variety of reasons with respect to the legal requirement of novelty and nonobviousness. For instance, Allison, Lemley, and Schwartz (2014) shows that a patent can be invalidated because of no patentable subject matter (success rate: 54%), prior art requirement of section 102 (success rate: 20%), obviousness issue of section 103 (success rate: 20%), indefiniteness issue of section 112 (success rate: 17%), lack of enablement of section 112 (success rate: 13%), inadequate written description of section 112 (success rate: 15%).

States in the post-war era is illustrated in Figure 3.

FIGURE 3: PATENT LAW ENFORCEMENT IN THE UNITED STATES
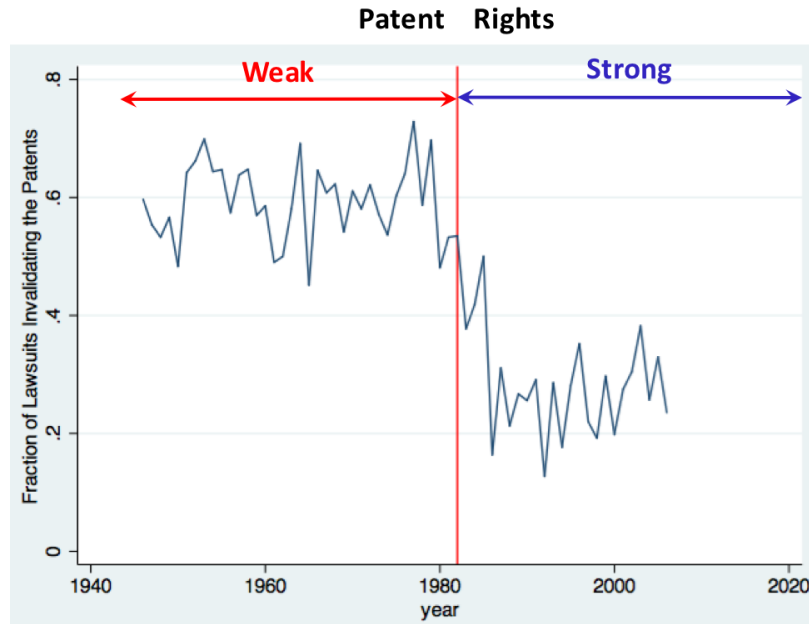
Legal Reform In 1982

The vertical axis of Figure 3 is the fraction of cases in which the patents are invalidated by the court, the proxy of the strength of patent rights. As shown in this figure, the patent law enforcement in the United States has experienced dramatic changes in the last several decades. While the legal environment in the post-war era used to be fairly stable before 1982, there was a precipitous plummet in the fraction of cases invalidating the patents around 1982. In consequence, patent law enforcement moved to a new legal regime after 1982 with remarkably lower invalidation rate and stronger patent rights.

This change of the legal environment was due to a sweeping legal reform on the court system in 1982. Before 1982, all patent-related cases were initiated at one of the ninety-four district courts across the country, and the litigants may further appeal to the regional appellate courts if they disagree with the court decisions. In practice, however, the interpretation and enforcement of the patent law were highly inconsistent across circuit courts before 1982,

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leading to a regionally fragmented legal system[3].

To address the issue of inconsistent patent law enforcement across the nation, in 1982 the Congress established a single appellate court to hear all the appeals of patent-related cases: the Court of Appeals of the Federal Circuit (CAFC). After this legal reform, this new court turned out to hold a salient pro-patent attitude. To see the impact of this legal reform, Table 1 performs a before-and-after comparison. Reported in this table is the fraction of cases invalidating the patents across circuit courts. A revealed in Table 1, there is a decrease in patent invalidation in every circuit court. This indicates stronger patent rights after the legal reform. In addition, there is a decrease in the dispersion, so the litigants are treated more equally around the country. This implies increasing legal uniformity across the nation.

TABLE 1: PATENT LAW ENFORCEMENT

Circuit Headquarter	Fraction of Cases Invalidating the Patents (%)	
	Before 1982	After 1982
Boston	0.68	0.22
Chicago	0.56	0.32
Cincinnati	0.60	0.28
Denver	0.42	0.16
New Orleans	0.41	0.23
New York City	0.67	0.26
Philadelphia	0.76	0.29
Richmond	0.54	0.22
San Francisco	0.59	0.29
St. Louis	0.67	0.37

Nevertheless, these changes in patent invalidation rates can be attributed to changing types of cases brought to the court. To address this concern, Table 2 conducts a regression analysis to examine the impact of this legal reform on patent law enforcement, controlling

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[3]In principle the Supreme Court could step in to ensure legal uniformity across circuit courts. In practice, however, the patent-related cases were rarely heard at the Supreme Court, and, hence, the discrepancy of patent law enforcement across circuits persisted over the years.

for the characteristics of the patents and the litigants involved in the lawsuits. The unit of observation in this regression are the individual patents reviewed by the court between 1946 and 2006. The dependent variable is a dummy for patent invalidation (which takes the value of one if this patent is invalidated by the court), and the main explanatory variable is a dummy for the legal reform (which takes the value of one if the court decision is made after 1982).

As demonstrated in Table 2, a patent is more likely to be invalidated after the legal reform in 1982, even after controlling for the characteristics of the patents and the litigants involved in the lawsuits[4]. In addition, this regression also uncovers a number of patterns of patent litigation: a patent is less likely to be invalidated when it constitutes a more important invention (as captured by more forward citations the patent receives), and when the patent has survived more years before being brought to the court. In contrast, it is more likely to be invalidated when the patentee is challenged in the court as the defendant (in contrast to the scenario where the patentee sues for infringement as the plaintiff).

The bottom line of this regression is, the same type of patent is less likely to be invalidated after the legal reform. This is suggestive evidence for a more pro-patent legal environment and stronger patent rights after the legal reform.

In light of these dramatic changes in the court system, the next question is, how do the the firms respond to this changing legal environment?

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[4]To be specific, this regression controls for the number of forward citations received by the patent (the truncation issue is addressed following Hall, Jaffe and Trajtenberg (2001)), the age of the patent at the lawsuit, the number of claims of the patent, dummies for the technology class of the patent, dummies for the circuit court where the final court decision is made, whether the patentee is the plaintiff or the defendant, and whether the patentee is U.S. or non-U.S. inventor.

	1{ Patent Invalidated by the Court }		
	(1)	(2)	(3)
CAFC (= 1 after 1982)	−0.786***	−0.743***	−0.619***
	(0.0851)	(0.105)	(0.139)
ln(num of citations received)			-0.0806**
			(0.0326)
age of patent at the lawsuit		-0.0184***	-0.0358**
		(0.00649)	(0.0151)
patentee as the defendant		0.156**	0.285**
		(0.0738)	(0.127)
ln(num of claims)		-0.0614***	-0.0167
		(0.0173)	(0.0338)
foreign patentee		-0.104***	0.0466
		(0.0376)	(0.0681)
technology class	N	Y	Y
circuit court	N	Y	Y
Observations	5,172	4,043	1,234

Notes: Probit regression of patent invalidation. The control variables are the number of forward citations received by the patent, the age of the patent at the lawsuit, the number of claims of the patent, dummies for the technology class of the patent, dummies for the circuit court, whether the patentee is the plaintiff or the defendant, and whether the patentee is U.S. or non-U.S. inventor. Robust standard errors in parentheses (clustered at the level of circuit court). *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

1.2.2. Specialization In the Innovating Sectors

The dramatic change in the legal environment underlined in the last section has a far-reaching impact on the industrial-organizational structures in the innovating sectors. To

uncover this impact, this section develops an empirical measure of innovation specialization based on the technological distance of patent. As shown in this section, there is increasing specialization in the innovating sectors in the United States in the recent decades, in the sense that the innovators invent in more closely related technological fields. In addition, as demonstrated in the regression analysis, this increasing innovation specialization is in part due to strengthened patent rights.

To gauge the degree of specialization in the innovating sectors, a metric of technological distance between patents is applied as a measure of firms' business scope with respect their innovating activities. The underlying rationale is, when a firm invents in more closely related technological fields, this signals a more focused business scope strategy and a higher level of innovation specialization.

The measure of technological distance[5] follows the metric developed in Akcigit, Celik, and Greenwood (2016). This distance metric is based on the citation links between the patents. To be specific, the technological distance between two patent classes X and Y is defined as:

$$d(X,Y) = 1 - \frac{\#(X \cap Y)}{\#(X \cup Y)}, \ d(X,Y) \in [0,1].$$

In the expression above, $d(X,Y)$ is the technological distance between two patent classes X and Y, $\#(X \cap Y)$ is the number of patents that cite both X and Y, and $\#(X \cup Y)$ is the number of patents that cite either X or Y. The rationale underlying this distance metric is straightforward: the more X and Y are cited together, the closer they are. By construction, this measure is always between 0 and 1, and a lower distance implies the patents are closer to each other.

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[5]In the baseline empirical analysis, technological fields are measured at the level of 3-digit International Patent Classification (IPC).

FIGURE 4: INCREASING INNOVATION SPECIALIZATION
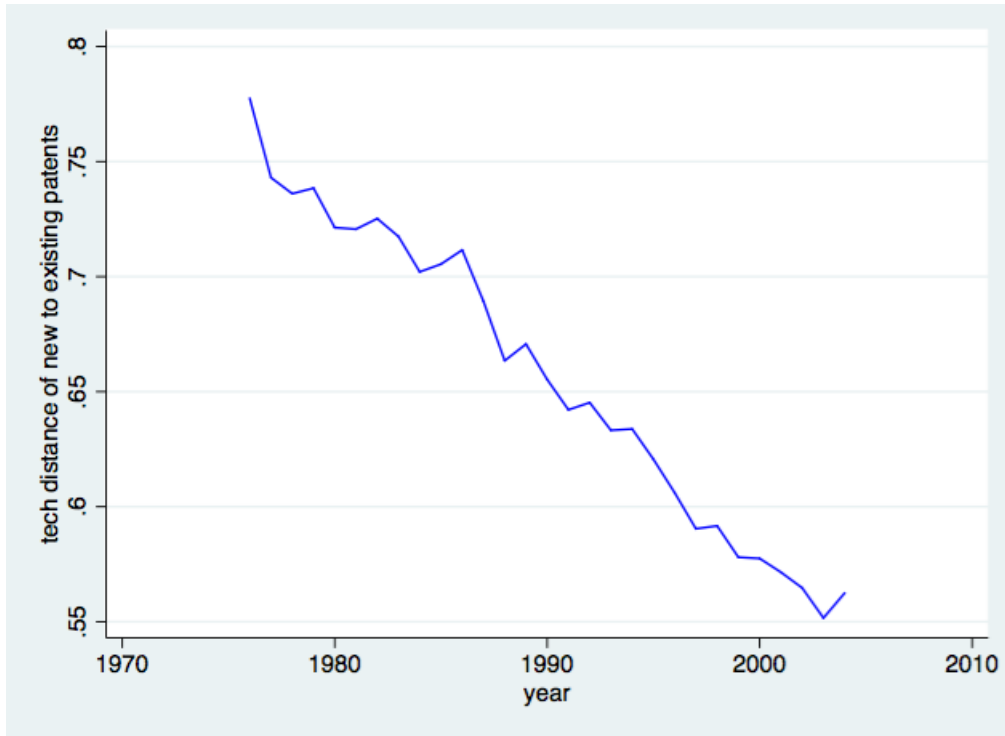
Based on this measure, the evolution of the business scope of the average U.S. innovator is tracked in Figure 4. The vertical axis in Figure 4 is the average technological distance between the new patent and the existing patent portfolio of the innovators[6]. As revealed in Figure 4, the average U.S. innovator is inventing in more closely related technological fields, which implies shrinking business scope and increasing specialization in the recent decades[7].

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[6]To be specific, for each innovator in each year, the technological distance between the new patent it obtains and its existing patent portfolio at the beginning of the year is calculated. The vertical axis is the average technological distance across the innovators in each year.

[7]Echoing these findings, Chesbrough (2006) documented increasing specialization in the semiconductor and life science industry. Back in the 1960s, the semiconductor industry was dominated by only two corporations: IBM and AT&T. Both of them included in their operations the entire production process from design to manufacturing. The landscape of this industry, however, started to change in the 1970s. Intel and Texas Instruments were created and they specialized in the production of chips, ushering in the emerging markets of intermediate inputs for semiconductors. By the 1980s, a revolutionary separation was introduced in the semiconductor industry, leading to a dichotomy of this industry into the "fabs", specializing in the fabrication of semiconductors (e.g., TSMC), and the "fabless", specializing in the design function. The design tools were further stripped in the 1990s, as epitomized by Qualcomm and ARM Holdings. Both of them offered their intellectual property underlying the design tools and effectively created a market for the design itself. The specialization in the semiconductor industry was by no means a unique experience. In the 1970s, pharmaceutical manufacturers, such as Pfizer and Johnson and Johnson, used to keep a whole army of staff, from the R&D team developing new drugs to the marketing division promoting their prod-

1.3. Model

1.3.1. Environment

Consider an economy where time is discrete. There are 3 types of agents in this economy: the household, the final good producer, and the intermediate goods firms. The final goods are assembled by combining a range of intermediate goods with labor. The intermediate goods are produced by the intermediate goods firms, and there is a constant marginal cost of production. The source of growth in this economy is expanding varieties of intermediate goods à la Romer (1990).

Patent

In this economy, the production technology of every intermediate input is embodied in a patent. In addition, there are 2 types of patents: the active patents and the expired patents. The number of active patents is N, and the owners of the active patents enjoy exclusive rights over their intellectual property. The number of expired patents is $\widetilde{N}$, and everyone has free access to the technologies embodied in these patents. An active patent can expire for 2 reasons. In practice, the term of the patent is limited. This is captured by stochastic survival with probability σ. In addition, the active patents can be involved in litigation, and they can be invalidated by the court. Once they are invalidated, they join the pool of the expired patents. This is a crucial channel for the court system to change the landscape of technology. Through this channel, the expired patents are introduced to capture the impact of the legal institution.

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ucts. In the 1980s, the development of new drugs began to be led by biotechnology firms specializing in the discovery of new compounds. The pharmaceutical producers acquired the patents of the compounds from these biotech firms, conducted clinical trials and then offered the new drugs to the market. Turning to the 1990s, independent research organizations specializing in performing clinical trials (e.g., Millennium Pharmaceuticals) were created, and they focused on testing the safety and efficacy of the new drugs.

Intermediate Goods Firm

The intermediate goods firms own all the patents, and they can produce the intermediate goods associated with the patents they hold. The boundaries between these firms can be identified by the range of intermediate goods they produce in-house. These intermediate goods are produced to serve 2 types of customers. They are sold to the final goods producer at price q, and they are used to produce the final goods. They are also sold to other intermediate goods firms at price p, and they are used in the R&D process to discover new varieties of intermediate goods. The prices p and q are different, because the underlying demand functions are different, as will be shown later. There is no resale between these 2 market segments. In addition, the intermediate goods firms can discover new varieties of intermediate goods from R&D, and they file a new patent for every new variety of intermediate good they discover. They can keep the new patents they develop, or they can sell them to other intermediate goods firms. They can also buy new patents, and all patents are traded at their intrinsic value (i.e., the present value of the future payoff).

R&D: Input and Output

The R&D process in this economy is illustrated in Figure 5.

FIGURE 5: R&D: SETUP

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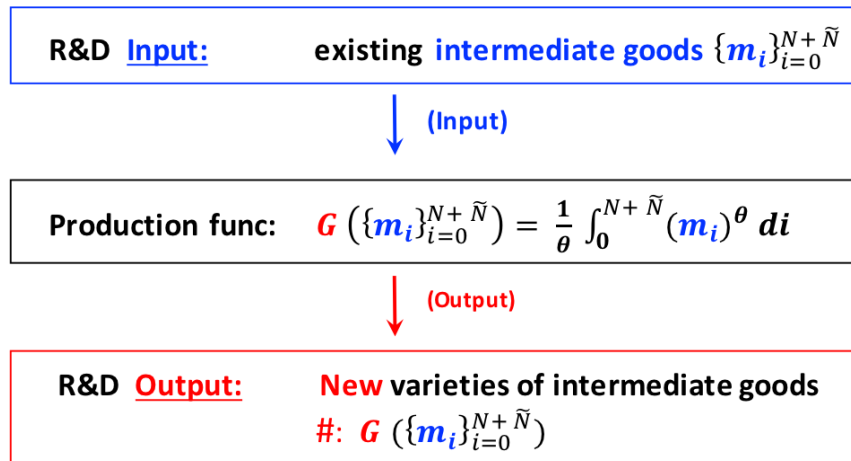
The input of R&D is a basket of the existing intermediate goods. These inputs are used in the corporate lab to discover new varieties of intermediate goods. By investing a basket of intermediate goods $\{m_i\}_{i=0}^{N+\widetilde{N}}$, an intermediate goods firm can discover new varieties of intermediate goods in the amount of $G(\{m_i\}_{i=0}^{N+\widetilde{N}}) = \frac{1}{\theta} \int_0^{N+\widetilde{N}} (m_i)^\theta di$.

These new varieties of intermediate goods are the output of R&D. They can be the blue prints of new semiconductors, or they can be the chemical structure of new drugs and medicine. The intermediate goods firms file a patent for each new variety of the intermediate goods they discover. This is how technology advances in this economy.

R&D Inputs: Sources and Costs

Where do the R&D inputs come from? They come from 3 sources, as depicted in Figure 6.

FIGURE 6: R&D INPUTS: SOURCES AND COSTS
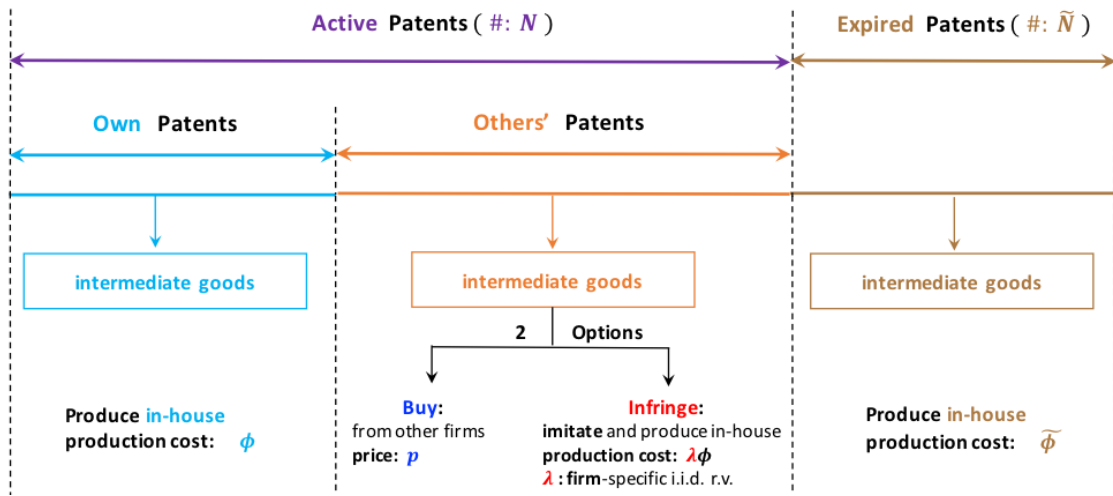
For some of the intermediate goods, the underlying production technology is embodied in the expired patents. Everyone has free access to these inputs, and they can produce them in-house at the cost $\widetilde{\phi}$. In contrast, for some of the intermediate goods, the underlying production technology is embodied in the active patents. These patents have owners. For instance, a semiconductor has many components. Some of these components are associated with the patents owned by Intel, and some components are based on the patents of Texas

Instruments. To develop a new semiconductor, Intel needs to combine the components based on its own patents, with the components patented by Texas Instruments. For the first type of the components, Intel can simply produce in-house, and there's a production cost, ϕ. In contrast, to obtain the components based on the patents of Texas Instruments, Intel has 2 options. He can buy these components from Texas Instruments, and he needs to pay the price p. Alternatively, Intel can infringe on the patents of Texas Instruments, by imitating its product and producing in-house. That is to day, Intel can produce the same component of Texas Instruments, but their production cost can be different. The production cost of Texas Instruments is ϕ, and the cost of Intel, the imitator (or the infringer), is $\lambda\phi$. λ is firm-specific random variable, and it captures heterogeneous firm capabilities to imitate. In addition, λ is uniformly distributed on $[0, \bar{\lambda}]$, and it is independently and identically distributed across firms and across periods.

Legal Institution

The legal institution to resolve the disputes are outlined in Figure 5. When the patent of an intermediate goods firm is infringed by other firms, it can sue them in the court. In response, the alleged infringers will challenge the validity of the patent, and the court will make a decision on whether this patent is valid or invalid. If the patent is adjudicated to be valid, the patent owner will receive a legal settlement. If the patent is adjudicated to be invalid, this patent will expire. In addition, a patent is more likely to be invalidated when more firms challenge the validity of this patent in the court. In order to invalidate a patent, the number of firms needed to challenge the patent follows an exponential distribution with parameter τ. That is to say, when a patent is challenged by f firms, this patent will be invalidated with probability $1 - e^{-\tau f}$.
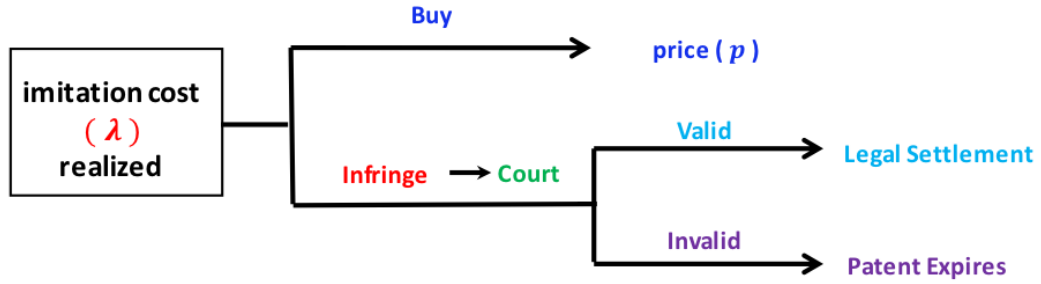
Measure of Specialization

In this economy, the production technology of every intermediate good is embodied in a patent. The number of patents a firm own determines the range of intermediate goods that it produces in-house, in contrast to the intermediate goods purchased from the external market. The model features representative firm. Every firm holds the same number of patents: s. Hence, the number of intermediate goods a firm produces in-house is also s. The business scope of the firm can be captured by s, because s pins down the boundary between the intermediate goods produced in-house, and the intermediate goods purchased from the external market.

In addition, the level of integration of the firm is captured by $\frac{s}{N}$, the fraction of intermediate goods the firm produces in-house. An increase in $\frac{s}{N}$ implies a higher level of integration, and a lower level of specialization. s is a key choice of the firm[8], and the number of the firms, F, is pinned down by $\frac{N}{s}$. The number of the firms will settle down along the balanced growth path.

Management Costs

It is costly to manage the business in this economy. To be specific, the management cost for a firm is $\Omega(\frac{s}{N}, w) = \frac{1}{\xi+1} \left(\frac{s}{N}\right)^{\xi+1} w$. The management cost is increasing in the wage rate,

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[8]More precisely speaking, s is a key state variable of the firm. Every firm starts with s at the beginning of the period, and it chooses s' for the next period.

w. Imagine every firm needs to hire managers to run the business. In addition, another key factor in the management cost is $\frac{s}{N}$, the fraction of intermediate goods the firm produces in-house. The management cost is increasing and convex in $\frac{s}{N}$. Imagine the management monitoring is subject to diminishing return to scope. The convexity of Ω implies scope diseconomies and benefits of specialization. When a firm pursues a more focused business strategy with lower $\frac{s}{N}$, the firm can enhance its management efficiency because of the convexity of the management cost function. The degree of scope diseconomies and the benefit of specialization is governed by the magnitude of the convexity (ξ).

Timing of Events

As bird view of the major decisions of the intermediate goods firms, the timing of events is illustrated in Figure 8. At the beginning of the period, the production cost under imitation (λ) is realized. Based on their λ, the firms will decide whether to buy the intermediate goods from other firms or infringe on their patents. Then the firms will produce the intermediate goods, and decides how much to spend in R&D. The legal disputes will be settled in the court, and the firms will make payment to each other based on the court decisions. If a firm's patents are infringed and it wins the lawsuit, this firm will receive a legal settlement from the violator. Meanwhile, if a firm infringes on others' patent and loses the case, it will have to pay the compensation to the patentee. Lastly, the firms can trade patents before the end of the period, so that every firm can adjust its business scope to any level as it desires.

Based on this setup, the decisions of the agents are delineated as follows.

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FIGURE 8: TIMING OF EVENTS
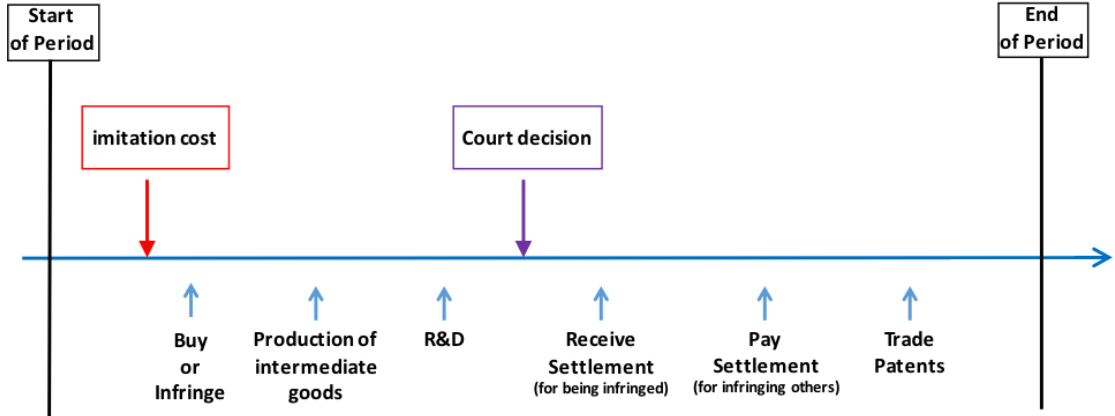
1.3.2. Households

There is a representative household in this economy. The household is endowed with one unit of labor. Labor supply of the household is inelastic. The household owns all the firms and earns income from wages and the dividends collected from the firms. The objective of the household is to maximize the lifetime utility with discount rate β for future. The preference of the household is characterized by CRRA utility with momentary utility function: $U(c) = \frac{c^{1-\epsilon}}{1-\epsilon}$, where c refers to current consumption and ϵ is the coefficient of relative risk aversion. Since this setup is entirely standard, the household problem is not explicitly delineated.

1.3.3. Final Goods Producer

The final goods in this economy are assembled by combining labor with a range of intermediate inputs. To be specific, the final goods are produced under the following production technology:

$$Y = \frac{1}{\alpha} \int_0^{N+\widetilde{N}} x_i^{\alpha} di \ L^{1-\alpha}$$

In this production function, L is the labor input and $1 - \alpha$ is the share of labor. x_i is the quantity of intermediate input i, and the varieties of the intermediate inputs are ranging from $[0, N+\widetilde{N}]$. Recall N and $\widetilde{N}$ are the number of active and expired patents, respectively,

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and $N + \widetilde{N}$ is the total number of intermediate inputs (or patents). There is a constant marginal production cost $\widetilde{\phi}$ for the intermediate inputs based on the expired patents (i.e., $i \in [0, \widetilde{N}]$). For the intermediate inputs associated with the active patents (i.e., $i \in [0, N]$), the final goods producer is facing a price q charged by the owner of the active patents. Therefore, the final goods producer is facing the problem:

$$\max_{\{\{x_i\}_0^{N+\widetilde{N}},\ L\}} \frac{1}{\alpha} \int_0^{N+\widetilde{N}} x_i^\alpha di\ L^{1-\alpha} - \int_0^N q\ x_i di - \int_N^{N+\widetilde{N}} \widetilde{\phi}\ x_i di - wL$$

The first term in the problem of the final goods producer is the output, the second term is the expenditures on the intermediate inputs associated with active patents, the third term is the production cost for intermediate inputs associated with expired patents, and the last term is the labor cost. The problem of the final goods producer delivers the demand function $q(x_i)$ for the intermediate input associated with active patent i and the wage rate of labor[9].

1.3.4. Intermediate Goods Firms

The intermediate goods firms are the core players in this economy, and their decisions are delineated in this section.

Buy or Infringe?

To begin with, an intermediate goods firm is facing the following decision: to obtain the intermediate goods associated with others' patents, should it buy their products or infringe on their patents? The key determinant for the decisions to buy or infringe is the firm's production cost under imitation (λ). A firm can enjoy lower R&D cost if it decides to infringe on others' patent, because it can imitate their product and produce in-house, instead of buying their product from the market. On the other hand, an infringer may lose the lawsuit when being sued in the court. In consequence, the infringer will have to pay the

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[9]Recall there is an inelastic supply of labor with one unit.

legal settlement. From this perspective, an infringer is facing the trade-off between lower R&D cost versus paying a legal settlement when losing the lawsuit. In equilibrium, the firms will follow a cutoff strategy to infringe (i.e., a firm will infringe if and only if its imitation cost λ is below a threshold)[10]. This threshold for the infringement decision will be precisely pinned down later in the section of equilibrium characterization.

Operating Profits

An intermediate goods firm can produce the intermediate goods associated with its patent, and sell them to both the final goods producer and other intermediate goods.

To be specific, the price an intermediate goods firm sets for the final goods producer is determined in the following problem:

$$\pi^X = \max_{\{q,\ x\}} (q - \phi) X(q)$$

$$\text{s.t. } X(q): \text{ demand of final goods producers}$$

An intermediate goods firm seeks to maximize the profits obtained from the final goods producer, and the equilibrium operating profit is denoted by π^X. There is a constant marginal production cost ϕ for the intermediate inputs based on the active patents (i.e., $i \in [0, N]$). q is the price the intermediate goods firm chooses to charge the final goods producer, $X(q)$ is the quantity demanded by the final goods producer given the price q.

In addition, the intermediate goods firms also need the products of each other, because these intermediate inputs will be used for R&D. Each firm has two options to access to the inputs based on others' patents: it can buy their products, or infringe on their patents by imitating their products. The decisions of other firms to buy or infringe depend on the price charged by the intermediate goods firm. To be specific, the price p a patent holder

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[10]Recall the imitation cost λ is a firm-specific variable. Hence, when a firm decides to infringe, it infringes on everything outside her own patent portfolio.

charges other patent holders is determined as follows:

$$\pi^M = \max_{\{p,\, m\}} \ (p - \phi) \times M(p) \times (1 - \widetilde{\lambda})F$$

s.t. $M(p)$: demand of other patent holder

s.t. $\widetilde{\lambda}(p)$: fraction of firms that infringes

An intermediate goods firm seeks to maximize the profits obtained from other intermediate goods firms, and the equilibrium operating profit is denoted by π^M. In the expression above, p is the price the intermediate goods firm charges, and $M(p)$ is the quantity demanded given the price p. The demand function $M(p)$ hinges on the return from R&D. Given the price p set by an intermediate goods firm, a fraction $1 - \widetilde{\lambda}$ of the firms will buy her products, and a fraction $\widetilde{\lambda}$ of the firms will infringe on her patent. Hence, the number of buyers is $(1 - \widetilde{\lambda})F$, where F is the total number of intermediate goods firms in this economy. As will be shown later, $\widetilde{\lambda}$ is increasing in p, the price charged by the intermediate goods firm. Hence, when a firm charges a lower price, more people will decide to buy its products instead of infringing. In addition, $\widetilde{\lambda}$ is increasing in τ, the odds for the patent to be invalidated by the court. When the patents are more likely to be invalidated, more firms will decide to infringe instead of buying. This is a crucial channel for the court system to change the firm decisions.

Patent Survival Rate

In this economy, a patent may expire for two reasons. First, the term of the patent is limited and this is captured by stochastic survival with probability σ.[11] In addition, a patent can be involved in litigation when it is infringed, and the patent may be invalidated when being challenged in the court. The number of alleged infringers challenging the patent is $\widetilde{\lambda}F$, so

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[11]In the quantitative analysis, σ will be specified to match the term of the patent in practice.

the patent will be adjudicated to be valid [12] with probability $e^{-\tau\widetilde{\lambda}F}$. Hence, the survival rate of patent is $\eta = \sigma e^{-\tau\widetilde{\lambda}F}$.

Legal Settlement Received

Though the intermediate goods firm does not receive any payment from the infringers, it can bring them to the court and sue them for infringement. In each lawsuit, the patent owner can win and receive the legal settlement with probability $e^{-\tau\widetilde{\lambda}F}$. The amount of the settlement upon victory is a fraction μ of the price of the patent, P.[13] However, it is costly to sue people, and the cost of litigation is a fraction ψ of the price of the patent, P. Hence, the net legal settlement received by an intermediate good firm is $z \times s = (e^{-\tau\widetilde{\lambda}F}\mu P - \psi P) \times \widetilde{\lambda}F \times s$.

Patent Trading

At the end of the period, all firms can adjust their business scope to any level as they desire. This adjustment can be achieved by trading their patents with each other. There is a centralized market for patent trading, where every patent can be traded at the intrinsic value (i.e., the present value of the future payoff). To be specific, the price of the patent is:

$$P_t = \sum_{j=0}^{\infty} \frac{\eta^j \left(\pi_{t+j} + z_{t+j}\right)}{R^j}$$

The price of a patent at period t is denoted by P_t.[14] The price of a patent is the present value of the expected payoff in future, and the future payoffs of a patent is discounted by the gross interest rate, R. In each period, the patent can survive with probability η. Conditional on survival, there are two sources of income for a patent: operating profits collected from the buyers (π) and legal settlement received from the infringers (z).

---

[12]In order to invalidate a patent, the number of firms needed to challenge the patent follows an exponential distribution with parameter τ. That is to say, when a patent is challenged by f firms, this patent will be invalidated with probability $1 - e^{-\tau f}$.

[13]The value of the patent (P) will be delineated momentarily in the next section.

[14]The price of patent will settle down along the balanced growth path.

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Given the price P', every firm chooses the number of patents to trade, h, and its expenditure on patent purchase (or revenue from patent sale) is $H = hP'$. A positive choice of h implies patent purchase, and a negative choice of h implies patent sale.

Value Function

Combining the payoffs of the patent holders delineated in the previous sections, the value function of the intermediate goods firm is characterized as follows:

$$V(s; N, \widetilde{N}; \lambda) = \max_{\{ \text{ Buy (B), Infringe (I) } \}} \{V^B(s; N, \widetilde{N}), V^I(s; N, \widetilde{N}; \lambda)\}$$

There are four state variables for the intermediate goods firm: the number of patents it owns (s), its production cost under imitation (λ), the total number of active patents (N), and the total number of expired patents $(\widetilde{N})$. In addition, every intermediate goods firm is facing two states of the world: she can be a buyer if she decides to buy the products of other intermediate goods firms, or she can be an infringer if she infringes on others' patent by imitating their products and producing in-house.

If an intermediate goods firm decides to buy the products of other firms, its value function is characterized as follows:

$$V^B(s; N, \widetilde{N}) =$$

$$\max_{\left\{q,\, p,\, \{m_i\}_{i=0}^{N+\widetilde{N}}, h, s'\right\}} \left\{ \begin{array}{l} \pi^X(q) \times s + \pi^M(p) \times s + z \times s - I\left(\{m_i\}_{i=0}^{N+\widetilde{N}}\right) - h \times P' \\[2mm] -\Omega(\frac{s}{N}, w) + \frac{1}{R}\mathbb{E}_\lambda\{V(s'; N', \widetilde{N}'; \lambda')\} \end{array} \right\}$$

$$\text{s.t. } s' = \eta s + G\left(\{m_i\}_{i=0}^{N+\widetilde{N}}\right) + h$$

There are four choices of the intermediate goods firm: the price it charges the final goods producer (q), and the price it charges other intermediate goods firms (p), how much it spends on R&D $(\{m_i\}_{i=0}^{N+\widetilde{N}})$, and how many patents to buy or sell (h). The current payoff of the

intermediate goods firm is its operating profits obtained from producing the intermediate goods ($\pi^X(q) \times s + \pi^M(p) \times s$), plus the legal settlement received for being infringed ($z \times s$), minus its R&D expenditures ($I\left(\{m_i\}_{i=0}^{N+\widetilde{N}}\right)$), minus its purchase or sale of patents ($h \times P'$), minus its management costs ($\Omega(\frac{s}{N}, w)$). The law of motion for the number of patents a firm has is: $s' = \eta s + G\left(\{m_i\}_{i=0}^{N+\widetilde{N}}\right) + h$. At the beginning of the period, every firm starts with the same number of patents, s, and a fraction η of the patents can survive. There will be new patent developed from R&D, $G\left(\{m_i\}_{i=0}^{N+\widetilde{N}}\right)$. At the end of the period, the firm can choose the number of patents to trade, h.

Analogously, if an intermediate goods firm decides to infringe, its value function is characterized as follows:

$$
V^I(s, \lambda, N, \widetilde{N}) =
$$
$$
\max_{\left\{q, \, p, \, \{m_i\}_{i=0}^{N+\widetilde{N}}, h, s'\right\}}
\left\{
\begin{array}{l}
\pi^X(q) \times s + \pi^M(p) \times s + z \times s - I\left(\{m_i\}_{i=0}^{N+\widetilde{N}}, \lambda\right) - h \times P' \\[2mm]
-\Delta - \Omega(\frac{s}{N}, w) + \frac{1}{R}\mathbb{E}_\lambda\{V(s'; N', \widetilde{N}'; \lambda')\}
\end{array}
\right\}
$$
$$
\text{s.t. } s' = \eta s + G\left(\{m_i\}_{i=0}^{N+\widetilde{N}}\right) + h
$$

When an intermediate goods firm decides to infringe, it imitates others' products and produces them in-house. Because of this, its R&D expenditures depend on its imitation cost, λ. On the other hand, it has to pay the legal settlement (Δ) if it loses the lawsuit in the court.

The key determinant for the decisions to buy or infringe is the firm's production cost under imitation (λ). This decision is static because λ is independently and identically distributed across periods. In addition, the optimal pricing and R&D decisions are designed to be static in this model, so the only dynamic choice is s', the choice for the scope of business in the next period. In equilibrium, every firm will choose the same s', because every firm has the

same marginal cost[15] and the same marginal benefit [16] of holding one more patent in the next period.

1.3.5. Equilibrium

Cutoff Strategy to Infringe

As revealed in the previous discussion, the key determinant for the decisions to buy or infringe is the firm's production cost under imitation (λ). Given λ, a potential infringer is facing the trade-off between lower R&D cost and paying a legal settlement when losing the lawsuit. In equilibrium the firms will follow a cutoff strategy to infringe in the following form:

Proposition 1. (*Cutoff Strategies to Infringe*) A firm infringes if and only if its production cost under imitation $\lambda \leq \widehat{\lambda}$, where $\widehat{\lambda}$ is determined by $V^I(s; N, \widetilde{N}; \widehat{\lambda}) = V^B(s; N, \widetilde{N})$.

Conditional on the decision to buy, the value function of the firm no longer depends on its production cost under imitation (λ). In contrast, the value function of the infringer is strictly decreasing in its λ. Therefore, in equilibrium a firm infringes if and only if its production cost under imitation (λ) is lower than $\widehat{\lambda}$, and the threshold $\widehat{\lambda}$ is determined where the value function of the buying coincides with the value function of infringing. Since λ is uniformly distributed on $[0, \bar{\lambda}]$, the fraction of firms that infringes is $\widetilde{\lambda} = \dfrac{\widehat{\lambda}}{\bar{\lambda}}$.

Balanced Growth Path

Proposition 2. (*Balanced Growth Path*) There exists a balanced growth path[17] along which:

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[15]The marginal cost of holding one more patent is P', and every firm is facing the same price.

[16]The marginal benefit of holding a patent is how much a firm expects her payoff can be boosted by holding one more patent in the next period, and this expected boost in payoff depends on whether she buys or infringes in the next period. In addition, since λ is i.i.d. across periods, every firm has the same expectation for the next period, and this does not depend on the current status of buying and infringing, so the marginal benefit of holding one more patent is the same for every firm.

[17]The balanced growth path can be solved by five variables from five equations. The five variables are: the price set for other intermediate goods producer, the cutoff of imitation cost to infringe, the price of the patent, the number of the firms, and the growth rate of the number of active patents.

1. The number of the firms (F) is constant.

2. The number of active patents (N) and expired patents $(\widetilde{N})$ grow at the same rate.

3. Output (Y) , comsumption (c) , wages (w) , and the number of patents held by each firm (s) all grow at the same rate as the number of active patents (N).

1.4. Quantitative Analysis

The theoretical model established in the previous section offers an apparatus to conduct thought experiments and policy evaluations. To achieve this objective, the parameters of this model are calibrated to match the data in this section. The quantitative analysis delivers three major implications of the model: (1) stronger patent rights induce specialization, (2) specialization enhances firm performance, and (3) specialization contributes to economic growth.

1.4.1. Calibration

The value of the parameters in this model are reported in Table 3. These parameters fall into three groups: parameters underlying the preferences of the households, parameters governing the technology of production and R&D, and parameters characterizing the legal system.

Seven of these parameters are determined by a priori information, and the detailed identification strategies are delineated as follows.

1. *CRRA parameter for households*, ϵ. The CRRA parameter is determined by taking the average values of the estimates in Kaplow (2005).

2. *Long-run interest rate*, R. The long-run interest rate in the United States follows Cooley and Prescott (1995).

3. *Discount factor for households*, β. The discount factor for households is deduced from the interest rate (R), the CRRA parameter for households (ϵ), and the growth rate

of the economy[18].

4. *Capital share*, α. The capital share is based on the U.S. National Income and Product Accounts, as reported in Corrado, Hulton and Sichel (2009).

5. *Imitation cost*, $\bar{\lambda}$. The imitation cost is gathered from Mansfield, Schwartz, and Wagner (1981).

6. *Patent survival rate*, σ. The term of patents in the United States is 17 years, so σ is taken to be $11/(1+17)$.

7. *Litigation cost*, ψ. The litigation cost is based on the report of the American Intellectual Property Law Association (2011)[19].

The other six parameters, ϕ (production cost of intermediate inputs associated with the active patent), $\widetilde{\phi}$ (production cost of intermediate inputs associated with the expired patent), θ (degree of complementarities in R&D), ξ (convexity of the management cost), τ (parameter for the odds to invalidate a patent) and μ (legal settlement for infringement) are calibrated to match five data targets. The model moments are contrasted with the data targets in Table 4. The sources of the data targets are discussed as follows.

1. *Level of integration.* This is the average fraction of technological field[20] a firm invents in between 1982 and 2006. The patenting information is obtained from the NBER patent dataset project.

2. *Ratio of expenditure on R&D inputs purchased from the market to in-house R&D.* This is the ratio of royalties to R&D taken from the report of the National Science Foundation (2013).[21]

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[18]To be specific, the discount factor for households (β) is pinned down from the Euler equation of the households.

[19]As reported by the American Intellectual Property Law Association, the average litigation cost is 2.8 million dollars when the value at risk is between 1 million and 25 million dollars.

[20]The technological fields are measured at the level of 3-digit International Patent Classification (IPC).

[21]More details can be found in Shackelford (2013).

3. *Fraction of cases invalidating the patents.* This is based on the records of the United States Patents Quarterly (USPQ).

4. *Litigation cost to R&D ratio.* The ratio of litigation cost to R&D is gathered from Lerner (1995).

5. *R&D expenditures to GDP ratio.* The ratio of R&D expenditures to GDP is calculated from the U.S. National Income and Product Accounts.

6. *Growth of real GDP per capita.* This is the average growth rate of real GDP per capita between 1982 and 2006, as calculated from the U.S. National Income and Product Accounts.

Table 3: Parameter Values

Parameter	Value	Description	Identification
β	0.98	discount factor for households	A priori information
ϵ	2.00	CRRA parameter for households	A priori information
R	0.06	interest rate	A priori information
α	0.60	capital share	A priori information
ϕ	140	production cost for intermediate goods associated with active patents	Calibration
$\widetilde{\phi}$	110	production cost for intermediate goods associated with expired patents	Calibration
θ	0.44	concavity of inputs in R&D function	Calibration
$\bar{\lambda}$	1.30	imitation cost	A priori information
ξ	2.71	convexity of management cost function	Calibration
σ	0.94	patent survival rate	A priori information
τ	0.61	parameter for the odds to invalidate a patent	Calibration
ψ	0.21	litigation cost	A priori information
μ	0.68	legal settlement for infringement	Calibration

Target	Source	Model (%)	Data (%)
level of integration ($\frac{s}{N}$)	USPTO	1.4	1.4
ratio of expenditure on R&D inputs purchased from the market to in-house R&D	NSF	13.0	13.0
fraction of cases invalidating the patents	US Patents Quarterly	29.0	29.0
litigation cost to R&D ratio	Lerner (1995)	27.0	27.0
R&D to GDP ratio	NIPA	3.3	2.9
growth of real GDP per capita	NIPA	2.1	2.1

1.4.2. Stronger Patent Rights Induce Specialization

The impact of patent rights on firm boundaries is depicted in Figure 9. The horizontal axis of Figure 9 are the odds for the patent to be invalidated by the court, a proxy for the strength of patent rights. The dashed line in Figure 9 is the fraction of firms that infringe on others' patents, and the solid line is the fraction of intermediate goods the firm produces in-house.

The scope of business is captured by the fraction of intermediate goods the firm produces in-house, as shown on the left vertical axis of Figure 9. As revealed in this figure, when the patents are more likely to be invalidated by the court, a higher fraction of the firms will decide to infringe on others' patents instead of buying their products. In response, the firms will expand their business scope and produce a higher fraction of the intermediate goods in-house.

This is because a major benefit of broader business scope in this model is to fend off potential infringement. The expansion of business scope is achieved by merger and acquisition. After acquiring some suppliers and customers, a firm will be facing a smaller number of trading partners. The firm is less likely to be infringed, because the odds to be infringed are increasing in the number of trading partners the firm deals with. In this way, a firm can

fend off infringement by expanding its business scope and acquiring its trading partners. In the extreme, imagine a firm acquires all the patents of all the other firms, then there would be a single firm in this economy. All the transactions will be internalized and there will be no issues of infringement. From this perspective, when a firm is concerned about being infringed, it can expand its business scope to fend off potential infringement and litigation.

FIGURE 9: PATENT RIGHTS AND FIRM BOUNDARIES
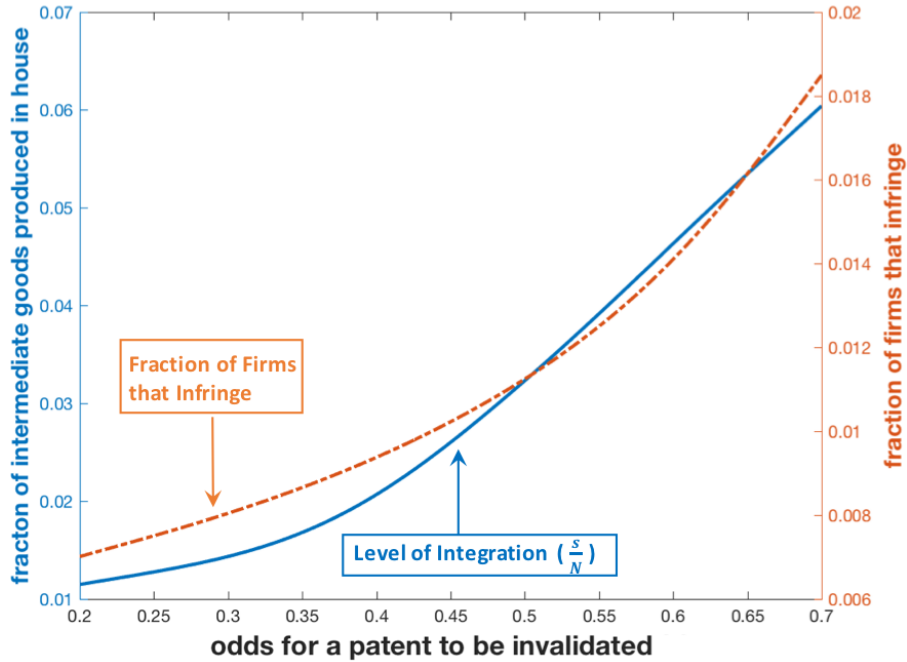
In addition, this benefit is increasing in the odds for the patent to be invalidated. This is because the patents of a firm are more likely to be infringed and invalidated with weaker patent rights. Hence, the consequences of infringement and litigation are more severe, and the benefit of expanding the business scope to fend off infringement increases. Therefore, weaker patent rights can discourage specialization and stronger patent rights can induce specialization.

1.4.3. Specialization Enhances Firm Performance

Figure 10 evaluates how the business scope decision of a firm affects its performance. The horizontal axis in Figure 10 is the fraction of intermediate goods each firm produces in-house. A higher fraction of intermediate goods produced in-house implies a broader business scope. The dashed line in Figure 10 is the ratio of management costs to sales, and the solid line is the marginal boost of patents to firm value.

FIGURE 10: SPECIALIZATION AND FIRM PERFORMANCE
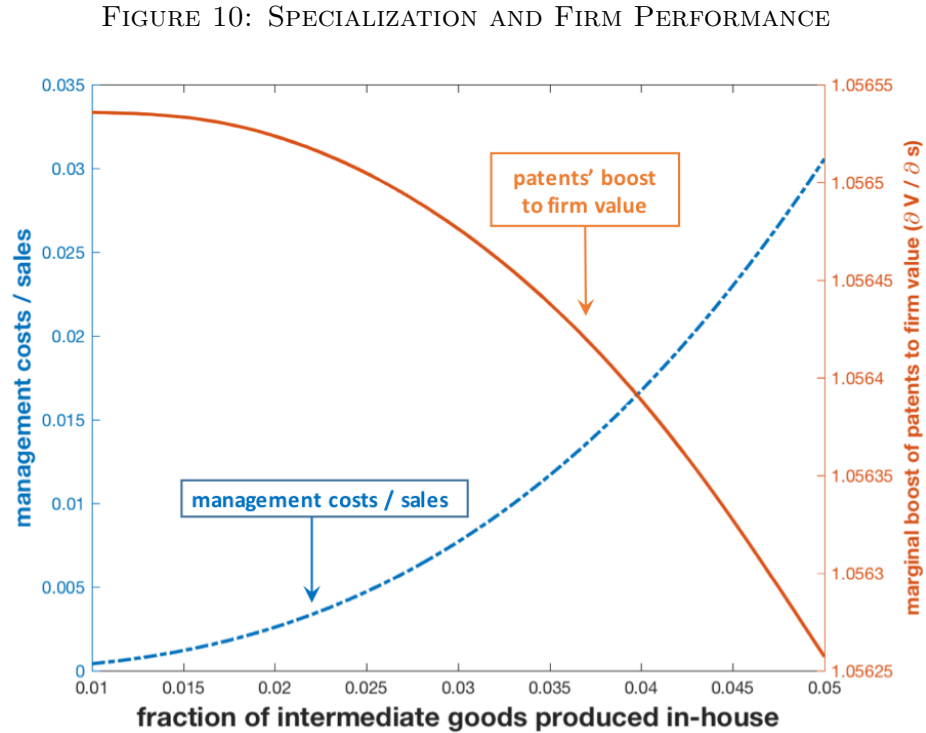
As demonstrated in Figure 10, broader business scope leads to higher management costs. Increasing costs originate from the convexity of the management costs function with respect to the fraction of intermediate goods the firm produces in-house. In consequence, the marginal boost of patents is increasingly lower when a firm has a broader business scope. Hence, specialization enhances firm performance by improving firms' management efficiency.

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1.4.4. Specialization Contributes to Economic Growth

Figure 11 examines the relationship between specialization and economic growth. The horizontal axis in Figure 11 is the fraction of intermediate goods each firm produces in-house. The dashed line in Figure 11 is the ratio of R&D to GDP, and the solid line is the growth rate of GDP per capita.

FIGURE 11: SPECIALIZATION AND ECONOMIC GROWTH
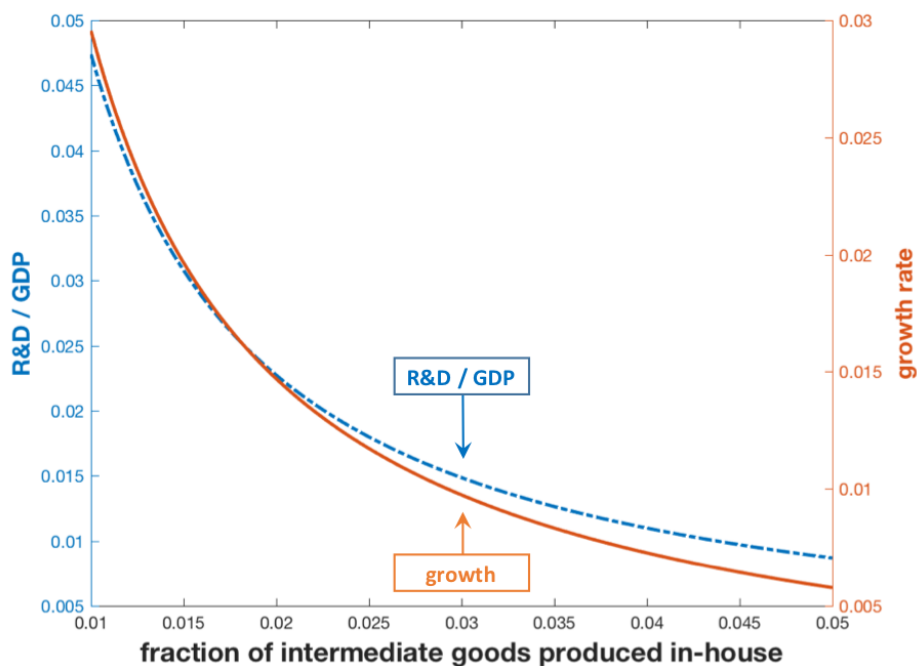
As shown in Figure 11, when the firms pursue a more focused business scope strategy by producing a lower fraction of intermediate goods in-house, the economy will experience faster economic growth. When each firm produces a lower fraction of intermediate goods in-house, there will be a higher number of firms in this economy, each pursuing a more focused business scope strategy with a narrower business scope. Each firm faces more buyers for its products, and its patents generate more revenue. Hence, the patents become more valuable, and the firms have stronger R&D incentives to develop more patents. As demonstrated in Figure 11, the economy features higher ratio of R&D to GDP, and, thus, it grows faster.

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1.5. Empirical Evidence

As highlighted in the previous section, the model delivers three major implications: (1) stronger patent rights induce specialization, (2) specialization enhances firm performance, and (3) specialization contributes to economic growth. These implications are tested by the empirical analysis in this section.

1.5.1. Business Scope and Technological Distance

In the empirical analysis, the business scope of the firm is measured as the technological distance between the new patents the firm develops to the existing patent portfolio of the firm. Figure 12 illustrates how this notion of technological distance is mapped into the model. Every firm has s patents in the model. These existing patents of the firm are indexed from 0 to s. In addition, imagine the firm develops new patents in the amount of ϵ. These new patents are indexed from s to $s + \epsilon$. Define the distance between 2 patents i and j, as the difference between their indices in absolute value, i.e., $d(i,j) = |i - j|$. In this scenario, the average distance of new patents to the firms' existing patent portfolio is $\dfrac{s+\epsilon}{2}$.[22]. Hence, the new patents will be more far away from the existing patent portfolio of the firm, when the firm has a larger business scope, s. This is how the notion of business scope in the empirical analysis and the model are connected with each other.

FIGURE 12: BUSINESS SCOPE AND TECHNOLOGICAL DISTANCE

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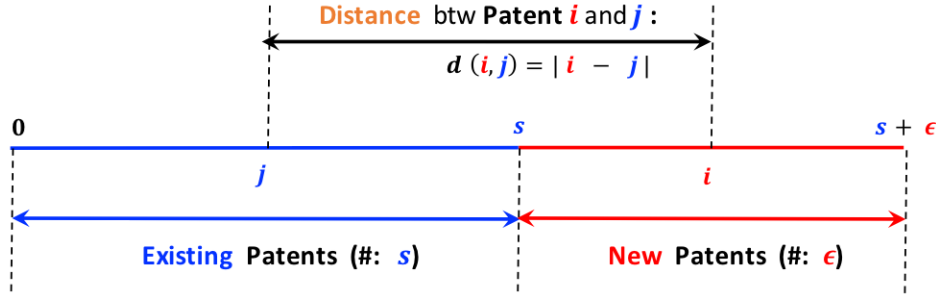
[22]To be specific, for each new patent $i \in (s, s+\epsilon]$, the distance of patent i to firm's existing patent portfolio is $\dfrac{\int_{j=0}^{s} d(i,j)dj}{s} = i - \dfrac{s}{2}$ Hence, The average distance of new patents to firm's existing patent portfolio is $\dfrac{\int_{i=s}^{s+\epsilon}(i - \frac{s}{2})di}{\epsilon} = \dfrac{s+\epsilon}{2}$.

1.5.2. Stronger Patent Rights Induce Specialization

As outlined in the introduction section, the recent decades have witnessed both stronger patent rights and increasing innovation specialization over time. Are they related to each other? To address this question, this section conducts a regression analysis to test the hypothesis that stronger patent rights can induce specialization.

To begin with, the impact of patent rights can be different across firms. To the extent the patent rights has an effect on the business scope decisions of the firms, this effect should be stronger for firms facing higher exposure to patent litigation. Based on this idea, the regression analysis is designed as follows.

$$y_{i,j,t} = \alpha \text{ invalidation rate }_{j,t} + \beta \text{ invalidation rate }_{j,t} \times \text{ litigation exposure }_{i,t} + X_{i,j,t}\delta + \epsilon_{i,t}$$

This regression is based on the U.S. public firms between 1976 and 2006. The dependent variable is the technological distance of the new patents a firm obtains in each year, to the existing patent portfolio of the firm at the beginning of the year. To be more specific, $y_{i,j,t}$ is the technological distance of the new patent to existing patent portfolio of firm i located in circuit court j. The main explanatory variable in this regression is the "invalidation rate", and it refers to the fraction of cases invalidating the patents in the circuit court where the firm is located. This is a proxy for the strength of patent rights in the firm's local environment.[23]

In addition, to capture potential heterogeneous effect across firms, there is an interaction term between the patent invalidation rate and the exposure to litigation at the firm level (i.e., "litigation exposure"). The measure of patent litigation exposure follows Mezzanotti (2017). As shown in Mezzanotti (2017), firms are facing different risks to litigation because they are inventing in different technological fields with different intensities of litigation. From this perspective, the litigation intensity of a technological class is measured as the

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[23]This patent invalidation rate varies both within a circuit over time, and across the circuits at each point in time.

fraction of patents litigated in this class.[24] Based on this measure of litigation intensity, the exposure to litigation at the firm level is the weighted average of the litigation intensities of all the technological classes the firm invents in, and the weight is the share of the firms' patents in each technological class.

$X_{i,j,t}$ refers to the control variables in this regression. To be specific, this regression controls for firm employment, firm age (and age squared), the patent stock of the firm (with quality adjustment),[25] year trend, industry effect, circuit court effect, and firm fixed effect.

TABLE 5: PATENT RIGHTS AND SPECIALIZATION

	ln(Tech Distance)		
	OLS		IV
	(1)	(2)	(3)
ln(invalidation rate)	0.0121**	0.0738***	0.386***
	(0.00516)	(0.0232)	(0.0648)
ln(invalidation rate) × ln(litigation exposure)		0.0166***	0.0919***
		(0.00609)	(0.0155)
Observations	17,547	17,547	17,547
R-squared	0.6377	0.6379	

Notes: Compustat, firm-level regressions. The control variables are firm employment, firm age (and age squared), the patent stock of the firm (with quality adjustment), year trend, industry effect, circuit court effect, and firm fixed effect. Standard errors are in parentheses. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level. The IV is a dummy variable for the legal reform in 1982, which takes the value of 1 after 1982.

Based on this setup, the regression results are presented in Table 5. As shown on the first row of Table 5, the estimate of the regression coefficient of the patent invalidation rate is positive. Hence, when the patents are less likely to be invalidated in a circuit court,

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[24]In the baseline empirical analysis, technological fields are measured at the level of 3-digit International Patent Classification (IPC).

[25]Quality adjustment of patent is performed by weighting the patent counts by the number of citations the patent receive, and the truncation issue is addressed following Hall, Jaffe and Trajtenberg (2001).

firms located in this circuit tend to innovate in more closely related technological fields. In addition, the estimate of the regression coefficient of the interaction term is also positive, so the impact of patent rights is more pronounced for firms facing higher exposure to patent litigation. To address potential endogeneity issues, the instrument variable (IV) approach is adopted in the last column of Table 5. The IV is a dummy variable for the legal reform in 1982, which takes the value of 1 after 1982. As shown in the last column of Table 5, the conclusions are robust in this IV regression. Therefore, these regressions support the hypothesis that stronger patent rights can induce innovation specialization.

1.5.3. Specialization Enhances Firm Performance

Does it matter for the firms to have a broad or narrow business scope? It does matter. It matters at the firm level because the business scope strategy is key to the fate of the firm, and it matters at the country level because of its impact on the economic growth of nations.

A telling example is the rise and fall of Yahoo. Yahoo was already a giant in both search engine and e-commerce when Google and e-Bay were still fledging start-ups. However, Yahoo has never been clear in where it should focus. Gradually it was defeated by Google in search engine, and it was dwarfed by e-Bay in e-commerce. Yahoo is not alone, and the story of Yahoo is echoed in the regression in Table 6. This regression is based on the U.S. public firms between 1976 and 2006, and it evaluates the impact of specialization on firm performance along three dimensions: the number of patents obtained per R&D dollar, the TFP of the firm,[26] and its market value.

Regression (1) is a patent production function, where the input is R&D of a firm and the output is patent that the firm obtains. To capture the impact of specialization on the R&D productivity of the firm, an interaction term between firm R&D and "technological distance" is introduced in regression (1). "Technological distance" here refers to the technological distance of the new patents a firm obtains each year, with its existing patent portfolio at the beginning of the year. As shown in the second row of regression (1), the interaction

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[26]The TFP estimation is based on Levinsohn and Petrin (2003).

term between R&D and technological distance has a negative regression coefficient. Hence, a firm will harvest more patents from the same R&D dollar when it invents in technological fields that are closer to its existing patent portfolio.

TABLE 6: SPECIALIZATION AND FIRM PERFORMANCE

	ln(patent, quality adj.)	ln(TFP)	ln(market value)
	(1)	(2)	(3)
ln(R&D)	0.186***		
	(0.0139)		
ln(R&D) × ln(tech distance)	−0.0163***		
	(0.00529)		
ln(patent stock)	0.812***	0.0823***	0.105***
	(0.0104)	(0.0228)	(0.0195)
ln(patent stock, dist-adj.)		−0.0376**	−0.0833***
		(0.0191)	(0.0167)
Observations	22,214	7,222	25,590
R-squared	0.804	0.951	0.918

Notes: Compustat, firm-level regressions. The control variables are firm employment, firm age (and age squared), the patent stock of the firm (with quality adjustment), year effect, industry effect, and firm fixed effect. Standard errors are in parentheses. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

Regression (2) and (3) assess the relationship between the firms' patent stock and their TFP and market value. As shown in regression (2) and (3), while the patent stock has a positive contribution to firm TFP and market value, the distance-adjusted patent stock has a negative contribution.[27] Therefore, the patent will have a stronger boost to TFP and market value when a firm invents in more closely related technological fields. The ratio of the second regression coefficient to the first one in regression (2) and (3) captures the loss of larger technological distance, or, the gain of a more focused strategy. The bottom

[27]When constructing the distance-adjusted patent stock, every patent is weighed by its distance to the existing patent portfolio of the firm when this patent is developed. The setup of regression (2) and (3) follows Akcigit, Celik, and Greenwood (2016).

line of Table 6 is that innovation specialization (or a more focused business scope strategy) enhances firm performance, and it is embraced by the shareholders.

1.5.4. Specialization Contributes to Growth

Built on the firm-level evidence, this section extends the analysis to a cross-country study. To evaluate the impact of innovation specialization on economic growth, a cross-country panel regression is conducted and the results are presented in Table 7.

The regression in Table 7 covers 88 countries between 1987 and 2006. These countries account for 98% of world GDP during this sampling period. This 2-decade sample is divided into 4 periods, each lasting for 5 years. The dependent variable in this regression is the average growth rate of each country in each period, and the main explanatory variable is the average technological distance between new patents and the existing patent portfolio of the innovators in each country in each period.[28] The regression in Table 7 controls for the 2 main factors demonstrated to be important in the empirical literature: the initial GDP per capita, and the Barro and Lee (2013) human capital index at the beginning of each period. Since the propensity of the innovators in a country to patent in the U.S. can depend on its trade relation with the U.S., the export to the U.S. and the import from the U.S. of each country are included as control variables. In addition, country dummies are added in this regression to control for country fixed effect, and period dummies are included to control for the global aggregate shocks to all countries.

An IV regression is performed to address potential endogeneity issues. The IV is the average technological distance between new patents and existing patent portfolio of the innovators for each country in the pre-sampling period,[29] following the strategy pioneered in Barro and Lee (1994). The pre-sampling period is one period before the sample starts (i.e., from 1982 to 1986).

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[28]To be specific, for each innovator in each country in each year, the technological distance between the new patent it obtains and its existing patent portfolio at the beginning of the year is calculated. The explanatory variable in this regression is the average technological distance across the innovators in each country in each period.

[29]The country dummies have to be dropped because the IV is country-specific and time-invariant.

TABLE 7: SPECIALIZATION CONTRIBUTES TO GROWTH

	growth of GDP per capita	
	OLS	**IV**
	(1)	(2)
tech distance	−4.548**	−5.692*
	(2.062)	(3.361)
Observations	267	185
R-squared	0.531	

Notes: Country-level regressions. The control variables are the initial GDP per capita and the Barro and Lee (2013) human capital index, the export to the U.S. and the import from the U.S. of each country, the country dummies, and the period dummies. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level. The IV is the average technological distance between new patents and existing patent portfolio of the innovators for each country in the pre-sampling period.

As demonstrated by the negative estimates in Table 7, when the innovators in a country invent in more closely related technological fields, this implies a higher level of specialization and the country tends to experience faster economic growth. Hence, innovation specialization can contribute to economic growth. In addition, the estimate suggests an economically large impact of specialization on growth. For instance, if the technological distance declines from the Japanese level to the U.S. level, the annual growth rate in Japan would have been higher by 62 basis point. Japan's GDP per capita would have been 13% higher after two decades.

1.6. Optimal Strength of Patent Rights

The optimal strength of patent rights is characterized by counterfactual analysis in this section, and it is compared with the actual patent law enforcement in practice. Through the lends of the model, the pro-patent legal reform in 1982 was welfare-enhancing, but it

was too extreme.

1.6.1. Patent Rights and Welfare

The welfare profile against the strength of patent rights is characterized in Figure 13. The horizontal axis of Figure 13 are the odds for the patent to be invalidated by the court, and the vertical axis is the consumption equivalent variation of the households. To be specific, the benchmark scenario is the economy with the optimal strength of patent rights, and the vertical axis is the fraction of consumption the households are willing to pay, if the strength of patent rights is changed from the optimal level to a suboptimal level.

In addition, two scenarios are portrayed and contrasted in Figure 13. In the first scenario, the business scope decision is an endogenous choice made by the firms, as delineated in the model. To unveil the role of the business scope decisions, the scope of business is fixed exogenously in the second scenario.[30] As shown in this figure, in both scenarios the welfare profile is hump-shaped with respect to the strength of patent rights. Part of this hump arises from this classic trade-off in the literature: stronger patent rights encourage R&D, but spur monopoly pricing by patent owners. The contribution of this paper is to introduce one additional impact of patent rights: the impact on firms' business scope strategies. In response to stronger patent rights, firms shrink their scope of business and specialize. This enhances firm performance and constitutes an additional benefit of patent rights. Hence, the optimal patent rights will be stronger when its impact on firm boundaries is taken into account. The optimal odds for patent invalidation is 42% with endogenous business scope, and it is 50% with the exogenous business scope. Hence, the optimal strength of patent rights would be weaker, if the impact of patent rights on firms' business scope decisions is ignored. In consequence, there will be a welfare loss of 6% in terms of consumption equivalent variation.

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[30]In this scenario, the business scope is fixed at the calibration level. The model is calibrated to match the U.S. economy after the legal reform in 1982, so the business scope is fixed at the level of the post-1982 economy.

FIGURE 13: OPTIMAL STRENGTH OF PATENT RIGHTS
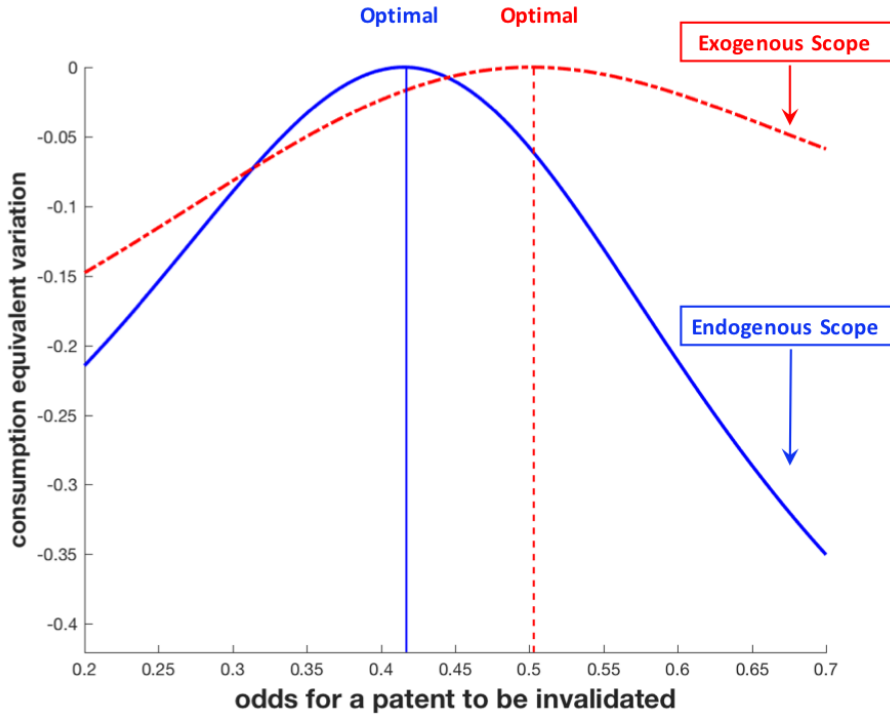
1.6.2. Legal Reform in 1982

Is the actual patent law enforcement optimal? To address this question, the optimal strength of patent rights implied by the model is contrasted with the actual law enforcement in Figure 14. As shown in Figure 14, the legal system before 1982 was characterized by a weak patent regime where the patents were invalidated in 60% of the cases. In contrast, the patent rights have been substantially strengthened after the reform in 1982. The legal reform in 1982 was definitely welfare-enhancing. This reform contributes to a welfare gain of 11.3% in terms of consumption equivalent variation. Through the lens of the model, however, this reform has been too extreme. Swinging back the legal pendulum and weakening patent rights to the optimal level will contribute to a welfare gain of 13.7% in terms of consumption equivalent variation.

FIGURE 14: OPTIMAL VS. ACTUAL STRENGTH OF PATENT RIGHTS
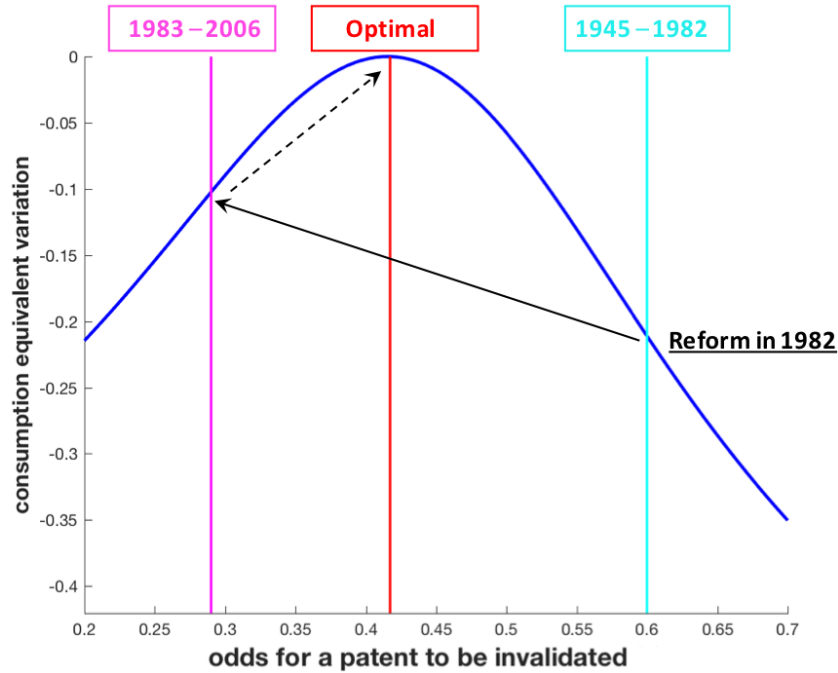
1.7. Conclusion

Stronger intellectual property rights induce specialization and contribute to economic growth. In the United States, a sweeping legal reform in 1982 created a more pro-patent legal environment with stronger patent rights. This pro-patent legal reform fostered specialization in the innovating sectors and enhanced firm performance. Around the world, countries experience faster economic growth when their innovating sectors are characterized by a higher level of specialization.

An endogenous growth model with endogenous firm boundaries is developed to disentangle the relationship between legal institutions, firm boundary decisions, and economic growth. Patent law enforcement is a crucial element of the model, and litigation concern plays a key role in the firm boundary decisions.

The model is matched with stylized facts of firm boundaries and patent litigation, and it delivers three major implications: (1) stronger patent rights induce specialization, (2)

specialization enhances firm performance, and (3) specialization contributes to economic growth. These implications are supported by the empirical analysis. When the patent rights are strengthened in a circuit court, firms located in this circuit tend to innovate in more closely related technological fields. When a firm invents in technological fields that are closer to its existing patent portfolio, it will harvest more patents from the same R&D dollar, and its patent stock will have a stronger boost to its TFP and market value. At the country level, a nation will experience faster economic growth when its innovators invent in more closely related technological fields.

Furthermore, the optimal strength of patent rights is characterized through the lens of the model. There is a classic trade-off for optimal patent rights in the literature: stronger patent rights encourage R&D, but spur monopoly pricing by patent owners. The contribution of this paper is to incorporate the impact of patent rights on firm boundaries, and by extension, the industrial-organizational structures in the innovating sectors. In response to stronger patent rights, firms shrink their scope of business and specialize. This enhances firm performance and constitutes an additional benefit of patent rights. Hence, the optimal patent rights will be stronger when its impact on firm boundaries is taken into account. In addition, the optimal strength of IPRs is contrasted with the actual patent law enforcement in practice. Through the lends of the model, the pro-patent legal reform in 1982 was welfare-enhancing, but it was too extreme. Swinging back the legal pendulum and weakening patent rights can improve welfare.

CHAPTER 2 : Financing Ventures

This chapter is co-authored with Jeremy Greenwood and Juan M. Sanchez.[1]

2.1. Introduction

> "I think the development of the venture capital system has been an example of something
> which is a successful improvement in risk-bearing. It doesn't exactly remove the risks at
> the beginning, but at least creates greater rewards at a slightly later stage and therefore
> encourages, say, small companies to engage in technologically risky enterprises. If you
> like innovation, you expect 50 percent to 60 percent failure. In a sense if you don't get
> that, you're not trying hard enough. Venture capital has done much more, I think, to
> improve efficiency than anything." Kenneth J. Arrow, 1995

The importance of venture capital in the U.S. economy has skyrocketed over the past 50 years. Investment by venture capitalists was roughly $303 million in 1970. This soared to $54 billion by 2015 (both numbers are in $2009). The rise in venture capital (VC) financing is shown in the right-hand-side panel of Figure 15. While the share of VC funding in total investment is still relatively small, around 2 percent in 2015, its punch far exceeds its weight. The fraction of public firms that have been backed at some time by venture capitalists is now around 20 percent, compared with just 4 percent in 1970–see the left-hand-side panel of Figure 15. (See the Data Appendix for the sources of all data used in the paper.) Such firms presently account for about 20 percent of market capitalization. The capitalization line lies below the fraction of firms line because VC-backed companies tend to be more recent entrants that are younger and smaller in size, whereas their non-VC-backed counterparts tend to be established incumbents. Today venture capitalists are a significant player in job creation and technological innovation. Public firms that were once backed by venture capitalists currently make up a significant fraction of employment and an even larger share of R&D spending, as opposed to virtually nothing in 1970, as the

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[1]This version of the paper is based on research in progress and hence is preliminary and incomplete.

left-hand-side panel of Figure 16 makes clear. The right-hand side of the figure displays their enormous contribution to the generation of patents, both in raw and quality-adjusted terms. The employment share of VC-backed firms is far less than the R&D (and patents) share. This is because VC-backed companies are more R&D intensive than their non-VC-backed counterparts. For instance, Google (a VC-backed company) has far fewer employees than General Motors (a non-VC-backed company), but Google invests a lot more in R&D than General Motors.

FIGURE 15: THE RISE OF VENTURE CAPITAL, 1970 TO 2015
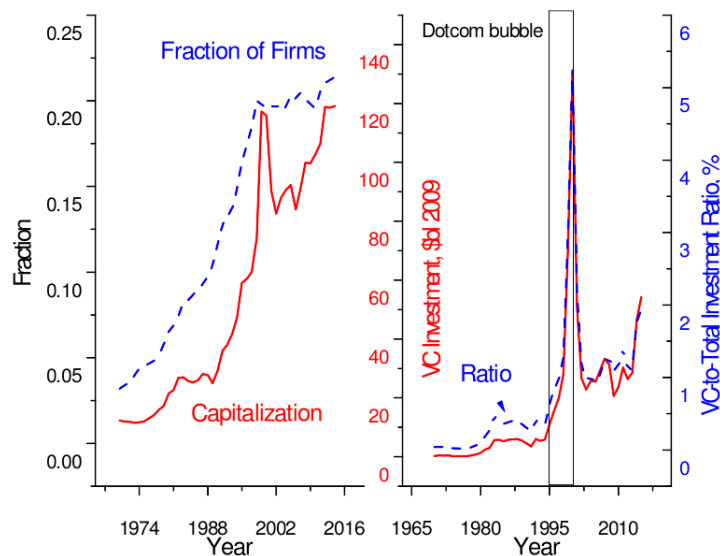
Notes: The right-hand-side panel shows investment by venture capitalists. The left-hand-side panel plots both the fraction of public firms financed by venture capitalists and the share of VC-backed public firms in market capitalization.

The VC industry has been an incubator of numerous technological giants in the information and communication technology sector as well as the biotechnology sector, plus an array of star innovators in the service industry. Former VC-backed firms are household names. Table 8 lists the top 30 VC-backed public companies by market capitalization. Figure 17 plots the relative significance of the words "banks" and "venture capital," as reflected by their usage in English language books. As shown, the term venture capital was virtually unused in 1930. The relative significance of venture capital vis-à-vis banks has increased considerably

since then.

FIGURE 16: THE SHARE OF VC-BACKED FIRMS IN EMPLOYMENT, R&D SPENDING, AND PATENTS
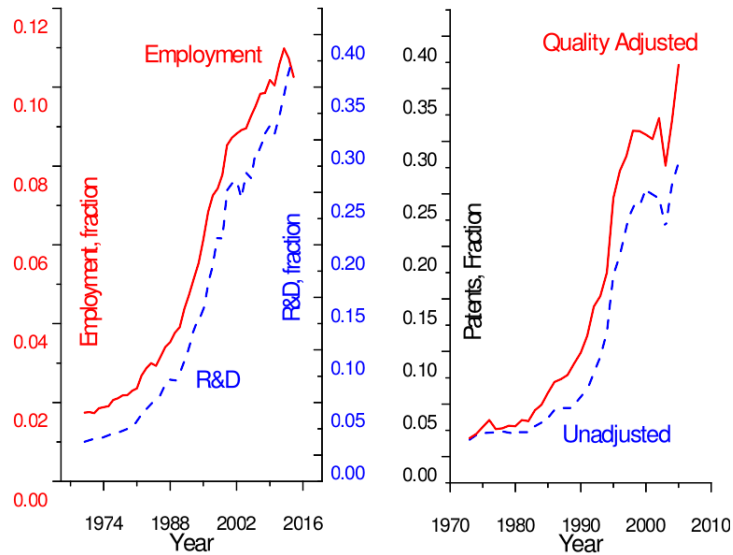
Notes: The data in the left-hand-side panel is from 1970 to 2014, while that in the right-hand-side panel spans 1973 to 2005.

TABLE 8: TOP 30 VC-BACKED COMPANIES

1	Apple Inc	11	Amgen Inc.	21	Fedex Corp.
2	Cisco Systems Inc.	12	Yahoo Inc.	22	Juniper Networks Inc.
3	Microsoft Corp.	13	Genentech Inc.	23	Nextel Communications Inc.
4	Alphabet Inc.	14	Celgene Corp.	24	Gap Inc.
5	Facebook Inc.	15	Ebay Inc.	25	Viacom Inc.
6	Oracle Corp.	16	Compaq Computer Corp.	26	Veritas Software Corp.
7	Amazon.Com Inc.	17	Starbucks Corp.	27	Salesforce.Com Inc.
8	Sun Microsystems Inc.	18	Micron Technology Inc.	28	Alexion Pharmaceuticals Inc.
9	Gilead Sciences Inc.	19	Applied Materials Inc.	29	Adobe Systems Inc.
10	Dell Inc.	20	Regeneron Pharmaceuticals	30	Twitter Inc.

Notes: The table shows the top 30 VC-backed companies by market capitalizaton. These companies are identified by matching firm names in VentureXpert and CompuStat.
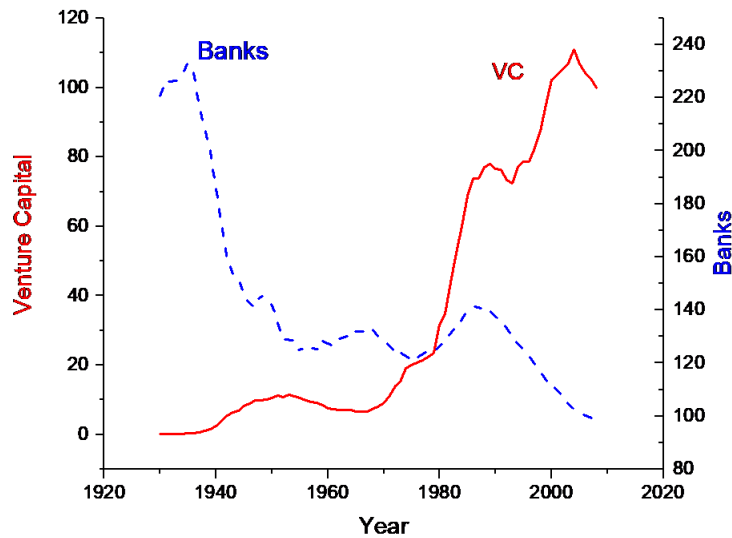
Notes: The figure plots the use of the words "banks" and "venture capital," relative to all words in English language books, using the Google Ngram Viewer. For each series, the value in 2008 is normalized to 100.

To address the importance of VC in the U.S. economy, an endogenous growth model is developed. At the heart of the growth model is a dynamic contract between an entrepreneur and a venture capitalist. The venture capitalist invests in the entrepreneur's startup as an active participant. The venture capitalist provides seed money for initial research. The project then enters a funding-round cycle. At the beginning of each cycle the venture capitalist evaluates the worthiness of the project. Those projects that pass the evaluation are given funds for development. The contract is designed so that it is not in the entrepreneur's interest to divert funds away from their intended purpose. The venture capitalist can imperfectly monitor at a cost the entrepreneur's use of funds, which helps to ensure incentive compatibility. Those ventures that are successful during a fund round are floated on the stock market. The contract specifies for each funding round the evaluation strategy to gauge the project's worthiness, the amount of VC invested in development, the level of

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monitoring to avoid malfeasance, and the shares of each party in the proceeds from a potential IPO. The predicted features of the contract are compared with some stylized facts about venture capital: (i) the success and failure rates by funding round, (ii) investment by funding round, (iii) the value of an IPO by duration of the project, and (iv) the venture capitalist's share of equity by funding round. Despite the importance of VC, the majority of U.S. firms are not financed through this channel. So, the analysis includes a traditional sector that produces the majority of output using capital that can be thought of as financed through regular banks. The key participants in a VC partnership receive the majority of their compensation in the form of stock options and convertible equity. As such, they are subject primarily to capital gains taxation. The analysis examines how innovative activity is affected by the capital gains tax rate.

Dynamic contract models have now been used for some time to study consumption/savings cum effort decisions with moral hazard. An early example is Phelan and Townsend (1991), with more recent work being represented by Karaivanov and Townsend (2014). Dynamic contract frameworks that focus on firms, and VC in particular, are rarer. Bergemann and Hege (1998), Clementi and Hopenhayn (2006), and Cole, Greenwood, and Sanchez (2016) develop contracting structures that share some similarities with the one presented here. In Bergemann and Hege (1998) a venture capitalist also learns about a project's type, good or bad, over time. The odds of a good project's success are a linear function of investment. The entrepreneur can secrete some of the funds intended for investment, so there is a moral hazard problem. Given the linear structure of their model, which generates corner solutions, analytical results obtain. In an extension, the venture capitalist can monitor investment or not. If he monitors, then any irregularities are uncovered with certainty. The analysis is done in partial equilibrium. While illuminating some economics about VC, it would be hard to take their streamlined structure to the data. While not focusing on VC, the Clementi and Hopenhayn (2006) model also reformulates as one where an entrepreneur can secrete investment. The lender cannot monitor the borrower. Again, the analysis is done in partial equilibrium.

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The current paper borrows Cole, Greenwood, and Sanchez's (2016) flexible-monitoring technology. The more the venture capitalist invests in auditing, the higher the odds that he will detect any irregularities. The venture capitalist can also invest in evaluating a project in each funding round to learn about its type, good or bad–something not allowed in Bergemann and Hege (1998). This feature is important because it allows the odds that a project is good to rise over funding rounds. This works to generate an upward-sloping investment profile by funding round. The odds of a good project's success are an increasing, concave function of investment in development. Additionally, VC is taken to be a competitive industry; this is similar to Cole, Greenwood, and Sanchez's (2016) assumption that financial intermediation, more generally, is competitive.

The current analysis is done within the context of an endogenous growth model. Cole, Greenwood, and Sanchez (2016) focus on the impact that financial intermediation, more broadly defined, has on cross-country technological adoption and income levels. As in Akcigit, Celik, and Greenwood (2016), the current work has a distribution of competitive firms operating in general equilibrium. This distribution is continually shifting rightward with technological progress in the economy. A new entrepreneur decides how far to push his productivity relative to the frontier; this is somewhat reminiscent of Parente (1994). The position of the frontier is determined by a classic Romer (1986) type externality. The last three papers noted have no startups. None of the above papers compares the predictions of their models with the VC process in the United States. And none of them examines how innovative activity is affected by the rate of capital gains taxation.

There is, of course, work on VC that does not take a dynamic contract perspective. Silveira and Wright (2016) build a canonical search model of the process where entrepreneurs are matched with venture capitalists, something abstracted from here. Upon meeting, the parties bargain in Nash fashion over each one's investment and how to split the proceeds. Jovanovic and Szentes (2013) focus on a setting where the incubation period for a project is unknown. Unlike entrepreneurs, venture capitalists have deep pockets and can weather

supporting a project over a prolonged period of time, if they so choose. A contract specifies the initial investment by the venture capitalist and some fixed split of the profits. The analysis focuses on characterizing and measuring the excess return earned by venture capitalists, due to the latters' scarcity. A tractable stylized Schumpeterian model of VC that has analytical solutions is developed by Opp (2018). He estimates that the welfare benefits of VC are worth 1 to 2 percent of aggregate consumption, despite the fact that VC investment is highly procyclical, which operates to trim the estimates. In his analysis, entrepreneurs do not choose how far to launch their endeavour ahead of the pack. Also, the likelihood of success does not depend on the level of development funding. Since the innovation process is essentially static, there is no investment over time in learning about the project's quality. Given the static nature of the R&D investment, he does not model the stage financing process; i.e., the success rates, failure rates, investment rates, equity shares, and values of an IPO by funding round.

2.2. The Rise of Venture Capital as Limited Partnerships

Financing cutting-edge technologies has always been problematic.[2] It is difficult to know whether new ideas are viable, if they will be saleable, and how best to bring them to market. Also, it is important to ensure that entrepreneurs' and investors' incentives are aligned. Traditional financial institutions, such as banks and equity/securities markets, are not well suited to engage in this sort of finance. Historically, the introduction of new technologies was privately financed by wealthy individuals. Investors were plugged into networks of inventive activity in which they learned about new ideas, vetted them, and drew on the expertise needed to operationalize them. These financiers could be considered similar to today's "angel investors."

The Brush Electric Company provided such a network for inventors and investors in Cleveland around the turn of the 20th century. Electricity was one of the inventions born during

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[2]This section draws *heavily* on Lamoreaux, Levenstein, and Sokoloff (2007) for the period prior to World War II and on Kenney (2011) for the period after.

the Second Industrial Revolution. Individuals linked with the Brush Electric Company network spawned ideas for arc lighting, liquefying air, smelting ores electrically, and electric cars and trolleys, among other things. The shops at Brush were a meeting place for inventors; they could develop and debug new ideas with help from others. Investors connected with the Brush network learned about promising new ideas from the scuttlebutt at the shops. They became partners/owners in the firms that they financed. Interestingly, in the Midest at the time, prolific inventors (those with more than 15 patents) who were principals in companies were much more likely to keep their patents or assign them to the companies where they were principals as opposed to other inventors, who typically sold them to businesses where they had no concern. This aligned the incentives of innovators and investors.

World War II and the start of the Cold War ushered in new technologies, such as jets, nuclear weapons, radars, and rockets. There was a splurge of spending by the Defense Department. A handful of VC firms were formed to exploit the commercialization of scientific advances. American Research and Development (ARD), founded by General Georges Doriot and others, was one of these. ARD pulled in money from mutual funds, insurance companies, and an initial public stock offering. The founders knew that it was important for venture capitalists to provide advice to the fledging enterprises in which they were investing. In 1956 ARD invested $70,000 in Digital Equipment Corporation (DEC) in exchange for a 70 percent equity stake. ARD's share was worth $38.5 million when DEC went public in 1966, which represented an annual return of 100 percent. While this investment was incredibly successful, the organizational form of ARD did not come to dominate the industry. The compensation structure of ARD made it difficult for the company to retain the VC professionals needed to evaluate startups and provide the guidance necessary for success.

An alternative organizational form came to emblemize the industry; viz., the limited partnership. This form is exemplified by the formation of Davis and Rock in 1961. These partnerships allowed VC professionals to share in the gains from startups along with the

entrepreneurs and investors. Limited partnerships served to align venture capitalists' interests with those of entrepreneurs, investors, and key employees. Money was put in only at the beginning of the partnership. The general partners received management fees as a salary plus a share of the capital gains from the investments, say 40 percent, with the limited partners earning 60 percent. The limited partners had no say in the decisions of the general partners. The partnerships were structured for a limited length of time, say 7 to 10 years. The returns from the partnership were paid out to the investors only when the partnership was dissolved–there were no dividends, interest payments, etc. Therefore, the returns upon dissolution were subject only to capital gains taxation at the investor level. The VC industry also rewarded founders, CEOs, and key employees using stock options. Thus, they too were subject to capital gains taxation and not taxation on labor income. The short time horizon created pressure to ensure a venture's rapid success.

Banks and other financial institutions are not well suited to invest in cutting-edge new ventures. While banks are good at evaluating systematic lending risk, they have limited ability to judge the skill of entrepreneurs, the worth of new technologies, and the expertise to help commercialize them. The Glass-Steagll Banking Act of 1933 prohibited banks from taking equity positions in industrial firms–the act was repealed in 1999. Allstate Insurance Company created a private placements program in the 1960s to undertake VC type investments. It abandoned the program because it could not compensate the VC professionals enough to retain them. The Employee Retirement Income Security Act of 1974 prevented pension funds (and dissuaded other traditional fiduciaries) from investing in high-risk ventures. The act was reinterpreted in the 1980s to allow pension funds to invest in VC operating companies, which provided a fillip for the VC industry.

2.3. Empirical Evidence on Venture Capital and Firm Performance

How does VC affect firm growth and technological innovation? The VC industry is a successful incubator of high-tech and high-growth companies. VC-backed public companies have higher R&D-to-sales ratios than their non-VC-backed counterparts. Following an

IPO, they also grow faster in terms of employment and sales. VC-backed companies are embraced as "golden geese" by the investors. They are valued higher than their non-VC-backed counterparts around the time of an IPO. In addition, VC is a potent apparatus for financing technological innovation. VC funding is positively associated with patenting activity by firms. Moreover, patenting depends more on VC funding in those industries where the dependence on external financing is high.

2.3.1. Venture Capital and Firm Growth

Regression analysis is now conducted to evaluate the performance of VC-backed and non-VC-backed firms along four dimensions for the year after an IPO: the R&D-to-sales ratio, the growth rate of employment, the growth of sales revenue, and the market value of firms. The results are presented in Table 9. The regressions are based on an unbalanced panel of U.S. public companies between 1970 and 2014. To compare VC-backed companies with their non-VC-backed counterparts, a VC dummy is entered as an independent variable that takes the value of 1 if the company is funded by VC before its IPO. In all regressions, industry dummies, year dummies, and a year dummy for the IPO are included. In addition, a cross term is added between the VC dummy and the number of years since the firm's IPO.

As shown by the first row of regression coefficients, VC-backed companies are more R&D intensive and grow faster than their non-VC-backed counterparts. On average the R&D-to-sales ratio of a public VC-backed company is higher than its non-VC-backed counterpart by 5.2 percentage points, and it grows faster–by 4.9 percentage points in terms of employment and 7.0 percentage points in terms of sales revenue. These superior performances translate into higher market values: VC-backed companies are valued 37.3 percent higher than their non-VC-backed counterparts. The difference in performance, however, gradually dwindles over the years, as shown by the negative signs of the regression coefficients in the second row. As a consequence, the performances of VC- and non-VC-backed public companies tend to converge in the long run, though the speed of convergence is fairly low, as revealed by the magnitude of the regression coefficients in the second row.

TABLE 9: VC- VERSUS NON-VC-BACKED PUBLIC COMPANIES

Dependent variable	R&D / Sales	Employment growth	Sales growth	ln(Firm value)
VC (= 1, if backed by VC)	0.0521***	0.0490***	0.0696***	0.373***
	(0.00169)	(0.00206)	(0.00270)	(0.0141)
VC ×years since IPO	-0.000780***	-0.00304***	-0.00406***	-0.0110***
	(0.000132)	(0.000165)	(0.000215)	(0.00110)
ln(employment)	-0.0133***	-0.00567***	-0.00641***	0.851***
	(0.000248)	(0.000254)	(0.000335)	(0.00170)
Observations	84,116	148,834	149,672	168,549
R-squared	0.383	0.084	0.108	0.737

Notes: All specifications include year dummies, industry dummies (at the 4-digit SIC), and a year dummy for the IPO. Standard errors are in parentheses and significance at the 1 percent level is denoted by ***.

2.3.2. Venture Capital and Innovation

Regression analysis now assesses the role of VC in encouraging technological innovation; specifically, the impact of VC funding on patent performance at an annual periodicity is evaluated, both at the firm and industry levels. The regression analysis is based on all companies funded by venture capitalists between 1970 and 2015. These VC-funded patentees are identified by matching firm names in VentureXpert and PatentsView.

Firm-Level Regressions. In the firm-level regression analysis, the primary independent variable is (the natural logarithm of) annual VC funding, while the dependent variable is a measure of patenting performance, both in the year and the year after the firm receives the funding. The primary independent variable may suffer from both measurement error and selection issues. So, in some of the regressions, two instrumental variables are used. The first instrumental variable (IV) is the (maximum) rate of capital gains taxation in the state where the VC-funded company is located. The second IV is a Rajan and Zingales (1998) type measure of the dependence on external finance of the industry in which the firm

operates. This measure reflects the extent to which outside funds are used in the industry for expenditures on property, plant and equipment, R&D, advertising, and employee training. Both of these datums are exogenous at the level of a startup. In all of the regressions, controls are added for the number of patents held by the firm at the beginning of the year, the age of the firm, and the total amount of private and federally funded R&D of the industry in which the firm operates. Additionally, both a year and industry dummy are entered. Last, since both innovation and VC activities are remarkably clustered in California or Massachusetts, a "cluster dummy" for a firm headquartered in California and Massachusetts is included.

The results of the regression analysis are reported in Table 10. Panel A of Table 10 conducts the analysis along the extensive margin analysis; i.e., it examines whether the firm obtains any patents after receiving VC funding. In regressions (1) and (2), the dependent variable is a dummy that takes the value of 1 if the firm files any successful patent applications at the U.S. Patents and Trademark Office (USPTO) within one year after it receives funding. Regressions (3) and (4) focus on the "breakthrough" patents, a measure pioneered by Kerr (2010). Breakthrough patents refers to those in the right tail of the citation distribution. Here the dependent variable in regressions (3) and (4) is a dummy variable that takes the value of 1 if the firm files any patents in the top 10 percent of the citation distribution in its cohort (i.e., those patents with the same technological class and same application year). Panel B of Table 10 turns to the intensive margin. In regressions (5) and (6) the dependent variable is the natural logarithm of the number of patents. The natural logarithm of the number of patents is weighted by citations in regressions (7) and (8).

As shown by the positive regression coefficients of VC funding in Panel A, a firm is more likely to file a patent and come up with a breakthrough patent the larger is the funding from a VC, although the impact of VC funding is somewhat smaller in spurring breakthrough patents than ordinary patents. According to the IV estimates in regressions (6) and (8), a 10 percent increase in VC funding will induce a 3.6 percent boost in patenting one year after

funding, and this number goes up to 6.7 percent when the number of patents is adjusted for quality. In addition, across all the regressions in Table 10, the estimates are consistently higher in the IV regressions.

TABLE 10: VC FUNDING AND PATENTING: FIRM-LEVEL REGRESSIONS

Panel A: Extensive Margin Analysis				
Dependent variable	1{*Patent > 0*}		1{*"Breakthrough patent" > 0*}	
	Probit	IV	Probit	IV
	(1)	(2)	(3)	(4)
ln(VC funding)	0.141***	0.682***	0.133***	0.635***
	(0.0108)	(0.0590)	(0.0112)	(0.0979)
Observations	9,166	8,132	9,149	8,122
Panel B: Intensive Margin Analysis				
Dependent variable	ln*(Patent)*		ln*(Patent, quality adj)*	
	OLS	IV	OLS	IV
	(5)	(6)	(7)	(8)
ln(VC funding)	0.115***	0.363*	0.155***	0.674*
	(0.00907)	(0.187)	(0.0164)	(0.356)
Observations	5,828	5,207	5,032	4,519
R-squared	0.244		0.123	

Notes: See the main text for a description of the dependent and independent variables. Standard errors are in parentheses. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

Industry-Level Regressions. The above firm-level regressions are now recast at the 4-digit SIC industry level. The main explanatory variable is the (natural logarithm of the) aggregate amount of VC investment across all industries between 1970 and 2015. The dependent variable is the (natural logarithm of the) number of patents filed by all VC-backed companies in the industry one year after they receive VC funding. To capture the hetero-

geneous dependence on external finance across industries, a cross term is added between aggregate VC funding and the industry's dependence on external finance. This specification emulates Rajan and Zingales (1998) in the sense that they exploit the variation of financial development across countries, whereas the current analysis taps into fluctuations of aggregate VC investment across time. As in the firm-level regressions, the main independent variable may suffer from both measurement error and selection issues. An instrumental variable is used to address this. The IV follows Kortum and Lerner (2000) and is based on the deregulation of pension funds in 1979, as highlighted in Section 2.2. To be specific, a "deregulation dummy," which takes the value of 1 after 1979, is used as an instrumental variable. In all of the industry-level regressions, controls are added for the total amounts of private R&D and federally funded R&D in the industry. A 2-digit industry dummy variable is also included. Since the deregulation dummy is used as an IV, year dummies cannot be used, so common shocks to all industries are controlled for by adding NBER recession dummies as a proxy for the business cycle and the federal funds rate as a proxy for the tightness of monetary policy.

The industry-level regressions are presented in Table 11. As can be seen from the first row of the regression coefficients, the positive signs on aggregate VC funding complement the findings at the firm level. VC investment contributes positively to patenting performance at the industry level. According to the IV estimate in column 2, at the median level of financial dependence across industries, a 10 percent increase in aggregate VC funding will induce a 1.51 percent boost in industry-level patenting within a year. This elasticity is 0.194 in the prepackaged software industry, which accounted for 23 percent of VC investment. In addition, the impact of VC is heterogeneous across industries, as revealed by the cross term between VC funding and the dependence on external finance (see the second row). Since the regression coefficients on the cross terms turn out to be positive, the impact of the fluctuations in aggregate VC investment is more pronounced the higher is the industry's dependence on external finance. For industries in the top quartile of financial dependence,

the elasticity is 0.339, versus 0.111 in the bottom quartile.[3] As complementary evidence on the cyclicality of VC activities, Khan and Petratos (2016) document that VC entry (the number of startups) and exit (the number of IPOs and M&As) are nearly three and five times, respectively, as volatile as business fixed investment.

TABLE 11: VC FUNDING AND PATENTING: INDUSTRY-LEVEL REGRESSIONS

Dependent variable	ln(Patent)		ln(Patent, quality adj)	
	OLS	IV	OLS	IV
ln(agg VC funding)	0.200***	0.151***	0.129***	0.115*
	(0.0381)	(0.0569)	(0.0454)	(0.0681)
ln(agg VC funding) × ind financial dependence	0.1854***	0.1852***	0.192***	0.191***
	(0.00965)	(0.00976)	(0.0117)	(0.0118)
Observations	1,971	1,971	1,890	1,890
R-squared	0.378		0.362	

Notes: See the main text for a description of the dependent and independent variables. Standard errors are in parentheses. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

2.4. The Setting

At center of the analysis is the interplay between an entrepreneur and a venture capitalist, which is governed by an incentive-compatible financial contract. Entrepreneurs have ideas, but no money, while venture capitalists have money, but no ideas. Each period new entrepreneurs bring ideas of their choosing to a venture capitalist to obtain funding. The parties sign a partnership agreement that has finite duration. Note that most VC entreprises are operated as partnerships. The share of corporate venture programs in total U.S. VC investment is low, averaging just 9 percent between 1995 and 2015. Also, corporate VC faces many of same challenges as VC partnerships; viz., the uncertainty about a project's

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[3]To be conservative, the number for the upper quartile excludes an unrealistic high elasticity for the insurance carrier industry, where there are only two VC-funded firms.

quality and the moral hazard problem connected with lending.

At the time the contract is signed, the venture capitalist provides seed money to research initially the idea. After this initial research is finished, the project enters a funding-round cycle that may last for many periods. Some ideas brought by entrepreneurs to the venture capitalist are good, others are bad. Only a good idea has a payoff, and even then this might not happen. Neither party knows whether an idea is good or bad. So, at the beginning of each funding round the venture capitalist evaluates the project at a cost in an attempt to detect whether the venture is bad. Bad projects are terminated. Projects that aren't known to be bad are given development money. The probability of success within a funding round is an increasing function of the level of investment in development undertaken by the entrepreneur. How much of the money the entrepreneur actually uses for development is private information. The venture capitalist can imperfectly monitor development investment at a cost in an attempt to detect any malfeasance. When malfeasance is detected, the venture capitalist drops the venture. If successful, the project will be floated on the stock market or sold to another firm, which yields a reward that will be a function of the idea's type. The reward is split between the entrepreneur and venture capitalist as specified by the partership agreement. Any profits from floating a VC-funded enterprise are subject to capital gains taxation. All revenue from capital gains taxation is rebated back to the populace in lump-sum transfer payments. If the project is not successful, then it enters another funding round, provided the contract has not expired, and the funding cycle goes on. The timing of events within a generic funding round is shown in Figure 18.
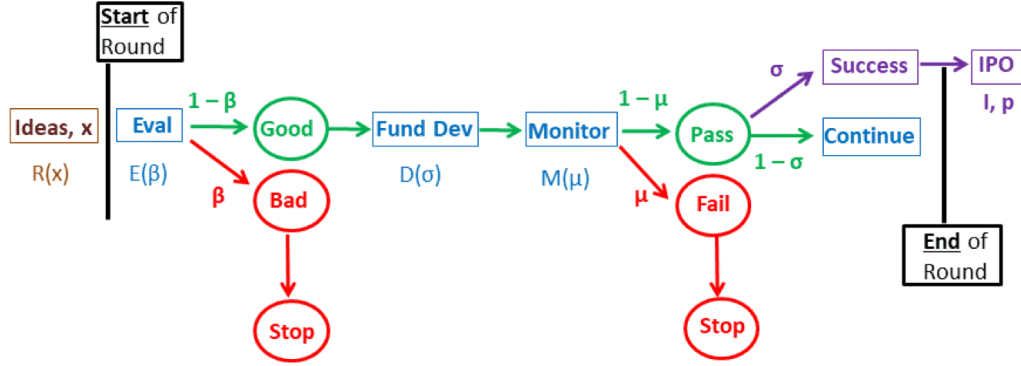
Notes: The research underlying the idea occurs at the very beginning of the funding cycle (round zero) and is shown to the left of generic funding round.

The analysis focuses on a balanced-growth path. The aggregate level of productivity in a period is denoted by $\mathbf{x}$, which represents the aggregate state of the economy. Along a balanced-growth path, $\mathbf{x}$ will grow at the gross rate $\mathbf{g_x} > 1$ so that

$$\mathbf{x}' = \mathbf{g_x}\mathbf{x}.$$

The gross growth rate of aggregate productivity, $\mathbf{g_x}$, is an endogenous variable in equilibrium. It will be a function of the efficiency of the VC system. The discussion now proceeds by detailing the stages portrayed in Figure 18.

2.4.1. The Research Stage–Starting a New Venture

Each period a flood of new entrepreneurs in the amount $\mathfrak{e}$ approach venture capitalists to obtain funding for their ideas. An entrepreneur incurs an opportunity cost in the amount $w\mathfrak{o}$ to run a project, where w is the wage rate for labor. The component $\mathfrak{o}$ of this cost is distributed across potential entrepreneurs according to the non-normalized distribution function, $O(\mathfrak{o})$. This distribution function $O(\mathfrak{o})$ is assumed to be Pareto so that

$$O(\mathfrak{o}) = 1 - (v/\mathfrak{o})^{\nu}, \ \text{with } \nu, v > 0. \tag{2.1}$$

Only those potential entrepreneurs who expect the payoff from a startup to exceed their opportunity cost, $w\mathfrak{o}$, will approach a venture capitalist for funding. This criterion will determine the number of funded entrepreneurs, $\mathfrak{e}$.

A new entrepreneur is free to choose the type of startup, x, that he wants to develop. In particular, when deciding on the project, the entrepreneur picks x subject to a research cost function of the form

$$i = R(\frac{x}{\mathbf{x}}, \mathfrak{e}) = w(\frac{x}{\mathbf{x}})^{\iota}\mathfrak{e}^{-\xi}/\chi_R,$$

where $i \geq 0$ is the initial investment in researching the project. The entrepreneur can choose how far ahead the productivity of his firm, x, is from the average level of productivity in the economy, $\mathbf{x}$. The more ambitious he is, or the higher x is relative to $\mathbf{x}$, the greater will be the research cost, which rises in convex fashion. The cost of research, $R(x/\mathbf{x}, \mathfrak{e})$, rises with the level of wages, w, which will be a function of the aggregate level of productivity, $\mathbf{x}$. (Think about $R(x/\mathbf{x}, \mathfrak{e})/w$ as representing the cost in terms of labor.) This structure provides a mechanism for endogenous growth in the model. The cost of researching the project is decreasing in the number of startups, $\mathfrak{e}$. The more new entrepreneurs there are pushing the frontier forward, the easier it will be for any particular entrepreneur to research his project due to spillover effects.

2.4.2. The Evaluation Stage

Out of the pool of *new* entrepreneurs, the fraction ρ will have good ideas, implying that the fraction $1 - \rho$ have bad ones. The venture capitalist can potentially discover a bad project by evaluating it. Assume that the venture capitalist can detect within each funding round a bad project with probability β, according to the cost function, $E(\beta; \mathbf{x})$, where E is an increasing, convex function in β. Specifically,

$$E(\beta; \mathbf{x}) = w(\frac{1}{1 - \beta} - 1)\beta/\chi_E.$$

The productivity of the evaluation process is governed by χ_E. Note that the marginal cost of evaluating starts at zero when $\beta = 0$ and goes to infinity as β approaches 1. The cost of evaluating rises with the level of wages, w. Think about χ_E as capturing the efficiency of investment in evaluation. Projects that are detected to be bad are thrown out.

2.4.3. The Development Stage

Ventures that pass the evaluation stage are given development funding. The level of funding depends upon the common prior (held by the entrepreneur and venture capitalist) that the project is good, which evolves across funding rounds. The odds of success during a funding round depend on the entrepreneur's investment in development. In particular, a probability of success, σ, can be secured by undertaking development investment of the amount $D(\sigma; \mathbf{x})$, where D is an increasing, convex function in σ. The development cost function $D(\sigma; \mathbf{x})$ is given the form

$$D(\sigma; \mathbf{x}) = w(\frac{1}{1 - \sigma} - 1)\sigma/\chi_D.$$

The development cost function $D(\sigma; \mathbf{x})$ has a similar form to that for $E(\beta; \mathbf{x})$.

There is also a fixed cost, ϕ_t, connected with developing a startup project in round t. This fixed cost rises with the level of wages in the economy. In particular,

$$\phi_t = w\mathbf{g}_w^{t-1}\phi(t),$$

where $\mathbf{g}_w > 1$ is the gross growth rate in wages (which will be a function of $\mathbf{g_x}$). Additionally, the fixed cost changes by the round of the project, as reflected by the function $\phi(t)$. The shape of the function $\phi(t)$ will be parameterized using a polynomial that is pinned down from the U.S. VC funding-round data.

2.4.4. The Monitoring Stage

The venture capitalist provides in a funding round the amount $D(\sigma; \mathbf{x})$ for development. The entrepreneur may decide to spend some smaller amount $D(\widetilde{\sigma}; \mathbf{x}) \leq D(\sigma; \mathbf{x})$ and siphon

off the difference, $D(\sigma; \mathbf{x}) - D(\widetilde{\sigma}; \mathbf{x})$. The entrepreneur uses the difference in funds for his own consumption. By diverting funds the entrepreneur reduces the odds of success in the current funding round; i.e., $\widetilde{\sigma} \leq \sigma$. The venture capitalist can dissuade this fraud by engaging in monitoring. Assume that the venture capitalist can pick the odds μ of detecting fraud in a venture during round t according to the strictly increasing, convex cost function, $M_t(\mu; \mathbf{x})$, where

$$M_t(\mu; \mathbf{x}) = w\mathbf{g}_w^{t-1}(\frac{1}{1 - \mu} - 1)\mu/\chi_{M,t}.$$

The cost of monitoring rises with wages in the economy. Additionally, monitoring costs change by the round of the project, as reflected by the term $\chi_{M,t}$; again, $\chi_{M,t}$ represents the productivity of this auditing process in round t. Presumably, as the venture capitalist becomes more familiar with the project, $\chi_{M,t}$ will rise with t. This feature implies that the incentive problem will become less severe over time and helps to generate an upward-sloping funding profile. A polynomial for $\chi_{M,t}$ will be fit to the U.S. VC funding-round data. While motivated by the prototypical costly-state-verification paradigms of Townsend (1979) and Williamson (1986), the monitoring technology employed here is different. In those frameworks, getting monitored is a random variable–in Williamson (1986) only those entrepreneurs declaring a bad outcome are monitored, while in Townsend (1979) some fraction of such entrepreneurs are. The audit will detect any fraud with certainty. By contrast, here everybody gets monitored, but the detection of any fraud is a probabilistic event.

2.4.5. The Success Stage–Floated Firms

A startup of type x turns into a going concern with productivity x, if successful. A successful VC-backed firm produces output, o, according to the production process

$$o = x^\zeta k^\kappa l^\lambda, \text{ with } \zeta + \kappa + \lambda = 1, \tag{2.2}$$

where k and l are the amounts of capital and labor used in production. This structure is borrowed from Akcigit, Celik, and Greenwood (2016). It results in the firm earning pure profits that are linear in its productivity, x. The lure of capturing these profits is what motivates entrepreneurs and venture capitalists. Labor is hired at the wage rate, w, and capital at the rental rate, r. The firm's per period net takings are

$$
\begin{aligned}
T(x; \mathbf{x}) &= \max_{k,l} \{ x^\zeta k^\kappa l^\lambda - rk - wl \} \\
&= x(1 - \kappa - \lambda) [(\frac{\kappa}{r})^\kappa (\frac{\lambda}{w})^\lambda]^{1/\zeta}.
\end{aligned}
\tag{P1}
$$

Clearly, as wages rise, which will be a function of the aggregate level of productivity, $\mathbf{x}$, net takings will shrink for a given level of the firm's productivity, x. Operating firms last stochastically with the time-invariant survival rate, $\mathfrak{s}$.

A successful VC-backed project is sold for $I(x; \mathbf{x})$, either through an IPO or an M&A, just before production starts. The (gross) reward for a successful IPO is

$$
I(x; \mathbf{x}) = \sum_{t=1}^{\infty} (\mathfrak{s}\delta)^{t-1} T(x; \mathbf{g}_\mathbf{x}^{t-1} \mathbf{x}),
\tag{2.3}
$$

where δ is the market discount factor. If the startup is successful, the entrepreneur must pay the venture capitalist the amount p. So the entrepreneur will reap the amount $I(x; \mathbf{x}) - p$, which is taxed at the capital gains rate, τ. If a project is not successful, it moves back to the evaluation stage, assuming that the contract has not expired.

2.5. The Financial Contract

The financial contract between the entrepreneur and the venture capitalist is cast now. VC is a competitive industry so the entrepreneur shops around to secure the financial contract with the best terms. Venture capitalists covers the cost of research, evaluation, development, and monitoring. They raise the money to do this from savers, to whom they promise a gross rate of return of $1/\delta$. There are no profits on VC activity in equilibrium. The profits that

accrue to the entrepreneur are subject to the rate of capital gains taxation, τ. The analysis presumes that there is a maximum of T rounds of potential funding. The timing of events for the contract is shown in Figure 18. The research for the idea is done at the start of the funding-round cycle or in round zero. At the beginning of a generic funding round, the venture capitalist evaluates projects and purges the ones that are found to be bad. Goods projects are then given an injection of cash for development. The venture capitalist monitors the use of these funds. If malfeasance is detected, the project is terminated. Some projects will be successful. These are floated in the next period on the stock market. The unsuccessful projects then start another funding round (assuming the number of funding rounds doesn't exceed T).

Let β_t represent the odds of detecting a bad project in round t and σ_t denote the probability of success for a good project. Now suppose that a unit measure of new entrepreneurs approach a venture capitalist for funding. As the funding rounds progress, the numbers of good and bad projects will evolve as shown in Table 12. For example, of the people initially applying for funding, the number ρ will have good projects and $1 - \rho$ will have bad ones. The venture capitalist will evaluate the applicants and eliminate $(1 - \rho)\beta_1$ bad projects, so that $(1 - \rho)(1 - \beta_1)$ bad ones will still remain. Of the good projects, the number $\rho\sigma_1$ will be successful. So, at the beginning of the second round there will be $\rho(1 - \sigma_1)$ good projects in the pool. After the second-round evaluation, $(1 - \rho)(1 - \beta_1)(1 - \beta_2)$ bad projects will still be around. Table 12 elaborates how the number of good and bad projects evolves over funding rounds.

Round	Number good	Number bad
1	ρ	$(1-\rho)(1-\beta_1)$
2	$\rho(1-\sigma_1)$	$(1-\rho)(1-\beta_1)(1-\beta_2)$
3	$\rho(1-\sigma_1)(1-\sigma_2)$	$(1-\rho)(1-\beta_1)(1-\beta_2)(1-\beta_3)$
$\vdots$	$\vdots$	$\vdots$
t	$\rho\Pi_{j=1}^{t-1}(1-\sigma_j)$	$(1-\rho)\Pi_{j=1}^{t}(1-\beta_j)$

Notes: The table shows how the number of good and bad projects change across funding rounds assuming that the venture capitalist starts with a unit mass of ventures.

The odds of a project being good in round t are

$$\Pr(\text{Good}|\text{Round} = t) = \frac{\rho\Pi_{j=1}^{t-1}(1-\sigma_j)}{\rho\Pi_{j=1}^{t-1}(1-\sigma_j) + (1-\rho)\Pi_{j=1}^{t}(1-\beta_j)}. \tag{2.4}$$

As time goes by, more and more bad projects are purged from the pool. The number of goods projects will also fall due to the successes. Thus, the odds of being good can rise or fall with the funding round, depending on which type of projects are exiting the pool the fastest, at least theoretically. If the odds of being good in the current round are $\rho\Pi_{j=1}^{t-1}(1-\sigma_j)$, then the odds of being good and still being around in the next round are $\rho\Pi_{j=1}^{t-1}(1-\sigma_j) \times (1-\sigma_t)$. The odds of being good and still being around $t+i$ rounds ahead are $\rho\Pi_{j=1}^{t-1}(1-\sigma_j) \times \Pi_{j=t}^{t+i-1}(1-\sigma_j)$.

The contract between the entrepreneur and the venture capitalist will specify for the length of the relationship: (i) the precision of evaluation, as given by the β_t's; (ii) the investments in development as reflected by the σ_t's; (iii) the exactness of monitoring as measured by the μ_t's; and (iv) the payments that an entrepreneur who finds success in round t must make to the intermediary, or the p_t's. The contract is summarized by the outcome of the following

maximization problem in sequence space:

$$C(x; \mathbf{x}) = \max_{\{\beta_t, \sigma_t, \mu_t, p_t\}} (1 - \tau) \sum_{t=1}^{T} \rho \Pi_{j=1}^{t-1} (1 - \sigma_j) \delta^t \sigma_t [I(x; \mathbf{g}_{\mathbf{x}}^t \mathbf{x}) - p_t], \qquad \text{(P2)}$$

subject to:

1. The round-t incentive constraints

$$\Pr(\text{Good}|\text{Round} = t) \times (1 - \tau) \times \{\delta \sigma_t [I(x; \mathbf{g}_{\mathbf{x}}^t \mathbf{x}) - p_t]$$

$$+ (1 - \sigma_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1} (1 - \sigma_j) \delta^{i+1-t} \sigma_i [I(x; \mathbf{g}_{\mathbf{x}}^i \mathbf{x}) - p_i]\}$$

$$\geq (1 - \mu_t) \max_{\widetilde{\sigma}_t} \Big(D(\sigma_t) - D(\widetilde{\sigma}_t)$$

$$+ \Pr(\text{Good}|\text{Round} = t) \times (1 - \tau) \times \{\delta \widetilde{\sigma}_t [I(x; \mathbf{g}_{\mathbf{x}}^t \mathbf{x}) - p_t]$$

$$+ (1 - \widetilde{\sigma}_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1} (1 - \sigma_j) \delta^{i+1-t} \sigma_i [I(x; \mathbf{g}_{\mathbf{x}}^i \mathbf{x}) - p_i]\} \Big),$$

$$(2.5)$$

for $t = 1, \cdots, T$, where $\Pr(\text{Good}|\text{Round} = t)$ is given by (2.4);

2. The round-0 zero-profit condition

$$\rho \sum_{t=1}^{T} \Pi_{j=1}^{t-1} (1 - \sigma_j) \delta^t \sigma_t p_t - \sum_{t=1}^{T} [\rho \Pi_{j=1}^{t-1} (1 - \sigma_j) + (1 - \rho) \Pi_{j=1}^{t} (1 - \beta_j)] \delta^{t-1} [D(\sigma_t) + \phi_t + M_t(\mu_t)]$$

$$- \sum_{t=1}^{T} [\rho \Pi_{j=1}^{t-1} (1 - \sigma_j) + (1 - \rho) \Pi_{j=1}^{t-1} (1 - \beta_j)] \delta^{t-1} E(\beta_t) - R(\frac{x}{\mathbf{x}}, \mathbf{\mathfrak{e}}) = 0.$$

$$(2.6)$$

The objective function in (P2) reflects the fact that VC is a competitive industry. A contract must maximize the expected return for the entrepreneur, subject to two constraints. The term $I(x; \mathbf{g}_{\mathbf{x}}^t \mathbf{x}) - p_t$ gives the payoff to the entrepreneur should the enterprise be floated

in round t. The payoff could come from executing stock options or convertible shares. It is taxed at the capital gains rate, τ. The maximized value of objective function, $C(x; \mathbf{x})$, specifies the worth of the financial contract for the entrepreneur. Note for use in the next section that this expected discounted payoff is a function of the entrepreneur's idea, x.

Equation (2.5) is the incentive compatibility constraint for a round-t project. The left-hand side gives the expected return to the entrepreneur when he undertakes the level of development investment linked with σ_t. The first term in brackets are the Bayesian odds of having a good project at the beginning of round t, conditional on the entrepreneur still dealing with the venture capitalist. The right-hand side gives the return when the entrepreneur deviates and picks the level of development linked with $\widetilde{\sigma}_t$. The level of development represented by $\widetilde{\sigma}_t$ maximizes the value of the deviation. The return from deviating will only materialize if the entrepreneur is not caught cheating, which has the odds $1 - \mu_t$; if caught cheating, which occurs with probability μ_t, then the contract is terminated and the entrepreneur receives nothing. The incentive constraint has a dynamic element to it. If the entrepreneur invests less in development today, he lowers the odds that a good project will be successful in the current period. He increases the probability that a success, if it happens, will occur in the future; thus, an intertemporal tradeoff is involved.

The last equation, or (2.6), is the zero-profit constraint. Observe that there is a fixed cost, ϕ_t, connected with operating a startup project in round t. Last, the venture capitalist must cover the initial development cost, $R(x/\mathbf{x}, \mathfrak{e})$. Since VC is a competitive industry, the expected returns from lending will exactly offset the expected costs.

Now, it is easy to see that the ability of the venture capitalist to monitor the entrepreneur is important. Focus on the incentive constraint (2.5). If $\mu_t = 1$, say because the cost of monitoring is zero, then the left-hand side of the constraint will always exceed the right-hand side. This transpires no matter what the solution for $\widetilde{\sigma}_t$ is, as dictated by the right-hand side of (2.5). In this situation, the first-best solution to problem (P2) can be obtained. Alternatively, suppose $\mu_t = 0$, because the cost of monitoring is infinite. Then, the incentive-

compatible contract specifies that $\sigma_t = \widetilde{\sigma}_t$. To see this, pull the $D(\sigma_t)$ term over onto the left-hand side of (2.5). Note that the terms on the left- and right-hand sides are then the same, except that they involve σ_t on the left and $\widetilde{\sigma}_t$ on the right. But $\widetilde{\sigma}_t$ maximizes the right-hand side, implying that the right-hand side must then equal the left-hand side. This can only be the case if $\sigma_t = \widetilde{\sigma}_t$, which greatly limits the contract and may result in an allocation far from first-best. So if no monitoring is done, then the incentive constraint holds tightly. Why can't the incentive constraint be slack? Suppose it is slack, implying that the associated Lagrange multiplier is zero. Then, no monitoring will be done because it would have no benefit and is costly. But, as just discussed, when $\mu_t = 0$, the constraint must hold tightly–a contradiction. Therefore, the incentive constraint (2.5) always binds.

Lemma 1 *(The venture capitalist constantly monitors the entrepreneur) The incentive constraint (2.5) holds tightly for all funding rounds with $0 < \mu_t < 1$.*

Remark 1 *(One-shot versus multi-shot deviations) The incentive constraints in (2.5) prevent one-shot deviations from occuring in any funding round. Lemma 4 in the Theory Appendix establishes that this is equivalent to using a single consolidated round-0 incentive constraint with multi-shot deviations.*

Remark 2 *(Self financing) If an entrepreneur has any funds, he should invest them all. This does not change the generic form of the contract problem. The entrepreneur's funds can merely be subtracted from the expected present value of the fixed costs, or the ϕ_t's, in (2.6). (See Cole, Greenwood, and Sanchez (2016, Lemmas 1 and 6)). What matters is how much the entrepreneur borrows, net of his own investment. The entrepreneur's funds can be incorporated in problem (P2) by simply transforming the fixed costs.*

2.6. The Choice of Idea

The entrepreneur is free to pick the type of venture, x, that he pitches to the venture capitalist. He selects the one that maximizes his expected discounted profits. Therefore, x

will solve

$$V(\mathbf{x}) = \max_x C(x; \mathbf{x}), \tag{P3}$$

where the value of the entrepreneur's contract, or $C(x; \mathbf{x})$, is specified by problem (P2). The faster profits rise with a venture's type, x, the higher will be the value of x picked by the entrepreneur. So if better intermediation implies that profits rise more steeply with x, then VC will increase growth. Note that the cost of researching x, or $R(x/\mathbf{x}, \mathfrak{e})$, is embedded in the zero-profit condition (2.6) connected with problem (P2). This problem will give a decision rule of the form

$$x = X(\mathbf{x})\mathbf{x}.$$

The function $V(\mathbf{x})$ gives an entrepreneur's expected discounted payoff from a startup.

2.7. The Flow of New Startups

Recall that an entrepreneur incurs an opportunity cost in the amount $w\mathfrak{o}$ to run a project. Therefore, only those new entrepreneurs with $w\mathfrak{o} \leq V(\mathbf{x})$ will choose to engage in a startup. Now, $\mathfrak{o}$ is distributed according the cumulative distribution function $O(\mathfrak{o})$. Therefore, $O(V(\mathbf{x})/w)$ entrepreneurs will approach the venture capitalist for funding. Consequently, the number of new entrants, $\mathfrak{e}$, is given by

$$\mathfrak{e} = O(V(\mathbf{x})/w). \tag{2.7}$$

2.8. Non-VC Sector

Most firms are not funded by venture capitalists. To capture this, suppose there are always $\mathfrak{m}$ firms operating that were not funded by VC. All firms in the non-VC sector are same. These non-VC firms produce using a production function that is identical to a VC firm with one exception: their productivity differs. Specifically, they produce in line with

$$o = z^\zeta k^\kappa l^\lambda, \text{with } \zeta + \kappa + \lambda = 1,$$

where z represents their productivity. Suppose that

$$z = \omega \mathbf{x}, \text{ with } \omega < 1.$$

Thus, firms in the non-VC segment of the economy are on average less productive than the ones in the VC segment, but will be dragged along by the latter. The non-VC firm's profit maximization problem is

$$\max_{k,l} \{ z^\zeta k^\kappa l^\lambda - rk - wl \}. \tag{2.8}$$

One can think about these firms as raising the funds for capital through traditional intermediation at the gross interest rate $1/\delta$. VC-funded firms also raise capital this way after they are floated. On this, Midrigan and Xu (2014) argue that producing establishments can quickly accumulate funds internally and thus rapidly grow out of any borrowing constraints. Therefore, modeling producing firms as having frictionless access to capital markets may not be grossly at variance with reality.

2.9. Balanced-Growth Equilibrium

The analysis focuses on analyzing a balanced-growth path for the model. Along a balanced-growth path, the rental rate on capital, r, is some fixed number. In particular, the rental rate on capital will be

$$r = 1/\delta - \eth, \tag{2.9}$$

where δ is the market discount factor and $\eth$ is the depreciation factor on capital. In balanced growth, the market discount factor, δ, in turn is given by

$$\delta = \widehat{\delta} \mathbf{g}_w^{-\varepsilon}, \tag{2.10}$$

where $\widehat{\delta}$ is the representative agent's discount factor and ε denotes his coefficient of relative risk aversion.[4]

The idea distribution for VC-backed firms will now be characterized. To this end, let $\mathfrak{n}_t$ represent the number of VC-backed firms that are operating with an idea, x_{-t}, that was generated t periods ago. Attention will now be turned to specifying the number $\mathfrak{n}_t$. Now, no firms will operate in the VC-backed sector with productivity level x, since this type is not operational yet. Each period, $\mathfrak{e}$ new entrepreneurs will be funded by the venture capitalist. Hence, $\mathfrak{n}_1 = \mathfrak{e}\rho\sigma_1$ firms will operate with an idea generated one period ago, x_{-1}. Likewise, there will be $\mathfrak{n}_2 = \mathfrak{e}\rho\sigma_1\mathfrak{s} + \mathfrak{e}\rho(1-\sigma_1)\sigma_2$ firms operating with a two-period-old idea, x_{-2}. So, the number of firms operating with an idea, x_{-t}, from $t \leq T$ periods ago is

$$\mathfrak{n}_t = \mathfrak{e} \sum_{i=1}^{t} \rho\Pi_{j=1}^{i-1}(1-\sigma_j)\sigma_i\mathfrak{s}^{t-i}, \text{ for } t = 1, \cdots, T. \tag{2.11}$$

The venture capitalist only funds entrepreneurs for T periods. Consequently, the number of operational firms with an idea from more than T periods ago is

$$\mathfrak{n}_{T+j} = \mathfrak{s}^j\mathfrak{n}_T, \text{ for } j \geq 1. \tag{2.12}$$

The total number of VC-backed firms in the economy, $\mathfrak{n}$, is given by

$$\mathfrak{n} = \sum_{t=1}^{T} \mathfrak{n}_t + \sum_{t=T+1}^{\infty} \mathfrak{n}_t = \sum_{t=1}^{T} \mathfrak{n}_t + \frac{\mathfrak{n}_T\mathfrak{s}}{1-\mathfrak{s}}.$$

In a stationary equilibrium the distribution function over VC-funded firms using an age-t idea will remain constant; that is, $\mathfrak{n}'_t = \mathfrak{n}_t$. It is easy to from (2.11) that this will be true provided that $\mathfrak{e}$ and the σ_j's are constant.

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[4]That is, in the background there is a representative consumer/worker who inelastically supplies one unit of labor and has a utility function (in period 1) of the form

$$\sum_{t=1} \widehat{\delta}^{t-1}c_t^{1-\epsilon}/(1-\epsilon),$$

where c_t is his period-t consumption.

In balanced growth the wage rate, w, will grow at some constant gross rate, $\mathbf{g}_w$. To determine this growth rate, note that a VC-funded firm with productivity level x will hire labor in the amount

$$l(x; w) = \left(\frac{\kappa}{r}\right)^{\kappa/\zeta} \left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta} x, \tag{2.13}$$

where again w and r are the wage and rental rates, respectively. For a non-VC-funded firm, just replace the x with a z in the above formula. In general equilibrium, the labor market must clear each period. Suppose that there is one unit of labor available in aggregate. To calculate the aggregate demand for labor, sum over all operating firms' demands for labor, both in the VC- and non-VC-backed sectors. Equilibrium in the labor market requires that

$$\sum_{t=1}^{T} \mathfrak{n}_t l(x_{-t}; w) + \sum_{t=T+1}^{\infty} \mathfrak{n}_t l(x_{-t}; w) + \mathfrak{m} l(z; w) = 1,$$

where $\mathfrak{m}$ is the measure of firms in the non-VC sector. Along a balanced-growth path, the productivity of the latest idea will grow at rate $\mathbf{g_x}$. Therefore, the above condition can be recast as

$$\sum_{t=1}^{T} \mathfrak{n}_t l(x_{-1}\mathbf{g_x}^{1-t}; w) + \sum_{t=T+1}^{\infty} \mathfrak{n}_t l(x_{-1}\mathbf{g_x}^{1-t}; w) + \mathfrak{m} l(\omega \mathbf{x}; w) = 1.$$

Using equations (2.13) and (2.12), this can be expressed as

$$\left(\frac{\kappa}{r}\right)^{\kappa/\zeta} \left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta} [x_{-1}(\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}}{1 - (\mathfrak{s}/\mathbf{g_x})}) + \mathfrak{m}\omega \mathbf{x}] = 1.$$

The solution for wages, w, obtained from the above labor-market clearing condition, is

$$w = \lambda \left(\frac{\kappa}{r}\right)^{\kappa/(\zeta+\lambda)} \underbrace{[x_{-1}(\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}}{1 - (\mathfrak{s}/\mathbf{g_x})}) + \mathfrak{m}\omega \mathbf{x}]^{\zeta/(\zeta+\lambda)}}_{=\mathfrak{n}\mathbf{x}}, \tag{2.14}$$

where aggregate productivity, $\mathbf{x}$, is

$$\mathbf{x} \equiv \frac{x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}/(1 - (\mathfrak{s}/\mathbf{g_x}))]}{\sum_{t=1}^{T} \mathfrak{n}_t + \mathfrak{n}_T \mathfrak{s}/(1 - \mathfrak{s})} = \frac{x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}/(1 - (\mathfrak{s}/\mathbf{g_x}))]}{\mathfrak{n}}.$$

As can be seen, wages rise with the aggregate level of productivity, $\mathbf{x}$, which grows at rate $\mathbf{g_x}$. Therefore, wages will grow at the gross growth rate $\mathbf{g_x}^{\zeta/(\zeta+\lambda)}$, so that

$$\frac{w'}{w} \equiv \mathbf{g}_w = \mathbf{g_x}^{\zeta/(\zeta+\lambda)}.$$

Attention is now turned to determining the growth rate in aggregate productivity, $\mathbf{g_x}$. All new entrepreneurs will pick the same type of project, x. Now

$$\mathbf{g_x} = \mathbf{x}'/\mathbf{x} = x'/x.$$

Recall that

$$x = X(\mathbf{x})\mathbf{x},$$

and

$$\mathbf{x} = x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}}{1 - (\mathfrak{s}/\mathbf{g_x})}]/\mathfrak{n}.$$

Therefore,

$$\mathbf{g_x} = \frac{x}{x_{-1}} = X(\mathbf{x})[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}}{1 - (\mathfrak{s}/\mathbf{g_x})}]/\mathfrak{n}. \qquad (2.15)$$

This is a nonlinear equation in $\mathbf{g_x}$.

It is easy to see that the aggregate capital stock and output grow at the same rate as wages. The demand for capital by a type-x VC-backed firm is

$$k(x; w) = (\frac{\kappa}{r})^{(1-\lambda)/\zeta}(\frac{\lambda}{w})^{\lambda/\zeta}x.$$

From this it is easy to deduce that $k(\boldsymbol{g}_x x; \mathbf{g}_w w) = \mathbf{g}_w k(x; w)$. The same is true for a non-VC backed firms; just replace x with z to get $k(\boldsymbol{g}_x z; \mathbf{g}_w w) = \mathbf{g}_w k(z; w)$. Let the aggregate capital stock in the current period be represented by $\mathbf{k}$ and that for next period by $\mathbf{k}'$. Then $\mathbf{k}' = \sum_{t=1}^{\infty} \mathfrak{n}_t k(\mathbf{g_x} x_{-t}; \mathbf{g}_w w) + \mathfrak{m} k(\boldsymbol{g}_x z; \mathbf{g}_w w) = \mathbf{g}_w [\sum_{t=1}^{\infty} \mathfrak{n}_t k(x_{-t}; w) + \mathfrak{m} k(z; w)] = \mathbf{g}_w \mathbf{k}$, so that the aggregate capital stock grows at gross rate $\mathbf{g}_w$. A similar argument can be used to

show that aggregate output grows at the same rate.

Definition *(Balanced-Growth Path) For a given subjective discount factor and coefficient of relative risk aversion, $\widehat{\delta}$ and ε, a balanced-growth path consists of (i) a financial contract, $\{\beta_t, \sigma_t, \mu_t, p_t\}$, between entrepreneurs and the venture capitalist; (ii) a set of labor inputs for VC- and non-VC-funded firms, $l(x; w)$ and $l(z; w)$; (iii) values for the contract, an IPO, and a startup, $C(x; \mathbf{x})$, $I(x; \mathbf{x})$, and $V(\mathbf{x})$; (iv) a project type, x, for new entrepreneurs; (v) a flow in of new entrepreneurs, $\mathfrak{e}$; (vi) a rental rate for capital, r, and a market discount factor, δ; (vii) an idea distribution for VC-funded firms, $\{\mathfrak{n}_t\}_{t=1}^{\infty}$; (viii) a wage rate, w; and (ix) a gross growth rate of aggregate productivity, $\mathbf{g_x}$, such that:*

1. *The financial contract, $\{p_t, \sigma_t, \mu_t, \beta_t\}$, solves problem (P2), given the function $I(x; \mathbf{x})$ and x, $\mathbf{g_x}$, and $\mathbf{x}$. The solution to this problem gives the expected return to a new entrepreneur from the contract, $C(x; \mathbf{x})$.*

2. *The VC-funded firm maximizes its profits, given x, r, and w, as specified by problem (P1). This determines the value of its IPO, $I(x; \mathbf{x})$, as presented in (2.3). The solution to the firm's maximization problem gives the rule for hiring labor (2.13). Analogously, a non-VC-funded firm maximizes its profits, given x, r and w, as specified by problem (2.8).*

3. *A new entrepreneur picks the project type, x, to solve problem (P3), given the value of the contract, $C(x; \mathbf{x})$, as a function of x and $\mathbf{x}$. This determines the expected value of a startup, $V(\mathbf{x})$.*

4. *The flow of new entrepreneurs, $\mathfrak{e}$, is regulated by (2.1) and (2.7), taking as given the value of the startup, $V(\mathbf{x})$.*

5. *The rental rate on capital, r, and the market discount factor, δ, are governed by (2.9) and (2.10), given $\mathbf{g}_w$.*

6. *The idea distribution for VC-funded firms, $\{\mathfrak{n}_t\}_{t=1}^{\infty}$, is specified by (2.11) and (2.12).*

7. *The market-clearing wage rate, w, is given by (2.14) and grows at the gross rate $\mathbf{g}_w = \mathbf{g_x}^{\zeta/(\zeta+\lambda)}$.*

8. *Aggregate productivity, $\mathbf{x}$, grows at the gross rate $\mathbf{g_x}$ specified by (2.15).*

The lemma below establishes that the setup will have a balanced-growth path.

Lemma 2 *(Balanced Growth) Let $x' = \mathbf{g_x}x$ and $\mathbf{x}' = \mathbf{g_x}\mathbf{x}$, for all time. In the contract specified by (P2), the new solution will be given by $\sigma'_t = \sigma_t, \mu'_t = \mu_t, \beta'_{t+1} = \beta_{t+1}, \widetilde{\sigma}'_t = \widetilde{\sigma}_t,$ $p'_t = \mathbf{g}_w p_t$, and $C(x'; \mathbf{x}') = \mathbf{g}_w C(x; \mathbf{x})$. The gap between the frontier, x, and average*

productivity, **x**, *as measured by* $x/\mathbf{x}$, *is time invariant. The flow of new entrepreneurs,* ɛ, *is a constant.*

Proof. See Theory Appendix. ∎

2.10. Calibration

As discussed in Section 2.2, VC partnerships are of a limited duration, usually between 7 to 10 years. So, the analysis assumes that an entrepreneur's contract with a venture capitalist has 7 potential funding rounds each lasting 1.5 years. Thus, partnerships are structured to last at most 10.5 years. The decreasing returns to scale parameter in the production function (P1) is taken from Guner, Ventura, and Xu (2008), which requires setting $\zeta = 0.20$. The exponents for the inputs are picked so that capital earns 1/3 of nonprofit income and labor receives 2/3. The survival rate of a firm is selected so that on average a publicly listed firm lives 25 years, as in the U.S. economy. The depreciation rate on capital, $1 - \eth$, is taken to be 7 percent. Last, Henrekson and Sanandaji (2016) report that the key personnel connected with VC startups are taxed at a 15 percent capital gains rate. So, set $\tau = 0.15$.[5]

The model is calibrated to match several facts. Over the period 1948 to 2015, U.S. GDP per hours worked grew at 1.8 percent per year. This fact is targeted in the calibration procedure. The long-run interest rate is set to 4 percent, a typical value. A standard value of 2 is assigned for the coefficient of relative risk aversion. The market discount factor is the reciprocal of the equilibrium interest rate, and it will change as the growth rate of the economy, $\mathbf{g}_w$, changes. At the calibrated equilibrium, the representative agent's annual discount factor is determined by the formula to $\widehat{\delta} = (1 - .04)/(1.018)^{-2}$; cf. (2.10). This yields a yearly interest rate of 4 percent.

The remaining parameters are jointly calibrated. Still, some data targets play a much more central role in identifying a parameter. To calibrate the two elasticities of the research cost

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[5]The capital gains tax rate has varied across time in the United States. The 15 percent rate was instituted under President Bush in 2003. The maximum rate rose to 20 percent in 2012 under President Obama.

function, ι and ξ, the following regression is run using VentureXpert data:

$$\ln(\text{IPO value}) = \underset{(0.154)}{0.390^{**}} \times \ln(\text{VC funding})$$

$$+ \underset{(0.088)}{0.176^{**}} \times \ln(\text{Agg VC funding}) + \text{Controls, obs} = 1{,}145,$$

$$(2.16)$$

where the controls are the $\ln(\# \text{ of employees})$, VC firm age at IPO, a 2-digit SIC industry dummy variable, and a cluster dummy for whether the venture capitalist was located in California or Massachusetts. Three instrumental variables are also used: capital gains taxes (which vary across states and time), dependence on external finance (which varies across industries), and the deregulation dummy. The first coefficient gives the impact of a firm's VC funding on its IPO value, while the second shows the effect of aggregate VC funding on the IPO value. The first coefficient is used to identify a value for ι and the second for ξ.

To identify ι, the impact of a change in firm-level VC funding on its IPO value is calculated for the model. This calculation is broken down into two steps. First, the elasticity of $I(x; \mathbf{x})$ with respect to x is computed. Second, the elasticity of VC funding with respect to x is totted up numerically. This is done in partial equilibrium to match the results of the firm-level regression. The ratio of these two elasticities gives the elasticity of market value with respect to VC funding. Thus, the following object is computed for the model:

$$\text{IPO Value Elasticity} = \frac{d \ln \text{IPO} / d \ln x}{d \ln(\text{VC Funding}) / d \ln x}.$$

Ideally, this should have a value of 0.390. A similar procedure is used to calculate an IPO elasticity with respect to aggregate VC funding, which has a target value of 0.176.

Another key elasticity in the model is the shape parameter, ν, for the Pareto distribution governing the opportunity cost of entrepreneurship. This regulates the inflow of entrepreneurs. Henrekson and Sanandaji (2016) report that a one percent increase in a country's effective tax rate on VC activity leads to a one percent decline in the VC investment-to-GDP ratio. This elasticity is targeted to recover the shape parameter, ν. The scale

parameter, υ, is normalized to 0.2.

The process for the efficiency of round-t monitoring, $\chi_{M,t}$, is taken to be a cubic:

$$\chi_{M,t} = \log(a_0 + a_1 \times t + a_2 \times t^2 + a_3 \times t^3).$$

This process requires specifying three parameters, namely a_0, a_1, a_2 and a_3. Additionally, the monitoring parameters are selected to match the venture capitalist's share of equity by the duration of the project (this pattern is taken up below). The more efficient monitoring is, the higher will be the venture capitalist's share of equity, as will be seen.

The time profile for the fixed cost, $\phi(t)$, will be governed by the quartic

$$\phi(t) = \exp(b_0 + b_1 \times t + b_2 \times t^2 + b_3 \times t^3 + b_4 \times t^4).$$

Five parameters, b_0, b_1, b_2, b_3, and b_4, govern this process. The pattern of VC investment by funding round (discussed below) determines these parameters.

Bernstein, Giroud, and Townsend (2016) estimate the impact of a venture capitalist's time cost for monitoring on his investment. To do this, they examine the effect of changes in airline routes that reduce the commuting time a venture capitalist spends visiting a startup. They find that the introduction of a new airline route (the treatment) leads to a 4.6 to 5.2 percent increase in VC investment. The average reduction in travel time is significant. The lead investor visits the company site roughly 20 times per year and spends approximately 12 hours traveling and 5 hours at the company per visit, which amounts to 100 contact hours annually.[6] On average, a treatment saves roughly 2 hours per trip, or 40 hours per year of a venture capitalist's time. Accordingly, the treatments correspond to fairly large reductions in monitoring costs: a reduction of 2 hours per trip translates into a 12.4 percent reduction in monitoring costs. Bernstein, Giroud, and Townsend (2016) argue that most of the resources spent by a venture capitalist on monitoring is time. So, assume that monitoring is done using labor in the model. The efficiency of monitoring impinges on the

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[6]The time spent visiting the company is quoted in the unpublished version of Bernstein, Giroud, and Townsend (2016).

success rate of startups. So, matching, in partial equilibrium, the Bernstein, Giroud, and Townsend's (2016) treatment effect helps to tie down the fraction of good ideas, ρ.

Next, projects that are funded by venture capitalists have an average success rate per funding round of 1.1 percent and a failure rate of 4.7 percent. The calibration procedure attempts to match these two statistics. To construct these statistics for the model, note that the success rate in period t is just the number of IPOs divided by the mass of surviving firms:

$$\text{SUCCESS RATE}_t = \frac{\text{IPOs}_t}{\text{SURVIVING FIRMS}_t} = \frac{\sigma_t \rho \Pi_{j=1}^{t-1}(1 - \sigma_j)}{\rho \Pi_{j=1}^{t-1}(1 - \sigma_j) + (1 - \rho)\Pi_{j=1}^{t}(1 - \beta_j)}.$$

The analogous definition for the failure rate in funding round t is

$$\text{FAILURE RATE}_t = \frac{\text{FAILURES}_t}{\text{SURVIVING FIRMS}_t} = \frac{\beta_t(1 - \rho)\Pi_{j=1}^{t-1}(1 - \beta_j).}{\rho \Pi_{j=1}^{t-1}(1 - \sigma_j) + (1 - \rho)\Pi_{j=1}^{t}(1 - \beta_j).}.$$

On average, a VC-backed company is 57.2 log points larger in terms of employment than a non-VC-backed firm. This is a calibration target. For the model, the employment ratio is

$$\text{EMPLOYMENT RATIO} = \frac{\left(\frac{\kappa}{r}\right)^{\kappa/\zeta}\left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta} \mathfrak{n}\mathbf{x}/\mathfrak{n}}{\left(\frac{\kappa}{r}\right)^{\kappa/\zeta}\left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta} \mathfrak{m}\omega\mathbf{x}/\mathfrak{m}} = \frac{1}{\omega}.$$

The upshot of the calibration procedure is now discussed. The parameter values used in the calibration are presented in Table 13. First, the model matches the average success and failure rates very well, as shown in Table 14. And the model replicates perfectly the ratio of VC-backed employment to non-VC backed employment. The two IPO elasticities are duplicated, and the model is extremely close to matching the Henrekson and Sanandaji (2016) tax rate elasticity. The monitoring-cost treatment effect lies within the range of estimates reported by Bernstein, Giroud, and Townsend (2016).

Next, note how investment in a project by a venture capitalist increases with the funding round (see the top panel of Figure 19). This time profile is a calibration target. Given the limited life span of a VC partnership, there is considerable pressure to bring a project

to fruition as quickly as possible. This is true in the model too, which displays the same increasing profile of funding. Two features help to generate this. The first is that bad projects get purged over time through the evaluation process. The second is that the cost of monitoring drops as the venture capitalist becomes more familiar with project, which reduces the incentive problem. Without these features, funding would fall over time. Last, since investment increases over time, one would expect that the venture capitalist's share of the enterprise will too. The bottom panel of Figure 19 illustrates this. The model does very well on this account. Again, the calibration procedure focuses on this feature of the data.

The time profiles for the success and failure rates are *not* targeted in the calibration procedure. As shown in the middle panel of Figure 20, in the data the odds of success decline by funding round or with the passage of time. While the model captures the average success across funding rounds very well, it has some difficulty mimicking the time profile. This may be because in the model a failed venture has no scrap value, which increases the pressure to succeed. Data on the scrap value of failed ventures are, unfortunately, not readily available. Failure rates also decline with time, and the model does very well on this dimension.

Now turn to the bottom panel of Figure 20. Observe that the value of an IPO drops with the incubation time for the project. In the model, as time passes, the value of a project declines because aggregate productivity catches up with the productivity of the entrepreneur's venture; "the thrill is gone," so to speak. It is a bit surprising that the framework can match almost perfectly this feature of the data, which is not targeted.

Finally, it is trivial to recalibrate the model for the situation where there are no spillovers in the research cost function. This obviously involves setting $\xi = 0$. The only thing that needs to be adjusted to recapture the benchmark calibration is the research efficiency parameter, χ_R. Absolutely nothing else changes. The values for ξ and χ_R in the economy without spillovers are shown in parentheses in Table 13.

TABLE 13: PARAMETER VALUES

| Parameter value	Description	Identification
Firms		
$\kappa = 1/3 \times 0.80$	Capital's share	Standard
$\lambda = 2/3 \times 0.80$	Labor's share	Standard
$1 - \mathfrak{d} = 0.07$	Depreciation rate	Standard
$\mathfrak{s} = 0.96$	Firm survival rate	Expected life of Compustat firms
$\chi_R = 59.9\ (10.9)$	Research efficiency, x	Growth rate
$\iota = 2.56$	Research cost elasticity, x	Regression (2.16)
$\xi = 0.46\ (0)$	Research cost elasticity, $\mathfrak{e}$	Regression (2.16)
$\nu = 0.0150$	Pareto shape parameter	H&S (2016) tax elasticity
$\upsilon = 0.02$	Pareto scale parameter	Normalization
Consumers		
$\varepsilon = 2$	CRRA	Standard
$\widehat{\delta} = 0.994$	Discount factor	4% risk-free rate
VC		
$T = 7$	Number of funding rounds	Partnership length (10.5 years)
$\rho = 0.21$	Fraction of goods ideas	BG&S treatment effect (2016)
$\chi_D = 0.012$	Development efficiency, σ	Average success rate
$\chi_E = 0.18$	Evaluation efficiency, β	Average exit rate
$a = \{-1.12, -0.12, 0.321, -0.018\}$	Monitoring efficiency, μ	Equity share by round
$b = \{-1.0, 1.69, -0.533,$	Fixed costs, ϕ	VC funding by round
$\qquad 0.081, -0.004\}$		
$\tau = 0.15$	Capital gains tax rate	H&S (2016)
Non-VC		
$\mathfrak{m} = 1.7$	Number non-VC firms	Relative empl. non-VC firms
$\omega = 0.56$	Relative prod of non-VC firms	Relative size of non-VC firms

Notes: The parameter values used in the baseline simulation.

TABLE 14: CALIBRATION TARGETS

| Target	Source	Data	Model
Economic growth	BEA	1.80	1.87
Cash Multiple	Gompers et al. (2016, Table 12)	3.8	3.78
Success Rate	Puri and Zarutskie (2012, Table VI.B)	1.1	1.39
Failure Rate	Puri and Zarutskie (2012, Table VI.B)	4.7	5.35
VC Inv/GDP	Henrekson and Sanandaji (2016)	0.2	0.2
VC funding	Crunchbase	Figure 19	
Equity Share	Crunchbase	Figure 19	
IPO Value Elasticity–firm level	Regression (2.16)	0.39	0.39
IPO Value Elasticity–aggregate	Regression (2.16)	0.176	0.176
Tax Elasticity of VC Inv/GDP	Henrekson and Sanandaji (2016)	-1.0	-0.9
Monitoring-Cost Treatment	Bernstein et al. (2016, Tables IAVI & IAVII)	4.6 to 5.3	5.2
Employment ratio	Puri and Zarutskie (2012, Table IV)	57.2	57.2

Notes: All numbers, except for the cash multiple, are in percentages.

FIGURE 19: INVESTMENT AND EQUITY SHARE BY FUNDING ROUND – DATA AND MODEL
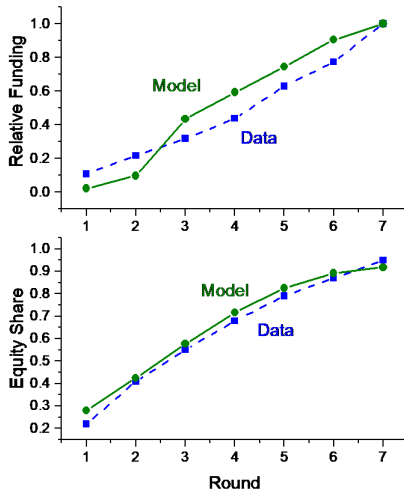
Notes: The upper panel shows the venture capitalist's investment by funding round. Funding in the last round is normalized to 1.0. The lower panel charts the venture capitalist's share of equity by funding round.
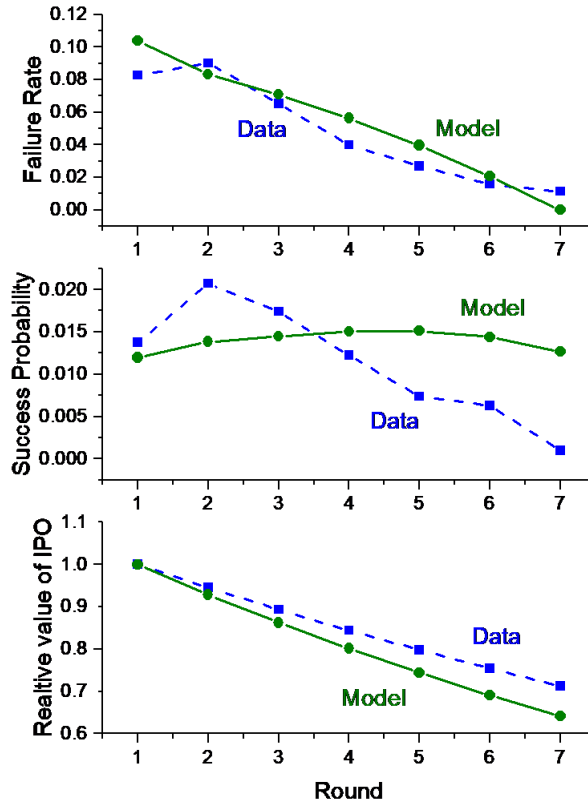
Notes: The value of an IPO that occurs during first funding round is normalized to 1.0. None of these profiles is targeted in the calibration.

2.11. Thought Experiments

The analysis stresses the ability of venture capitalist to evaluate, develop, and monitor startup projects. The importance of these three factors is now investigated one by one and then the efficiencies of each debased in tandem to approximate the success rate of non-VC methods of finance.

2.11.1. Changes in Monitoring Efficiency, $\chi_{M,t}$

How important is the venture capitalist's ability to monitor the use of funds by entrepreneurs? Figure 21 shows the general equilibrium impact of improving the efficiency of monitoring

in the model. To undertake this thought experiment, the monitoring efficiency profile, $\{\chi_{M,1}, \cdots, \chi_{M,7}\}$, is changed by scalar, which takes the value of 1 for the baseline calibration. As monitoring efficiency improves, there is an increase in the average odds of detecting fraud across funding rounds (see the top panel). The venture capitalist's share of equity rises, on average, because it is now easier for him to ensure that funds are not diverted. Compliance with the contract can be still be guaranteed when the entrepreneur is given a lower share of an IPO. As a result of improved monitoring, the venture capitalist can increase investment, which is reflected by a higher share of VC investment in GDP (see the middle panel). The venture capitalist must still earn zero profits. Part of the increased return to the venture capitalist is soaked up by letting the new entrepreneur be more ambitious about his choice of technique, which raises the initial cost of research, $R(x/\mathbf{x}, \mathfrak{e})$; the rest of the increased return is absorbed by increased investment. So, the economy's growth rate moves up, which results in a welfare gain (measured in terms of consumption; see the bottom panel).[7]

2.11.2. Changes in Evaluation Efficiency, χ_E

The importance of efficiency in evaluation is examined now, where χ_E is normalized to 1 for the baseline calibration. The results are displayed in Figure 22. As evaluation becomes more efficient, the odds of detecting a bad project increase. Hence, the average failure rate across funding rounds moves up (see the top panel). The success rate rises, both due to the purging of bad projects and the resulting increased VC investment. The purging of bad projects dominates the exit of good ones so that the fraction of good projects in the last round increases with χ_E. The fact that it is more profitable to invest is reflected by upward movement in the VC investment-to-GDP ratio. Economic growth and welfare move up in tandem as evaluation efficiency improves (see the bottom panel).

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[7]See Akcigit, Celik, and Greenwood (2016, Section 5.1) for details on how the welfare gain is computed. In the current work, the initial level of consumption across balanced-growth paths is held fixed, though, as opposed to aggregate productivity.

FIGURE 21: EFFICIENCY IN MONITORING, $\chi_{M,t}$
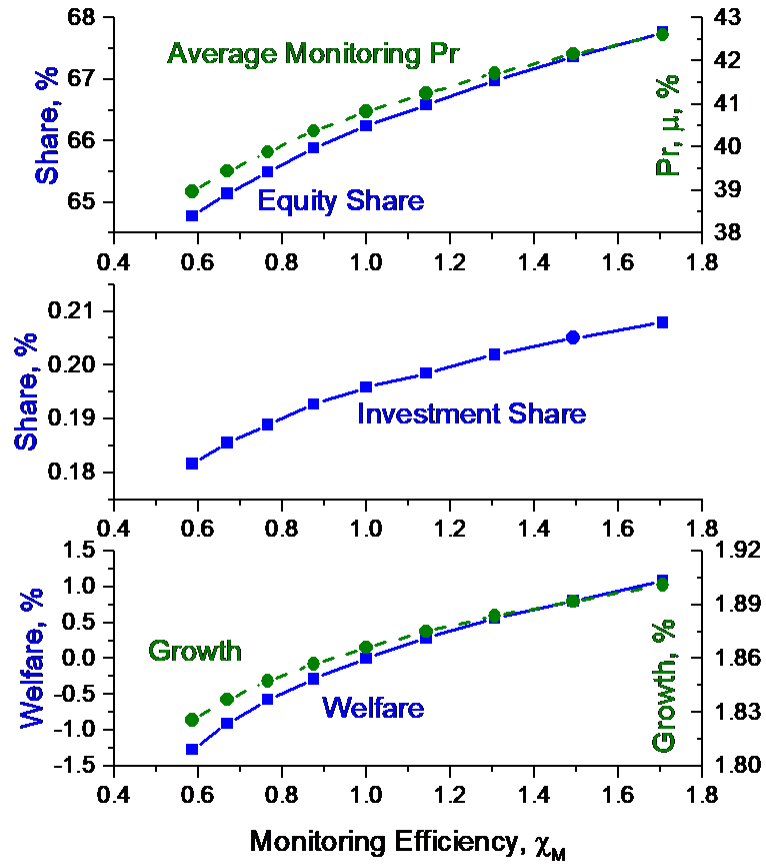
Notes: The top panel shows how the average probability of detecting fraud and the venture capitalist's share of equity vary with efficiency in monitoring. The middle panel illustrates how the VC investment-to-GDP ratio responds. Growth and welfare are displayed in the bottom panel.
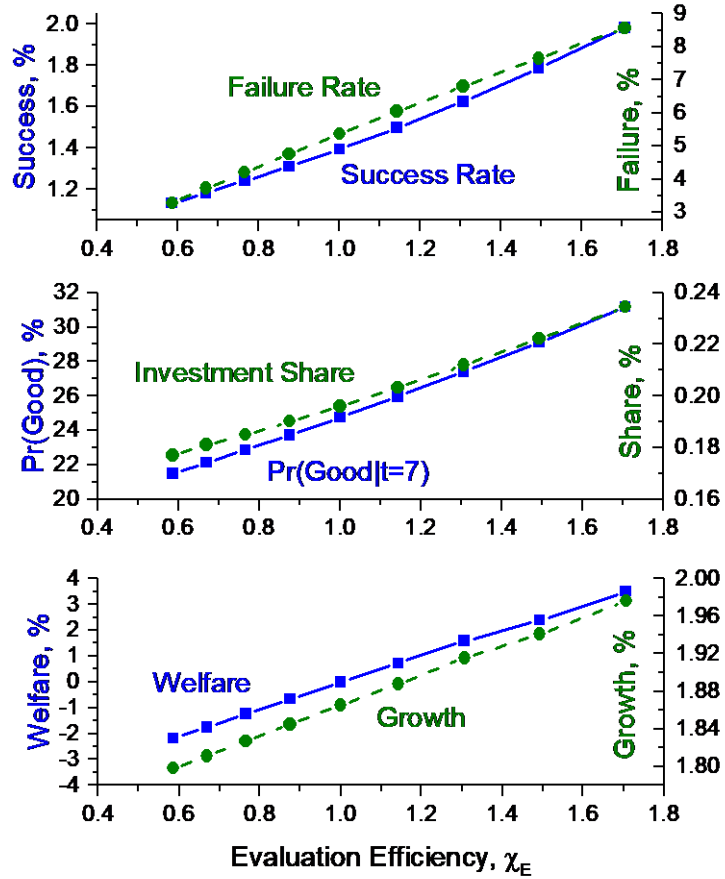
Notes: The top panel shows how the average failure and success rates across funding rounds vary with efficiency in evaluation. The middle panel illustrates how the odds of a project being good in the seventh round and the VC investment-to-GDP ratio respond. Growth and welfare are illustrated in the bottom panel.

2.11.3. Changes in Development Efficiency, χ_D

Finally, impact of changes in development efficiency is studied. Again, χ_D is normalized to 1 for the baseline calibration. As it becomes less expensive to develop a project, the odds of success improve. The failure rate also rises because fewer good projects remain in the pool over time. The VC investment-to-GDP ratio moves up, as it is more profitable to fund a project. Last, economic growth and welfare rise with development efficiency.
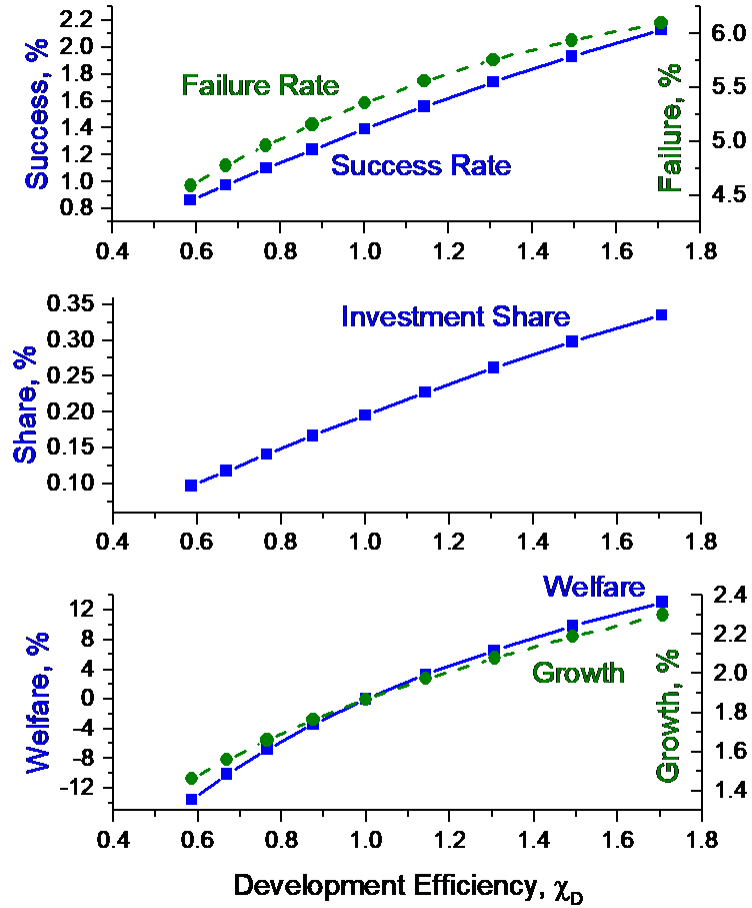
Notes: The top panel shows how the average failure and success rates across funding rounds vary with efficiency in development. The middle panel illustrates how the VC investment-to-GDP ratio responds. Growth and welfare are illustrated in the bottom panel.

2.11.4. *Debasing Venture Capital–An Approximation to Non-VC Forms of Financing*

Venture capitalists lend development and evaluation expertise to startups that alternative forms of finance, such as angel investors, banking, and more recently crowdfunding, do not. Arguably, venture capitalists are also better at monitoring projects. Wealthy people have always been willing to lend seed money to startups, as discussed in Section 2.2. This is what angel investors do today. The sheer size of financing needed as a startup evolves goes well

beyond an angel investor's pockets. The average investment per deal of an angel investor was $510,000 in 2014. In contrast, the average venture capitalist invests $4 million and $14 million in seed-stage and later-stage deals. These investments are 8 times and 28 times larger than those of angel investors. VC organizations feature substantially higher levels of professionalism and specialization than angel investors: All the roles of a VC organization (e.g., evaluation, development, and monitoring) are rolled up into one single angel investor.

To approximate alternative forms of finance, some empirical evidence from Puri and Zarutskie (2012) is used. They track the performance of VC- and non-VC-financed firms using the Longitudinal Business Database (LBD). They identify firms in the LBD as VC-financed if they can be matched to the VentureSource and VentureXpert databases. They match each VC-financed firm to a non-VC-financed firm based on four characteristics: age, 4-digit SIC code, geographical region, and employment size. They find that VC-financed and non-VC-financed firms are observationally identical at the time the former first receive VC financing. Based on this comparison, they report that the ratio of the success rate of non-VC-financed firms to the success rate of observationally identical VC-financed firms is 0.23.[8]

To approximate more traditional forms of finance in the model, the efficiency of development, evaluation, and monitoring are all debased in an equiproportional manner to render the same average success-rate ratio for a startup. Note that for this ratio to be comparable with its empirical counterpart, this recalibration is done in partial equilibrium. The 0.23 ratio is reproduced by reducing in tandem development, evaluation, and monitoring efficiency to 39 percent of their original values.[9] The upshot of this exercise is shown in Table 15. Alternative forms of finance have a much lower success rate (1.4 versus 0.5 percent) than do VC-financed projects. Note that the ratio of 0.5/1.4 is larger than 0.23 because there are general equilibrium effects that partially offset the reduction in financing efficiency. When the efficiency of VC is debased, the failure rate drops from 5.4 to 1.8 percent. The financier's share of the project declines considerably. Since monitoring is less efficient, a larger share

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[8]Venture capitalists could match with better firms based on unobservable characteristics. Given that startups are very young with little in terms of employment and patents, it might difficult to control empirically for this selection effect. To the extent that such selection effects are important, the results in this section would constitute an upper bound for the effect of VC financing.

[9]The results are quite similar when only development efficiency is debased.

of the project must be given to the entrepreneur to ensure that he will invest all of the development funds. The drop off in the success rate and the financier's share of equity lead to less investment. As a result, growth falls. This generates a large welfare loss.

TABLE 15: AN ALTERNATIVE FORM OF FINANCE

Variable	Baseline	Debased economy
Success	1.4	0.5
Failure	5.4	1.8
VC Inv/GDP	0.20	0.03
Equity Share	66.2	41.6
Growth	1.86	1.05
Welfare loss	0	28.8

Notes: All numbers are in percentages.

2.12. Capital Gains Taxation

Most VC-funded firms in the United States are setup as partnerships. CEOs, central employees, founders, and investors are paid in terms of convertible equity and stock options. These financial assets payoff only under certain well-specified contingencies and serve to align the incentives of key participants. Interestingly, the returns on convertible equity and stock options are taxed in the United States at the capital gains rate, which is 15 percent. The IRS lets companies assign artificially low values to these instruments when they are issued. So, effectively, participants are only subject to taxation at the time of an acquisition/IPO. In other countries the rate of taxation on VC-funded startups is much higher. For example, it is 30 percent in France, 47.5 percent in Germany, and 72 percent in Italy. In a cross-country regression analysis, Henrekson and Sanandaji (2016, Table 4) report a strong negative correlation between capital gains tax rates and VC investment as a percentage of GDP. The elasticity of the tax rate on VC activity is about -1.0, as mentioned earlier.
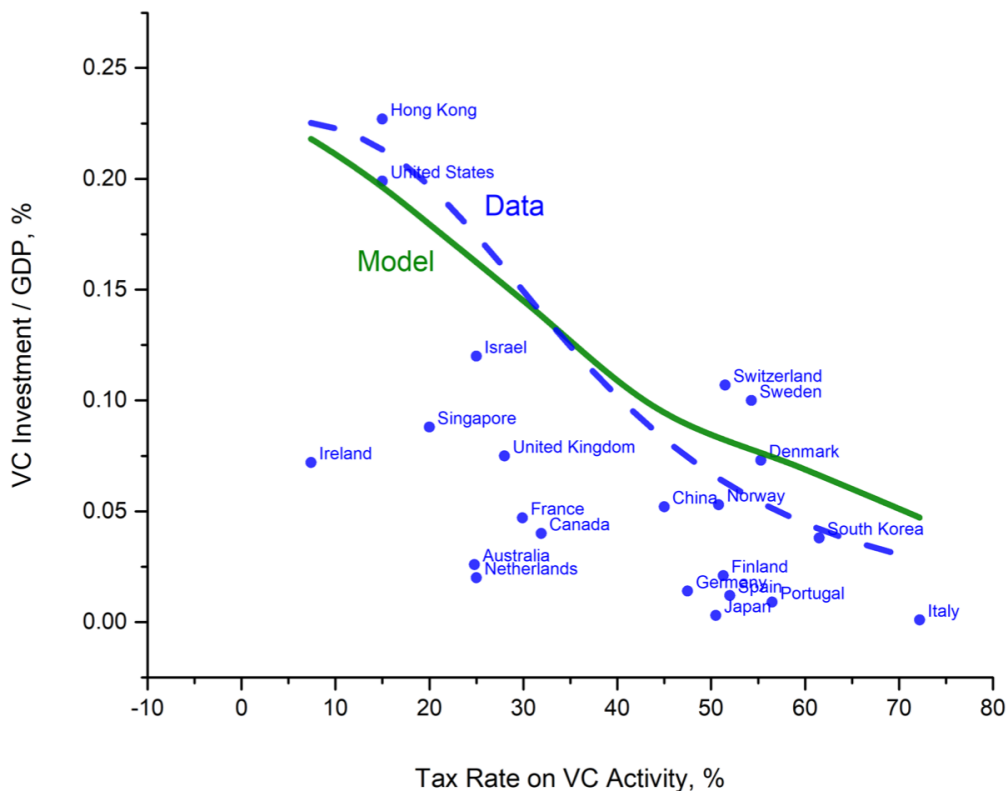
Notes: The numbers are expressed as percentages.

Figure 24 shows how VC investment as a percentage of GDP tends to fall with the tax rate on VC activity. The data are from Henrekson and Sanandaji (2016). To obtain the tax rates on VC activity, they asked the local offices of PricewaterhouseCoopers in 22 countries to calculate the effective tax rate for a representative VC startup. As the figure shows, the fitted lines for the data and model match each other quite well. As the capital gains tax rate rises, not surprisingly, the share of VC investment in GDP declines. It drops from about 0.22 percent, when capital gains are taxed at a 7.4 percent rate, to 0.047 percent, when capital gains are taxed at a 74 percent rate. Note that the share of VC investment in GDP is very small, both in the data and model. Yet in the model, VC investment drives all of growth.

The impact of capital gains taxation in the model is also illustrated in Figure 25. As the capital gains tax rate rises, not surprisingly economic growth declines (see the top panel). As the capital gains tax rate moves up from -15 percent (a subsidy) to 60 percent, economic growth in the benchmark economy falls from 2.04 percent to 1.41 percent. As the figure illustrates, when there is no externality in the research cost function, the effect is muted. This transpires because as the tax rate is hiked, the number of VC-funded firms drops. With an externality present, this raises the cost of research. The effects on growth might appear small, but lowering the capital gains tax rate from 60 percent to 15 percent produces a long-run welfare gain of 15.4 percent, when ignoring transitional dynamics. Going further from 15 percent to -15 percent generates an additional welfare gain of 5.5 percent, all measured in terms of consumption. The welfare gains are smaller when the externality is absent.

FIGURE 25: IMPACT OF CAPITAL GAINS TAXATION
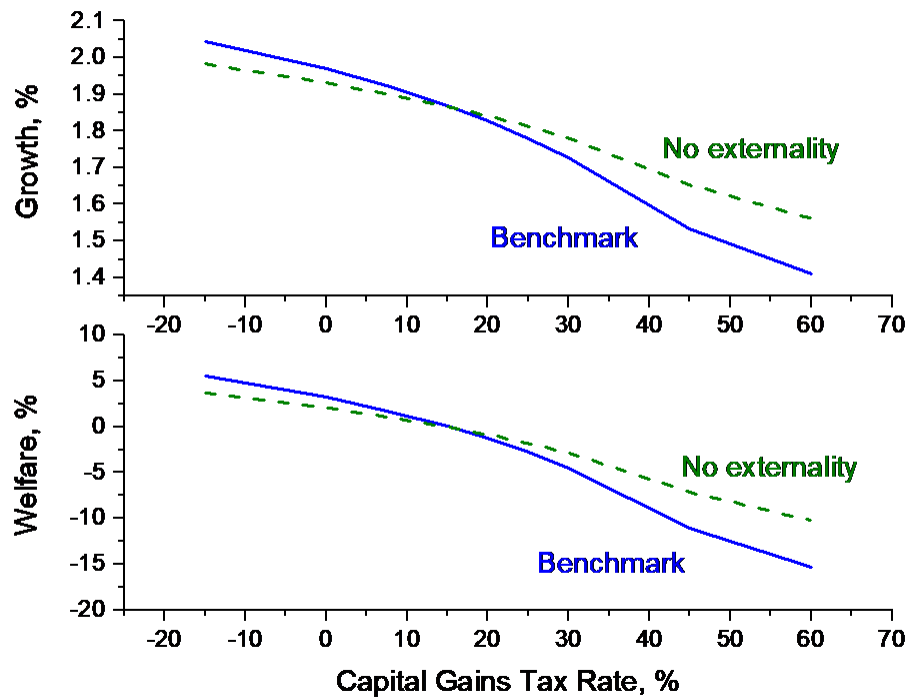
Notes: The upper panel shows the impact of capital gains taxation on economic growth, both for the benchmark economy and the one where there are no externalities in research. The lower panel illustrates the same for welfare.

2.13. What about Growth?

Is the recent rise in VC investment reflected in growth statistics? The answer to this question is nuanced. On the one hand, at the country level VC investment appears to be positively linked with economic growth. A scatter plot between economic growth and VC investment for G7 countries is shown in the upper panel of Figure 26. These are developed nations. As the figure shows, there is a clear positive association between these two variables. The analysis is extended to G20 countries in the bottom panel of the figure. Now the scatter plot includes some poorer countries, where VC investment isn't so prevalent. There is still a positive association, but not surprisingly it is weaker.

To conduct a more formal analysis, some regression analysis is conducted with a sample of 37 economies over the period 1995 to 2014. The sample covers 99 percent of world VC investment and 88 percent of world GDP. In addition, the two-decade sampling period is divided into four sub-periods, each lasting five years. A country is included in the sample if its share of world VC investment between 1995 and 2014 is not less than 0.05 percent.[10] The dependent variable in the regression analysis is the median growth rate of real GDP per capita in each period, while the main explanatory variable is the natural logarithm of the median VC investment-to-GDP ratio. The regressions include the initial levels of real GDP per capita and the Barro and Lee (2013) human capital index. These control variables are the two main factors demonstrated in the empirical literature to be important for economic growth. Moreover, period dummies are included to control for aggregate shocks to all countries. An IV approach is also taken to address the endogeneity issues. Two IVs are used. The first, which follows the strategy pioneered in Barro and Lee (1994), is the median VC investment-to-GDP ratio for each country during the decade preceding the sample period (i.e., 1985 to 1994). The second is a dummy variable for the legal origin of the country, which is equal to 1 for common-law countries. The idea is that common-law legal systems foster better financial development than civil-law legal systems, because of higher judicial independence from the government and the flexibility of the courts to adapt

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[10]An exception is Bermuda, which accounted for 0.18 percent of world VC investment. Bermuda is excluded because it is a tax haven. Companies set up offices there, while undertaking virtually no business activity, just to avoid corporate income taxation.

to changing conditions (see Beck, Demirguc-Kunt, and Levine (2005)).

FIGURE 26: ECONOMIC GROWTH AND VC INVESTMENT, 1995-2014
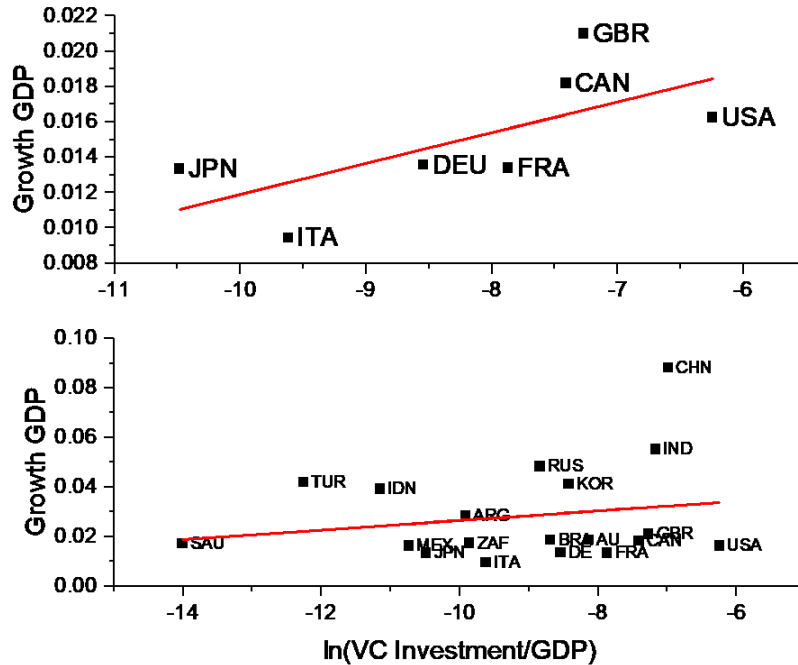
Notes: The upper panel shows the relationship between VC investment and growth in G7 countries, while the bottom panel does the same for the G20.

The main regression results are reported in Table 16. As the table shows, VC and growth are positively correlated. The IV estimate for the G7 countries in the last regression in Panel A shows that a 10 percent increase in the VC investment-to-GDP ratio is connected with a 0.024 percentage point increase in growth. This may seem small, but it implies that increasing the VC investment-to-GDP ratio from the Japanese level (0.003 percent) to the U.S. level (0.19 percent) would increase growth by 1.01 percentage points.[11]

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[11]Relatedly, Sampsa and Sorenson (2011) estimate, using a panel of U.S. metropolitan statistical areas, that VC positively affects startups, employment, and regional income.

Dependent variable		Growth of GDP		
	OLS		IV	
		Pre ln(VC Inv/GDP)	Legal origin	Both
Panel A: G7				
ln(VC Inv/GDP)	0.186**	0.253***	0.227**	0.240***
	(0.0782)	(0.0910)	(0.0899)	(0.0816)
Observations	28	28	28	28
R-squared	0.695			
Panel B: 37-Country Sample				
	0.228**	1.156**	0.421*	0.463*
	(0.112)	(0.501)	(0.254)	(0.260)
Observations	148	120	148	120
R-squared	0.295			

Notes: See the main text for a description of the dependent and independent variables. Pre ln(VC Inv/GDP) refers to the pre-sample VC investment-to-GDP ratio. Standard errors are in parentheses. ***, **, and * denote significance at the 1, 5 and 10 percent levels.

On the other hand, the impact of VC may not be readily apparent in growth statistics for several reasons. First, technological revolutions, such as the Information Age, may cause disruptions in an economy. Old forms of businesses are displaced by new forms. Online retailing is displacing brick and mortar stores, for example. Greenwood and Yorukoglu (1997) discuss how the dawning of the First and Second Industrial Revolutions were associated with productivity slowdowns and suggest that the same phenomena characterize the Information Age. Second, measuring investment and output in the information age is difficult. Think about the introduction of cell phones, as discussed in Hulten and Nakumura (2017). Cell phones substitute for traditional land lines, audio players, cameras, computers, navigation systems, and watches, inter alia. Cell phones have free apps. Between 1988 and 2015, land lines fell from 1.7 to 0.3 percent of personal consumption expenditures. Since cell phones constitute 0.15 percent of personal consumption expenditures, this would be measured as a

drop or slowdown in GDP. An iPhone 5 would have cost more than \$3.56 million to build in 1991.[12] Likewise, global camera production dropped from 120 million units to 40 million over the 2007 to 2014 period. Additionally, investment may be in intangibles, such as software, R&D, retraining workers, reconfiguring products and organizational forms, and branding new products. Corrado, Hulten, and Sichel (2009) estimate that investment in such intangibles is now as large as that in tangibles. Including intangible investment in GDP accounting could increase estimates of growth by 10 to 20 percent. McGrattan and Prescott (2005) argue that, after taking intangibles into account, the 1990s was a boom period. Third, technologies flow across national boundaries. So even countries that don't innovate will experience growth from the adoption of new technologies. Out of France, Germany, Japan, the United Kingdom, and the United States, Eaton and Kortum (1999) find that only the United States derived most of its growth from domestic innovation. Comin and Hobijn (2010) document that diffusion lags for new technologies have shrunk over time. Fourth, firms may park the profits from new innovation offshore to avoid taxation. Accounting for this could increase productivity growth by 0.25 percentage points over the 2004 to 2008 period, according to Guvenen et al. (2017).

2.14. Conclusion

Venture capital is important for economic growth. Funding by venture capitalists is positively associated with patenting activity. VC-backed firms have higher IPO values when they are floated. Following flotation they have higher R&D-to-sales ratios. VC-backed firms also grow faster in terms of employment and sales.

An endogenous growth model of the VC process is constructed and taken to the data. In the framework, entrepreneurs bring ideas to venture capitalists for funding. Venture capitalists provide seed money to research the ideas. After this projects enter a funding-round cycle. During each round, projects are: (i) evaluated to assess their ongoing viability; (ii) those that pass are then provided with VC to develop the project; (iii) the use of funds is monitored

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[12]This 2017 guesstimate was done by Bret Swanson, who calculates that the flash memory, processor, and broadband communications of an iPhone 5 would have cost \$1.44, \$0.62, and \$1.5 million in 1991. The cost of these three components adds up to \$3.56 million. On top of that, considering the other components (camera, iOS operating system, motion detectors, display, apps, etc), it would have cost more than \$3.56 million to build an iPhone 5 in 1991.

is done to ensure that there is no malfeasance; and (iv) successful projects are floated on the stock market or sold to other businesses. The evaluation plan, development funding, the monitoring strategy, and the equity share of the venture capitalist are governed by a dynamic contract between the entrepreneur and a venture capitalist. The model is capable of matching several stylized facts of the VC process by funding round. In particular, it mimics the funding-round profiles for the success and failure rates of projects, the injections of VC for development, the venture capitalist's share of equity, and the value of an IPO by the time it takes to go to market. This is done while matching the share of VC-backed firms in total employment and the average size of a VC-backed firm relative to a non-VC-backed one.

The key personnel involved with starting up the enterprises funded by venture capitalists are rewarded in the form of convertible equity and stock options. In the United States, venture capitalists are subject only to capital gains taxation. The rate at which VC-funded startups are taxed in the United States is low relative to other developed countries. Does this promote innovative activity? The analysis suggests that raising the tax on VC-funded startups from the U.S. rate of 15 percent to the Portugese rate of nearly 60 percent would shave 0.4 percentage points off growth and lead to a long-run consumption-equivalent welfare loss of 15.4 percent.

APPENDIX

A.1. Appendix for Chapter 1

Data Sources

Detailed information of the patents granted in the United States Patent and Trademark Office (USPTO) between 1976 and 2006 is offered in the NBER-USPTO Utility Patents Grant Data (PDP).[1] Accounting information of the publicly held corporations is available in Compustat North American Fundamentals (Annual) retrieved from Wharton Research Data Services. The matching between patent assignees and corporate entities are developed in the NBER Patent Data Project.

Litigation information of a patent is obtained by combining three data sets: Lex Machina Database on Patent Litigations, Derwent Litalert Database on Patent Litigations, and UGA Patent Litigation Datafile. For patents involved in litigation after 2000, Lex Machina Database on Patent Litigations is the most thorough data base.[2] Information on litigated patents before 2000 relies on Derwent Litalert Database on Patent Litigations.[3] However, neither Lex Machina Database nor Derwent Litalert Database provides information on the detailed court decisions. To address this issue, UGA Patent Litigation Datafile is applied. This dataset covers all patent-related cases where at least one court decision (district or appellate court) is recorded in the United States Patents Quarterly (USPQ). USPQ maintained the same publisher since 1929, and it includes the entire opinions for all appellate court decisions and a large sample of district court decisions (especially when they constitute important precedents for later judicial review)[4].

In the cross-country study, the national account information of countries is based on the

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[1]Source link to NBER-USPTO Utility Patents: https://sites.google.com/site/patentdataproject/Home
[2]Source link: https://lexmachina.com/
[3]More detailed information can be found in Galasso, Schankerman, and Serrano (2013).
[4]Source link of UGA Patent Litigation Datafile: http://people.terry.uga.edu/jlturner/patentlitigationdata/

World Development Indicator of the World Bank.[5] The Barro and Lee (2013) human capital index is contained in the Penn World Table.[6] The import and export of each country with the U.S. is gathered from Schott (2008).[7]

A.2. Appendix for Chapter 2

A.2.1. Data Appendix

Figures

- *Figure 15: The rise of venture capital, 1970 to 2015.* Investment by venture capitalists is obtained from the VentureXpert database of Thomson ONE. The fraction of public firms backed by VC companies is created by matching firm names in VentureXpert and CompuStat; the latter are available from Wharton Research Data Services.[8]

- *Figure 16: The share of VC-backed companies in employment, R&D spending, and patents.* The employment and R&D shares of VC-backed public companies are calculated by matching firm names in VentureXpert and CompuStat, as in Figure 15. The share of patents for VC-backed public companies is computed by matching firm names in VentureXpert and the NBER Patent Data Project.[9]

- *Figure 19: Investment and equity share by funding round.* Investment in each funding round is based on the VC-funded deals in Crunchbase between 1981 and 2015. Crunchbase has better funding-round information than VentureXpert. The vertical axis is the mean of funding in a round across all deals, from round 1 (i.e., series A) to round 7 (i.e., series G). Funding is converted into millions of constant $2009 using the GDP deflator. The mean duration of a funding round in Crunchbase is 1.4 years, which is taken to 1.5 years here. The share of equity transferred to the venture capi-

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talist in each funding round is calculated as the ratio of VC funding in each round to the post-money valuation of the company after the VC investment. For each funding round the mean equity share across all deals is calculated. The vertical axis is the cumulated share of equity transferred to the venture capitalist.

- *Figure 20: The odds of success and failure by funding round and the value of an IPO by the duration of funding.* The underlying data source is Puri and Zarutskie (2012, Table VI.B, p. 2271). The success rate refers to firms that have an IPO or that are acquired by another firm. The acquisitions in Puri and Zarutskie (2012) are converted into successes by multiplying by 0.629. This is based on the fact that the cash multiple for acquisitions is 37.1 percent lower than for IPOs, as reported in Achleitner et al. (2012). In addition, the success and failure rates by funding round are obtained by interpolating the original annual data using a cubic spline to get a periodicity of 1.5 years. Puri and Zarutskie (2012, Table V) classify a firm "as having failed if it disappears from the LBD in its entirety." The value of an IPO, as a function of the duration of VC funding, derives from regression (2) in Table 17 (discussed in Section A.2.1).

- *Figure 24: The cross-country relationship between the tax rate on VC activity and the VC investment-to-GDP ratio.* The source for the cross-country data is Henrekson and Sanandaji (2016, Table 1) .

- *Figure 26: Economic growth and VC investment.* VC investment and the growth rate of real GDP per capita are based on VentureXpert of Thomson ONE and the World Development Indicators of the World Bank, respectively.

Tables

- *Table 8: Top 30 VC-Backed Companies.* As in Figure 15, the list of VC-backed public companies is gathered by matching firm names in VentureXpert and CompuStat.

- *Table 9: VC versus Non-VC-Backed Public Companies.* The VC-backed public companies are singled out by matching firm names in VentureXpert and CompuStat. Since the R&D-to-sales ratio and growth rates can be very volatile across firms, the top and bottom 5 percent of the outliers are trimmed in this regression. The results are robust to changing the trimming threshold (at the 1 percent versus 5 percent level).

- *Table 10: VC and Patenting, Firm-Level Regressions.* The VC-funded patentees are identified by matching firm names in VentureXpert and PatentsView.[10] The capital gain taxes are accessed from TAXSIM, an NBER tax simulation program.[11] In calculating the dependence on external finance, 30 percent of selling, general, and administrative expenses is taken as intangible investment. The industry-level of private and federally funded R&D is collected from the Business R&D and Innovation Survey by the National Science Foundation.[12] A truncation adjustment for citations is made following Bernstein (2015). The industry dummies in this regression are at the 2-digit SIC level.

- *Table 11: VC and Patenting, Industry-Level Regressions.* The product of the deregulation dummy and dependence on external finance is used as the IV for the cross term between VC funding and dependence on external finance. The industry panel is based on the 4-digit SIC. The industry dummies in this regression are at 2-digit SIC level.

- *Table 16: VC Investment and Growth, Cross-Country Regressions.* The full sample covers 37 economies between 1995 and 2014. As in Figure 26, VC investment is from VentureXpert and the GDP growth rate is from the World Development Indicators. The Barro and Lee (2013) human capital index is a measure of educational attainment at the country level. The IVs are the median VC investment-to-GDP ratio (in natural logarithm) for each country between 1985 and 1994, and a dummy variable for legal

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[10]Source link of PatentsView: http://www.patentsview.org/download/.
[11]Source link of TAXSIM: http://users.nber.org/~taxsim/state-rates/.
[12]Source link of BRDIS: https://www.nsf.gov/statistics/srvyindustry/#tabs-2.

origin (equal to 1 for common-law countries) à la Beck, Demirguc-Kunt, and Levine (2005).

Duration of VC Funding and the Value of an IPO

The relationship between the firm's value at an IPO and the number of years it received funding from the venture capitalist is examined using regression analysis. The regressions are based on public companies funded by venture capitalists between 1970 and 2015. These VC-backed companies are identified by matching firm names in CompuStat and VentureXpert. The dependent variable in the regressions is the natural logarithm of the market value of the firms at IPO (in $2009). A three-year average is used for market value because of the notorious volatility of share prices following an IPO. IPOs are excluded when they take more than 11 years for the firms to go public after receiving the first VC funding. This is for two reasons: (i) the sampling period is formulated to be consistent with the model where the maximum duration for each VC investment is 10.5 years, and (ii) only 4.5 percent of the observations occur after 11 years with the data being very noisy. The main explanatory variable is the number of years between the firm's first VC funding and the date of its IPO. The findings are shown in Table 17. The first coefficient in regression (2) is used in Figure 20 to plot the decline in the value of an IPO across successive funding rounds.

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Dependent variable	ln(*Firm value at IPO, real*)	
	1	2
years btw first VC funding and IPO	-0.0470***	-0.0385***
	(0.0161)	(0.0146)
firm age at IPO		-0.0246***
		(0.00495)
# of employees at IPO (log)		0.709***
		(0.0375)
year dummy for IPO	N	Y
industry effect	N	Y
Observations	1,042	1,006
R-squared	0.008	0.627

Notes: Standard errors are in parentheses. ***, **, and * denote significance at the 1, 5 and 10 percent levels.

A.2.2. Theory Appendix

Proofs for Lemmas 2 and 4 are supplied in turn here. Lemma 2 establishes the existence of a balanced-growth path. Lemma 4 shows that solving the contract problem (P2) subject to a sequence of one-shot incentive constraints is equivalent to solving it subject to a single consolidated round-0 incentive constraint that allows for multi-shot deviations. This is proved using Lemma 3 as an intermediate step.

Balanced Growth

Lemma 2 *(Balanced Growth) There exists a balanced-growth path of the form outlined in Section 2.9.*

Proof. Suppose that $\{p_t, \sigma_t, \mu_t, \beta_t\}$ solves the old problem for x and $\mathbf{x}$. It will be shown that $\{\mathbf{g}_w p_t, \sigma_t, \mu_t, \beta_t\}$ solves the new one for $x' = \mathbf{g_x} x$ and $\mathbf{x'} = \mathbf{g_x} \mathbf{x}$. First, observe that if $x' = \mathbf{g_x} x$ and $\mathbf{x'} = \mathbf{g_x} \mathbf{x}$, then $I(x'; \mathbf{g_x^t x'}) = \mathbf{g}_w I(x; \mathbf{g_x^t x})$. This occurs because $T(x'; \mathbf{x}_t') = \mathbf{g}_w T(x; \mathbf{x}_t)$. This can be seen from (P1) because x will rise by $\mathbf{g_x}$ and wages by $\mathbf{g}_w$. If $p_t' = \mathbf{g}_w p_t$, then it is immediate from the objective function in (P2) that $C(x'; \mathbf{x'}) = \mathbf{g}_w C(x; \mathbf{x})$. Now, consider the incentive constraint (2.5). At the conjectured solution, the left-hand side will inflate by the factor $\mathbf{g}_w$. So will the right-hand side because $D(\sigma_t') - D(\widetilde{\sigma}_t') = \mathbf{g}_w[D(\sigma_t) - D(\widetilde{\sigma}_t)]$, since all costs are specified as a function of w. Therefore, the new solution still satisfies the incentive constraint. Move now to the zero-profit constraint (2.6). Again, the left-hand side will inflate by the factor $\mathbf{g}_w$, since $p_t' = \mathbf{g}_w p_t$, $\phi_t' = \mathbf{g}_w \phi_t$, $D(\sigma_t') = \mathbf{g}_w D(\sigma_t)$, $M_t(\mu_t') = \mathbf{g}_w M_t(\mu_t)$, $E(\beta_t') = \mathbf{g}_w E(\beta_t)$, and $R(x'/\mathbf{x'}, \mathbf{e}) = \mathbf{g}_w R(x/\mathbf{x}, \mathbf{e})$. This is trivially true for the right-hand side. Hence, the zero-profit constraint holds at the new allocations. It is easy to deduce from the right-hand side of (2.5) that the old solution for $\widetilde{\sigma}_t$ will still hold. This can be seen by using the above argument while noting that $D_1(\widetilde{\sigma}_t') = \mathbf{g}_w D_1(\widetilde{\sigma}_t)$. To sum up, at the conjectured new solution, the objective function and the constraints all scale up by the same factor of proportionality $\mathbf{g}_w$. By cancelling out this factor of proportionality, the new problem reverts back to the old one. Likewise, it is easy to deduce that if x solves problem (P3) for $\mathbf{x}$, then $x' = \mathbf{g_x} x$ solves it when $\mathbf{x'} = \mathbf{g_x} \mathbf{x}$. The occurs because problem (P3) also scales up by the factor of proportionality $\mathbf{g}_w$. When $x/\mathbf{x}$ remains constant along a balanced-growth path, then the initial research cost of the project will rise at the same rate as wages, $\mathbf{g}_w$. Additionally, $V(\mathbf{x})$ will grow the same rate as wages, w, so from (2.7) it is apparent that $\mathbf{e}$ will remain constant. $\blacksquare$

One-Shot Deviations versus Multi-Shot Deviations

This is an intermediate step toward solving Lemma 4. To this end, it will be shown that if the incentive constraint (2.5) holds for round t, when the entrepreneur has not deviated up to and including round $t-1$, then it will also hold when he follows some arbitrary path of deviations up to and including round $t-1$. Let α_t represent that the probability that a

project is good at round t as defined by (2.4). These odds evolve recursively according to

$$\alpha_{t+1} = \frac{(1 - \sigma_t)\alpha_t}{(1 - \sigma_t)\alpha_t + (1 - \beta_{t+1})(1 - \alpha_t)},$$

where $\alpha_1 = \rho/[\rho + (1-\rho)(1-\beta_1)]$. For use in proving Lemma 3, note that α_{t+1} is increasing in α_t and decreasing in σ_t. This implies that if the entrepreneur deviates in round t, so that $\widetilde{\sigma}_t < \sigma_t$, he will be more optimistic about the future, as α_{t+1} will be higher. This increases the value of the α's for future rounds as well. With this notation, the round-t incentive constraint (2.5) then reads

$$\alpha_t(1 - \tau)\{\delta\sigma_t[I(x; \mathbf{g_x^t x}) - p_t] + (1 - \sigma_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g_x^i x}) - p_i]\}$$

$$\geq (1 - \mu_t)\max_{\widetilde{\sigma}_t}\bigg(D(\sigma_t) - D(\widetilde{\sigma}_t)$$

$$+ \alpha_t(1 - \tau)\{\delta\widetilde{\sigma}_t[I(x; \mathbf{g_x^t x}) - p_t] + (1 - \widetilde{\sigma}_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g_x^i x}) - p_i]\}\bigg).$$

Lemma 3 *If the incentive constraint (2.5) holds for round t, when the entrepreneur has not deviated up to and including in round $t - 1$, then it will also hold when he follows some arbitrary path of deviations up to and including in round $t - 1$.*

Proof. Suppose that the entrepreneur deviates in some manner before round t. Let $\widehat{\alpha}_t$ be the prior associated with this path of deviation. Since the $\widetilde{\sigma}$'s will be less that than the σ's, it follows that $\widehat{\alpha}_t > \alpha_t$. Let $\widehat{\sigma}_t$ be the *optimal* round-t deviation associated with $\widehat{\alpha}_t$. Now,

$$\alpha_t(1 - \tau)\{\delta\sigma_t[I(x; \mathbf{g_x^t x}) - p_t] + (1 - \sigma_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g_x^i x}) - p_i]\}$$

$$\geq (1 - \mu_t)\bigg(D(\sigma_t) - D(\widehat{\sigma})$$

$$+ \alpha_t(1 - \tau)\{\delta\widehat{\sigma}_t[I(x; \mathbf{g_x^t x}) - p_t] + (1 - \widehat{\sigma}_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g_x^i x}) - p_i]\}\bigg),$$

because $\widetilde{\sigma}_t$ is maximal when the prior is α_t, while $\widehat{\sigma}_t$ is not. Next, replace α_t with $\widehat{\alpha}_t$ to get

$$\widehat{\alpha}_t\{\delta\sigma_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t] + (1-\sigma_t)\sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x}) - p_i]\}$$

$$\geq (1-\mu_t)\bigg(D(\sigma_t) - D(\widehat{\sigma}_t)$$

$$+ \widehat{\alpha}_t\{\delta\widehat{\sigma}[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t] + (1-\widehat{\sigma}_t)\sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x}) - p_i]\}\bigg),$$

since $\widehat{\alpha}_t > \alpha_t$. Last, if the prior is $\widehat{\alpha}_t$, then $\widehat{\sigma}_t$ is maximal, so the above equation can be rewritten as

$$\widehat{\alpha}_t(1-\tau)\{\delta\sigma_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t] + (1-\sigma_t)\sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x}) - p_i]\}$$

$$\geq (1-\mu_t)\max_{\widehat{\sigma}_t}\bigg(D(\sigma_t) - D(\widehat{\sigma}_t)$$

$$+ \widehat{\alpha}_t(1-\tau)\{\delta\widehat{\sigma}_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t] + (1-\widehat{\sigma}_t)\sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x}) - p_i]\}\bigg).$$

∎

The Consolidated Round-0 Incentive Constraint

The consolidated round-0 incentive constraint is

$$(1-\tau)\sum_{t=1}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t] \geq$$

$$\max_{\{\widetilde{\sigma}_t\}_{t=1}^{T}}\{\sum_{t=1}^{T}\delta^{t-1}[\rho\Pi_{j=1}^{t-1}(1-\widetilde{\sigma}_j) + (1-\rho)\Pi_{j=1}^{t}(1-\beta_j)]$$

$$\times(1-\mu_t)[D(\sigma_t) - D(\widetilde{\sigma}_t)]$$

$$+(1-\tau)\sum_{t=1}^{T}\rho\Pi_{j=1}^{t-1}(1-\widetilde{\sigma}_j)\delta^t\widetilde{\sigma}_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t]\}.$$

$$(A.1)$$

Lemma 4 *(Equivalence of contracts) A contract $\{\beta_t, \sigma_t, \mu_t, p_t\}$ solves problem (P2) subject*

to the sequence of one-shot incentive constraints (2.5) if and only if it solves (P2) subject to the consolidated time-0 incentive constraint (A.1).

Proof. (*Necessity*) Suppose that an allocation satisfies the one-shot incentive compatibility constraints (2.5) but that it violates the consolidated one (A.1). This implies that at some round in the problem with the consolidated constraint it pays to deviate and pick a $\widetilde{\sigma}_t \neq \sigma_t$. Pick the last round of deviation (which may be T). It must be true that $\widetilde{\sigma}_t$ solves the maximization problem

$$(1 - \mu_t) \max_{\widetilde{\sigma}_t} \bigg(D(\sigma_t) - D(\widetilde{\sigma}_t)$$

$$+ \widehat{\alpha}_t(1 - \tau)\{\delta\widehat{\sigma}[I(x; \mathbf{g}_{\mathbf{x}}^t\mathbf{x}) - p_t] + (1 - \widetilde{\sigma}_t)\sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g}_{\mathbf{x}}^i\mathbf{x}) - p_i]\} \bigg),$$

where $\widehat{\alpha}_t$ is the prior associated with the path of σ's up to round $t - 1$, which may include previous deviations. But, as was shown in Lemma 3, this is less than the value of sticking with the contract or

$$\widehat{\alpha}_t(1 - \tau)\{\delta\sigma_t[I(x; \mathbf{g}_{\mathbf{x}}^t\mathbf{x}) - p_t] + (1 - \sigma_t)\sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g}_{\mathbf{x}}^i\mathbf{x}) - p_i]\},$$

when the round-t one-shot incentive constraint (2.5) holds, as assumed.

(*Sufficiency*) Suppose $\{\sigma_t\}_{t=1}^{T}$ satisfies the consolidated incentive constraint, but violates

the one-shot incentive constraint at round k. Then, using (2.4) and (2.5), it follows that

$$\rho\Pi_{j=1}^{k-1}(1-\sigma_j)\delta^{k-1}(1-\tau)\{\delta\sigma_k[I(x;\mathbf{g}_{\mathbf{x}}^k\mathbf{x})-p_k]$$

$$+(1-\sigma_k)\sum_{t=k+1}^{T}\Pi_{j=k+1}^{t-1}(1-\sigma_j)\delta^{t+1-k}\sigma_t[I(x;\mathbf{g}_{\mathbf{x}}^t\mathbf{x})-p_t]\}$$

$$=(1-\tau)\sum_{t=k}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_{\mathbf{x}}^t\mathbf{x})-p_t]$$

$$<\delta^{k-1}(1-\mu_k)\bigg([\rho\Pi_{j=1}^{k-1}(1-\sigma_j)+(1-\rho)\Pi_{j=1}^{k}(1-\beta_j)][D(\sigma_k)-D(\widetilde{\sigma}_k)]$$

$$+\rho\Pi_{j=1}^{k-1}(1-\sigma_j)(1-\tau)\{\delta\widetilde{\sigma}_k[I(x;\mathbf{g}_{\mathbf{x}}^k\mathbf{x})-p_k]$$

$$+(1-\widetilde{\sigma}_k)\sum_{t=k+1}^{T}\Pi_{j=k+1}^{t-1}(1-\sigma_j)\delta^{t+1-k}\sigma_t[I(x;\mathbf{g}_{\mathbf{x}}^t\mathbf{x})-p_t]\}\bigg).$$

$$(\text{A.2})$$

The left-hand side gives the payoff in the contract at the optimal solution from round k on, when using the consolidated incentive constraint, while the right-hand side represents the payoff from a one-shot deviation at round k. Now the objective function for the contract can be written as

$$(1-\tau)\sum_{t=1}^{k-1}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_{\mathbf{x}}^t\mathbf{x})-p_t]+(1-\tau)\sum_{t=k}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_{\mathbf{x}}^t\mathbf{x})-p_t].$$

Evaluate this at the optimal solution for the contract when using (A.1) instead of (2.5). Next, in this objective function, replace the payoff from round k on, as represented by the left-hand side of (A.2), with the payoff from the one-shot deviation as given by the right-hand side. This deviation would increase the value of the objective function for the entrepreneur, which is a contradiction. ∎

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