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The Effect Of Dynamic Pricing And Revenue Management On Agent Behavior And Customer Perception

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Abstract
My dissertation extends the traditional fields of revenue management and dynamic pricing to newer markets. Specifically, my first two chapters explore the revenue management strategies and their impacts in the airline industry in the presence of loyalty programs. The first chapter solves the optimal revenue management algorithms when the firm is rewarding frequent customers with free capacity. Using a game-theoretic Littlewood model, we show that limiting award capacity can increase profits by enhancing loyalty award values; airlines can benefit from transitioning from mileage-based programs to revenue-based programs by simplifying its revenue management algorithm and allowing 100% award availability. The second chapter investigates customers' evaluations of loyalty program points. By fitting a Multinomial Logit model on DB1B data set, we calibrate customers' valuations for loyalty points at the issuance and redemption. We have two main conclusions: consumers are rational about the value of miles at issuance, but underestimate and overspend miles at redemption; higher award availability and more award choices lead to higher values of Loyalty points. Finally, my third chapter examines the impact of dynamic pricing in the ride-sharing economy. By using actual Uber pricing and partner data, the paper shows that ride-sharing platforms can efficiently signal market conditions, stimulate desirable agents' behavior, and reduce marketplace frictions through dynamic pricing.

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THE EFFECT OF DYNAMIC PRICING AND REVENUE MANAGEMENT ON AGENT BEHAVIOR AND CUSTOMER PERCEPTION

Xingwei Lu

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THE EFFECT OF DYNAMIC PRICING AND REVENUE MANAGEMENT ON AGENT BEHAVIOR AND CUSTOMER PERCEPTION

Dedicated to My Parents

ABSTRACT

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My dissertation extends the traditional fields of revenue management and dynamic pricing to newer markets. Specifically, my first two chapters explore the revenue management strategies and their impacts in the airline industry in the presence of loyalty programs. The first chapter solves the optimal revenue management algorithms when the firm is rewarding frequent customers with free capacity. Using a game-theoretic Littlewood model, we show that limiting award capacity can increase profits by enhancing loyalty award values; airlines can benefit from transitioning from mileage-based programs to revenue-based programs by simplifying its revenue management algorithm and allowing 100% award availability. The second chapter investigates customers' evaluations of loyalty program points. By fitting a Multinomial Logit model on DB1B data set, we calibrate customers' valuations for loyalty points at the issuance and redemption. We have two main conclusions: consumers are rational about the value of miles at issuance, but underestimate and overspend miles at redemption; higher award availability and more award choices lead to higher values of Loyalty points. Finally, my third chapter examines the impact of dynamic pricing in the ride-sharing economy. By using actual Uber pricing and partner data, the paper shows that ride-sharing platforms can efficiently signal market conditions, stimulate desirable agents' behavior, and reduce marketplace frictions through dynamic pricing.

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CHAPTER 1 : REVENUE MANAGEMENT WITH LOYALTY PROGRAMS

This paper studies loyalty programs in firms such as airlines and hotels, where limited capacity is commonplace and revenue management is crucial. Based on Littlewood's classic two-type model, our model additionally reserves some capacity for rewards and allows customers to choose between paying with cash and redeeming with points. We have three conclusions. First, we show that revenue management algorithms need to be adjusted to include award liability, i.e. the cost of issuing points to customers. However, the adjustment can be neglected if the number of issued points is proportional the customers' purchasing price. Second, the optimal award capacity is constrained by a fixed level of redemption probability in loyalty points. However, the redemption probability can be as high as 100% if the number of redeemed points is proportional to the price. Finally, several airlines (American, United and Delta) recently switched from rewarding customers based on their purchasing quantity (volume-based) to rewarding them based on their purchasing expense (expense-based). Other airlines (Southwest and JetBlue) decide both the issuance and redemption based on the purchasing price (point-based programs). We compare the pros and cons of these program schemes. We show that volume-based schemes enhance profits but generate accounting challenges. Expense-based schemes maintain profitability while eliminating accounting challenges. Point-based schemes lose these profits in return for high customer satisfaction, with a 100% award availability.

1.1. Introduction

Loyalty programs are ubiquitous among service firms such as airlines, hotels and rental businesses. Well-known examples include American Airlines AAdvantage, Hilton HHonors, and Hertz Gold Plus, all of which reward frequent patronage with free services. Whether they are free flights or free hotel stays, loyalty rewards all take up capacity, which is a constrained resource in service firms. While firms strive to fulfill their obligation of giving out rewards to eligible customers, they have to accept the reality that every reward may potentially displace a cash-paying customer. There is a constant tug-of-war between reward and cash customers vying for the same pool of capacity. This problem immediately calls to mind revenue management techniques, which have been developed to sell the right product to the right customer at the right price. Our goal in this paper is to study optimal revenue management strategies in the presence of loyalty programs.

There is substantial variation in the amount of capacity that firms set aside for loyalty program members. Consequently, reward availability differs widely across firms. In a survey on reward seat availability of 20 airlines, Southwest Airlines enjoys the first place with 100% availability, while US Airways is at the bottom, trailing with an reward availability of 35% (Ideaworks, 2015). Because of the limited reward availability, on average, about 15% to 20% of issued miles are never redeemed (Gerchick, 2013). Similarly, the hotel industry also exhibits some variation in reward availability. In a survey of seven hotel loyalty programs (BoardingArea, 2015), Marriott Rewards tops the chart with an availability rate of 99%, while Choice Privileges ends up at the bottom with an availability rate of 81%. Such variation suggests that there is no simple formula to the question of how much capacity should be allocated to loyalty rewards. We shall examine this issue in this paper.

To facilitate reward transactions, loyalty points have emerged as a virtual currency. Typically, customers earn points for their purchases and subsequently redeem points for rewards. When an reward is issued, the firm receives no cash income and merely retrieves a bulk of the faux currency that was previously issued. Despite the apparent lack of dividends, it appears that loyalty rewards somehow still pay off. For example, frequent flyer program members are willing to spend 2% to 12% more for similar itineraries provided by the program carrier than by other airlines (Brunger, 2013), 67% of travelers report that hotel loyalty programs are highly influential in their choices (Cognizant, 2014), and restaurant loyalty programs increase visits by 35% (Loyalogy, 2014). One possible theory for the increased profit is that loyalty points do carry value to customers. In fact, casual assessments tend to put the value of most loyalty points at between \$0.01 and \$0.02 each; for example, an AAdvantage mile is estimated to be worth $0.017 (BoardingArea, 2014). The value of points hinge greatly upon the value and availability of the award the points can redeem for. In this spirit, we analyze how revenue management rules impact the value of points and customer decisions.

The issuance and redemption of loyalty points can be fixed or price-dependent. For example, consider Traveler A who commutes between Philadelphia and San Francisco every month. A full-fare roundtrip ticket is priced at $800, but a discounted airfare at $300 is occasionally offered. If he is enrolled in a volume-based program, such as the old version of American Airlines' AAdvantage program, he would be rewarded the same number of miles (5,030) for each ticket regardless of the price. However, in 2016, American Airlines started to reward customers based on how much they pay. Specifically, the new program issues 5 miles for every dollar customers spent. Hence, the customer earns 4,000 miles for a full-fare ticket and 1,500 miles for a discount-fare ticket. We refer to the new program as expense-based program. American Airlines was not the first to make this change. In fact, United and Delta Airlines abandoned the traditional mileage program and switched to an expense-based program in 2014. The media reaction to such design change is mixed. Supporters argue that the change is more fair to high-paying customers (Forbes, 2013) and may help slow an ongoing trend of mileage devaluation (Airline Weekly, 2015). However, critics believe that leisure customers are more responsive to loyalty program incentives (New York Times, 2014, Bloomberg, 2014).

Note that in both volume-based and expense-based programs, the redemption of loyalty points are price-independent. Specifically, Traveler A is required the same number of frequent flyer miles (usually 250,000 miles) to redeem any roundtrip ticket regardless of its price. However, this is not the case for JetBlue's and Southwest's loyalty programs, in which the number of miles needed for an award is proportional to the price. For example, if the traveler is enrolled Southwest Rapid Rewards program, he is required to pay 70 miles for every dollar he redeems for. Therefore, the traveler pays 56,000 miles for a full-fare ticket ($800) and pays 21,000 for a discount-fare ticket ($300). We refer to these programs as point-based programs.

This paper aims to study the interaction between revenue management and loyalty programs. Specifically, we focus on the following three questions.

1. How to characterize revenue management decisions in the presence of loyalty programs?

2. How should firms determine the amount of capacity to set aside for loyalty awards?

3. What are pros and cons of each type of loyalty programs?

To answer these questions, we gathered empirical evidence from participants about their perceptions of loyalty programs. Based on the evidence, we incorporate loyalty programs into Littlewood's (1972) model of quantity-based revenue management. In the classic model, the firm sells a limited capacity by allocating it between low-paying customers already seeking to buy and high-paying customers who may not arrive; in our model, we add loyalty awards as a third use of the firm's capacity. Customers choose between paying cash and redeeming awards to maximize utility. We solve for the customers' medium of purchase and the firm's revenue management decisions in equilibrium under three different program designs. We have three conclusions.

First, we show that revenue management algorithms need to be adjusted by including award liability into prices. The award liability reflects the expected opportunity cost of fulfilling future redemptions of loyalty points. Nevertheless, this adjustment becomes redundant when the issuance of points is proportional to prices (expense-based and point-based schemes). In such cases, revenue management decisions are prescribed as if there is no loyalty programs.

Second, the optimal award capacity is constrained by quantity sold to customers. The reason is that the firm needs to restrict the redemption probability of loyalty points and limit their values, so that the customers prefer to use points immediately rather than hoard them for future use, since "future use" may never materialize. However, if the number of redeemed points is proportional to the price (point-based schemes), a 100% award availability rate can be optimal, which can be explained below. The specific redemption rule creates a fixed conversion rate between cash and point. On the demand side, customers have no incentives to hoard points for future uses. On the supply side, the firm can treat award customers and cash-paying customers equally, and provide an award availability rate as high as 100%.

Finally, we compare the three types of program schemes. Volume-based schemes enhance profits but generate accounting challenges; expense-based programs maintain profitability while eliminating accounting challenges; point-based programs give up these profits in return for customer satisfaction with a 100% award availability. As explained in the previous point, a low redemption probability induces customers to spend loyalty points more immediately. In fact, customers spend more than they are willing to pay in cash. Consequently, when giving out loyalty awards and collecting payment in the form of loyalty points, the firm can extract a higher payment than what could have been possible with cash. In contrast, point-based programs create a fixed conversion rate between points and cash, which does not breed overspending behavior of loyalty points. Hence, point-based programs are less profitable but also less restrictive - firms need not maintain a low redemption probability and find it optimal to accept any redemption requests.

1.2. Literature Review

There has been extensive research on loyalty programs in the marketing literature. Readers can refer to Bijmolt et al. (2010) and Breugelmans et al. (2014) for recent reviews. This body of work seeks to measure the effect of loyalty programs using sales data and results are mixed. Early studies (e.g., Sharp and Sharp, 1997) did not find significant evidence of increased purchase frequency. There were also results suggesting that loyalty programs, even if profitable, do not derive benefit from frequent buyers: loyalty programs have the least impact on these customers (Lal and Bell, 2003), and yet they are the ones most likely to claim rewards (Liu, 2007). However, Bolton et al. (2000) showed that members of loyalty programs discount or overlook negative service experiences. In another study, Taylor and Neslin (2005) demonstrated both a points-pressure effect (customers buy more as they get closer to earning rewards) and a rewarded-behavior effect (customers buy more after savoring the benefit of rewards). In terms of methodology, Lewis (2004) introduced a structural modeling framework to model repeated purchase decisions as a dynamic program and found that the loyalty program being studied was successful in increasing purchases for a substantial fraction of customers. While the above papers focused on the frequency reward component, Kopalle et al (2012) also considered the customer tier component (e.g., silver or gold status) of loyalty programs; using a dynamic structural model, they found that customers buy more as they approach the next tier. Similar to most papers above, we focus on the frequency reward component of loyalty programs, but we ask a new set of question: how should firms adjust the value of loyalty points through capacity allocations and pricing strategies? How would these decisions change under volume-based and expense-based programs? These aspects can have a significant impact on customer behavior and firm profits in industries such as airlines and hotels when price fluctuations are commonplace.

Our work is also related to the literature on consumer behavior in the context of loyalty programs. Many papers have studied how consumers perceive and value loyalty points as an independent currency. Using a reference dependence framework, Drèze and Nunes (2004) developed a mental accounting model where customers evaluate different currencies (i.e., cash and loyalty points) in separate accounts; Stourm et al. (2015) recently extended this mental accounting model to explain why many customers stockpile loyalty points even though the firm does not reward such behavior. In another study, van Osselaer et al. (2004) showed that loyalty points are an overvalued currency and create an illusion of progress. In a similar vein, Kivetz et al. (2006) and Nunes and Drèze (2006) showed that artificial advancement (e.g., replacing a 10-stamp coffee card with a 12-stamp card that starts with 2 stamps already filled in) increases customer effort; the former study also found evidence of purchase acceleration as customers come closer to earning rewards. These results suggest that customers place an explicit value on each loyalty point even though loyalty points are only a medium (i.e., a means to an end); see Hsee et al. (2003) on the medium effect. Finally, Raghubir and Srivastava (2002) and Wertenbroch et al. (2007) found that consumers' valuation of an unfamiliar currency (such as loyalty points) is biased towards the face value; a possible explanation is that consumers anchor on the nominal face value and do not adjust sufficiently for the exchange rate when making decisions. Sayman and Hoch (2014) showed that buyers are willing to pay a price premium for loyalty points, and the premium is less than the normative levels. Motivated by these behavioral studies, our theoretical model takes the view that each loyalty point is a unit of currency valued at the nominal face value of goods that it can be redeemed for.

It is useful to put our work in the context of existing research that elucidates the economic function of loyalty programs. The bulk of this research focuses on the switching costs generated by loyalty programs (for example, travelers who have accumulated many miles at an airline will not be keen to switch to another airline). Consequently, loyalty programs soften price competition and facilitate tacit collusion; see Kim et al. (2001), Singh at al. (2008) and Fong and Liu (2011) for models along these lines. Another economic explanation for loyalty programs is price discrimination. Since frequency rewards such as buy-n-get-one-free are a type of quantity discounts, loyalty programs can facilitate price discrimination between frequent and occasional customers (Hartmann and Viard, 2008), or between "cherry pickers" who buy from lowest-priced stores and single-store-shoppers (Lal and Bell, 2003), or between heavy and light users (Kim at al, 2001). Next, it has also been demonstrated that loyalty programs enable firms to profit from the agency relationship between employers and employees. Typically, employers pay for business trips but employees reap the benefits from loyalty rewards; see Cairns and Galbraith (1990) and Basso et al. (2009). In another study, Kim et al. (2004) showed that loyalty programs can help regulate capacity in face of demand uncertainty: when demand is low, firms can offer loyalty rewards to reduce excess capacity and ease the pressure to slash prices. Although we have limited capacity in our model, our results do not rely on this mechanism because in our model, capacity is allocated for redemption before demand uncertainty is realized. Instead, our analysis highlights a new function of loyalty programs: since loyalty points are appraised at face value, they enable firms to extract surplus when customers redeem points on items that they are unwilling to pay cash to buy.

Another stream of related literature is the revenue management literature on capacity controls. In most models, the firms allocates capacity to different booking classes, and when a lower-priced booking class is sold out, customers can only purchase at a higher-priced booking class. Such capacity allocation decisions trace back to Littlewood (1972), who showed using a two-class model that current bookings should be accepted as long as their revenue exceed the expected value of future bookings. This work has been extended to multiple booking classes (e.g., Wollmer, 1992, Brumelle and McGill, 1993, Robinson, 1995) in arbitrary order of arrival (e.g., Lee and Hersh, 1993, Lautenbacher and Stidham, 1999). Now known as Littlewood's rule, the original model has also been the basis for the expected marginal seat revenue heuristics, which were proposed by Belobaba (1989) and widely used in revenue management practice (see comprehensive reviews by McGill and van Ryzin, 1999, Bitran and Caldentey, 2003, Elmaghraby and Keskinocak, 2003, and the reference book by Talluri and van Ryzin, 2004). Subsequent revenue management models of capacity controls incorporate additional complexities such as buy-up behavior (Belobaba and Weatherford, 1996), customer substitution (Shumsky and Zhang, 2004), choice between parallel flights (Zhang and Cooper, 2005), and competition (Netessine and Shumsky, 2005). Our analysis in this paper is based on Littlewood's rule, but our research takes a different perspective: instead of allocating capacity to lower-priced classes, we are interested in allocating capacity for redemption of loyalty rewards. In fact, given that this is a central concern in any capacity-constrained firm running a loyalty program, we are surprised that there has been little to no work on understanding the interactions between capacity and loyalty rewards.

More recently, over the last decade, the literature on revenue management and dynamic pricing has paid more attention to strategic customer behavior (see Netessine and Tang, 2009, for a review). When making purchase decisions, customers adopt a forward-looking perspective and take future price changes and potential stock-outs into consideration (see, e.g., Su, 2007, Liu and van Ryzin, 2008, Aviv and Pazgal, 2008). Such a dynamic customer perspective is particularly important in the context of loyalty programs because frequency rewards earned over multiple purchases are inherently dynamic. In our model, not all loyalty points will be redeemed and capacity may not always be available for redemption; such factors influence the value of loyalty points and are incorporated using modeling approaches in Su and Zhang (2008) and Cachon and Swinney (2009). The literature has also studied consumer stockpiling of purchases (Su, 2010, Besbes and Lobel, 2015), which may be relevant for accumulation of loyalty points, but we do not consider stockpiling in this paper.

Recently, we are thrilled to see a few papers in the operations literature on the optimal design of loyalty programs. Sun and Zhang (2015) examine the expiration terms of customer reward programs and find that a finite expiration term can increase firm profits, even without accelerating consumer purchases. Our model does not specify the expiration terms, so it applies to the airline and hotel industry where unused points can rollover. Chun and Ovchinnikov (2015) study the customer tier component of loyalty programs (eg, requirements to reach gold status), while we focus on the frequency reward component of loyalty programs (eg, requirements for a free flight). Methodologically, to capture customers' intertemporal decisions, Sun and Zhang (2015) develop a full dynamic programming model, while Chun and Ovchinnikov(2015) allow customers to choose how many flight to fly over a year. In contrast, our model simplifies the decision dynamics by incorporating the value of the loyalty program currency, which is endogenously determined by firm and customers' strategic behavior.

1.3. Behavioral Evidence of Consumer Model

Fundamentally, loyalty programs create a new option for customers: buying with points. The starting point for any model-building activity is to understand how customers perceive this option. Consider a customer who is eligible to redeem for an award. The first question we address here is to find the plausible model for his redemption behavior.

At the most basic level, the model will involve (at least) a binary choice between buying with cash (pay the current price) and buying with points (pay the loyalty points). The goal of this section is to establish the boundary of a plausible model for that binary decision.

Consider a customer who makes the binary decision: cash or points. It is clear that the customer chooses to pay points when the current cash price is high and vice versa. Hence, there is a maximum price that he will pay to keep his points. This maximum price can be seen as a proxy for the customer's "value of points". Note that if the customer chooses to pay the maximum price, he automatically hoards the points for future purchases. Hence, the "value of points" hinges on their future purchasing power. The question is how the customer determines this "purchasing power". Specifically, we ask the following questions. What are the metrics the customers rely on for the evaluation of the "purchasing power"? Is the "purchasing power" determined dynamically or one-shot? In the rest of this section, we shall discuss each of the two questions separately.

First, intuitively, the "purchasing power of points" should be dependent on the customer's expectation about the future prices that these points can redeem for. In fact, there is ample evidence in the literature supporting this theory. Thaler (1985) proposes that consumers consider not only the benefits from the good they might buy but also the perceived merits of the deal: whether the actual price is higher or lower than they expect. Take the airline industry for example. When travelers decide the time to redeem their frequent flyer miles, they are not only concerned about the benefit and convenience generated from the flight ticket, but also whether the purchase is a "good bargain". In Thaler's study, participants imagined sitting on a beach with a friend who had just offered to bring them back a bottle of their favorite beer. When told that beer would be purchased from a fancy hotel, participants authorized their friend to spend \$2.65, but when told the retailer was a run-down grocery store, they were willing to pay just \$1.50. In other words, the expectation to pay became the willingness to pay, i.e. the customer's maximum price to keep the beer is linked to his expectation to pay in that store.This is referred to as the Reference

Price Theory (Weaver and Frederick, 2012). In the context of airlines, it is likely that the customer's maximum price to keep the points are linked to the customer's expectation to pay for a future redeemable flight ticket. If this is true, the effect of expectation (reference price) should be reflected in customers' behavior: when the expectation of future prices is higher, points become more worthy and the maximum price customers pay to keep points is higher.

Second, the value of points may be determined either dynamically or one-shot. If it is determined dynamically, it must be state-dependent, i.e. the value of points varies with how many points the customer has already accumulated. Otherwise, it is state-independent. We shall discuss each case separately. In our previous airline example, a customer trying to redeem frequent flyer miles must be aware of the potential trips that he can accumulate miles from or redeem miles for in the future, and have rational beliefs about their future utilities and market prices in advance. To fully capture this process, a dynamic program over multiple periods is required. In each period, the customer may incur a need to take a flight. In this dynamic program, the customers' state variable is the amount of miles they accumulated in their frequent flyer account, and their decision variable is whether to redeem the miles or pay cash for each trip. Even if a full-fledged dynamic program is implausible, as long as there is any dynamic consideration (even if imperfectly so), there will be some state dependence, and decisions will depend on mileage balance. Intuitively, it is quite obvious that with a higher mileage balance, the customer is more likely to use miles more freely.

On the other hand, evidence suggests that some customers may view the evaluation of points as a one-shot decision. Specifically, many frequent flyers follow a simple rule of thumb: they form an implicit estimation of the average value of one mile, and conclude that it is a good deal to use miles when the value of a mile given the current price exceeds the average value of a mile. For example, Tripadvisor.com calculated that each mile is worth ¢1.4. This was derived from dividing the average domestic roundtrip ticket price $350 by the required 25,000 miles. Using this baseline, customers can calculate whether any ticket is worth using the miles for. A frequent flyer (Smarttravel) has the following examples: "Cashing in 25,000 miles for a ticket that could be purchased for \$100 yields just ¢0.4. On the other hand, redeeming 100,000 miles for a business-class ticket to Europe priced at almost \$11,000 yields a nominal per-mile value of ¢11, slightly less with the hassle factor adjustment." Following this rule of thumb, a customer should purchase the \$100 ticket in cash and the \$11,000 ticket in miles. Some frequent flyers have developed online spreadsheets to calculate when to redeem miles, while the exact baseline can be adjusted to different airlines, programs, or even the customers' own calibration. Instead of evaluating miles dynamically, customers stick to a fixed value of miles when making the binary decision. Nevertheless, they are still strategic in the sense that they balance the tradeoff between using the miles right away (which gives a value of ¢0.4 cent and ¢11 cents per mile in the previous examples respectively), and hoarding the miles for later use (which yields an expected value of ¢1.4 per mile). Note that the Tripadvisor.com example not only supports the one-shot evaluation hypothesis, but also echoes the reference theory: the evaluation of points depends on the average price \$350 of a flight ticket, which sets customers' expectation of the future price.

Both the "reference price" theory and the "dynamic vs one-shot" hypotheses are plausible, we need to run a study. In the study, we shall test whether the customers' redemption decisions are price-dependent (reference theory) and state-dependent (dynamic vs one-shot).

Experiment Design Participants (N = 510) were recruited on Amazon Mechanical Turk. They were asked to complete a survey. In the beginning of the survey, they answered the screening question whether they are members of any frequent flyer programs. Then they considered the hypothetical situation to choose a flight destination that they would like to redeem using miles. They were first told the following: "Imagine that you have accumulated enough miles for a free round-trip flight anywhere in the continental US. Where is your most likely destination?" Choosing the destination initially pins down the flight before any manipulations come in.

They then examined the option to pay the average cash price for the chosen flight. In the high (low) average price segment, participants were asked: “Next, imagine that a trusted friend familiar with airlines tells you that the average price of a round-trip flight is \$800 (\$300). If you did not have any miles, are you willing to pay \$800 (\$300) for this flight?”

Finally, they examined the options between paying cash and redeeming miles for the flight. They were given information about their mileage balance (200,000 or 50,000), the average price of the redeemable flight (\$800 or \$300 as in the previous question), and the current price (ranging from \$0 to \$1000 with increment of \$100), which might differ from the average price. In the high (low) average price/mileage balance group, participants are asked “You have 200,000 (50,000) miles in your account and you can use 25,000 miles to pay for the flight. The average price for the flight is \$800 (\$300) but the actual price you find might be higher or lower. At each price below, do you prefer to pay money or use miles?” The participants then chose from a list of current prices and identified the maximum price they would like to pay to keep their miles, i.e. their willingness to pay for the miles (WTP). Specifically, the midpoint of the two prices where the customer switched from “money” to “miles” is used to calculate WTP.

Therefore, the experiment should be able to answer the following question related to the binary decision: whether it is state-dependent and subject to reference-price effects.

Results We restrict our analysis on 244 participants who responded that they were enrolled in some frequent flyer programs. These participants have had previous experience with the accumulation and redemption of loyalty points and their behavior will most reflect the loyalty program members’ decisions in practice. The key statistics are summarized in Table 1:

Table 1: Summary of Statistics

Group	G1	G2	G3	G4
Average Price	\$800	\$300	\$800	\$300
Mileage Balance	200k	200k	50k	50k
Number of Participants	60	64	59	61
Average WTP	\$413.3	\$306.3	\$428.8	\$342.6
Standard Deviation	\$182.7	\$154.2	\$170.2	\$133.5

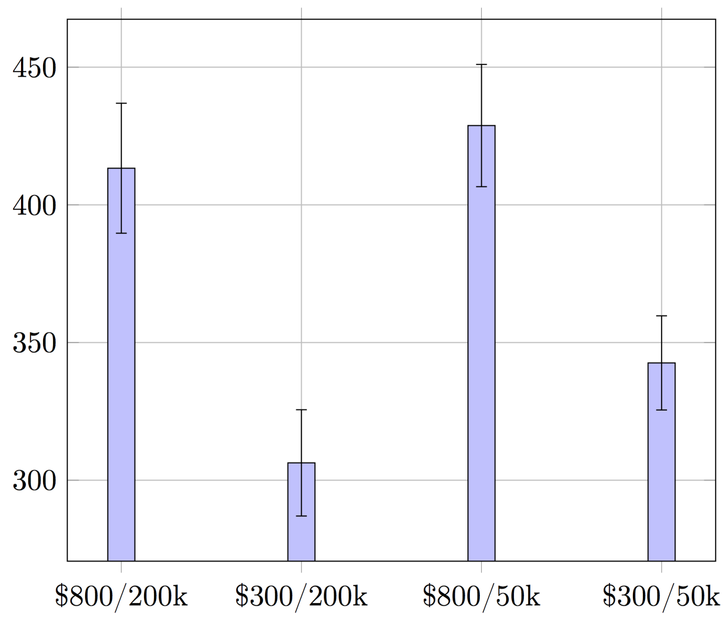

Figure 1: Willingness to Pay

First, we find strong evidence of the reference effects. We conduct two t-tests on WTP between the \$800 and \$300 average price segments. Across the 200k mileage balance groups, the p-value is 0.0006 (t = 3.5157, df = 115.757); across the 50k mileage balance groups, the p-value is 0.0026 (t = 3.0795, df = 109.918). Both suggest strong statistical difference in customers' WTP.

Second, we do not observe evidence of state-dependent decisions. Similarly, we conduct t-tests between the 200k and 50k mileage balance segments, across the \$800 and \$300 average price groups separately. The p-values are 0.6333 (t = -0.4783, df = 116.665) and 0.1605 (t = -1.412, df = 121.897), showing no significant effects of mileage balances.

Finally, we run a regression with all demographic information controlled (gender, age, frequent flyer program, true mileage balance, travel frequency, etc.). The effect of average price on WTP is significant (p=6.62e-06); by increasing the average price from \$300 to \$800, the WTP increased by \$97.453. However, the effect of mileage balance is not statistically significant (p=0.1058).

Consumer Behavior Model Based on the evidence, we need to incorporate the following properties of consumer decisions when building a coherent model: i) consumers evaluate points in a one-shot manner; ii) consumers evaluate points according to their expectations of future prices. We shall describe the consumer behavioral model briefly below.

Consider a loyalty program member i who has accumulated enough points to redeem a free unit that he evaluates at v_i. He chooses between the following three options: i) pay the current cash price p and earn N points (I); ii) use M points to redeem for a free unit (A); iii) leave the market (O).

The customer then evaluates points as a one-shot decision. We shall use w as the value of a point. Then the customer's utility from a purchase is

$$u(I) = v_i + Nw - p,$$

from a redemption is

$$u(A) = v_i - Mw,$$

from leaving the market is exactly 0. The customer compares these three options and determines a preference rule denoted by a_i.

Finally, we describe how the value of loyalty points w depends on the firm's and customers' decisions. The empirical study suggests strong effect of reference price, i.e. customers evaluate points according to the average price of an award. We will incorporate that into our model. Specifically, We characterize the market value of a point as

$$w = \frac{R \cdot \sigma}{M},$$

where R is the market value of an award unit, σ is the probability that each point will eventually be redeemed, and M is the number of miles required for an award. The market value R is the net price at the time of redemption: i.e., if the prevailing price at the time of redemption is $\tilde{p}$, then the market value of the award unit is $\tilde{p} - Nw$; we subtract Nw where N is the average number of points issued to customers with a cash-paid purchase. The setup is consistent with the reference price ($\tilde{p}$) effects. The redemption probability σ is simply the ratio of the average number of redeemed points over the average number of issued points. (For example, if an airline issues twice as many miles as are redeemed, then the chances that each mile will be redeemed is 50% on average.)

Note that the value R depends on the firm's pricing decisions, while the value σ depends on the firm's award capacity decisions. Hence, the value of a point w is endogenous. In the next section, we will describe the firm's model in detail.

1.4. Firm's Model and Equilibrium

Our model builds on Littlewood's (1972) model of quantity-based revenue management. In the classic model, there is a firm that sells a fixed capacity of K units over two time periods. In period one, the firm faces an infinite population of low-type customers, each with valuation v_L for a unit of capacity. In period two, the firm faces a random population of $X \sim F(\cdot)$ high-type customers, each with valuation v_H, where $v_H > v_L$. The firm sells q_L units to low-types and reserves q_H units for high-types, where $q_L + q_H = K$. Given the decision above, the expected profit is $v_L \cdot q_L + v_H \cdot E[q_H \wedge X]$, which demonstrates a tradeoff between the guaranteed but lower revenue from low-types and the higher but uncertain revenue from high-types. The profit-maximizing decision, known as Littlewood's rule, is

$q_H^0 = \min\{K, \bar{q}\}$, where $\bar{q} = \bar{F}^{-1}(\frac{v_L}{v_H})$. We use q_H^0, q_L^0, π^0 to denote the optimal decisions and profit in the classic model.

Loyalty Program Now, we introduce loyalty programs into Littlewood's setting. Assume that with loyalty programs, the firm charges prices p_L and p_H to low and high types. For each unit purchased at p_i, the firm issues N_i loyalty points to the customer; once a customer accumulates M_i points, the customer may redeem those points for a free unit priced at p_i, $i = L, H$.

This general setup is applicable to the three types of loyalty programs of interest: volume-based programs, expense-based programs, and point-based programs. For example, in a volume-based program (old AAdvantage) that issues 5,030 miles for a round-trip US coast-to-coast flight and requires 25,000 miles for a free flight, we have $N_L = N_H = 5,030$ and $M_L = M_H = 25,000$. In an expense-based program (new AAdvantage) that issues 5 miles for every dollar spent by a customer and requires 25,000 miles for a free flight, we have $N_i = 5 \cdot p_i$, $i = L, H$ and $M_L = M_H = 25,000$. In a point-based program (Southwest Rapid Rewards) that issues 6 miles for every dollar spent by a customer and requires 70 miles for every dollar redeemed by a customer, we have $N_i = 6 \cdot p_i$ and $M_i = 70 \cdot p_i$, $i = L, H$.

We consider a setting where the firm sells the same capacity of K units repeatedly over time. In the airline example, miles earned on a current flight may be redeemed for a future flight. Nonetheless, each individual flight is managed similarly to the classic model as described below.

Firm Decisions With loyalty programs, the firm divides the capacity of K units into three instead of two pools. As before, the firm chooses a protection level q_H (number of units to reserve for high-types) and a booking limit q_L (number of units to sell to low-types), but now the firm also sets aside q_A units for award redemption. We use $q = (q_A, q_L, q_H)$ to denote the capacity allocation decision, where the three components are nonnegative and add up to K. In addition, the firm chooses prices p_L and p_H to charge to low and high types, and we denote $p = (p_L, p_H)$. The firm's objective, as before, is to maximize total expected profit.

Profit Function Having specified the firm's decisions (p, q) and customers' preference rules $a = (a_L, a_H)$, we are now ready to write down the profit function:

$$\pi(p, q, a) = p_L \cdot s_L(q, a) + p_H \cdot s_H(q, a),$$

where s_L and s_H denote the expected number of units sold at the two prices. The former is $s_L = q_L$ if low-types buy and zero otherwise. The latter is $s_H = E[q_H \wedge X]$ if high-types buy before redemption, $s_H = E[q_H \wedge (X - q_A)^+]$ if high-types buy after redemption, and zero otherwise. Finally, we also use $s_A(q, a)$ to denote the expected number of award units redeemed, even though they do not contribute to revenue and thus do not enter the profit function directly. There are four possible values for s_A: if no customers redeem, $s_A = 0$; if low types redeem, $s_A = q_A$; if high types redeem as their first choice, $s_A = E[q_A \wedge X]$; if high types redeem after purchases, $s_A = E[q_A \wedge (X - q_H)^+]$. Given s_L, s_H and s_A, we can express the redemption probability as follows

$$\sigma(p, q, a) = \frac{M s_A}{N_L s_L + N_H s_H},$$

as before, M is the average number of points charged for a redemption.

Timeline and Equilibrium The chronology of events is as follows. First, the firm chooses a revenue management strategy (p, q), which is observed by all. Then, low-types arrive and choose their preference rule a_L; this is followed by sales to and/or redemptions by low-types, after which they leave the market. Finally, high-type demand X is realized, they observe the entire history and choose their preference rule a_H, and then sales to and redemptions by high-types occur. Given this timeline, we use backward induction to solve for the sub-game perfect equilibria, which can be defined as follows.

Definition An equilibrium (p^*, q^*, a^*) satisfies the following conditions.

1. (Customer optimality) Given any (p, q), customers choose $a^*(p, q)$ to maximize utility.

2. (Firm optimality) The firm chooses (p^*, q^*) to maximize expected profit:

$$p^*, q^* = \arg\max \pi(p, q, a^*(p, q)) \tag{1}$$

1.5. Volume-Based Loyalty Programs

We first consider volume-based programs. In such programs, the firm issues the same number of points to each purchase and requires the same number of points for each redemption, regardless of the prices. For simplicity, we shall denote $N = N_L = N_H$ and $M = M_L = M_H$.

By solving the customers' and firm's problems, we characterize the equilibrium below.

Proposition 1. *In the equilibrium,*

(i) Low types buy q_L^*, then redeem q_A^*; high types buy q_H^*.

(ii) $p_i^* = v_i + Nw^*$;

(iii) There exists $\bar{K}$ such that

(a) if $K \leq \bar{K}$, then $q_L^* = 0$ and q_A^*, q_H^* satisfy

$$q_A^* + q_H^* = K, \quad q_A^* = \frac{\sigma^* N}{M} E[q_H^* \wedge X].$$

(b) If $K > \bar{K}$, then $q_H^* = \underline{q}$ and q_A^*, q_L^* satisfy

$$q_A^* + q_L^* + \underline{q} = K, \quad q_A^* = \frac{\sigma^* N}{M}\{q_L^* + E[\underline{q} \wedge X]\}.$$

Here, $\underline{q} = \bar{F}^{-1}(\frac{p_L^* - c}{p_H^* - c})$ and $c = \frac{\sigma^* N}{M + \sigma^* N} p_L$.

(iv) The equilibrium profit is $v_L \cdot q_L^* + v_H \cdot q_A^* + v_H \cdot E[q_H^* \wedge X]$.

(v) $R^* = v_H$, $\sigma^* = \frac{v_L}{v_H}$, $w^* = \frac{v_L}{M}$.

Proposition 1(i) summarizes equilibrium customer behavior (Figure 1). First, low types prefer to pay the low price but are willing to redeem awards when the low-price capacity runs out; this is intuitive because at sufficiently low prices, customers would seize the deal and save points for future use. Second, high types are willing to pay the high price. Hence, the following sequence of events occur in equilibrium: 1) upon arrival, low types purchase at the low price p_L; 2) low-price capacity q_L runs out, so the prevailing price rises to p_H; 3) low types who have not received a unit redeem awards using their points; 4) award capacity q_A runs out, 5) all remaining low types leave the market; 6) high types arrive and buy at the high price. Given this chronology, the prevailing price is the high price when redemptions occur, so it is not surprising that the market value of awards is $R^* = v_H$ as indicated in Proposition 1(iv).

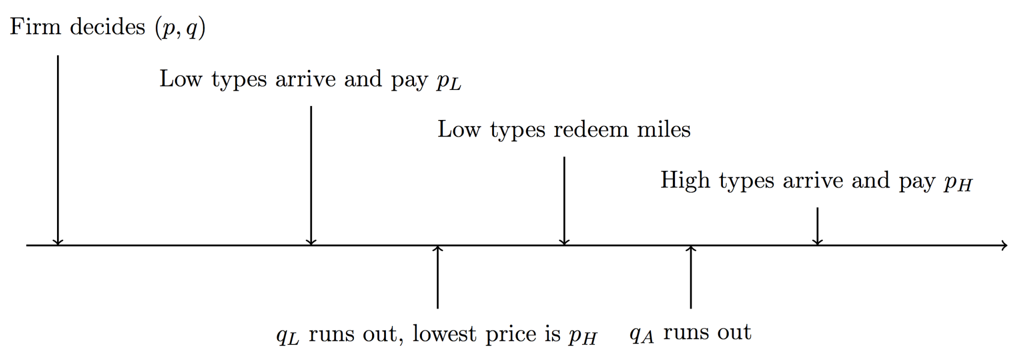

Figure 2: Equilibrium Timeline

Next, we discuss the firm's equilibrium decisions. As shown in Proposition 1(ii), the firm selects prices $p_i^* = v_i + Nw^*$ to extract maximum customer surplus. The prices consist of two parts: the value of the unit (v_i) and the value of issued points (Nw^*). With these prices, Proposition 1(iii) then summarizes the equilibrium capacity allocation q^*. To understand this result, we first introduce a critical fractile $\underline{q}$. We can rewrite the equation of $\underline{q}$ as follows:

$$(p_H^* - c)\bar{F}(\underline{q}) = p_L^* - c.$$

This is similar to the critical fractile $\bar{q}$ used in Littlewood's rule in our baseline model: the left hand side is the expected revenue from reserving an additional unit for the high type customers; the right hand side is the certain revenue from giving this unit to the low types. However, there is an additional cost c associated with the revenues. This is the cost of issuing points. For each unit sold, the firm issues N points, each of which is redeemed with probability σ^* for $\frac{1}{M}$ unit of capacity. In other words, each sold unit is associated with the liability of fulfilling $\frac{\sigma^* N}{M}$ units worth of loyalty rewards. Altogether, the firm needs a total of $1 + \frac{\sigma^* N}{M}$ units to sell to and award the customer, and the awarded capacity is a fraction $\frac{\sigma^* N}{M+\sigma^* N}$ of this total capacity. For each unit of awarded capacity, there is an opportunity cost of p_L^*: if this unit is not awarded, then it can be sold to a low type customer at price p_L^*. Hence, the liability of issuing points is $c = \frac{\sigma^* N}{M+\sigma^* N} p_L^*$.

Using the critical fractile $\underline{q}$, we can interpret the optimal capacity allocation q^* in Proposition 1. The critical fractile $\underline{q}$ can be viewed as a protection level for high-type demand. Protecting $\underline{q}$ units for high-types generates a total of $NE[\underline{q} \wedge X]$ points and a corresponding award liability of $\frac{\sigma^* N}{M} E[\underline{q} \wedge X]$ units. If the award liability exceeds remaining capacity $K - \underline{q}$, as in case (i), the firm does not sell at the low price (i.e., $q_L^* = 0$) and lowers the protection level q_H^* below $\underline{q}$ so that the corresponding award liability can be covered by available capacity K. In case (ii), the protection level $q_H^* = \underline{q}$ and the corresponding award liability do not take up the entire capacity K. Then, the remaining capacity is allocated to low-price capacity q_L^* and the corresponding award liability $\frac{\sigma^* N}{M} q_L^*$. In other words, we have $q_L^* + \frac{\sigma^* N}{M} q_L^* + \underline{q} + \frac{\sigma^* N}{M} E[\underline{q} \wedge X] = K$, as indicated in the proposition.

Finally, Proposition 1(iv) gives the equilibrium profit, which is similar to the expected profit $v_L \cdot q_L^* + v_H \cdot E[q_H^* \wedge X]$ in the classic model, but has an additional term $v_H \cdot q_A^*$. This additional term suggests that the firm receives v_H from each redeemed unit. Even though the firm does not receive revenue from award redemptions, it can charge a price premium for issuing loyalty points. When the price premium is accounted for, it is as if the firm sells award units at the high-type valuation v_H to low-type customers. In other words, when low types redeem awards, the firm effectively uses its loyalty program to extract the high-type valuation v_H from these low-type customers. This follows directly from the reference effects: low types refer the value of points according to the price at the redemption.

This result echoes the current accounting protocols of loyalty programs. Under the "Deferred Revenue" accounting criterion, firms are required to defer the revenue associated with issued points to the point of redemption. The most prevailing way of calculating the deferred revenue is by the "fare value" of the redeemable reward, i.e., the price at which the award is redeemed. Put it in a simple way, the firm recognizes a revenue equivalent to the current price at the redemption point, and this revenue must have been be deducted/deferred previously at the time of issuing these points. This is exactly what Proposition 1(iv) indicates: instead of recognizing the whole revenue p_i when selling q_i and 0 for rewarding q_A, the firm can record v_i for q_i, recognize an additional revenue for q_A, and yield the same total revenue.

To induce these low types to redeem points, Proposition 1(v) ensures that the redemption probability is below a limit $\frac{v_L}{v_H}$, i.e., the chances that points can eventually be used are low enough that low types are better off redeeming instead of hoarding them. Ultimately, the constraint in (2) determines the amount of capacity the firm should reserve for awards: q_A should be increased to the point where the redemption rate reaches the upper limit.

In summary, loyalty programs enhance profits by extracting high revenues from low types. The revenue management strategy must be adjusted to account for the cost and benefit of issuing points. Specifically, the redemption probability under the optimal revenue management strategy has to be low to prevent customers from hoarding points for future use.

1.6. Expense-Based Loyalty Programs

The previous section investigates volume-based loyalty programs, under which customers receive the same number of points from each purchase, no matter how much they pay. In practice, some companies use expense-based programs, under which customers receive points based on the amount of money they spend. This section extends our results to expense-based programs.

Consider American Airlines that provides an expense-based loyalty program: it issues 5 miles for each dollar spent and requires 25,000 miles for a free flight. Then, the total number of points issued per unit depends on the price paid. For instance, a customer gets 4,000 miles for a ticket priced at \$800, but only gets 1,500 miles for a ticket discounted to \$300 on the same flight. Let n be the number of points issued per dollar spent; here, $n = 5$. As before, we use M to denote the number of points required for a free unit; in our example, $M = 25,000$, so customers are entitled to a free flight after spending \$5,000 (e.g., 7 full-fare flights or 17 discounted flights). This results in $N_i = np_i$ and $M_i = M$, where $i = L, H$.

With expense-based loyalty programs, most of our results remain unchanged. We begin with the following proposition.

Proposition 2. *In the equilibrium,*

(i) Low types buy q_L^*, then redeem q_A^*; high types buy q_H^*.

(ii) $p_i^* = \frac{v_i}{1-nw^*}$;

(iii) There exists $\bar{K}$ such that

(a) if $K \leq \bar{K}$, then $q_L^* = 0$ and q_A^*, q_H^* satisfy

$$q_A^* + q_H^* = K, \quad q_A^* = \frac{\sigma^* n p_H^*}{M} E[q_H^* \wedge X].$$

(b) *If* $K > \bar{K}$*, then* $q_H^* = \underline{q}$ *and* q_A^**,* q_L^* *satisfy*

$$q_A^* + q_L^* + \bar{q} = K, \quad q_A^* = \frac{\sigma^* n p_L^*}{M} q_L^* + \frac{\sigma^* n p_H^*}{M} E[\bar{q} \wedge X].$$

Here, $\bar{q} = \bar{F}^{-1}(\frac{v_L}{v_H})$*.*

(iv) *The equilibrium profit is* $v_L \cdot q_L^* + v_H \cdot q_A^* + v_H \cdot E[q_H^* \wedge X]$*;*

(v) $R^* = v_H$*,* $\sigma^* = \frac{v_L}{v_H}$*,* $w^* = \frac{v_L}{M}$*.*

A quick glance at Proposition 2 reveals several similarities to Proposition 1. First, Proposition 2(i) shows that with expense-based programs, it remains optimal to induce low-types to buy before redeeming, resulting in awards being valued at the high valuation, i.e., $R^* = v_H$, as in volume-based programs. Second, the profit function in Proposition 2(iv) remains unchanged and shows that, by extracting a price premium for loyalty points, the firm again receives v_H from each of the q_A redeemed units. In other words, whether they are volume-based or expense-based, loyalty programs enable the firm to extract the high valuation from each unit redeemed by a low-type. Finally, Proposition 2(v) shows that the redemption probability $\sigma^* = \frac{v_L}{v_H}$ and the value of loyalty points $w^* = \frac{v_L}{M}$ match our earlier results for volume-based programs.

However, there are a couple of differences between Proposition 1 and Proposition 2. First, to extract all consumer surplus, the firm chooses $p_i^* = v_i + np_i^* w$, which gives $p_i^* = \frac{v_i}{1-nw^*}$. As a result, the price premium for loyalty points enters as a multiplicative factor $\frac{1}{1-nw^*}$ in expense-based programs, instead of an additive term Nw^* in volume-based programs. Second, the exact form of revenue management decisions differ from volume-based programs. Nevertheless, the optimal capacity allocation rule for expense-based programs follows the same logic as before. We start with a protection level $\bar{q}$ for high-types, à la Littlewood. If this protection level and the associated award liability exceeds capacity, then all units are priced high and the protection level is adjusted downward to meet capacity. Otherwise, the firm protects $q_H^* = \bar{q}$ units for high-types and additionally allocates q_L^* units for sale at the low price so that all capacity is used up, after considering all associated award liability. Consequently, the rule of thumb described in the previous section still applies: the optimal award capacity can be achieved by matching the redemption probability to σ^*.

One noteworthy result in Proposition 2 is that the protection level $\bar{q}$ is identical to that in the classic model. This is because in expense-based loyalty programs, both the price premium and the cost of issuing points are proportional to the valuation of the customer who made the purchase. As a result, the critical fractile $\bar{q}$ determining the protection level is simply the ratio of customer valuations, as in the classic model. This finding leads us to our next result. The result suggests that Expense-based programs not only retains the property of inducing low types to pay the high price, but also simplifies the calculation of revenue management. An expense-based program protects the same number of units for high-types as in the classic Littlewood model, suggesting that capacity allocation decisions can be made without considering loyalty programs. The firm only needs to collect historical information about prices to make decisions on the protection levels.

1.7. Point-Based Loyalty Programs

In the previous section, we considered price-dependent *issuance* of loyalty points. In this section, we investigate price-dependent *issuance* and *redemption* in point-based programs. In such programs, for every unit priced at p_i, a cash-paying customer earns np_i points, and a point-paying customer pays mp_i points, i.e., $N_i = np_i$ and $M_i = mp_i$. Point-based programs have been adopted by Southwest Airlines and JetBlue Airlines.

Proposition 3. *In the equilibrium,*

(i) Low types redeem q_A^*, then buy q_L^*; high types buy q_H^*.

(ii) $p_i^* = \frac{v_i}{1-nw^*}$;

(iii) There exists $\bar{K}$ such that

(a) *If* $K \leq \bar{K}$*, then* $q_L^* = 0$ *and* q_A^**,* q_H^* *satisfy*

$$q_A^* + q_H^* = K, \quad q_A^* = \frac{\sigma^* n}{m} E[q_H^* \wedge X].$$

(b) *If* $K > \bar{K}$*, then* $q_H^* = \bar{q}$ *and* q_A^**,* q_L^* *satisfy*

$$q_A^* + q_L^* + \bar{q} = K, \quad q_A^* \in [0, \frac{n}{m}(q_L^* + \frac{v_H}{v_L} E[\bar{q} \wedge X])].$$

Here, $\bar{q} = \bar{F}^{-1}(\frac{v_L}{v_H})$*.*

(iv) *The equilibrium profit is* $v_H \cdot q_A^* + v_H \cdot E[q_H^* \wedge X]$ *if* $K \leq \bar{K}$*, is* $v_L \cdot q_L^* + v_H \cdot E[q_H^* \wedge X]$ *otherwise.*

(v) $R^* = v_H$*,* $\sigma^* = \frac{v_L}{v_H}$*,* $w^* = \frac{v_L}{v_L n + v_H m}$ *if* $K \leq \bar{K}$*;* $R^* = v_L$*,* $\sigma^* \leq 1$*,* $w^* = \frac{\sigma^*}{\sigma^* n + m}$ *otherwise.*

Proposition 3 reveals several unique properties of point-based programs. First, customers always prefer redeeming awards rather than paying cash. When the number of points needed for redemption is proportional to the cash price, there is a fixed conversion rate between points and cash. Therefore, customers have no incentives to hoard points for future redemptions and they will always use points whenever an award unit is available.

Second, Proposition 3(iv) suggests that the firm can only extract the high value from award capacity when it fully closes the low price capacity. Note that low types always redeem before purchasing. If the low price capacity is offered, when customers redeem, the prevailing price is low and they evaluate points according to the that price. Hence, the firm cannot extract high values from issuing points to them. On the contrary, if the low price capacity is closed, they can only redeem at the high price and points have high values. Therefore, only when the low price capacity is closed do redemptions eventually generate high values for the firm.

Finally, Proposition 3(iii) and (v) indicate that when the firm opens up some low price capacity, it becomes more flexible in choosing the award capacity q_A. Specifically, the limit on the redemption probability of points is extended to be as high as 100%. Previously, in volume-based and expense-based programs, the firm needs to restrict the redemption probability below 100% to induce immediate redemptions by customers. Now in point-based programs, customers never hoard points for future use. As a result, the firm can allow 100% redemption probabilities. Moreover, since now redemptions happen at the low price, they generate exactly the same revenues as the low price capacity. Indifferent between selling and rewarding the low types, the firm has a more flexible rewarding rule, i.e. it can split the Littlewood booking limit $K - \bar{q}$ arbitrarily between the award capacity q_A and low price capacity q_L, as long as the redemption rate is below 100%. Specifically, the 100% redemption probability can be optimal. In such cases, the firm does not need to distinguish between cash-paying customers and reward customers. Instead, the firm can put them in the same booking bucket. Consequently, a customer can always redeem an award whenever the unit of capacity can be purchased using cash. Our result is consistent with practice - the 100% availability rate is exactly what the highly-praised point-based programs of Southwest Airlines and JetBlue Airlines are well-known of.

1.8. Numerical Examples

In this section, we will provide some numerical examples to clarify the mechanics of the revenue management strategies. Our primary data source is Airline Origin and Destination Survey (DB1B) conducted by Bureau of Transportation Statistics. It is a 10% sample of airline tickets from reporting carriers. Data includes origin, destination, price and other itinerary details of passengers transported.

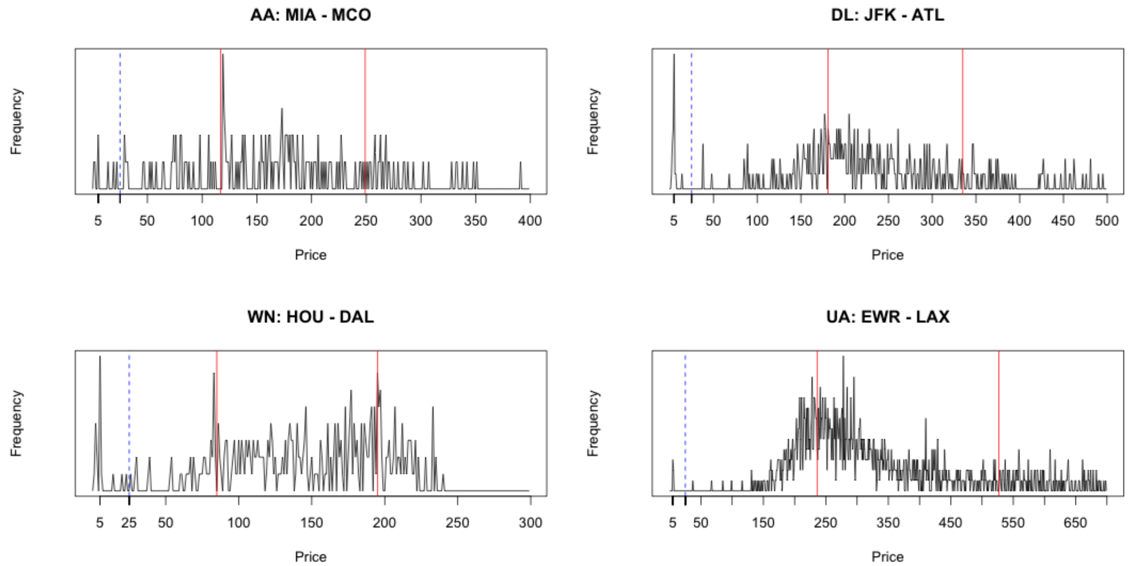

Figure 3: Price Distribution

We select 16 routes over the third quarter of 2015 across four airlines for analysis. At that period of time, American Airlines offers a mileage-based (volume-based) program, both United Airlines and Delta Airlines offers an expense-based program, and Southwest Airlines offers a point-based program. All these routes are direct flights, with a market share of non-stop passengers greater than 70% (mostly 100%) such that competition is rare. The following approaches are used for model calibrations: (i) the number of standard economy class seats on the plane is used for capacity K; (ii) the first quantile price is used to calibrate low valuation v_L and third quantile price is used to calibrate high valuation v_H (See Figure 3. The two red lines corresponds to the first quantile and third quantile price. Data suggests that the first quantile price has the highest density in the price distribution, which may approximate a mass probability of low type customers; we understands that such simplifications of the valuation distribution have limitations, but hope to proceed with the numerical examples to generate insights of the theoretical model); (iii) a quarter of the economy class seats is used for the mean of the high type demand, i.e. $E[X] = K/4$, since only 25% of all passengers paid the high value price v_H; (iv) we assume X follows the normal distribution, with coefficient of variation ranging from 0.5 to 2; (v) the award policy for United and Delta is used to calibrate N_i and M_i in both the volume-based and expense-based programs, while the award policy for Southwest is used to calibrate for point-based programs; (vi) we use the fraction of tickets sold under \$25 as the fraction of award tickets (illustrated in blue dashed line in Figure 3), and multiply that by K for the award space q_A in practice (the tax paid for redemption ranges from \$5 to \$10, and the probability of paid price between \$10 and \$25 is less than 1%).

Table 2 summarizes the results of the 16 routes, by calculating the optimal RM strategies and profit improvements of loyalty programs ($\Delta\Pi$). The routes are listed in ascending order of flying distance. Note that there is a pattern of increasing $\Delta\Pi$ of the volume-based program and a decreasing-increasing $\Delta\Pi$ of the expense-based program. We will discuss three routes in detail below.

(1) BOS - CVG (Delta): Table 2 has three implications. First, it indicates that Delta is better off with the expense-based program for this route. Because of the high price and short distance of the route, an expense-based program issues more miles to customers compared to the mileage-based programs. Since each mile yields high valuations, the airline can benefit from "selling" these miles to customers. Second, Delta Airlines over-reward its customers under the expense-based program. While it gives 7.6 seats to its award passengers, the optimal reward level is only around 5.8. This can be due to the simple approximation of demand distribution. Finally, although the point-based program does not generate higher revenues directly, it can significantly enhance award space and potentially lead to higher customer satisfaction. (The same logic applies to Route (2) - (6).)

(15) SFO to CLT (American): compared to Route (1), Route (15) has much longer distance. As a result, a mileage based program issues more miles to customers. For this specific route, American Airlines benefit from its mileage-based programs more, since putting more miles into circulation allows the firm to gain higher profits. In fact, the

number of capacity rewarded to customers (3.1 seats) is close to the optimal level in the model (2.8 seats). Finally, note that although the mileage-based program is slightly better, the expense-based program also enhances the profits over 5.7%. (The same logic applies to Route (7) - (16).)

(9) DAL to PHL (Southwest): this is a medium-haul flight. As in the case of flight (15), the volume-based program is more profitable. However, the point-based program allows significantly more award space, thus Southwest is able to apply a simple rule of thumb: treat award customers exactly the same as cash passengers. Note that the q_A in data is greater than the optimal q_A under the point-based program. This may be due to two reasons: (i) biased sample of the price distribution; (ii) the ratio of award redemptions is relatively lower in other routes, to make up for the additional redeemed miles in this specific route.

Table 2: Numerical Examples

		Route Data						Littlewood	Volume-based			Expense-based			Point-based		
		Distance	v_L	v_H	K	q_A	Market%	q_H	q_A	q_H	$\Delta\Pi$	q_A	q_H	$\Delta\Pi$	q_A	q_H	$\Delta\Pi$
(1)	DL: BOS - CVG	187	240	503	108	7.6	100%	28.6	0.7	28.4	0.6%	5.8	28.6	5.3%	10.9	28.6	0%
(2)	AA: MIA - MCO	192	117	249	108	3.0	100%	29.0	0.7	29.0	0.7%	2.7	29.0	2.5%	11.0	29.0	0%
(3)	WN: HOU - DAL	239	85	195	143	7.5	100%	41.5	1.1	41.3	0.8%	2.5	41.5	2.0%	14.9	41.5	0%
(4)	UA: DCA -CLE	310	203	411	44	5.0	100%	11.2	0.5	11.1	1.0%	2.0	11.2	4.2%	4.4	11.2	0%
(5)	DL: MSP -CLE	622	244	447	120	9.3	100%	26.5	3.0	26.1	1.9%	7.1	26.5	4.6%	11.6	26.5	0%
(6)	DL: JFK - ATL	760	181	335	108	10.5	95%	24.3	3.2	23.8	2.4%	4.6	24.3	3.4%	10.5	24.3	0%
(7)	WN: DAL - MSP	853	75	150	143	20.0	100%	35.8	4.4	35.0	2.8%	2.4	35.8	1.5%	14.2	35.8	0%
(8)	WN: HOU - SLC	1214	172	217	143	19.9	100%	6.6	10.1	6.2	1.9%	7.9	6.6	1.4%	13.2	6.6	0%
(9)	WN: DAL - PHL	1295	112	195	175	17.7	100%	35.5	9.3	34.4	3.7%	4.8	35.5	1.9%	16.8	35.5	0%
(10)	UA: PHX - IAD	1956	219	503	90	14.0	91%	26.2	5.2	25.1	6.5%	4.1	26.2	5.2%	9.4	26.2	0%
(11)	DL: SEA - CVG	1965	262	480	132	17.5	100%	29.2	9.8	27.9	5.9%	8.4	29.2	5.0%	12.8	29.2	0%
(12)	DL: EWR - SLC	1969	245	397	120	9.2	100%	21.1	10.2	20.1	5.0%	7.8	21.1	3.9%	11.3	21.1	0%
(13)	DL: DTW - LAX	1979	247	577	108	14.0	75%	31.9	6.2	30.5	6.6%	5.6	31.9	6.0%	11.3	31.9	0%
(14)	AA: SEA - CLT	2279	275	500	138	11.9	100%	30.3	11.9	28.8	7.0%	9.2	30.3	5.2%	13.4	30.3	0%
(15)	AA: SFO - CLT	2296	274	540	36	3.1	100%	8.8	2.8	8.4	7.1%	2.3	8.8	5.7%	3.6	8.8	0%
(16)	UA: EWR -LAX	2454	236	527	108	16.5	78%	30.5	8.0	28.9	8.0%	5.5	30.5	5.5%	11.1	30.5	0%

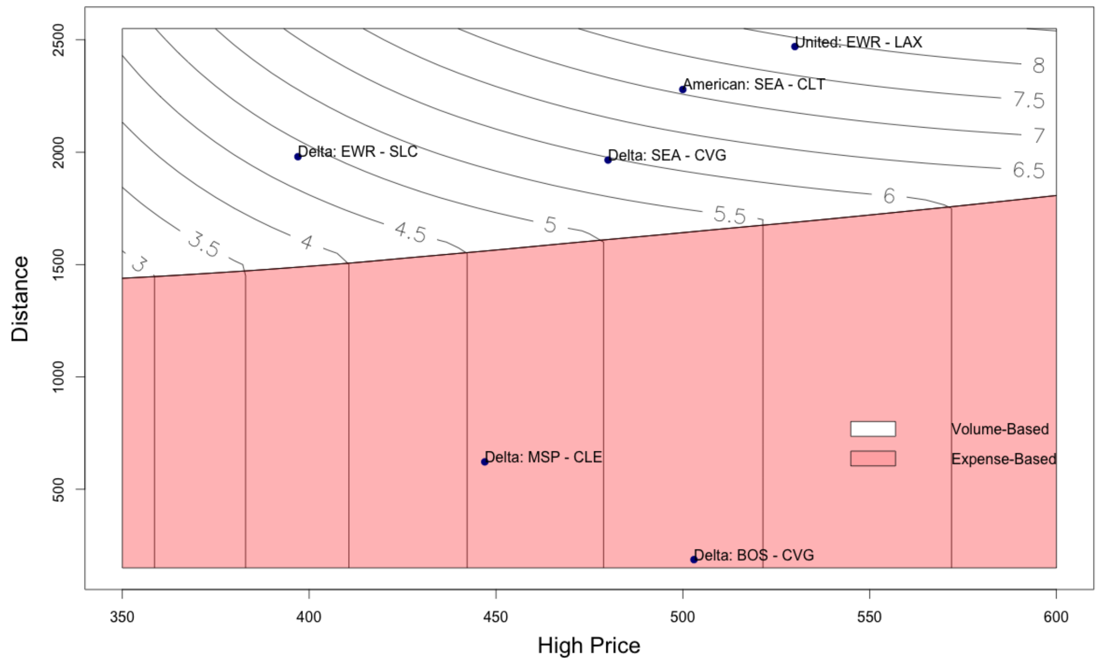

Figure 4: Profit Benefit of Loyalty Programs (Unit: %)

Finally, we plot the six high-fare routes (with a low price ranging from \$240 to \$274) in Figure 4. The horizontal axis is the high price v_H, and the vertical axis is the flying distance. The contour lines indicate the percentage of profit improvement of volume-based and expense-based programs over the classic Littlewood model. Moreover, the shaded areas suggests that expense-based programs are more profitable than volume-based programs.

In Figure 4, as the flying distance increases or high price increases, the loyalty programs become more profitable. This is because the firm is able to issue more miles in such scenarios. Specifically, the volume-based program is more profitable than the expense-based when the flying distance is longer, as for the four routes above the line. In contrast, for short-haul expensive flights (bottom Delta routes), expense-based programs are more profitable.

1.9. Conclusions

In this paper, we study loyalty programs in industries such as airlines and hotels where capacity is limited. Starting with Littlewood's classic revenue management model with two customer types (e.g., leisure and business travelers, representing low-paying and high-paying types), we incorporate loyalty rewards (i.e., non-paying types) and obtain the following results. First, loyalty points lead to adjustment of revenue management decisions by including the liability of points. Second, optimal award capacity is constrained by quantity sold under fixed redemption of points, but the 100% award availability can be optimal when the number of redeemed points is proportional to the price of the award. Finally, we compared the three different programs schemes: volume-based and expense-based programs extract high values from low type customers; expense-based programs simplifies the calculation of revenue management; point-based programs allow 100% award availability.

This research can be extended in several directions. First, just as how Littlewood's original model was generalized to multiple demand classes, which led to the development of heuristics and algorithms for practical implementation (e.g., Belobaba, 1998), our methods can be extended to more general demand patterns. Second, although we have focused on the frequency rewards component of loyalty programs, most programs in practice also have the customer tier component that offer precious metal status as incentives. Considering both components at the same time may uncover interesting interactions (e.g., Kopalle, 2012). Third, loyalty program transactions are closely related to the finance and accounting functions of the firm. It is interesting to study firms' flexibility in reporting loyalty rewards and the corresponding regulatory implications (see related work by Chun et al., 2015). We hope that our suggestions above will motivate further work on revenue management with loyalty programs.

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1.10. Appendix: Proofs

First we shall formulate consumers' decisions. Each type-i customer may pay the price p_i (I), redeem a free award unit (A) or leave the market (O). Customers indicate their preferences by choosing $a_i \in \{O, I, A, IA, AI\}$. In the last two options, the customer is open to both redeeming and buying: if $a_i = IA$, the customer prefers paying cash and uses points only when units for sale run out; if $a_i = AI$, the customer resorts to paying cash only after award capacity runs out. We denote $a = (a_L, a_H)$.

Proof of Proposition 1. Before solving for the equilibrium, we eliminate a couple of dominated or unreasonable strategies:

- Eliminate pricing strategies p in which $p_i > v_i + Nw$. If $p_i > v_i + Nw$, type i will never buy products at p_i. There is no need to create two prices. This is the same as setting $q_i = 0$ for some $i \in L, H$.
- Eliminate value of a point w that $w > v_H/M$. If $w > v_H/M$, then $u(A) = v_i - Mw < 0 = u(Q)$, no one redeems. Then redemption probability $\sigma = 0$, we must have $w = 0$.

Therefore, we shall assume $p_i \leq v_i + Nw$ and $w \leq v_H/M$.

We use backward induction to solve for the equilibrium. The proof consists of 2 steps.

1. Customers' best responses. A type i customer' utility from purchasing is $u(I) = v_i + Nw - p_i$, given $p_i \leq v_i + Nw$, we have $u(I) \geq 0$. Thus the customers' strategy space is reduced to $a \in \{I, IA, AI\}$. His utility from redemption is $u(A) = v_i - Mw$. There are four possible customer behavior: (a) if $p_L \geq (M+N)w$, then $a_L^* = AL$, $a_H^* = AH/HA$, low types first redeem q_A then buy q_L, high types buy q_H; (b) if $p_L \leq (M+N)w$ and $Mw \leq v_L$, then $a_L^* = LA$, $a_H^* = AH/HA$, low types first buy q_L then redeem q_A, high types buy q_H; (c) if $p_L \leq (M+N)w \leq p_H$ and $Mw > v_L$, then $a_L^* = L$, $a_H^* = AH$, low types buy q_L, high types first redeem q_A then buy q_H; (d) if $(M+N)w \geq p_H$, then $a_L^* = L$, $a_H^* = HA$, low types buy q_L, high types first buy q_H

then redeem q_A.

2. Firm's optimal strategy. The firm can use (p, q) to generate different values of w, and induce the four types of customer behavior as described above. In each case, we can derive the closed form functions of π, R and σ.

   (a) If $p_L \geq (M+N)w$, $\pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge X]p_H$, $R(p,q,a^*(p,q)) = p_L - Nw$, $\sigma(p,q,a^*(p,q)) = \frac{Mq_A}{N(q_L+E[q_H \wedge X])}$.

   (b) If $p_L \leq (M+N)w$ and $Mw \leq v_L$, $\pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge X]p_H$, $R(p,q,a^*(p,q)) = p_H - Nw$, $\sigma(p,q,a^*(p,q)) = \frac{Mq_A}{N(q_L+E[q_H \wedge X])}$.

   (c) If $p_L \leq (M+N)w \leq p_H$ and $Mw > v_L$, $\pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge (X - q_A)^+]p_H$, $R(p,q,a^*(p,q)) = p_H - Nw$, $\sigma(p,q,a^*(p,q)) = \frac{ME[q_A \wedge X]}{N(q_L+E[q_H \wedge (X-q_A)^+])}$.

   (d) if $(M+N)w \geq p_H$, $\pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge X]p_H$, $R(p,q,a^*(p,q)) = p_H - Nw$, $\sigma(p,q,a^*(p,q)) = \frac{ME[q_A \wedge (X-q_H)^+]}{N(q_L+E[q_H \wedge X])}$.

   We next show that (b) dominates (a), (c) and (d). First, we prove that in case (a), (c) and (d), the optimal profit is smaller than the case without LP (π^0). Note that $Mw = R\sigma$, where $\sigma = \frac{Ms_A}{N(s_L+s_H)}$. This gives $Nw(s_L + s_H) = Rs_A$. Plug this in the profit function we have

$$\begin{aligned} \pi &= \textstyle\sum_{i=L,H} p_i s_i \\ &\leq \textstyle\sum_{i=L,H} (v_i + Nw) s_i \\ &= \textstyle\sum_{i=L,H} v_i s_i + Rs_A \end{aligned}$$

   For case (a), $R = p_L - Nw \leq v_L$, so $\pi \leq \max_q v_L(q_A + q_L) + v_H E[q_H \wedge X] = \pi^0$. For case (c) and (d), $R = p_H - Nw \leq v_H$. Thus, $\pi \leq \max_q v_L q_L + v_H E[(q_H + q_A) \wedge X] = \pi^0$.

   Second, we prove that (b) gives a higher profit than π_0. We plug the corresponding

π, σ and R into (1):

$$\begin{aligned} \max_{p,q} \quad & \pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge X] p_H \\ s.t. \quad & Mw = (p_H - Nw)\tfrac{Mq_A}{N(q_L + E[q_H \wedge X])} \\ & p_H \leq v_H + Nw \\ & p_L \leq (M+N)w \\ & Mw \leq v_L \\ & q_A + q_L + q_H \leq K \end{aligned}$$

The first constraint is from the definition of w, the second to fourth constraints are to induce the corresponding customer behavior, and the last constraint is from limited capacity. We can think of w as a new decision variable for the firm that is subject to these constraints.

We show that the optimal solutions have $p_i^* = v_i + Nw^*$, $w^* = \frac{v_L}{M}$. Using Kuhn-Tucker method, let $\mathcal{L} = q_L p_L + E[q_H \wedge X] p_H + \lambda(Mw - (p_H - Nw)\frac{Mq_A}{N(q_L + E[q_H \wedge X])}) - \mu(p_H - v_H - Nw) - \eta(p_L - (M+N)w) - \alpha(Mw - v_L) - \beta(q_A + q_L + q_H - K)$, where $\mu, \eta, \alpha, \beta \geq 0$. Then $\frac{\partial \mathcal{L}}{\partial q_A} = 0$ suggests $\lambda < 0$. $\frac{\partial \mathcal{L}}{\partial p_H} = 0$ suggests $\mu_H > 0$, by complementary slackness, $p_H = v_H + Nw$; $\frac{\partial \mathcal{L}}{\partial p_L} = 0$ suggests $\eta_L > 0$, by complementary slackness, $p_L = (M+N)w$. The program becomes

$$\begin{aligned} \max_{p,q,w} \quad & \pi(p,q,a^*(p,q)) = q_L Mw + E[q_H \wedge X] v_H + q_A v_H \\ s.t. \quad & v_H q_A = N(q_L + E[q_H \wedge X])w \\ & Mw \leq v_L \\ & q_A + q_L + q_H \leq K \end{aligned}$$

The new Lagrangian is $\mathcal{L} = q_L Mw + E[q_H \wedge X] v_H + q_A v_H - \lambda(v_H q_A - N(q_L + E[q_H \wedge X])w) - \alpha(Mw - v_L) - \beta(q_A + q_L + q_H - K)$. $\frac{\partial \mathcal{L}}{\partial q_L} = Mw + \lambda uw - \beta$, $\frac{\partial \mathcal{L}}{\partial q_A} = v_H - \lambda v_H - \beta$, $\frac{\partial \mathcal{L}}{\partial q_L} = \frac{\partial \mathcal{L}}{\partial q_A} = 0$ gives $\lambda = \frac{v_H - Mw}{v_H - Nw} > 0$. $\frac{\partial \mathcal{L}}{\partial w} = 0$ suggests that $\alpha > 0$; by complementary slackness, we have $w^* = \frac{v_L}{M}$.

Therefore, $p_i^* = v_i + Nw^*$, $w^* = \frac{v_L}{M}$, the problem becomes:

$$
\begin{aligned}
\max_q \quad & v_L q_L + v_H(q_A + E[q_H \wedge X]) \\
s.t. \quad & M v_H q_A = v_L N(q_L + E[q_H \wedge X]) \\
& q_A + q_L + q_H \leq K
\end{aligned}
\tag{1.1}
$$

Note that if the firm chooses $q_L' = q^0$, q_A' and q_H' such that $\frac{Mq_A'}{N(q^0+E[q_H' \wedge X])} = \frac{v_L}{v_H}$, all conditions are satisfied, which yields a profit of $\pi' = q^0 v_L + q_A' v_H + E[q_H' \wedge X] \geq q^0 v_L + E[q_H \wedge X] = \pi^0$. Thus the optimal solution in (b) must gives a profit higher than π^*.

Rewrite (1) in terms of p^* and σ^*:

$$
\begin{aligned}
\max_q \quad & q_L p_L + E[q_H \wedge X] p_H \\
s.t. \quad & M q_A = \sigma^* N(q_L + E[q_H \wedge X]) \\
& q_A + q_L + q_H \leq K
\end{aligned}
$$

The new Lagrangian is $\mathcal{L} = q_L p_L + E[q_H \wedge X]p_H + \lambda(Mq_A - \sigma^* N(q_L + E[q_H \wedge X])) - \beta(q_A + q_L + q_H - K)$, where $\beta \geq 0$. Take derivatives with respect to q:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q_A} &= M\lambda - \beta \\
\frac{\partial \mathcal{L}}{\partial q_L} &= p_L - \lambda\sigma^* N - \beta \\
\frac{\partial \mathcal{L}}{\partial q_H} &= \bar{F}(q_H)(p_H - \lambda\sigma^* N) - \beta
\end{aligned}
$$

If $\frac{\partial \mathcal{L}}{\partial q_A} = \frac{\partial \mathcal{L}}{\partial q_L} = \frac{\partial \mathcal{L}}{\partial q_H} = 0$, then $\lambda = \frac{p_L}{M+\sigma^* N}$, $\beta = \frac{Mp_L}{M+\sigma^* N}$, $q_H = \underline{q} = \bar{F}^{-1}(\frac{\frac{Mp_L}{M+\sigma^* N}}{p_H - \frac{\sigma^* N p_L}{M+\sigma^* N}})$. Plugging these back in the condition $q_A + q_L + q_H = K$ and $Mq_A = \sigma^* N(q_L + E[q_H \wedge X])$, we get $q_L^* = \frac{M}{M+\sigma^* N}(K - \underline{q} - \frac{\sigma^* N}{M} E[\underline{q} \wedge X])$, $q_A^* = \frac{\sigma^* N}{M+\sigma^* N}(K - \underline{q} + E[\underline{q} \wedge X])$. However, if $K - \underline{q} - \frac{\sigma^* N}{M} E[\underline{q} \wedge X] < 0$, q_L is negative. Thus the condition $\frac{\partial \mathcal{L}}{\partial q_L} = 0$

may not hold for the optimal solutions. In such cases, we have $q_L = 0$, $q_A^* + q_H^* = K$, $q_A^* = \frac{\sigma^* N}{M} E[q_H^* \wedge X]$.

$\square$

Proof of Proposition 2. Similarly to the proof of Proposition 1, we shall assume $p_i \leq \frac{v_i}{1-nw}$, and $w \leq v_H/M$.

We use backward induction to solve for the equilibrium. The proof consists of 2 steps.

1. Customers' best responses. A type i customer' utility from purchasing is $u(I) = v_i + p_i nw - p_i$, given $p_i \leq \frac{v_i}{1-nw}$, we have $u(I) \geq 0$. Thus the customers' strategy space is reduced to $a \in \{I, IA, AI\}$. His utility from redemption is $u(A) = v_i - Mw$. There are four possible customer behavior: (a) if $p_L \geq \frac{Mw}{1-nw}$, then $a_L^* = AL$, $a_H^* = AH/HA$, low types first redeem q_A then buy q_L, high types buy q_H; (b) if $p_L \leq \frac{Mw}{1-nw}$ and $Mw \leq v_L$, then $a_L^* = LA$, $a_H^* = AH/HA$, low types first buy q_L then redeem q_A, high types buy q_H; (c) if $p_L \leq \frac{Mw}{1-nw} \leq p_H$ and $Mw > v_L$, then $a_L^* = L$, $a_H^* = AH$, low types buy q_L, high types first redeem q_A then buy q_H; (d) if $\frac{Mw}{1-nw} \geq p_H$, then $a_L^* = L$, $a_H^* = HA$, low types buy q_L, high types first buy q_H then redeem q_A.

2. Firm's optimal strategy. The firm can use (p, q) to generate a desired w that induces the four types of customer behavior as described above. In each case, we can derive the closed form functions of π, R and σ.

   (a) If $p_L \geq \frac{Mw}{1-nw}$, $\pi(p, q, a^*(p, q)) = q_L p_L + E[q_H \wedge X] p_H$, $R(p, q, a^*(p, q)) = p_L(1 - nw)$, $\sigma(p, q, a^*(p, q)) = \frac{Mq_A}{n\pi(p,q,a^*(p,q))}$.

   (b) If $p_L \leq \frac{Mw}{1-nw}$ and $Mw \leq v_L$, $\pi(p, q, a^*(p, q)) = q_L p_L + E[q_H \wedge X] p_H$, $R(p, q, a^*(p, q)) = p_H(1 - nw)$, $\sigma(p, q, a^*(p, q)) = \frac{Mq_A}{n\pi(p,q,a^*(p,q))}$.

   (c) If $p_L \leq \frac{Mw}{1-nw} \leq p_H$ and $Mw > v_L$, $\pi(p, q, a^*(p, q)) = q_L p_L + E[q_H \wedge (X - q_A)^+] p_H$, $R(p, q, a^*(p, q)) = p_H(1 - nw)$, $\sigma(p, q, a^*(p, q)) = \frac{ME[q_A \wedge X]}{n\pi(p,q,a^*(p,q))}$.

(d) if $\frac{Mw}{1-nw} \geq p_H$, $\pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge X]p_H$, $R(p,q,a^*(p,q)) = p_H(1-nw)$, $\sigma(p,q,a^*(p,q)) = \frac{ME[q_A \wedge (X-q_H)^+]}{n\pi(p,q,a^*(p,q))}$.

We next show that (b) dominates (a), (c) and (d). First, we prove that in case (a), (c) and (d), the optimal profit is smaller than the case without LP (π^0). Note that $Mw = R\sigma$, where $\sigma = \frac{Ms_A}{n\pi}$. This gives $nw\pi = Rs_A$. Plug this in the profit function we have

$$
\begin{aligned}
\pi &= \textstyle\sum_{i=L,H} p_i s_i \\
&\leq \textstyle\sum_{i=L,H} \frac{v_i}{1-nw} s_i \\
&= \textstyle\sum_{i=L,H} v_i s_i + nwp_i s_i \\
&= \textstyle\sum_{i=L,H} v_i s_i + nw\pi \\
&= \textstyle\sum_{i=L,H} v_i s_i + Rs_A
\end{aligned}
$$

For case (a), $R = p_L(1-nw) \leq v_L$, so $\pi \leq \max_{q_A,q_L,q_H} v_L(q_A+q_L)+v_H E[q_H \wedge X] = \pi^0$. For case (c) and (d), $R = p_H(1-nw) \leq v_H$. Thus, $\pi \leq \max_{q_A,q_L,q_H} v_L q_L + v_H E[(q_H+q_A) \wedge X] = \pi^0$.

Second, we prove that (b) gives a higher profit than π_0. We plug the corresponding a^*, π, σ and R into (1):

$$
\begin{aligned}
\max_{p,q} \quad & \pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge X]p_H \\
s.t. \quad & Mw = p_H(1-nw)\frac{Mq_A}{n(q_L p_L + E[q_H \wedge X]p_H)} \\
& p_i \leq \frac{v_i}{1-nw} \\
& p_L \leq \frac{Mw}{1-nw} \\
& Mw \leq v_L \\
& q_A + q_L + q_H \leq K
\end{aligned}
$$

Similarly, we can think of w as a new decision variable that is subject to those constraints. We shall show that the optimal solutions have $p_i^* = \frac{v_i}{1-nw^*}$, $w^* = \frac{v_L}{M}$.

Using Kuhn-Tucker method similar to the proof of Proposition 1, we have $w^* = \frac{v_L}{M}$, $p_i^* = \frac{v_i}{1-nw^*}$.

Therefore, the problem becomes:

$$\begin{aligned} \max_q \quad & v_L q_L + v_H(q_A + E[q_H \wedge X]) \\ s.t. \quad & M v_H q_A = v_L n(v_L q_L + v_H(q_A + E[q_H \wedge X])) \\ & q_A + q_L + q_H \le K \end{aligned} \tag{1.2}$$

Note that if the firm chooses $q'_L = q^0$, q'_A and q'_H such that $\frac{Mq'_A}{n(v_L q_L + v_H(q_A + E[q_H \wedge X]))} = \frac{v_L}{v_H}$, all conditions are satisfied, which yields a profit of $\pi' = q^0 v_L + q'_A v_H + E[q'_H \wedge X] \ge q^0 v_L + E[q_H \wedge X] = \pi^0$. Thus the optimal solution in (b) must gives a profit higher than π^*.

Rewrite (2) in terms of p^* and σ^*:

$$\begin{aligned} \max_{p_H, q_j} \quad & q_L p_L + E[q_H \wedge X] p_H \\ s.t. \quad & M q_A = \sigma^* n(q_L p_L + E[q_H \wedge X] p_H) \\ & q_A + q_L + q_H \le K \end{aligned}$$

The new Lagrangian function is $\mathcal{L} = q_L p_L + E[q_H \wedge X] p_H + \lambda(M q_A - \sigma^* n(q_L p_L + E[q_H \wedge X] p_H)) - \beta(q_A + q_L + q_H - K)$, where $\beta \ge 0$. Take derivatives with respect to q we have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial q_A} &= \lambda M - \beta \\ \frac{\partial \mathcal{L}}{\partial q_L} &= p_L(1 - \lambda \sigma^* n) - \beta \\ \frac{\partial \mathcal{L}}{\partial q_H} &= \bar{F}(q_H) p_H (1 - \lambda \sigma^* n) - \beta \end{aligned}$$

If $\frac{\partial \mathcal{L}}{\partial q_A} = \frac{\partial \mathcal{L}}{\partial q_L} = \frac{\partial \mathcal{L}}{\partial q_H} = 0$, then $\lambda = \frac{\sigma^* n p_L}{M + \sigma^* n p_L}$, $\beta = \frac{M \sigma^* n p_L}{M + \sigma^* n p_L}$, $q_H = \bar{q} = \bar{F}^{-1}(\frac{v_L}{v_H})$. Plugging these back in the constraints $q_A + q_L + q_H = K$ and $M q_A = \sigma^* n(q_L p_L +$

$E[q_H \wedge X]p_H)$, we get $q_L^* = \frac{M}{M+\sigma^* np_L^*}(K - \bar{q} - \frac{\sigma^* np_H^*}{M} E[\bar{q} \wedge X])$, $q_A^* = \frac{\sigma^* n}{M+\sigma^* np_L^*}(p_L^*(K - \bar{q}) + p_H^* E[\bar{q} \wedge X])$. However, if $K - \bar{q} - \frac{\sigma^* np_H^*}{M} E[\bar{q} \wedge X] < 0$, q_L is negative. In such cases, we have $q_L^* = 0$, q_A^* and q_H^* satisfies $q_A^* + q_H^* = K$, $q_A^* = \frac{\sigma^* np_H^*}{M} E[q_H^* \wedge X]$.

□

Proof of Proposition 3. Similarly to the proofs of Proposition 1 and 2, we shall assume $p_i \leq \frac{v_i}{1-nw}$, and $w \leq \frac{1}{m+n}$.

We use backward induction to solve for the equilibrium. The proof consists of 2 steps.

1. Customers' best responses. A type i customer' utility from purchasing is $u(I) = v_i + p_i nw - p_i$, given $p_i \leq \frac{v_i}{1-nw}$, we have $u(I) \geq 0$. Thus the customers' strategy space is reduced to $a \in \{I, IA, AI\}$. His utility from redemption is $u(A) = v_i - mp_i w$. Since $(m+n)w \leq 1$, we have two possible customer behavior: (a) if $(m+n)w \leq 1$, then $u(A) \geq u(I)$ for $I = L, H$, $a_L^* = AL$, $a_H^* = AH$, low types first redeem q_A then buy q_L, high types buy q_H. (b) if $(m+n)w = 1$, then $u(I) < u(A)$ for $I = L, H$ and $p_i = p_L, p_H$, $a_L^* = L$, $a_H^* = HA$, low types first buy q_L, high types buy q_H then redeem q_A; (b)

2. Firm's optimal strategy. Rewrite the firm's profit as

$$
\begin{aligned}
\pi &= \sum_{i=L,H} p_i s_i \\
&\leq \sum_{i=L,H} \frac{v_i}{1-nw} s_i \\
&= \sum_{i=L,H} v_i s_i + nwp_i s_i \\
&= \sum_{i=L,H} v_i s_i + nw\pi \\
&= \sum_{i=L,H} v_i s_i + Rs_A
\end{aligned}
$$

   The firm can use (p, q) to generate a desired w that induces the two types of customer behavior as described above. In each case, we can derive the closed form functions of π, R and σ.

(a) If $(m+n)w \leq 1$, $a_L^* = AL$, $a_H^* = AH$, $\pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge X]p_H$.

There are two cases. If $q_L > 0$, $R(p,q,a^*(p,q)) = p_L(1-nw)$, $\sigma(p,q,a^*(p,q)) = \frac{mp_L q_A}{n\pi(p,q,a^*(p,q))}$. We have

$$\begin{aligned} \pi &\leq \textstyle\sum_{i=L,H} v_i s_i + R s_A \\ &= \textstyle\sum_{i=L,H} v_i s_i + p_L(1-nw)s_A \\ &\leq \textstyle\sum_{i=L,H} v_L(q_A + q_L) + v_H E[q_H \wedge X] \\ &\leq \pi^0 \end{aligned}$$

The Littlewood profit is attainable at $p_i = \frac{v_i}{1-nw}$, $q_H = q_H^0$, $q_A + q_L = q_L^0$ and $q_A^* \in [0, \frac{n}{m}(q_L + \frac{v_H}{v_L}E[q_H^0 \wedge X])]$ (so the redemption rate $\sigma \leq 1$).

If $q_L = 0$, $R(p,q,a^*(p,q)) = p_H(1-nw)$, $\sigma(p,q,a^*(p,q)) = \frac{mp_H q_A}{n\pi(p,q,a^*(p,q))}$. The optimal profit in this case is

$$\begin{aligned} \max_{p,q} \quad & \pi(p,q,a^*(p,q)) = E[q_H \wedge X]p_H \\ s.t. \quad & Mw = p_H(1-nw)\tfrac{p_H m q_A}{nE[q_H \wedge X]p_H} \\ & p_H \leq \tfrac{v_H}{1-nw} \\ & (m+n)w \leq 1 \\ & q_A + q_H \leq K \end{aligned}$$

By solving this program, we have $p_H = \frac{v_H}{1-nw}$, $q_A = \frac{v_L n}{v_H m}E[q_H \wedge X]$, $w = \frac{v_L}{v_L n + v_H m}$. The profit is $\pi^a = v_H(q_A + E[q_H \wedge X])$.

(b) If $(m+n)w = 1$, $a_L^* = L$, $a_H^* = HA$, $\pi(p,q,a^*(p,q)) = q_L p_L + E[q_H \wedge X]p_H$,

$R(p,q,a^*(p,q)) = p_H(1-nw)$, $\sigma(p,q,a^*(p,q)) = \frac{mp_H E[(X-q_H)^+ \wedge q_A]}{n\pi(p,q,a^*(p,q))}$. We have

$$\begin{aligned} \pi &\leq \textstyle\sum_{i=L,H} v_i s_i + R s_A \\ &= \textstyle\sum_{i=L,H} v_i s_i + p_H(1-nw)s_A \\ &\leq \textstyle\sum_{i=L,H} v_L q_L + v_H E[(q_H + q_A) \wedge X] \\ &\leq \pi^0 \end{aligned}$$

The Littlewood profit is attainable at $p_i = \frac{v_i}{1-nw}$, $q_L = q_L^0$, $q_A + q_H = q_H^0$ and $E[(X-q_H)^+ \wedge q_A] \in [0, \frac{n}{m}(\frac{v_L}{v_H} q_L^0 + E[q_H \wedge X])]$ (so the redemption rate $\sigma \leq 1$).

As a final step, we need to compare π^a from closing the low price capacity and and π^0 from the Littlewood benchmark. Note that when K is small, $q_L^0 = 0$, $\pi^0 = v_H E[K \wedge X] \leq v_H((K-q_H) + E[q_H \wedge X]) = \pi^a$. It suffices to show that when $q_L^0 > 0$, there exists $\bar{K}$, such that $\pi^a \geq \pi^0$ iff $K \leq \bar{K}$. Note that when $q_L^0 \geq 0$, $\frac{\partial \pi^0}{\partial K} = v_L$, $\frac{\partial^2 \pi^0}{\partial K^2} = 0$. In contrast, $\frac{\partial \pi^a}{\partial K} = v_H \frac{1 + \frac{nv_L}{mv_H}}{\frac{1}{\bar{F}(q_H)} + \frac{nv_L}{mv_H}}$, $\frac{\partial^2 \pi^a}{\partial K^2} = \frac{\partial \pi^a}{\partial \bar{F}(q_H)} \frac{\partial \bar{F}(q_H)}{\partial q_H} \frac{\partial q_H}{\partial K} \leq 0$. Therefore, $\frac{\partial^2 \pi^0 - \pi^a}{\partial K^2} < 0$, and $\pi^0 - \pi^a$ when $q_L = 0$, so there exists $\bar{K}$ such that $\pi^a \geq \pi^0$ iff $K \leq \bar{K}$. $\frac{\partial \pi^a}{\partial K} = v_H$,

$\square$

CHAPTER 2 : LOYALTY PROGRAMS AND CONSUMER CHOICE: EVIDENCE FROM AIRLINE INDUSTRY

We study customers' valuation of loyalty program points. By using airlines survey data, we compute customers' willingness to pay (WTP) for points at issuance and willingness to accept (WTA) at redemption. We demonstrate that compared to the objective value of miles, customers over-evaluate miles both at issuance (by 139%) and redemption (by 346%). The huge difference may result from overconfidence of the redemption value and redemption probability. We also show that airlines can improve profits (up to 7%) by simply manipulating program designs.

2.1. Introduction

Loyalty programs have been ubiquitous in modern industries. A frequent-flyer program (FFP) is a loyalty program offered by an airline. Such programs are designed to encourage airline customers to accumulate "miles" which may then be redeemed for air travel or other rewards.

The miles earned and redeemed under FFPs may be price-dependent or independent. The most traditional program type is mileage-based programs, in which both the issuance and redemption are price-independent: while the number of the issued miles equal to the flying distance of the customer, the number of redeemed miles is fixed for domestic flight. For example, consider a customer who flies from Philadelphia to San Francisco. Under a mileage-based program, he earns 2,515 miles (the distance) no matter how much he pays; when he wishes to redeem a domestic flight, he spends 25,000 miles. Contrary to mileage-based programs, fare-based programs determine both the number of issued miles and the number of redeemed miles proportional to the price the customer pays. In the previous example, suppose the airline changes to a fare-based program which issues 5 miles for every dollar the customer pays and requires 60 miles for every dollar he redeem. If the price is $500, then the customer earns 25,00 miles for a cash-paid ticket, and redeems 300,000 miles for an award ticket; if the price is \$300, then the numbers reduce to 1,500 miles and 180,000 miles respectively. Finally, a third type of program is mixed-program, in which the issuance is proportional to the price (like in a fare-based program) and the redemption is fixed (like in a mileage-based program).

Ever since American Airlines launched its AAdvantage mileage program in 1981, almost every airlines started with a mileage-based program. However, the industry has witnessed a recent trend of changing toward price-dependent issuance and redemption. For example, JetBlue (True Blue) and Southwest Airlines (Rapid Rewards) changed to fare-based programs around 2010. United Airlines and Delta Airlines offered their new mixed programs in 2014; American Airlines followed in 2016.

These program changes imposed an impact on the value of frequent flyer miles, which have emerged as a virtual currency. Typically, customers earn miles for their purchases and subsequently redeem them for rewards. It has been shown that frequent flyer program members are willing to spend 2% to 12% more for similar itineraries provided by the program carrier than by other airlines (Brunger, 2013). In fact, casual assessments tend to put the value of most loyalty points at between \$0.01 and \$0.02 each; for example, an AAdvantage mile is estimated to be worth \$0.017 (BoardingArea, 2014). Despite the heating discussions about the value of frequent flyer miles in different programs, it remains unclear how customers evaluate miles at their purchasing and redemption decisions.

Our research aims to investigate how customers evaluate this virtual currency given different program designs and rewarding rules. Specifically, we look at three levels of the values:

Market value of miles Since miles can be used to redeem for flight ticket, their market value can be calculated based on the market price of the awarded ticket. For instance, suppose for airline i, the every price of a ticket redeemed by 25,000 miles is \$500, then the market value of a mile is \$0.02. This reflects the objective value of miles (OBJ).

Value of miles at issuance When customers accumulate miles by paying cash, they may be willing to pay additionally for more issued miles. We are interested in the amount of cash price the customers are willing to pay for the issued miles. This reflects customers' subjective willingness to pay (WTP) as buyers of miles.

Value of miles at redemption When customers spend miles to redeem award, they compare the mile price to the cash price. We are interested in the customers' disutility of spending miles as opposed to spending cash. This reflects customers' subjective willingness to accept (WTA) as sellers of miles.

The goal is to calculate each level of the values, and study how they influence customers' choice decisions at both the purchasing point and redemption point. While many frequent flyers calculated that each mile is worth 1-2 cents based on the value of a redeemable award, the question remains unclear whether customers make decisions according to this value. We shall calibrate both the objective and the subjective values of miles from real customer transaction data. Moreover, we will address the following questions.

- How do the WTP and WTA of miles compare to the objective market values (OBJ)?
- Do operational decisions (award availability, award choices and award rules) impact how customers evaluate miles and consequently their decision-making processes?
- Do miles carry different values in different airlines and at different locations? What is the impact of program changes on the values?

Finally, given the value of miles, we are also interested in its interpretations on the airlines' profits. Specifically, the number of issued miles varies across program designs and may result in different impacts on revenues. Which program design is most profitable? Do airlines benefit from changing to fare-based issuance of frequent flyer miles? We shall address these questions in the study.

Our work is related to the literature on consumer behavior in the context of loyalty programs. Many papers have studied how consumers perceive and value loyalty points as an independent currency. Using a reference dependence framework, Drèze and Nunes (2004) developed a mental accounting model where customers evaluate different currencies (i.e., cash and loyalty points) in separate accounts; Stourm et al. (2015) recently extended this mental accounting model to explain why many customers stockpile loyalty points even though the firm does not reward such behavior. In another study, van Osselaer et al. (2004) showed that loyalty points are an overvalued currency and create an illusion of progress. In a similar vein, Kivetz et al. (2006) and Nunes and Drèze (2006) showed that artificial advancement (e.g., replacing a 10-stamp coffee card with a 12-stamp card that starts with 2 stamps already filled in) increases customer effort; the former study also found evidence of purchase acceleration as customers come closer to earning rewards. These results suggest that customers place an explicit value on each loyalty point even though loyalty points are only a medium (i.e., a means to an end); see Hsee et al. (2003) on the medium effect. Finally, Raghubir and Srivastava (2002) and Wertenbroch et al. (2007) found that consumers' valuation of an unfamiliar currency (such as loyalty points) is biased towards the face value; a possible explanation is that consumers anchor on the nominal face value and do not adjust sufficiently for the exchange rate when making decisions. Sayman and Hoch (2014) showed that buyers are willing to pay a price premium for loyalty points, and the premium is less than the normative levels. Motivated by these behavioral studies, our theoretical model takes the view that each loyalty point is a unit of currency valued at the nominal face value of goods that it can be redeemed for.

2.2. Data

The primary database we used is Airline Origin and Destination Survey (DB1B) conducted by Bureau of Transportation Statistics. It is a 10% sample of airline tickets from reporting carriers. Data includes origin, destination and other itinerary details of passengers transported.

To compare the effect of program change in early 2015, we used data in 2014 and 2015. Each quarter has more than 1 million itineraries over around 10 thousand routes operated by 27 carriers. We restricted our analysis to round-trip itineraries with an origin at large/medium hubs (with a ranking smaller or equal to 61 in Passenger Boarding (Enplanement) and All-Cargo Data for U.S. Airports). This accounts for 87.7% of total data.

Table 4-4 summarize the statistics of major airlines in the dataset in both years. Between the year of 2014 and 2015, the industry witnessed an overall price drop and demand increase, with two exceptions. United Airlines increased its price at the cost of demand loss. Southwest decreased its price but still had demand loss. Note that these numbers were calculated by aggregating all itineraries and routes.

Besides DB1B database, two other databases were used. First, the census population data for the origin cities was utilized to calculate the total market base. Second, the airline on-time performance data was used to get information about the take-off time of each flight, as well as the delay frequency and duration.

Table 3: Summary Statistics (2014)

Airlines	American	Alaska	United	Delta	Southwest	JetBlue	Frontier
Routes	2607	169	1081	2267	4082	240	426
Total Passengers	1663509	315565	1025765	1488061	3061011	515823	260145
Average Passengers	164.5	539.4	246.3	177.5	173.8	538.7	199.0
lightgrayAverage Price	225.0	178.8	252.4	238.6	179.3	192.0	137.1
yellow Average Distance	1156.8	1112.6	1363.8	1106.5	880.6	1241.0	1014.5
greenAverage Miles Awarded	1156.8	1112.6	1363.8	1106.5	1075.8	1152.0	1014.5
Origin Flight Share	14.8%	15.8%	17.8%	17.3%	40.2%	19.5%	3.5%
Award Fraction	6.3%	5.7%	9.2%	8.7%	11.5%	3.9%	2.9%

Table 4: Summary of Statistics (2015)

Airlines	American	Alaska	United	Delta	Southwest	JetBlue	Frontier
Routes	2636	140	1214	2230	4763	267	401
Total Passengers	1673525	349751	1071056	1529566	2675637	553929	338724
Average Passengers	175.1	672.6	245.3	180.1	170.1	553.6	281.1
lightgrayAverage Price	209.2	180.5	241.9	232.7	175.8	190.1	106.6
yellowAverage Distance	1163.1	1120.7	1349.6	1108.0	908.3	1249.9	1072.5
greenAverage Miles Awarded	1163.1	1120.7	1209.5	1163.5	1054.8	1140.6	1072.5
Origin Flight Share	14.1%	15.4%	33.8%	21.8%	30.3%	19.4%	2.7%
Award Fraction	5.4%	5.4%	8.9%	8.2%	12.0%	6.9%	1.5%

2.3. Methods

To analyze the problem, we aggregate itineraries over each flight route and calculate route-specific attributes (average price, standard deviation of prices, number of passengers, number of stops, mileage distance, etc.), resulting in over 10 thousand observations every quarter. (The standard deviation of prices is to capture price fluctuation in each route due to revenue management.)

2.3.1. Market Value of a Mile

The market value of a mile can be calculated by dividing the average market price of the redeemed awards by the number of miles needed for an award. For example, American Airlines requires 25,000 miles for a round-trip award ticket from Philadelphia to San Francisco. The average price of a direct flight ticket is \$334.96, resulting in an average value of 1.34 cent per mile; in contrast, the average price of a two-stop flight ticket is \$298.75, resulting in a value of each mile to be 1.19 cent. Using the similar method and aggregation, we can calibrate the value of a mile redeemed for every route, every airline and every year.

2.3.2. Value of an Issued Mile

In this subsection, we calculate the value of issued miles to customers. Consider customer i who plans to travel from Philadelphia to San Francisco (route j), he face the following options: i) 8am nonstop flight for \$800 by American Airlines, which issues 2,515 miles; ii) 2pm one-stop flight for \$500 by Southwest Airlines, which issues 3,000 miles; iii) 11pm one-stop flight for \$300 by Delta Airlines, which issues 1,500 miles. The customer's final decisions should take all information into account, i.e. flight time, fare price, and the number of issued miles. We shall analyze how each factor influences the customers' choice decisions.

For every route j offered by carrier c, its flight time is captured by a vector $takeofftime_{jc} = [takeofftime^t_{jc}]$, where $takeofftime^t_{jc}$ is the number of flights during time t_{th} hour of the day, $t = 1, \cdots, 24$. The prices are captured in variables $price_{jc}$ and $pricesd_{jc}$, which measure the mean and stand deviation of prices respectively. The number of issued miles is captured by $milesearned_{jc}$. Here, for mileage-based programs, we have $milesearned_{jc} = distance_j$, where $distance_j$ is the flying distance of route j; for fare-based programs, we have $milesearned_{jc} = issuerate_c \cdot price_{jc}$, where $issuerate_c = 6$ for Southwest Airlines, $issuerate_c = 5$ for United Airlines and Delta Airlines. (We didn't consider JetBlue here because the issuing policy varies by purchasing channels.)

Note that the effect of the issuing miles may be dependent on the future values of the miles, which further relies on their easiness of redemption. Without loss of generosity, we use two factors to capture the easiness of redemption: redemption rate and redemption choices. Hence, we shall include the interaction of these factors into the regression function as well.

The award redemption rate can be estimated using origin-carrier (OC) pairs. Specifically, we use $fracaward_{oc}$ as a proxy for reward redemption rate, where $fracaward_{oc}$ is the fraction of reward passengers over all passengers for all the routes from an origin o by carrier c. For example, at Philadelphia International Airport, American Airlines rewards 5.96% of its tickets, so we have $fracaward_{oc} = 0.0596$. Note that this metric is the joint result of both award supply and award demand. Nevertheless, for most Airlines except Southwest, the award availability is below 90% (IdeaWorks Survy); hence, $fracaward_{oc}$ is more of a valid metric for award supply than demand. Even in the case of Southwest, when $fracaward_{oc}$ purely reflects award demand, it should still be positively correlated with award redemption rate.

Similarly, we use OC pairs to calibrate redemption choices. One hypothesis is the following: if a higher fraction of future flights at the customers' home airport o is carried by airline c, the customer has more opportunities to use the miles issued by c. Hence, the expected future value of miles becomes higher, and the customer is more likely to be influenced by the issuance of miles. To capture this effect, we use $flightshare_{oc}$ as a proxy for the proportion of future flights carried by c, where $flightshare_{oc}$ equals to the fraction of flight routes offered by c over all the flight routes from origin o.

We use Logit model to estimate the impact of these metrics. Specifically, and the market share of route j from origin o to destination d by carrier c is given by

$$s_{jcod} = \frac{exp\{u_{jcod}\}}{\sum_{j',c'} exp\{u_{j'c'od}\}} \tag{2.1}$$

where u_{jcod} is the customer's expected utility:

$$\begin{aligned} u_{jcod} = \quad & \beta_0 + \beta_1 \cdot price_j + \beta_2 \cdot pricesd_j + \beta_3 \cdot distance_j + \beta_4 \cdot stops \\ & + \beta_5 \cdot milesearned_{jc} + \beta_6 \cdot fracaward_{oc} + \beta_7 \cdot flightshare_{oc} \\ & + \beta_8 \cdot milesearned_{jc} \cdot fracaward_{oc} + \beta_9 \cdot milesearned_{jc} \cdot flightshare_{oc} \end{aligned} \tag{2.2}$$

we also control for time, ticket carrier, flight quality (takeoff time, origin airport and destination airport), and program type in the regression.

Here, we are interested to see whether the coefficients from β_5 to β_{10} are significant. Besides, we can calculate the value of an issued mile to customers to be $w^I(\gamma) = \frac{\beta_5 + \beta_8 fracaward_{oc} + \beta_9 flightshare_{oc}}{\beta_1}$
.

However, note that the estimate of β_1 and β_2 may suffer from endogeneity issues. Specifically, if airline c forecasts the demand to decrease for route j, it may drop its prices to attract more travelers. As a result, the loss of passengers (or decrease in consumer demand) may seem to be a result of the price drop. Hence, β_1 is biased downward and consequently the value of miles are biased upward. It is also likely that that as demand increases, the potential revenue becomes more attractive and competition becomes more severe, the airlines drop prices to promote their own routes and β_1 is biased upward. Similarly, the airlines may apply different dynamic pricing strategies when expected demand varies, thus resulting in a biased estimate of β_2. To control for this endogeneity, we create a set of instrument variables, including the mean and standard deviation of the prices the airline charges at other origin airports, which is referred to as Hausman-type price instruments (Hausman, 1996). For example, consider a route from Philadelphia to San Francisco by American Airlines, we use the pricing information of all AA flights going to San Francisco from places other than Philadelphia as instruments. It captures the characteristics of the airlines' general pricing schemes, but does not include any other information from the route "market" (since the "market" is based on the origin - Philadelphia). We run a two step least squares regressions.

2.3.3. Value of a Redeemed Mile

In this subsection, we calculate the value of a redeemed mile. Consider a customer who has accumulated enough miles to redeem for a free flight ticket with carrier c. At each travel opportunity, he chooses between redeeming his miles and paying cash to save the miles for future use. For example, suppose he flies from Philadelphia and San Francisco. The current cash price is \$500, while the required miles for a redemption is 25,000. He compares the two options.

Using Logit model, the fraction of customers who pay cash for route j is given by

$$c_j = \frac{exp\{u_j^c\}}{exp\{u_j^c\} + exp\{u_j^m\}} \tag{2.3}$$

the rest is the proportion of customers who pay miles:

$$m_j = \frac{exp\{u_j^m\}}{exp\{u_j^c\} + exp\{u_j^m\}} \tag{2.4}$$

Here, u_j^c and u_j^m are the customers' expected utility from using cash (\$500) and miles (25,000 miles) for route j, respectively. Apparently, if the price in cash is high, the customers' utility from paying cash (u_j^c) is lower, if the price in miles or the value of miles is high, then the customers' utility from using the miles (u_j^m) is low. The value of miles depends on the chances of a future redemption using those miles, which might be dependent of the redemption rate ($fracredeemed_{oc}$) and redemption choices ($flightshare_{oc}$).

Thus, we can model these effects in a simple regression

$$\begin{aligned}\log \frac{m_j}{1-m_j} = \quad & u_j^m - u_j^c = \gamma_0 + \gamma_1 \cdot price_j + \gamma_2 \cdot pricesd_j + \gamma_3 \cdot distance_j + \gamma_4 \cdot stops \\ & + \gamma_5 \cdot milesredeemed_{jc} + \gamma_6 \cdot fracaward_{oc} + \gamma_7 \cdot flightshare_{oc} \\ & + \gamma_8 \cdot milesredeemed_{jc} \cdot fracaward_{oc} + \gamma_9 \cdot milesredeemed_{jc} \cdot flightshare_{oc}\end{aligned} \tag{2.5}$$

We are interested to estimate γ_1, γ_5, γ_8 and γ_9 and test whether they are significantly different from 0. Intuitively, $\gamma_1 \geq 0$ (higher cash prices, more redemption), $\gamma_5 \leq 0$ (higher mile prices, fewer redemption), $\gamma_8 \leq 0$ and $\gamma_9 \leq 0$ (easier redemptions, higher value of miles). Moreover, the value of one mile should be equal to $\frac{|\gamma_7 + \gamma_8 \cdot fracaward_{oc} + \gamma_9 \cdot flightshare_{oc}|}{\gamma_1}$.

However, note that the effect of $fracaward_{oc}$ does not only impact the customers' future redemptions, but also reflect the convenience of the current redemption as well. Specifically, if $fracaward_{oc}$ is high, the award capacity of the route is likely to be higher and the customer find it easier to redeem his miles. This results in a higher m_j and a positive γ_8, which biases the value of miles downward. Hence, the effect of future award availability on the value of miles cannot be accurately predicted by $fracaward$, so we exclude it from regression. Besides, the value of a redeemed mile may also be route-dependent, i.e., for those routes with higher values, the redeemed miles carry higher values. To capture this effect, we use a variable $destprice_d$, which equals to the average price of all routes to the same destination from other origins. This variables is a proxy for the value of the route, without using any market price information of the origin airport. Hence, the regression equation is updated below.

$$\begin{aligned}\log \frac{m_j}{1-m_j} = \quad & u_j^m - u_j^c = \gamma_0 + \gamma_1 \cdot price_j + \gamma_2 \cdot pricesd_j + \gamma_3 \cdot distance_j + \gamma_4 \cdot stops \\ & + \gamma_5 \cdot milesredeemed_{jc} + \gamma_6 \cdot destprice_d + \gamma_7 \cdot flightshare_{oc} \\ & + \gamma_8 \cdot milesredeemed_{jc} \cdot destprice_d + \\ & \gamma_9 \cdot milesredeemed_{jc} \cdot flightshare_{oc} + \gamma_{10} \cdot milesredeemed_{jc} \cdot fracaward_{oc}\end{aligned} \tag{2.6}$$

Similarly, the estimate of γ_1 may suffer from endogeneity issues. Specifically, if the route is more "touristic" (i.e., it has more leisure customers who wish to redeem miles), the firm may adjust prices as follows. First, the firm anticipates more redemptions, hence fewer capacity sold for cash. It increases cash prices to sell to a smaller group of high-end cash-paying customers. This inflates the estimate γ_1. Second, the firm anticipates higher demand for redemptions. To maintain the volume of cash sales, it decreases cash prices to induce more cash purchases. This deflates the estimate of γ_1.

Both endogeneity problems come from the unobserved demand for the route. We control this by including origin and destination in the regression equation. Besides, we also use the same set of instrument variables for the mean and standard deviation of prices.

Finally, note that we model the issuance and redemption separately. In the previous section, customers can purchase with cash from all airlines, but can not redeem miles. The simplification can be justified by two assumptions: (i) the customers are business travelers who only spend cash for the trip; (ii) the customers haven't accumulated enough miles for an award redemption. In contrast, this section allows customers to redeem miles, but does not consider the customers' outside option of using cash for other airlines. This assumption can be justified by the following reasons. First, since the customer has accumulated enough miles for a redemption, he must be a "frequent flyer" with carrier c. It is very likely that c is his preferred carrier. Research has shown that customers are willing to pay up to 12% more for similar itineraries provided by their major program carrier than by other airlines (Brunger, 2013). The customer may prefer to spending more with carrier c rather than switching to other carriers. Second, for all the origin airports we have, the dominant carriers offer around 49.29% of all flights, while the second popular carrier only carries 17.15% of all flights. Hence, competition is not very intense and we can assume that customers stick with one carrier once they accumulate enough miles. By these assumptions, we restrict the customers' choices to the simple binary decisions.

2.4. Results: Value of Miles

2.4.1. Market Value of Miles (Objective Value)

The average price of a redeemed ticket is $207.52, resulting in a value of one mile around 0.83 cent. This value is well below the 1.4 cent per mile benchmark calculated by Tripadvisor.com. Decomposing the redemptions into each airlines and both years yields the following table. (The values of Southwest and JetBlue miles are calculated by using program rules.)

Table 5: Market Value of a Mile (Unit: Cent)

Airlines	American	Alaska	United	Delta	Southwest	JetBlue
2014	0.98	0.77	1.07	1.04	1.67	1.40
2015	0.80	0.81	1.05	0.98	1.49	1.40

The most valuable miles are from Southwest Airlines. Their program allowed 60 miles to account for one dollar at redemption, yielding one mile to be worth of 1.67 cent. In 2015, Southwest devalued its miles by increasing the exchange rate to 70:1 from April. This resulted in an average value of 1.49 cent per mile throughout the year. The least valuable miles are from Alaska Airlines, due to low prices of the redeemed tickets ($192.14 in 2014 and $201.40 in 2015).

Moreover, we can also calculate the value of miles for every route. The most valuable redemption is flying directly from New York (EWR) to Dallas (DFW) by American Airlines, yielding a value of 2.25 cent per mile. The least valuable redemption is flying directly from Las Vegas (LAX) to Salt Lake City (SLC), yielding a value of 0.64 cent per mile.

2.4.2. Value of an Issued Mile (WTP)

We first run model (2.1) - (2.2) and summarize the results in Table 6.

The key observations are as follows:

1. Higher prices lead to lower customer utility. More revenue management (higher standard deviation of prices) leads to a higher chance of finding a bargain and hence higher customer utility.

2. Miles are worthless when award space and award choices are non-existent ($fracaward$ and $flightshare$ are standardized). Miles become a scam and may generate negative utility to customers. Only when there is any redemption options do customers value miles.

3. The average value of an issued mile is 3.64 cent.

4. Award availability ($fracaward$) has a negative impact on customer utility (perhaps due to the reduced availability for cash purchases), but a positive impact on the value of frequent flyer miles (customers enjoy the easiness of redemption).

5. Award choices ($flightshare$) has a positive impact on customer utility (since customers are likely to be members of dominant programs), but do not have any impact on the value of frequent flyer miles.

Table 6: Consumer Choice Model (Issued Miles)

Dependent Variable: Consumer Utility				
	OLS Regression		2SLS Regression	
	Model 1	Model 2	Model 3	Model 4
price	-0.0239^{***}	-0.0265^{***}	-0.0429^{***}	-0.0197^{***}
	(0.0009)	(0.0011)	(0.0043)	(0.0040)
pricesd	0.0372^{***}	0.0357^{***}	0.0632^{***}	0.0722^{***}
	(0.0013)	(0.0013)	(0.0060)	(0.0059)
stops	-21.3500^{***}	-21.0000^{***}	-21.1100^{***}	-20.5200^{***}
	(0.1254)	(0.1285)	(0.1627)	(0.1637)
distance	-0.0023^{***}	-0.0027^{***}	-0.0018^{***}	-0.0040^{***}
	(0.0001)	(0.0001)	(0.0002)	(0.0002)
milesearned		0.0004^{***}		0.0007^{***}
		(0.0001)		(0.0002)
fracaward		-3.2750		-35.6800^{***}
		(3.1800)		(3.9670)
flightshare		12.2500^{***}		-0.2018
		(0.6260)		(2.2230)
milesearned · fracaward		0.0007		0.0215^{***}
		(0.0039)		(0.0055)
milesearned · flightshare		-0.0003		-0.0004
		(0.0005)		(0.0007)
ticketcarrier	X	X	X	X
origin	X	X	X	X
destination	X	X	X	X
year	X	X	X	X
quarter	X	X	X	X
ontimes	X	X	X	X
takeofftime	X	X	X	X
farebased	X	X	X	X
R^2	0.7370	0.7405	0.7355	0.7305
Adj. R^2	0.7365	0.7400	0.7351	0.7300

6. The value of miles in cash can be predicted below with and without IV (we exclude JetBlue because the issuance rules depend on the purchasing channel):

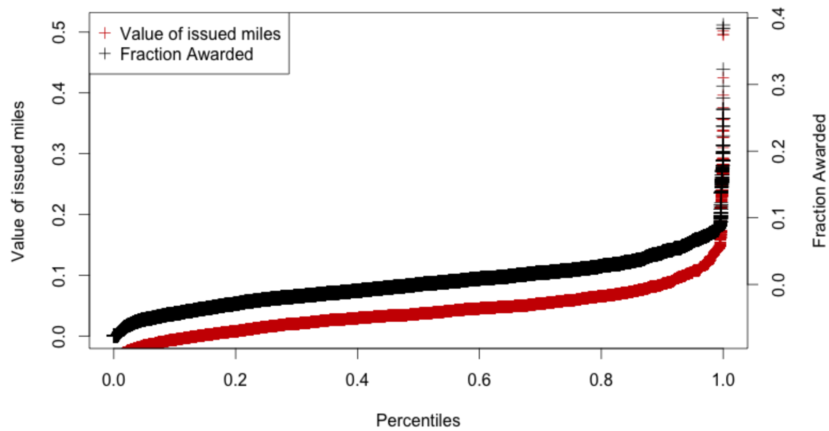

Figure 5: Value of Issued Miles

Table 7: Value of an Issued Mile without IV (Unit: Cent)

(Market Value in Parenthesis)

Airlines	American	Alaska	United	Delta	Southwest
2014	1.68 (0.98)	1.65 (0.77)	1.72 (1.07)	1.72 (1.04)	1.51 (1.67)
2015	1.68 (0.80)	1.67 (0.81)	1.52 (1.05)	1.65 (0.98)	1.64 (1.49)

Table 8: Value of an Issued Mile with IV (Unit: Cent)

(Market Value in Parenthesis)

Airlines	American	Alaska	United	Delta	Southwest
2014	1.95 (0.98)	0.47 (0.77)	3.76 (1.07)	3.69 (1.04)	5.94 (1.67)
2015	1.00 (0.80)	0.80 (0.81)	3.28 (1.05)	2.99 (0.98)	6.69 (1.49)

Key observations from Table 8:

(a) Southwest miles have the highest value (5.94-6.69cent), almost four times of the 1.4cent benchmark.

(b) Alaska miles have the lowest value (0.47 cent) in 2014, but its value improved in

2015 due to enhanced award capacity (a 6.4% improvement from 4.67% award capacity in 2014 to to 4.97% award capacity in 2015).

(c) The most valuable miles are Southwest Airlines at ABQ (12.22 cent per mile), followed by Delta Airline miles at CVG (8.27cent per mile). (CVG is a hub airport of Delta Airlines).

(d) The least valuable miles are Alaska Airline miles at BDL, with 0% award capacity.

2.4.3. Value of a Redeemed Mile (WTA)

We first run the models and summarize the results in Table 9.

Table 9: Consumer Choice Model (Redeemed Miles)

Dependent Variable: $u^c - u^m$				
	OLS Regression		2SLS Regression	
	Model 1	Model 2	Model 3	Model 4
price	0.1956***	0.3055***	0.3158***	0.2498***
	(0.0077)	(0.0086)	(0.0200)	(0.0131)
pricesd	0.1238***	0.0896***	0.3549***	0.4330***
	(0.0110)	(0.0110)	(0.0265)	(0.0260)
stops	16.9500***	22.4900***	23.3800***	25.7400***
	(1.0200)	(1.0430)	(1.0890)	(1.0820)
distance	0.0089***	0.0099***	−0.0040**	0.0067***
	(0.0006)	(0.0007)	(0.0012)	(0.0013)
milesredeemed		−0.0052***		−0.0059***
		(0.0002)		(0.0004)
flightshare		18.3400***		−49.0500**
		(5.5540)		(16.7400)
destprice		164.0000***		294.5000***
		(47.2900)		(62.4600)
milesredeemed · flightshare		−0.0012***		−0.0022***
		(0.0003)		(0.0004)
milesredeemed · destprice		−0.0025***		−0.0025***
		(0.0005)		(0.0006)
ticketcarrier	X	X	X	X
origin	X	X	X	X
destination	X	X	X	X
year	X	X	X	X
quarter	X	X	X	X
ontimes	X	X	X	X
takeofftime	X	X	X	X
farebased	X	X	X	X
R^2	0.3371	0.3526	0.3182	0.3376
Adj. R^2	0.3357	0.3511	0.3168	0.3361

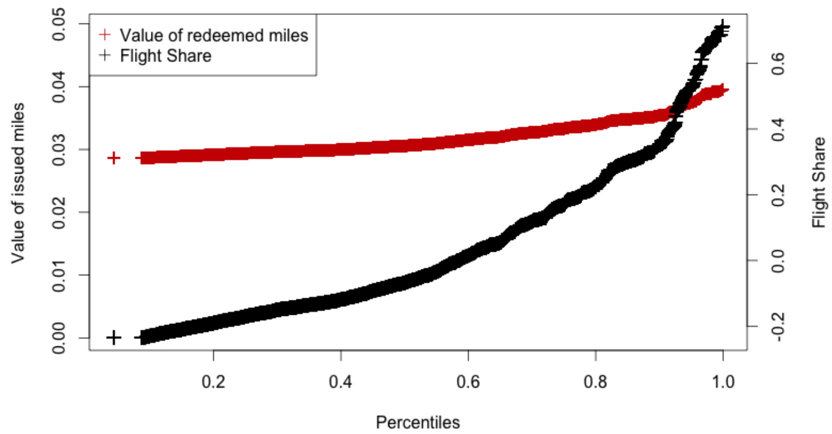

Figure 6: Value of Redeemed Miles

Key observations from the table:

1. Higher cash prices lead to more award redemptions; higher mile prices lead to fewer award redemptions.

2. The value of redeemed miles (WTA) is significantly positive even when there is no chance of using the miles in the future, i.e. $flightshare = 0$.

3. Higher chance of using miles in the future ($flightshare$) leads to higher value of redeemed miles. Higher values of the route destination (*destprice*) leads to higher value of redeemed miles.

4. The average value of a redeemed mile is 2.35 cent. The value of miles in cash can be predicted in the Table 10-11 (we does not include JetBlue because the redemption rules vary on flight-to-flight basis):

Table 10: Value of a Redeemed Mile without IV (Unit: Cent)

(Market Value in Parenthesis)

Airlines	American	Alaska	United	Delta	Southwest
2014	1.71 (0.98)	1.70 (0.77)	1.73 (1.07)	1.72 (1.04)	1.79 (1.67)
2015	1.71 (0.80)	1.69 (0.81)	1.79 (1.05)	1.74 (0.98)	1.75 (1.49)

Table 11: Value of a Redeemed Mile with IV (Unit: Cent)

(Market Value in Parenthesis)

Airlines	American	Alaska	United	Delta	Southwest
2014	2.29 (0.98)	2.29 (0.77)	2.33 (1.07)	2.32 (1.04)	2.49 (1.67)
2015	2.28 (0.80)	2.27 (0.81)	2.47 (1.05)	2.36 (0.98)	2.40 (1.49)

Key observations from Table 11:

(a) Southwest miles have the highest value (2.49 cent) in 2014. While it is still higher than the 1.4 cent benchmark, it is much lower than the WTP (over 5 cent).

(b) United miles have the highest value (2.47 cent) in 2015, due to improved flight share by over 15%.

(c) The most valuable redemption is by flying with American Airlines from Charlotte (CLT) to Cincinnati (CVG) (3.25 cent per mile). However, if that American Airlines mile is used to fly Baltimore (BWI) to Oakland (OAK), it only yields 1.89 cent in value.

(d) Another example is Alaska Airlines. Using one mile from Salt Lake City to San Jose gives a value 2.02 cent, while from Seattle to New York gives 2.75 cent per mile.

2.4.4. *Comparisons between WTP, WTA and Objective Values*

In the previous subsections, we calculated both the objective market values (OBJ) and the subjective values of miles at issuance (WTP) and redemption (WTA). The tables suggest that WTP$\geq$ and WTA$\geq$ OBJ. We will further calculate these values for every route (given origin, destination and stops) and compare them using paired one-tail t-tests.

- H_1: WTA>OBJ.
  - Without IV: The t-test is significant ($t = 386.8825, df = 72386, p - value < 2.2e - 16$) and gives a mean difference of 0.53 cent at 95% confidence interval.
  - With IV: The t-test is significant ($t = 825.6316, df = 72386, p - value < 2.2e - 16$) and gives a mean difference of 1.16 cent at 95% confidence interval.
- H_2: WTP>OBJ.
  - Without IV: The t-test is significant ($t = 222.3091, df = 72386, p - value < 2.2e - 16$) and gives a mean difference of 0.42 cent at 95% confidence interval.
  - With IV: The t-test is significant ($t = 236.8925, df = 72386, p - value < 2.2e - 16$) and gives a mean difference of 2.87 cent at 95% confidence interval.

Those t-tests support our hypotheses. Customers over-evaluate miles at issuance and underspend miles at redemption, compared to the objective value of miles.

2.5. Counterfactual Analysis

Recently, several airlines changed from mileage-based issuance of miles to fare-based issuance of miles. For instance, Southwest Airlines changed to fare-based program in 2011; Delta switched to mixed-program in 2015, United followed in the same year and American in the next year. In this section, we shall address the profitability of such program modifications.

For those airlines, a major change is the number of miles issued to customers. Those miles carry value. Therefore, if the airline issues more miles, then it can potentially charge a higher price and keep the same level of customer utility (hence the same level of market share as suggested by the Logit model). In contrast, if the airline issues fewer miles in the new program, it needs to decrease its price to make up for it. For instance, table 8 indicates that a United Airlines mile is worth 3.28 cent in 2015. If United issues 100 fewer mile to customers on average under the new program, it must decrease its price by $3.28 so customers would still purchase the same amount of tickets. Since the average fare of an United ticket is $266, the price drop would result in a decrease of 1.23% of its total profit.

The analysis can be decomposed into every route as follows. For every route, we first calculate the miles earned under the hypothetical program (for example, United has fare-based issuance in 2015, then the hypothetical program is fare-independent issuance, i.e., its old program in 2014, which issues one mile for every mile flown). Then, given the change in the issued miles, we calibrate the price that maintains the same level of market share (or same level of customer utility) in the hypothetical program. This could be done by utilizing the regression results in Table 6. Finally, we aggregate the revenue changes of all routes, to determine whether the hypothetical program is more profitable for this airline.

For example, United Airlines changed to mixed-program in 2015 and issued 1758.8 miles on average for a round trip ticket from PHL to SFO. In contrast, if it kept its mileage-based program, it would have issued 2645.8 miles on average for the same route - 887 more miles. Since each mile is valued at 3.28 cent, United can actually charge an additional $29.1 to keep the same market share. Consider another route from EWR to IAD. The current United Airlines' program issued 1033.6 miles while the hypothetical program would have issued 212 miles. Under the hypothetical program, United needed to decrease its price by $26.9 to maintain its market share. In the same manner, we calculated the hypothetical price change for every route and aggregate them together to study the impact of program change on United Airlines revenues, as well as other airlines. The results are summarized below.

Table 12: Revenue Benefit from Fare-based Issuance: Without IV

Airlines	Southwest	Delta	United	American
Revenue Benefit (2014)	1.95%	0.55%	-0.92%	-0.49%
Revenue Benefit (2015)	1.69%	0.34%	-0.92%	-1.32%

Table 13: Revenue Benefit from Fare-based Issuance: With IV

Airlines	Southwest	Delta	United	American
Revenue Benefit (2014)	7.08%	0.83%	-1.50%	-1.13%
Revenue Benefit (2015)	6.06%	0.37%	-1.78%	-1.09%

Table 12-13 suggest that the first movers (Southwest and Delta) benefited from the fare-based programs, not the followers. Specifically, Southwest improved its revenues by 6.06% under the fare-based program in 2015, and Delta improved by 0.37%. In fact, if Delta changed its program in the earlier year, it could have enhanced its profit by 0.85%. By simply manipulating the program deigns, the airlines were able to improve profits.

However, United lost a profit of 1.78% due to the program change. American would have lost 1.09% of its total revenues by changing its program in 2015 (we do not have data in 2016).

The contrasting results are due to the following reasons. As shown in table 3 and 4, Southwest and Delta offers flight routes of shorter distance. By changing from mileage-based issuance to fare-based issuance, they are able to issue more miles to their frequent flyers. Frequent flyers, earning more miles, are willing to accept higher prices. In such way, the airlines enhance their profits. Specifically, the benefit is even more significant for Southwest, since its average flight share (40%) at its operating airports is much higher in its airports of operations, which leads to higher value of the issued miles. In contrast, United and American operate longer-distance flights. Hence, they already issue many miles and cannot benefit from changing to the new program.

Issuing more miles lead to higher profits. However, it remains unclear whether the results hold when the number of issued miles approaches infinity, since the current analysis is limited by the range of issued miles in the data. It is likely that as more miles are put into circulation, a redemption probability that is too low leads to reduced values of miles eventually.

2.6. Conclusions

We have four conclusions:

First, frequent flyer miles carry values to customers only when they can be put into good use. Both the award capacity and award choices have a significant impact on the value of miles. At the average level of award capacity, a mile is valued between 2-4 cents in customers' decisions, which can be two times higher than the Tripadvisor.com benchmark. Southwest miles are most valuable, yielding around 6 cent per mile; JetBlue miles are least valuable at 1 cent per mile. Even within the same airlines, miles are valued differently by customers from different regions. For example, an American Airlines mile is worth 6.52 cent at JFK but is worthless at BUR.

Second, a redeemed mile is valued differently from an issued mile by customers. The average value of a mile at redemption is 2.35 cent. Redeemed miles have higher values when they are used from origin airports where their issuer provides more future flight options, and to destination airports with higher flight fares. For instance, an American Airlines mile redeemed from CLT to CVG is worth 3.25 cent; in comparison, if it is redeemed from BWI to OAK, the value decreases to 1.92 cent, due to lower market share at BWI and lower prices toward Oakland.

Third, compared to the objective value of miles, customers over-evaluate miles both at issuance (by 139%) and redemption (by 346%). The huge difference may result from overconfidence of the redemption value and redemption probability.

Finally, airlines can improve profits (up to 7%) by simply manipulating program designs. Specifically, short-distance carriers (Southwest and Delta) benefit from fare-based issuance of miles, while long-distance carriers (American and United) find mileage-based issuance more attractive. Airlines should pay careful attention to their flight attributes and design its loyalty programs in order to issue more miles into circulation.

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CHAPTER 3 : THE EFFECT OF DYNAMIC PRICING ON UBER'S DRIVER-PARTNERS

We study the effects of dynamic pricing (so-called "surge pricing") on Uber's driver-partners. Using a natural experiment arising from a surge pricing service outage for a portion of Uber's driver-partners over 10 major cities, and a difference-in-differences approach, we study the effect of showing the surge heatmap 1) on drivers' decisions to relocate to areas with higher or lower prices and 2) on drivers' earnings. We demonstrate that the ability to see the surge heatmap has a statistically significant impact on both outcomes, explaining 10%-60% of Uber drivers' self-positioning decisions and attracting drivers toward areas with higher surge prices, and increasing drivers' earnings on surged trips by up to 70%.

3.1. Introduction

The study of dynamic pricing at Uber and within other ride-sharing platforms has typically focused at an aggregate spatial level. This literature argues that rider-side pricing influences the number of riders wishing to take trips (10), and driver-side pricing along with this volume of riders taking trips influences the availability of drivers to successfully fulfill those trips (5; 8). This literature then seeks to understand how different pricing methodologies influence outcome like social welfare and the firm's profit (8; 3).

The way in which pricing influences availability of drivers, however, is multifaceted: This influence may occur through (1) changing the numbers of drivers driving somewhere within the city (by influencing drivers' decisions to sign up for Uber, or when to drive within the week); and (2) where to drive conditioned on having chosen to drive. The influence of price on *where* drivers drive if it indeed occurs, presumably happens through influencing drivers' expectations about the price and trip volume at a particular location in the near future. This influence on expectations may be effected through the *average* or typical price at a particular location and time of day/week as observed over longer timescales (days or weeks), and/or through the signaling effect of the current price on price and trip volume in other nearby locations over a much shorter timescale (minutes or hours).

Much of this influence (when to drive, and influencing where to drive through average price) can be effected by driver pricing schemes that are not dynamic. These could be effected by changing the driver-side price slowly over time, perhaps advertising in advance what the driver-side prices would be. Only the short-run signaling effect need be effected via a dynamic pricing scheme. Moreover, anecdotal evidence is ambiguous on the whether this short-run signaling effect is significant. While some drivers do report responding to the real-time value of the surge multiplier, others advise against "chasing surge" (6), suggesting that variability in surge prices and the costs of changing one's location makes reacting too strongly to surge prices disadvantageous.

If the short-run effect of dynamic pricing on signaling is not significant, and all of driver-side pricing's effect on driver availability occurs through slower timescales, one could imagine an alternate design from today's ride-sharing platforms using a static or slowly-varying driver-side pricing scheme intended to replicate the average-case dynamics of today's dynamic scheme, together with a more dynamic rider-side pricing scheme that reacts to short-term fluctuations in demand. This would be of particular interest because the dynamic nature of Uber's surge pricing has generated a great deal of attention, particularly within the popular press (18**?** ; 12), but also within the academic literature (3).

On the other hand, if the short-run effect of dynamic pricing on signaling is significant, then this alternatively suggests that supply-side dynamic pricing schemes can be useful mechanisms for reducing friction in labor markets, and may have implications for other two-sided markets where labor is constrained in space, in time, by skill-set, or by specialty.

Moreover, while intertemporal substitution of labor has been studied in ridesharing and taxi markets in the context of income targeting (9) and long-run supply elasticity (13), spatial elasticity in ridesharing markets has been under-investigated. The most closely-related paper to our knowledge (4) estimates taxi drivers' "spatial equilibrium" behavior in NYC and the impact of pricing policy. While both their papers and ours study similar settings using MNL models, their paper focuses mainly on the impact of ex-ante beliefs of location values on drivers' movement decisions, without the consideration of dynamic pricing or other real-time information. In contrast, our paper focuses on the impact of real-time spatial pricing on drivers' decisions.

In this paper, we study this question empirically using a natural experiment in which Uber's surge prices ceased to be visible for drivers using phones on the iOS operating system in 10 of its largest markets. Using a difference-in-differences approach (2; 7), and controlling for a number of confounding factors, we provide evidence that dynamic surge prices do have a significant effect on drivers' self-positioning decisions, causing drivers to drive toward nearby areas with higher surge values. We also show that having access to real-time information from the surge heatmap increased earnings for unaffected drivers, controlling for systematic differences between drivers using iOS and Android phones. This suggests that dynamic pricing is useful as a real-time signaling tool for reducing frictions in the ridesharing labor market, better aligning drivers' locations with riders' desire to take trips.

The use of a natural experiment is critical for answering the question of whether information provided by the surge heatmap causes drivers to relocate over short timescales. This is because endogeneity is particularly problematic for understanding causality in the surge heatmap's relationship to drivers' repositioning decisions. Drivers learn the areas of their city where demand tends to outstrip supply, and they tend to drive toward those areas to benefit from shorter wait times between trips and higher surge multiplies. If drivers make these repositioning decisions exclusively based on their private knowledge, and not based on the real-time information present in the surge heatmap, then we would nevertheless expect to see a correlation between the surge heatmap's multipliers and drivers' repositioning decisions. Disentangling this effect from the causal impact of the heatmap on drivers' movements would be challenging.

The natural experiment allows us to disentangle these effects. Using a difference-in-differences approach within a multinomial logit discrete choice model, we can compare the relocation decisions and earnings on the outage weekend among iOS drivers with an estimate for what these decisions and earnings would have been without the outage based on data from other weekends on all drivers, and data from the outage weekend from unaffected Android drivers.

In addition to the work cited above on ridesharing, our work is also related to the larger literature on spatial mobility in labor markets (16), and in particular to empirical analyses of the spatial elasticity of labor supply (15, Chapter 9,), (14). While related, this literature has typically focused on spatial mobility over longer time and spatial scales. More generally, our work can be viewed within the larger literature on informational and physical frictions in labor markets (16). This literature often focuses on search (17), and within this context, the surge heatmap can be seen as an aid that reduces the cost of search for drivers in the ridesharing market.

The rest of the paper is organized as follows. Section 2 starts with a description of methodology, including Uber's pricing system, the natural experiment, a driver behavior model and its difference-in-differences estimation. Section 3 dives into the estimation results and their implications of the effects of dynamic pricing on both driver movement and earnings. Finally, Section 4 summarizes with conclusions.

3.2. Methodology

We will analyze a natural experiment in which an outage made the surge heatmap unavailable for drivers using the iOS driver app in many of Uber's largest cities over one weekend. To analyze this experiment, we use a multinomial logit model over the driver's direction of motion. This MNL model uses utility determined by a factor model over the change in surge multiplier in each direction of movement, the visibility of the surge heatmap, the driver's operating system, a time indicator controlling for changes in driver movement between the outage and non-outage weeks, and driver covariates intended to control for differences in behavior between iOS and Android drivers. Under the assumption of zero coefficients for the two-way interaction between the operating system and time terms, and for the three-way interaction between operating system, time, and surge multiplier change, this analysis is able to identify the impact of making the surge heatmap visible to drivers, with results presented in section 3.3.

We now describe this methodology in detail, first summarizing background on Uber's surge pricing system (section 3.2.1), then summarizing the MNL model (section 3.2.3) and factor utility model (section 3.2.4), and finally describing the natural experiment and how the MNL and factor utility model was applied to analyze it (section 3.2.5).

3.2.1. Background on Uber's Surge Pricing System

Uber operates a two-sided market in which individuals wishing to take a trip ("riders") are matched with other individuals willing to drive them for a fee ("drivers"). The rider requests a trip via a smartphone application, the "rider app", and the driver accepts or rejects dispatch requests via another smartphone application, the "driver app". We focus on the UberX service in which a single rider or party of riders occupies a car, and do not discuss other Uber products.

At the time when the natural experiment we analyze occurred, both the price paid by the rider and the fee earned by the driver for participating in the UberX service were both set via a "surge multiplier" and the "unsurged fare". The unsurged fare was computed from the time and distance traveled by the driver with the rider in the vehicle via a fixed city-specific linear functon, while the surge multiplier was computed dynamically as described below. The rider price was then obtained by multiplying the unsurged fare by the surge multiplier. The driver's earnings for the trip were then calculated by removing a fixed commission from the total amount paid by the rider.

Cities are partitioned into non-overlapping uniform hexagons, each with an edge length of 0.2 miles (0.32km). Each hexagon is assigned its own surge multiplier, which is recalculated every 2 minutes, and is applied to all trips starting in that hexagon over that two minute period.

Uber sets surge multipliers dynamically and algorithmically. This algorithm sets surge multiplier based on the number of riders in the process of using the rider app to make trip requests in a geographically localized area, the number of driver partners in or near that area who have made themselves available to conduct trips via the driver app, as well as some additional factors.

The surge pricing algorithm is designed to balance supply and demand in real time: when the number of trip requests seem likely to exceed the number of trips that nearby cars can fulfill, it increases the surge multiplier to ensure that only riders that place a high value on taking a trip do so, and to attract drivers to the undersupplied area.

It is reasonable to expect that drivers would prefer to be in hexagons with high surge multipliers, both because a higher surge multiplier results in a larger payment to the driver holding fixed a trip's time and distance, and because a higher surge multiplier typically indicates that the ratio of riders to drivers is high and thus a driver's waiting time for a trip will be short. Uber also prefers drivers to move to areas with higher surge multipliers, because their presence in high-demand areas allows more riders to take trips with lower waiting times.

To support this movement toward surging areas, the driver app shows a visualization called the "surge heatmap" (see Figure 7) that displays the current surge multiplier in each hexagon. Drivers can see this surge heatmap when they have indicated in the driver app they will consider dispatch requests and they are not currently servicing a request. We call such drivers in the "open" state. Drivers who are unwilling to consider dispatch requests (either because they have indicated so in the app, or the app is turned off) are considered to be in the "offline" state. Once a rider is matched with an open driver, the driver is given directions on where to meet the rider and the surge heatmap is no longer visible. We call drivers on their way to pick up a rider "en route," and drivers who are driving with a rider in the car “on-trip.”

In the following sections we describe mathematical methodology used to model driver behavior in response to this surge heatmap and other information they may have, in preparation for describing and analyzing a natural experiment pertinent to the question of whether drivers’ movement decisions are influenced by this surge heatmap.

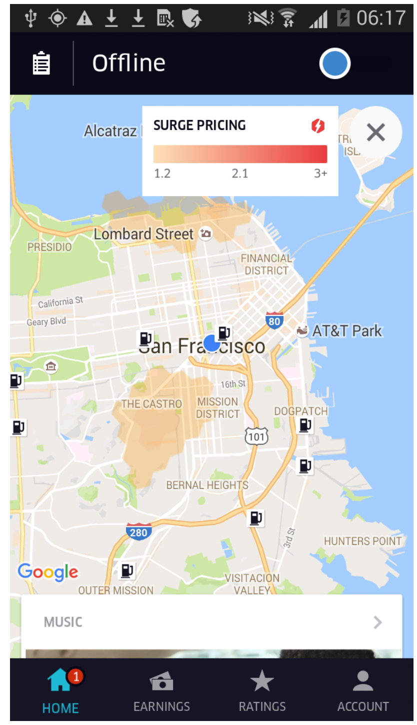


Figure 7: Screenshot of the surge heatmap in the Uber driver app. The surge heatmap shows the current value of the surge multiplier in each hexagon to driver partners.

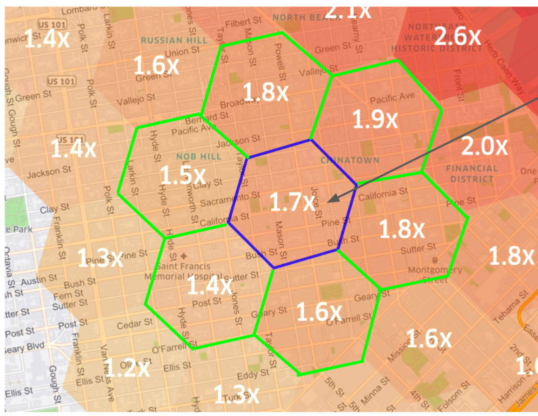
3.2.2. Data Overview

We used data from three data sources: 1) surge heatmap data that stores the time series of surge values in each hexagon across the cities; 2) driver location data that records each driver’s hexagon location at the beginning of each minute; 3) driver metric data that contains basic driver information, including the operating system of their phones. Table 1 provides summary facts for this data sets in the 10 largest cities.

Combining the three data sets we create a data set ready for analysis (Table 2). Each row describes the information related to one driver-minute, including driver, time, driver’ current hexagon, next hexagon he moved to, as well as the price information of all the surrounding hexagons and driver iOS information.

3.2.3. A Model of Driver Behavior

Consider a driver d in the open state at minute t in hexagon i, deciding whether to stay or move. We are interested in this driver’s desired direction of motion. To study this, we record his state (open, en-route, on-trip, or offline) at minute $t+1$, and if he is open, en-route or on-trip we record his location. For drivers that are open at minute $t+1$, we determine whether each driver remains in the same hexagon i (indicating this lack of motion by $j=0$), has moved to one of the 6 immediately adjacent hexagons (indicating these directions of motion by $j \in \{1,2,\ldots,6\}$), or has moved to some other hexagon. We model the selection j as a choice made by a driver. This choice is illustrated below in Figure 8.



7 options with Prices

	Immediate	Smoothed
0	1.7x (+0.0x)	1.69x (+0.0x)
1	1.9x (+0.2x)	2.02x (+0.33x)
2	1.8x (+0.1x)	1.83x (+0.14x)
3	1.6x (-0.1x)	1.57x (-0.12x)
4	1.4x (-0.3x)	1.33x (-0.36x)
5	1.5x (-0.2x)	1.37x (-0.32x)
6	1.8x (+0.1x)	1.84x (+0.15x)

Figure 8: The figure illustrates a driver at an origin hexagon i (outlined in blue) choosing which hexagon to move to next (adjacent hexagons are outlined in green). We model this choice as being correlated with the change in smoothed surge multiplier (shown at right) between the origin hexagon and the 3 hexagons in the chosen direction of motion.

Drivers that are not open at minute $t+1$ or that move to a hexagon outside i and its 6 immediate neighbors are treated making choices that are unobserved. The most frequent cause for not being open at time $t+1$ is a driver's being dispatched, placing them in en-route state. Drivers can cross two hexagons in 1 minute if they are on a highway or another arterial road permitting high-speed travel. The 7 values for j we consider include over 90% of driver movements among drivers that remained open.

We model drivers' choices using a multinomial logit (MNL) discrete choice model (1). Specifically, we model an open driver's utility of moving in direction j from hexagon i at minute t as

$$u(t,i,j,d) + \xi_{t,i,j,d}.$$

Here, $\xi_{t,i,j,d}$ is an independent Gumbel distributed random variable. $u(t, i, j, d)$ represents the driver's perceived utility of making movement direction j when in hexagon i at time t for a driver with a set of driver features d. We will discuss the form of $u(t, i, j, d)$ in detail below.

While this model, conditioned on having an observation (which includes, for example, the condition that the driver is not dispatched), and given a particular utility function, the probability of observing choice j is,

$$P(j|t, i, d) = \frac{\exp(u(t, i, j, d))}{\sum_{j'=0}^{6} \exp(u(t, i, j', d))}.$$

A number of factors may contribute to a driver's choice of j. We are interested most importantly in the causal effect of the surge multiplier, but also the confounding effect that motion may be correlated with the surge multiplier because drivers tend to move toward high-demand areas and high-demand areas tend to surge. To allow our model to capture the dependence of motion on such factors, we include in our utility $u(t, i, j, d)$ the difference in smoothed surge multiplier $\Delta p(t, i, j)$ between the origin hexagon i and for the hexagon in direction j. We will address possible confounding through the natural experiment discussed below.

More precisely, the term $\Delta p(t, i, j)$ is computed by first computing a "smoothed" surge multiplier for each hexagon, obtained by averaging the surge multipliers in the hexagons in three concentric rings. This provides a smoothed surge price for the origin $p(t, i)$ and for the hexagons in the 6 directions $p(t, j)$. Then $\Delta p(t, i, j) = p(t, j) - p(t, i)$ is the difference in these prices. The values of the smoothed surge multipliers and corresponding $\Delta p(t, i, j)$ are presented in the column "Smoothed" in the table in Figure 8. The values in the "Immediate" column are based on the non-smoothed surge multipliers from only a single hexagon. The motivation of using smoothed multipliers is to capture the attraction of surged hexagons that are further away than one ring. When we did not smooth multipliers, we saw similar results on all metrics.

Another factor that may contribute to a driver's choice of j is whether the heatmap was visible when the driver was making his or her decision. This will be encoded through a factor invisible(d, t), which takes a value of 0 for drivers d and times t for which the surge heatmap was visible to open drivers (as is typical), and takes a value of 1 when the heatmap is hidden, as it was in the outage in our natural experiment.

While we discuss in more detail the specific functional form assumed for $u(t, i, j, d)$ in the next section, and discuss assumptions following from that functional form there, we note and briefly discuss assumptions we have made thus far:

A1. $\xi_{t,i,j,d}$ are independent across t, i, j, d. This assumes implicitly and in particular that driver d's decisions are not directly influenced by other drivers in the immediate area. It does, however, allow a driver's decisions to be influenced indirectly by other nearby drivers, through the impact their presence has on surge multipliers and waiting times.

A2. The dependence of $u(t, i, j, d)$ on hexagon i and direction j is only through the price difference $\Delta p(t, i, j)$.

A3. Drivers' movement decisions are not influenced by surge multipliers further than the 4th ring of hexagons from their current hexagon. This is a distance of approximately 1.5 miles.

A4. Observations of drivers movements are censored independently of their unobserved movement decisions.

A5. We assume that the probability distribution describing driver's choices' has the functional form of an MNL model.

We now discuss the functional form of $u(t, i, j, d)$ in detail.

3.2.4. The Utility Function

Without loss of generality, we normalize the utility of staying in the same hexagon, $u(t, i, 0, d)$, to be 0. The utility of moving in direction j, $u(t, i, j, d)$, then is the change in the driver's utility relative to staying. To model the value of $u(t, i, j, d)$ for $j > 0$, we use a factor model containing the following features:

- the difference in surge multiplier $\Delta p(t, i, j)$ discussed above
- a collection of driver metrics that depend on d: the operating system, the driver's age, and the driver's tenure on the Uber platform. We discuss and motivate these choices below. We indicate these here in a generic way with a vector $x(d)$ with components $x_k(d)$, where k starts at 0.
- the binary indicator invisible(d, t) that is 1 if the heatmap is hidden to drivers and 0 otherwise
- a binary time indicator $T(t)$. Within the analysis of the natural experiment, we will apply our factor model to data collected over two weeks. This indicator will take the value 1 for the week when the outage occurred, and 0 in the previous week.

This model has the following specific form:

$$\begin{aligned} u(t, i, j, d) = &\beta_0 + \beta_1 \cdot \Delta p(t, i, j) \\ &+ T(t) \cdot [\beta_2 + \beta_3 \cdot \cdot \Delta p(t, i, j)] \\ &+ \text{invisible}(d, t)[\beta_4 + \beta_5 \cdot \Delta p(t, i, j)] \\ &+ \sum_k x_k(d) \cdot [\beta_{6+3k} \cdot + \beta_{7+3k} \cdot \Delta p(t, i, j) + \beta_{8+3k} \cdot T(t) + \beta_{9+3k} \cdot \Delta p(t, i, j) \cdot T(t)] \end{aligned} \tag{3.1}$$

The first row of coefficients includes a constant term β_0, which one can interpret as the value of moving out of the current hexagon if all other factors are 0 (recall that this utility is the value for all $j > 0$, and the utility at $j = 0$ is fixed to 0). It also includes a term β_1 that represents the desirability of moving toward increasing surge multiplier. The second and third rows contains similar terms, but now interacted with the time indicator $T(t)$ in the second row and the surge visibility invisible(d, t) in the third. The fourth row also contains similar terms interacted with each driver feature, and also the interaction of this driver feature with the time indicator and the price difference.

Taken collectively, the sum $\beta_1 + \beta_3 \cdot T(t) + \beta_5 \cdot \text{invisible}(d,t) + \sum_k (\beta_{7+3k} + \beta_{9+3k} \cdot T(t)) \cdot x_k(d)$ represents the dependence of the utility to the surge multiplier gradient, including both dependence due to causal factors (drivers being attracted to areas with higher surge multiplier) and due to non-causal confounding factors (drivers wishing to move toward areas that are good to drive in, that happen to also have higher surge gradients). The coefficient of β_5 determines how this sensitivity changes when the surge heatmap is hidden, and it is on this coefficient that we will focus when using the natural experiment to understand the causal relationship between the surge heatmap and driver movement.

We take note of the model form (3.1) as an assumption.

A6. We assume that drivers' utility is modeled by the functional form (3.1).

3.2.5. Description of the Natural Experiment

During the weekend of November 4th to 6th in 2016, cities served by one of Uber's data centers suffered from a technical outage in the surge pricing system. These cities included New York City, Boston, Chicago, Washington DC, and many other cities in the United States and around the world. In the affected cities, drivers using the driver app on an iOS phone (so-called "iOS drivers") received a blank map with no surge information. Drivers using the driver app on an Android phone (so-called "Android drivers") could see the surge heatmap as usual. The outage only affected iOS drivers' ability to *see* the surge heatmap, but did not change the way in which they were paid.

The dispatch screen shown to drivers when they are offered a trip indicates the surge multiple, and this was working normally. Thus, while some drivers at some times were likely unaware that there was an outage and simply thought no areas were surging, many drivers would have quickly become aware that the surge heatmap was not working, especially those positioned in parts of the city that were surging.

The natural experiment enables us to study the impact of the lack of visibility of the surge heatmap on drivers by, roughly speaking, comparing the difference between iOS and Android drivers on the outage week and another non-outage week, while controlling for systematic differences between these two groups. If lack of visibility has an impact, then this difference between iOS and Android drivers should change significantly during the outage week. For the weekend unaffected by the outage, we gathered data from the weekend of 10/22 to 10/24. This skips the immediately previous Halloween weekend, since Halloween is one of Uber's busiest days and causes unusual activity.

3.2.6. Difference-in-Differences Estimation (DID)

To apply the previously discussed model within our natural experiment, we explicitly write our list of driver metrics as $x(d) = (\mathrm{iOS}(d), \mathrm{age}(d), \mathrm{tenure}(d))$. Here, $\mathrm{iOS}(d)$ is a binary variable that is 1 if the driver uses an iOS phone; $\mathrm{age}(d)$ is a continuous variable storing the driver's age; and $\mathrm{tenure}(d)$ is a continuous variable storing the number of years that have passed since the driver signed up to drive with Uber. The choice of $\mathrm{iOS}(d)$ allows us to compare drivers that experienced the outage from those that did not, while the two covariates are present to control for systematic differences between iOS and Android drivers, as discussed below.

We then note that the surge heatmap is only hidden during the outage week for iOS drivers. Thus, $\mathrm{invisible}(d,t) = \mathrm{iOS}(d) \times T(t)$.

We finally make the following additional assumption, in light with the parallel trend assumption ((7)) typically made in applications of DID methodology.

A7. The coefficients on the interaction terms $\text{iOS}(d) \cdot T(t)$ and $\text{iOS}(d) \cdot T(t) \cdot \Delta p(t,i,j)$ are 0.

Assumption A7 assumes that the difference between iOS and Android drivers stays the same week over week (except for the outage), and as we change the surge multiplier gradient $\Delta p(t,i,j)$. To help ensure that A7 is met, we include $\text{age}(d)$ and $\text{tenure}(d)$ and their interactions with $T(t)$, $\Delta p(t)$ and $T(t) \cdot \Delta p(t)$ in our regression. This is discussed in more detail in the next section.

Applying these three modeling choices to the utility model (3.1), we obtain:

$$\begin{aligned}
u(t,i,j,d) = \beta_0 &+ \beta_1 \cdot \Delta p(t,i,j) \\
&+ T(t) \cdot [\beta_2 + \beta_3 \cdot \Delta p(t,i,j)] \\
&+ \text{invisible}(d,t) \cdot [\beta_4 + \beta_5 \cdot \Delta p(t,i,j)] \\
&+ \text{tenure}(d) \cdot [\beta_6 + \beta_7 \cdot \Delta p(t,i,j) + \beta_8 \cdot T(t) + \beta_9 \cdot \Delta p(t,i,j) \cdot T(t)] \\
&+ \text{age}(d)[\beta_{10} + \beta_{11} \cdot \Delta p(t,i,j) + \beta_{12} \cdot T(t) + \beta_{13} \cdot \Delta p(t,i,j) \cdot T(t)] \\
&+ \text{iOS}(d) \cdot [\beta_{14} + \beta_{15} \cdot \Delta p(t,i,j)]
\end{aligned}$$

To estimate the model parameters, we use maximum likelihood estimation with the likelihood implied by this factor model for the utility and the MNL model over driver decisions j. To create confidence intervals and perform hypothesis tests, we use a bootstrapping approach (11).

Within this model and estimation method, the sensitivity to the surge gradient is given by

$$\begin{aligned}
&\beta_1 + \beta_3 \cdot T(t) + \beta_5 \cdot \text{invisible}(d,t) + \beta_7 \cdot \text{tenure}(d) + \beta_9 \cdot \text{tenure}(d) \cdot T(t) + \\
&\beta_{11} \cdot \text{age}(d) + \beta_{13} \cdot \text{age}(d) \cdot T(t) + \beta_{15} \cdot \text{iOS}(d).
\end{aligned} \tag{3.2}$$

With this in mind, we wish to test the following hypotheses about this sensitivity in our analysis:

- that equation (3.2) is positive (indicating that drivers tend to move toward surge) for typical values of $T(t)$, tenure(d), age(d), and iOS(d) when invisible$(d, t) = 0$
- β_5 is negative, showing that sensitivity of movement to surge is reduced when the heatmap is not visible
- that equation (3.2) remains non-negative when invisible$(d, t) = 1$, showing that lack of visibility of the surge heatmap does not cause drivers to move *away* from surge

Additionally, the coefficient associated with staying in the same place when $\Delta p = 0$ is given by

$$\begin{aligned} &\beta_0 + \beta_2 \cdot T(t) + \beta_4 \cdot \text{invisible}(d,t) + \beta_6 \cdot \text{tenure}(d) + \beta_8 \cdot \text{tenure}(d) \cdot T(t) + \\ &\beta_{10} \cdot \text{age}(d) + \beta_{12} \cdot \text{age}(d) \cdot T(t) + \beta_{14} \cdot \text{iOS}(d) \end{aligned} \tag{3.3}$$

We wish to test the hypotheses that:

- this coefficient is negative for typical values of $T(t)$, tenure(d), age(d), and iOS(d) when invisible$(d, t) = 0$.
- this coefficient remains negative when invisible$(d, t) = 1$.

3.2.7. Differences Across Operating Systems and Time

Our approach relies on the assumption (A7) that iOS$(d) \cdot T(t)$ and iOS$(d) \cdot T(t) \cdot \Delta p(t, i, j)$ have zero coefficients. To study this assumption, we study the difference between the two weekends in our analysis, and the difference between iOS and Android drivers. We find differences both across weekends and across groups, which are mitigated by controlling for time, operating system, and other driver covariates in our analysis.

Figure 9: Surge Multipliers Over Time: The left-hand plot shows the percentage of drivers' earnings that were due to surge on the outage weekend and the previous non-outage weekend in each of the cities in our analysis. The right-hand plot similarly shows the percentage of surged trips between the two weekends and across cities.

Differences Across Time

Market conditions are determined by the imbalance between demand and supply. On the demand side, large events, weather and traffic conditions influence customers' need for a rider; on the supply side, incentive campaigns, competitors' strategies and other opportunity costs affect drivers' driving hours. There is no guarantee that market conditions over any two weekends are similar. Indeed, demand and supply patterns behave differently over the two weekends studied, and the surge pricing algorithms adjusts for the change correspondingly, as shown by the statistics in Figure 9.

In general, the previous weekend was more supply-constrained and consequently surged more. For example, 41% of trips in Boston had a surge multiplier strictly larger than 1 in the previous weekend while only 12% of trips in the outage weekend did. Similarly, surge impacted drivers' income to a different extent over the two weekends. Surge income constituted 17% of all drivers' income during the past week, but only 5% during the outage weekend.

Differences between the iOS and Android Drivers

Drivers' choice of the phone's operating system (OS) might reflect differences in demographics, which are correlated with their driving habits. To test this, we collected data on drivers' age and tenure (years since singing up for Uber) along with their phones' operating systems. Data exhibited extreme diversity: drivers' age ranged from 18 to 82, and tenure varied from just a few days to over six years.

Figure 10 shows that iOS drivers are on average younger than Android drivers in all cities.

However, the ordering of their tenure differs across cities. Specifically, an average iOS drivers has a longer tenure in Chicago (CHI), Boston (BOS), Washington D.C.(DC), Hong Kong (HK), Moscow (MOW) and New Jersey (NJ), while an average android Android driver has a longer tenure in New York (NYC), Atlanta (ATL), and Dallas (DAL).

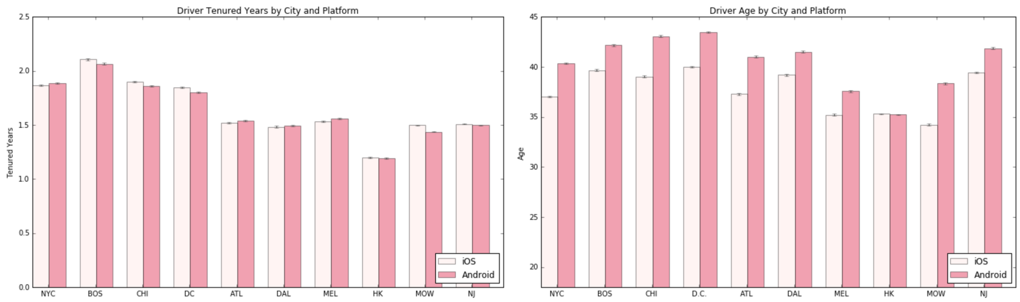


Figure 10: Differences by Operating System: The figures show the tenure (left) and age (right) for iOS and Android Drivers, by city. Confidence intervals for the mean value are shown using the standard deviation of the sample mean.

To address these differences, we include tenure and age as covariates in our factor model, their interactions with $T(t)$ and $\Delta p(t,i,j)$, as well as the covariate $\text{iOS}(d)$.

To further verify Assumption A7, we include in the appendix the results of a DiD analysis with two regular weekends, which concludes that the coefficient on $\text{iOS}(d) \cdot T(t) \cdot \Delta p(t,i,j)$ is 0.

3.3. Results

3.3.1. Impact of Surge on Driver Movement

Maximum likelihood estimates along with confidence intervals for model coefficients are listed in Table 3. This table consists of two parts i) coefficients described in (3.3) that represent the disutility of moving away from the current hexagon when $\Delta p(t,i,j) = 0$; and ii) coefficients described in (3.2) that represent the sensitivity of movement to the surge gradient $\Delta p(t,i,j)$.

The table leads to several conclusions. First, it shows that in all cities, drivers incur a disutility for driving out of the current hexagon ($\beta_0 < 0$). Moreover, if they choose to drive out, they derive a higher utility when they are driving toward hexagons with higher surge values ($\beta_1 > 0$). As the estimated value of β_0 is large and negative compared to the other coefficients in (3.3), if we compute (3.3) for another combination of factors the coefficient will remain negative. A similar statement tends to hold for β_1 and (3.2), although not universally.

Second, surge information impacted driver movement, even when controlling for confounding factors. Lack of visibility of the surge heatmap caused drivers to be less sensitive to surge differences: β_5 is significantly negative for all cities except for Washington D.C. and New Jersey. Without the real-time knowledge of seeing the surge heatmap, iOS drivers had a weaker signal of where to drive.

Table 3 quantifies the exogenous effect (β_5) and endogenous effect ($\beta_1 + \beta_3 + \beta_{15}$) of surge for iOS drivers respectively. Therefore, we can measure the actual value of the real time surge information on movement, out of the total surge-movement effect, as

$$e = \frac{-\beta_5}{\beta_1 + \beta_3 + \beta_{15}}$$

The exogenous effect of the heatmap accounted for 10% to 60% of the movement effect (Figure 11). Consistent with discussion below, the effect is the lowest in the large cities with more experienced drivers and higher surge (New York, Boston, Chicago).

Third, in all of the 10 cities, more experienced drivers were less likely to drive out of the current hexagon ($\beta_6 < 0$). Here are two potential explanations: 1) experienced drivers understood that real-time demand conditions may change rapidly, and an imbalance that was causing surge may indeed dissipate in the 2 minutes that pass between surge calculations. For this reason, they chase surge less often than inexperienced drivers; 2) experienced drivers, afraid of being dispatched out of a low-surge hexagon, turned off their *Open* status (go *Offline*) to be indispatchable, then drive toward high-surge hexagon and become *Open*

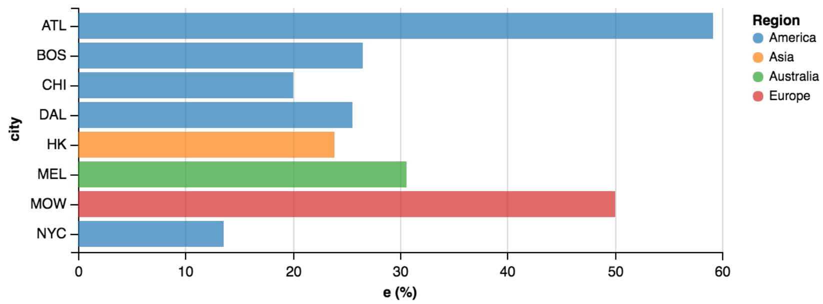


Figure 11: Impact of Surge Information on Movement (%)

Removed Cities with insignificant coefficient (DC, NJ)

upon arrival. When they were *Open*, they did not need to move further. While experienced drivers moved less often, their movement might be more or less sensitive to the surge values in the direction of movement (β_7). Interestingly, in big cities (Boston, Chicago, Washington D.C.), experienced drivers are less attracted by surge; in small cities (Melbourn, Hong Kong, Moscow and New Jersey), the effect was reversed.

3.3.2. Impact of Surge on Driver Earnings

We now ask whether access to the surge heatmap improves drivers' earnings using a similar DID analysis. Specifically, we model driver d's change in hourly earnings from the previous weekend as

$$\text{Earnings}^{\text{outage}}(d) - \text{Earnings}^{\text{previous}}(d) = \alpha_0 + \alpha_1 \cdot \text{iOS}(d) + \alpha_2 \cdot \text{age}(d) + \alpha_3 \cdot \text{tenure}(d) + \eta_d \quad (3.4)$$

Since iOS drivers were not able to "chase surge" in the outage weekend, we might expect a drop in earnings compared to Android drivers ($\alpha_1 < 0$). This drop is purely due to the movement activity by utilizing the information on the map, instead of any surge difference on trips at the same dispatched locations. Table 4 summarizes the results of this analysis.

Indeed, iOS drivers earned significantly less in most cities. In the two exceptions (Atlanta and New Jersey), surge constitute less than 3% of all driver earnings during the outage weekend, explaining why no significant difference was detected between iOS and Android drivers' earnings in these two areas.

On average, the absence of surge information reduced driver earnings by 20 to 80 cents per hour. This amount does not seem striking, constituting only 1% to 4% of drivers' total earnings (Figure 12). However, recall: 1) the outage did not change the surged earnings the drivers could get; 2) surged earnings only constituted 2% to 12% of all driver earnings, i.e., 40 cents to $3 per hour depending on the city. In fact, dividing the earnings' difference (α_1) due to the outage by the total surged earnings, we can caliberate the effects of self-positioning on surge earnings, which ranges from 10% to as high as 70% (Figure 13). The absence of the heatmap reduced driver earnings in the small cities the most, possibly because these drivers had less experience and they relied more heavily on the heatmap to make positioning decisions.

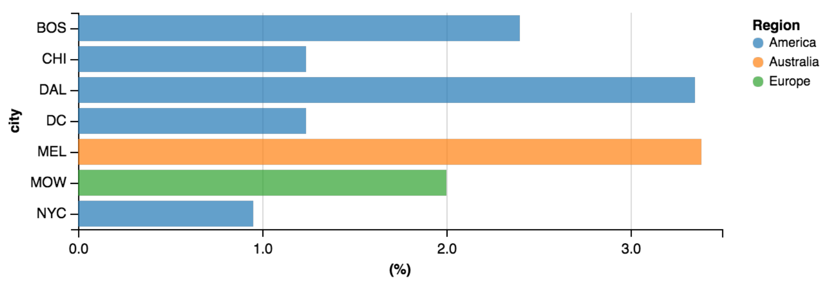


Figure 12: Effects of Self-Positioning on Total Earnings (%)

Removed Cities with insignificant coefficient (ATL, HK, NJ)

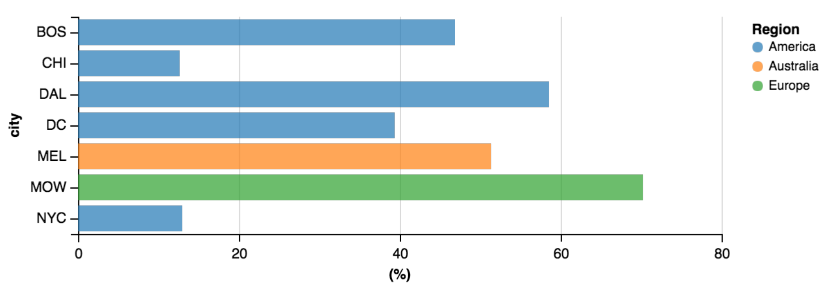


Figure 13: Effects of Self-Positioning on Surged Earnings (%)

Removed Cities with insignificant coefficient (ATL, HK, NJ)

3.4. Conclusions

This paper studies the short-run effect of dynamic pricing on Uber's driver partners' self-positioning decisions and earnings. We first built up a driver positioning model in a Multinomial Logit setup. Then we used data on a natural experiment covering 135,800 active drivers over two weekends and performed a difference-in-differences estimation that resolved endogeneity issues. The results suggest drivers rely heavily on the real-time dynamic pricing information to make self-positioning decisions, and the effect is lower (10% 30%) in big cities with professional drivers and higher (30% -60%) in small cities and with less experienced drivers. By utilizing this information, drivers can identify potential earning opportunities and significantly improve their surged earnings by up to 70%.

The results imply strong evidence that dynamic pricing is useful as a real-time signaling tool for drivers to make self-positioning decisions that aligns with rider's willingness to pay, reducing frictions in the ride-sharing labor market, better aligning drivers locations with riders desire to take trips.

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Appendix: DID Model without Driver Metrics

We also included a DID model with fewer predictors. The results are summarized in Table 5. Similarly, the model shows significant effects of the heatmap ($iOS \cdot week \cdot \Delta p$) for all cities except for Washington DC and New Jersey.

DID Model of Two Regular Weekends

Our conclusion of the impact of dynamic pricing on driver movement hinges on Assumption A7 (The Parallel Trend Assumption), i.e., iOS and Android drivers do not exhibit diverse trends in their sensitivity to prices, after controlling for different driver metrics (Age and Tenure). To test this theory, we ran a similar DID analysis over two regular weekends, including the previous weekend in the main analysis (10/22 to 10/24) and the weekend before (10/15 to 10/17).

The results are summarized in Table 6. For almost all cities, the coefficient on $iOS \cdot week \cdot p$ is insignificant at 95% confidence interval with mixed signs. The only exception is New Jersey, which had no significant results in the outage weekend analysis. This could possibly due to random noise, or some experiment running in the city that had different builts in the platforms.

Table 14: Summary Statistics for 10 Largest Cities.

	City Metrics			Driver Metrics			
	Trips	Surge Percentage	Average Surge	iOS Drivers	iOS Supply Hours	Android Drivers	Android Supply Hours
NYC	754753	0.16	1.07	35057	17539	34982	18627
BOS	381959	0.13	1.02	8012	8838	7533	9600
CHI	534294	0.19	1.07	11010	11496	13690	11539
DC	478290	0.07	1.01	5279	2251	5293	2550
ATL	215983	0.03	1.01	4769	6512	7902	4198
DEN	152086	0.17	1.13	2794	2684	4026	1996
MEL	214744	0.15	1.05	6465	3891	4002	6104
HK	117862	0.11	1.06	2269	2921	7401	982
MOW	137170	0.07	1.06	1263	5010	6987	1068
NJ	232656	0.08	1.06	8009	4918	7967	3463

This table provides statistics on city pricing metrics and driver supply metrics related to trips and driver movements taken in the 10 cities between November 4, 2016 and November 6, 2016. The surge percentage and average surge values are based on the surge data. The number of active iOS and Android drivers and their supply hours (number of hours on the Uber platform) are collected through the movement data.

Table 15: Example of Data.

Row	Driver	Time	OS	Current Hexagon	1-Ring Surge Values	Next Hexagon
0	Josie	9:00	iOS	a0	(1.0,1.0,1.2,1.2,1.3,1.0, 1.2)	a0
1	Josie	9:01	iOS	a0	(1.2,1.0,1.2,1.3,1.4,1.0, 1.2)	a1
2	Josie	9:02	iOS	a1	(1.2,1.0,1.2,1.3,1.4,1.0, 1.2)	a2
3	Josie	10:25	iOS	a10	(1.8,2.2,1.7,1.6,1.5,1.5, 1.6)	a12
4	Josie	10:26	iOS	a12	(1.8,2.2,1.7,1.6,1.5,1.5, 1.6)	a12
	...	...	...	...	...	...
5	Mark	10:25	iOS	b38	(1.0,1.0,1.0,1.0,1.0,1.0,1.0)	b39
6	Mark	10:26	iOS	b39	(1.0,1.2,1.0,1.0,1.0,1.0,1.0)	b40
	...	...	...	...	...	...

Each row is one driver-minute, recording the driver's location, Operating system, movement in the next minute and the spatial prices of the surrounding hexagons.

Table 16: Driver Movement Results for 10 Largest Cities.

	NYC	BOS	CHI	DC	ATL	DAL	MEL	HK	MOW	NJ
$const$	$\mathbf{-2.94}^{***}$	$\mathbf{-2.46}^{***}$	$\mathbf{-2.48}^{***}$	$\mathbf{-2.64}^{***}$	$\mathbf{-2.44}^{***}$	$\mathbf{-2.71}^{***}$	$\mathbf{-2.73}^{***}$	$\mathbf{-2.52}^{***}$	$\mathbf{-2.69}^{***}$	$\mathbf{-3.11}^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.01)
age	-0.00	$\mathbf{-0.17}^{***}$	$\mathbf{-0.10}^{***}$	$\mathbf{0.18}^{***}$	$\mathbf{0.32}^{***}$	$\mathbf{0.69}^{***}$	$\mathbf{1.01}^{***}$	-0.20	$\mathbf{0.20}^{***}$	$\mathbf{1.19}^{***}$
	(0.01)	(0.03)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.16)	(0.03)	(0.02)
$tenured\_days$	$\mathbf{-0.00}^{**}$	$\mathbf{-0.06}^{***}$	$\mathbf{-0.03}^{***}$	$\mathbf{-0.04}^{***}$	$\mathbf{-0.20}^{***}$	$\mathbf{-0.23}^{***}$	$\mathbf{-0.04}^{***}$	$\mathbf{-0.36}^{***}$	$\mathbf{-0.12}^{***}$	$\mathbf{-0.15}^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)
$week$	-0.00	$\mathbf{-0.04}^{***}$	-0.00	$\mathbf{-0.12}^{***}$	$\mathbf{-0.06}^{***}$	$\mathbf{-0.16}^{***}$	$\mathbf{-0.14}^{***}$	$\mathbf{-0.05}^{***}$	$\mathbf{-0.02}^{**}$	-0.00
	(0.00)	(0.01)	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)
iOS	$\mathbf{0.03}^{***}$	$\mathbf{-0.04}^{***}$	$\mathbf{-0.04}^{***}$	0.00	$\mathbf{-0.13}^{***}$	$\mathbf{-0.03}^{**}$	$\mathbf{0.07}^{***}$	$\mathbf{-0.04}^{***}$	0.01	$\mathbf{0.33}^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$age \cdot week$	-0.01	-0.03	-0.01	$\mathbf{-0.05}^{**}$	$\mathbf{0.03}^{*}$	$\mathbf{-0.24}^{***}$	$\mathbf{0.86}^{***}$	0.04	-0.04	$\mathbf{0.11}^{**}$
	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)	(0.04)	(0.03)	(0.24)	(0.03)	(0.03)
$tenured\_days \cdot week$	0.00	$\mathbf{-0.03}^{***}$	-0.00	-0.01	$\mathbf{-0.02}^{***}$	$\mathbf{0.02}^{**}$	-0.01	0.01	$\mathbf{0.04}^{***}$	$\mathbf{-0.03}^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$iOS \cdot week$	-0.01	0.00	$\mathbf{0.03}^{***}$	$\mathbf{-0.04}^{***}$	$\mathbf{0.09}^{***}$	$\mathbf{0.08}^{***}$	0.01	-0.01	-0.01	$\mathbf{0.02}^{*}$
	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Δp	$\mathbf{1.25}^{***}$	$\mathbf{1.99}^{***}$	$\mathbf{3.30}^{***}$	$\mathbf{1.97}^{***}$	$\mathbf{2.43}^{***}$	$\mathbf{2.57}^{***}$	$\mathbf{3.20}^{***}$	$\mathbf{1.69}^{***}$	$\mathbf{3.52}^{***}$	$\mathbf{5.61}^{***}$
	(0.05)	(0.04)	(0.07)	(0.07)	(0.09)	(0.12)	(0.09)	(0.08)	(0.08)	(0.35)
$tenured\_days \cdot \Delta p$	-0.04	$\mathbf{-0.11}^{***}$	$\mathbf{-0.36}^{***}$	$\mathbf{-0.23}^{***}$	-0.12	$\mathbf{-0.18}^{*}$	$\mathbf{0.51}^{***}$	$\mathbf{0.65}^{*}$	$\mathbf{0.76}^{***}$	$\mathbf{0.77}^{*}$
	(0.03)	(0.03)	(0.01)	(0.03)	(0.06)	(0.07)	(0.12)	(0.27)	(0.13)	(0.30)
$age \cdot \Delta p$	-0.02	$\mathbf{-2.52}^{***}$	-0.30	0.15	-0.23	-0.16	$\mathbf{3.77}^{***}$	$\mathbf{1.03}^{***}$	-1.44	-0.79
	(0.18)	(0.21)	(0.24)	(0.16)	(0.52)	(0.31)	(0.56)	(0.06)	(0.74)	(1.72)
$iOS \cdot \Delta p$	0.10	$\mathbf{0.18}^{***}$	0.03	-0.04	$\mathbf{0.34}^{*}$	0.24	$\mathbf{0.56}^{***}$	-0.10	$\mathbf{0.44}^{**}$	$\mathbf{-1.15}^{*}$
	(0.06)	(0.05)	(0.07)	(0.07)	(0.16)	(0.17)	(0.10)	(0.15)	(0.16)	(0.53)
$week \cdot \Delta p$	$\mathbf{0.31}^{***}$	$\mathbf{-0.18}^{*}$	$\mathbf{-1.66}^{***}$	$\mathbf{-1.94}^{***}$	-0.10	$\mathbf{1.09}^{***}$	$\mathbf{-0.79}^{***}$	$\mathbf{0.57}^{***}$	0.08	0.36
	(0.07)	(0.08)	(0.09)	(0.07)	(0.15)	(0.13)	(0.10)	(0.12)	(0.11)	(0.71)
$tenured\_days \cdot week \cdot \Delta p$	0.04	0.01	$\mathbf{0.28}^{***}$	0.05	$\mathbf{0.41}^{***}$	$\mathbf{0.26}^{**}$	0.11	-0.45	-0.42	$\mathbf{-0.74}^{*}$
	(0.03)	(0.05)	(0.03)	(0.05)	(0.08)	(0.10)	(0.19)	(0.31)	(0.24)	(0.34)
$age \cdot week \cdot \Delta p$	0.00	$\mathbf{1.44}^{*}$	0.02	$\mathbf{0.62}^{**}$	0.43	0.03	-0.11	$\mathbf{0.97}^{***}$	-1.22	-0.15
	(0.22)	(0.67)	(0.31)	(0.21)	(0.41)	(0.30)	(0.92)	(0.02)	(0.84)	(1.88)
$iOS \cdot week \cdot \Delta p$	$\mathbf{-0.29}^{***}$	$\mathbf{-0.57}^{***}$	$\mathbf{-0.39}^{***}$	-0.08	$\mathbf{-1.45}^{***}$	$\mathbf{-0.95}^{***}$	$\mathbf{-1.05}^{***}$	$\mathbf{-0.72}^{*}$	$\mathbf{-1.03}^{***}$	-0.19
	(0.06)	(0.08)	(0.07)	(0.09)	(0.30)	(0.20)	(0.17)	(0.31)	(0.26)	(0.93)
N	$4.7e6$	$1.7e6$	$2.8e6$	$1.2e6$	$1.1e6$	$5.1e5$	$9.1e5$	$6.8e5$	$6.4e5$	$9.8e5$

:* p-value = 0.05   *:* p-value = 0.01   ****:* p-value = 0.001

Dependent variables are drivers' hourly earning difference over the two weekends

Table 17: Driver Earnings Results for 10 Largest Cities.

	NYC	BOS	CHI	DC	ATL	DAL	MEL	HK	MOW	NJ
const	0.23	$\mathbf{-5.91}^{***}$	$\mathbf{0.61}^{***}$	$\mathbf{-2.56}^{***}$	$\mathbf{-2.42}^{***}$	0.02	$\mathbf{-5.24}^{***}$	$\mathbf{0.45}^{***}$	$\mathbf{-0.36}^{*}$	−0.22
	(0.22)	(0.26)	(0.18)	(0.17)	(0.20)	(0.31)	(0.33)	(0.09)	(0.15)	(0.18)
iOS	$\mathbf{-0.31}^{*}$	$\mathbf{-0.57}^{***}$	$\mathbf{-0.27}^{**}$	$\mathbf{-0.24}^{*}$	0.06	$\mathbf{-0.61}^{***}$	$\mathbf{-0.75}^{***}$	0.01	$\mathbf{-0.31}^{**}$	−0.03
	(0.14)	(0.15)	(0.10)	(0.10)	(0.12)	(0.18)	(0.19)	(0.06)	(0.10)	(0.21)
age	0.01	$\mathbf{0.63}^{***}$	0.06	$\mathbf{0.14}^{***}$	$\mathbf{0.16}^{***}$	$\mathbf{0.11}^{*}$	$\mathbf{0.90}^{***}$	0.08	0.01	0.07
	(0.03)	(0.06)	(0.04)	(0.03)	(0.04)	(0.05)	(0.07)	(0.12)	(0.03)	(0.07)
tenure	$\mathbf{-0.31}^{***}$	$\mathbf{-1.17}^{***}$	−0.09	$\mathbf{-0.32}^{***}$	$\mathbf{-0.49}^{***}$	$\mathbf{-0.46}^{***}$	$\mathbf{-0.91}^{***}$	$\mathbf{-0.19}^{**}$	$\mathbf{-0.33}^{***}$	−0.17
	(0.07)	(0.08)	(0.05)	(0.05)	(0.07)	(0.10)	(0.15)	(0.07)	(0.07)	(0.18)
N	$3.4e4$	$1.5e4$	$2.7e4$	$2.2e4$	$1.4e4$	$9.4e3$	$9.6e3$	$1.0e4$	$7.8e3$	$2.1e4$

*: p-value = 0.05 **: p-value = 0.01 ***: p-value = 0.001

Dependent variables are drivers' hourly earning difference over the two weekends

Table 18: Driver Movement Results for 10 Largest Cities without Driver Metrics (Appendix)

	NYC	BOS	CHI	DC	ATL	DAL	MEL	HK	MOW	NJ
$const$	$\mathbf{-2.94}^{***}$	$\mathbf{-2.46}^{***}$	$\mathbf{-2.48}^{***}$	$\mathbf{-2.64}^{***}$	$\mathbf{-2.43}^{***}$	$\mathbf{-2.64}^{***}$	$\mathbf{-2.71}^{***}$	$\mathbf{-2.51}^{***}$	$\mathbf{-2.69}^{***}$	$\mathbf{-3.10}^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.01)	(0.00)
$week$	0.00	$\mathbf{-0.03}^{***}$	−0.01	$\mathbf{-0.12}^{***}$	$\mathbf{-0.05}^{***}$	$\mathbf{-0.19}^{***}$	$\mathbf{-0.17}^{***}$	$\mathbf{-0.04}^{***}$	−0.01	$\mathbf{-0.01}^{*}$
	(0.00)	(0.01)	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)
iOS	$\mathbf{0.03}^{***}$	$\mathbf{-0.05}^{***}$	$\mathbf{-0.05}^{***}$	−0.01	$\mathbf{-0.17}^{***}$	$\mathbf{-0.12}^{***}$	$\mathbf{0.10}^{***}$	$\mathbf{-0.06}^{***}$	−0.01	$\mathbf{0.31}^{***}$
	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$iOS \cdot week$	−0.01	0.00	$\mathbf{0.03}^{***}$	$\mathbf{-0.03}^{***}$	$\mathbf{0.13}^{***}$	$\mathbf{0.15}^{***}$	0.01	−0.01	0.00	$\mathbf{0.06}^{***}$
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)
Δp	$\mathbf{1.28}^{***}$	$\mathbf{2.02}^{***}$	$\mathbf{3.29}^{***}$	$\mathbf{1.80}^{***}$	$\mathbf{2.34}^{***}$	$\mathbf{2.42}^{***}$	$\mathbf{3.41}^{***}$	$\mathbf{1.67}^{***}$	$\mathbf{3.51}^{***}$	$\mathbf{5.86}^{***}$
	(0.04)	(0.03)	(0.06)	(0.10)	(0.16)	(0.18)	(0.13)	(0.09)	(0.12)	(0.34)
$iOS \cdot \Delta p$	0.05	$\mathbf{0.14}^{*}$	−0.10	−0.00	0.39	0.31	$\mathbf{0.58}^{***}$	−0.09	$\mathbf{0.48}^{**}$	$\mathbf{-1.43}^{***}$
	(0.07)	(0.06)	(0.10)	(0.12)	(0.24)	(0.24)	(0.17)	(0.13)	(0.16)	(0.36)
$week \cdot \Delta p$	$\mathbf{0.31}^{***}$	−0.17	$\mathbf{-1.64}^{***}$	$\mathbf{-1.73}^{***}$	−0.06	$\mathbf{1.11}^{***}$	$\mathbf{-1.25}^{***}$	$\mathbf{0.56}^{***}$	0.16	0.20
	(0.05)	(0.10)	(0.07)	(0.11)	(0.33)	(0.23)	(0.17)	(0.15)	(0.15)	(0.53)
$iOS \cdot week \cdot \Delta p$	$\mathbf{-0.24}^{**}$	$\mathbf{-0.63}^{***}$	$\mathbf{-0.31}^{***}$	−0.21	$\mathbf{-1.65}^{***}$	$\mathbf{-1.01}^{***}$	$\mathbf{-0.86}^{***}$	$\mathbf{-0.76}^{**}$	$\mathbf{-0.90}^{***}$	0.41
	(0.08)	(0.16)	(0.09)	(0.13)	(0.48)	(0.24)	(0.19)	(0.25)	(0.18)	(0.56)
N	$4.7e6$	$1.7e6$	$2.8e6$	$1.2e6$	$1.1e6$	$5.1e5$	$9.1e5$	$6.8e5$	$6.4e5$	$9.8e5$

*: p-value = 0.05 **: p-value = 0.01 ***: p-value = 0.001

Dependent variables are drivers' hourly earning difference over the two weekends

Table 19: Driver Movement Results over Two Regular Weekends. (Appendix)

	NYC	BOS	CHI	DC	ATL	DAL	MEL	HK	MOW	NJ
const	$\mathbf{-2.80}^{***}$	$\mathbf{-2.55}^{***}$	$\mathbf{-2.53}^{***}$	$\mathbf{-2.71}^{***}$	$\mathbf{-2.62}^{***}$	$\mathbf{-2.54}^{***}$	$\mathbf{-2.63}^{***}$	$\mathbf{-2.55}^{***}$	$\mathbf{-2.69}^{***}$	$\mathbf{-2.84}^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)	(0.01)
age	-0.01	$\mathbf{-0.07}^{**}$	0.00	$\mathbf{0.15}^{***}$	$\mathbf{-0.15}^{***}$	$\mathbf{0.27}^{***}$	$\mathbf{0.13}^{**}$	$\mathbf{-0.89}^{***}$	$\mathbf{0.25}^{***}$	$\mathbf{0.38}^{***}$
	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.04)	(0.04)	(0.21)	(0.06)	(0.02)
tenured_days	$\mathbf{-0.02}^{***}$	$\mathbf{-0.02}^{***}$	$\mathbf{-0.06}^{***}$	$\mathbf{-0.03}^{***}$	$\mathbf{-0.01}^{**}$	$\mathbf{-0.03}^{**}$	$\mathbf{-0.02}^{**}$	$\mathbf{-0.21}^{***}$	0.00	$\mathbf{-0.03}^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
week	$\mathbf{-0.14}^{***}$	$\mathbf{0.10}^{***}$	$\mathbf{0.06}^{***}$	$\mathbf{0.08}^{***}$	$\mathbf{0.19}^{***}$	$\mathbf{-0.10}^{***}$	$\mathbf{-0.05}^{***}$	$\mathbf{0.03}^{***}$	-0.01	$\mathbf{-0.26}^{***}$
	(0.00)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)
iOS	-0.00	-0.00	0.01	-0.01	$\mathbf{0.05}^{***}$	$\mathbf{0.09}^{***}$	-0.01	$\mathbf{0.05}^{***}$	0.00	$\mathbf{0.04}^{***}$
	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$age \cdot week$	0.00	$\mathbf{-0.12}^{***}$	$\mathbf{-0.11}^{***}$	$\mathbf{0.03}^{*}$	$\mathbf{0.47}^{***}$	$\mathbf{0.41}^{***}$	$\mathbf{0.89}^{***}$	$\mathbf{0.70}^{***}$	-0.05	$\mathbf{0.79}^{***}$
	(0.01)	(0.03)	(0.03)	(0.01)	(0.03)	(0.04)	(0.05)	(0.19)	(0.07)	(0.04)
$tenured\_days \cdot week$	$\mathbf{0.01}^{***}$	$\mathbf{-0.04}^{***}$	$\mathbf{0.03}^{***}$	$\mathbf{-0.01}^{*}$	$\mathbf{-0.19}^{***}$	$\mathbf{-0.19}^{***}$	$\mathbf{-0.03}^{**}$	$\mathbf{-0.16}^{***}$	$\mathbf{-0.12}^{***}$	$\mathbf{-0.12}^{***}$
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$iOS \cdot week$	$\mathbf{0.03}^{***}$	$\mathbf{-0.04}^{***}$	$\mathbf{-0.05}^{***}$	0.01	$\mathbf{-0.18}^{***}$	$\mathbf{-0.11}^{***}$	$\mathbf{0.08}^{***}$	$\mathbf{-0.09}^{***}$	0.01	$\mathbf{0.29}^{***}$
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
Δp	$\mathbf{1.14}^{***}$	$\mathbf{2.02}^{***}$	$\mathbf{2.86}^{***}$	$\mathbf{2.21}^{***}$	$\mathbf{2.62}^{***}$	$\mathbf{2.20}^{***}$	$\mathbf{3.68}^{***}$	$\mathbf{1.71}^{***}$	$\mathbf{2.44}^{***}$	$\mathbf{1.99}^{***}$
	(0.08)	(0.05)	(0.17)	(0.19)	(0.09)	(0.23)	(0.16)	(0.25)	(0.13)	(0.38)
$tenured\_days \cdot \Delta p$	-0.08	$\mathbf{0.15}^{***}$	$\mathbf{0.64}^{***}$	$\mathbf{0.29}^{*}$	$\mathbf{0.59}^{***}$	-0.09	$\mathbf{0.74}^{***}$	0.09	$\mathbf{0.69}^{***}$	$\mathbf{0.84}^{***}$
	(0.08)	(0.05)	(0.18)	(0.13)	(0.13)	(0.18)	(0.12)	(0.41)	(0.19)	(0.20)
$age \cdot \Delta p$	-0.24	$\mathbf{-1.05}^{**}$	$\mathbf{-4.47}^{***}$	$\mathbf{-3.01}^{**}$	-0.44	-0.27	$\mathbf{-7.86}^{***}$	$\mathbf{1.01}^{***}$	$\mathbf{-2.77}^{*}$	-1.33
	(0.36)	(0.34)	(0.81)	(1.07)	(1.22)	(0.14)	(1.38)	(0.18)	(1.11)	(0.77)
$iOS \cdot \Delta p$	-0.00	$\mathbf{0.16}^{*}$	0.45	0.02	0.29	$\mathbf{0.68}^{*}$	-0.12	0.03	$\mathbf{1.06}^{*}$	-0.04
	(0.09)	(0.08)	(0.24)	(0.21)	(0.23)	(0.34)	(0.18)	(0.58)	(0.47)	(0.55)
$week \cdot \Delta p$	0.13	0.04	$\mathbf{0.61}^{**}$	-0.14	-0.04	0.41	-0.09	0.04	$\mathbf{1.16}^{***}$	$\mathbf{3.85}^{***}$
	(0.09)	(0.07)	(0.22)	(0.21)	(0.15)	(0.29)	(0.26)	(0.21)	(0.17)	(0.52)
$tenured\_days \cdot week \cdot \Delta p$	0.03	$\mathbf{-0.28}^{***}$	$\mathbf{-0.95}^{***}$	$\mathbf{-0.50}^{***}$	$\mathbf{-0.79}^{***}$	-0.00	-0.20	0.33	0.00	0.12
	(0.08)	(0.05)	(0.17)	(0.15)	(0.15)	(0.17)	(0.10)	(0.59)	(0.24)	(0.29)
$age \cdot week \cdot \Delta p$	0.26	$\mathbf{-1.46}^{**}$	$\mathbf{3.87}^{***}$	$\mathbf{3.24}^{**}$	0.19	-0.09	$\mathbf{11.69}^{***}$	$\mathbf{1.01}^{***}$	1.76	-0.15
	(0.36)	(0.47)	(1.09)	(1.19)	(1.25)	(0.09)	(1.39)	(0.17)	(1.17)	(0.46)
$iOS \cdot week \cdot \Delta p$	0.06	0.00	-0.36	-0.02	-0.18	-0.13	0.42	-0.09	-0.66	$\mathbf{-1.45}^{*}$
	(0.11)	(0.11)	(0.25)	(0.20)	(0.32)	(0.26)	(0.30)	(0.54)	(0.55)	(0.72)
N	$3.8e6$	$1.1e6$	$2.2e6$	$1.6e6$	$6.8e5$	$3.2e5$	$5.2e5$	$6.1e5$	$4.2e5$	$7.5e5$

:* p-value = 0.05 *:* p-value = 0.01 ****:* p-value = 0.001

Dependent variables are drivers' hourly earning difference over the two weekends