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Abstract

Should a firm's price respond dynamically to shifts in demand? With dynamic pricing the firm can exploit high demand by charging a high price, and can cope with low demand by charging a low price to more fully utilize its capacity. However, many firms announce their price in advance and do not make adjustments in response to market conditions, i.e., they use static pricing. Therefore, with static pricing the firm may find that its price is either lower or higher than optimal given the observed market condition. Nevertheless, we find that when consumers are strategic and can anticipate such pricing behavior, a firm may actually be better off with static pricing. Dynamic pricing can be ineffective because it imposes pricing risk on consumers - given that it is costly to visit the firm, an uncertain price may cause consumers to avoid visiting the firm altogether. We show that the advantage of dynamic pricing over static pricing. However, the superiority of dynamic pricing can be restored if the firm sets a modest base price and then commits only to reduce its price, i.e., it never raises its price in response to strong demand. Hence, a successful implementation of dynamic pricing tempers the magnitude of price adjustments.

Disciplines

Advertising and Promotion Management | Behavioral Economics | Business | Business Administration, Management, and Operations | Business Analytics | Business Intelligence | Marketing | Operations and Supply Chain Management | Sales and Merchandising

Comments

This is an unpublished manuscript.

Dynamic versus Static Pricing
in the Presence of Strategic Consumers

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Should a firm's price respond dynamically to shifts in demand? With dynamic pricing the firm can exploit high demand by charging a high price, and can cope with low demand by charging a low price to more fully utilize its capacity. However, many firms announce their price in advance and do not make adjustments in response to market conditions, i.e., they use static pricing. Therefore, with static pricing the firm may find that its price is either lower or higher than optimal given the observed market condition. Nevertheless, we find that when consumers are strategic and can anticipate such pricing behavior, a firm may actually be better off with static pricing. Dynamic pricing can be ineffective because it imposes pricing risk on consumers - given that it is costly to visit the firm, an uncertain price may cause consumers to avoid visiting the firm altogether. We show that the advantage of static pricing relative to dynamic pricing can be substantially larger than the advantage of dynamic pricing over static pricing. However, the superiority of dynamic pricing can be restored if the firm sets a modest base price and then commits only to reduce its price, i.e., it never raises its price in response to strong demand. Hence, a successful implementation of dynamic pricing tempers the magnitude of price adjustments.

1 Introduction

Uncertainty in demand suggests that firms can benefit from dynamic pricing. With dynamic pricing a firm delays its pricing decisions until after market conditions are revealed so that the firm can adjust prices accordingly - when demand is ample, set a high price, and when demand is weak, set a low price. Yet, despite the apparent advantages, many firms do not adjust prices to respond to market conditions. For example, movie theaters charge a fixed price, regardless of whether the movie turned out to be a hit or a flop. Restaurants do not adjust their menu prices depending on whether it is a busy or a slow night. Sports teams keep their seat prices fixed, regardless of how well the team is performing, or if the weather on a particular game day turns out to be good or bad.[1]

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[1] A few exceptions exist. The San Francisco Giants of the MLB and the Dallas Stars of the NHL are experimenting with dynamic pricing techniques using a software developed by the Austin-based start-up, QCue (Branch 2009). It

Several explanations have been provided for why firms may not adjust prices in response to changing demand conditions (a phenomenon which is sometimes referred to as *price stickiness* or *price rigidity*). Firms may incur *menu costs* to change prices (Mankiw 1985): if it is costly to change prices, firms naturally hesitate to change prices frequently. Menu costs were originally thought of as the physical costs for changing prices, such as the cost to reprint restaurant menus. They can also be interpreted to be managerial costs (information gathering and decisions-making) or customer costs (communication and negotiation of new prices) (Zbaracki *et al.* 2004). Alternatively, sticky prices may be due to consumer psychology: consumers dislike price changes, especially if they perceive the changes to be "unfair" (e.g., Hall and Hitch 1939; Kahneman *et al.* 1986; Blinder *et al.* 1998).

While menu costs and consumer psychology may play a role in pricing decisions, we present an alternative explanation. A key component of our theory is that consumers incur "visit costs" - before consumers attempt to make purchases, they must incur a cost to consider the purchase. For example, a consumer must drive to a baseball park or must take the time to call a restaurant, etc. Consequently, dynamic and static pricing impose different risks on consumers. With a dynamic pricing strategy a consumer risks incurring the visit cost only to discover that the price charged is more than she wants to pay, i.e., dynamic pricing imposes a price risk on consumers. With static pricing a consumer may discover after visiting the firm that the firm has no capacity left to sell, i.e., static pricing imposes rationing risk on consumers. We find that it can be better to impose on consumers rationing risk (via static pricing) than pricing risk (via dynamic pricing). Furthermore, the advantage of static pricing can be substantial whereas the advantage of dynamic pricing is less significant.

The limitation with dynamic pricing is not that the firm may choose to lower its price when it observes weak demand - consumers like price cuts and are therefore more willing to visit a firm that is known for cutting its price. The drawback with dynamic pricing is that the firm may choose a high price when demand is abundant - why incur a visit cost when you may also have to pay a high price? This suggests a hybrid approach - the firm starts with a modest base price and commits only to reduce the price from that level. This "constrained" dynamic pricing strategy is better for the firm because it blends the demand-supply matching benefits of pure dynamic pricing with the incentives of static pricing. Hence, dynamic pricing can be a good strategy for the firm as long as the firm is not too aggressive in its price adjustments. Otherwise, static pricing may be the better

has been reported that for Giants' tickets, "the price change will most likely be 25 cents to $1" (Muret 2008), where tickets range from $8 to $41. These price changes do not appear very significant and it is not yet clear how using the software affects these teams' revenues.

Table 1. Summary of Consumer Types.

Segment	Number	Value	Visit cost
High type	$X \sim F\left(\cdot\right)$	v_h	c
Low type	∞	v_l	0

alternative, despite its rigidity to respond to changing demand conditions.

2   Model Description

A single firm with k units of capacity sells to two types of consumers, all of whom require one unit of capacity to be served. There is a potential number of X high-value consumers, where X is a non-negative random variable that is drawn from a cumulative distribution function $F\left(\cdot\right)$, pdf $f\left(\cdot\right)$, complimentary cdf $\overline{F}\left(\cdot\right) = 1 - F\left(\cdot\right)$ and mean $\mu = \mathbb{E}[X]$. The high-value consumers are non-atomistic. They have value v_h for the firm's service. They must incur a positive cost, $c < v_h$, to "visit" the firm (e.g., the time and effort to walk to a movie theater) to purchase the service. The visit cost need not be an explicit cost. It can also be interpreted as a mental cost to consider an alternative or an opportunity cost – the cost of forgoing an outside option when choosing to consider visiting the firm. All of the realized high-type consumers must decide whether or not to visit the firm and if they do not, then they receive zero net value. We allow them to adopt mixed strategies: let $\gamma \in [0,1]$ be the probability that a high-type consumer visits the firm.

Low-value consumers are the second type of consumers. There is an ample number of them, and each of them has v_l value for the firm's service. These consumers do not incur a cost to visit the firm, which implies that the firm can always sell its entire capacity by charging v_l. Therefore, an alternative interpretation of v_l is that it is the maximum price that guarantees the firm can sell its entire capacity regardless of market conditions.[2] Table 1 summarizes the consumer types.

The firm seeks to maximize revenue and consumers seek to maximize their net value, the value of the service minus visit costs and the price paid to the firm. The sequence of events is as follows: (1) the firm chooses a pricing strategy, which is a set of prices $\mathcal{A}$, $\mathcal{A} \subseteq \mathfrak{R}^+$; (2) the number of high-type consumers, X, is realized; (3) high-type consumers choose a visit strategy, γ, knowing the firm's pricing strategy, $\mathcal{A}$, but not the realization of X; (4) the firm observes γ and X and chooses a price, p^*, from among those in $\mathcal{A}$; (5) all high-type consumers who visited the firm plus

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[2]Our results continue to hold qualitatively even if low-type consumers incur a positive visit cost, as long as this cost is sufficiently low. In this case, the firm cannot guarantee selling its entire capacity by charging v_l. However, if the visit cost of low type consumers is low enough, there exists a positive price that makes all low type consumers visit and therefore guarantees that the entire capacity can be sold.

the low-type consumers observe p^* and decide to purchase if p^* is no greater than their value for the service; and (6) if there are more than k high-type consumers who want to purchase, the k units are randomly rationed among them, i.e., they have priority over the low-types (our allocation rule). In step (4) the firm chooses the revenue maximizing price from the set $\mathcal{A}$ given γ and X : $p^* = \arg\max_{p \in A} \{R(p, x, \gamma)\}$, where x is the realization of X and

$$
R(p, x, \gamma) = \left\{ \begin{array}{cc} pk & p \leq v_l \\ p \min\{\gamma x, k\}, & if \ v_l < p \leq v_h \\ 0 & p > v_h \end{array} \right. .
$$

Note, the firm sells units only at a single price (the firm does not have the ability to price discriminate). If fewer than k units of capacity are sold, the remainder earns zero revenue. Our rationing rule (that the high types have priority over the low types) has also been adopted by Su and Zhang (2008) and Tereyağoğlu and Veeraraghavan (2009). This allocation rule simplifies the analysis, but is not critical for our results. In fact, we later argue that any other allocation may only strengthen our main result.

We do not *a priori* restrict the number of prices in $\mathcal{A}$. They can be thought of as commonly established price points in the market. We say the firm uses a *static pricing* strategy when the firm includes only a singe price in its set, $\mathcal{A} = \{p_s\}$. Given that there is only one choice in $\mathcal{A}$, consumers know exactly what the price will be before they choose whether or not to visit. We say the firm uses a *dynamic pricing* strategy when $\mathcal{A}$ includes two or more prices that could be observed for some realizations of γ and X. Even though consumers are charged only one price, the price is dynamic in the sense that it is chosen from a set of possible prices based on updated information (the realization of demand). Section 4 studies the static pricing strategy and section 5 studies a particular dynamic pricing strategy, $\mathcal{A} = \mathfrak{R}^+$. Section 6 compares these two strategies and section 7 considers a broader set of dynamic pricing strategies.

3 Related Literature

There is an extensive literature on dynamic pricing with exogenous demand, i.e., situations in which the pricing strategy does not influence how many customers visit the firm, when they consider purchasing or their valuations for the firm's service. See Elmaghraby and Keskinicak (2003) for a review. With exogenous demand the question is not whether dynamic pricing is better than static pricing (it clearly is) but rather how to implement dynamic pricing (when to change prices and by how much), how much better is dynamic pricing and under what conditions is dynamic pricing substantially better. However, dynamic pricing does not clearly dominate static pricing

when consumers are strategic.

Several papers discuss dynamic versus static pricing in the context of multi-period models with strategic consumers. These consumers pose a challenge to the firm because they can time when they purchase - they will not buy at a high price if they can anticipate that the price will be substantially lower later on. With dynamic pricing the firm cannot commit to not lower its price, whereas with static pricing the firm commits to a price path that does not include substantial price reductions. It is precisely this commitment that confers an advantage to static pricing over dynamic pricing, as shown formally by Besanko and Winston (1990). However, their model has no uncertainty in either the number of consumers or their valuations, nor a capacity constraint. An important virtue of dynamic pricing is that it enables the firm to better match its supply to its uncertain demand. Hence, there is a tradeoff between committing to limited price reductions (thereby encouraging consumers to buy early on at a high price) and responding to updated demand information so as to maximize revenue given constrained capacity. This tension is explored by Dasu and Tong (2006), Aviv and Pazgal (2008) and Cachon and Swinney (2009). In models with fixed capacity, Dasu and Tong (2006) and Aviv and Pazgal (2008) find that neither scheme dominates and the performance gap between them is generally small. Cachon and Swinney (2009) allow the firm to adjust its capacity and finds that dynamic pricing is generally better. The key differences between our model and these papers is that our consumers incur visit costs and our firm only chooses a single price. Hence, consumers do not consider *when* to buy (there is only a single opportunity to buy), but rather they consider whether to incur a cost to visit the firm. Consequently, in our model static pricing is not used to prevent strategic waiting but rather to encourage consumers to participate in the market. However, like those other papers, our firm has limited capacity and potential demand is uncertain, so dynamic pricing is better than static pricing at matching supply to demand.

Like Cachon and Swinney (2009), Liu and van Ryzin (2008) and Su and Zhang (2008) allow the firm to control capacity to prevent strategic waiting for discounts - with less inventory consumers face greater rationing risk if they wait. However, in these papers the firm implements a static pricing policy, and they do not consider dynamic pricing.

As in our model, in Dana and Petruzzi (2001), Çil and Lariviere (2007), Alexandrov and Lariviere (2008) and Su and Zhang (2009) consumers incur a visit cost before they can transact with the firm. Dana and Petruzzi (2001) have fixed prices and focus instead on how visit costs influence the firm's capacity choice. Çil and Lariviere (2007) studies the allocation of capacity across two market segments and Alexandrov and Lariviere (2008) study why firms may offer reservations. Prices are

exogenously fixed in both of those papers. In Su and Zhang (2009) a firm chooses a price and a capacity before observing potential demand. Consumers observe the firm's price before choosing whether to incur a visit cost, but they do not observe the firm's capacity nor the number of consumers in the market. In contrast, in our model the firm chooses a set of potential prices before observing potential demand and then chooses its actual price (constrained by initial decision) after observing potential demand. Furthermore, in our model the firm's capacity is fixed and known to consumers. Hence, our model is suitable for comparing static versus dynamic pricing whereas Su and Zhang (2009) focus on capacity commitments and availability guarantees (and cannot compare static versus dynamic pricing).

Van Mieghem and Dada (1999) study price postponement, which is related to our dynamic pricing strategy - the firm chooses a price after learning some updated demand information. However, they do not consider strategic consumer behavior (their demand is exogenous), so price postponement is always beneficial in their setting, unlike in our model.

Other papers that compare between different pricing schemes when consumers are strategic include single versus priority pricing (Harris and Raviv 1981), subscription versus per-use pricing (Barro and Romer 1987; Cachon and Feldman 2010), and markdown regimes with and without reservations (Elmaghraby et al. 2006). In all of these papers the firm selects its pricing strategy before learning some updated demand information, whereas in our study we allow the firm to choose a price after potential demand is observed.

4 Static Pricing Strategy

With a static pricing strategy, the firm chooses a single price, p, to include in $\mathcal{A}$ before observing demand, so consumers know that the price will indeed be p before deciding whether or not to visit the firm. All high-value consumers who visit the firm receive a net value equal to $v_h - p - c$ if they obtain a unit, and if they do not obtain a unit, their net value is $-c$. A customer visits the firm if net utility is not negative, i.e., if

$$\phi\left(v_h - p\right) \geq c, \tag{1}$$

where ϕ is the customer's expectation for the probability of getting a unit conditional on visiting the firm. ϕ is determined by the underlining potential demand distribution, X, the high-value customers' strategy, γ, and the rationing rule used to allocate scarce capacity. All else being equal, as γ increases, more high-type customers will visit the firm, thereby reducing the chance that any

one of them will get a unit. In particular,

$$\phi = \frac{S_{\gamma X}(k)}{\gamma \mu} = \frac{S(k/\gamma)}{\mu},$$

(2)

where $S_D(q) = \mathbb{E}_D[\min\{D, q\}]$ is the sales function and $S(\cdot)$ is shorthand for $S_X(\cdot)$. Note that $S(k/\gamma)/\mu$ is the firm's fill rate, or the fraction of high-customer demand who visits the firm that the firm is able to satisfy. This probability accounts for the observation that, conditional on being in the market, a consumer is more likely to be in a market with a large number of consumers (and therefore have a low chance to get a unit) than in a market with a few number of consumers (and therefore have a high chance to get a unit). See Deneckere and Peck (1995) and Dana (2001) for a more detailed discussion of why the fill rate correctly expresses the probability of receiving a unit given our allocation rule.

With finite capacity, $\phi < 1$ is surely possible, i.e., under static pricing consumers face a rationing risk when they visit the firm.

Definition 1 *A high-type consumer faces a* rationing risk *if there is a chance that the consumer will not be able to obtain the unit at a price which is strictly lower than the consumer's value for the unit.*

A symmetric equilibrium strategy for high-type consumers is a $\widehat{\gamma} \in [0, 1]$ such that $\widehat{\gamma}$ is optimal for each consumer given that all other consumers choose $\widehat{\gamma}$ as their strategy. If p is low enough, there is an equilibrium in which all high-type consumers visit the firm, i.e., $\widehat{\gamma} = 1$. From (1) and (2), that occurs if

$$\frac{S(k)}{\mu}(v_h - p) \geq c$$

or

$$p \leq v_h - \frac{\mu c}{S(k)} = \overline{p}.$$

If $p > \overline{p}$, the unique symmetric equilibrium has $\widehat{\gamma} < 1$, where $\widehat{\gamma}$ is the unique solution to

$$S(k/\widehat{\gamma}) = \frac{\mu c}{v_h - p},$$

(3)

or, alternatively,

$$p = v_h - \frac{\mu c}{S(k/\widehat{\gamma})}.$$

In this case, for every price p there exists a unique $\widehat{\gamma}(p)$ that satisfies 3 and is decreasing in p. Using (3) the firm's revenue function can be written as a function of $\widehat{\gamma}$ alone. Define $R_s^h(\widehat{\gamma})$ as the

firm's revenue function from only high-type customers:

$$
\begin{aligned}
R_s^h\left(\widehat{\gamma}\right) &= S_{\widehat{\gamma}X}\left(k\right)\left(v_h - \frac{\mu c}{S\left(k/\widehat{\gamma}\right)}\right) \tag{4}\\
&= \widehat{\gamma}S\left(k/\widehat{\gamma}\right)v_h - \widehat{\gamma}\mu c
\end{aligned}
$$

Observe that the first term in 4 is the expected value high-type consumers receive, accounting for the possibility of rationing and the second term is the sure visit costs they incur. Hence, $R_s^h\left(\widehat{\gamma}\right)$ is the high-type consumers' total welfare. Consequently, restricting attention to only high-type consumers, the firm chooses a price that both maximizes its revenue as well as consumer welfare. That is, by charging a single price the firm is able to extract all consumer welfare.

The next lemma finds the equilibrium fraction of high-type consumers who visit the firm under static pricing, γ_s. If c is sufficiently low, all customers visit the firm. Otherwise, a fraction of the high-type customers visit. (This and all subsequent proofs are provided in the appendix.)

Lemma 1 *With static pricing, the firm's revenue function from high-type consumers, $R_s^h\left(\widehat{\gamma}\right)$, is concave. Let $\gamma_s = \arg\max R_s^h\left(\widehat{\gamma}\right)$: (i) if $v_h\int_0^k xf\left(x\right)dx \geq \mu c$, then $\gamma_s = 1$ and $p_s^h = \overline{p}_s$; otherwise (ii) γ_s is the unique solution to*

$$
v_h\int_0^{k/\gamma_s} xf\left(x\right)dx = \mu c \tag{5}
$$

and

$$
p_s^h = v_h - \frac{\mu c}{S\left(k/\gamma_s\right)}.
$$

Instead of choosing p_s^h and selling only to high type consumers, the firm also has the option to choose $p_s \leq v_l$, in which case the firms sells all its capacity and its revenue is $p_s k$. Clearly, $p_s = v_l$ is optimal among the prices that guarantee full utilization.

The firm's optimal price, p_s, is either p_s^h or v_l. It can be shown that $R_s^h\left(\gamma_s\right) = kv_h\overline{F}\left(k/\gamma_s\right)$. Thus, $p_s = p_s^h$ when $v_h\overline{F}\left(k/\gamma_s\right) \geq v_l$, otherwise $p_s = v_l$. In the former case, revenue is independent of v_l, whereas in the latter case it is linearly increasing in v_l.

5 Dynamic Pricing Strategy

With a dynamic pricing strategy the firm chooses its price, from a set of possible options, after observing γ (the fraction of consumers who visit the firm) and x (the realization of high-type demand). We consider in this section a particular dynamic pricing strategy in which the firm imposes no *a priori* constraint on the price it can choose, $\mathcal{A} = \mathbb{R}^+$. Given this strategy, the firm's optimal price is either v_h or v_l : demand is inelastic in $p \leq v_l$ and $v_l < p \leq v_h$, so it is optimal to

set a price equal to the maximum of one of those two ranges. Note, consumer and firm behavior would not change if the pricing strategy were $\mathcal{A} = \{v_l, v_h\}$. Section 7 considers other dynamic pricing strategies.

Given $\mathcal{A} = \mathbb{R}^+$, the firm can price at $p = v_l$ and earn revenue $v_l k$. Alternatively, it can price at $p = v_h$ and earn revenue $v_h \gamma x$. Consequently, the firm chooses $p = v_l$ when

$$x \leq \frac{v_l}{v_h} \frac{k}{\gamma}, \tag{6}$$

which has probability $F(v_l k/(v_h \gamma))$ and chooses $p = v_h$, otherwise.

Observe that high value consumers only earn positive utility if the price is v_l and they are able to obtain the unit. In all other cases, consumers get zero surplus. Thus, to find the high-type consumer's surplus from visiting the firm, we let ψ be the high-type consumer's expectation for the probability that the firm charges v_l and he is able to get a unit. A high value consumer is indifferent towards visiting the firm if

$$\psi (v_h - v_l) = c. \tag{7}$$

As in the discussion of Section 4, in equilibrium, the belief about the probability ψ has to be consistent with the actual probability. Given our rationing rule, because v_l is charged only when $\gamma x \leq \frac{v_l}{v_h} k < k$ (from 6), high type consumers are *guaranteed* to get the unit when the price is v_l. (They may not be able to get the unit if the price if v_h, but in this case, their surplus is zero.) Thus, according to Definition 1, under dynamic pricing consumers do not face a rationing risk. However, they do face a price risk.

Definition 2 *A high-type consumer faces a* price risk *if the consumer does not know which price will be charged when the consumer chooses whether to visit the firm.*

With dynamic pricing, high-type consumers know that if they visit the firm, they will be able to obtain the unit if the price is low (no rationing risk), but they do not know what price will be charged. With other allocation rules, the high-type consumer may not be guaranteed to obtain a unit conditional that the price is low, i.e., the high-type consumer may also face a rationing risk. Consequently, with other allocation rules high-type consumers may be less inclined to visit the firm and the firm's revenue could be lower than what is achieved with our allocation rule.

The actual probability a high-type consumer obtains a unit at $p = v_l$ therefore is the probability that the firm charges that price, conditional that the high type consumer is in the market. Because the market size, X, is uncertain, conditional on his presence in the market, a high type

9

consumer's demand density is $xf(x)/\mu$ (following Deneckere and Peck 1995). Therefore, this consumer anticipates that the price will be v_l with probability

$$\psi = \frac{\int_0^{\frac{v_l}{v_h}\frac{k}{\gamma}} xf(x)\,dx}{\mu}. \tag{8}$$

Note that $\psi \leq F(v_l k/(v_h\gamma))$: if a high-type customer is in the market, the probability that demand is low (which implies that the price charged is v_l) is lower than the unconditional probability.

If $v_l < v_h - c$, then there exists some γ that satisfies (8). If $v_l \geq v_h - c$, then $\gamma = 0$ is the optimal strategy for consumers: if the utility from visiting is less than the lowest possible price, the consumer never visits. As that case is not interesting, we assume $v_l < v_h - c$. Let γ_d be the fraction of high-type consumers who visit the firm in equilibrium under dynamic pricing. The following lemma characterizes γ_d.

Lemma 2 *The fraction of high-type consumers who visit the firm in equilibrium, γ_d, is unique. Furthermore, (i) $\gamma_d = 1$, if*

$$\int_0^{\frac{v_l}{v_h}k} xf(x)\,dx \geq \frac{\mu c}{v_h - v_l};$$

and (ii) otherwise, γ_d is the solution to

$$(v_h - v_l)\int_0^{\frac{v_l}{v_h}\frac{k}{\gamma_d}} xf(x)\,dx = \mu c. \tag{9}$$

Observe, that while the value of v_l did not factor into the solution of γ_s, it definitely affects the fraction of high-type consumers who visit the firm under dynamic pricing.

Lemma 3 *The following limits hold: (i) $\lim_{v_l \to 0} \gamma_d(v_l) = 0$; and (ii) $\lim_{v_l \to v_h - c} \gamma_d(v_l) = 0$. Furthermore, if $F(\cdot)$ is an increasing generalized failure rate (IGFR) distribution, the fraction of consumers who visit the firm in equilibrium under dynamic pricing, $\gamma_d(v_l)$, is quasi-concave.*

Lemma 3 shows that when v_l is either very low or very high, high-type consumers do not visit the firm under dynamic pricing. If $v_l \to v_h - c$, consumers know that whether the price charged is v_l or v_h, they will obtain no utility from the product, and therefore they decide not to visit. When $v_l \to 0$ high-type consumers can potentially obtain the highest surplus. However, consumers anticipate that in this case there is little chance that the firm will choose $p = v_l$. Hence, they decide not to visit the firm.

The firm's revenue with dynamic pricing is

$$
\begin{aligned}
R_d^* &= F\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right)v_l k + v_h\gamma_d\int_{\frac{v_l}{v_h}\frac{k}{\gamma_d}}^{\frac{k}{\gamma_d}} xf(x)\,dx + \overline{F}\left(\frac{k}{\gamma_d}\right)v_h k \\
&= v_h k - v_h\gamma_d\int_{\frac{v_l}{v_h}\frac{k}{\gamma_d}}^{\frac{k}{\gamma_d}} F(x)\,dx \\
&= v_l k + v_h\gamma_d\left(S\left(\frac{k}{\gamma_d}\right) - S\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right)\right),
\end{aligned}
$$

where γ_d is characterized in Lemma 2. The next lemma characterizes the revenue function at the boundaries of v_l.

Lemma 4 *The following limits hold: (i) $\lim_{v_l\to 0} R_d = 0$; and (ii) $\lim_{v_l\to v_h - c} R_d = (v_h - c)\,k$.*

6   Comparison between Static and Dynamic Pricing

Holding the consumer's strategy, γ, fixed dynamic pricing is clearly superior - after observing the realization of demand, γx, the firm can decide whether it makes sense to choose a high price, v_h, and possibly not fully utilize its capacity, or to choose a low price, v_l, and sell all of its capacity. Static pricing does not give the firm the flexibility to optimally respond to realized demand. However, the consumer's strategy is not fixed - it depends on the set of potential prices the firm initially chooses, $\mathcal{A}$. With static pricing consumers face a rationing risk but not a price risk, whereas with dynamic pricing they face a price risk but not a rationing risk. According to the Theorem 1, the price risk associated with $\mathcal{A} = \mathbb{R}^+$ leads to lower potential demand than the rationing risk of static pricing. Hence, with static pricing the firm enjoys higher potential demand but the inability to optimally respond to it, whereas with dynamic pricing the firm has flexibility to respond but receives less potential demand.

Theorem 1 *The fraction of consumers who visit the firm under dynamic pricing is lower than under static pricing.*

To understand the difference between the visiting behavior in equilibrium under the two pricing schemes, observe that, in equilibrium, the fraction of consumers who visit the firm under static pricing is given by (5) and can be written as

$$
v_h\frac{\int_0^{\frac{k}{\gamma_s}} xf(x)\,dx}{\mu} = c, \tag{10}
$$

Table 2. Parameter values used in the numerical study.

Parameter	Values
Demand distribution	Gamma
μ	1
σ	$\{0.25\mu, 0.5\mu, \mu, 1.5\mu, 2\mu\}$
k	$\{0.1\mu, 0.5\mu, \mu, 2\mu, 5\mu\}$
c	$\{0.01, 0.1, 0.25, 0.5, 0.75, 0.9, 0.99\}$
v_h	1

The left-hand side of (10) is the utility of a high-type consumer when the firm chooses a market clearing price, which is zero if demand is less than capacity and v_h otherwise. Hence, (10) can be interpreted in terms of prices - it is as if the consumer expects that the price will be zero if demand is less than capacity (generating a utility of v_h) and that the price will be v_h if capacity is binding (generating a utility of zero). With dynamic pricing, the fraction of consumers who visit the firm is given by (9) and can be written as

$$(v_h - v_l) \frac{\int_0^{\frac{v_l}{v_h} \frac{k}{\gamma_d}} x f(x)\, dx}{\mu} = c. \tag{11}$$

Now the consumer expects that the price will be $v_l \geq 0$ if demand is less than $(v_l/v_h)(k/\gamma_d)$, i.e., there is a smaller chance of a smaller discount than with static pricing, implying that fewer consumers choose to visit with dynamic pricing.

Although static pricing generates higher demand, it does not always charge the highest price, so it may not yield the highest revenue. The following theorem states that static pricing indeed generates higher revenue than dynamic pricing when v_l is sufficiently low because high-type consumers anticipate that the firm is unlikely to charge v_l when it is low and therefore they decide not to visit the firm. With static pricing consumers always anticipate some surplus from visiting, so some visit and some revenue can be gained. When v_l is sufficiently high, the two schemes generate the same revenue because they both charge v_l and always sell all of their capacity.

Theorem 2 *There exists a $\widetilde{v}_l$, such that $R_s^*(v_l) > R_d^*(v_l)$ for all $v_l < \widetilde{v}_l$. Further, $R_s^*(v_h - c) = R_d^*(v_h - c) = (v_h - c)\, k$.*

To obtain additional results comparing R_s^* to R_d^*, we construct 175 instances using all combinations of μ, σ, k, c and v_h in Table 2. For all instances, we observe that R_d^* increases monotonically with v_l, despite that fact that fewer high-type consumers visit the firm as v_l gets large (i.e., the
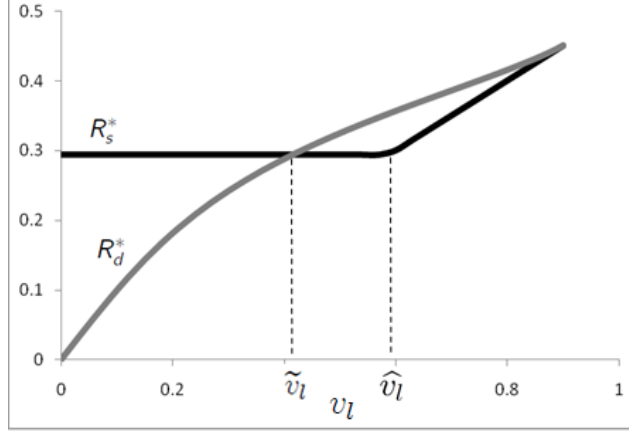
Figure 1. Revenue functions under static (R_s^*) and dynamic (R_d^*) pricing as a function of v_l for $X \sim Gamma\,(1,1)$, $v_h = 1$, $k = 0.5$, $c = 0.1$.

higher per unit revenue from an increase in v_l dominates any reduction in high-type consumer demand). Therefore, for the remainder of our analysis, we assume revenue with dynamic pricing is increasing in v_l.[3] Given (A1), it can be shown that there exists a unique $\widetilde{v}_l$ such that $R_s^*(v_l) > R_d^*(v_l)$ for all $v_l < \widetilde{v}_l$ and $R_s^*(v_l) \leq R_d^*(v_l)$ for all $v_l \geq \widetilde{v}_l$.

Assumption 1 *(A1) Revenue with dynamic pricing is increasing in v_l, i.e., $R_d^{*\prime}(v_l) > 0 \quad \forall v_l$.*

Figure 1 illustrates the revenue functions under both pricing schemes as a function of v_l for one of the instances in our study. The advantage of static pricing is greatest when $v_l = 0$. The advantage of dynamic pricing is greatest when $v_l = \widehat{v}_l$, where $\widehat{v}_l = v_h F\,(k/\gamma_s)$ (i.e., $\widehat{v}_l$ is the smallest v_l for which the firm charges $p_s = v_l$ under static pricing). We define $\Delta_s = R_s^*(0) - R_d^*(0) = R_s^*(0)$ and $\Delta_d = R_d^*(\widehat{v}_l) - R_s^*(\widehat{v}_l)$ and compare these measures in our sample. The results, reported in Table 3, are consistent with the observation from Figure 1: the advantage of R_s^* over R_d^* is indeed more significant than the advantage of R_d^* over R_s^*. In all cases $\Delta_s > \Delta_d$ and at best Δ_d is at most 70.4% of Δ_s. On average, Δ_d is only a little more than a tenth of Δ_s.

The value $\widetilde{v}_l$ provides another measure of the relative advantage of static over dynamic pricing: a large value of $\widetilde{v}_l$ indicates that static pricing is superior to dynamic pricing over a large set of parameters. We first consider how capacity, k, influences $\widetilde{v}_l$. With static pricing, as k decreases, the probability to obtain the unit decreases, so consumers face a higher rationing risk. With dynamic pricing, as k decreases, the firm is less likely to charge v_l, so consumers face a higher

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[3]It is difficult to analytically show that the dynamic pricing function is increasing in v_l because (i) the function $\gamma_d\,(v_l)$ is not monotone in v_l; and (ii) usual methods (such as the Envelope Theorem) cannot be applied on the dynamic pricing revenue function because it is not obtained through optimization, but rather is a consequence of equilibrium behavior.

Table 3. Summary statistics of the maximum benefit of using dynamic pricing, Δ_d, relative to the maximum benefit of using static pricing, Δ_s (in %).

	Δ_d/Δ_s
average	11.86%
standard deviation	16.02%
minimum	$6.8 \cdot 10^{-3}\%$
maximum	70.40%

price risk. Under both pricing schemes, the decrease of k negatively affects consumers visiting behavior. For all instances of Table 2, we observe that $\widetilde{v}_l$ increases when the level of capacity decreases implying that the price risk effect is stronger than the rationing risk, i.e., static pricing is favored over dynamic pricing as capacity decreases. Now consider how the visit cost, c, influences $\widetilde{v}_l$.

Lemma 5 *The following hold: (i) When $c = 0$, $\gamma_s = \gamma_d = 1$ and $R_s^*(v_l) \leq R_d^*(v_l) \; \forall v_l$; and (ii) $\lim_{c \to v_h} \gamma_s = \lim_{c \to v_h} \gamma_d = 0$ and $R_s^*(v_l) = R_d^*(v_l) = v_l k \; \forall v_l$.*

Lemma 5 shows that when the cost to visit the firm is either negligible or very high, dynamic pricing dominates static pricing. When $c \to 0$, all high-type consumers visit the firm regardless of the pricing strategy. In this case, dynamic pricing naturally performs better. When $c \to v_h$, the visit cost is so high that high-type consumers do not visit the firm. Thus, under both pricing schemes the firm is better off charging v_l and selling all its capacity (i.e., the two schemes are equivalent). Finally, we observe that as the visit cost, c, increases, $\widetilde{v}_l$ first increases and then decreases. Therefore, the range of v_l for which static pricing dominates is the largest for intermediate values of c. Figure 2 illustrates this, by plotting $\widetilde{v}_l/(v_h - c)$ as a function of c for different capacity levels, where $X \sim Gamma(1,1)$ and $v_h = 1$. Note that the value of $\widetilde{v}_l/(v_h - c)$ measures the fraction below which static pricing performs better than dynamic pricing. Each line represents the value of $\widetilde{v}_l/(v_h - c)$ for a different capacity level. For example, when $k = 2$ and $c = 0.2$, static pricing is strictly better than dynamic pricing in 30% of the v_l parameter range.

Finally, consider how the coefficient of variation affects the value of $\widetilde{v}_l$. Assuming that the number of high-type consumers is Gamma distributed provides a simple way to numerically test how a change in the coefficient of variation affects $\widetilde{v}_l$. The coefficient of variation is defined as $CV = \sigma/\mu$, where σ is the standard deviation of X. For all instances of Table 2, we observe that $\widetilde{v}_l$ increases as CV decreases. This suggests that pricing dynamically becomes more favorable (in the sense that the range for which dynamic pricing dominates increases) when the high-type consumer
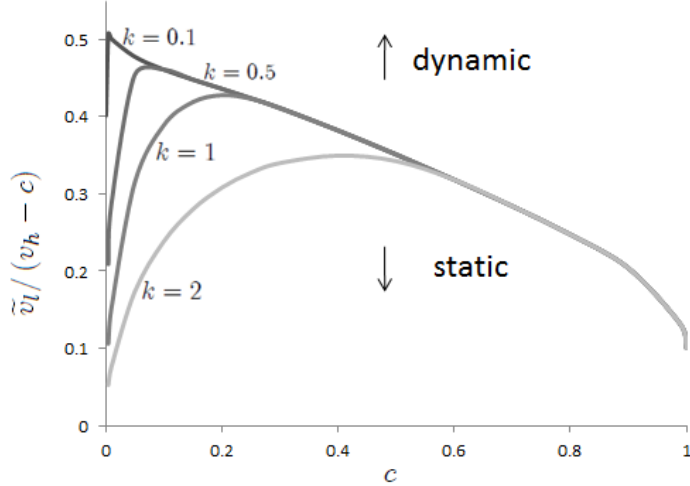
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Figure 2. The threshold $\tilde{v}_l/(v_h - c)$ as a function of c for $X \sim Gamma\,(1,1)$, $v_h = 1$ and different values of k.

demand uncertainty rises. To summarize, static pricing is more likely to be better than dynamic pricing (in the sense that $\tilde{v}_l$ is large relative to $v_h - c$) for low values of capacity and demand uncertainty and for intermediate values of visit cost.

7   Generalized dynamic pricing

The previous section demonstrates that static pricing can perform better than dynamic pricing when v_l is low relative to v_h. But we considered one particular dynamic pricing strategy, $\mathcal{A} = \mathbb{R}^+$. This section considers whether there exists a better dynamic pricing strategy. To this end, we now allow the firm to choose which prices to include in the set $\mathcal{A}$. Recall that static and dynamic pricing are special cases of this scheme: under static pricing the firm selects a single price $\mathcal{A} = \{p_s\}$ and under dynamic pricing the firm selects all possible prices, $\mathcal{A} = \mathbb{R}^+$.

Theorem 3 *For every $\mathcal{A}$, there exists a subset $\mathcal{B} = \{p_l, p_h\}$ where $p_l \in \mathcal{A}, p_h \in \mathcal{A}$ such that*

$$\max_{p \in \mathcal{A}} \{R\,(p, x, \gamma)\} = \max_{p \in \mathcal{B}} \{R\,(p, x, \gamma)\}\,.$$

Furthermore, $p_l = \sup_{p \in \mathcal{A}} \{p \leq v_l\}$ and $p_h = \sup_{p \in \mathcal{A}} \{p \leq v_h\}$.

Theorem 3 demonstrates that within the general set of pricing strategies, it is sufficient for the firm to consider only pricing strategies in which the firm commits to at most two prices before demand is realized. To explain, recall that there are two types of consumers and the firm must choose the optimal price among the preannounced feasible set, $\mathcal{A}$, after observing the realization of

15

demand. Thus, no matter how many prices are in $\mathcal{A}$, after observing demand, either the firm will choose $p_l \in \mathcal{A}$, where p_l is the highest price in $\mathcal{A}$ that low-type consumers will buy at, or the firm will choose $p_h \in \mathcal{A}$, where p_h is the highest price in $\mathcal{A}$ that high-type consumers will buy at. High-type consumers anticipate this and thus, their equilibrium joining behavior under set $\mathcal{A}$ is equivalent to their equilibrium joining behavior under set $\mathcal{B} = \{p_l, p_h\}$. As an example, the dynamic pricing strategy $\mathcal{A} = \mathfrak{R}^+$ is equivalent to the dynamic pricing strategy $\mathcal{B}_d = \{v_l, v_h\}$. Therefore, we can restrict attention to the subset of the pricing schemes $\mathcal{A}$, in which the firm preannounces at most two prices.

Denote the allowable prices under static and dynamic pricing by $\mathcal{B}_s = \{p_s\}$ and $\mathcal{B}_d = \{v_l, v_h\}$, respectively. Moreover, let $R_{\{p_l, p_h\}}$ be the revenue function when the set of prices $\{p_l, p_h\}$ is announced, $p_l \leq v_l$ and $v_l < p_h \leq v_h$. The revenue function is given by:

$$R_{\{p_l, p_h\}} = p_l k + p_h \gamma_g \left(S\left(\frac{k}{\gamma_g}\right) - S\left(\frac{p_l}{p_h}\frac{k}{\gamma_g}\right) \right),$$

where γ_g is given by

$$\frac{v_h - p_h}{\mu} S\left(\frac{k}{\gamma_g}\right) + \frac{p_h - p_l}{\mu} \int_0^{\frac{p_l}{p_h}\frac{k}{\gamma_g}} x f(x)\,dx = c$$

Theorem 4 *The following properties hold:*

1. $R_{\{v_l, p_s\}} \geq R_s^*$.

2. $R_{\{v_l, p_h\}} \geq R_{\{p_l, p_h\}} \; \forall p_l \leq v_l$.

The first statement of Theorem 4 implies that static pricing is always dominated by a dynamic strategy in which the firm announces $\{v_l, p_s\}$. Relative to static pricing, $\mathcal{B}_s$, with that scheme more consumers visit the firm (because they anticipate that they may be charged v_l) and the firm gains the capability to choose the better price to respond to demand conditions. Thus, when the firm can reduce its price, dynamic pricing can actually work better for both consumers and the firm. In fact, the second part of the theorem suggests that the problem with $\mathcal{B}_d = \{v_l, v_h\}$ is not with the lower price: holding the high price fixed, the firm's best low price is the highest possible low price, v_l. (Note, this is not immediately obvious because the high-type consumers are more likely to visit with $p_l < v_l$ than with $p_l = v_l$.) Thus, the concern with $\mathcal{B}_d = \{v_l, v_h\}$ is with the high price, v_h. While $p_h < v_h$ generates lower revenue for the firm per sale than $p_h = v_h$, more high-type consumers are likely to visit with $p_h < v_h$. Hence, revenue with $p_h < v_h$ may be higher than with $p_h = v_h$.

If $\gamma_d = 1$, then it is not possible to improve upon $\mathcal{B}_d = \{v_l, v_h\}$: all high-type consumers join and revenue is maximized in all realizations of demand. However, among all 175 instances of Table 2,when $\gamma_d < 1$, we *always* find there exists a price $p_h < v_h$ such that $R_{\{v_l, p_h\}} > R_d^*$. In other words, the dynamic pricing strategy $\mathcal{B}_d = \{v_l, v_h\}$ can be improved by committing to leave the high-type consumers with some surplus no matter which price is chosen - the problem with the dynamic pricing strategy $\mathcal{B}_d = \{v_l, v_h\}$ is that the high price can be too high.

We are now in a position to define a better dynamic pricing strategy. Let $p_h^* = \arg\max_{p_h} R_{\{v_l, p_h\}}$ and $\mathcal{B}_g = \{v_l, p_h^*\}$. That is, p_h^* is the optimal high price and $R_{\{v_l, p_h^*\}}$ is the maximum revenue that can be achieved under the generalized scheme. We refer to $\mathcal{B}_g$ as constrained dynamic pricing because the firm *a priori* constrains itself to not charge the highest possible price - when demand is high the firm may prefer to charge v_h, but due to its initial commitment, it is restricted to choose $p_h^* \leq v_h$. In addition to earning more revenue, the key distinction between $\mathcal{B}_g$ and $\mathcal{B}_d$ is that with $\mathcal{B}_g$ the firm must be able to commit to choose a price that everyone knows may be sub-optimal once demand is realized whereas such a commitment is not necessary with $\mathcal{B}_d$. Without that ability to commit, the firm is relegated to choose the only dynamic pricing strategy that is sub-game perfect, $\mathcal{B}_d$.

Whether a firm can commit to $\mathcal{B}_g = \{v_l, p_h^*\}$ may depend on how it is implemented. One way to implement $\mathcal{B}_g$ is to announce v_l as the "list price" (or "regular price") and commit to charge the list price or to charge the moderately higher price, p_h^*. More naturally, the firm can announce p_h^* as the list price and commit to charge either that list price or a lower price (and the lower price will be v_l). It seems plausible that firms, through repeated dynamics, may be able to commit to only mark down their prices. In fact, this policy (sometimes referred to as *asymmetric price adjustments*), is both empirically observed and theoretically assumed (e.g., Aviv and Pazgal 2008; Liu and Van Ryzin 2008; Su and Zhang 2008). Our theory provides an explanation for this effect beyond "consumers dislike price increases" - by committing to leave consumers with some surplus in all states, the firm is ensuring that a sufficient number of consumers will actually make the effort to visit the firm.

Static pricing, $\mathcal{B}_s = \{p_s\}$, also requires a commitment on the part of the firm (to neither mark up or mark down). Figure 3 illustrates that the commitment to not mark up is more important than the commitment to not mark down, as $\mathcal{B}_g = \{v_l, p_h^*\}$ generates higher revenue than both static, $\mathcal{B}_s$, and dynamic pricing, $\mathcal{B}_d$.

It is straightforward to show that $\lim_{v_l \to 0} R_{\{v_l, p_h^*\}} = \lim_{v_l \to 0} R_s^*(v_l)$ and that $\lim_{v_l \to v_h - c} R_{\{v_l, p_h^*\}} = \lim_{v_l \to v_h - c} R_s^*(v_l) = \lim_{v_l \to v_h - c} R_d^*(v_l)$. In addition, for each parameter combination, our numeri-
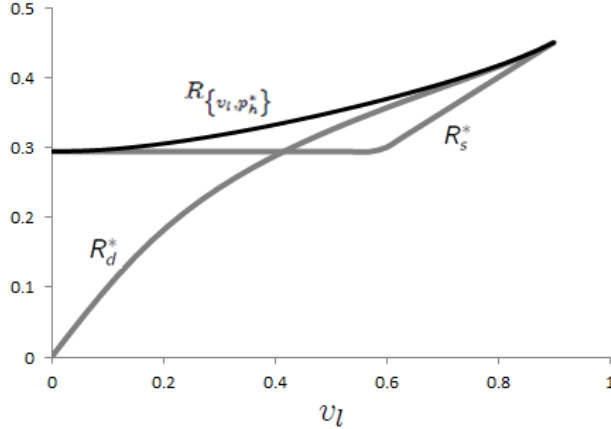
Figure 3. Revenue functions under static (R_s), dynamic (R_d) and the generalized ($R_{\{v_l, p_h^*\}}$) pricing schemes as a function of v_l for $X \sim Gamma(1,1)$, $v_h = 1$, $k = 0.5$, $c = 0.1$.

Table 4. Summary statistics of the maximum benefit of using either static or dynamic pricing relative to the maximum benefit of using the constrained dynamic pricing policy (in %).

	$R_s^*(\widetilde{v}_l)/R_{\{\widetilde{v}_l, p_h^*\}}$	$\max\{R_s^*(v_l), R_d^*(v_l)\}/R_{\{v_l, p_h^*\}}$
average	77.4%	93%
standard deviation	13.0%	10.7%
minimum	58.2%	58.2%
maximum	99.7%	100%

cal results found that implementing the constrained dynamic pricing policy is most beneficial when $v_l = \widetilde{v}_l$ (i.e., the v_l where $R_s^* = R_d^*$). Column 2 in Table 4 documents the ratio $R_s^*(\widetilde{v}_l)/R_{\{\widetilde{v}_l, p_h^*\}}$, where note that $R_s^*(\widetilde{v}_l) = R_d^*(\widetilde{v}_l)$. This ratio measures the worst case performance of the best simple pricing scheme relative to the optimal generalized scheme. We find that in the worst case, either static or dynamic pricing yields only 77.4% of the revenue generated by constrained dynamic pricing. As this is the worst case scenario, we are also interested in the average benefit for different values of v_l. To this end, for each instance in 2 we consider the eleven v_l such that the ratio of v_l to v_h is taken from the following set:

$$\{0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}.$$

If a resulting v_l exceeds $v_h - c$, we exclude it from the analysis, which leaves us with 925 instances. Column 3 in Table 4 reports the relative advantage of constrained dynamic pricing over the two simpler policies and indicates that on average, the simpler policies yield 7% less revenue than constrained dynamic pricing.
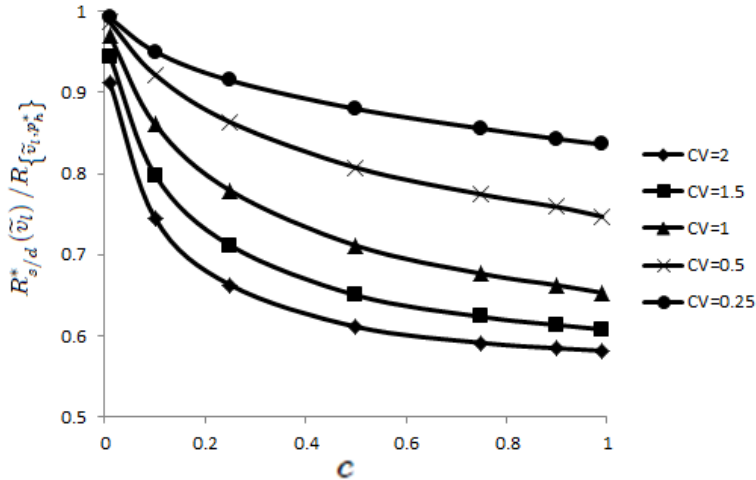
18

Figure 4. Worst case performance of dynamic or static pricing relative to the constrained dynamic pricing policy as a function of the visit cost, c, for different values of the coefficient of variation, CV.

Furthermore, we find that the relative advantage of constrained dynamic pricing is greatest when the visit cost, c, or the demand uncertainty (measured by the coefficient of variation, $CV = \sigma/\mu$) are high, as illustrated in Figure 4.

8   Conclusion

We explain why a firm may prefer static pricing over dynamic pricing when consumers are strategic and decide whether to consider to purchase based on the firm's chosen pricing strategy. By charging a static price a firm imposes a rationing risk on consumers whereas a firm that changes prices dynamically imposes a price risk on consumers. Imposing a rationing risk on consumers can dominate, especially when consumers' valuations for the product are highly variable. The problem with dynamic pricing is that the firm may charge a high price that leaves consumers with zero surplus, so the firm can improve its revenues by implementing a pricing strategy that leaves consumers with a positive surplus in all states of demand. Overall, we conclude that even though dynamic pricing responds better to demand conditions, charging a static price can be the preferable pricing strategy when consumers are strategic. However, constrained dynamic pricing is an even better strategy - charge either a reasonable list price or mark down from that list price, but never mark up.

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Appendix

Proof of Lemma 1. First, note that the expected sales function is given by

$$S\left(k/\gamma\right) = \int_0^{k/\gamma} xf\left(x\right)dx + \frac{k}{\gamma}\overline{F}\left(\frac{k}{\gamma}\right)$$

and that

$$S'\left(k/\gamma\right) = \frac{dS\left(k/\gamma\right)}{d\gamma} = -\frac{k}{\gamma^2}\overline{F}\left(\frac{k}{\gamma}\right)$$

Differentiating $R_s^h\left(\gamma\right)$ with respect to γ, we get:

$$\begin{aligned}
\zeta_s\left(\gamma\right) &= \frac{dR_s^h\left(\gamma\right)}{d\gamma} = v_h\left(S\left(k/\gamma\right) + \gamma S'\left(k/\gamma\right)\right) - \mu c \\
&= v_h\int_0^{k/\gamma} xf\left(x\right)dx - \mu c.
\end{aligned}$$

$R_s^h\left(\gamma\right)$ is concave because $\zeta_s\left(\gamma\right)$ is decreasing in γ. The optimal γ_s may be 1 (a corner solution) if $\zeta_s\left(1\right) \geq 0$ (result (i)) or interior, in which case solving the first-order condition $\zeta_s\left(\gamma\right) = 0$ gets the result (ii). Note that $\gamma_s \neq 0$, because we assume that $v_h > c$. ∎

Proof of Lemma 2. Under dynamic pricing, the indifferent consumer solves

$$\frac{\int_0^{\frac{v_l}{v_h}\frac{k}{\gamma}} xf\left(x\right)dx}{\mu}\left(v_h - v_l\right) = c. \tag{12}$$

As the left-hand-side (LHS) strictly decreases with γ and the right-hand-side (RHS) is constant, there either exists a unique $\gamma \in [0,1]$ which solves (12), or, if $\left(v_h - v_l\right)\int_0^{\frac{v_l}{v_h}k} xf\left(x\right)dx > \mu c$, there does not exist a γ which solves (12), in which case $\gamma_d = 1$. ∎

Proof of Lemma 3. Limit calculations: (i) Let $h'\left(x\right) = xf\left(x\right)$ so that $h\left(\xi\right) = \int_0^\xi xf\left(x\right)dx$. Therefore, from the Fundamental Theorem of Calculus, $\int_0^{\frac{v_l}{v_h}\frac{k}{\gamma_d}} xf\left(x\right)dx = h\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right) - h\left(0\right) = h\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right)$ (because $h\left(0\right) = 0$). Note that $h'\left(\xi\right) > 0$ and thus invertible. From (12), we can write $h\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right) = \mu c/\left(v_h - v_l\right)$ and

$$h^{-1}\left(\frac{\mu c}{v_h - v_l}\right) = \frac{v_l}{v_h}\frac{k}{\gamma_d}.$$

Rearranging, we get:

$$\gamma_d = \frac{\frac{v_l}{v_h}k}{h^{-1}\left(\frac{\mu c}{v_h - v_l}\right)}.$$

$h^{-1}\left(\frac{\mu c}{v_h - v_l}\right) > 0$, since $\mu c/\left(v_h - v_l\right) > 0$, $h\left(0\right) = 0$ and $h'\left(x\right) > 0$. Thus, taking the limit, we get $\lim_{v_l \to 0}\gamma_d = 0$. (ii) Rearranging (12) and letting $v_l \to v_h - c$, we get that for (12) to hold, we must have $\lim_{v_l \to v_h - c}\int_0^{\frac{v_l}{v_h}\frac{k}{\gamma_d(v_l)}} xf\left(x\right)dx = \mu$, which implies that $\lim_{v_l \to v_h - c}\gamma\left(v_l\right) = 0$.

To show that $\gamma_d(v_l)$ is quasi-concave, write:

$$\mathcal{F} = (v_h - v_l) \int_0^{\frac{v_l}{v_h}\frac{k}{\gamma_d}} x f(x)\, dx - \mu c.$$

Note that if condition (12) holds, $\mathcal{F} = 0$. Differentiating $\mathcal{F}$ and applying the Implicit Function Theorem, we get:

$$\frac{\partial \mathcal{F}}{\partial v_l} = \left(\frac{v_h - v_l}{v_l}\right)\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right)^2 f\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right) - \int_0^{\frac{v_l}{v_h}\frac{k}{\gamma_d}} x f(x)\, dx,$$

$$\frac{\partial \mathcal{F}}{\partial \gamma_d} = -\left(\frac{v_h - v_l}{\gamma_d}\right)\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right)^2 f\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right)$$

and

$$\frac{\partial \gamma_d}{\partial v_l} = \frac{\gamma_d}{v_l}\left(1 - \frac{v_l \int_0^y x f(x)\, dx}{(v_h - v_l) y^2 f(y)}\right) \tag{13}$$

$$= \frac{\gamma_d}{v_l}\left(1 + \frac{v_l}{v_h - v_l}\left(\frac{\overline{F}(y)}{y f(y)} - \frac{\int_0^y \overline{F}(x)\, dx}{y^2 f(y)}\right)\right),$$

where $y = \frac{v_l}{v_h}\frac{k}{\gamma_d}$. Observe first that γ_d/v_l is decreasing in v_l (and therefore that y is increasing in v_l). To see this, note that

$$\frac{\gamma_d}{v_l} = \frac{\frac{k}{v_h}}{h^{-1}\left(\frac{\mu c}{v_h - v_l}\right)},$$

which is decreasing in v_l because h^{-1} is increasing. Equating (13) to zero and rearranging, we get:

$$-\frac{v_h - v_l}{v_l} = \left(\frac{1}{y}\right)\left(\frac{\overline{F}(y)}{y f(y)}\right)\left(y - \frac{\int_0^y \overline{F}(x)\, dx}{\overline{F}(y)}\right).$$

Note that the LHS is increasing. The first term on the RHS is decreasing and the second terms is decreasing as well, because F is IGFR. Differentiating the third term with respect to y, we get:

$$1 - \frac{\overline{F}^2(y) + f(y)\int_0^y \overline{F}(x)\, dx}{\overline{F}^2(y)} = -\frac{f(y)\int_0^y \overline{F}(x)\, dx}{\overline{F}^2(y)} < 0,$$

and therefore it is decreasing as well. Thus, the RHS is decreasing. Together with the fact that $\gamma_d = 0$ in the limits and that $\gamma_d \geq 0$, we get the desired result. ∎

Proof of Lemma 4. The results immediately follow from the limits of Lemma 3. ∎

Proof of Theorem 1. To establish the result, assume first that γ_s and γ_d are interior. Denote the LHS of (5) and the LHS of (9) by $\tau_s(\gamma)$ and $\tau_d(\gamma)$, respectively. Observe that $\tau_s(\gamma) > \tau_d(\gamma)$ $\forall \gamma$. Furthermore, $\tau_s'(\gamma) < 0$ and $\tau_d'(\gamma) < 0$. Since the RHS of both conditions is the same, the

result follows. Also note that the conditions for boundary solutions are such that

$$v_h \int_0^k xf(x)\, dx > (v_h - v_l) \int_0^{\frac{v_l}{v_h}k} xf(x)\, dx,$$

implying that we must have that $\gamma_s \geq \gamma_d(v_l)\ \forall v_l$. ∎

Proof of Theorem 2. $R_s^*(v_l) = k \max\left\{v_h \overline{F}(k/\gamma_s), v_l\right\}$. Therefore, $R_s^*(v_h - c) = k \max\left\{v_h \overline{F}(k/\gamma_s), v_h - c\right\}$. To show that $R_s^*(v_h - c) = k(v_h - c)$, it remains to show that $v_h - c \geq v_h \overline{F}(k/\gamma_s)$. Combining with (10), it remains to show that

$$F(k/\gamma_s) \geq \frac{\int_0^{\frac{k}{\gamma_s}} xf(x)\, dx}{\mu}. \tag{14}$$

The LHS represents the probability that demand is less than k/γ_s, where the RHS represents the same probability conditional on a high-type consumer being in the market and hence (14) must hold.

$$
\begin{aligned}
\lim_{v_l \to v_h - c} R_d^*(v_l) &= \lim_{v_l \to v_h - c} F\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right) v_l k + \lim_{v_l \to v_h - c} v_h \gamma_d \int_{\frac{v_l}{v_h}\frac{k}{\gamma_d}}^{\frac{k}{\gamma_d}} xf(x)\, dx + \lim_{v_l \to v_h - c} \overline{F}\left(\frac{k}{\gamma_d}\right) v_h k \\
&= (v_h - c)\, k,
\end{aligned}
$$

where the last equality follows because $\lim_{v_l \to v_h - c} \gamma_d = 0$. In addition, $R_s^*(0) > 0$ and $R_d^*(0) = 0$.

Finally, differentiating $R_d^*(v_l)$ with respect to v_l, we get:

$$\frac{dR_d^*(v_l)}{dv_l} = kF\left(\frac{v_l}{v_h}\frac{k}{\gamma_d}\right) + v_h \int_{\frac{v_l}{v_h}\frac{k}{\gamma_d}}^{\frac{k}{\gamma_d}} xf(x)\, dx \frac{d\gamma_d}{dv_l},$$

and

$$\lim_{v_l \to v_h - c} \frac{dR_d^*(v_l)}{dv_l} < k$$

since $\lim_{v_l \to v_h - c} d\gamma_d/dv_l < 0$. ∎

Proof of Lemma 5. (i) When $c = 0$, condition (i) of Lemma 1 and condition (i) of Lemma 2 hold, and therefore $\gamma_s = \gamma_d = 1$. Furthermore, $R_s^*(v_l) = \max\{v_l k, v_h S(k)\}$ and $R_d^*(v_l) = v_l k + v_h S(k) - v_h S\left(\frac{v_l}{v_h}k\right)$. Suppose first that $v_l k \geq v_h S(k)$. Then, $R_s^*(v_l) = v_l k$ and $R_d^*(v_l) \geq R_s^*(v_l)$. Otherwise, suppose that $v_l k < v_h S(k)$. Then, $R_s^*(v_l) = v_h S(k)$ and $R_d^*(v_l) = v_l k + R_s^*(v_l) - v_h S\left(\frac{v_l}{v_h}k\right)$. $R_d^*(v_l) \geq R_s^*(v_l)$, because $\frac{v_l}{v_h}k \geq S\left(\frac{v_l}{v_h}k\right)$ (from the definition of the expected sales function). (ii) Following the same steps of Lemma 3, we get that

$$\gamma_s = \frac{k}{h^{-1}\left(\frac{\mu c}{v_h}\right)}.$$

Therefore, the $\lim_{c \to v_h} \gamma_s$ exists and is unique. To find the limit, observe that, when $c \to v_h$, $\lim_{c \to v_h} c\mu / v_h = \mu$ and we must have

$$\int_0^{\frac{k}{\gamma_s}} x f(x) \, dx = \mu,$$

which implies that $\lim_{c \to v_h} \frac{k}{\gamma_s} = \infty$ or $\lim_{c \to v_h} \gamma_s = 0$. Furthermore, since $\gamma_s \geq \gamma_d(v_l) \, \forall v_l$ and $\gamma_d \in [0,1]$, $\lim_{c \to v_h} \gamma_d(v_l) = 0$. For the revenues, when $\gamma_s = 0$, $R_s^*(v_l) = \max\{v_l k, 0\} = v_l k$ and when $\gamma_d = 0$, $R_d^*(v_l) = v_l k$. $\blacksquare$

Proof of Theorem 3. Suppose that the firm preannounced a set of prices $\mathcal{A}$ and that based on this set, γx high-type consumers visit the firm. Partition the set $\mathcal{A}$ to two disjoint sets, $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$, such that $\mathcal{A}_1 = \{p \in \mathcal{A} | p \leq v_l\}$ and $\mathcal{A}_2 = \{p \in \mathcal{A} | p > v_l\}$. Given that γx high-type consumers visited, the firm can choose to serve only high-type consumers, by choosing a price $p \in \mathcal{A}_2$ (if exists) or to serve both consumer types, by choosing a price $p \in \mathcal{A}_1$ (if exists). Suppose there exist two prices, $p^1 \in \mathcal{A}_2$ and $p^2 \in \mathcal{A}_2$, where $p^1 > p^2$. Because the choice of a price among $\mathcal{A}_2$ will not affect γ, setting p^1 strictly dominates p^2. Similarly, suppose there exist two prices, $p^3 \in \mathcal{A}_1$ and $p^4 \in \mathcal{A}_1$, where $p^3 > p^4$. Because the firm is guaranteed to sell k units by choosing any price among $\mathcal{A}_1$, setting p^3 strictly dominates p^4. $\blacksquare$

Proof of Theorem 4. For the two general prices (p_l, p_h), such that $p_l \leq p_h$, $p_l \leq v_l$ and $p_h \leq v_h$ the revenue function is given by

$$R(p_l, p_h) = p_l k + p_h \gamma \left(S\left(\frac{k}{\gamma}\right) - S\left(\frac{p_l}{p_h}\frac{k}{\gamma}\right) \right),$$

where γ is given by

$$\frac{v_h - p_h}{\mu} S\left(\frac{k}{\gamma}\right) + \frac{p_h - p_l}{\mu} \int_0^{\frac{p_l}{p_h}\frac{k}{\gamma}} x f(x) \, dx = c. \tag{15}$$

(1) $p_s = \max\{v_l, p^h\}$. If $p_s = v_l$, then $\mathcal{B}_s = \mathcal{B}$. If $p_s = p^h$, then (15) implies that $\gamma_s \leq \gamma$ and $R(v_l, p_s) \geq R_s$, because $\gamma_s \leq \gamma$ and because $\max_{p \in \mathcal{B}}\{R(p, x, \gamma)\} \geq \max_{p \in \mathcal{B}'}\{R(p, x, \gamma)\}$ if $\mathcal{B}' \subseteq \mathcal{B}$;

(2) First note that from the assumption that R_d is increasing in v_l, we get that

$$\frac{dR_d}{dv_l} = \frac{\partial R_d}{\partial v_l} + \frac{\partial R_d}{\partial \gamma_d}\frac{\partial \gamma_d}{\partial v_l} \tag{16}$$

$$= kF(y) + v_h \int_y^{\frac{k}{\gamma_d}} x f(x) \, dx \cdot \gamma_d \left(\frac{1}{v_l} - \frac{c\mu}{(v_h - v_l)^2 y^2 f(y)} \right) \geq 0,$$

where $y = \frac{v_l}{v_h}\frac{k}{\gamma_d}$. To prove the property, we need to show that $dR(p_l, p_h)/dp_l \geq 0$. Let $z = \frac{p_l}{p_h}\frac{k}{\gamma}$. Differentiating, we get:

$$\frac{\partial R(p_l, p_h)}{\partial p_l} = kF(z),$$

25

$$\frac{\partial R\left(p_l, p_h\right)}{\partial \gamma} = p_h \int_z^{\frac{k}{\gamma}} x f\left(x\right) dx$$

and from the Implicit Function Theorem,

$$\frac{\partial \gamma}{\partial p_l} = \gamma \cdot \frac{\left(p_h - p_l\right) \frac{z}{p_l} z f\left(z\right) - \int_0^z x f\left(x\right) dx}{\left(p_h - p_l\right) z^2 f(z) + \left(v_h - p_h\right) \frac{k}{\gamma} \overline{F}\left(\frac{k}{\gamma}\right)}.$$

As $\frac{\partial R(p_l, p_h)}{\partial p_l} \geq 0$ and $\frac{\partial R(p_l, p_h)}{\partial \gamma} \geq 0$, the result follows immediately if $\frac{\partial \gamma}{\partial p_l} \geq 0$. It remains to show that $dR\left(p_l, p_h\right)/dp_l \geq 0$ if $\frac{\partial \gamma}{\partial p_l} < 0$. Note that because $\left(v_h - p_h\right) \frac{k}{\gamma} \overline{F}\left(\frac{k}{\gamma}\right) \geq 0$, when $\frac{\partial \gamma}{\partial p_l} < 0$,

$$
\begin{aligned}
\frac{\partial \gamma}{\partial p_l} &\geq \gamma \left(\frac{1}{p_l} - \frac{\int_0^z x f\left(x\right) dx}{\left(p_h - p_l\right) z^2 f(z)}\right) \\
&= \gamma \left(\frac{1}{p_l} - \frac{c\mu - \left(v_h - p_h\right) S\left(\frac{k}{\gamma}\right)}{\left(v_h - v_l\right)^2 z^2 f\left(z\right)}\right) \\
&\geq \gamma \left(\frac{1}{p_l} - \frac{c\mu}{\left(v_h - v_l\right)^2 z^2 f\left(z\right)}\right).
\end{aligned}
$$

Note that the last term is equivalent to the derivative $\frac{\partial \gamma_d}{\partial p_l}$ of dynamic pricing in (16), where $v_l = p_l$ and $v_h = p_h$. Thus, if R_d is increasing, it must be that $\frac{\partial R(p_l, p_h)}{\partial p_l} \geq 0$. ∎