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Abstract
E-commerce has proved to be fertile ground for new business models, which may be patented (for up to 20 years) and have potentially far-reaching impact on the e-commerce landscape. One such electronic market is the reverse-auction model popularized by Priceline.com. There is still uncertainty surrounding the survival of such new electronic markets currently available on the Internet. Understanding user behavior is necessary for better assessment of these sites' survival. This paper adds to economic analysis a formal representation of the emotions evoked by the auction process, specifically, the excitement of winning if a bid is accepted, and the frustration of losing if it is not. We generate and empirically test a number of insights related to (1) the impact of expected excitement at winning, and frustration at losing, on bids across consumers and biddings scenarios; and (2) the dynamic nature of the bidding behavior—that is, how winning and losing in previous bids influence their future bidding behavior.

Keywords
online auction design, electronic markets, pricing, behavioral decision models, experimental economics

Disciplines
Behavioral Economics | Business | Business Administration, Management, and Operations | Business Analytics | Business Intelligence | Cognition and Perception | Cognitive Psychology | E-Commerce | Experimental Analysis of Behavior | Marketing | Organizational Behavior and Theory

Emotional Bidders -- An Analytical and Experimental Examination of Consumers' Behavior in Reverse Auction

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ABSTRACT

E-commerce has proved to be fertile ground for new business models, which may be patented (for up to 20 years) and have potentially far-reaching impact on the e-commerce landscape. However, there is still uncertainty surrounding the survival of various electronic markets currently available on the Internet. One such electronic market is the so-called reverse auctions model, pioneered in e-commerce by Priceline.com. Some have hailed it as the new pricing paradigm that may replace the traditional one-price-for-all practice, while others consider it just another marketing gimmick. These conflicting evaluations, however, are either based on anecdotal evidence or on the face value of the business model. As a result, it has been a trial and error experience for all four parties concerned (investors, customers, auction site itself, and auction site's partners – i.e., sellers of various products). This paper examines formally the bidding behavior of an individual customer engaged in such reverse auction. The paper explicitly models both emotional as well as economic aspects that are essential to understand the customer's bidding strategy and feelings. Specifically, we address and model the excitement of winning (bid is accepted), the frustration of losing (bid is rejected), and the money saved (compared to best price from alternative channels). We generate a number of interesting insights related to: (1) the bidding strategies of new (potential) customers. For example, what type of individuals will unlikely to bid, and for those who will bid, how the bidding strategies (prices) change across individuals; (2) the dynamic bidding behavior of existing customers-for example, how winning and losing in previous biddings influence their future bids, and (3) the managerial implications for the reverse auction sites-for example, how to optimize an individualized subsidy program based on the expected lifetime value of a customer. We conducted experiments designed specifically to test the theoretical insights obtained.

1. INTRODUCTION

Corporations are investing billions of dollars in electronic markets. E-commerce may exceed $1.2 trillion worldwide by the end of 2002.The Internet has generated $300 billion for the U.S. economy. The unique characteristics of the Internet have proved to be fertile ground for some new business models. The Federal Circuit Court of Appeals ruled in July 1998 that computerized “business methods” can be patented. Numerous applications have been received at the U.S. Patent and Trademark Office (Wall Street Journal, March 29, 2000). Some of the patents approved so far include the “one-click” process for Internet purchases (granted to Amazon.com in September 1999), “reverse” auctions patented to Priceline.com (August 1998), and, more recently, group purchasing model (Accompany received Notice of Allowance for their 1st U.S. patent on March 21, 2000). Given that a patent is valid for approximately 20 years, it is crucial that both firms as well as regulators (consumer advocates) understand the potential impact of each business model. At this time, however, it is not clear which of the new Internet business models will succeed. Investors have few reliable sources to value firms based on new business models (as partially reflected in their volatile stock prices). Analysis done in this area is typically based on anecdotal evidence that often leads to conflicting conclusions, as was the case following the launch of Priceline.com. Some customers claimed that their bids were never accepted, while others lauded the site for its great deals. Basic understanding of the potential of an Internet site and its business model requires, among other things, a thorough analysis of consumer behavior.

In this paper, we focus on reverse auctions sites. In a reverse auction sites, the sellers bid for the buyer` s business. We propose an analytical model of a consumer interacting with a reverse auction site, and we formally study the implications on the consumer behavior as well as on the management of the reverse auction site. Our objective is to understand the fundamentals of the buyer-seller behavior in this context. Hence, our analytical model is supplemented with empirical analyses. For exposition purposes, we will use Priceline.com (Five High Ridge Park, Stamford, CT 06905; website:

http://www.priceline.com; ticket symbol: PCLN) as an example. The basic idea behind Priceline' s business model is to allow consumers to name their own price. Priceline.com then decides which offer to accept and at what spread. As of March 29, 2000, customers could bid on the following product categories at Priceline.com: airline tickets, hotel rooms, rental cars, new cars and truck, home financing, Webhouse Club (for groceries), and Perfect YardSale. According to Priceline's own estimation, more than two million people have used Priceline. For the nine-month period ended 9/30/99, revenues totaled $313.2M, compared to $16.2M in the previous nine months.

To motivate and understand better the structure of the model developed here, a brief description of the reverse auction model used by Priceline is in order (see, for example, Priceline WebHouse Club, Harvard Business School Case, 2000). Let's take, as a case in point, a customer who wants to purchase an airline ticket (the flagship product category offered by Priceline). First, the customer will submit her bid to Priceline. Contrary to common belief, what Priceline does next is not to shop individual bid to airlines, instead, it compares the bid to a database of lowest fares already known to itself. If the bid price is higher than the lowest fare (i.e., cost to Priceline), it will then accept the bid and retain the spread (bid - lowest fare) as its profit. Priceline will sometimes accept bids below the available lowest fares. Some of the negative spreads in these cases are subsidized directly by Priceline (e.g., in the form of coupons given to new customers) to attract and strengthen relationships with new customers. Priceline also adopts a "cross-subsidy" promotion program in certain categories. For example, airline ticket purchase may be subsidized by a third party (Discover Card will subsidize $60 towards your bid if you sign up at Priceline site). Individual customer does not know the lowest fares that Priceline commands, and the airlines do not know the actual bid submitted by an individual customer. As a result, Priceline functions as a market maker instead of a pure facilitator of (reverse) auction.

The jury is still out on whether the business model patented by Priceline will be successful in the long run. The debate becomes even hotter after Priceline closed its WebHouse operation (grocery and gasoline) in October 2000. The proponents of Priceline model attribute the event to the lack of financial support and unwillingness of investors to bear the huge upfront costs associated with subsidizing new customers. The naysayers use it as a further proof of the fundamental flaw of Priceline business model. We expect the insights from our paper to shed light on this controversy and, more importantly, help to enhance the current model used by Priceline.

The model developed here relies on three literature streams. The first literature stream is the economic theory. Utility maximization serves as the foundation of this paper, where the optimization is done by the consumer based on the expected utility theory (von Neumann and Morgenstern, 1944). Since we study reverse-auction, auction theory is relevant to our paper (see Fudenberg and Tirole, 1998; Wilson, 1992; McAfee and McMillan, 1987 for comprehensive reviews). There are two main areas in auction literature, private value auctions (bidders know the value of the item to themselves with certainty) and common value auctions (the value of the item is the same for every bidders, but each bidder has different information about the potential value, e.g., bidding on the right of developing an oil field). The auction literature can also be divided into two streams based on the market structure. The first stream addresses one-sided auctions, where there are many buyers and one seller and many sellers and one buyer. The second literature stream addresses two-sided auctions with many sellers and many buyers. The branch of auction literature that is most closely related to our paper is the private value one-sided auction with many sellers and one buyer.

The one-sided auction with many sellers and one buyer may take at least two formats. The commonly studied form in economics is the scenario where sellers compete by submitting bids while the buyer waits passively for the best bid. Theoretically, this could take any one format commonly observed for many-buyer-one-seller auctions, namely, first price, second-price, English, and Dutch auctions, and the theoretical insights are similar as a result. The second format, in which a buyer takes the active role of submitting a price and the sellers decide whether they will accept the price or not, has received less attention by researchers. The reverse auction populated by Internet (e.g., Priceline) refers to this format. But for the reverse auctions to be implemented on the Internet, suppliers are in general blind to individual bids partially due to the fact that they are mostly operated by a third party site. Instead, suppliers determine a minimum acceptable bid price *a priori* for a given time for all consumers, and this information enters the price database controlled by the third party reverse auction site. As a result, the individual consumer is a price taker and his/her action has no effect on the minimum price a supplier is willing to accept.

The implication of above is that consumers in a reverse auction site are solving a fundamentally different problem compared to bidders in traditional auctions. Instead of trying to balance the probability of outbidding all competitors and the consequently realized utility (if the consumer wins), the consumer in a reverse auction site will try to balance the probability of bidding above the minimum price which varies from period to period and is unknown and unaffected by the individual consumer.

The second literature stream that inspires our work relies on behavioral-based analytical modeling. In general, economists make the assumption that consumers are perfectly rational decision makers unaffected by emotion. This is a reasonable approximation since traditional auctions are participated by either business entities or very experienced individuals. The reverse-auction populated by Internet, however, has made it possible for anybody to participate. While a consumer is normally asked whether he will or will not buy a product at a given price (a relative simple decision), reverse auction forces a consumer to identify the best price for a product (a much highly involved decision). Anecdotal evidence has indicated that the rational decision maker approximation is no longer acceptable. As a result, we must explicitly incorporate relevant behavioral and psychological components in our proposed utility function. A number of examples of this behavioral-based analytical modeling approach have been reported in the literature (see, for instances, Cross 1969; England, 1975; Pruitt, 1981). Balakrishnan and Eliashberg (1995), for instance, studied buyer-seller interactions, where each party makes offers and counter-offers for the same object. Their model employs behavioral constructs such as power, concession, aspiration and time pressure. We adopt here, in a similar fashion, two important behavioral constructs (that are salient in man-machine interactions on the Internet), namely, excitement and frustration. Some initial attempts to include such factors in the context of (passively) watching an entertainment experience have been reported in Eliashberg and Sawhney (1994).

The third literature stream that we rely on is the rich pricing (specifically, behavioral pricing) literature. Since the key decision variable in the reverse auction site is price, we need to incorporate various price concepts that are both relevant to economic theory and behavioral constructs. The behavioral pricing literature has suggested, with some empirical support, that consumers have a few subjective perceptions of price that affect their choices. These subjective perceptions are called reference prices. Reference price in general refers to the price that a consumer uses as a standard against which the observed price is compared. Five different operationalizations of reference price have been proposed (see Winer, 1988, for a comprehensive review): (1). Fair or just price. This refers to what a good ought to cost. Rao and Gautschi (1982) used the term *evoked price* in place of just price, (2). Reservation Price. This is commonly used in economics (Scherer, 1980, p17) and refers to the price just low enough to overcome resistance to buy a good. Alternatively, Rao and Gautschi (1982) defined it as the highest price a consumer is willing to pay for a good, (3). Lowest acceptable price. This is the price below which signals inferior quality, (4). Expected price. Emery (1970) hypothesizes that consumers also forecast future levels (as a reference price) to make current consumption decisions, and (5). Perceived price. This is the price one is expected to pay for a good in the current period (Zeithaml and Graham, 1983). It can be calculated based on prices most frequently charged, last period price, average price of similar products, or price of the brand usually bought. Of the five operationalizations, perceived price is most relevant to the current period market price and is likely to be independent of individuals. This characteristic of perceived price makes it an ideal candidate for our model. In additional to perceived price, the reservation price is also employed in our model.

Built upon these three research streams, we are able to generate a number of interesting theoretical insights related to (1) the bidding strategies of new (potential) customers, for example, what type of individuals will unlikely to bid, and for those who will bid, how the bidding strategies (prices) change across individuals. (2) the dynamic bidding behavior of existing customers. How winning and losing in previous biddings influence future bids. (3) managerial implications for the reverse auction sites, for example, how to optimize an individualized subsidy program based on the expected lifetime value of a customer. The theoretical insights are then tested empirically. We observe strong empirical support for some of the propositions we derived.

This research aims to contribute both to the academic literature as well as to the business practice of the Internet. The questions investigated are unique to the Internet and, to our knowledge; they have not been studied previously. We expect that our research will lead to new knowledge to be added to the extant academic literature. We also expect that the insights from this research will provide a rigorous base for analyzing business models such as the one employed by Priceline. The rest of the paper is organized as follows. The model is developed and described in Section 2. It focuses on customers who have already decided to purchase a product (either through a regular channel or reverse auction). Theoretical insights obtained from the model are discussed in Section 3. In Section 4, empirical testing of the theoretical insights is described and the results are presented. We summarize and discuss avenues for future research in Section 5.

2. MODEL DEVELOPMENT

We model the market for a given product (e.g., diapers, airline ticket) over time at the individual customer level, where each customer purchases exactly one unit of the product during each period. A customer has the option of purchasing the product through regular channels at the market price (best price from non-auction channels) or he/she could come to a reverse auction site to bid for the same product. The market price is operationalized as the perceived price. Hence, it is common knowledge among customers. Only one bid is allowed in each period, a customer will purchase the product from regular channels if his/her bid is rejected in that period. The customer is assumed to have knowledge concerning the distribution of the true cost, but not the actual cost of the product to the reverse auction site, which may vary from period to period. Each customer has a reservation price (maximum price he is willing to pay) for the product.

We focus on customers who have already decided that they will buy the product, either through regular channel or the reverse auction site. This implies that their reservation price is higher than the market price in this case.

The following constructs and notations are used:

C_l : The lower bound of the site's cost, as expected by a bidder.

C_u : The upper bound of the site's cost, as expected by a bidder.

C_t : Cutoff price (minimum acceptable bid) in period t.

$P_{o,t}$: The price offered by a bidder in period t (bid number).

$P_{m,t}$: The market price for the product, or perceived price for the product. This is the lowest price a consumer could pay to buy the product from non-auction channels in period t.

t : The number of periods after a customer starts to use the reverse auction site.

$P_{r,t}$: The reservation price of a bidder in period t.

$U_t(P_{o,t})$: The utility a bidder derived from participating in the reverse auction at period t.

$u_{1,t}(P_{o,t})$: The utility derived from the money saved from a winning bid in period t.

$u_{2,t}(P_{o,t})$: The emotional utility in period t.

$u_{2,t,accept}(P_{o,t})$: The excitement induced by acceptance of a bid in period t.

$u_{2,t,reject}(P_{o,t})$: The frustration induced by rejection (losing) of a bid in period t.

α_t : Excitement coefficient at period t, represents how sensitive a bidder is to winning. It is affected by prior bidding outcomes and bounded between α_{min} and α_{max}.

β_t : Frustration coefficient at period t, represents how sensitive a bidder is to losing. It is affected by prior bidding outcomes and bounded between β_{min} and β_{max}.

θ : Monetary coefficient represents how much a bidder values money.

$\varphi(C_t)$: The PDF of the true cost of the product to the site.

The decision rule for the reverse auction site (market maker) is straightforward. In the absence of subsidy, the site will only accept a bid if it is below its cost (lowest price offered to the site by suppliers of goods) and reject it otherwise.

Market Maker (reverse auction site):

$$\text{If} \quad P_{o,t} - C_t \begin{cases} \geq 0 & \text{then} \quad \text{accept bid} \\ < 0 & \text{then} \quad \text{reject bid} \end{cases} \qquad \text{---(1)}$$

The decision rule for a customer is more complicated. Due to the nature of the reverse auction and, in particular, since customers do not know what exactly happens behind the website (recall a customer does not know the prices each supplier is willing to accept at a given time), customers (bidders) become very involved emotionally in this "name-your-own-price" process. Richins (1997) undertook six studies and thoroughly assessed the domain of consumption-related emotions. The author identified sixteen relevant clusters of emotion. A careful examination shows that two of them, anger (frustration) and excitement, are most likely dominate the emotion evoked during reverse auction. Excitement will be induced when a bid is accepted and frustration will be induced when a bid is rejected. As a result, the utility function of a customer should include these emotion utility components. This akin to transaction utility (Thaler, 1985).

We define the utility function for an individual as following:

$$U_t(P_{o,t}) = u_{1,t}(P_{o,t}) + u_{2,t}(P_{o,t}) \qquad \text{---(2)}$$

where the monetary utility $u_{1,t}(P_{o,t})$ is

$$u_{1,t}(P_{o,t}) = \begin{cases} \theta(P_{m,t} - P_{o,t}) & \textit{if bid is accepted} \\ 0 & \textit{if bid is rejected} \end{cases} \qquad \text{---(3)}$$

and the emotional utility $u_{2,t}(P_{o,t})$ is

$$u_{2,t}(P_{o,t}) = \begin{cases} u_{2,t,accept}(P_{o,t}) & \textit{if bid is accepted} \\ u_{2,t,reject}(P_{o,t}) & \textit{if bid is rejected} \end{cases} \qquad \text{---(4)}$$

The expected utility associated with bidding $P_{o,t}$ can be represented as:

$$\mathbf{E}[U_t(P_{o,t})] = \mathbf{E}[u_{1,t}(P_{o,t})] + \mathbf{E}[u_{2,t}(P_{o,t})]$$
$$= \int_{C_l}^{P_{o,t}} u_{1,t}(P_{o,t})\varphi(C_t)dC_t + \int_{C_l}^{P_{o,t}} u_{2,t,accept}(P_{o,t})\varphi(C_t)dC_t - \int_{P_{0,t}}^{C_u} u_{2,t,reject}(P_{o,t})\varphi(C_t)dC_t \quad \text{---(5)}$$

We develop further the functional forms of the emotional constructs employed here, based on theories of consumer behavior. From a theoretical perspective, Weber's Law (see Winer, 1988) of psychophysics relates proportional changes in a stimulus to a psychological response,

$$\frac{\Delta S}{S} = K \quad \text{---(6)}$$

where S is the stimulus and K is the response. This suggests the magnitude of our emotional constructs (excitement or frustration) should be correlated to the percentage, rather than absolute, difference in price. The literature on emotion also supports this notion. It has been widely accepted that "the intensity of emotion depends on the relationship between an event and some frame of reference against which the event is evaluated" (Frijda, 1988). In other words, the absolute stimulus (potential money that could be saved through bidding at the reverse auction) elicits emotion through its relative magnitude as compared to the reference (in the case of reverse auction site, this will be the market price). This finding has been termed Law of Comparative Feeling (Frijda, 1988, Frijda, 1999).

Three observations with regard to reverse auction further support this theoretic premise. (1), The actual amount of a bid affect the magnitude of the emotional utility induced. A customer tends to be more excited if he/she wins with a relatively low bid. For example, a customer will be more excited about a \$100 winning bid than a \$300 winning bid for a round trip airline ticket between Boston and Philadelphia, assuming the ticket sells for \$400 through a regular travel agency. By the same token, a customer tends to be more frustrated if he/she loses with a relatively high bid. (2), The previous effect is moderated by an internal reference point, most likely, an individual's reservation price for that product.

Expanding on the airline ticket example, a businessperson who is willing to pay $1000 for the ticket is likely to be less excited than a leisure traveler who plans to spend at most $400. (3), Each individual will respond to the same outcome differently, even when they have the same reservation prices for the product.

Thus, based the above observations, we propose:

$$u_{2,t,accept}(P_{o,t}) = \alpha_t \frac{P_{m,t} - P_{o,t}}{P_{r,t}} \quad , \qquad \text{---(7)}$$

and

$$u_{2,t,reject}(P_{o,t}) = \beta_t \frac{P_{o,t}}{P_{r,t}} \qquad \text{---(8)}$$

Observation (1) is captured through $P_{m,t}$-$P_{o,t}$ in (7) and $P_{o,t}$ in (8), both are scaled by the reservation price $P_{r,t}$ (observation (2)). As Winer (1988) pointed out, consumers are characterized by a multidimensional price vector, and the above operationalizations fully reflect this spirit. Reservation price is used for the scaling effect (denominator) since it captures how valuable the product is to a customer (the magnitude of the stimulus itself). The change in stimulus is based on, however, the difference between bid price and perceived (market) price. This is due to the presence of an alternative market in which a customer could expect to buy the product for sure at the market (perceived) price. In the absence of this market, the difference should be between bid price and the reservation price. The coefficients α and β represent the different individual responses to the same possible outcome.

We invoke the traditional assumption that a potential bidder (customer) expects the cost of the product to the site is uniformly distributed between $[C_l, C_u]$. Others also made this assumption in similar situations (e.g., Holt and Sherman, 1994). Hence, the probability of a bid that will be accepted could be represented as

$$\int_{C_l}^{P_{o,t}} \varphi(C_t) dC_t = \frac{P_{o,t} - C_l}{C_u - C_l} \qquad \text{---(9)}$$

If we substitute equations (3), (6), (7) and (8) back into equation (5), the bidder's expected utility can now be represented as:

$$\mathbf{E}\left[U_t(P_{o,t})\right] = \theta(P_{m,t} - P_{o,t})\frac{P_{o,t} - C_l}{C_u - C_l} + \alpha_t \frac{P_{m,t} - P_{o,t}}{P_{r,t}}\frac{P_{o,t} - C_l}{C_u - C_l} - \beta_t \frac{P_{o,t}}{P_{r,t}}\frac{C_u - P_{o,t}}{C_u - C_l} \quad \text{---(10)}$$

A potential bidder is also subject to two sanity checks before he/she participates in a reverse auction site. First, the price to be submitted by him/her must be smaller than the best alternative (market) price and larger than the minimum cost to the site. Second, he/she must have a positive expected utility for this optimal bid. Thus, a potential customer's problem at a given time is to find a bidding price that will maximize the sum of her monetary utility and emotional utility, subject to the two constraints stated above.

Mathematically, the bidder (customer) seeks to:

$$\max_{P_{o,t}} \quad \mathbf{E}\left[U_t(P_{o,t})\right] \quad \text{---(11)}$$

subject to

$$C_l \le P_{o,t} \le P_{m,t} \le C_u \quad \text{---(12)}$$

$$\mathbf{E}\left[U_t(P_{o,t})\right] \ge 0 \quad \text{---(13)}$$

It is also clear that the excitement and frustration coefficients (α, β) are not static based on observations of bidders' behavior. Similar to the sensitization (increased pleasure) observed from, for example, successive use of marijuana or high-quality wine (Groves and Thompson, 1973), people engaged in reverse auction exhibits similar behavioral pattern. The dynamic bidding behavior is essentially driven through the dynamic changes in α_t and β_t, based on prior bidding outcomes. While it is reasonable to assume that a bidder, initially at least, will get more excited after each win, and more frustrated after each loss, there are countering forces due to fading effect (the impact of a win becomes smaller as time passes) and inverted U type of adaptation effect (bidder gets less excited after winning too many bids). In both cases, we expect to see α_t and β_t decrease at some time point. In this paper, we will not attempt to make any specific prediction on how, in general, α_t and

β_t change as a result of previous bidding outcome. Instead, we will focus on understanding how a subject's bid changes dynamically over time as her α_t and β_t change.

We thus make the following simple assumptions:

If a bid is accepted in period t-1, then:

$$\beta_t = \beta_{t-1} \qquad \text{and}$$

$$\alpha_t > \alpha_{t-1} \text{ or } \alpha_t < \alpha_{t-1} \text{ or } \alpha_t = \alpha_{t-1} \qquad \text{--- (14)}$$

If a bid is rejected in period t-1, then:

$$\alpha_t = \alpha_{t-1} \qquad \text{and}$$

$$\beta_t > \beta_{t-1} \text{ or } \beta_t < \beta_{t-1} \text{ or } \beta_t = \beta_{t-1} \qquad \text{--- (15)}$$

3. THEORETICAL INSIGHTS

In this section, we present the analytical results broken down into three subgroups. We first discuss the insights related to new customers and their optimal bidding strategy in sub-section 3.1. existing customers' bidding strategies over time (dynamic bidding behavior) as a function of prior bidding history are discussed in sub-section 3.2., and implications for managing the reverse auction site in sub-section 3.3.

3.1. Results related to new customers' bidding strategy

The bidder's problem, as presented in equations (11)-(13), is solved (see Appendix). We find that new customers (i.e., bidders who bid for the first time) will either stay out of the reverse auction site, equivalent to $P^*_{o,1} = 0$, or they will bid at an interior optimal price within the bounds specified in equation (12). This optimal bidding price is

$$P_{o,1}{}^* = \frac{C_u - (P_{m,1} + C_l)X_1}{2(1 - X_1)} \qquad \text{---(16)}$$

where

$$X_1 = \frac{\theta P_{r,1} + \alpha_1}{\beta_1} \qquad \text{---(17)}$$

X_1 captures the individuality of a bidder: a bidder with higher X_1 tends to enjoy winning more, values the product higher, has higher utility for money, and/or not very sensitive to losing.

Proposition 1. Each reverse auction site has a unique threshold defined as:

$$X' = \left(\frac{\sqrt{P_{m,1}(C_u - C_l)} + \sqrt{C_l(C_u - P_{m,1})}}{P_{m,1} - C_l} \right)^2 \qquad \text{---(18)}$$

A potential customer will never bid in a reverse auction site where her $X_1 \le X'$, she will bid at the price specified in equation (16) if $X_1 > X'$. Furthermore, this threshold (X') increases as C_u increases, C_l increases, or $P_{m,1}$ decreases.

Proof: See Appendix.

As noted earlier, X_1 represents individuality; it captures who the customer is prior to his/her first bidding. The right hand side of equation (18), on the other hand, captures the threshold level attractiveness of the reverse auction site, which is a function of cost distribution and the (alternative) market price. The bidder will bid only if his/her individual characteristics exceed the threshold. As expected, the threshold increases as the site itself becomes less attractive (high upper bound or high lower bound) or the regular market becomes more attractive (lower market price).

Our analyses reveals that reverse auction sites can be classified into three types based on their inherent attractiveness. They are defined as follows:

Type 1 Sites: these are sites where $C_u - C_l > P_{m,1}$, i.e., $\frac{1}{C_u - C_l} < \frac{1}{P_{m,1}}$

Type 2 Sites: these are sites where $C_u - C_l < P_{m,1}$, i.e., $\frac{1}{C_u - C_l} > \frac{1}{P_{m,1}}$

Type 3 Sites: these are sites where $C_u - C_l = P_{m,1}$, i.e., $\frac{1}{C_u - C_l} = \frac{1}{P_{m,1}}$

Conceptually, an optimal bid is essentially driven by two competing factors. Higher bid will increase the probability of winning, but at the same time, decrease the magnitude of the utility when the outcome is revealed, whether it's a win (since less money will be saved and the excitement will be lower) or a loss (more frustration). This tradeoff drives the intuitive rationale behind the division of three types of reverse auction sites. $\frac{1}{C_u - C_l}$ represents what could be gained in terms of increased probability of winning by bidding \$1 more (first factor), and $\frac{1}{P_m}$ represents the decrease in percentage of the utility by bidding \$1 more (second factor).

We characterize the bidding prices for the three different types of sites below.

Proposition 2. For potential customers whose $X_1 > X'$, the optimal bidding prices can be characterized as:

- **Strictly concave and increasing over X_1, and bounded between $\frac{C_u - (P_{m,1} + C_l)X'}{2(1-X')}$ and $\frac{P_{m,1} + C_l}{2}$ in Type 1 sites;**
- **Strictly convex and decreasing over X_1, and bounded between $\frac{P_{m,1} + C_l}{2}$ and $\frac{C_u - (P_{m,1} + C_l)X'}{2(1-X')}$ in Type 2 sites;**
- **Constant over X_1 and equals to $\frac{P_{m,1} + C_l}{2}$ in Type 3 sites.**

Proof: See appendix.

Lemma 1. Compared to the normative result of bidding at $\frac{P_{m,1} + C_l}{2}$ (when the transaction utility is discounted), bidders in Type 1 sites tend to underbid while

bidders in Type 2 sites tend to overbid. Bidders in Type 3 sites or those with very high X (regardless of the type of the sites) will bid close to $\frac{P_{m,1}+C_l}{2}$.

Proof. Follow directly from Proposition 2.

The results in Propositions 1 and 2 have been graphically represented in Figure 1.

Insert Figure 1 here

3.2. Results related to the dynamic bidding strategy of existing customers

In order to optimize customer retention, it is critical to understand their customer's dynamic bidding strategies over time. In particular, the management of the site needs to know how prior bidding results influence (if at all) future bidding strategy. To address this issue, we will examine the impact of winning (accepted bids) and losing (rejected bids) on the customer's future bids in this subsection.

Lemma 2. Assuming the reservation price, P_r, is constant over time, the optimal bidding prices submitted by a given customer are:

- **Strictly concave and increasing over X_1 in Type 1 sites, and will always be lower than $\frac{P_{m,t}+C_l}{2}$. The customer will stop bidding if his/her $\frac{C_u-(P_{m,t}+C_l)X_t}{2(1-X_t)}$ drops below $\frac{C_u-(P_{m,t}+C_l)X'}{2(1-X')}$.**
- **Strictly convex and decreasing over X_1 in Type 2 sites. It will always be larger than $\frac{P_{m,t}+C_l}{2}$. The customer will stop bidding if his/her $\frac{C_u-(P_{m,t}+C_l)X_t}{2(1-X_t)}$ becomes larger than $\frac{C_u-(P_{m,t}+C_l)X'}{2(1-X')}$.**

- **Constant over X_1 and equals to $\frac{P_{m,1}+C_l}{2}$ in Type 3 sites.**

Proof: Follow directly from Proposition 2.

Proposition 3 below provides further insights into the dynamic behavior of the customer.

Proposition 3. Assuming the reservation price, $P_{r,t}=P_r$, is constant over time,

1) **In Type 1 sites –**
   - **If the customer's last period's bid is accepted, she will increase her current period bid if she becomes more sensitive about winning auction, maintain the same bid if her sensitivity remains the same, and decrease her bid if she becomes satiated and less sensitive about winning.**
   - **If the customer's last period's bid is rejected, she will increase her current period bid if she becomes less sensitive to losing, maintain the same bid if her sensitivity to losing remains the same, and decrease her bid if she becomes more sensitive to losing.**
2) **In Type 2 sites –**
   - **If the customer's last period's bid is accepted, she will decrease her current period bid if she becomes more sensitive about winning auction, maintain the same bid if her sensitivity remains the same, and increase her bid if she becomes satiated and less sensitive about winning.**
   - **If the customer's last period's bid is rejected, she will decrease her current period bid if she becomes less sensitive to losing, maintain the same bid if her sensitivity to losing remains the same, and increase her bid if she becomes more sensitive to losing.**
3) **In type 3 sites –**
   - **The customer will always bid $\frac{P_m+C_l}{2}$ regardless the results of prior biddings and how her sensitivities about winning/losing change.**

Proof: This follows directly from equation (17) and proposition 2.

3.3. Implications for managing the reverse auction site

In this subsection, we will examine how a reverse auction site can improve its profitability. In general, a site can improve its profit taking either at the aggregate or the individual level. We discuss actionable managerial recommendations for each level.

One form of aggregate level is to change the perception of the site for **all** individual customers. The parameters that are within the control of a site are essentially the perceived distribution of the cost to the site. It may be possible to change the perceived distribution bounds through various marketing (communication) effects. For instance, the site should try to convince the customers that its upper cost bound is lower than the actual bound. Proposition 4 provides formal guidance to the communication message that the site management should adopt.

Proposition 4. As long as a customer's optimal strategy is to bid, he/she will always bid higher as the lower bound of a site's cost distribution (C_l) increases, or the upper bound (C_u) decreases.

Proof: See Appendix.

If the true cost to the site is the same, higher bid price is generate higher profit. Thus we could infer

Lemma 3. A reverse auction site could improve its profit by increasing its perceived lower bound (C_l) or decreasing its perceived upper bound (C_u), subject that such action does not make the customers optimal not to bid.

Proof: This directly follows from Proposition 4.

This observation is particularly interesting because these managerial actions could be achieved with minimum cost, neither require the site to subsidize a customer's bid.

Profit can also be improved at individual level by influencing the bidding path of a given individual. Two strategies can be taken. One is to prevent a customer from leaving the site permanently. As Lemma 2 has shown, a customer will stop bidding once her X becomes smaller than the threshold as a result of losing. A site should therefore identify customers what are approaching the boundaries and provide individualized subsidy to ensure that these customers stay with the site. Alternatively, a bid could be artificially rejected (turning down an above-cutoff-price bid) or accepted (accepted a below-cutoff-price bid, for example, via subsidy). This will affect customers' X such that their future bids will be higher. As we have seen in Proposition 3, this strategy will be highly dependent on the individual, since different individuals will vary in terms of their X parameters.

4. EMPIRICAL TESTING AND ANALYSES

In this section we describe an experiment designed to test the major insights obtained in Section 3. Eighty-seven undergraduate business students at a major university participated in this experiment. We will first describe the experimental procedure, followed by the analyses of the data.

4.1. Experimental procedure

Subjects participated in the experiment were assigned to one of six sessions (each session had an average of 15 subjects). The experiment was conducted using pencil-and-paper format and the cutoff price (C_t) in each round was publicly generated using Excel spreadsheet function projected onto the screen in the front of the room.

The subjects were told that this would be a moneymaking opportunity. They were told that there was a huge demand for a Springbreak package (a trip to Florida for two, with all expenses included), and that people are willing to pay $1160 for this package. The subjects could choose one of two options to buy the package and sell it at $1160, and pocket the difference. The two options are: 1) Purchase the package from a travel agency for $1100. 2) Submit a bid to a reverse auction site for the package. If the bid is rejected, they could still go to the travel agency and buy it for $1100. Each subject participated in 20 rounds, and had the opportunity to purchase one package in each round. In the first 10 rounds, they had to go to a Type 1 auction site, in the next 10 rounds; they will go to a Type 2 auction site. The parameters of the two sites are shown in Table 1.

Insert Table 1 here

In order to ensure that the subjects are incentive compatible with the experiment and that they are serious about the bidding, subjects were told that some of them would be chosen to receive a cash award. For those chosen, one out of the twenty rounds would be randomly selected to determine the cash award as follows: The difference between what the student could sell the package for ($1160) and the price the student paid for it during that round (his/her bid price if he/she participated in the auction and won, $1100 otherwise), divided by 10. The reward thus ranged from $6 to $106.

At each round, subjects will be asked to make following decision:

1. participate or not in the reverse auction site;
2. if he/she decides to participate, what is his/her bid, and what is his/her expected excitement if the bid is accepted, or expected frustration if the bid is rejected.

At the end of the experiment, each subject was given a hypothetical bidding scenario for which his excitement (on a scale from 1 to 10) was measured and he/she was asked to fill the blank "I am really excited and had a lot of fun doing it, it's almost as if you had paid me $_______ to do this". Similarly, frustration and its monetary equivalence are measured. These relationships are used to convert measurements of excitement and frustration into monetary equivalences in the 20 rounds.

4.2. Experimental results

While the subjects presumably were motivated properly for this experiment, it is likely that some students would not have participated in such auctions, if they were in the real world. To parse out these effects, we have deleted those subjects who, according to the theoretical prediction (Proposition 1) would not have participated in reverse auction. To do this, we calculated the average X for each individual in 10 rounds (in either Type 1 site or Type 2 site) and kept only those whose average X were above the threshold X'. As a result, we were left with 69 subjects for the Type 1 site and 23 subjects for the Type 2 site. All analyses described below are based on these subjects.

To test Proposition 1, we first calculated the theoretical optimal bid for each subject in each round (based on equation (16)). We then calculated the correlation between actual bids and the optimal bids for each subject over the 10 rounds he/she participated in each reverse auction site. The distribution of the correlations across subjects is graphically represented in Figure 2.

Insert Figure 2 here

The average correlation across subjects is 0.3064. This provides partial support for Proposition 1 concerning the optimal bidding price. The test of who will bid and who will not is inappropriate in this case since, as discussed above, all subjects were likely to bid even though they would not have bid in the real world.

Proposition 2 characterizes the concavity and convexity of bidding prices under different conditions. In order to test this proposition, we have chosen a general quadratic function:

$$P_o = \gamma_0 + \gamma_1 X + \gamma_2 X^2 \qquad \text{---(19)}$$

We test concavity/convexity by checking the empirical sign of γ_2 via regression. Proposition 2 also predicts the behavior of the first order derivative. To address this, we have chosen to test the simple correlation between the bid and X. This empirical testing was also used for Lemma2. The procedure is thus as follows: 1) calculate the average bid and average X for each subject over the 10 rounds he/she participated in each site. 2) regress (or test the correlation) the average bid price against the average X across all subjects in a site. The results are summarized in Table 2.

Insert Table 2 here

As can be seen from the Table, bid is concave and increases over X in Type 1 site. The prediction for Type 2 sites is, in general, also supported, although the coefficient γ_2 is close to 0 (indicating a monotonic relationship). The relationship between the average X and the average bid are presented in Figure 3.

Insert Figures 3a and 3b here

Since there are too many data points (for instance, 69 pairs for the Type 1 site), Figure 3a and 3b are condensed version of the data. For Type 1 site, we assigned individuals into groups of 10 based on their X (subjects with lowest 10 Xs are in group 1, subjects with the next lowest 10 Xs are in group 2, and so on), and then calculated group average X and bid. For type 2 sites, the size of the group was 5 instead of 10 since the total number of data points is smaller (23).

One very important result of the paper is that we predict subjects will either underbid or overbid in Type 1 or 2 sites, respectively, compared to classic (without transaction utility) theoretical predictions. To test this result, we calculated the average bid over all subjects submitted in each round, and the results are plotted in Figure 4.

Insert Figures 4a and 4b here

As can be seen in Type 1 site, the average bids in all 10 round are below the classic prediction of $600 (Figure 4a). These results thus provide support for including the transaction utility for making prediction. For Type 2 site (Figure 4b), the average bids in 7, out of 10, rounds are above the classic prediction of $950, providing some (albeit weaker) support for the prediction.

Previous tests have focused on cross-individual bidding behavior. Our Lemma 2 focuses on dynamic bidding behavior (within subjects). To test this, we employed the same empirical procedures used for testing Proposition 2 (for concavity/convexity, increase/decrease). In this case, we performed one analysis for each subject, using his/her bids in the 10 rounds against his/her corresponding Xs. The results are summarized in Table 3.

Insert Table 3 here

For Type 1 site, the empirical evidence is quite strong. 74% of subjects have concave relationship (bid over X), and 92% have positive relationship (bid over X)(see Table 3a). On the other hand, we could not find support for subjects bidding in Type 2 site (Table 3b). To illustrate the results, we have chosen one subject and plotted his bid over X for both sites (see Figure 5)

Insert Figures 5a and 5b here

Finally, we tested the prediction implied by Proposition 3. The evidence for Type 1 site is very strong. It is quite clear that subjects increase their bids if X becomes larger, decrease their bids if X becomes smaller, and maintain the same bid if X does not change (see Table 4a).

Insert Table 4 here

As predicted, subjects do not necessarily increase or decrease their X after a win (or lose, for that matter). It can be seen, however, that subjects are unlikely to maintain the same X (thus the same bid price) after either win or lose in Type 1 site. The results for Type 2 site are not conclusive. Especially, when subjects have lost in the previous round, they behave as if they were in Type 1 site, contrary to the prediction.

5. SUMMARY AND DISCUSSION

In this paper, we have developed and tested empirically an analytical model with both economic and behavioral constructs to capture the bidding behavior in reverse auction sites. We have generated insights concerning both new (or potential) customers as well as existing customers. Equally important, we have shown that there are at least two ways to increase a reverse auction site's profit, one through changing the perception of the site, the other is through optimal subsidy strategy at the individual level.

There are some aspects of the paper that might be worth additional research effort in the future. For instance, we have assumed in our model that a bidder is a myopic optimizer. She chooses a bidding strategy (price) that will maximize the expected payoff at that period. Although we believe it captures the actual decision making process of many people, it might be interesting, at least theoretically, to relax this assumption in future research.

As can be seen from the proof of Proposition 1, boundary-bidding strategies (bid at the best alternative price (R), or the lower bound of cost (C_l)) have been eliminated because the expected utilities for these two strategies in the model are negative. This, however, may change if we make slightly different assumptions. Allowing a base level excitement regardless of how much money is saved will allow the utility of bidding at R to potentially become positive. Similarly, if a customer expects the reverse auction site might subsidize her bid, the utility for bidding at C_l may also become positive. These are interesting variations in bidding strategies, which could be easily tested in our experiments (by checking whether any subjects bid at these boundaries).

APPENDIX: PROOFS OF PROPOSITIONS 1, 2, AND 4

Proposition 1.

Proof:

The 1st period problem is the same for any period. For simplicity, we will drop the subscript for period, t, in the following proof. The results hold for any period t.

The bidder's problem could be examined by analyzing the first and second order conditions of equation (10). Constraint 1 (boundary constraint) and 2 (expected utility) are essentially sanity checks (a bidder should expect positive utility if any bid submitted, and she should not bid more than the price she will pay through an alternative channel, nor below the minimum cost of the site). We choose, for simplicity and clarity, not to use the Lagrangean method and instead, compare the results of unconstrained optimization with the constraints in the end.

Step 1. Unconstrained optimization.

First order condition:

$$\frac{\partial \mathbf{E}U}{\partial P_o} = \frac{2(\beta - \alpha - \theta P_r)P_o + (\theta P_r + \alpha)(P_m + C_l) - \beta C_u}{P_r(C_u - C_l)} \quad \text{---(A1)}$$

Let $$P_o^{'} = \frac{\beta C_u - (\theta P_r + \alpha)(P_m + C_l)}{2(\beta - \alpha - \theta P_r)} \quad \text{---(A2)}$$

If $\beta < \alpha + \theta P_r$, then

$$\frac{\partial \mathbf{E}U}{\partial P_o} > 0 \quad \text{if} \quad P_o < P_o^{'} \quad \text{---(A3)}$$

$$\frac{\partial \mathbf{E}U}{\partial P_o} < 0 \quad \text{if} \quad P_o > P_o^{'} \quad \text{---(A4)}$$

$$\frac{\partial \mathbf{E}U}{\partial P_o} = 0 \quad \text{if} \quad P_o = P_o^{'} \quad \text{---(A5)}$$

If $\beta > \alpha + \theta P_r$, then

$$\frac{\partial \mathbf{E}U}{\partial P_o} > 0 \quad \text{if} \quad P_o > P_o^{'} \qquad \text{---(A6)}$$

$$\frac{\partial \mathbf{E}U}{\partial P_o} < 0 \quad \text{if} \quad P_o < P_o^{'} \qquad \text{---(A7)}$$

$$\frac{\partial \mathbf{E}U}{\partial P_o} = 0 \quad \text{if} \quad P_o = P_o^{'} \qquad \text{---(A8)}$$

If $\beta = \alpha + \theta P_r$, then

$$\frac{\partial \mathbf{E}U}{\partial P_o} > 0 \quad \text{if} \quad P_m + C_l > C_u \qquad \text{---(A9)}$$

$$\frac{\partial \mathbf{E}U}{\partial P_o} < 0 \quad \text{if} \quad P_m + C_l < C_u \qquad \text{---(A10)}$$

$$\frac{\partial \mathbf{E}U}{\partial P_o} = 0 \quad \text{if} \quad P_m + C_l = C_u \qquad \text{---(A11)}$$

Second order condition:

$$\frac{\partial^2 \mathbf{E}U}{\partial P_o^{\,2}} = \frac{2(\beta - \alpha - \theta P_r)}{P_r(C_u - C_l)} \qquad \text{---(A12)}$$

Thus,

$$\frac{\partial^2 \mathbf{E}U}{\partial P_o^{\,2}} < 0 \quad (\mathbf{E}\text{U is concave}) \quad \text{if} \quad \beta < \alpha + \theta P_r \qquad \text{---(A13)}$$

$$\frac{\partial^2 \mathbf{E}U}{\partial P_o^{\,2}} > 0 \quad (\mathbf{E}\text{U is convex}) \quad \text{if} \quad \beta > \alpha + \theta P_r \qquad \text{---(A14)}$$

$$\frac{\partial^2 \mathbf{E}U}{\partial P_o^{\,2}} = 0 \quad (\mathbf{E}\text{U is monotomous}) \quad \text{if} \quad \beta = \alpha + \theta P_r \qquad \text{---(A15)}$$

The results of the above-unconstrained optimization are summarized in the following table with the constraints.

Condition from Unconstrained Optimization	Constraint #1	Constraint #2	Optimal Bidding Strategy Under Constraints
$\beta < \alpha + \theta P_r$	$C_l \le P_o' \le P_m$	$EU(P_o') > 0$	P_o'
		$EU(P_o') \le 0$	0
	$P_o' < C_l$	$EU(C_l) > 0$	C_l
		$EU(C_l) \le 0$	0
	$P_o' > P_m$	$EU(P_m) > 0$	P_m
		$EU(P_m) \le 0$	0
$\beta = \alpha + \theta P_r$	$P_m + C_l > C_u$	$EU(P_m) > 0$	P_m
		$EU(P_m) \le 0$	0
	$P_m + C_l = C_u$	$EU(C_l) > 0$	any price between C_l and P_m
		$EU(C_l) \le 0$	0
	$P_m + C_l < C_u$	$EU(C_l) > 0$	C_l
		$EU(C_l) \le 0$	0
$\beta > \alpha + \theta P_r$	$C_l \le P_o' \le P_m$	$EU(P_m) \ge EU(C_l)$ and $EU(P_m) > 0$	P_m
		$EU(P_m) \ge EU(C_l)$ and $EU(P_m) \le 0$	0
		$EU(P_m) < EU(C_l)$ and $EU(C_l) > 0$	C_l
		$EU(P_m) < EU(C_l)$ and $EU(C_l) < 0$	0
	$P_o' < C_l$	$EU(P_m) > 0$	P_m
		$EU(P_m) \le 0$	0
	$P_o' > P_m$	$EU(C_l) > 0$	C_l
		$EU(C_l) \le 0$	0

Step 2. Examining optimal bidding strategies under two constraints.

This table could be greatly simplified because the utilities (constraint 2) at the boundaries (constraint 1) are always negative under current model assumption (see section 5 for further discussion).

$$EU(C_l) = -\beta \frac{C_l}{P_r} < 0 \qquad \text{---(A16)}$$

$$EU(P_m) = -\beta \frac{P_m}{P_r} \frac{C_u - P_m}{C_u - C_l} < 0 \qquad \text{---(A17)}$$

Thus,

Condition from Unconstrained Optimization	Constraint #1	Constraint #2	Optimal Bidding Strategy Under Constraints
$\beta < \alpha + \theta P_r$	$C_l \le P_o' \le P_m$	$EU(P_o') > 0$	P_o'
		$EU(P_o') \le 0$	0
	$P_o' < C_l$		0
	$P_o' > P_m$		0
$\beta = \alpha + \theta P_r$			0
$\beta > \alpha + \theta P_r$			0

Before we further analyze the constraints, let's first combine the variables and let

$$X = \frac{\alpha + \theta P_r}{\beta} \qquad \text{---(A18)}$$

and thus $$P_o' = \frac{C_u - (P_m + C_l)X}{2(1-X)} \qquad \text{---(A19)}$$

Let's first examine constraint #1 when $\beta < \alpha + \theta P_r$, i.e., $X > 1$,

$$P_o' < C_l \Leftrightarrow \frac{C_u - (P_m + C_l)X}{2(1-X)} < C_l \Leftrightarrow X < 1 - \frac{P_m - C_u + C_l}{P_m - C_l} \qquad \text{---(A20)}$$

which is only possible when $C_u > P_m + C_l$.

$$P_o' > P_m \Leftrightarrow \frac{C_u - (P_m + C_l)X}{2(1-X)} > P_m \Leftrightarrow X < 1 + \frac{P_m - C_u + C_l}{P_m - C_l} \qquad \text{---(A21)}$$

similarly, this is only possible when $C_u < P_m + C_l$.

Now we will need to examine constraint #2.

$EU(P_o')$

$= \frac{\beta}{(C_u - C_l)P_r}$

$\left\{X\left[P_m - \frac{C_u - (P_m + C_l)X}{2(1-X)}\right]\left[\frac{C_u - (P_m + C_l)X}{2(1-X)} - C_l\right] - \frac{C_u - (P_m + C_l)X}{2(1-X)}\left[C_u - \frac{C_u - (P_m + C_l)X}{2(1-X)}\right]\right\}$

$= \frac{\beta}{4(C_u - C_l)P_r(X-1)}\left\{(P_m - C_l)^2 X^2 + [4P_m C_l - 2C_u(P_m + C_l)]X + C_u{}^2\right\}$

---(A22)

Let

$$Y = (P_m - C_l)^2 X^2 + [4P_m C_l - 2C_u(P_m + C_l)]X + C_u{}^2 \quad \text{---(A23)}$$

Since $X > 1$,

$$Y > 0 \Leftrightarrow EU(P_o') > 0 \quad \text{---(A24)}$$

Solve Y=0, we could obtain:

$$X' = \frac{C_u(P_m + C_l) - 2P_m C_l + 2\sqrt{P_m C_l (C_u - P_m)(C_u - C_l)}}{(P_m - C_l)^2} \quad \text{---(A25)}$$

$$X'' = \frac{C_u(P_m + C_l) - 2P_m C_l - 2\sqrt{P_m C_l (C_u - P_m)(C_u - C_l)}}{(P_m - C_l)^2} \quad \text{---(A26)}$$

Alternatively, they could be expressed as:

$$X' = \left(\frac{\sqrt{P_m(C_u - C_l)} + \sqrt{C_l(C_u - P_m)}}{P_m - C_l}\right)^2 \quad \text{---(A27)}$$

$$X'' = \left(\frac{\sqrt{P_m(C_u - C_l)} - \sqrt{C_l(C_u - P_m)}}{P_m - C_l}\right)^2 \quad \text{---(A28)}$$

It is straightforward to show that Y is convex over X.

The results from equations (A20)-(A25) are incorporated into the table below:

Condition from Unconstrained Optimization	Constraint #1	Constraint #2	Optimal Bidding Strategy Under Constraints
$X > 1$	$X > 1 + \frac{P_m - C_u + C_l}{P_m - C_l}$ **and** $X > 1 - \frac{P_m - C_u + C_l}{P_m - C_l}$	$X > X'$ *or* $X < X''$	P_o'
		$X' \geq X \geq X''$	0
	$X < 1 - \frac{P_m - C_u + C_l}{P_m - C_l}$		0
	$X < 1 + \frac{P_m - C_u + C_l}{P_m - C_l}$		0
$X = 1$			0
$X < 1$			0

The last step is to reconcile the conditions as captured in the first three columns in the above table.

Since

$$\begin{aligned} X''-1 &= \left(\frac{\sqrt{P_m(C_u - C_l)} - \sqrt{C_l(C_u - P_m)}}{P_m - C_l}\right)^2 - 1 \\ &= \frac{\left(\sqrt{P_m(C_u - C_l)} - \sqrt{C_l(C_u - P_m)}\right)^2 - \left(P_m - C_l\right)^2}{\left(P_m - C_l\right)^2} \\ &= \frac{C_u\left(P_m + C_l\right) - P_m^{\,2} - C_l^{\,2} - 2\sqrt{C_l P_m (C_u - C_l)(C_u - P_m)}}{\left(P_m - C_l\right)^2} \\ &= \left(\frac{\sqrt{P_m(C_u - P_m)} - \sqrt{C_l(C_u - C_l)}}{P_m - C_l}\right)^2 \geq 0 \end{aligned} \qquad \text{---(A29)}$$

So, $X' > X'' \geq 1$ ---(A30)

We could also rearrange the terms of constraint #1:

$$1 + \frac{P_m - C_u + C_l}{P_m - C_l} = \frac{2P_m - C_u}{P_m - C_1} = \frac{C_u(P_m + C_l) - 2P_m C_l - 2P_m(C_u - P_m)}{\left(P_m - C_1\right)^2} \qquad \text{---(A31)}$$

$$1 - \frac{P_m - C_u + C_l}{P_m - C_l} = \frac{C_u - 2C_l}{P_m - C_1} = \frac{C_u(P_m + C_l) - 2P_m C_l - 2C_l(C_u - C_l)}{\left(P_m - C_1\right)^2} \qquad \text{---(A32)}$$

By comparing to equation (A24), it is clear that

$$X' > 1 + \frac{P_m - C_u + C_l}{P_m - C_l} \qquad \text{---(A33)}$$

$$X' > 1 - \frac{P_m - C_u + C_l}{P_m - C_l} \qquad \text{---(A34)}$$

Further analysis is divided into two scenarios:

1). If $C_u \geq P_m + C_l$,

Then $1 - \frac{P_m - C_u + C_l}{P_m - C_l} > 1 > 1 + \frac{P_m - C_u + C_l}{P_m - C_l}$ ---(A35)

and

$$\begin{aligned}
&1 - \frac{P_m - C_u + C_l}{P_m - C_l} - X'' \\
&= \frac{C_u(P_m + C_l) - 2P_m C_l - 2C_l(C_u - C_l)}{(P_m - C_1)^2} - \frac{C_u(P_m + C_l) - 2P_m C_l - 2\sqrt{P_m C_l (C_u - P_m)(C_u - C_l)}}{(P_m - C_l)^2} \\
&= \frac{-2C_l(C_u - C_l) + 2\sqrt{P_m C_l (C_u - P_m)(C_u - C_l)}}{(P_m - C_l)^2} \\
&= \frac{2\sqrt{C_l(C_u - C_l)}\left(\sqrt{P_m(C_u - P_m)} - \sqrt{C_l(C_u - C_l)}\right)}{(P_m - C_l)^2}
\end{aligned}$$

---(A36)

Since $P_m(C_u - P_m) - C_l(C_u - C_l) = (P_m - C_l)(-P_m - C_l + C_u) \geq 0$

Thus $1 - \frac{P_m - C_u + C_l}{P_m - C_l} \geq X''$ ---(A37)

Combining all the results, the inequality could be summarized as following:

$$X' > 1 - \frac{P_m - C_u + C_l}{P_m - C_l} \geq X'' \geq 1 > 1 + \frac{P_m - C_u + C_l}{P_m - C_l} \qquad \text{---(A38)}$$

2). If $C_u < P_m + C_l$,

Then $1 + \frac{P_m - C_u + C_l}{P_m - C_l} > 1 > 1 - \frac{P_m - C_u + C_l}{P_m - C_l}$ ---(A39)

and

$$
\begin{aligned}
&1+\frac{P_m-C_u+C_l}{P_m-C_l}-X'' \\
&=\frac{C_u(P_m+C_l)-2P_mC_l-2P_m(C_u-P_m)}{\left(P_m-C_1\right)^2}-\frac{C_u(P_m+C_l)-2P_mC_l-2\sqrt{P_mC_l(C_u-P_m)(C_u-C_l)}}{(P_m-C_l)^2} \\
&=\frac{-2P_m(C_u-P_m)+2\sqrt{P_mC_l(C_u-P_m)(C_u-C_l)}}{(P_m-C_l)^2} \\
&=\frac{2\sqrt{P_m(C_u-P_m)}\left(\sqrt{C_l(C_u-C_l)}-\sqrt{P_m(C_u-P_m)}\right)}{(P_m-C_l)^2}
\end{aligned}
$$

---(A40)

Since $C_l(C_u-C_l)-P_m(C_u-P_m)=(P_m-C_l)(P_m+C_l-C_u)>0$

Thus $$1+\frac{P_m-C_u+C_l}{P_m-C_l}\geq X''$$ ---(A41)

Combining all the results, the inequality could be summarized as following:

$$1-\frac{P_m-C_u+C_l}{P_m-C_l}<1\leq X''<1+\frac{P_m-C_u+C_l}{P_m-C_l}<X'$$ ---(A42)

As the result, the optimal strategy, P_o^*, for a bidder could be summarized as following:

If	Optimal Bidding Strategy (Price, P_o^*)
$X\leq X'=\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)^2$	0
$X>X'=\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)^2$	$\frac{C_u-(P_m+C_l)X}{2(1-X)}$

The nature of the threshold X' could be analyzed as following:

$$
\begin{aligned}
\frac{\partial X'}{\partial C_u}&=\partial\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)^2\Big/\partial C_u \\
&=2\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)\left(\frac{\frac{P_m}{2\sqrt{P_m(C_u-C_l)}}+\frac{C_l}{2\sqrt{C_l(C_u-P_m)}}}{P_m-C_l}\right)>0
\end{aligned}
$$ ---(A43)

$$\frac{\partial X'}{\partial C_l} = \partial\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)^2 \Big/ \partial C_l$$

$$= 2\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)\left(\frac{\left(\frac{-P_m}{2\sqrt{P_m(C_u-C_l)}}+\frac{C_u-P_m}{2\sqrt{C_l(C_u-P_m)}}\right)(P_m-C_l)+\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{(P_m-C_l)^2}\right)$$

$$= 2\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)\left(\frac{\left(\frac{-P_m(P_m-C_l)}{2\sqrt{P_m(C_u-C_l)}}\right)+\sqrt{P_m(C_u-C_l)}+\left(\frac{(C_u-P_m)(P_m-C_l)}{2\sqrt{C_l(C_u-P_m)}}\right)+\sqrt{C_l(C_u-P_m)}}{(P_m-C_l)^2}\right)$$

$$= 2\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)\left(\frac{\left(\frac{-P_m(P_m-C_l)+2P_m(C_u-C_l)}{2\sqrt{P_m(C_u-C_l)}}\right)+\left(\frac{(C_u-P_m)(P_m-C_l)}{2\sqrt{C_l(C_u-P_m)}}\right)+\sqrt{C_l(C_u-P_m)}}{(P_m-C_l)^2}\right)$$

$$> 0$$

---(A44)

$$\frac{\partial X'}{\partial P_m} = \partial\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)^2 \Big/ \partial P_m$$

$$= 2\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)\left(\frac{\left(\frac{C_u}{2\sqrt{P_m(C_u-C_l)}}+\frac{-C_l}{2\sqrt{C_l(C_u-P_m)}}\right)(P_m-C_l)-\sqrt{P_m(C_u-C_l)}-\sqrt{C_l(C_u-P_m)}}{(P_m-C_l)^2}\right)$$

$$= 2\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)\left(\frac{\left(\frac{C_u(P_m-C_l)}{2\sqrt{P_m(C_u-C_l)}}\right)-\sqrt{P_m(C_u-C_l)}+\left(\frac{-C_l(P_m-C_l)}{2\sqrt{C_l(C_u-P_m)}}\right)-\sqrt{C_l(C_u-P_m)}}{(P_m-C_l)^2}\right)$$

$$= 2\left(\frac{\sqrt{P_m(C_u-C_l)}+\sqrt{C_l(C_u-P_m)}}{P_m-C_l}\right)\left(\frac{\left(\frac{C_u(P_m-C_l)-2P_m(C_u-C_l)}{2\sqrt{P_m(C_u-C_l)}}\right)-\left(\frac{C_l(P_m-C_l)}{2\sqrt{C_l(C_u-P_m)}}\right)-\sqrt{C_l(C_u-P_m)}}{(P_m-C_l)^2}\right)$$

$$< 0$$

---(A45)

QED.

Proposition 2.

Proof:

For $P_o^* = \frac{C_u-(P_m+C_l)X}{2(1-X)}$

FOC:

$$\frac{\partial P_o^*}{\partial X} = \frac{-2(P_m + C_l)(1-X) - \left[C_u - (P_m + C_l)X\right](-2)}{4(1-X)^2}$$

$$= \frac{C_u - (P_m + C_l)}{2(1-X)^2} \begin{cases} > 0 & if \quad C_u > P_m + C_l \\ = 0 & if \quad C_u = P_m + C_l \\ < 0 & if \quad C_u < P_m + C_l \end{cases} \qquad \text{---(A46)}$$

SOC:

$$\frac{\partial^2 P_o^*}{\partial X^2} = \frac{C_u - (P_m + C_l)}{(1-X)^3} \begin{cases} < 0 & if \quad C_u > P_m + C_l \\ = 0 & if \quad C_u = P_m + C_l \\ > 0 & if \quad C_u < P_m + C_l \end{cases} \qquad \text{---(A47)}$$

It is straightforward to show that

$$P_o^* = \frac{P_m + C_l}{2} \text{ when } \quad X \to \infty \text{ or } C_u = P_m + C_l \qquad \text{---(A48)}$$

QED.

Proposition 4.

Proof:

$$\frac{\partial P_o^*}{\partial C_u} = \frac{1}{2(1-X)} < 0 \qquad \text{---(A49)}$$

$$\frac{\partial P_o^*}{\partial C_l} = \frac{-X}{2(1-X)} > 0 \qquad \text{---(A50)}$$

QED.

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Table 1. Experimental Design, Parameters for the Two Auction Sites

	P_r	P_m	C_u	C_l
Site 1 (Type 1)	$1160	$1100	$1250	$100
Site 2 (Type 2)	$1160	$1100	$1850	$800

Table 2. Testing Relationships between Bid and X (across subject)

	Type 1 Site	Type 2 Site
Correlation Between Bid and X*	0.1122	-0.0127
Coefficient for X^2 in Quadratic Fitting (γ_2)**	-0.0001	0.0000

* Positive correlation indicates subject with higher X will bid higher

** Negative coefficient for X^2 indicates bid is a concave function of X

Table 3a. Testing Relationships between Bid and X (within subject), Type 1

	Positive (>0)	Negative (<0)
Correlation Between Bid and X*	92% (57)	8% (5)
Coefficient for X^2 in Quadratic Fitting (γ_2) **	26% (16)	74% (46)

* Positive correlation indicates subject will increase bid as his X increases
** Negative coefficient for X^2 indicates bid is a concave function of X
Note: There are 7 subjects who are excluded from this analysis due to limited bids or inability to calculate X

Table 3b. Testing Relationships between Bid and X (within subject), Type 2

	Positive	Negative
Correlation Between Bid and X*	83% (15)	17% (3)
Coefficient for X2 in Quadratic Fitting (γ_2) **	39% (7)	61% (11)

* Positive correlation indicates subject will increase bid as his X increases
** Negative coefficient for X2 indicates bid is a concave function of X
Note: There are 5 subjects who are excluded from this analysis due to limited bids or inability to calculate X

Table 4a. Dynamic Bidding Behavior in Type 1 Site

A. Change of X and bid in round n+1 if bid was accepted in round n:

	Increase Bid	Same Bid	Decrease Bid	Total
X Increases	51	9	27	87
X Same	1	18	1	20
X Decreases	9	4	91	104
Total	61	31	119	211

B. Change of X and bid in round n+1 if bid was rejected in round n:

	Increase Bid	Same Bid	Decrease Bid	Total
X Increases	161	17	9	187
X Same	5	49	0	54
X Decreases	40	20	109	169
Total	206	86	118	410

Table 4b. Dynamic Bidding Behavior in Type 2 Site

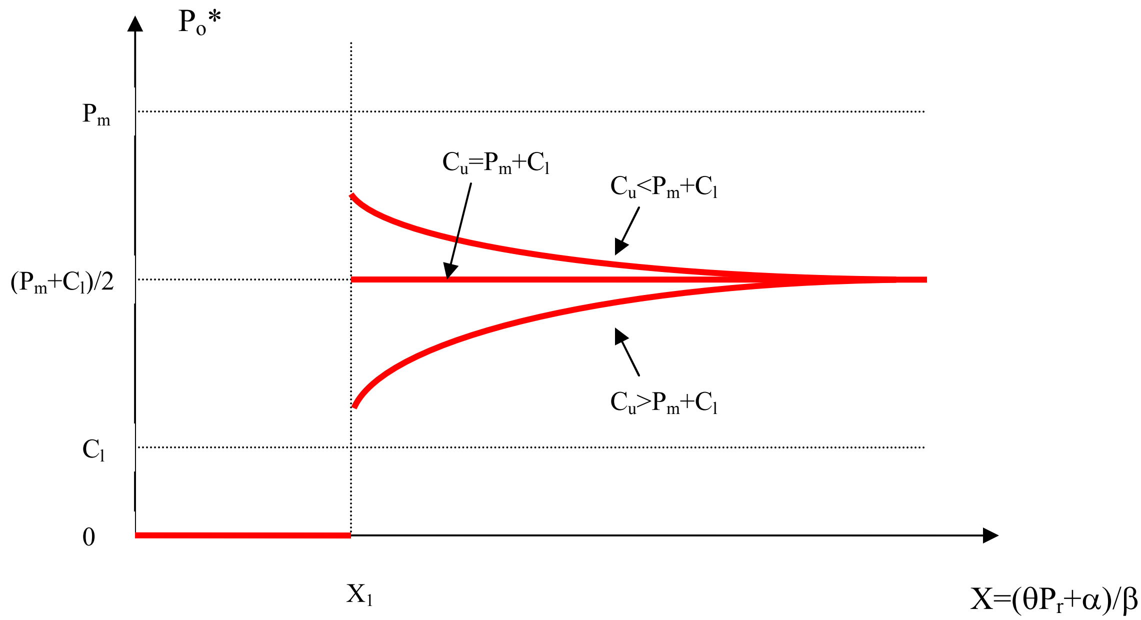
A. Change of X and bid in round n+1 if bid was accepted in round n:

	Increase Bid	Same Bid	Decrease Bid	Total
X Increases	4	3	3	10
X Same	0	6	0	6
X Decreases	1	1	7	9
Total	5	10	10	25

B. Change of X and bid in round n+1 if bid was rejected in round n:

	Increase Bid	Same Bid	Decrease Bid	Total
X Increases	40	9	9	58
X Same	1	57	0	58
X Decreases	19	11	36	66
Total	60	77	45	182

Figure 1. Optimal Bidding Strategy (Price)
As a Function of a Bidder's Individual Characteristics



The thicker line represents the optimal bidding strategies for different bidders under different conditions.

Figure 2. Correlation Between Actual Bids and Predicted Bids (within subjects)
Average correlation: 0.3064

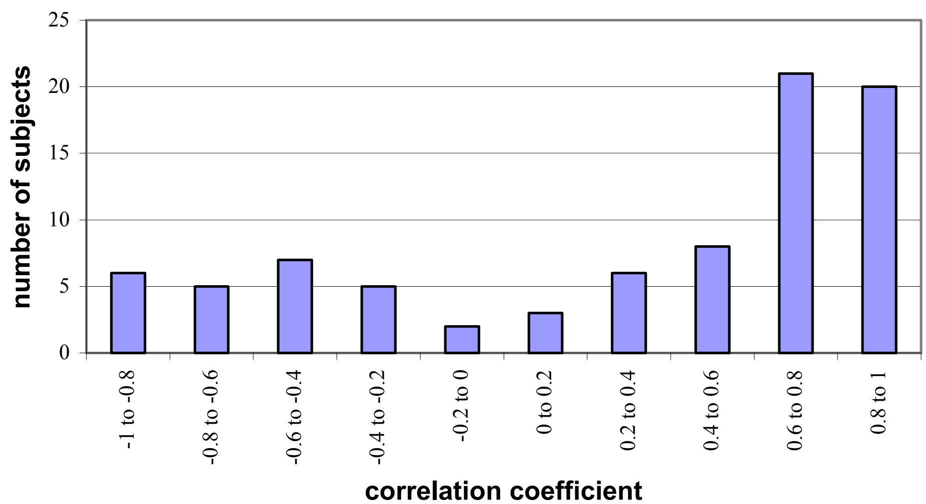


Figure 3a. Characterizing the bid as a function of X (across subjects)
Type 1 Site

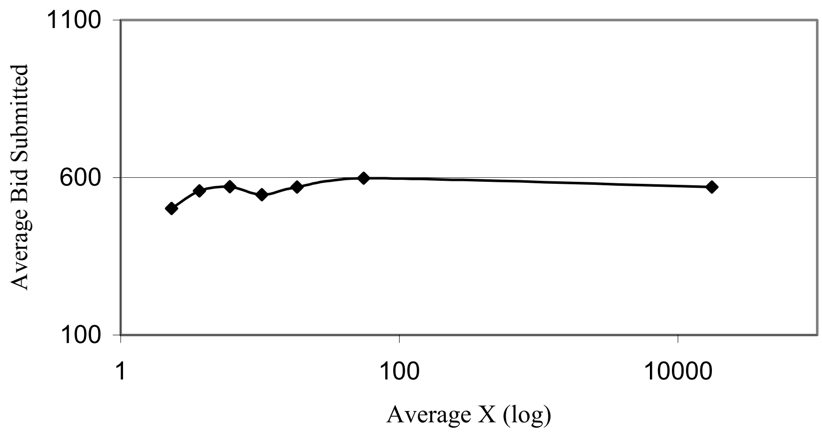


Figure 3b. Characterizing the bid as a function of X (across subjects)
Type 2 Sites

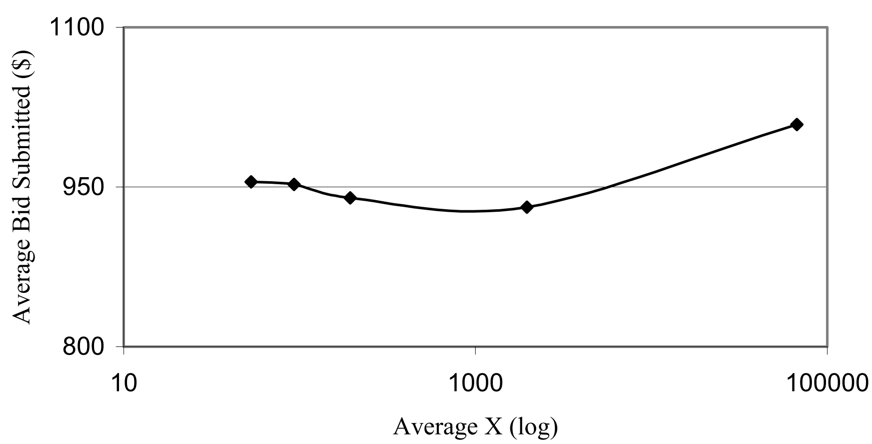


Figure 4a. Average Bids Submitted in Each Auction Round
Type 1 Site

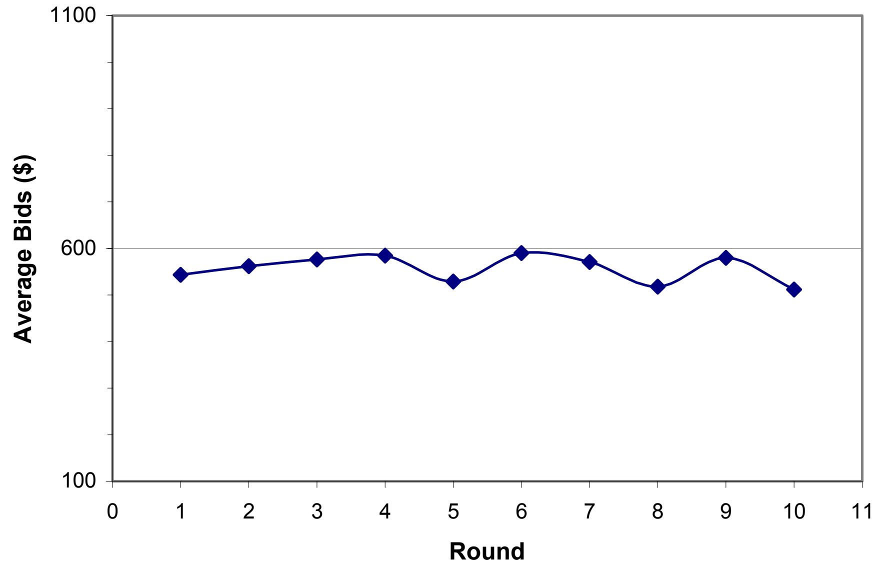


Figure 4b. Average Bids Submitted in Each Auction Round
Type 2 Sites

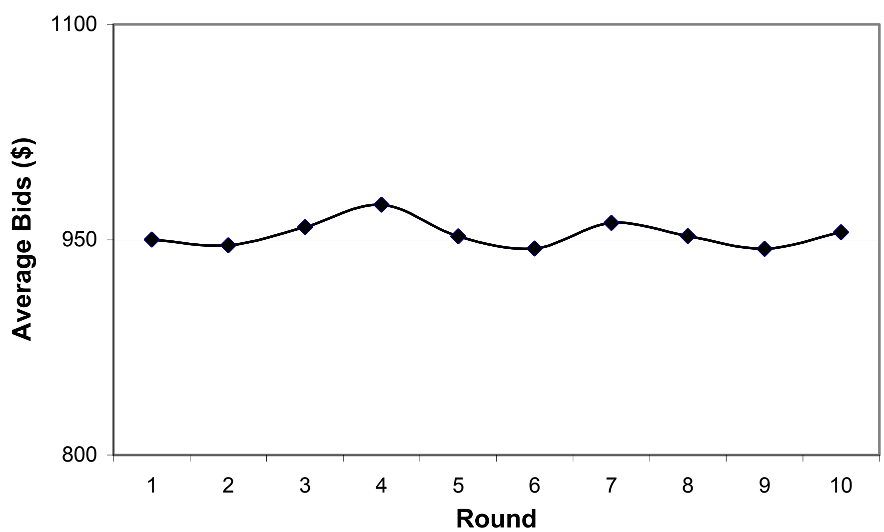


Figure 5a. A Graphic Representation of Bid as a Function of X for One Subject in Type 1 Site

Subject: gregk, Coefficient: -0.07597, Correlation: 0.9763

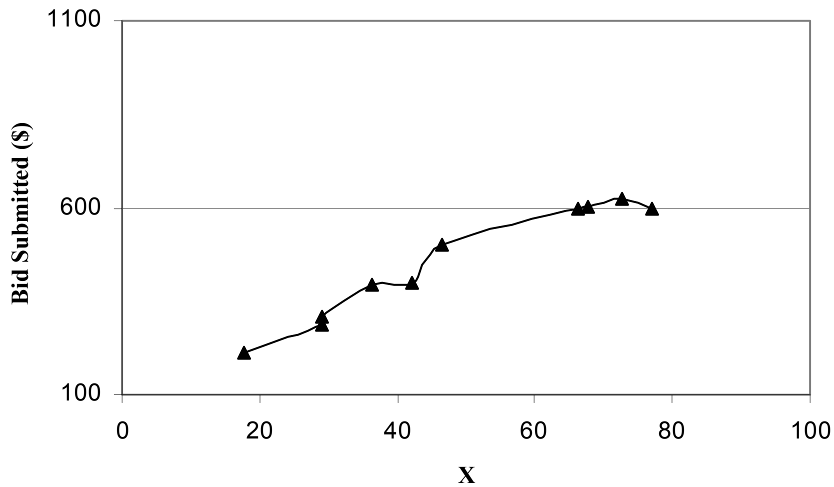


Figure 5b. A Graphic Representation of Bid as a Function of X for One Subject in Type 2 Site

Subject: gregk, Coefficient: 0.0004, Correlation: 0.6334