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Resolving Informational Inefficiencies In Bilateral Trade

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Abstract
Asymmetric information can impede socially efficient trade in bilateral transactions. This dissertation consists of three essays that address how informational inefficiencies can be resolved in three different settings. These approaches to resolve the informational inefficiencies can help explain many characteristics of real estate markets, mergers and acquisitions, OTC markets, etc.

The first chapter considers the two-person bargaining problem under asymmetric information as in Myerson and Satterthwaite (1983). The famous Myerson and Satterthwaite's theorem shows that direct bilateral bargaining is generally inefficient. We show that an informed broker, acting as an intermediary between a buyer and seller, can achieve efficient trade. We provide a sufficient condition on the broker's information to achieve an efficient outcome. In the broker's optimal mechanism, while a more informed broker extracts a higher surplus, trade is still more efficient; consequently, the buyer and seller can be better off trading with a more informed broker.

In the second chapter, co-authored with Vincent Glode and Christian Opp, we analyze optimal voluntary disclosure by a privately informed buyer who faces a seller endowed with market power in a bilateral transaction. While disclosures reduce the buyer's informational advantage, they may increase his ex ante information rents by mitigating the seller's incentives to resort to inefficient screening. We show that when disclosures are restricted to be ex post verifiable, the buyer always finds it optimal to design a partial disclosure plan that implements socially efficient trade in equilibrium.

In the third chapter, co-authored with Vincent Glode and Christian Opp, we consider the limiting result of Glode and Opp (2016). We show that, generically, if efficient trade can be implemented by an incentive-compatible mechanism in direct bilateral trading, it can also be achieved in a sequential trading game with a sufficiently long chain of heterogeneously informed intermediaries.

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RESOLVING INFORMATIONAL INEFFICIENCIES IN BILATERAL TRADE

To my wife and my son, Fan and Kevin

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ABSTRACT

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George Mailath

Asymmetric information can impede socially efficient trade in bilateral transactions. This dissertation consists of three essays that address how informational inefficiencies can be resolved in three different settings. These approaches to resolve the informational inefficiencies can help explain many characteristics of real estate markets, mergers and acquisitions, OTC markets, etc.

The first chapter considers the two-person bargaining problem under asymmetric information as in Myerson and Satterthwaite (1983). The famous Myerson and Satterthwaite's theorem shows that direct bilateral bargaining is generally inefficient. We show that an informed broker, acting as an intermediary between a buyer and seller, can achieve efficient trade. We provide a sufficient condition on the broker's information to achieve an efficient outcome. In the broker's optimal mechanism, while a more informed broker extracts a higher surplus, trade is still more efficient; consequently, the buyer and seller can be better off trading with a more informed broker.

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1.1. Introduction

Asymmetric information can lead to market failure. In particular, the seminal work of Myerson and Satterthwaite (1983) shows that regardless of the bargaining game between a buyer and a seller, there is always a positive probability of inefficient outcomes under very mild conditions. However, real world negotiations often occur through third-party "brokers". Examples include realtors in real estate transactions, investment banks in mergers and acquisitions, brokers in bond markets, etc. Moreover, brokers are likely to be informed about their market. A realtor, experienced in the real estate industry, is well informed about the intrinsic value of a property, which is likely correlated with the seller's and buyer's private valuation. A financial intermediary is informed about the supply and demand of financial assets, and therefore knows more about the reservation value of the seller and buyer.

In this paper, I study a problem in which a broker acts as an intermediary in the bargaining between a buyer and a seller in the presence of private information. I assume the broker is informed, in the sense that he observes imperfect signals about the buyer's and seller's valuations. My main result establishes that bargaining through an informed broker can improve welfare by alleviating inefficiencies caused by asymmetric information. I first examine the possibility of efficient trade. If the broker is uninformed, the problem reduces to Myerson and Satterthwaite (1983): Efficient trade is impossible. At the other extreme, if the broker is perfectly informed, an efficient outcome can be achieved.[1] I characterize a sufficient condition on what in-

[1] In this case, the broker can buy from the seller at her cost and resell to the buyer at his valuation. Trade is efficient, and the broker receives all of the surplus.

formation the broker must have for efficient trade to be implemented. Importantly, full efficiency can be achieved with a broker who has imperfect information.

The second part of this paper concerns optimal mechanisms when the broker's information is characterized by a partition structure.[2] Bargaining through a more informed broker can achieve higher expected gains from trade. If the broker chooses a mechanism, a more informed broker can extract more surplus, but the buyer and seller can also receive ex ante higher information rents because trade is more efficient. As a result, bargaining through an informed broker can Pareto dominate bargaining through an uninformed broker.

The general intuition that an informed broker can improve efficiency is as follows. The presence of private information implies that the buyer and seller may misreport their valuations. An uninformed mechanism designer needs to provide proper incentives for them to truthfully report. In Myerson and Satterthwaite (1983), the mechanism designer cannot distinguish a buyer who has a valuation v and reports $\hat{v}$ from another buyer who has a valuation v' but also reports $\hat{v}$. In contrast, consider bargaining through an informed broker. Since the broker's information depends on whether the buyer's valuation is v or v', he can treat the same reports from different types of buyers differently. The broker's correlated information can help detect misreporting behaviors, therefore making it easier to induce truthful revelation.

The idea of using correlated information in mechanism design was introduced by Crémer and McLean (1985). In an auction environment, they show that a seller can extract all of the surplus under certain conditions on the correlated information structure across buyers' valuations.[3] McAfee and Reny (1992) extend the result to

[2] We say that the broker's information is characterized by a partition structure, if his information partitions the types space into several subintervals.

[3] Crémer and McLean (1988) consider a general setting with correlated information.

the continuous type case, and show that a mechanism designer can have more power in a correlated information setting. There are two main differences in my paper. First, I show that an efficient outcome, rather than higher efficiency, can be achieved by a broker who has imperfect information. In my paper, I keep the assumption that the buyer's valuation and the seller's are independent. The key ingredient to achieve an efficient outcome is the correlated structure between the broker's information and the negotiators' valuations.[4] Second, in contrast to McAfee and Reny (1992), I show that full efficiency can be achieved in a broader set of information structures. For instance, if the broker's information is characterized by a partition structure, which fails to satisfy the McAfee and Reny's condition, the broker can use cross subsidization to facilitate efficient trade as I will show in Section 1.2.

Brokers often play multiple roles in facilitating the exchange of a good. For instance, in the sale of a house, the realtor helps in the search process by matching a seller and a buyer; at the same time, the realtor also intermediates negotiations between the two parties. A large literature has demonstrated that brokers can alleviate search frictions, therefore improving welfare (e.g., Rubinstein and Wolinsky (1987)). My paper does not consider the search channel, and focuses instead on how an informed broker can help in the bargaining. When the bargaining is mediated by a broker, one might expect trade efficiency to be reduced, because a broker's commission comes from the gains from trade and the broker's incentive may not be aligned with the negotiators'.[5] Despite these downsides, I show that involving an informed broker can

[4]McAfee and Reny (1992) show that an efficient mechanism can exist when the buyer's private valuation is correlated with the seller's. But the mechanism may require a large payment. In the case where there is an exogenous bound on the payment, it is unclear whether efficient trade can be sustained.

[5]Bazerman et al. (1992) is the first paper that distinguishes between neutral third parties and non-neutral third parties. A critical issue in the case of non-neutral third parties is that brokers act to maximize their own personal gain. Consequently, the incentive structure may affect the information transmitted in the negotiation process. Similarly, Yavas, Thomas, and Sirmans (2001) find that realtors may be harmful in negotiations. On the other hand, a neutral third party–for example, a

improve trade efficiency when direct bilateral bargaining cannot reach an efficient outcome.[6]

Relatedly, Glode and Opp (2016) study the efficiency of trade in a one-sided incomplete information setting in which an uninformed seller deals with a privately informed buyer. They show that introducing informed brokers in a sequential trading game can expand the set of distributions of the buyer's and seller's private valuations under which efficient trade is possible. In subsequent work, Glode, Opp, and Zhang (2016) extend the analysis by showing that generically, if efficient trade can be implemented by an incentive-compatible mechanism in direct bilateral trading, it can also be achieved in a sequential trading game with a sufficiently long chain of heterogeneously informed brokers. The major difference between my paper and their papers is that in showing that an efficient outcome can be achieved, I consider *all* mechanisms, rather than a specific game in which the asset owner makes a take-it-or-leave-it offer.[7] Therefore, my paper differs from their work in three key ways. First, in their setting the broker may end up with the asset, but in my setting he optimally never retains the asset. So my paper can address the applications such as how, without the possibility of holding the asset, a realtor improves trade efficiency between a buyer and a seller or how investment banks facilitate mergers and acquisitions. Second, the buyer knows the seller's cost in their bargaining environment, but in my setting he does not. I consider a two-sided asymmetric information setting and

mediator–tries to help the parties to see the negotiation from a more rational perspective (Neale and Bazerman, 1983). That is, the mediator can help the parties make decisions that maximize their own utility.

[6]Yavas, Thomas, and Sirmans (2001) examine an experiment in which the broker has imperfect information. However, in their setting, there is no inefficiency caused by asymmetric information in direct bargaining because the buyer's valuation is assumed to always be greater than the seller's cost. My paper offers a rational framework and points out a positive effect of bargaining through a broker from the perspective of efficiency–which is likely to be a first-order concern, especially in a market in which bargaining frictions caused by asymmetric information are substantial.

[7]In Glode and Opp (2016) and Glode, Opp, and Zhang (2016), one consequence of take-it-or-leave-it offer is that inefficiencies are partially driven by market power, which is absent in my setting.

show that an informed broker can help facilitate trade.[8] If I restrict my analysis to their environment (Section 1.6.1), I show that efficient trade can be achieved for a larger set of distributions of the buyer's and seller's private valuations. Third, they study partition information structures. In my setting, an efficient outcome can be achieved with more general information structures.

The rest of the paper proceeds as follows. In Section 1.2, I provide a simple example to demonstrate how an informed broker can facilitate efficient trade. In Section 3, I present a model with an informed broker. In Section 4, I consider the possibility of efficient trade, and present a condition on the broker's information such that efficient trade is implementable. In Section 5, I study the optimal mechanisms. Section 6 discusses two alternative assumptions and relationship to the literature. Section 7 concludes.

1.2. Illustrative Example

In this section, I provide a simple example to understand the main results in the paper. A seller owns an object that a buyer is interested in. The seller has a cost c of supplying the object, while the buyer values it at v. Both the cost c and the value v are private information, and are drawn independently from the uniform distribution over the $[0, 1]$ interval. The efficient allocation of the object is for the parties to trade whenever $v > c$. The total expected gains from trade in the efficient allocation is 1/6.
[9]

[8] In the working paper version, Glode and Opp (2015) show that a similar result holds with two-sided incomplete information. But they focus on the sequential trading game in which the asset owner has market power (i.e. the asset owner makes a take-it-or-leave-it offer).

[9] If the object is allocated efficiently, the gains from trade are $v - c$ if $v > c$, and 0 otherwise. Thus, the expected surplus is

$$\int_0^1 \int_0^1 (v-c)1_{v>c}\, dv\, dc = \frac{1}{6},$$

where $1_{v>c}$ is the indicator function and is equal to 1 if $v > c$, and 0 otherwise.

A seminal result in the bargaining literature is the Myerson and Satterthwaite (MS) theorem. The MS theorem shows that there are no budget balanced bargaining mechanisms that implement efficient trade. That is, there are always inefficiencies, regardless of the particular bargaining game that is played between the parties. For example, consider the bargaining game in which the buyer makes a take-it-or-leave-it offer to the seller. The buyer would never offer his true value v, because if he does, he will have to pay v whenever there is a transaction, being left with a surplus of 0. If, instead, the buyer shades his bid–offering $v/2$ for example–he receives a positive expected surplus. This means that the buyer always bids less than his true value, so that trade does not happen, in some cases when $v > c$.

A key assumption in MS's analysis is budget balance: that the mechanism does not lose money on average. There are mechanisms that lose money but guarantee that trade is efficient. For example, consider the Vickrey-Clarke-Groves mechanism (VCG). The buyer and seller announce v and c. There is trade if $v > c$. In that case, the buyer pays c, and the seller receives v. This mechanism is efficient, because there is trade whenever $v > c$. Moreover, both the buyer and the seller have incentives to declare their true valuations, because they cannot affect the payments that they make or receive. However, this mechanism is not budget balanced. The mechanism designer loses the price difference $v - c$ whenever there is trade, so that the average deficit of the mechanism equals the total gains from trade, 1/6.[10]

The main contribution of this paper is to show that an informed broker can facilitate efficient trade while running a surplus. To see this, suppose the transaction is mediated by a broker. The broker knows which of the following three subintervals v belongs to: $[0, 1/3)$, $[1/3, 2/3)$, $[2/3, 1]$. Similarly, he knows which of the three subinter-

[10]Williams (1999) provides a nice proof of MS theorem by showing that the VCG mechanism always runs a deficit, and any efficient mechanism loses at least as much money as VCG.

vals the seller's valuation c belongs to. As shown in Figure 1, the broker knows which of the nine regions the valuations pair belongs to. I show that efficient trade can be sustained by the following mechanism. The broker first announces which region is true, then the buyer and seller play one of the following three mechanisms: fixed price mechanism, VCG, or no trade. In the top left region, that is, the seller has a valuation lower than 1/3 and the buyer has a valuation higher than 2/3, the broker buys from the seller at a price of 1/3 and resells it to the buyer at a price of 2/3. Trade always occurs and the broker earns a profit of 1/3. In the bottom left region, the VCG mechanism is used; in this case, efficient trade can be achieved and the broker provides a subsidy. The required subsidy is 1/18, because the problem is a scaled version of the original MS problem.[11] Similarly, in the other two diagonal regions, efficient trade is implemented and the broker provides a subsidy of 1/18 in each region. In the other two regions above the diagonal line, efficient trade is implemented at a fixed price of 1/3 and 2/3, respectively. Lastly, in all of the other three regions, no trade is efficient. Overall, an efficient outcome is achieved, and the broker's ex ante expected profit is positive.[12] This example shows that an informed broker can use cross subsidization to implement efficient trade.

In order to get the full efficiency result, the broker must be sufficiently informed. Returning to the example, suppose the broker only knows that the buyer's and seller's valuations belong to either $[0, 1/2)$ or $[1/2, 1]$. In this case, the broker cannot earn profits in the top left region to subsidize trade in the diagonal regions, so efficient

[11] The required subsidy to achieve an efficient outcome is

$$\int_0^{\frac{1}{3}} \int_0^{\frac{1}{3}} (v-c)1_{v>c} \cdot 3 \cdot 3 \, dv \, dc = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18},$$

where $1_{v>c}$ is the indicator function and is equal to 1 if $v > c$, and 0 otherwise.

[12] The broker's ex ante expected profit is $\left(\frac{1}{3} - 3 \cdot \frac{1}{18}\right) \cdot \frac{1}{9} = \frac{1}{54}$.

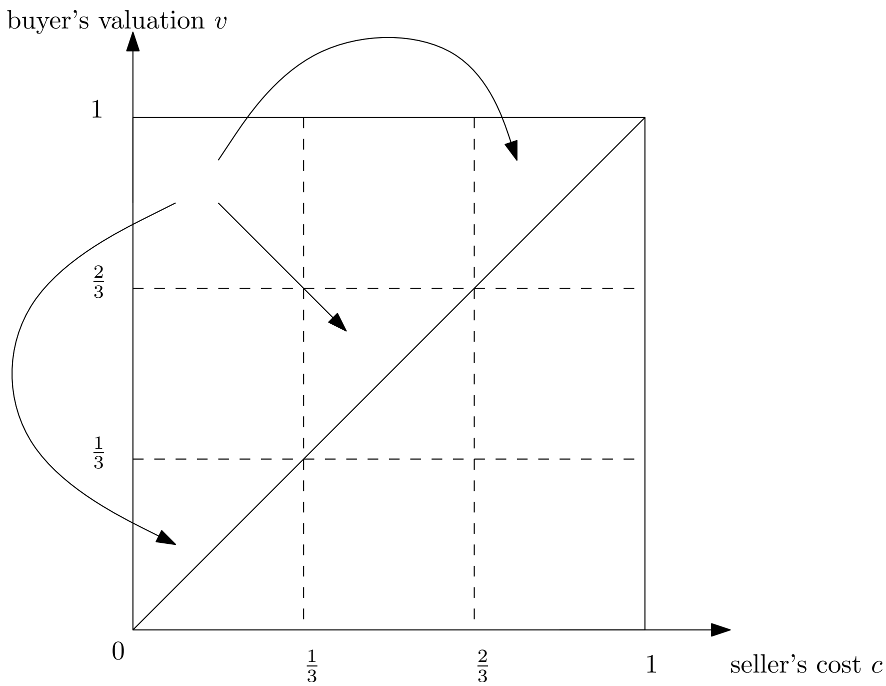


Figure 1: This figure illustrates that efficient trade can be achieved with an informed broker, when both the buyer's and seller's private valuations are drawn independently from the uniform distribution over the $[0, 1]$ interval, and the broker knows which of the following three subintervals v and c belong to: $[0, 1/3)$, $[1/3, 2/3)$, $[2/3, 1]$. I show that the broker may use profits from the top left region to subsidize trade in the diagonal regions to implement efficient trade.

trade cannot be achieved.[13] When the broker's information is given by a partition structure, Proposition 1.5 provides a necessary and sufficient condition on the broker's information such that an efficient outcome can be achieved. Moreover, if the broker's information divides the interval into finer partitions, efficient trade is easier to sustain (Corollary 1.7).

A key assumption underlying the cross subsidization mechanism is that the broker can commit to subsidize trade in the diagonal regions. If he does not have commitment power, then he will not participate in the game after he learns that signals are in the diagonal regions. However, as I will show in Proposition 1.1, whenever efficient

[13] In the top left region, the buyer's valuation is in $[1/2, 1]$ and the seller's valuation is in $[0, 1/2)$. The broker can implement efficient trade by setting a price of $\frac{1}{2}$, but he earns a zero profit.

trade can be implemented by cross subsidization, I can construct another mechanism that implements efficient trade without requiring the broker's commitment. The intuition of the construction is as follows. Consider a buyer whose valuation belongs to $[2/3, 1]$. In the cross subsidization mechanism, with probability 2/3, he gets the asset for sure and pays a fixed price of 2/3; with probability 1/3, he gets the asset when his valuation is greater than the seller's cost, and he pays the seller's cost if there is a trade. In the top left region, any fixed price below 2/3 can induce the buyer to truthfully report. This indicates that the incentive constraint is slack. In the top right region, the seller receives some subsidy from the broker. I can modify the buyer's payments in the top left and top right regions to achieve efficient trade without losing incentive compatibility. Consider a new mechanism, in which the buyer and seller simultaneously report their valuations to the broker.[14] In the new mechanism, the buyer pays a lower price in the top left region and a higher price in the top right region, so that his expected payment is unchanged. Since the buyer reports his valuation without conditioning on the region, he would truthfully report, as in the cross subsidization mechanism. Since he pays a higher price in the top right region, the subsidy needed from the broker in the top right region can be reduced. By properly adjusting the payments, efficient trade can be achieved without the broker's losing money in some regions.

I explore the issues raised by the example in greater detail. I also study the case in which the broker's information is not given by a partition structure. Proposition 1.1 shows that it is sufficient to study the case in which the broker has commitment power to derive the condition to sustain efficient trade. When I restrict to a broker who has

[14]We can interpret the cross subsidization mechanism as a two-stage game, in which the buyer and seller play the corresponding mechanism conditional on the broker's announcing region. In contrast, since I focus on Bayesian implementation, I construct the new mechanism in a way that the buyer and seller report their valuations without knowing which region is true.

commitment power, the condition is related to the least expensive way to induce the buyer and seller to participate in an efficient mechanism (Proposition 1.2). So the condition can be read as the total gains from trade being greater than the minimal information rents left to the buyer and seller. If the broker is informed, information rents are strictly less than that with an uninformed broker. It is easier, therefore, to implement efficient trade with an informed broker.

1.3. Model

1.3.1. Environment

Consider a trading problem in which there are one seller (she), one buyer (he), and one broker. The seller holds an indivisible object that is valuable for both her and the buyer. The broker does not intend to own the object in the trade, as the broker does not enjoy consumption from the object directly.[15]

Let v and c denote the value of the object to the buyer and the seller, respectively. I assume that these two valuations are independent random variables. The buyer and seller know his/her own valuation with certainty, but other agents know the valuation only probabilistically. The buyer's valuation, v, is represented by a probability density function f, which is positive in the range $[\underline{v}, \overline{v}]$. The corresponding cumulative density function is F. The seller's cost, c, is represented by a probability density function g, which is positive in the range $[\underline{c}, \overline{c}]$. The corresponding cumulative density function is G. I assume that $\underline{v} \geq 0, \underline{c} \geq 0$. For the result to be interesting, I also assume that $[\underline{v}, \overline{v}]$ and $[\underline{c}, \overline{c}]$ have a non-empty intersection, in which case direct bilateral bargaining is necessarily inefficient, as in Myerson and Satterthwaite (1983).

[15]Even if I allow the possibility that the broker may retain the object, the broker would not get it in the case of either looking for mechanisms that achieve full efficiency or mechanisms that optimize the broker's (seller's) profit.

Suppose the transaction is mediated by a broker. The broker does not know the buyer's or seller's exact private valuations, but he may have some information about them. For example, a realtor, experienced in the real estate industry, is informed about the intrinsic value of a property, which is likely correlated with the seller's private valuation. A financial intermediary is informed about the supply (demand) of some financial asset, and therefore can partially assess the reservation value of a seller (buyer). Hence, I assume that the broker receives informative signals about each agent's valuation. Prior to trading, suppose the broker *privately* receives a signal $b \in B$ of the buyer's valuation and a signal $s \in S$ of the seller's valuation. I assume that (v, b) is independent of (c, s). The broker's signals can be expressed in terms of conditional cumulative distribution functions, $F^I(b|v)$ and $G^I(s|c)$. Let $f^I(b|v)$ and $g^I(s|c)$ denote the associated conditional probability density functions. I do not impose assumptions on the cardinality of B and S. The broker's information structure is denoted as I. The distributions (F and G) and the conditional distributions (F^I and G^I) are common knowledge.

Finally, all individuals are risk neutral. Both the buyer and the seller have additively separable utility for money and the object. The broker only derives utility from money.

1.3.2. Direct mechanism

The seller, the buyer, and the broker are going to participate in some bargaining game. Rather than explicitly modeling the process, I will study a direct mechanism in which the probability of trade and payment schedules are determined as a function of the agents' reported valuations and reported signals.

By the Revelation Principle (see, e.g., Myerson (1979) and Myerson (1981)), the restriction to the direct mechanism is without loss of generality[16]: Any outcome associated with an equilibrium of some bargaining game will also be an equilibrium outcome of some revelation mechanism in which the buyer and seller truthfully report their valuations and the broker truthfully reports signals.

Let $p(v, c, b, s)$ be the probability that the object is transferred from the seller to the buyer, if v and c are the reported valuations of the buyer and the seller, and b and s are the broker's reported signals. I will refer to the probability of trade, $p(v, c, b, s)$, as an *allocation rule*. I assume no outsider (other than the buyer, the seller, and the broker) can provide funds; thus, there are only two payment schedules: the payment from the buyer to the broker and from the broker to the seller, denoted as $t_B(v, c, b, s)$ and $t_S(v, c, b, s)$, respectively. The broker's payoff is $t_0(v, c, b, s) = t_B(v, c, b, s) - t_S(v, c, b, s)$. Thus, a direct mechanism can be represented as

$$(p(v, c, b, s), t_B(v, c, b, s), t_S(v, c, b, s)).$$

I define

$$\bar{t}_B(v, \hat{v}) := \int_{\underline{c}}^{\overline{c}} \int_S \int_B t_B(\hat{v}, c, b, s)\, dF^I(b|v)\, dG^I(s|c)\, dG(c), \tag{1.1}$$

$$\overline{p}_B(v, \hat{v}) := \int_{\underline{c}}^{\overline{c}} \int_S \int_B p(\hat{v}, c, b, s)\, dF^I(b|v)\, dG^I(s|c)\, dG(c), \tag{1.2}$$

$$U_B(v, \hat{v}) := v \cdot \overline{p}_B(v, \hat{v}) - \bar{t}_B(v, \hat{v}), \tag{1.3}$$

$$U_B(v) := U_B(v, v). \tag{1.4}$$

[16]While the restriction to the direct mechanism is without loss of generality, the restriction on the outcome to be probability of trade and payment schedules is not. For example, I cannot cover the scenario in which agents are bargaining through time and agents have different discounting factors. However, if agents have same discounting factors, the restriction to the outcome functions is also without loss of generality. See Mailath and Postlewaite (1990) for further discussion.

In other words, if the buyer's valuation is v and he reports $\hat{v}$, $\overline{t}_B(v,\hat{v})$ is the expected payment from the buyer to the broker, $\overline{p}_B(v,\hat{v})$ is the expected probability of trade, $U_B(v,\hat{v})$ is the buyer's expected utility, and $U_B(v)$ is the buyer's expected payoff.

Similarly, for the seller, I define

$$\overline{t}_S(c,\hat{c}) := \int_{\underline{v}}^{\overline{v}} \int_S \int_B t_S(v,\hat{c},b,s)\, dF^I(b|v)\, dG^I(s|c)\, dF(v), \tag{1.5}$$

$$\overline{p}_S(c,\hat{c}) := \int_{\underline{v}}^{\overline{v}} \int_S \int_B p(v,\hat{c},b,s)\, dF^I(b|v)\, dG^I(s|c)\, dF(v), \tag{1.6}$$

$$U_S(c,\hat{c}) := \overline{t}_S(c,\hat{c}) - c \cdot \overline{p}_S(c,\hat{c}), \tag{1.7}$$

$$U_S(c) := U_S(c,c). \tag{1.8}$$

If the seller's valuation is c and she reports $\hat{c}$, $\overline{t}_S(c,\hat{c})$ is the expected payment she receives, $\overline{p}_S(c,\hat{c})$ is the expected probability of trade, $U_S(c,\hat{c})$ is the seller's expected utility, and $U_S(c)$ is the seller's expected payoff.

After the broker receives a signal of b for the buyer's valuation, the broker updates the distribution of the buyer's valuation. I denote the cumulative distribution as $F_0^I(v|b)$. After receiving s, the broker updates the distribution of the seller's valuation. I denote the cumulative distribution as $G_0^I(c|s)$. Thus, if the broker reports $(\hat{b},\hat{s})$ after receiving signals of (b,s), his expected payoff is

$$U_0(b,s,\hat{b},\hat{s}) := \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} (t_B(v,c,\hat{b},\hat{s}) - t_S(v,c,\hat{b},\hat{s}))\, dG_0^I(c|s) dF_0^I(v|b).$$

Define $U_0(b,s) := U_0(b,s,\hat{b},\hat{s})$ to be the broker's expected profit after receiving signals of (b,s).

For a mechanism to induce honest reporting, it must be *incentive-compatible* (IC):

$$v = arg\max_{\hat{v}} U_B(v, \hat{v}), \tag{1.9}$$

$$c = arg\max_{\hat{c}} U_S(c, \hat{c}), \tag{1.10}$$

$$(b, s) = arg\max_{(\hat{b},\hat{s})} U_0(b, s, \hat{b}, \hat{s}). \tag{1.11}$$

I also require that each individual wants to be included in the bargaining game. There are two possible individual rationality constraints that I can impose. The stronger condition requires that each individual earns at least a nonnegative profit after any realization of valuations and signals. This is an *ex post individually rational* (EXPIR) condition:

$$v \geq t_B(v, c, b, s) \geq t_S(v, c, b, s) \geq c, \text{ if } p(v, c, b, s) > 0.$$

The weaker condition requires that each individual has a non-negative expected gain after each individual learns the private valuation or signals. This is an *interim individually rational* (INTIR) condition:

$$U_B(v) \geq 0, \quad U_S(c) \geq 0, \quad U_0(b, s) \geq 0.$$

I write that an allocation rule p is *implementable* if there exists payment schedules (t_B, t_S) such that (p, t_B, t_S) is incentive-compatible and interim individually rational.

The broker's ex ante expected payoff is given by the buyer's expected payment minus the seller's expected payment:

$$U_0 := \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \int_B \int_S (t_B(v, c, b, s) - t_S(v, c, b, s))\, dG^I(s|c) dF^I(b|v) dG(c) dF(v).$$

If $U_0 \geq 0$, I say that the mechanism is non subsidized. I write that an allocation rule p is *implementable under broker commitment* if there exist payment schedules (t_B, t_S) such that (p, t_B, t_S) is non subsidized and gives the buyer and seller proper incentives to truthfully report and a nonnegative interim expected payoff. In this case, if the broker has commitment power, then the allocation rule can be implemented.

Ex post efficient trade requires

$$p(v, c, b, s) = \begin{cases} 1 & \text{if } v > c, \\ 0 & \text{if } v < c, \end{cases}$$

i.e., trade should occur as long as the buyer's reported valuation is greater than the seller's reported valuation, regardless of the broker's signals.

1.4. Implementability of Efficient Trade

1.4.1. Equivalence

My primary focus is the case in which ex post efficient trade is implementable. I first provide a useful equivalence result.

Proposition 1.1. *If an allocation rule is implementable under broker commitment, then it is implementable.*

Proof. See appendix. □

The proof of Proposition 1.1 is constructive. I provide a sketch here and the details can be found in the Appendix. Suppose $(p(v, c, b, s), t_B(v, c, b, s), t_S(v, c, b, s))$ is the mechanism that requires the broker's commitment. I can construct another mechanism that implements the same allocation rule. Let

$$t'_B(v,c,b,s) = \mathbb{E}_{c,s} t_B(v,c,b,s) + \mathbb{E}_{v,b} t_S(v,c,b,s) - \mathbb{E}_{v,c,b,s} t_S(v,c,b,s),$$
$$t'_S(v,c,b,s) = \mathbb{E}_{c,s} t_B(v,c,b,s) + \mathbb{E}_{v,b} t_S(v,c,b,s) - \mathbb{E}_{v,c,b,s} t_B(v,c,b,s),$$

where $\mathbb{E}_x$ represents the expectation by integrating over x. Consider the following new mechanism: $(p(v,c,b,s), t'_B(v,c,b,s), t'_S(v,c,b,s))$.

The intuition of the construction is as follows. Consider the environment in Figure 1 in Section 2. Roughly speaking, in the new mechanism, the buyer whose valuation belongs to $[2/3, 1]$ pays a lower price in the top left region and a higher price in the top right region. The buyer's expected payment is unchanged, so he would truthfully report, as in the cross subsidization mechanism. Formally in the proof, the buyer's objective function under $t'_B(v,c,b,s)$ is exactly same as his objective function under $t_B(v,c,b,s)$. In addition, the broker's payoff is a nonnegative constant under the new mechanism. Thus, the new mechanism can implement the same allocation rule without the broker's commitment.

The definition of implementability imposes more conditions than that of implementability under broker commitment. Proposition 1.1 establishes an important equivalence result: To study the condition for implementability, it is sufficient to study the condition for implementability under broker commitment.

1.4.2. Implementability

In this subsection, I provide a general sufficient condition for ex post efficient trade to be implementable. Consider the efficient allocation rule

$$p(v, c, b, s) = \begin{cases} 1 & \text{if } v > c, \\ 0 & \text{if } v < c. \end{cases}$$

Thus, a buyer who reports $\hat{v}$ expects the probability of trade to be $G(\hat{v})$, i.e., $\overline{p}_B(v, \hat{v}) = G(\hat{v})$. Similarly, a seller who reports $\hat{c}$ expects the probability of trade to be $1 - F(\hat{c})$, i.e., $\overline{p}_S(c, \hat{c}) = 1 - F(\hat{c})$. I define minimal information rent to the buyer as

$$\begin{aligned} R_B^I :&= \inf_{\widetilde{t}_B(\cdot,\cdot,\cdot,\cdot)} \left\{ \int_{\underline{v}}^{\overline{v}} \widetilde{U}_B(v, v)\, dF(v) \right\} \\ s.t.\ \widetilde{U}_B(v, \hat{v}) &= vG(\hat{v}) - \int_{\underline{c}}^{\overline{c}} \int_S \int_B \widetilde{t}_B(\hat{v}, c, b, s)\, dF^I(b|v)\, dG^I(s|c)\, dG(c), \\ \widetilde{U}_B(v, v) &\geq \widetilde{U}_B(v, \hat{v}), \forall v, \hat{v}, \\ \widetilde{U}_B(v, v) &\geq 0, \forall v. \end{aligned}$$

Recall from equation (1.3) that the first constraint represents the utility of a buyer who has true valuation v and reports $\hat{v}$ when the payment is given by $\widetilde{t}_B$. Thus, the minimal information rent to the buyer, R_B^I, is the minimal ex ante expected utility he obtains from any ex post efficient mechanisms that give him proper incentive to truthfully report and at least a nonnegative expected payoff. Note that I take infimum instead of minimum over all possible payment schedules, because the minimum value may not necessarily be achieved. I say that the infimum problem of R_B^I is *achievable* if there exists $\widetilde{t}_B$ that gives the buyer ex ante expected utility exactly equal to R_B^I.

Similarly, I can define minimal information rent to the seller as

$$
\begin{aligned}
R_S^I :=& \inf_{\widetilde{t}_S(\cdot,\cdot,\cdot,\cdot)} \left\{ \int_{\underline{c}}^{\overline{c}} \widetilde{U}_S(c,c)\, dG(c) \right\} \\
s.t.\ \widetilde{U}_S(c,\hat{c}) =& \int_{\underline{v}}^{\overline{v}} \int_S \int_B \widetilde{t}_S(v,\hat{c},b,s)\, dF^I(b|v)\, dG^I(s|c)\, dF(v) - c(1-F(\hat{c})), \\
& \widetilde{U}_S(c,c) \geq \widetilde{U}_S(c,\hat{c}), \forall c, \hat{c}, \\
& \widetilde{U}_S(c,c) \geq 0, \forall c.
\end{aligned}
$$

The minimal information rent to the seller, R_S^I, is the minimal ex ante expected utility that she obtains from any ex post efficient mechanisms that give her proper incentive to truthfully report and at least a nonnegative expected payoff. I say that the infimum problem of R_S^I is *achievable* if there exists $\widetilde{t}_S$ that gives the seller ex ante expected utility exactly equal to R_S^I.

Minimal information rents to the buyer and seller only depend on the fundamentals of the problem (i.e., the distributions of valuations and signals). In particular, it does not require any knowledge of payments in the bargaining game that they will play.

I use notation with a tilde, $\widetilde{t}_i$, in the context of minimal information rents. I use notation without the tilde, t_i, in the context of a direct mechanism.

Denote the total surplus from an ex post efficient mechanism by

$$
W := \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} (v-c) 1_{v>c}\, dG(c)\, dF(v).
$$

I now state the main result.

Proposition 1.2. *A sufficient condition for ex post efficient trade to be implementable* *is*

$$R_B^I + R_S^I < W.$$

A necessary condition is

$$R_B^I + R_S^I \leq W.$$

Furthermore, if the infimum problems of R_B^I and R_S^I are achievable, then a necessary and sufficient condition for ex post efficient trade to be implementable is

$$R_B^I + R_S^I \leq W.$$

Proof. See appendix. □

Proposition 1.2 provides a condition under which ex post efficient trade exists with an informed broker. If the distributions of the buyer's and the seller's valuations are fixed, R_B^I and R_S^I only depend on the distributions of the broker's signals. So the conditions in Proposition 1.2 impose the condition on the distributions of the broker's signals.

In direct bilateral bargaining, Myerson and Satterthwaite (1983) prove a strong negative result: that inefficiencies always occur with a positive probability in any equilibrium under any possible bargaining game. Proposition 1.2 says that introducing an informed broker leads to a significantly different result: An equilibrium with the efficient outcome exists in some bargaining game. I do not claim that the efficient outcome will always be obtained. For example, as I discuss in Section 1.5, the mechanism that maximizes the broker's ex ante expected utility is not necessarily ex post efficient. Even if the efficient outcome can arise as an equilibrium in a bargaining game, I do not claim that it is the only equilibrium. The result should be interpreted as follows: Under the sufficient condition there exists a bargaining game with the efficient outcome such that the strong negative result in Myerson and Satterthwaite (1983) breaks.

Bargaining frictions in Myerson and Satterthwaite (1983) are naturally nested in my model with an uninformed broker. Minimal information rents with an uninformed broker are exactly given by the information rents the buyer and seller earn in the VCG mechanism. I show that with an informed broker, the minimal information rents are strictly reduced. The channel that bargaining through an informed broker can resolve the inefficiencies is solely through the broker's information.

The intuition that an informed broker can improve efficiency is as follows. The presence of private information implies that the buyer (seller) may misreport their valuations. In Myerson and Satterthwaite (1983), the mechanism designer cannot distinguish a buyer who has a valuation v and reports $\hat{v}$ from another buyer who has a valuation v' but also reports $\hat{v}$. Put differently, any buyer who reports $\hat{v}$ will be treated identically. In contrast, consider bargaining through an informed broker. Since the broker's information depends on whether the buyer's valuation is v or v', the mechanism can treat the same reports from different types of buyers differently. The broker's correlated information can help to detect misreporting behaviors, and therefore make it easier to induce truthful revelation.

It is worth noting that the sufficient condition requires strict inequality, while the necessary condition involves weak inequality. This is because the infimum problems of R_B^I and R_S^I may not be achievable. The last part of the proposition says that if I know the infima are achievable, then I have the necessary and sufficient condition. The proposition suggests, therefore, that the condition with the strict inequality cannot be further extended.

Lastly, recall that the definition of implementability only requires INTIR instead of EXPIR. Thus, if the sufficient condition in Proposition 1.2 holds, I can construct an efficient mechanism–but it may not satisfy EXPIR. If we further require EXPIR in implementability, the condition on the broker's information would be more restricted. In the extreme case, if the broker's signals are perfect–i.e., the broker knows the buyer's and the seller's valuations perfectly–then an efficient mechanism that is EXPIR exists: The broker buys from the seller at a cost equal to her valuation, and resells to the buyer at a price equal to his valuation, whenever trade is efficient. Under this mechanism, the buyer and seller would choose to participate and earn a rent of zero. So, the set of conditions for the existence of an EXPIR efficient mechanism is non-empty.[17]

To apply the proposition, consider the illustrative example in Section 1.2. I will show in Lemma 1.4 that $R_B^I = R_S^I = \frac{2}{27}$. Total gains from trade $W = \frac{1}{6}$. Since $W - R_B^I - R_S^I = \frac{1}{54} > 0$, an efficient mechanism exists. In Section 4.4, I study the case in which the broker's information has a partition structure. In Section 4.5, I study the information structure that can fully extract surplus, as in McAfee and Reny (1992). First, however, I present some simple facts about minimal information rents.

[17]If the type spaces are discrete, then the Lesbegue measure of the set of conditions is strictly positive. To see this, consider the following simple setting. Let θ represent the informativeness of the broker's signal, where $\theta = 0$ represents an uninformed broker and $\theta = 1$ represents a perfectly informed broker. In the discrete type case, both incentive-compatible (IC) constraints and ex post individual rational (EXPIR) constraints are linear in payments. Thus, finding EXPIR efficient mechanisms is equivalent to solve a system of linear constraints. Let $R_1(\theta)$ denote the region given by the IC constraints. Let R_2 denote the region defined by the EXPIR constraints. If $\theta = 1$, there are many choices of payments to satisfy both IC and EXPIR. It is not hard to show that $R_1(1) \cap R_2$ has a nonempty interior region. By continuity, there exists $\bar{\theta}$ such that for any $\theta > \bar{\theta}$, $R_1(\theta) \cap R_2 \neq \emptyset$. In other words, the set of conditions for the existence of an EXPIR efficient mechanism is non-empty, and has a positive measure.

1.4.3. Some facts about minimal information rents

To determine whether ex post efficient trade is implementable, it is crucial to know the values of the minimal information rents to the seller and the buyer. Therefore, I present some simple facts about minimal information rents.

Denote $\emptyset$ as the null information structure in which the broker is uninformed, i.e., signals are not informative. Let $R_B^{\emptyset}$ and $R_S^{\emptyset}$ denote the minimal information rents to the buyer and the seller that are associated with the null information structure.

Lemma 1.3. *The minimal information rents associated with the null information structure are*

$$R_B^{\emptyset} = \int_{\underline{v}}^{\overline{v}} G(t)(1 - F(t))dt,$$
$$R_S^{\emptyset} = \int_{\underline{c}}^{\overline{c}} G(t)(1 - F(t))dt.$$

The total gain from trade can be expressed as

$$W = \int_{\underline{c}}^{\overline{v}} G(t)(1 - F(t))dt.$$

For any information structure I,

$$0 \leq R_B^I \leq R_B^{\emptyset} \leq W \text{ and } 0 \leq R_S^I \leq R_S^{\emptyset} \leq W.$$

If the broker has perfect information, then it is clear that the minimal information rent would be zero. The lemma states that for any information structure, the minimal information rent is bounded by the two extreme cases: the perfect information and the null information. In next section, if the broker's information is given by partition of an interval, I show that the minimal information rents with an informed broker are strictly less than that with an uninformed broker.

As an application, it is straightforward to see that as long as $(\underline{v}, \overline{v})$ and $(\underline{c}, \overline{c})$ have a nonempty intersection,

$$R_B^{\emptyset} + R_S^{\emptyset} > W.$$

This is the result in Myerson and Satterthwaite (1983)–i.e., if the broker is uninformed, ex post efficient trade can not be implemented without a subsidy.

1.4.4. Partition structure

In this subsection, I study a particular type of information structure that the broker may have: a partition of an interval. Glode and Opp (2016) and Glode, Opp, and Zhang (2016) study this information structure. Suppose the broker knows which of the following intervals v belongs to:

$$[v_0, v_1), [v_1, v_2), \cdots, [v_{n-1}, v_n],$$

and which of the following intervals c belongs to:

$$[c_0, c_1], (c_1, c_2], \cdots, (c_{m-1}, c_m],$$

where $\underline{v} = v_0 < v_1 < \cdots < v_n = \overline{v}$, $\underline{c} = c_0 < c_1 < \cdots < c_m = \overline{c}$, and $m, n \geq 1$. The set of all possible signals can be expressed as $B = \{1, 2, \cdots, n\}$ and $S = \{1, 2, \cdots, m\}$. For instance, if $v \in [v_0, v_1)$, then the broker's signal is $b = 1$. If $m = n = 1$, then the broker is uninformed. Denote the broker's information structure by I.

The following lemma characterizes the minimal information rents to the buyer and the seller.

Lemma 1.4. *If the broker's information is given by a partional structure, then*

$$R_B^I = \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} \int_{v_{k-1}}^{v} G(t)dtdF(v),$$
$$R_S^I = \sum_{k=1}^{m} \int_{c_{k-1}}^{c_k} \int_{c}^{c_k} 1 - F(t)dtdG(c).$$

In addition, as long as the broker has some information, the minimal information rents will be reduced:

1. $R_B^I < R_B^\emptyset$ *if and only if* $n > 1$.

2. $R_S^I < R_S^\emptyset$ *if and only if* $m > 1$.

Proof. See appendix. □

From Lemma 1.4, I can calculate the minimal information rents for any partition structure. In addition, if the broker's signal about the buyer's valuation is informative, the minimal buyer's information rent is strictly less than the minimal information rent associated with the null information structure.

Suppose the valuations of both the buyer and the seller are drawn from the uniform distribution over $[0, 1]$ interval. If the broker is uninformed, I find $R_B^\emptyset = R_S^\emptyset = \frac{1}{6}$. From the lemma, I can calculate the minimal information rents to the buyer and the seller in the illustrative example in Section 2: $R_B^I = R_S^I = \frac{2}{27}$. It is clear that $R_B^I < R_B^\emptyset$ and $R_S^I < R_S^\emptyset$.

Recall that Proposition 1.2 provides a condition for the existence of an efficient mechanism. I then obtain a closed-form condition under which the broker's information is given by a partition structure. I summarize the result in the following proposition.

Proposition 1.5. *If the broker's information is given by a partition structure, I, then the necessary and sufficient condition for ex post efficient trade to be implementable is*

$$0 \leq \sum_{b=1}^{n} \sum_{s=1}^{m} \int_{v_{b-1}}^{v_b} \int_{c_{s-1}}^{c_s} 1_{v>c} \cdot f(v) \cdot g(c) \cdot \left(v - c - \frac{F(v_b) - F(v)}{f(v)} - \frac{G(c) - G(c_{s-1})}{g(c)} \right) dc\, dv \tag{1.12}$$

Proof. See appendix. □

It is intuitive to interpret the expression in (1.12). First, $v - \frac{F(v_b)-F(v)}{f(v)}$ is similar to the "virtual valuation" term $v - \frac{1-F(v)}{f(v)}$. The difference comes from the fact that the broker can update the distribution of v and infer the buyer's valuation from the cumulative distribution function $\hat{F}$ given by $\hat{F}(v) = \frac{F(v)-F(v_{b-1})}{F(v_b)-F(v_{b-1})}$, once the broker knows which interval v belongs to. The associated probability density function is $\hat{f}(v) = \frac{f(v)}{F(v_b)-F(v_{b-1})}$. Now the virtual valuation term becomes

$$v - \frac{1 - \hat{F}(v)}{\hat{f}(v)} = v - \frac{F(v_b) - F(v)}{f(v)}. \tag{1.13}$$

Similarly, $c + \frac{G(c)-G(c_{s-1})}{g(c)}$ is the "virtual cost" conditional on the broker knowing c belongs to $(c_{s-1}, c_s]$. So the right-hand side of (1.12) is the expected value, whenever we want the two parties to trade, of simultaneously buying from the seller and reselling to the buyer. It must be nonnegative so that the broker is willing to intermediate the bargaining.

If $m = n = 1$, i.e., the broker is uninformed, (1.12) becomes

$$0 \leq \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} 1_{v>c} \cdot f(v) \cdot g(c) \cdot \left(v - c - \frac{1 - F(v)}{f(v)} - \frac{G(c)}{g(c)} \right) dc\, dv,$$

which is exactly the condition in Myerson and Satterthwaite (1983).

Recall that

$$I = \{[v_0, v_1), \cdots, [v_{n-1}, v_n]\} \times \{[c_0, c_1], \cdots, (c_{m-1}, c_m]\}.$$

The broker's information partitions the types space into mn regions. Suppose I_1 and I_2 are two information structures. It is natural to say that I_1 is more *informative* than I_2 if and only if for any region in I_2, there exists weakly finer regions in I_1, and there exists at least one region such that I_1 has a strictly finer partition than I_2. It is straightforward to show that a more informative structure leads to a smaller value of minimal information rent. I summarize the result in the following lemma.

Lemma 1.6. *If I_1 is more informative than I_2, then $R_B^{I_1} \leq R_B^{I_2}$ and $R_S^{I_1} \leq R_S^{I_2}$. The equality can hold if and only if I_1 does not create a finer partition on the valuation of the buyer (seller) than I_2.*

Directly applying Proposition 1.2, I have the following corollary.

Corollary 1.7. *If there exists an ex post efficient mechanism in which the broker's information is I_2, and I_1 is more informative than I_2, then there also exists an ex post efficient mechanism in which the broker's information is I_1.*

Corollary 1.7 says that the condition is less constraining when the broker's signals are more informative.[18]

Proposition 1.8. *For any F and G, there exists a partition information structure I such that the efficient allocation rule can be implemented when the broker's information is I.*

Proof. See appendix. □

Proposition 1.8 says that for any given distributions, there always exists a fine enough partition such that an efficient outcome can be achieved with an informed broker.

I now provide some examples of when the broker's information is given by a partition structure. Assume $v \sim U[0,1], c \sim U[0,1]$, and subintervals are equally distributed. Using Lemma 1.4, I can calculate the minimal information rents:

$$R_B^I = \frac{3n-1}{12n^2}, R_S^I = \frac{3m-1}{12m^2}.$$

From the formula, it is easy to see that R_B^I goes to 0 at a rate of $1/n$. Total gains from trade $W = 1/6$. Suppose that $m = n = 1$, which is the case in Myerson and Satterthwaite (1983) with an uninformed broker. Then $R_B^I + R_S^I = 1/3 > W$. So ex post efficient trade is not implementable.

Now consider $m = 1$ and $n > 1$. Then $R_B^I + R_S^I - W = \frac{3n-1}{12n^2} > 0$. Ex post efficient trade is not implementable, but the subsidy needed from the broker to reach full efficiency is approaching to 0 as $n \to \infty$.

Suppose $m = n = 2$, that is, the broker knows which of $[0, 1/2)$ or $[1/2, 1]$ the buyer's

[18] Corollary 1.7 also holds in the setting with general information structures, where "more informative" is defined by Blackwell's order (see Blackwell (1951) and Blackwell (1953)).

and seller's valuations fall into. Since $W - (R_B^I + R_S^I) = -1/24 < 0$, there is no ex post efficient mechanism.

Next, I give an example in which ex post efficient trade exists when the broker is informed. Suppose that $m = n = 3$. Then $W - (R_B^I + R_S^I) = 1/54$. Furthermore, by Corollary 1.7, if the broker's information is finer than $m = n = 3$, ex post efficient trade is implementable.

If $m = n = 4$, $W - (R_B^I + R_S^I) = 5/96$. Thus, ex post efficient trade is implementable.

1.4.5. Condition for full surplus extraction

In this subsection, I show that minimal information rents are closely related to full surplus extraction, as in Crémer and McLean (1988) and McAfee and Reny (1992). Since the type spaces in my setting are continuous, I assume that the signal sets B and S are compact subsets of $\mathbb{R}$ in order to apply the theorem in McAfee and Reny (1992). Without loss of generality, let $B = [\underline{v}, \overline{v}]$ and $S = [\underline{c}, \overline{c}]$. I also assume that both of the joint cumulative distributions, $F^I(\cdot,\cdot)$ and $G^I(\cdot,\cdot)$, have a continuous probability density function.

McAfee and Reny (1992) give a necessary and sufficient condition for (almost) full rent extraction, which I first describe. I then show that if their (almost) full rent extraction condition holds, then minimal information rents are zero.

Let $\Delta(B)$ denote the set of probability measures on B. Recall that $f^I(\cdot|\cdot)$ is the conditional probability density function. I say that $f^I(\cdot|\cdot)$ satisfies the McAfee and Reny condition if, for every v_0 and every $\mu \in \Delta(B)$, $\mu(\{v_0\}) \neq 1$ implies that $f^I(\cdot|v_0) \neq \int_{\underline{v}}^{\overline{v}} f^I(\cdot|v)\mu(dv)$. Similarly, I say that $g^I(\cdot|\cdot)$ satisfies the McAfee and Reny condition if, for every s_0 and every $\mu \in \Delta(S)$, $\mu(\{s_0\}) \neq 1$ implies that

$g^I(\cdot|s_0) \neq \int_{\underline{c}}^{\overline{c}} g^I(\cdot|s)\mu(ds)$. I say that the broker's information structure satisfies the McAfee and Reny condition if both f^I and g^I satisfy the (almost) full rent extraction condition.

Proposition 1.9. *If the broker's information structure satisfies the McAfee and Reny condition, then $R^I_B = R^I_S = 0$ and ex post efficient trade is implementable.*[19]

Proof. See appendix. $\square$

To understand why $R^I_B = 0$, I first explain the McAfee and Reny condition. It is easier to explain its discrete counterpart, in which the condition becomes the Crémer and McLean (1988) condition. Basically, the Crémer and McLean condition says that the vector of conditional probabilities corresponding to any possible value is not in the convex hull of the vectors of conditional probabilities corresponding to other possible types. This condition on the information structure is a spanning condition. Roughly speaking, Crémer and McLean (1988) show that we can design a "side bet" with the following properties: It gives the buyer a large negative payoff if he does not report truthfully; it gives the truth-telling buyer the exact amount of payoff such that he is willing to participate. The spanning condition is exactly what I need to invert a system of linear equations to design the desired "side bet".

Similar to McAfee and Reny (1992), I consider first a truth-telling mechanism in the bargaining stage (second stage) supplementing a "pre"-mechanism (first stage). In the second stage, the buyer pays the seller's reported valuation and the seller receives the buyer's reported valuation, if the buyer's reported valuation is greater

[19] Since $W \geq R^{\emptyset}_S$, I obtain the following results by directly applying Proposition 1.2: If the broker is uninformed about the seller's valuation and the signals about the buyer's valuation, $f^I(\cdot|\cdot)$, satisfies the McAfee and Reny condition, then $R^I_B = 0$ and ex post efficient trade is implementable.

than the seller's. Honesty is a dominant strategy for both the buyer and the seller. I then supplement the following first-stage mechanism. The broker offers a set of participation fee schedules $\{z_n^B(\cdot)\}$ for the buyer. Prior to the bargaining, the buyer chooses one of the participation fee schedules $z_n^B(\cdot)$, which requires the buyer to pay $z_n^B(b)$ if the broker receives a signal of b. Since the broker's information structure satisfies the McAfee and Reny condition, I can design the participation fee schedules such that the buyer earns a rent of no greater than ϵ, $\forall\epsilon > 0$.

Finally, let's consider the buyer's payment schedule, $\widetilde{t}_B(v,c,b,s) = z_n^B(b) + c1_{v>c}$, where $z_n^B(b)$ is the side bet and $c1_{v>c}$ is the second-stage payment. Since the side bet depends on the broker's signal and not on the seller's reported type, the payment schedule induces the buyer to truthfully report. In addition, $\widetilde{t}_B(v,c,b,s)$ yields an information rent of no greater than ϵ to the buyer. Consequently, the minimal buyer's information rent $R_B^I = 0$.

Similarly, the minimal seller's information rent $R_S^I = 0$. Applying Proposition 1.2, efficient trade is implementable. Furthermore, since the McAfee and Reny condition holds generically (McAfee and Reny, 1992), I conclude that efficient trade can be sustained with a generic informed broker. However, there is a caveat. If a signal contains very little information, a large penalty in payment will be required because of the side bet. In my model, a risk-neutral player does not care about a large penalty, as long as the expected profit is nonnegative. However, in reality, agents may be budget constrained, and in many cases a large penalty is not feasible. If I restrict an exogenous bound on ex post payments, then efficient trade can be implemented only if the broker's signals contain sufficient information.

Example 1.1. *Suppose that $v, c \sim U[0,1]$ and $F^I(b|v) = \left(\frac{b}{v}\right)^\theta$ for $b \in [0,v]$ and $G^I(c|s) = \left(\frac{s}{c}\right)^\theta$ for $s \in [0,c]$, where $\theta \geq 0$ is a parameter. For any $\theta > 0$, the* *broker's information structure satisfies the McAfee and Reny condition. If $\theta = 0$, the information structure is completely uninformative. Now consider $\theta > 0$; we can show that $R^I_B = R^I_S = 0$. Consider $\widetilde{t}_B(v, c, b, s) = \frac{\theta+1}{\theta} vb$, then $\widetilde{U}_B(v, \hat{v}) = v\hat{v} - \int_0^v \frac{\theta+1}{\theta} \hat{v} b\theta \left(\frac{b}{v}\right)^{\theta-1} \frac{1}{v} db = 0$. Consider $\widetilde{t}_S(v, c, b, s) = \frac{\theta+1}{\theta}(1-c)s$, then $\widetilde{U}_S(c, \hat{c}) = -c(1-\hat{c}) + \int_0^v \frac{\theta+1}{\theta}(1-\hat{c})s\theta \left(\frac{s}{c}\right)^{\theta-1} \frac{1}{c} db = 0$. Thus, $R^I_B = R^I_S = 0$.*

I now construct an ex post efficient mechanism. Consider payment schedules $t_B(v, c, b, s) = \frac{\theta+1}{\theta} vb + \frac{\theta+1}{\theta}(1-c)s - \frac{1}{6}$ and $t_S(v, c, b, s) = \frac{\theta+1}{\theta} vb + \frac{\theta+1}{\theta}(1-c)s - \frac{1}{3}$. We can verify that the buyer and the seller would truthfully report and earn a profit of zero. The broker's payoff is $\frac{1}{6}$ after any realization.

As we can see, as $\theta \to 0$, the payments go unbounded. If there is a bound on ex post payments, then for small θ, an ex post efficient mechanism may not exist. Put differently, the broker needs to be sufficiently informed so that efficient trade can be sustained.

If the broker has commitment power, efficient trade is implementable for generic informative signals. However, if there is an exogenous bound on ex post payments, then efficient trade may be implementable with broker commitment, but may not be implementable otherwise. Let $\Omega_1(I)$ denote all possible mechanisms that implement the efficient trade with the broker's commitment, and $\Omega_2(I)$ denote all possible mechanisms that implement the efficient trade without the broker's commitment, if the broker's signals are given by an information structure I. Let $a_i(I)$ be the lower bound on the maximal possible absolute value of the payment schedules in $\Omega_i(I)$, i.e.,

$$a_i(I) = \inf_{(p, t_B, t_S) \in \Omega_i(I)} \max \left(\max_{(v,c,b,s)} |t_B(v, c, b, s)|, \max_{(v,c,b,s)} |t_S(v, c, b, s)| \right).$$

Since $\Omega_2(I) \subset \Omega_1(I)$, $a_2(I) \geq a_1(I)$. If a bound on ex post payments are imposed, for instance, the payment must be between $a_2(I)$ and $a_1(I)$, then efficient trade is implementable with broker commitment, but is not implementable otherwise.[20]

1.5. Optimal Mechanisms

In the previous sections, I have focused on when trading with an informed broker can help achieve efficient trade. In particular, I studied the mechanism design problem without specifying who chooses the mechanism. In many applications, the mechanism is chosen by one of the agents. For example, if the broker is a monopoly, then it is reasonable to consider the broker's optimal mechanism. If the seller's asset is scarce, then it is reasonable to consider the seller's optimal mechanism.

1.5.1. *The broker's ex ante optimal mechanism*

Suppose the broker chooses a mechanism before he learns the realizations of signals.[21] I first consider the broker's information is given by a partition structure:

$$I = \{[v_0, v_1), \cdots, [v_{n-1}, v_n]\} \times \{[c_0, c_1], \cdots, (c_{m-1}, c_m]\}.$$

To study the broker's ex ante optimal mechanism, I need the condition under which an allocation rule can be implemented. Recall that $p(v, c, b, s)$ denotes the probability that the asset should be transferred to the buyer. Let $p_B(v, b)$ denote the probability that a buyer, who reports v, receives the asset when the broker observes a signal of

[20]Suppose I define some measure of informativeness. I conjecture that if I is less and less informative, then $\frac{a_1(I)}{a_2(I)} \to 0$. In other words, the cost of no commitment power is aggravated if the broker's signals only contain very limited information. I leave this for future research.

[21]If the broker chooses a mechanism after he acquires his information, the problem is an informed principal problem (Myerson, 1983). I study the case in which the broker maximize his ex ante expected profit; I do not study the informed principal problem here. In the case where the broker's information is given by a partition structure, the mechanism that maximizes either the broker's or the seller's ex ante expected profits is also the solution to the informed principal problem. See Yilankaya (1999) for an elegant proof.

b for the buyer. Let $p_S(c, s)$ denote the probability that a seller, who reports c, sells the item when the broker observes a signal of s for the seller. The following lemma establishes the condition under which an allocation rule p can be implemented.

Lemma 1.10. *If $p(\cdot, \cdot, \cdot, \cdot)$ is any function mapping into $[0, 1]$ and $p(v, c, b, s) = 0$ if $v \notin [v_{b-1}, v_b)$ or $c \notin (c_{s-1}, c_s]$, then there exists t_B, t_S such that (p, t_B, t_S) is incentive-compatible and individually rational if and only if the following conditions hold.*

1. *$p_B(v, b)$ is weakly increasing in v for $v \in [v_{b-1}, v_b)$.*
2. *$p_S(c, s)$ is weakly decreasing in c for $c \in (c_{s-1}, c_s]$.*
3.

$$0 \leq \sum_{b=1}^{n} \sum_{s=1}^{m} \int_{v_{b-1}}^{v_b} \int_{c_{s-1}}^{c_s} p(v, c, b, s) * f(v) * g(c) * \left(v - c - \frac{F(v_b) - F(v)}{f(v)} - \frac{G(c) - G(c_{s-1})}{g(c)} \right) dc\, dv. \tag{1.14}$$

Proof. See appendix. □

The proof is constructive, and is similar to the proof of Theorem 1 in Myerson and Satterthwaite (1983). As a special case of this lemma, if p is the efficient allocation rule, then equation (1.14) becomes equation (1.12) in Proposition 1.5.

From the proof of Lemma 1.10, the right-hand side of (1.14) represents the broker's utility. If the broker can choose any mechanism, he would maximize the right-hand side of (1.14). I introduce the following regularity condition so that I do not need to worry about the monotonicity conditions of $p_B(v, b)$ and $p_S(c, s)$.

Assumption 1.1. *For any $b \in B, s \in S$, suppose $v - \frac{F(v_b) - F(v)}{f(v)}$ and $c + \frac{G(c) - G(c_{s-1})}{g(c)}$*

are monotone increasing functions on $[v_{b-1}, v_b)$ *and* $(c_{s-1}, c_s]$*, respectively.*

This assumption is often assumed in the literature to bypass the monotonicity conditions (Myerson, 1981), including Myerson and Satterthwaite (1983), Williams (1987), and Yilankaya (1999), among many others. The assumption holds for wide classes of distributions. For instance, if v and c are standard uniform distributions, Assumption 1.1 holds.

Proposition 1.11. *If Assumption 1.1 holds and the broker's information is given by* I*, then a broker would choose the following allocation rule* p*:*

$$p(v,c,b,s) = \begin{cases} 1 & \textit{if } v - \frac{F(v_b)-F(v)}{f(v)} > c + \frac{G(c)-G(c_{s-1})}{g(c)}, \\ 0 & \textit{otherwise.} \end{cases}$$

I provide an example in which $v, c \sim U[0,1]$ and the broker knows which of the following three subintervals v and c belong to: $[0, 1/3), [1/3, 2/3), [2/3, 1]$. From Proposition 1.11, the broker would choose the following allocation rule p:

$$p(v,c,b,s) = \begin{cases} 1 & \text{if } v - c > \frac{b-s+1}{6} \text{ and } b \geq s, \\ 0 & \text{otherwise.} \end{cases}$$

Figure 2 illustrates the region of trade.

As we can see from Figure 2, trade is not efficient. But compared to the uninformed broker's optimal mechanism, the probability of trade is strictly higher. In general, I have the following result.

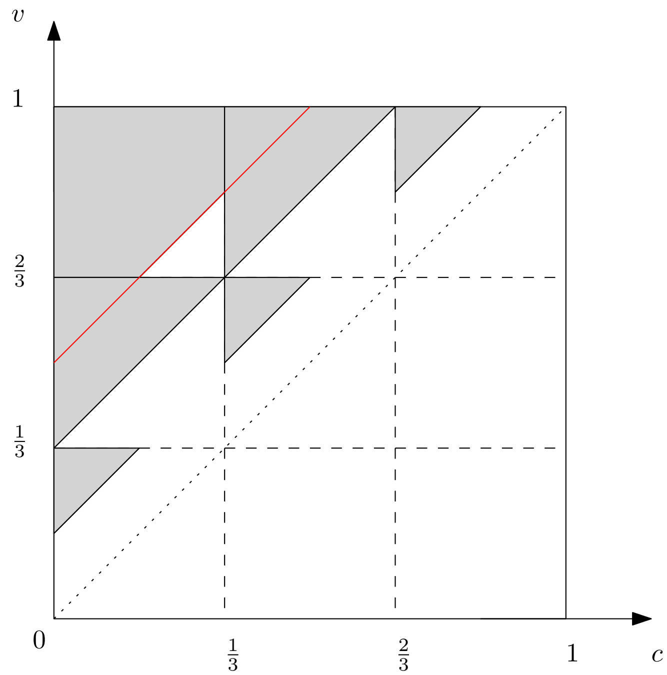


Figure 2: This figure shows the mechanism that maximizes the broker's profit when v and c are drawn independently from the uniform distribution over $[0, 1]$ interval, and the broker knows which of the following three subintervals v and c belong to: $[0, 1/3), [1/3, 2/3), [2/3, 1]$. The gray area represents the region of trade. Recall from Myerson and Satterthwaite (1983) that when the broker is uninformed, the mechanism that maximizes the broker's profit is that the asset is traded if and only if $v > c + \frac{1}{2}$.

Corollary 1.12. *In the broker's optimal mechanism, a broker who has more informative signals induces a higher probability of trade than does a broker with less informative signals.*

So even though an informed broker can not facilitate efficient trade in the broker's optimal mechanism, we still have a higher trading probability than the case of an uninformed broker. In the extreme case in which the broker is perfectly informed about the buyer's and seller's valuations, the monopolistic broker can extract all of the surplus by offering the seller's valuation to buy from the seller and offering the buyer's valuation to sell to the buyer.

Lastly, I consider the case in which the broker's information structure satisfies the McAfee and Reny condition.

Proposition 1.13. *If the broker's information structure satisfies the McAfee and Reny condition, then the broker's ex ante optimal mechanism is the one that gives the broker all of the surplus. In particular, the broker can charge a fixed commission fee of W and the efficient outcome is achieved.*

Proof. See appendix. □

Proof sketch: Since the broker can earn at most all of the trading surplus, to prove the proposition I construct an efficient mechanism that gives all of the surplus to the broker. Similar to the proof of Proposition 1.2, the mechanism features a fixed commission fee of W.

If the broker's information structure satisfies the McAfee and Reny condition, an efficient outcome is achieved under the broker's optimal mechanism. However, as the broker's signals contain less and less information, the payment schedule involves a large penalty that may not be feasible in reality. If there is an exogenous bound on the payment, then efficient trade may not be implemented.

1.5.2. The seller's ex ante optimal mechanism

In this subsection, I study the seller's ex ante optimal mechanism. Since the roles of the buyer and the seller are symmetric in my model, the buyer's ex ante optimal mechanism can be similarly derived.

Assume the broker's information is given by a partition structure. The seller's ex ante expected payoff is

$$\sum_{i=1}^{m} \int_{c_{i-1}}^{c_i} U_s(c)dG(c).$$

Proposition 1.14. *If the broker's information is given by a partition structure, then the seller's ex ante optimal mechanism is given by one in which the seller makes a take-it-or-leave-it offer conditional on the seller's knowing the broker's signals.*

Proof. See appendix. □

The proof is similar to Williams (1987), who studies the case in which the broker is uninformed and finds that having the seller make a take-it-or-leave-it offer is the seller's ex ante optimal mechanism. Figure 3 illustrates this result when $v, c \sim U[0, 1]$ and the broker knows which of the following three subintervals v and c belong to: $[0, 1/3), [1/3, 2/3), [2/3, 1]$.

Corollary 1.15. *When a seller is choosing a mechanism to maximize profit, a broker who has more informative signals induces a higher probability of trade than does a broker with less informative signals.*

The corollary says that, even if an efficient mechanism does not exist, the probability of trade with a more informative broker is higher than bargaining through a less informative broker.

1.6. Discussion

1.6.1. Common value

In this subsection, I consider a common value setting and show that a broker's information can help improve efficiency.

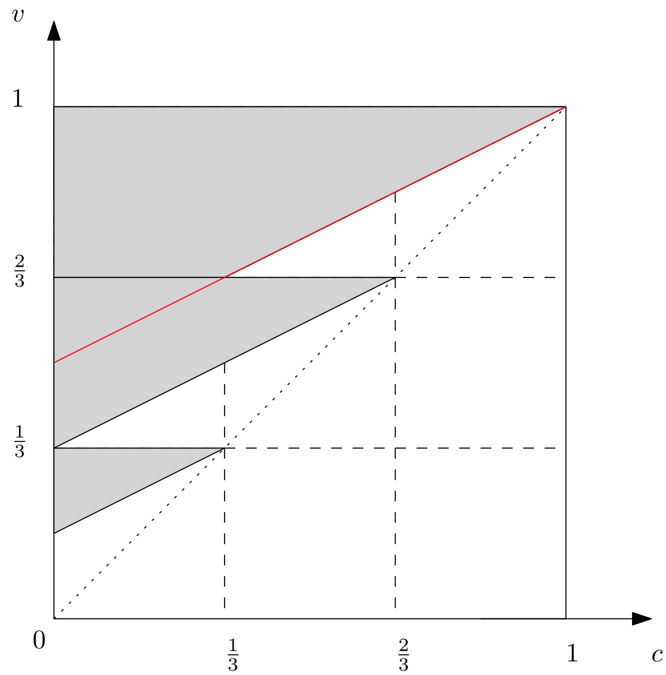


Figure 3: This figure shows the mechanism that maximizes the seller's ex ante expected profit when v and c are drawn independently from the uniform distribution over $[0, 1]$ interval, and the broker knows which of the following three subintervals v and c belong to: $[0, 1/3), [1/3, 2/3), [2/3, 1]$. The gray area represents the region of trade. The red line represents $v = \frac{1+c}{2}$. Recall from Myerson and Satterthwaite (1983) that when the broker is uninformed, the mechanism that maximizes the seller's ex ante expected profit is given by the region $v > \frac{1+c}{2}$.

Consider an environment in which the buyer's valuation is v and the seller's valuation is $c(v)$. v is a random variable, represented by a probability density function f, which is positive in the range $[\underline{v}, \overline{v}]$. The corresponding cumulative density function is F. The buyer knows the realization of v, but the seller does not. To compare with Glode and Opp (2016) and Glode, Opp, and Zhang (2016), I assume that $c(v) < v, \forall v$, i.e., trade is always efficient. It is common knowledge that trade should happen, but because of adverse selection, there is a condition to sustain efficient trade.[22]. The condition is $v_L \geq \mathbb{E}c(v)$.[23] Now I consider the bargaining problem with an informed broker. The following proposition summarizes the result.

[22]See Samuelson (1984) for a general analysis.

[23]See the proof of Proposition 3 in Glode, Opp, and Zhang (2016).

Proposition 1.16. *Suppose $v_L = v_0 < v_1 < v_2 < \cdots < v_n = v_H$ and the broker knows which of the following subintervals v belongs to: $[v_0, v_1)$, $\cdots$, $[v_{n-1}, v_n]$. If $\sum_{i=1}^{n} v_{i-1}(F(v_i) - F(v_{i-1})) \geq \mathbb{E}c(v)$, then efficient trade can be achieved with the informed broker.*

Proof. See appendix. □

Note that $\sum_{i=1}^{n} v_{i-1}(F(v_i) - F(v_{i-1})) > v_L$. So the set of distributions of the buyer's valuation that leads to an efficient outcome is strictly wider with an informed broker than that under direct trade. Therefore, by considering all mechanisms, efficient trade can be achieved when it may not possible in the sequential trading game considered in Glode and Opp (2016) and Glode, Opp, and Zhang (2016).

Another observation is that as the broker's signal becomes more informative, then

$$\sum_{i=1}^{n} v_{i-1}(F(v_i) - F(v_{i-1})) \to \mathbb{E}v.$$

Since $\mathbb{E}v > \mathbb{E}c(v)$, for any $c(\cdot)$ there exists n and $v_L = v_0 < v_1 < v_2 < \cdots < v_n = v_H$ such that $\sum_{i=1}^{n} v_{i-1}(F(v_i) - F(v_{i-1})) \geq \mathbb{E}c(v)$. In other words, if the broker's signal partitions the types space into fine enough subintervals, efficient trade can be achieved.

1.6.2. Discrete types

In this subsection, I consider a setting in which the type spaces are discrete. Matsuo (1989) characterizes the conditions under which efficiency is obtained for distributions with two values. Kos and Manea (2009) extend to the general discrete distribution. In this section, I show how to use minimal information rent to characterize the necessary and sufficient condition for efficient trade. I focus on the case in which the broker is uninformed, but the method can be similarly extended to the case in which the broker is informed.

Suppose the buyer's valuation could be $V_1 < V_2 < \cdots < V_n$ and the seller's valuation could be $C_1 < C_2 < \cdots < C_m$. As in Matsuo (1989) and Kos and Manea (2009), I assume that $V_i \neq C_j, \forall i, j$ to simplify the analysis. Denote the CDF for the buyer and the seller as F and G, and denote the probability mass function as $f_i = \mathbb{P}(V_i)$ and $g_i = \mathbb{P}(C_i)$.

Proposition 1.17. *Suppose the types are discrete; then an ex-post efficient mechanism can be implemented with an uninformed broker if and only if*

$$\sum_{i=1}^{n}(1 - F(V_{i-1})(G(V_i) - G(V_{i-1})V_i \geq \sum_{i=1}^{m} G(C_i)(F(C_{i+1}) - F(C_i))C_i,$$

where I use notation $F(V_0) = G(V_0) = 0$ and $F(C_{m+1}) = 1$.

Proof. See appendix. □

For $m = n = 2$, the condition reduced to $f_2 g_2 V_2 - f_2 C_2 + g_1 V_1 - g_1 f_1 C_1 \geq 0$. This is exactly the condition in Matsuo (1989). For general m and n, using a minimal information rents approach can help identify the condition to achieve efficient trade. In addition, if the broker is informed, a condition can be similarly derived. [24]

[24] An earlier draft of this paper discusses the situation in which the broker's information is characterized by a partition. I obtain a result similar to Lemma 1.4 by analyzing the information rents that induce agents not to misrepresent information in the same partition.

1.6.3. Relationship to the literature

This paper's main contribution is that trading with an informed broker can improve welfare by alleviating inefficiencies caused by asymmetric information. The paper is related to the literature on how brokers can facilitate trade. Most papers focus on alleviating search frictions, e.g., Rubinstein and Wolinsky (1987) and Duffie, Gârleanu, and Pedersen (2005), among many others. Another strand of literature consider transaction cost. For example, an intermediary can economize on fixed costs (Townsend, 1978). Shi and Siow (2014) demonstrate that an intermediary can reduce showing costs. My paper focuses on how a broker's information can help.

Several other papers show that a broker's information can improve efficiency. Biglaiser (1993) demonstrates that the trade efficiency can be improved by the involvement of perfectly informed middlemen who care about their reputation. Lizzeri (1999) shows a monopolistic certification intermediary, who reveals only whether the quality is above some minimal standard, can improve efficiency. Unlike these models, in my paper the broker does not have perfect information; he is not a certifier, but he helps by intermediating the negotiation. Glode and Opp (2016) show that trade efficiency can be improved by involving an informed broker in a sequential trading game. My paper does not restrict on the trading mechanism.

Following Myerson and Satterthwaite (1983), there is a large literature on how inefficiency can be eliminated in different settings. My paper also contributes to this literature. Myerson and Satterthwaite (1983) show that if the broker can provide a subsidy, then an efficient outcome can be achieved. The broker, in my setting, does not provide a subsidy to achieve efficient trade. Wilson (1985), Rustichini et al. (1994), Reny and Perry (2006), and Cripps and Swinkels (2006) show the inefficiency asymptotically disappears when the number of market participants becomes large. If the good is divisible, Cramton, Gobbons, and Klemperer (1987) show that if agents initially own roughly equal shares of the item, then there exists an efficient mechanism. Unlike these models, I consider bilateral bargaining over an indivisible good, as in Myerson and Satterthwaite (1983). McAfee and Reny (1992) show that an efficient mechanism can exist when the buyer's private valuation is correlated with the seller's, but the broker acts as an budget balancer. In my setting, an efficient mechanism can exist without the broker's losing money after any realization. By relaxing the assumption that utilities are risk neutral, Chatterjee and Samuelson (1983) and Garratt and Pycia (2016) show that efficient trade is possible. Wolitzky (2016) studies maxmin expected utility maximizers and shows that efficient trade can be implemented under the assumption that the agents know each other's expected valuation of the good. In contrast to these models, my paper features a third-party broker in bilateral trading and shows that an informed broker can implement an efficient outcome without subsiding.

1.7. Conclusion

I have studied a bargaining model with an informed broker. I show that a broker's information can improve welfare by alleviating inefficiencies caused by asymmetric information. I provide a sufficient condition such that efficient trade can be achieved. Thus, introduction of an informed broker leads to a significantly different result than that found in Myerson and Satterthwaite (1983), who show the strong negative result that an efficient outcome is impossible without an outsider's subsidy. The positive result I obtain highlights a function brokers perform in facilitating trade.

It would be interesting to understand how imposing a bound on ex post payments affects the performance of bargaining. If there is no such bound, efficient trade can be implemented with a generic informed broker when his information satisfies the McAfee and Reny condition. But ex post payments may be very large when the broker's signals contain little information. In reality, a large payment may not be feasible because of limited liability or budget constraints. If there is a bound on ex post payments, the broker's signals must be sufficiently informative to implement efficient trade. By properly defining a measure of informativeness, it would be interesting to characterize the condition under which there is a bound on the payment. Similarly, in the broker's optimal mechanism, the bound also affects his profit; the full extraction result in Proposition 1.13 cannot hold. It would be interesting to understand the optimal mechanism in the presence of a bound on ex post payments.

CHAPTER 2 : Voluntary Disclosure in Bilateral Transactions[1]

2.1. Introduction

Asymmetric information might harm agents by disrupting socially efficient trade (Akerlof, 1970; Myerson and Satterthwaite, 1983; Glosten and Milgrom, 1985). But why would agents *let* their private information be an impediment to trade in the first place? In this paper, we study the incentives of a privately informed agent to share his information with a counterparty endowed with market power prior to a bilateral transaction.

Recent work by Bergemann, Brooks, and Morris (2015) highlights that information available for price discrimination plays a crucial role in determining the total surplus and its allocation in the classic monopoly pricing problem with private values, raising the important question "what forms of price discrimination will endogenously arise, and for whose benefit." We consider an environment with both private- and common-value uncertainty and analyze one natural channel determining the information a seller with market power can use for price discrimination — the privately informed buyer can make voluntary ex post verifiable disclosures prior to the transaction, as in Grossman (1981), Milgrom (1981), and Shin (2003).

Ex post verifiability is a common restriction in the literature that is imposed to ensure that disclosures are not subject to commitment and incentive problems, even in one-shot interactions.[2] If erroneous disclosures can be verified with probability one and are penalized — perhaps by a regulator or courts — the sender optimally designs signals

[1]This is a joint work with Vincent Glode and Christian Opp.

[2]The early literature analyzing these types of "persuasion games" is surveyed by Milgrom (2008). Since these games focus on ex post verifiable disclosures, they significantly differ from "cheap talk games" popularized by Crawford and Sobel (1982).

that are always truthful. Moreover, the sharing of verifiable information appears to be relevant in many important economic contexts with hard information, such as the trading of financial securities, corporate takeovers, and supply chain transactions.[3]

In this environment, we obtain a surprisingly strong result: the informed agent always designs a partial disclosure plan that yields socially efficient trade in equilibrium. Moreover, this disclosure plan improves both his surplus and that of the counterparty with market power. Whereas possessing superior information allows the informed agent to extract information rents, sharing information reduces the extent to which the agent is being inefficiently screened by his counterparty. We show that the agent is always willing and able to design ex post verifiable signals such that he privately benefits from giving up part of his informational advantage in order to preempt inefficient screening.

We also characterize optimal disclosure plans, which generally pool multiple disjoint intervals of valuations. Although the informed agent benefits from disclosing some information, he finds it privately suboptimal to disclose all information as doing so completely eliminates his information rents. While we initially consider an environment where the disclosure plan is designed before any uncertainty is realized—as is common in models of Bayesian persuasion (Rayo and Segal, 2010; Kamenica and Gentzkow, 2011; Goldstein and Leitner, 2015; Ely, 2017)–we also consider the case where disclosure is chosen after the agent obtains private information (Grossman,

[3]See Boyarchenko, Lucca, and Veldkamp (2016) and Di Maggio, Franzoni, Kermani, and Sommavilla (2016) for empirical evidence consistent with broker-dealers sharing private (deal-flow) information among themselves and with clients, Hong, Kubik, and Stein (2005) and Pool, Stoffman, and Yonkers (2015) for empirical evidence consistent with information sharing among socially connected mutual fund managers, Heimer and Simon (2012) for empirical evidence of information sharing among foreign exchange traders, Eckbo and Langohr (1989) and Brennan (1999) for empirical evidence of information sharing among bidders and target companies in corporate takeovers, and Zhou and Benton (2007) for empirical evidence of information sharing among firms part of the same supply chain.

1981; Milgrom, 1981; Verrecchia, 1983; Shin, 2003). We show that, in line with our earlier analysis, partial disclosure leading to socially efficient trade also characterizes all informed-agent-preferred equilibria of this interim disclosure game.

Given the standard set of assumptions we consider and the strong predictions we obtain — trade is always efficient — our paper also contributes to the literature by characterizing economically relevant conditions that must be *violated* for asymmetric information and market power to impede the efficiency of trade. Our paper thus has important implications for regulating information disclosure in bilateral transactions with imperfect competition and asymmetric information problems, such as corporate takeovers, real estate transactions, and over-the-counter trading.[4] In an environment like ours, a regulator does not need to mandate what information agents should disclose nor does he need to produce additional information for uninformed market participants. All the regulator needs to do is to enforce the truthfulness of disclosure by disciplining agents who send signals that ex post prove to violate their own disclosure standards.

Related Literature. Our paper contributes to the theoretical literature that studies optimal information sharing among traders. An important result in this literature goes back to Grossman (1981) and Milgrom (1981) who show that, when disclosures are restricted to be ex post verifiable, an agent may find it optimal to fully reveal his private information to his counterparties. However, unlike in our model the agent

[4] For empirical evidence that these types of bilateral transactions often feature imperfect competition, see Ambrose, Highfield, and Linneman (2005), Glaeser, Gyourko, and Saks (2005), Boone and Mulherin (2007), King, Osler, and Rime (2012), Atkeson, Eisfeldt, and Weill (2013), Li and Schürhoff (2014), Begenau, Piazzesi, and Schneider (2015), Hendershott et al. (2015), Di Maggio, Kermani, and Song (2016), Li, Taylor, and Wang (2016), and Siriwardane (2016). For empirical evidence that these types of bilateral transactions often involve heterogeneously informed traders, see Eckbo, Giammarino, and Heinkel (1990), Garmaise and Moskowitz (2004), Green, Hollifield, and Schürhoff (2007), Hollifield, Neklyudov, and Spatt (2014), Jiang and Sun (2015), Menkhoff et al. (2016), and Stroebel (2016).

making the disclosure decision is not being screened by counterparties with market power — either all traders take the price as given Milgrom (1981) or it is the informed agent who sets the price Grossman (1981). In those environments, an informed seller always benefits from improving his customers' perception of product quality (which, as we show, is not necessarily the case once counterparties have market power). He then finds it optimal to fully disclose his private information, since any information he withholds is interpreted to be unfavorable Grossman and Hart (1980); Milgrom and Roberts (1986). Verrecchia (1983) modifies this setting by adding disclosure costs and shows that full disclosure may not be optimal, whereas Admati and Pfleiderer (2000) show that a firm may still pick a socially optimal disclosure plan despite disclosure costs if that firm is a monopolist that captures all gains to trade. Shin (2003), Acharya, DeMarzo, and Kremer (2011), and Guttman, Kremer, and Skrzypacz (2014) show that full disclosure also becomes suboptimal once there is uncertainty about the existence of private information. Unlike in these settings, the information designer in our model is the responder to an ultimatum offer and his private information is thus his only source of profits. As a result, his optimal disclosure plan is partial — despite the existence of his private information being common knowledge and disclosure being costless — yet we show that it always yields socially efficient trade in equilibrium. Our framework therefore speaks to how voluntary information sharing can eliminate inefficient rationing in classic monopoly pricing problems.

Monopoly pricing is also studied in Bergemann, Brooks, and Morris (2015) who analyze how signals providing monopolists with additional information for price discrimination affect total surplus and its allocation. Bergemann, Brooks, and Morris (2015) show in a setting with private value uncertainty that general information structures (including randomization) exist such that total surplus can be increased to any level less than or equal to the one from efficient trade, and any allocation of the incremental surplus is attainable. Information available for price discrimination thus critically determines efficiency and the allocation of surplus, raising the question what information a monopolist will obtain endogenously. Our paper sheds light on this question, but also has other objectives. We analyze a setting with both private- and common-value uncertainty and show that when information disclosure by the informed agent is (a) voluntary and (b) ex post verifiable (with randomization not being possible), precise predictions for both total surplus and its allocation obtain: (i) total surplus is unique and equal to the surplus generated by efficient trade (whether the disclosure plan is designed at an ex ante or interim stage), and (ii) the surplus allocation is unique and both agents weakly benefit from the optimal disclosure plan.

More broadly, our focus on market power also relates our paper to Gal-Or (1985) who models oligopolistic firms that can commit ex ante to sharing noisy signals of their private information about the uncertain demand for their products. Since sharing information increases the correlation of firms' output decisions, thereby lowering their expected profits, the unique symmetric pure-strategy equilibrium is characterized by no information sharing among firms. Lewis and Sappington (1994) investigate in a setting without disclosure whether an uninformed seller with market power would like to help his prospective buyer(s) acquire private information about the value of the asset [who assume that trading occurs through an auction] Eső and Szentes (2007). Under general conditions, the seller in Lewis and Sappington (1994) either wants his prospective buyer(s) to be fully informed or completely uninformed about how much the asset is worth to them. Finally, Roesler and Szentes (2017) solve for a buyer's optimal information acquisition in a monopoly setting without disclosure and show that the buyer finds it optimal to limit his information acquisition, avoiding that the monopolist seller inefficiently screens him Glode, Green, and Lowery (2012).

The next section presents the classic problem of a monopolist who inefficiently screens a privately informed agent. In Section 2.3, we study the agent's incentives to share some of his private information with the monopolist and how the resulting disclosure plan affects the efficiency of trade. Section 2.4 shows that our main insights survive when the agent designs his disclosure plan after obtaining private information rather than before. The last section concludes. All proofs are collected in Appendix A.

2.2. The Bilateral Transaction

The monopolist seller of an asset (or good) chooses the price he will quote to a prospective buyer (or customer) in a take-it-or-leave-it offer.[5] The seller is uncertain about how much the buyer is willing to pay for the asset but knows that the buyer's valuation of the asset, which we denote by v, has a cumulative distribution function (CDF) denoted by $F(v)$. The buyer only accepts to pay a price p in exchange for the asset if $v \geq p$; otherwise, the seller must retain the asset, which is worth $c(v) \geq 0$ to him. The CDF $F(v)$ is continuous and differentiable and the probability density function (PDF), denoted by $f(v)$, takes strictly positive values everywhere on the support $[v_L, v_H]$.[6] The function $c(v)$ is assumed to be weakly increasing and continuous. Both agents are risk neutral and the functions $F(v)$ and $c(v)$ are common knowledge.[7]

Whenever the buyer's valuation is greater than the seller's — perhaps due to heterogeneity in preferences, in inventories, or in liquidity needs — trade would create a social surplus. However, the seller may find it privately optimal to use his market

[5] The buyer/seller roles could be reversed without affecting our results.

[6] As should become clear later, our results would also hold if the support of v was unbounded above, whereas if it was unbounded below, our results would hold as long as $\lim_{v \to -\infty} v - c(v) \leq 0$.

[7] See Hirshleifer (1971), Diamond (1985), and Kurlat and Veldkamp (2015), among many others, for the costs and benefits of disclosure linked with traders' risk aversion.

power and inefficiently screen the informed buyer, thereby jeopardizing the gains to trade $[v-c(v)]$. We assume that whenever indifferent between two strategies, an agent picks the one that maximizes the social surplus in the resulting subgame-perfect Nash equilibrium.

The seller's expected payoff from quoting a price p is given by:

$$\Pi(p) = [1 - F(p)]p + F(p)\mathbb{E}[c(v)|v < p]. \tag{2.1}$$

When picking a price, the seller considers the trade-off between the probability that a sale occurs and the profit he gets if a sale occurs. The seller's marginal profit of increasing the price p is:

$$\Pi'(p) = [1 - F(p)] - f(p)p + f(p)\mathbb{E}[c(v)|v < p] + F(p)\frac{\partial}{\partial p}\mathbb{E}[c(v)|v < p], \tag{2.2}$$

which simplifies to:

$$\Pi'(p) = [1 - F(p)] - f(p)[p - c(p)]. \tag{2.3}$$

The first term on the right-hand side of equation (2.3) is the seller's marginal expected benefit from collecting a higher price when trade occurs. The second term is the marginal expected cost from reducing the probability of trade and destroying gains to trade. We impose the following condition on the surplus from trade $[v - c(v)]$:

Assumption 2.1. *The surplus from trade $[v - c(v)]$ crosses zero at most once (from below).*

This condition is implied by any of the following assumptions common in the literature: (i) the seller's valuation for the asset is a constant; (ii) the seller's valuation for the asset is a fraction of v ; (iii) the surplus from trade $[v - c(v)]$ is a constant;

(iv) the ratio of the above-mentioned cost and benefit of marginally increasing the price, i.e., $\frac{f(v)}{1-F(v)}[v-c(v)]$, is strictly increasing in v.[8] Any one of these fairly standard assumptions is sufficient to obtain our results.

Assumption 2.1 implies that we can designate a cutoff $\hat{v} \in [v_L, v_H]$ such that trade is socially efficient when it occurs if and only if $v \geq \hat{v}$. Since it is possible under Assumption 2.1 that $[v-c(v)]$ remains at zero for a positive-measure subset of $[v_L, v_H]$ and then becomes positive for higher values of v, we define the relevant cutoff as $\hat{v} \equiv \inf\{v \in [v_L, v_H] : v > c(v)\}$. Since $f(v)$ is strictly positive everywhere on the support $[v_L, v_H]$, the maximum price the seller can quote and still maintain socially efficient trade is $p = \hat{v}$. As a result, trade can be efficient only if:

$$\Pi'(\hat{v}) \leq 0. \tag{2.4}$$

This necessary condition for efficient trade can be interpreted as follows. Efficient trade requires that $\hat{v} - c(\hat{v}) \geq \frac{1-F(\hat{v})}{f(\hat{v})}$, which means that either the gains to trade are large, or that the seller's beliefs about v are concentrated (i.e., the density $f(v)$ is high enough) when the surplus from trade becomes positive. If instead $\Pi'(\hat{v}) > 0$, the seller inefficiently screens the buyer and jeopardizes the gains to trade. Moreover, the seller never quotes a price $p < \hat{v}$, because quoting a price $p = \hat{v}$ yields strictly higher profits.[9]

[8] See, e.g., Glode and Opp (2016) and Glode, Opp, and Zhang (2016) who specifically impose this condition, Fuchs and Skrzypacz (2015) who define a "strictly regular environment" in a similar way and Myerson (1981) who similarly assumes that bidders' virtual valuation functions are strictly increasing.

[9] Specifically, if the seller quotes a price $p < \hat{v}$, his expected payoff can be written as:

$$\Pr(v \geq \hat{v})p + \Pr(p \leq v < \hat{v})p + \Pr(v < p)\mathbb{E}[c(v)|v < p].$$

In contrast, if the seller quoted a price $\hat{v}$, his payoff would increase by $\hat{v} - p > 0$ when $v \geq \hat{v}$, by $c(v) - p \geq 0$ when $p \leq v < \hat{v}$ and would remain the same when $v < p$.

Also note that, since we assume that whenever indifferent, an agent picks the strategy that maximizes social surplus, we can rule out any equilibrium where the seller inefficiently mixes between quoting multiple prices $p_n \in [v_L, v_H]$. If he were to mix over several prices, the seller would have to be indifferent between mixing and quoting any of these prices with probability one (taking into account the buyer's best response to each price). The tie-breaking rule implies that the seller instead plays the pure strategy of quoting the price that socially dominates all other prices. Similarly, we can rule out equilibria where the buyer inefficiently mixes between accepting and not accepting a price quote. A tie-breaking rule based on social optimality thus ensures that we can restrict our attention to pure-strategy subgame-perfect Nash equilibria in our model.

We further illustrate the seller's incentives to set inefficient prices through a simple parameterized example that we will revisit later.

Example 2.1. *Suppose the buyer values the asset at $v \sim U[1,2]$ and the seller values it at a constant $c \leq 1$. The surplus from trade is then always positive (i.e., $\hat{v} = 1$) and trade is efficient if and only if it occurs with probability 1. The seller's optimization problem when picking a price can be written as:*

$$\max_{p \in [1,2]} \Pi(p) = \Pr(v \geq p)p + \Pr(v < p)c = (2-p)p + (p-1)c. \tag{2.5}$$

Since $\Pi'(1) = c$, the seller quotes a price $p = 1$ whenever $c \leq 0$ and the buyer always accepts, implying that trade is efficient. However, when $c \in (0,1]$ the seller finds it optimal to quote a price $p = 1 + \frac{c}{2}$, which destroys the surplus from trade with probability $\frac{c}{2}$.

The example above shows a simple case where $\hat{v} = v_L$, that is, the surplus from trade is positive for any realization of v. In cases like that, efficient trade requires that $v_L - c(v_L) \geq \frac{1}{f(v_L)}$. For cases where $\hat{v} \in (v_L, v_H)$ however, efficient trade can never be sustained in equilibrium since $\hat{v} - c(\hat{v}) = 0 < \frac{1-F(\hat{v})}{f(\hat{v})}$, implying that the seller always finds it optimal to quote a price that is at least marginally higher than the efficient price $p = \hat{v}$. This situation arises, for example, whenever the seller values the asset at a constant $c \in (v_L, v_H)$.

2.3. Information Disclosure prior to Trading

In this section, we analyze the buyer's decision to share a subset of his information with the seller before trade occurs. Clearly, if trade is already socially efficient without disclosure, the buyer is only paying $\hat{v}$ for the asset and information disclosure is suboptimal — with additional information the seller might raise the price he quotes, but he would never lower it. Thus, for the remainder of the paper we focus on situations where trade would be socially inefficient if the buyer did not disclose any of his private information. Sharing information might hurt the buyer since possessing private information yields informational rents, but it might also reduce the seller's incentives to charge inefficient mark-ups that reduce the expected gains from trade.

For now, we assume that the agent must design his disclosure plan prior to acquiring private information, and that he can commit to not manipulating the signal later, as is common in models of Bayesian persuasion. Assuming that the buyer is uninformed at the time of the information design facilitates our analysis as it eliminates the existence of signaling concerns. We relax this assumption in Section 2.4. We also restrict our attention to ex post verifiable disclosures or signals, as in Grossman (1981), Milgrom (1981), and Shin (2003).

In practice, the ex ante design of such disclosure plans is likely relevant in economic contexts with hard information. In a variety of industries, information is shared automatically between firms via information technology (IT) systems according to pre-determined algorithms. For example, firms in the same supply chain are typically connected to a common IT system that automatically shares information about inventories and production problems. Similarly, in the context of financial markets, hedge funds systematically share financial data (e.g., holdings and performance data) with broker-dealers and clients, which reduces information asymmetries about trading motives, such as liquidity needs (see, also, footnote 3).

In the following, we formally define *ex post verifiability* in the context of our model.

Definition 2.1. *A signal s whose realization belongs to a set S is called "ex post verifiable" if it can be represented by a function $g : [v_L, v_H] \rightarrow S$ such that $g^{-1}(s) \equiv \{v : g(v) = s\} \in \mathcal{B}([v_L, v_H])$, where $\mathcal{B}([v_L, v_H])$ denotes the Borel algebra on $[v_L, v_H]$.*

This definition implies that for any signal $s \in S$, $g^{-1}(s)$ is a Borel set in $[v_L, v_H]$. Since a Borel set of $[v_L, v_H]$ must be characterized by unions of intervals, designing a disclosure plan implies combining partitions to inform the seller about possible realizations of v. If the buyer sends a signal, the seller must be able to confirm once uncertainty about v is resolved that the true realization of v was indeed possible given the signal sent. Signals that are subject to additional random shocks (due to "noise" components or randomization) are thus ruled out by ex post verifiability. This restriction is common in the literature on disclosure Verrecchia (2001); Milgrom (2008); Beyer, Cohen, Lys, and Walther (2010) and strikes us as a natural one given the assumption that the "sender" of the information does not manipulate his signal, as is common in the literature on persuasion games. In a more general setting with verifiable disclosures, erroneous disclosures could be penalized heavily, providing the sender with the incentives to indeed send signals that are truthful, even when manipulation is a feasible action.[10]

Before going further, we summarize the timeline in our baseline model. First, the buyer designs a disclosure plan to send ex post verifiable signals to the seller. Then the buyer learns his private valuation and the seller receives a signal consistent with the chosen disclosure plan. Finally, the seller quotes a price and the buyer decides whether to accept or not. We can now state our main result.

Proposition 2.1. *If the buyer can commit to any disclosure plan that sends ex post verifiable signals to the seller, he designs a partial disclosure plan that yields socially efficient trade.*

Proposition 2.1 states that if private information can only be shared in a verifiably truthful manner, the incentives of the privately informed buyer, who is being screened by the seller, are aligned with social welfare. By sharing a subset of his information with the seller, the buyer is making sure that he will be quoted more efficient prices, which leads to incremental surplus from trade that is shared between both agents. An optimal disclosure plan thereby ensures that the buyer can increase his information rents even though it reduces his informational advantage. The proposition also reveals that it is never optimal for the buyer to share all his information with the seller, as such a disclosure plan would drive the buyer's rents to zero. Unlike in Grossman (1981) where full disclosure is optimal, the informed trader in our model does not have market power and can only extract rents if he conceals some information from his counterparty. We now return to our earlier parameterized example to illustrate this new result.

[10] Due to the absence of noise, penalties would then remain off-equilibrium — penalties would only be triggered if the sender intentionally violates the standards set by his own disclosure plan.

Example 2.2. *As in Example 2.1, we assume the buyer values the asset at* $v \sim U[1,2]$ *and the seller values it at a constant* $c \leq 1$*. We have already shown that since* $c \in (0,1]$ *the seller quotes a price* $p = 1 + \frac{c}{2}$*, which destroys the gains to trade with probability* $\frac{c}{2}$*. The buyer acquires the asset whenever* $v \geq p$ *and he collects an expected profit of:*

$$\Pr\left(v \geq 1 + \frac{c}{2}\right)\left[E\left(v|v \geq 1 + \frac{c}{2}\right) - \left(1 + \frac{c}{2}\right)\right] = \frac{(2-c)^2}{8}. \tag{2.6}$$

Consider what happens if the buyer promises to share some of his information with the seller, for example, by disclosing whether $v \in \left[1, 1 + \frac{c}{2}\right)$ *or* $v \in \left[1 + \frac{c}{2}, 2\right]$*. The seller's optimization problem when quoting a price to the buyer now depends on the realization of the signal. If the seller learns that* $v \geq 1 + \frac{c}{2}$*, his optimization problem becomes:*

$$\max_{p \in \left[1+\frac{c}{2},2\right]} \Pr\left(v \geq p|v \geq 1 + \frac{c}{2}\right)p + \Pr\left(v < p|v \geq 1 + \frac{c}{2}\right)c = \left(\frac{2-p}{1-\frac{c}{2}}\right)p + \left(\frac{p-(1+\frac{c}{2})}{1-\frac{c}{2}}\right)c, \tag{2.7}$$

and if instead he learns that $v < 1 + \frac{c}{2}$*, it becomes:*

$$\max_{p \in \left[1,1+\frac{c}{2}\right)} \Pr\left(v \geq p|v < 1 + \frac{c}{2}\right)p + \Pr\left(v < p|v < 1 + \frac{c}{2}\right)c = \left(\frac{1+\frac{c}{2}-p}{\frac{c}{2}}\right)p + \left(\frac{p-1}{\frac{c}{2}}\right)c. \tag{2.8}$$

In the first case, it is easy to verify that the seller finds it optimal to quote $p_h = 1+\frac{c}{2}$*, just as he did without disclosure. However, in the second case, the seller finds it optimal to quote* $p_l = \max\{\frac{1}{2} + \frac{3}{4}c, 1\}$*. Under this disclosure plan, the buyer collects an expected profit of:*

$$\begin{aligned} &\Pr\left(v \geq 1 + \frac{c}{2}\right)\left[E\left(v|v \geq 1 + \frac{c}{2}\right) - \left(1 + \frac{c}{2}\right)\right] \\ + \ &\Pr\left(p_l \leq v < 1 + \frac{c}{2}\right)\left[E\left(v|p_l \leq v < 1 + \frac{c}{2}\right) - p_l\right]. \end{aligned} \tag{2.9}$$

The first term is equal to the expected profit the buyer would collect without disclosure. The second term is the additional expected profit the buyer is able to collect with *disclosure, which is strictly positive whenever $c > 0$. Thus, the buyer is strictly better off under this disclosure plan than without any disclosure. Moreover, if $c \leq \frac{2}{3}$ the seller quotes $p_l = 1$ when $v < 1 + \frac{c}{2}$, which implies that trade is efficient regardless of the signal realization.*

If $c > \frac{2}{3}$ however, the seller quotes $p_l = \frac{1}{2} + \frac{3}{4}c$ when $v < 1 + \frac{c}{2}$, which leads to a higher probability of trade than without disclosure, but still causes trade to break down with positive probability. The proof of Proposition 2.1 shows that in cases like this, a similar reasoning can be applied again to construct an alternative disclosure plan that splits the region of inefficient trade $\left[1, 1 + \frac{c}{2}\right)$ into $\left[1, \frac{1}{2} + \frac{3}{4}c\right)$ and $\left[\frac{1}{2} + \frac{3}{4}c, 1 + \frac{c}{2}\right)$, such that the buyer is strictly better off and trade is more efficient than under the first disclosure plan. Hence, given any proposed disclosure plan that does not yield efficient trade, it is always possible to construct a more efficient disclosure plan that strictly dominates from the buyer's perspective.

Before solving for the optimal disclosure plan in our parameterized example, we analyze the tradeoff the buyer faces when designing the signals he will share with the seller under a general CDF $F(\cdot)$.

Consider the decision of a buyer to pool or separate two generic intervals in a disclosure plan $g(v)$. Let $A \equiv [a_L, a_H)$ and $B \equiv [b_L, b_H)$ denote these two intervals, where $b_L \geq a_H$ and $a_L \geq \hat{v}$. Specifically, the buyer decides whether his disclosure plan $g(v)$ should generate separate signals, s_a when $v \in A$ and s_b when $v \in B$, or whether it should pool these regions, such that there is only one signal s_{ab} indicating that $v \in A \cup B$. Since we know from Proposition 2.1 that the optimal disclosure plan always leads to efficient trade, we now focus on a situation where the seller quotes an efficient price after receiving any of the two separating signals. When the disclosure plan generates signals separating the two intervals, the signal-dependent price quote, which we denote as $x(s)$, must satisfy the following two conditions to allow for efficient trade: $x(s_a) = a_L$ and $x(s_b) = b_L$. The buyer then expects to collect a surplus of:

$$\int_{a_L}^{a_H} (v - a_L)dF(v) + \int_{b_L}^{b_H} (v - b_L)dF(v). \tag{2.10}$$

If instead the disclosure plan generates a pooling signal s_{ab}, the buyer's expected surplus depends on the seller's response to the disclosure, that is, the price he quotes after receiving a signal that $v \in A \cup B$. If this price is weakly greater than $x(s_b) = b_L$ then the buyer is clearly strictly better off sending separating signals. Yet, if the seller responds to the disclosure by quoting a price $p \in [a_L, a_H)$,[11] the buyer expects to collect a surplus of:

$$\int_{p}^{a_H} (v - p)dF(v) + \int_{b_L}^{b_H} (v - p)dF(v). \tag{2.11}$$

The net benefit of pooling can then be written as:

$$\begin{aligned} & \left[\int_{p}^{a_H} (v - p)dF(v) + \int_{b_L}^{b_H} (v - p)dF(v)\right] - \left[\int_{a_L}^{a_H} (v - a_L)dF(v) + \int_{b_L}^{b_H} (v - b_L)dF(v)\right] \\ = \; & (b_L - p)[F(b_H) - F(b_L)] - (p - a_L)[F(a_H) - F(p)] - \int_{a_L}^{p} (v - a_L)dF(v). \end{aligned} \tag{2.12}$$

By pooling the regions, the buyer lowers the price he is being quoted when $v \in B$, but he might also increase the price he is quoted when $v \in A$.

When a pooling signal s_{ab} is generated by the disclosure plan, the condition for

[11] When $a_H < b_L$, it is never optimal for the seller to quote a price $p \in [a_H, b_L)$ as quoting b_L always dominates any of these prices — a price quote of b_L is accepted with the same probability, but the price paid is strictly higher.

efficient trade is:

$$a_L - c(a_L) \geq \frac{\Pr(v \in A \cup B)}{f(a_L)}, \tag{2.13}$$

which is strictly more restrictive than the corresponding condition when a separating signal s_a is generated:

$$a_L - c(a_L) \geq \frac{\Pr(v \in A)}{f(a_L)}. \tag{2.14}$$

As a result, the seller is more likely to deviate to a price $p > a_L$ once the regions A and B are pooled. Yet, as long as pooling the two regions still allows for efficient trade, the benefit of pooling from equation (2.12) simplifies to:

$$(b_L - a_L)[F(b_H) - F(b_L)] > 0, \tag{2.15}$$

implying that the buyer is strictly better off sending a pooling signal s_{ab}.

For similar reasons, the buyer may benefit from pooling disjoint intervals that are "far" away from each other. When regions A and B are far apart, the buyer receives large information rents from obtaining an asset at a price of $p = a_L$ when valuing the asset at $v \in B$ with positive probability. An optimal disclosure plan might therefore pool intervals of v that are separated by gaps. On the other hand, this type of pooling tends to increase the seller's incentives to quote a high, inefficient price.

Overall, when designing a disclosure plan, the buyer thus aims to pool multiple intervals of v in order to minimize the requested price, but he is also concerned with the seller's potential response to screen him, which leads to inefficient rationing. In order to fully characterize the buyer's optimal disclosure plan, we return to our parameterized example.

Example 2.3. *In earlier analyses of the parameterized example with $v \sim U[1, 2]$,* *we analytically showed that the buyer is better off disclosing a subset of his private information whenever trade is otherwise inefficient. However, solving for the buyer's optimal disclosure plan is a complex problem that involves functional optimization that generally does not admit closed-form solutions. We thus rely on numerical methods and discretize the interval* $[1, 2]$ *equally using* $n = 11$ *points denoted by* v_i. *(We discuss below how our results generally apply to discrete distributions.) Given each possible disclosure plan, the optimal price quote* $x(s)$ *is equal to one of the* n *possible realizations for* v. *The buyer's optimization problem effectively aims to pool possible sets of* v_i *to minimize the expected transaction price while ensuring that trade remains socially efficient. Since the choice variables are integers and the system is linear, this is an integer linear programming problem. Figure 1 shows a solution to the optimal disclosure problem when* $c = 0.5$.

The buyer finds it optimal to split the interval $[1, 2]$ *into two combinations of sub-intervals. Figure 1 also shows that the signal structure involves gaps between these subintervals, allowing the buyer to pay low prices and to extract large information rents. When the seller receives a signal that* v *belongs to the lower/darker combination of sub-intervals, he responds by quoting a price* $p = 1$. *Panel (a) of Figure 2 illustrates the seller's pricing problem conditional on receiving the low signal. The seller's conditional expected payoff from owning the asset is maximized by quoting either a price* $p = 1$, *or a less efficient price* $p = 1.5$. *When the seller instead receives a signal that* v *belongs to the higher/paler combination of sub-intervals, quoting a price* $p = 1.2$ *maximizes his conditional expected payoff as shown in Panel (b) of Figure 2. In both cases, these price quotes are equal to the lowest possible realizations of* v, *given the signal, and as a result the buyer always accepts them.*

The low signal is generated with probability 0.36 *and results in the buyer paying* 1

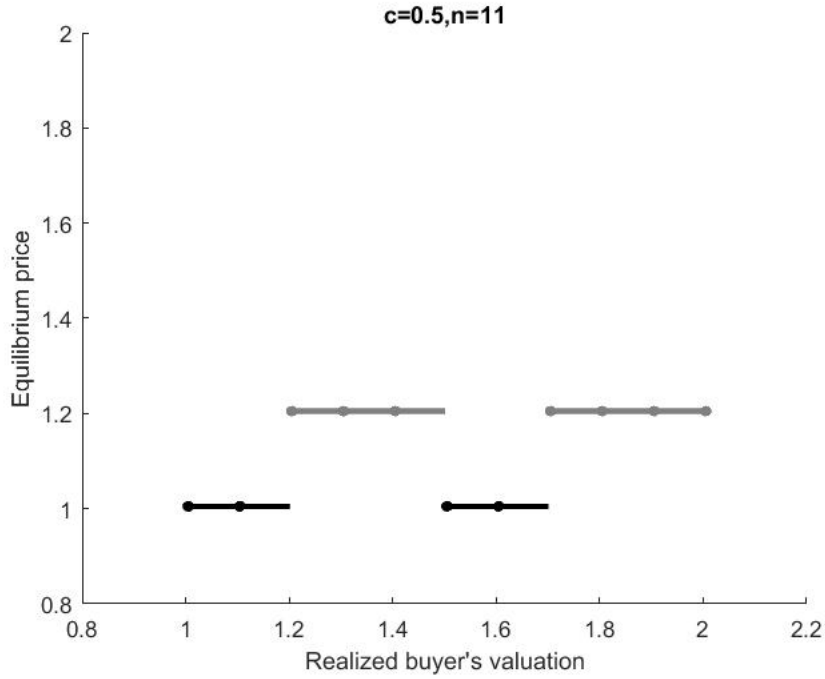
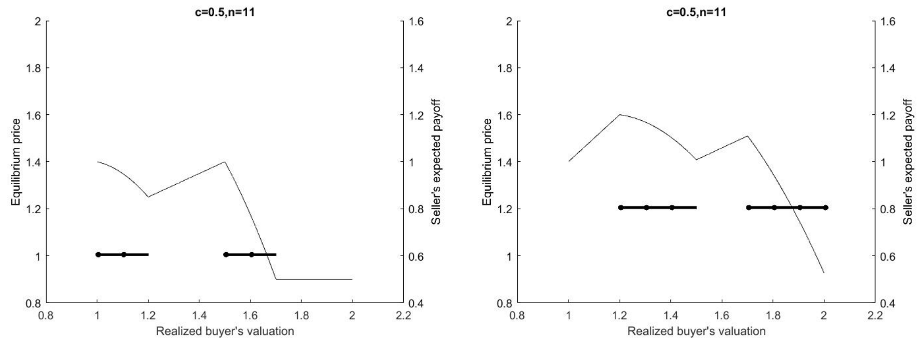


Figure 4: **Optimal disclosure plan when $c = 0.5$ and $v \sim U[1, 2]$.** In this parameterization, the buyer finds it optimal to release two signals. When the realization of v belongs to the lower (and darker) combination of sub-intervals the seller receives a signal that leads him to quote a price 1. Otherwise, the seller receives a signal that leads him to quote a price of 1.2. Trade is socially efficient under this disclosure plan.

in exchange for an asset worth, on average, 1.3 *to him. The high signal arises with probability* 0.64 *and results in the buyer paying* 1.2 *in exchange for an asset worth, on average,* 1.6 *to him. Overall, the buyer collects an expected surplus of* 0.37*, whereas the seller collects an expected surplus of* 0.63 *under this disclosure plan. The expected social surplus is* 1*, since trade occurs with probability* 1 *and the seller values the asset at* $c = 0.5$*. In contrast, without disclosure the seller would have quoted an inefficient price* $p = 1.25$*, resulting in an expected surplus of* 0.56 *for the seller and an expected surplus of* 0.28 *for the buyer. Thus, both agents benefit from the buyer's optimal disclosure plan. Note that there exist alternative disclosure plans that deliver identical payoffs to all agents. Hence the equilibrium disclosure plan is not unique.*



(a) Low signal. (b) High signal.

Figure 5: **Seller's expected payoff under the disclosure plan illustrated in Figure 4.** The seller's expected payoff from quoting a price $p \in [1, 2]$ is plotted as a thin solid line in both panels (values are indicated on the right axis). These expected payoffs depend on the signal, and thus vary across the two panels. When the seller receives the first signal (left panel), he maximizes his expected payoff by quoting a price $p = 1$. When he receives the second signal (right panel), he maximizes his expected payoff by quoting a price $p = 1.2$.

As featured in the example above, our restriction that disclosure plans must be ex post verifiable still allows for the design of signals that pool multiple disjoint intervals.[12] Such disclosure plans can allow the buyer to minimize the average price paid while preventing the seller from quoting prices that cause inefficient rationing. We note, however, that the proof of Proposition 2.1 does not rely on allowing for disjoint intervals, or equivalently, non-monotonic signal structures. If the buyer can only design ex post verifiable signals associated with connected intervals, our result that the buyer's optimal disclosure plan always leads to socially efficient trade still holds.

Example 2.4. *In Appendix B.2, we characterize in closed form the optimal disclosure plan for a buyer constrained to pick ex post verifiable signals associated with connected intervals. Using these derivations, we can show that the disclosure plan in Figure 3 is a solution to the optimal disclosure problem when* $c = 0.5$.

[12] Non-monotonicity is also a property of the signal function of the optimal disclosure plan in Goldstein and Leitner (2015) who study an information design problem for a regulator who observes banks' stress test results and wants to maximize banks' funding.

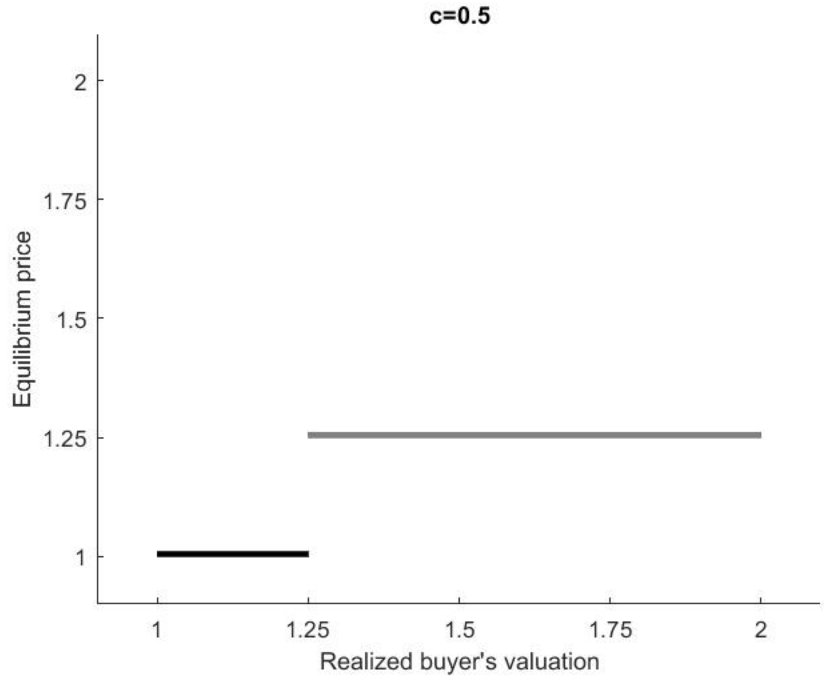


Figure 6: **Optimal disclosure plan with connected intervals when $c = 0.5$ and $v \sim U[1, 2]$.** In this parameterization, the buyer finds it optimal to release two signals. When the realization of v belongs to the lower (and darker) sub-interval the seller receives a signal that leads him to quote a price of 1. Otherwise, the seller receives a signal that leads him to quote a price of 1.25. Trade is socially efficient under this disclosure plan.

The optimal disclosure plan splits the interval $[1, 2]$ *into two sub-intervals:* $v \in [1, 1.25)$ *and* $v \in [1.25, 2]$*. After both signals, the seller quotes the lowest possible realization of* v *given the disclosure and the buyer always accepts. Overall, the buyer collects an expected surplus of* 0.31*, whereas the seller collects an expected surplus of* 0.69 *under this disclosure plan. Again, both agents benefit from the buyer's optimal disclosure plan. Moreover, the expected social surplus is* 1*, since trade occurs with probability* 1 *and the seller values the asset at* $c = 0.5$*.*

It is also worth emphasizing that the proof of Proposition 2.1 can easily be adapted to discrete distributions of v. As mentioned above, our main insight is that whenever a disclosure plan leads to inefficient trade, the buyer can design an alternative disclosure plan that improves the efficiency of trade and allows him to extract additional surplus from states where trade would have failed under the original disclosure plan. This intuition still holds with a discrete distribution of v, except that there may exist cases where the alternative, more efficient disclosure plan makes the buyer only weakly better off, rather than strictly better off as in our baseline model with a continuous distribution.[13] Under the tie-breaking rule mentioned above (i.e., whenever indifferent an agent takes the action that maximizes social surplus), the buyer's optimal disclosure plan still always leads to socially efficient trade when the distribution of v is discrete. For similar reasons, however, the optimal disclosure plan may then be fully revealing for some parameterizations (for example, when v can only take one of two values and trade would be inefficient without disclosure). In Appendix B.2 we provide a simple numerical example of optimal disclosure with a discrete distribution.

2.4. Interim Disclosure

In the previous section, we assumed that the buyer designs the disclosure plan prior to obtaining private information. We now discuss the robustness of our results to "interim" disclosure, that is, disclosure that is chosen after the buyer obtains private information, but before the realization of v becomes publicly observable. Specifically, the timeline of the sequential game we now study is as follows. First, the buyer privately observes v. Second, he designs an ex post verifiable signal that he sends to the seller. Finally, the seller quotes a price and the buyer decides whether to accept or reject. Below we show that in any "buyer-preferred" equilibrium, information disclosure is partial and leads to socially efficient trade, just as in the baseline setting

[13]Specifically, while the second term of equation (A7) in the proof of Proposition 2.1 is strictly positive when v is continuously distributed with strictly positive density everywhere on the support, this term may occasionally take a value of 0 when the distribution is discrete.

(see Proposition 2.1).

We start by describing each agent's strategy profile. Consistent with earlier notation, let $g(v)$ denote the signal that the buyer sends when his valuation for the asset is v. In the context of interim disclosure (where the buyer does not commit ex ante to a mapping between realizations of v and signals), ex post verifiability requires that any signal $s = g(v)$ is itself a Borel set in $[v_L, v_H]$ and that $v \in g(v)$ for any v. Since $g(v)$ is now designed by the buyer after he observes v, we can interpret $g(v)$ as the pure-strategy message that the buyer sends in this signaling game Bertomeu and Cianciaruso (2016). Upon receiving a signal s, the seller forms a belief about the buyer's types (i.e., valuations), which we denote as $\mu(s) \in \Delta([v_L, v_H])$.[14] Then the seller quotes a price $x(s)$ that maximizes his expected profit, and the buyer decides whether to accept. A buyer's optimal strategy in that last stage is simply to accept the offer if and only if the quoted price is weakly less than his true valuation. For ease of exposition, we do not introduce extra notation for this final stage and directly impose that the buyer follows this dominant strategy.

To summarize, we now consider a signaling game where the buyer sends a message and the seller chooses an action based on that message. We dub this signaling game as the *interim disclosure game*. We can now state the definition of an equilibrium in this setting.

Definition 2.2. *A $(g(\cdot), \mu(\cdot), x(\cdot))$ profile forms a perfect Bayesian equilibrium of the interim disclosure game if:*

1. *For every possible signal s, $x(s)$ solves $\max_p \pi(p, s)$, where $\pi(p, s)$ denotes the seller's expected profit if he quotes a price p and the buyer's valuation is drawn*

[14] We use $\Delta([v_L, v_H])$ to denote the set of all possible probability distributions on $[v_L, v_H]$.

from $\mu(s)$.

2. *For every $v \in [v_L, v_H]$, $g(v)$ solves $\max_s \max(v - x(s), 0)$, where $v \in g(v)$.*

3. *For every s in the range of g (that is, every Borel set s that can be disclosed in equilibrium), the seller's belief function $\mu(s)$ is obtained by applying Bayes' rule given the particular signal s.*

Since beliefs are unrestricted following off-equilibrium deviations, there exist beliefs such that the seller (who has market power) drives the buyer's information rents to zero following any off-equilibrium deviation in disclosure. This leads to the existence of multiple perfect Bayesian equilibria with various degrees of information revelation, as opposed to a unique equilibrium with full revelation as in Grossman (1981) and Milgrom (1981) [for a broader discussion of equilibrium multiplicity when the information designer picks a signal structure after acquiring private information]Perez-Richet (2014). For instance, either full disclosure, partial disclosure, or no disclosure can be supported in equilibrium if the seller has the following beliefs: if for any s not in the range of g (that is, whenever s is an off-equilibrium signal), the belief $\mu(s)$ assigns probability 1 to type $\bar{v}(s)$, where $\bar{v}(s) \equiv \sup s$ (recall that s is a Borel set).[15]

Given the multiplicity of equilibria, we focus on buyer-preferred equilibria in order to capture the spirit of our previous setting where the buyer moved first. What it means for the buyer to "prefer" an equilibrium is now complicated by the fact that he can be of many types when designing the disclosure plan. Thus, we define as buyer-preferred equilibria the set of equilibria that are not dominated among buyer types (in the Pareto sense) by another equilibrium based on their interim payoffs.

[15] An equilibrium is said to feature full disclosure if $\mu(g(v))$ assigns probability 1 to type v, whereas it is said to feature no disclosure if $g(v) = [v_L, v_H]$ for all $v \in [v_L, v_H]$, and thus $\mu([v_L, v_H])$ is equal to $F(v)$, the prior distribution of v.

As in Riley (1979), we are treating different informed-agent types as distinct players and looking for equilibria in which none of these types can be strictly worse off, while all the other types are as well off as in an alternative equilibrium. We conclude our analysis by stating the main result for this section.

Proposition 2.2. *In any buyer-preferred equilibrium of the interim disclosure game, the buyer's optimal disclosure is partial and yields socially efficient trade.*

In Appendix B.3, we show the robustness of this result to an alternative equilibrium refinement known as Grossman-Perry-Farrell.[16]

2.5. Conclusion

We analyze the optimal disclosure strategy of a privately informed agent who faces a counterparty endowed with market power in a bilateral transaction. While disclosures reduce the agent's informational advantage, they may increase his information rents by mitigating the counterparty's incentives to inefficiently screen the agent. We show that when disclosures are restricted to be ex post verifiable, the privately informed agent always finds it optimal to design a partial disclosure plan that implements socially efficient trade in equilibrium. Moreover, our analysis shows how the optimal disclosure plan maximizes the privately informed agent's rents by potentially pooling multiple disjoint intervals.

Our paper speaks to the fundamental origins of asymmetric information problems that impede efficient trade under imperfect competition. The type of information agents privately observe (i.e., hard vs. soft, verifiable vs. unverifiable, transaction-specific vs. valuable in multiple transactions) and the enforceability of truthfulness

[16]We adopt the terminology "Grossman-Perry-Farrell" from Gertner, Gibbons, and Scharfstein (1988), Lutz (1989), and Bertomeu and Cianciaruso (2016).

(due to the regulatory and legal environments) greatly matter for determining the extent to which efficient trade is impeded despite voluntary disclosure.

In particular, if information is ex post verifiable and if truthfulness is enforced, bilateral trade should be socially efficient even though the buyer and seller are asymmetrically informed when they meet. Moreover, our model assumes that traders' private information pertains only to the bilateral transaction considered and has no value outside of bargaining. If these conditions are violated however, social efficiency might require the involvement of informed intermediaries Biglaiser (1993); Li (1998); Glode and Opp (2016); Zhang (2016), or some other external intervention. Our insights thus have important implications for regulating information disclosure in bilateral transactions. In our model, a regulator would not need to mandate what information agents must disclose, nor would it need to produce additional information for uninformed market participants. The regulator should instead focus on enforcing the truthfulness of disclosures by disciplining agents who send signals that ex post prove to violate their own disclosure standards. Agents would then have incentives to share their private information in ways that maximizes the social efficiency of trade.

CHAPTER 3 : On the Efficiency of Long Intermediation Chains[1]

3.1. Introduction

Long intermediation chains (i.e., sequential trading of an asset by several intermediaries) can be observed in many decentralized markets. For example, Li and Schürhoff (2014) report that 10% of municipal bond transactions involve a chain of 3 or more intermediaries. In the market for securitized products, Hollifield, Neklyudov, and Spatt (2014) find that transactions sometimes involve up to 10 intermediaries. Shen, Wei, and Yan (2015) show that the average transaction in the corporate bond market involves 1.81 intermediaries and that chains in the 99th percentile involve, on average, 7.53 intermediaries.

In this paper, we study a classic problem in economics where an agent uses his market power to inefficiently screen a privately informed counterparty. We already know from Glode and Opp (2016) that it is possible to improve the efficiency of trade by involving moderately informed intermediaries, each endowed with their own market power, as part of an intermediation chain in which each trader's information set is similar to those of his direct counterparties.[2] In this paper, we show a stronger result: whenever there exist incentive-compatible mechanisms that can implement efficient trade between a buyer and a seller, there also exist (except for a knife-edge case) intermediation chains that achieve the same result.

We initially consider a standard bilateral trading situation where one agent has market power in pricing the asset and his counterparty is privately informed about the

[1]This is a joint work with Vincent Glode and Christian Opp.

[2]Zhang (2016) also shows that mechanisms that use a third party's information about traders' private valuations can help implement efficient trade, which is otherwise impossible without subsidies in the setting with two-sided asymmetric information of Myerson and Satterthwaite (1983).

(private or common) value of the asset. When the surplus of trade is small relative to the degree of information asymmetry, inefficient screening leads to destruction of the surplus. We first highlight how the allocation of market power is a key driver of this inefficiency. For example, if we added competition among uninformed agents pricing the asset, efficient trade would be sustained in a greater parametric region. More broadly, incentive-compatible mechanisms that simply eliminate the market power problem would also facilitate efficient trade.

We then consider the involvement of multiple intermediaries who trade the asset sequentially, as part of an intermediation chain in which each trader's information set is similar, although not identical, to those of his direct counterparties. We show that a long enough intermediation chain can eliminate all inefficiencies associated with imperfect competition. When market power leads to inefficient trade, we typically expect that adding sequential layers of intermediation would reduce efficiency due to problems of double marginalization (e.g., Spengler (1950) and more recently Gofman (2014)). However, Glode and Opp (2016) show that if the intermediaries are partially informed, the reduction of incentives to screen in every stage of the intermediation chain can, somewhat paradoxically, improve efficiency. Yet, Glode and Opp (2016) do not evaluate under which general conditions intermediation chains — if not restricted in their length— can achieve full efficiency. In this paper, we show that the mechanism uncovered by Glode and Opp (2016), when extended to long enough chains of intermediaries with precisely defined information sets, can generically replicate the implementation of full efficiency by any bilateral incentive-compatible mechanism (Hurwicz (1972)).

Our new result that sequentially involving a large number of heterogeneously informed intermediaries may eliminate all trading inefficiencies caused by imperfect competition and asymmetric information sheds light on the earlier evidence of long intermediation chains in many decentralized markets. More broadly, it may also explain why the U.S. financial system, which used to follow a traditional, centralized model of financial intermediation, shifted in recent decades toward a more complex, market-based model characterized by "the long chain of financial intermediaries involved in channeling funds" (Adrian and Shin (2010)). [3]

3.2. The Inefficiency of Trade

We initially consider a standard bilateral transaction between two risk-neutral agents as in Glode and Opp (2016). The monopolist seller of an asset (or good) must choose the price he will quote to a potential buyer (or customer) as a take-it-or-leave-it offer. The seller is, however, uncertain about how much the buyer is willing to pay for the asset. In particular, the seller only knows that the buyer's valuation of the asset, which we denote by v, has a cumulative distribution function (CDF) denoted by $F(v)$. This CDF is continuous and differentiable and the probability density function (PDF), denoted by $f(v)$, takes strictly positive values everywhere on the support $[v_L, v_H]$. The buyer only accepts to pay the seller's quoted price p if $v \geq p$; otherwise, the seller must retain the asset, which is worth $c(v)$ to him. The function $c(v)$ is assumed to be weakly increasing, continuous, and to satisfy $c(v) < v$ for all $v \in [v_L, v_H]$. The functions $c(\cdot)$ and $F(\cdot)$ are common knowledge.

Since the buyer always values the asset more than the seller does, trade creates a surplus for any realization of v and is therefore efficient if and only if the buyer obtains the asset with probability 1. However, the seller may find it privately optimal to use his market power and inefficiently screen the informed buyer, thus jeopardizing

[3] See also Kroszner and Melick (2009), Cetorelli, Mandel, and Mollineaux (2012), and Pozsar et al. (2013).

the gains to trade.[4]

3.2.1. Direct trade

A subgame-perfect Nash equilibrium in this bilateral transaction consists of a price that the seller quotes and an acceptance rule for each possible buyer type v that are mutual best responses in every subgame. The seller's expected payoff by quoting a price p is thus given by:

$$\Pi(p) = [1 - F(p)]p + F(p)\mathbb{E}[c(v)|v < p]. \tag{3.1}$$

By picking a price, the seller trades off his payoff when a sale occurs and the probability that a sale occurs. The seller's marginal profit of increasing the price p is:

$$\Pi'(p) = [1 - F(p)][1 - H(p)], \tag{3.2}$$

where we define the $H(\cdot)$ function as:

$$H(v) = \frac{f(v)}{1 - F(v)}[v - c(v)], \forall v \in [v_L, v_H). \tag{3.3}$$

We impose the following regularity condition on the function $H(\cdot)$ to guarantee that the marginal profit function $\Pi'(\cdot)$ crosses zero from above at most in one point. The condition thus ensures that we obtain a unique subgame-perfect Nash equilibrium under direct trade.

Assumption 3.1. *$H(v)$ is strictly increasing in v for $v \in [v_L, v_H)$.*

[4] Most of the analysis in this paper would remain unchanged if we instead considered an alternative setting where the buyer/seller roles were reversed. An uninformed buyer would make an ultimatum offer to a privately informed seller and, in some cases, the buyer's market power would lead to inefficient trading. See Glode and Opp (2015) for a related discussion.

Assumption 3.1 is closely related to the definition of a strictly regular environment by Fuchs and Skrzypacz (2015) as well as a standard assumption in auction theory that bidders' virtual valuation functions are strictly increasing (Milgrom (1981)).[5] When the gains to trade are independent of v, that is, when $v - c(v) = \Delta > 0$ for all $v \in [v_L, v_H]$, Assumption 3.1 simplifies to imposing that the hazard rate function $h(v) \equiv f(v)/[1 - F(v)]$ is strictly increasing.

Socially efficient trade requires that the seller quotes a price that is accepted by the buyer with probability 1. The maximum price that maintains efficient trade is thus $p = v_L$ and direct trade is efficient if and only if $\Pi'(v_L) \leq 0$. For later derivations, it is helpful to rewrite this last condition as summarized in the following proposition.

Proposition 3.1. *With a monopolistic seller, efficient trade can be achieved if and only if:* $v_L \geq c(v_L) + \frac{1}{f(v_L)}$.

If instead $v_L < c(v_L) + \frac{1}{f(v_L)}$, the monopolistic seller quotes an inefficient price $p > v_L$ that sets $\Pi'(p) = 0$ and jeopardizes the surplus from trade.

3.2.2. Market power

Before discussing a solution to this problem of inefficient trade, it is important to emphasize that the seller's market power is a key driver of potentially inefficient

[5]To see this, we define the function $\varphi(p)$ as the derivative of the seller's expected payoff with respect to the probability of trade when quoting a price p:

$$\varphi(p) \equiv \Pi'(p) \frac{dp}{d(1 - F(p))} = p - c(p) - \frac{1 - F(p)}{f(p)}.$$

The function $\varphi(p)$ represents the difference between the buyer's virtual valuation and the seller's marginal valuation when $v = p$. If we assume constant gains to trade $v - c(v) = \Delta > 0$, a strictly increasing $\varphi(\cdot)$ simplifies to a strictly increasing hazard rate and is thus equivalent to Assumption 3.1. With general definitions of $c(v)$, these two conditions are mathematically different, yet they yield the same results in our model for the case of direct trade. As will become clear later, imposing Assumption 3.1 will, however, yield an additional useful property when we introduce intermediaries and analyze their impact on trade efficiency.

behavior. Suppose that instead of having a monopolistic seller, we have two identical, competing sellers. Each seller quotes a price to the buyer who is only willing to acquire one unit of the asset. The buyer observes both prices before deciding whether to buy from a seller. In this scenario, a classic result is that (Bertrand) competition will drive both sellers to quote prices equal to their marginal cost. Since we are interested in conditions where efficient trade can be sustained (i.e., the buyer obtains the asset with probability 1), each seller's valuation of the asset is given by $\mathbb{E}[c(v)]$ as the buyer's decision to accept does not provide additional information on v in this case. Thus, the condition for efficient trade is $v_L \geq \mathbb{E}[c(v)]$, i.e., the lowest-type buyer accepts the seller's quoted price, which is $\mathbb{E}[c(v)]$. We summarize the result in the following proposition.

Proposition 3.2. *With two competing sellers, efficient trade can be achieved if and only if:* $v_L \geq \mathbb{E}[c(v)]$.

We can compare this condition to that when there is only one seller who has market power and derive the following result.

Lemma 3.3. *If* $v_L \geq c(v_L) + \frac{1}{f(v_L)}$*, then* $v_L > \mathbb{E}[c(v)]$.

Thus, the condition for efficient trade is strictly less restrictive when the seller does not possess market power. Thus, in cases where $\mathbb{E}[c(v)] \leq v_L < c(v_L) + \frac{1}{f(v_L)}$, competing sellers behave efficiently but a monopolistic seller inefficiently screens the buyer and jeopardizes the gains to trade.

Example 3.1. *Suppose the buyer values the asset at* $v \sim U[1, 2]$ *and the seller values the asset at a constant* $c < 1$*. If the seller has market power, his optimization problem* *when picking a price is:*

$$\max_{p\in[1,2]} \Pi(p) = Pr(v \geq p)p + Pr(v < p)c = (2-p)p + (p-1)c. \tag{3.4}$$

When $\Pi'(1) \leq 0$, the seller quotes a price $p = 1$ that is always accepted by the buyer. Thus, trade is efficient if and only if $c \leq 0$. If the seller does not have market power, the condition for efficient trade becomes $c \leq 1$ as Bertrand competition drives the seller's quoted price to c, which is strictly less than the lowest possible buyer valuation. So when $c \in (0, 1]$, the seller's market power yields inefficient trading outcomes.

We have shown that efficient trade is easier to achieve if the seller does not have market power. Below we show that the condition $v_L \geq \mathbb{E}[c(v)]$ is also the necessary and sufficient condition for an efficient, incentive-compatible mechanism to exist.

Proposition 3.4. *An incentive-compatible mechanism that achieves efficient trade exists if and only if: $v_L \geq \mathbb{E}[c(v)]$.*

Thus, increasing seller competition sustains efficient trade as much as any incentive-compatible mechanism would in our model. In cases where the asset is in scarce supply, however, adding competition on the seller side can be impossible. Moreover, in many contexts market power is not something that can simply be reallocated. In the next section, we generically show however that, whenever trading inefficiencies can be eliminated by the two solutions above, there exist long intermediation chains that can equivalently sustain efficient trade.

3.3. Intermediation chains

We now consider the involvement of M intermediaries, indexed by m based on their position in a trading chain. To simplify the notation, we label the seller as trader 0 and the buyer as trader $(M+1)$. All intermediaries are risk-neutral and value the asset at $c(v)$ just like the seller does. To keep the model tractable despite the presence of several intermediation rounds, we assume that in every transaction the asset holder makes an ultimatum offer to his counterparty. In addition, we propose the following signal structure that allows proving our main existence result and maintains the tractability of the analysis: each intermediary observes a signal that partitions the domain $[v_L, v_H]$ into sub-intervals and intermediary $(m+1)$'s signal creates a strictly finer conditional partition than intermediary m's signal. Nesting sequential traders' information sets eliminates signaling concerns and implies a generically unique subgame perfect Nash equilibrium in our model, even though there are $(M+1)$ bargaining problems among $(M+2)$ heterogeneously informed agents.

In particular, the information structure is modeled as follows. Suppose $v_L = v_0 < v_1 < v_2 < \cdots < v_M < v_{M+1} = v_H$ and

- Intermediary 1 knows whether v belongs to $[v_L, v_M)$ or $[v_M, v_H]$.
- Intermediary 2 knows whether v belongs to $[v_L, v_{M-1})$, $[v_{M-1}, v_M)$, or $[v_M, v_H]$.
- $\cdots$
- Intermediary M knows whether v belongs to $[v_L, v_1)$, $\cdots$, $[v_{M-1}, v_M)$, or $[v_M, v_H]$.

Before deriving our main results, it is useful to state the following lemma.

Lemma 3.5. *If Assumption 3.1 is satisfied under distribution $F(v)$, it is also satisfied under any truncated version of that distribution.*

Lemma 3.5 is the reason why we imposed a regularity condition on $H(\cdot)$ rather than on $\varphi(\cdot)$. Unlike with a strictly increasing $H(\cdot)$ function, a strictly increasing $\varphi(\cdot)$ function does not guarantee that an analogous property holds for the truncated version of $F(v)$. As in the case with direct trade, Assumption 3.1 guarantees that the marginal profit function for each intermediary crosses zero (from above) at most once when quoting a price to the buyer and that we have, generically, a unique subgame perfect Nash equilibrium under intermediated trade.

The following proposition characterizes our main existence result.

Proposition 3.6. *If $v_L > \mathbb{E}[c(v)]$, there exists an $\bar{M}$ such that the involvement of $M \geq \bar{M}$ intermediaries sustains efficient trade.*

We now return to our parameterized example to illustrate this result.

Example 3.2. *As in Example 1, the buyer values the asset at $v \sim U[1,2]$ and the seller values it at a constant $c(v) = c$. We focus on the case with $c \in (0,1)$ where direct trade is inefficient due to the seller's market power (the knife-edge case with $c = 1$ violates the condition in Proposition 3.6). We denote $\epsilon \equiv \frac{1}{M+1}$ and construct a chain of M intermediaries who are informed as follows:*

- *Intermediary 1 knows whether v belongs to $[1, 2-\epsilon)$ or $[2-\epsilon, 2]$.*
- *Intermediary 2 knows whether v belongs to $[1, 2-2\epsilon)$, $[2-2\epsilon, 2-\epsilon)$, or $[2-\epsilon, 2]$.*
- $\cdots$
- *Intermediary M knows whether v belongs to $[1, 2-M\epsilon)$, $\cdots$, $[2-2\epsilon, 2-\epsilon)$, or $[2-\epsilon, 2]$.*

For $0 \leq m \leq M-1$, we first observe that if trader m knows that $v \in [1, 2-m\epsilon)$, *he must prefer quoting a price $p = 1$ over $p = 2 - (m + 1)\epsilon$ to his better informed counterparty, who knows whether v belongs to $[1, 2 - (m + 1)\epsilon)$, or to $[2 - (m + 1)\epsilon, 2 - m\epsilon]$, for trade to be efficient. We thus need:*

$$1 \geq \left(\frac{\epsilon}{2 - m\epsilon - 1}\right)[2 - (m + 1)\epsilon] + \left(1 - \frac{\epsilon}{2 - m\epsilon - 1}\right)c. \tag{3.5}$$

As we can see given that $\epsilon \equiv \frac{1}{M+1}$, a larger number of intermediaries implies that deviating to the inefficient price $p = 2 - (m + 1)\epsilon$ becomes less profitable. Moreover, we can show that the condition above simplifies to $c \leq 1 - \epsilon$, which holds as long as $M \geq \bar{M} \equiv \frac{c}{1-c}$. For any other signal trader m receives, he knows that his counterparty is identically informed and trade is efficient. Each intermediary m then extracts an expected surplus of $(1 - m\epsilon)\epsilon$ when trade is efficient.

Now consider trader M who knows that $v \in [1 + i\epsilon, 1 + (i + 1)\epsilon)$ for some $i = 0, 1, 2, \cdots, M$. Using the same reasoning as under direct trade, we know that trader M will prefer to quote a price $p = 1 + i\epsilon$ over any inefficient price $p > 1 + i\epsilon$ as long as $1 + i\epsilon \geq c + \frac{1}{1/\epsilon}$, which always holds if $c \leq 1 - \epsilon$, or equivalently if $M \geq \bar{M} \equiv \frac{c}{1-c}$.

Overall, this chain of M intermediaries sustains efficient trade if $c \leq 1 - \frac{1}{M+1}$. Hence, sufficiently long intermediation chains (i.e., with $M \geq \bar{M}$ intermediaries) can sustain efficient trade whenever $c < 1$.

We have shown that if $v_L > \mathbb{E}[c(v)]$ the sequential involvement of intermediaries can eliminate all inefficiencies caused by the monopolistic seller's incentives to screen his privately informed counterparty. This solution to the problem involves multiple intermediaries who are each endowed with their own market power, once they acquire the asset. The key idea is that a trader who holds the asset faces, for high realizations of v, a symmetrically informed counterparty, which makes efficient trade trivial to achieve, and for low realizations of v, he faces a steep trade-off between trading efficiently at conservative prices and trading inefficiently at slightly higher prices. This long intermediation chain thus limits each trader's incentives to inefficiently screen his better informed counterparty and it promotes efficient behavior by all agents involved.

Our final result shows that the sufficient condition for long intermediation chains to yield trade efficiency is also a necessary condition.

Proposition 3.7. *If involving M intermediaries, whose information are characterized by the above partitions with $v_L = v_0 < v_1 < v_2 < \cdots < v_M < v_{M+1} = v_H$, can implement efficient trade, then it must be that $v_L > \mathbb{E}[c(v)]$.*

Except for the knife-edge case where $v_L = \mathbb{E}[c(v)]$, a long, yet finite, chain of intermediaries can support efficient trade despite imperfect competition as long as there exists an efficient incentive-compatible mechanism, or equivalently as long as efficient trade is possible under perfect competition. Intermediation can thus be as good as competition in improving trade efficiency, despite the additional monopoly problem it entails.

Finally, note that the implementation of a socially optimal intermediation chain could be formalized in our model by adding a network-formation game similar to that analyzed in Glode and Opp (2016). This game would precede the trading game discussed above, and characterize order-flow agreements that traders commit to before information is obtained and trading occurs. Consistent with Glode and Opp (2016), any intermediation chain sustaining efficient trade can be part of a "coalition-proof equilibrium" of this network-formation game, given appropriate ex ante transfers that incentivize traders to commit to specific counterparties. Such order-flow agreements are very common in financial markets and either take the form of explicit agreements involving cash payments, or implicit arrangements promising profitable IPO allocations or subsidies on other services Blume (1993); Chordia and Subrahmanyam (1995); Reuter (2006); Nimalendran, Ritter, and Zhang (2007).

3.4. Conclusion

We study a classic problem in economics where an agent uses his market power to inefficiently screen a privately informed counterparty. We show that trading through long chains of heterogeneously informed intermediaries can generically eliminate all trading inefficiencies due to imperfect competition and asymmetric information. If efficient trade can be achieved by adding competition that shuts down the inefficient use of market power or, more broadly, by allowing for incentive-compatible mechanisms that simply eliminate market power problems, it can also be achieved by setting up a trading network that takes the form of a sufficiently long intermediation chain.

The uninformed agent's market power plays an important role in our environment — it is the seller's ability to potentially appropriate additional rents by charging higher prices that creates the social inefficiency that intermediation chains might help alleviate. When the seller has no ability to seek additional rents in the first place (because there are multiple sellers making simultaneous offers to a unique buyer), this inefficiency is assumed away. The mechanism we propose, however, differs from interventions aimed at increasing competition. Specifically, the intermediaries we involve in the chain are each endowed with monopoly power once they obtain the asset, potentially creating problems of double marginalization (Spengler (1950)). Moreover, if instead of adding heterogenously informed monopolists, we added several monopolists who are either uninformed like the seller or perfectly informed like the buyer, intermediation chains would not improve the efficiency of trade relative to direct trade. In this case, most pairs of counterparties would be trading without an information asymmetry but whenever an uninformed trader would have to quote a price to a perfectly informed counterparty, trade would still break down, much like under direct trade. To improve trading efficiency via the involvement of *homogeneously informed* traders, traders need to compete simultaneously rather than sequentially. This form of mechanism thus relies on different forces than the intermediation chains we consider here.

More generally, if we allowed for any mechanism, adding moderately informed agents could help the seller extract more surplus and improve the efficiency of trade (Zhang (2016)). In particular, if multiple informed traders were to bid simultaneously for the seller's asset, the seller could extract information from these competing bidders, leaving less information rents to these agents Crémer and McLean (1988). This channel is, however, absent in our setting where trade is bilateral and the asset moves through each trader sequentially. The seller does not extract any information from competing bidders, but rather faces a single intermediary who is less informed than the expert buyer. This smaller information gap between the seller and his counterparty can strengthen the seller's incentives to quote an efficient price. Similarly, our channel also differs from the sequential skimming channel analyzed in Fudenberg, Levine, and Tirole (1985) where the seller quotes gradually decreasing prices to the buyer, ensuring that an impatient buyer with a high valuation for the asset would accept an early/higher price instead of waiting for a later/lower price. Instead of dynamically learning the buyer's valuation, in our model the seller faces a moderately informed intermediary who provides him with strong incentives to quote a low, efficient price (which is accepted in equilibrium). Overall, our solution to the problem features decentralized, sequential trading among many heterogeneously informed agents and is thus different from these other types of mechanisms.

APPENDIX A: Efficient Bargaining Through a Broker

A.1. Omitted Proofs

Proof of Proposition 1.1: I only need to show that if an allocation rule, p, is implementable when the broker has commitment power, then it is also implementable when the broker does not have commitment power. Suppose the direct mechanism when the broker has commitment power is (p, t_B, t_S). I now construct the direct mechanism when the broker does not have commitment power.

Let

$$t_1(v, b) = \int_{\underline{c}}^{\overline{c}} \int_S t_B(v, c, b, s)\, dG^I(s|c)\, dG(c),$$

and

$$t_2(c, s) = \int_{\underline{v}}^{\overline{v}} \int_B t_S(v, c, b, s)\, dF^I(b|v)\, dF(v).$$

Define

$$t'_B(v, c, b, s) = t_1(v, b) + t_2(c, s) - A_1,$$
$$t'_S(v, c, b, s) = t_1(v, b) + t_2(c, s) - A_2,$$

where

$$A_1 = \int_{\underline{c}}^{\overline{c}} \int_S t_2(c, s)\ dG^I(s|c)\, dG(c)$$
$$A_2 = \int_{\underline{v}}^{\overline{v}} \int_B t_1(v, b)\ dF^I(b|v)\, dF(v).$$

Consider a direct mechanism (p, t'_B, t'_S). First, A_1 represents the total expected payment from the broker to the seller, and A_2 represents the total expected payment from the buyer to the broker. Under this mechanism,

$$
\begin{aligned}
U_B(v,\hat{v}) &= vG(\hat{v}) - \int_{\underline{c}}^{\overline{c}} \int_S \int_B t_B(\hat{v},c,b,s)\, dF^I(b|v)\, dG^I(s|c)\, dG(c) \\
&= vG(\hat{v}) - \int_{\underline{c}}^{\overline{c}} \int_S \int_B \left(t_1(\hat{v},b) + t_2(c,s) - A_1\right)\, dF^I(b|v)\, dG^I(s|c)\, dG(c) \\
&= vG(\hat{v}) - \int_B t_1(\hat{v},b)\, dF^I(b|v) - \int_{\underline{c}}^{\overline{c}} \int_S t_2(c,s)\; dG^I(s|c)\, dG(c) + A_1 \\
&= vG(\hat{v}) - \int_B \int_{\underline{c}}^{\overline{c}} \int_S t_B(\hat{v},c,b,s)\, dG^I(s|c)\, dG(c)\, dF^I(b|v) \\
&= vG(\hat{v}) - \int_{\underline{c}}^{\overline{c}} \int_S \int_B t_B(\hat{v},c,b,s)\, dF^I(b|v)\, dG^I(s|c)\, dG(c).
\end{aligned}
$$

The buyer has the exact same objective function as in the direct mechanism (p, t_B, t_S). Similarly, the seller has the exact same objective function as in the direct mechanism (p, t_B, t_S). Since the broker's ex ante payoff is non negative in (p, t_B, t_S) (i.e., the mechanism is non subsidized), I know $A_2 \geq A_1$. So the broker's payoff is $A_2 - A_1$ after any realizations in the new mechanism (p, t'_B, t'_S). Thus, (p, t'_B, t'_S) can implement the same allocation rule without requiring the broker's commitment.

Proof of Proposition 1.2: I first show the necessary condition. Suppose an ex post efficient mechanism (p, t_B, t_S) can be implemented, in which p is the efficient allocation rule. Then the buyer's expected utility is given by $\int U_B(v)dF(v)$, which is greater than or equal to R_B^I because of the definition of minimal information rent. The seller's expected utility is given by $\int U_S(c)dG(c)$, which is greater than or equal to R_S^I because of the definition of minimal information rent. Now, since the total surplus, W, must be at least $\int U_B(v)dF(v) + \int U_S(c)dG(c)$, I have $W \geq R_B^I + R_S^I$.

Next, I show the sufficiency. From the definition of infimum, for any $\epsilon > 0$, I know that there exists $\widetilde{t}_B$ such that the buyer's expected utility is less than $R_B^I + \epsilon$, and there exists $\widetilde{t}_S$ such that the seller's expected utility is less than $R_S^I + \epsilon$. By choosing ϵ small enough, the direct mechanism $(p, \widetilde{t}_B, \widetilde{t}_S)$ can implement efficient trade with the broker's commitment. By applying Proposition 1.1, efficient trade is implementable.

Lastly, if the infima are achievable, I can let $\epsilon = 0$ in the above sufficiency proof. Then I have a necessary and sufficient condition.

Proof of Lemma 1.3: Suppose the broker is uninformed. At the minimal information rent, $U_B(\underline{v}) = 0$ and

$$U_B(v) = \int_{\underline{v}}^{v} G(t)dt.$$

Thus,

$$\begin{aligned}
R_B^{\emptyset} &= \int_{\underline{v}}^{\overline{v}} U_B(v)dF(v) \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{v} G(t)dtdF(v) \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{v} G(t)f(v)dtdv \\
&= \int_{\underline{v}}^{\overline{v}} \int_{t}^{\overline{v}} G(t)f(v)dvdt \\
&= \int_{\underline{v}}^{\overline{v}} (1 - F(t))G(t)dt.
\end{aligned}$$

Similarly, I can show that

$$R_S^{\emptyset} = \int_{\underline{c}}^{\overline{c}} (1 - F(t))G(t)dt.$$

The total gains from trade are:

$$
\begin{aligned}
W &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} 1_{v>c}(v-c)f(v)g(c)dcdv \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} 1_{v>c}vf(v)g(c)dcdv - \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} 1_{v>c}cf(v)g(c)dcdv \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{v} vf(v)g(c)dcdv - \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{v} cf(v)g(c)dcdv \\
&= \int_{\underline{v}}^{\overline{v}} vf(v)G(v)dv - \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{v} cdG(c)f(v)dv \\
&= \int_{\underline{v}}^{\overline{v}} vf(v)G(v)dv - \int_{\underline{v}}^{\overline{v}} [vG(v) - \int_{\underline{c}}^{v} G(c)dc]f(v)dv \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{v} G(c)f(v)dcdv = \int_{\underline{c}}^{\overline{v}} \int_{c}^{\overline{v}} G(c)f(v)dvdc \\
&= \int_{\underline{c}}^{\overline{v}} G(c)(1-F(c))dc.
\end{aligned}
$$

Finally, I show that $R_B^I \leq R_B^{\emptyset}$. Consider $\widetilde{t}_B(v,c,b,s) = \int_{\underline{v}}^{v} tg(t)dt + A$, where $A = \underline{v}G(\underline{v})$. The associated $\widetilde{U}_B(v,v') = vG(v') - \int_{\underline{v}}^{v'} tg(t)dt - A$. It is easy to see that $v = arg\min_{v'} \widetilde{U}_B(v,v')$. I also have $\widetilde{U}_B(v,v) = vG(v) - \int_{\underline{v}}^{v} tg(t)dt - A = vG(v) - \int_{\underline{v}}^{v} tdG(t) - A = \underline{v}G(\underline{v}) + \int_{\underline{v}}^{v} G(t)dt - A \geq 0$. Thus, $R_B^I \leq \int_{\underline{v}}^{\overline{v}} \widetilde{U}_B(v,v)dF(v) = \int_{\underline{v}}^{\overline{v}} \left(\underline{v}G(\underline{v}) + \int_{\underline{v}}^{v} G(t)dt - A\right) dF(v) = \int_{\underline{v}}^{\overline{v}} \left(\int_{\underline{v}}^{v} G(t)dt\right) dF(v) = R_B^{\emptyset}$. Similarly, I can show that $R_S^I \leq R_S^{\emptyset}$.

Proof of Lemma 1.4: Define the constrained minimal information rent to the buyer:

$$
\begin{aligned}
r_B^I := &\inf_{\widetilde{U}_B(\cdot)} \left\{ \int \widetilde{U}_B(v)\, dF(v) \right\} \qquad \text{(A.1)} \\
s.t.\ &\widetilde{U}_B(v) \geq (v-\hat{v})G(\hat{v}) + \widetilde{U}_B(\hat{v}), \forall v, \hat{v} \in [v_{b-1}, v_b) \text{ and } v > \hat{v}, \\
&\widetilde{U}_B(v) \geq 0, \forall v.
\end{aligned}
$$

It is straightforward to see that

$$r_B^I = \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} \int_{v_{k-1}}^{v} G(t)dtdF(v).$$

I next show that $r_B^I = R_B^I$. Recall from (1.1) that if $v_1, v_2 \in [v_{b-1}, v_b)$

$$\begin{aligned}
\bar{t}_B(v_1, v_2) &= \int_{\underline{c}}^{\bar{c}} \int_S \int_B t_B(v_2, c, b, s)\, dF^I(b|v_1)\, dG^I(s|c)\, dG(c) \\
&= \int_{\underline{c}}^{\bar{c}} \int_S \int_B t_B(v_2, c, b, s)\, dF^I(b|v_2)\, dG^I(s|c)\, dG(c) \\
&= \bar{t}_B(v_2, v_2).
\end{aligned}$$

Then

$$\begin{aligned}
U_B(v_1) - U_B(v_2) &\geq (v_1 G(v_2) - t(v_1, v_2)) - (v_2 G(v_2) - t(v_2, v_2)) \\
&= v_1 G(v_2) - v_2 G(v_2) \\
&= (v_1 - v_2) G(v_2).
\end{aligned}$$

Thus, $r_B^I \leq R_B^I$. It is left to show that $R_B^I \leq r_B^I$. Consider the following:

$$\widetilde{t}_B(v, c, b, s) = \begin{cases} \int_{v_{b-1}}^{v} tdG(t) + v_{b-1} G(v_{b-1}) & \text{if } v \in [v_{b-1}, v_b), \\ A & \text{otherwise,} \end{cases}$$

where A is a sufficiently large number that is left to be determined. Since A is large, the buyer would not report some type $\hat{v} \notin [v_{b-1}, v_b)$. I can verify that the buyer would choose to report $\hat{v} = v$. Similar to the proof of Lemma 1.3, I can show that the buyer's IC and IR hold. I can verify that the associated buyer's utility is exactly given by r_B^I. Thus,

$$R_B^I \leq r_B^I.$$

So, $R_B^I = r_B^I$ and the infimum problem of R_B^I can be achieved by the above $\widetilde{t}_B$.

Now suppose the broker has some information about the buyer's valuation, i.e., $n > 1$.

$$\begin{aligned}
R_B^I &= \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} \int_{v_{k-1}}^{v} G(t)dtdF(v) \\
&< \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} \int_{\underline{v}}^{v} G(t)dtdF(v) \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{v} G(t)dtdF(v) \\
&= R_B^{\emptyset}.
\end{aligned}$$

Similarly, I can show that $R_S^I = \sum_{k=1}^{m} \int_{c_{k-1}}^{c_k} \int_{c}^{c_k} 1 - F(t)dtdG(c)$ and $R_S^I < R_B^{\emptyset}$ if $m > 1$.

Proof of Proposition 1.5: Recall that

$$\begin{aligned}
r_B^I &= \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} \int_{v_{k-1}}^{v} G(t)dtdF(v), \\
r_S^I &= \sum_{k=1}^{m} \int_{c_{k-1}}^{c_k} \int_{c}^{c_k} 1 - F(t)dtdG(c), \\
W &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} 1_{v>c}(v-c)dG(c)dF(v).
\end{aligned}$$

Note that

$$W = \sum_{b=1}^{n} \sum_{s=1}^{m} \int_{v_{b-1}}^{v_b} \int_{c_{s-1}}^{c_s} 1_{v>c}(v-c)dG(c)dF(v).$$

Since the infimum problems are achievable, the necessary and sufficient condition can be written as $W \geq R^I_B + R^I_S$. Interchanging the integral in R^I_B and R^I_S, one can rewrite the condition as

$$0 \leq \sum_{b=1}^{n} \sum_{s=1}^{m} \int_{v_{b-1}}^{v_b} \int_{c_{s-1}}^{c_s} 1_{v>c} * f(v) * g(c) * \\ \left(v - c - \frac{F(v_b) - F(v)}{f(v)} - \frac{G(c) - G(c_{s-1})}{g(c)} \right) dc\, dv.$$

Proof of Proposition 1.8: It is sufficient to show that for any $\epsilon > 0$, I can choose n and v_is such that $R^I_B < \epsilon$. Let $v_{k+1} = v_k + \frac{1}{n}$. Note that

$$\begin{aligned} R^I_B &= \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} \int_{v_{k-1}}^{v} G(t) dt dF(v) \\ &\leq \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} \int_{v_{k-1}}^{v} 1 dt dF(v) \\ &= \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} (v - v_{k-1}) dF(v) \\ &\leq \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} (v_k - v_{k-1}) dF(v) \\ &= \frac{1}{n} \sum_{k=1}^{n} \int_{v_{k-1}}^{v_k} dF(v) \\ &= \frac{1}{n}. \end{aligned}$$

Thus, I can choose $n > \frac{1}{\epsilon}$. Similarly, I can choose m and c_js such that $R^I_S < \epsilon$. Since total gains from trade $W > 0$, for any F and G I can choose $I = \{[v_0, v_1), \cdots, [v_{n-1}, v_n]\} \times \{[c_0, c_1], \cdots, (c_{m-1}, c_m]\}$ such that the efficient allocation rule can be implemented with an informed broker.

Proof of Proposition 1.9: Let $\pi(v) = \int_{\underline{v}}^{v} G(t)dt$. By the full extraction condition in McAfee and Reny (1992), I know there exists an almost full extraction. In other words, if $\forall \epsilon > 0$, there exists $z_1(\cdot), \cdots, z_{n_B}(\cdot)$ such that

$$0 \leq \int_{\underline{v}}^{v} G(t)dt - \min_{n} \int_{b} z_n(b)dF^I(b|v) \leq \epsilon.$$

Let $x(v, b) = z_{n^*(v)}(b)$ where $n^*(v) = arg\min_n \int_b z_n(b)dF^I(b|v)$. Consider $t_B(v, c, b, s) = c1_{v>c} + x(v, b)$.

$$\begin{aligned}
U_B(v, \hat{v}) &= vG(\hat{v}) - \int_{\underline{v}}^{\hat{v}} t\, dG(t) - \int x(\hat{v}, b)dF^I(b|v) \\
&\leq vG(v) - \int_{\underline{v}}^{v} t\, dG(t) - \int x(\hat{v}, b)dF^I(b|v) \\
&= \int_{\underline{v}}^{v} G(t)\, dt - \int x(\hat{v}, b)dF^I(b|v) \\
&= \int_{\underline{v}}^{v} G(t)\, dt - \int z_{n^*(\hat{v})}(b)dF^I(b|v) \\
&\leq \int_{\underline{v}}^{v} G(t)\, dt - \int z_{n^*(v)}(b)dF^I(b|v) \\
&\leq \epsilon.
\end{aligned}$$

The first two inequalities can be achieved if $\hat{v} = v$. Thus, IC constraints hold. In addition, $0 \leq U_B(v) \leq \epsilon$.

So $R_B^I = 0$. Similarly, I can show that $R_S^I = 0$. Since $W > 0$, applying Proposition 3.6, I know that ex post efficient trade is implementable.

Proof of Lemma 1.10: Suppose that p satisfies all conditions. I construct a payment scheme such that the direct mechanism can be implemented. I consider $t_B = t_S = t$, where

$$
\begin{aligned}
&t(v,c,b,s)\\
=&\begin{cases}
0 & \text{if } v \notin [v_{b-1}, v_b) \text{ and } c \notin (c_{s-1}, c_s],\\
A & \text{else if } v \notin [v_{b-1}, v_b),\\
-A & \text{else if } c \notin (c_{s-1}, c_s],\\
\int_{v_{b-1}}^{v} t\, dp_B(t,b) + \int_{c}^{c_s} t\, d[-p_S(c,s)] + \alpha(b,s) & \text{otherwise,}
\end{cases}
\end{aligned}
$$

where A is sufficiently large and $\alpha(b,s) = v_{b-1}p_B(v_{b-1},b) - c_s p_S(c_s,s)$. I now show that the buyer's and the seller's incentive constraints hold. For example, for the buyer who has valuation v and the broker observes b, the buyer may consider reporting $\hat{v} \in [v_{b-1}, v_b)$. Suppose that $\hat{v} < v$, then

$$
\begin{aligned}
&v(p_B(v,b) - p_B(\hat{v},b))\\
=&v\int_{\hat{v}}^{v} dp_B(t,b)\\
\geq&\int_{\hat{v}}^{v} t\, dp_B(t,b)\\
=&\int_{\underline{v}}^{\overline{c}}\int_S t_B(v,c,b,s) - t_B(\hat{v},c,b,s)dG^I(s|c)dG(c).
\end{aligned}
$$

So if $\hat{v} < v$,

$$U_B(v,v) \geq U_B(v,\hat{v}).$$

Similarly, one can show that if $\hat{v} > v$, $U_B(v,v) \geq U_B(v,\hat{v})$. Thus, the buyer's IC constraints hold. I can also show that the seller's IC constraints hold and verify that IR constraints hold.

Thus, the above constructed payment scheme is consistent with the allocation rule p.

Proof of Proposition 1.13: Since $R_B^I = R_S^I = 0$ and full rent extractions can be achieved, the proof follows immediately from the sufficiency part of Proposition 1.2.

Proof of Proposition 1.14: Suppose that $c \in (c_{s-1}, c_s]$, then $U_S(c) = U_S(c_s) + \int_c^{c_k} p_S(t,s)dt$. The seller's ex ante expected utility is

$$\begin{aligned}\int_{\underline{c}}^{\overline{c}} U_S(c)dG(c) &= \sum_{s=1}^{m} \int_{c_{s-1}}^{c_s} \left(U_S(c_s) + \int_c^{c_k} p_S(t,s)dt \right) dG(c) \\ &= \sum_{s=1}^{m} U_S(c_s)(G(c_s) - G(c_{s-1}) + \sum_{s=1}^{m} \int_{c_{s-1}}^{c_s} p_S(t,s)dtdG(c).\end{aligned}$$

To maximize the seller's ex ante expected utility, the seller would retain all of the broker's profit from the Proof of Lemma 1.10:

$$\begin{aligned}&\sum_{b=1}^{n}\sum_{s=1}^{m}\int_{v_{b-1}}^{v_b}\int_{c_{s-1}}^{c_s} p(v,c,b,s) * f(v) * g(c) * \left(v - c - \frac{F(v_b) - F(v)}{f(v)} - \frac{G(c) - G(c_{s-1})}{g(c)} \right) dc\, dv \\ &= \sum_{s=1}^{m} U_S(c_s)(G(c_s) - G(c_{s-1}) + \sum_{b=1}^{n} U_B(v_b)(F(v_b) - F(v_{b-1}) + U_0.\end{aligned}$$

Thus,

$$\begin{aligned}\sum_{s=1}^{m} U_S(c_s)(G(c_s) - G(c_{s-1}) \le &\sum_{b=1}^{n}\sum_{s=1}^{m}\int_{v_{b-1}}^{v_b}\int_{c_{s-1}}^{c_s} p(v,c,b,s) * f(v) * g(c) * \\ &\left(v - c - \frac{F(v_b) - F(v)}{f(v)} - \frac{G(c) - G(c_{s-1})}{g(c)} \right) dc\, dv.\end{aligned}$$

The equality can be achieved if $U_0 = U_B(v_b) = 0, \forall b$. Now, plugging in the seller's ex ante expected utility, I obtain

$$\begin{aligned}\int_{\underline{c}}^{\overline{c}} U_S(c)dG(c) \le &\sum_{b=1}^{n}\sum_{s=1}^{m}\int_{v_{b-1}}^{v_b}\int_{c_{s-1}}^{c_s} p(v,c,b,s) * f(v) * g(c) * \\ &\left(v - c - \frac{F(v_b) - F(v)}{f(v)} \right) dc\, dv.\end{aligned}$$

Thus, to maximize the seller's ex ante expected utility, I would choose

$$p(v, c, b, s) = \begin{cases} 1 & \text{if } v - c - \frac{F(v_b) - F(v)}{f(v)} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Consider that the seller makes a take-it-or-leave-it offer if the seller knows the broker's signals. Recalling equation (1.13), the "virtual valuation" term becomes $v - \frac{F(v_b) - F(v)}{f(v)}$. One can show that the above allocation rule is equivalent to having the seller make a take-it-or-leave-it offer conditional on the broker's signals.

Proof of Proposition 1.16: Consider the following mechanism:

$$p(i, \hat{v}) = \begin{cases} 1 & \text{if } \hat{v} \in [v_{i-1}, v_i), \\ 0 & \text{otherwise.} \end{cases}$$

and

$$t(i, \hat{v}) = \begin{cases} v_{i-1} & \text{if } \hat{v} \in [v_{i-1}, v_i), \\ 0 & \text{otherwise.} \end{cases}$$

Here I let the payment from the buyer to the broker be equal to the payment from the broker to the seller; hence I use t to denote the payment. The broker makes a profit of 0, so he does not have an incentive to misreport his signal.

Consider the buyer with private information v. Since trade only happens if the value the buyer reports is consistent with the broker's signal, the buyer would choose $\hat{v} \in [v_{i-1}, v_i)$ if $v \in [v_{i-1}, v_i)$. Then the payment is v_{i-1} for sure, and the buyer's IC and IR constraints are satisfied.

Finally, consider the seller. The seller's expected revenue is given by $\sum_{i=1}^{n} v_{i-1}(F(v_i) -$

$F(v_{i-1})$), i.e., payment v_{i-1} with probability of $F(v_i) - F(v_{i-1})$. The seller's IR requires that $\sum_{i=1}^{n} v_{i-1}(F(v_i) - F(v_{i-1})) \geq \mathbb{E}c(v)$, which is the assumption.

Thus, if $\sum_{i=1}^{n} v_{i-1}(F(v_i) - F(v_{i-1})) \geq \mathbb{E}c(v)$, the mechanism can implement efficient trade.

Proof of Proposition 1.17: Similar to the proof of Lemma 1.4, I can show that R_B^I equals to the constrained minimal information rents as in program (A.1) for the discrete case. Thus,

$$\begin{aligned}
R_B^I &= \sum_{i=1}^{n}(F(V_i) - F(V_{i-1}))\sum_{k=2}^{i}(V_k - V_{k-1})G(V_{k-1}),\\
R_S^I &= \sum_{i=1}^{m}(G(C_i) - G(C_{i-1}))\sum_{k=i}^{m-1}(C_{k+1} - C_k)(1 - F(C_{k+1})),\\
W &= \sum_{i=1}^{n}\sum_{j=1}^{m} 1_{V_i > C_j}(V_i - C_j)(F(V_i) - F(V_{i-1}))(G(C_j) - G(C_{j-1})).
\end{aligned}$$

Collecting terms on V_is and C_js, I can simplify the condition to

$$\sum_{i=1}^{n}(1 - F(V_{i-1})(G(V_i) - G(V_{i-1})V_i \geq \sum_{i=1}^{m} G(C_i)(F(C_{i+1}) - F(C_i))C_i,$$

where I define $F(V_0) = G(V_0) = 0$ and $F(C_{m+1}) = 1$.

APPENDIX B: Voluntary Disclosure in Bilateral Transactions

B.1. Omitted Proofs

Proof of Proposition 2.1: By contradiction, suppose that the buyer's optimal disclosure plan is represented by $g(\cdot)$, which does not implement efficient trade. Just like in the setup without disclosure, we can rule out any case where trade is inefficient, given the disclosure plan, due to trade occurring for some $v < \hat{v}$ where $c(v) > v$. Suppose the seller receives a signal s_0 and quotes a price $p < \hat{v}$. His expected payoff is then:

$$\begin{aligned} & \Pr(v \geq p|s_0)p + \Pr(v < p|s_0)\mathbb{E}[c(v)|v < p, s_0] \\ = \; & \Pr(v \geq \hat{v}|s_0)p + \Pr(p \leq v < \hat{v}|s_0)p + \Pr(v < p|s_0)\mathbb{E}[c(v)|v < p, s_0]. \quad \text{(A1)} \end{aligned}$$

If the seller quoted a price $\hat{v}$ instead, his payoff would increase by $\hat{v} - p > 0$ when $v \geq \hat{v}$, by $c(v) - p \geq 0$ when $p \leq v < \hat{v}$ and it would remain the same when $v < p$. This deviation is strictly profitable for the seller, implying that in equilibrium the seller always quotes prices weakly greater than $\hat{v}$.

Now suppose that trade is inefficient, given the disclosure plan, because trade fails for some $v > \hat{v}$ where $c(v) < v$. We show that there exists another disclosure plan that yields a higher profit for the buyer. Denote by $x(s)$ the price the seller would quote upon receiving a signal s. Then there exists $s_0 \in S$, such that if the signal is s_0, the seller quotes a price $x(s_0) > \hat{v}$ and $x(s_0) > \inf\{v \in [v_L, v_H] : g(v) = s_0\}$. A buyer whose valuation belongs to $\{v : g(v) = s_0\} \cap (\hat{v}, x(s_0))$ would refuse to pay the seller's quoted price $x(s_0)$, leading to inefficient trade.

Now, consider the following disclosure plan where $S' = S \cup \{s'\}$ for some $s' \notin S$ and

$$\tilde{g}(v) = \begin{cases} g(v) & \text{if } g(v) \neq s_0 \\ s_0 & \text{else if } g(v) = s_0, v \geq x(s_0) \\ s' & \text{otherwise.} \end{cases} \tag{A2}$$

By definition, the disclosure plan $\tilde{g}(\cdot)$ would also be ex post verifiable. We now show that $\tilde{g}(\cdot)$ would give the buyer a strictly higher ex ante expected profit. First, note that if $s \neq s_0$, the seller would still quote a price $x(s)$. Second, if $s = s_0$, the seller would also quote $x(s_0)$ under the alternative disclosure plan $\tilde{g}(\cdot)$ as long as he receives a signal s_0. To see this, it is sufficient to establish the following lemma.

Lemma B.1. *Suppose that the seller would quote a price x if the buyer's valuation was drawn from a distribution with CDF $H(v)$. Let $H_0(v)$ denote the distribution $H(v)$ truncated from below at x, i.e., $H(v|v \geq x)$. Then the seller would also quote a price x if the buyer's valuation was drawn from $H_0(v)$.*

Proof. We argue by contradiction. Suppose the seller would instead quote a price y if the buyer's valuation was drawn from $H_0(\cdot)$. Then $y > x$ since the support of $H_0(\cdot)$ is bounded below at x. Thus,

$$(1 - H_0(y))y + H_0(y)\mathbb{E}_H[c(v)|x \leq v < y] > x \tag{A3}$$

where the subscript H in $\mathbb{E}_H$ reminds the distribution of v. Note that $H_0(y) = \frac{H(y)-H(x)}{1-H(x)}$ and we thus have:

$$(1 - H(y))y + (H(y) - H(x))\mathbb{E}_H[c(v)|x \leq v < y] > (1 - H(x))x. \tag{A4}$$

Since

$$
\begin{aligned}
(H(y) - H(x))\mathbb{E}_H[c(v)|x \le v < y] &= \int_x^y c(v)dH(v) \\
&= \int_{v_L}^y c(v)dH(v) - \int_{v_L}^x c(v)dH(v),
\end{aligned}
$$

we can rewrite the inequality as:

$$
(1 - H(y))y + \int_{v_L}^y c(v)dH(v) > (1 - H(x))x + \int_{v_L}^x c(v)dH(v) \tag{A5}
$$

or equivalently,

$$
(1 - H(y))y + H(y)\mathbb{E}_H[c(v)|v < y] > (1 - H(x))x + H(x)\mathbb{E}_H[c(v)|v < x]. \tag{A6}
$$

This inequality contradicts our initial statement that the seller would quote a price x if the buyer's valuation was drawn from $H(\cdot)$. $\square$

Given this lemma, we know that the seller would quote a price at $x(s_0)$ upon receiving a signal s_0 under the alternative disclosure plan $\tilde{g}(\cdot)$.

Finally, suppose the seller would quote a price at z if he receives a signal s'. Since quoting $x(s_0)$ yields zero profit in this case, it must be that $z \in [\inf g^{-1}(s_0), x(s_0))$. As a result, the buyer's ex ante expected profit under the alternative disclosure plan $\tilde{g}(\cdot)$ is given by:

$$
\sum_{s \in S} \underbrace{\int_{g^{-1}(s) \cap [x(s), v_H]} (v - x(s))dF(v)}_{\text{Profit from } s \in S} + \underbrace{\int_{g^{-1}(s_0) \cap [z, x(s_0))} (v - z)dF(v)}_{\text{Profit from } s'} \tag{A7}
$$

while the profit under the disclosure plan $g(\cdot)$ is only the first term. Since $x(s_0) > z$, the second term is strictly positive. So the buyer earns a strictly higher profit under the disclosure plan $\tilde{g}(\cdot)$ than that under $g(\cdot)$. This is a contradiction to the optimality of $g(\cdot)$.

Thus, the optimal disclosure plan must result in socially efficient trade. We also know that the optimal disclosure plan must reveal the buyer's information only partially. Otherwise, the seller quotes the buyer a price $p = v$ for all realizations of v and the buyer obtains no surplus. A full disclosure plan is therefore weakly dominated by a no-disclosure plan that leads to inefficient trade, which is then strictly dominated by a partial disclosure plan that leads to efficient trade, consistent with the arguments above. $\square$

Proof of Proposition 2.2: To show that trade is socially efficient in any buyer-preferred equilibrium $(g(\cdot), \mu(\cdot), x(\cdot))$, we argue by contradiction. Suppose there exists a signal $s_0 = g(v)$ for some $v \in [v_L, v_H]$ such that $x(s_0) > \hat{v}$ and $x(s_0) > \inf\{v \in [v_L, v_h] : g(v) = s_0\}$. A buyer whose valuation belongs to $\{v : g(v) = s_0\} \cap (\hat{v}, x(s_0))$ would refuse to pay the seller's quoted price $x(s_0)$, leading to inefficient trade. Let $s' \equiv \{v \in s_0 : \hat{v} \leq v < x(s)\}$. Consider the following candidate equilibrium $(\tilde{g}(\cdot), \tilde{\mu}(\cdot), \tilde{x}(\cdot))$, where

$$\tilde{g}(v) = \begin{cases} g(v) & \text{if } v \notin s_0 \\ s' & \text{else if } v \in s' \\ s_0 \backslash s' & \text{otherwise .} \end{cases} \tag{A8}$$

We obtain $\tilde{\mu}(s')$ and $\tilde{\mu}(s_0 \backslash s')$ using Bayes' rule at s' and $s_0 \backslash s'$, respectively. For any signal outside the range of s_0, $\tilde{\mu}(s) = \mu(s)$. For any other signal s, $\tilde{\mu}(s)$ assigns probability 1 to $\bar{v}(s) \equiv \sup s$. Let $\tilde{x}(s_0)$ solves $\max_p \pi(p, s_0)$, where $\pi(p, s_0)$ denotes the seller's profit if she quotes a price p and the buyer's valuation is drawn from $\tilde{\mu}(s_0)$. It is clear that we are indeed in an equilibrium, since deviating to any other disclosure yields a profit of 0 for the buyer. Now consider the buyer's interim payoffs in this alternative equilibrium. For buyer types $v \notin s_0$ and $v \in s_0 \backslash s'$, they receive payoffs identical to those from the original equilibrium $(g(\cdot), \mu(\cdot), x(\cdot))$. However, for buyer types in s', they receive weakly higher payoffs. Moreover, a buyer type $x(s_0)-\epsilon$, where ϵ is a small positive number, receives a strictly higher payoff, since he made zero profit in the original equilibrium. Overall, if trade is not efficient in an equilibrium, then it is Pareto dominated among buyer types by a more efficient equilibrium. Consequently, in any buyer-preferred equilibrium of the interim disclosure game, trade must be socially efficient.

To show that a buyer-preferred equilibrium does not feature full disclosure, where each buyer type is quoted $p = v$ and makes zero profit, it is sufficient to construct an equilibrium where some buyer types receive positive payoffs (as no buyer type can do worse than zero profit given their right to reject a price quote). Consider the equilibrium induced by the ex ante disclosure plan we solved for in Proposition 2.1 of Section 2.3. Formally, suppose $g(\cdot)$ is the ex post verifiable disclosure plan chosen by the buyer in the ex ante disclosure game and let the interim disclosure plan follow $g(v)$ for all $v \in [v_L.v_H]$. Now, let $\mu(\cdot)$ be a belief function obtained using Bayes' rule on the equilibrium path and that assigns probability 1 to the highest type for any signal off the equilibrium path. Lastly, $x(s)$ maximizes the seller's profit based on the belief $\mu(s)$. The profile $(g(\cdot), \mu(\cdot), x(\cdot))$ is clearly an equilibrium of the interim disclosure game. In this equilibrium, the buyer receives profits identical to those in the ex ante disclosure. Thus, this equilibrium featuring partial disclosure Pareto dominates among buyer types any equilibrium with full disclosure. □

B.2. Alternative Environments

Discrete Distributions of v. As pointed out in the main text, the proof of Proposition 2.1 can easily be adapted to discrete distributions of v and yield similar results. The main difference is that there may now exist cases where the alternative, more efficient disclosure plan being considered makes the buyer only weakly better off, rather than strictly better off as in our baseline model. Specifically, while the second term of equation (A7) in the proof of Proposition 2.1 is strictly positive when v is continuously distributed with strictly positive density everywhere on the support, this term may occasionally take a value of 0 with a discrete distribution. If we apply the tie-breaking rule that assumes that whenever indifferent between disclosure plans the buyer chooses the one that maximizes social surplus, our result that the buyer's optimal disclosure plan always leads to socially efficient trade also holds with discrete distributions. For similar reasons, however, it is now possible that the optimal disclosure plan is fully revealing for some parameterizations (for example, when v can only take one of two values and trade would be inefficient without disclosure).

We now present a simple parameterized example with a discrete distribution and solve for the optimal disclosure plan. To make this example analogous to that in the main text, we assume that the buyer's valuation $v \in \{1, 1.5, 2\}$ with equal probabilities. We also assume that the seller values the asset at $c \leq 1$. For trade to be efficient without disclosure, we need the seller to prefer quoting a price 1 over a price 1.5 or a price 2, that is, we need:

$$1 \geq \frac{2}{3} \cdot 1.5 + \frac{1}{3} c, \tag{B1}$$

and

$$1 \geq \frac{1}{3} \cdot 2 + \frac{2}{3} c. \tag{B2}$$

Taken together, these two conditions are satisfied only when $c \leq 0$. As in the continuous example of the main text, we focus on cases where $c \in (0, 1]$ and trade is inefficient without disclosure. Analogously to the continuous setting, the buyer must choose a disclosure plan that trades off the benefits from pooling realizations of v and extracting information rents with the downside of strengthening the seller's incentives to screen as the range of possible realizations gets wider. The buyer can choose among these five disclosure plans:

1. The buyer separately discloses $\{\{1\}, \{1.5\}, \{2\}\}$ (i.e., full disclosure), collects a profit of 0, and trade is always socially efficient.

2. The buyer separately discloses $\{\{1, 1.5\}, \{2\}\}$ (i.e, partial disclosure). For efficient trade to occur, the seller must quote a price 1 upon receiving a signal that $v \in \{1, 1.5\}$:
$$1 \geq \frac{1}{2} \cdot 1.5 + \frac{1}{2}c, \tag{B3}$$
which yields a profit of $\frac{1}{6}$ to the buyer. Otherwise, the buyer collects a profit of 0.

3. The buyer separately discloses $\{\{1\}, \{1.5, 2\}\}$ (i.e, partial disclosure). For efficient trade to occur, the seller must quote a price 1.5 upon receiving a signal that $v \in \{1.5, 2\}$:
$$1.5 \geq \frac{1}{2} \cdot 2 + \frac{1}{2}c, \tag{B4}$$
which yields a profit of $\frac{1}{6}$ to the buyer. Otherwise, the buyer collects a profit of 0.

4. The buyer separately discloses $\{\{1, 2\}, \{1.5\}\}$ (i.e, partial disclosure). For efficient trade to occur, the seller must quote a price 1 upon receiving a signal that

$v \in \{1, 2\}$:

$$1 \geq \frac{1}{2} \cdot 2 + \frac{1}{2}c, \tag{B5}$$

which yields a profit of $\frac{1}{3}$ to the buyer. Otherwise, the buyer collects a profit of 0.

5. The buyer always discloses $\{\{1, 1.5, 2\}\}$ (i.e, no disclosure). If the seller quotes a price 1.5:

$$\frac{2}{3} \cdot 1.5 \geq \frac{1}{3} \cdot 2 + \frac{2}{3}c, \tag{B6}$$

it yields a profit of $\frac{1}{6}$ to the buyer. Otherwise, the buyer collects a profit of 0.

For $c \in (0, 1]$ the no-disclosure strategy implies that the seller screens the buyer. Disclosing some information can benefit the buyer by reducing the seller's incentives to quote a high, inefficient price. Due to the associated profit of $\frac{1}{3}$, the buyer prefers to disclose $\{\{1, 2\}, \{1.5\}\}$ whenever it leads to efficient trade. However, this requires that $c \leq 0$, which is violated in our scenario. In other words, this disclosure plan does not sufficiently weaken the seller's incentives to screen the buyer and thus yields a profit of 0 for the buyer. When $c \in (0.5, 1)$, however, the buyer is strictly better off disclosing $\{\{1\}, \{1.5, 2\}\}$ as any other disclosure plan, including no disclosure, yields a profit of 0. His choice of disclosure then leads to socially efficient trade. If $c \in (0, 0.5]$ instead, the buyer is indifferent among three disclosure plans, including no disclosure, that each yield a profit of $\frac{1}{6}$. Our tie-breaking rule then ensures that he picks a disclosure plan that maximizes the social surplus, which means he either discloses $\{\{1, 1.5\}, \{2\}\}$ or $\{\{1\}, \{1.5, 2\}\}$, as either one of these disclosure plans yields efficient trade and therefore strictly dominates no disclosure from a social standpoint.

Connected Intervals. Our restriction that disclosure plans must be ex post verifiable allows for the design of signals that pool multiple disjoint intervals (e.g., see Figure 1 for the optimal disclosure plan in the uniform-distribution example). However, the proof of Proposition 2.1 does not rely on this possibility — thus, if the buyer can only design ex post verifiable signals associated with connected intervals (or, equivalently, weakly monotonic signal functions), our result that the buyer's optimal disclosure plan always leads to socially efficient trade still holds.

We now return to our simple parameterized example from the main text and solve for the optimal disclosure plan with connected intervals. The buyer values the asset at $v \sim U[1, 2]$ and the seller values it at a constant c. As before, we focus on the case where $c \in (0, 1]$, that is, trade is inefficient without disclosure.

We start by establishing the following result.

Lemma B.2. *Suppose $v \sim U[a, b]$ where $1 \leq a < b \leq 2$ and $\frac{b+c}{2} > a$, such that trade is inefficient without disclosure. Among the disclosure plans that split the interval into two sub-intervals $[a, x)$ and $[x, b]$, the buyer optimally chooses the interior cutoff $x^* = \frac{b+c}{2}$.*

Proof. If the seller learns from the disclosure that $v \in [a, x)$, the seller finds it optimal to quote:

$$p_1 = \max\{a, \frac{x+c}{2}\}. \tag{B7}$$

If the seller instead learns that $v \in [x, b]$, the seller finds it optimal to quote:

$$p_2 = \max\{x, \frac{b+c}{2}\}. \tag{B8}$$

We prove the lemma separately for the following two exhaustive cases.

- Case 1: $\frac{b+c}{2} \geq 2a - c$.

  - If $\frac{b+c}{2} \leq x \leq b$, then $p_1 = \frac{x+c}{2}$ and $p_2 = x$. The buyer's ex ante expected profit is $\frac{(b-x)^2}{2} + \frac{(x-c)^2}{8}$, which is maximized at $x^* = \frac{b+c}{2}$. The maximal value is $\frac{5(b-c)^2}{32}$.

  - If $2a - c \leq x \leq \frac{b+c}{2}$, then $p_1 = \frac{x+c}{2}$ and $p_2 = \frac{b+c}{2}$. The buyer's ex ante expected profit is $\frac{(b-c)^2}{8} + \frac{(x-c)^2}{8}$, which is maximized at $x^* = \frac{b+c}{2}$.

  - If $a \leq x \leq 2a - c$, then $p_1 = a$ and $p_2 = \frac{b+c}{2}$. The buyer's ex ante expected profit is $\frac{(b-c)^2}{8} + \frac{(x-a)^2}{2}$, which is maximized at $x^* = 2a - c$ but the value is no greater than $\frac{5(b-c)^2}{32}$ because $\frac{b+c}{2} \geq 2a - c$.

  Thus, in this case the optimal $x^* = \frac{b+c}{2}$.

- Cases 2: $\frac{b+c}{2} \leq 2a - c$.

  - If $a \leq x \leq \frac{b+c}{2}$, then $p_1 = a, p_2 = \frac{b+c}{2}$.The buyer's ex ante expected profit is $\frac{(x-a)^2}{2} + \frac{(b-c)^2}{8}$, which is maximized at $x^* = \frac{b+c}{2}$. The maximal value is $V_B \equiv \frac{(b-2a+c)^2}{8} + \frac{(b-c)^2}{8}$.

  - If $\frac{b+c}{2} \leq x \leq 2a - c$, then $p_1 = a, p_2 = x$.The buyer's ex ante expected profit is $\frac{(x-a)^2}{2} + \frac{(b-x)^2}{2}$, which is maximized at $x^* = \frac{b+c}{2}$.

  - If $x \geq 2a - c$, then $p_1 = \frac{x+c}{2}, p_2 = x$. The buyer's ex ante expected profit is $\frac{(b-x)^2}{2} + \frac{(x-c)^2}{8}$, which is maximized at end points. If $x = b$, the value is $\frac{(b-c)^2}{8} < V_B$. If $x = 2a - c$, the value is $\frac{(b-2a+c)^2}{2} + \frac{(a-c)^2}{2}$. We next show that $\frac{(b-2a+c)^2}{2} + \frac{(a-c)^2}{2} \leq \frac{(b-2a+c)^2}{8} + \frac{(b-c)^2}{8}$, which is equivalent to $3(b - 2a + c)^2 + 4(a - c)^2 \leq (b - c)^2$. Recall that $\frac{b+3c}{4} \leq a < \frac{b+c}{2}$. Note $3(b - 2a + c)^2 + 4(a - c)^2$ as a function of a achieves maximum at end points of a, which we can show is equal to $(b-c)^2$. Thus, if $x \geq 2a - c$, the buyer's ex ante expected profit is no greater than V_B.

Thus, the optimal $x^* = \frac{b+c}{2}$ in this case as well. $\square$

We denote a buyer's connected disclosure plan as:

$$\{[v_0, v_1), [v_1, v_2), \cdots, [v_M, v_{M+1}]\}, \tag{B9}$$

where $v_0 = 1$ and $v_{M+1} = 2$. Since the lemma above holds for any interval as long as $\frac{b+c}{2} > a$, we conclude that any cutoff $\{v_i\}$ must satisfy $v_i = \frac{v_{i+1}+c}{2}, \forall i = 1, \cdots, M$. Note that v_0 does not have to be equal to $\frac{v_1+c}{2}$, but we need $v_0 \geq \frac{v_1+c}{2}$, otherwise trade is inefficient, which contradicts our result in Proposition 2.1. To solve for the optimal disclosure plan, we define the following sequence. Let $k_0 = 2$ and for $i \geq 1$,

$$k_i = \begin{cases} \frac{k_{i-1}+c}{2} & \text{if } k_{i-1} > 1 - 2c \\ 1 & \text{otherwise.} \end{cases} \tag{B10}$$

It is straightforward to show that this sequence is weakly decreasing. For any $c \in (0, 1)$, this sequence will reach 1 in finite steps whereas if $c = 1$ we need a countable, infinite set of intervals to span the whole support of v. Let $M + 1$ be the smallest subscript i such that $k_i = 1$. Reversing the sequence, we define $v_i = k_{M+1-i}, \forall i = 0, \cdots, M + 1$. Such a disclosure plan is optimal for the buyer when he is restricted to disclosing signals associated with connected intervals.

B.3. Alternative Equilibrium Refinement

For the interim disclosure game of Section 2.4, we showed that in any "buyer-preferred" equilibrium information disclosure is partial and leads to socially efficient trade. In this Appendix, we show the robustness of this result to an alternative equilibrium refinement known as Grossman-Perry-Farrell, based on the perfect sequential equilibrium of Grossman and Perry (1986) and the neologism-proof equilibrium of Farrell (1993).

Recall that $\mu(s)$ is the seller's equilibrium belief about the buyer's valuation, upon receiving a signal s. Denote by $U(v, s, \mu(s))$ the buyer's utility if his valuation is v, he sends a message s, and the seller quotes an optimal price given the belief $\mu(s)$. For any signal s that is a Borel set in $[v_L, v_H]$ (including off-equilibrium messages), denote by μ_s the distribution of v conditional on $v \in s$. (Recall that we restrict the sets of signals to be Borel sets in the interim disclosure game, to be consistent with ex post verifiability.)

As in Bertomeu and Cianciaruso (2016), we first define a *self-signaling* set.

Definition B.1. *Given a pure-strategy perfect Bayesian equilibrium of the interim disclosure game,* $(g(\cdot), \mu(\cdot), x(\cdot))$*, a nonempty Borel set* $s \in [v_L, v_H]$ *is a self-signaling set if*

$$s = \{v \in s : U(v, s, \mu_s) > U(v, g(v), \mu(g(v)))\}. \tag{C1}$$

Notice that only buyers whose valuation $v \in s$ can send the signal s because ex post verifiability requires that the true valuation belongs to the chosen signal. A self-signaling set s contains all buyer types who could be strictly better off by sending the signal s rather than playing according to the considered perfect Bayesian equilibrium.

A deviation from an equilibrium consists of a message announcing "my type is in s."[1] The deviation is credible if s is self-signaling. An equilibrium survives the refinement if it does not admit any credible deviation.

Definition B.2. *A pure-strategy perfect Bayesian equilibrium of the interim disclosure game, $(g(\cdot), \mu(\cdot), x(\cdot))$, is a Grossman-Perry-Farrell equilibrium if there are no self-signaling sets.*

We now derive our result.

Proposition B.3. *In any Grossman-Perry-Farrell equilibrium of the interim disclosure game, the buyer's optimal disclosure is partial and yields socially efficient trade.*

Proof. To show that trade is socially efficient in any Grossman-Perry-Farrell equilibrium $(g(\cdot), \mu(\cdot), x(\cdot))$, we argue by contradiction. Suppose there exists a signal $s_0 = g(v)$ for some $v \in [v_L, v_H]$ such that $x(s_0) > \hat{v}$ and $x(s_0) > \inf\{v \in [v_L, v_h] : g(v) = s_0\}$. A buyer whose valuation belongs to $\{v : g(v) = s_0\} \cap (\hat{v}, x(s_0))$ would refuse to pay the seller's quoted price $x(s_0)$, leading to inefficient trade. Let $s_1 \equiv \{v \in s_0 : \hat{v} \leq v < x(s_0)\}$. Suppose the seller would quote a price x_1 under the belief μ_{s_1}.

Now consider the following set: $s_2 \equiv \{v \in s_0 : x_1 < v < x(s_0)\}$. From Lemma 1 in Appendix A, we know that the seller would also quote a price x_1 under the belief μ_{s_2}. Thus, $U(v, s_2, \mu_{s_2}) > 0, \forall v \in s_2$. Recall that all types of buyers in s_2 does not trade

[1]Unlike Farrell (1993) who allows for the possibility of any type of senders announcing "my type is in s", we assume only buyer types whose true valuation $v \in s$ can do so, consistent with our restriction of verifiably truthful disclosure.

in the equilibrium $(g(\cdot), \mu(\cdot), x(\cdot))$, thus $U(v, g(v), \mu(g(v))) = 0, \forall v \in s_2$. So, all types of buyers in s_2 are strictly better off by announcing "my type is in s_2." Therefore, s_2 is a self-signaling set, contradicting the conjecture that a Grossman-Perry-Farrell equilibrium can feature inefficient trade.

To show that full disclosure cannot be a feature of a Grossman-Perry-Farrell equilibrium, it is sufficient to construct a self-signaling set. Suppose the seller would quote a price x' under the prior belief. Then, it is clear that $(x', v_H]$ is a self-signaling set, since these buyer types would be strictly better off being quoted a price x' than a price equal to their respective valuation v. $\square$

APPENDIX C: On the Efficiency of Long Intermediation Chains

C.1. Omitted Proofs

Proof of Proposition 3.1: Directly follows from $\Pi'(v_L) \leq 0$. $\square$

Proof of Proposition 3.2: Directly follows from the arguments that precede the proposition. $\square$

Proof of Lemma 3.3: The condition $v_L \geq c(v_L) + \frac{1}{f(v_L)}$ can be rewritten as $H(v_L) \geq 1$. By Assumption 3.1, $H(v_L) \geq 1$ implies that $H(v) > 1, \forall v \in (v_L, v_H)$. Using the definition of $H(\cdot)$, we obtain $v - c(v) > \frac{1-F(v)}{f(v)}$. Taking expectation on each side, we have $\mathbb{E}v - \mathbb{E}[c(v)] > \mathbb{E}\frac{1-F(v)}{f(v)} = \int_{v_L}^{v_H}(1-F(v))dv = (1-F(v))v|_{v_L}^{v_H} - \int_{v_L}^{v_H} vd(1-F(v)) = \mathbb{E}v - v_L$. Thus, $v_L > \mathbb{E}[c(v)]$. $\square$

Proof of Proposition 3.4: Without loss of generality, we can consider a direct mechanism (Milgrom (1981)). In our setting, only the buyer holds private information. Thus, in the direct mechanism, the buyer reports his value of v and this report directly determines the outcome. In the direct mechanism, we need to specify $(p(v), t(v))$, where p is the probability that the asset is transferred from the seller to the buyer, and t is the transfer payment from the buyer to the seller, if v is the buyer's reported valuation.

Since we assume $c(v) < v$ for all $v \in [v_L, v_H]$, trade always creates a surplus. Thus, the mechanism is efficient if and only if the buyer obtains the asset with probability 1. We must therefore consider mechanisms where $p(v) = 1, \forall v \in [v_L, v_H]$.

In order to implement efficient trade, it must give the buyer proper incentives to report his true valuation for the asset. The buyer's expected profit from reporting $\hat{v}$ is given by

$$p(\hat{v})v - t(\hat{v}) = v - t(\hat{v}) \tag{C1}$$

Then the buyer always wants to report $\hat{v} = arg\min_v t(v)$. So it must be the case that the transaction price is a constant, which we denote by t. The buyer always pays t for the asset and v_L must therefore be greater than or equal to t for the lowest type buyer to be willing to trade. On the other hand, the seller is willing to participate if and only if $t \geq \mathbb{E}[c(v)]$. Thus, we need $v_L \geq \mathbb{E}[c(v)]$.

To prove the sufficiency, consider a direct mechanism where the probability of trade $p(v) = 1$ and the transfer payment $t(v)$ is a constant $\mathbb{E}[c(v)]$. Under this mechanism, the buyer does not have any profitable deviations from truth-telling and all individually rational constraints are satisfied. □

Proof of Lemma 3.5: See proof of Lemma 1 in the online appendix for Glode and Opp (2016). □

Proof of Proposition 3.6: Since the PDF $f(\cdot)$ is continuous and strictly positive on the compact set $[v_L, v_H]$, there exists $a > 0$ such that $f(v) \geq a, \forall v \in [v_L, v_H]$. Since $v - c(v) > 0$ for all $v \in [v_L, v_H]$, there exists $b > 0$ such that $v - c(v) \geq b, \forall v \in [v_L, v_H]$. Since $v_L > \mathbb{E}c(v)$, we have

$$\frac{v_H - v_L}{v_H - \mathbb{E}c(v)} < 1. \tag{C2}$$

We can choose A such that

$$\max\left(1 - ab, \frac{v_H - v_L}{v_H - \mathbb{E}c(v)}\right) < A < 1. \tag{C3}$$

We then choose M such that $A^M \leq ab$, which exists since $A < 1$ and $A^M \to 0$ as $M \to +\infty$. We construct the corresponding cutoffs v_m's such that for any m that satisfies $1 \leq m \leq M$ we have:

$$F(v_m) = A^{M+1-m}. \tag{C4}$$

We are left to show that this intermediation chain implements efficient trade.

Trade between trader M and the buyer. For any signal that trader M receives, $[v_i, v_{i+1})$, for $i = 0, 1, \cdots, M$, efficient trade requires that he quotes a price $p = v_i$. Similarly to the case of direct bilateral trade, the seller makes a take-it-or-leave-it offer to his counterparty whose valuation now follows the PDF $\frac{f(x)}{F(v_{i+1})-F(v_i)}$ where $v_i \leq x < v_{i+1}$. To implement efficient trade, we thus need given Lemma 3.5:

$$v_i \geq c(v_i) + \frac{F(v_{i+1}) - F(v_i)}{f(v_i)} \tag{C5}$$

Since $\min f(v) \geq a$ and $\min(v - c(v)) \geq b$, it is sufficient to show that:

$$ab \geq F(v_{i+1}) - F(v_i) \tag{C6}$$

For $1 \leq i \leq M$, we know that $F(v_{i+1}) - F(v_i) = A^{M-i}(1 - A) \leq 1 - A < ab$. When $i = 0$, $F(v_{i+1}) - F(v_i) = F(v_1) - F(v_0) = A^M \leq ab$ by the definition of M. Thus (C5) always holds and trade occurs with probability 1 between trader M and the buyer.

Trade between trader m and trader $m + 1$, where $0 \leq m < M$. Consider trader m, who knows that $v \in [v_L, v_{M-m+1})$. Trader m knows that the signal received by trader $(m + 1)$ either locates v in $[v_L, v_{M-m})$ or in $[v_{M-m}, v_{M-m+1})$. To have efficient trade, we need trader m to quote a price $p = v_L$ instead of $p = v_{M-m}$. Thus, we need:

$$v_L \geq \left[1 - \frac{F(v_{M-m})}{F(v_{M-m+1})}\right] v_{M-m} + \frac{F(v_{M-m})}{F(v_{M-m+1})} \mathbb{E}[c(v)|v < v_{M-m}], \tag{C7}$$

which simplifies to:

$$\frac{F(v_{M-m})}{F(v_{M-m+1})} \geq \frac{v_{M-m} - v_L}{v_{M-m} - \mathbb{E}[c(v)|v < v_{M-m}]}. \tag{C8}$$

This last condition holds since:

$$\begin{aligned} \frac{F(v_{M-m})}{F(v_{M-m+1})} = A > & \frac{v_H - v_L}{v_H - \mathbb{E}c(v)} \\ \geq & \frac{v_{M-m} - v_L}{v_{M-m} - \mathbb{E}[c(v)]} \\ \geq & \frac{v_{M-m} - v_L}{v_{M-m} - \mathbb{E}[c(v)|v < v_{M-m}]}. \end{aligned} \tag{C9}$$

For any other signal trader m may receive, i.e., $v \notin [v_L, v_{M-m+1})$, he finds it optimal to quote a price equal to the lowest bound of the interval since trader $m+1$ has the same information as him and he is expected to quote a price equal to the lowest bound of the interval to his counterparty (for trade to be efficient). Thus, trade also occurs with probability 1 between traders m and $m+1$. $\square$

Proof of Proposition 3.7: If $M = 0$, then $H(v_L) \geq 1$ and by Lemma 3.3, we know that $v_L > \mathbb{E}[c(v)]$.

Now suppose that $M \geq 1$. If the chain implements efficient trade, we first show that $H(v_M) \geq 1$. Intermediary M, if he knows that $v \in [v_M, v_H]$, must quote a price $p = v_M$ to achieve efficiency. Similarly to the direct trading game where the seller was making a take-it-or-leave-it offer to the buyer, we need:

$$v_M \geq c(v_M) + \frac{1 - F(v_M)}{f(v_M)}, \tag{C10}$$

which reduces to $H(v_M) \geq 1$. We also need the seller to quote $p = v_L$ to intermediary 1, that is:

$$v_L \geq (1 - F(v_M))v_M + F(v_M)\mathbb{E}[c(v)|v < v_M] \tag{C11}$$

Recall that: $\Pi(p) = (1 - F(p))p + F(p)\mathbb{E}[c(v)|v < p]$. Condition (C11) can thus be rewritten as: $v_L \geq \Pi(v_M)$. Moreover, since $H(p) > H(v_M) \geq 1$ for $p > v_M$, we have $\Pi'(p) < 0$ for $p > v_M$ and therefore $\Pi(v_M) > \Pi(v_H) = \mathbb{E}[c(v)]$, which implies that $v_L > \mathbb{E}[c(v)]$. $\square$

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